Title: Dynamical Systems - Review (PHYS 607) - Lecture 6

Date: Jan 11, 2010 09:00 AM

URL: http://pirsa.org/10010018

Abstract:

RG equation

"universal function" $g(z)$

$$g(z) = -\alpha\, g\left(g\left(-z/\alpha\right)\right)$$

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Solve RG equation

Let $g(z) = A_0 + A_2 z \ldots$

$A_0 + A_2 z^2 + A_4 z^4 + \ldots$

RG equation

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Solve RG equation

Let $g(v) = A_0 + A_2 v^2 + A_4 v^4 + \ldots$

$$A_0 + A_2 z^2 + A_4 z^4 + \ldots = -\alpha \Big[A_0 + A_2$$

RG equation

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Solve RG equation

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$$A_0 + A_2 z^2 + A_4 z^4 + \ldots = -\alpha\Big[A_0 + A_2\Big(A_0 + A_2\left(\tfrac{z}{\alpha}\right)$$

RG equation

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$$+ A_4\left(A_0 + A_2\left(\tfrac{z}{\alpha}\right)^2 + A_4\left(\tfrac{z}{\alpha}\right)^4 + \dots\right.$$

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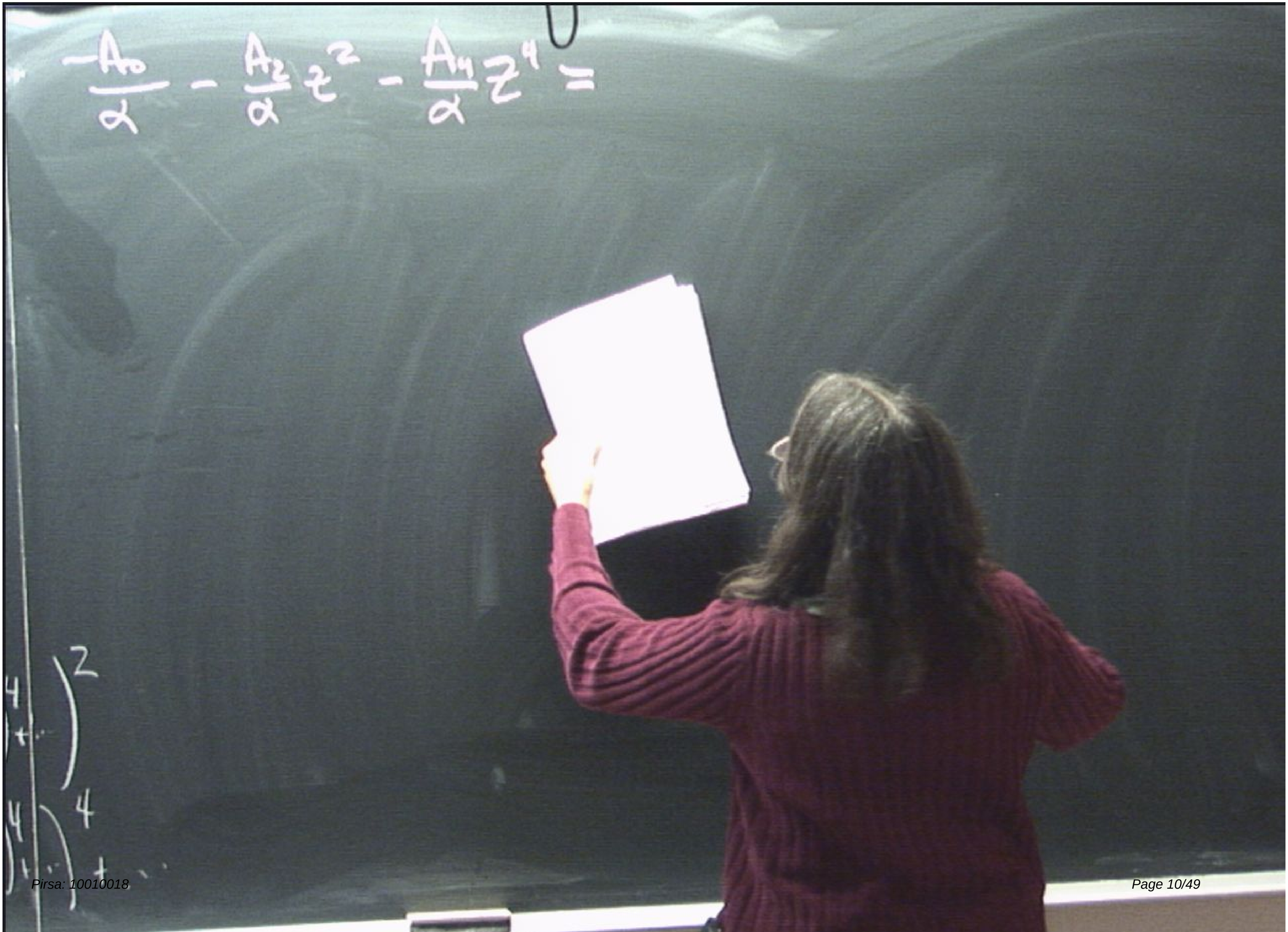
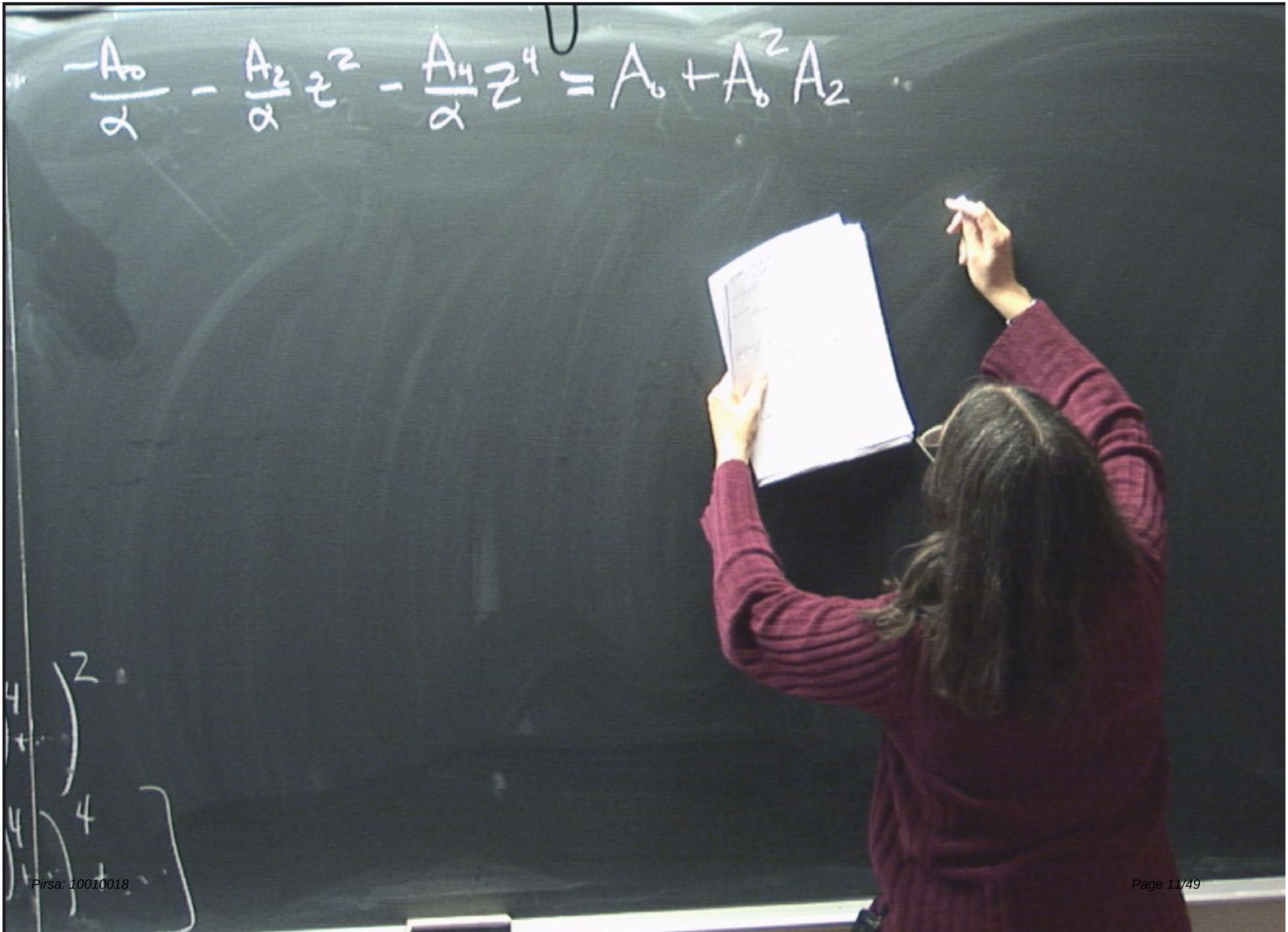
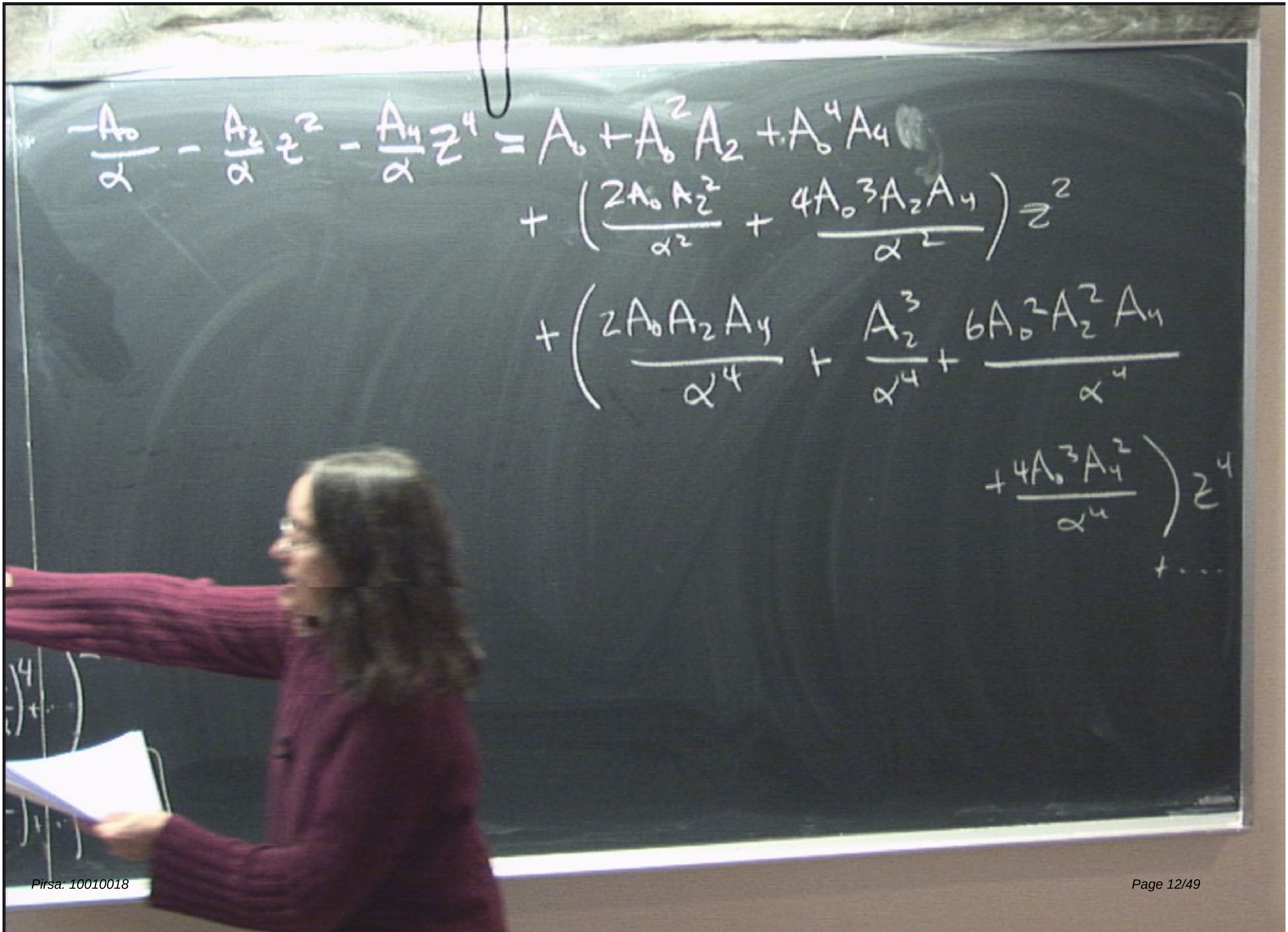

$-\frac{A_0}{\alpha} - \frac{A_2}{\alpha} z^2 - \frac{A_4}{\alpha} z^4 =$


$-\frac{A_0}{\alpha} - \frac{A_2}{\alpha} z^2 - \frac{A_4}{\alpha} z^4 = A_0 + A_0^2 A_2$


$-\frac{A_0}{\alpha} - \frac{A_2}{\alpha} z^2 - \frac{A_4}{\alpha} z^4 = A_0 + A_0^2 A_2 + A_0^4 A_4$
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$+ \left(\frac{2A_0 A_2 A_4}{\alpha^4} + \frac{A_2^3}{\alpha^4} + \frac{6A_0^2 A_2^2 A_4}{\alpha^4}$
$+ \frac{4A_0^3 A_4^2}{\alpha^4} \right) z^4$
$+ \dots$

$$-\frac{A_0}{\alpha} - \frac{A_2}{\alpha}z^2 - \frac{A_4}{\alpha}z^4 = A_0 + A_0^2 A_2 + A_0^4 A_4 + \left(\frac{2A_0A_2^2}{\alpha^2} + \frac{4A_0^3A_2A_4}{\alpha^2}\right)z^2 + \left(\frac{2A_0A_2A_4}{\alpha^4} + \frac{A_2^3}{\alpha^4} + \frac{6A_0^2A_2^2A_4}{\alpha^4} + \frac{4A_0^3A_4^2}{\alpha^4}\right)z^4 + \ldots$$

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Simplify:

$$z^0: \; -\frac{1}{\alpha} = 1 + A_0 A_2 + A_0^3 A_4$$

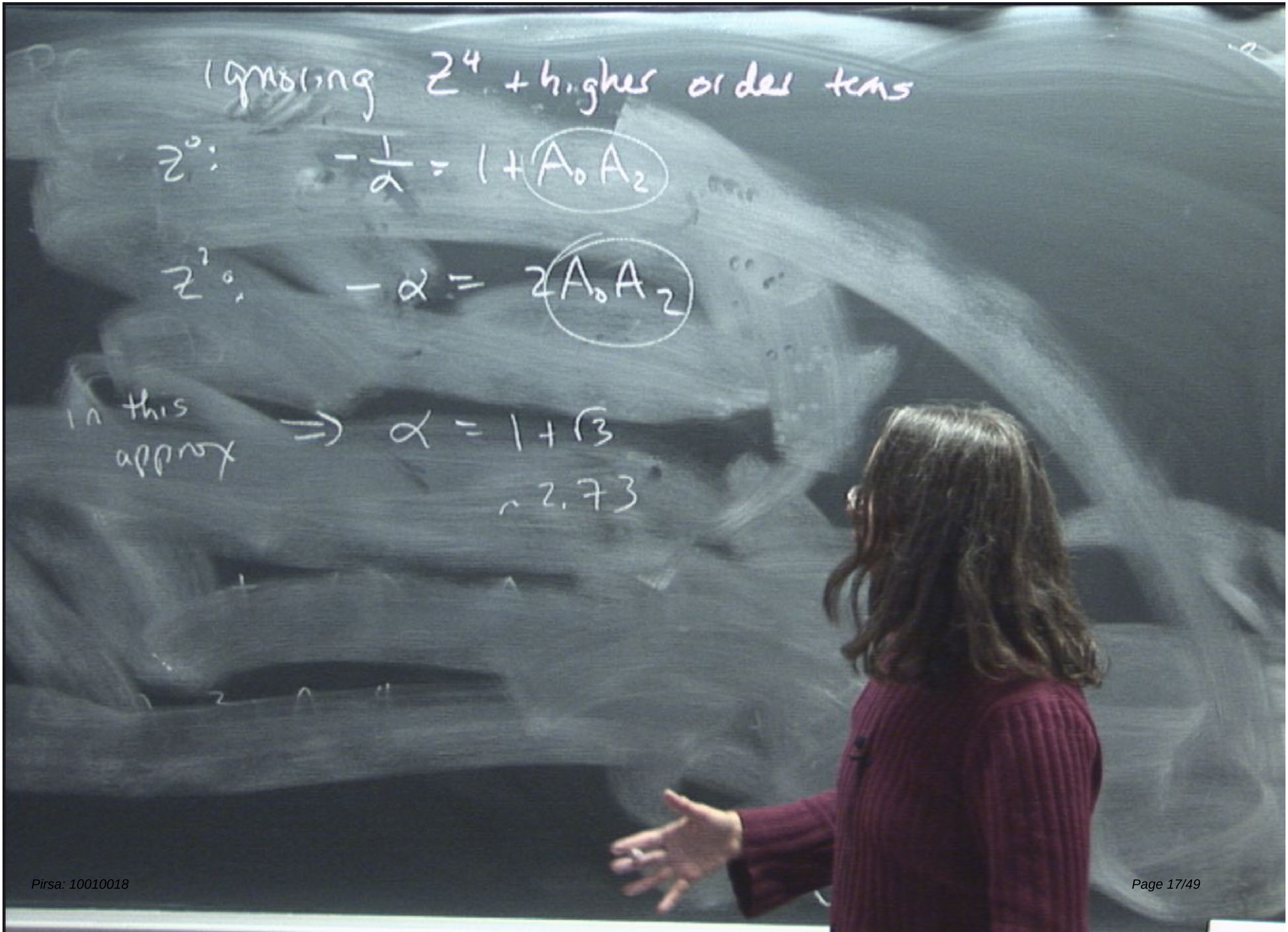
$$z^2: \; -\alpha = 2A_0 A_2 + 4A_0^3 A_4$$

$$z^4: \; -\frac{A_4}{\alpha} = \frac{-2A_0 A_2 A_4}{\alpha^4} + \frac{A_2^3}{\alpha^4} + \frac{6A_0^2 A_2^2 A_4}{\alpha^4} + \frac{4A_0^3 A_4^2}{\alpha^4}$$

ignoring z^4 + higher order terms

z^0: $-\frac{1}{\alpha} = 1 + A_0 A_2$

z^2: $-\alpha = 2 A_0 A_2$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in this approx $\Rightarrow \alpha = 1 + \sqrt{3}$
≈ 2.73

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Can w.l.o.g set $A_0 = 1$

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z^2: $-\alpha = 2 A_0 A_2$

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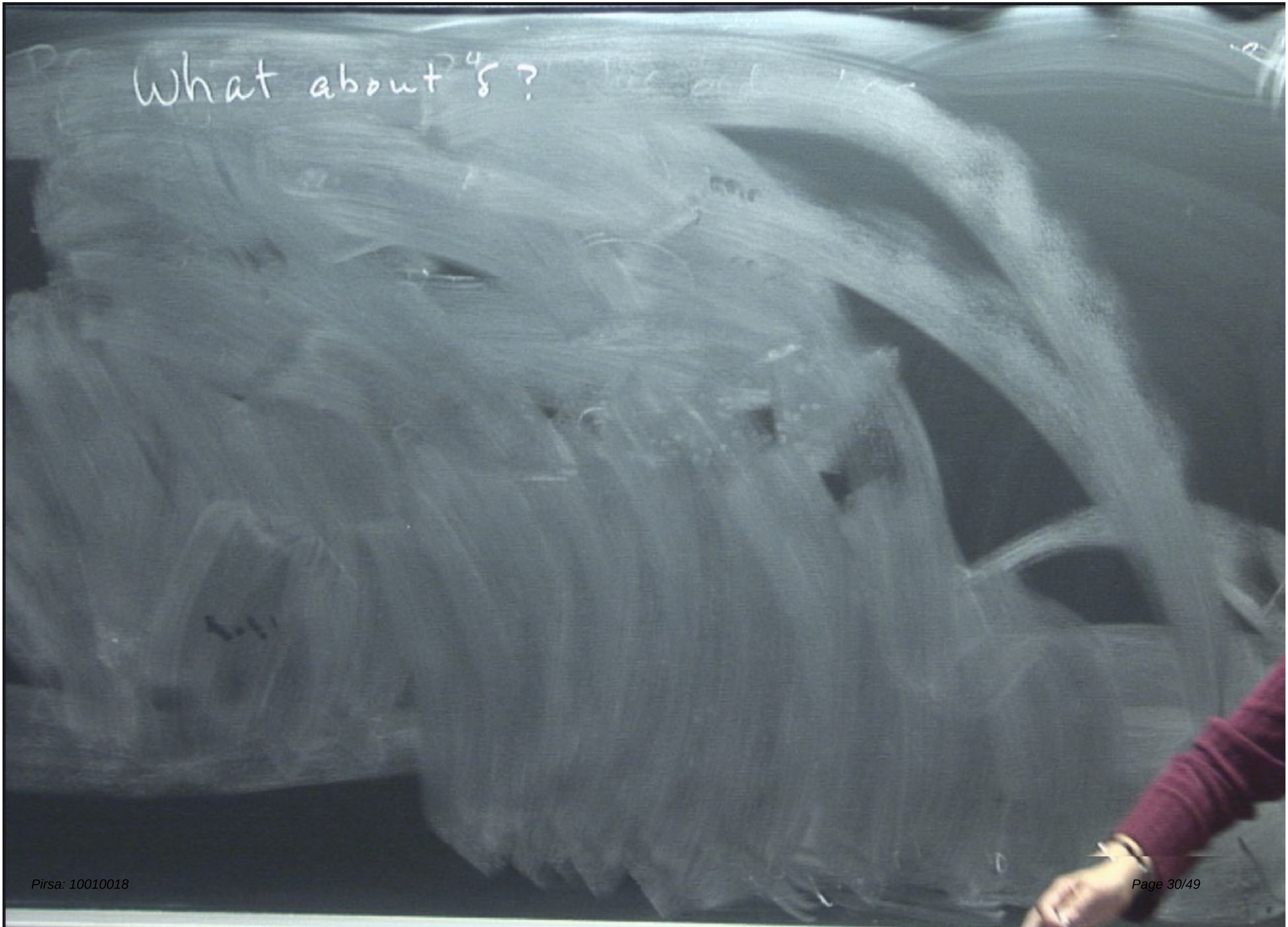
$$= \beta g(z/\beta) \rightarrow h(z)$$


What about ${}^4\delta$?

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δ describes approach to critical pt.

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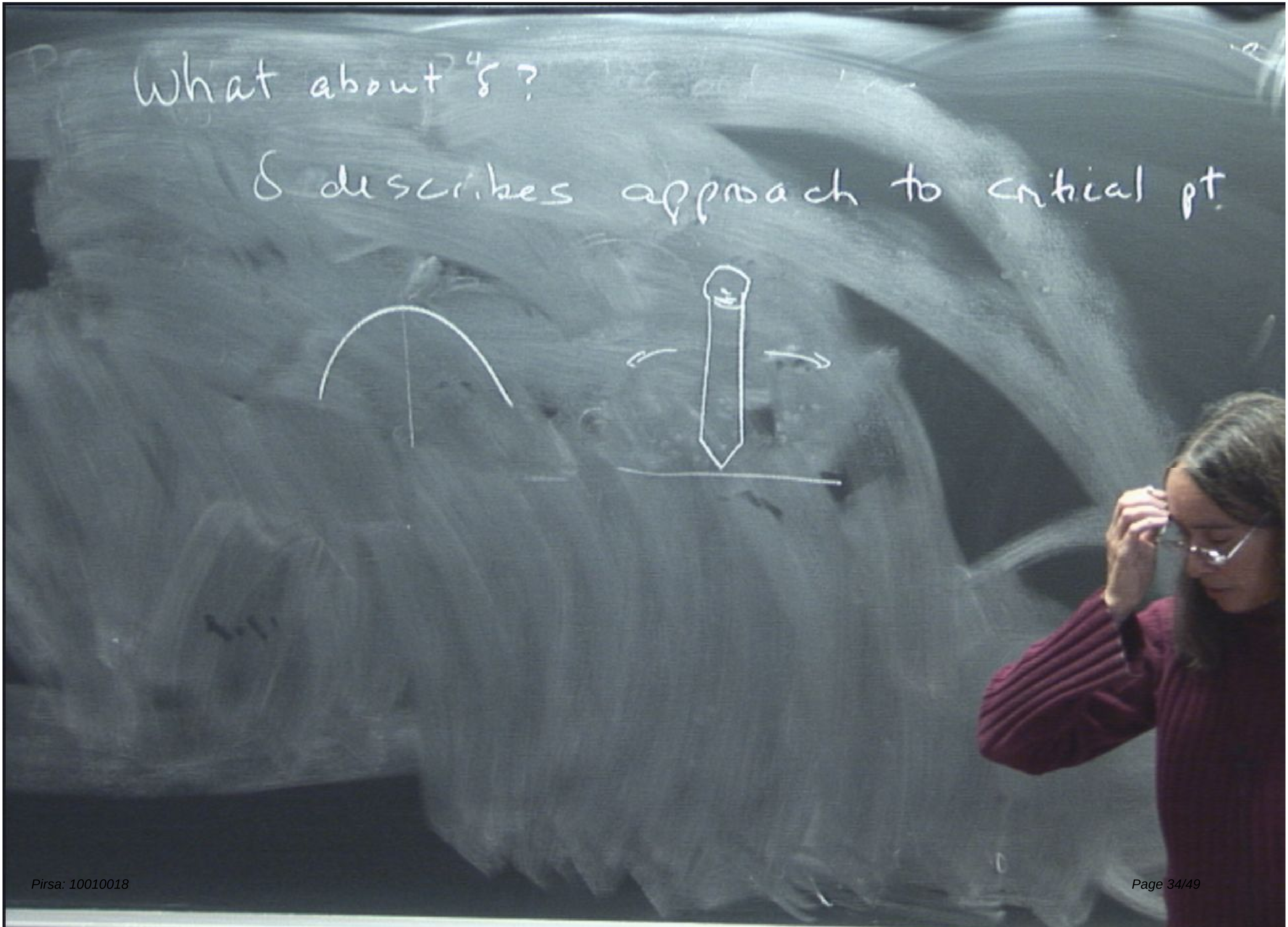
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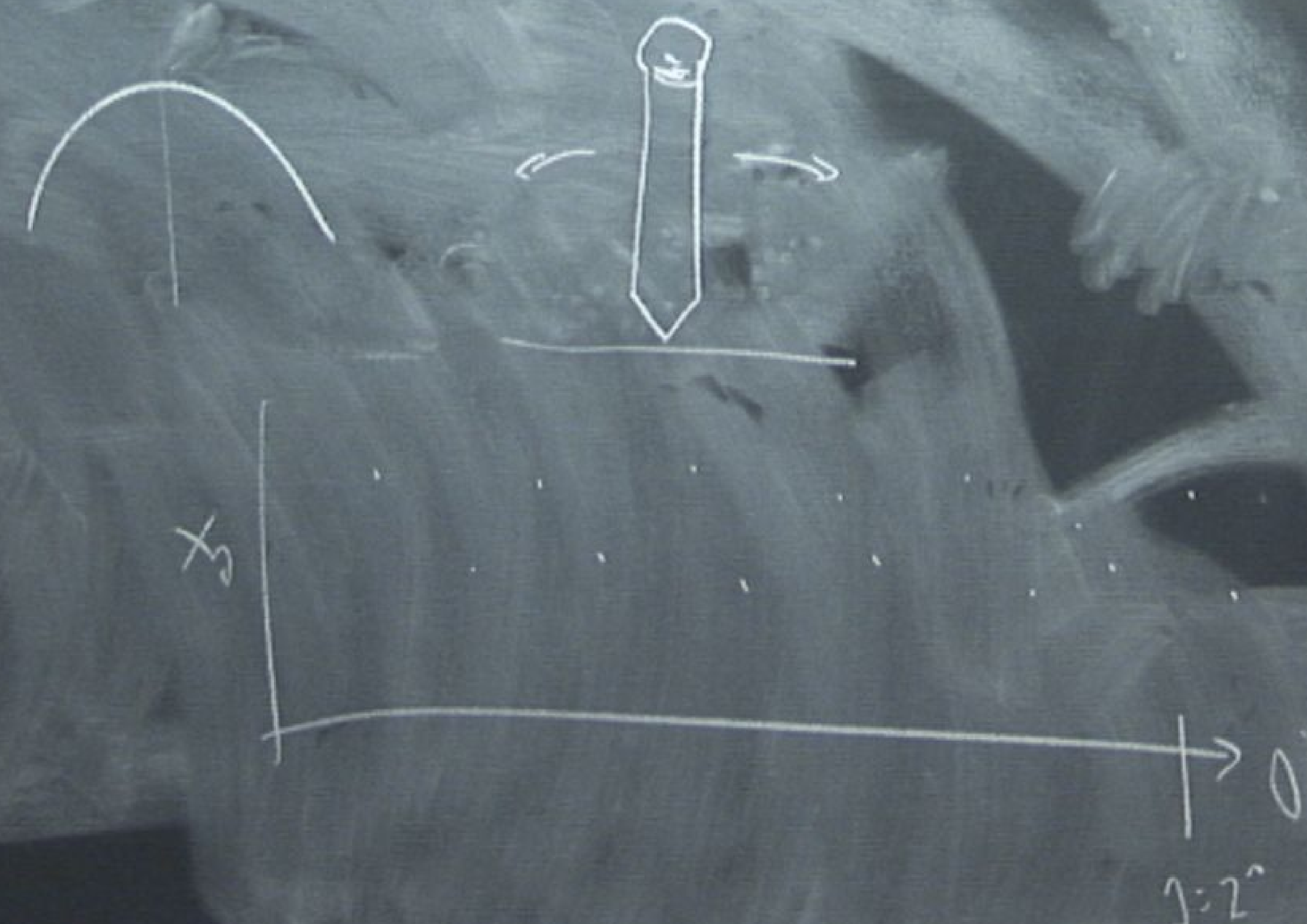

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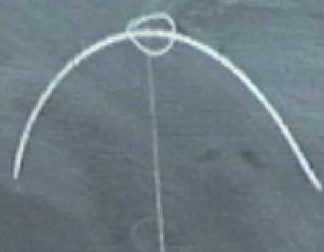

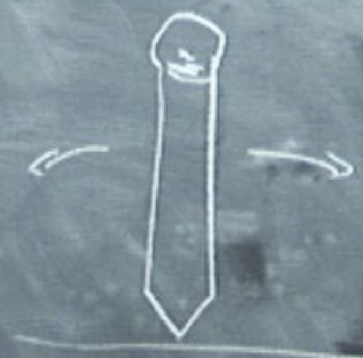

how fast do perturbations to $g(x)$ grow?

What about δ?

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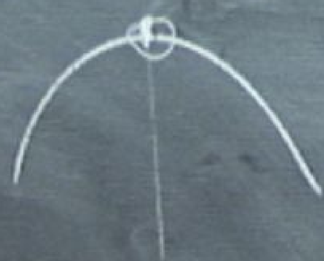

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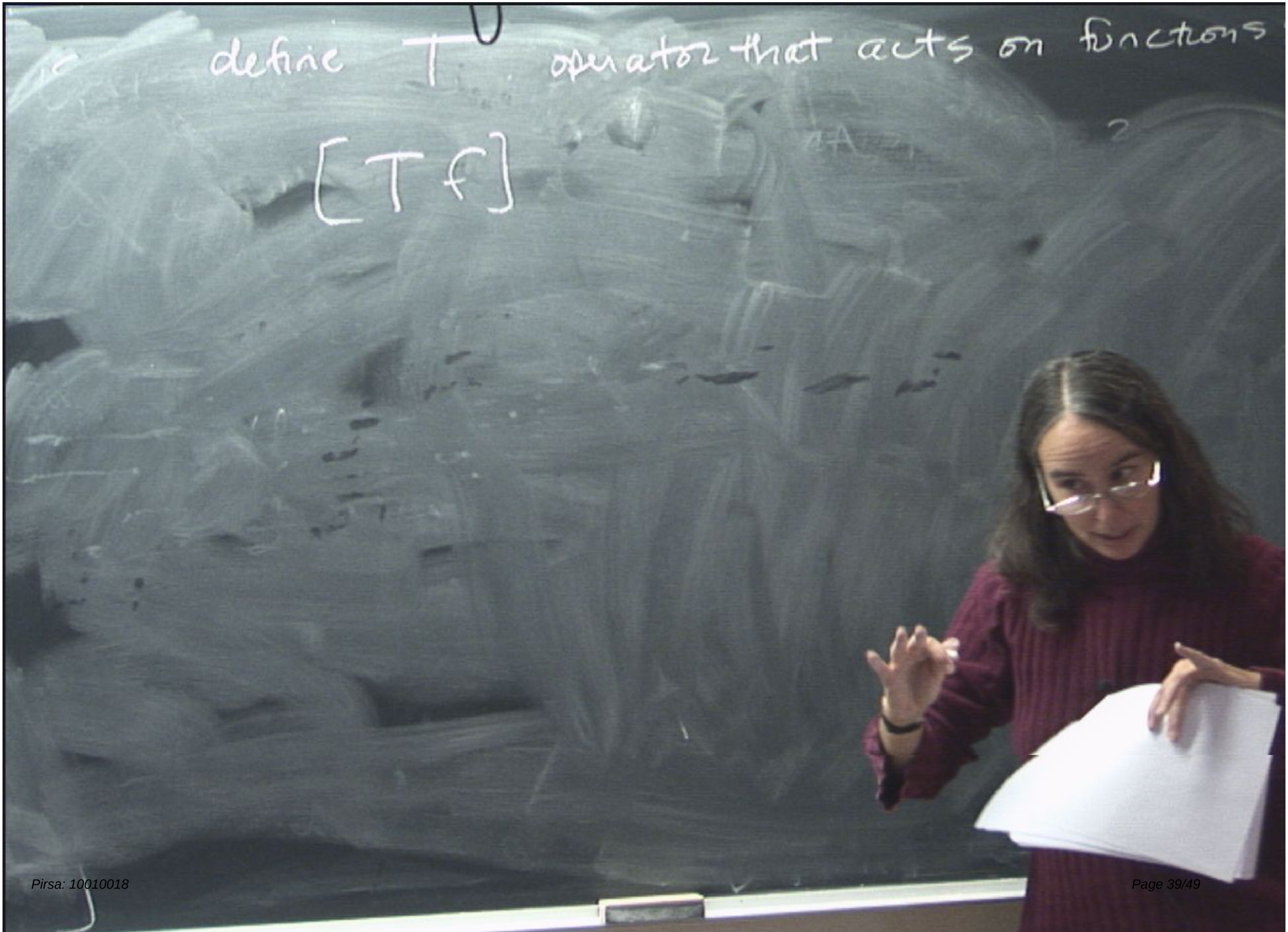
define T operator that acts on functions
[T f]

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consider functions $h(x)$ that are near $g(x)$:

$$h(x) = g(x) + \epsilon \eta(x)$$

define T: operator that acts on functions

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$$= -\alpha\left[g(h(-x/\alpha)) + \epsilon\eta\, h(-x/\alpha)\right]$$

define T: operator that acts on functions

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$$= -\alpha[g(g(-x/\alpha) + \epsilon\eta(-x/\alpha)) + \epsilon\eta(g(\tfrac{x}{\alpha} + \epsilon$$

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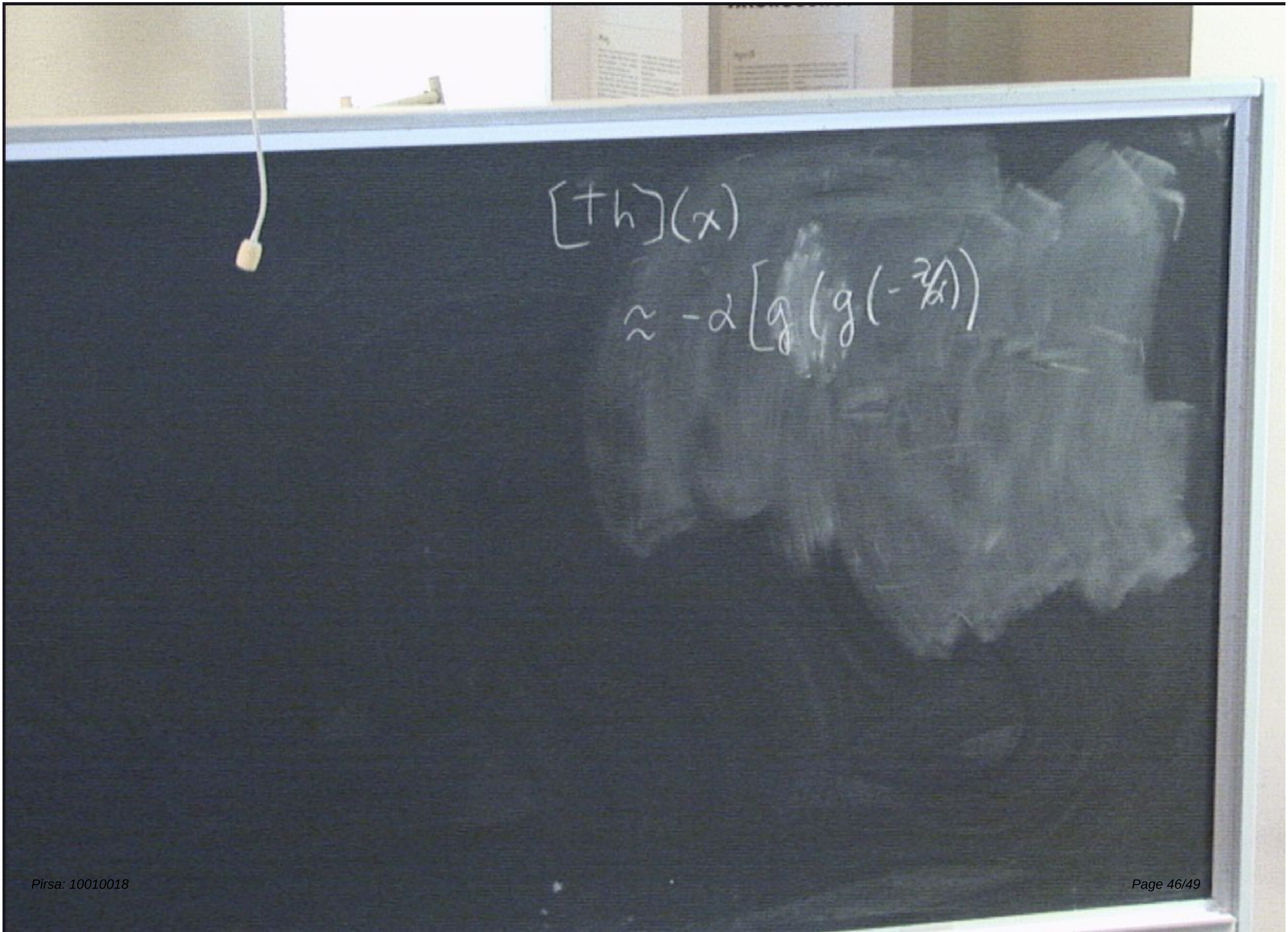
$$= -\alpha\left[g\left(h\left(-x/\alpha\right)\right) + \epsilon\eta\, h\left(-x/\alpha\right)\right]$$

$$= -\alpha\left[g\left(g\left(-x/\alpha\right) + \epsilon\eta\left(-x/\alpha\right)\right) + \epsilon\eta\left(g\left(\tfrac{x}{\alpha}\right) + \epsilon\eta\left(\tfrac{x}{\alpha}\right.\right.\right.$$


$[Th](x)$
$\approx -\alpha [g(g(-x/\alpha))$

$$[Th](x)$$

$$\approx -\alpha\left[g\left(g\left(-\tfrac{x}{\alpha}\right)\right) + \epsilon\,\eta\left(-\tfrac{x}{\alpha}\right)\frac{dg}{dx}\Big|_{g\left(-\frac{x}{\alpha}\right)} + \epsilon\,\eta\left(g\left(-\tfrac{x}{\alpha}\right)\right)\right] + O(\epsilon^2)$$

$$[Th](x)$$

$$\approx -\alpha\left[g\left(g\left(-z/\alpha\right)\right)+\epsilon\,\eta\left(\tfrac{-z}{\alpha}\right)\frac{dg}{dx}\bigg|_{g\left(\frac{-z}{\alpha}\right)}+\epsilon\,\eta\left(g\left(\tfrac{-z}{\alpha}\right)\right)\right]+O(\epsilon^2)$$

$$\approx g(z)-\alpha\epsilon\,\eta\left(g\left(-z/\alpha\right)\right)-\alpha\epsilon\,\frac{dg}{dx}\bigg|_{g(-z/\alpha)}\eta\left(-z/\alpha\right)$$

$$[Th](x)$$

$$\approx -\alpha\left[g\left(g\left(-z/\alpha\right)\right) + \epsilon\,\eta\left(-\frac{z}{\alpha}\right)\frac{dg}{dx}\bigg|_{g\left(-\frac{z}{\alpha}\right)} + \epsilon\,\eta\left(g\left(-\frac{z}{\alpha}\right)\right)\right] + O(\epsilon^2)$$

$$\approx g(z) - \alpha\epsilon\,\eta\left(g\left(-z/\alpha\right)\right) - \alpha\epsilon\,\frac{dg}{dx}\bigg|_{g(-z/\alpha)}\eta\left(-z/\alpha\right)$$