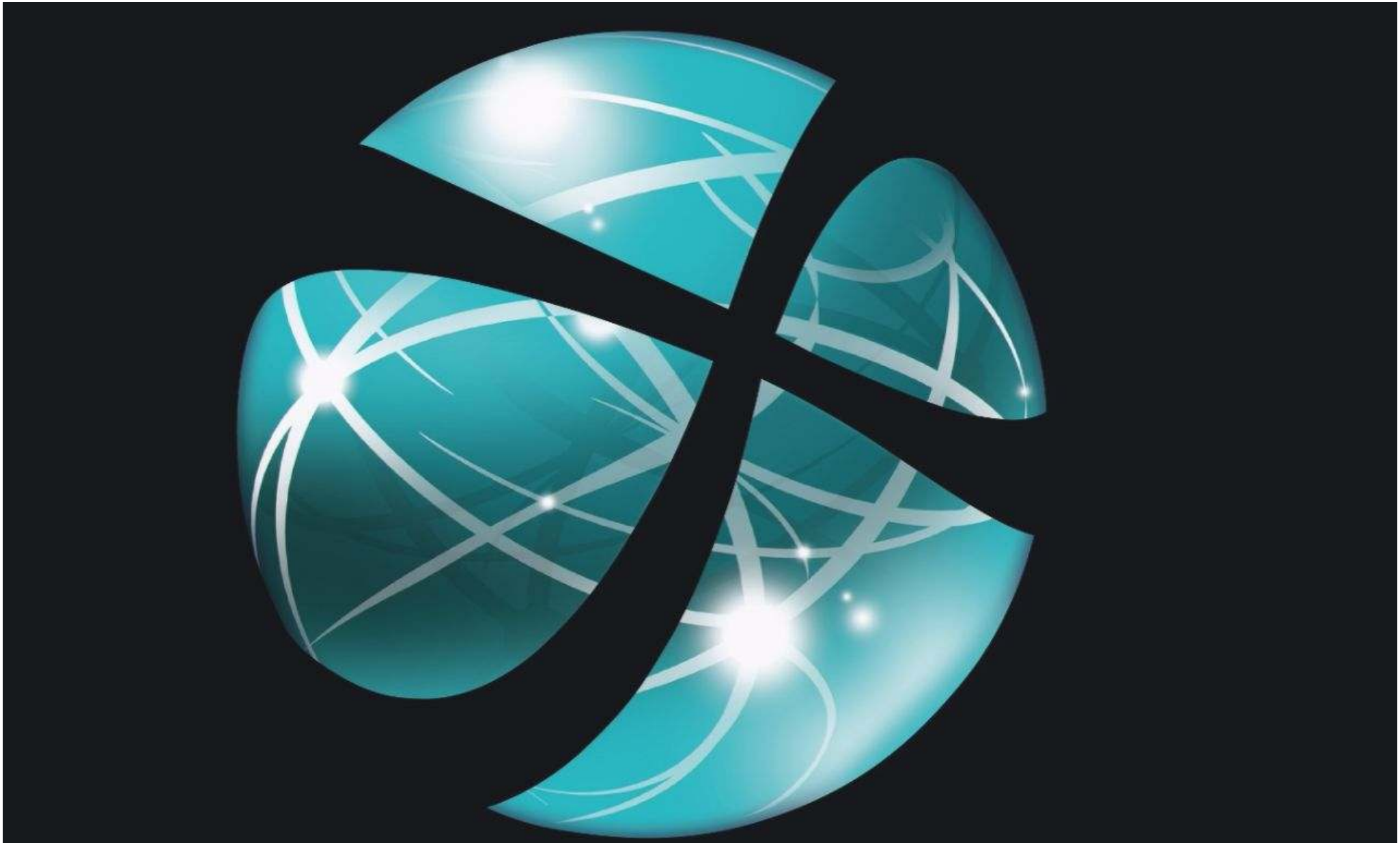
Title: Dynamical Systems - Review (PHYS 607) - Lecture 3

Date: Jan 06, 2010 09:00 AM

URL: http://pirsa.org/10010013

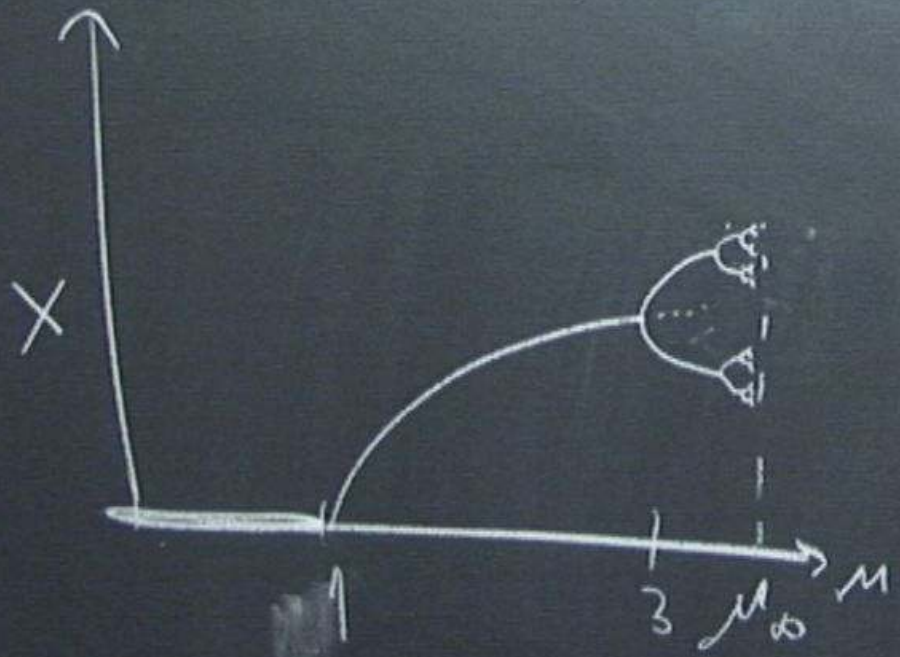
Abstract:


perimeter scholars
international

$f(x) = \mu x(1 - x_n)$

$x_{j+1} = f(x_j)$

bifurcation diagram



fixed points of

$f^{2^n}(x)$

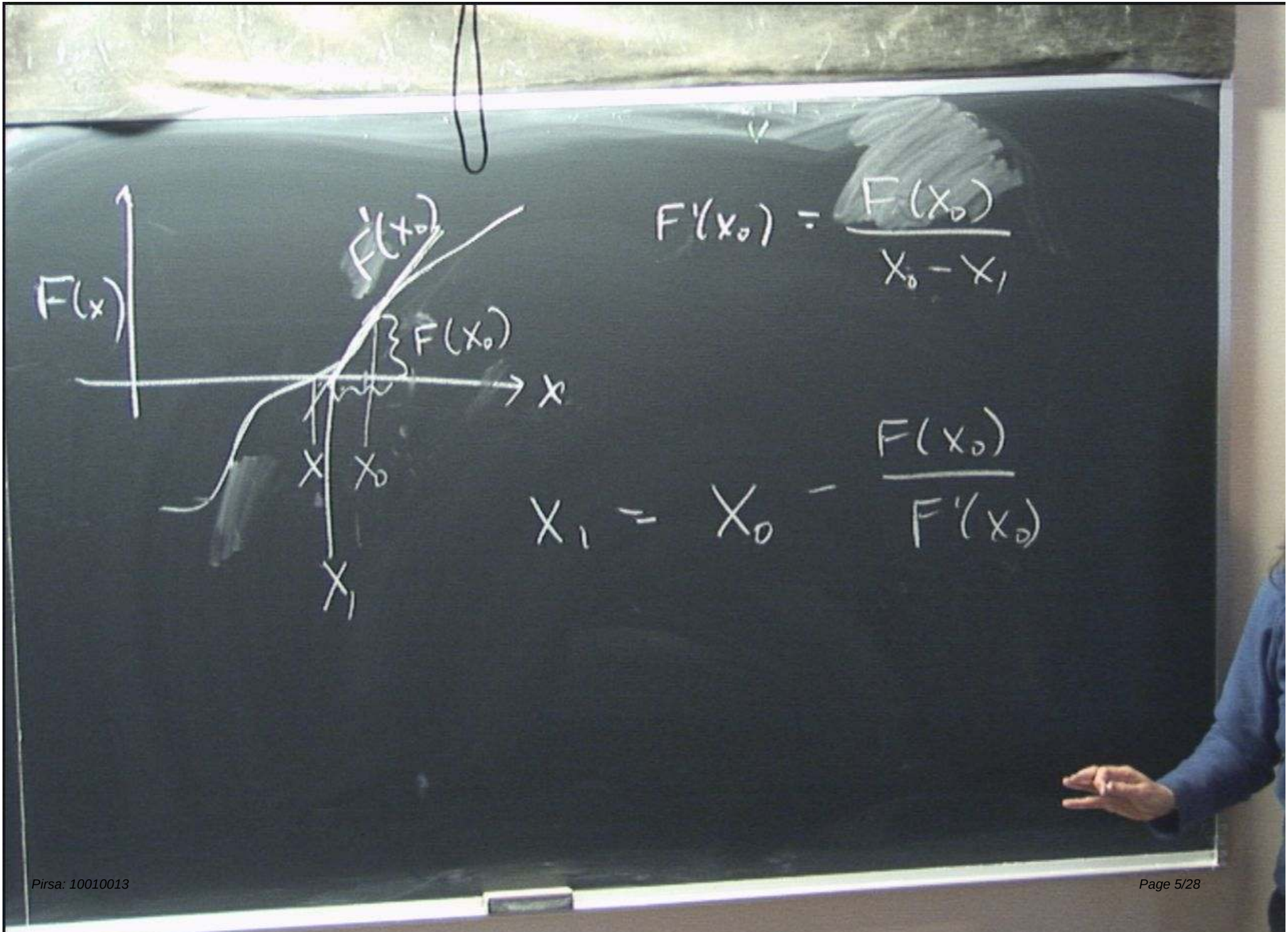
Solve equation

$F(x) = 0$

$F(x) = f^{2^n}(x)$

Newton-Raph

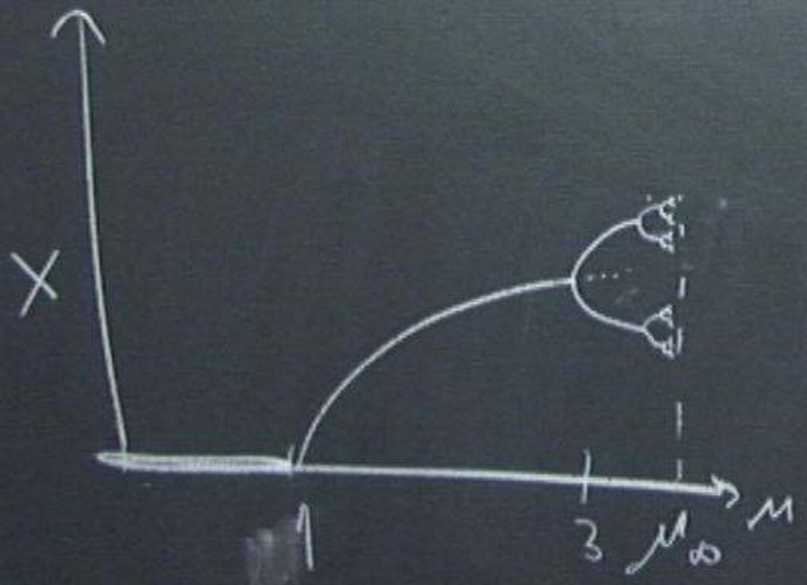
f(x)
f'(x0)
x
x
x0


F(x)
F'(x₀)
F(x₀)
x
x
x₀
x₁
F'(x₀) = F(x₀) / (x₀ − x₁)
x₁ = x₀ − F(x₀) / F'(x₀)

$f(x) = \mu x(1 - x_n)$

$x_{n+1} = f(x_n)$

furcation
iagram



fixed points of

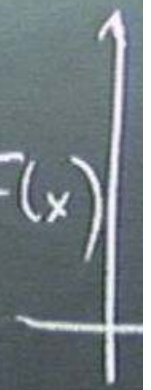
$f^{2^n}(x)$

Solve equation

$F(x) = 0$

$F(x) = f^{2^n}(x) - x$

Newton-Raphson



$f(x) = \mu x(1 - x_n)$

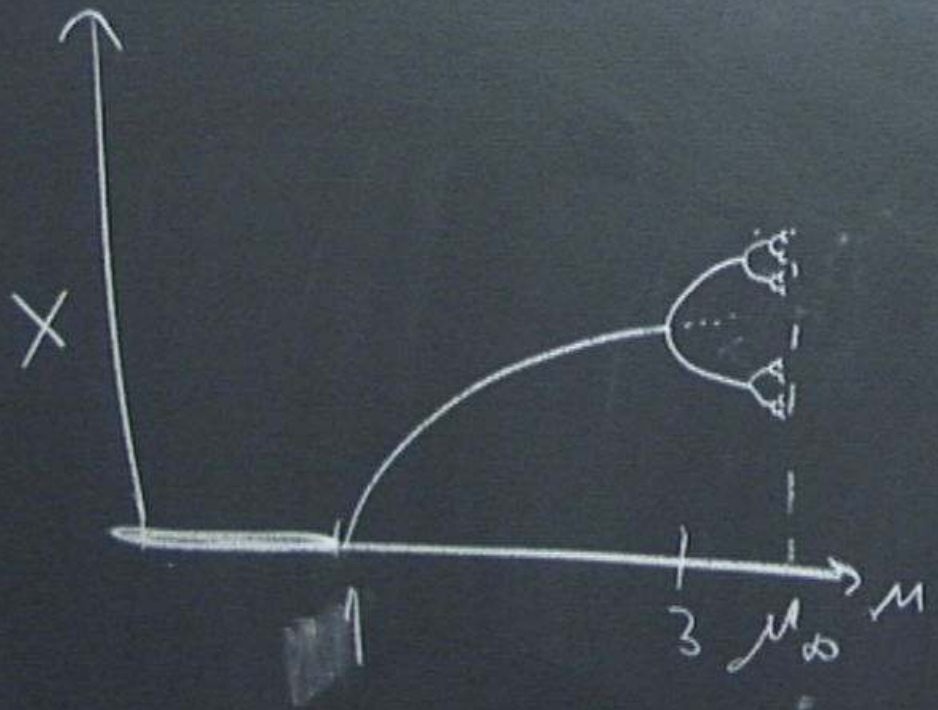
$x_{n+1} = f(x_n)$

Stability determined by

$\frac{d}{dx_0} f^{2^N}(x_0)$

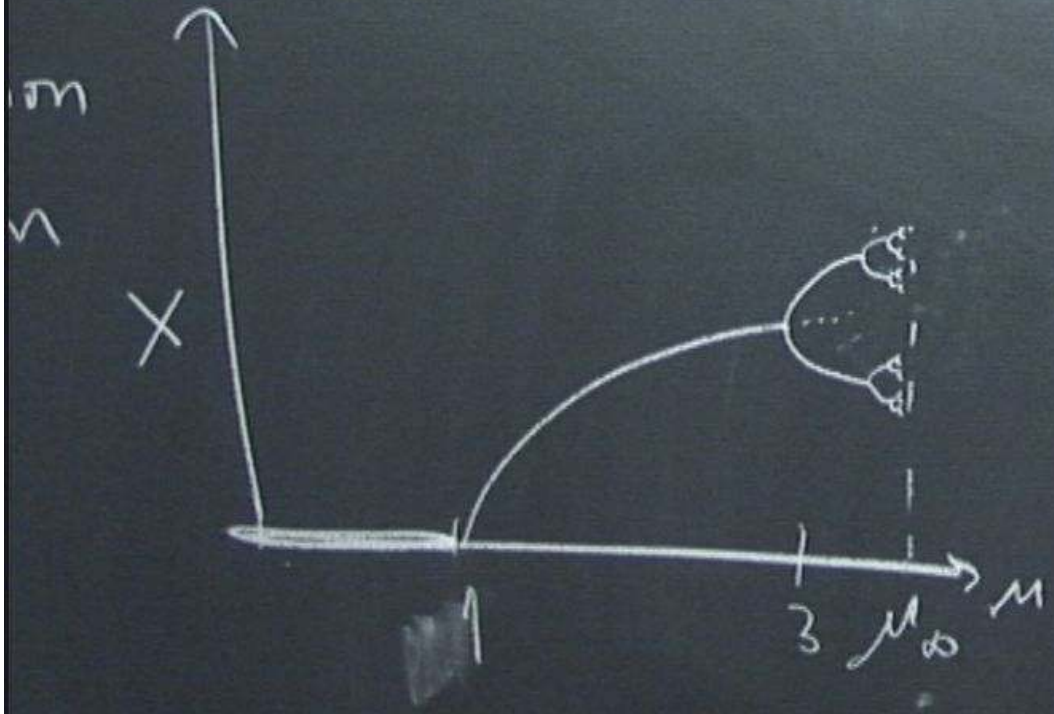
rcation

gram



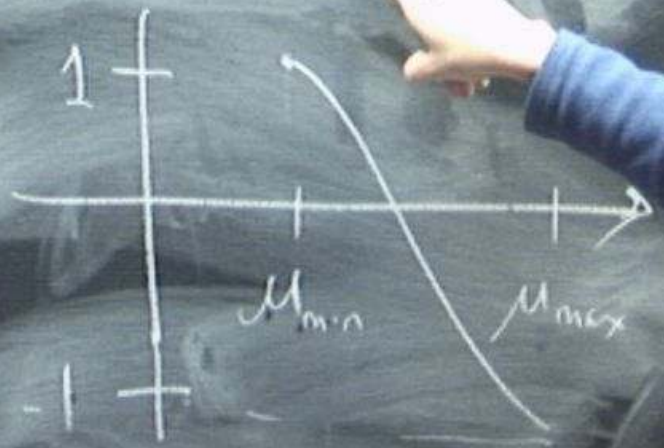
$f(x) = \mu x(1 - x_n)$

$x_{n+1} = f(x_n)$



Stability determined by

$$\frac{d}{dx_0} f^{2^N}(x_0) = \lambda$$

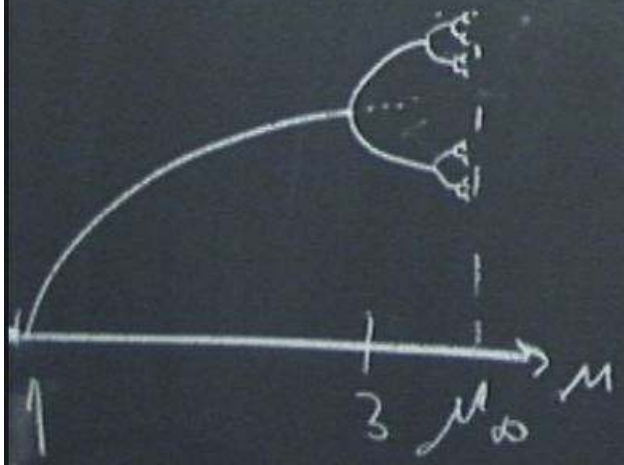


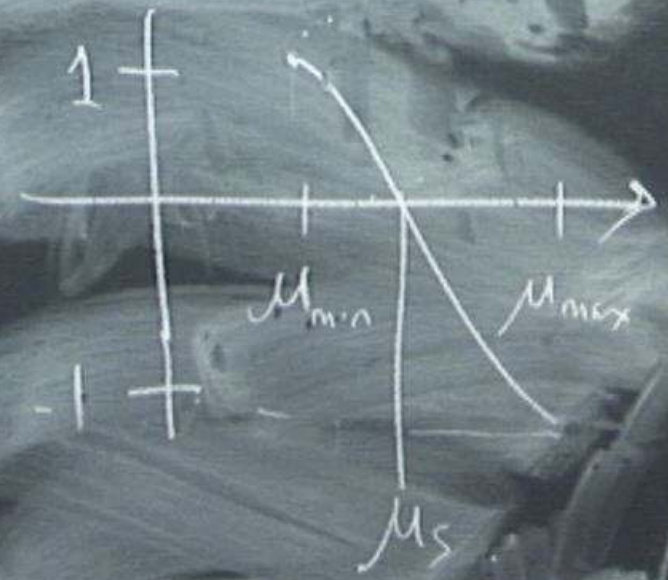
$x(1-x_n)$

$= f(x_n)$

Stability determined by

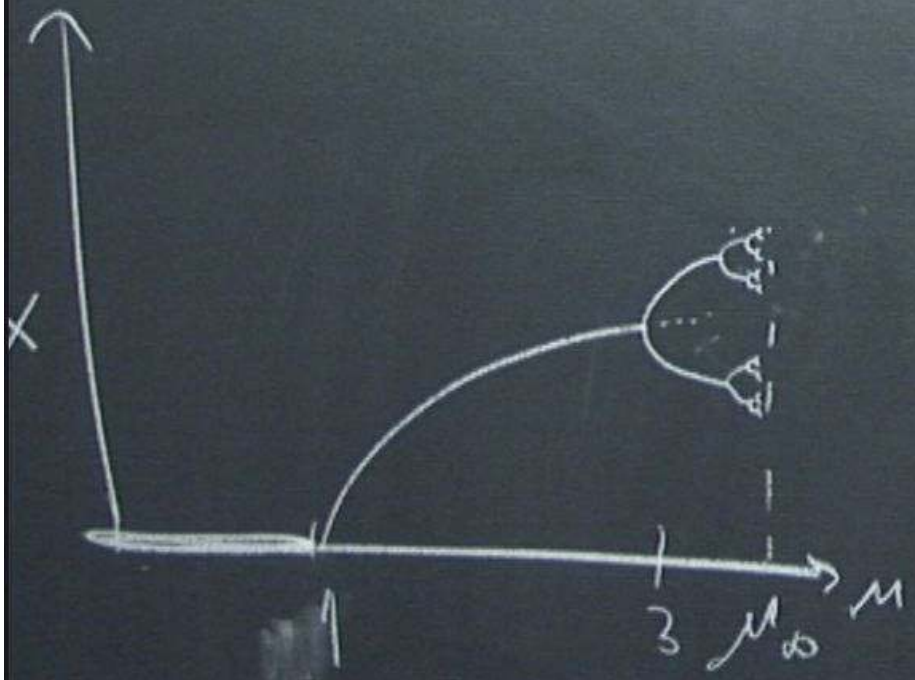
$$\frac{d}{dx_0} f^{2^N}(x_0) = \lambda$$





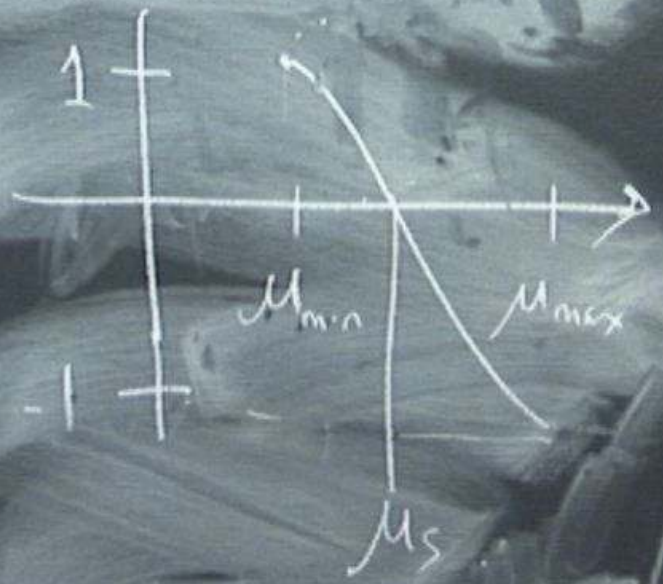
$f(x) = \mu x(1 - x_n)$

$x_{n+1} = f(x_n)$



Stability determined by

$$\frac{d}{dx_0} f^{2^N}(x_0) = \lambda$$



for each n [period $= 2^n$]

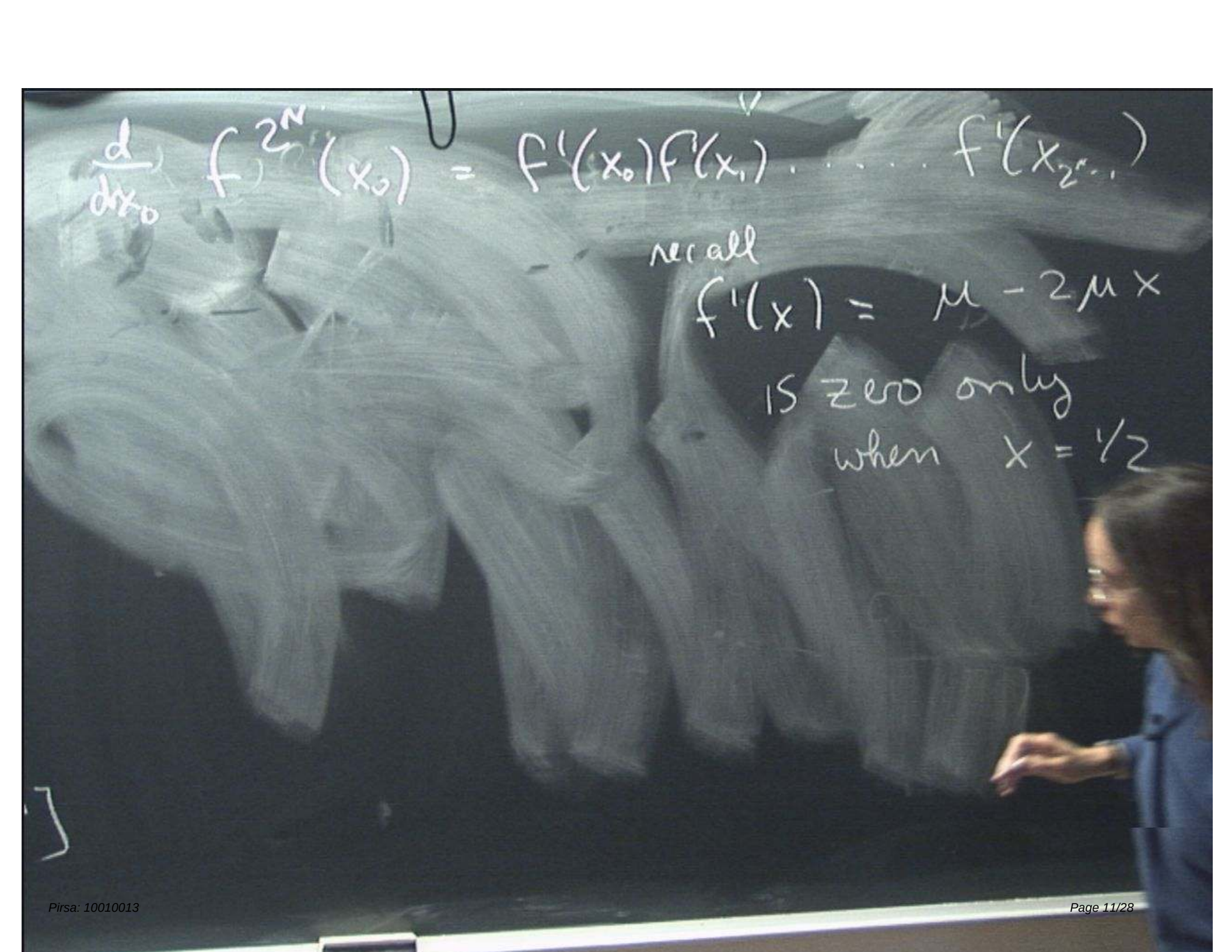
there is a μ for which $\lambda = 0$

$$\frac{d}{dx_0} f^{2^N}(x_0) = f'(x_0) f'(x_1) \cdots f'(x_{2^N-1})$$

recall

$$f'(x) = \mu - 2\mu x$$

is zero only when $x = 1/2$



$$\frac{d}{dx_0} f^{2^N}(x_0) = f'(x_0) f'(x_1) \cdots f'(x_{2^N-1})$$

recall

$$f'(x) = \mu - 2\mu x$$

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when $x = 1/2$

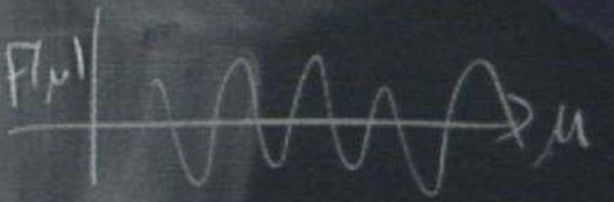
$$\frac{d}{dx_0} f^{2^n}(x_0) = f'(x_0) f'(x_1) \cdots f'(x_{2^n-1})$$

recall

$$f'(x) = \mu - 2\mu x$$

is zero only when $x = 1/2$

$$F(\mu) = f_\mu^N(1/2) - 1/2$$



$\frac{d}{d\mu} f^{2^n}(\frac{1}{2})$

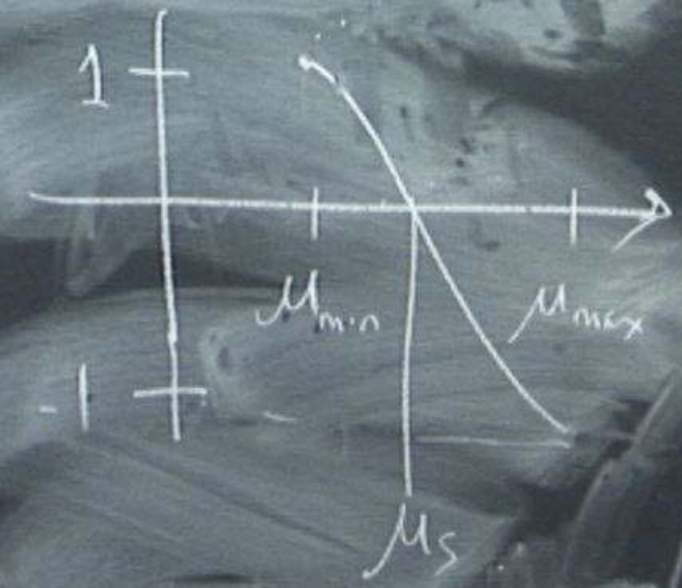
$x_0 = \frac{1}{2}$

$\frac{d}{d\mu} x_1 = \frac{d}{d\mu} \mu x_0(1-x_0)$

$= x_0(1-x_0)$

Stability determined by

$\frac{d}{dx_0} f^{2^n}(x_0) = \lambda$



for each n (period
there is a μ for which λ

$$\frac{d}{d\mu} f^{2^n}\left(\tfrac{1}{2}\right)$$

$$x_0 = \tfrac{1}{2}$$

$$\frac{d}{d\mu} x_1 = \frac{d}{d\mu} \mu x_0(1-x_0)$$

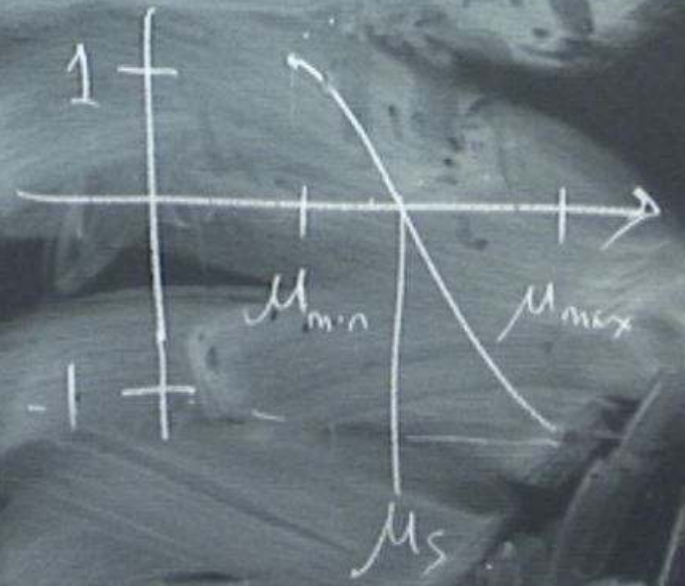
$$\frac{dx_1}{d\mu} = x_0(1-x_0)$$

$$\frac{d}{d\mu} x_2 = \frac{d}{d\mu} \mu x_1(1-x_1)$$

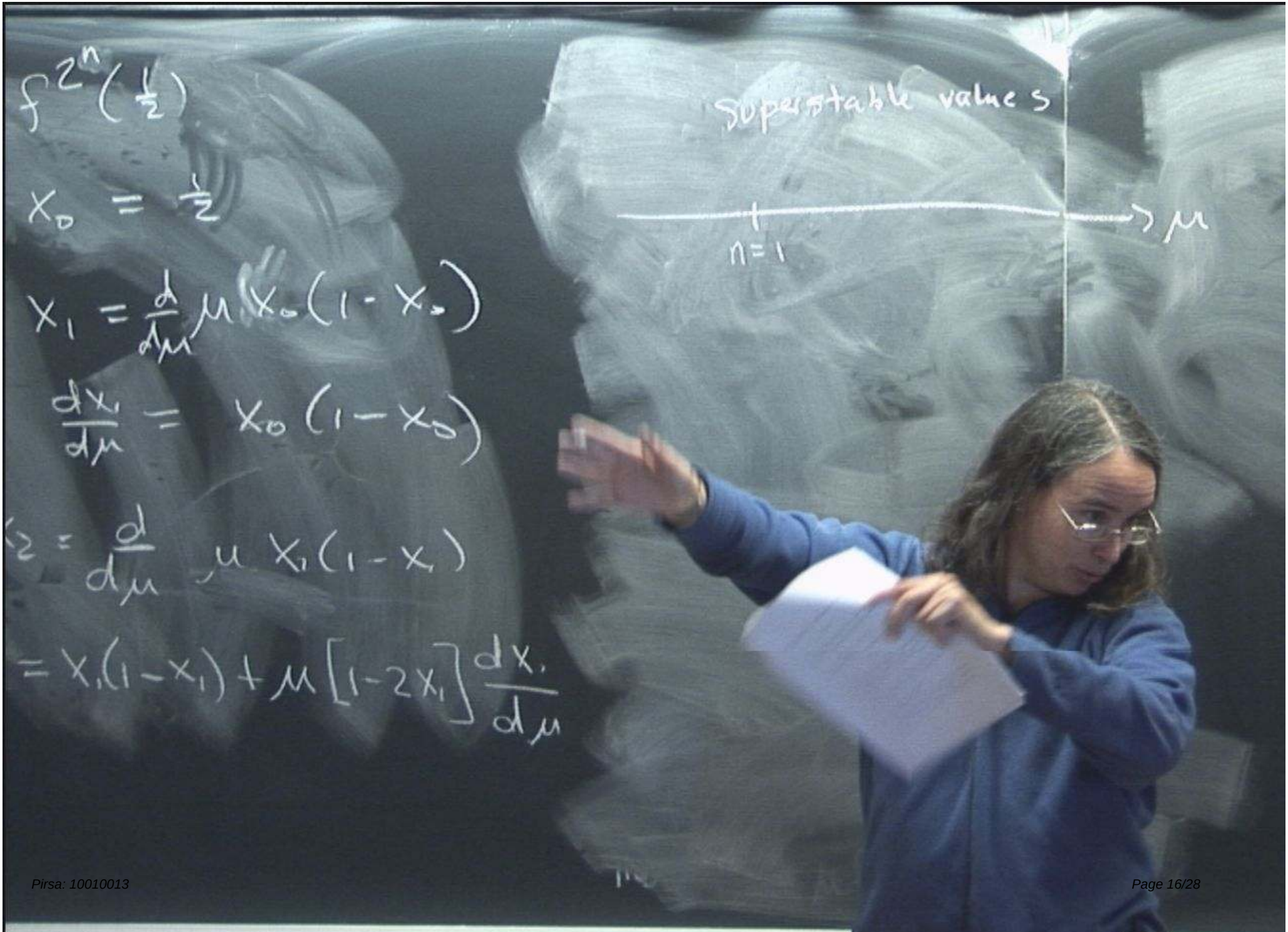
$$= x_1(1-x_1) + \mu\left[1-2x_1\right]\frac{dx_1}{d\mu}$$

Stability determined by

$$\frac{d}{dx_0} f^{2^n}(x_0) = \lambda$$



for each n (period) there is a μ for which λ


$f^{2^n}(\frac{1}{2})$
$x_0 = \frac{1}{2}$
$x_1 = \frac{d}{d\mu}\mu x_0(1-x_0)$
$\frac{dx_1}{d\mu} = x_0(1-x_0)$
$\frac{d}{d\mu}\mu x_1(1-x_1)$
$= x_1(1-x_1) + \mu[1-2x_1]\frac{dx_1}{d\mu}$
superstable values
n=1
μ

$f^{2^n}\left(\frac{1}{2}\right)$

$x_0 = \frac{1}{2}$

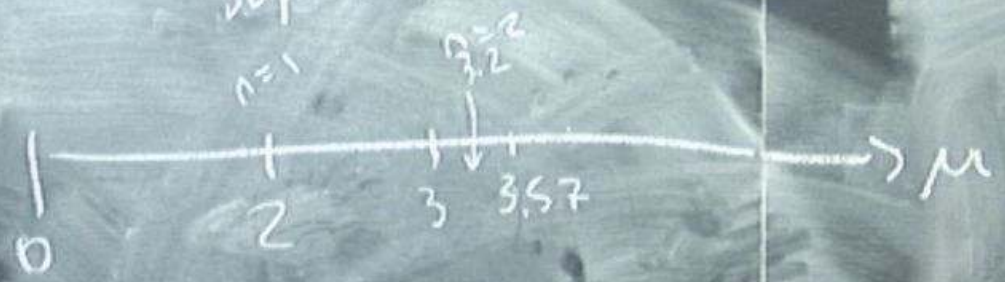
$x_1 = \frac{d}{d\mu}\mu x_0(1-x_0)$

$\frac{dx_1}{d\mu} = x_0(1-x_0)$

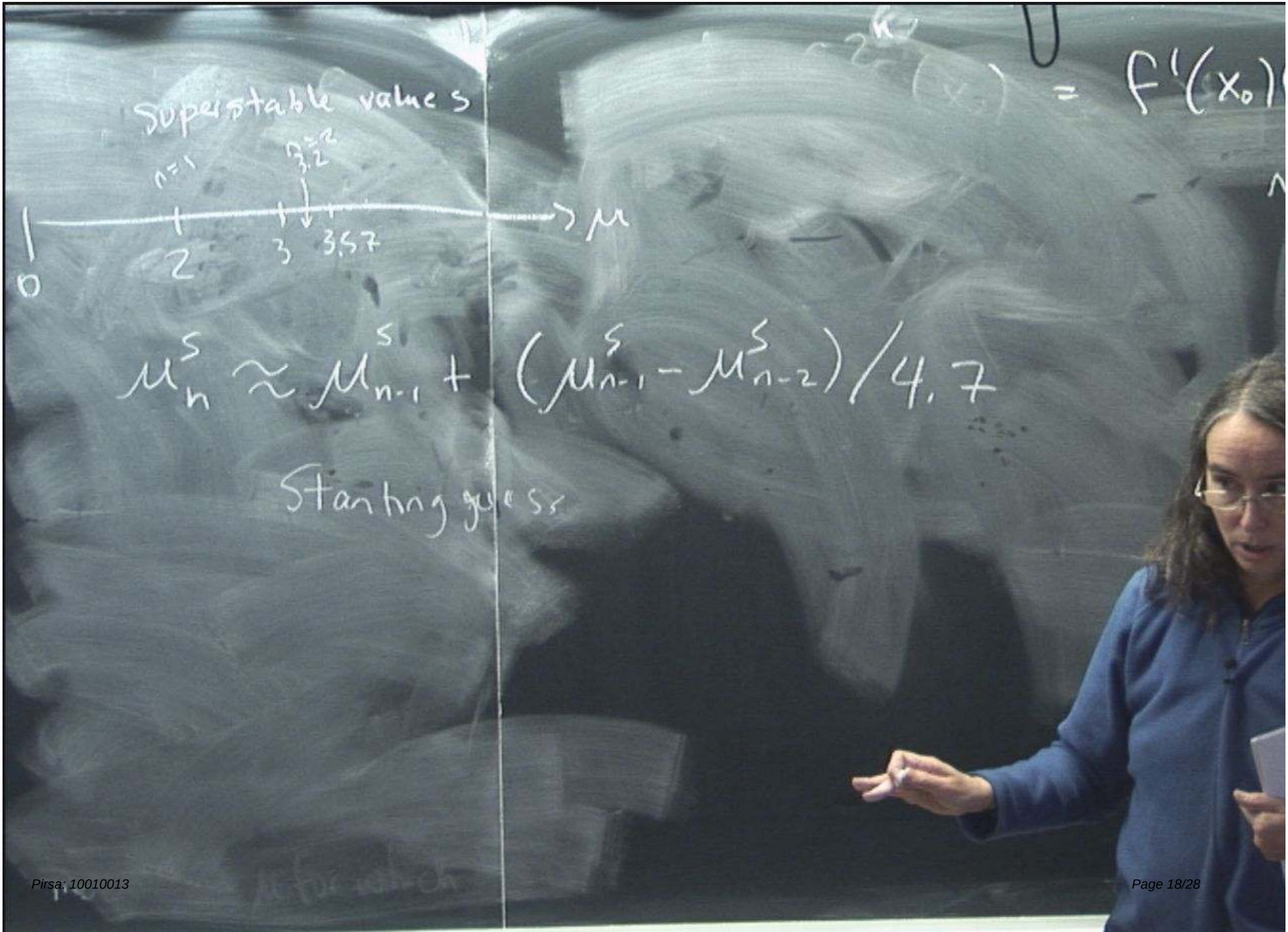
$x_2 = \frac{d}{d\mu}\mu x_1(1-x_1)$

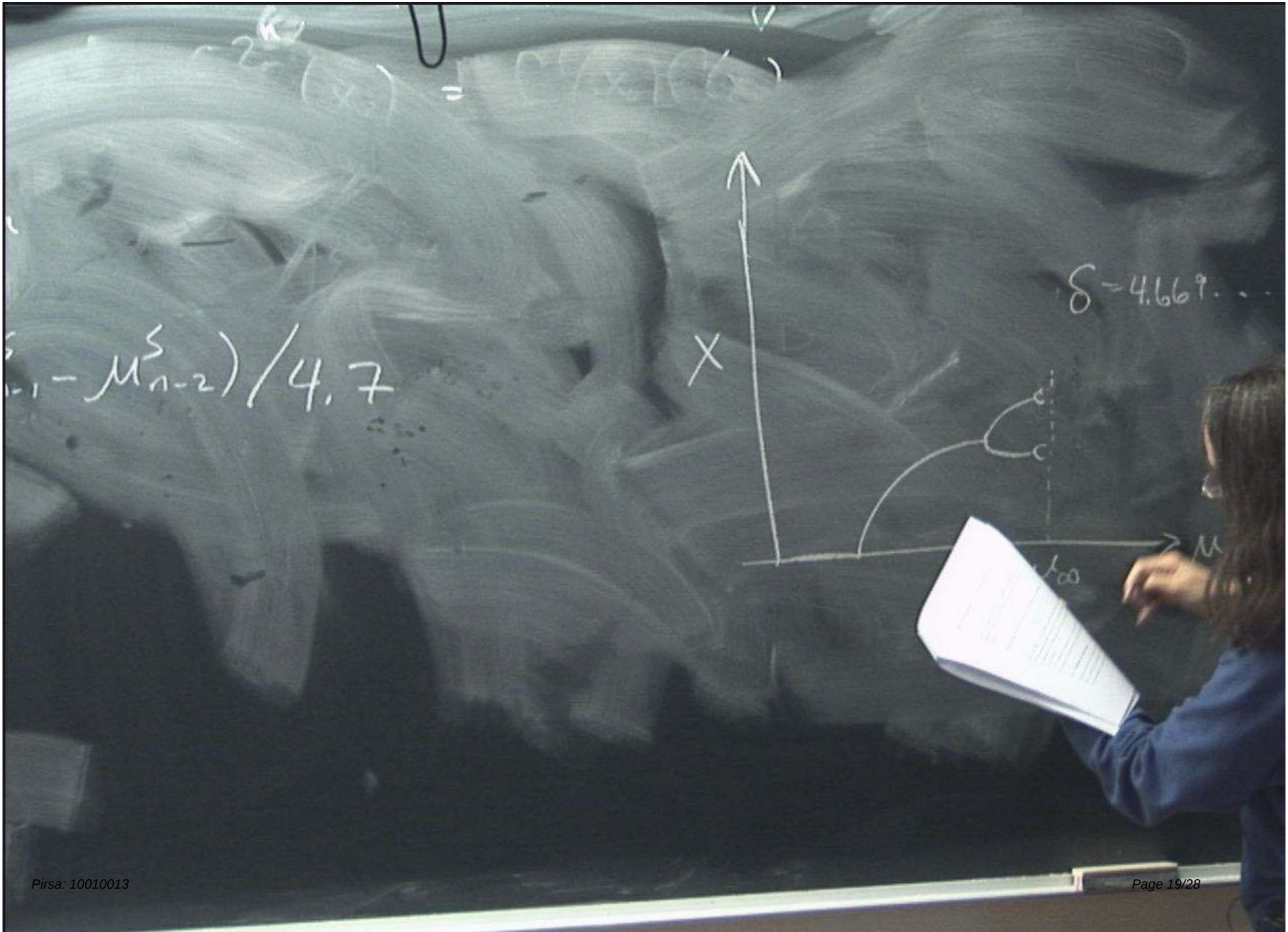
$= x_1(1-x_1) + \mu\left[1-2x_1\right]\frac{dx_1}{d\mu}$

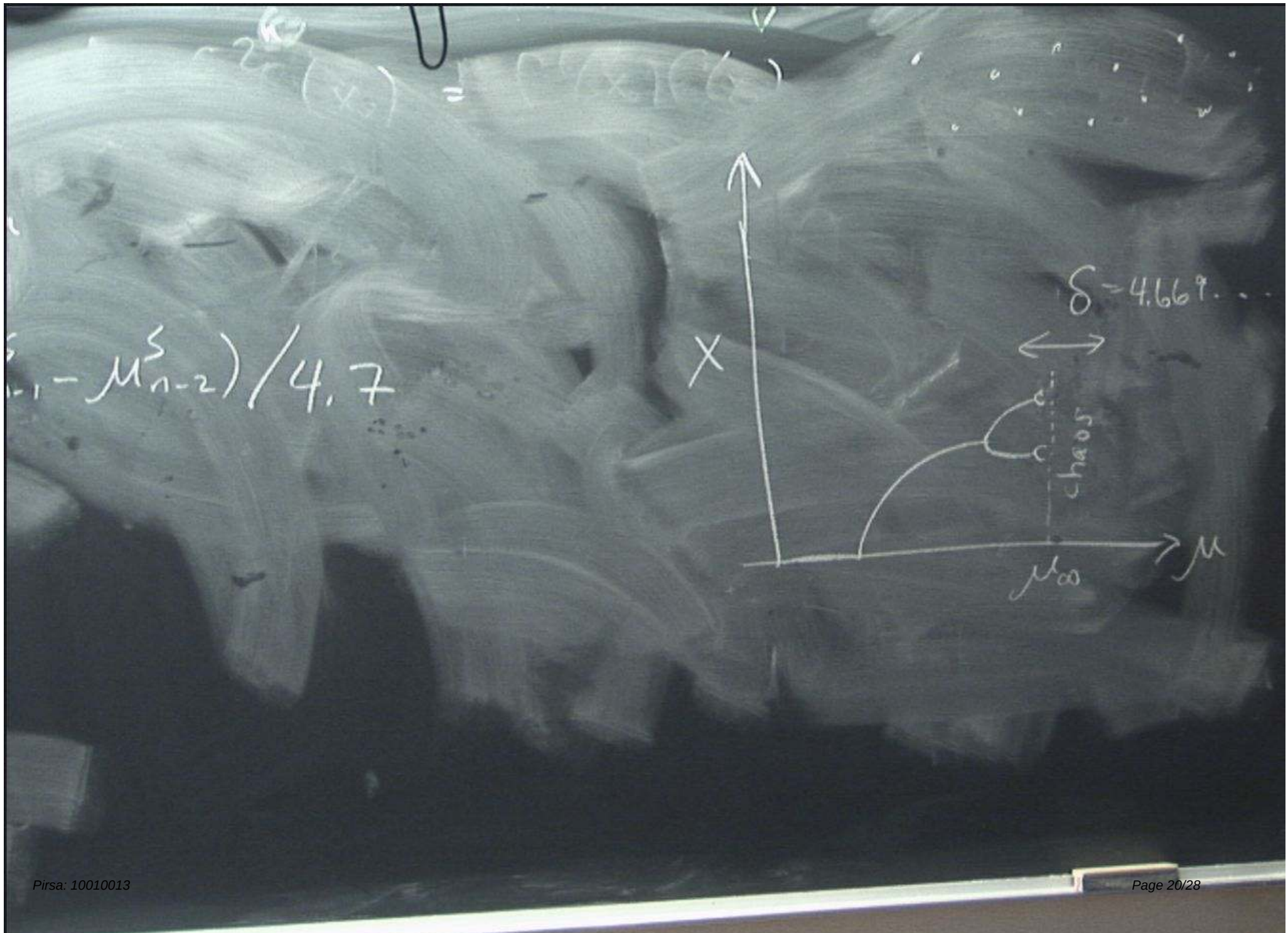
Superstable values

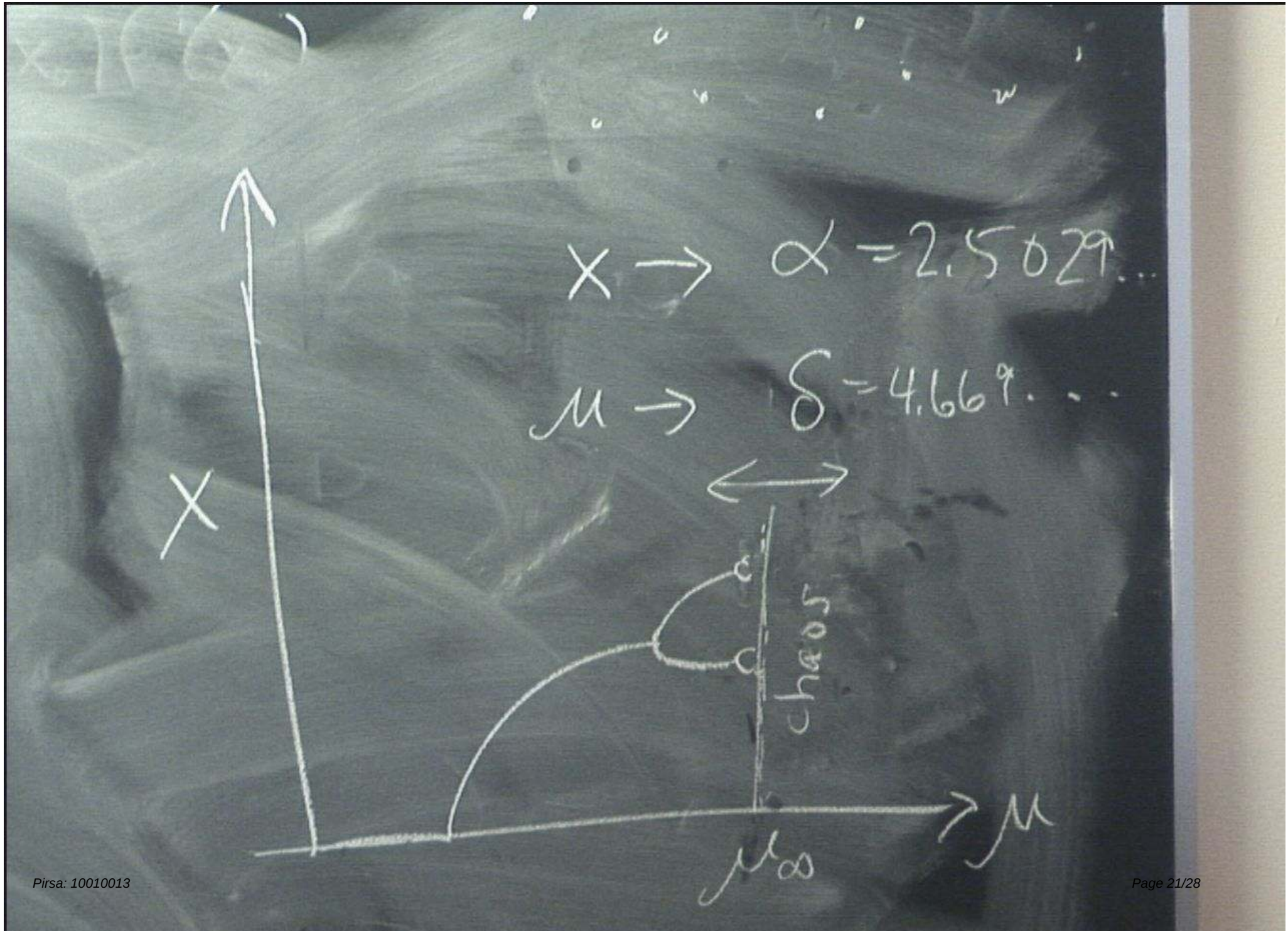


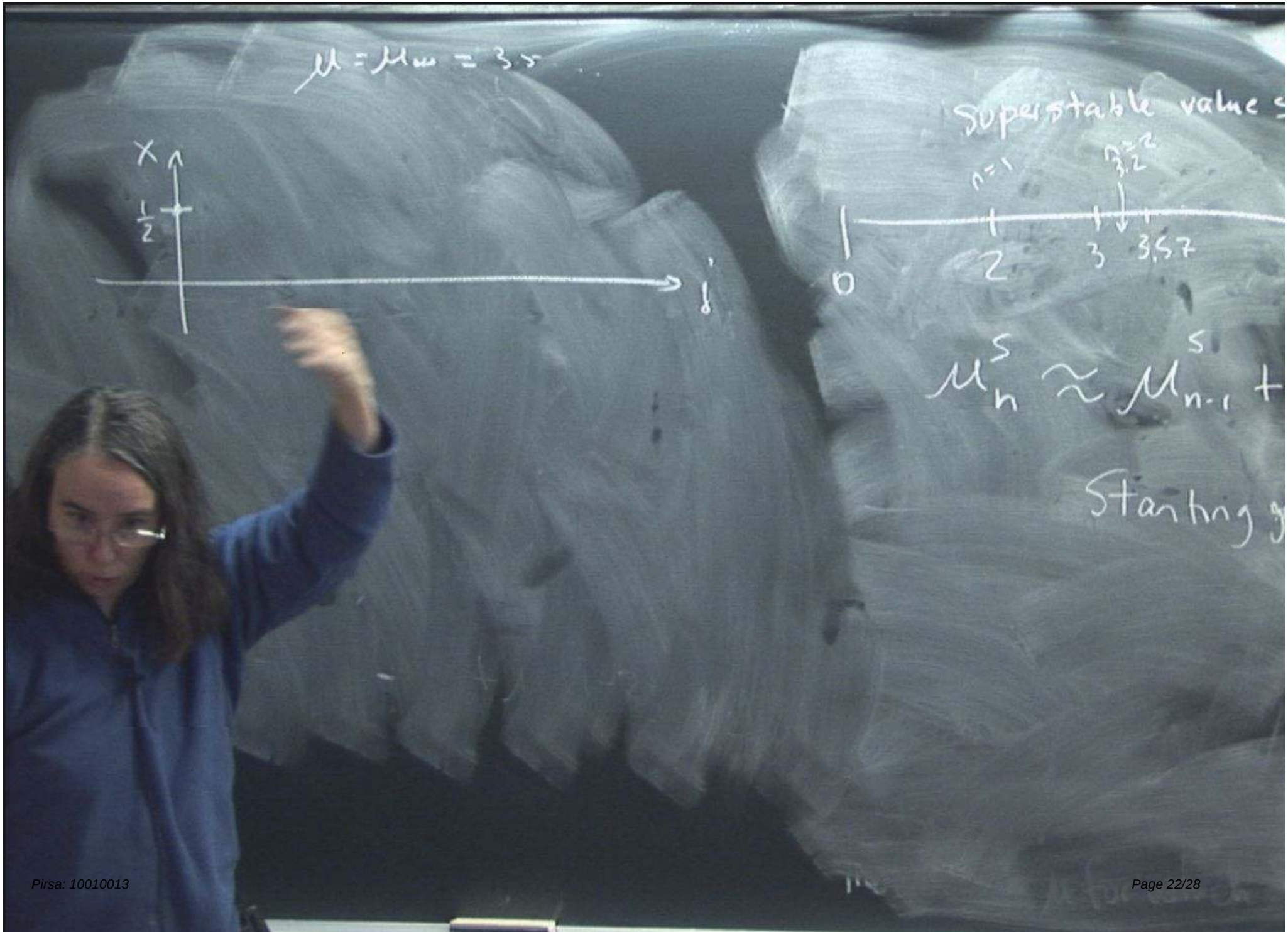
$\mu_n^s \approx \mu_{n-1}^s$

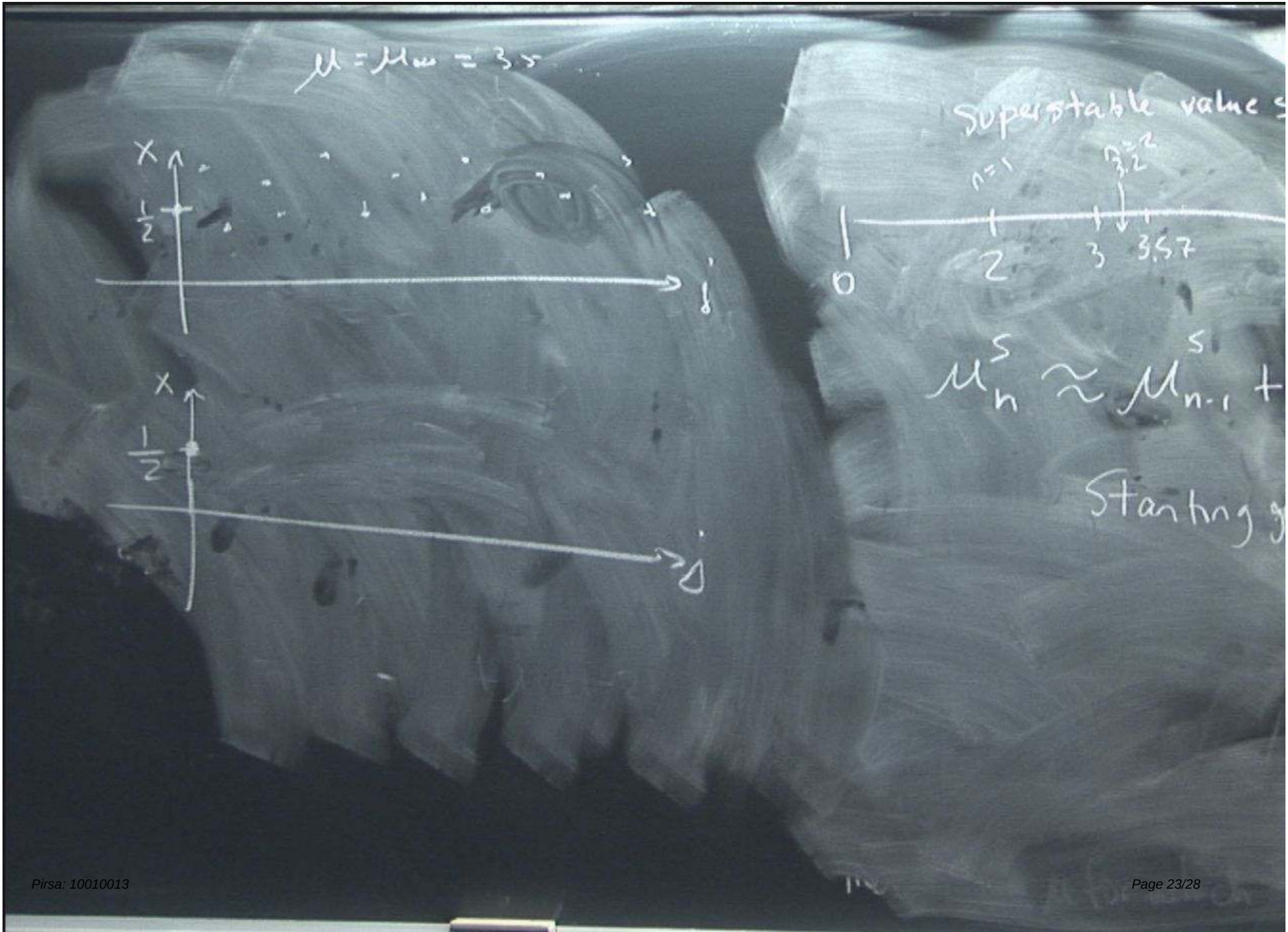

Superstable values
n=1
3.2
0
2
3
3.57
μ
$\mu_n^s \approx \mu_{n-1}^s + (\mu_{n-1}^s - \mu_{n-2}^s)/4.7$
Starting guess

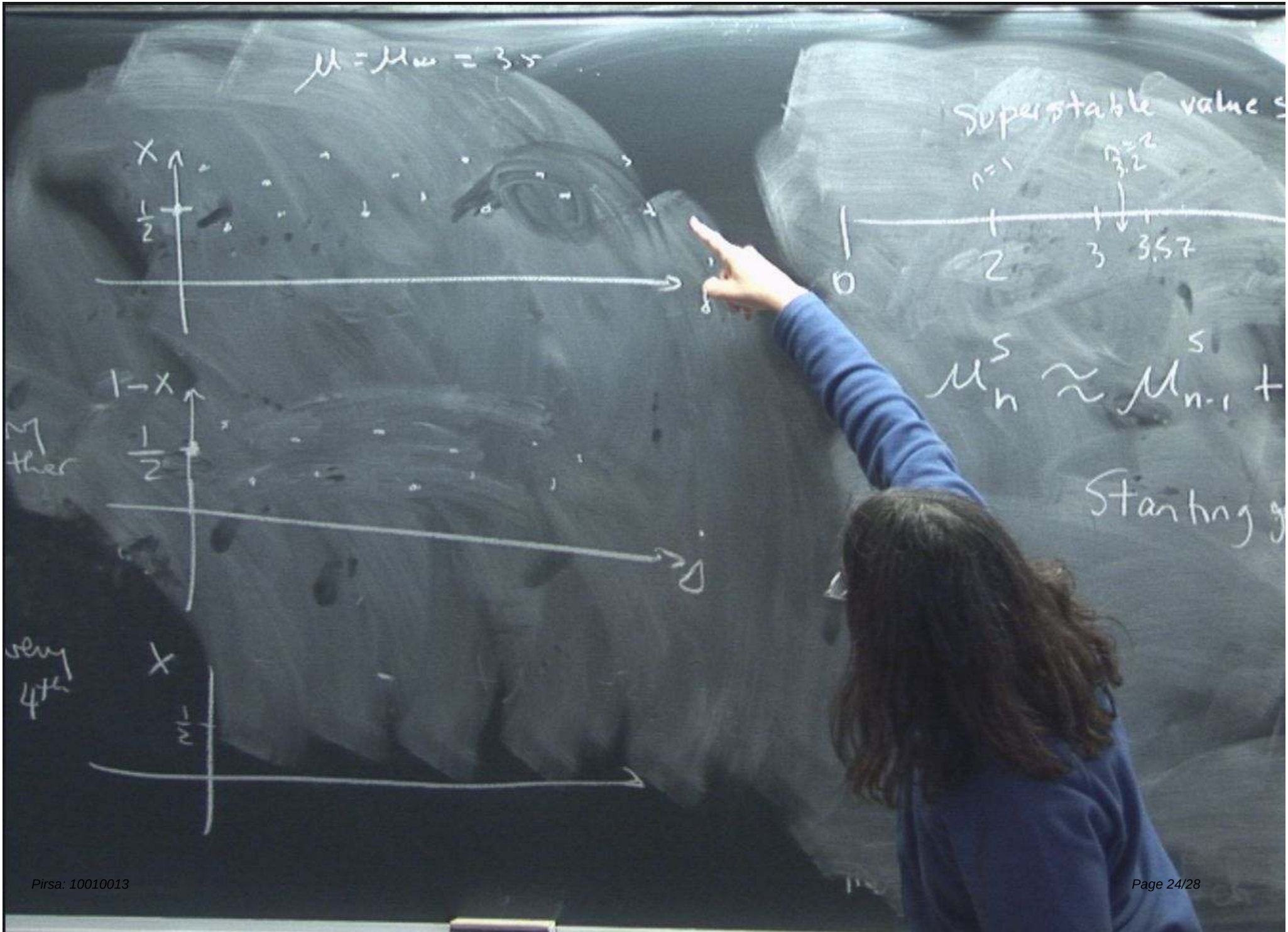

δ = 4.669...
X
μ

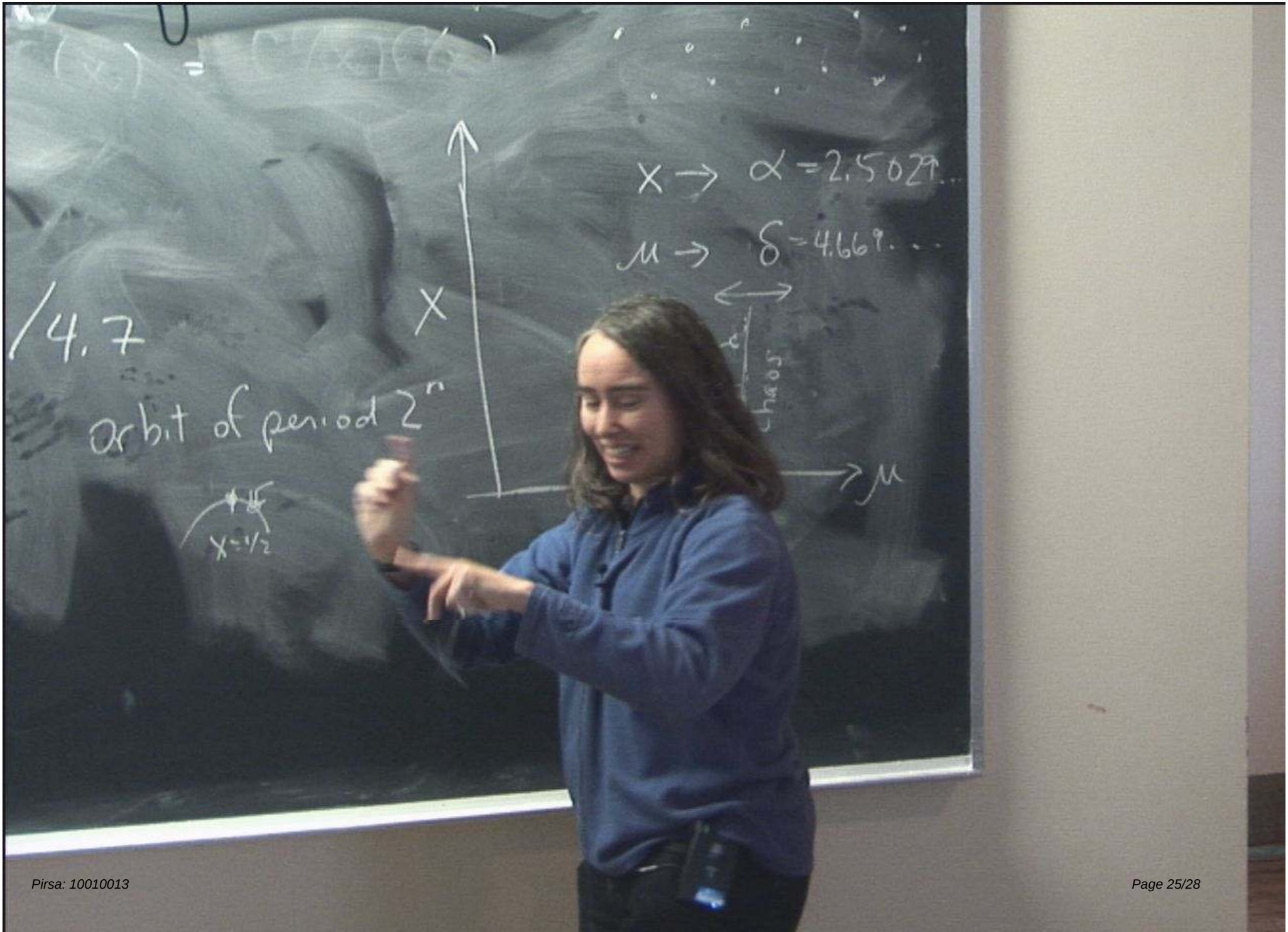

$\delta = 4.669\ldots$
x
chaos
μ
μ_∞

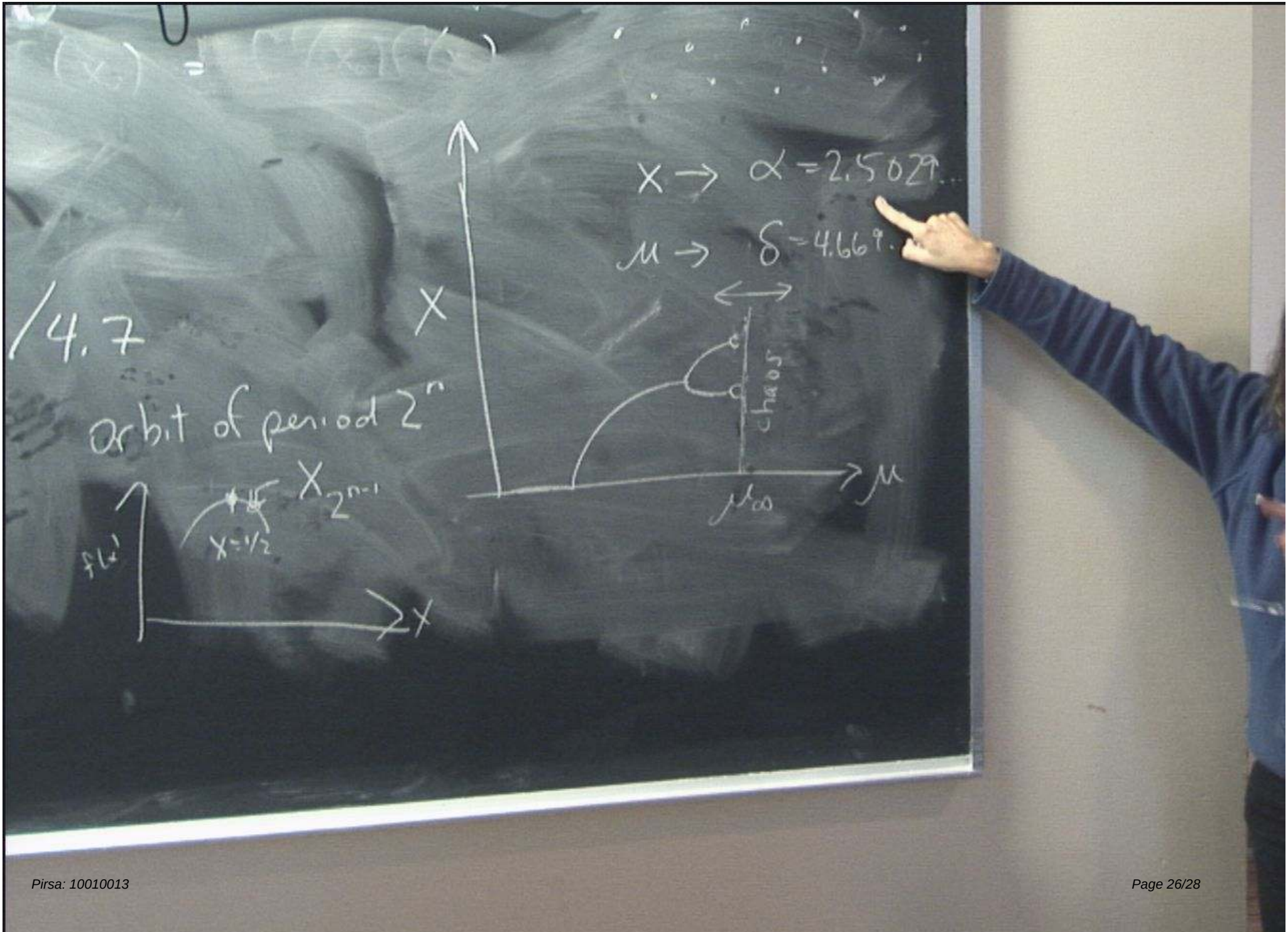

$x \to \alpha = 2.5029\ldots$
$\mu \to \delta = 4.669\ldots$
x
chaos
μ_∞
μ

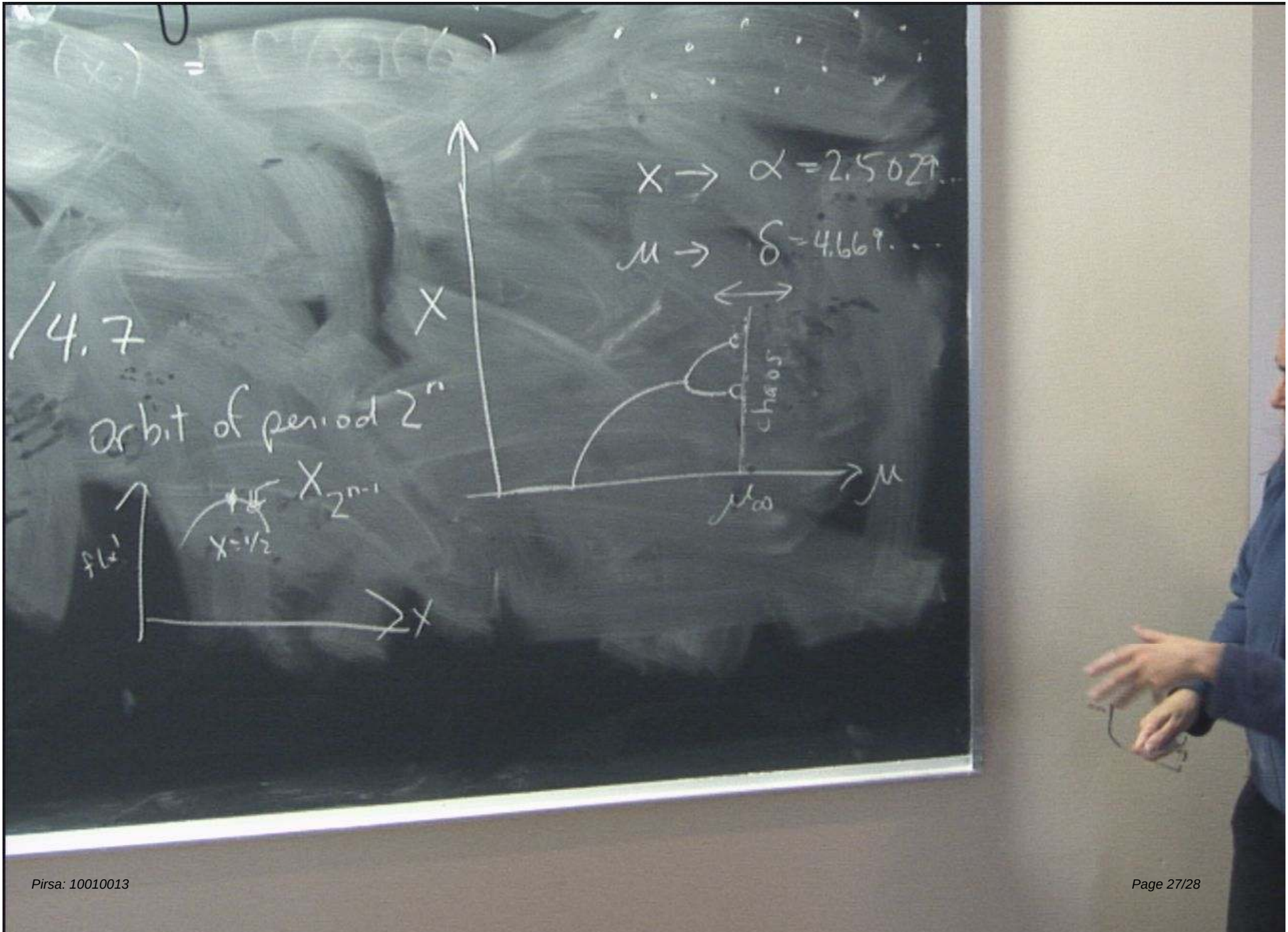

μ = μ∞ ≈ 3.5
x
1/2
1
Superstable value s
n=1
3.2
0
2
3
3.57
Starting

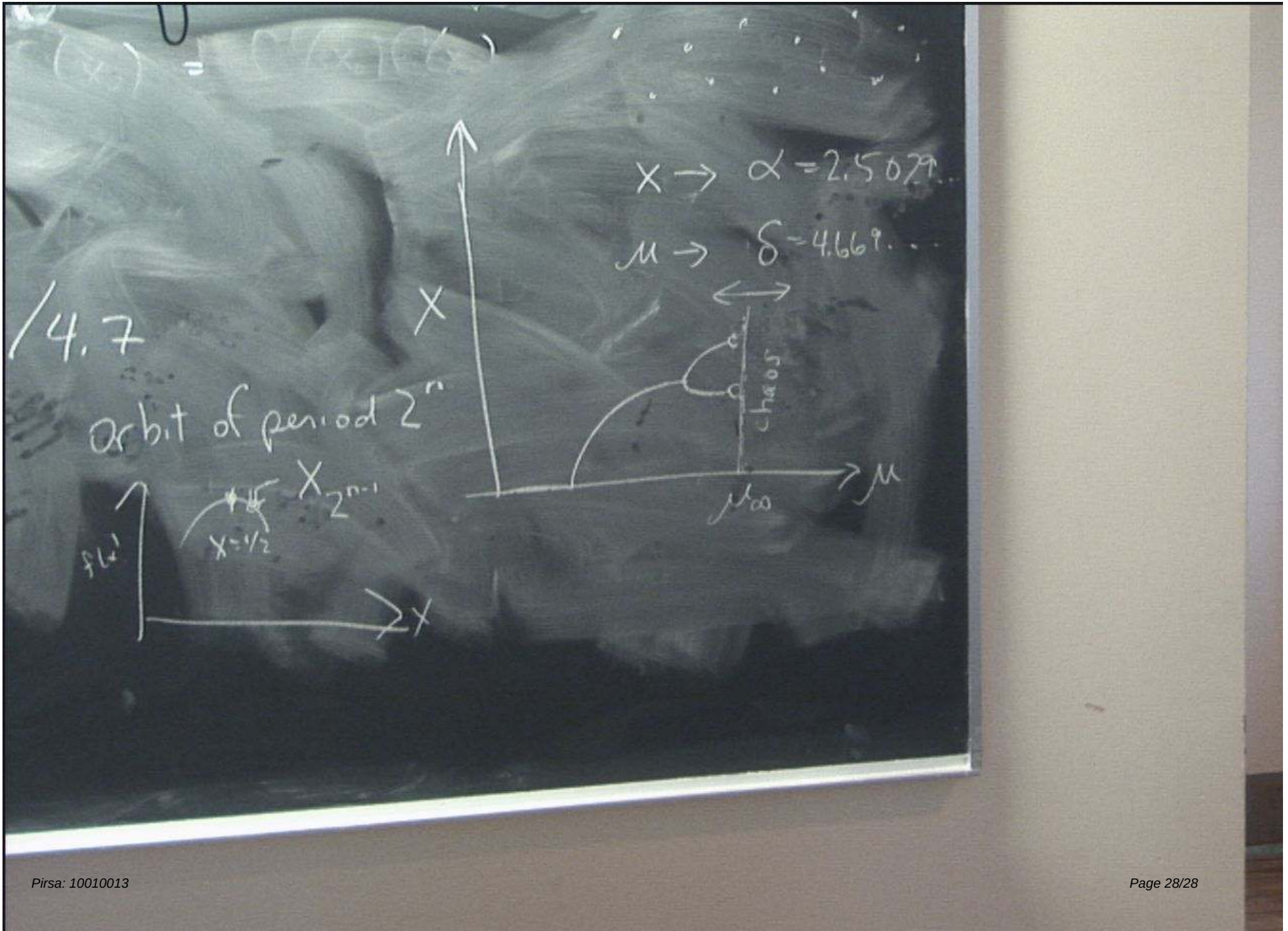

μ = μ∞ = 3.5
X
1/2
1
X
1/2
1
Superstable value s
n=1
3.2
0
2
3
3.57
$\mu_n^s \approx \mu_{n-1}^s +$
Starting g


μ = μ∞ = 3
Superstable value
n=1
3.2
0
2
3
3.57
X
1/2
1−X
1/2
ther
very
4th
X
1/2
Starting


x → α = 2.5029...
μ → δ = 4.669...
/4.7
X
orbit of period 2^n
chaos
μ
X=1/2


4.7
Orbit of period 2^n
$X_{2^{n-1}}$
$x=1/2$
$f(x)$
X
$X \to \alpha = 2.5029\ldots$
$\mu \to \delta = 4.669\ldots$
X
chaos
μ_∞
μ


$x \to \alpha = 2.5029\ldots$
$\mu \to \delta = 4.669\ldots$
4.7
X
Orbit of period 2^n
$X_{2^{n-1}}$
$f(x)$
$x = 1/2$
X
chaos
μ_∞
μ


4.7
Orbit of period 2^n
$X_{2^{n-1}}$
$x=1/2$
$f(x)$
X
$X \to \alpha = 2.5029\ldots$
$\mu \to \delta = 4.669\ldots$
X
chaos
μ_∞
μ