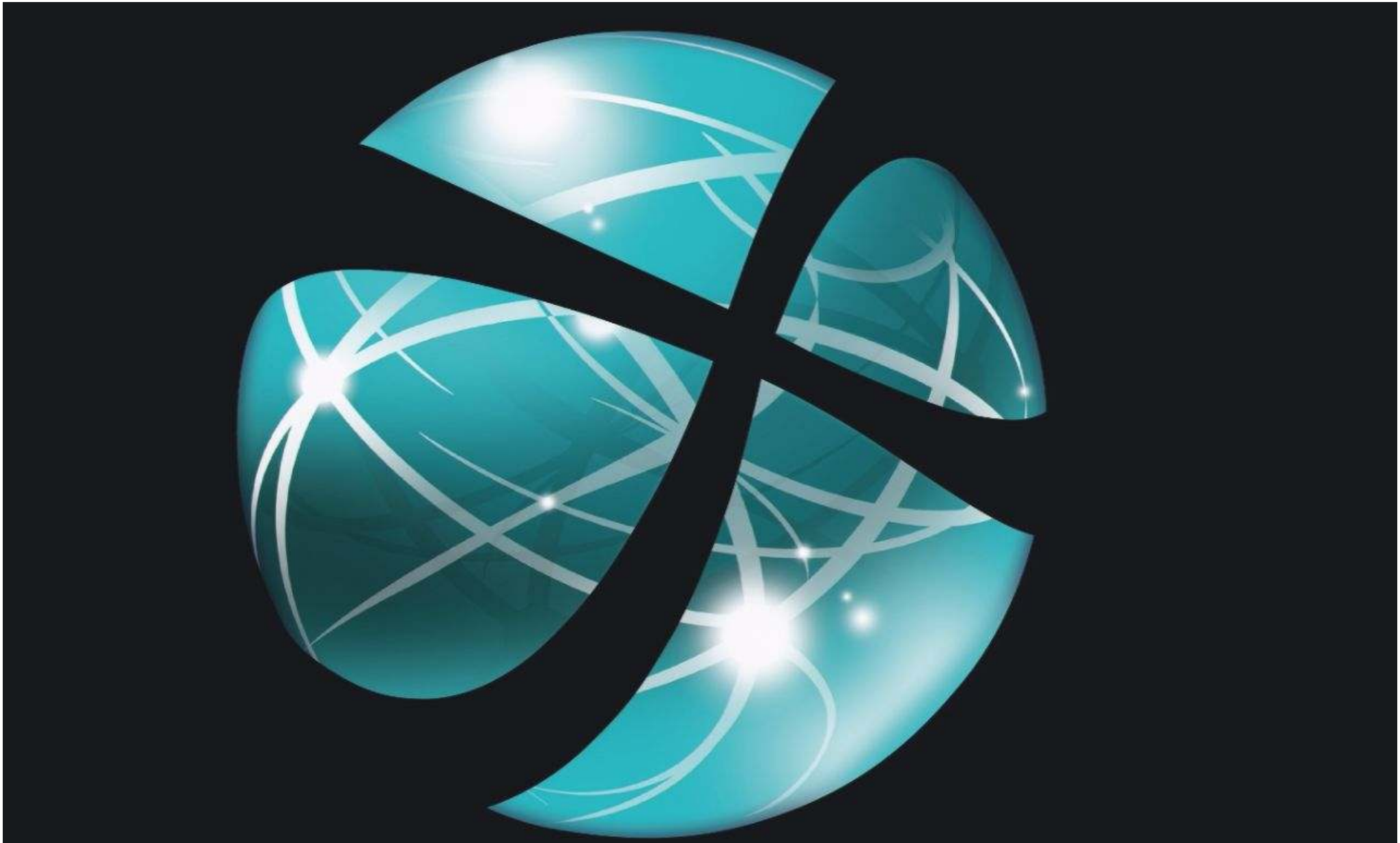
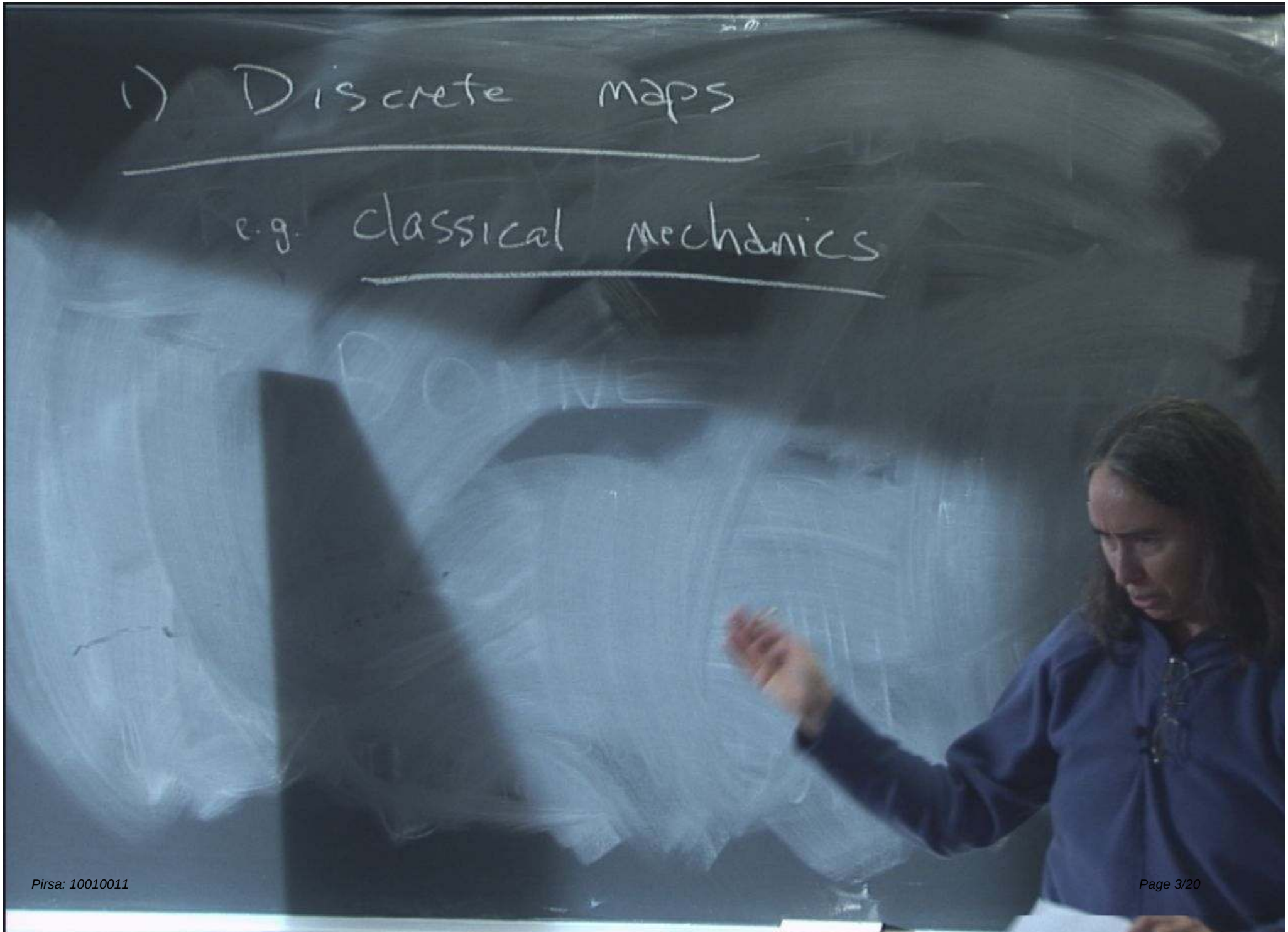
Title: Dynamical Systems - Review (PHYS 607) - Lecture 1

Date: Jan 04, 2010 09:00 AM

URL: http://pirsa.org/10010011

Abstract:


perimeter scholars
international


1) Discrete maps
e.g. classical mechanics

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e.g. classical mechanics

$\{x_i, p_i\} \longrightarrow \{x_i, p_i\}$

now

all future times

1) Discrete maps

e.g. classical mechanics

$\{x_i, p_i\} \longrightarrow \{x_i, p_i\}$

now → all future times

$x_i \longrightarrow x_i$

now → later.

1) Discrete maps

e.g. classical mechanics

$\{x_0, p_0\} \longrightarrow \{x_\infty, p_\infty\}$

now

all future tim

$X_0 \longrightarrow X_1$

now later

$X_{t+1} = f(X_t)$

given X_0, f

1) Discrete maps

e.g. classical mechanics

$\{x_0, p_0\} \longrightarrow \{x_t, p_t\}$

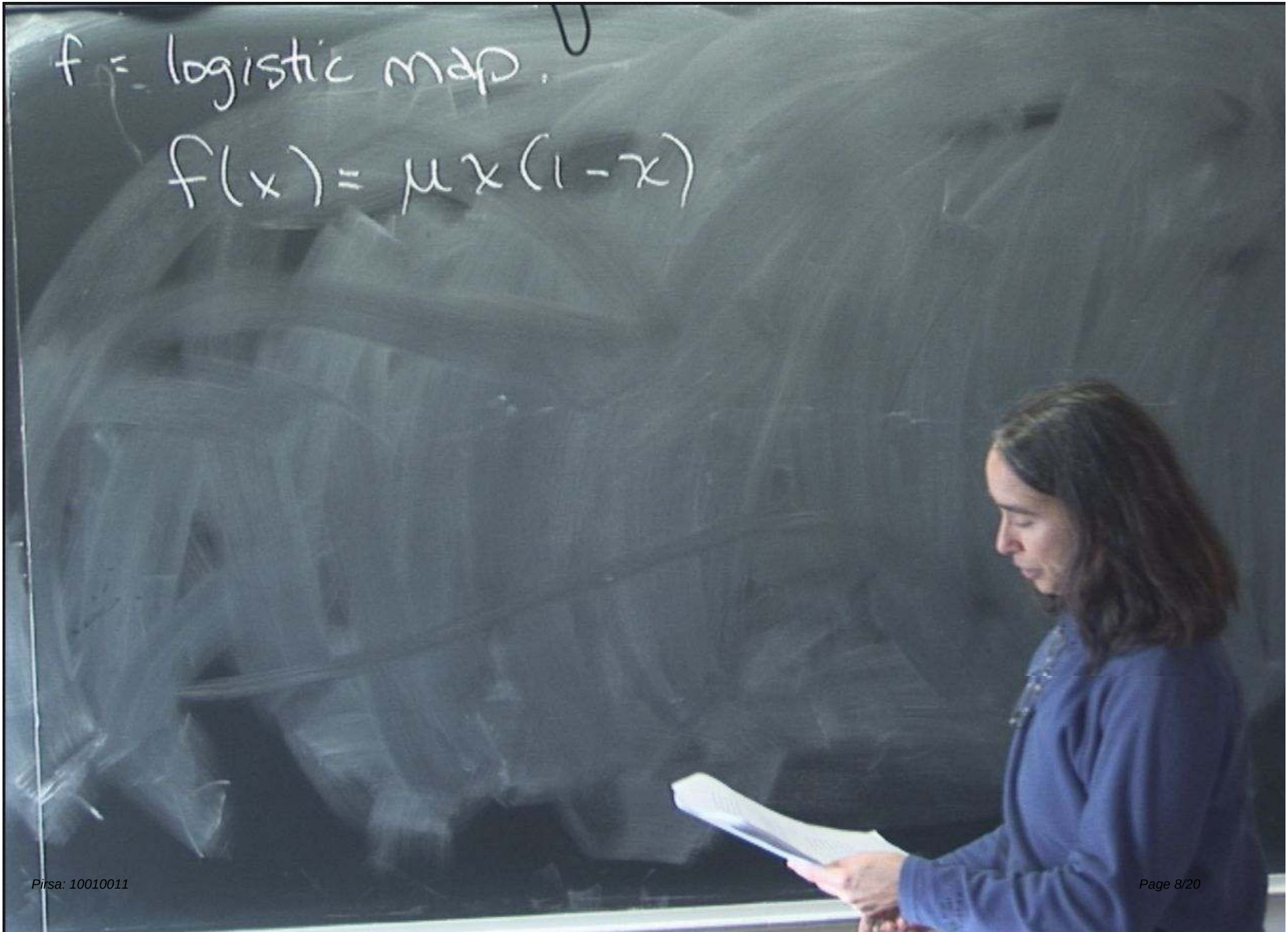
now — all future times

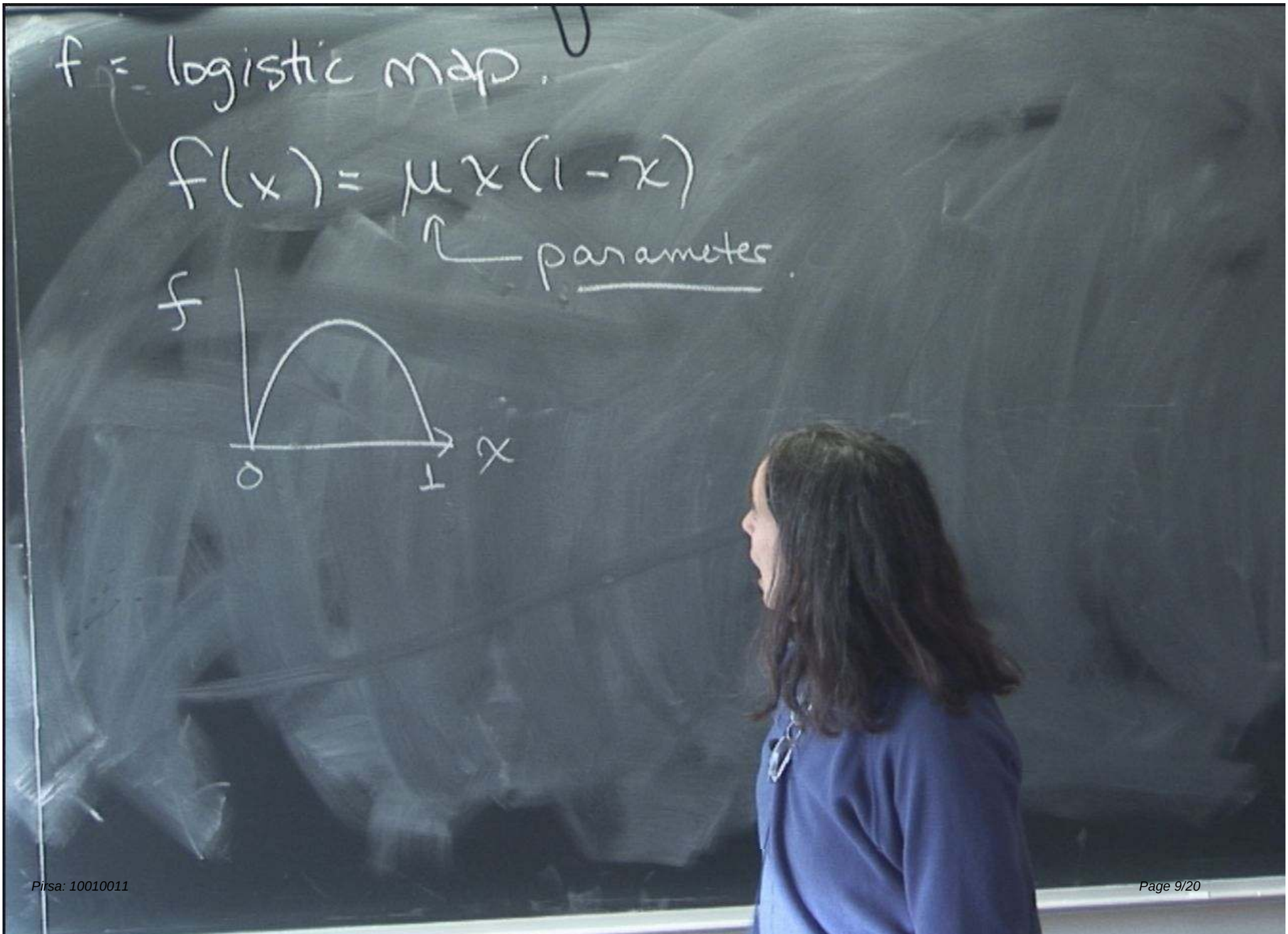
$X_n \longrightarrow X_{n+1}$

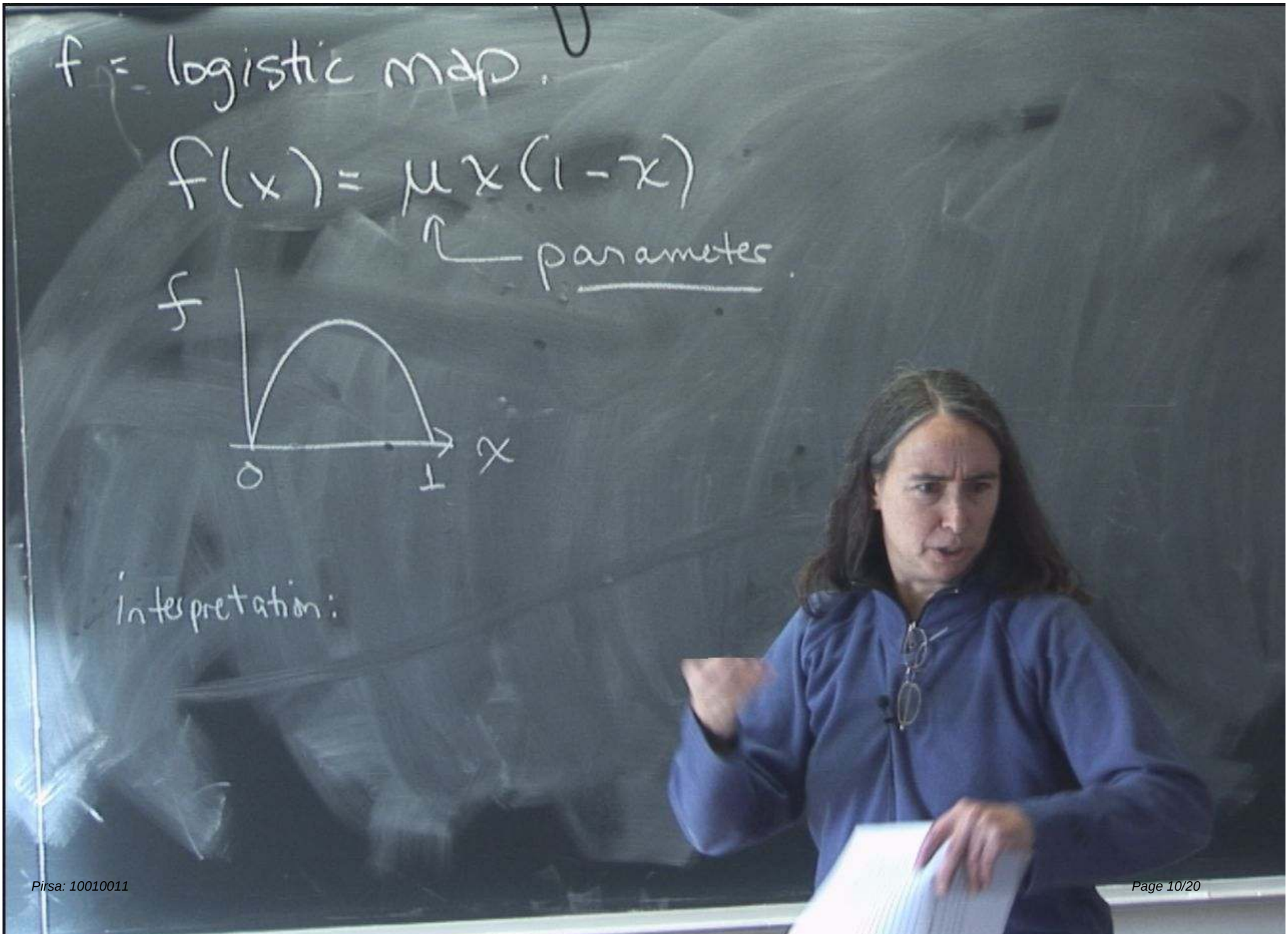
now — later

$X_{n+1} = f(X_n)$

given X_0, f


f = logistic map.
f(x) = μx(1-x)

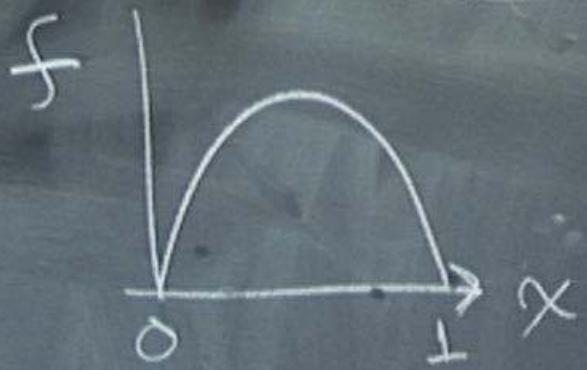

f = logistic map.
f(x) = μx(1-x)
parameter
f
x
0
1


f = logistic map.
f(x) = μx(1-x)
parameter
f
x
0
1
interpretation:

f = logistic map.

$$f(x) = \mu x(1-x)$$

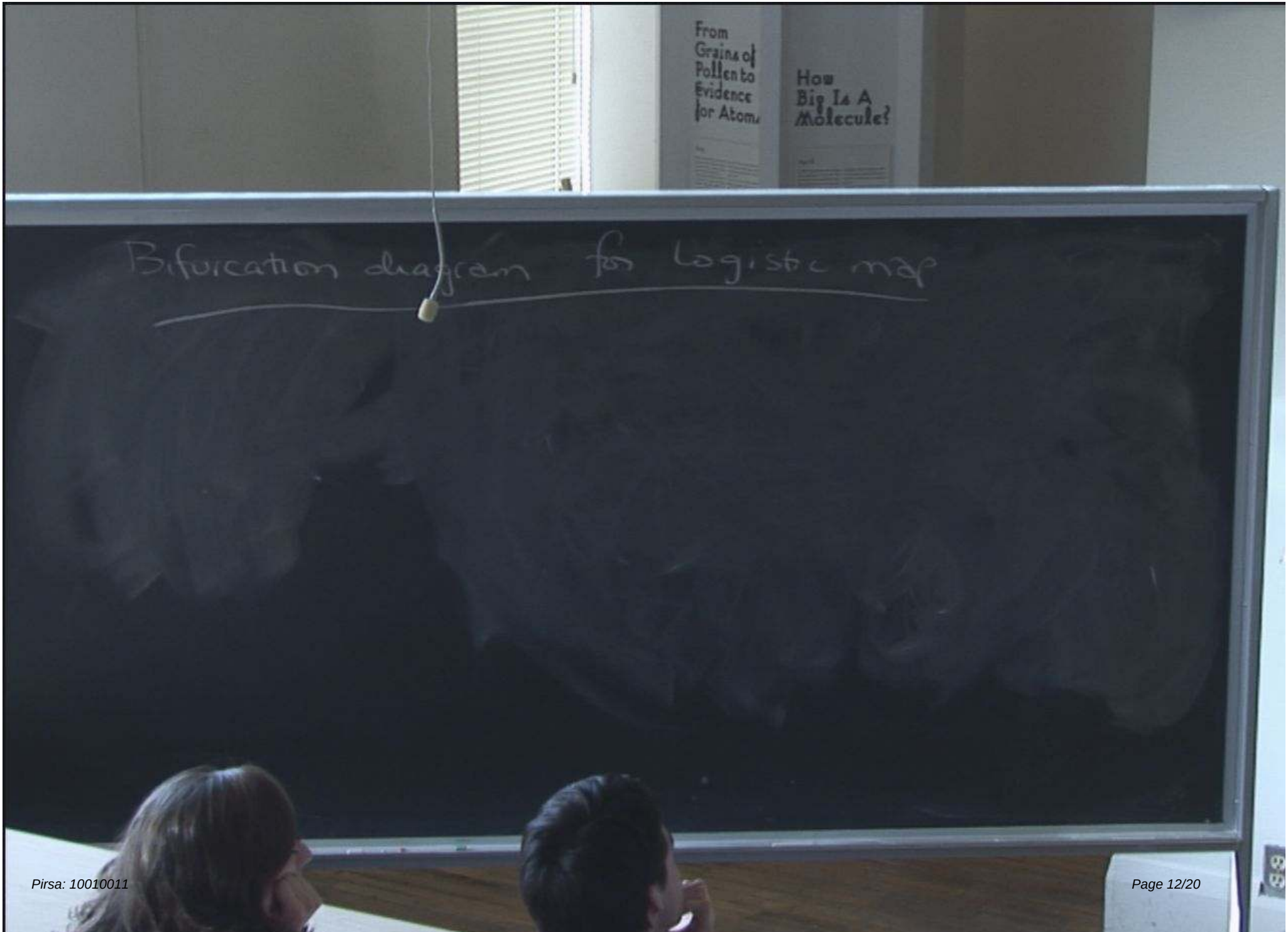
μ ← parameter

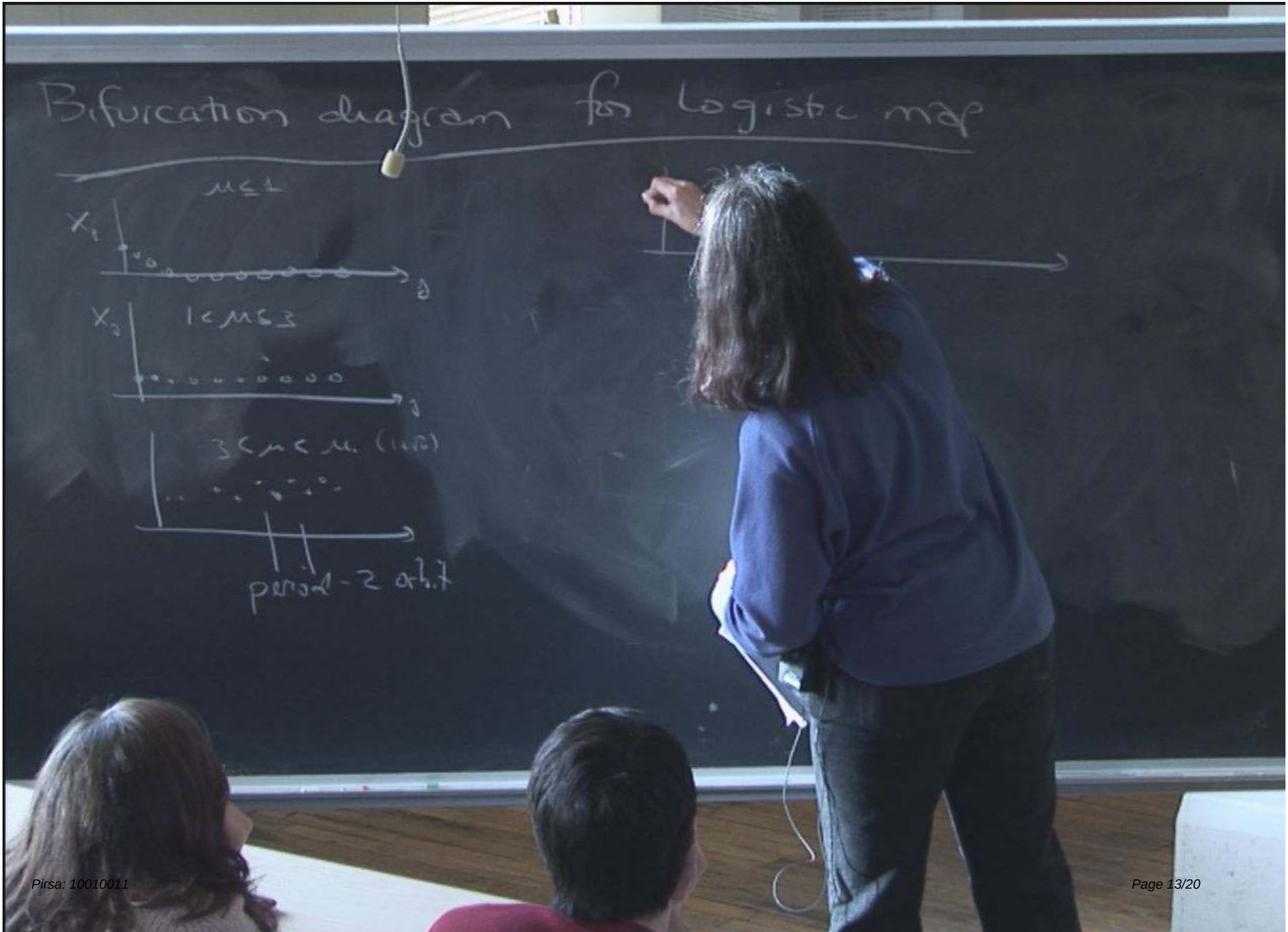


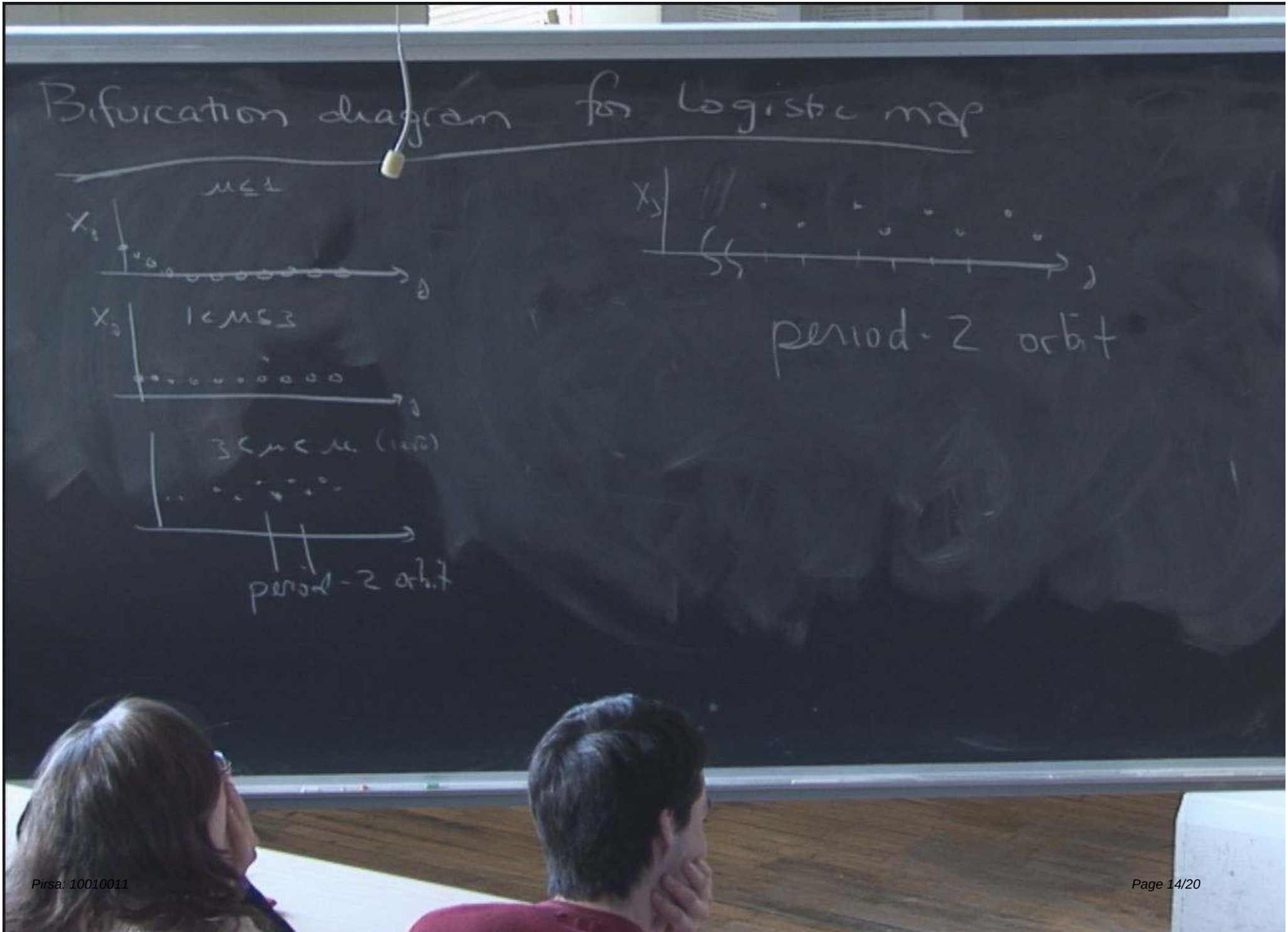
Interpretation: x_j = population of generation j / max population

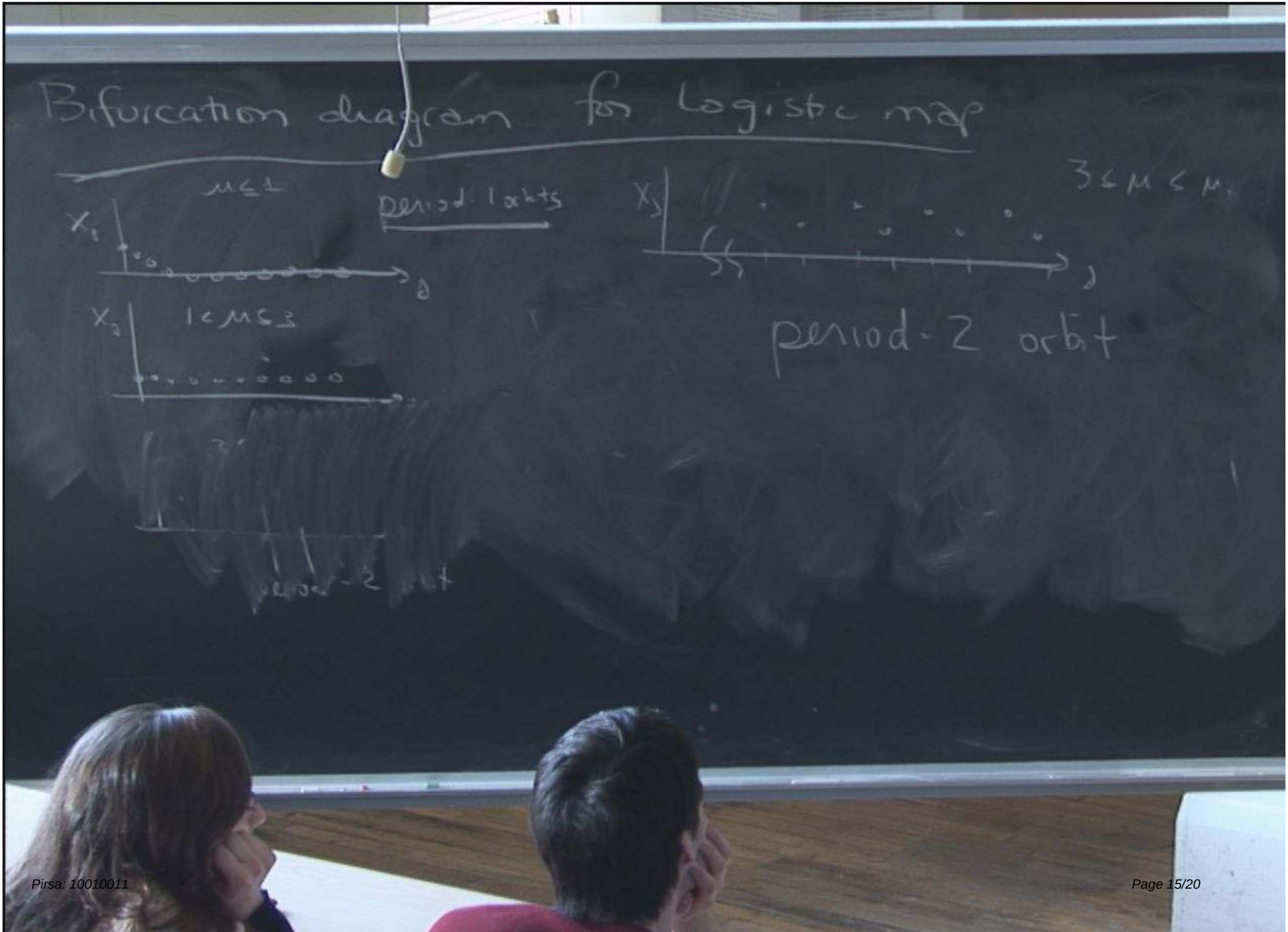
$x_{j+1} > x_j$ if x small

$x_{j+1} < x_j$ if x large


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Bifurication diagram for logistic map


Bifurcation diagram for logistic map
μ ≤ 1
1 < μ ≤ 3
period - 2 orbit

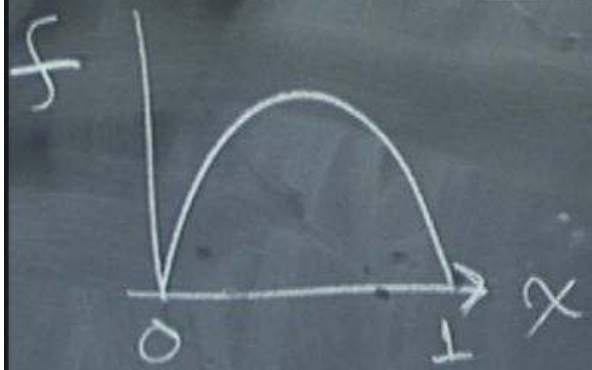

Bifurcation diagram for logistic map
μ ≤ 1
1 < μ ≤ 3
3 < μ < μ∞ (1+√6)
period - 2 orbit
period - 2 orbit


Bifurcation diagram for logistic map
μ≤1
period-1 orbits
1<μ≤3
3≤μ≤μ
period-2 orbit

logistic map.

$f(x) = \mu x(1-x)$

μ ← parameter



$x_{n+1} = f(x_n, x_{n-1}, x_{n-2})$

Interpretation: x_j = population of generation j / max population

$x_{j+1} > x_j$ if x small

$x_{j+1} < x_j$ if x large

ation diagram for logistic map

$\mu \leq 1$

period-1 orbits

$3 \leq \mu \leq \mu_\infty$

$1 < \mu \leq 3$

period-2 orbit

$f(x) = \mu x(1-x)$

when $\mu < 1$

$f(x) < \mu x$

$x_j \leq \mu^j x$

ation diagram for logistic map

$\mu \leq 1$

period-1 orbits

$3 \leq \mu \leq \mu_1$

$1 < \mu \leq 3$

period-2 orbit

$f(x) = \mu x(1-x)$

$x_1 \leq \mu x_0$

$x_2 \leq \mu x_1 \leq \mu^2 x$

when $\mu < 1$

$f(x) < \mu x$

$x_3 \leq \mu^3 x$

period-2 orbit

$f(x) = \mu x(1-x)$

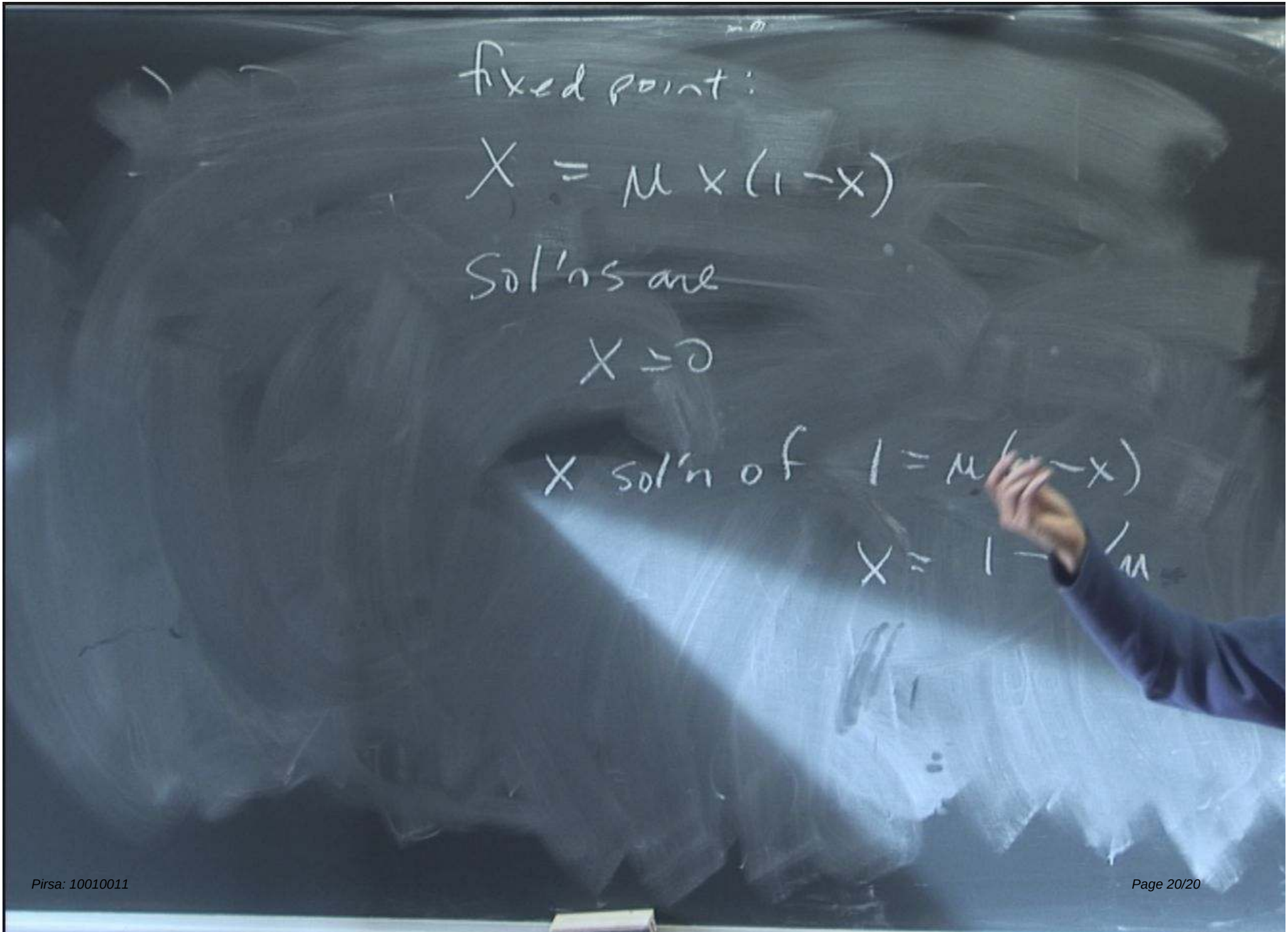
$x_1 \leq \mu x_0$

$x_2 \leq \mu x_1 \leq \mu^2 x_0$

when $\mu < 1$

$f(x) < \mu x$

$x_j \leq \mu^j x$


fixed point:
$X = \mu x(1-x)$
Sol'ns are
$X = 0$
X sol'n of $1 = \mu(1-x)$
$X = 1 - 1/\mu$