Title: Jets in Effective Field Theory

Date: Jan 29, 2010 02:30 PM

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Abstract: Final states involving hadronic jets are an important background to new physics processes in colliders, as well as a probe of QCD over a large range of energies. Because the physics of jets involves multiple energy scales, they are both complex theoretically and ideally suited to study using effective field theory techniques. In this talk I will discuss some recent progress in using effective field theory to describe the physics of jets.

# Jets in Effective Field Theory

(W. Cheung, ML and S. Zuberi, Phys.Rev.D80:114021, 2009)

Michael Luke
Department of Physics
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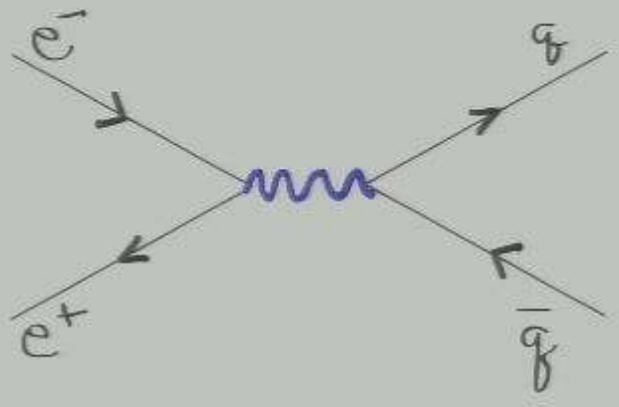
# Outline

1. Introduction: Jets, Factorization and Effective Field Theory
2. SCET
3. Phase space and jets
4. Prospects

# Jets in QCD



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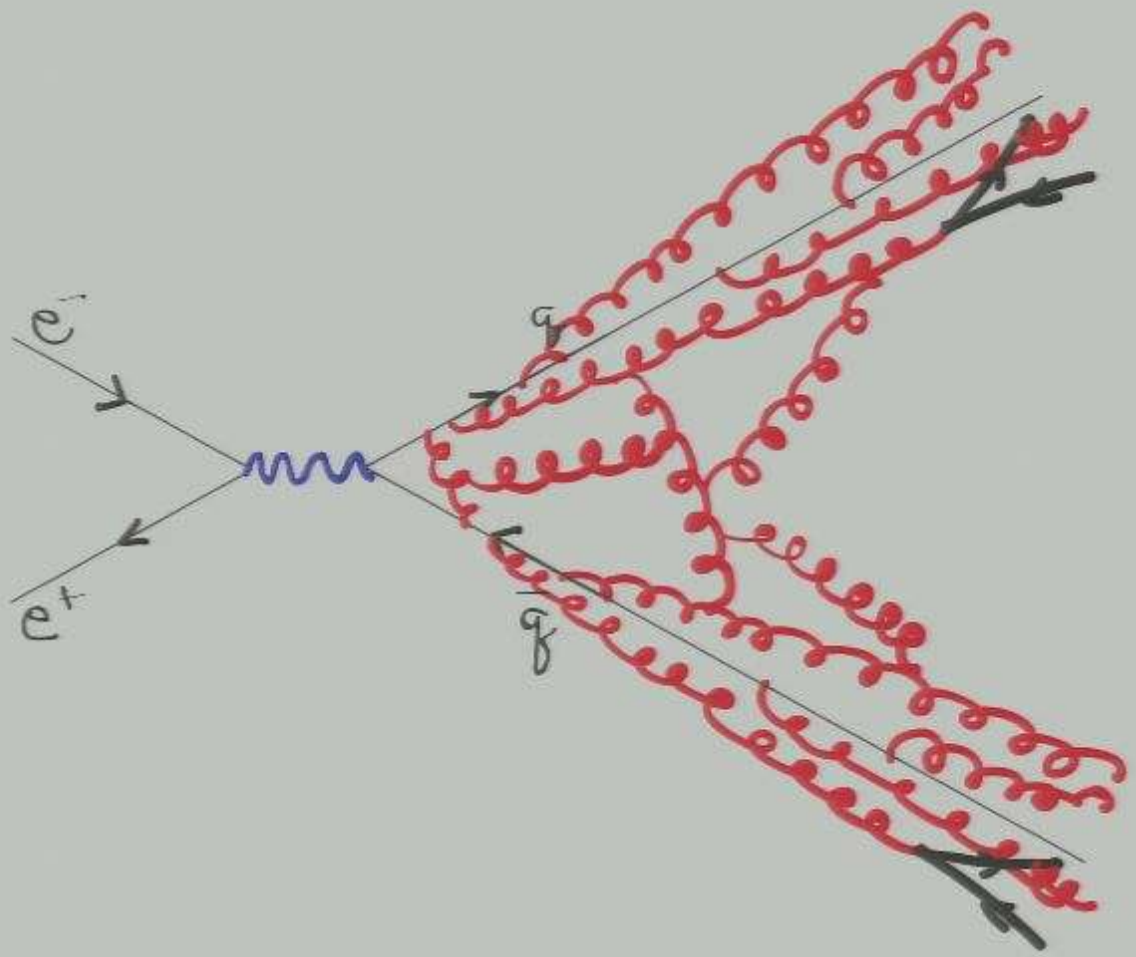

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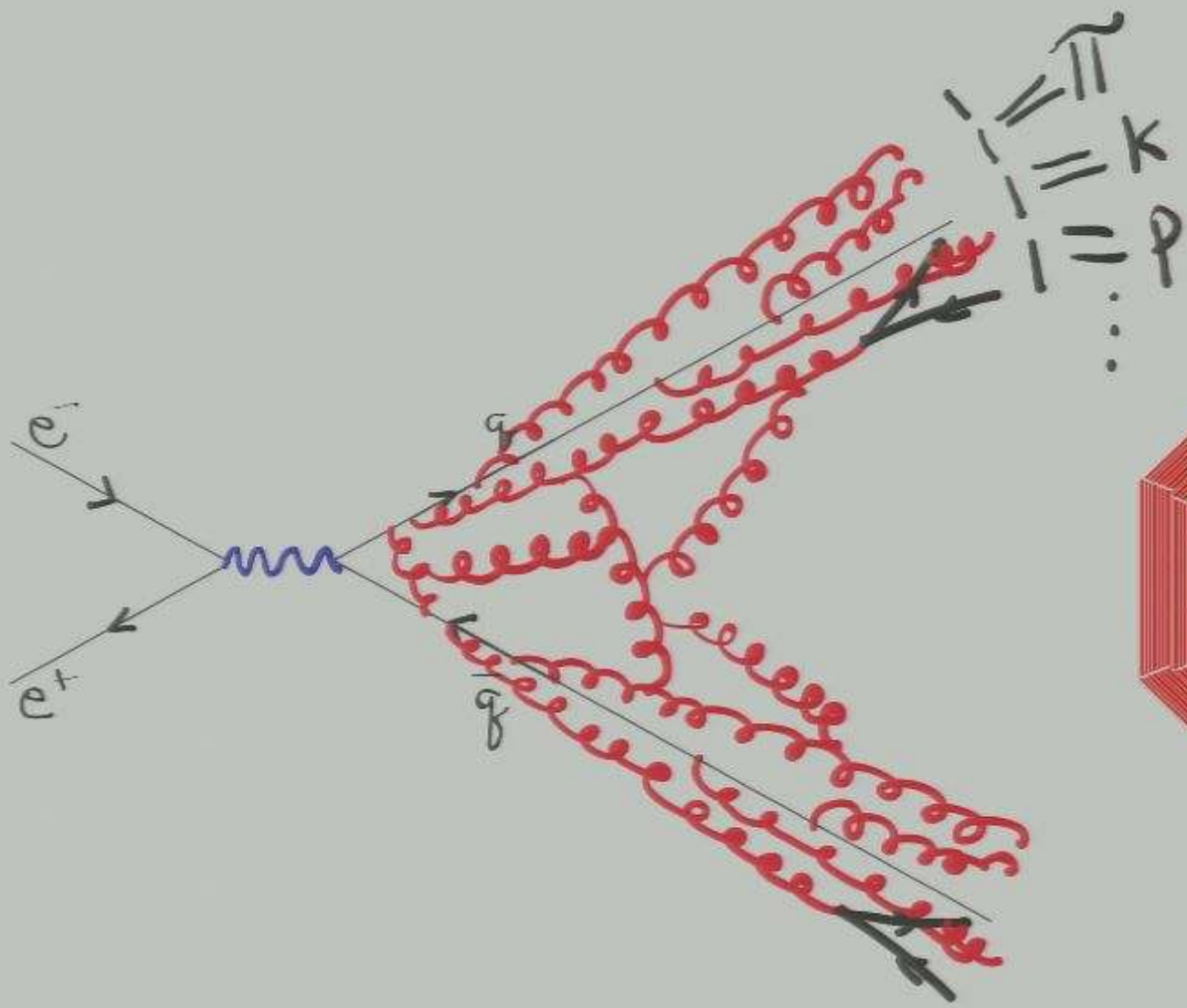


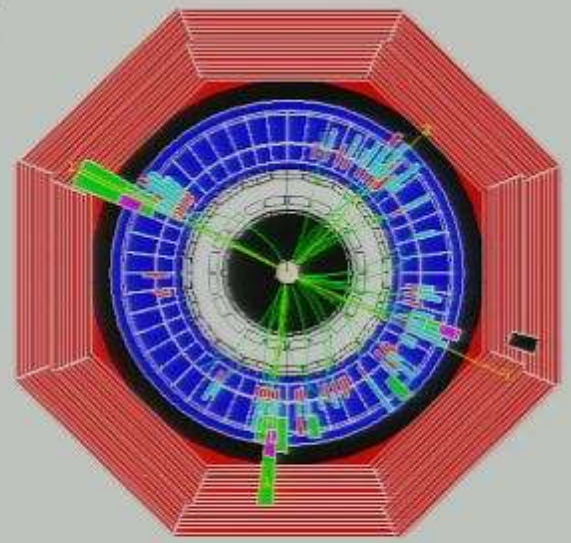

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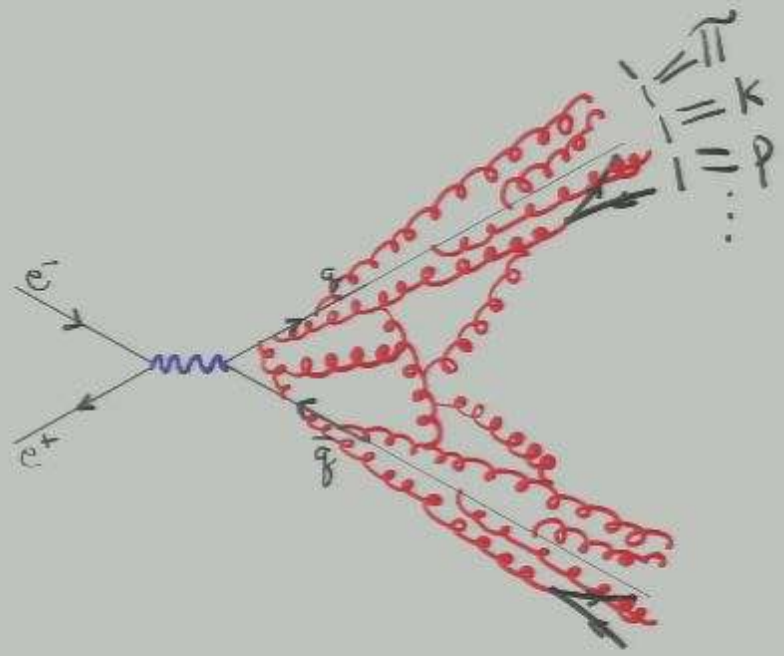

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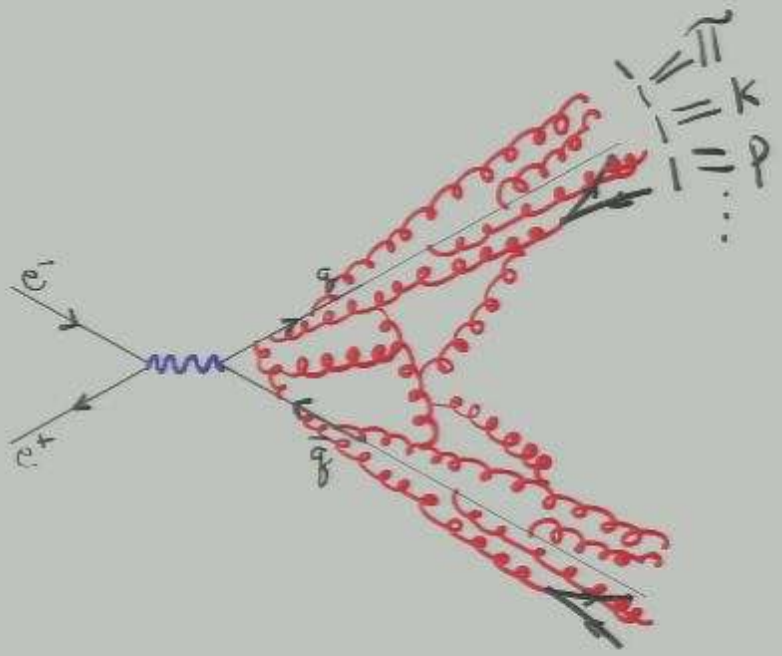
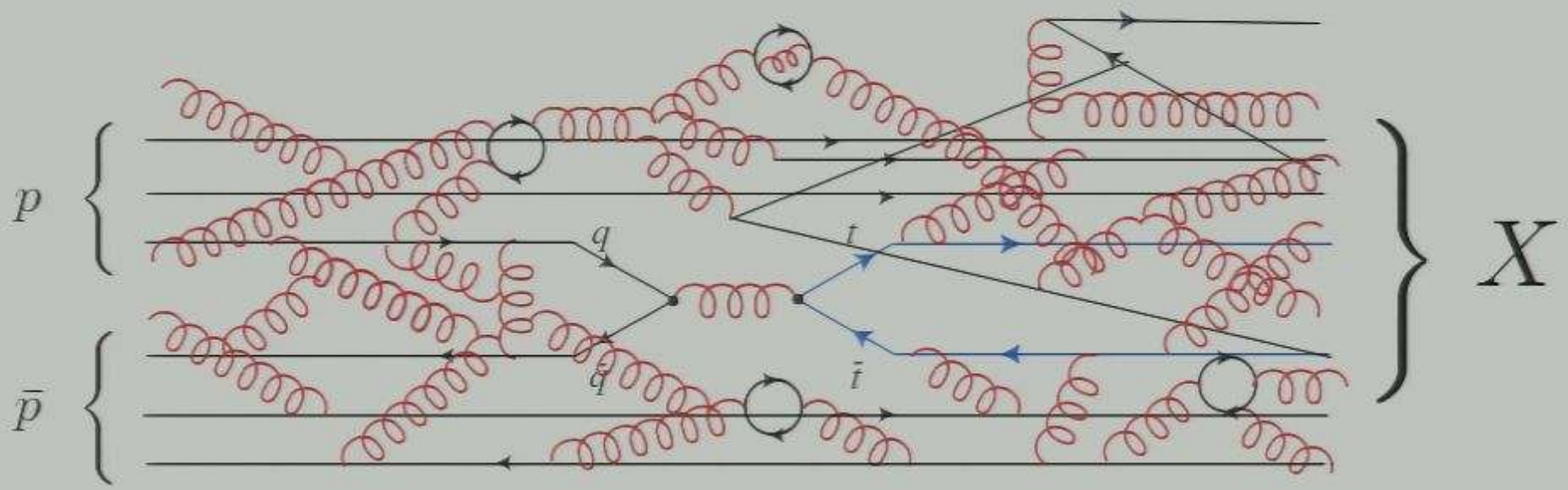



- jets in final states are backgrounds to new physics processes
- structure of jets contain signatures of hard scattering process - can allow us to distinguish SM origin from new physics

All collider QCD problems are inherently **multiscale**.
Traditional QCD approach relies on **factorization theorems**

ex: $p+\bar{p} \rightarrow t\bar{t} + X$



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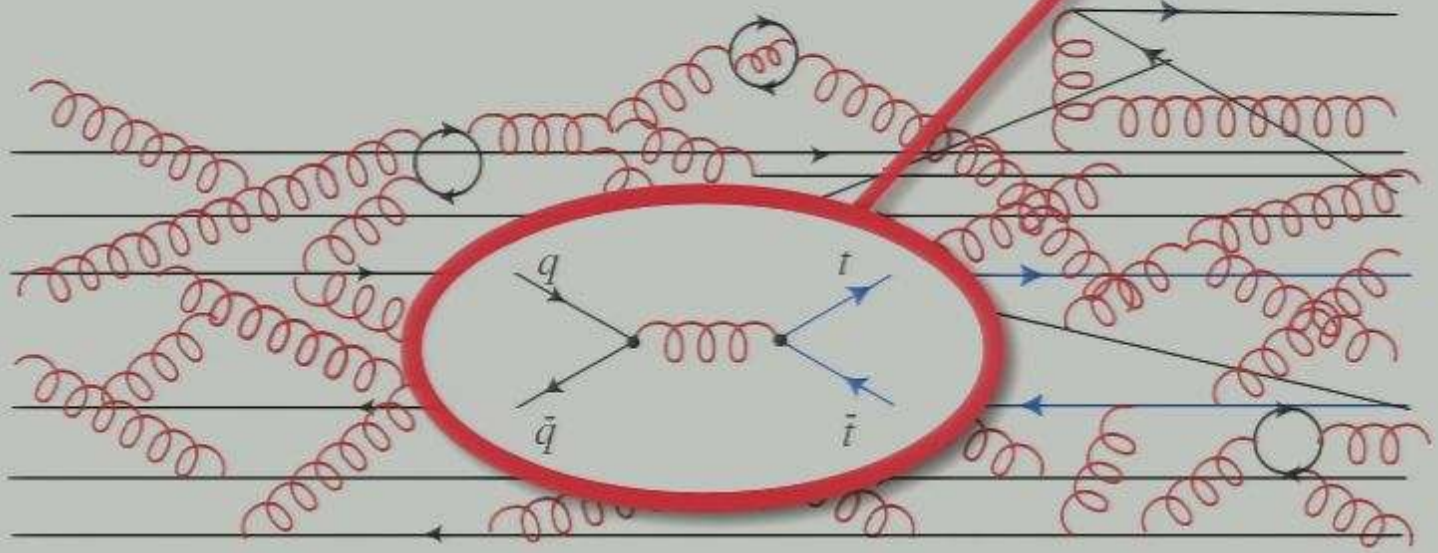
$$\sigma(p(P_1) + p(P_2) \to t\bar{t} + X)$$

$$= \int_0^1 dx_1 dx_2 \sum_f f_f(x_1) f_{\bar{f}}(x_2) \cdot \sigma(q_f(x_1 P) + \bar{q}_f(x_2 P) \to t\bar{t}) + \ldots$$




(Feynman, Bjorken)

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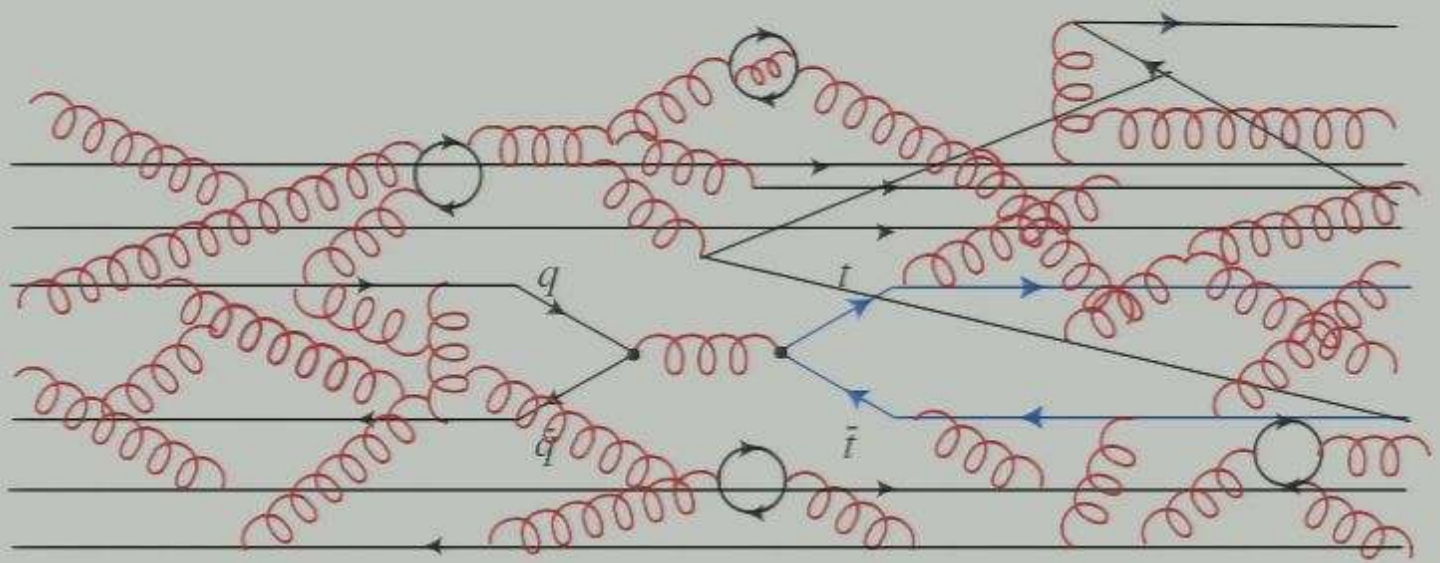


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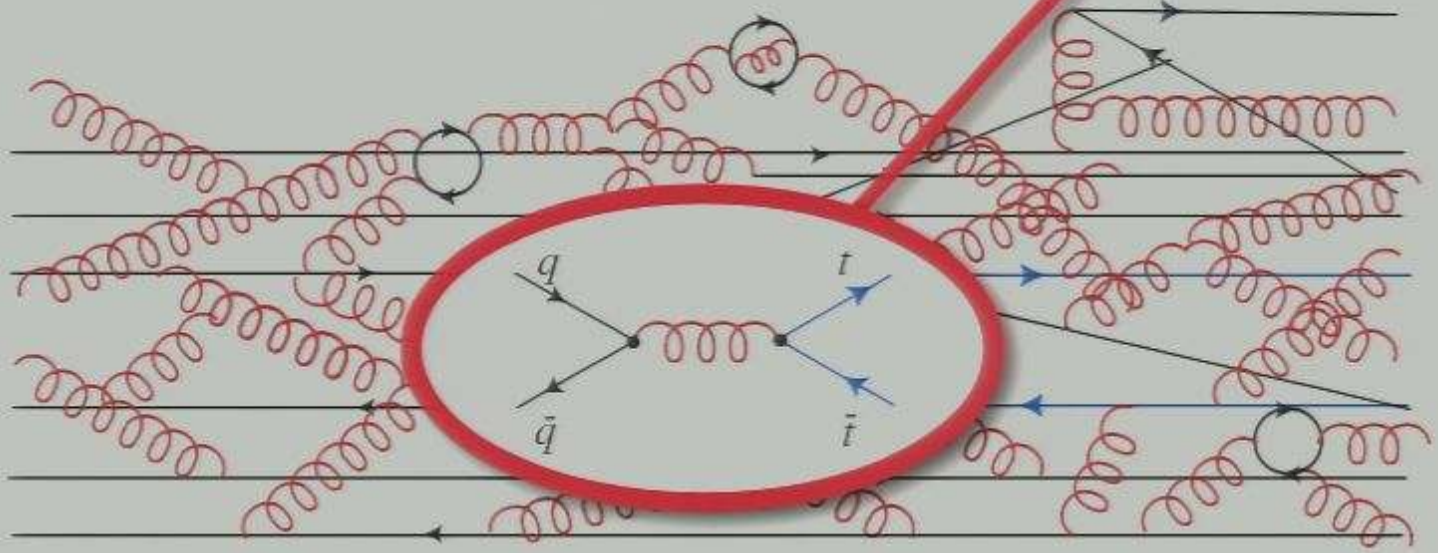




(Feynman, Bjorken)

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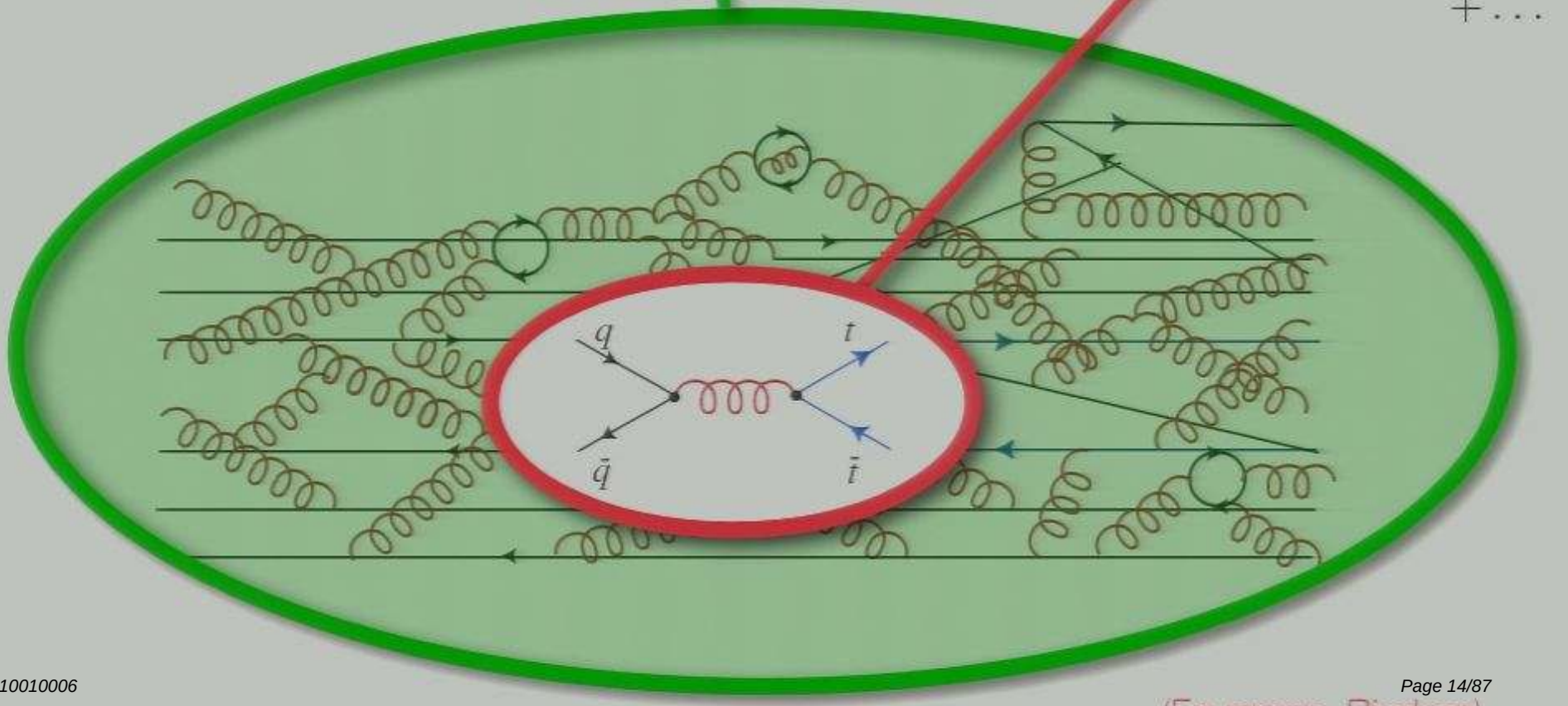


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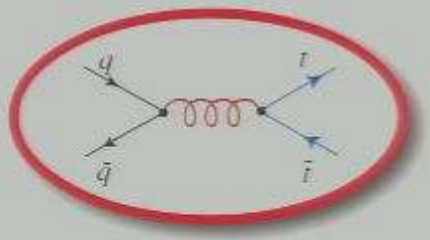
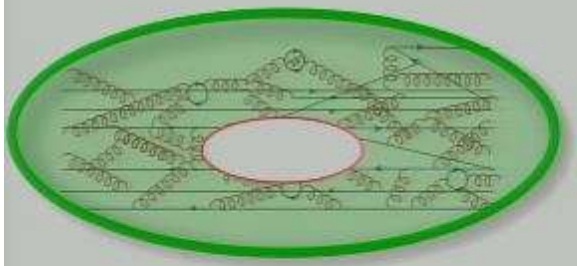



**SHORT DISTANCE**: cross section for free quarks (and gluons) - can calculate in perturbation theory

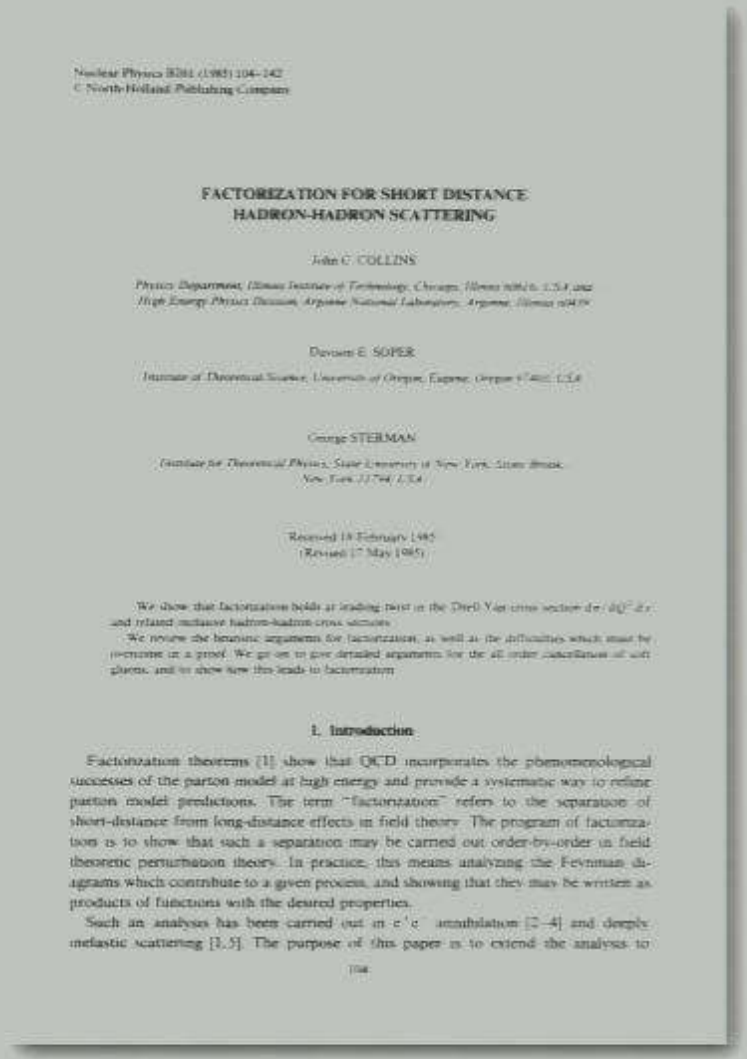


**LONG DISTANCE**: $f_f(x_1)$ :probability to find parton *f* with fraction $x_1$ of longitudinal momentum of proton ("parton distribution function") - property of the PROTON - can't calculate ... but UNIVERSAL (can measure in another process)

Factorization: short and long-distance contributions are separately well-defined (IR, collinear safe)

# The proofs of factorization are long and complicated

(and based on exhaustive analysis of Feynman diagrams ...)



Nuclear Physics B261 (1985) 104–142
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FACTORIZATION FOR SHORT DISTANCE HADRON-HADRON SCATTERING

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Received 18 February 1985
(Revised 7 May 1985)

We show that factorization holds at leading twist in the Drell-Yan cross section $d\sigma/dQ^2\,dy$ and related inclusive hadron-hadron cross sections.

We review the heuristic arguments for factorization, as well as the difficulties which must be overcome in a proof. We go on to give detailed arguments for the all order cancellation of soft gluons, and to show how this leads to factorization.

1. Introduction

Factorization theorems [1] show that QCD incorporates the phenomenological successes of the parton model at high energy and provide a systematic way to refine parton model predictions. The term "factorization" refers to the separation of short-distance from long-distance effects in field theory. The program of factorization is to show that such a separation may be carried out order-by-order in field theoretic perturbation theory. In practice, this means analyzing the Feynman diagrams which contribute to a given process, and showing that they may be written as products of functions with the desired properties.

Such an analysis has been carried out in $e^+e^-$ annihilation [2–4] and deeply inelastic scattering [1,5]. The purpose of this paper is to extend the analysis to

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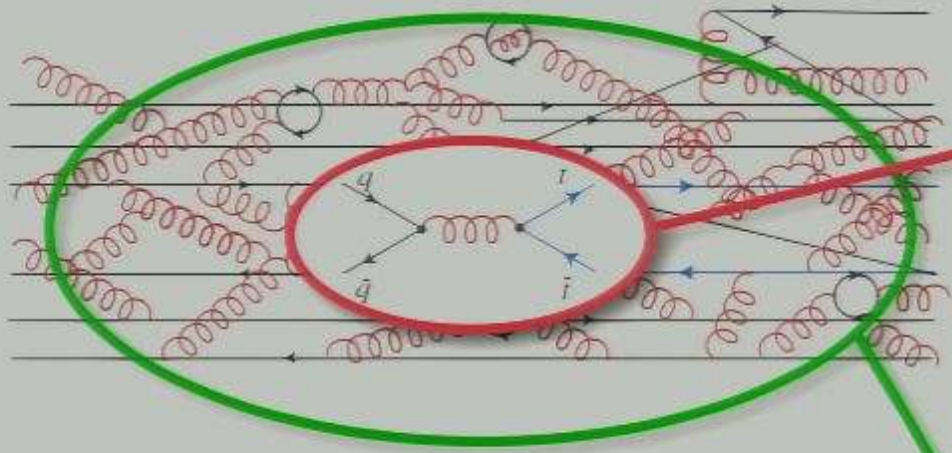
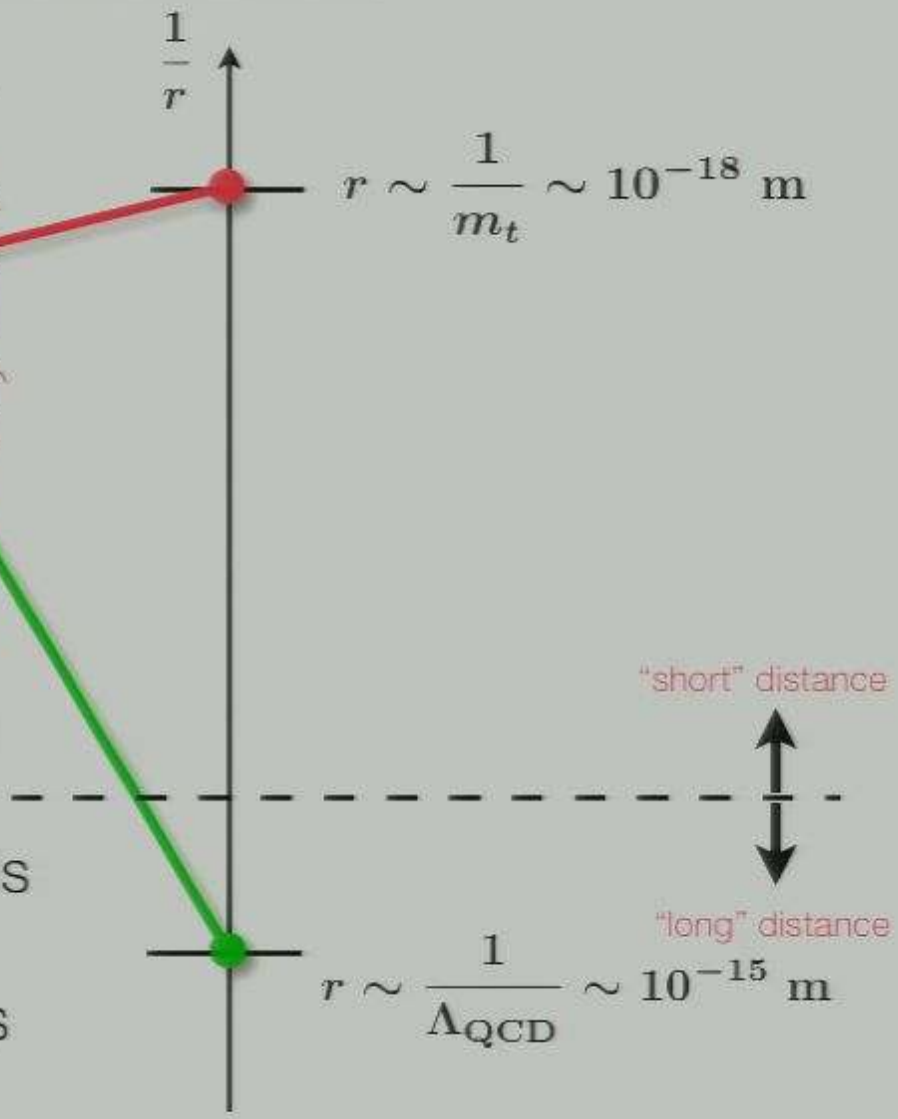
(Collins, Soper, Sterman, 1980's)

... but the physics is simple: **Separation of Scales**

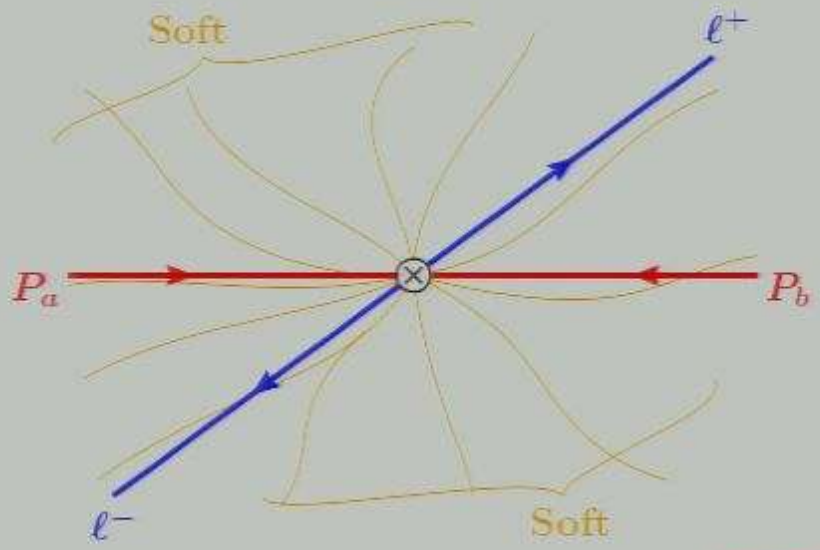


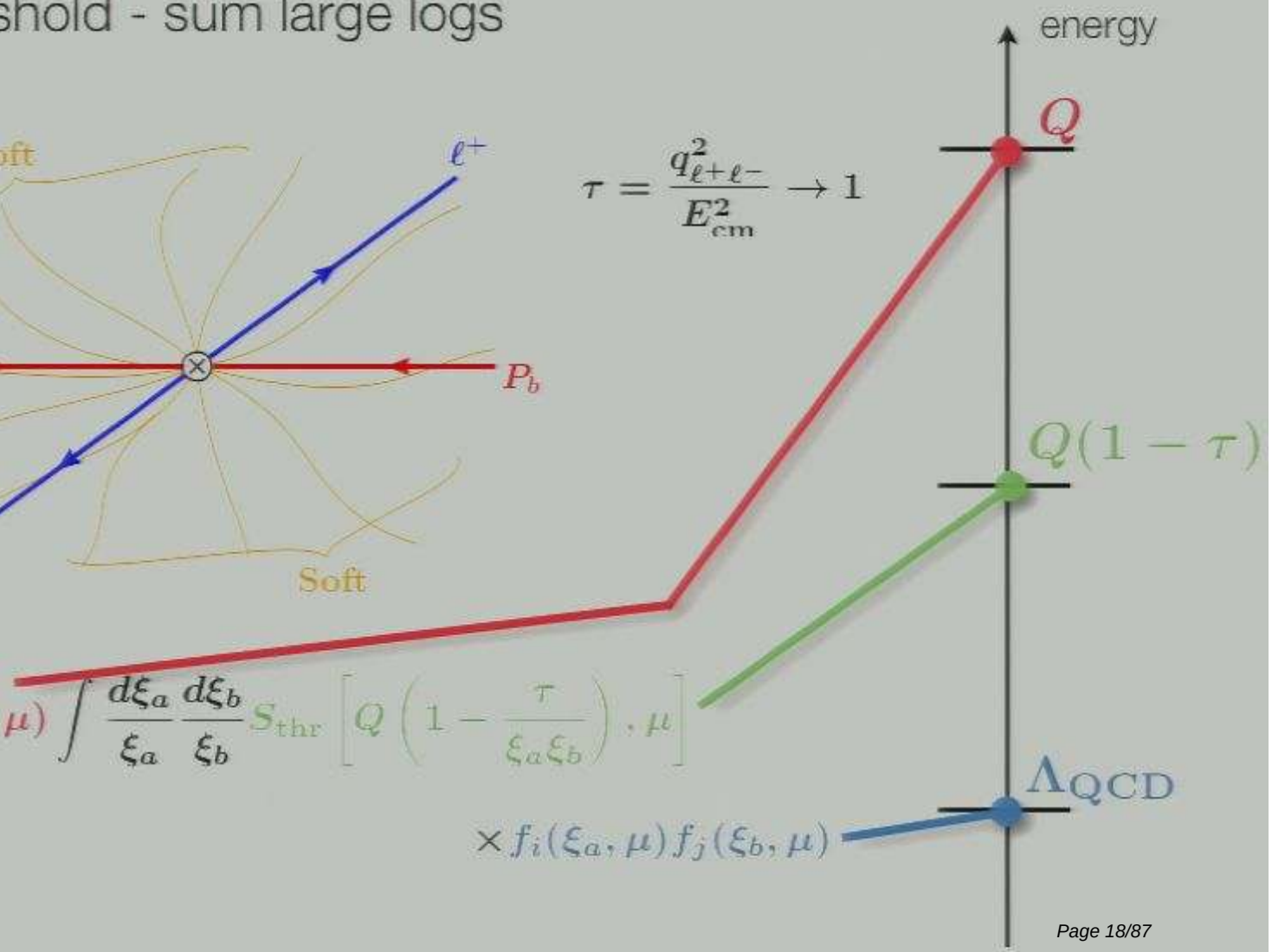

- top quark production is a short-distance process, hadronic physics is long-distance
- hadronic physics cannot resolve details of short-distance physics - hadronization is independent of details of scattering (so parton distributions are universal)

With restrictions on the final states, there are more scales in the problem, and factorization gets more complicated:

ex: DY near threshold - sum large logs



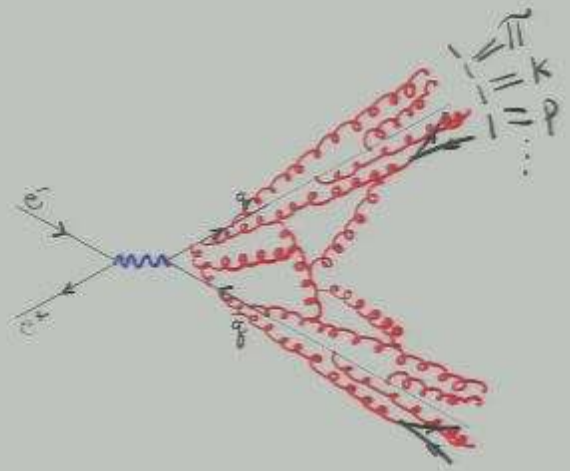

$$\tau = \frac{q^2_{\ell^+\ell^-}}{E^2_{\rm cm}} \to 1$$

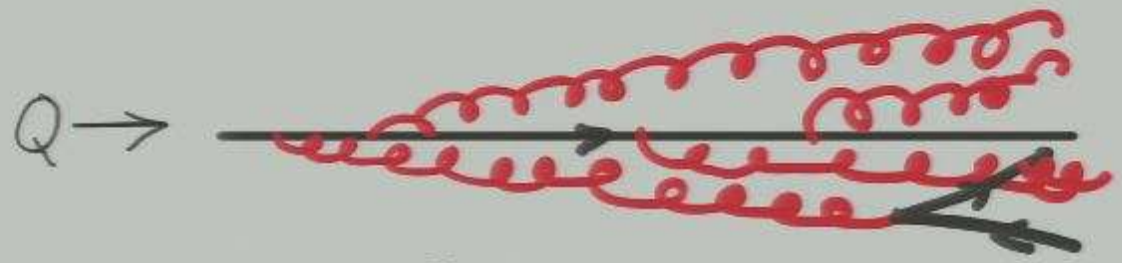



$$\frac{1}{\sigma_0}\frac{d\sigma}{dq^2} = Q\sum_{ij} H_{ij}(q^2,\mu)\int \frac{d\xi_a}{\xi_a}\frac{d\xi_b}{\xi_b} S_{\rm thr}\left[Q\left(1-\frac{\tau}{\xi_a\xi_b}\right),\mu\right] \times f_i(\xi_a,\mu) f_j(\xi_b,\mu)$$

With restrictions on the final states, there are more scales in the problem, and factorization gets more complicated:

ex: $e^+e^- \to$ jets




$E_J \sim Q$

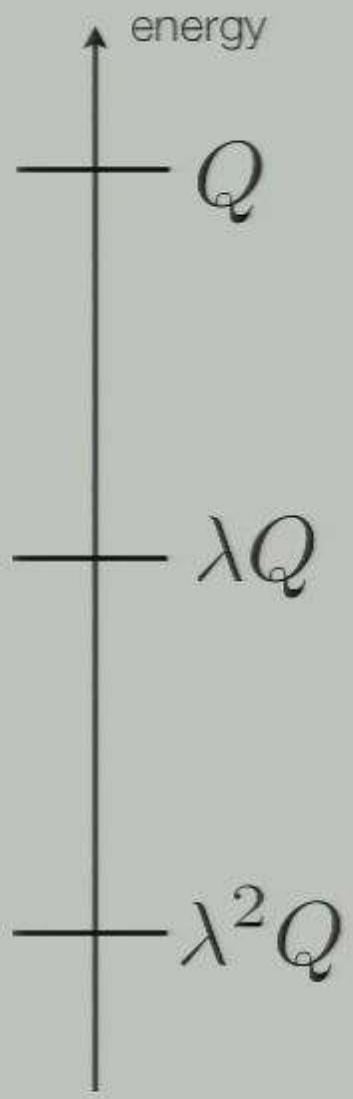
$p_J^2 \sim \lambda^2 Q^2 \ll E_J^2$

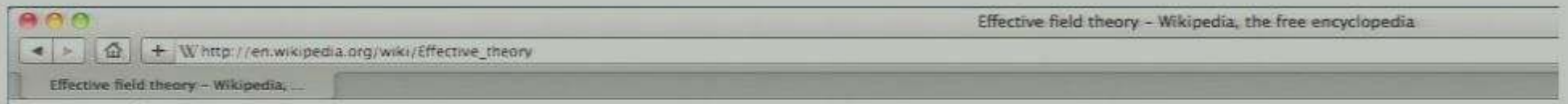
$\therefore p_J \sim Q(1, \lambda^2, \lambda)$

$+ \; - \; \perp$

(at least) 3 scales

$(+\Lambda_{QCD})$





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# Effective field theory

From Wikipedia, the free encyclopedia

(Redirected from Effective theory)

In physics, an **effective field theory** is an approximate theory (usually a quantum field theory) that includes appropriate degrees of freedom to describe physical phenomena occurring at a chosen length scale, while ignoring substructure and degrees of freedom at shorter distances (or, equivalently, at higher energies).

**Contents** [hide]

1 The renormalization group
2 Examples of effective field theories
2.1 Fermi theory of beta decay
2.2 BCS theory of superconductivity
2.3 Other examples
3 See also
4 References and external links

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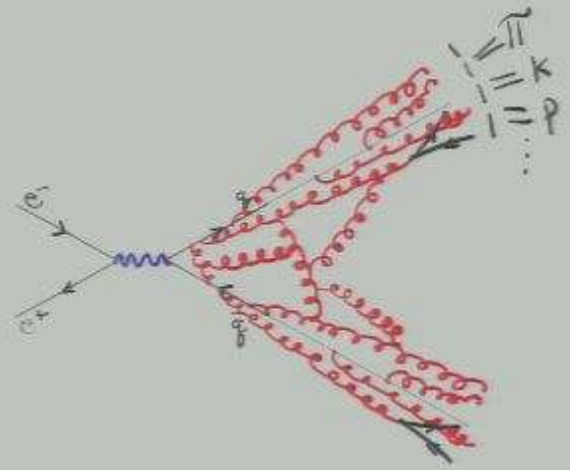


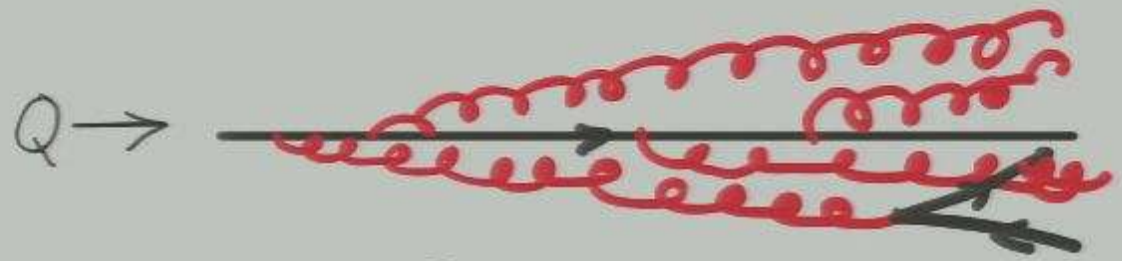


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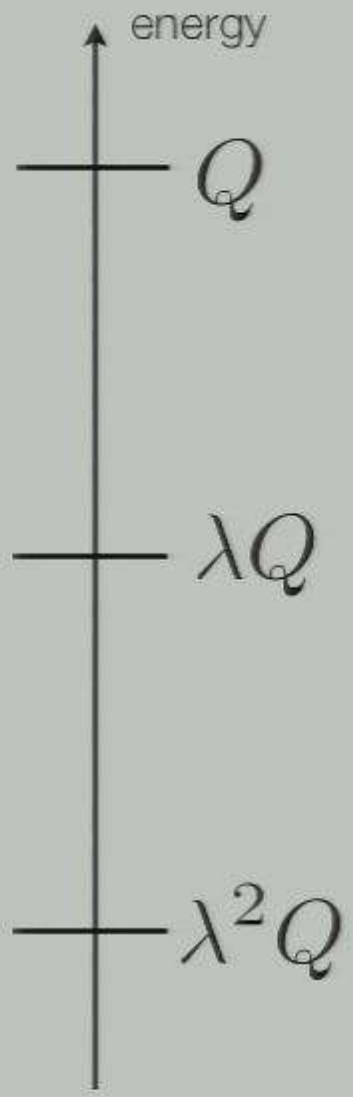

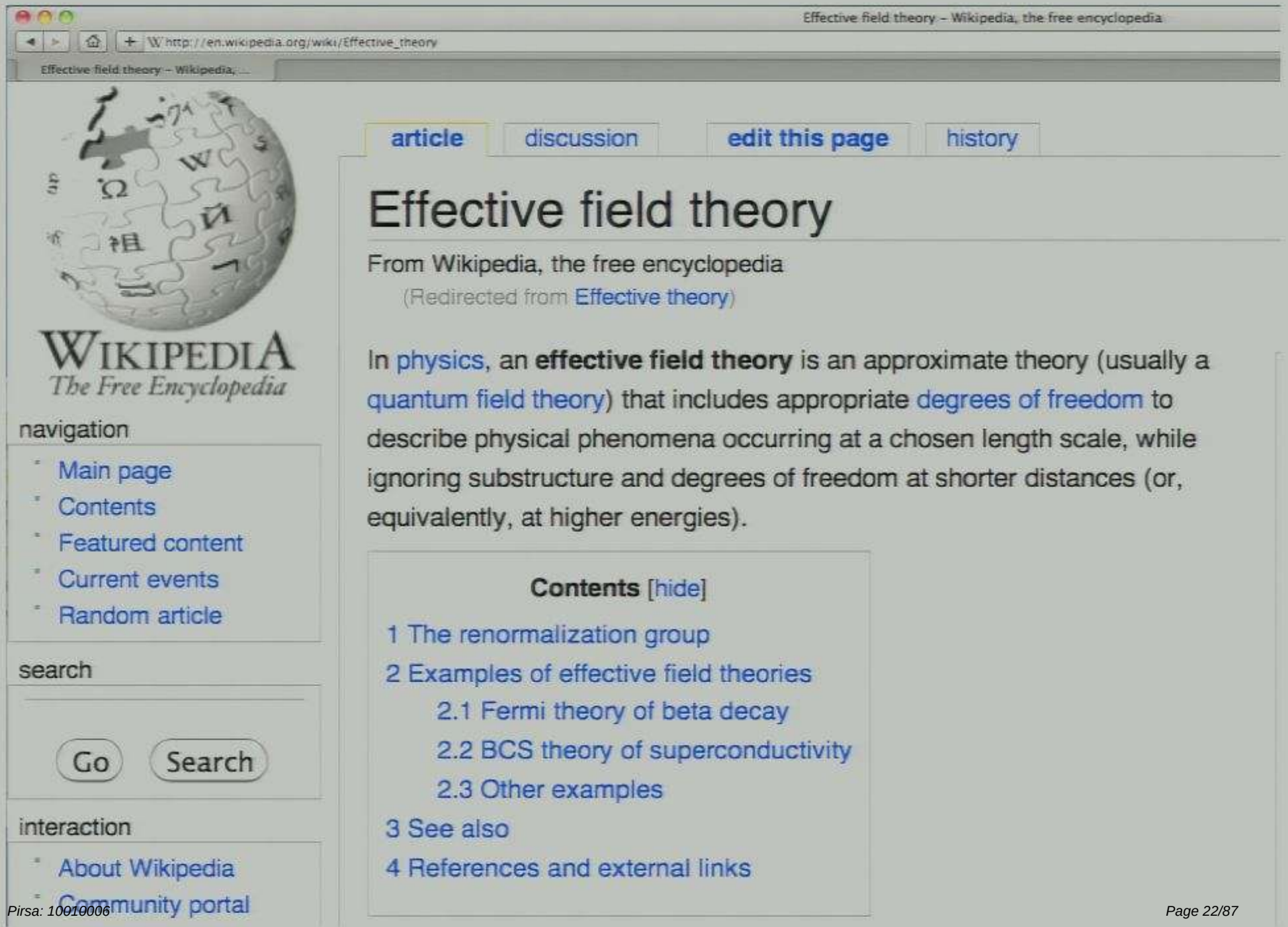
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Effective field theory
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(Redirected from Effective theory)
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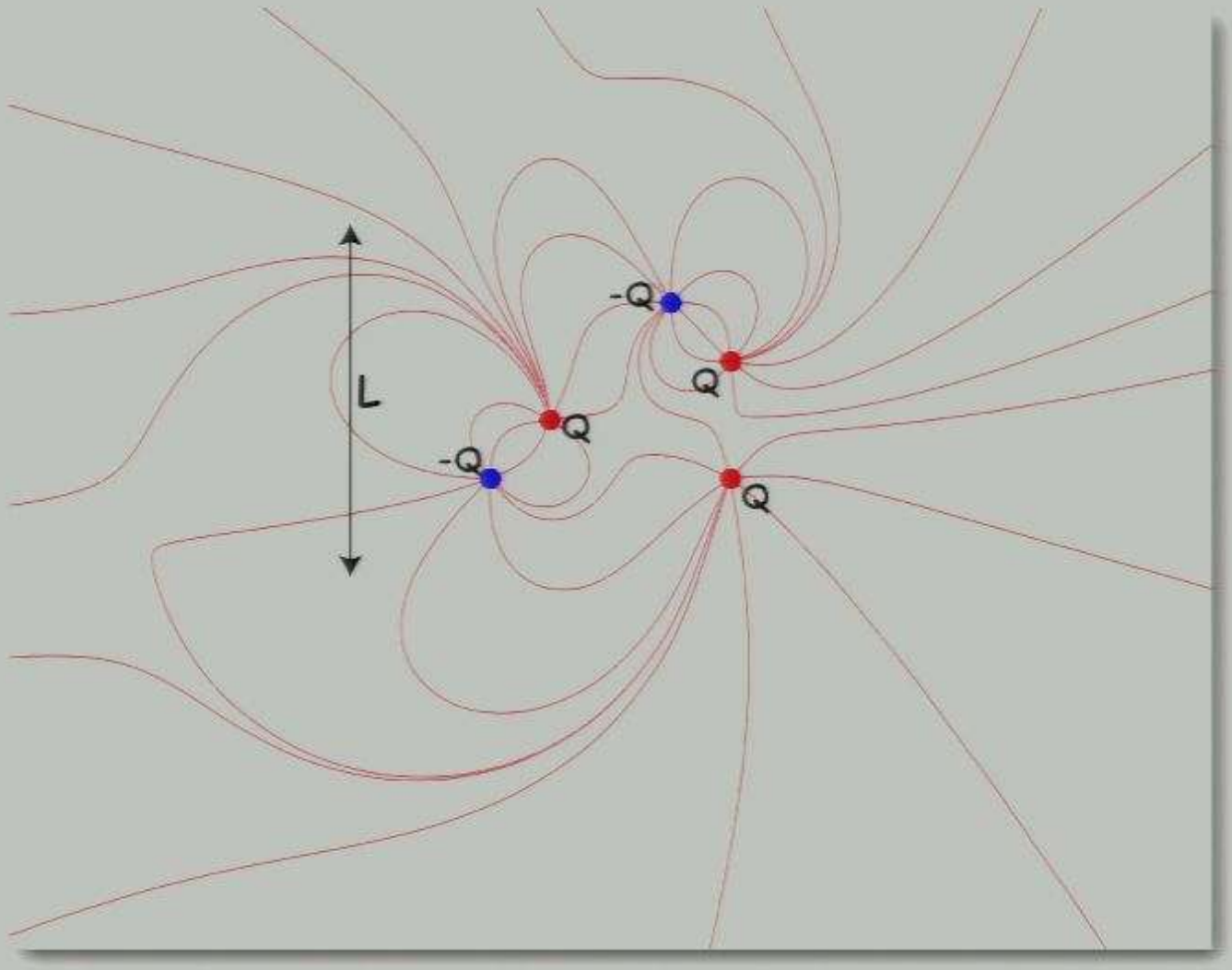
# Effective field theory

- is a TOOL to separate scales in a multiscale process - a "turn-the-crank" approach to factorization
- different momentum regions can be treated separately (perturbative, extracted from experiment, lattice, etc.)
- renormalization group can be used to sum logs of small parameters

We do this all the time in classical electrodynamics:




Physics at r~L is complicated - depends on details of charge distribution

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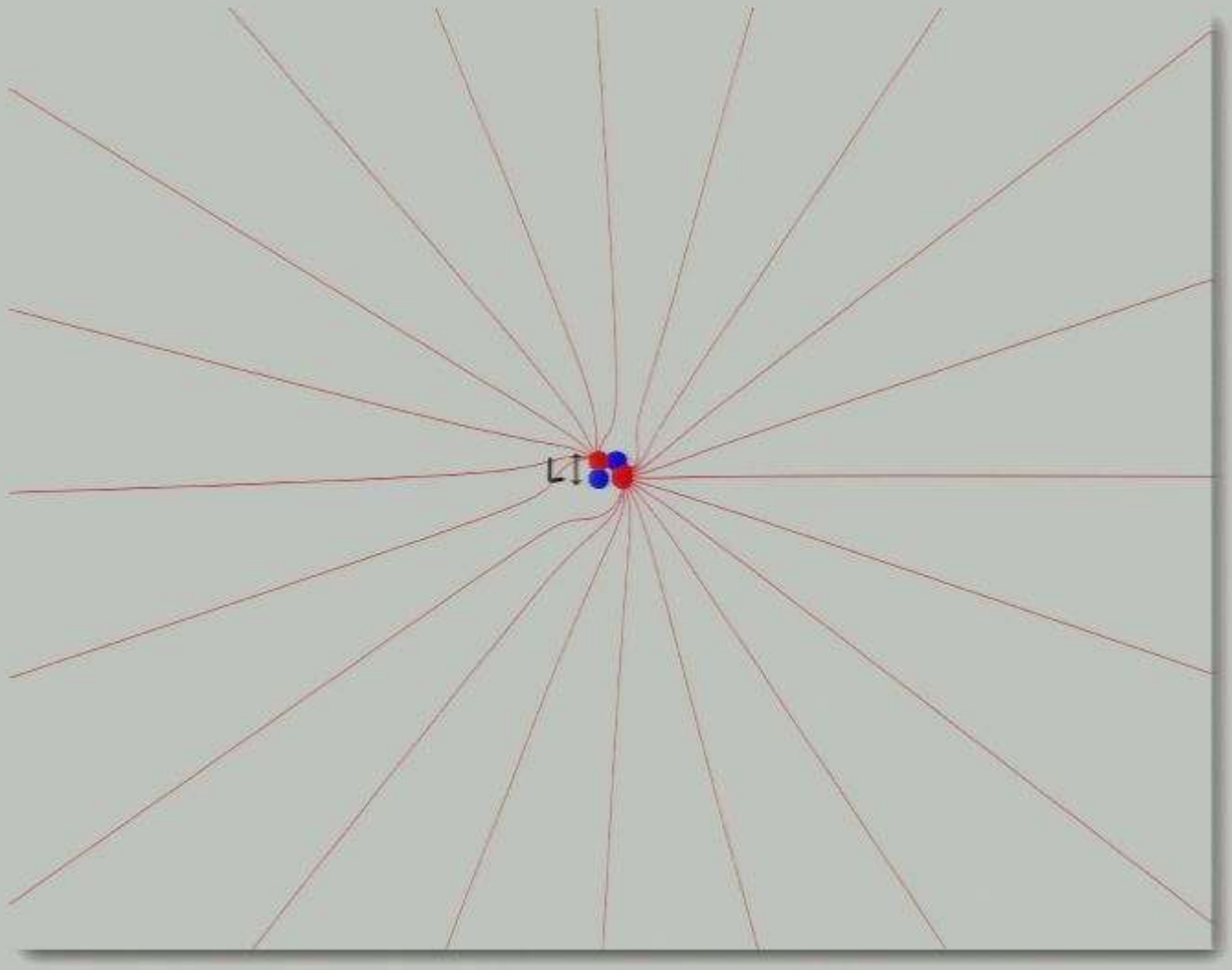
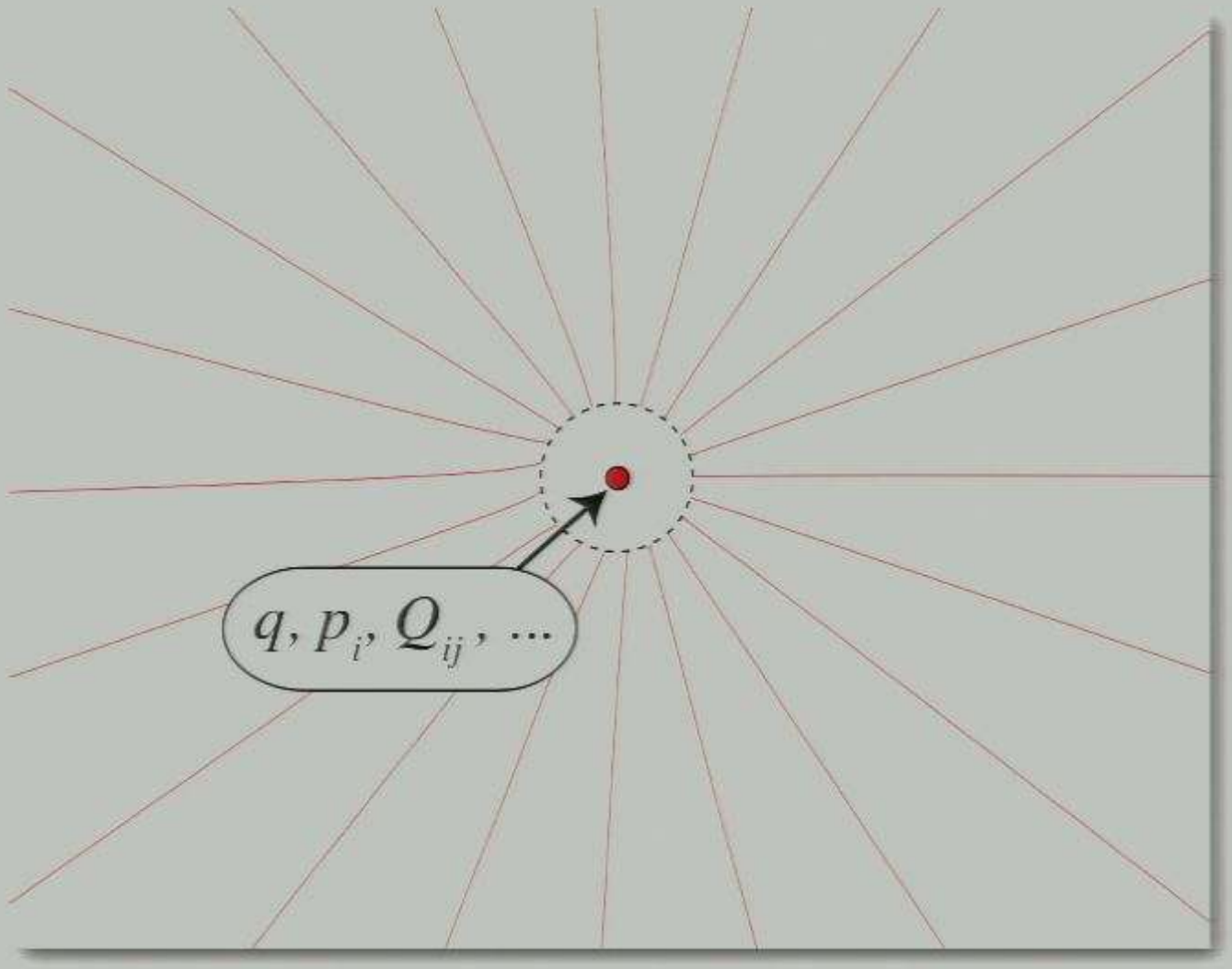



BUT ... if we are interested in physics at r>>L, things are much simpler ...

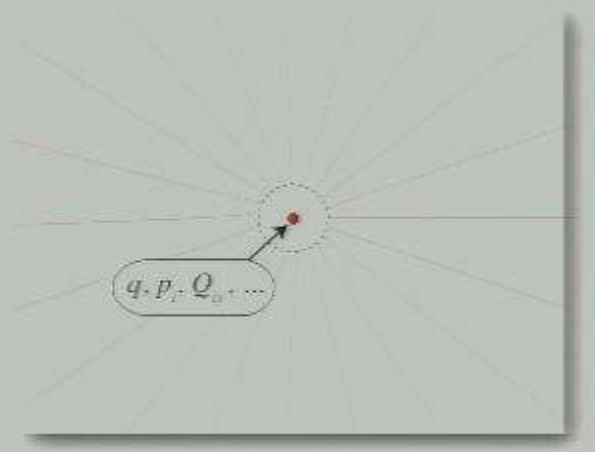
We do this all the time in classical electrodynamics:




... can replace complicated charge distribution by a POINT source with additional interactions (multipoles)...

Multipole expansion:

$$V(r) = \frac{q}{r} + \frac{\vec{p}\cdot\vec{x}}{r^3} + \frac{1}{2}Q_{ij}\frac{x_i x_j}{r^5} + \cdots$$




$q,\ p_i,\ Q_{ij},\ \cdots$ : short distance quantities (depend on details of charge distribution)

$\left\langle \frac{1}{r} \right\rangle, \left\langle \frac{x_i}{r^3} \right\rangle, \left\langle \frac{x_i x_j}{r^5} \right\rangle, \cdots$ : long distance quantities (independent of short distance physics)

FACTORIZATION!

higher multipole moments <-> new effective interactions from "integrating out" short distance physics .. effects are suppressed by powers of L/r

## Field Theory generalization: **Effective Field Theory**

-at low momenta p<<Λ, a theory can be described by an effective Hamiltonian where degrees of freedom at scale Λ have been "integrated out":

$$H_{\text{eff}} = H_0 + \sum_i \frac{C_i}{\Lambda^{n_i}} \mathcal{O}_i$$

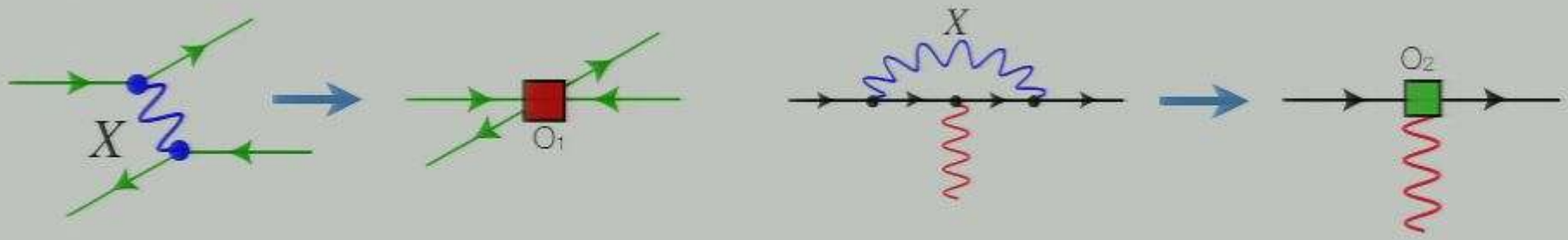
$H_0$: Hamiltonian in Λ->∞ limit

$\sum_i \frac{C_i}{\Lambda^{n_i}} \mathcal{O}_i$: corrections determined by matrix elements of operators $O_i$ - power counting determined by dimensional analysis

$C_n$'s : short distance quantities (in QCD: perturbatively calculable if $\Lambda >> \Lambda_{QCD}$)

$\langle \mathcal{O}_n \rangle$ 's : long distance quantities (in QCD: nonperturbative ... need to get them elsewhere)

(1) “Classic” (4-fermi theory and the like):



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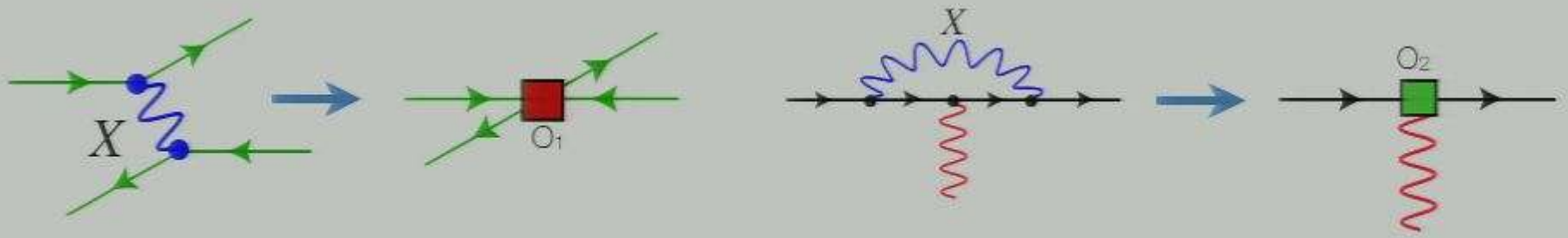
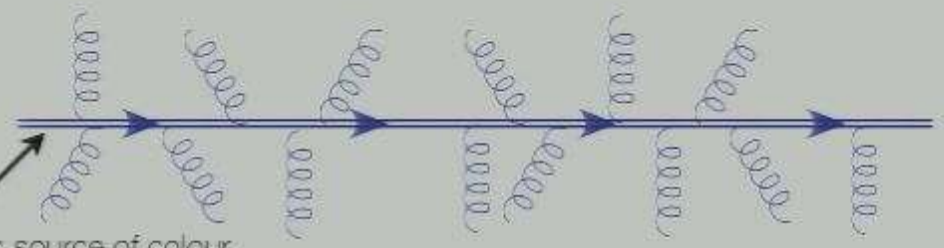


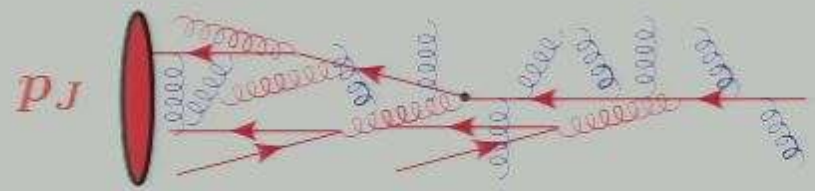

## (2) “Modern”: Heavy Quark Effective Theory (“HQET”)



QCD: heavy quark
HQET: Wilson line (static source of colour charge)

an EFT of heavy, coloured, stable objects - b, c quarks

## (3) “Post-Modern”: Soft-Collinear Effective Theory (“SCET”)




an EFT of energetic, light coloured particles - jets!

EFT has some advantages over traditionally pQCD approach:

- systematically improvable - can look beyond leading order
- simplifies proofs of factorization
- conceptually simpler framework, unifying pQCD ingredients of power counting, gauge invariance, RG evolution
- turn-the-crank!

Our goal (long-term): understand factorization in jet production in lepton and hadron colliders using SCET.

Simple "warm-up" question: can we use SCET to sum large logs in dijet rates?

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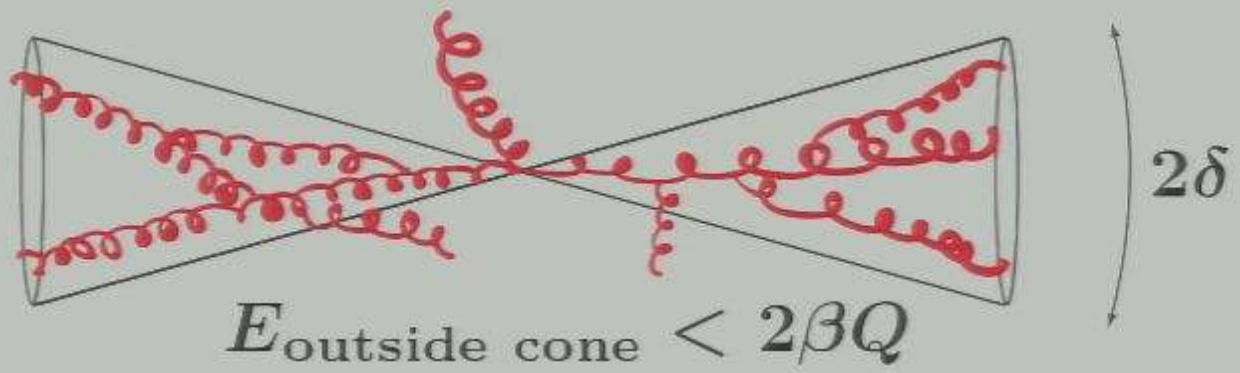
NB There is no unique definition of a jet - lots of choices on the market.

ex: **Sterman-Weinberg** jet definition ("cone" algorithm):



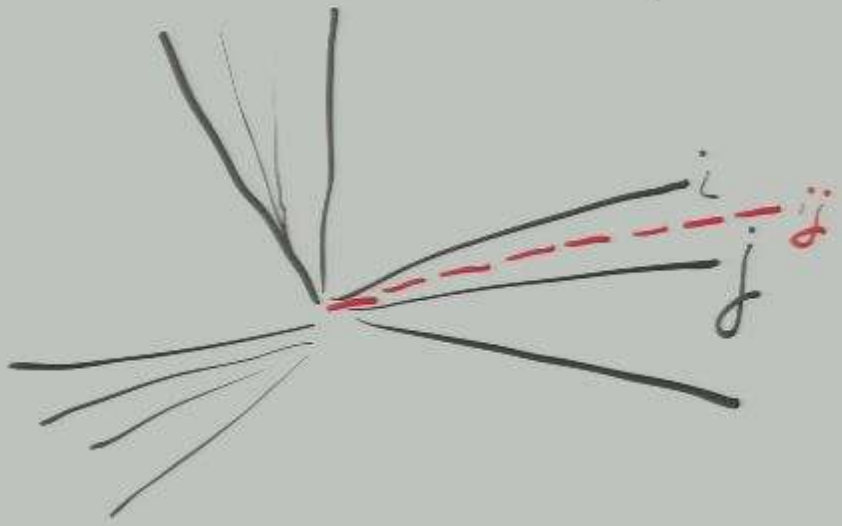

$$f_2^{\mathrm{SW}} \equiv \frac{\sigma^{2\text{ jet}}}{\sigma_0} = 1 + \frac{\alpha_s C_F}{\pi}\left(-4\ln 2\beta \ln\delta - 3\ln\delta + \dots\right)$$

for $\delta \ll 1$, jets are narrow and large logarithms can spoil perturbation theory. Can these be resummed using EFT and renormalization group?

Simple "warm-up" question: can we use SCET to sum large logs in dijet rates?

NB There is no unique definition of a jet - lots of choices on the market.

ex: **JADE**, **$k_T$**, **anti-$k_T$**, ... ("cluster" algorithms)




**JADE**: Calculate invariant mass of each pair of particles, look at smallest:
- if $M_{ij}^2 < jQ^2$ combine particles into a pseudoparticle, repeat
- if $M_{ij}^2 > jQ^2$ stop -> each pseudoparticle is a jet

(These are "exclusive" jet definitions, relevant for e+e- machines. For hadron colliders, want "inclusive" jet definitions)

**$k_T$**: same as JADE, but variable is

$$y_{ij} = M_{ij}^2 \min\left(\frac{E_i}{E_j}, \frac{E_j}{E_i}\right)$$

$$f_2 \sim 1 + \alpha_s \ln^2 j + \alpha_s^2 \ln^4 j + \ldots$$

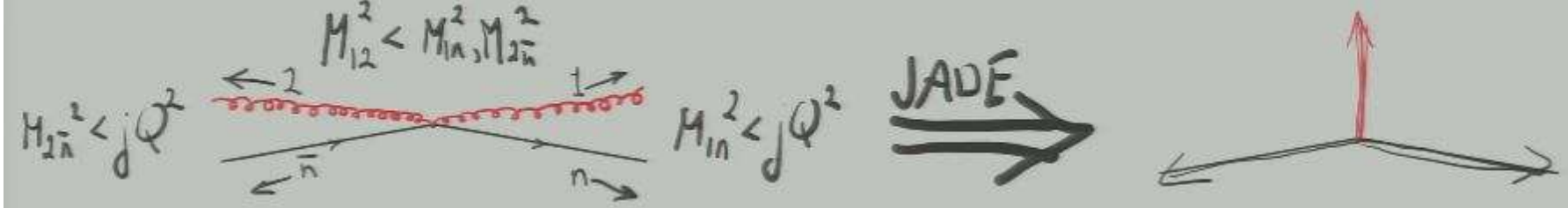
for j«1, jets are narrow - same problem

Current status:

SW: formal resummation of leading logs claimed, but unclear

(Mukhi & Sterman, 1982)

JADE at $O(\alpha_s^2)$:




Individually, gluons 1 and 2 would form jets with the quark and antiquark, respectively (this is the information in the O($\alpha_s$) result)

BUT there are regions of phase space where JADE makes a third jet out of the gluons ... this contributes to the rate at leading log ($O(\alpha_s^2 \ln^4 j)$) but we don't see it from the one-loop RGE! ($k_T$ was invented to avoid this).

Current status:

SW: formal resummation of leading logs claimed, but unclear

(Mukhi & Sterman, 1982)

JADE: no known way to resum ... leading logs do NOT exponentiate

(Brown & Stirling, 1990)

kT: leading logs exponentiate ... not clear how to sum subleading logs

(Brown & Sterling, 1992)

NB: there isn't any factorization theorem for jets in pQCD .. but we can still attack the problem in EFT

# Soft-Collinear Effective Theory ("SCET"*): the Essentials

What is the correct EFT to describe the dynamics of a very LIGHT, ENERGETIC quark?

$$p_q \sim (Q, \lambda^2 Q, \lambda Q)$$

*(Bauer, ML and Fleming, Phys.Rev.D63:014006,2000; Bauer, Fleming, Pirjol and Stewart, Phys.Rev.D63:114020,2001, ...)

(originally developed to describe B decays in jetty regions of phase space, but soon extended to traditional perturbative QCD problems)

# Soft-Collinear Effective Theory ("SCET"): the Essentials

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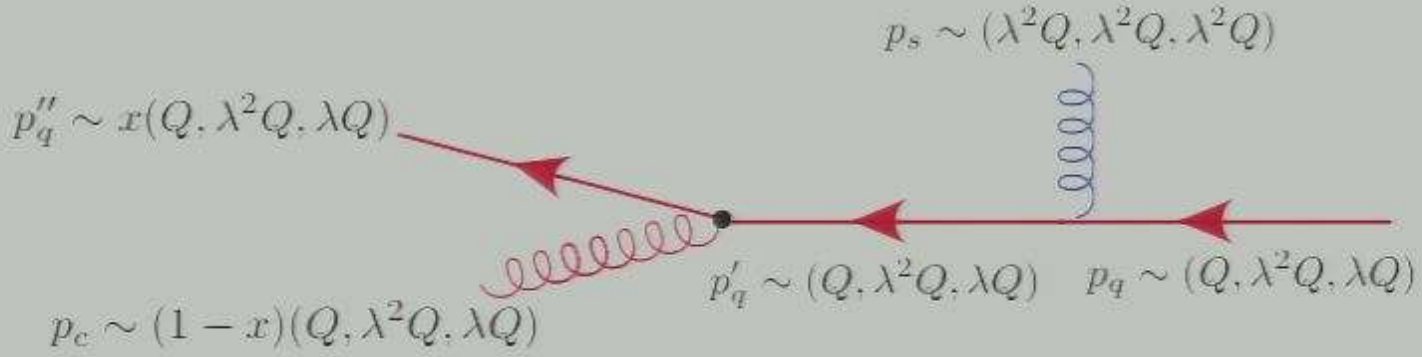


Interactions with soft gluons don't deflect the worldline of the energetic quark

BUT ... the quark can also split into two hard, collinear partons

## Soft-Collinear Effective Theory ("SCET"): the Essentials

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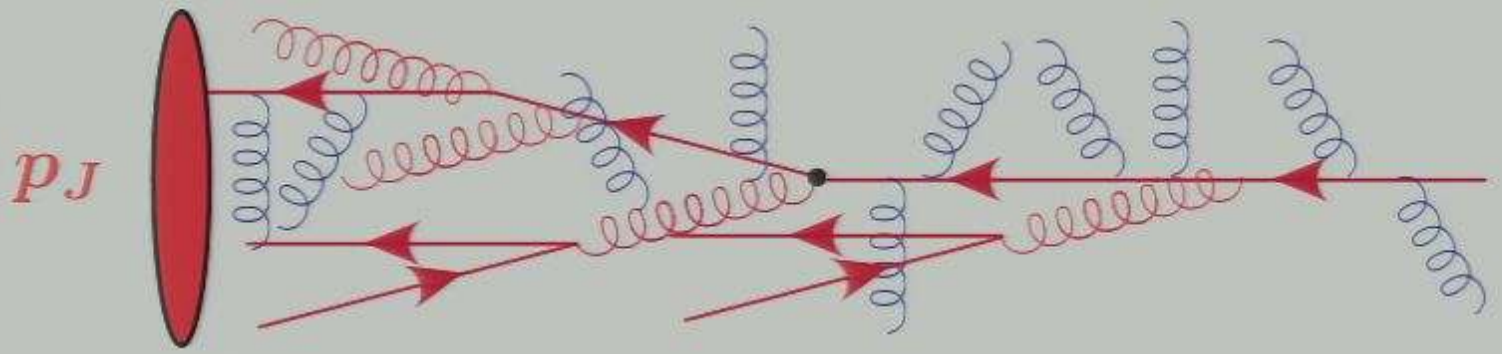



Interactions with soft gluons don't deflect the worldline of the energetic quark

BUT ... the quark can also split into two hard, collinear partons

- get a JET of final state particles

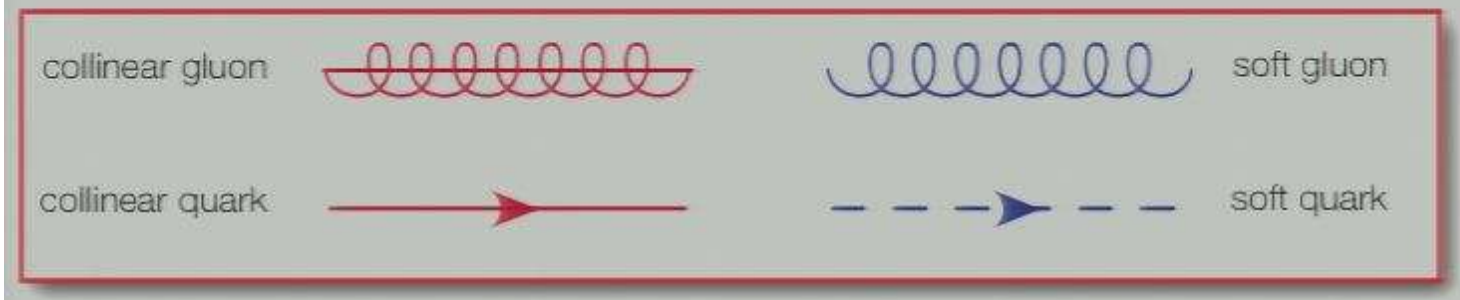
- jet energy is large, invariant mass is parametrically smaller

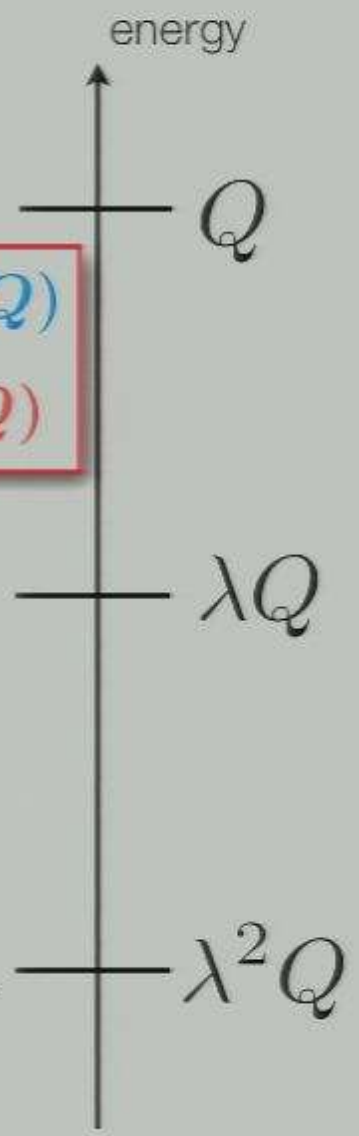
$$E_J \sim Q \qquad p_J^2 \sim \lambda Q \ll Q^2$$

# Soft-Collinear Effective Theory ("SCET"): the Essentials

"Soft" particles $p_s^\mu = (p^+, p^-, \vec{p}_\perp) \sim (\lambda^2 Q, \lambda^2 Q, \lambda^2 Q)$

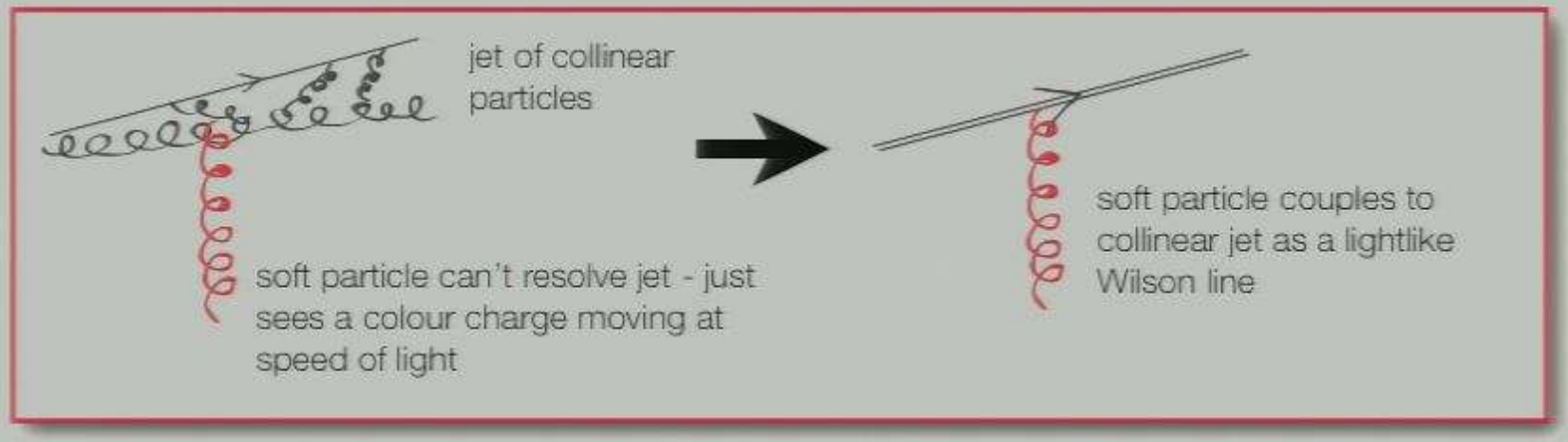
"Collinear" particles $p_c^\mu = (p^+, p^-, \vec{p}_\perp) \sim (Q, \lambda^2 Q, \lambda Q)$



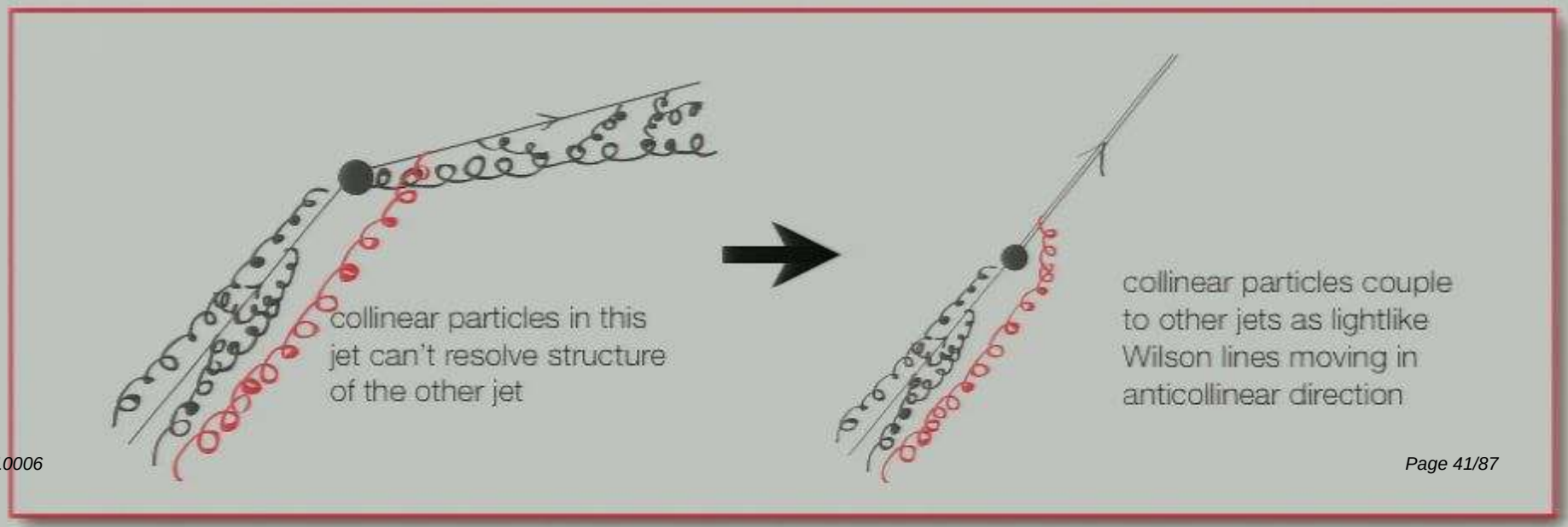




- need a separate field for each momentum scaling (a hallmark of "postmodern" EFT's)
- in situations with multiple collinear directions, need multiple collinear fields
- couplings are interesting, because each field "sees" the others in different ways ...

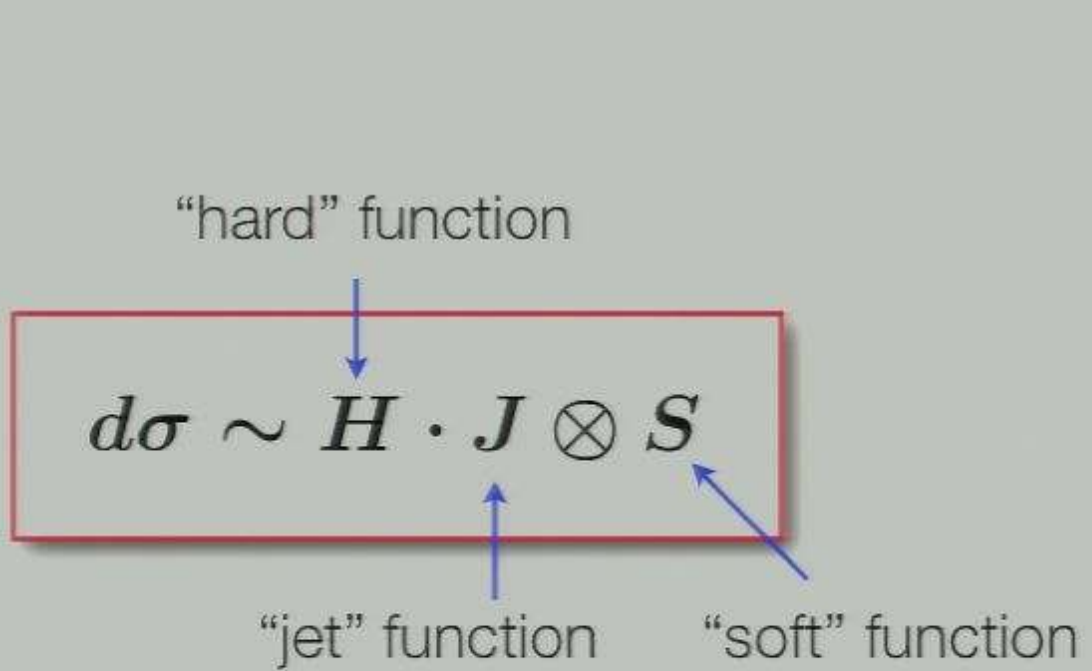
## Soft and collinear modes FACTORIZE in SCET:



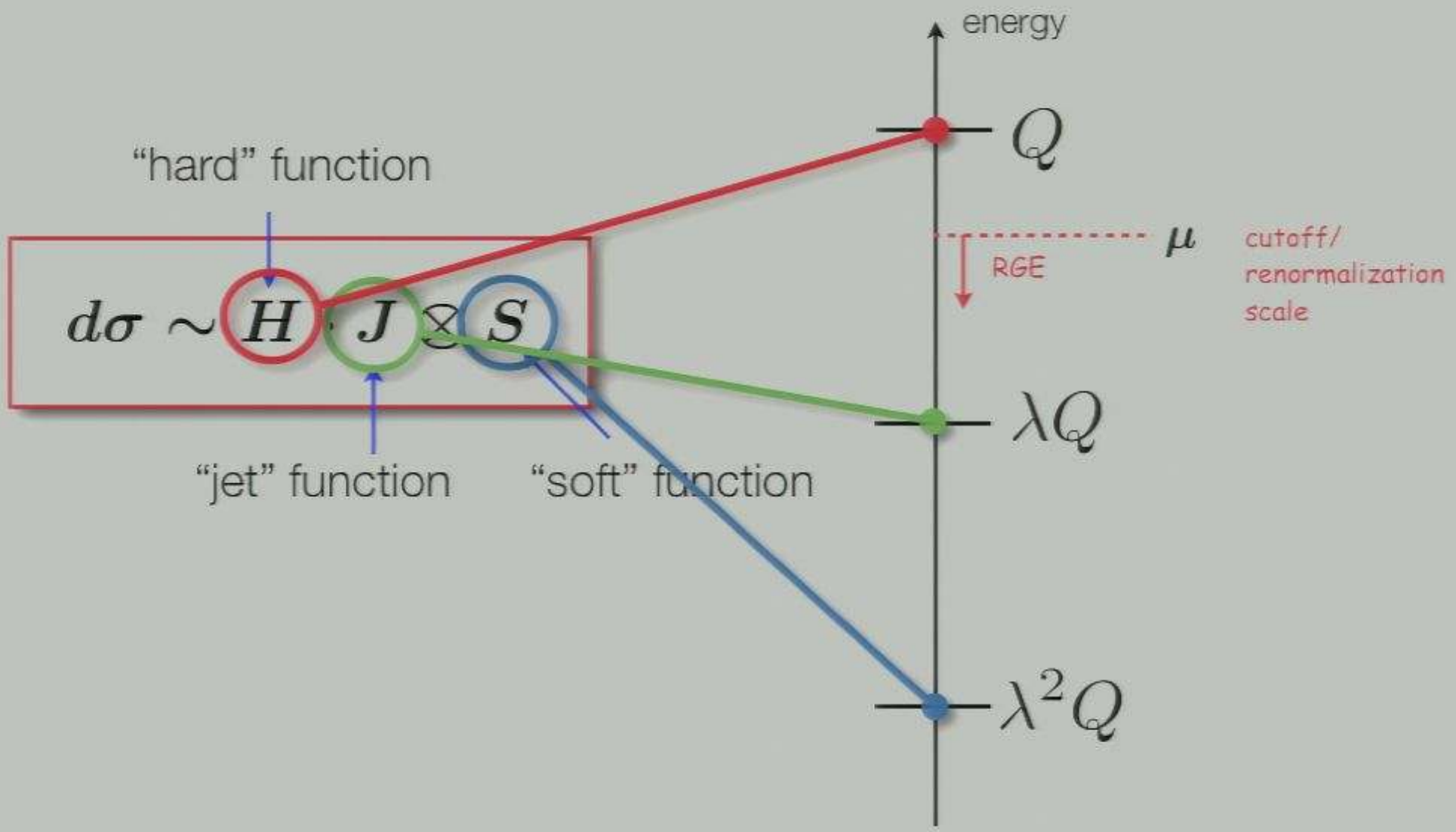

## Similarly, partons moving different collinear directions factorize:



Factorization at the level of the Lagrangian can be used to prove various factorization theorems:



(this form of factorization has been known since the 1980's, but now it is manifest in the Lagrangian)

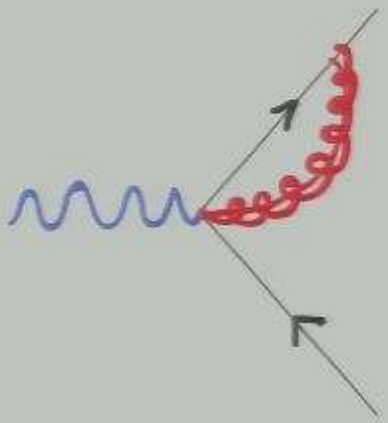



each of H, J and S depends on physics at a **single** scale - choose renormalization scale appropriately, using RGE to evolve to appropriate scales sums large logarithms in perturbation theory

# Technical aside ... zero-bin subtraction

Describing different momenta of the same (in QCD) field with separate fields can be subtle ... i.e. what is the difference between a $p \to 0$ collinear mode and a soft mode??

A: none! need to avoid double-counting



$$= \int \frac{d^4q}{(2\pi)^4} \mathcal{I}_n$$

"zero-bin"

# Technical aside ... zero-bin subtraction

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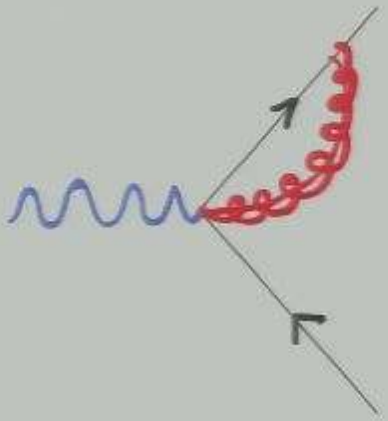
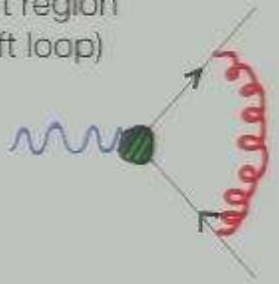


$$= \int \frac{d^4q}{(2\pi)^4} I_n$$

includes integration over soft region
(already accounted for in soft loop)

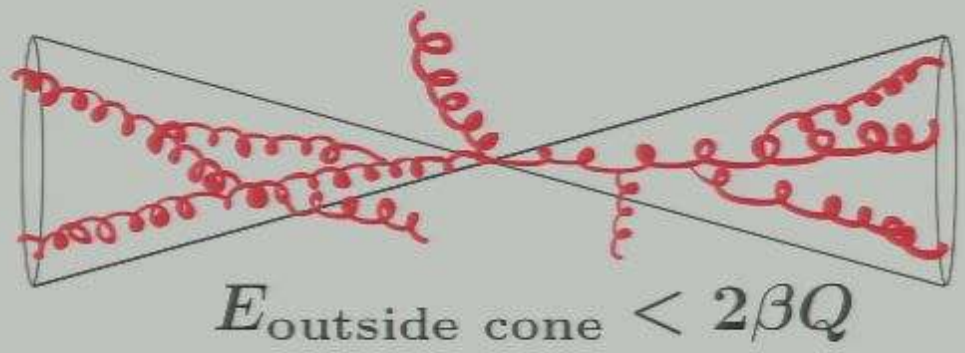


$$= \int \frac{d^4q}{(2\pi)^4} I_n - \int \frac{d^4q}{(2\pi)^4} \lim_{q \to \text{soft}} I_n$$

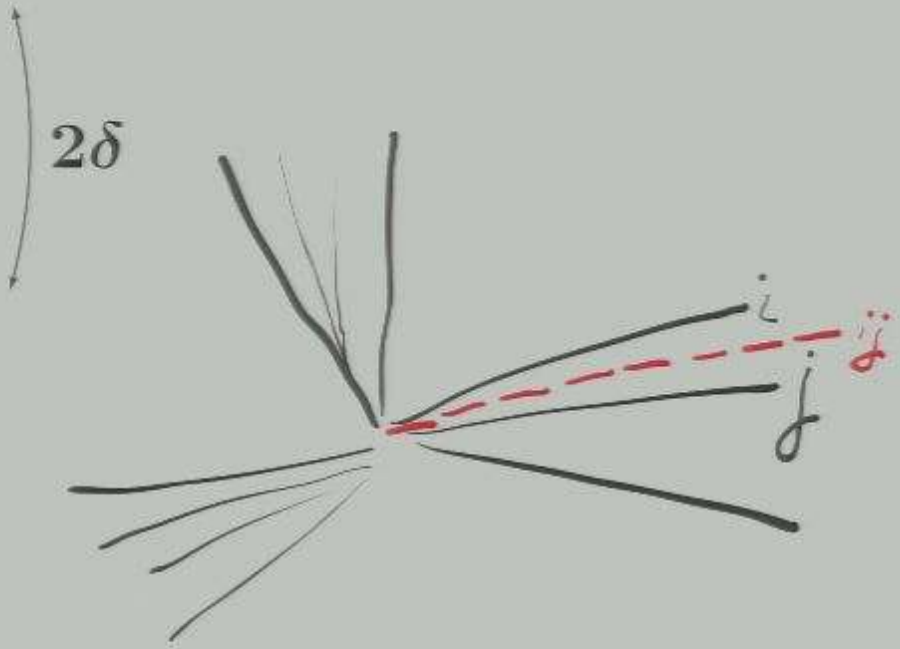
"zero-bin"

In most examples before this work, the zero-bin integral was scaleless and vanished in dimensional regularization, but it will be critical to getting phase space integrals right.

Back to $e^+e^- \to$ jets: how do we calculate this in SCET?

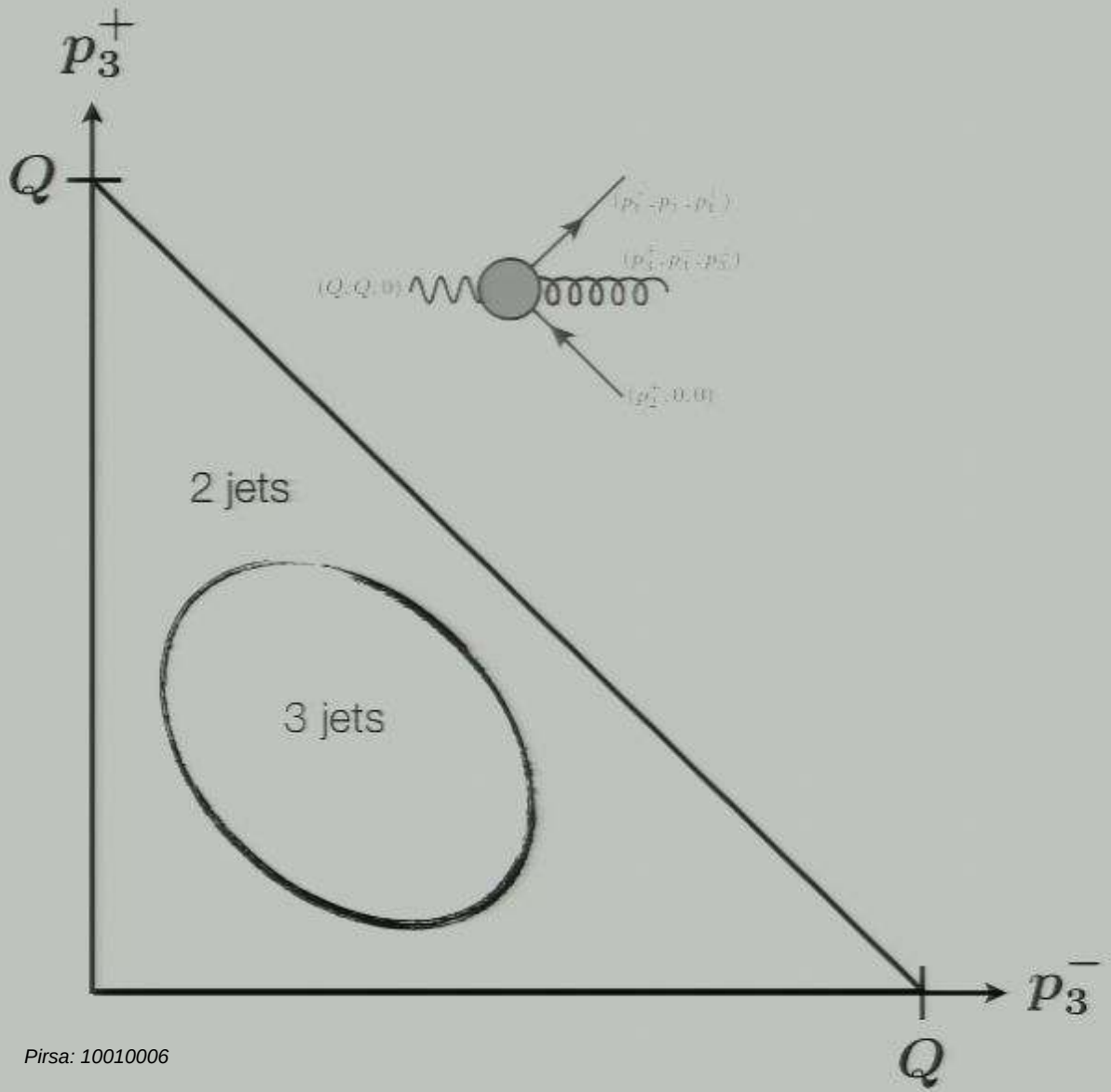


$$E_{\text{outside cone}} < 2\beta Q$$

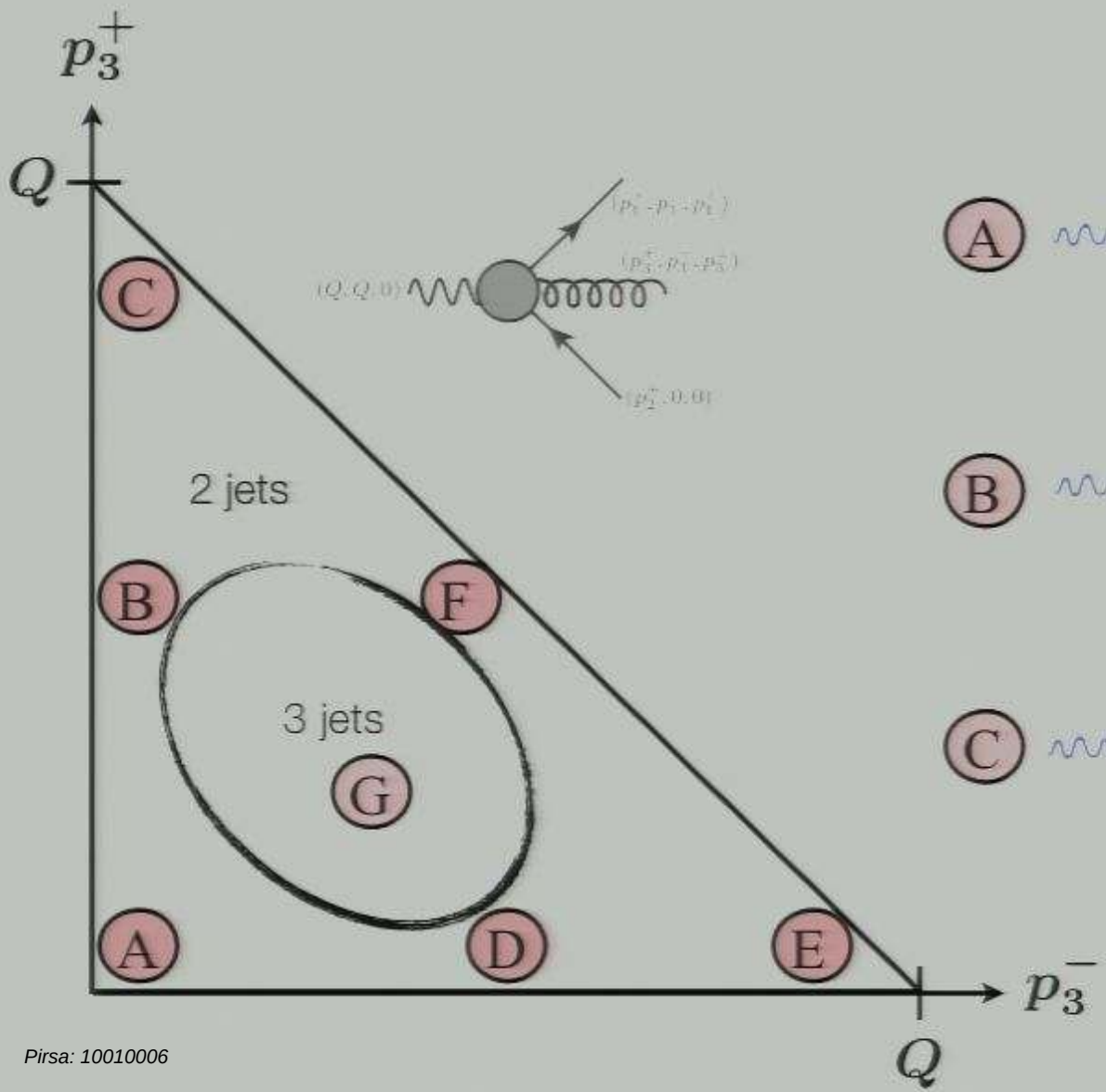



For definiteness, look at three different jet definitions: SW, JADE, $k_T$, calculate 2-jet rate in SCET at $O(\alpha_S)$

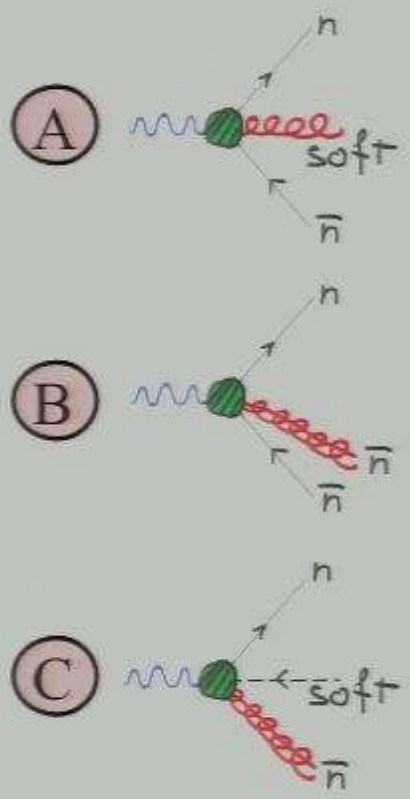
At $O(\alpha_s)$, a jet definition just determines the dijet region in 3-body phase space:

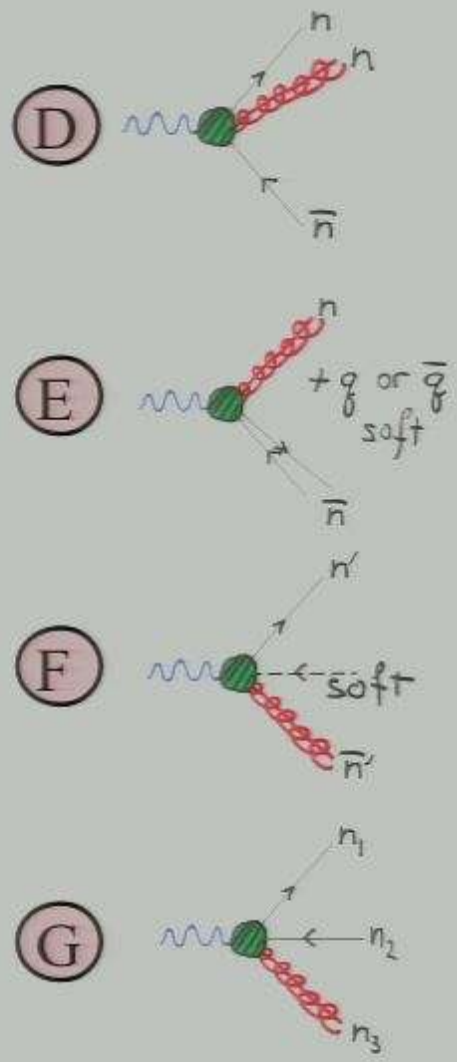


At $O(\alpha_s)$, a jet definition just determines the dijet region in 3-body phase space:

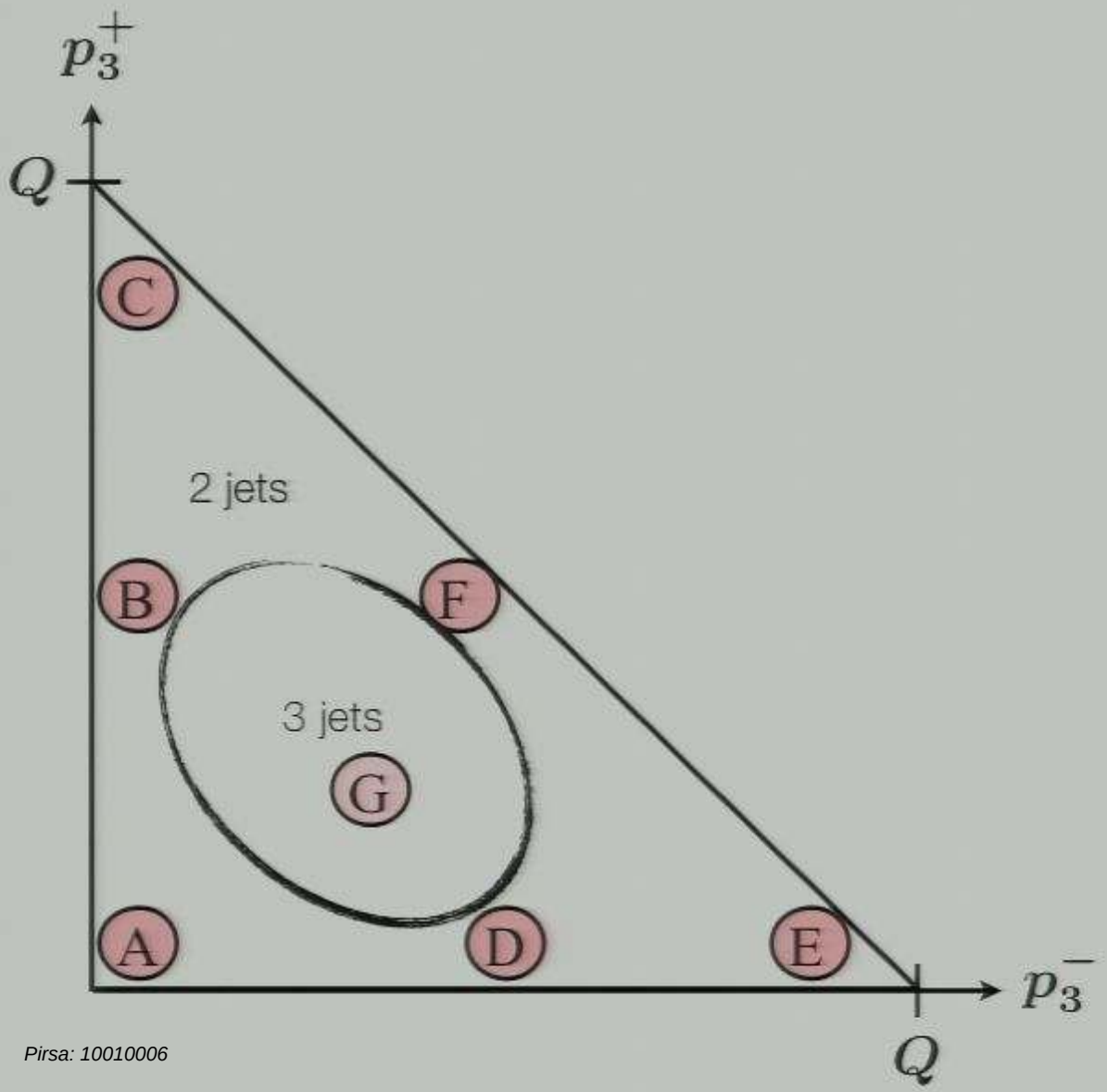




How do we do integrate over the 2-jet region in SCET?



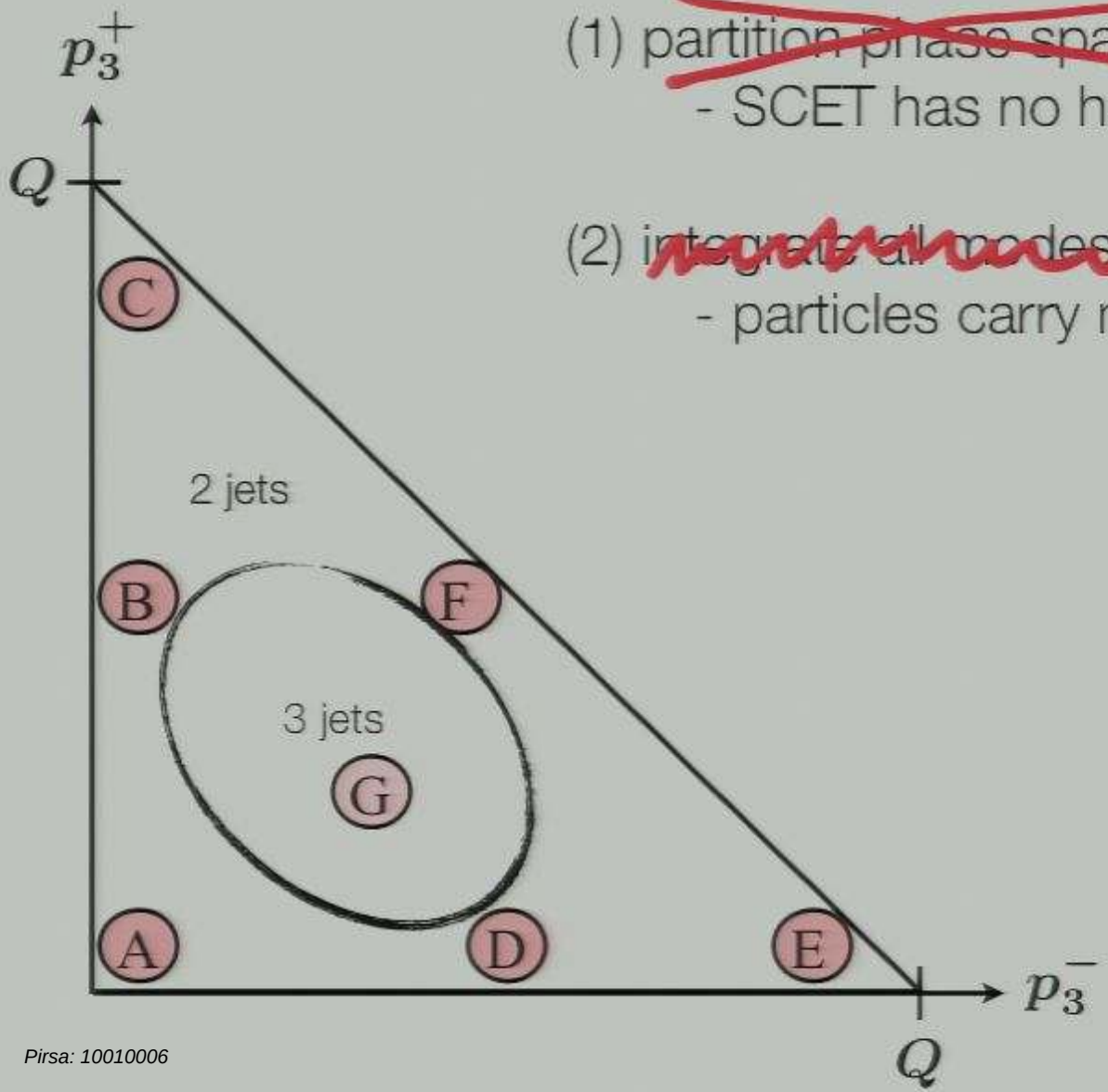
How do we do integrate over the 2-jet region in SCET?

(1) partition phase space?

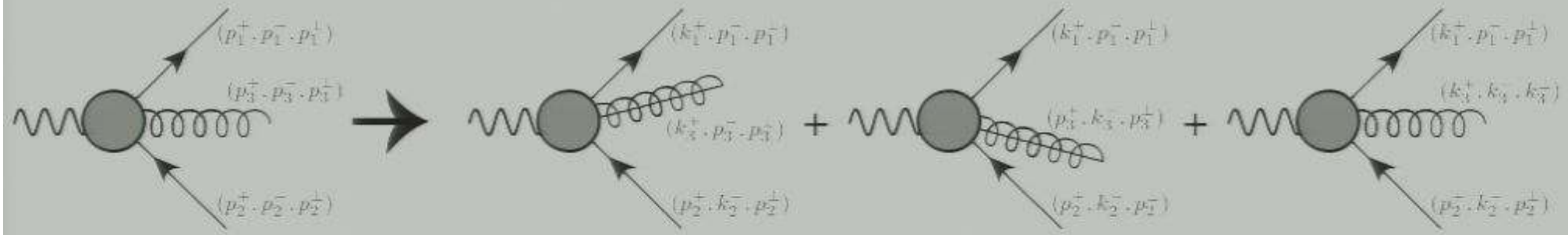
- SCET has no hard cutoff on momenta

(2) integrate all modes over all phase space?

- particles carry momenta above the cutoff!



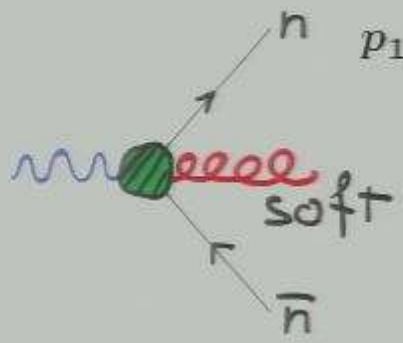
(3) as (2), but be consistent with power counting:




Momenta of different modes scale differently with λ:

$$p_i^{\pm} \sim Q \qquad p_i^{\perp} \sim \lambda Q \qquad k_i^{\mu} \sim \lambda^2 Q$$

Phase space constraints must be consistent with scaling:



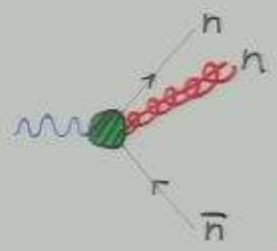

$$p_1 \sim Q(\lambda^2, 1, \lambda)$$

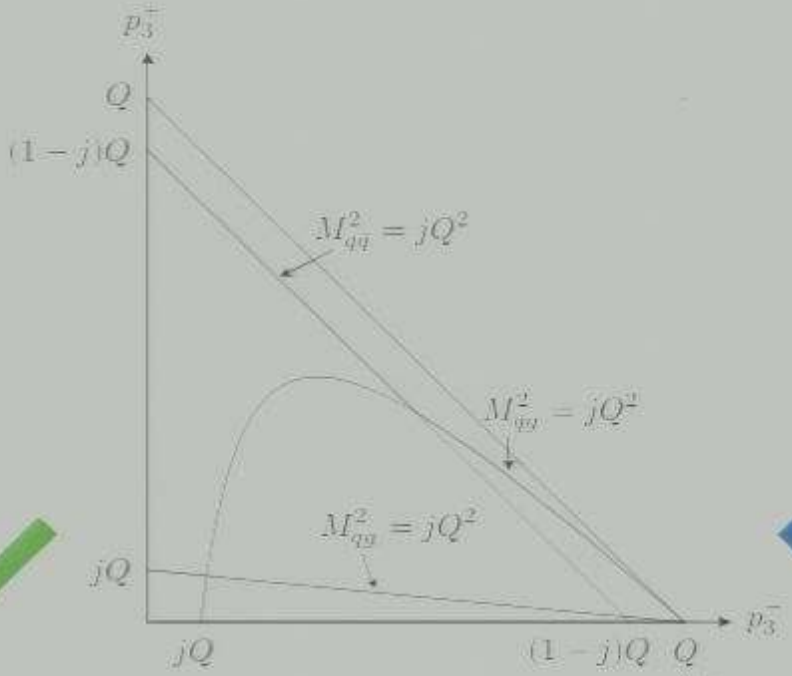
$$p_3 \sim Q(\lambda^2, \lambda^2, \lambda^2)$$

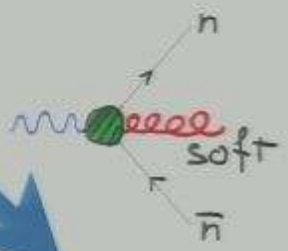
$$M_{13}^2 = (p_1 + p_3)^2 \sim \underbrace{p_1^- k_3^+}_{O(\lambda^2)} + O(\lambda^3)$$

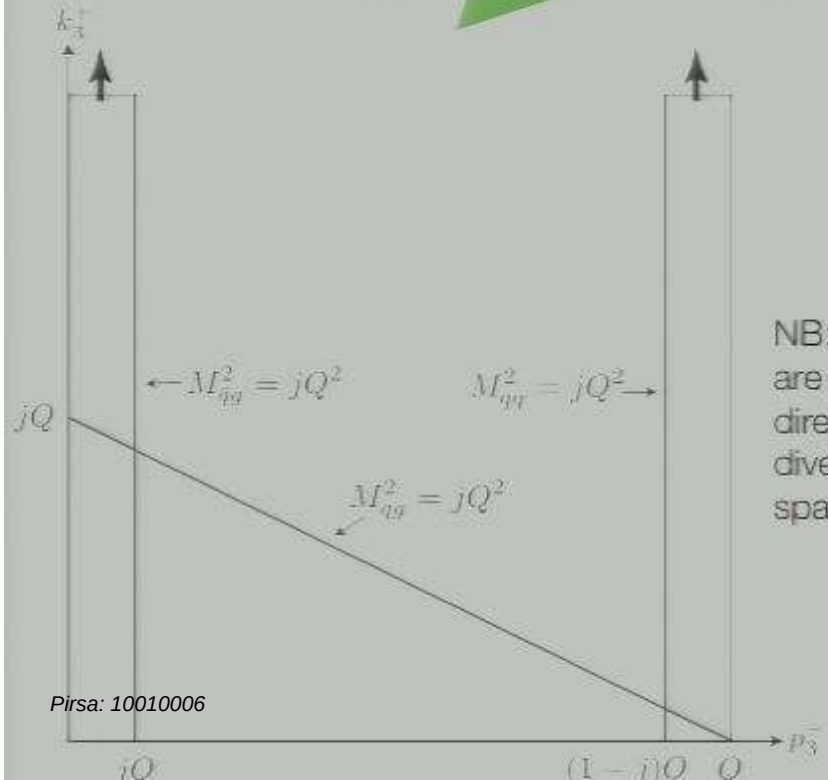
so QCD constraint $M_{13}^2 < jQ^2 \Rightarrow p_1^- k_3^+ < jQ^2$ in SCET

# ex: JADE

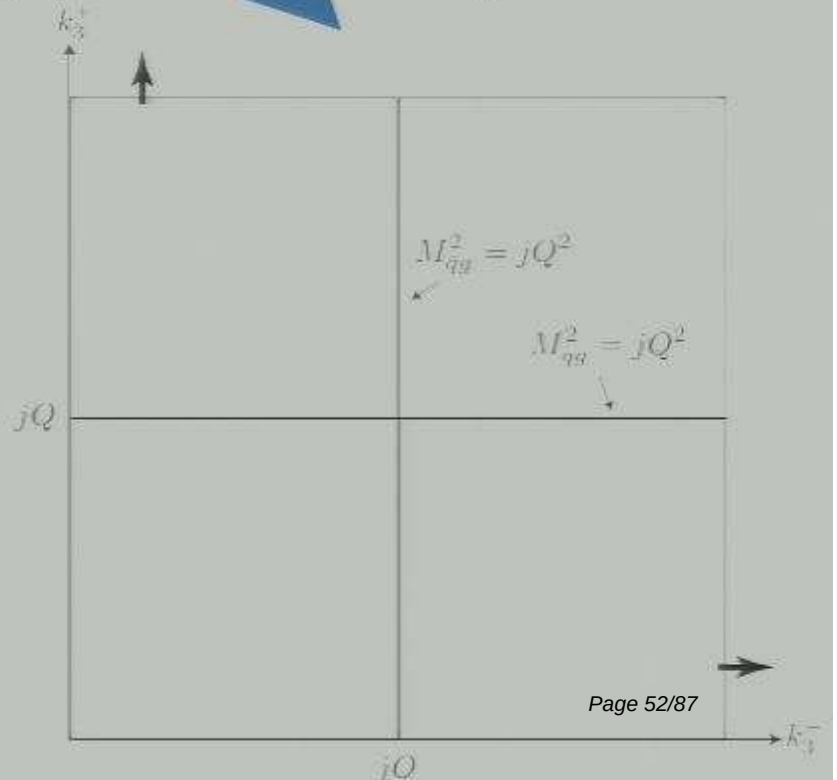


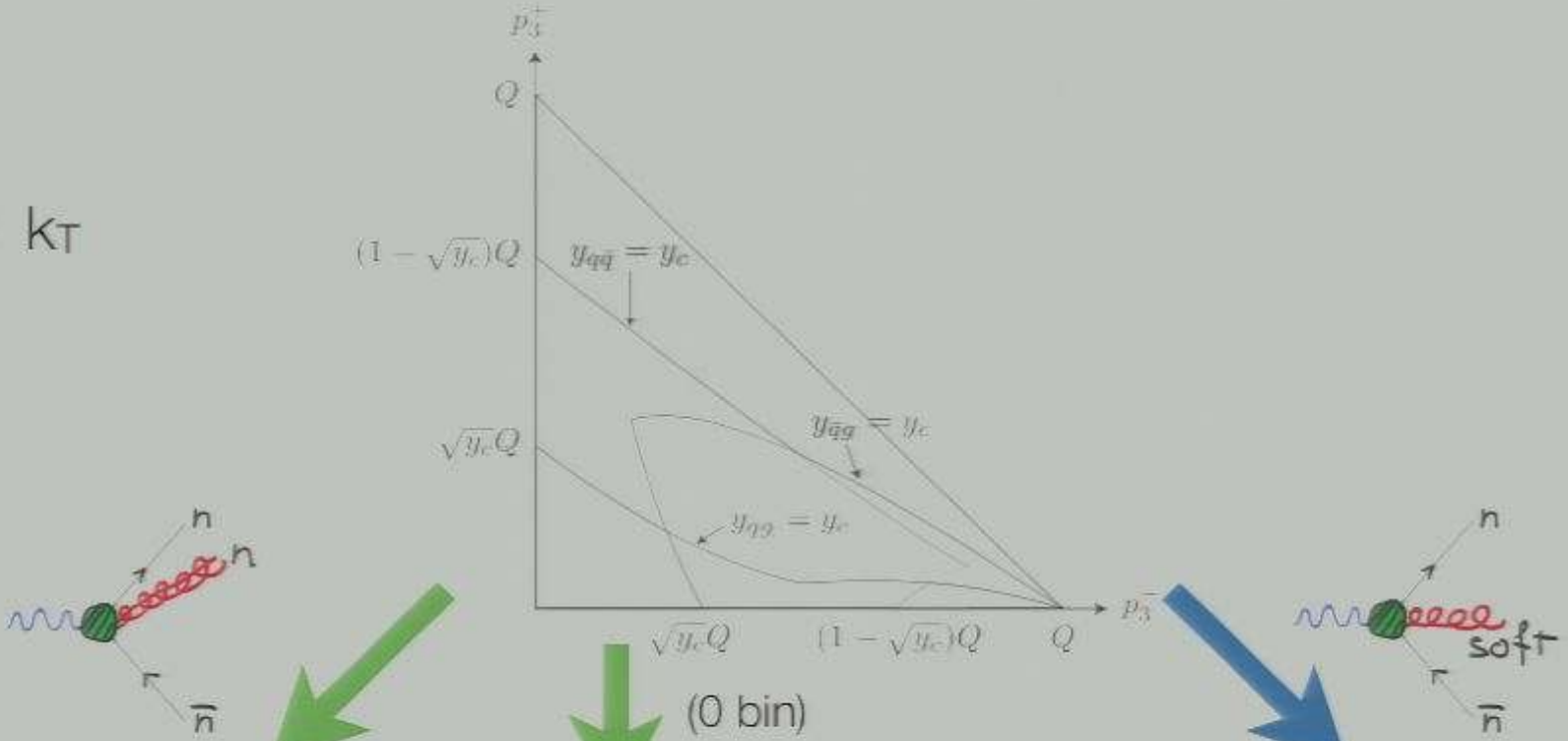



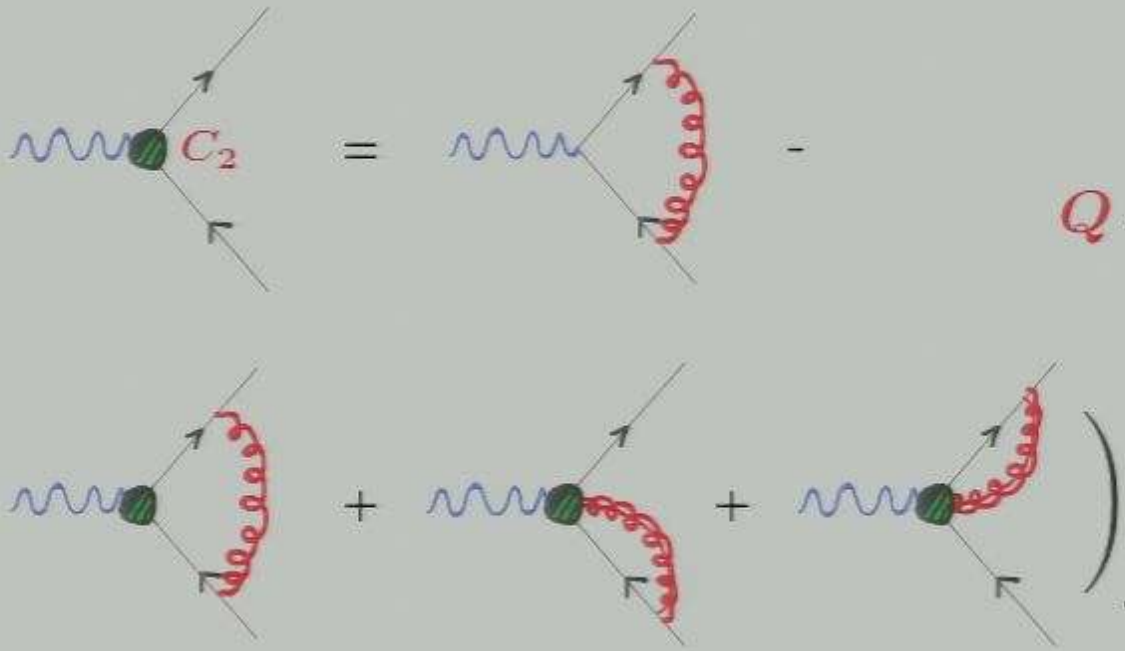

NB: phase space integrals are unbounded in some directions - get new UV divergences in phase space integrals



ex: $k_T$

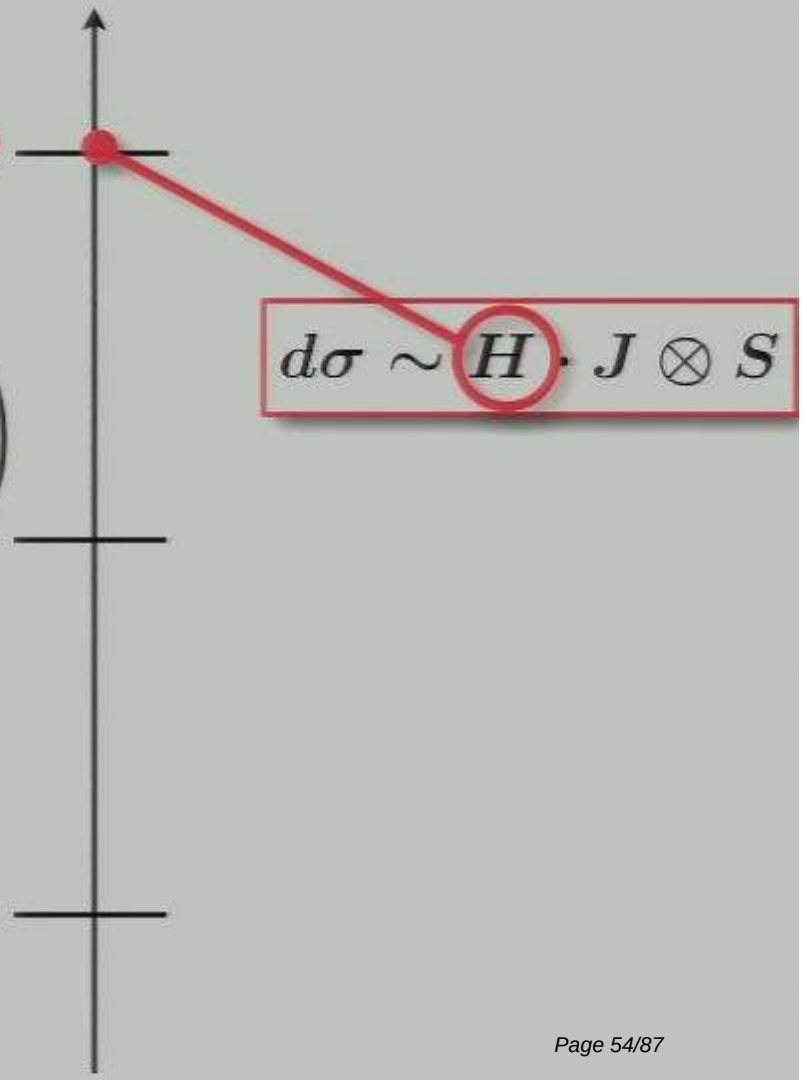

$p_3^+$
$Q$
$(1-\sqrt{y_c})Q$
$\sqrt{y_c}Q$
$y_{qq} = y_c$
$y_{\bar{q}\bar{q}} = y_c$
$y_{q\bar{q}} = y_c$
$\sqrt{y_c}Q$
$(1-\sqrt{y_c})Q$
$Q$
$p_3^-$
n
n
$\bar{n}$
n
soft
$\bar{n}$
(0 bin)
$k_3^-$
$y_{qq} = y_c$
$y_{qq} = y_c$
$y_{q\bar{q}} = y_c$
$p_3^-$
$k_3^-$
$y_{\bar{q}\bar{q}} = y_c$
$y_{qq} = y_c$
$k_3^-$
$k_3^-$
$y_{\bar{q}\bar{q}} = y_c$
$y_{q\bar{q}} = y_c$

# (1) Hard scale: matching onto SCET operator O2

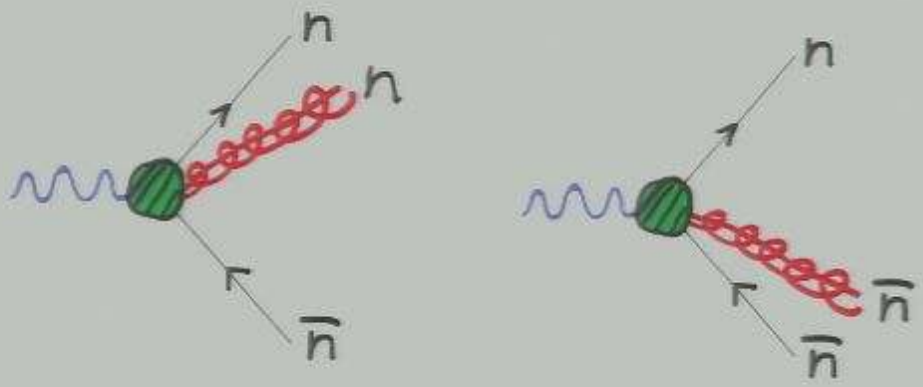



$$C_2 = 1 + \frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{2}\ln^2\frac{\mu^2}{-Q^2} - \frac{3}{2}\ln\frac{\mu^2}{-Q^2} - 4 + \frac{\pi^2}{12}\right)$$

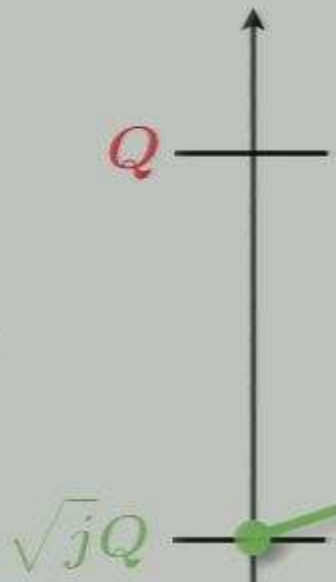
$$Z_2 = 1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{1}{\epsilon}\ln\frac{\mu^2}{-Q^2}\right)$$



# (2) Jet scale: emission of collinear gluons (incl. zero-bin subtraction)




(loop graphs are scaleless - vanish in dim. reg.)




$$d\sigma \sim H \cdot J \otimes S$$

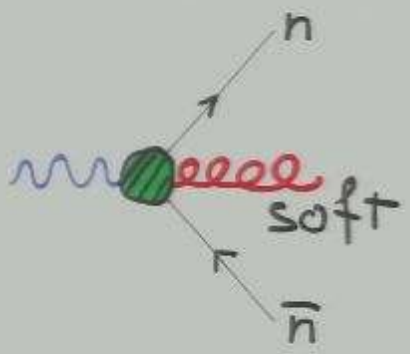
$$\frac{1}{\sigma_0}\tilde{\sigma}^n_{\text{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2\epsilon} + \frac{2}{\epsilon}\ln j + \frac{3}{2}\ln\frac{\mu^2}{jQ^2} + 2\ln\frac{\mu^2}{Q^2}\ln j - 3\ln^2 j - \frac{\pi^2}{3} + \frac{7}{2}\right)$$

$$\frac{1}{\sigma_0}\sigma^{0\text{ bin}}_{\text{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\mu^2}{j^2Q^2} - \ln^2\frac{\mu^2}{j^2Q^2} + \frac{\pi^2}{6}\right)$$

$$\frac{1}{\sigma_0}\sigma^n_{\text{JADE}} = \frac{1}{\sigma_0}(\tilde{\sigma}^n_{\text{JADE}} - \sigma^{0\text{ bin}}_{\text{JADE}})$$

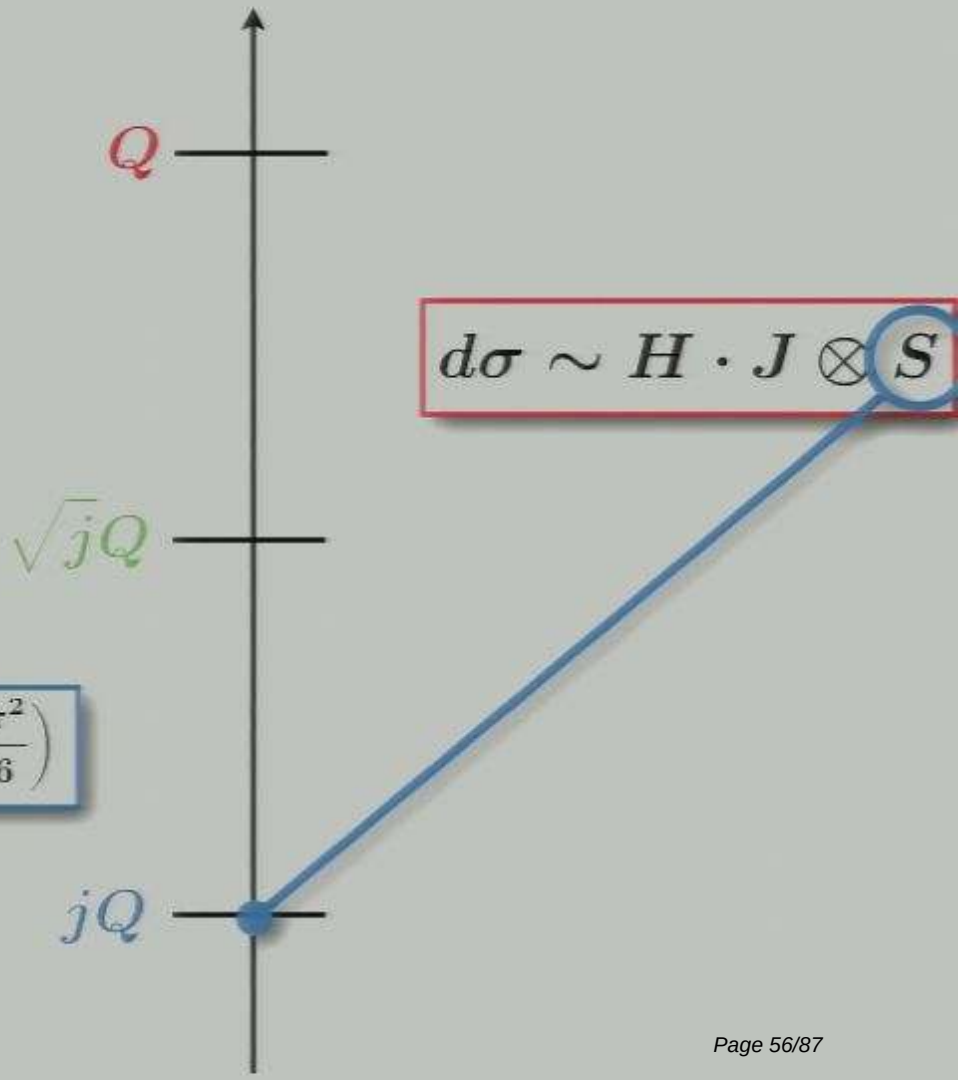
$$= \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{2}{\epsilon}\ln\frac{\mu^2}{jQ^2} + \frac{3}{2}\ln\frac{\mu^2}{jQ^2} + \ln^2\frac{\mu^2}{jQ^2} - \frac{\pi^2}{2} + \frac{7}{2}\right)$$

# (3) Soft scale: emission of soft gluons




(loop graphs are scaleless - vanish in dim. reg.)

$$\frac{1}{\sigma_0}\sigma^s_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\mu^2}{j^2Q^2} - \ln^2\frac{\mu^2}{j^2Q^2} + \frac{\pi^2}{6}\right)$$



# Combine the results - reproduce QCD result

$$C_2 = 1 + \frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{2}\ln^2\frac{\mu^2}{-Q^2} - \frac{3}{2}\ln\frac{\mu^2}{-Q^2} - 4 + \frac{\pi^2}{12}\right)$$

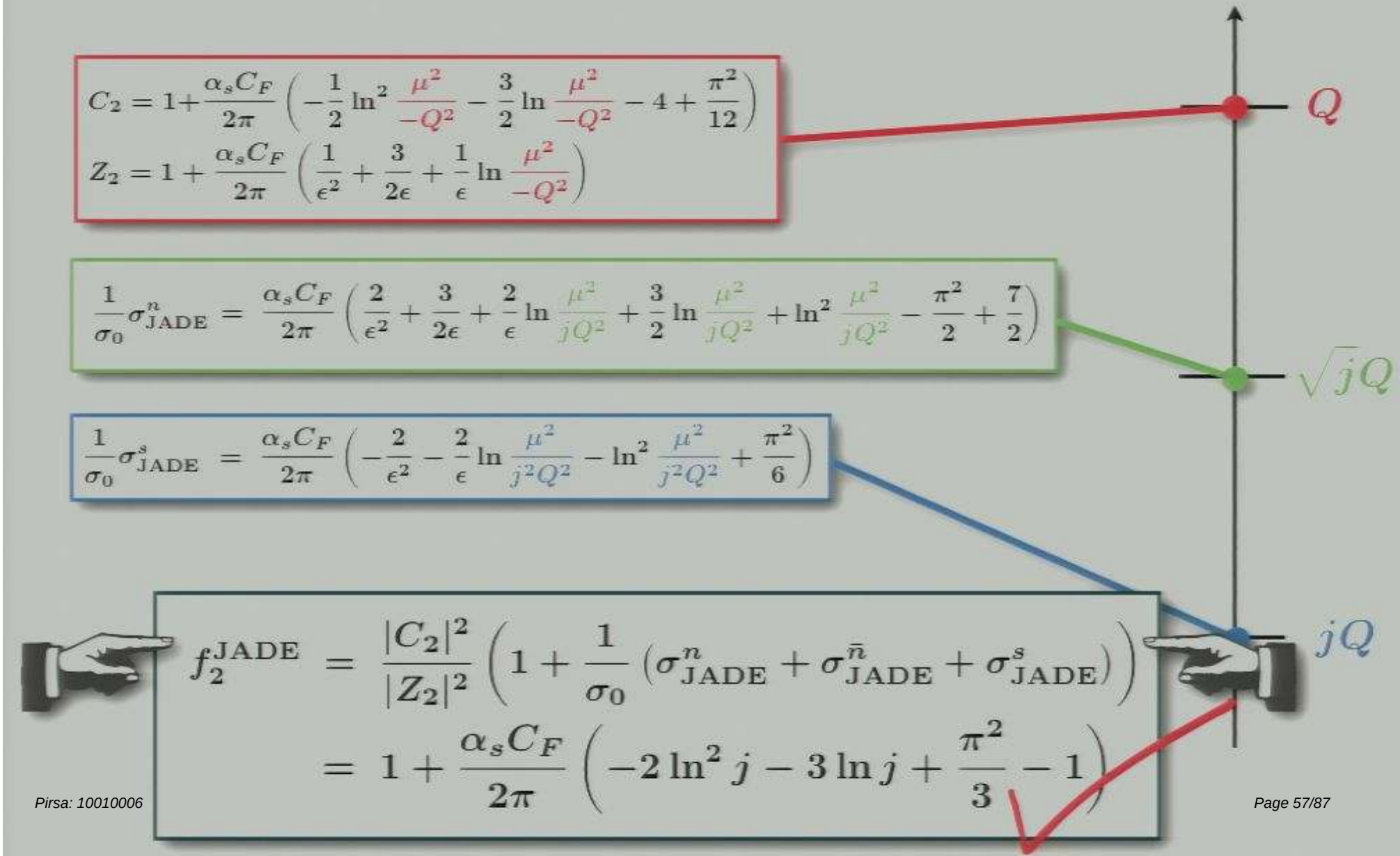
$$Z_2 = 1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{1}{\epsilon}\ln\frac{\mu^2}{-Q^2}\right)$$

$$\frac{1}{\sigma_0}\sigma^n_{\text{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{2}{\epsilon}\ln\frac{\mu^2}{jQ^2} + \frac{3}{2}\ln\frac{\mu^2}{jQ^2} + \ln^2\frac{\mu^2}{jQ^2} - \frac{\pi^2}{2} + \frac{7}{2}\right)$$

$$\frac{1}{\sigma_0}\sigma^s_{\text{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\mu^2}{j^2Q^2} - \ln^2\frac{\mu^2}{j^2Q^2} + \frac{\pi^2}{6}\right)$$

$$f_2^{\text{JADE}} = \frac{|C_2|^2}{|Z_2|^2}\left(1 + \frac{1}{\sigma_0}\left(\sigma^n_{\text{JADE}} + \sigma^{\bar{n}}_{\text{JADE}} + \sigma^s_{\text{JADE}}\right)\right)$$

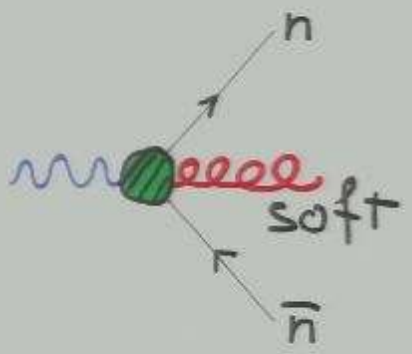
$$= 1 + \frac{\alpha_s C_F}{2\pi}\left(-2\ln^2 j - 3\ln j + \frac{\pi^2}{3} - 1\right)$$



Comments:

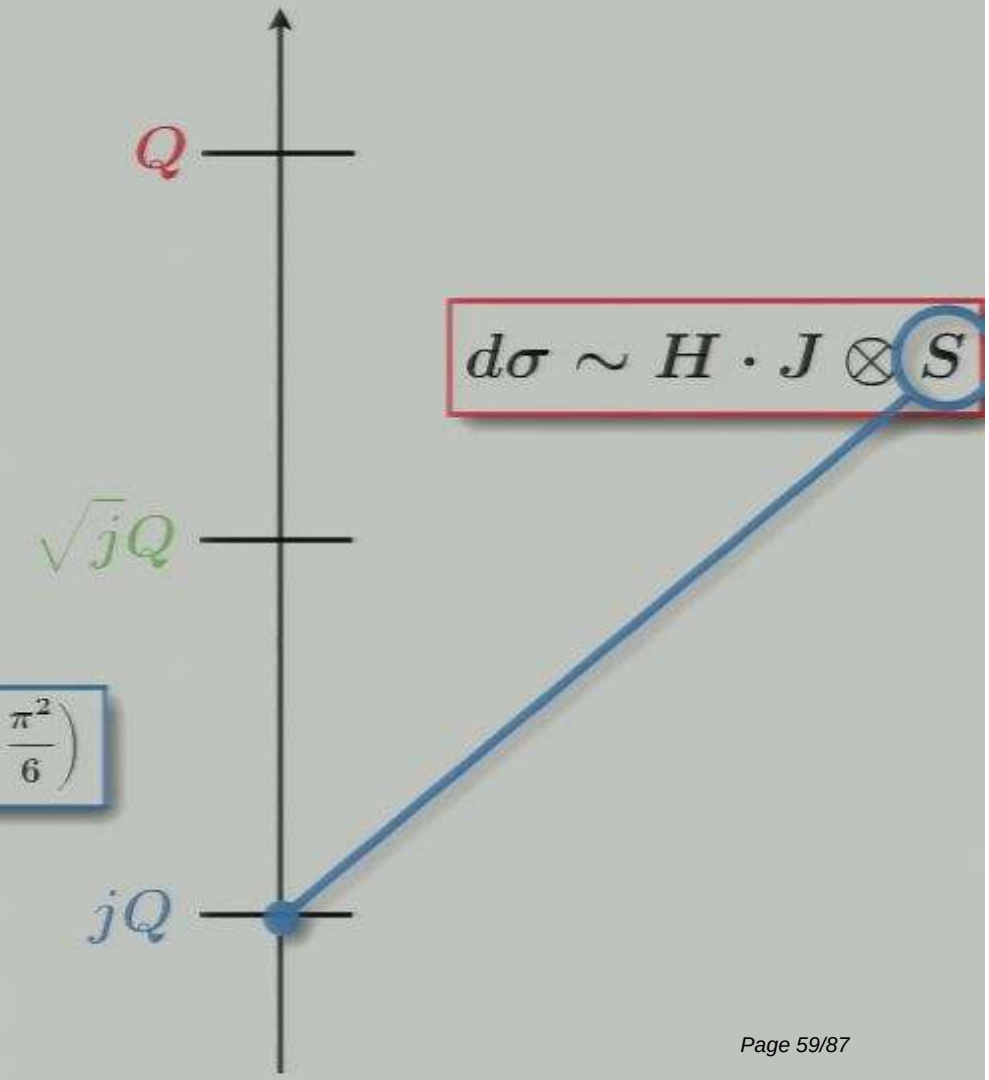
(1) zero-bin is non-trivial and required

# (3) Soft scale: emission of soft gluons

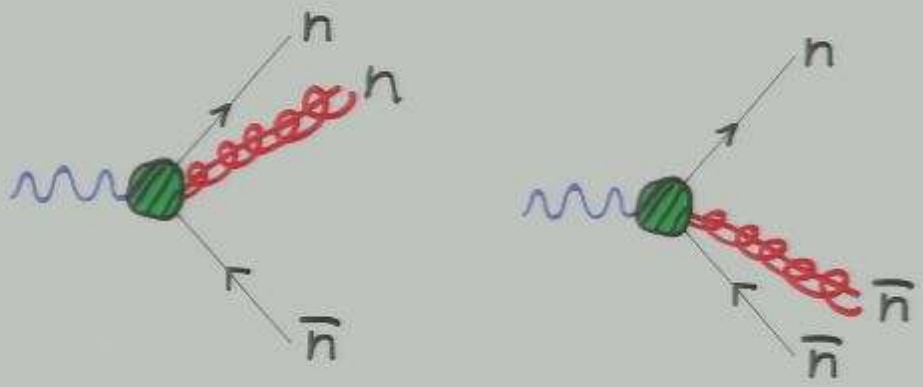



(loop graphs are scaleless - vanish in dim. reg.)

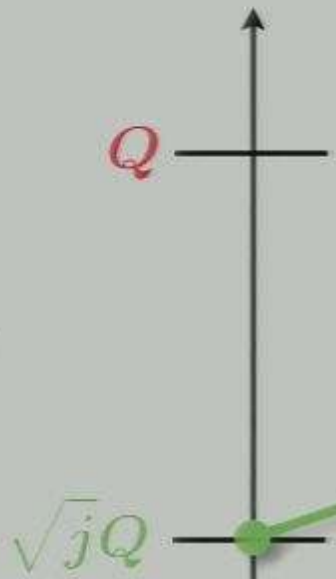
$$\frac{1}{\sigma_0}\sigma^s_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\mu^2}{j^2Q^2} - \ln^2\frac{\mu^2}{j^2Q^2} + \frac{\pi^2}{6}\right)$$



## (2) Jet scale: emission of collinear gluons (incl. zero-bin subtraction)




(loop graphs are scaleless - vanish in dim. reg.)




$$d\sigma \sim H \cdot J \otimes S$$

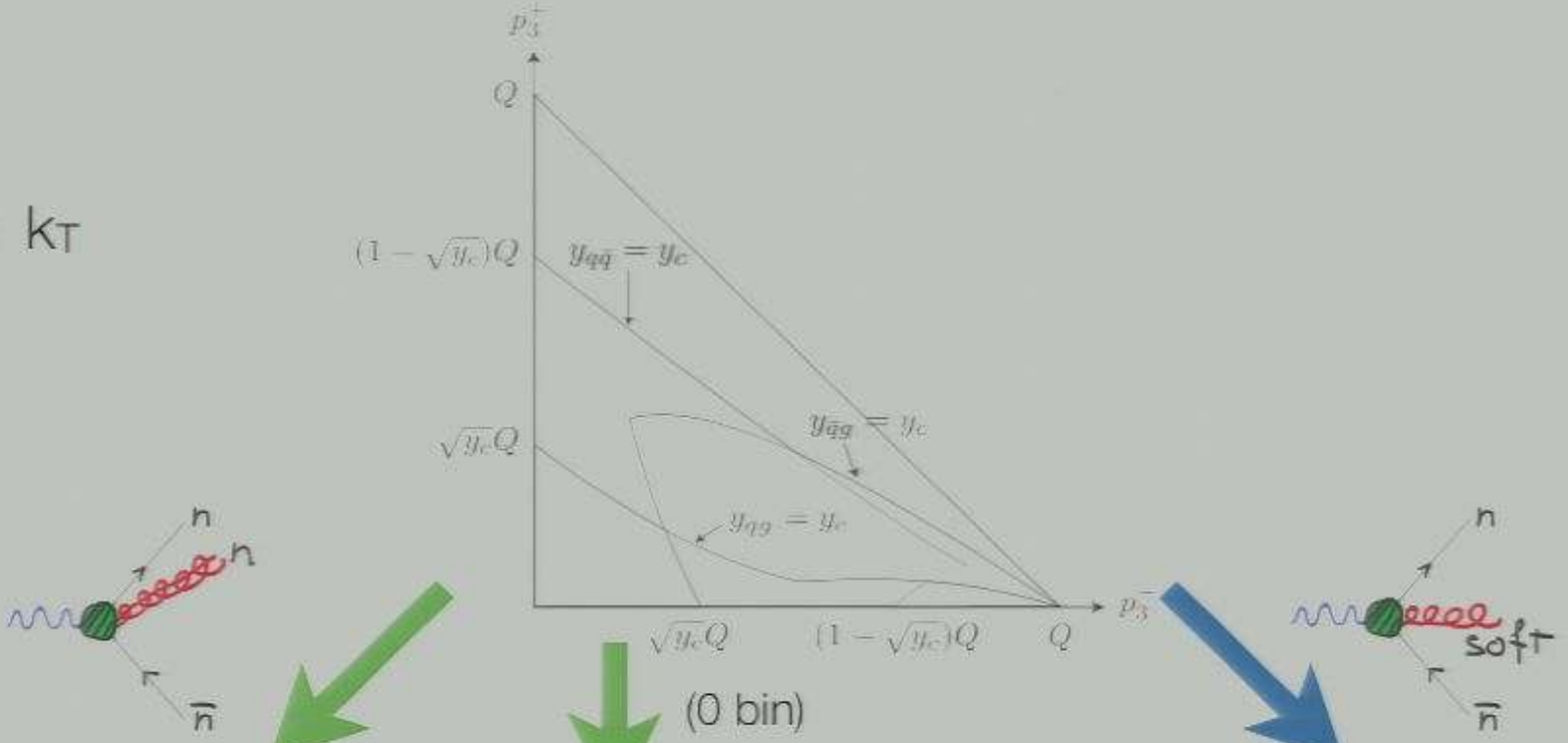
$$\frac{1}{\sigma_0}\tilde{\sigma}^n_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2\epsilon} + \frac{2}{\epsilon}\ln j + \frac{3}{2}\ln\frac{\mu^2}{jQ^2} + 2\ln\frac{\mu^2}{Q^2}\ln j - 3\ln^2 j - \frac{\pi^2}{3} + \frac{7}{2}\right)$$

$$\frac{1}{\sigma_0}\sigma^{0\text{ bin}}_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\mu^2}{j^2Q^2} - \ln^2\frac{\mu^2}{j^2Q^2} + \frac{\pi^2}{6}\right)$$

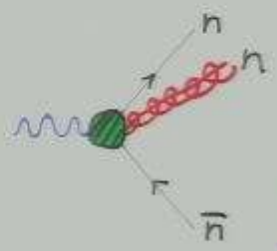
$$\frac{1}{\sigma_0}\sigma^n_{\rm JADE} = \frac{1}{\sigma_0}(\tilde{\sigma}^n_{\rm JADE} - \sigma^{0\text{ bin}}_{\rm JADE})$$

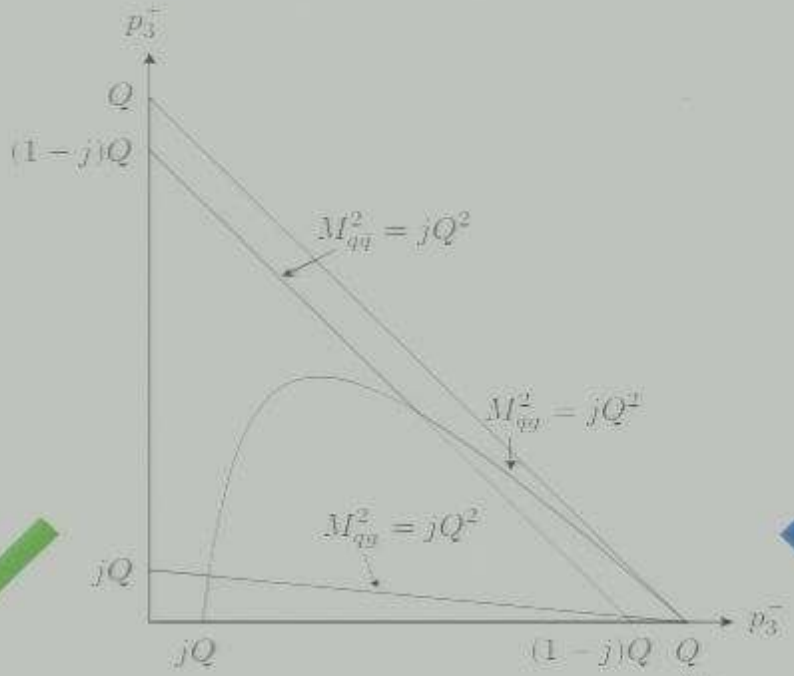
$$= \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{2}{\epsilon}\ln\frac{\mu^2}{jQ^2} + \frac{3}{2}\ln\frac{\mu^2}{jQ^2} + \ln^2\frac{\mu^2}{jQ^2} - \frac{\pi^2}{2} + \frac{7}{2}\right)$$

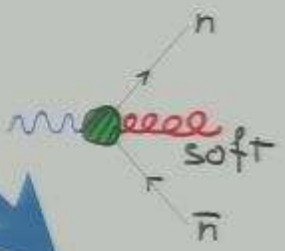
ex: $k_T$

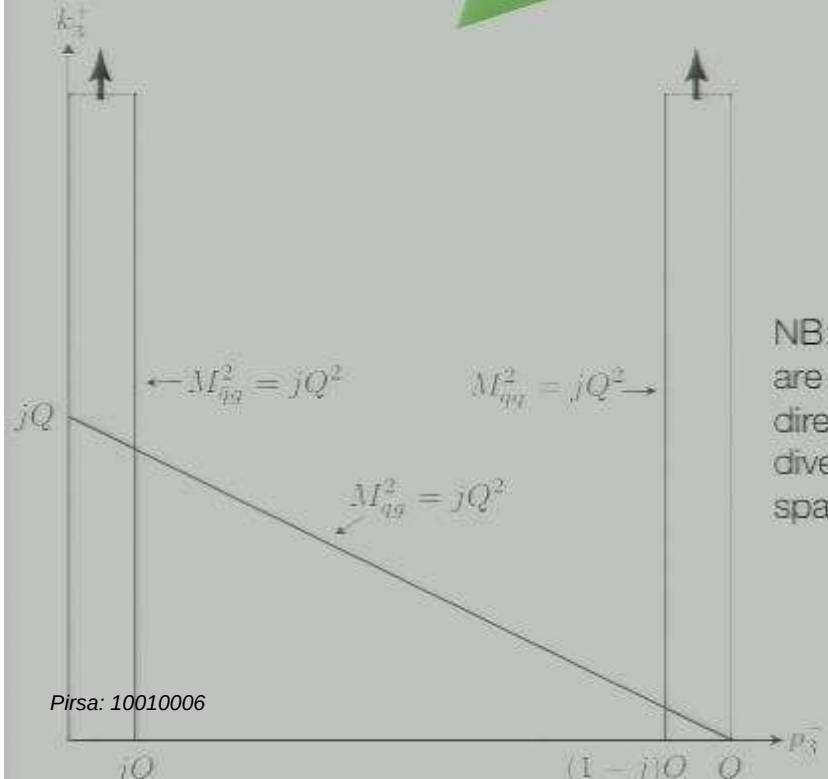

$p_3^+$
$Q$
$(1-\sqrt{y_c})Q$
$\sqrt{y_c}Q$
$y_{qq} = y_c$
$y_{\bar{q}\bar{q}} = y_c$
$y_{q\bar{q}} = y_c$
$\sqrt{y_c}Q$
$(1-\sqrt{y_c})Q$
$Q$
$p_3^-$
n
n
$\bar{n}$
n
soft
$\bar{n}$
(0 bin)
$k_3^-$
$y_{q\bar{q}} = y_c$
$y_{qq} = y_c$
$y_{\bar{q}\bar{q}} = y_c$
$y_{\bar{q}\bar{q}} = y_c$
$y_{q\bar{q}} = y_c$
$p_3^-$
$k_3^-$

# ex: JADE

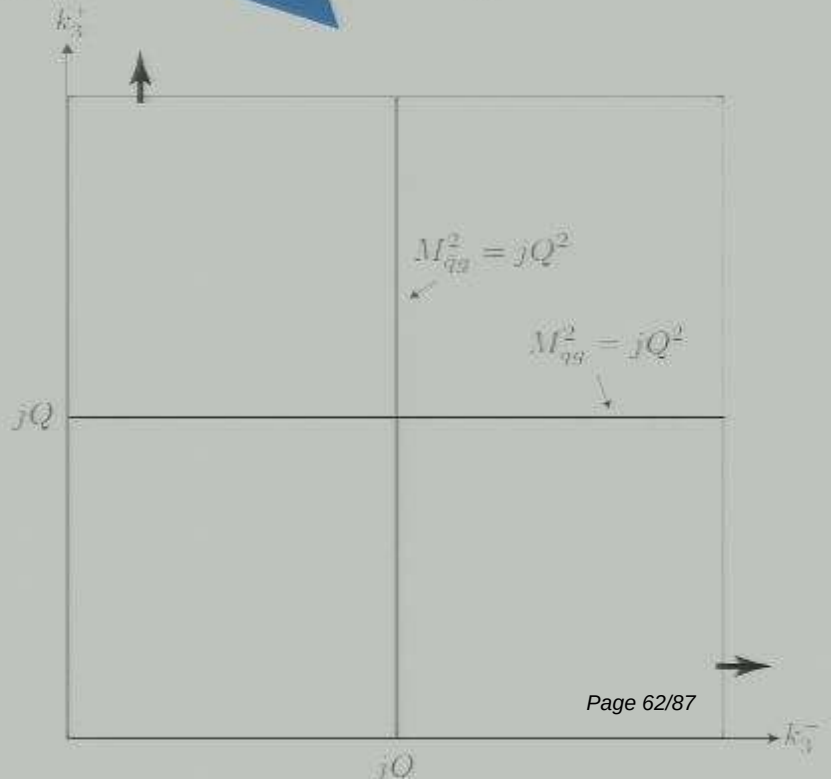


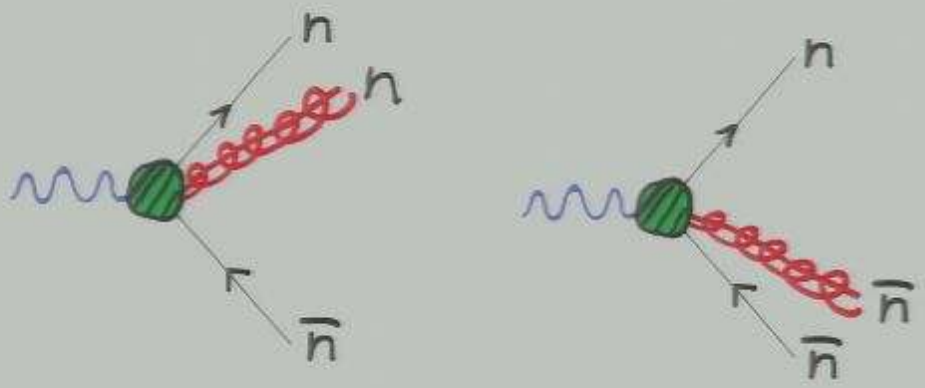



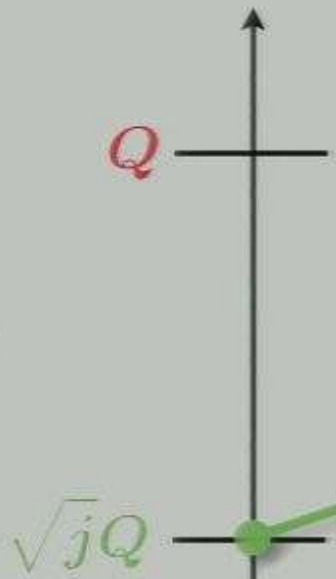

NB: phase space integrals are unbounded in some directions - get new UV divergences in phase space integrals



# (2) Jet scale: emission of collinear gluons (incl. zero-bin subtraction)




(loop graphs are scaleless - vanish in dim. reg.)




$$d\sigma \sim H \cdot J \otimes S$$

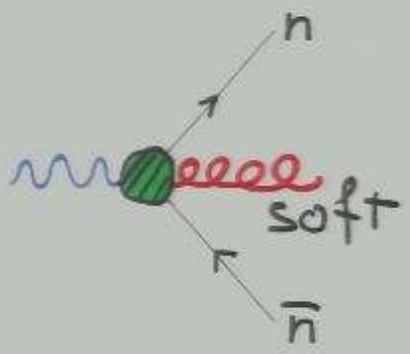
$$\frac{1}{\sigma_0}\tilde{\sigma}^n_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2\epsilon} + \frac{2}{\epsilon}\ln j + \frac{3}{2}\ln\frac{\mu^2}{jQ^2} + 2\ln\frac{\mu^2}{Q^2}\ln j - 3\ln^2 j - \frac{\pi^2}{3} + \frac{7}{2}\right)$$

$$\frac{1}{\sigma_0}\sigma^{0\text{ bin}}_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\mu^2}{j^2Q^2} - \ln^2\frac{\mu^2}{j^2Q^2} + \frac{\pi^2}{6}\right)$$

$$\frac{1}{\sigma_0}\sigma^n_{\rm JADE} = \frac{1}{\sigma_0}(\tilde{\sigma}^n_{\rm JADE} - \sigma^{0\text{ bin}}_{\rm JADE})$$

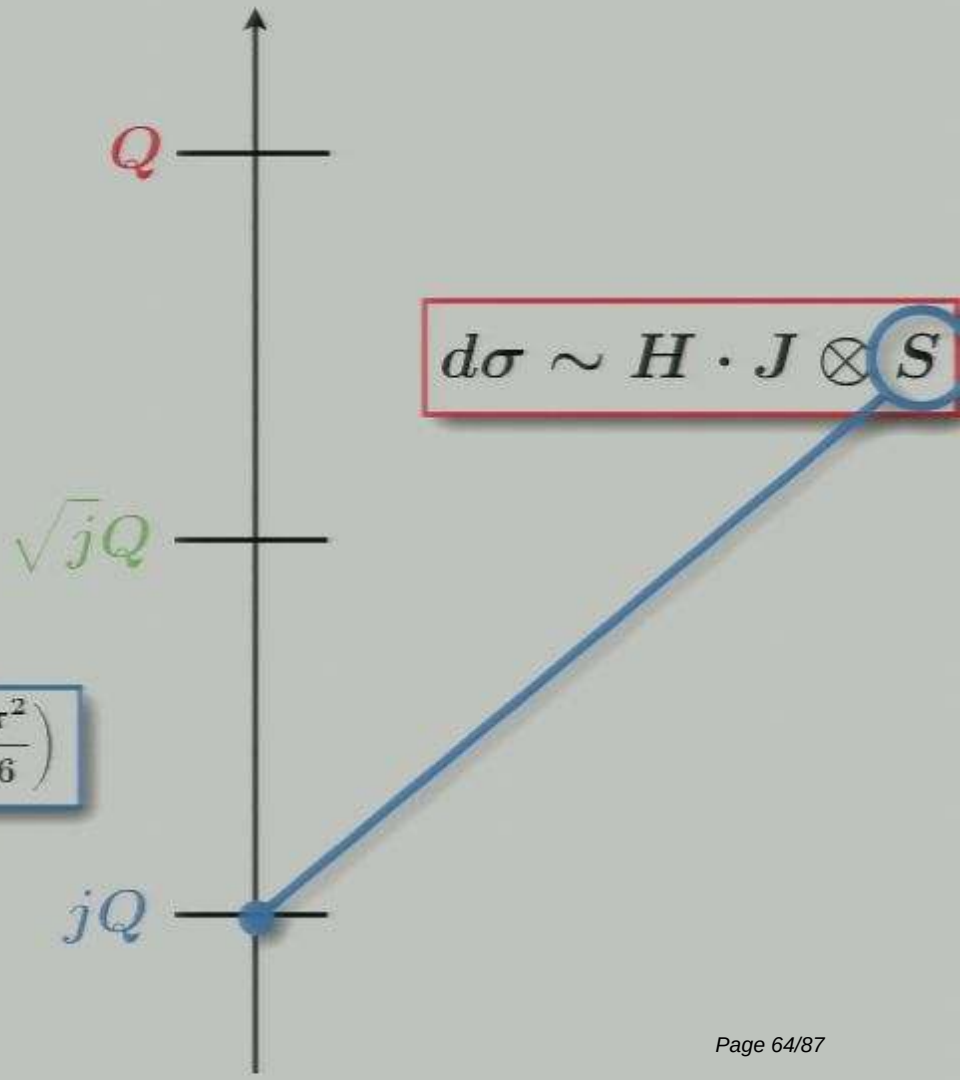
$$= \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{2}{\epsilon}\ln\frac{\mu^2}{jQ^2} + \frac{3}{2}\ln\frac{\mu^2}{jQ^2} + \ln^2\frac{\mu^2}{jQ^2} - \frac{\pi^2}{2} + \frac{7}{2}\right)$$

# (3) Soft scale: emission of soft gluons




(loop graphs are scaleless - vanish in dim. reg.)

$$\frac{1}{\sigma_0}\sigma^s_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\mu^2}{j^2Q^2} - \ln^2\frac{\mu^2}{j^2Q^2} + \frac{\pi^2}{6}\right)$$



# Combine the results - reproduce QCD result

$$C_2 = 1 + \frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{2}\ln^2\frac{\mu^2}{-Q^2} - \frac{3}{2}\ln\frac{\mu^2}{-Q^2} - 4 + \frac{\pi^2}{12}\right)$$

$$Z_2 = 1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{1}{\epsilon}\ln\frac{\mu^2}{-Q^2}\right)$$

$$\frac{1}{\sigma_0}\sigma^n_{\text{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{2}{\epsilon}\ln\frac{\mu^2}{jQ^2} + \frac{3}{2}\ln\frac{\mu^2}{jQ^2} + \ln^2\frac{\mu^2}{jQ^2} - \frac{\pi^2}{2} + \frac{7}{2}\right)$$

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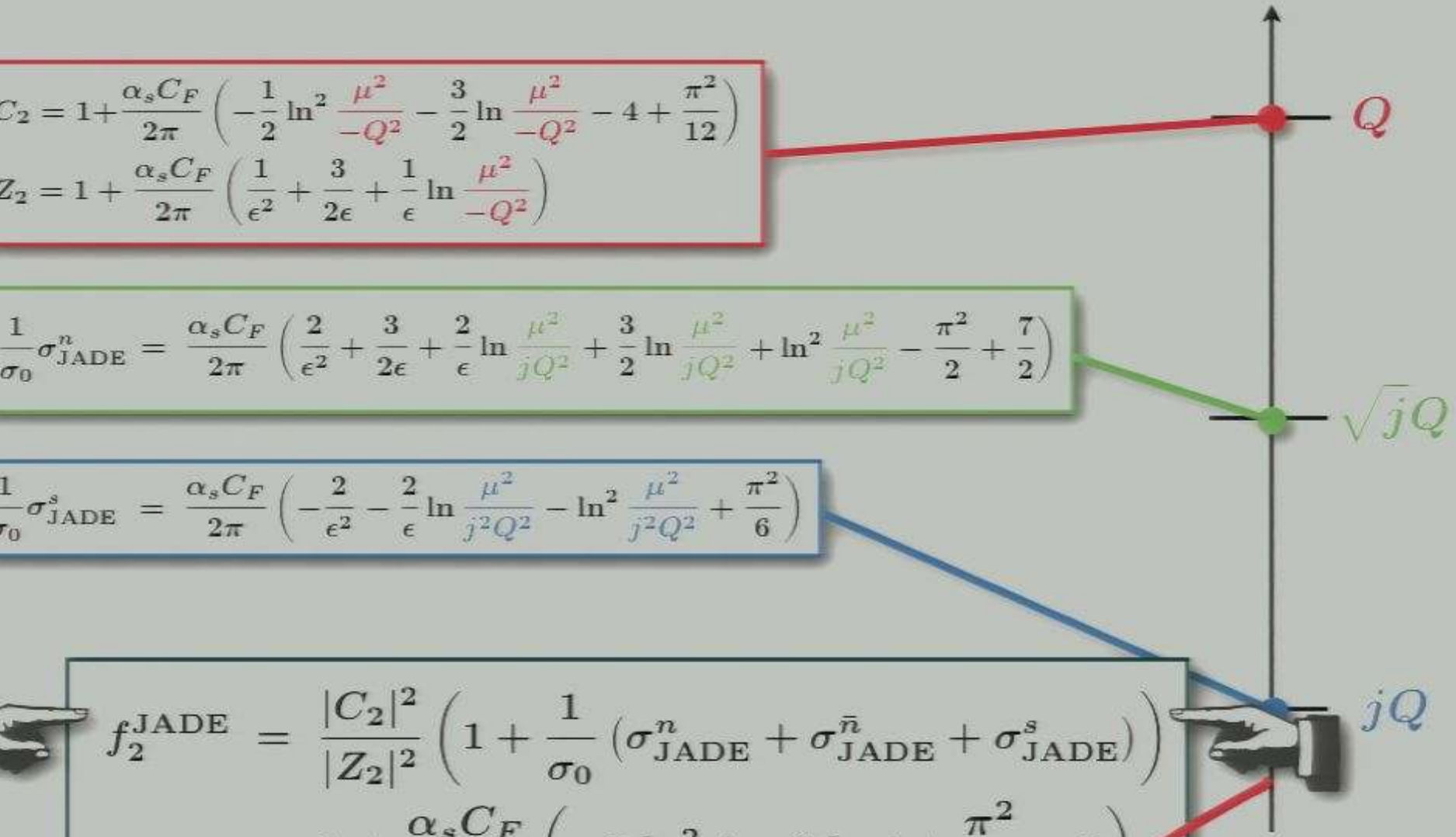
$$f_2^{\text{JADE}} = \frac{|C_2|^2}{|Z_2|^2}\left(1 + \frac{1}{\sigma_0}\left(\sigma^n_{\text{JADE}} + \sigma^{\bar{n}}_{\text{JADE}} + \sigma^s_{\text{JADE}}\right)\right)$$

$$= 1 + \frac{\alpha_s C_F}{2\pi}\left(-2\ln^2 j - 3\ln j + \frac{\pi^2}{3} - 1\right)$$



Comments:

(1) zero-bin is non-trivial and required

(2) UV divergences in soft and collinear phase space integrals cancel ... demonstrate with explicit IR regulator

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**UV divergences**

$$\frac{1}{\sigma_0}\sigma^n_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\epsilon}\ln\frac{p_1^2}{jQ^2} - \ln^2\frac{p_1^2}{Q^2} + 2\ln\frac{\mu^2}{Q^2}\ln\frac{p_1^2}{Q^2} + \frac{3}{2}\ln\frac{p_1^2}{Q^2}\right) + \dots$$

$$\frac{1}{\sigma_0}\sigma^s_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon}\left(\ln\frac{p_1^2}{jQ^2} + \ln\frac{p_2^2}{jQ^2}\right) + \left(\ln\frac{p_1^2}{Q^2} + \ln\frac{p_2^2}{Q^2}\right)^2 - 2\left(\ln\frac{p_1^2}{Q^2} + \ln\frac{p_2^2}{Q^2}\right)\ln\frac{\mu^2}{Q^2}\right) + \cdots$$

---

$$\frac{1}{\sigma_0}\sigma^R_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(2\ln\frac{p_1^2}{Q^2}\ln\frac{p_2^2}{Q^2} + \frac{3}{2}\ln\frac{p_1^2}{Q^2} + \frac{3}{2}\ln\frac{p_2^2}{Q^2}\right) + \dots$$

**UV divergences cancel in sum**

Comments:

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(3) the soft physics is more subtle than it appears ...

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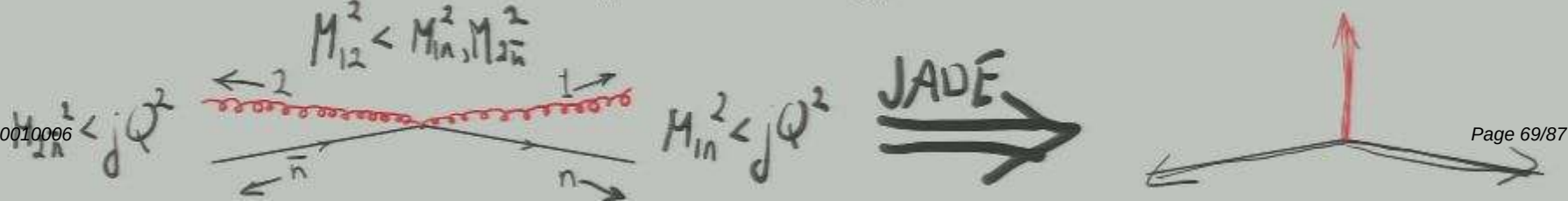
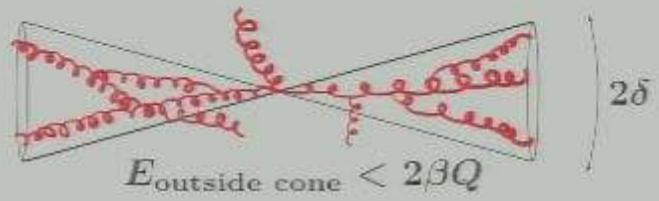
it appears we can use the RGE to renormalize H, J, S at the appropriate scales and sum leading logs in the dijet rate ...

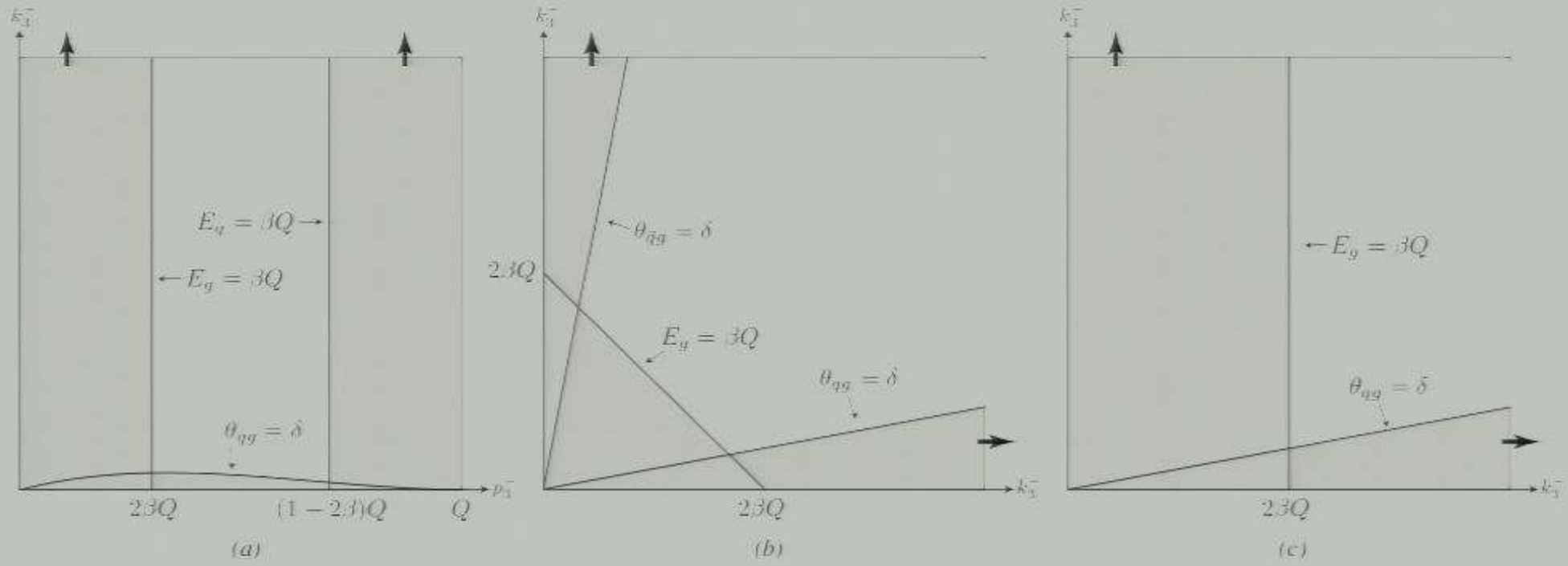
BUT this is known not to work for JADE! there are leading log effects that are not captured by $O(\alpha_s)$ calculation ("non-global logs"). Failure of factorization? (presumably) - need to understand further!



Other jet definitions (SW, $k_T$) are similar, but each introduces a new twist:

SW: phase space for zero bin is different from soft phase space




n-collinear soft zero-bin

Other jet definitions (SW, $k_T$) are similar, but each introduces a new twist:

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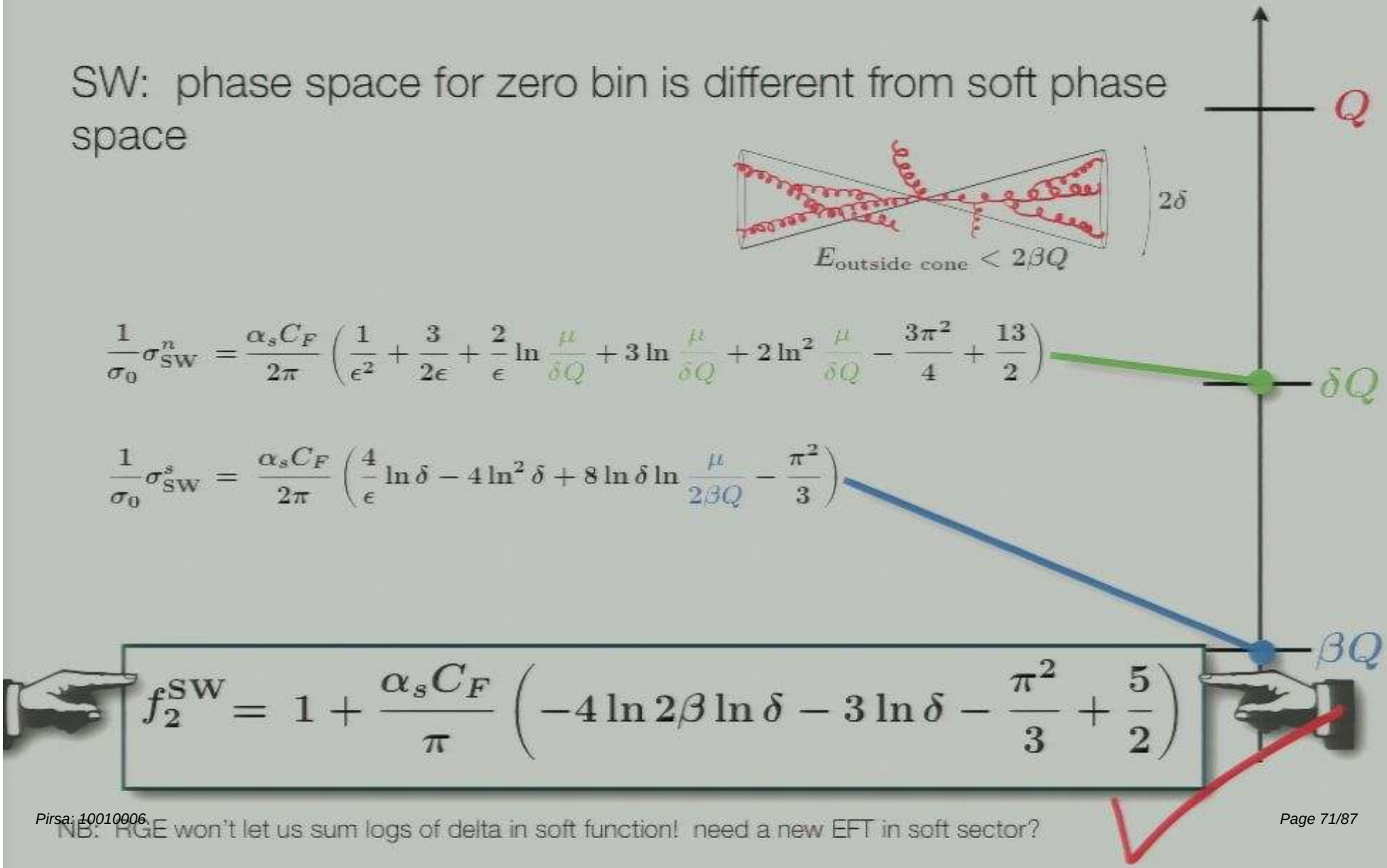
$$\frac{1}{\sigma_0}\sigma^n_{\rm SW} = \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}+\frac{2}{\epsilon}\ln\frac{\mu}{\delta Q}+3\ln\frac{\mu}{\delta Q}+2\ln^2\frac{\mu}{\delta Q}-\frac{3\pi^2}{4}+\frac{13}{2}\right)$$

$$\frac{1}{\sigma_0}\sigma^s_{\rm SW} = \frac{\alpha_s C_F}{2\pi}\left(\frac{4}{\epsilon}\ln\delta-4\ln^2\delta+8\ln\delta\ln\frac{\mu}{2\beta Q}-\frac{\pi^2}{3}\right)$$

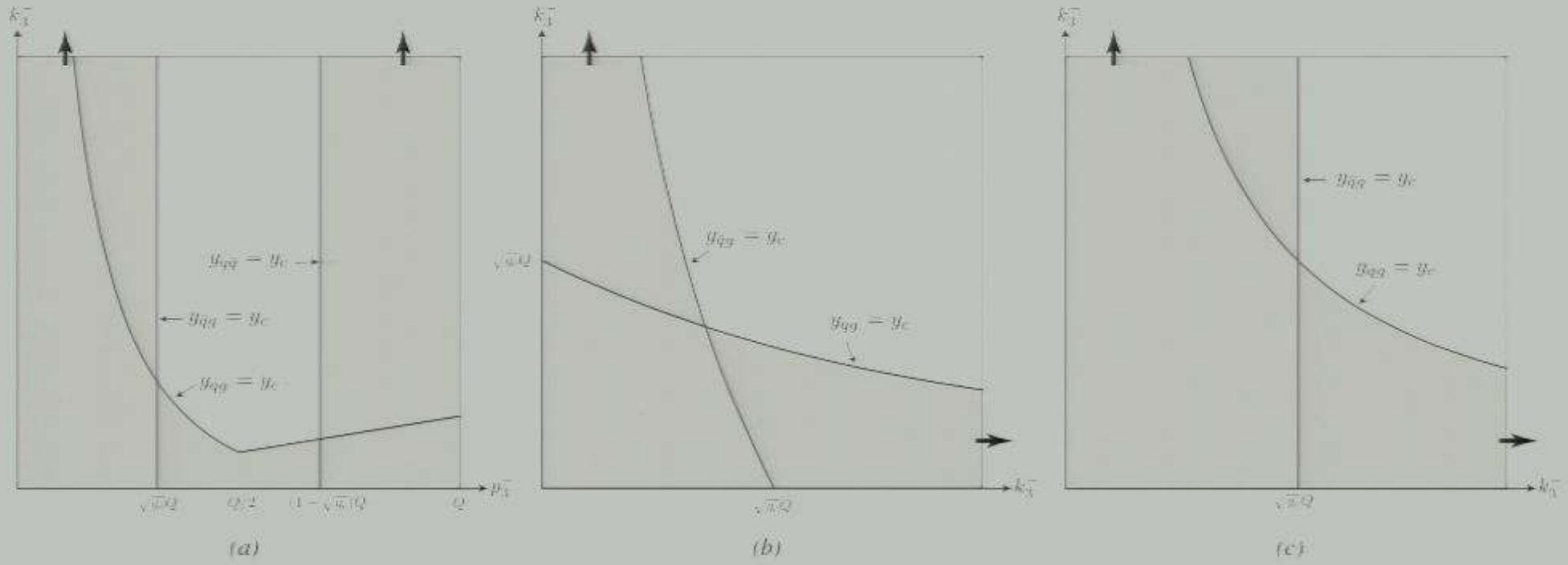
$$f_2^{\rm SW} = 1+\frac{\alpha_s C_F}{\pi}\left(-4\ln 2\beta\ln\delta-3\ln\delta-\frac{\pi^2}{3}+\frac{5}{2}\right)$$




NB: RGE won't let us sum logs of delta in soft function! need a new EFT in soft sector?

Other jet definitions (SW, $k_T$) are similar, but each introduces a new twist:

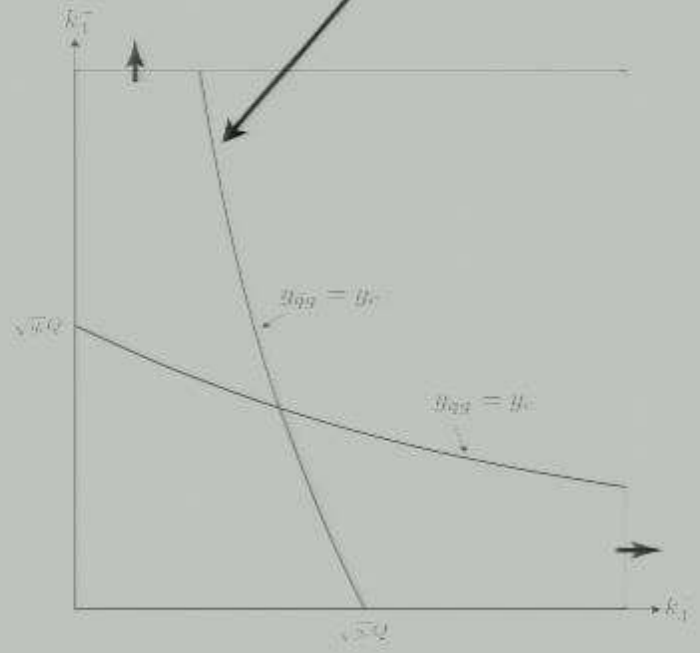
$k_T$: soft and jet functions are separately IR divergent




n-collinear soft zero-bin

$$\frac{d\sigma}{dk_3^-} \sim \frac{1}{\epsilon k_3^-}$$

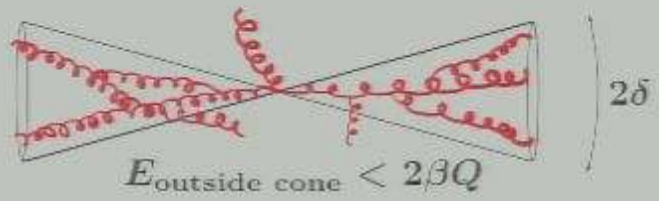
$k_3^-$ integral diverges in all dimensions! how is rate finite in EFT?




soft

Other jet definitions (SW, $k_T$) are similar, but each introduces a new twist:

SW: phase space for zero bin is different from soft phase space




$$\frac{1}{\sigma_0}\sigma^n_{\rm SW} = \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}+\frac{2}{\epsilon}\ln\frac{\mu}{\delta Q}+3\ln\frac{\mu}{\delta Q}+2\ln^2\frac{\mu}{\delta Q}-\frac{3\pi^2}{4}+\frac{13}{2}\right)$$

$$\frac{1}{\sigma_0}\sigma^s_{\rm SW} = \frac{\alpha_s C_F}{2\pi}\left(\frac{4}{\epsilon}\ln\delta-4\ln^2\delta+8\ln\delta\ln\frac{\mu}{2\beta Q}-\frac{\pi^2}{3}\right)$$

$$f_2^{\rm SW} = 1+\frac{\alpha_s C_F}{\pi}\left(-4\ln 2\beta\ln\delta-3\ln\delta-\frac{\pi^2}{3}+\frac{5}{2}\right)$$

NB: RGE won't let us sum logs of delta in soft function! need a new EFT in soft sector?

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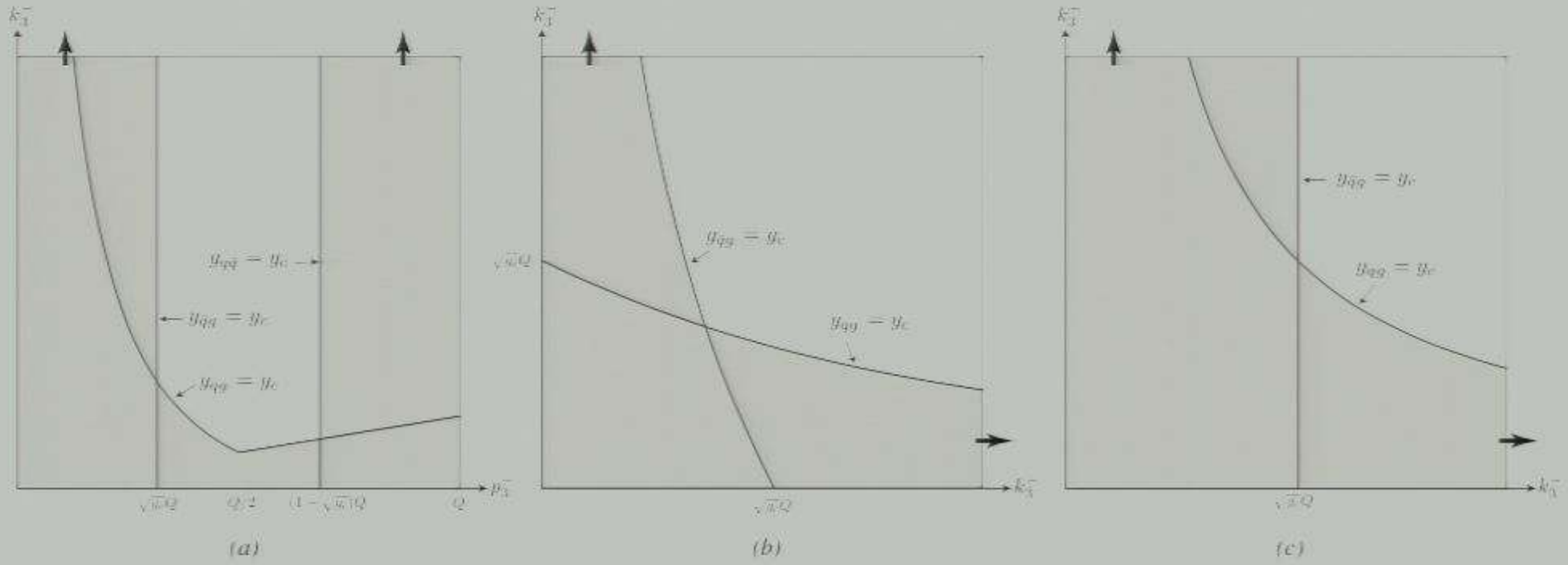


n-collinear soft zero-bin

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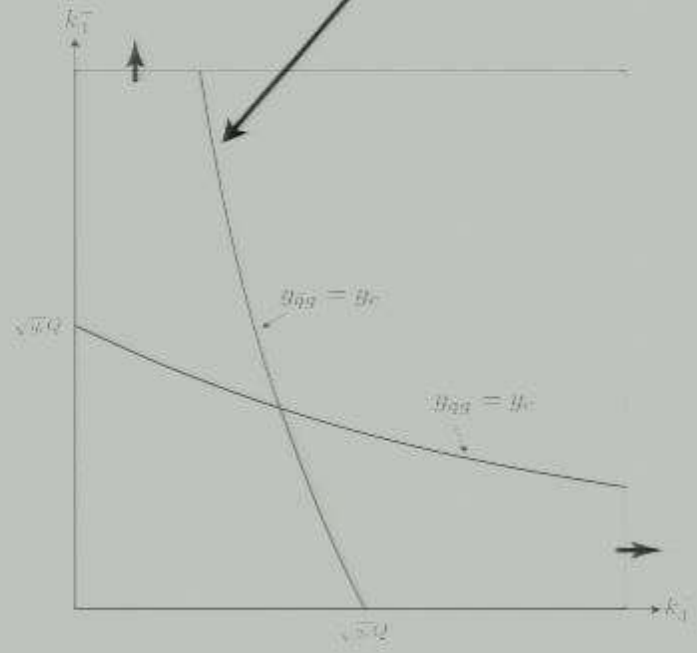

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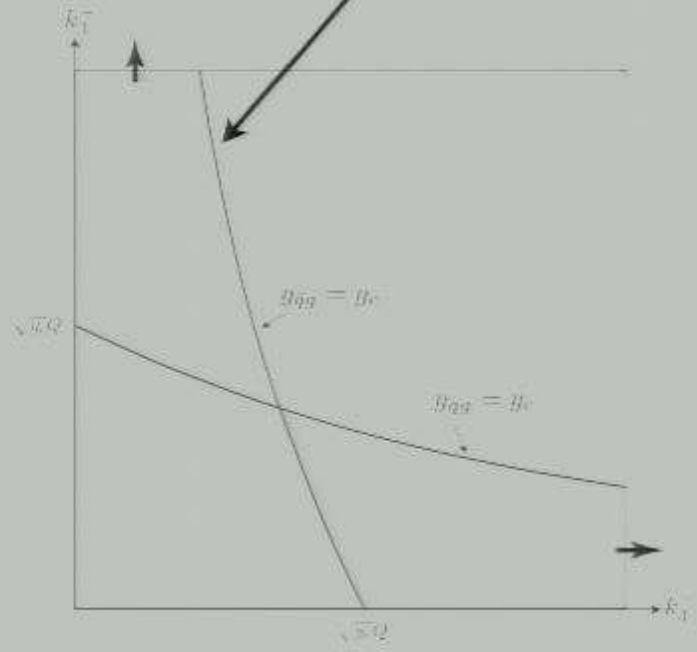
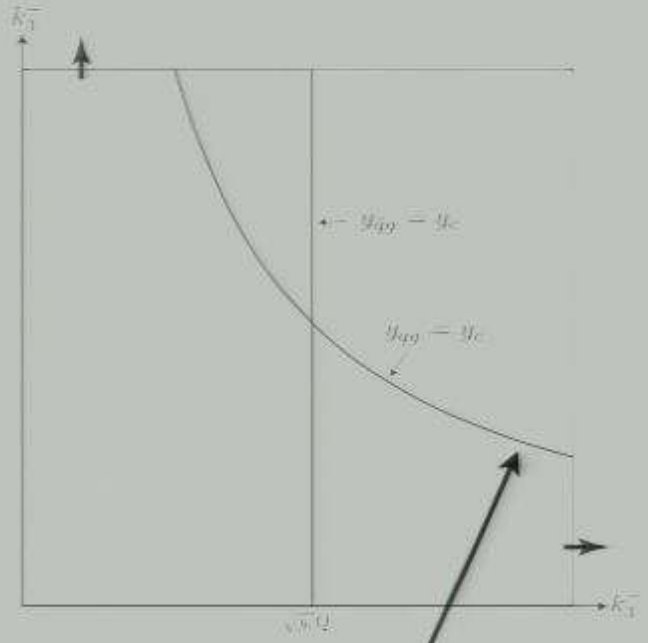



soft

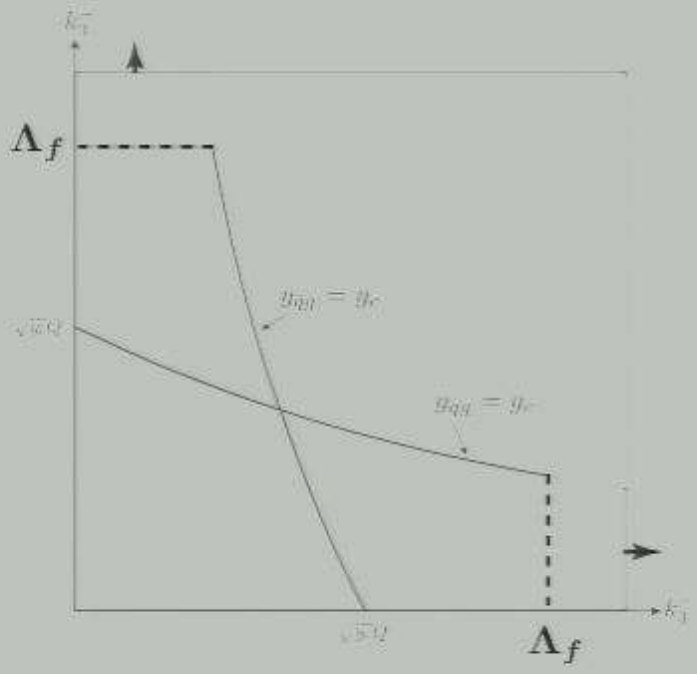



zero-bin has same asymptotic behaviour - divergence cancels between soft and (zero-bin) collinear - sum is FINITE

$$f_2^{k_\perp} = 1 + \frac{\alpha_s C_F}{2\pi}\left(-\ln^2 y_c - 3\ln y_c - 6\ln 2 + \frac{\pi^2}{6} - 1\right)$$

# jet and soft functions can't be separately defined for $k_T$ ... failure of factorization?

not necessarily ... the cancellation occurs between unphysical (arbitrarily high momentum) degrees of freedom in soft and collinear - is this an artifact of the UV regulator? (dim. reg.)




Introduce UV cutoff in +/- directions

$$|k_3^{\pm}| < \Lambda_f$$

$$\frac{1}{\sigma_0}\sigma^s_{k_\perp} = \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\frac{\mu^2}{\Lambda_f^2} - \ln^2\frac{y_c Q^2}{\Lambda_f^2} + \ln^2\frac{\mu^2}{\Lambda_f^2} - \frac{\pi^2}{3}\right)$$

$$\frac{1}{\sigma_0}\left(\sigma^s_{k_\perp} + \sigma^s_V\right) = -\frac{\alpha_s C_F}{2\pi}\left(\ln^2\frac{y_c Q^2}{\Lambda_f^2} + \frac{\pi^2}{6}\right)$$

so the form of factorization is UV-regulator dependent

The story thus far ...

- we have demonstrated consistent power counting for phase space integrals in SCET - nontrivial zero bins, cancellations of UV divergences between soft and collinear sectors

- soft logs don't resum at this stage - failure of factorization? presence of additional soft scales? - "non-global" logs (Dasgupta & Salam): can we get a handle on these in EFT?

- $k_T$ may factorize, but appears dependent on UV regulator

To go further, we need to understand factorization theorems for jet rates (in progress ...)

# Event Shapes in Jet production:

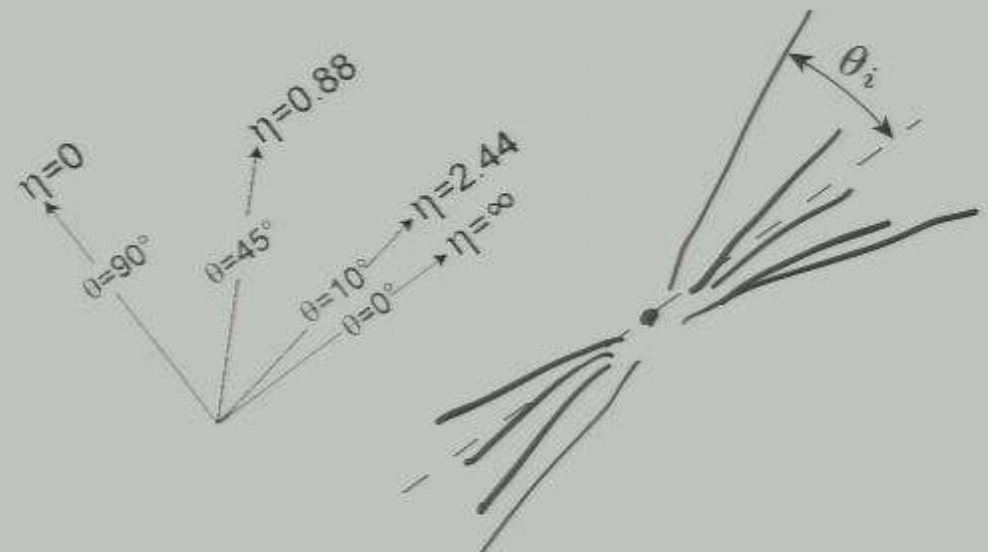
(Lee, Sterman; Lee, Hornig, Ovanesyan; Ellis, Vermilion, Walsh, Hornig, Lee)

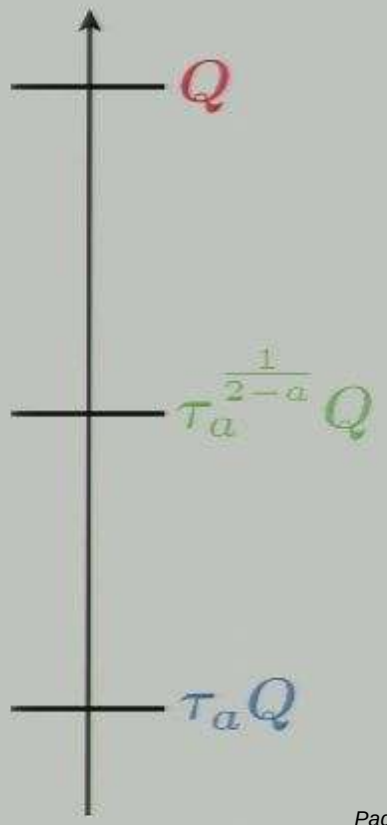
- probing structure of jets provides a powerful tool to distinguish light parton jets to those produced by heavy particle decays

- define event shape parameters which can probe structure of jets, calculable in QCD

$$\tau_a(X) = \frac{1}{Q}\sum_{i\in X} |\mathbf{p}_i^T| e^{-|\eta_i|(1-a)}$$

$a = 0$: "Thrust"
$a = 1$: "jet broadening"



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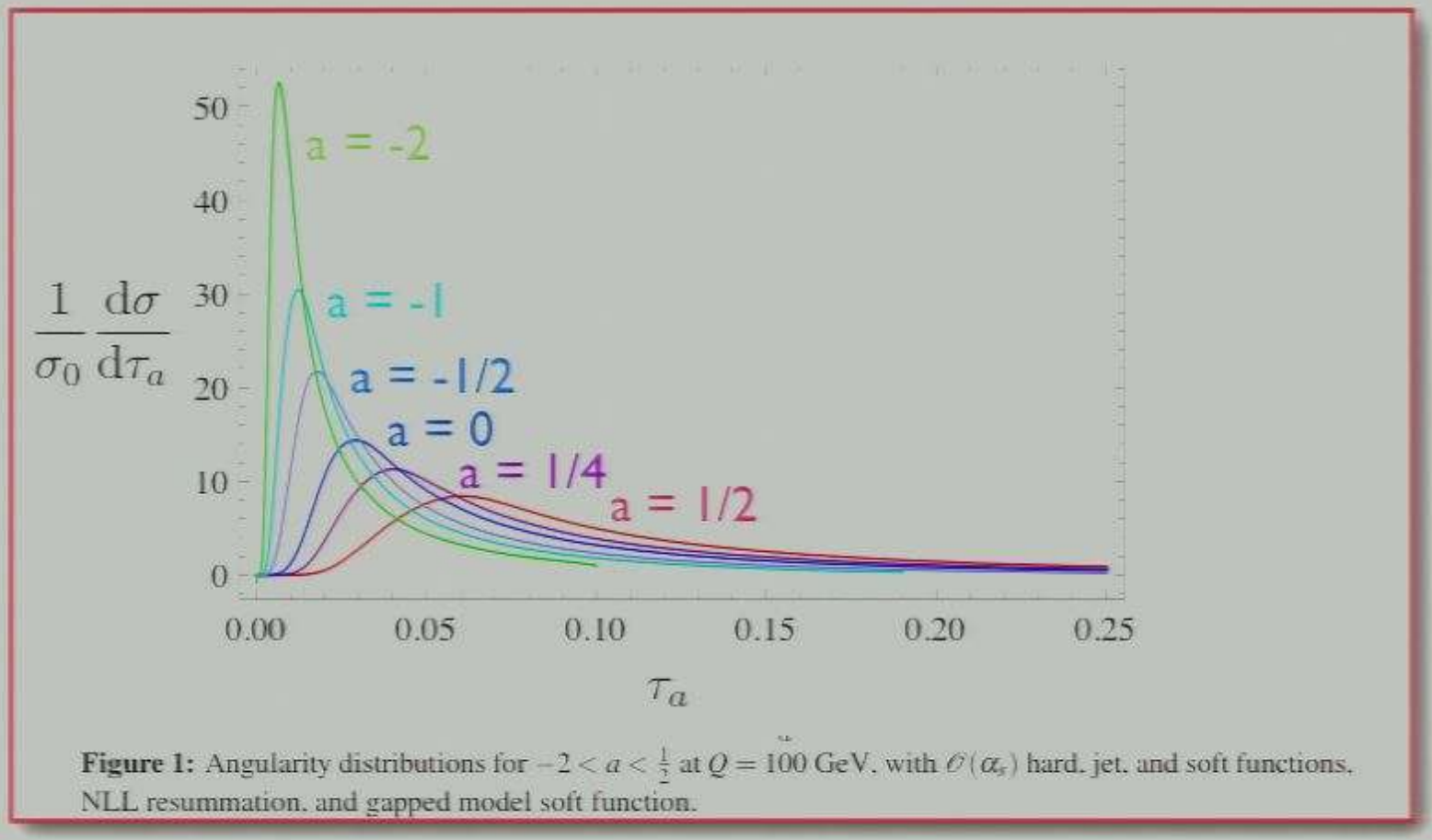


**Figure 1:** Angularity distributions for $-2 < a < \frac{1}{2}$ at $Q = 100$ GeV, with $\mathcal{O}(\alpha_s)$ hard, jet, and soft functions, NLL resummation, and gapped model soft function.

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Ellis, Vermilion, Walsh, Hornig, Lee (arXiv:1001.0014) have recently generalized this analysis to multijet final states: defined distributions for shapes of individual jets in various schemes, proved factorization (nontrivial!) for jet shape distributions and demonstrated renormalization group running - still have an issue with "non-global" logs

**scales**: jet energies, cut on angular size of each jet, measured values of jet shapes, other parameters introduced by jet algorithm - difficult to do in traditional QCD approach

**LHC and hadron colliders**: life is complicated by nontrivial initial state - incoming collinear fields in SCET

The parton model is only strictly applicable for fully inclusive final states ... less inclusive states introduce anything from large logs (resummation required) to new NP information. SCET is being used to study these more complex factorization theorems.

(Stewart, Tackman, Wallewijn, arXiv:0910:0467)

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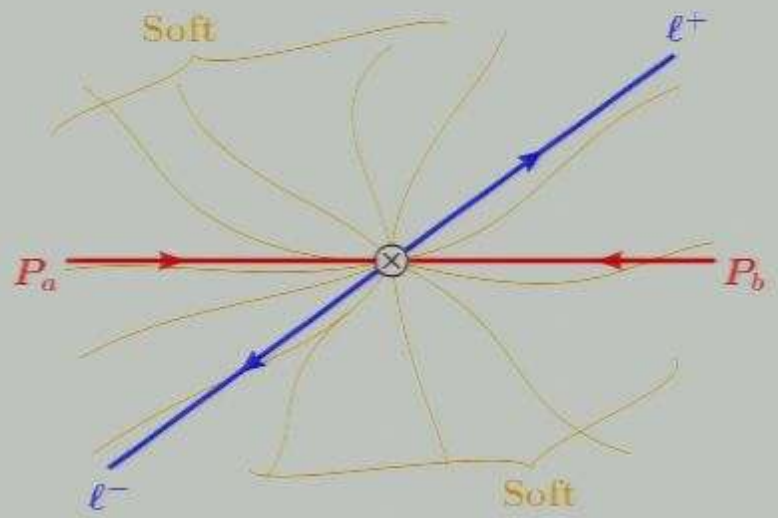
ex: Drell-Yan

b) threshold: new soft function required




$$\frac{1}{\sigma_0}\frac{d\sigma}{dq^2} = Q\sum_{ij} H_{ij}(q^2,\mu)\int \frac{d\xi_a}{\xi_a}\frac{d\xi_b}{\xi_b} S_{\rm thr}\left[Q\left(1-\frac{\tau}{\xi_a\xi_b}\right),\mu\right]\times f_i(\xi_a,\mu) f_j(\xi_b,\mu)$$

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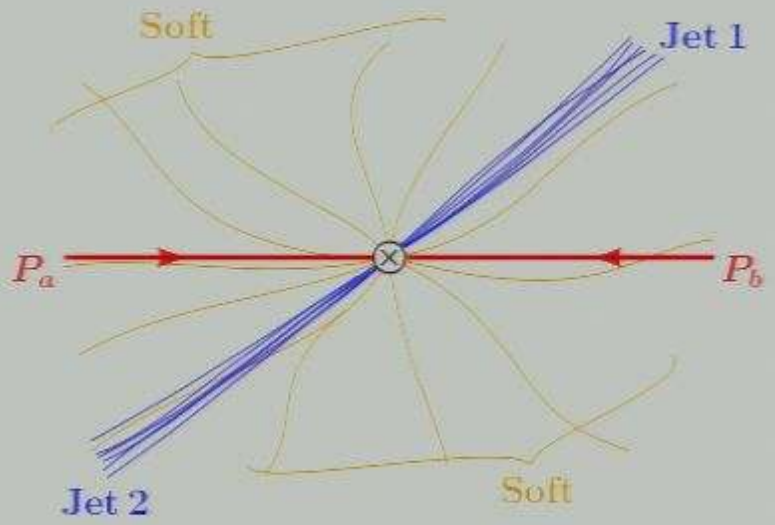
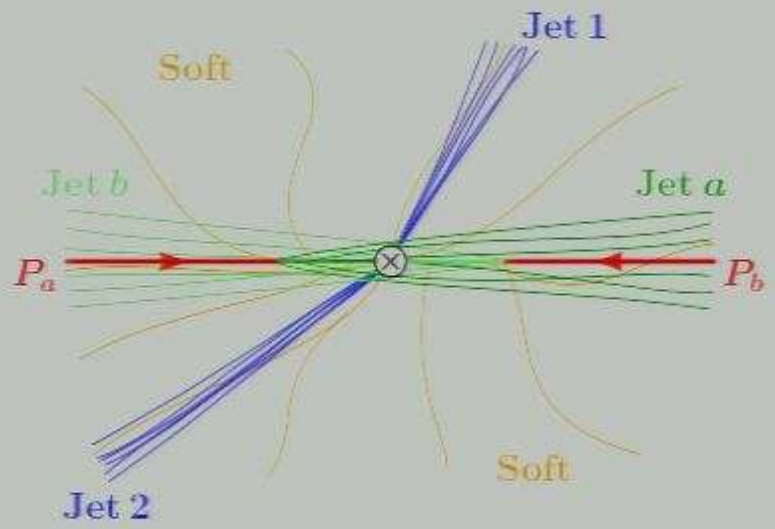
ex: dijet production - conjectured




d) as (b), with leptons replaced by jets




e) as (c), with leptons replaced by jets

There have been many recent applications of SCET to collider physics ... for example:

- electroweak processes & gauge boson production (Manohar, Kelley, Chiu, Fuhrer, Hoang)
- hard photon production in hadronic collisions (Becher, Schwartz)
- Higgs transverse momentum distribution (Mantry, Petriello)
- Drell-Yan (Neubert, Becher)
- t-$\bar{t}$ production - soft radiation and precision extraction of the top quark mass (Fleming, Hoang, Mantry, Stewart)

and lots more ...

# Summary:

Effective Field Theory provides a powerful new tool to study traditional pQCD problems, with distinct advantages over traditional pQCD methods.

We are working on understanding factorization and jet algorithms in this framework.

Lots of interesting work being done!