Title: Dynamical Systems - Review (PHYS 607) - Lecture 2

Date: Jan 05, 2010 09:00 AM

URL: http://pirsa.org/10010012

Abstract:

perimeter scholars
international

Logistic map

$$x_{j+1} = \mu x_j (1 - x_j)$$

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fixed points:

$$x^* = f(x^*)$$

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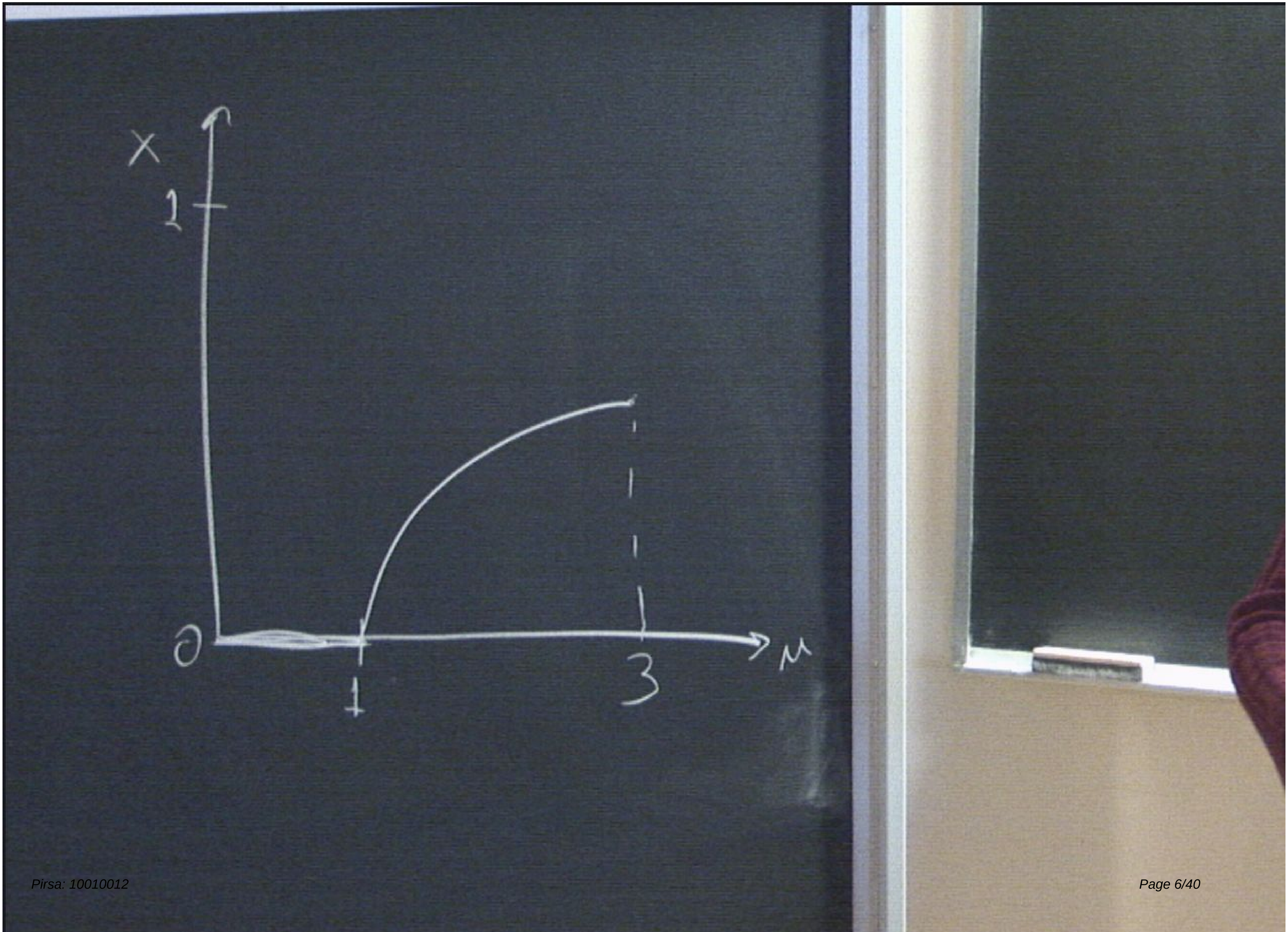
Solutions are

$$x^* = 0$$

$$x^* = 1 - 1/\mu$$


x
1
0
1
3
μ

Logistic map

$$x_{j+1} = \mu x_j (1 - x_j)$$

fixed points:

$$x^* = f(x^*)$$

$$x^* = \mu x^* (1 - x^*)$$

Solutions are

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Stability of fixed points

Logistic map

$$x_{j+1} = \mu x_j (1 - x_j)$$

fixed points:

$$x^* = f(x^*)$$

$$x^* = \mu x^* (1 - x^*)$$

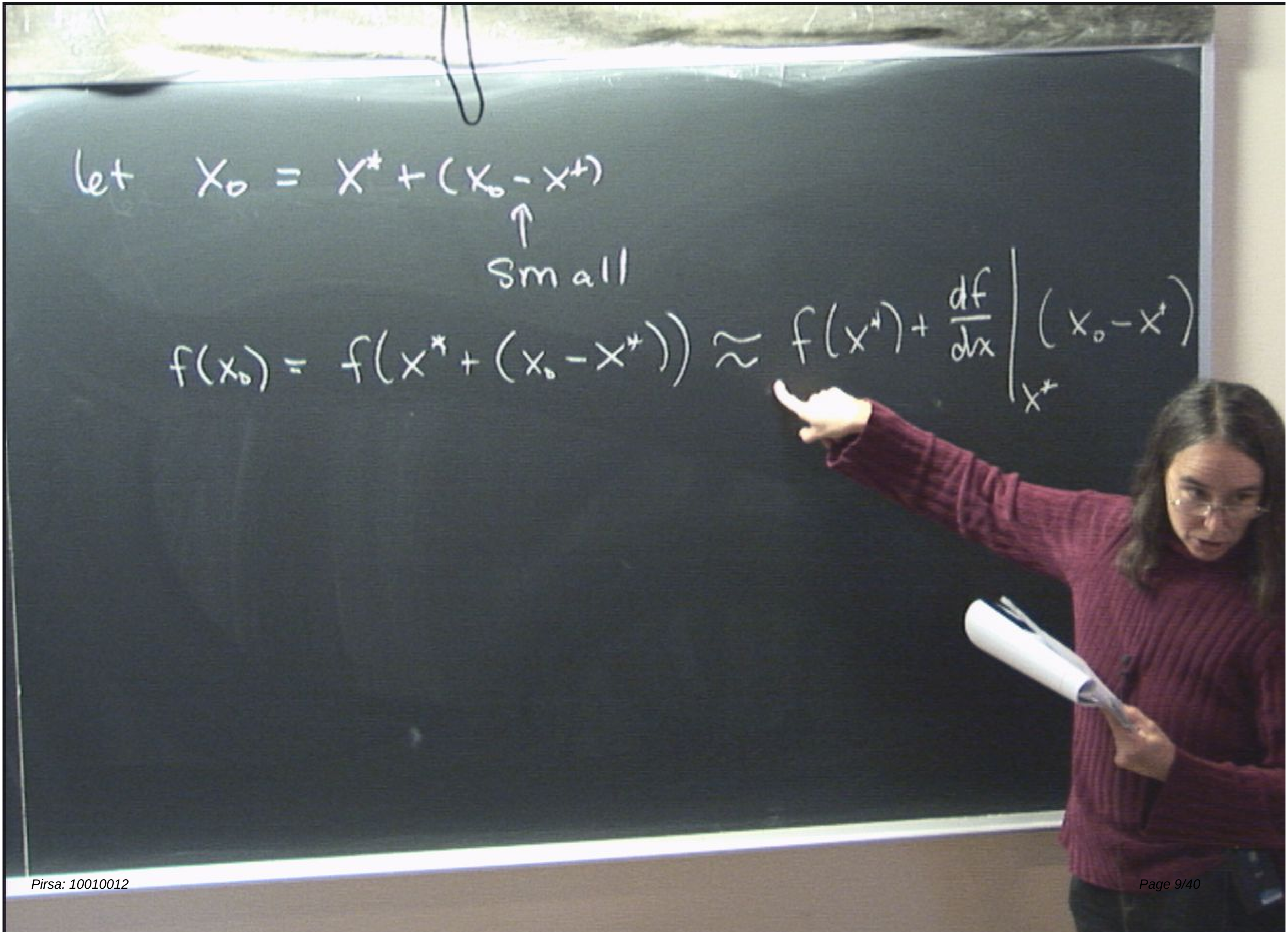
Solutions are

$$x^* = 0$$

$$x^* = 1 - 1/\mu$$

Stability of fixed points

given a fixed pt look at what happens if you start near it


let $x_0 = x^* + (x_0 - x^*)$
small
$f(x_0) = f(x^* + (x_0 - x^*)) \approx f(x^*) + \left.\frac{df}{dx}\right|_{x^*} (x_0 - x^*)$

let $x_0 = x^* + (x_0 - x^*)$

$$\underset{\text{small}}{\uparrow}$$

$$f(x_0) = f(x^* + (x_0 - x^*)) \approx f(x^*) + \left.\frac{df}{dx}\right|_{x^*} (x_0 - x^*)$$

$$x_1 = f(x_0) \qquad \delta x_1 = x_1 - x^*$$

let $$x_0 = x^* + (\underbrace{x_0 - x^*}_{\text{small}})$$

δx_0

$$f(x_0) = f(x^* + (x_0 - x^*)) \approx f(x^*) + \overbrace{\left.\frac{df}{dx}\right|_{x^*} (x_0 - x^*)}^{\delta x_1}$$

$$\text{if} \quad \left| \left.\frac{df}{dx}\right|_{x^*} \right| \leq 1$$

then

$$x_1 = f(x_0) \qquad \delta x_1 = x_1 - x^*$$

let $$x_0 = x^* + \underbrace{(x_0 - x^*)}_{\text{small}}$$

$$\delta x_0$$

$$f(x_0) = f(x^* + (x_0 - x^*)) \approx f(x^*) + \left.\frac{df}{dx}\right|_{x^*} \overbrace{(x_0 - x^*)}^{\delta x_0}$$

if $$\left| \left.\frac{df}{dx}\right|_{x^*} \right| \leq 1$$

then $$|\delta x_1| < |\delta x_0|$$

fixed pt is stable

$$x_1 = f(x_0) \qquad \delta x_1 = x_1 - x^*$$

let $x_0 = x^* + (x_0 - x^*)$

$\uparrow$ small

δx_0

$$f(x_0) = f(x^* + \underbrace{(x_0 - x^*)}_{\delta x_0}) \approx f(x^*) + \underbrace{\frac{df}{dx}\Big|_{x^*}(x_0 - x^*)}_{\delta x_1}$$

if $\left| \frac{df}{dx}\Big|_{x^*} \right| \leq 1$

then $|\delta x_1| < |\delta x_0|$

fixed pt is stable

$$f(x) = \mu x(1-x)$$

$$\frac{df}{dx} = \mu - 2\mu x$$

$$f(x^* = 0)$$

$$\left.\frac{df}{dx}\right|_0 = \mu$$

$$x^* = 1 - 1/\mu$$

$$\frac{df}{dx} = \mu - 2\mu(1 - 1/\mu)$$

$$= -\mu + 2$$

$$f(x) = \mu x(1-x)$$

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$$\frac{df}{dx} = \mu - 2\mu(1 - 1/\mu)$$

$$= -\mu + 2 \qquad -1 < -\mu + 2 < 1$$

$$-3 < -\mu < -1 \Rightarrow 1 < \mu < 3$$

$$f(x) = \mu x(1-x)$$

$$\frac{df}{dx} = \mu - 2\mu x$$

$$(x^* = 0)$$

$$\left.\frac{df}{dx}\right|_0 = \mu$$

$$x^* = 1 - 1/\mu$$

$$\frac{df}{dx} = \mu - 2\mu(1 - 1/\mu)$$

$$= -\mu + 2 \qquad -1 < -\mu + 2 < 1$$

$$-3 < -\mu < -1 \Rightarrow 1 < \mu < 3$$

Period-2 orbit

if $f(x_a) = x_b$

$f(x_b) = x_a$

then $f(f(x_a)) = x_a$

$f(f(x_b)) = x_b$

let f

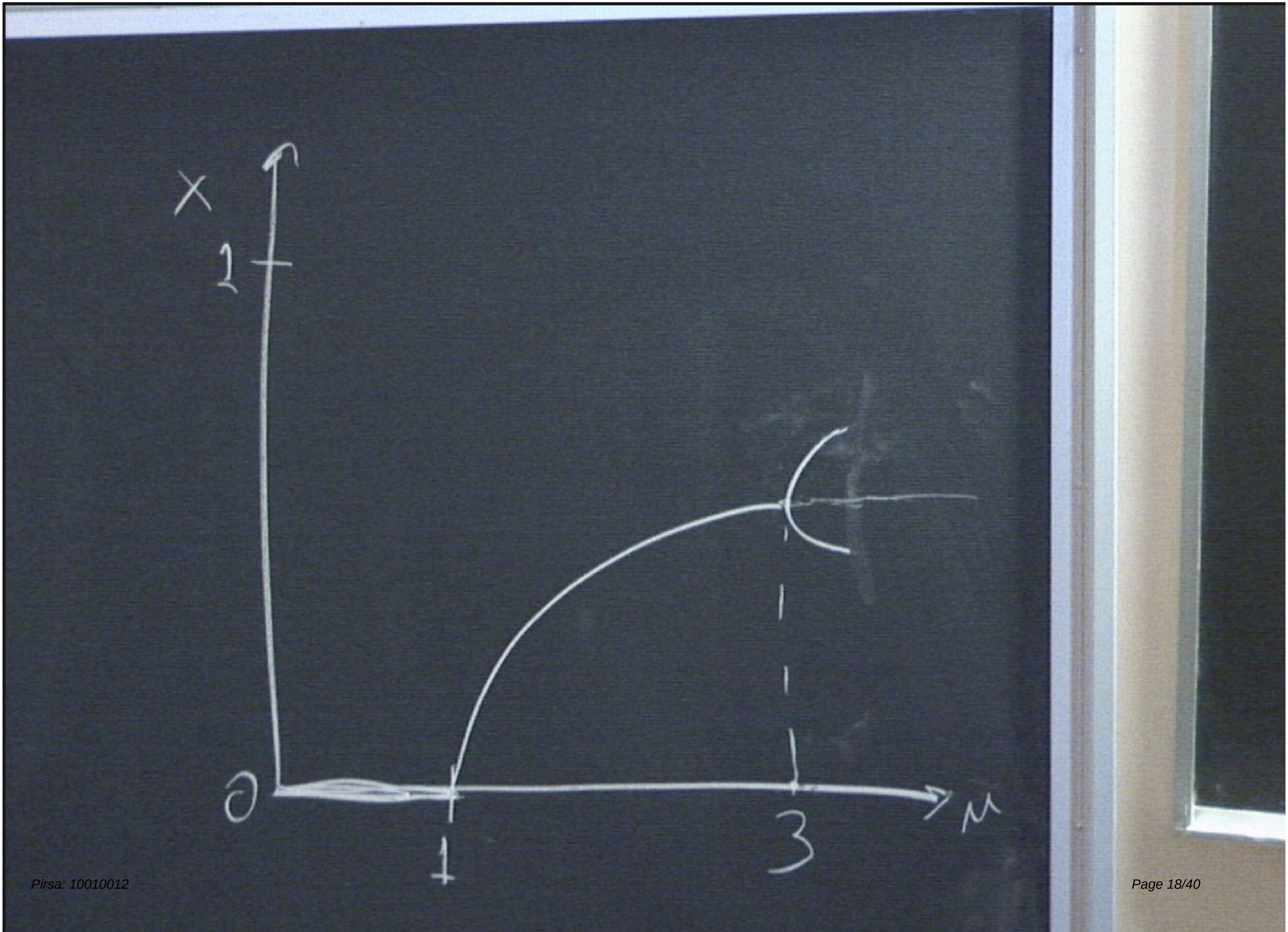
let $f(f(x)) = f^2(x)$

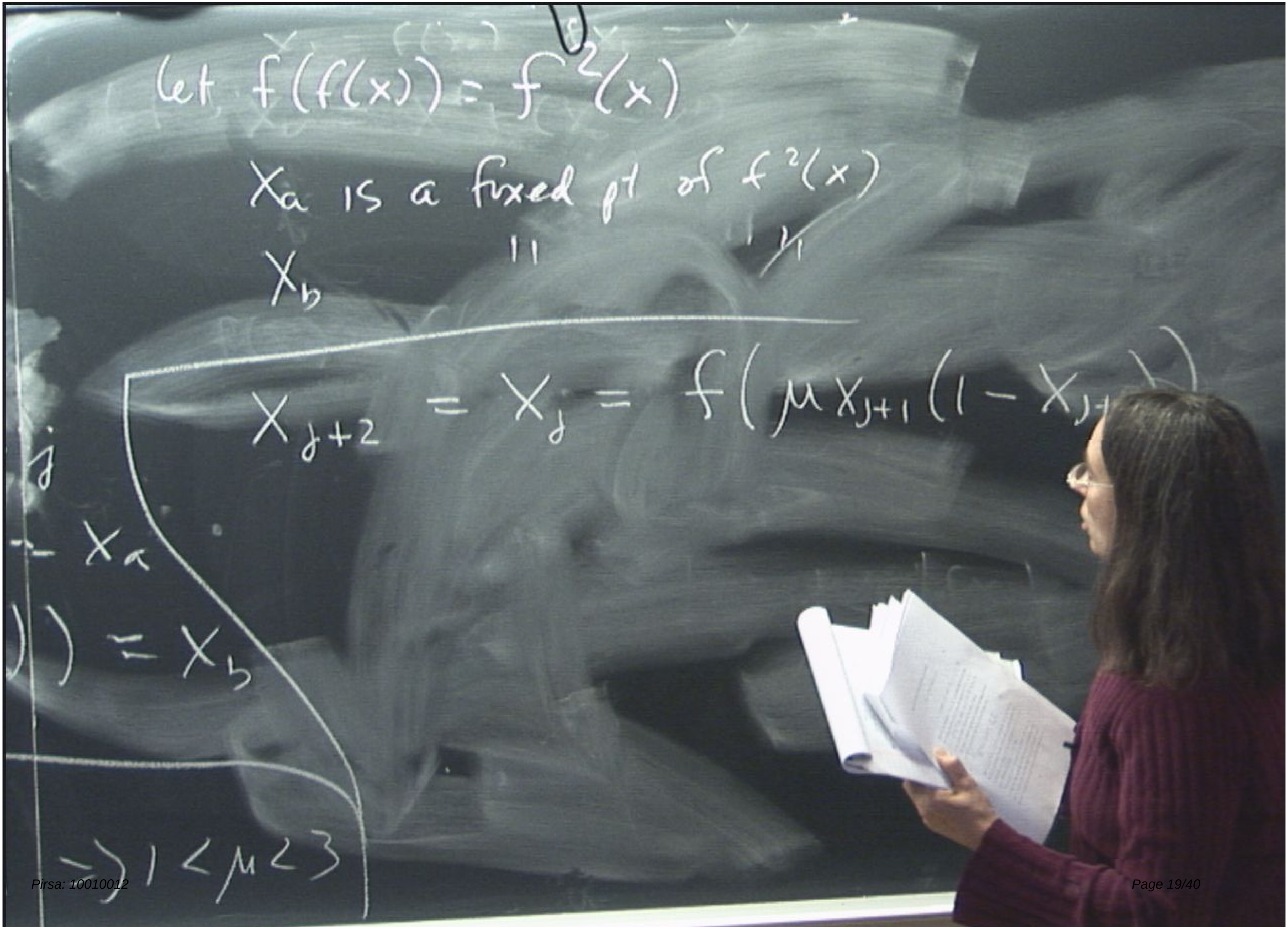
j

$= x_a$

$)) = x_b$

$\to 1 < \mu < 3$


x
1
0
1
3
μ


let $f(f(x)) = f^2(x)$
x_a is a fixed pt of $f^2(x)$
x_b " "
$x_{j+2} = x_j = f(\mu x_{j+1}(1 - x_{j+1}))$
$= x_a$
$)) = x_b$
$\to 1 < \mu < 3$

let $f(f(x)) = f^2(x)$

x_a is a fixed pt of $f^2(x)$

x_b " "

$$x_{j+2} = f(\mu x_{j+1}(1 - x_{j+1}))$$

$$x_j = \mu[\mu x_j(1-x_j)(1 - \mu x_j(1-x_j))]$$

x_a

$) = x_b$

$\Rightarrow 1 < \mu < 3$

let $f(f(x)) = f^2(x)$

x_a is a fixed pt of $f^2(x)$

x_b " "

$$x_{j+2} = f(\mu x_{j+1}(1 - x_{j+1}))$$

$$x_j = \mu[\mu x_j(1-x_j)(1 - \mu x_j(1-x_j))]$$

$= x_b$

$\Rightarrow$ fixed pts of f^2 satisfy either $x = 0$ or

$$0 = \mu^2 - 1 - \mu^2(1+\mu)x + 2\mu^3 x^2 - \mu^3 x^3$$

$1 < \mu < 3$

let $f(f(x)) = f^2(x)$

x_a is a fixed pt of $f^2(x)$

x_b " "

$$x_{j+2} = f(\mu x_{j+1}(1 - x_{j+1}))$$

$$x_j = \mu\left[\mu x_j(1-x_j)\left(1 - \mu x_j(1-x_j)\right)\right]$$

$\Rightarrow$ fixed pts of f^2 satisfy either $x = 0$ or

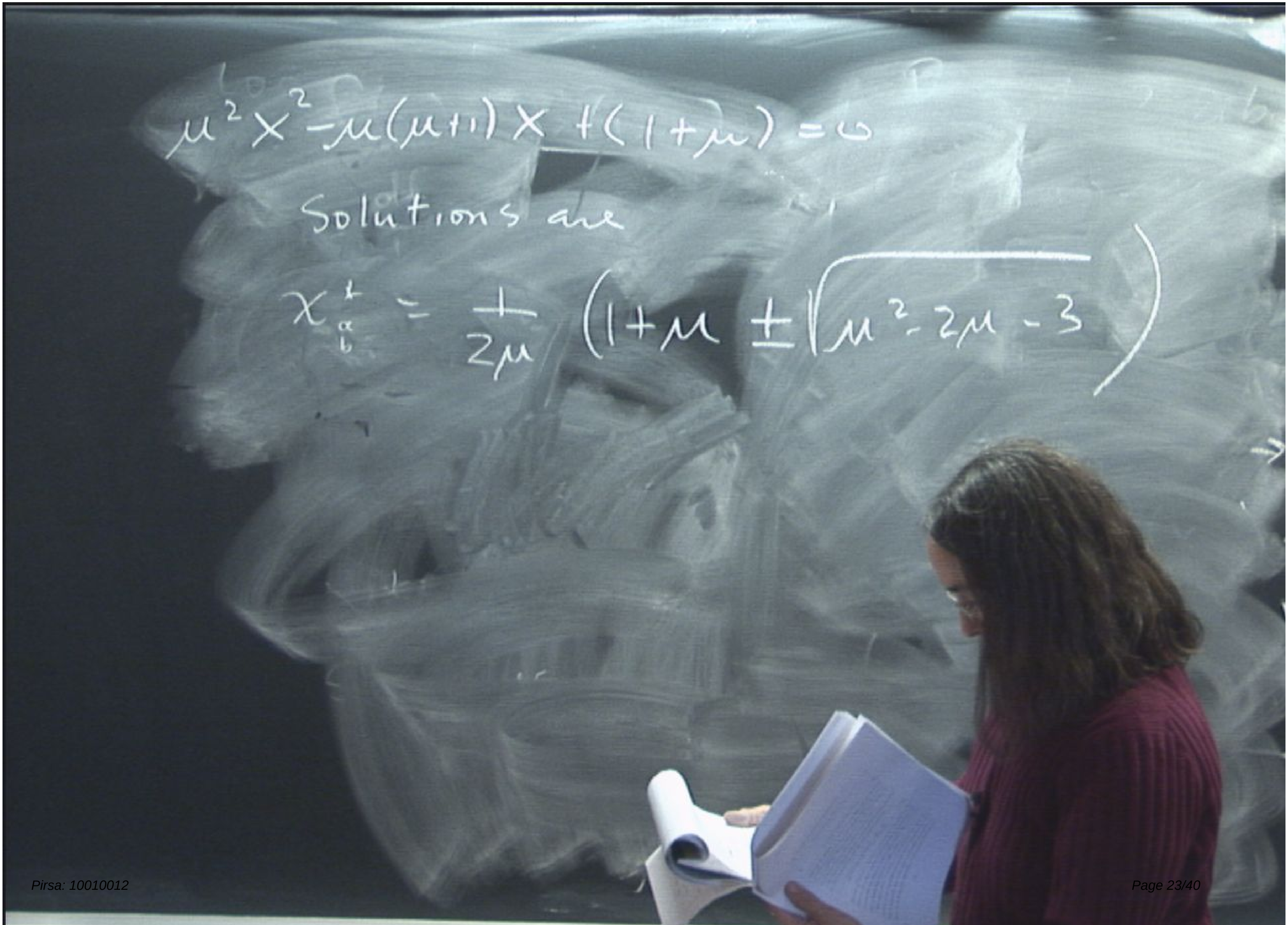
$$0 = \mu^2 - 1 - \mu^2(1+\mu)x + 2\mu^3 x^2 - \mu^3 x^3$$

$x^* = 1 - 1/\mu$ is solution

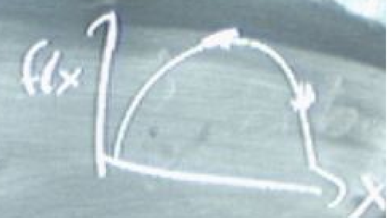
a

$= x_b$

$1 < \mu < 3$


$\mu^2 x^2 - \mu(\mu+1)x + (1+\mu) = 0$
Solutions are
$x^{\pm}_{a,b} = \frac{1}{2\mu}\left(1+\mu \pm \sqrt{\mu^2-2\mu-3}\right)$

$$\mu^2 x^2 - \mu(\mu+1)x + (1+\mu) = 0$$



Solutions are

$$x^*_{a,b} = \frac{1}{2\mu}\left(1+\mu \pm \sqrt{\mu^2-2\mu-3}\right)$$

Check stability:

$$\frac{d}{dx} f^2(x) = \frac{d}{dx} f(y) = \left.\frac{df}{dx}\right|_y \frac{dy}{dx}$$

let $y = f(x)$

$$\left.\frac{d}{dx} f^2(x)\right|_{x=x_0} = \left.\frac{df}{dx}\right|_{f(x_0)} \left.\frac{df}{dx}\right|_{x_0}$$

$$+\mu \pm \sqrt{\mu^2 - 2\mu - 3}\,)$$

$$\Rightarrow j$$

$$\frac{d}{dx} f(y) = \frac{df}{dx}\Big|_y \frac{dy}{dx}$$

$$) = x$$

$$f(x)$$

$$\frac{d}{dx} f^2(x)\Big|_{x=x_0} \frac{df}{dx}\Big|_{f(x_0)} \frac{df}{dx}\Big|_{x_0}$$

$$\Rightarrow 1 < \mu < 3$$

$$\frac{d}{dx} f(f(x_a))\Big|_{x_a} = \frac{df}{dx}\Big|_{x_b} \frac{df}{dx}\Big|_{x_a}$$

$f(x) =$

$\frac{df}{dx} =$

$) = x$

$\rightarrow 1 < \mu < 3$

$$f(x) = \mu x(1-x)$$

$$\frac{df}{dx} = \mu - 2\mu x$$

$$f(x) = \mu x(1-x)$$

$$\frac{df}{dx} = \mu - 2\mu x$$

$$\frac{d}{dx} f(f(x_a))\Big|_{x_a} = \frac{df}{dx}\Big|_{x_b} \frac{df}{dx}\Big|_{x_a}$$

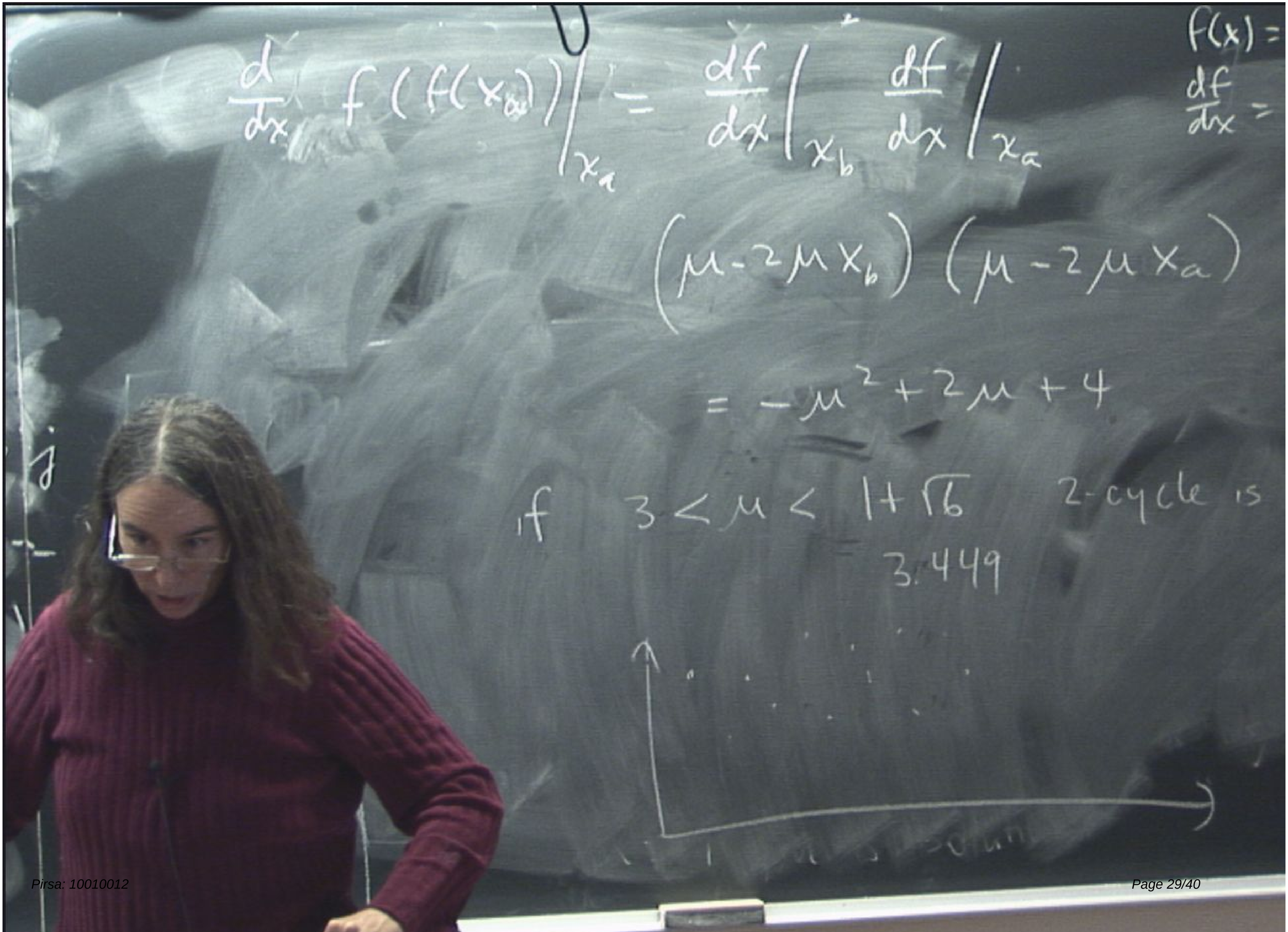
$$(\mu - 2\mu x_b)(\mu - 2\mu x_a)$$

$$= -\mu^2 + 2\mu + 4$$

if $3 < \mu < 1 + \sqrt{6}$ 2-cycle is stable

3.449

$< \mu < 3$


$\frac{d}{dx} f(f(x_a))\Big|_{x_a} = \frac{df}{dx}\Big|_{x_b} \frac{df}{dx}\Big|_{x_a}$
$f(x) =$
$\frac{df}{dx} =$
$(\mu - 2\mu x_b)(\mu - 2\mu x_a)$
$= -\mu^2 + 2\mu + 4$
if $3 < \mu < 1+\sqrt{6}$ 2-cycle is
3.449

$$f(x) = \mu x(1-x)$$

$$\frac{df}{dx} = \mu - 2\mu x$$

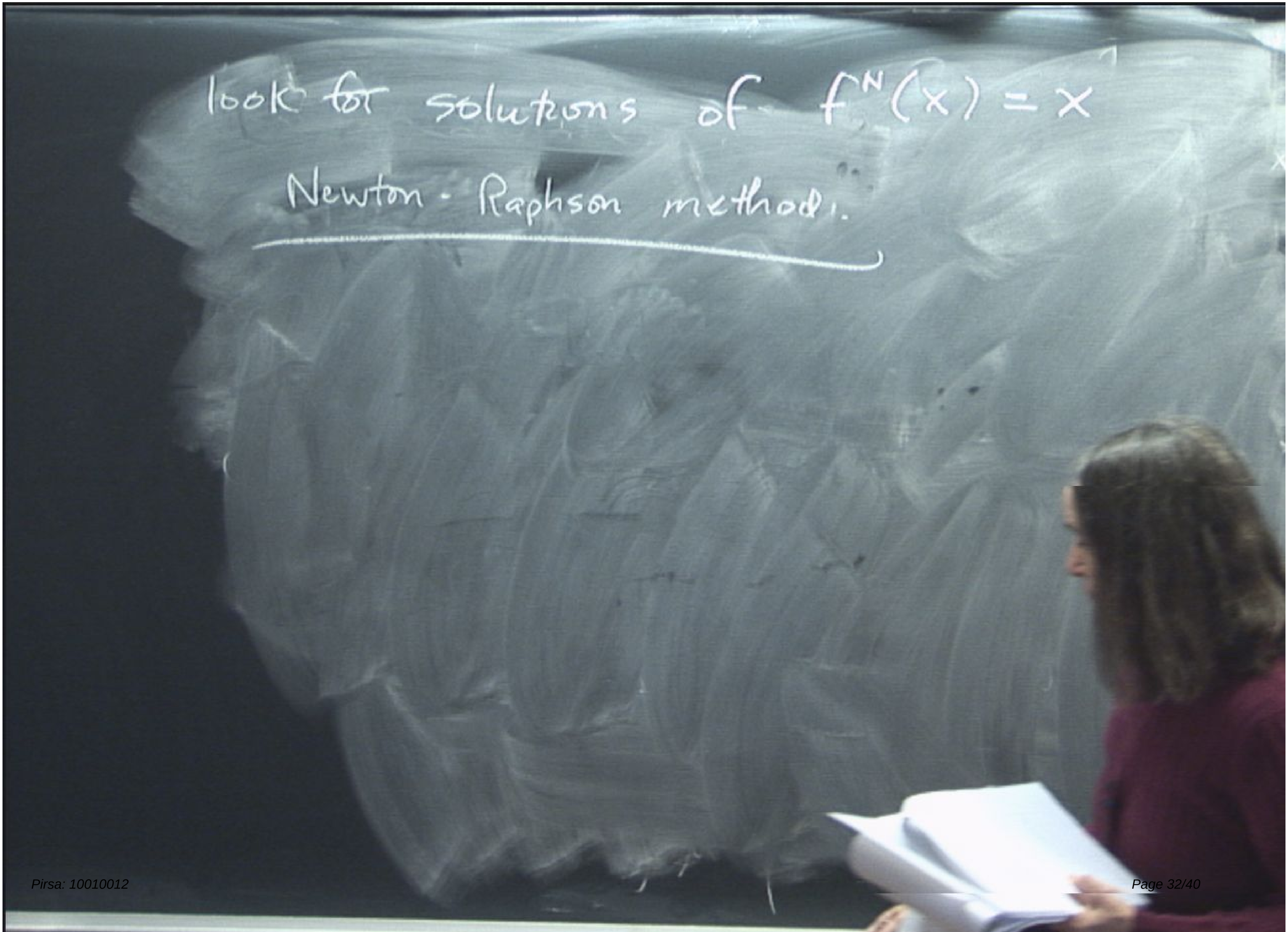
$$\frac{d}{dx} f(f(x_a))\Big|_{x_a} = \frac{df}{dx}\Big|_{x_b} \frac{df}{dx}\Big|_{x_a}$$

$$(\mu - 2\mu x_b)(\mu - 2\mu x_a)$$

$$= -\mu^2 + 2\mu + 4$$

if $3 < \mu < 1+\sqrt{6} = 3.449$ 2-cycle is stable

look for solutions of $f^N(x) = x$


look for solutions of $f^N(x) = x$
Newton - Raphson method.

look for solutions of $f^N(x) = x$

Newton - Raphson method.

$$F(x) = 0$$

$$\left[\, f^N(x) - x = 0 \,\right]$$

look ~~for~~ solutions of $f^N(x) = x$

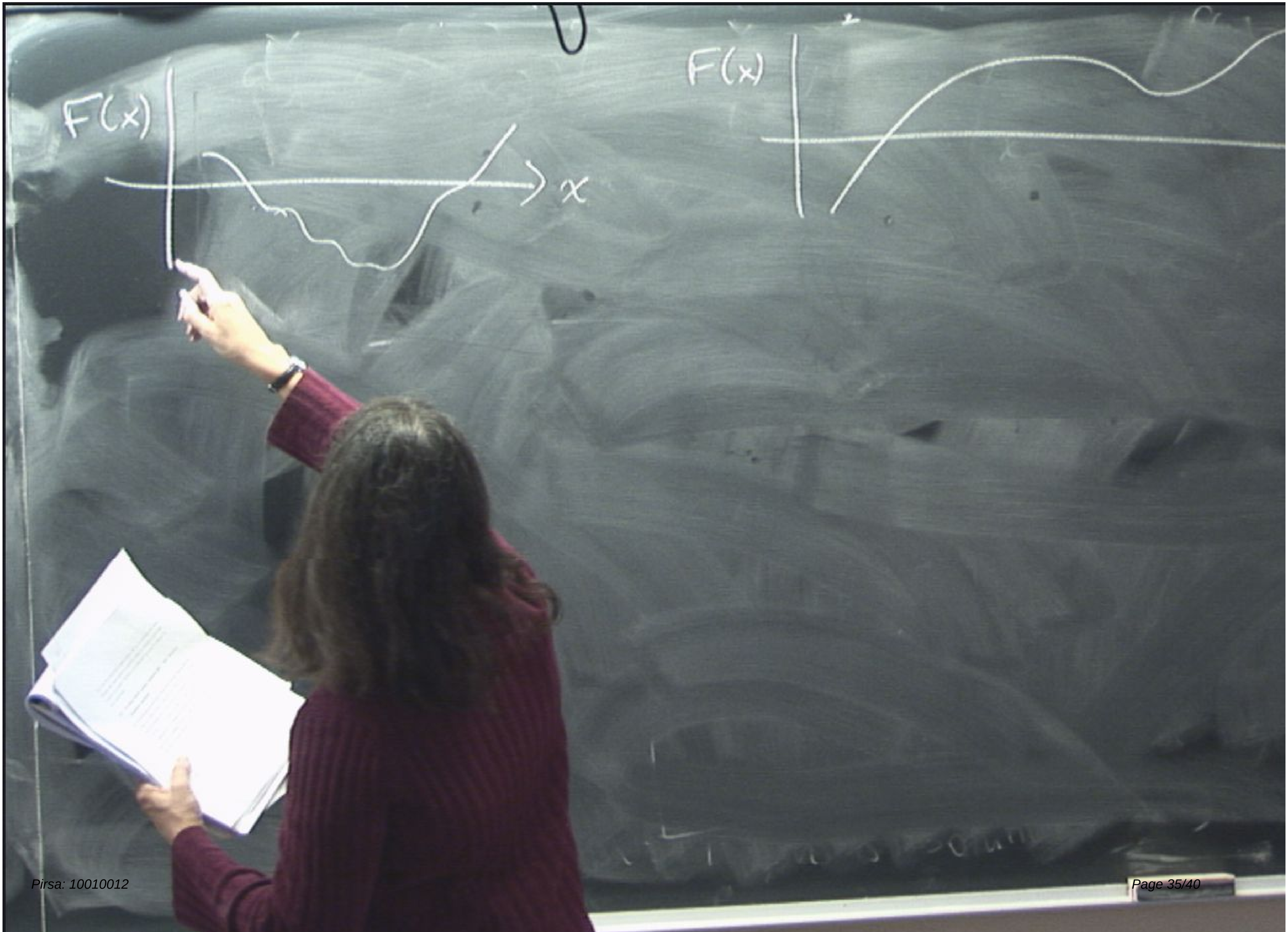
Newton - Raphson method.

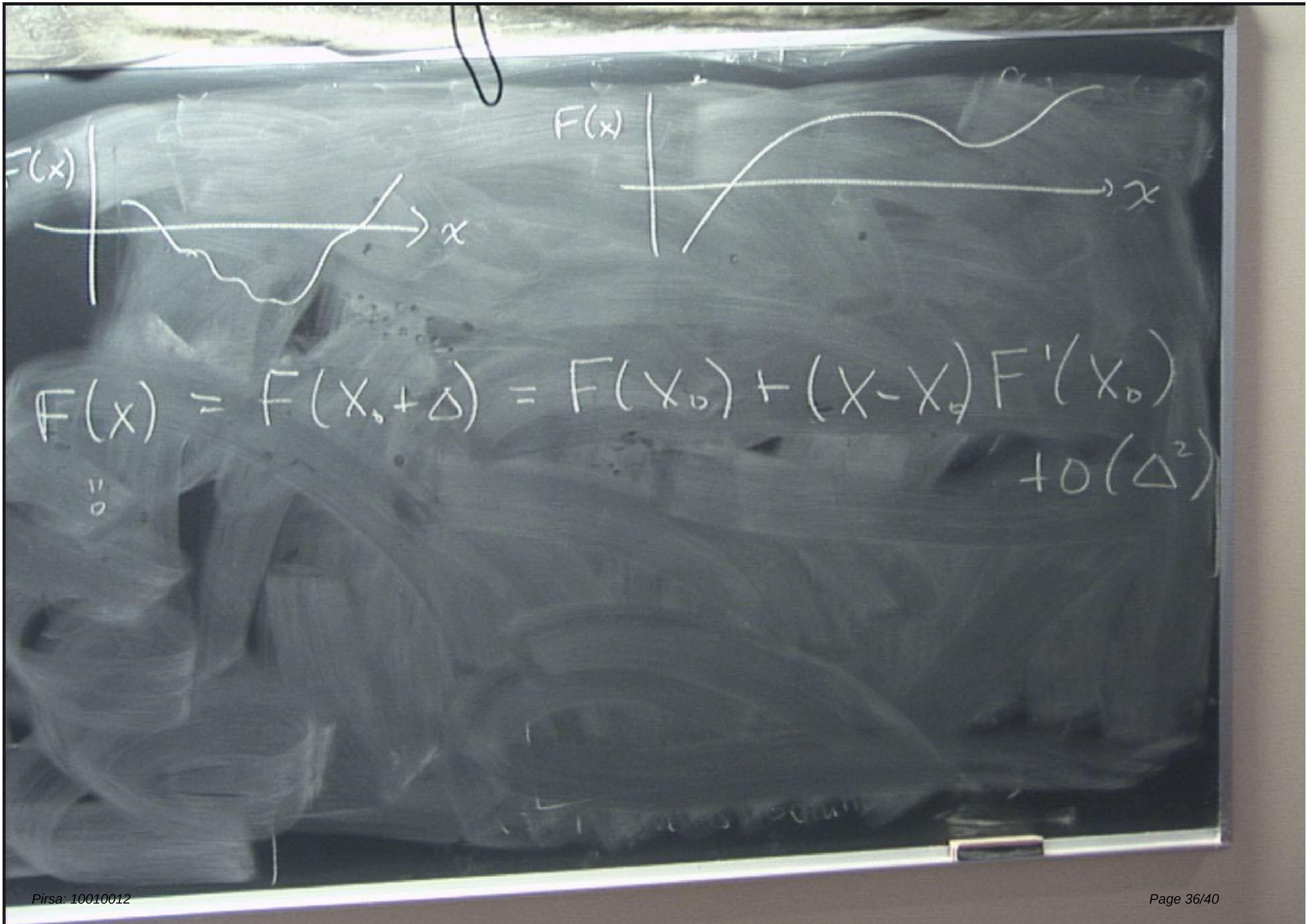
$$F(x) = 0$$

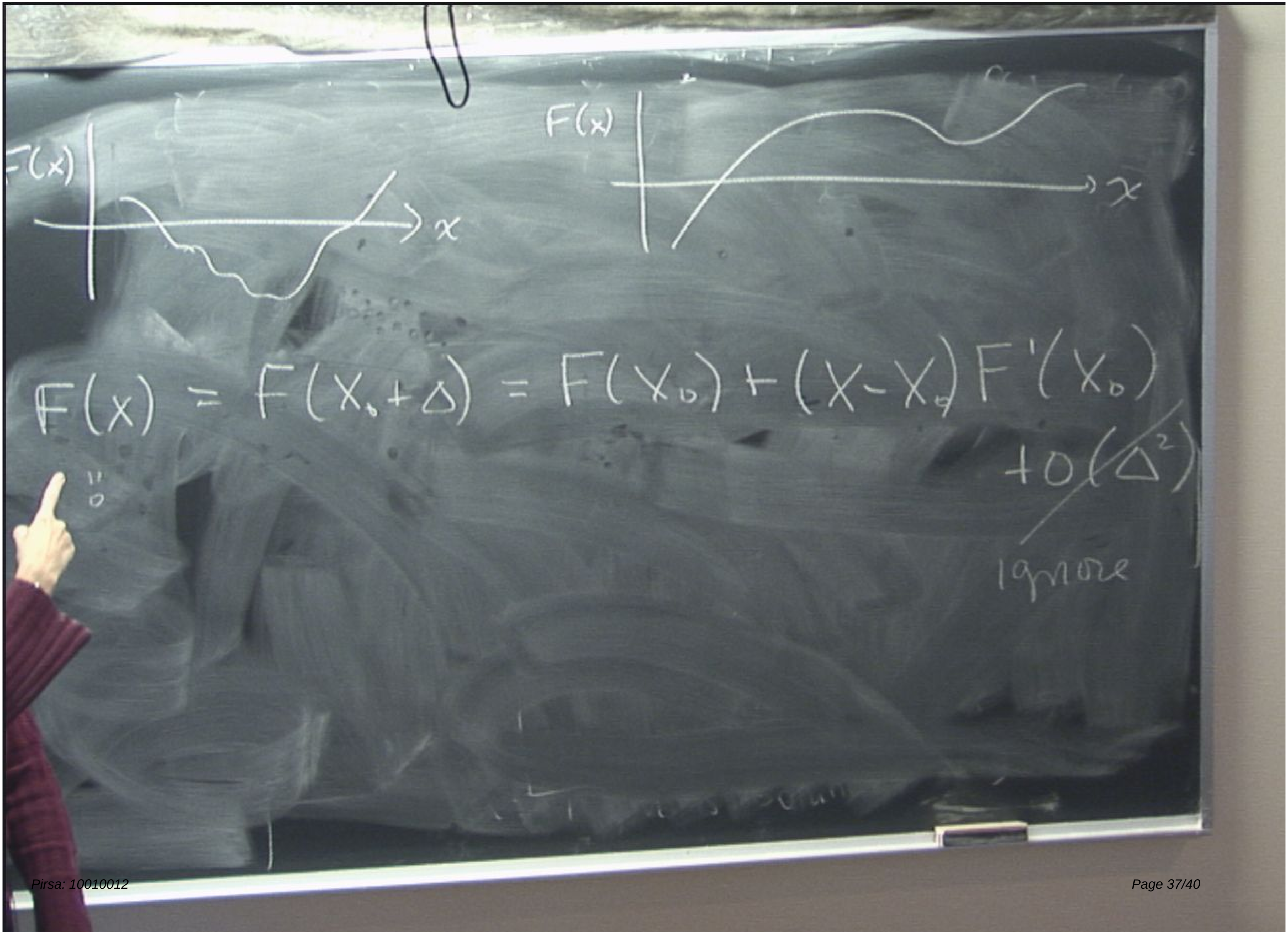
$$\left[f^N(x) - x = 0 \right]$$

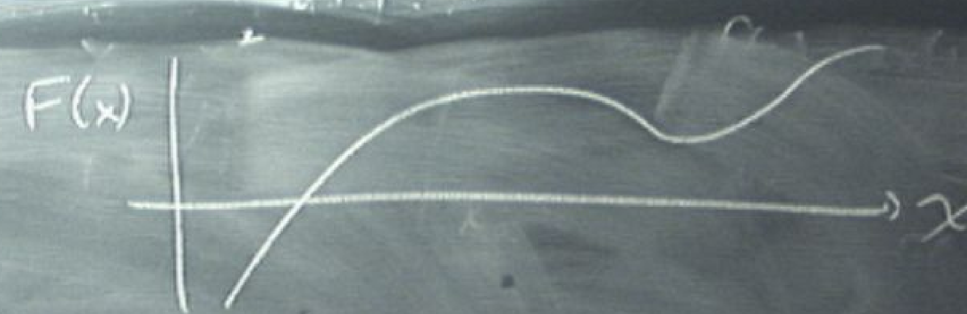
guess solution is near X_0

write $\underbrace{X}_{\text{solution}} = \underbrace{X_0}_{\text{guess}} + \underbrace{\Delta}_{\text{error}}$


F(x)
x
F(x)


$F(x)$
x
$F(x)$
x
$F(x) = F(x_0+\Delta) = F(x_0) + (x-x_0)F'(x_0) + O(\Delta^2)$


F(x)
x
F(x)
x
F(x) = F(x₀+Δ) = F(x₀) + (x−x₀) F'(x₀)
+O(Δ²)
ignore



$$F(x) = F(x_0+\Delta) = F(x_0) + (x-x_0)F'(x_0) + \cancel{O(\Delta^2)}$$

ignore

$$\parallel$$

$$0 = F(x_0) + (x-x_0)\left.\frac{dF}{dx}\right|_{x_0}$$

So our new guess is

$$x_1 = x_0 - \frac{F(x_0)}{F'(x_0)}$$

$$F(x) = F(x_0 + \Delta) = F(x_0) + (x - x_0) F'(x_0) + \cancel{O(\Delta^2)}$$

ignore

$$F(x) = 0$$

$$0 = F(x_0) + (x - x_0) \left.\frac{dF}{dx}\right|_{x_0}$$

So our new guess is

$$x_1 = x_0 - \frac{F(x_0)}{F'(x_0)}$$

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