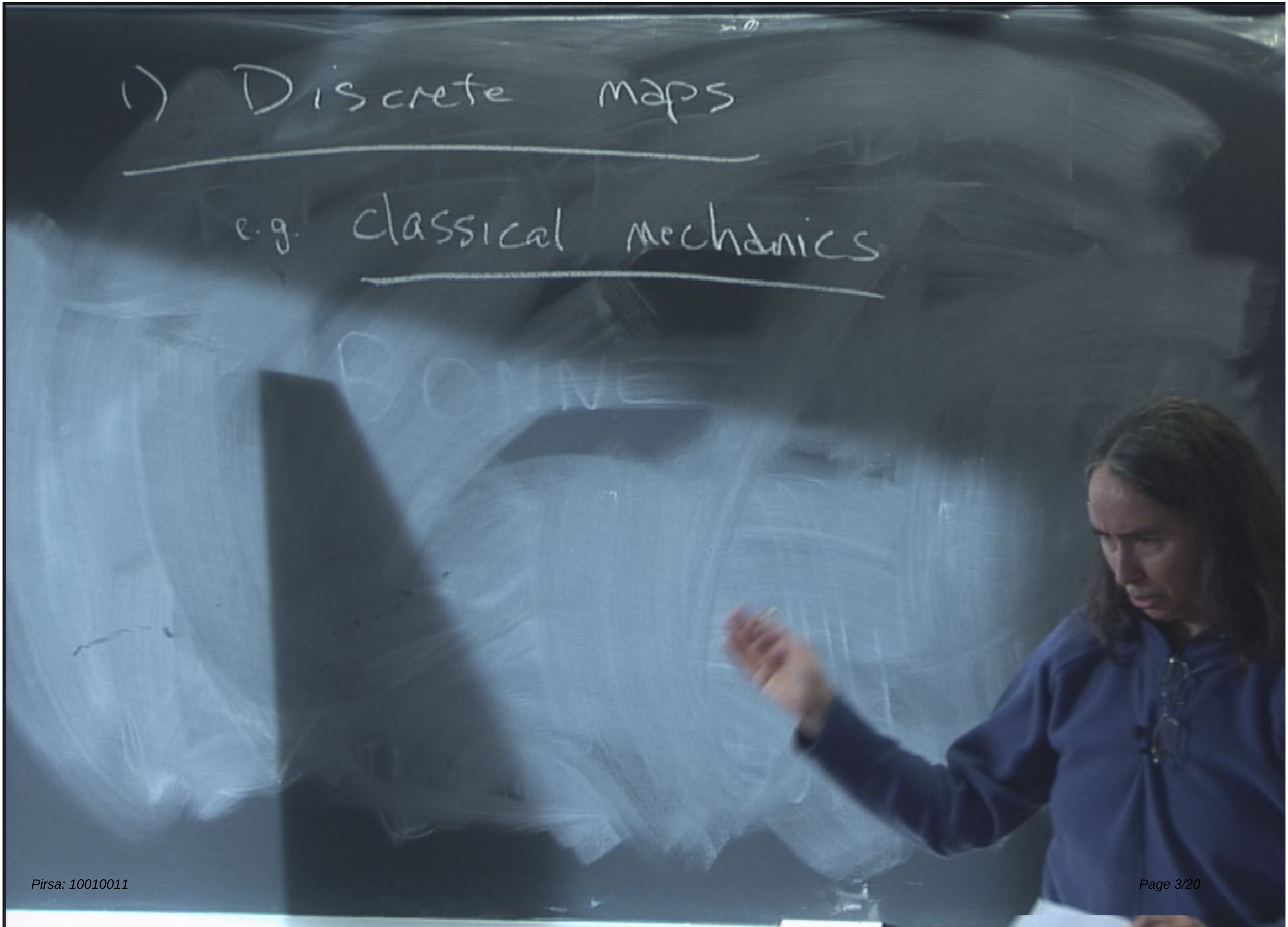
Title: Dynamical Systems - Review (PHYS 607) - Lecture 1

Date: Jan 04, 2010 09:00 AM

URL: http://pirsa.org/10010011

Abstract:

perimeter scholars
international


1) Discrete maps
e.g. classical mechanics

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$x_i \longrightarrow x_i$

now → later

$x_{i+1} = f(x_i)$

given x_0, f

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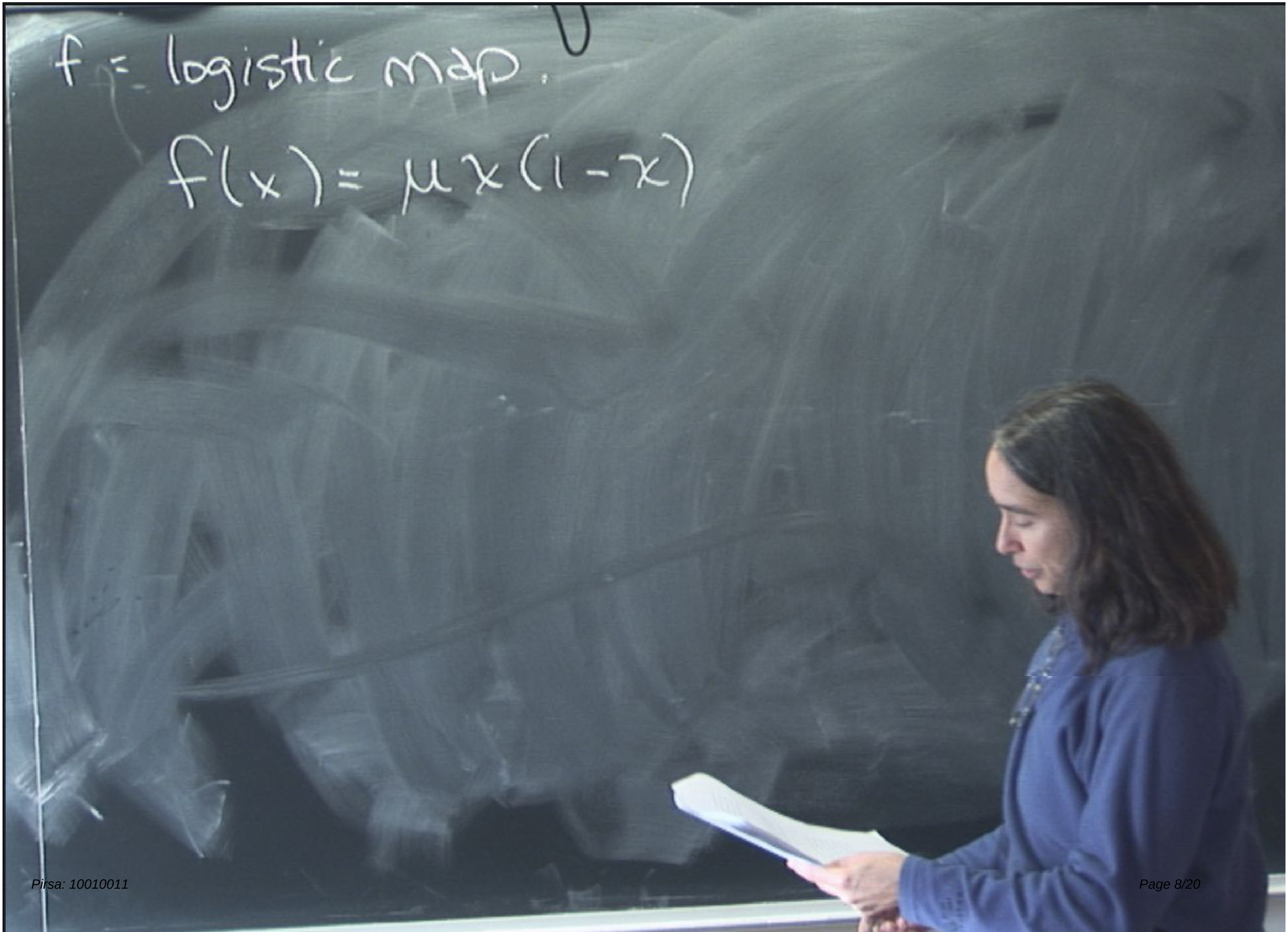
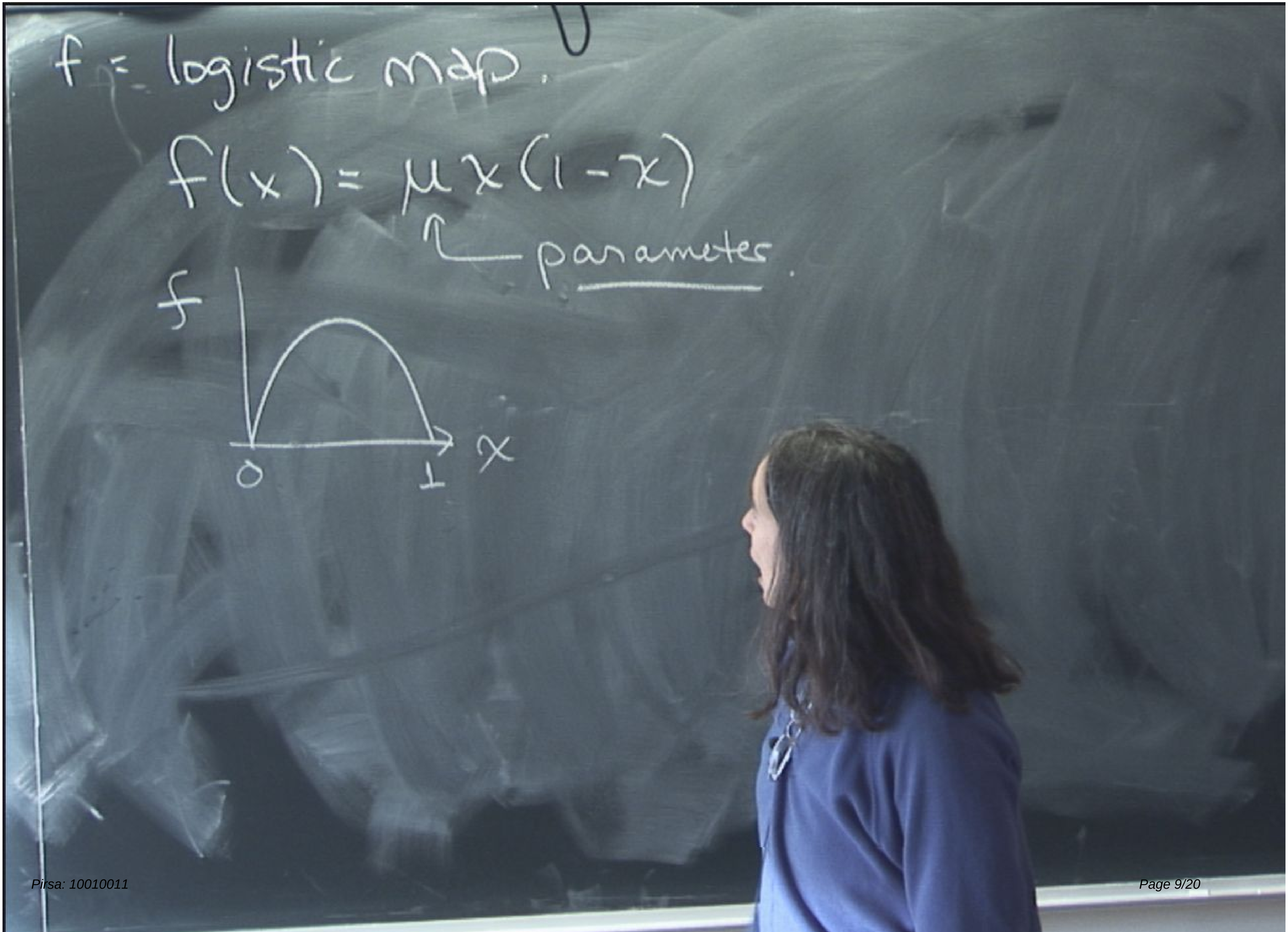

f = logistic map.
f(x) = μx(1-x)


f = logistic map.
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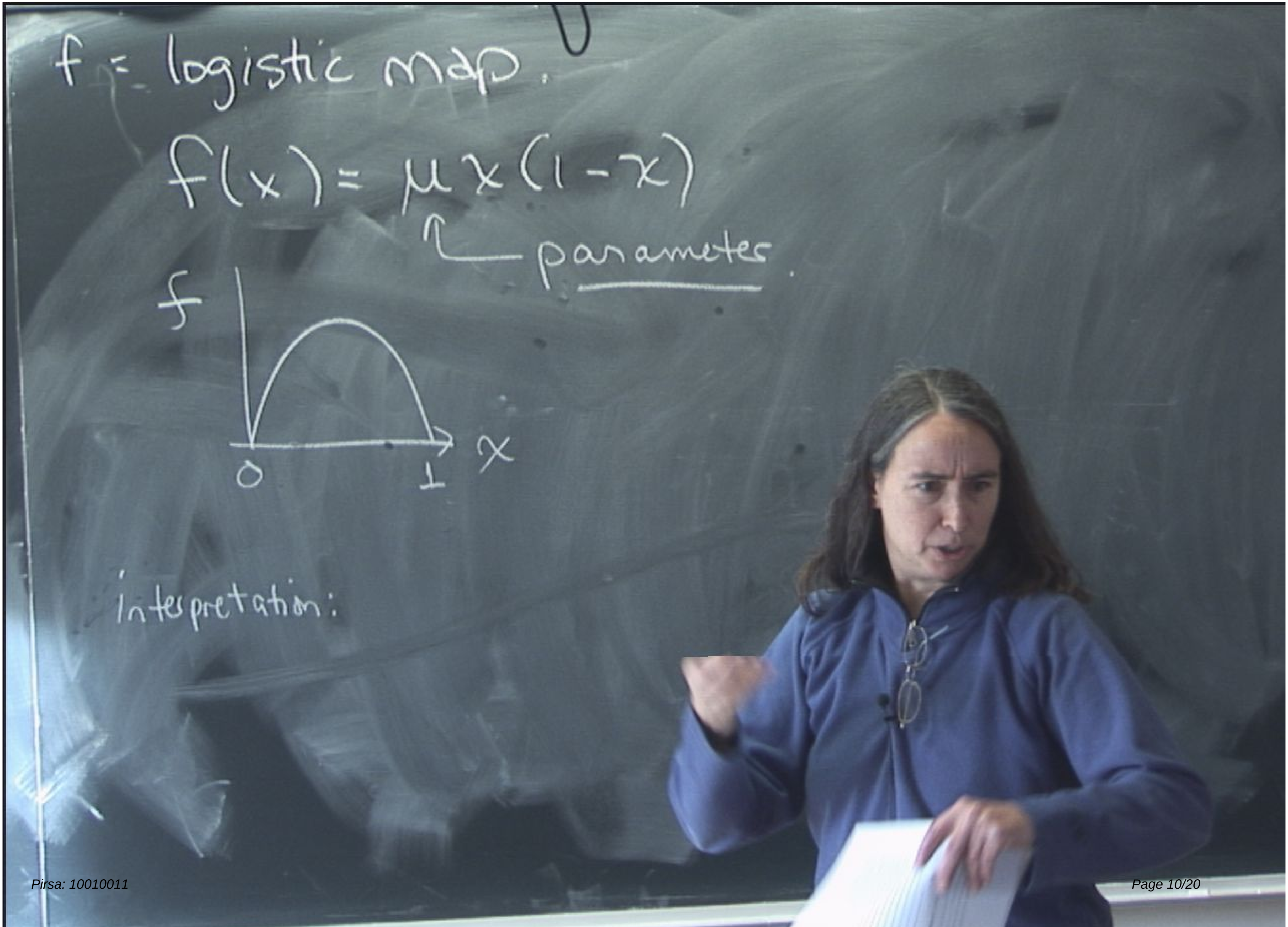
parameter
f
x
0
1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f
x
0
1
interpretation:

f = logistic map.

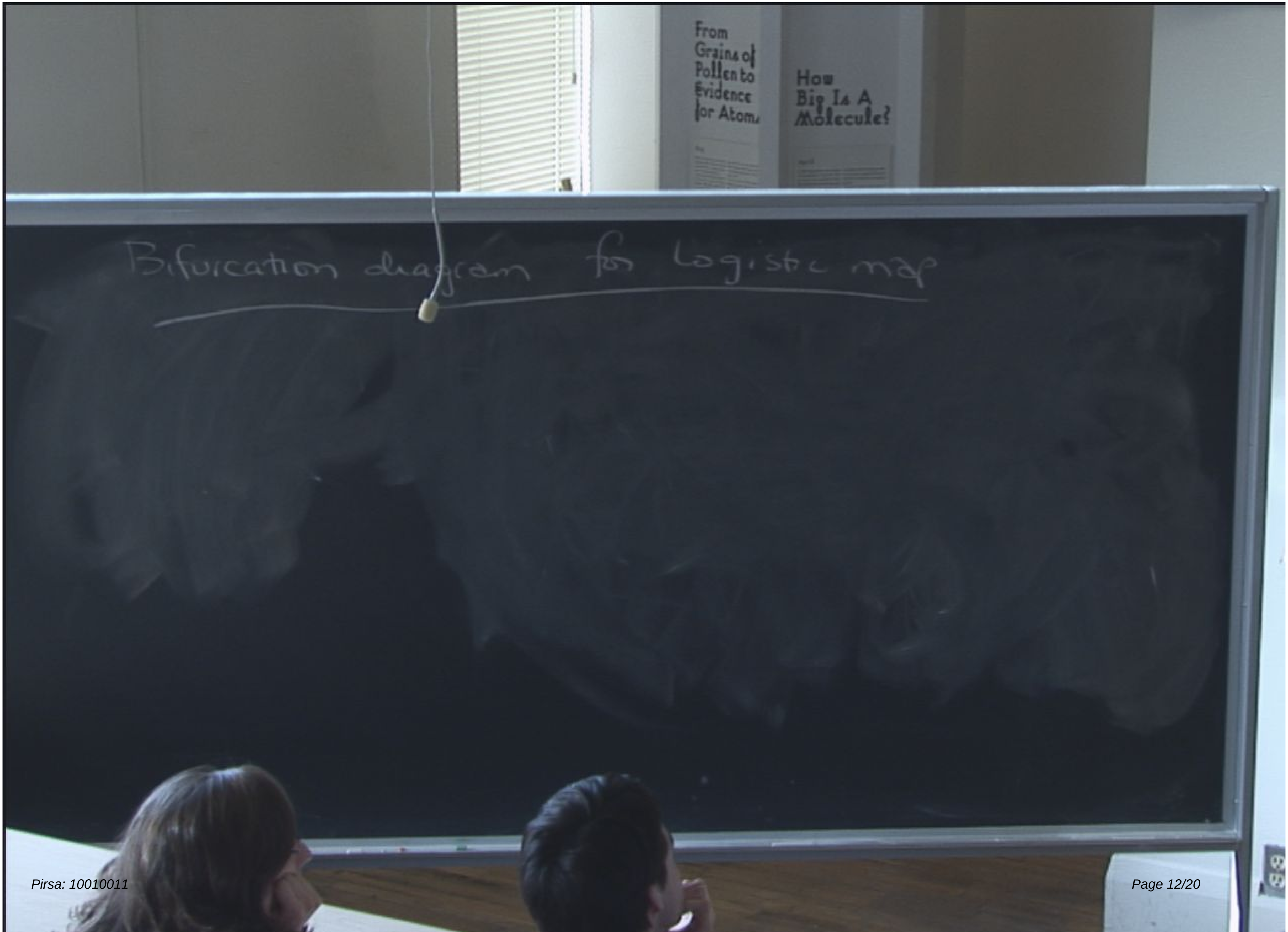
$$f(x) = \mu x(1-x)$$

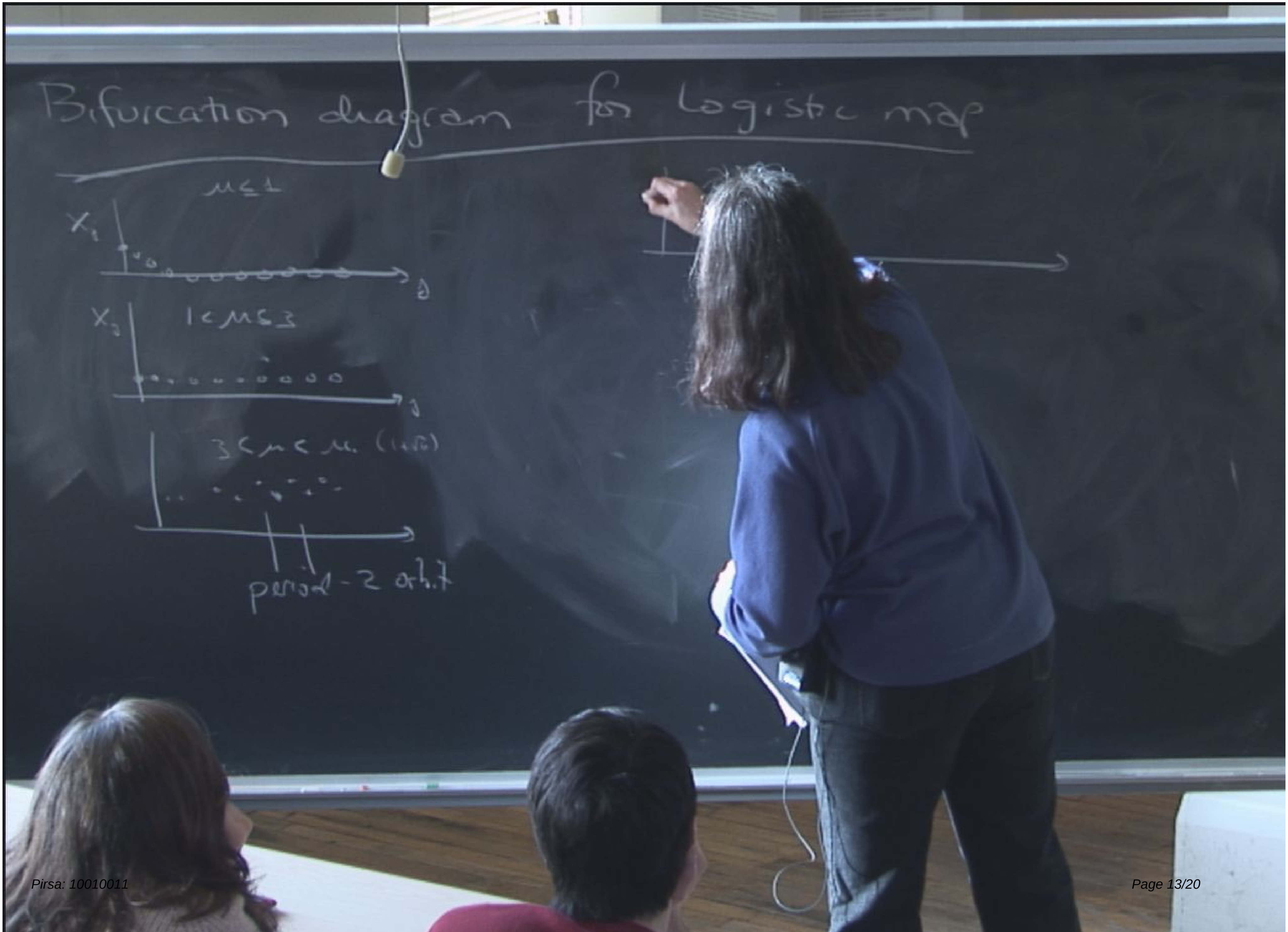
↑ parameter

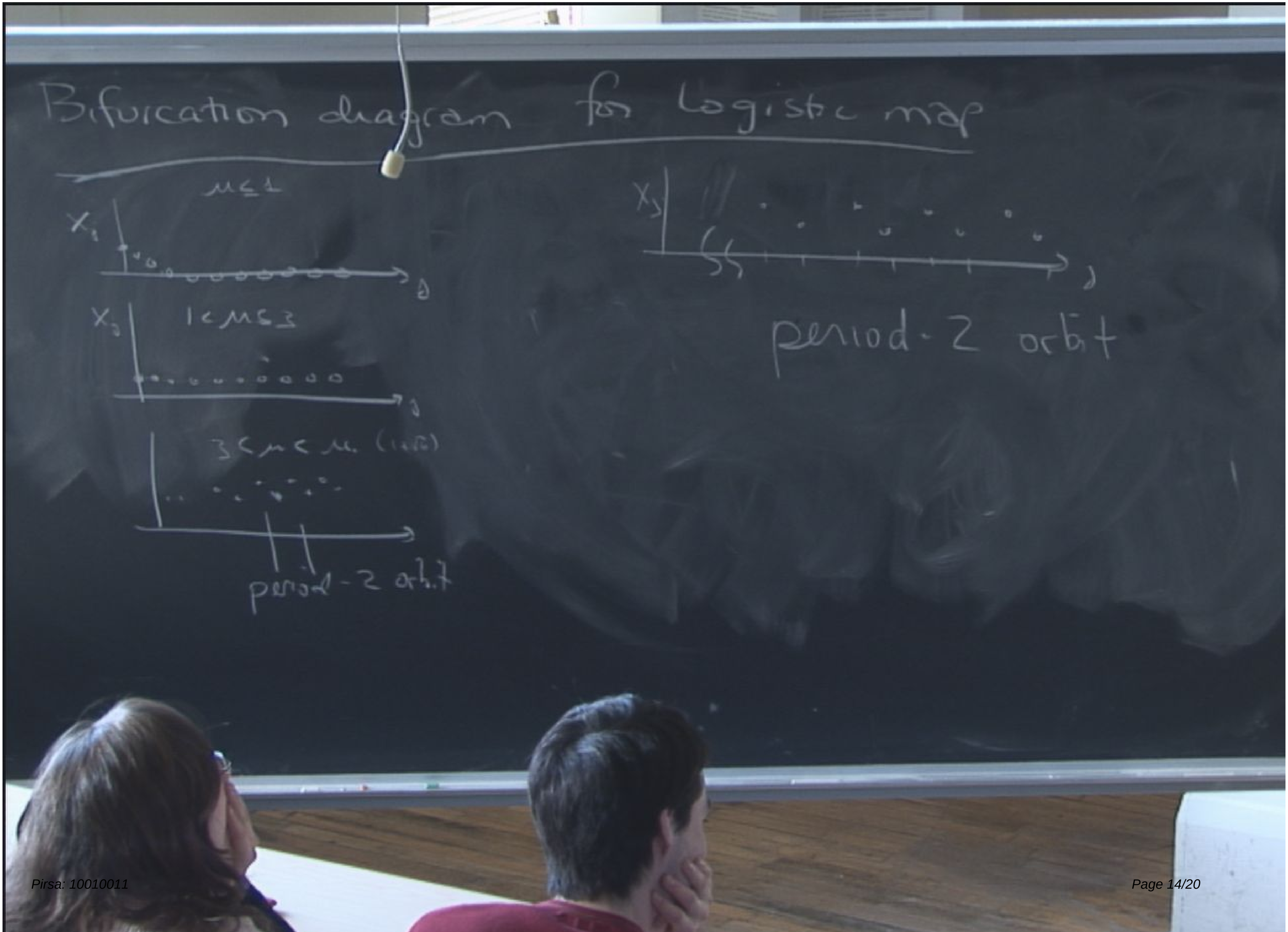
interpretation: x_j = population of generation j / max population

$$x_{j+1} > x_j \text{ if } x \text{ small}$$

$$x_{j+1} < x_j \text{ if } x \text{ large}$$


Bifurcation diagram for logistic map
From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?


Bifurcation diagram for logistic map
μ ≤ 1
1 < μ ≤ 3
period-2 orbit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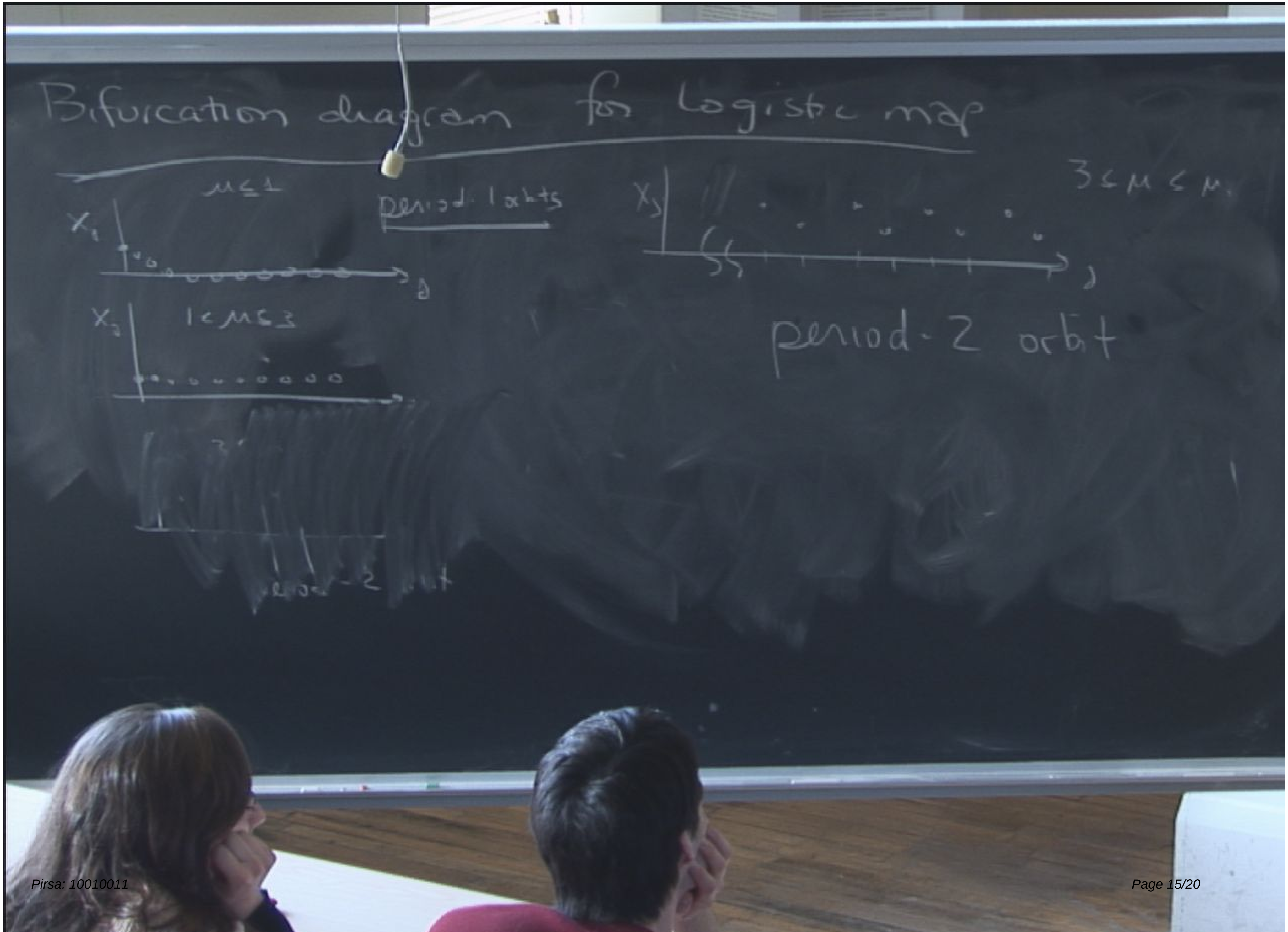
Bifurcation diagram for logistic map
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logistic map:

$$f(x) = \mu x(1-x)$$

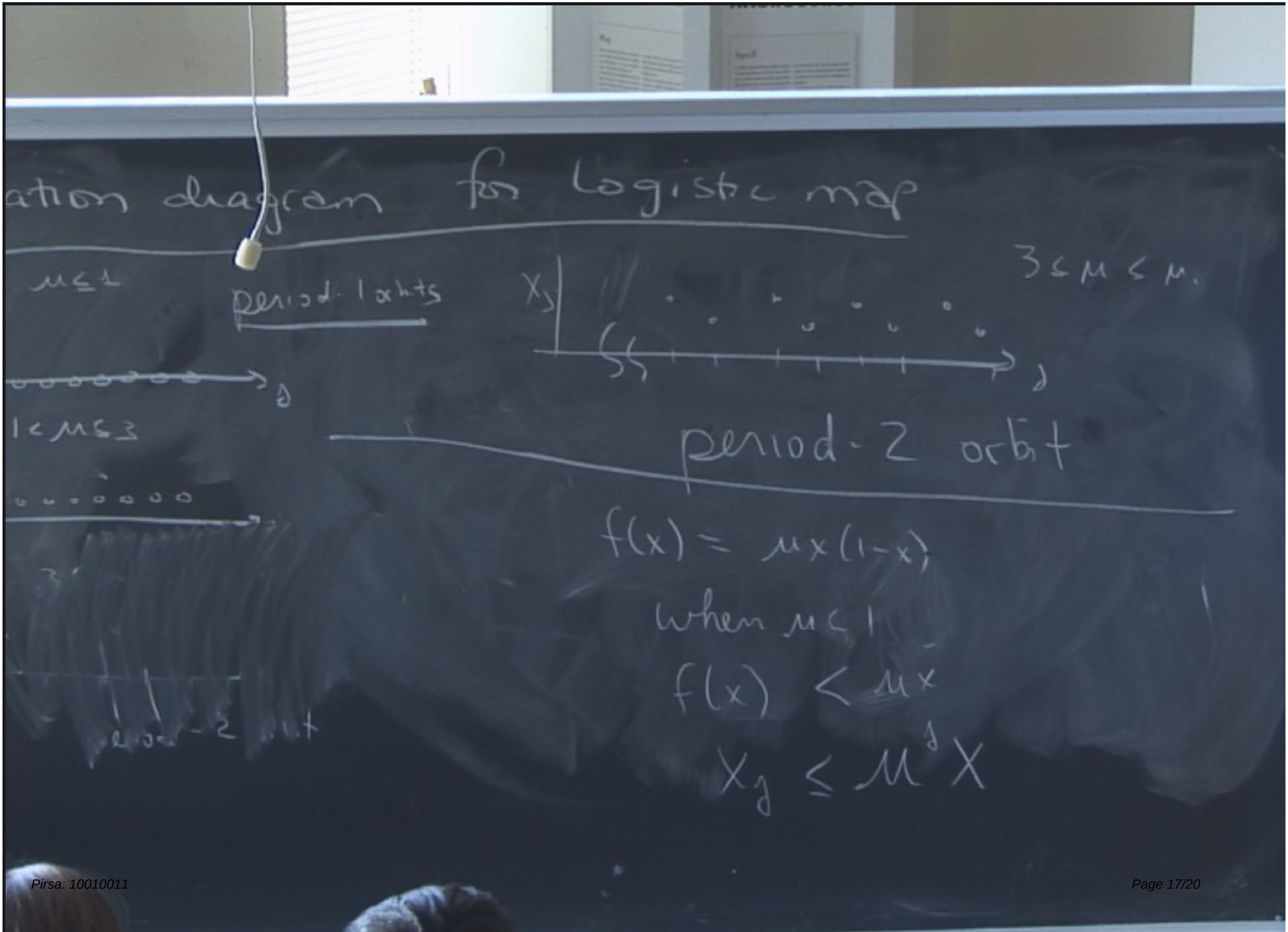
$\uparrow$ parameter

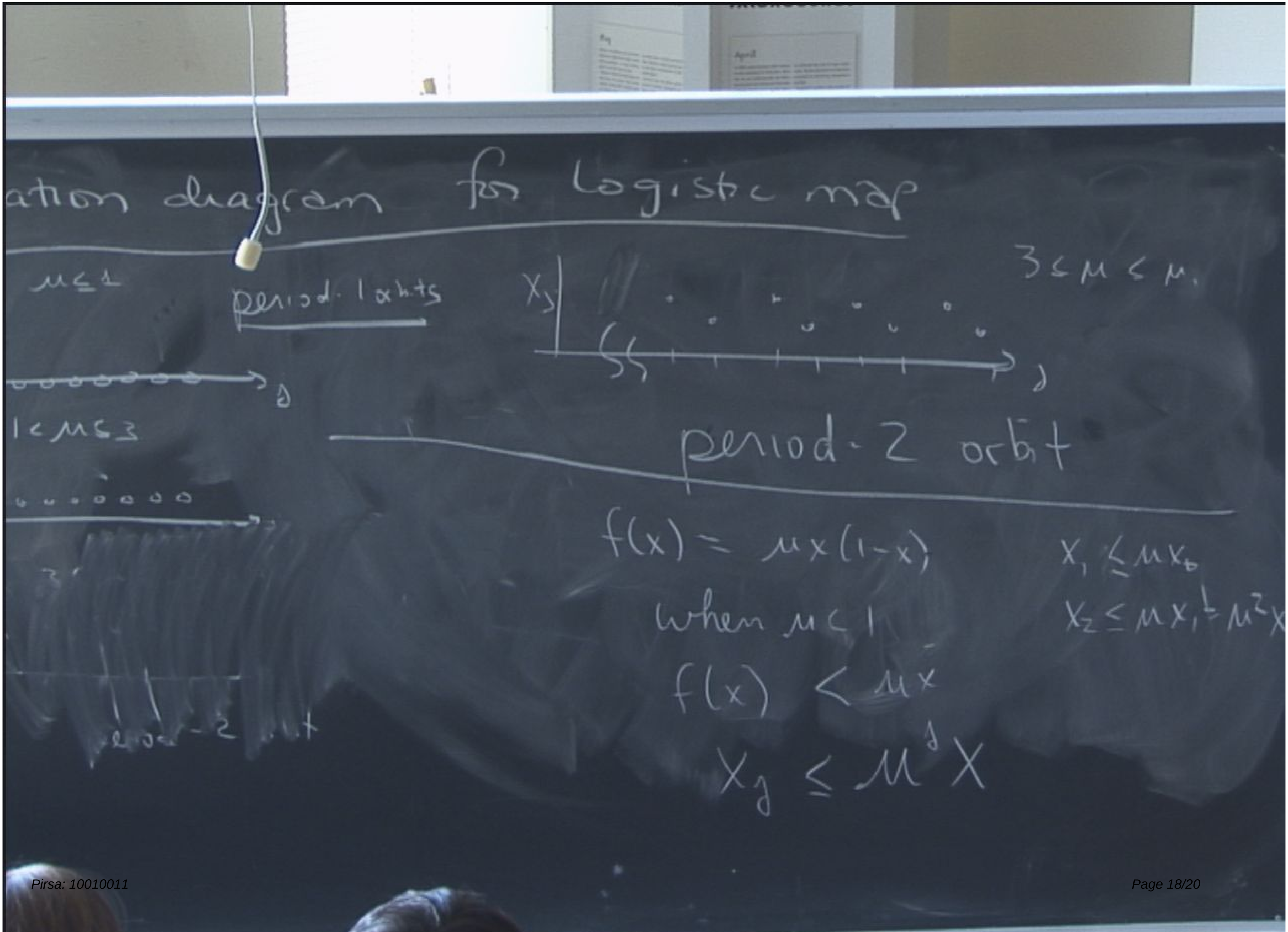
$$X_{n+1} = \tilde{f}(X_n, X_{n-1}, X_{n-2})$$

terpretation: x_j = population of generation j / max population

$x_{j+1} > x_j$ if x small

$x_{j+1} < x_j$ if x large


ation diagram for logistic map
μ ≤ 1
period-1 orbits
1 < μ ≤ 3
3 ≤ μ ≤ μ
period-2 orbit
f(x) = μx(1-x)
when μ ≤ 1
f(x) < μx
x_j ≤ μ^j x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μ ≤ 1
period-1 orbits
1 < μ ≤ 3
3 ≤ μ ≤ μ1
period-2 orbit
f(x) = μx(1-x)
when μ < 1
f(x) < μx
Xj ≤ μ^j X
X1 ≤ μX0
X2 ≤ μX1 ≤ μ^2X

period-2 orbit

$f(x) = \mu x(1-x)$

when $\mu < 1$

$f(x) < \mu x$

$x_j \leq \mu^j x$

$x_1 \leq \mu x_0$

$x_2 \leq \mu x_1 \leq \mu^2 x_0$

fixed point:

$$x = \mu x(1-x)$$

Sol'ns are

$$x = 0$$

x sol'n of $1 = \mu(1-x)$

$$x = 1 - 1/\mu$$