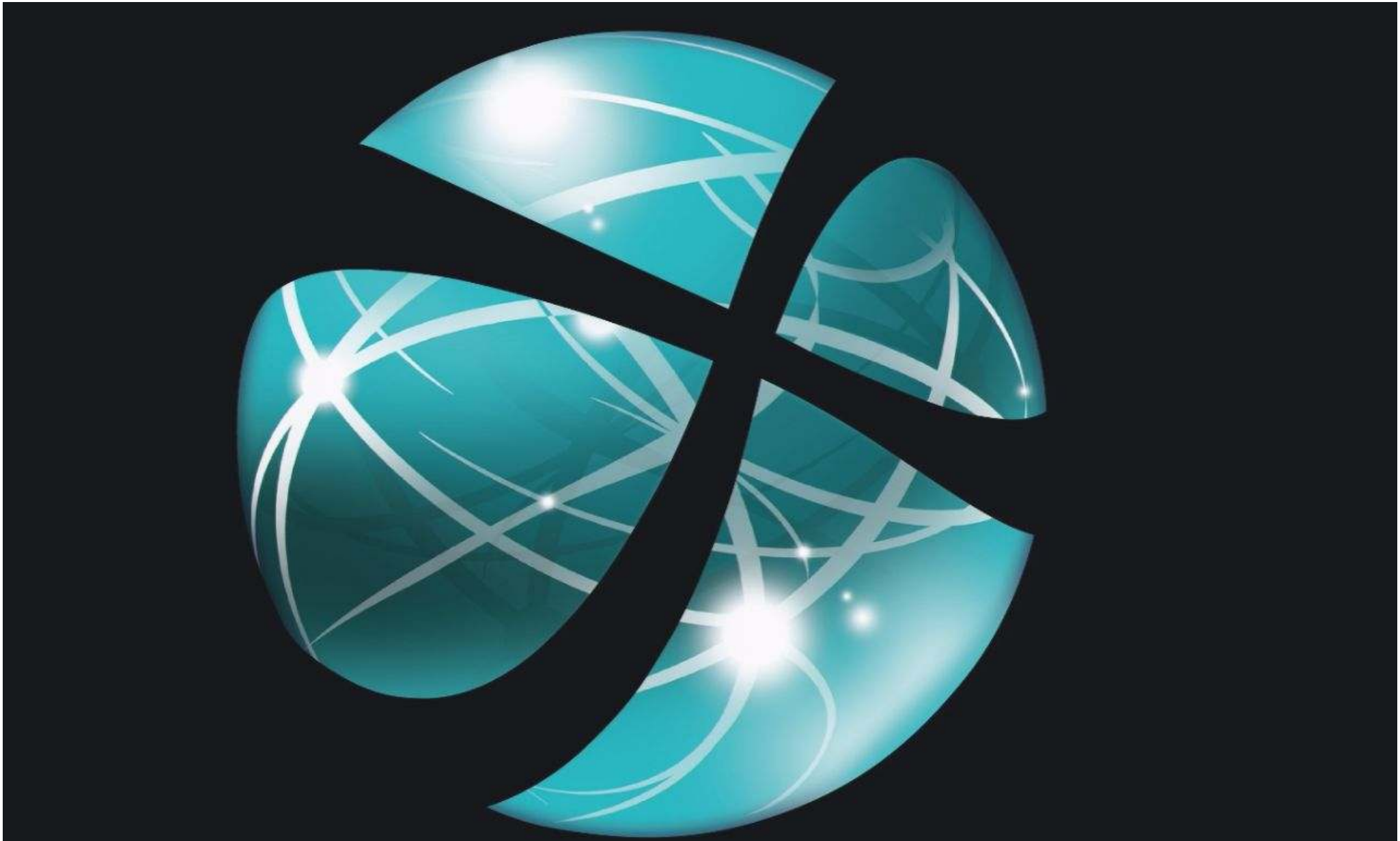
Title: Dynamical Systems - Review (PHYS 607) - Lecture 2

Date: Jan 05, 2010 09:00 AM

URL: http://pirsa.org/10010012

Abstract:


perimeter scholars
international

Logistic map

$$X_{j+1} = \mu X_j (1 - X_j)$$

Logistic map

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fixed points:

$$x^* = f(x^*)$$

$$x^* = \mu x^* (1 - x^*)$$

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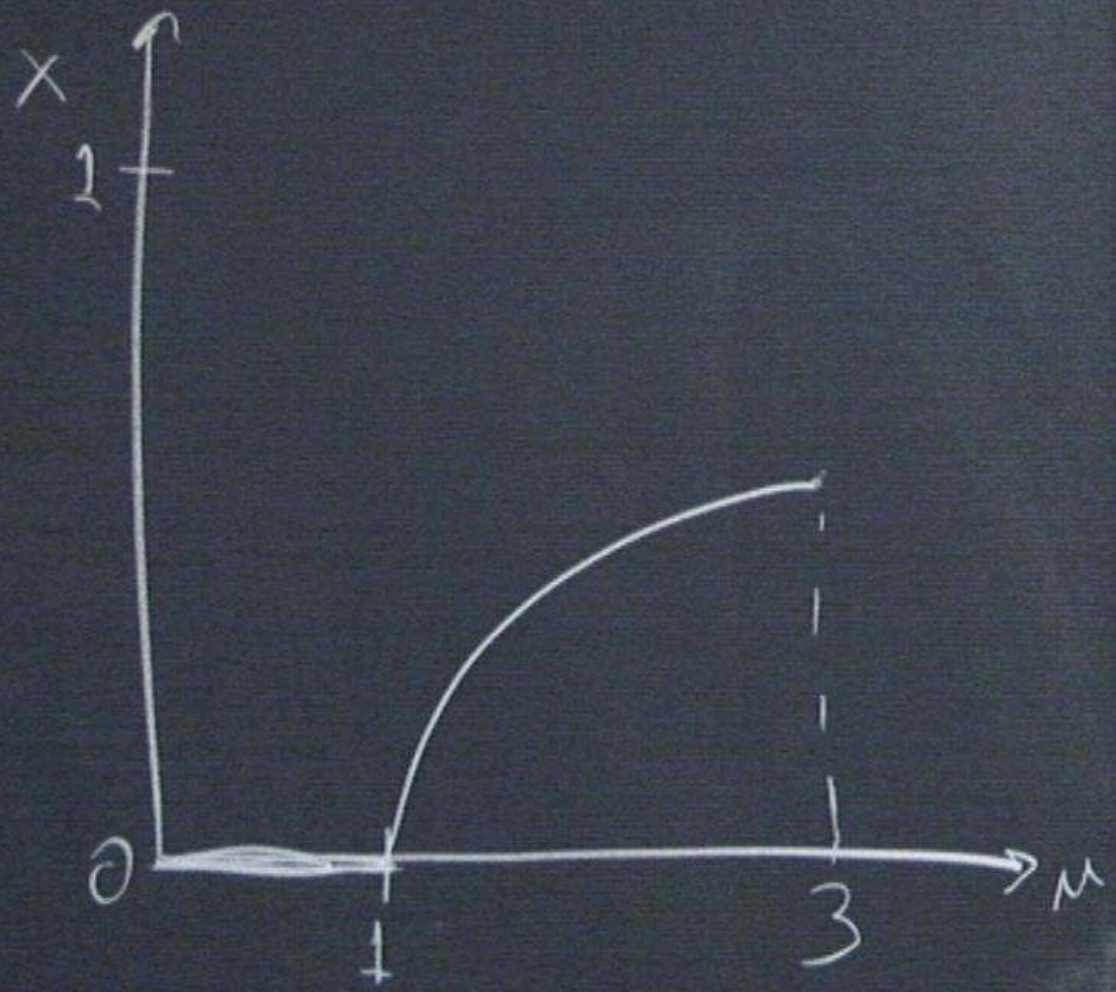
Solutions are

$$x^* = 0$$

$$x^* = 1 - 1/\mu$$


X
1
0
1
3
M

Logistic map

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fixed points:

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Stability of fixed points

Logistic map

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fixed points:

$$x^* = f(x^*)$$

$$x^* = \mu x^* (1 - x^*)$$

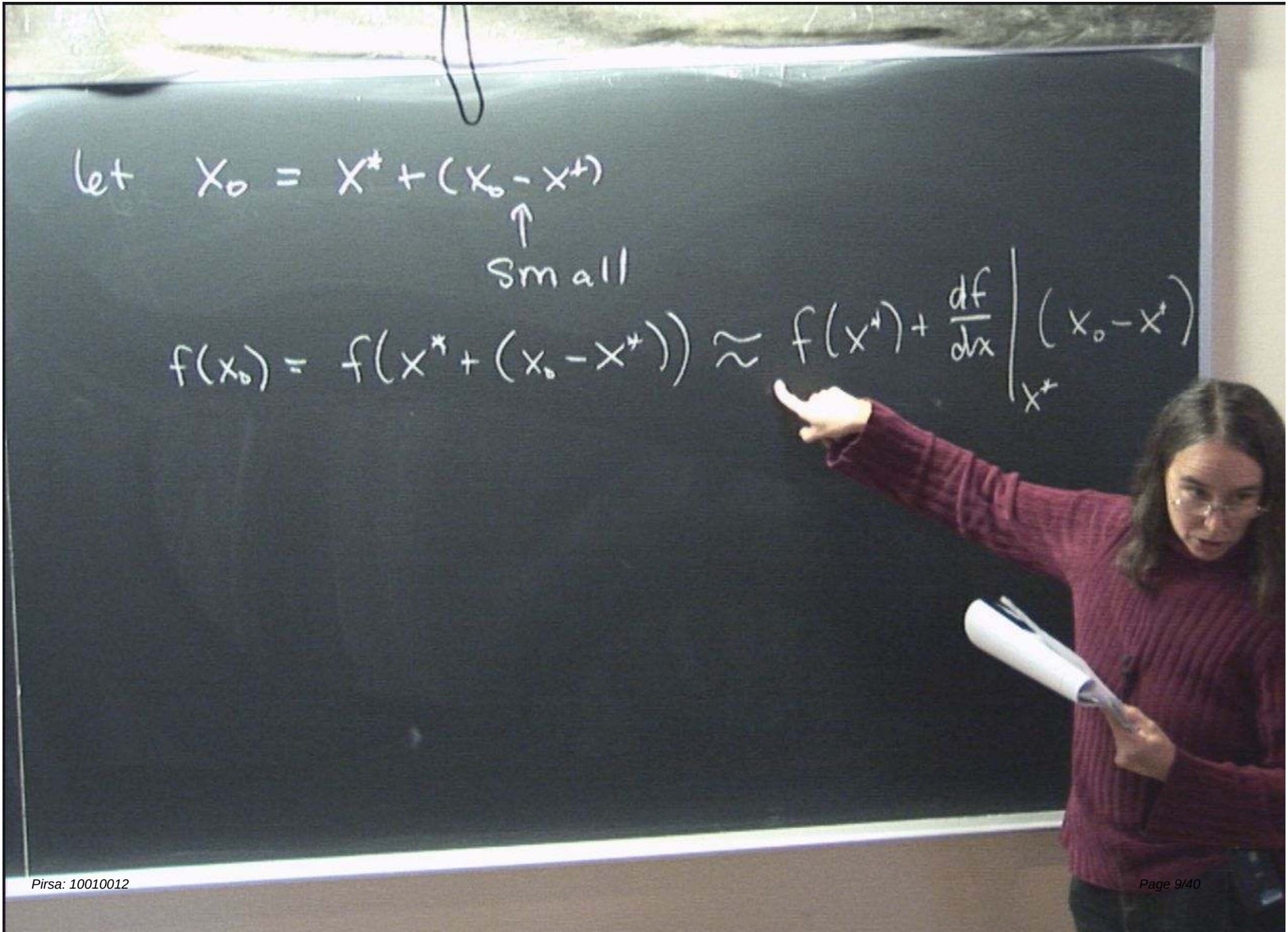
Solutions are

$$x^* = 0$$

$$x^* = 1 - 1/\mu$$

Stability of fixed points

given a fixed pt look at what happens if you start near it


let $x_0 = x^* + (x_0 - x^*)$
small
$f(x_0) = f(x^* + (x_0 - x^*)) \approx f(x^*) + \left.\frac{df}{dx}\right|_{x^*} (x_0 - x^*)$

let $x_0 = x^* + (x_0 - x^*)$

↑ small

$$f(x_0) = f(x^* + (x_0 - x^*)) \approx f(x^*) + \left.\frac{df}{dx}\right|_{x^*} (x_0 - x^*)$$

$x_1 = f(x_0)$ $\delta x_1 = x_1 - x^*$

let $x_0 = x^* + (x_0 - x^*)$

small $\uparrow$ δx_0 $\qquad$ δx_1

$$f(x_0) = f(x^* + (x_0 - x^*)) \approx f(x^*) + \left.\frac{df}{dx}\right|_{x^*} (x_0 - x^*)$$

if $\left|\left.\frac{df}{dx}\right|_{x^*}\right| \leq 1$

then

$x_1 = f(x_0)$ $\delta x_1 = x_1 - x^*$

let $x_0 = x^* + (x_0 - x^*)$

small $\uparrow$ δx_0 $\qquad$ δx_1

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if $\left|\left.\frac{df}{dx}\right|_{x^*}\right| \leq 1$

then $|\delta x_1| < |\delta x_0|$

fixed pt is stable

$x_1 = f(x_0)$ $\delta x_1 = x_1 - x^*$

let $x_0 = x^* + (x_0 - x^*)$

small $\uparrow$ δx_0 $\qquad$ δx_1

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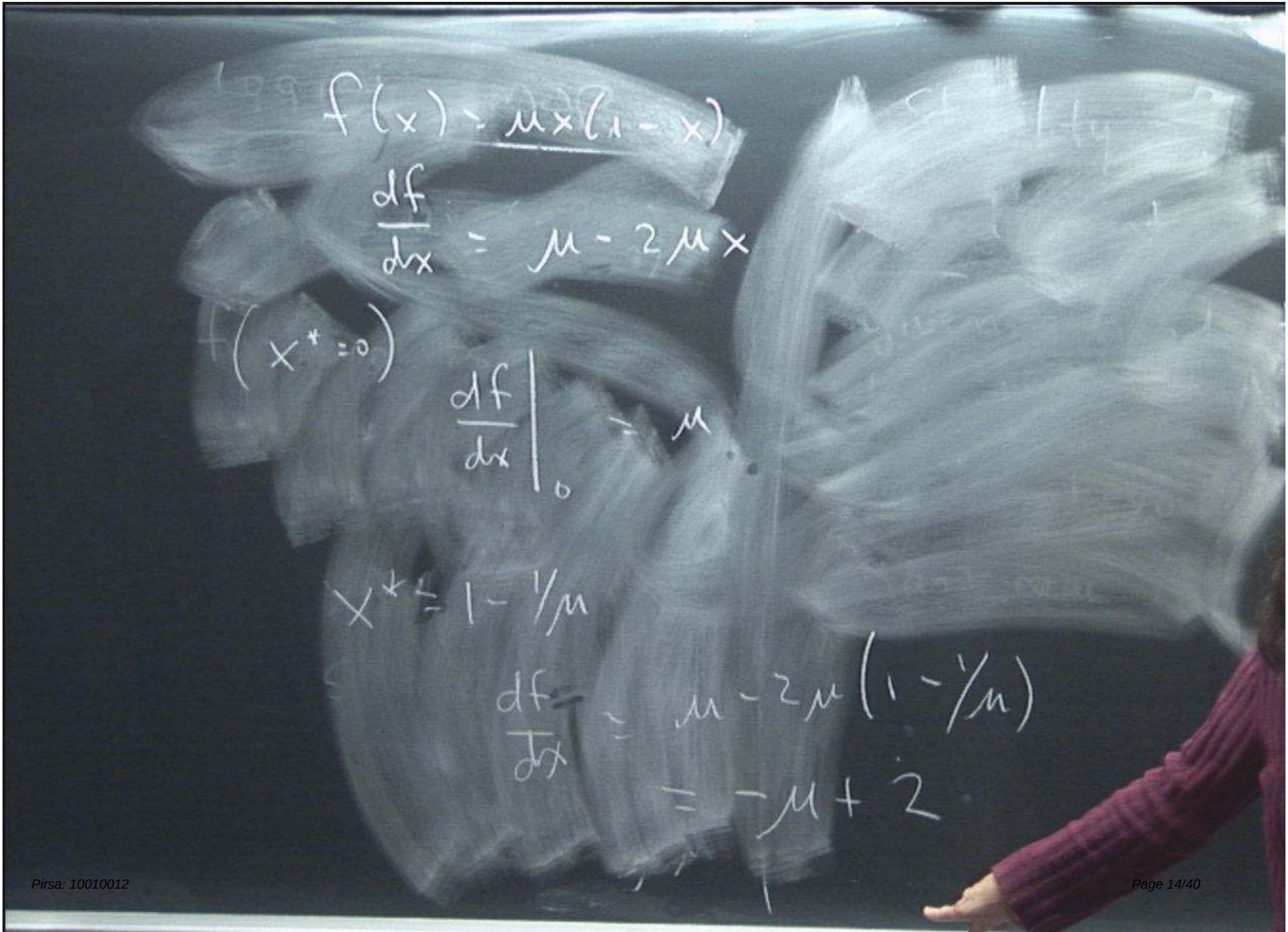
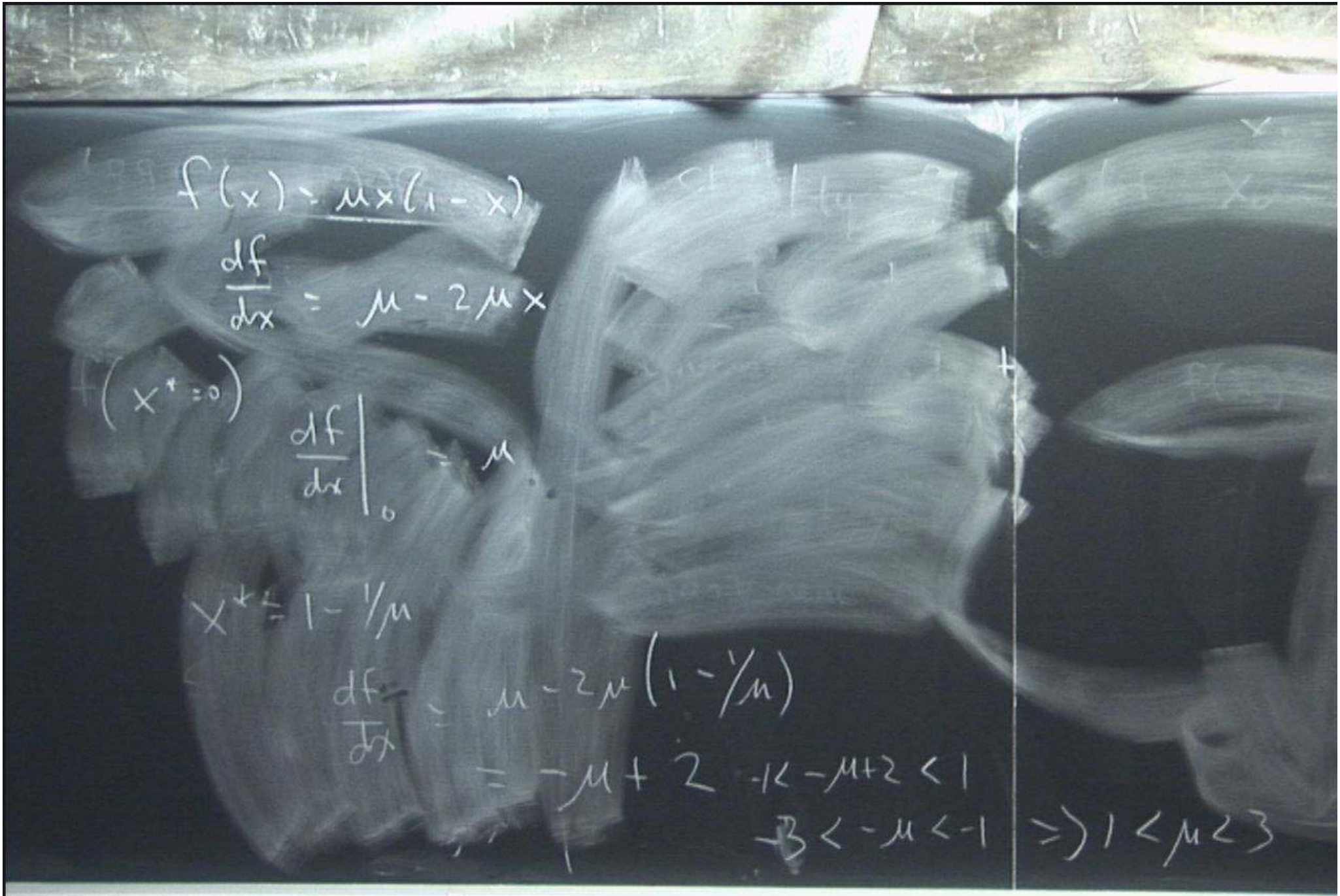
then $|\delta x_1| < |\delta x_0|$

fixed pt is stable


$f(x) = \mu x(1-x)$
$\frac{df}{dx} = \mu - 2\mu x$
$(x^* = 0)$
$\left.\frac{df}{dx}\right|_0 = \mu$
$x^* = 1 - 1/\mu$
$\frac{df}{dx} = \mu - 2\mu(1 - 1/\mu)$
$= -\mu + 2$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$= -\mu + 2$
$-1 < -\mu + 2 < 1$
$-3 < -\mu < -1$
$\Rightarrow 1 < \mu < 3$

$f(x) = \mu x(1-x)$

$\frac{df}{dx} = \mu - 2\mu x$

$(x^* = 0)$

$\left.\frac{df}{dx}\right|_0 = \mu$

$x^* = 1 - 1/\mu$

$\frac{df}{dx} = \mu - 2\mu(1 - 1/\mu)$

$= -\mu + 2$

$-1 < -\mu + 2 < 1$

$-3 < -\mu < -1$ $\Rightarrow 1 < \mu < 3$

Period - 2 orbit

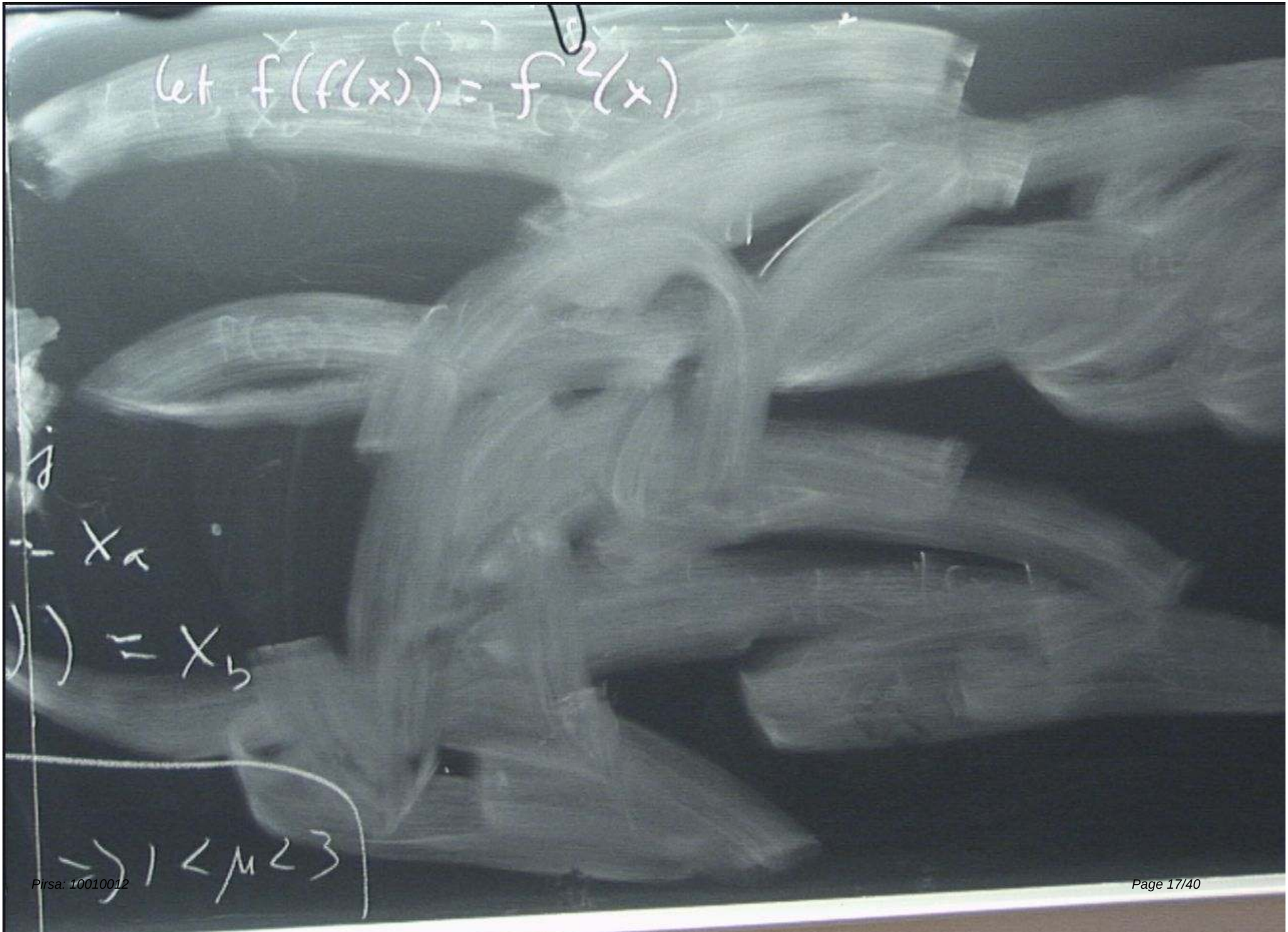
if $f(x_a) = x_b$

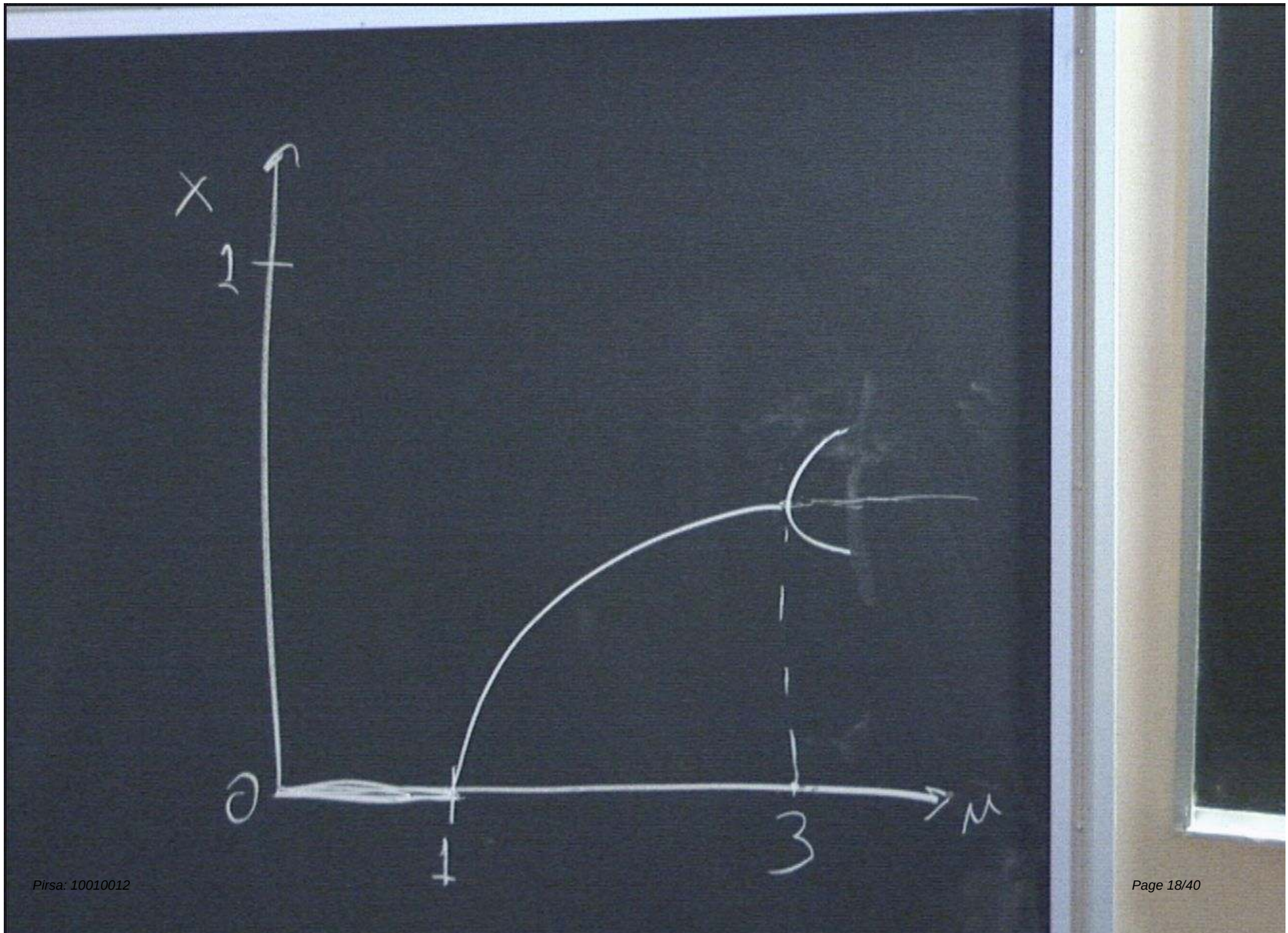
$f(x_b) = x_a$

then $f(f(x_a)) = x_a$

$f(f(x_b)) = x_b$

let f


let f(f(x)) = f^2(x)
$= x_a$
$)) = x_b$
$\Rightarrow 1 < \mu < 3$


X
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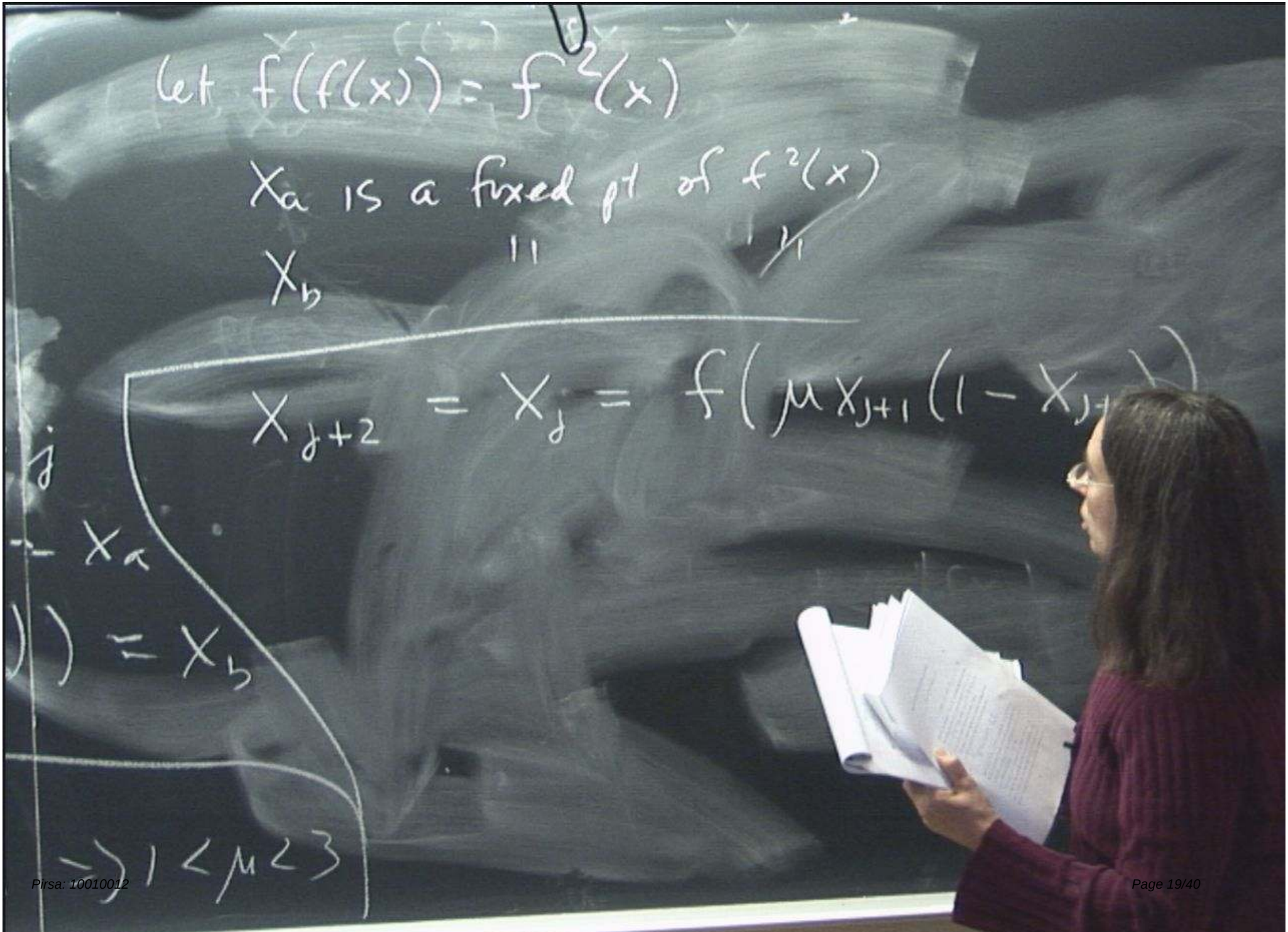
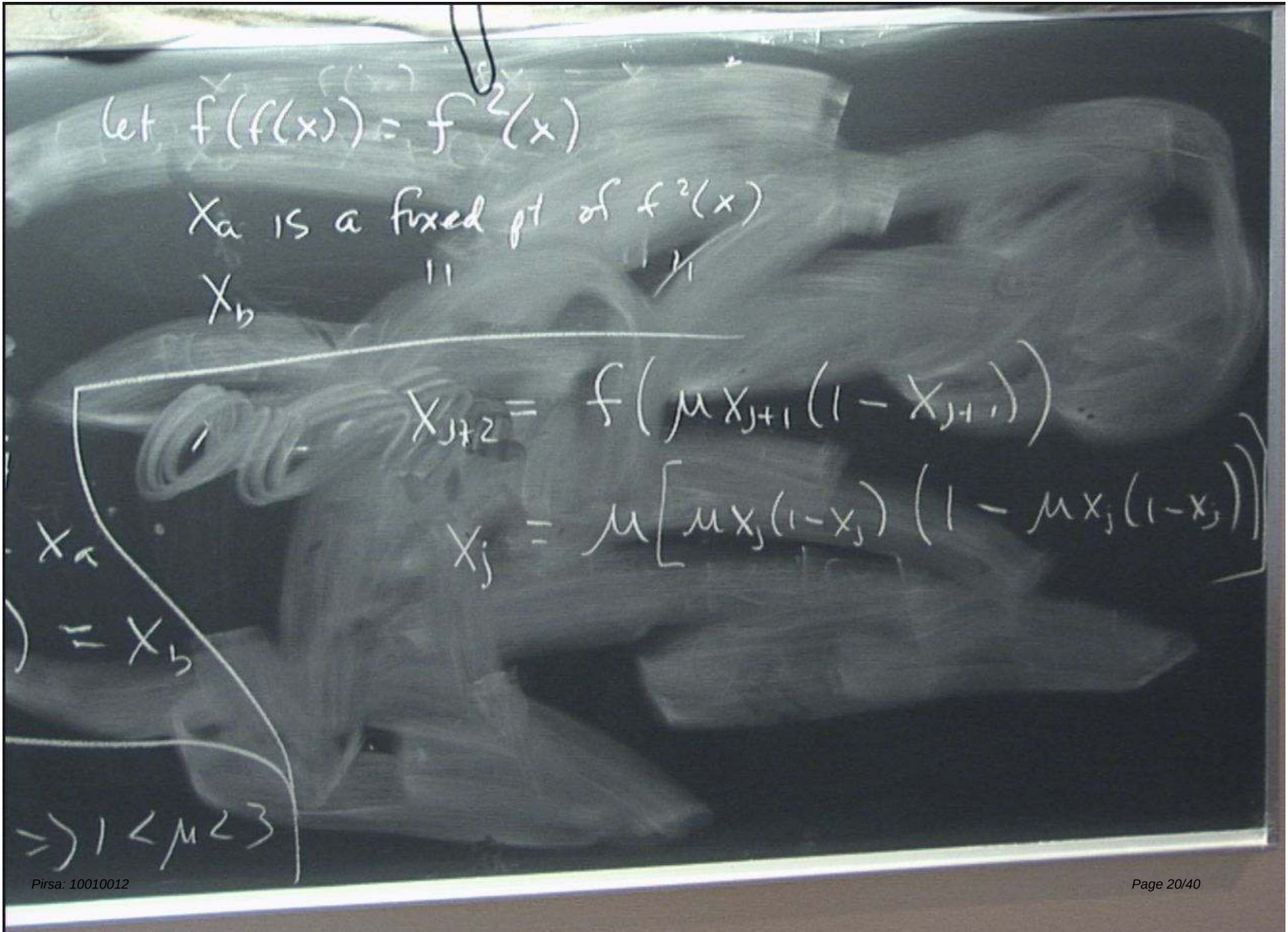
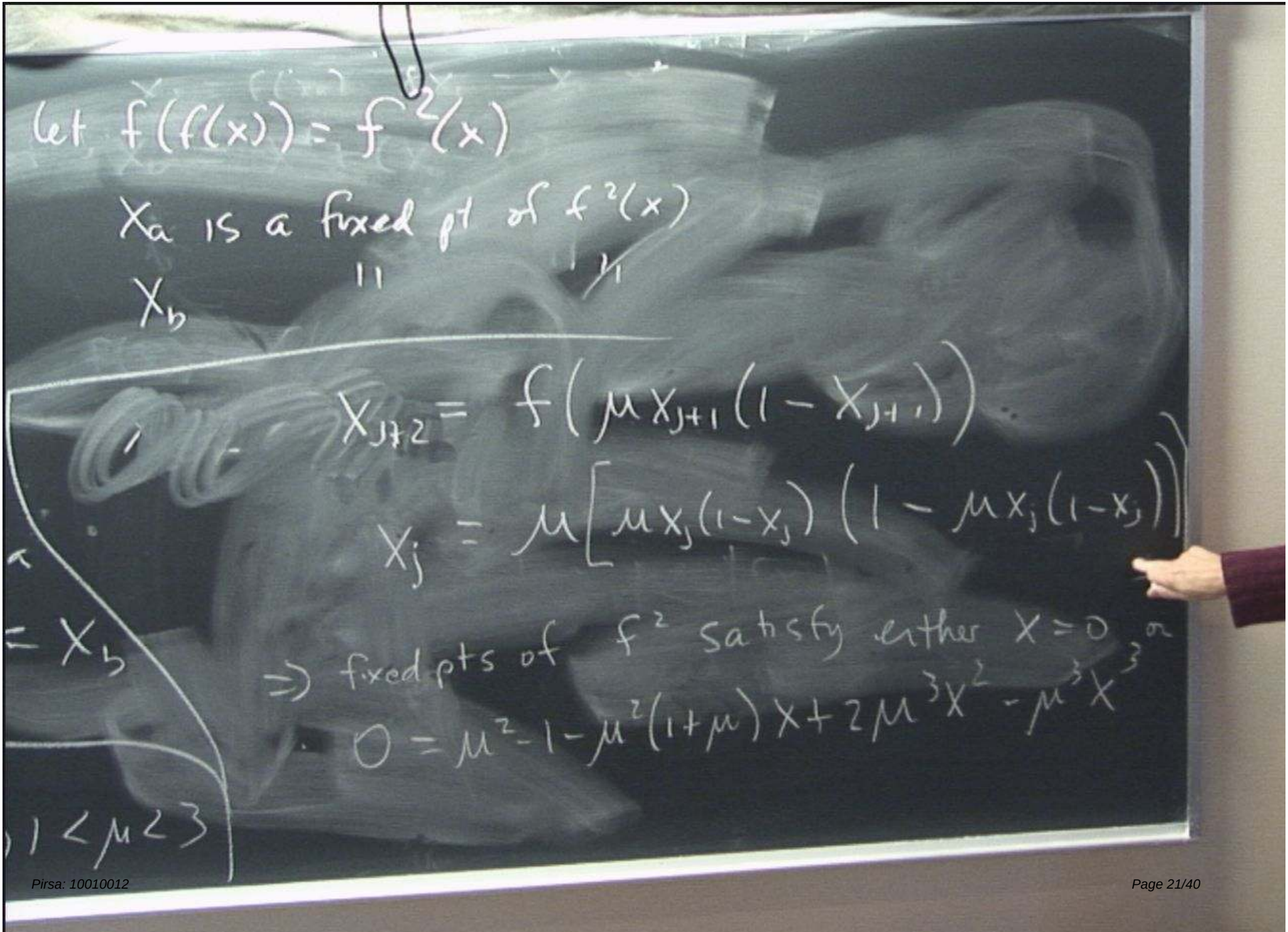

let f(f(x)) = f²(x)
Xa is a fixed pt of f²(x)
Xb
Xj+2 = Xj = f(μ Xj+1 (1 − Xj+1))
Xa
= Xb
1 < μ < 3


let f(f(x)) = f²(x)
x_a is a fixed pt of $f^2(x)$
x_b " "
$x_{j+2} = f(\mu x_{j+1}(1 - x_{j+1}))$
$x_j = \mu[\mu x_j(1-x_j)(1 - \mu x_j(1-x_j))]$
x_a
$= x_b$
$\Rightarrow 1 < \mu < 3$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x_a " "
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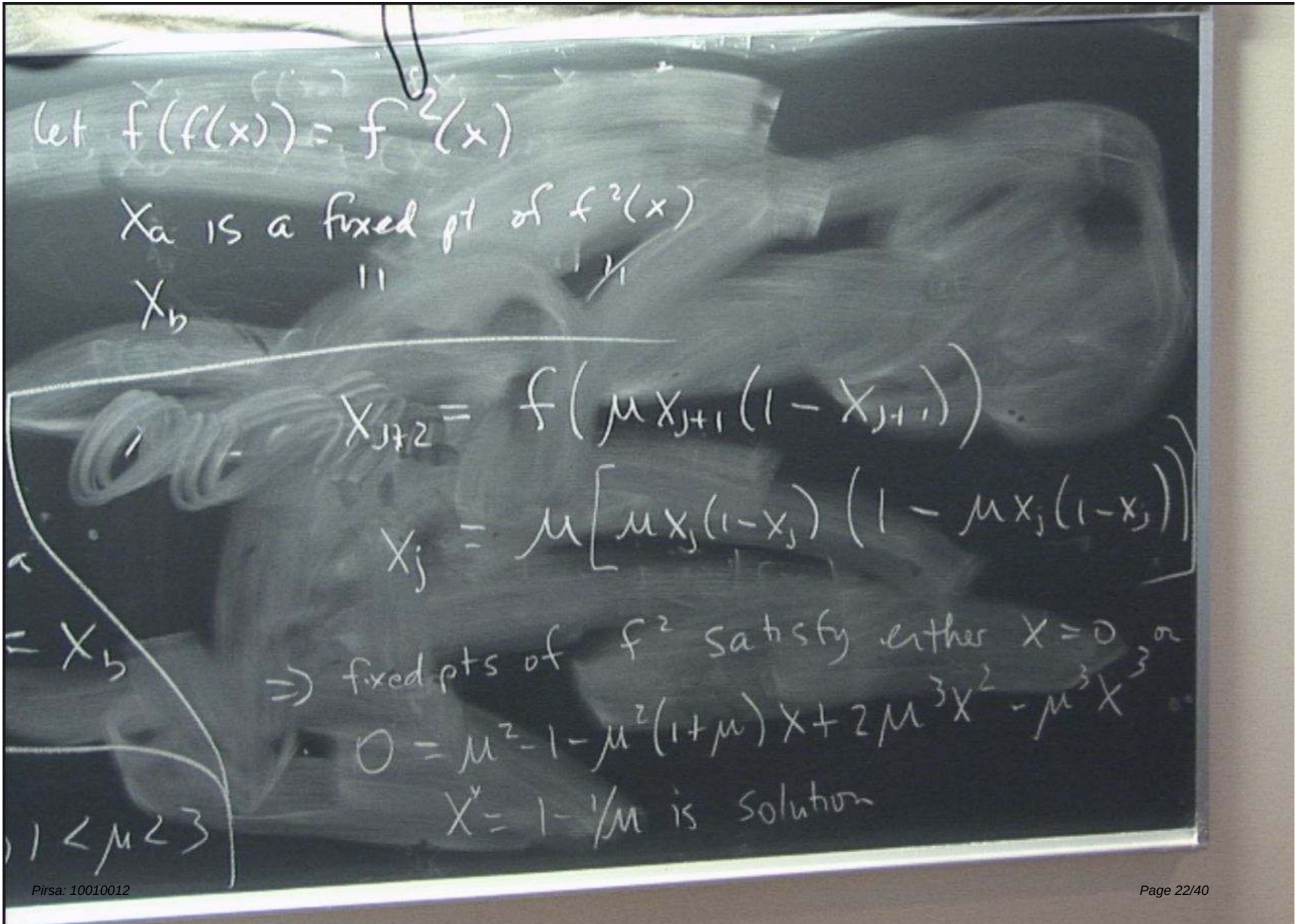
$x_{j+2} = f(\mu x_{j+1}(1-x_{j+1}))$
$x_j = \mu[\mu x_j(1-x_j)(1-\mu x_j(1-x_j))]$
$= x_b$
$\Rightarrow$ fixed pts of f^2 satisfy either $x=0$ or
$0 = \mu^2 - 1 - \mu^2(1+\mu)x + 2\mu^3 x^2 - \mu^3 x^3$
$1 < \mu < 3$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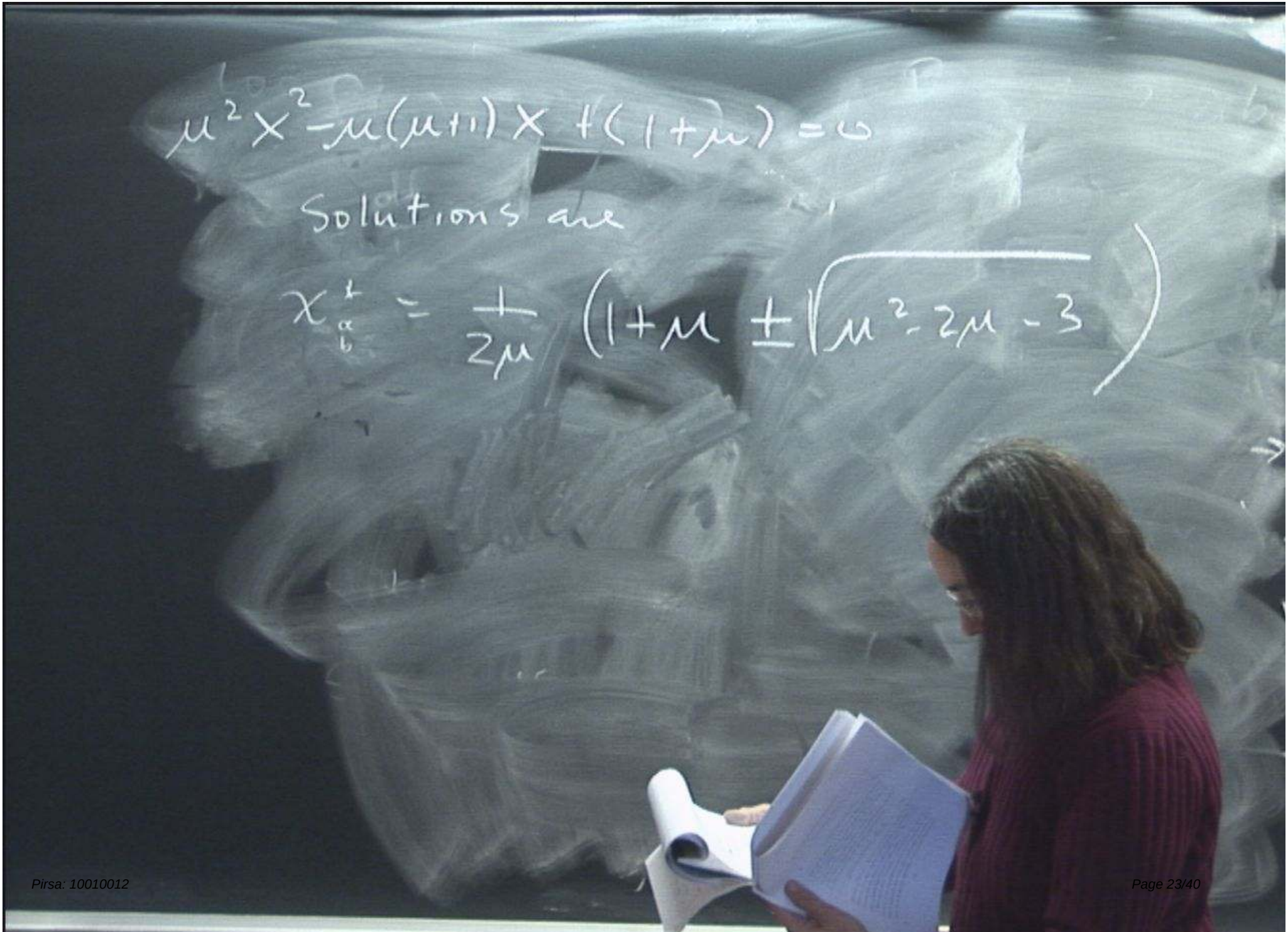
$\Rightarrow$ fixed pts of f^2 satisfy either $x = 0$ or
$0 = \mu^2 - 1 - \mu^2(1+\mu)x + 2\mu^3 x^2 - \mu^3 x^3$
$x^* = 1 - 1/\mu$ is solution
$1 < \mu < 3$


$\mu^2 x^2 - \mu(\mu+1)x + (1+\mu) = 0$
Solutions are
$x^{+}_{a} = \frac{1}{2\mu}\left(1+\mu \pm \sqrt{\mu^2 - 2\mu - 3}\right)$

$$\mu^2 x^2 - \mu(\mu+1)x + (1+\mu) = 0$$

Solutions are

$$x^{\pm}_{a,b} = \frac{1}{2\mu}\left(1+\mu \pm \sqrt{\mu^2 - 2\mu - 3}\right)$$

Check stability:

$$\frac{d}{dx} f^2(x) = \frac{d}{dx} f(y) = \left.\frac{df}{dx}\right|_y \frac{dy}{dx}$$

let $y = f(x)$

$$\left.\frac{d}{dx} f^2(x)\right|_{x=x_0} = \left.\frac{df}{dx}\right|_{f(x_0)} \left.\frac{df}{dx}\right|_{x_0}$$

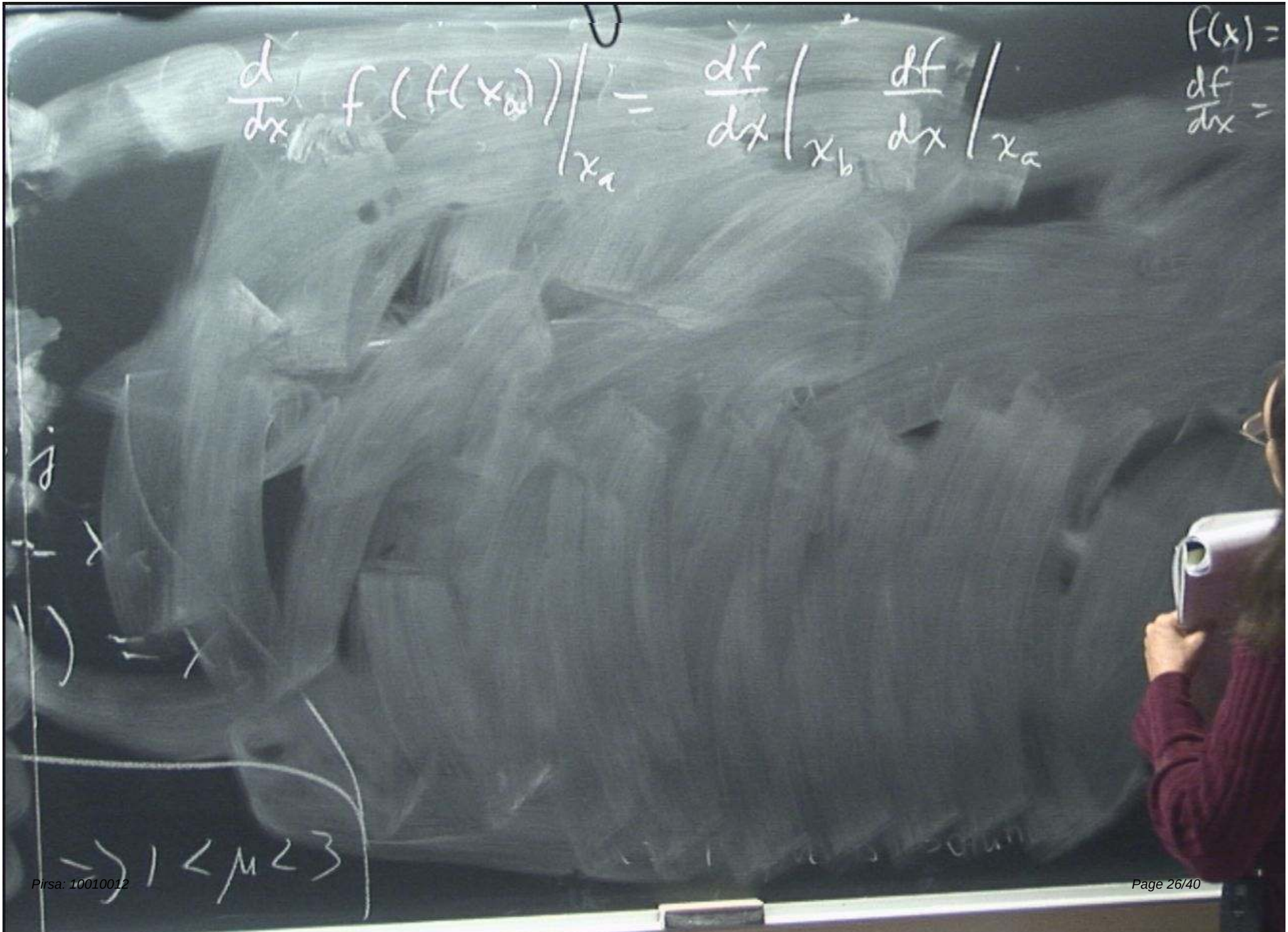
$$+\mu \pm \sqrt{\mu^2 - 2\mu - 3}$$

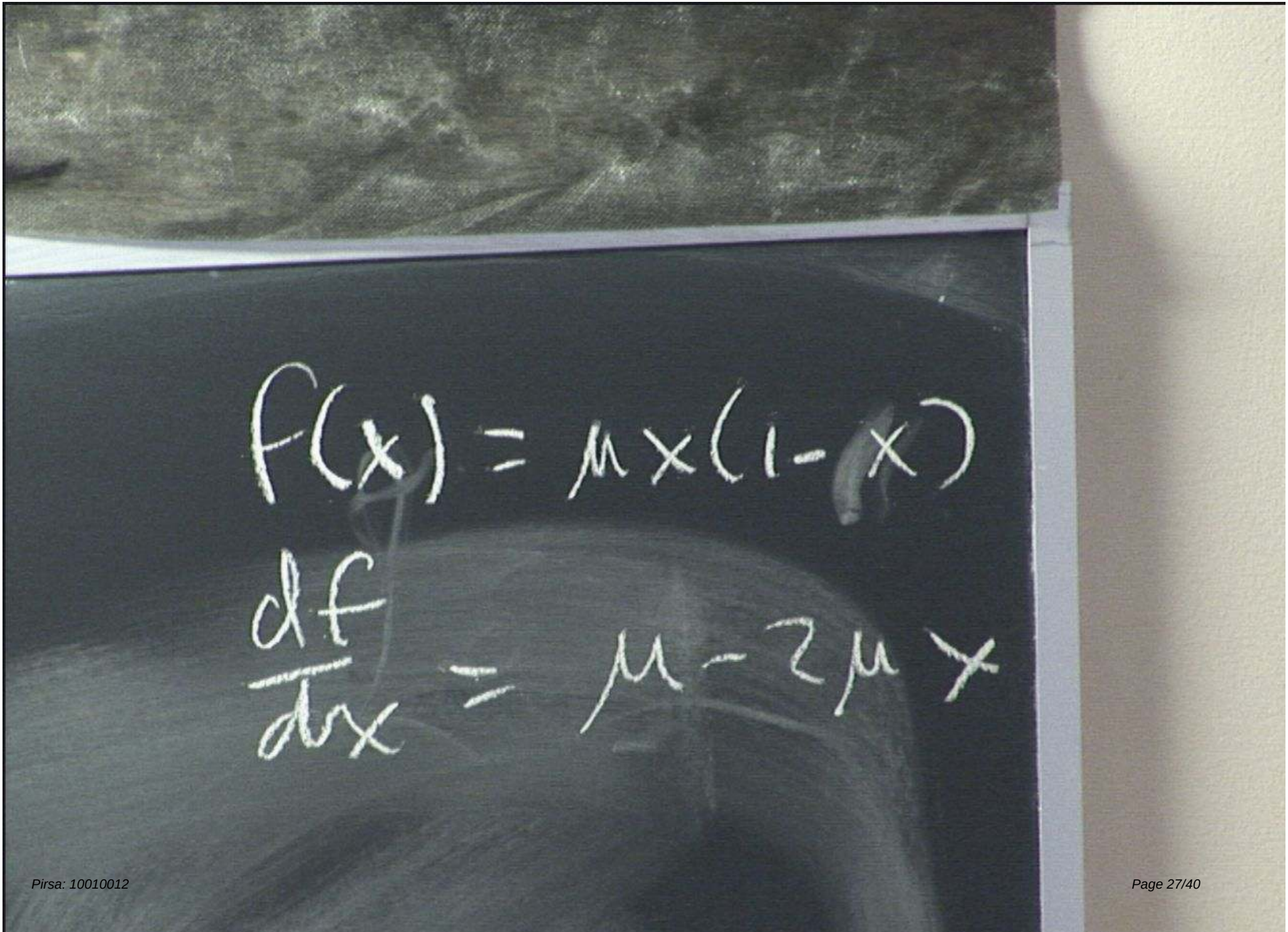
$$\frac{d}{dx} f(y) = \frac{df}{dx}\bigg|_y \frac{dy}{dx}$$

$$f(x)$$

$$\frac{d}{dx} f^2(x)\bigg|_{x=x_0} \frac{df}{dx}\bigg|_{f(x_0)} \frac{df}{dx}\bigg|_{x_0}$$

$$\Rightarrow 1 < \mu < 3$$


$\frac{d}{dx} f(f(x_a))\Big|_{x_a} = \frac{df}{dx}\Big|_{x_b} \frac{df}{dx}\Big|_{x_a}$
$f(x) =$
$\frac{df}{dx} =$
$1 < \mu < 3$


$f(x) = \mu x(1-x)$
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$$\frac{d}{dx} f(f(x_a))\Big|_{x_a} = \frac{df}{dx}\Big|_{x_b} \frac{df}{dx}\Big|_{x_a}$$

$f(x) = \mu x(1-x)$

$\frac{df}{dx} = \mu - 2\mu x$

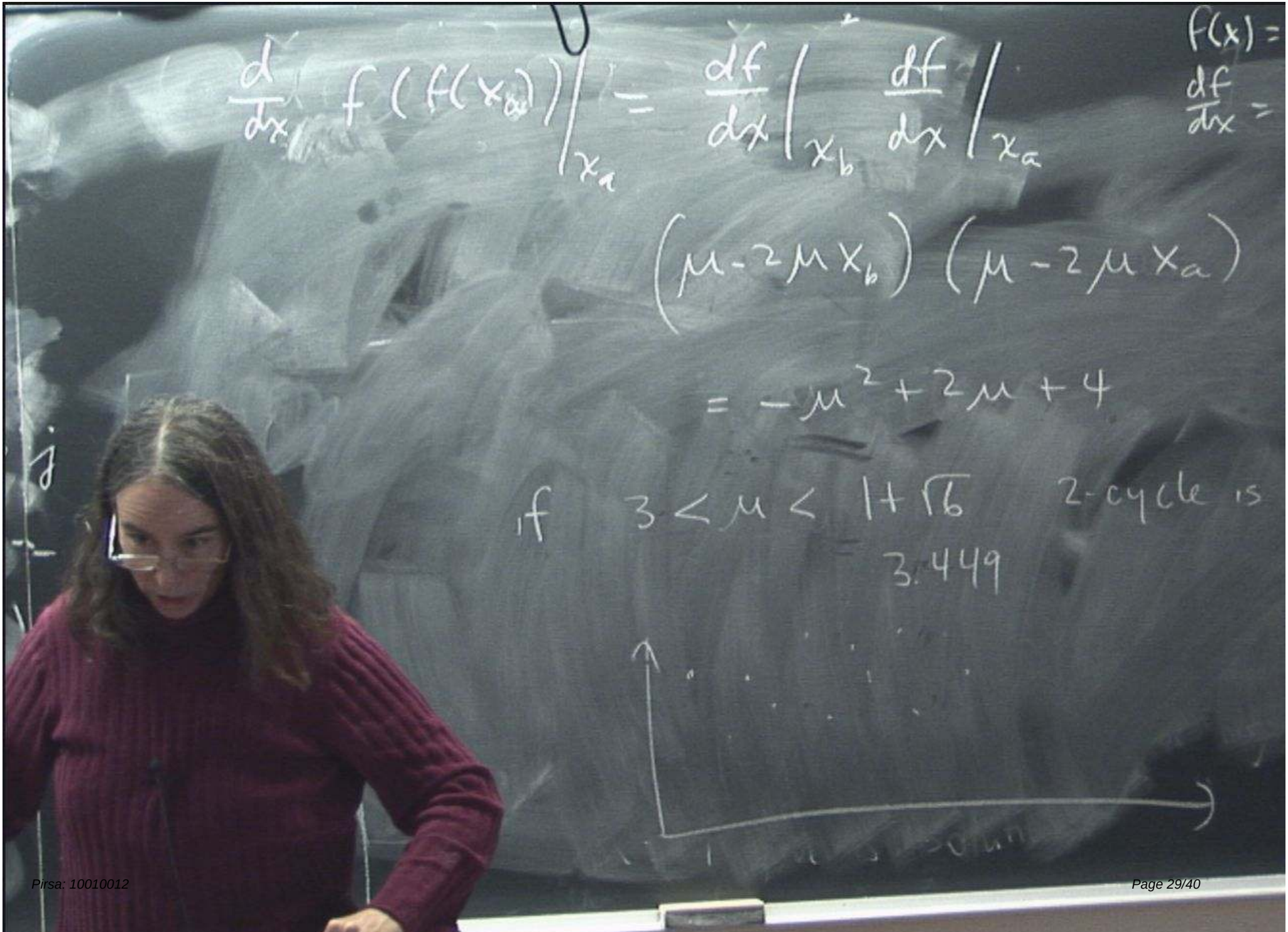
$$(\mu - 2\mu x_b)(\mu - 2\mu x_a)$$

$$= -\mu^2 + 2\mu + 4$$

if $3 < \mu < 1+\sqrt{6}$ 2-cycle is stable

3.449

$< \mu < 3$


$\frac{d}{dx_a} f(f(x_a))\Big|_{x_a} = \frac{df}{dx}\Big|_{x_b} \frac{df}{dx}\Big|_{x_a}$
$f(x) =$
$\frac{df}{dx} =$
$(\mu - 2\mu x_b)(\mu - 2\mu x_a)$
$= -\mu^2 + 2\mu + 4$
if $3 < \mu < 1+\sqrt{6}$ 2-cycle is
3.449

$$\frac{d}{dx_1} f(f(x_a))\Big|_{x_a} = \frac{df}{dx}\Big|_{x_b} \frac{df}{dx}\Big|_{x_a}$$

$f(x) = \mu x(1-x)$

$\frac{df}{dx} = \mu - 2\mu x$

$$(\mu - 2\mu x_b)(\mu - 2\mu x_a)$$

$$= -\mu^2 + 2\mu + 4$$

if $3 < \mu < 1+\sqrt{6}$ 2-cycle is stable

3.449

$< \mu < 3$

look for solutions of $f^N(x) = x$

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Newton-Raphson method.

look for solutions of $f^N(x) = x$

Newton-Raphson method:

$$F(x) = 0$$

$$[\, f^N(x) - x = 0 \,]$$

look for solutions of $f^N(x) = x$

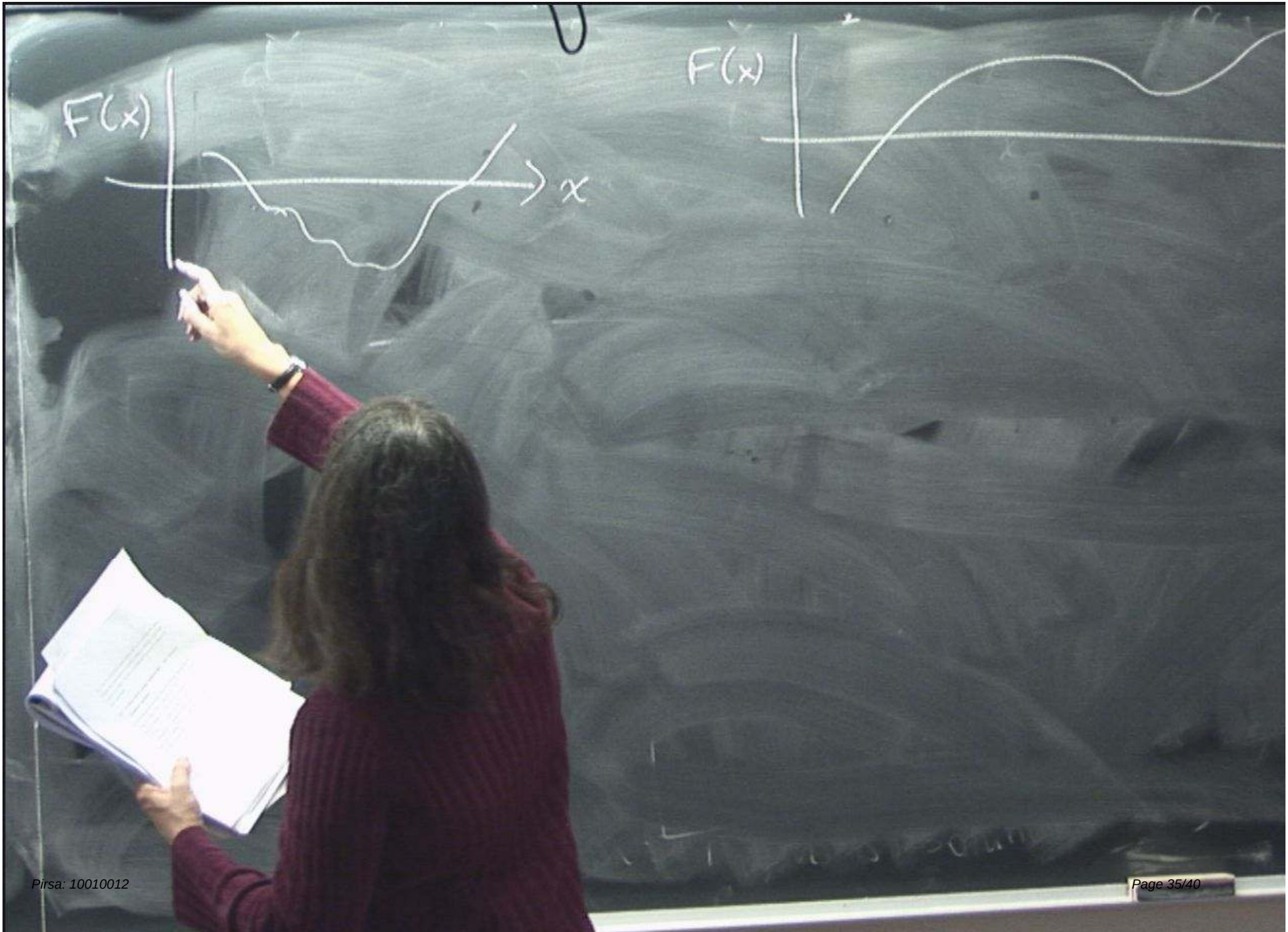
Newton-Raphson method:

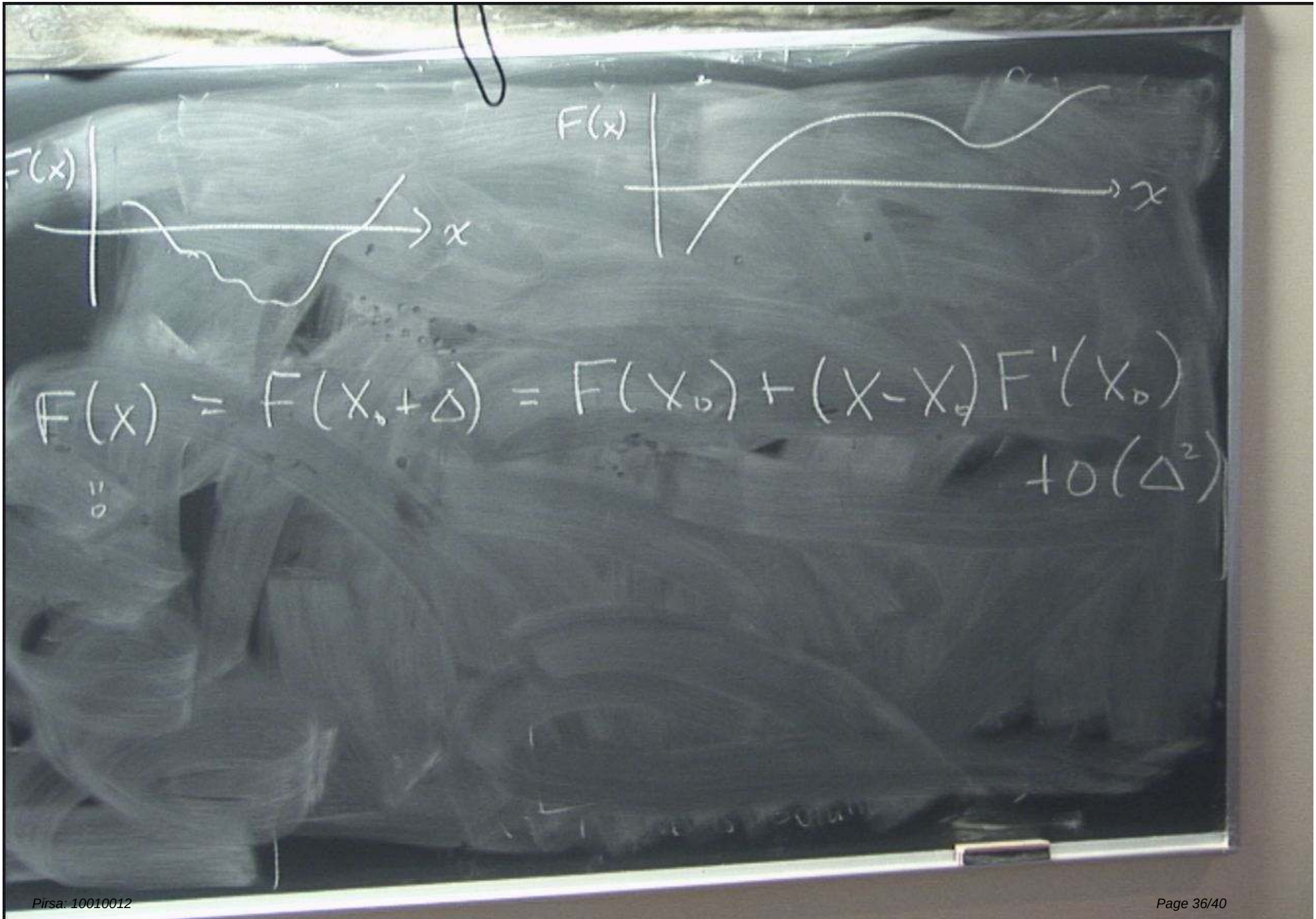
$$F(x) = 0$$

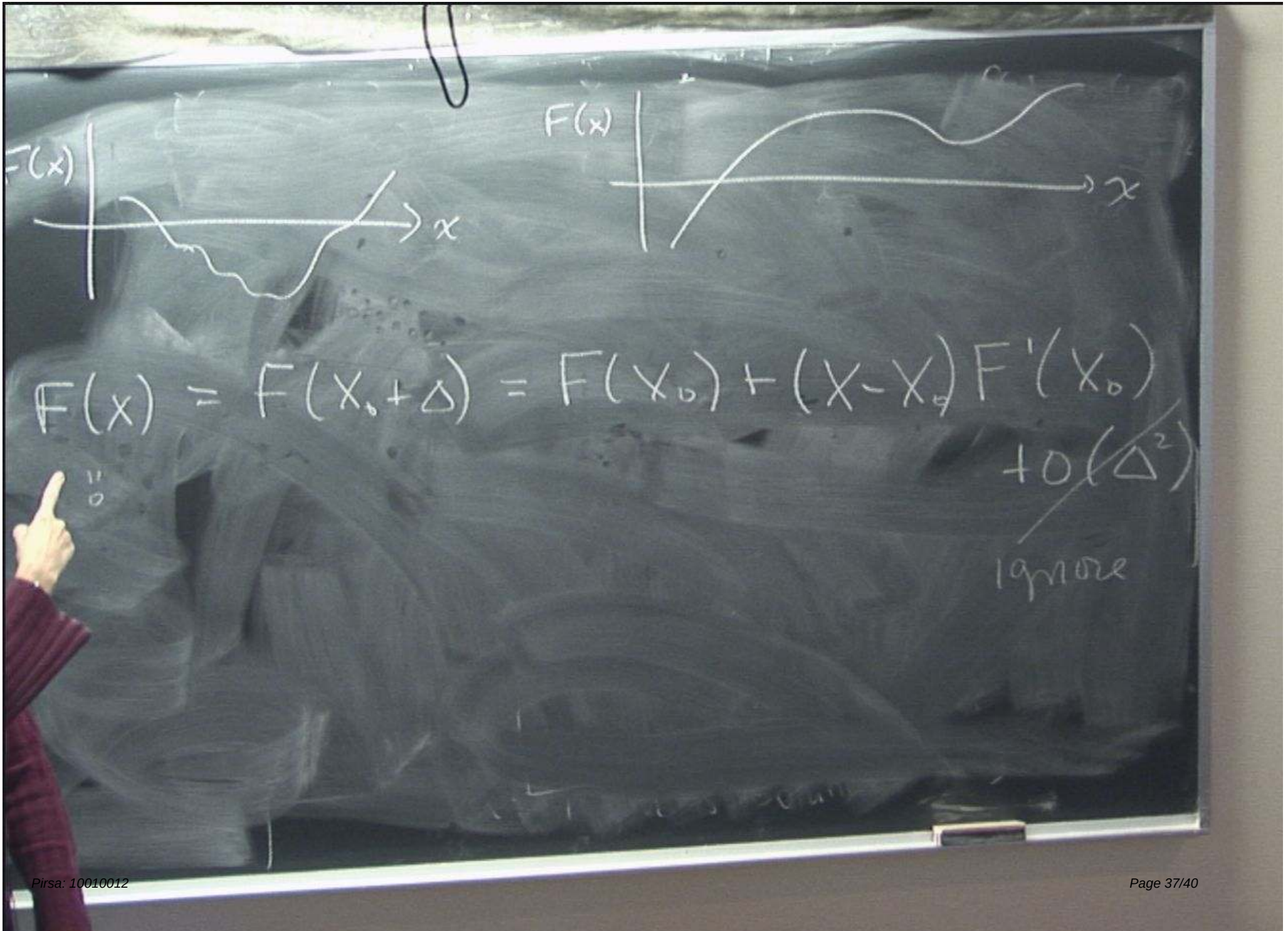
$$[\, f^N(x) - x = 0 \,]$$

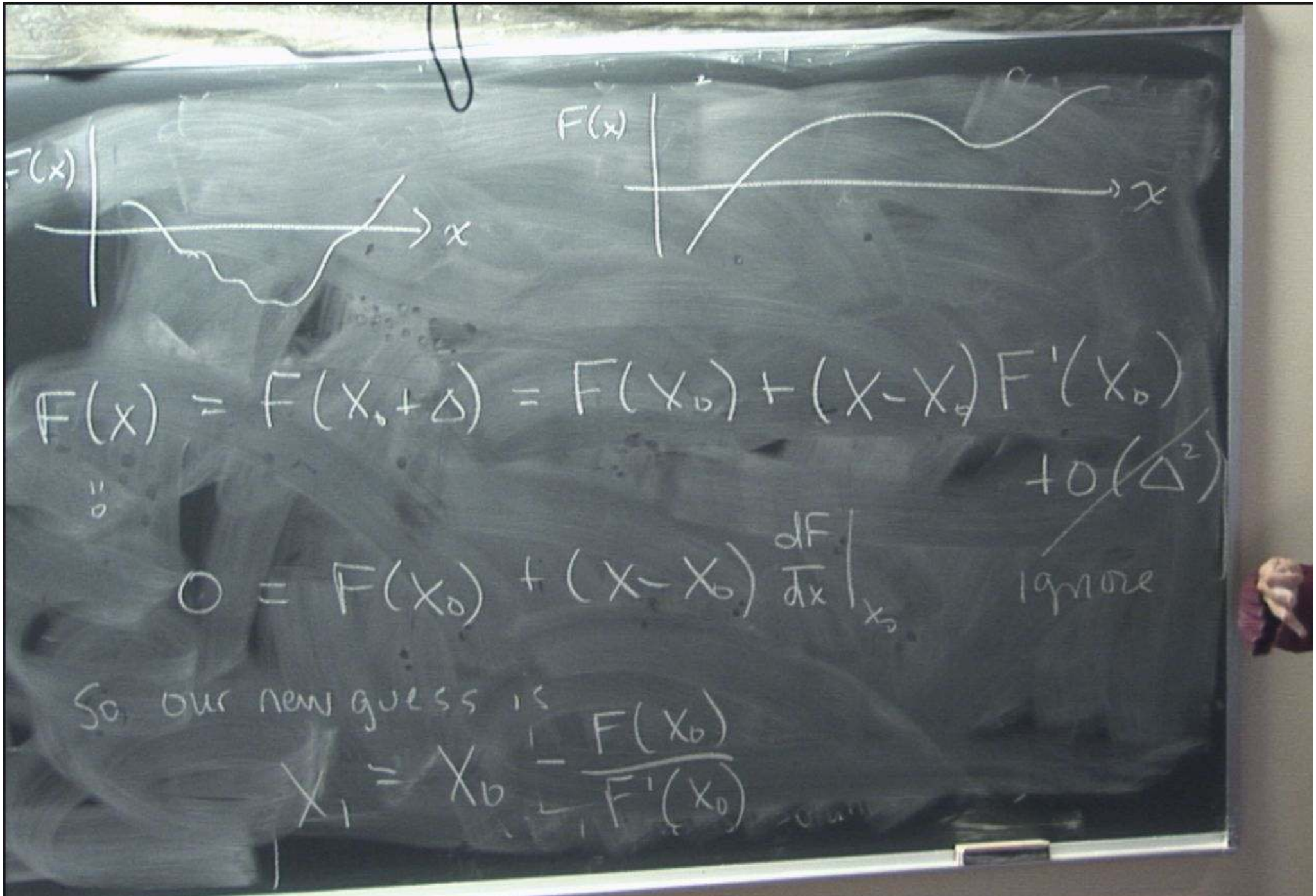
guess solution is near X_0

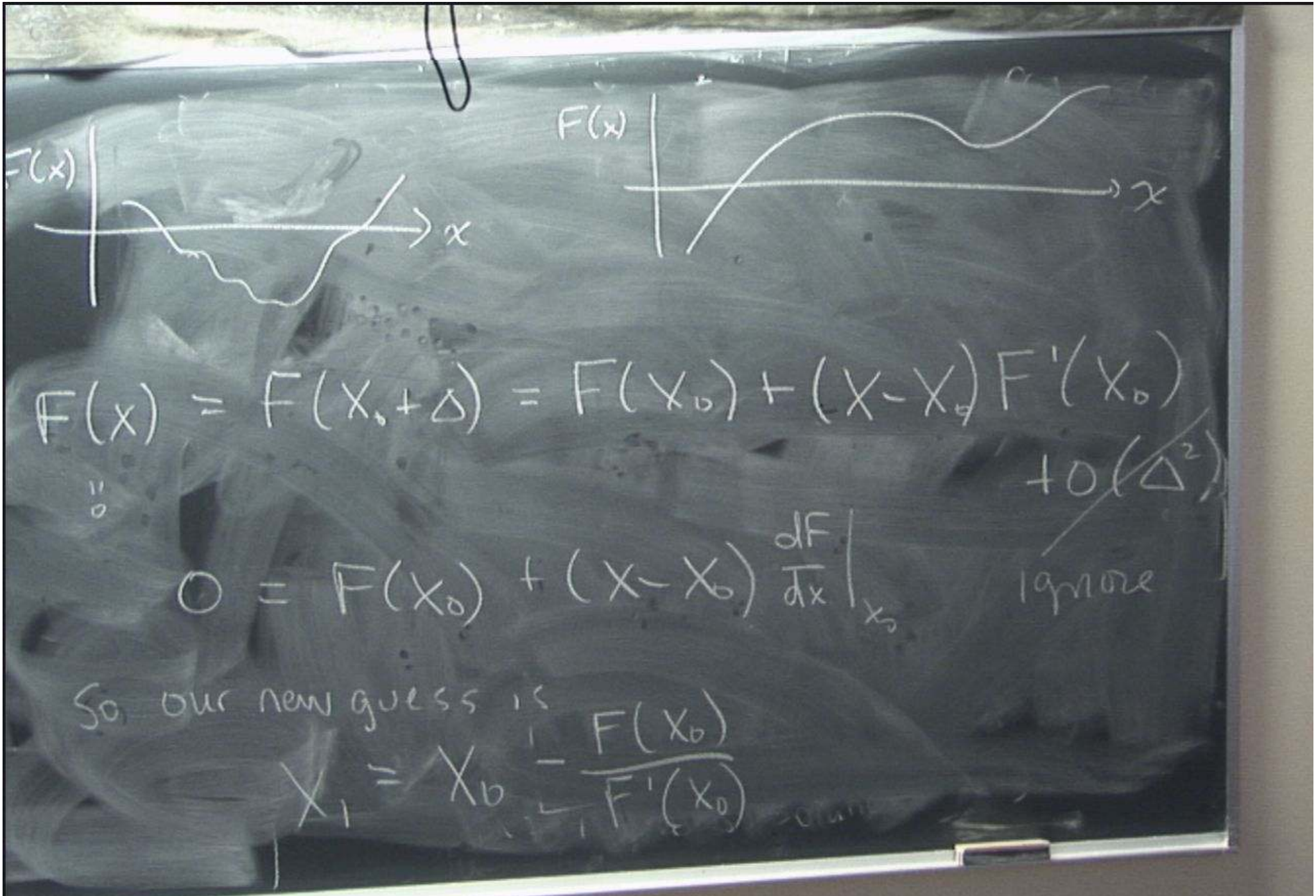
write $X_{\text{solution}} = X_{0\ \text{guess}} + \Delta_{\text{error}}$


F(x)
x
F(x)


F(x)
x
F(x)
x
F(x) = F(X₀ + Δ) = F(X₀) + (X − X₀) F'(X₀)
+ O(Δ²)
X₀


F(x)
x
F(x)
x
F(x) = F(X₀+Δ) = F(X₀) + (X-X₀) F'(X₀)
+ O(Δ²)
ignore


F(x)
x
F(x)
x
F(x) = F(X_0 + Δ) = F(X_0) + (X - X_0) F'(X_0) + O(Δ^2)
0 = F(X_0) + (X - X_0) dF/dx |_{X_0}
ignore
So our new guess is
X_1 = X_0 - F(X_0) / F'(X_0)


F(x)
x
F(x)
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F(x) = F(X_0 + Δ) = F(X_0) + (X - X_0) F'(X_0) + O(Δ^2)
0
0 = F(X_0) + (X - X_0) dF/dx |_{X_0}
ignore
So our new guess is
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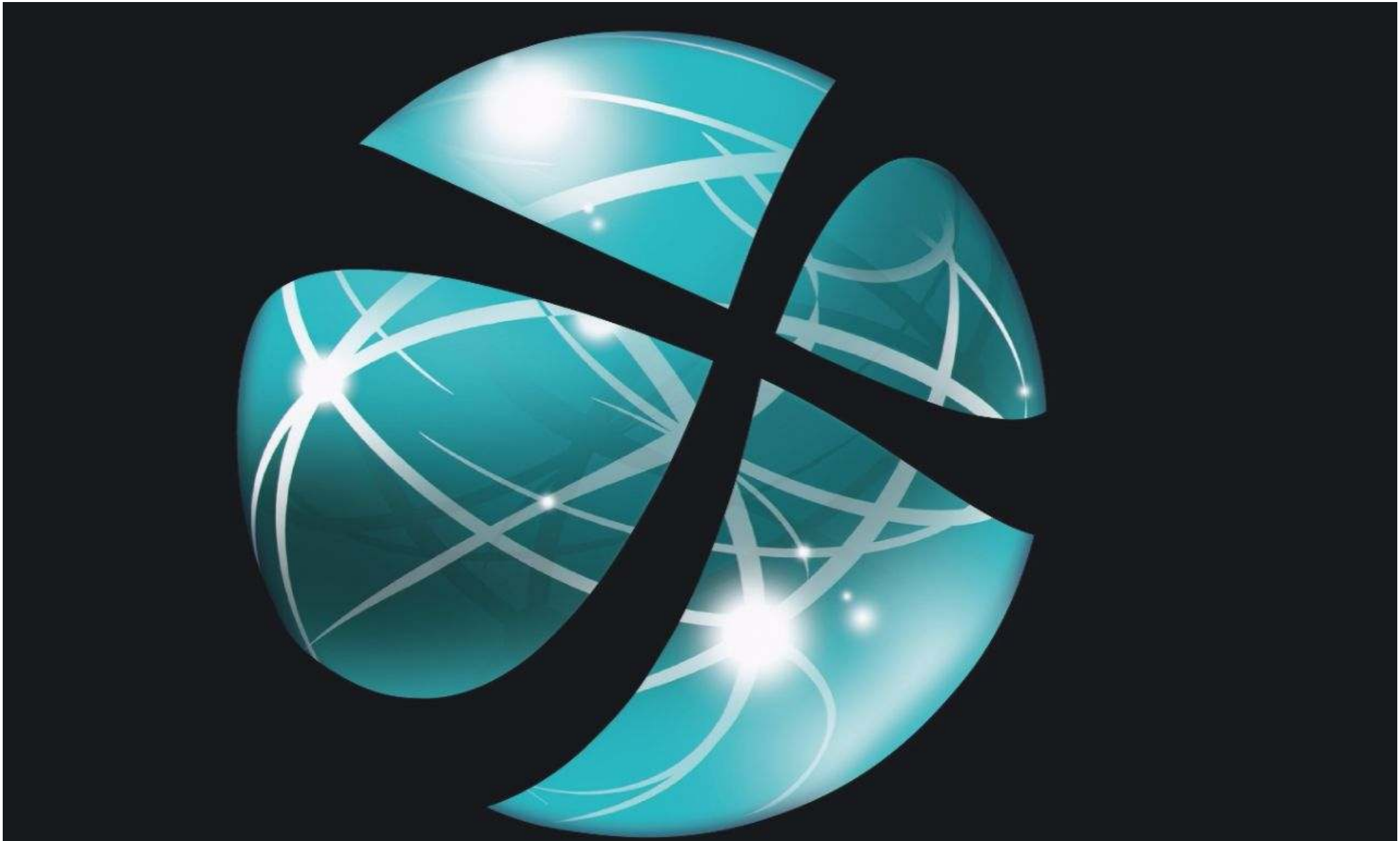
perimeter scholars
international