Title: Marginally Relevant Deformations in Lifshitz Holography

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Abstract: The simplest gravity duals for quantum critical theories with 'Lifshitz' scale invariance admit a marginally relevant deformation. We will explore the holographic renormalization of such theories, including this deformation. Additionally we explore how this holographic renormalization illuminates the physics of black holes in the qunatum critical regime.

Marginally Relevant Deformations in Lifshitz Holography

0912.2784, with M. Cheng and S. Hartnoll

Lifshitz Holography: What and Why

- We are interested in gravity solutions whose boundaries have anisotropic scaling between space and time:

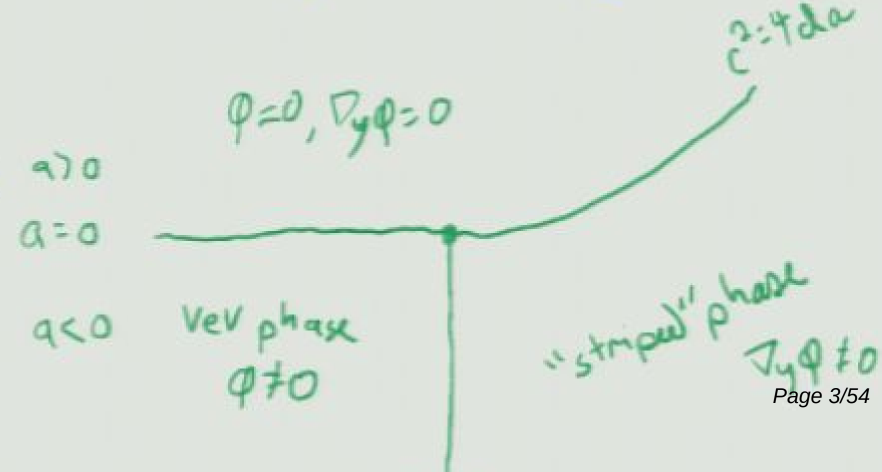
$$ds^2 \sim \ell^2 \left(-\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{dx^2 + dy^2}{r^2} \right)$$

- This scaling provides an interesting variation to the usual AdS-CFT story

- The field theory action

$$\frac{1}{2}\int dx\, dy \left[a\varphi^2 + b(\nabla_x\varphi)^2 + c(\nabla_y\varphi)^2 + d(\nabla_y^2\varphi)^2 + u\varphi^4 \right]$$

has a critical point at a=0, c=0 which exhibits such scaling with z=2, and also has an interesting phase diagram:



Field Theory Dual and Large N

- We are interested in systems where the time direction scales differently than space:

$$\mathcal{L} = \frac{1}{2}\int d\tau d^2x \left((\partial_\tau \phi)^2 + K(\nabla^2\phi)^2\right)$$

- To get large N, consider the dual action under $\star d\phi = d\mathbf{A}$:

$$\mathcal{L} = \frac{1}{2}\int d\tau d^2x \left(|\mathbf{B}|^2 + K|\nabla \times \mathbf{E}|^2\right)$$

- Lorentz invariance is explicitly broken

- The U(1) theory could be extended to SU(N)

Einstein-Proca Action

- Not talking about Hořava-Lifshitz gravity; instead, we use regular Einstein gravity and add matter to support solutions with the desired asymptotic scaling

- Action is Einstein gravity plus a massive vector field:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} \left[R + \frac{10}{\ell^2} \right] - \frac{1}{g^2} \left[\frac{1}{4} F^2 + \frac{2}{\ell^2} A^2 \right] \right)$$

- AdS is obviously a solution, with the vector field turned off

- There is also a solution with Lifshitz symmetry at z=2:

$$ds^2_{\rm Lif} = \ell^2 \left(-\frac{dt^2}{r^4} + \frac{dr^2}{r^2} + \frac{dx^2 + dy^2}{r^2} \right), \; A = \frac{g}{\sqrt{2}} \frac{\ell}{\kappa} \frac{dt}{r^2}.$$

- Simplest action which includes both AdS and Lifshitz solutions

Simplifying Isotropic Ansatz

- We work with the ansatz

$$
\begin{aligned}
ds^2 &= \ell^2 \left(-f(r)dt^2 + \frac{dr^2}{r^2} + p(r)(dx^2 + dy^2) \right) \\
A &= \frac{\ell}{\kappa} g\, h(r) dt \,.
\end{aligned}
$$

- Includes both AdS and Lifshitz fixed points (and no others)
- Action is tuned to produce Lifshitz scaling with z=2
- Can numerically find solutions which interpolate between AdS and Lifshitz
- Chargeless black branes can be described analytically; they asymptote to AdS

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Lifshitz Asymptotics

- Linearize equations of motion around the Lifshitz fixed point
- There are three solutions, two of which produce r^4 falloffs and one constant mode.
- Going beyond linearization, we see this constant mode becomes marginally relevant. Specifically, consider:

$$
\begin{aligned}
-\frac{\partial \rho}{\partial r} = q \;&=\; -2\left(1-\frac{1}{\log(\Lambda r)}-\frac{(1+\lambda)-5\log(-\log(\Lambda r))}{2\log(\Lambda r)^2}+\cdots\right)\\
&-\;\frac{2\sqrt{2}}{3}(\Lambda r)^4\log^2(\Lambda r)\left(\beta\left(1+\frac{-\frac{4}{3}+5\log(-\log(\Lambda r))}{\log(\Lambda r)}+\cdots\right)+\alpha\left(\frac{1}{\log(\Lambda r)}+\cdots\right)\right)+\mathcal{O}(r^8)
\end{aligned}
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- Λ, α, and β are the 3 parameters corresponding to the three solutions at the linearized level. λ appears due to breaking of scale invariance, and is just a symmetry transformation:

$$q(\Lambda r;\alpha,\beta;\lambda) = q\left(e^{\lambda'/2}\Lambda r; e^{-2\lambda'}(\alpha-\lambda'\beta), e^{-2\lambda'}\beta;\lambda+\lambda'\right)$$

Consequently, we fix λ by setting it to zero.

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Exploring the Asymptotics

Integrating the expression for p on the previous slide, we find

$$\frac{1}{\ell^2} g_{xx}(\rho) \;=\; P_0^2 \frac{\log^2(\Lambda r)}{(\Lambda r)^2}\left(1 + \frac{4 + 5\log(-\log(\Lambda r))}{\log(\Lambda r)} + \cdots\right)$$

When we do not set Λ =0, it appears the deviation from Lifshitz asymptotics grows as we approach the boundary of the spacetime.

However we will be able to explore holographic renormalization anyways:

- Introduce sliding scale $\mu \gg \Lambda$ such that $\mu r \sim 1$. for the energy scale we want to explore.
- Expand in $\left|\frac{\log \mu r}{\log \frac{\mu}{\Lambda}}\right| \ll 1$
- After rescaling t and x, in the regime where this double expansion is valid we can write

$$\frac{1}{\ell^2} g_{xx}(\rho) \;=\; \frac{1}{r^2}\left(1 - 2\frac{\log(\mu r)}{\log(\frac{\mu}{\Lambda})} + \cdots\right)$$

$$\log \Lambda r = \log \frac{\Lambda}{\mu} + \log \mu r$$

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New Counterterms

Previous counterterms did not allow for the marginal mode. (Saremi+Ross, Peet et al) Good counter terms should have

- finite action, energy
- well-defined variational principle

We can write such counterterms provided we allow the counterterms themselves to depend on $\log \Lambda r$. In fact we need only three counterterms:

$$\int_\partial \sqrt{-\gamma}\, \rho \left[c_0 + c_1\left(-\frac{\kappa^2}{g^2}A^2 - \frac{1}{2}\right) + c_2\left(-\frac{\kappa^2}{g^2}A^2 - \frac{1}{2}\right)^2 \right]$$

However the c_i are each an infinite series, e.g.

$$c_1 = 2 + \frac{16}{3\log(r\Lambda)} + \frac{-91 - 120\log(-\log(r\Lambda))}{9\log(r\Lambda)^2} + \cdots .$$

There is actually an ambiguity here, but we make the minimal choice, which will not affect the value of the energy or the action.

Defining the Energy

- Naively we define the stress-energy tensor via:

$$\delta \mathcal{F} = \frac{\sqrt{\gamma}}{2} \tau^{ab} \delta \gamma_{ab} + \mathcal{J}^a \delta A_a$$

- In the presence of non-scalar matter we must work in the vielbein frame; thus we define the stress-energy tensor as:

$$\delta \mathcal{F} = \sqrt{\gamma}\, \mathcal{T}^a{}_{\hat{a}}\, \delta e^{\hat{a}}_a + \mathcal{J}^{\hat{a}} \delta A_{\hat{a}} \quad , \quad \mathcal{T}^{ab} = \mathcal{T}^a{}_{\hat{a}} e^{b\hat{a}}$$

- The energy density thus becomes

$$\mathcal{E} = \sqrt{\sigma}\, k_a\, \xi_b\, \mathcal{T}^{ab} = \sqrt{\gamma}\, \tau^t{}_t + \mathcal{J}^t A_t$$

- $\mathcal{J}^t$ is not a conserved current because our vector is massive

Energy, Action, and J

Armed with our new counterterms we can now write expressions for the action, energy, and J near the Lifshitz fixed point, in terms of our expansion parameters.

$$\mathcal{F} = \left(\frac{\ell}{\kappa}\right)^2 \frac{\sqrt{2}}{9}(-5\beta + 6\alpha) \ ,$$

$$\mathcal{E} = -\left(\frac{\ell}{\kappa}\right)^2 \frac{\sqrt{2}}{9}(7\beta + 6\alpha) \ ,$$

$$\mathcal{J}^{\hat{t}} = \frac{1}{g}\frac{\ell}{\kappa}\left(\frac{27}{703} + 2a\right)\beta \ .$$

- All three quantities are zero at the fixed point
- The "a" present in J is due to the counterterm ambiguity
- Additionally we find another combination of functions which is r-invariant:

$$K = -\frac{1}{2}\sqrt{f}p\Big(-q + m + k\,x\Big) = -\frac{2\sqrt{2}}{9}(\beta + 6\alpha)$$

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Black Holes

- Begin with near horizon ansatz:

$$
\begin{aligned}
f(r) &= f_0\left(\left(1-\frac{r}{r_+}\right)^2+\left(1-\frac{r}{r_+}\right)^3+\frac{11+16h_0^2}{12}\left(1-\frac{r}{r_+}\right)^4+\cdots\right), \\
p(r) &= p_0\left(1+\frac{5-2h_0^2}{2}\left(1-\frac{r}{r_+}\right)^2+\frac{5-2h_0^2}{2}\left(1-\frac{r}{r_+}\right)^3+\cdots\right), \\
h(r) &= \sqrt{f_0}\left(h_0\left(1-\frac{r}{r_+}\right)+h_0\left(1-\frac{r}{r_+}\right)^2+\cdots\right).
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- f and h have double zeros at the horizon r_+, where p becomes a constant. h_0 is a free parameter.

- We work in the unitless quantity $\frac{r}{r_+}$ so we will need to consider unitless physical quantities

- f_0 and p_0 divide out of the diff. eqs. when we numerically integrate. We will fix them via matching to asymptotics

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Physical Quantities

- We now calculate the temperature, entropy density, and flux density:

$$
\begin{aligned}
T &= \frac{r_+}{2\pi}\sqrt{\frac{1}{2}\frac{d^2 f}{dr^2}}\Big|_{r=r_+}\,, \\
s &= 2\pi\left(\frac{\ell}{\kappa}\right)^2 p(r_+)\,, \\
\phi &= \frac{\ell g r_+}{\kappa}\left(\frac{p}{\sqrt{f}}\frac{dh}{dr}\right)\Big|_{r=r_+}\,.
\end{aligned}
$$

- Our r-invariant quantity K satisfies

$$K = \sqrt{f_0}p_0 = Ts\left(\frac{\kappa}{\ell}\right)^2$$

- Using expansions for K, E, F in terms of α and β , we find

$$\mathcal{F} = \mathcal{E} - Ts$$

Expected Relations Near Λ =0

- Since $\mathcal{F} = -\mathcal{P}$ and d$\mathcal{P}$=zε with d=z=2 at Λ=0, we expect

$$\mathcal{F}_0 = -\mathcal{E}_0.$$

- Combining with the first law we thus expect

$$\mathcal{E}_0 = -\mathcal{F}_0 = \frac{1}{2} T s_0$$

- Additionally we expect

$$\mathcal{J}_0^{\hat{t}} = 0.$$

Graphing the Physical Quantities

- Integrate black brane ansatz outward numerically, choosing $h_0 < 0.9714$.
- Match solution near boundary to asymptotics to set f_0, ρ_0, and read off Λ.
- Picking $h_0 \approx 0.9714$ results in $\Lambda = 0$.
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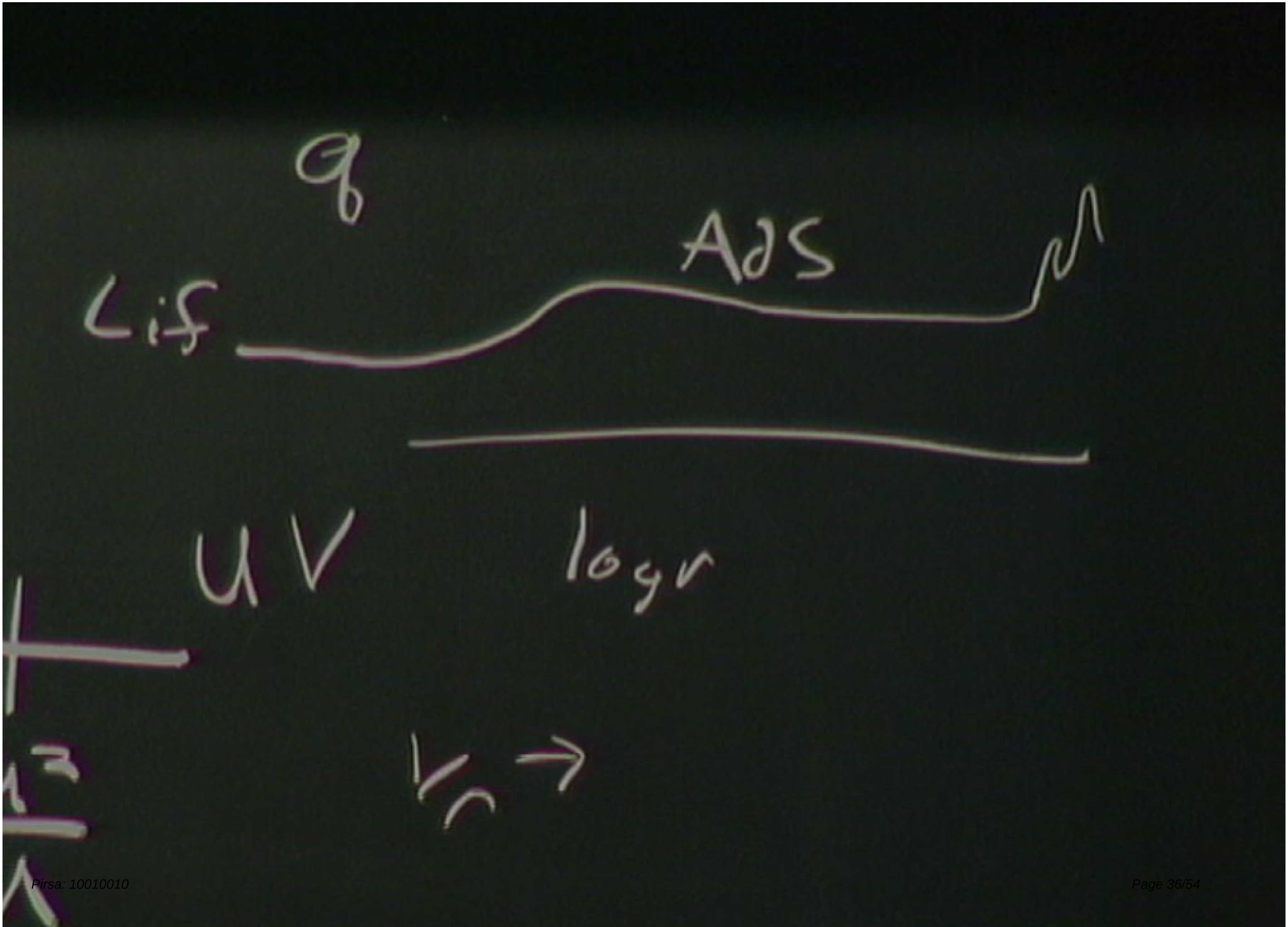
q
AdS
Lif
UV
log r

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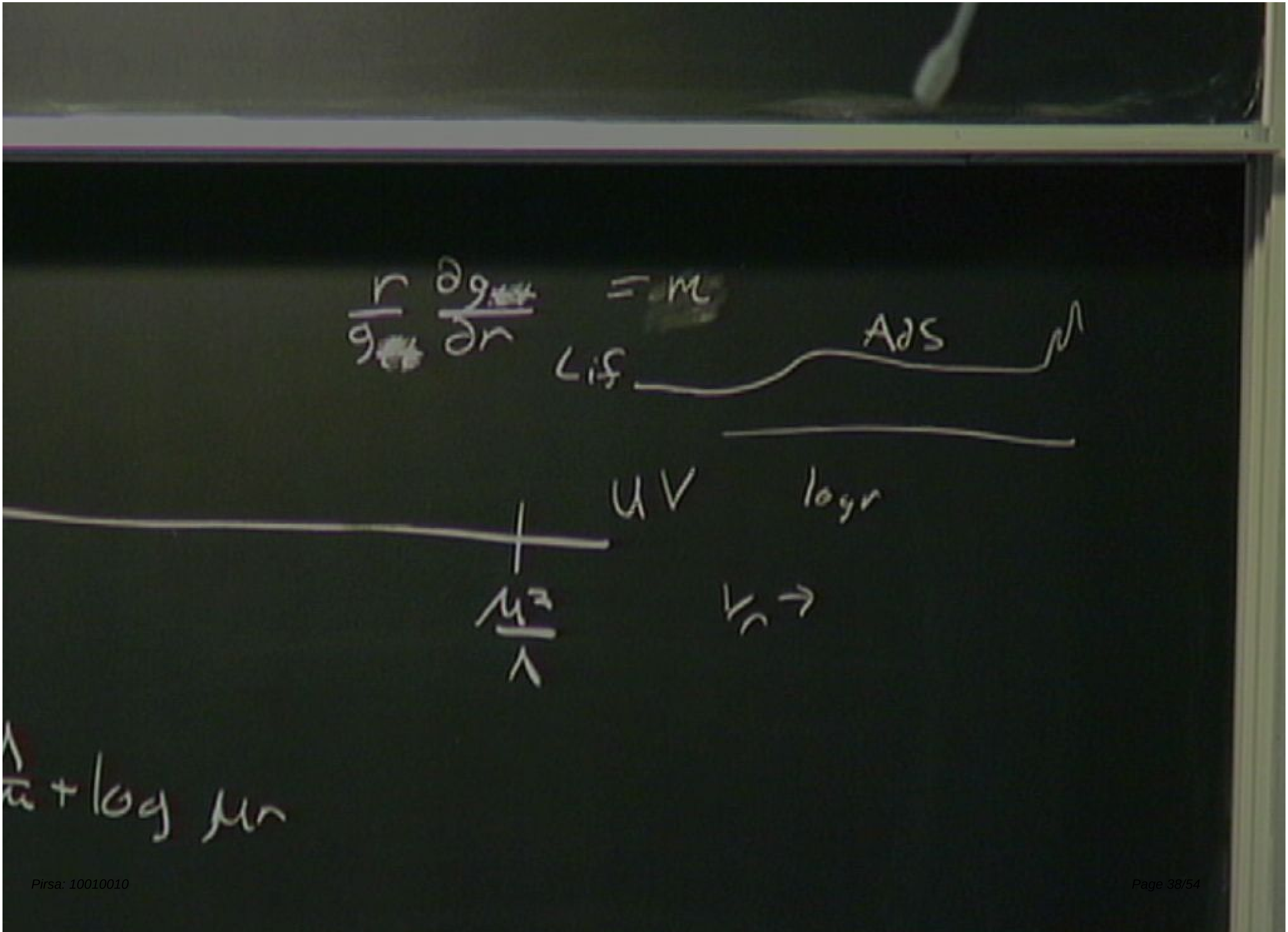
r/g ∂g/∂r = m
AdS
Lif
UV
log r
μ²/Λ
+ log μr

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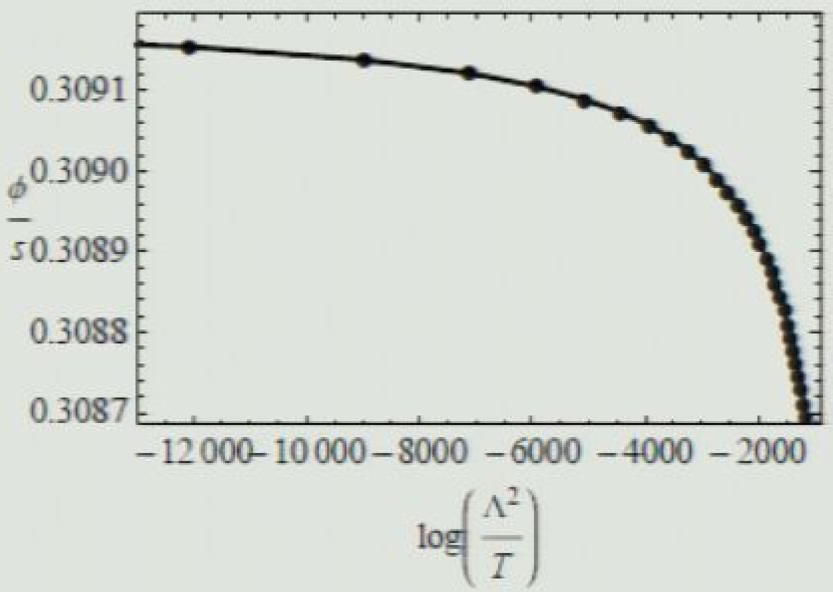

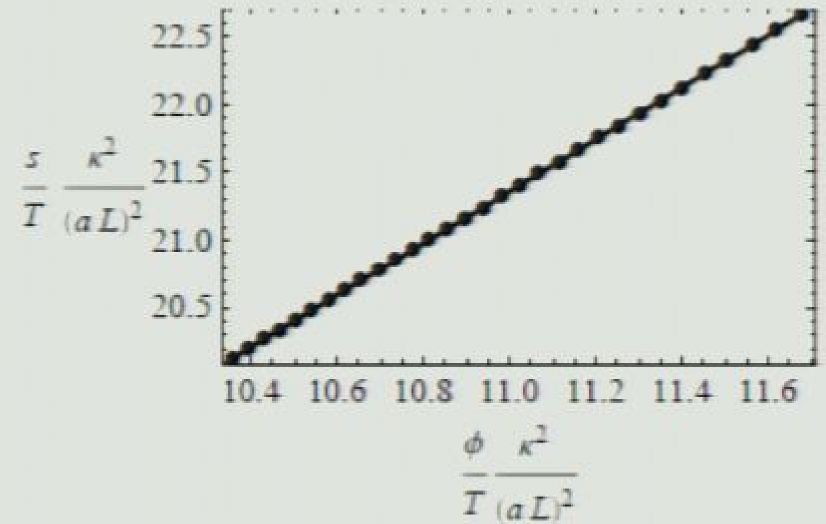

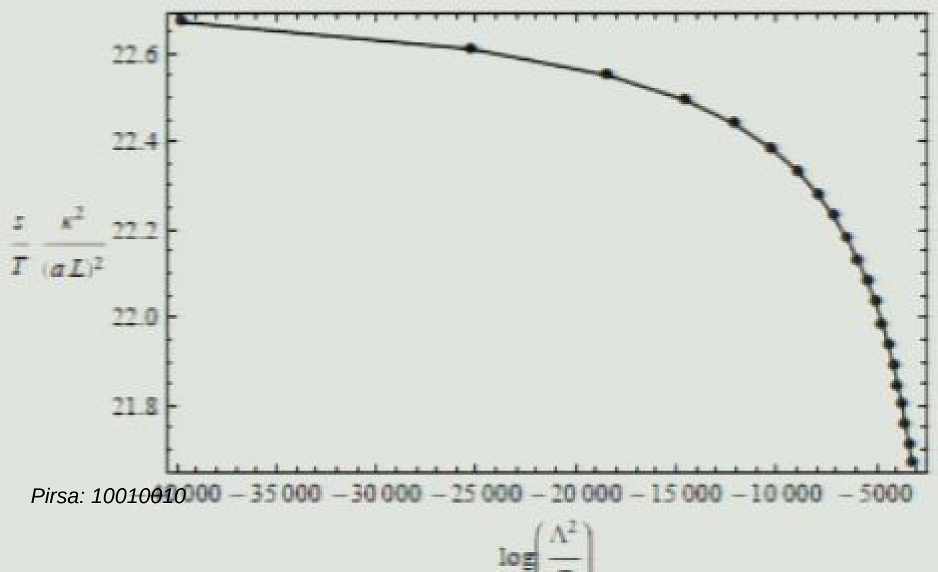

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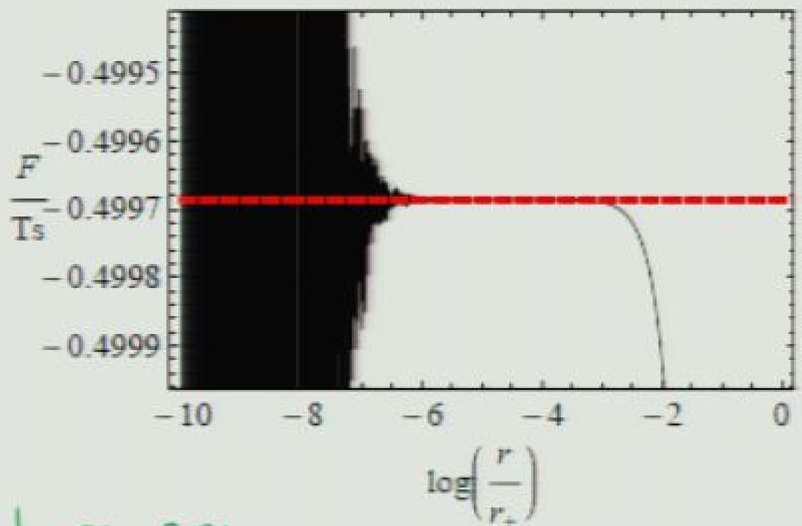
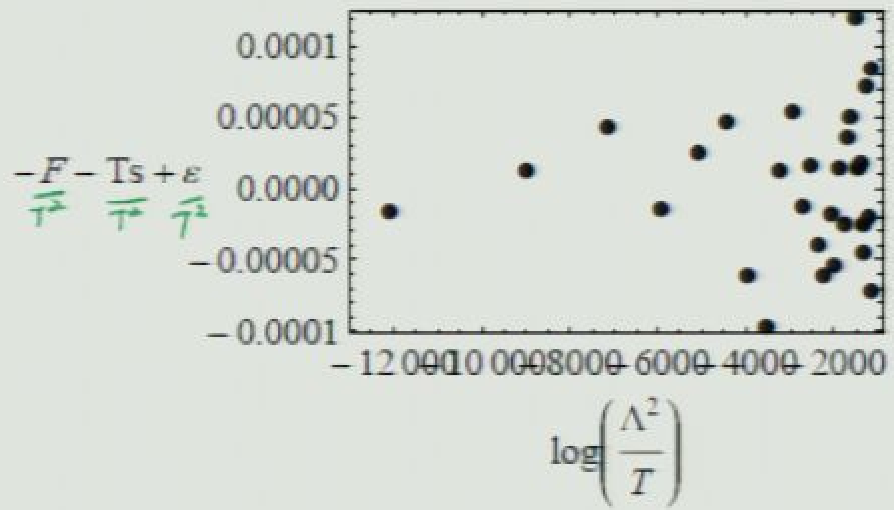
Finiteness + First Law Check



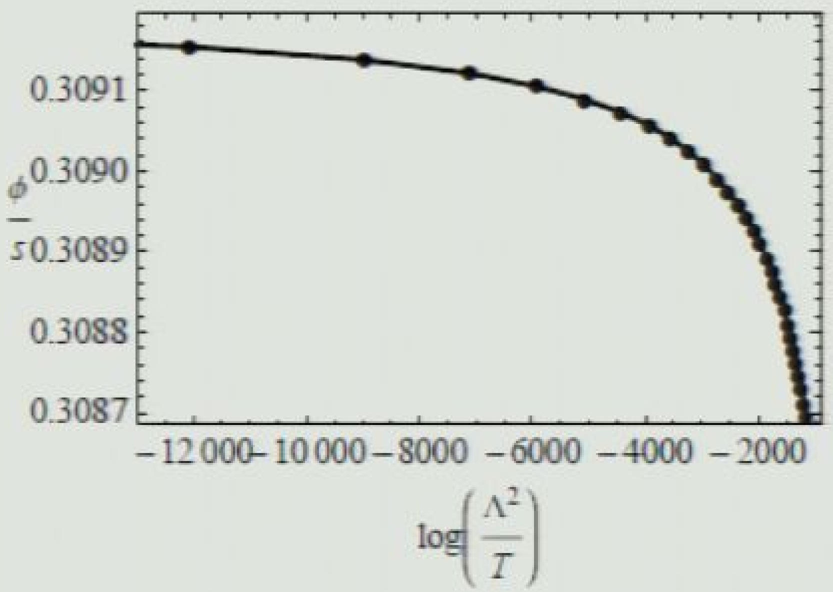


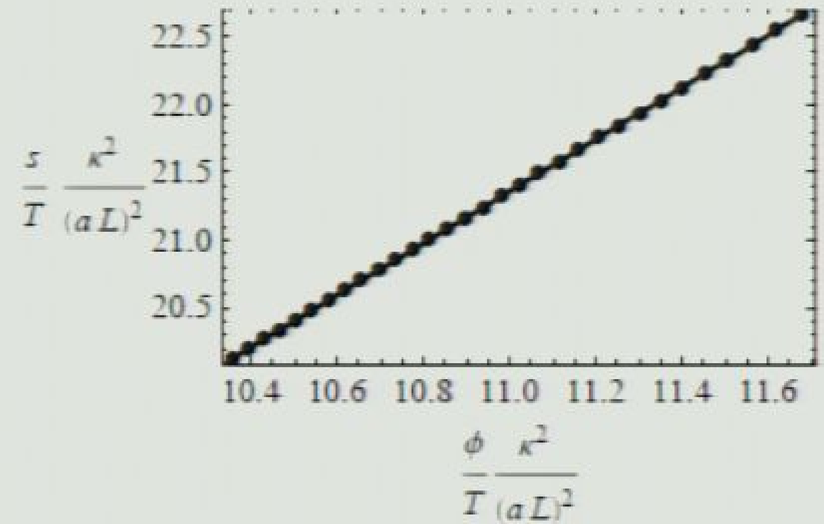
$h_0 \approx .971$, $\log \Lambda^2/T = -3210$

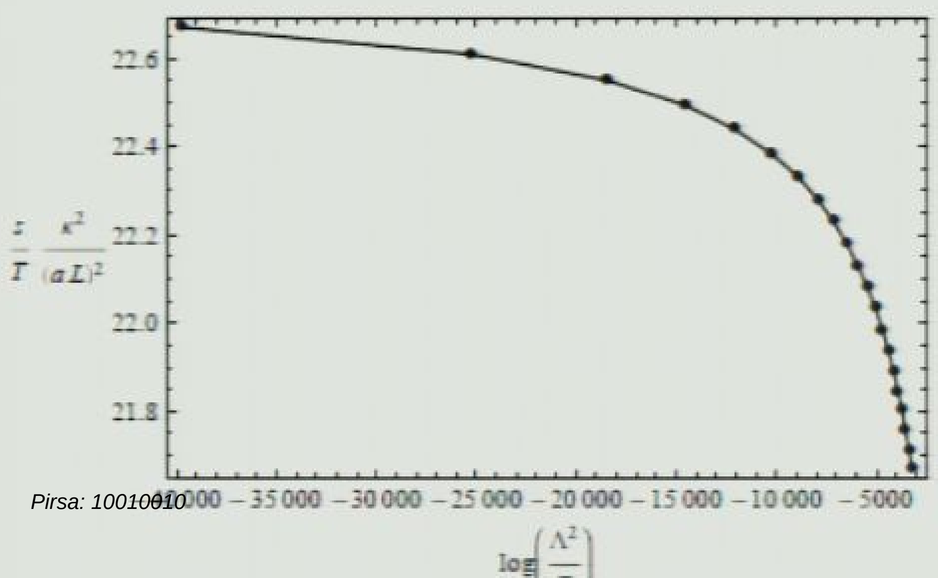
10 counterterm orders

each is c. 20, so this is 1 part in 10^5. not bad!

$\Phi + S$







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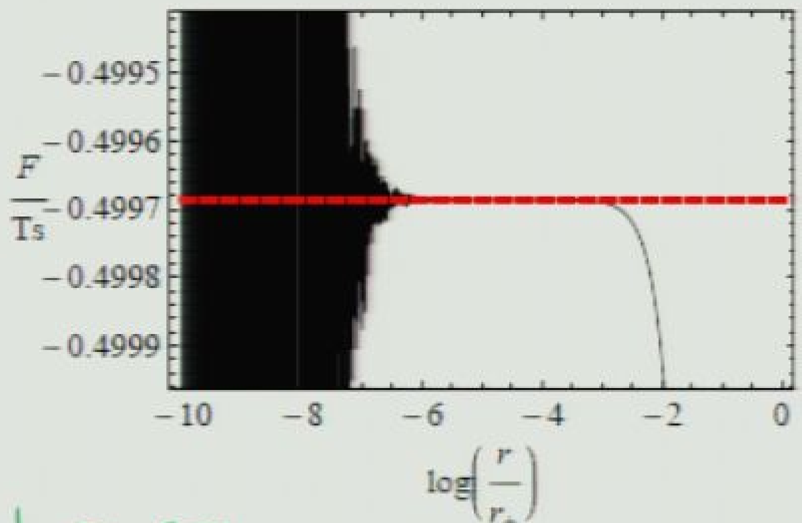

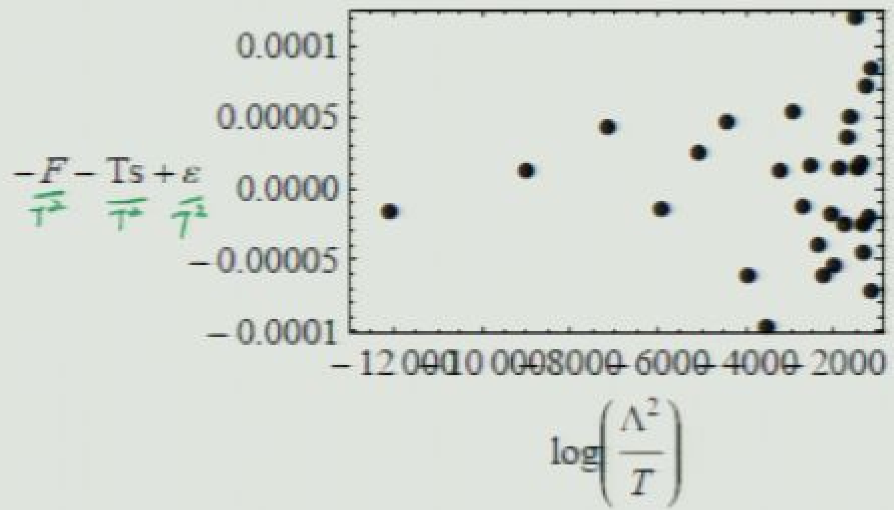

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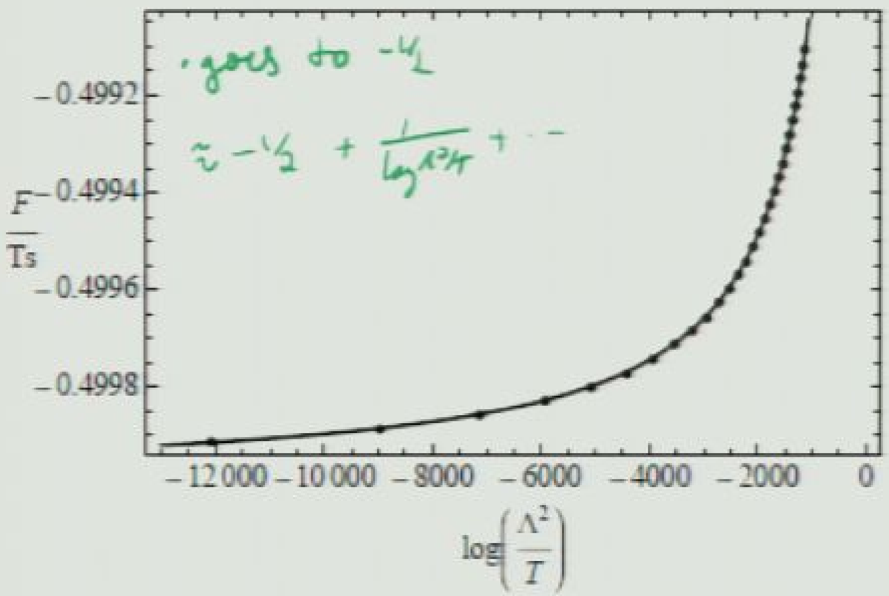
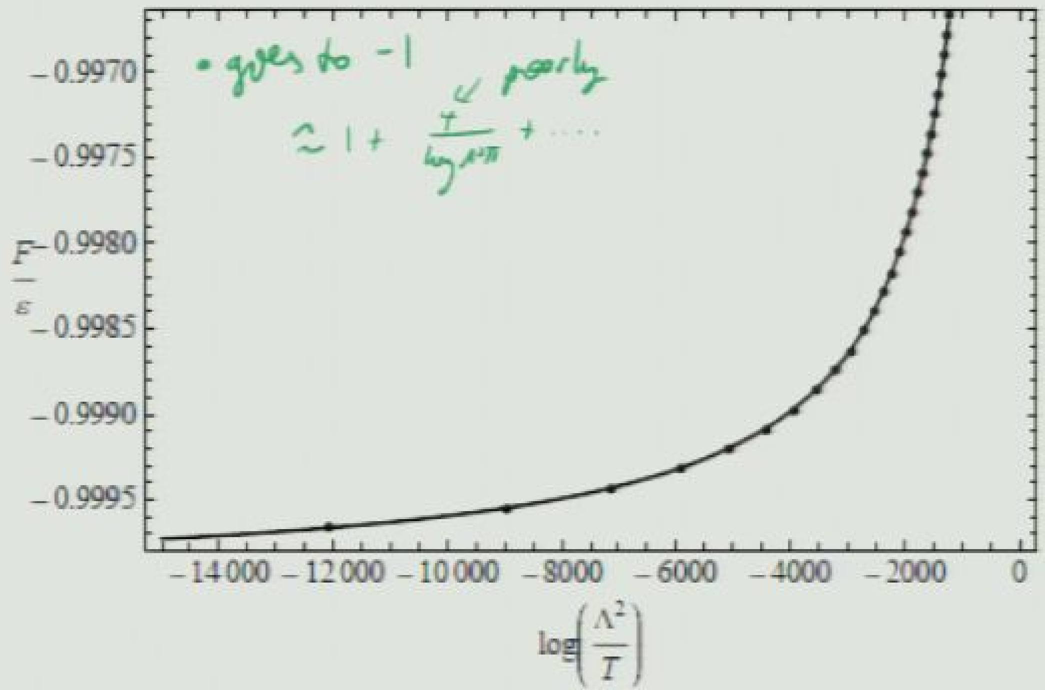
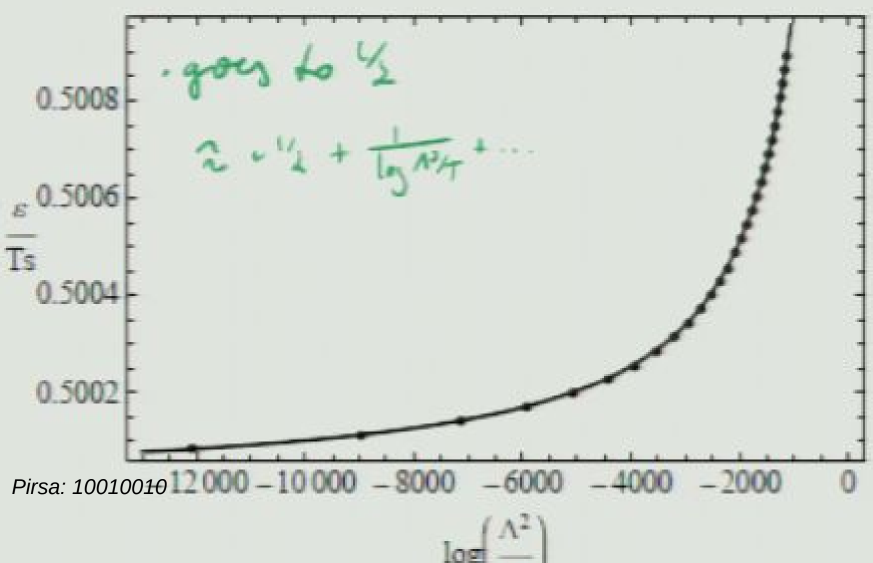
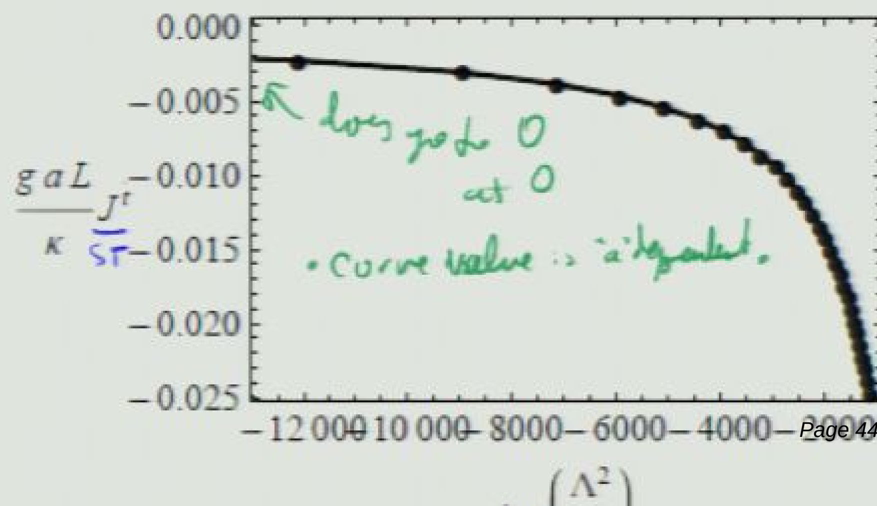
F, E, and $J^{\hat{t}}$

(h_0 = .9712 to .9698)









Conclusions

- One can control a marginally relevant mode holographically
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In the future we would like to:

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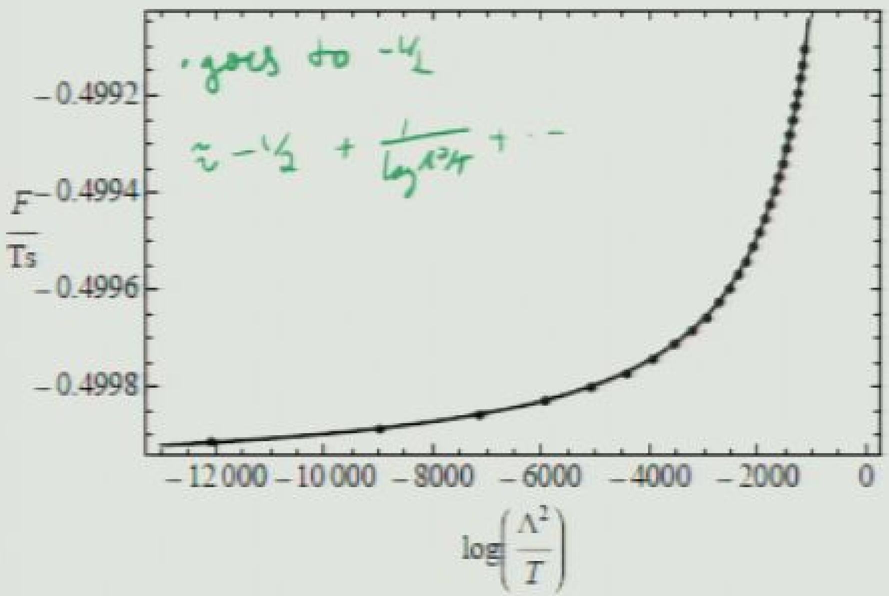

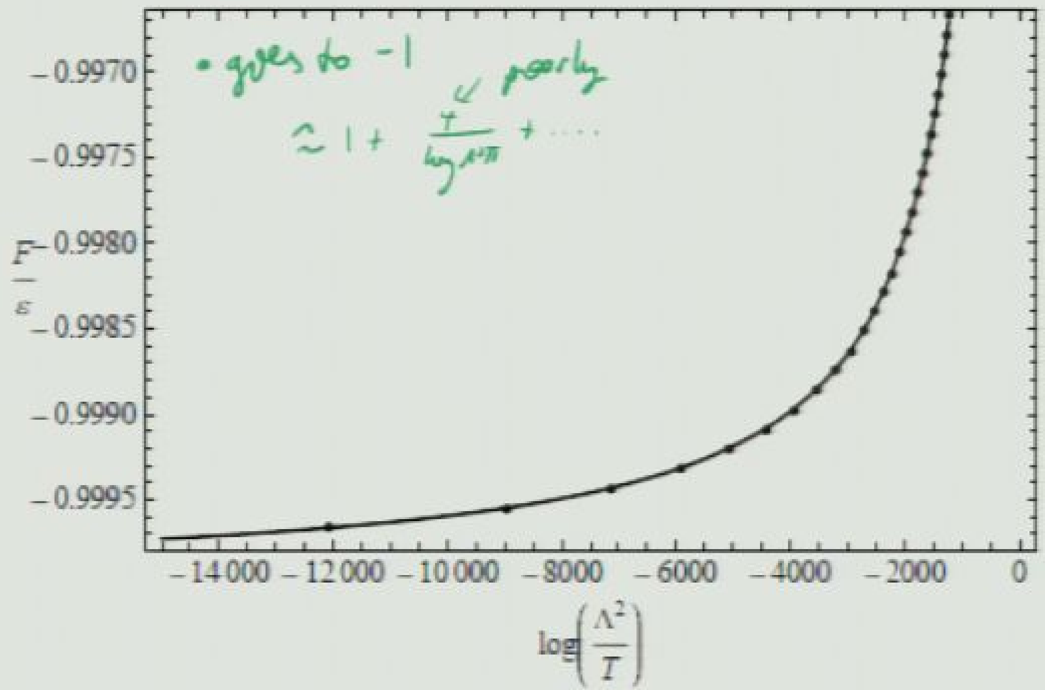

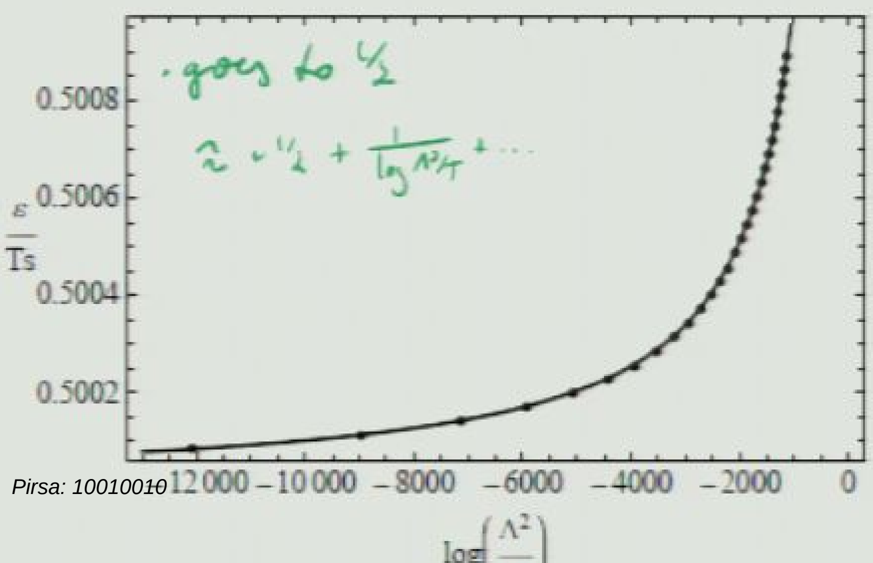

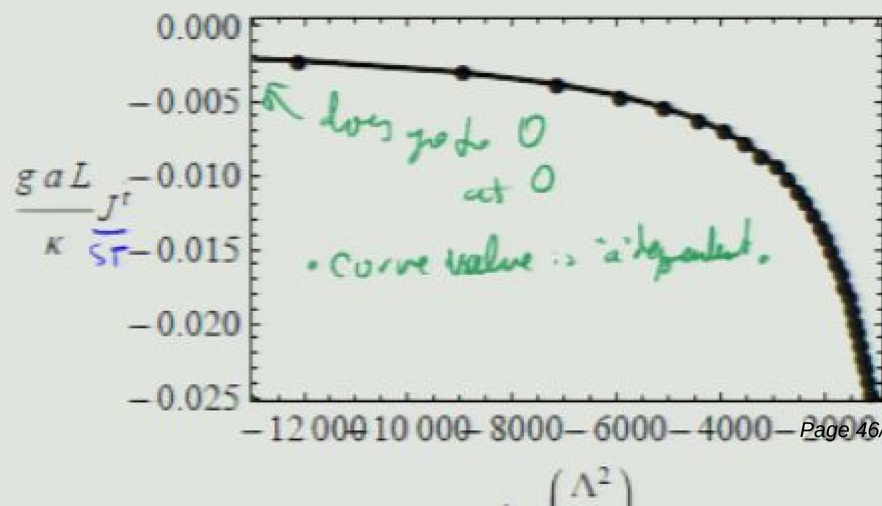

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Energy, Action, and J

Armed with our new counterterms we can now write expressions for the action, energy, and J near the Lifshitz fixed point, in terms of our expansion parameters.

$$
\begin{aligned}
\mathcal{F} &= \left(\frac{\ell}{\kappa}\right)^2 \frac{\sqrt{2}}{9}(-5\beta + 6\alpha)\ , \\
\mathcal{E} &= -\left(\frac{\ell}{\kappa}\right)^2 \frac{\sqrt{2}}{9}(7\beta + 6\alpha)\ , \\
\mathcal{J}^{\hat{t}} &= \frac{1}{g}\frac{\ell}{\kappa}\left(\frac{27}{703} + 2a\right)\beta\ .
\end{aligned}
$$

- All three quantities are zero at the fixed point
- The "a" present in J is due to the counterterm ambiguity
- Additionally we find another combination of functions which is r-invariant:

$$
K = -\frac{1}{2}\sqrt{f}p\Big(-q + m + k\,x\Big) = -\frac{2\sqrt{2}}{9}(\beta + 6\alpha)
$$

Simplifying Isotropic Ansatz

- We work with the ansatz

$$
\begin{aligned}
ds^2 &= \ell^2 \left(-f(r)dt^2 + \frac{dr^2}{r^2} + p(r)(dx^2 + dy^2) \right) \\
A &= \frac{\ell}{\kappa} g\, h(r) dt\,.
\end{aligned}
$$

- Includes both AdS and Lifshitz fixed points (and no others)
- Action is tuned to produce Lifshitz scaling with z=2
- Can numerically find solutions which interpolate between AdS and Lifshitz
- Chargeless black branes can be described analytically; they asymptote to AdS

Marginally Relevant Deformations in Lifshitz Holography

0912.2784, with M. Cheng and S. Hartnoll

Lifshitz Holography: What and Why

- We are interested in gravity solutions whose boundaries have anisotropic scaling between space and time:

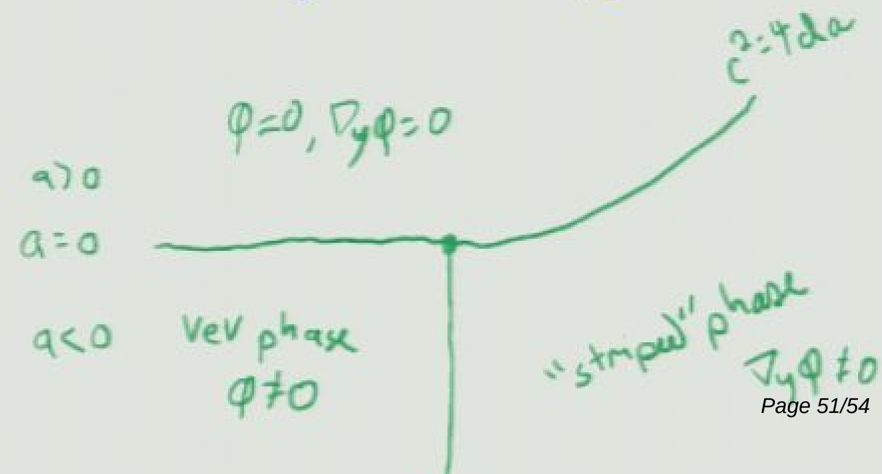
$$ds^2 \sim \ell^2 \left(-\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{dx^2 + dy^2}{r^2} \right)$$

- This scaling provides an interesting variation to the usual AdS-CFT story

- The field theory action

$$\frac{1}{2}\int dx\, dy \left[a\varphi^2 + b(\nabla_x \varphi)^2 + c(\nabla_y \varphi)^2 + d(\nabla_y^2 \varphi)^2 + u\varphi^4 \right]$$

has a critical point at a=0, c=0 which exhibits such scaling with z=2, and also has an interesting phase diagram:



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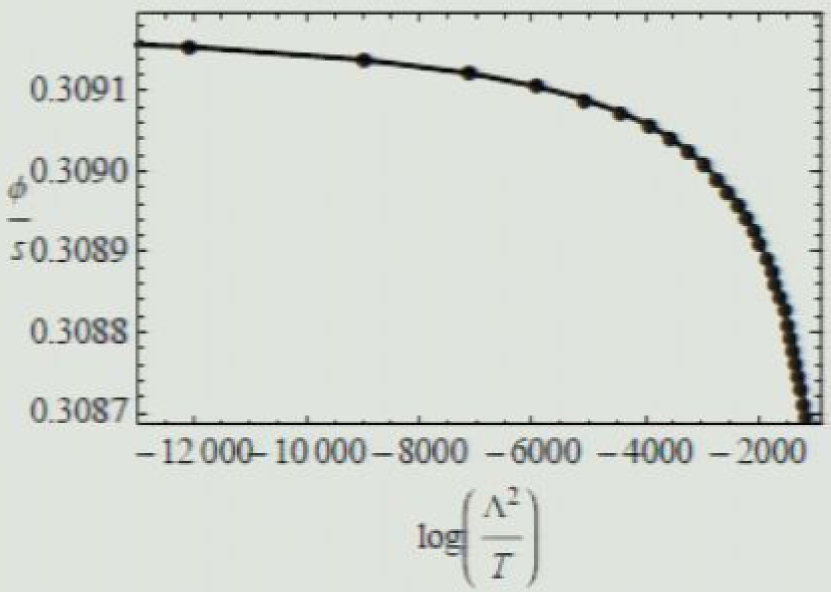

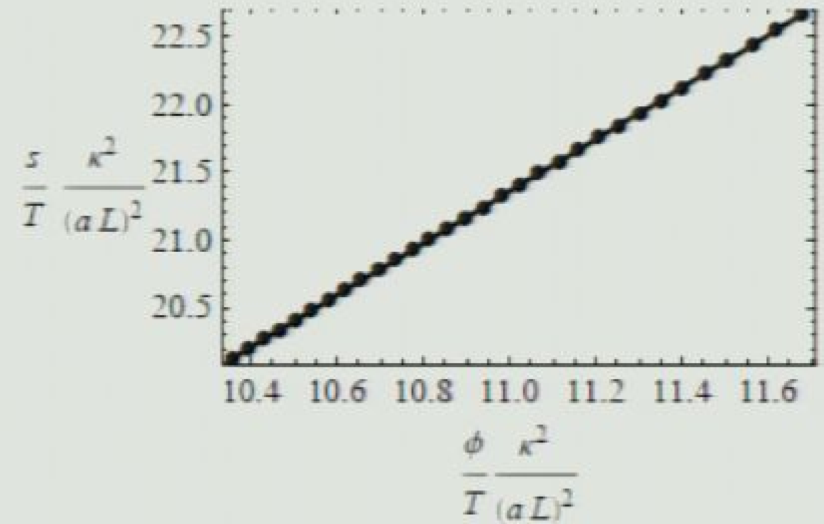

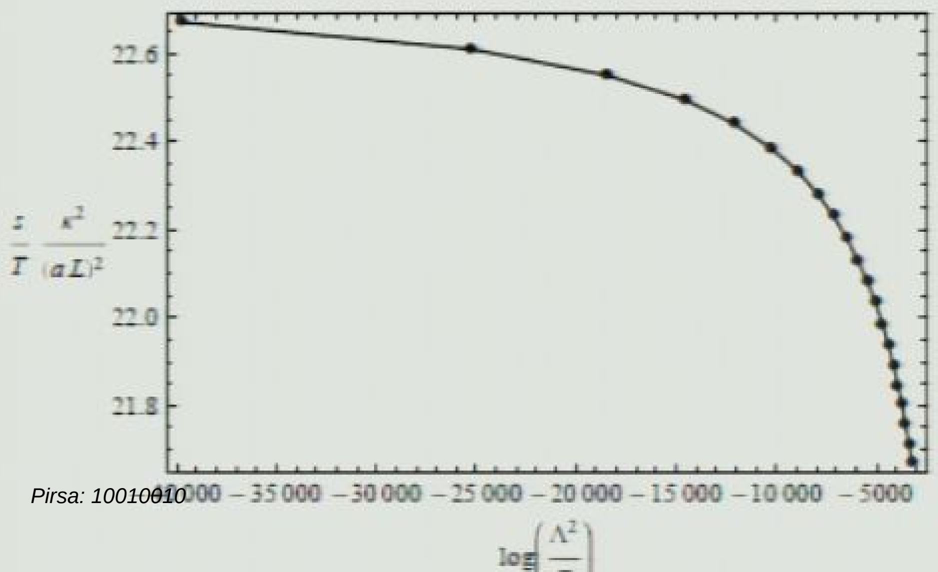

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