Title: Tests of Relativistic Gravity in Space: Brief History, Recent Progress and Possible Future Directions

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Abstract: Einstein’s general theory of relativity is the standard theory of gravity, especially where the modern needs of astronomy, astrophysics, cosmology and fundamental physics are concerned. As such, this theory is used for many practical purposes involving spacecraft navigation, geodesy, time transfer and etc. Series of recent experiments have successfully tested general relativity to a remarkable precision. Various experimental techniques were used to test relativistic gravity in the solar system namely spacecraft Doppler tracking, planetary ranging, lunar laser ranging, dedicated gravity experiments in space and many ground-based efforts. We will discuss the recent progress in the tests of relativistic gravity and motivation for the new generation of high-accuracy gravitational experiments in space. We also discuss the advances in our understanding of fundamental physics that are anticipated in the near future and evaluate the discovery potential of the recently proposed solar system gravitational experiments.

Solar System Tests of Relativistic Gravity:

Recent Progress and Possible Future Directions

Slava G. Turyshev

with special thanks to

Richard T. Baran, William Farr, William M. Folkner, Andre Girerd, Gary Gutt, Hamid Hemmati, Ruwan P. Somawardhana, Michael Shao, Robert Spero, and James G. Williams

Jet Propulsion Laboratory, California Institute of Technology
4800 Oak Grove Drive, Pasadena, CA 91009 USA

John J. Degnan
Sigma Space Corporation
Lanham, MD, USA

Thomas W. Murphy, Jr.
University of California, San Diego
La Jolla, CA, USA

Benjamin F. Lane
Charles Stark Draper Laboratory
Cambridge, MA, USA

Talk at the Perimeter Institute,
Waterloo, Canada, 28 January 2010

Outline: Solar System Tests of Gravity

The talk will cover:

- Theoretical Landscape in the 20th Century:
  - (brief…) History of the tests of general relativity
  - Frameworks used: the PPN formalism and Robertson-Mansouri-Sexl
  - Recent progress in the tests of general relativity
- Beginning of the 21st Century…:
  - Motivations for high-precision tests of gravity
  - What to expect in the near future? and some proposed experiments

- Main objective:
  - Remind where we came from and what lessons we learned

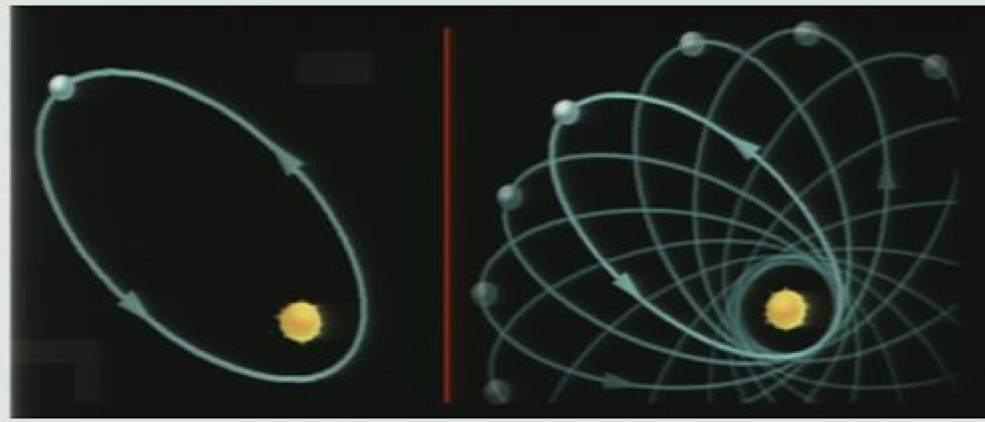
- Themes for discussion:
  - Are the solar system tests still useful?
  - Is there a discovery potential? Or what is the importance of new improved limits?
  - What tests are most valuable?

Triumph of Mathematical Astronomy in 19th Century

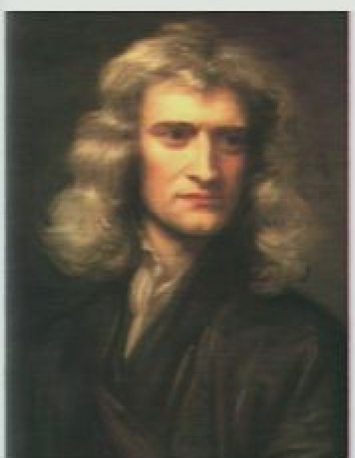
Discovery of Neptune: 1845

Urbain LeVerrier
(1811-1877)

- 1845: the search for Planet-X:
  - Anomaly in the Uranus' orbit → Neptune
  - Anomalous motion of Mercury → Vulcan



Newtonian Gravity   General Relativity



Sir Isaac Newton
(1643-1727)

- Anomalous precession of Mercury's perihelion :
  - 43 arcsec/cy can not be explained by Newton's gravity
- Before publishing GR, in 1915, Einstein computed the expected perihelion precession of Mercury
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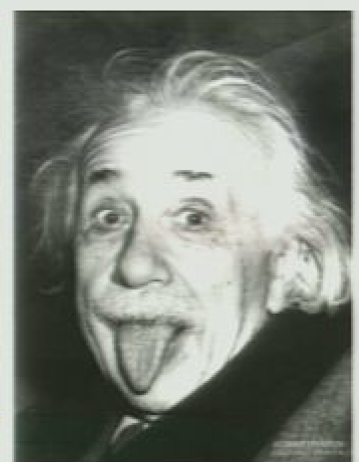
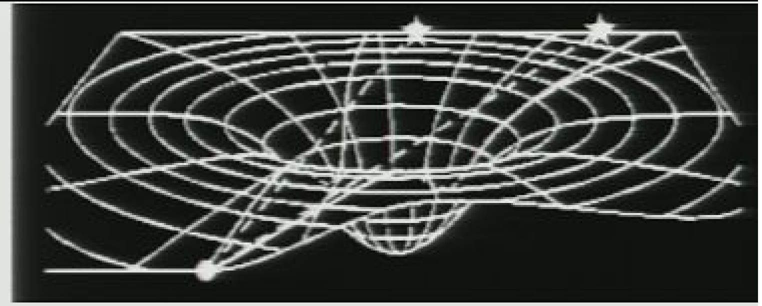
Almost in one year LeVerrier both confirmed the Newton's theory (Neptune) & cast doubt on it (Mercury's anomaly).



Albert Einstein
(1879-1955)

The First Test of General Theory of Relativity

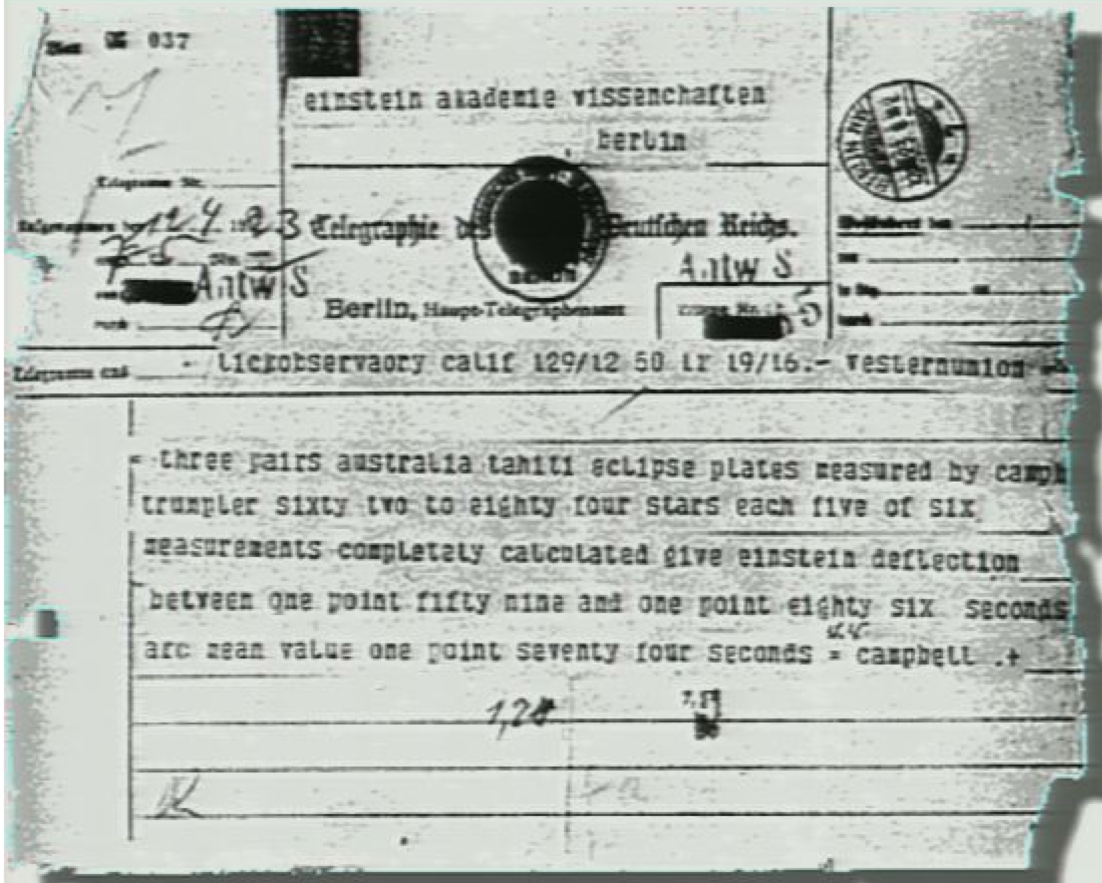


Gravitational Deflection of Light:

$$\theta_{gr}(b) = \frac{2(1+\gamma)GM_{\odot}}{bc^2} \simeq 8 \times 10^{-6}\left(\frac{1+\gamma}{2}\right)\left(\frac{R_{\odot}}{b}\right)$$

Solar Eclipse 1919: possible outcomes

Deflection = 0;
Newton = 0.87 arcsec;
Einstein = 2 x Newton = 1.75 arcsec



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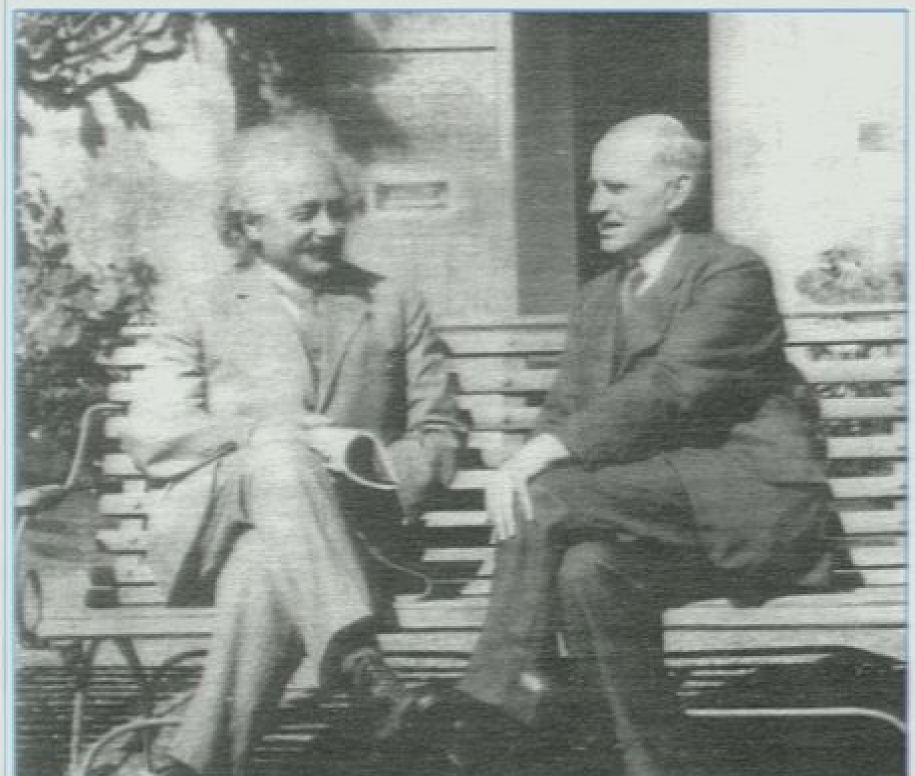
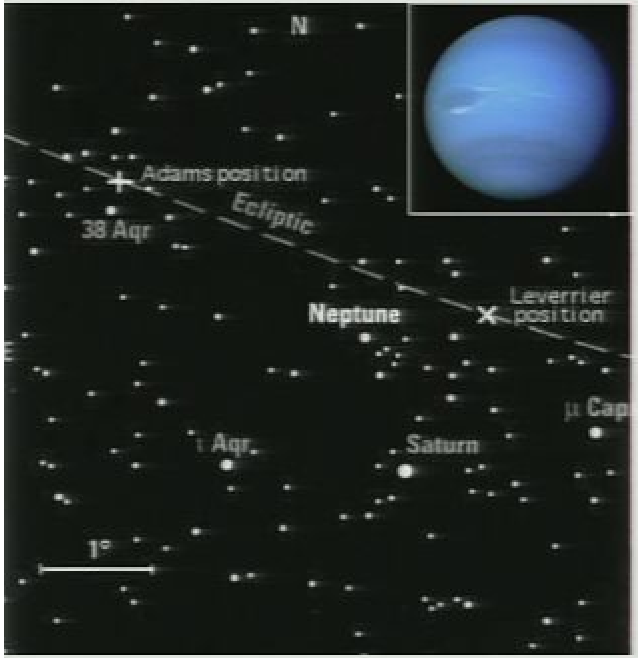
Eddington's telegram to Einstein, 1919



Einstein and Eddington, Cambridge, 1930

TESTS OF RELATIVISTIC GRAVITY IN SPACE

Triumph of Mathematical Astronomy in 19th Century



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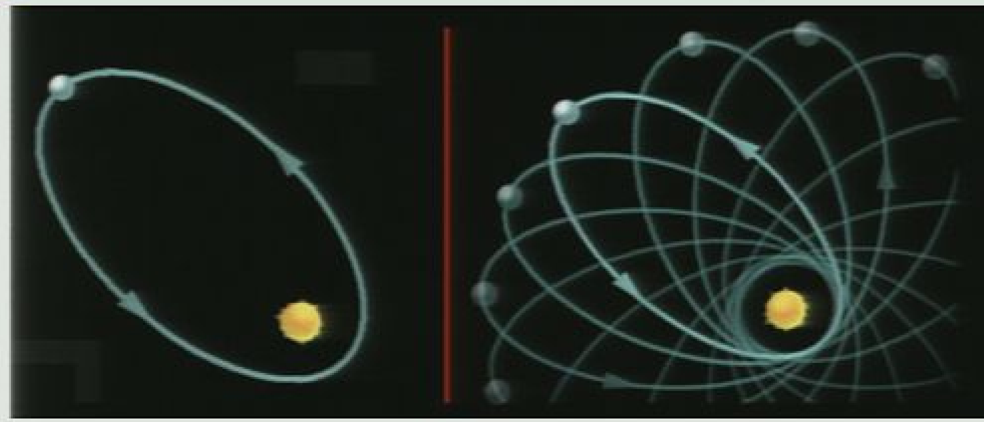

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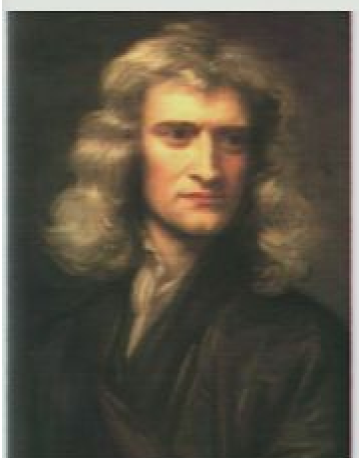

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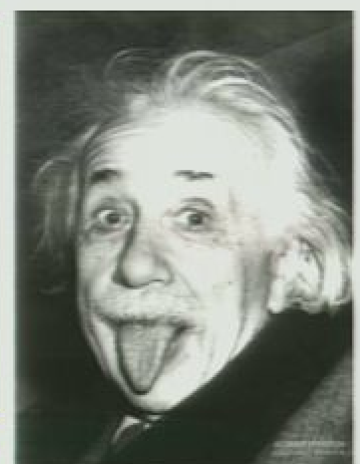

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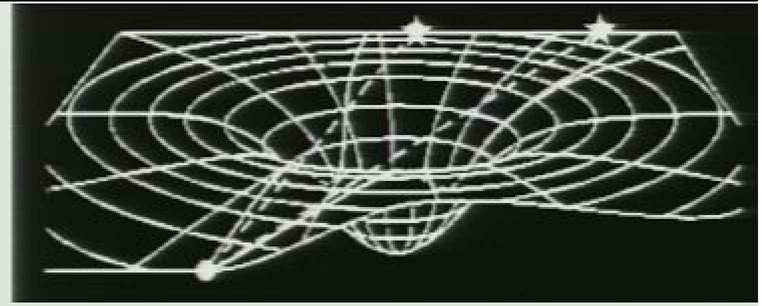

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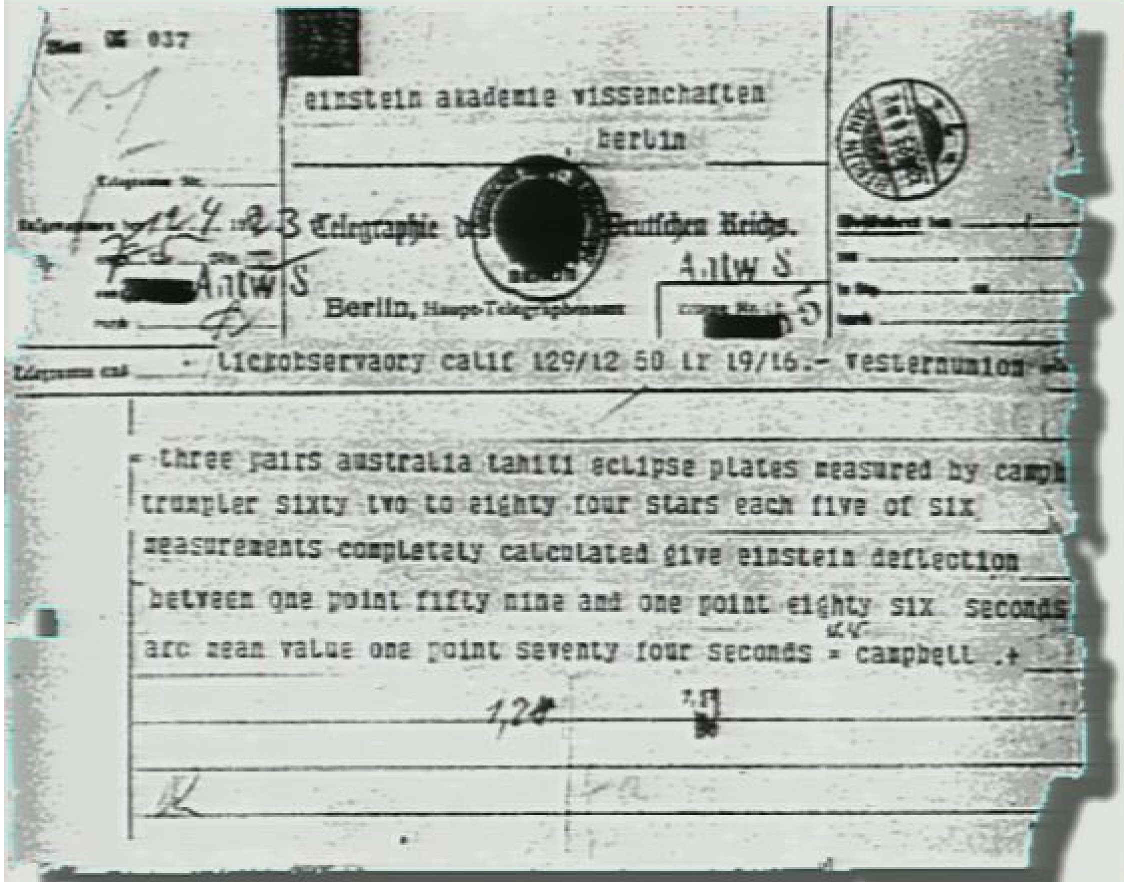
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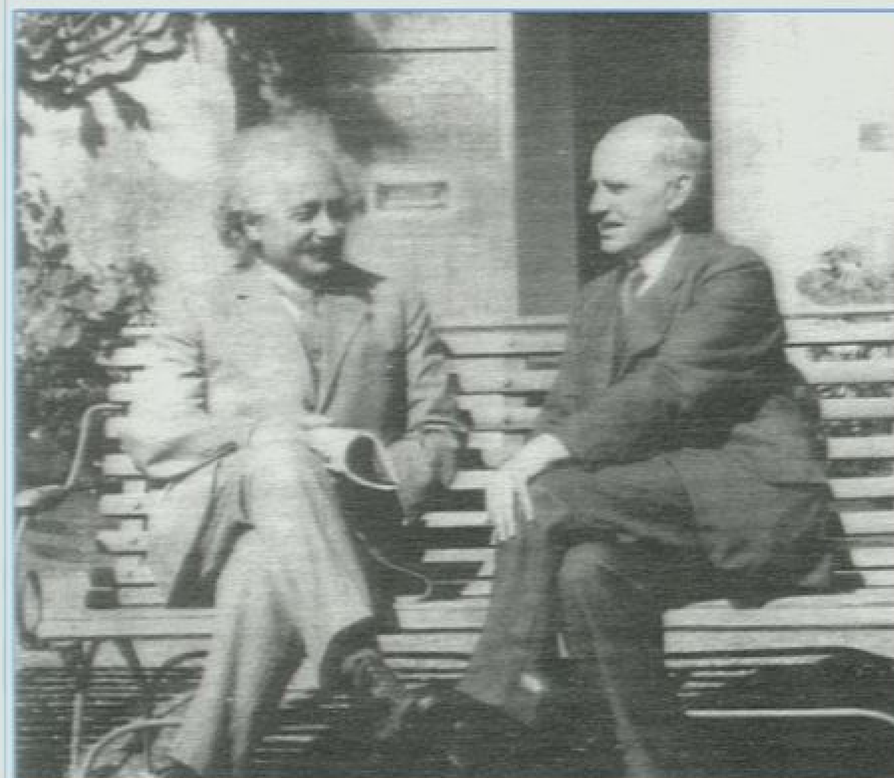

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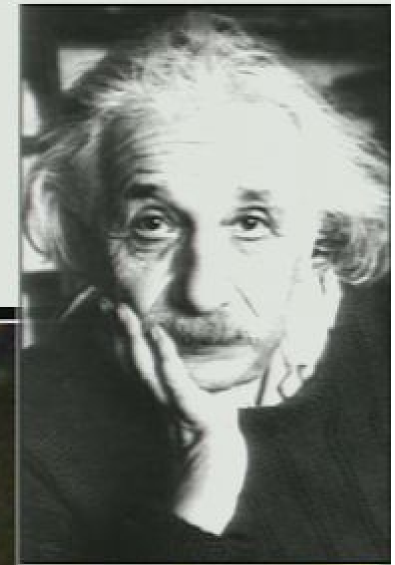
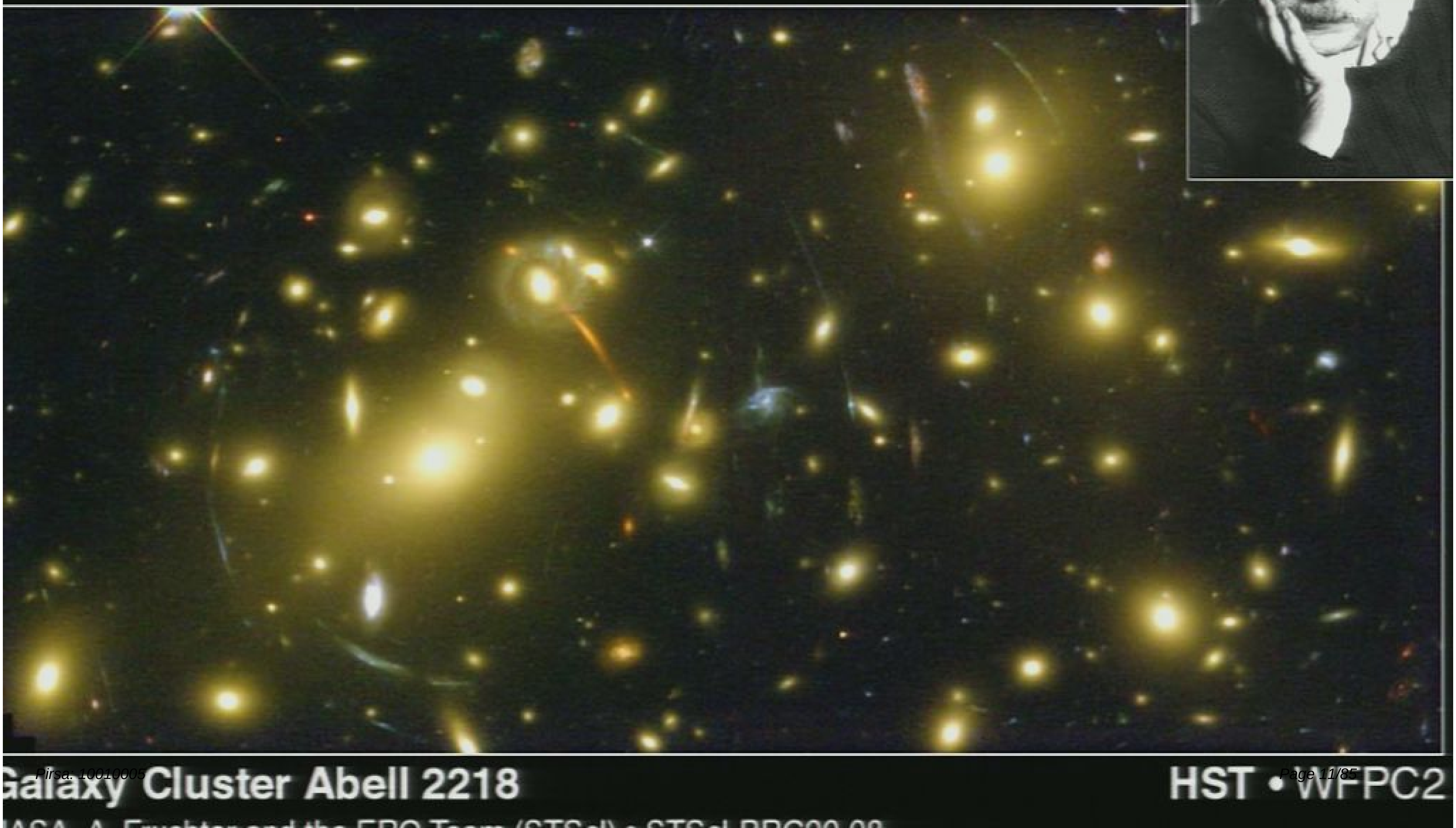
Gravitational Deflection of Light is a Well-Known Effect Today





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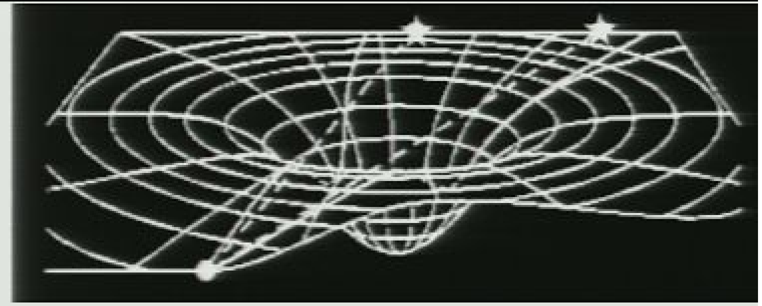

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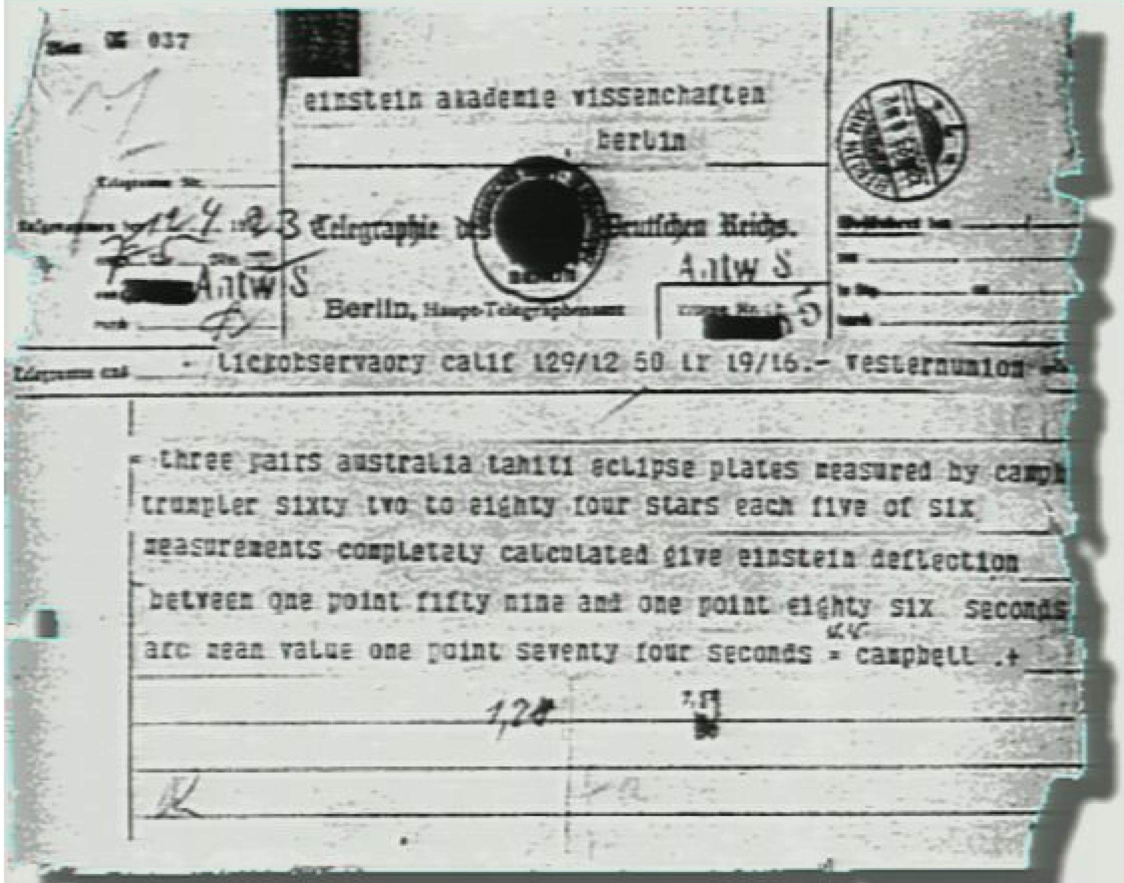

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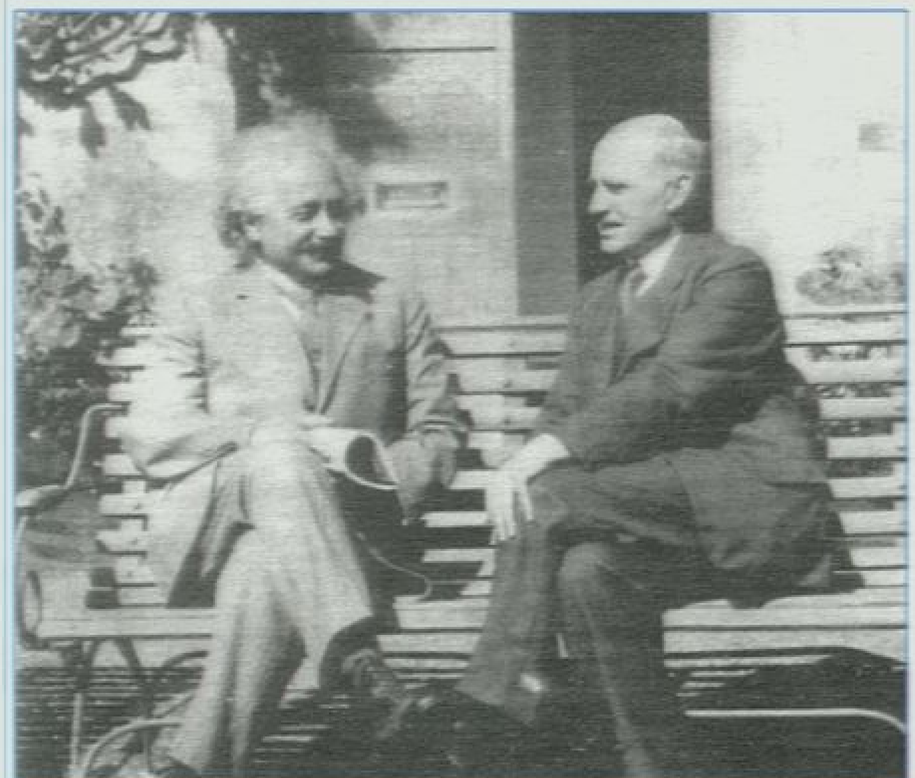

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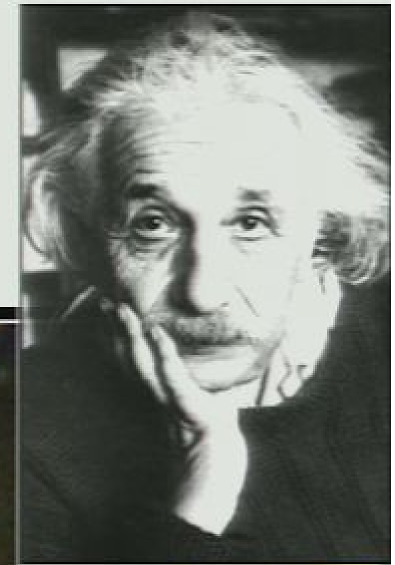

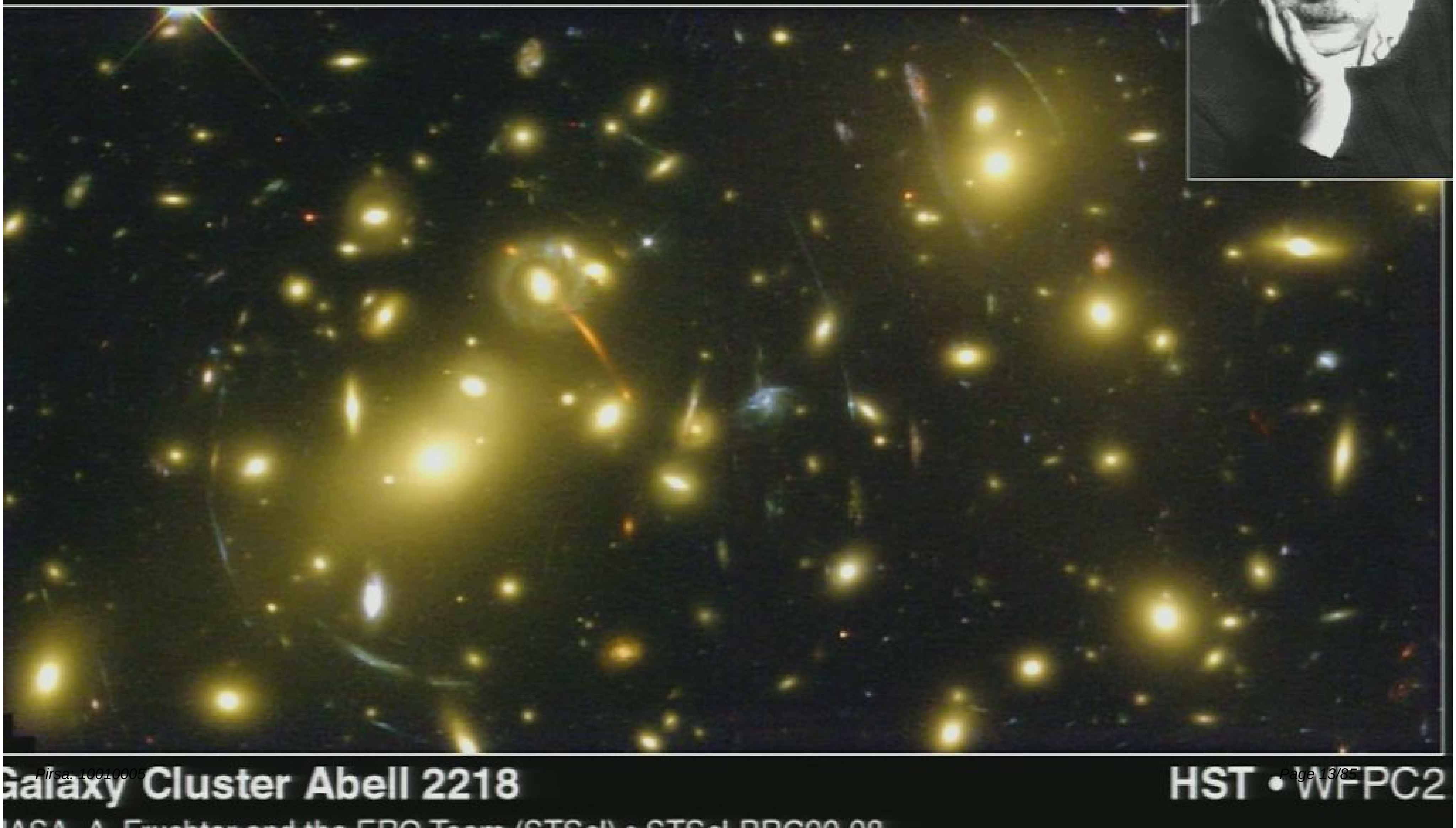

Theoretical Landscape of the 20th Century:
Competing Theories of Gravity

not a complete list…

Newton 1686	Poincaré 1890			
Einstein 1912	Nordstrøm 1912	Nordstrøm 1913	Einstein & Fokker 1914	Einstein 1915
Whitehead 1922	Cartan 1923	Kaluza & Klein 1932	Fierz & Pauli 1939	Birkhoff 1943
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Coleman 1983	Logunov 1987	Hehl 1997	Overlooked (20th century)	

- Some authors proposed more than one theory, e.g. Einstein, Ni, Lee, Nordtvedt, Papapetrou, Yilmaz, etc.
- Some theories are just variations of others
- Some theories were proposed in the 1910s/20s; many theories in the 1960s/70s
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Theory must be:

- **Complete**: not a law, but a theory. Derive experimental results from first principles
- **Self-consistent**: get same results no matter which mathematics or models are used
- **Relativistic**: Non-gravitational laws are those of Special Relativity
- **Newtonian**: Reduces to Newton's equation in the limit of low gravity and low velocities

JPL

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

NASA

Theories that fail already

Newton 1686 | **Poincaré 1890**

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- Newton (1686) - non-relativistic: implicit action at a distance - incompatible with special relativity
- Poincare (1890) and conformally flat theory of Whithrow-Morduch (1965) - incomplete: do not mesh with non-gravitational physics (Maxwell)
- Fierz & Pauli (1939) ["spin-2 field theory"] - inconsistent: field equations -> all gravitating bodies move along straight lines, equation of motion -> gravity deflects bodies
- Birkhoff (1943) - not Newtonian: demands *speed of sound = speed of light.*
- Milne (1948) – incomplete - no gravitational red-shift prediction
- Kustaanheimo-Nuotio (1967) – inconsistent – grav. redshift for photons but not for light waves

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that violate the Einstein's Equivalence Principle

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Einstein's Equivalence Principle (EEP):

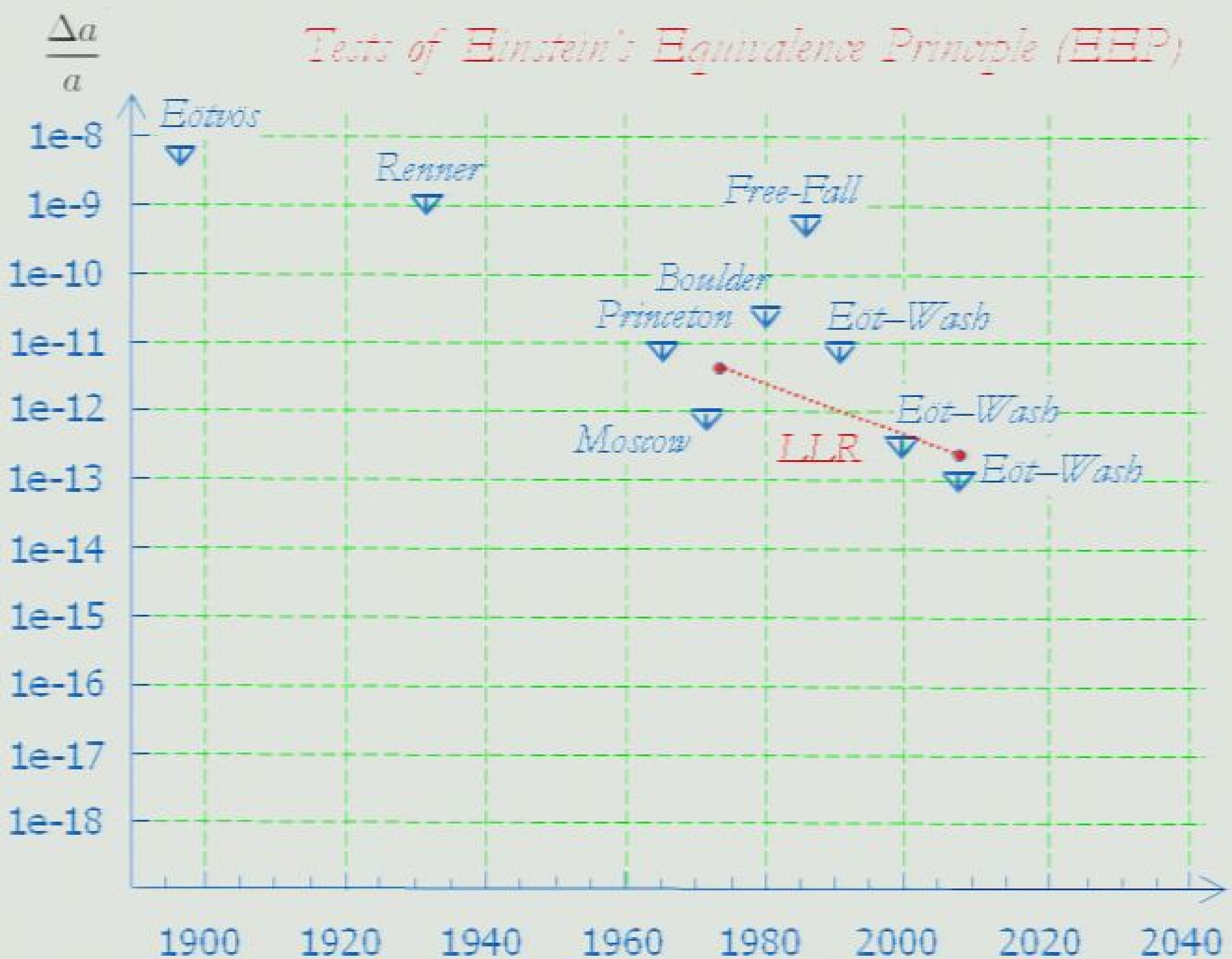
- Uniqueness of Free Fall
- Local Lorentz Invariance
- Local Position Invariance

Only metric theories are viable:

- Belinfante & Swihart (1975): not a metric theory
- Kaluza-Klein (1932): violates EEP
- Still too many theories around…

Empirical Foundations of General Relativity:

Confrontation Between the Theory and Experiment



Uniqueness of Free Fall
(≡ Weak Equivalence Principle):

$$\vec{F} = m_I \vec{a} = m_G \vec{g}$$
$$\Rightarrow m_I = m_G$$

All bodies fall with the same acceleration

Define the test parameter that signifies a violation of the **WEP**

$$\frac{\Delta a}{a} = \frac{(a_1 - a_2)}{\frac{1}{2}(a_1 + a_2)} = \left[\frac{m_G}{m_I}\right]_1 - \left[\frac{m_G}{m_I}\right]_2$$

Let Ω is the gravitational binding energy of a test body, then the test parameter that signifies a violation of the **SEP** is

- • funded projects
- • proposed projects
- *LLR*, *APOLLO*, and *PLR* testing the Strong Equivalence Principle (SEP)

$$\left[\frac{m_G}{m_I}\right]_{SEP} = 1 + \eta\left(\frac{\Omega}{mc^2}\right)$$

$$\frac{\Delta a}{a} = (4\beta - \gamma - 3)\left\{\left[\frac{\Omega}{mc^2}\right]_1 - \left[\frac{\Omega}{mc^2}\right]_2\right\}$$

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that violate Local Lorentz Invariance (LLI)

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Quasi-linear theories:

- Deser & Laurent (1968), Bollini, Giambiagi & Tiomno (1970) both predict existence of a preferred reference frame (i.e., $\xi=1$)
- Whitehead (1922) predicts time-dependence for ocean tides in violation of everyday experience

Empirical Foundations of General Relativity:

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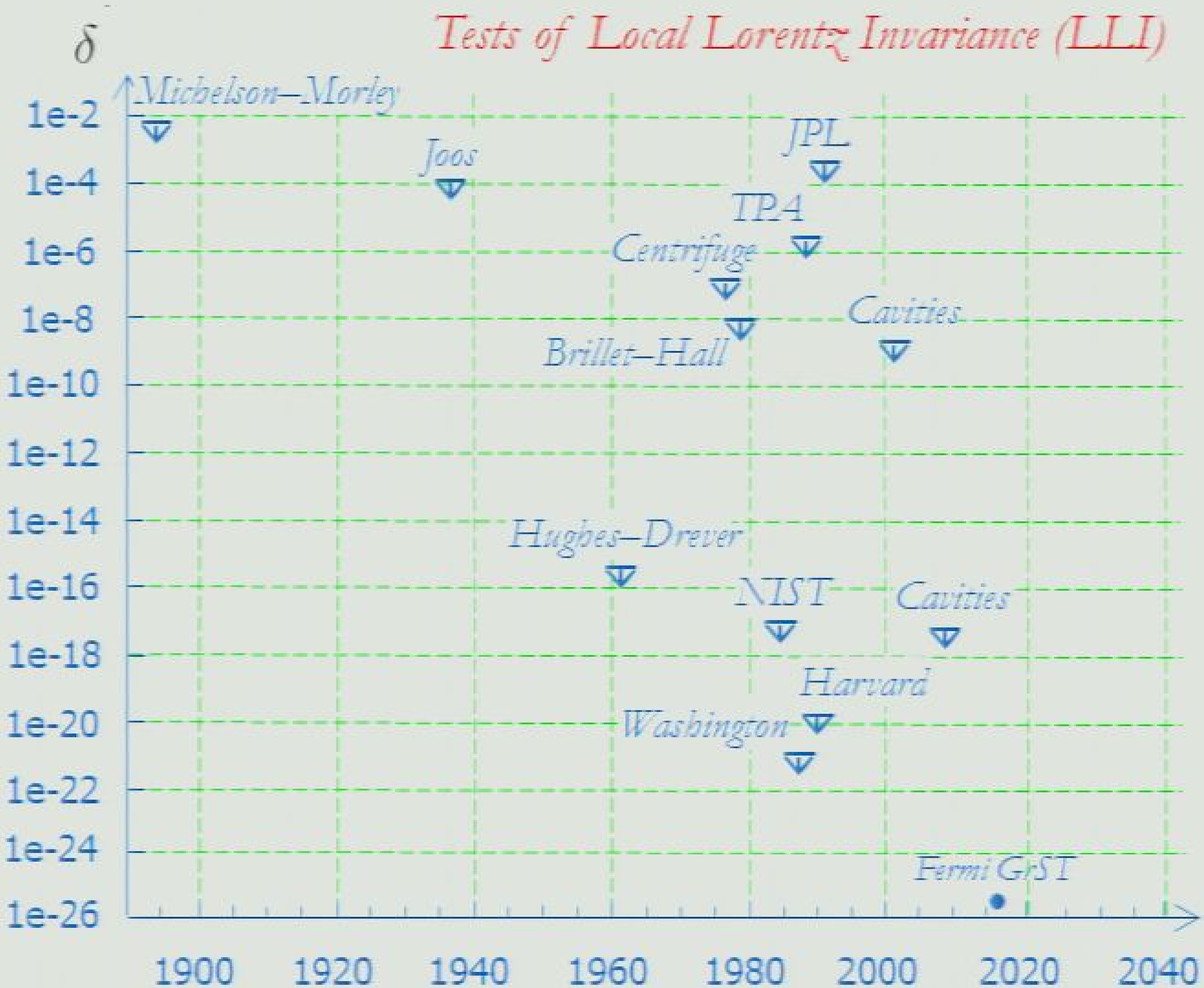

- Michelson-Morley, Joos, Brillet-Hall: Round-trip propagation
- Centrifuge, TPA, JPL: One-way propagation
- Rest: Hughes-Drever experiments

$$\delta \equiv \frac{c^2}{c_0^2} - 1$$

Local Lorentz Invariance:

- Extended frameworks by Kostelecky et al., Jacobson et al.

Future experiments:

- Clock comparisons
- Clocks vs microwave cavities
- Time of flight of high energy photons
- Birefringence in vacuum
- Neutrino oscillations
- Threshold effects in particle physics

Test of one-way speed of light:

- Important to fundamental physics, cosmology, astronomy and astrophysics

Laboratory tests of **Lorentz Invariance**:
search for preferred-frame effects

frame1 : $S(T, X)$ e.g. CMB $v_{sol} \approx 377$ km/s

frame2 : $s(t, x)$ laboratory $RA, dec = (11.2, -6.4^\circ)$

Mansouri & Sexl, 1977

$$dT = \frac{1}{a}(dt + \frac{v}{c^2}dx) \qquad a = 1 + \alpha\frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$

$$dX = \frac{1}{b}dx + \frac{v}{a}(dt + \frac{v}{c^2}dx) \qquad b = 1 + \beta\frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$

$$dY = \frac{1}{d}dy, \; dZ = \frac{1}{d}dz \qquad d = 1 + \delta\frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$

Special Theory of Relativity: $\alpha = -1/2, \; \beta = 1/2, \; \delta = 0$

Clock comparison experiments:

$$P_{MM} = (\tfrac{1}{2} - \beta + \delta)$$ Michelson-Morley: orientation dependence

$$P_{KT} = (\beta - \alpha - 1)$$ Kennedy-Thorndike: velocity dependence

$$P_{IS} = |\alpha + \tfrac{1}{2}|$$ Ives-Stillwell: contraction, dilation

Precision tests of Lorentz Invariance:

$$P_{MM} = -1.6(\pm 6.1) \times 10^{-12}$$ Eisele et al, PRL 103 (2009) 090401

$$P_{KT} = 3.1(\pm 6.9) \times 10^{-7}$$ Wolf et al, PRL 90 (2003) 060402

$$P_{IS} < 2.2 \times 10^{-7}$$ Saathoff et al, PRL 91 (2003) 190403

Tests of isotropy of the speed of light:

$$\Delta c_\theta / c \lesssim 1 \times 10^{-17}$$ Herrmann et al, PRD 80 (2009) 105011

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that violate Local Position Invariance (LPI)

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Stratified theories with time-orthogonal time slices all predict $\xi \neq 0$:

- Einstein (1912), Papapetrou (1954) (actually two theories)
- Yilmaz (1962), Whithrow & Morduch (1965)
- Page & Tupper (1968), Rosen (1971)
- Ni (1972), Coleman (1983)

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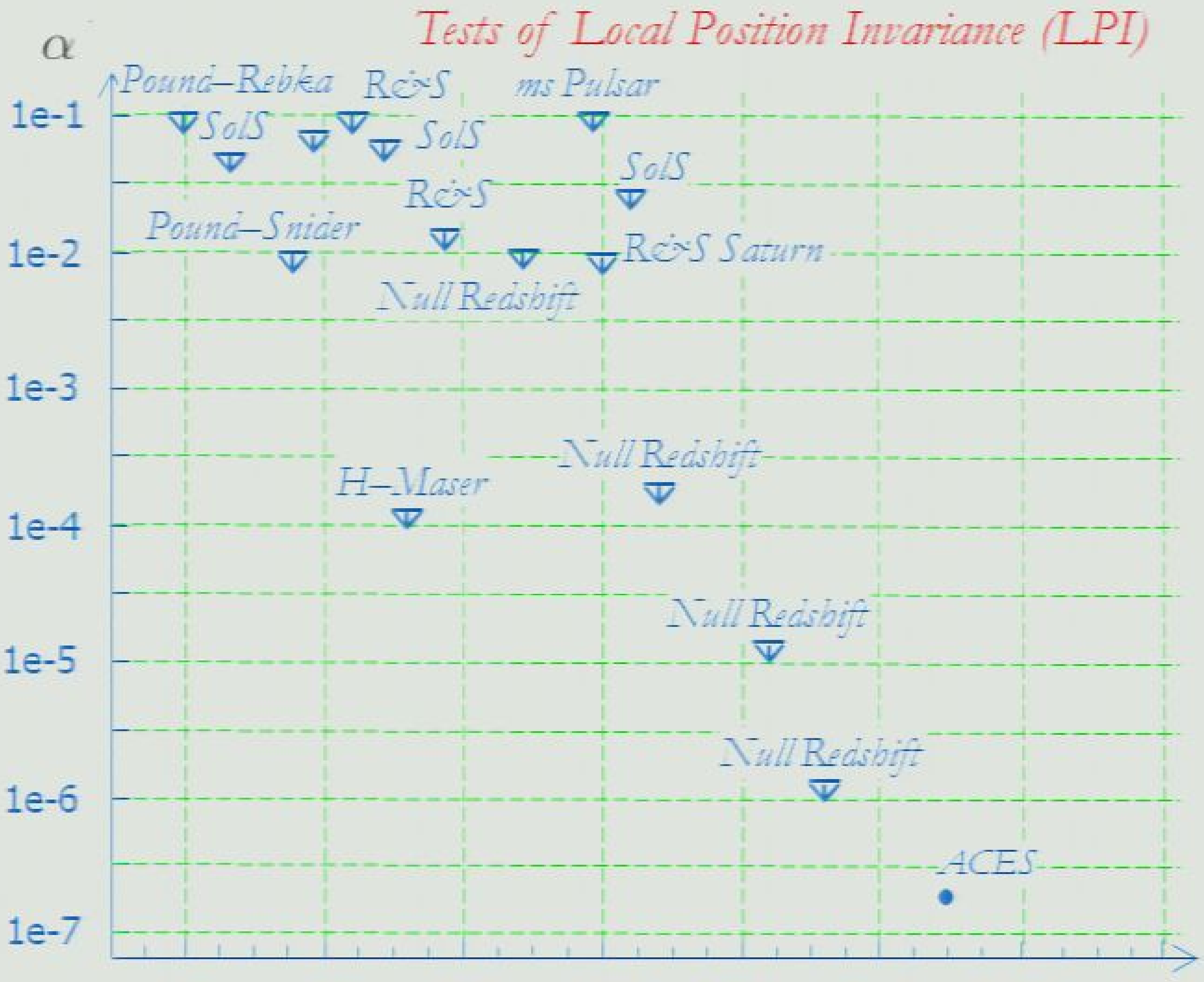

– SolS: Solar Spectra; – R&S: Rockets and Spacecraft

– Null Redshift: comparison of different atomic clocks

Gravitational redshift:

$$\frac{\Delta\nu}{\nu} = (1+\alpha)\frac{\Delta U}{c^2}$$

Local Position Invariance:

- The outcome of any local non-gravitational experiment is independent of where & when in the universe it is performed

Splits into:

- spatial invariance
- temporal invariance
- Current best result is by Ashby et al., Phys. Rev. Lett. 98, 070802 (2007)

$$|\alpha| < 1.4 \times 10^{-6}$$

- Yet unpublished test with matter wave ntereferometry by Mueller et al (2010):

$$|\alpha| < 7 \times 10^{-9}$$

General Theory of Relativity and its Alternatives…

$$\mathcal{S}_{\rm G}[g_{mn}] = \frac{c^4}{16\pi G_{\rm N}} \int d^4x \sqrt{-g} R$$ ← Action of general relativity

$R = g^{mn} R_{mn}$ ← Ricci scalar, Ricci tensor & Christoffel symbols

$$R_{mn} = \partial_k \Gamma^k_{mn} - \partial_m \Gamma^k_{nk} + \Gamma^k_{mn}\Gamma^l_{kl} - \Gamma^k_{ml}\Gamma^l_{nk} \qquad \Gamma^k_{mn} = \frac{1}{2} g^{kp}(\partial_m g_{pn} + \partial_n g_{pm} - \partial_p g_{mn})$$

$$\mathcal{S}_{\rm SM}[\psi, A_m, H; g_{mn}] = \int d^4x \Big[-\frac{1}{4} \sum \sqrt{-g} g^{mk} g^{nl} F^a_{mn} F^a_{kl} - \sum \sqrt{-g} \bar{\psi} \gamma^m D_m \psi$$
$$- \frac{1}{2} \sqrt{-g} g^{mn} \overline{D_m H} D_n H - \sqrt{-g} V(H) - \sum \lambda \sqrt{-g} \bar{\psi} H \psi - \sqrt{-g} \rho_{\rm vac} \Big]$$

Action of Standard Model

Variational principle: $\frac{\delta}{\delta g_{mn}} \otimes \Big[\mathcal{S}_{\rm tot}[\psi, A_m, H; g_{mn}] = \mathcal{S}_{\rm G}[g_{mn}] + \mathcal{S}_{\rm SM}[\psi, A_m, H; g_{mn}] \Big] \Rightarrow$

$$R_{mn} - \frac{1}{2} g_{mn} R + \Lambda g_{mn} = \frac{8\pi G_{\rm N}}{c^4} T_{mn} \qquad \Lambda = 8\pi G_{\rm N} \rho_{\rm vac}/c^4 \qquad \rho_{\rm vac} \approx (2.3 \times 10^{-3} eV)^4$$

$$S = \frac{c^3}{4\pi G} \int d^4x \sqrt{-g} \left[\frac{1}{4} f(\varphi) R - \frac{1}{2} g(\varphi) \partial_\mu \varphi \partial^\mu \varphi + V(\varphi) \right] + \sum_i q_i(\varphi) \mathcal{L}_i$$

Scalar-Tensor theories of gravity

$$f(\varphi) = \varphi, \qquad g(\varphi) = \frac{\omega}{\varphi}, \qquad V(\varphi) = 0.$$ Brans and Dicke (1961)

TESTS OF RELATIVISTIC GRAVITY IN SPACE

Parameterized Post-Newtonian (PPN) formalism

PPN Formalism: Eddington, Fock, Chandrasekhar, Dicke, Nordtvedt, Thorne, Will,…

$$\begin{aligned} g_{00} &= 1 - \frac{2}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}} + \frac{2\beta}{c^4}\Big[\sum_{j\neq i}\frac{\mu_j}{r_{ij}}\Big]^2 - \frac{1+2\gamma}{c^4}\sum_{j\neq i}\frac{\mu_j \dot{r}_j^2}{r_{ij}} + \\ &+ \frac{2(2\beta-1)}{c^4}\sum_{j\neq i}\frac{\mu_j}{r_{ij}}\sum_{k\neq j}\frac{\mu_k}{r_{jk}} - \frac{1}{c^4}\sum_{j\neq i}\mu_j\frac{\partial^2 r_{ij}}{\partial t^2} + \mathcal{O}(c^{-5}) \\ g_{0\alpha} &= \frac{2(1+\gamma)}{c^3}\sum_{j\neq i}\frac{\mu_j \dot{\mathbf{r}}_j^{\alpha}}{r_{ij}} + \mathcal{O}(c^{-5}) \\ g_{\alpha\beta} &= -\delta_{\alpha\beta}\Big(1 + \frac{2\gamma}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}} + \frac{3\delta}{2c^4}\Big[\sum_{j\neq i}\frac{\mu_j}{r_{ij}}\Big]^2\Big) + \mathcal{O}(c^{-5}) \end{aligned}$$

- Assumption: Local Lorentz Invariance (LLI) and local position invariance (LPI) hold, thus, preferred frame parameters $\alpha_1, \alpha_2, \alpha_3$ are not included…
- General case, there are 10 PPN parameters: $\gamma, \beta, \zeta, \alpha_1, \alpha_2, \alpha_3, \xi_1, \xi_2, \xi_3, \xi_4$
- γ are β the Eddington's parameterized post-Newtonian (PPN) parameters:

General relativity: $\gamma = \beta = 1$ Brans-Dicke theory: $\gamma = \frac{1+\omega}{2+\omega}, \ \beta = 1$

- δ is the post-PPN parameter – important for next generation of light propagation tests.

TESTS OF RELATIVISTIC GRAVITY IN SPACE

PPN Equations of Motion (a part of the model)

$$
\begin{aligned}
\ddot{\mathbf{r}}_i = & \sum_{j\neq i} \frac{Gm_j(\mathbf{r}_j-\mathbf{r}_i)}{r_{ij}^3}\Big\{ \left[\frac{m_G}{m_I}\right]_i - \frac{2(\beta+\gamma)}{c^2}\sum_{l\neq i}\frac{Gm_l}{r_{il}} - \frac{2\beta-1}{c^2}\sum_{k\neq j}\frac{Gm_k}{r_{jk}} + \\
& + \gamma\left(\frac{\dot{r}_i}{c}\right)^2 + (1+\gamma)\left(\frac{\dot{r}_j}{c}\right)^2 - \frac{2(1+\gamma)}{c^2}\dot{\mathbf{r}}_i\dot{\mathbf{r}}_j + \frac{\dot{G}\cdot t}{G} - \\
& - \frac{3}{2c^2}\left[\frac{(\mathbf{r}_i-\mathbf{r}_j)\dot{\mathbf{r}}_j}{r_{ij}}\right]^2 + \frac{1}{2c^2}(\mathbf{r}_j-\mathbf{r}_i)\ddot{\mathbf{r}}_j\Big\} + \\
& + \frac{1}{c^2}\sum_{j\neq i}\frac{Gm_j}{r_{ij}^3}\Big\{\left[\mathbf{r}_i-\mathbf{r}_j\right]\cdot\left[(2+2\gamma)\dot{\mathbf{r}}_i-(1+2\gamma)\dot{\mathbf{r}}_j\right]\Big\}(\dot{\mathbf{r}}_i-\dot{\mathbf{r}}_j) + \\
& + \frac{3+4\gamma}{2c^2}\sum_{j\neq i}\frac{Gm_j\ddot{\mathbf{r}}_j}{r_{ij}} + \sum_{m=1}^{3}\frac{Gm_m(\mathbf{r}_m-\mathbf{r}_i)}{r_{im}^3} + \sum_{c,s,m}\mathbf{F}_{\text{asteroids}}
\end{aligned}
$$

Possible EP violation

Possible temporal dependence of *G*

$$\left[\frac{m_G}{m_I}\right]_{\text{SEP}} = 1+\eta\left(\frac{\Omega}{mc^2}\right)$$

$$\eta = 4\beta-\gamma-3$$

$$\Omega_i = -\frac{G}{2}\int_i d^3x\rho_i U_i = -\frac{G}{2}\int_i d^3x d^3x' \frac{\rho_i(\mathbf{r})\rho_i(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

- In general theory of relativity $\gamma=\beta=1$, thus $\eta=0$ (this is not the case for scalar-tensor theories of gravity, for instance, where these parameters can have different values).

$$t_2-t_1 = \frac{r_{12}}{c} + (1+\gamma)\sum_i \frac{\mu_i}{c^3}\ln\left[\frac{r_1^i+r_2^i+r_{12}^i+\frac{(1+\gamma)\mu_i}{c^2}}{r_1^i+r_2^i-r_{12}^i+\frac{(1+\gamma)\mu_i}{c^2}}\right] + \mathcal{O}(c^{-5})$$

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that predict $\gamma = 0$ or -1 fail

Newton 1686 Poincaré 1890

Einstein 1912 **Nordstrøm 1912** **Nordstrøm 1913** **Einstein & Fokker 1914** Einstein 1915

Whitehead 1922 Cartan 1923 Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

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Belinfante & Swihart 1975 Lee et al. 1976 Bekenstein 1977 Barker 1978 Rastall 1979

Coleman 1983 Logunov 1987 Hehl 1997 Overlooked (20th century)

Parameterized Post-Newtonian Formalism (PPN):

- Solar system is the main arena to test weak gravity:
- Expand the metrics; identify various potentials
- They have 10 PPN parameters in front

$$\gamma, \beta, \zeta, \alpha_1, \alpha_2, \alpha_3, \xi_1, \xi_2, \xi_3, \xi_4$$

- Calculate those parameters & Compare with experiments

[2010: A need for Cosmological PPN?]

Conformally-flat theories fail test of time delay and deflection of light:

- Nordstrom (1912)
- Nordstrom (1913)
- Einstein & Fokker (1914)
- Littlewood & Bergmann (1956)
- Ni (1972)

Cassini 2002 Conjunction Experiment

Results of Cassini Conjunction Experiment:

- Spacecraft—Earth separation > 1 billion km
- Doppler/Range: X~7.14GHz & Ka~34.1GHz
- Result: $\gamma = 1 + (2.1 \pm 2.3) \cdot 10^{-5}$

Gravitational deflection of light (Einstein, 1915):

$$\theta_{gr}^{\odot}(b) = \frac{2(1+\gamma)GM_{\odot}}{bc^2} \simeq 8 \times 10^{-6} \left(\frac{1+\gamma}{2}\right)\left(\frac{R_{\odot}}{b}\right)$$

Gravitational delay of light signals (Shapiro, 1976):

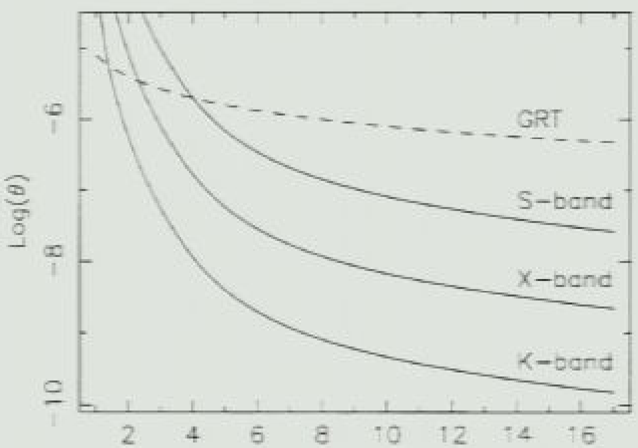
$$\Delta t = (1+\gamma)\frac{GM_{\odot}}{c^3} \ln\left[\frac{r_1 + r_2 + r_{12}}{r_1 + r_2 - r_{12}}\right] \simeq (1+\gamma)\frac{GM_{\odot}}{c^3} \ln\left[\frac{2r_1 r_2}{b^2}\right]$$

Gravitational frequency drift (Bertotti et al, 2003):

$$y_{gr}(b) \simeq -\frac{v}{c}\theta_{gr}^{\odot}(b) = \frac{2(1+\gamma)GM_{\odot}}{bc^2}\frac{v}{c} \simeq 8 \times 10^{-10}\left(\frac{1+\gamma}{2}\right)\left(\frac{R_{\odot}}{b}\right)$$

$$y = \frac{\Delta\nu}{\nu}$$

Modern-day DSN frequency stability $y \sim 10^{-15}$



TESTING RELATIVISTIC GRAVITY IN SPACE

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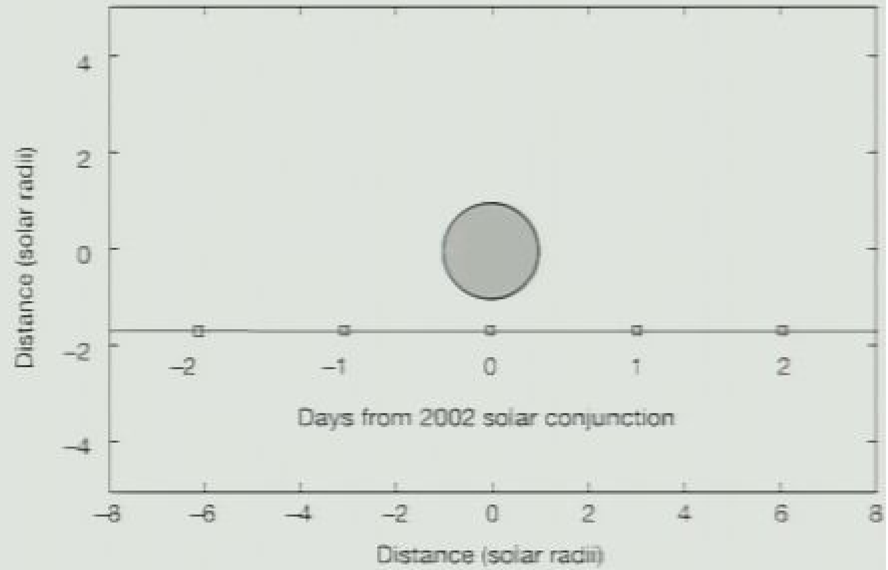

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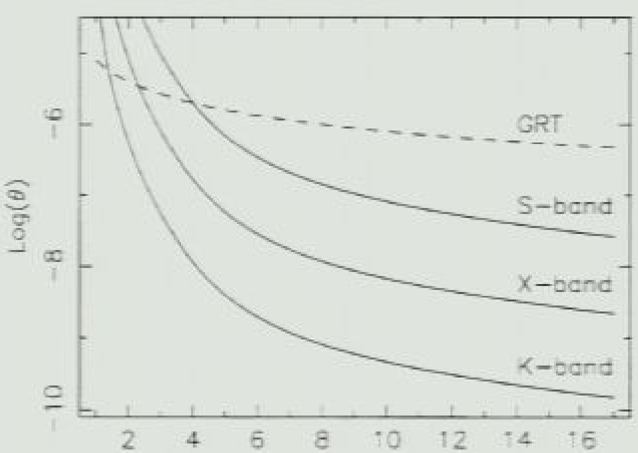

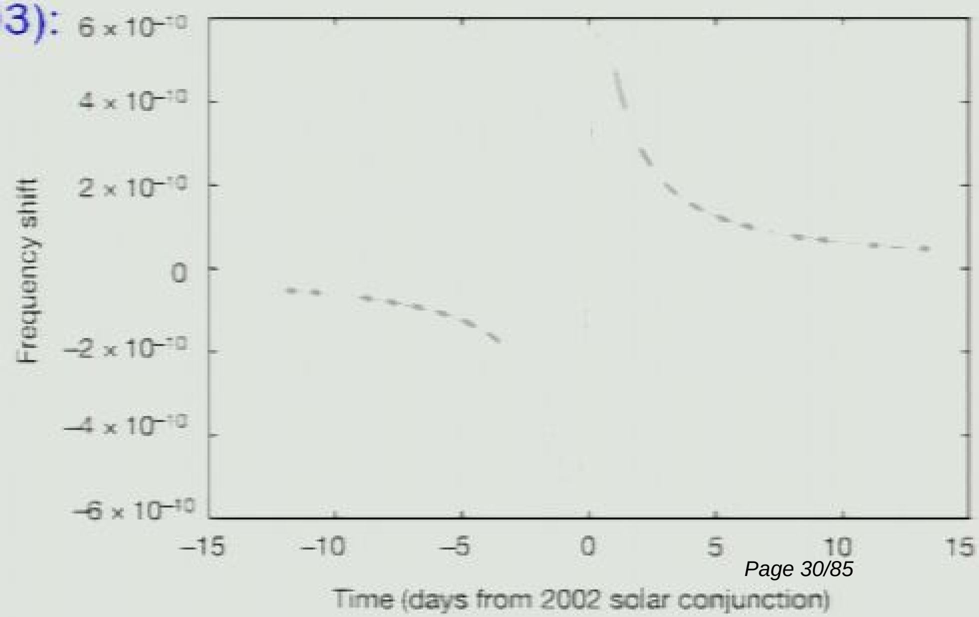

TESTS OF RELATIVISTIC GRAVITY IN SPACE

List of PPN Parameters for Competing Theories

Competing theories of Gravity	γ	β	ξ	α_1	α_2	α_3	ζ_1	ζ_2	ζ_3	ζ_4
Einstein (1915) GR	1	1	0	0	0	0	0	0	0	0
Scalar Field theories – Note: in Page-Tupper (1968): parameter *d* is defined as $\Delta = 1-\gamma$										
Einstein (1912) [not GR]	0	0	–	–4	0	–2	0	–1	0	0*
Whitrow-Morduch (1965)	0	–1	–	–4	0	0	0	–3	0	0*
Rosen (1971)	λ	$\frac{3}{4}+\frac{\lambda}{4}$	–	$-4(1-\lambda)$	0	–4	0	–1	0	0
Papetrou (1954a, 1954b)	1	1	–	–8	-4	0	0	2	0	0
Ni (1972) (stratified)	1	1	–	–8	0	0	0	2	0	0
Yilmaz (1958, 1962)	1	1	–	–8	0	–4	0	–2	0	–1*
Page-Tupper (1968)	γ	β	–	-4Δ	0	-2Δ	0	ζ_2	0	ζ_4
Nordström (1912, 1913)	–1	½	–	0	0	0	0	0	0	0*
Einstein-Fokker (1914)	–1	½	–	0	0	0	0	0	0	0
Ni (1972) (flat)	–1	$1-q$	–	0	0	0	0	ζ_2	0	0*
Whitrow-Morduch (1960)	–1	$1-q$	–	0	0	0	0	q	0	0*
Littlewood (1953), Bergman (1956)	–1	½	–	0	0	0	0	–1	0	0*

– Note: * The theory is incomplete, and ζ_4 can take one of two values. The value closest to zero is listed

List of PPN Parameters for Competing Theories

Competing theories of Gravity	γ	β	ξ	α_1	α_2	α_3	ζ_1	ζ_2	ζ_3	ζ_4
Einstein (1915) GR	1	1	0	0	0	0	0	0	0	0
Scalar-Tensor theories										
Bergmann (1968), Wagoner (1970)	$\frac{1+\omega}{2+\omega}$	β	0	0	0	0	0	0	0	0
Nordtvedt (1970), Bekenstein (1977)	$\frac{1+\omega}{2+\omega}$	β	0	0	0	0	0	0	0	0
Brans-Dicke (1961)	$\frac{1+\omega}{2+\omega}$	1	0	0	0	0	0	0	0	0
Vector-Tensor theories										
Hellings-Nordtvedt (1973)	γ	β	0	α_1	α_2	0	0	0	0	0
Will-Nordtvedt (1972)	1	1	0	0	α_2	0	0	0	0	0
Bimetric theories – Note: in Rosen (1975): parameter k_2 is defined as $k_2 = (c_0/c_1) - 1$										
Rosen (1975)	1	1	0	0	k_2	0	0	0	0	0
Rastall (1979)	1	1	0	0	α_2	0	0	0	0	0
Lightman-Lee (1973)	γ	β	0	α_1	α_2	0	0	0	0	0
Stratified theories										
Lee-Lightman-Ni (1974)	ac_0/c_1	β	ξ	α_1	α_2	0	0	0	0	0
Ni (1973)	ac_0/c_1	bc_0	0	α_1	α_2	0	0	0	0	0

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Unlikely Scalar-Tensor Theories

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 **Cartan 1923** Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 **Thiry 1948** Papapetrou 1954 **Jordan 1955** Littlewood & Bergmann 1956

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Page & Tupper 1968 **Bergmann 1968** Deser & Laurent 1968 **Nordtvedt 1970** **Wagoner 1970**

Bollini et al. 1970 Rosen 1971 **Will & Nordtvedt 1972** Ni 1972 **Hellings & Nordtvedt 1972**

Ni 1973 **Yilmaz 1973** **Lightman & Lee 1973** **Lee, Lightman & Ni 1974** **Rosen 1975**

Belinfante & Swihart 1975 **Lee et al. 1976** **Bekenstein 1977** **Barker 1978** **Rastall 1979**

Coleman 1983 **Logunov 1987** **Hehl 1997** **Overlooked (20th century)**

Scalar-Tensor theories are extremely constrained by Viking (1976) result on γ:

- Thiry (1948), Jordan 1955
- Brans & Dicke (1961): ω >6500 (Viking, 1976), ω >40,000 (Cassini, 2003)
- Bergmann (1968), Nordtvedt (1970)
- Wagoner (1970), Bekenstein (1977)
- Barker (1978)

JPL

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

NASA

Mercury's Perihelion: Theories that fail

Newton 1686 Poincaré 1890

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Stratified theories predict preferred frame effects on perihelion shift:

- Ni (1973)
- Lee, Lightman & Ni (1974)

$$\dot{\varpi} = (2+2\gamma-\beta)\frac{GM_\odot n_M}{c^2 a_M(1-e_M^2)} + \frac{3}{4}\left(\frac{R_\odot}{a_M}\right)^2 \frac{J_{2\odot} n_M}{(1-e_M^2)^2}(3\cos^2 i_M - 1), \quad ''/\mathrm{cy}$$

$$\dot{\varpi} = 42''.98\left[\frac{1}{3}(2+2\gamma-\beta) + 0.296\cdot J_{2\odot}\times 10^4\right]. \quad ''/\mathrm{cy}$$

$J_{2\odot} \simeq 2\times 10^{-7}$ from helioseismology; confirmed by Konopliv et al., 2008

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

GW & Binary Pulsar: Theories that fail

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 **Cartan 1923** Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

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Coleman 1983 **Logunov 1987** **Hehl 1997** **Overlooked (20th century)**

Bi-metric Theories predict a dipole radiation. Can't be…:

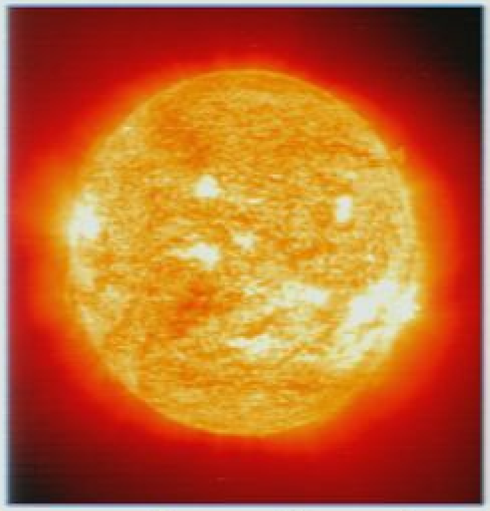
- Rosen (1975)
- Lee et al. (1976)
- Rastall (1979)
- Lightman & Lee (1973)

TESTS OF RELATIVISTIC GRAVITY IN SPACE

The Current Values of the PPN Parameters

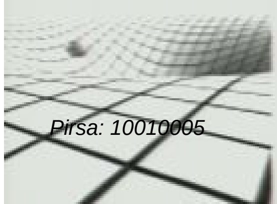
Para-meter	What it measured relative to General Relativity?	Current value	Effects	Experiments
γ	Measure of space curvature produced by unit mass	2.3×10^{-5}	Time delay, light deflection	Cassini tracking
β	Measure of non-linearity in gravitational superposition	1.1×10^{-4}	Nordtvedt effect, perihelion shift	Lunar laser ranging
ξ	Measure of existence of preferred location effects	1×10^{-3}	Earth tides	Gravimeter data
α_1	Measure the existence of preferred frame effects	1×10^{-4}	Orbit polarization	Lunar laser ranging
α_2		4×10^{-7}	Spin precession	Sun axis' alignment w/ ecliptic
α_3		4×10^{-20}	Self-acceleration	Pulsar spin-down statistics
ζ_1	Measure (plus α_3) of the failure of conservation laws of energy, momentum and angular momentum	2×10^{-2}	–	Combined PPN bounds
ζ_2		4×10^{-5}	Binary pulsar acceleration	Pulsar: PSR 1913+16
ζ_3		1×10^{-8}	Newton's 3rd law	Lunar acceleration
ζ_4		6×10^{-3}	–	Kreuzer experiment

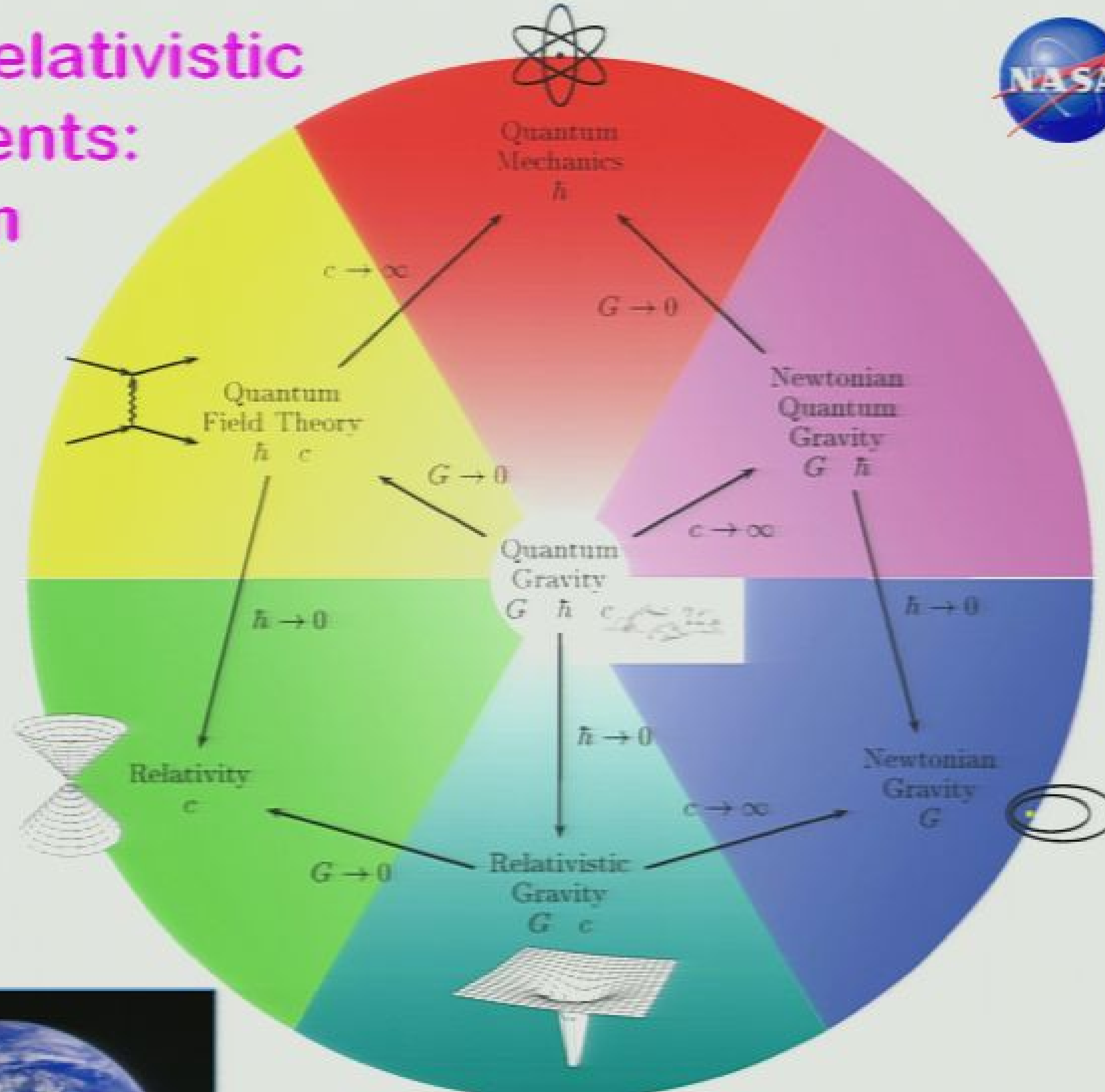
Laboratory for Relativistic Gravity Experiments: Our Solar System



Strongest gravity potential

$$\frac{GM_{Sun}}{c^2 R_{Sun}} \sim 10^{-6}$$





$$\frac{GM_{\oplus}}{c^2 R_{\oplus}} \sim 10^{-9}$$

Most accessible region for gravity tests in space:

- ISS, LLR, SLR, free-fliers

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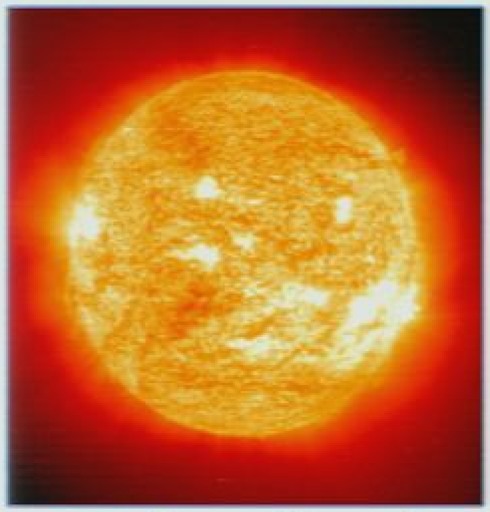

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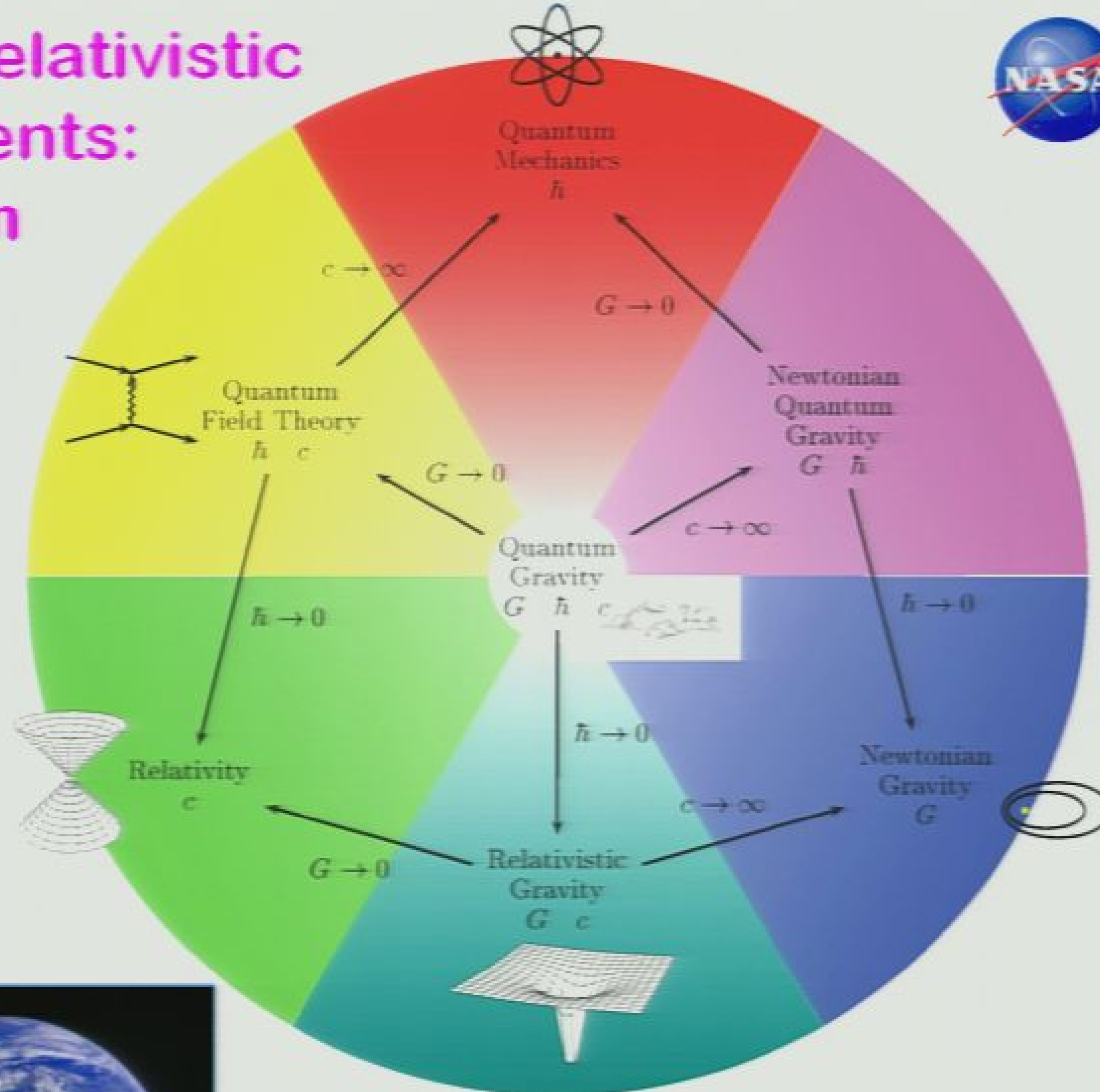

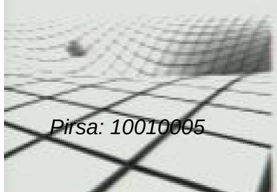

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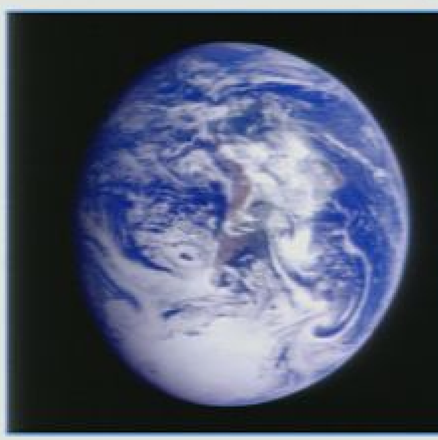

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40 Years of Solar System Gravity Tests

Techniques for Gravity Tests:

Radar Ranging:

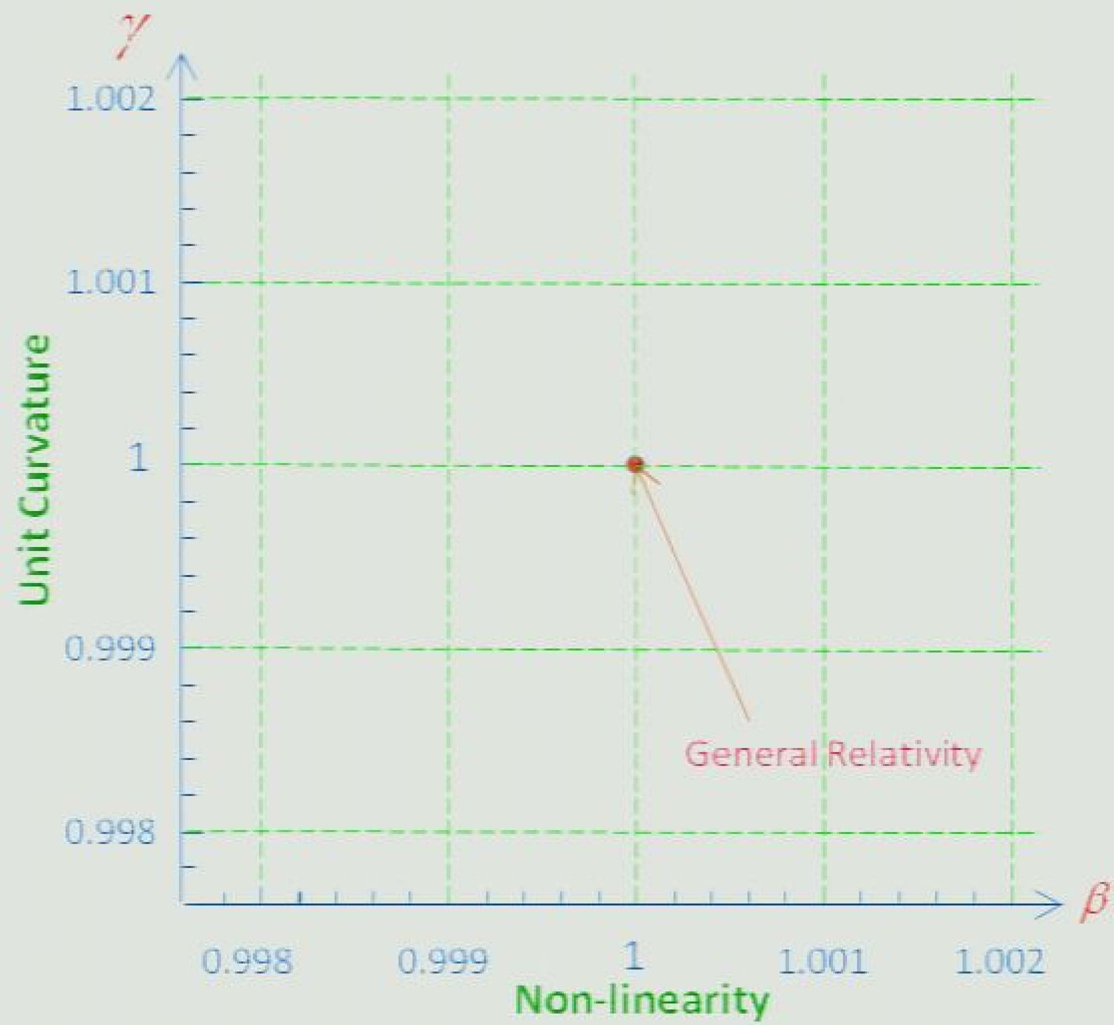
- Planets: Mercury, Venus, Mars
- s/c: Mariners, Vikings, Pioneers, Cassini, Mars Global Surveyor, Mars Orbiter, etc.
- VLBI, GPS, etc.

Laser:

- SLR, LLR, interplanetary, etc.

Dedicated Gravity Missions:

- **LLR (1969 - on-going!!)**
- GP-A, '76; LAGEOS, '76,'92; GP-B, '04; LARES, '10; LISA, 2020+(?)



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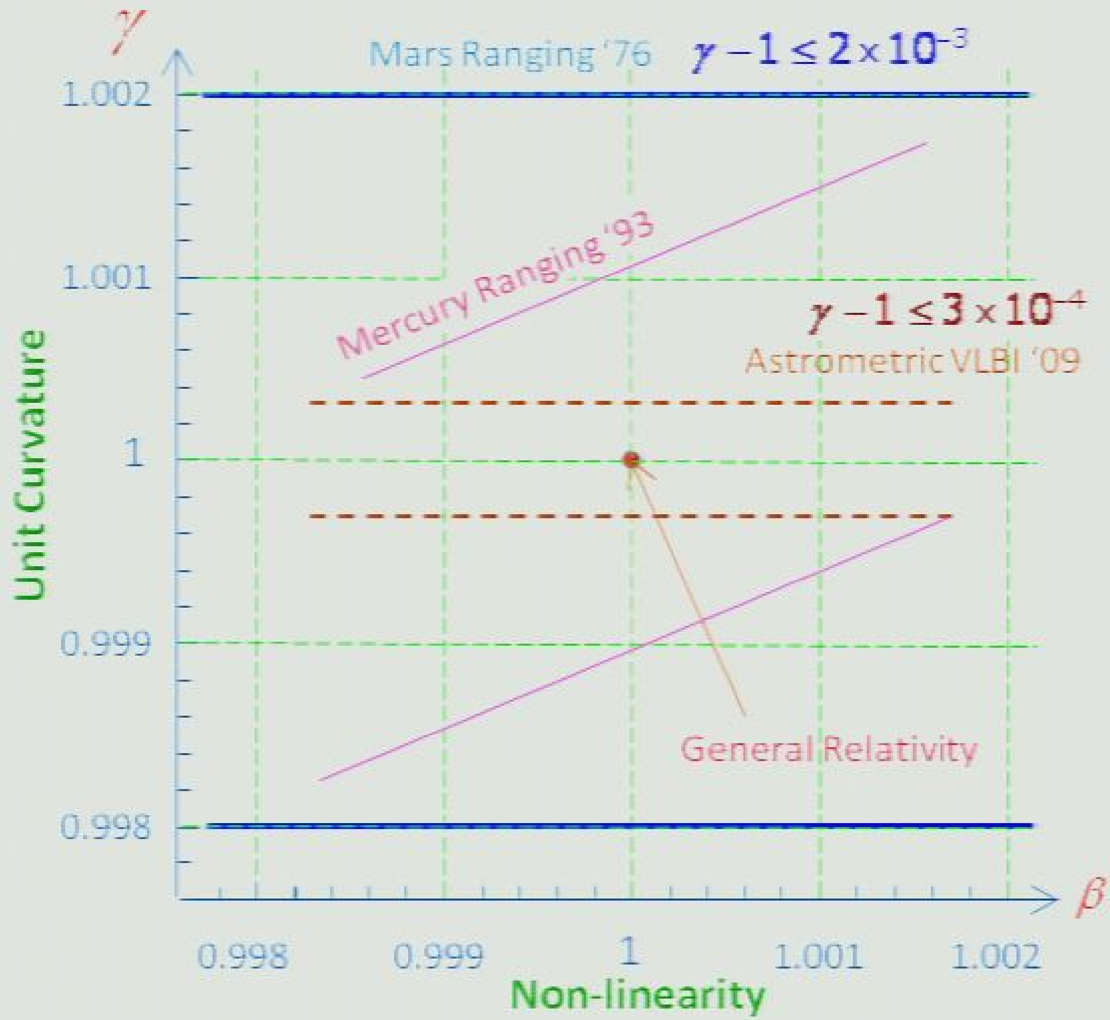
- GP-A, '76; LAGEOS, '76,'92; GP-B, '04; LARES, '10; LISA, 2020+(?)



TESTS OF RELATIVISTIC GRAVITY IN SPACE

40 Years of Solar System Gravity Tests

Techniques for Gravity Tests:

Radar Ranging:

- Planets: Mercury, Venus, Mars
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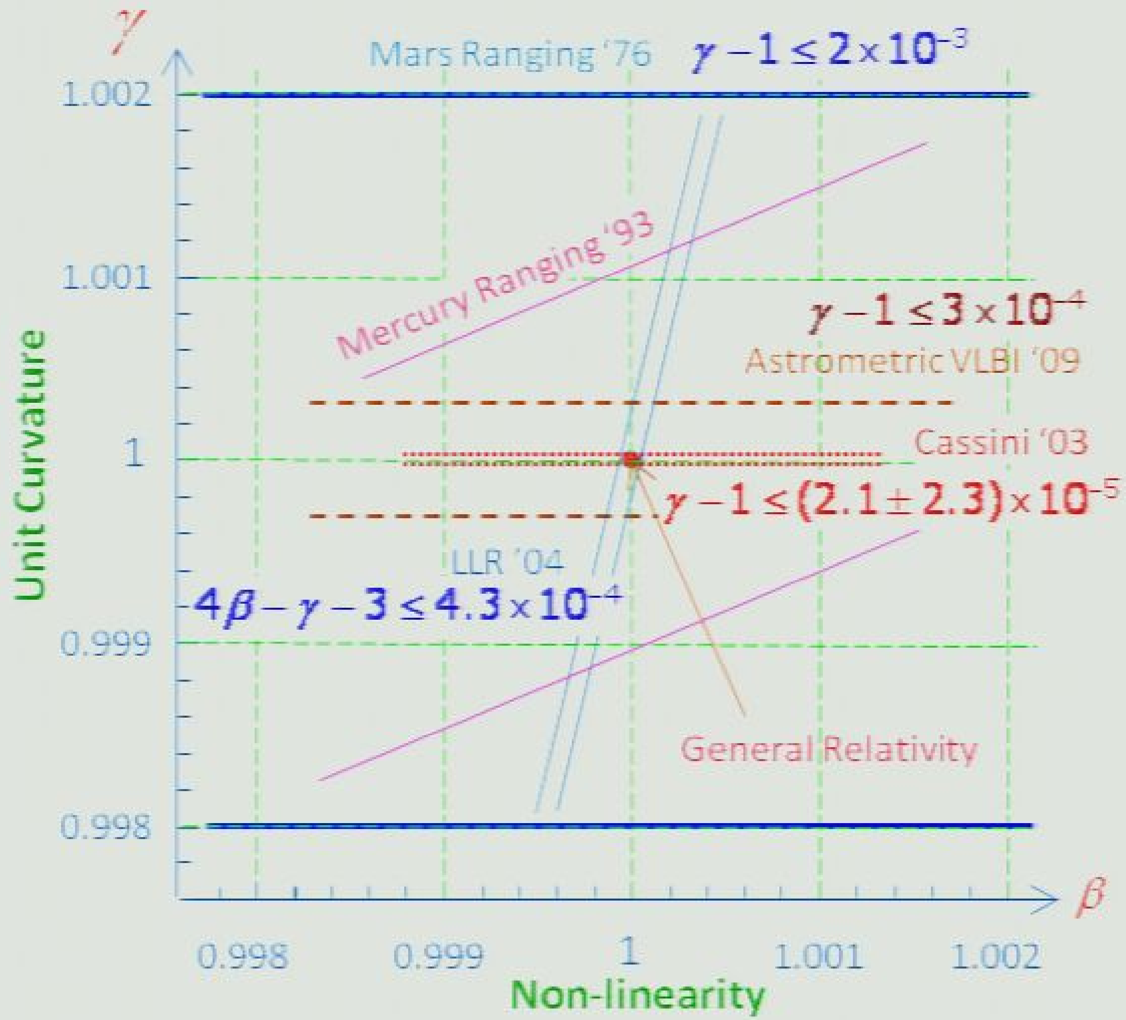

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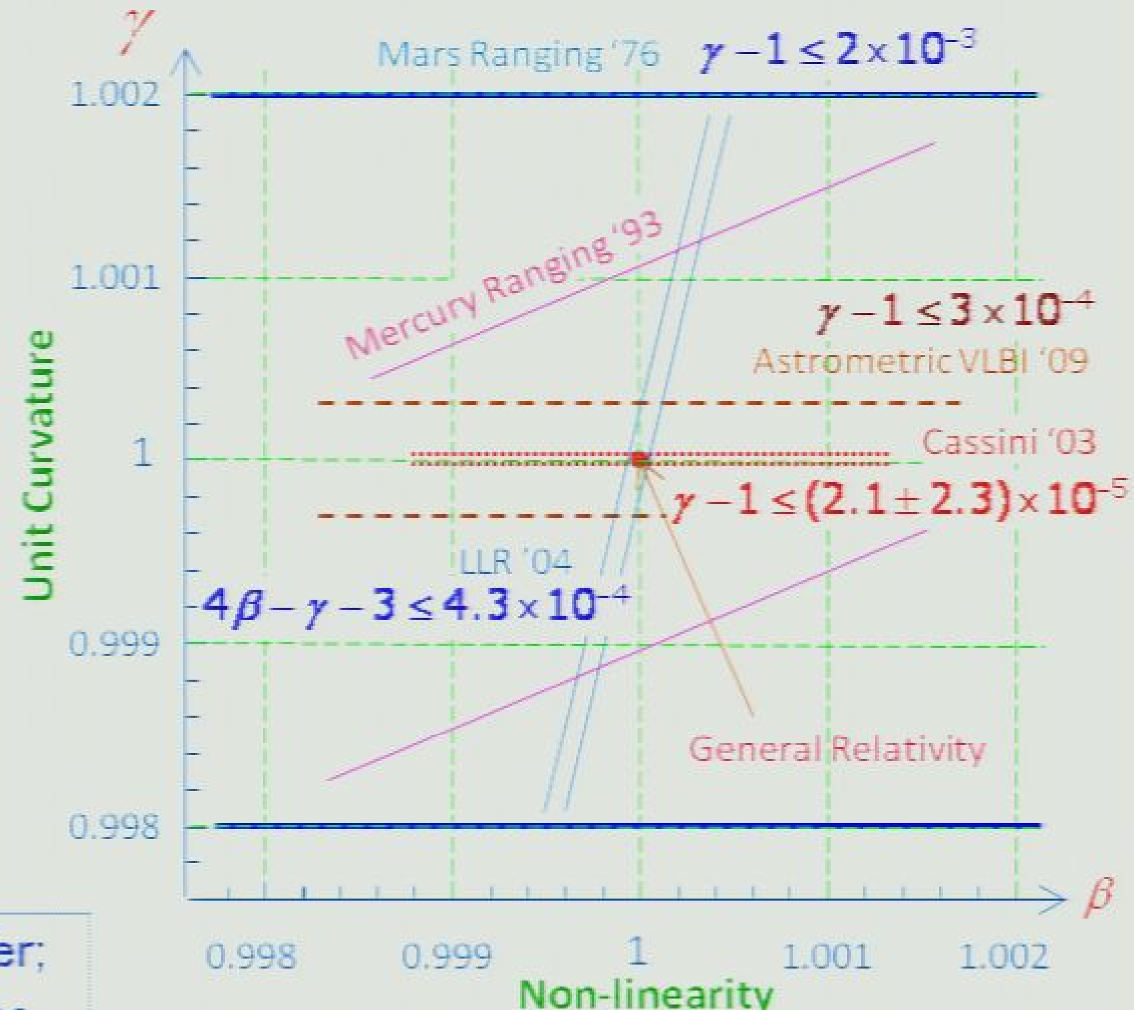
New Engineering Discipline – Applied General Relativity:
- Daily life: GPS, geodesy, time transfer;
- Precision measurements: deep-space navigation & astrometry (SIM, Gaia,....).



A factor of 100 in 40 years is impressive, but is not enough for the near future!

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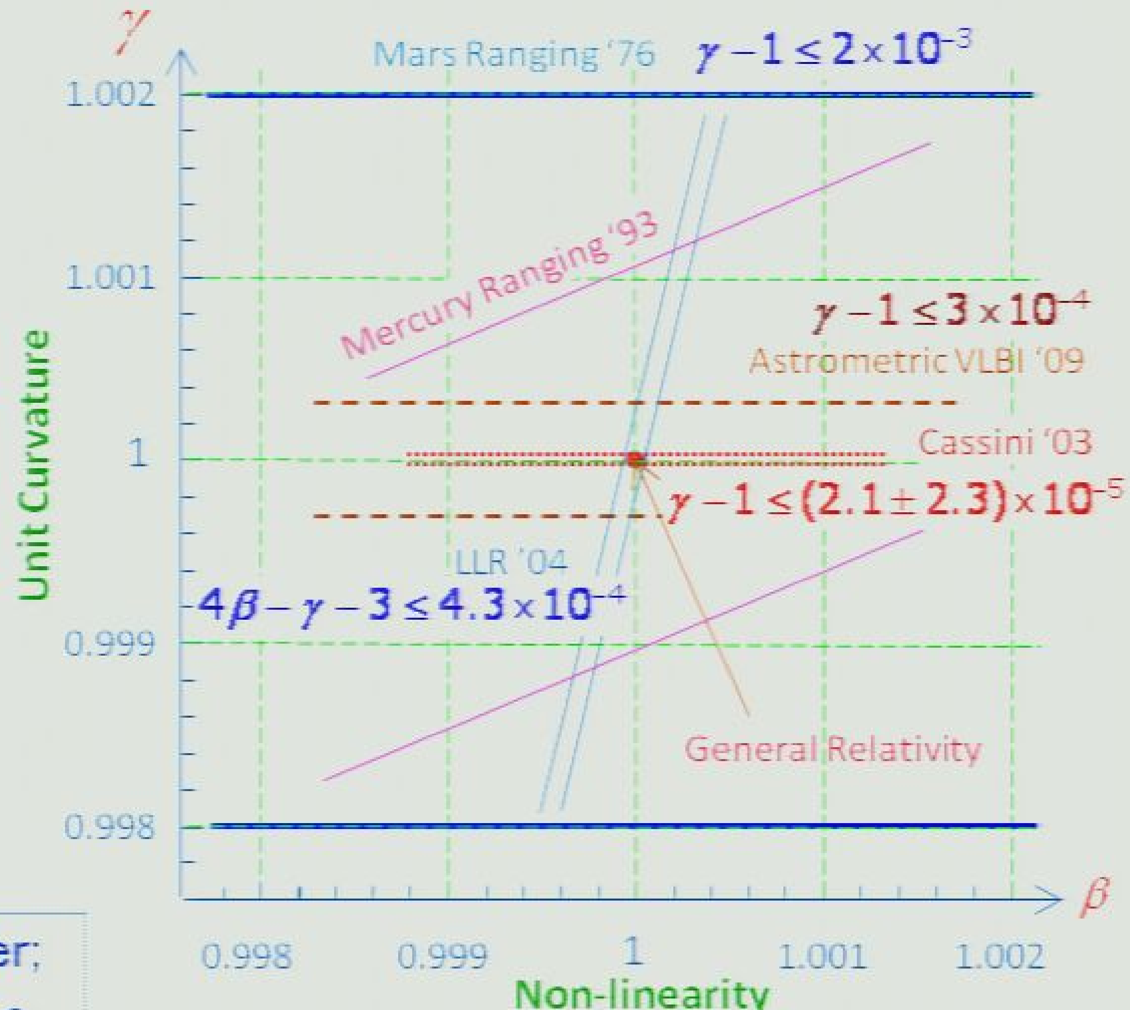

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Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Some Theories resist to fail

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 **Cartan 1923** Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 Thiry 1948 Papapetrou 1954 Jordan 1955 Littlewood & Bergmann 1956

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Bollini et al. 1970 Rosen 1971 **Will & Nordtvedt 1972** Ni 1972 **Hellings & Nordtvedt 1972**

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Belinfante & Swihart 1975 Lee et al. 1976 Bekenstein 1977 Barker 1978 Rastall 1979

Coleman 1983 **Logunov 1987** **Hehl 1997** **Overlooked (20th century)**

- Will & Nordtvedt (1972) and Hellings & Nordtvedt (1972) are vector-tensor theories. Deviations can be only significant in high energy regimes (e.g. Planck energy) to be in accord with current precision of experiments
- Yilmaz (1973) was mathematically inconsistent, but now is fixed. Does not predict black holes
- Logunov (1987) bi-metric theory with massive graviton; fits all the solar system and binary pulsar data, does not predict black holes, universe is flat and infinite, no Big Bang singularity, oscillates…
- Cartan (1923), Hehl (1997) introduces matter spin

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

"Aesthetics-Based" Conclusion for 20th Century

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- "Among all bodies of physical law none has ever been found that is simpler and more beautiful than Einstein's geometric theory of gravity"

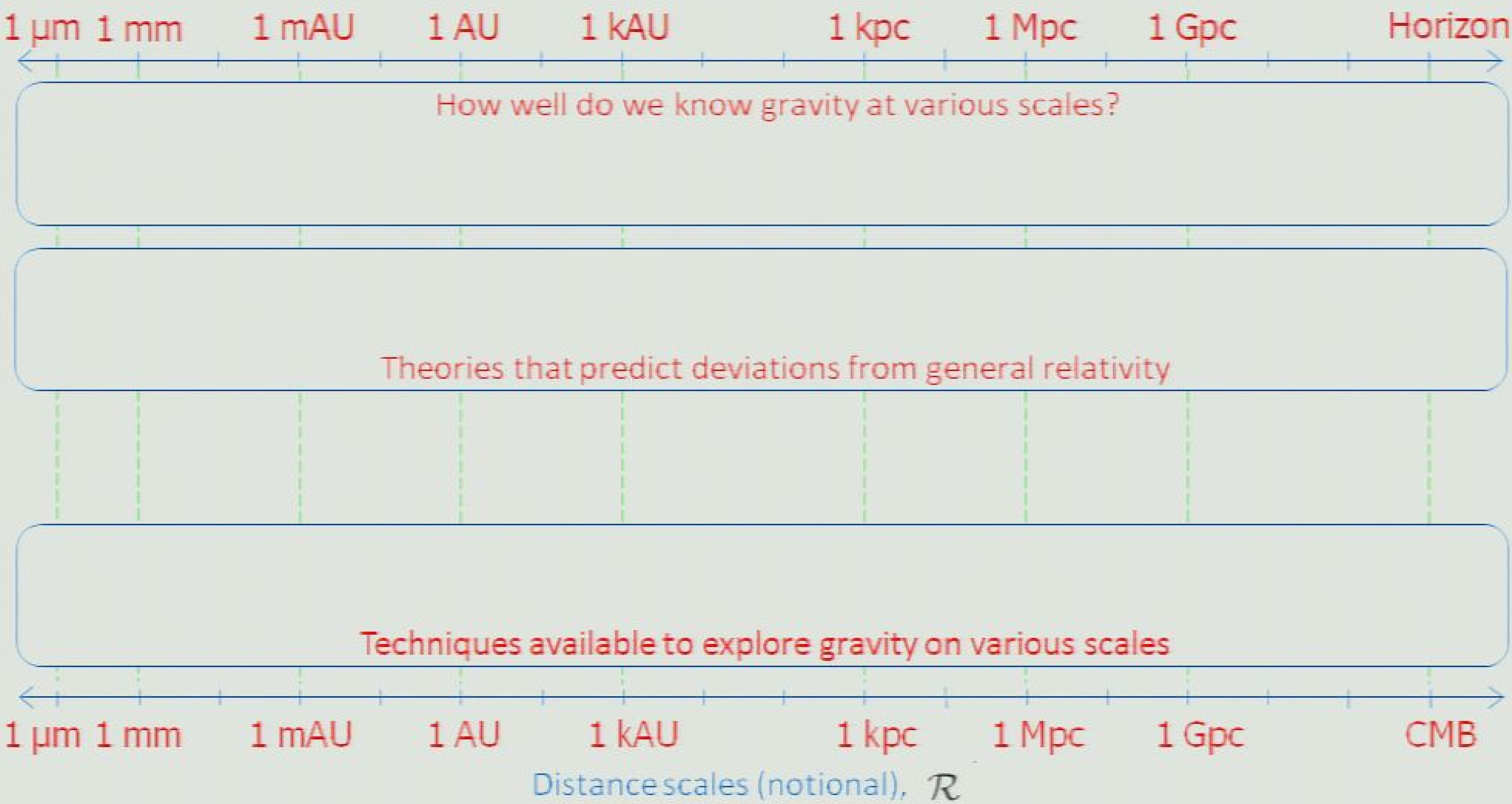
  – Misner, Thorne and Wheeler, 1973

- "[…] Unfortunately, any finite number of effects can be fitted by a sufficiently complicated theory. […] Aesthetic or philosophical motives will therefore continue to play a part in the widespread faith in Einstein's theory, even if all tests verify its predictions."

  – Malcolm MacCallum, 1976

Theoretical Landscape of the 21th Century:

How well do we know gravity?



Solar system experiments allow for improvements in our knowledge of gravity

Theoretical Landscape of the 21th Century:

How well do we know gravity?

1 µm 1 mm 1 mAU 1 AU 1 kAU 1 kpc 1 Mpc 1 Gpc Horizon

How well do we know gravity at various scales?

?? **Well-tested** **Reasonably well-tested** **No precise data** **Poorly tested:** **Just started:**

– Pioneer and flyby anomalies? *– Dark Matter?* *– Dark Energy?*

Theories that predict deviations from general relativity

Techniques available to explore gravity on various scales

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Distance scales (notional), $\mathcal{R}$

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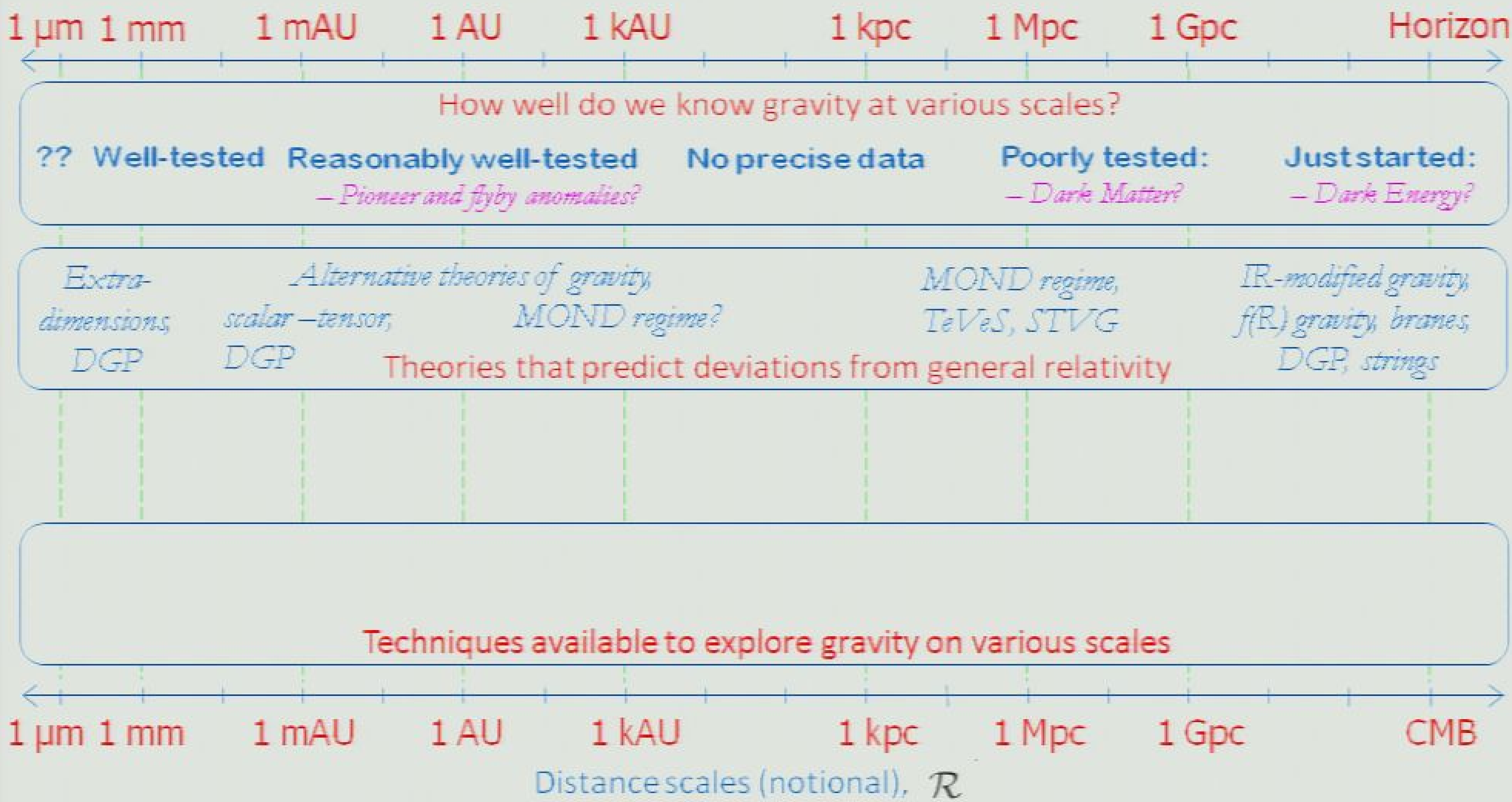

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Extra-dimensions, DGP

Alternative theories of gravity, scalar – tensor, DGP MOND regime?

MOND regime, TeVeS, STVG

IR-modified gravity, f(R) gravity, branes, DGP, strings

Theories that predict deviations from general relativity

Lab Tests *Space-Based Experiments* *Astronomy* *Astrophysics* *Cosmology*

Controlled Experiments *Astronomical Observations*

Techniques available to explore gravity on various scales

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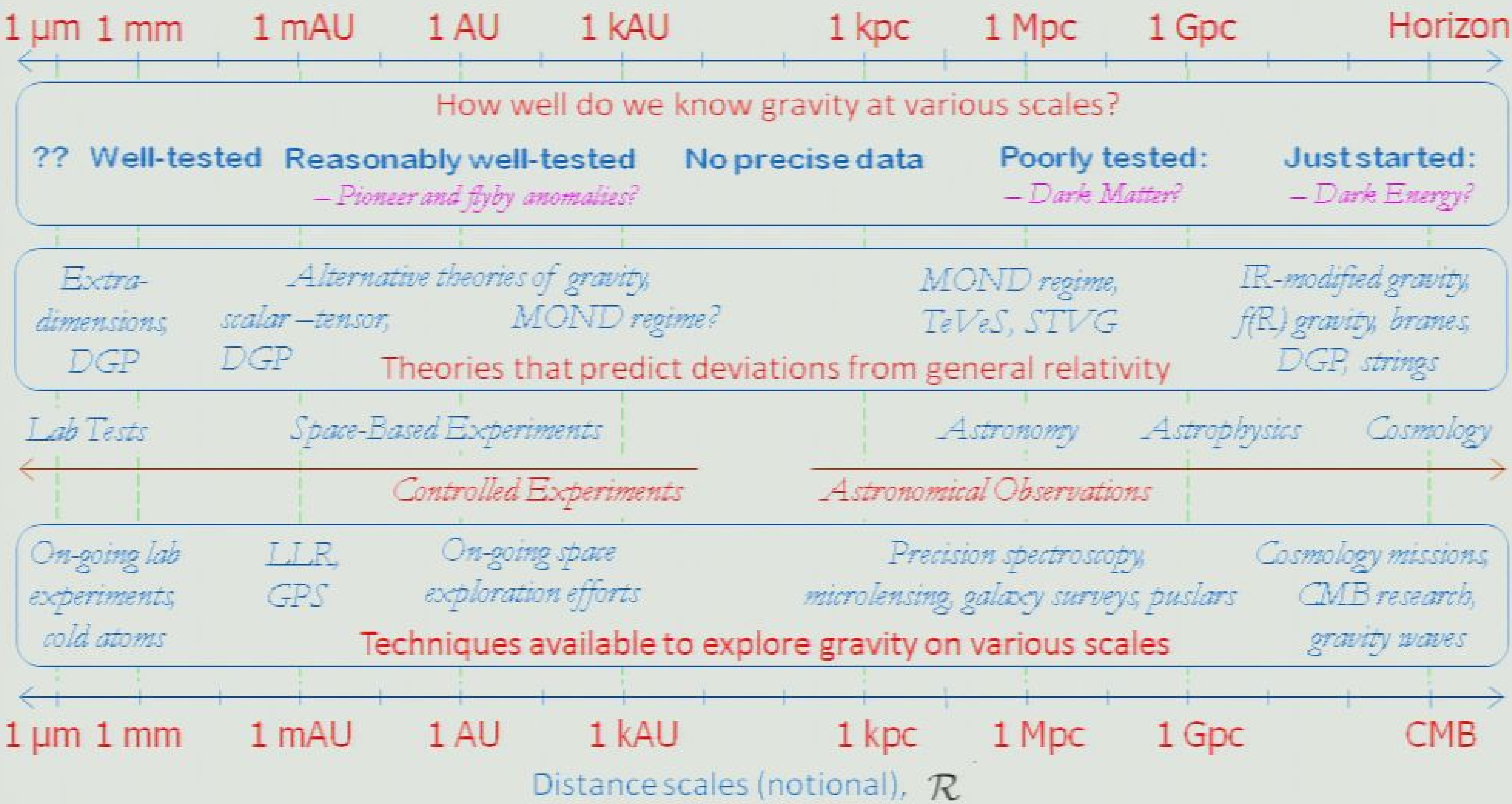

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Coleman 1983 Logunov 1987 Hehl 1997 Overlooked (20th century) **Scalar-Tensor Theories**

Arkani-Hamed, Dimopoulos & Dvali 2000 **Dvali, Gabadadze & Poratti 2003** **Strings theory?**

Bekenstein 2004 **Moffat 2005** **Multiple f(R) models 2003-07** **Bi-Metric Theories**

Need for new theory of gravity:

- Classical GR description breaks down in regimes with large curvature
- If gravity is to be quantized, GR will have to be modified or extended

Other challenges:

- Dark Matter
- Dark Energy
- Pioneer Anomaly...

Motivations for new tests of GR:

- GR is a fundamental theory
- Alternative theories & models
- New ideas & techniques require comprehensive investigations

Theoretical Landscape of the 21st Century: *(Re-emergence of the)* **Scalar Field**

Fundamental Physics Challenges:

- Appearance of space-time singularities;
- Classical description breaks down in large curvature domains;
- Quest for Quantum Gravity → Standard Model and/or GR modification;
- Dark Energy…

Alternative Theories of Gravity:

- Grand Unification Models, Standard Model Extensions;
- Inflationary cosmologies, strings, Kaluza-Klein theories;

Common element: scalar partners – dilaton, moduli fields...

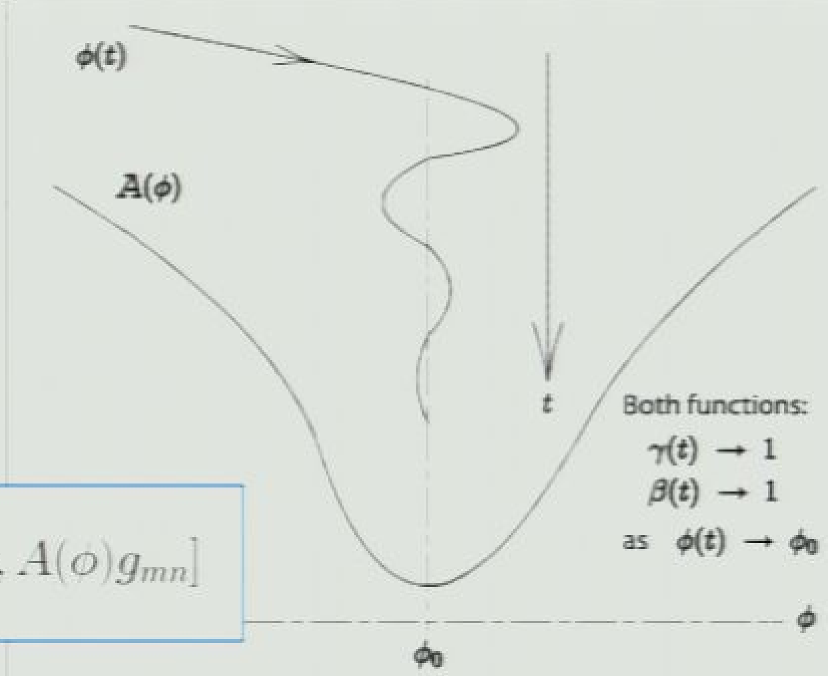
The scalar field is a pioneer, sent out to explore new worlds of physics!

"Gravity and the Tenacious Scalar Field"
Carl Brans, gr-qc/9705069

– Waves, Optics
– Electrodynamics
– Quantum Mechanics
– Scalar QED
– Field Theory
– Symmetry Breaking
– Dilatons, Moduli
– …

– Nordstrom's Scalar Gravity
– Kaluza-Klein Unification
– Dirac and Jordan's Cosmology
– Scalar-Tensor Gravity
– Inflation
– Quintessence
– TeVeS, STVG,
– …

Theoretical Motivation for New Gravity Tests



Long-range massless [or low-mass] scalar:

The low-energy limit of the String Theory in 'Einstein Frame' (Damour-Nordtvedt-Polyakov 1993) suggests:

$$S = -\frac{1}{16\pi G}\int dx^4\sqrt{-g}\Big(R - 2g^{mn}\nabla_m\phi\nabla_n\phi\Big) + S_M[\psi_M, A(\phi)g_{mn}]$$

Expansion $A(\varphi)$ around background value φ_0 of the scalar leads:

$$\ln A(\varphi) = \ln A(\varphi_0) + \alpha_0(\varphi - \varphi_0) + \frac{1}{2}k_0(\varphi - \varphi_0)^2 + \mathcal{O}(\Delta\varphi^3)$$

Slope α_0 measures the coupling strength of interaction between matter and the scalar.

$$\gamma - 1 = \frac{-2\alpha_0^2}{1+\alpha_0^2} \simeq -2\alpha_0^2$$

$$\beta - 1 = \frac{1}{2}\frac{\alpha_0^2 k_0}{(1+\alpha_0^2)^2} \simeq \frac{1}{2}\alpha_0^2 k_0 \simeq \frac{1}{4}(1-\gamma)k_0$$

Scenario for cosmological evolution of the scalar (Damour, Piazza & Veneziano 2002):

$$\gamma - 1 \sim 7.3 \times 10^{-7}\left(\frac{H_0}{\Omega_0^3}\right)^{\frac{1}{2}} \Rightarrow \gamma - 1 \sim 10^{-5} - 10^{-7}$$

The unit curvature PPN parameter γ is the most important quantity to test

TESTS OF RELATIVISTIC GRAVITY IN SPACE

Modified Gravity: f(R) theories

- A broad class of alternative theories

$$\mathcal{L}_\phi = -\frac{M^2}{2}\omega(\phi)(\partial\phi)^2 - V(\phi)$$

$$S = \int d^n x\sqrt{-g}\Big[\frac{1}{2}f(R,\phi) + \mathcal{L}_\phi(g_{\mu\nu},\phi,\partial\phi) + \mathcal{L}_m(g_{\mu\nu},\Psi)\Big]$$

$$(\partial\phi)^2 = \nabla_\mu\phi\nabla^\mu\phi \qquad F(R,\phi) = \partial f(R,\phi)/\partial R$$

Generalized gravity	$\frac{1}{2}f(R,\phi)$	$\mathcal{L}_\phi(\phi,\partial\phi)$	$p(R,\phi)$	φ	$V(\varphi)$
Nonlinear gravity	$\frac{1}{2}f(R)$	$\omega=0,\ V=0$	$p=F(R)$	$\sqrt{\frac{3}{2}}\ln F$	$\frac{FR-f}{2F^2}$
R^2-gravity	$\frac{1}{2}(R+\alpha R^2)$	$\omega=0,\ V=0$	$p=1+2\alpha R$	$\sqrt{\frac{3}{2}}\ln F$	$\frac{FR-f}{2F^2}$
$1/R$-gravity	$\frac{1}{2}(R-\mu^4/R)$	$\omega=0,\ V=0$	$p=1+\mu^4/R^2$	$\sqrt{\frac{3}{2}}\ln F$	$\frac{FR-f}{2F^2}$
Scalar-tensor theory	$\frac{1}{2}F(\phi)R$	$\omega(\phi),\ V(\phi)$	$p=F(\phi)$	$\int\sqrt{\frac{\omega}{F}+\frac{3}{2}\frac{F'^2}{F^2}}d\phi$	$\frac{V}{F^2}$
Brans-Dicke theory	ϕR	$\omega(\phi)=2\frac{\omega}{\phi},\ V=0$	$p=\phi$	$\int\sqrt{\frac{\omega}{F}+\frac{3}{2}\frac{F'^2}{F^2}}d\phi$	0
Dilaton	$\frac{1}{2}e^{-\phi}R$	$\omega(\phi)=e^{-\phi},\ V=0$	$p=e^{-\phi}$	$\frac{5}{2}\phi$	0
NMC scalar	$\frac{1}{2}(1+\xi\phi^2)R$	$\omega=1,\ V(\phi)$	$p=1+\xi\phi^2$	$\int\frac{\sqrt{1+\xi(6\xi-1)\phi^2}}{1-\xi\phi^2}d\phi$	$\frac{V}{1-\xi\phi^2}$
CC $(\xi=\frac{1}{6})$	$\frac{1}{2}(1+\frac{1}{6}\phi^2)R$	$\omega=1,\ V(\phi)$	$p=1+\frac{1}{6}\phi^2$	$\sqrt{6}\tanh^{-1}\frac{\phi}{\sqrt{6}}$	$\frac{V}{1-\frac{1}{6}\phi^2}$
Induced Gravity	$\frac{1}{2}\epsilon\phi^2R$	$\omega=1,\ V(\phi)$	$p=\epsilon\phi^2$	$\sqrt{6+\frac{1}{\epsilon}}\ln\phi$	$\frac{V}{\epsilon\phi^2}$
GR with a scalar	$\frac{1}{2}R$	$\omega=1,\ V(\phi)$	$p=1$	ϕ	V

ψ - are matter fields; ϕ - is a scalar field

too many references

Dark Energy Ideas: What it is there for us?

Modifications of Einstein Gravity

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_m[g_{\mu\nu}, \psi]$$

Carroll et al, PRD 70 (2004) 043528

...

Modification of PPN Gravity

$$\gamma - 1 = -\frac{f''(R)^2}{f'(R) + 2f''(R)^2}.$$

$$\beta - 1 = \frac{1}{4}\frac{f'(R)\cdot f''(R)}{2f'(R) + 3f''(R)^2}\frac{d\gamma}{dR}.$$

Analogy between scalar-tensor and higher-order gravity

Constraints on ... $f(R)$ from solar system experiments...

...tight restrictions on the form of the gravitational Lagrangian

Need for cosmological PPN formalism

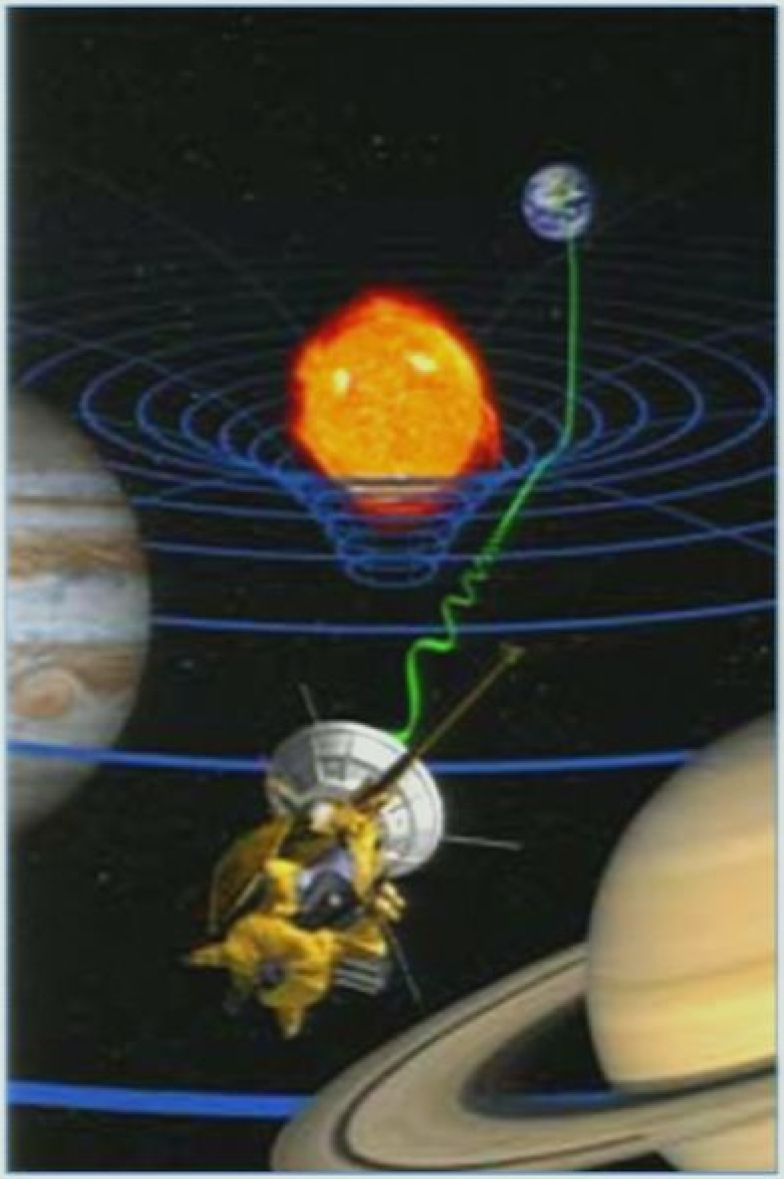
Capozziello, Stabile, Trosi, gr-qc/0603071

TESTING RELATIVISTIC GRAVITY IN SPACE

Cassini 2003: Where Do We Go From Here?

Cassini Conjunction Experiment:

- Spacecraft—Earth separation > 1 billion km
- Doppler/Range: X~7.14GHz & Ka~34.1GHz
- Result: $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$



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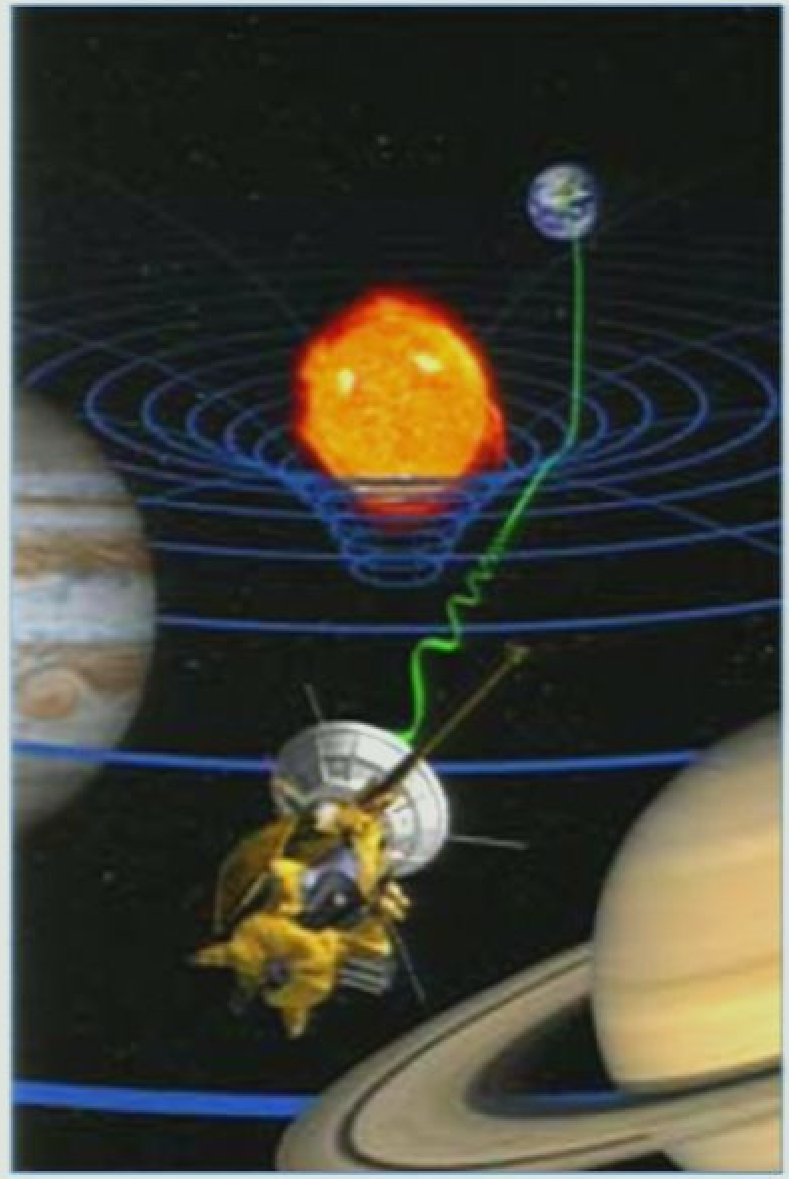
Possible with Existing Technologies?!

- VLBI [current $\gamma = 3 \times 10^{-4}$]: limited to $\sim 1 \times 10^{-4}$:
  - uncertainty in the radio source coordinates
- LLR [current $\eta = 4 \times 10^{-4}$]: in 5 years $\sim 3 \times 10^{-5}$:
  - mm accuracies [APOLLO] & modeling efforts
- μ-wave ranging to a lander on Mars $\sim 6 \times 10^{-6}$
- tracking of BepiColombo s/c at Mercury $\sim 2 \times 10^{-6}$
- Optical astrometry [current $\gamma = 3 \times 10^{-3}$]:

  SIM & Gaia $\sim 1 \times 10^{-6}$ (2016/18?)



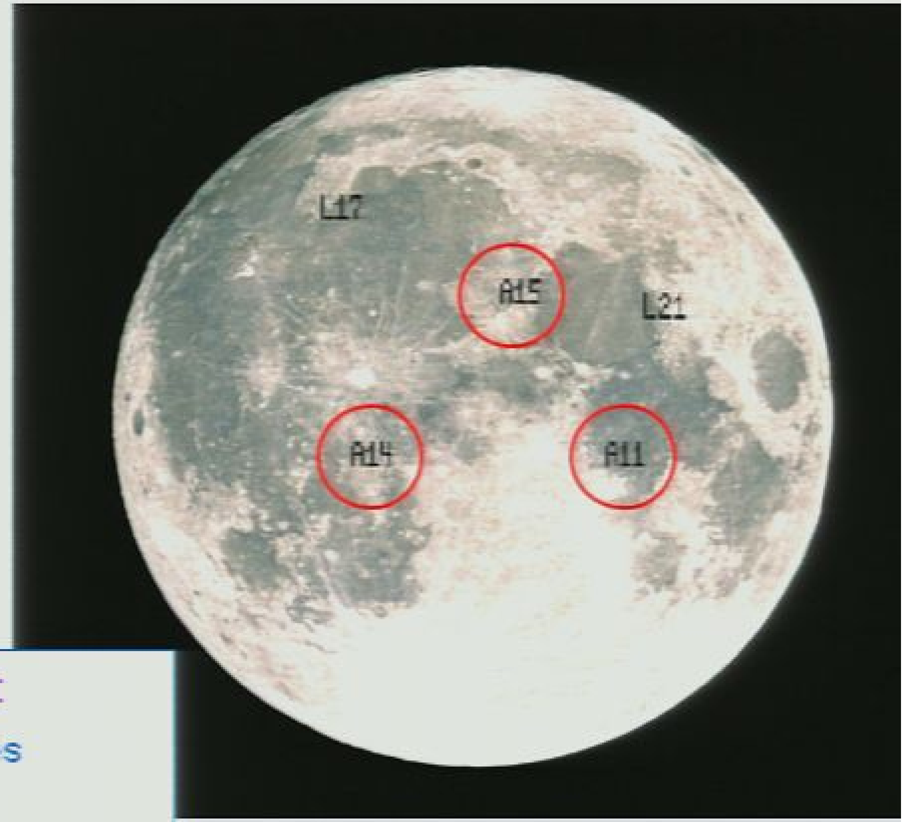
One needs a **dedicated mission** to explore accuracies better than 10^{-6} for both PPN parameters γ (and β). Interplanetary laser ranging is a possibility.

LUNAR LASER RANGING SCEINCE

Lunar Laser Ranging

It is all begun 40 year ago…

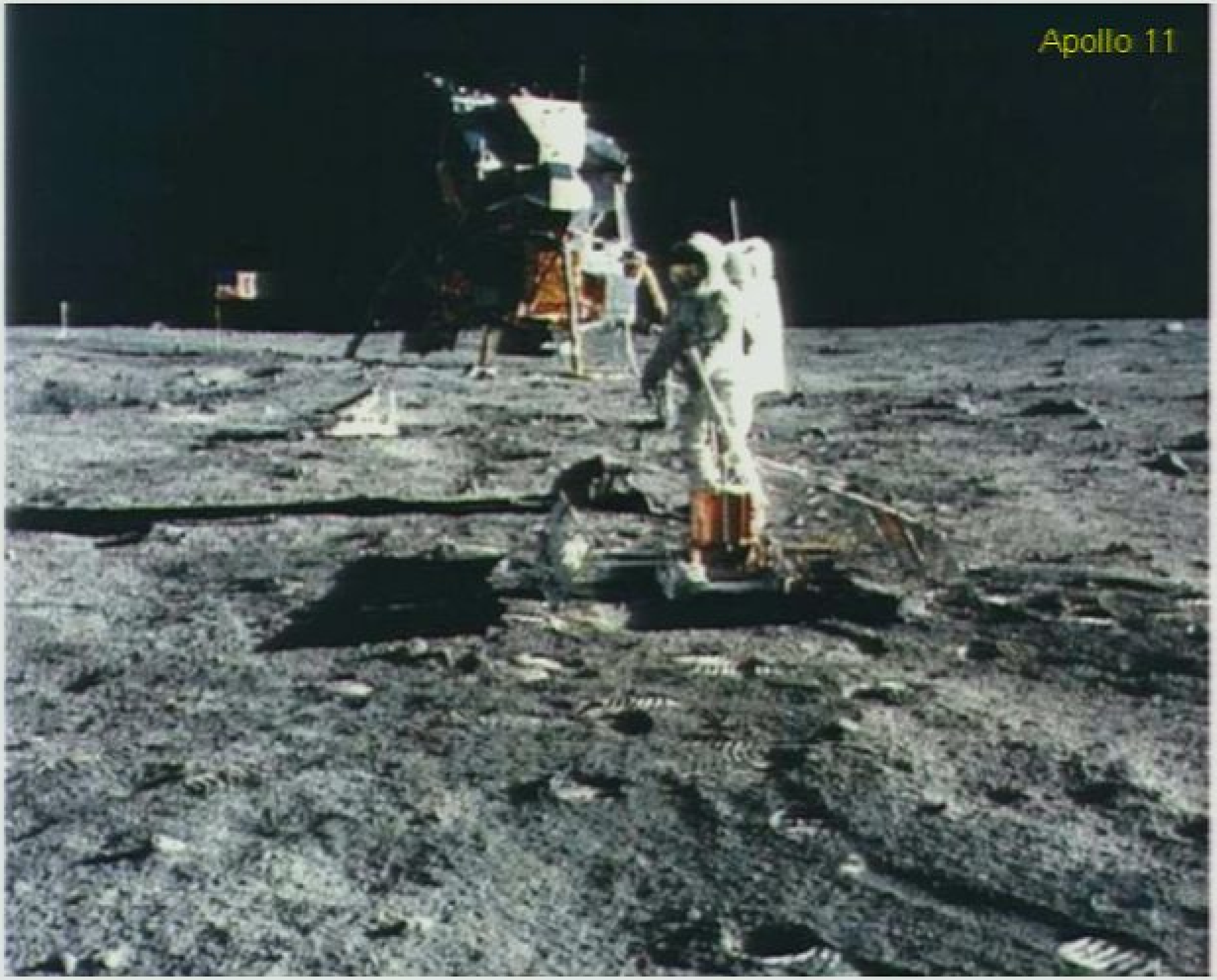
Laser Ranges between observatories on the Earth and retroreflectors on the Moon started by Apollo in 1969 and continue to the present



- 4 reflectors are ranged:
  - Apollo 11, 14 & 15 sites
  - Lunakhod 2 Rover
- LLR conducted primarily from 3 observatories:
  - McDonald (Texas, USA)
  - OCA (Grasse, France)
  - Haleakala (Hawaii, USA)
- New LLR stations:
  - Apache Point, (NM, USA)
  - Matera (Matera, Italy)
  - South Africa, former OCA LLR

Excellent Legacy of the Apollo Program

The Apollo 11 retroreflector initiated a shift from analyzing lunar position angles to ranges. Today LLR is the **only** continuing experiment since the Apollo-Era



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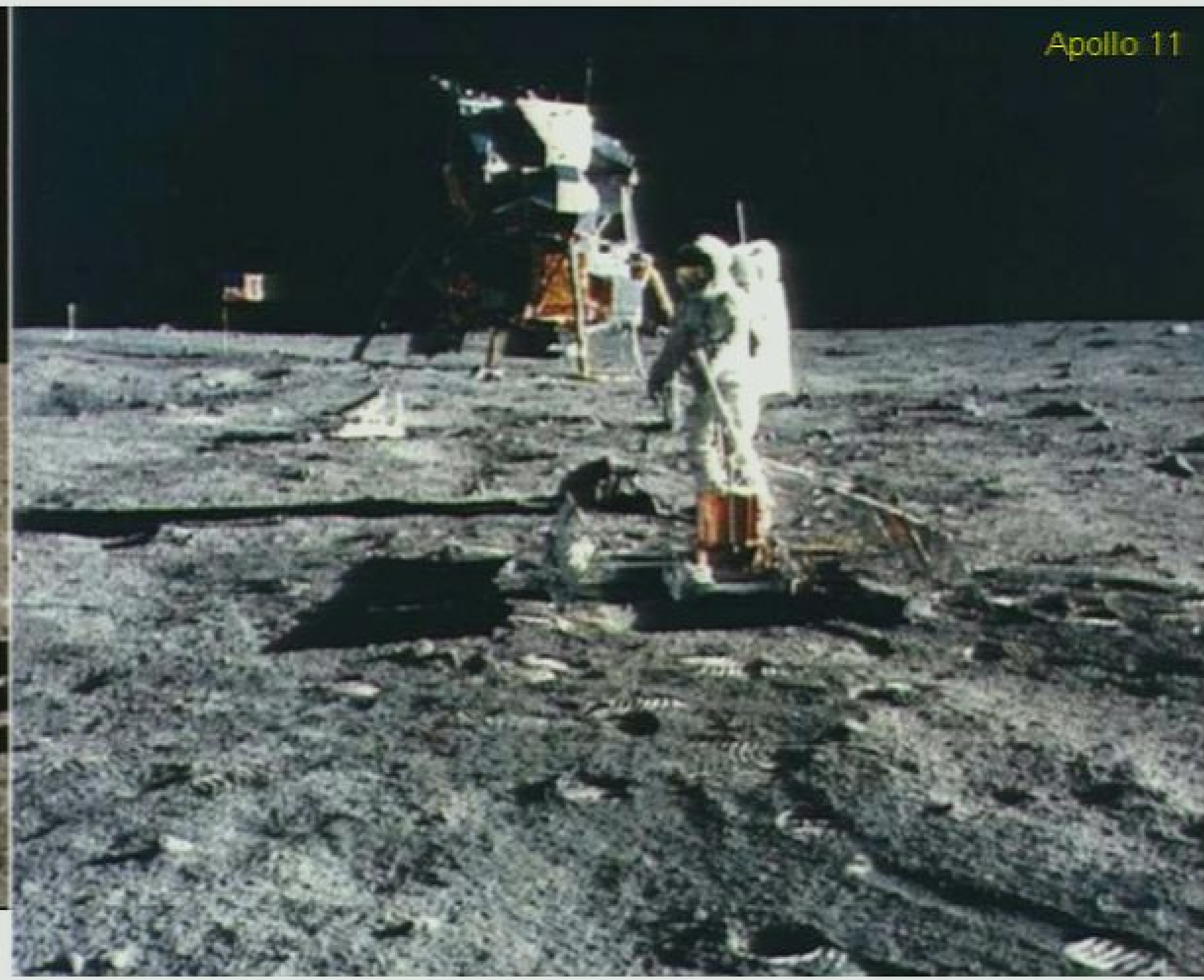

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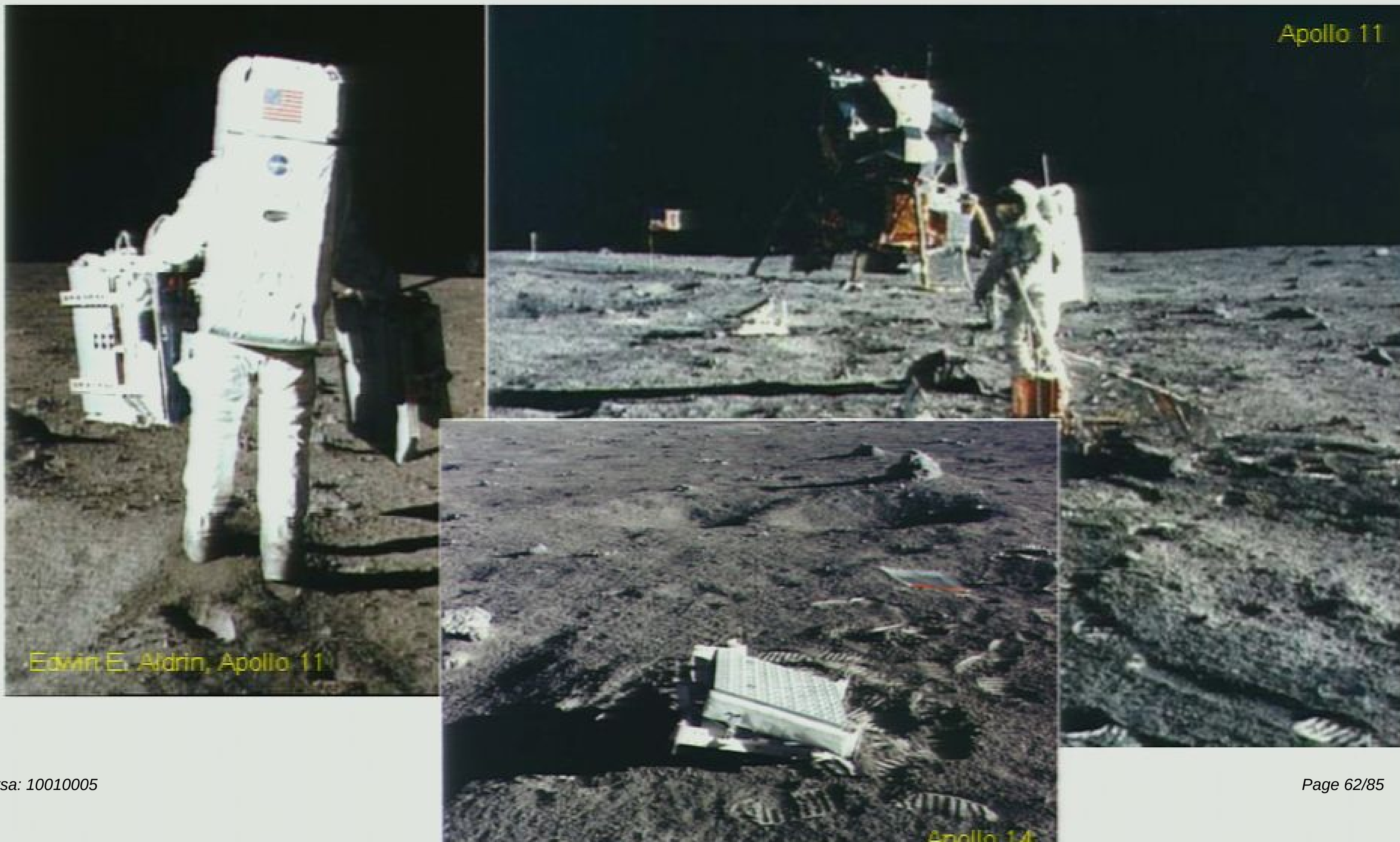
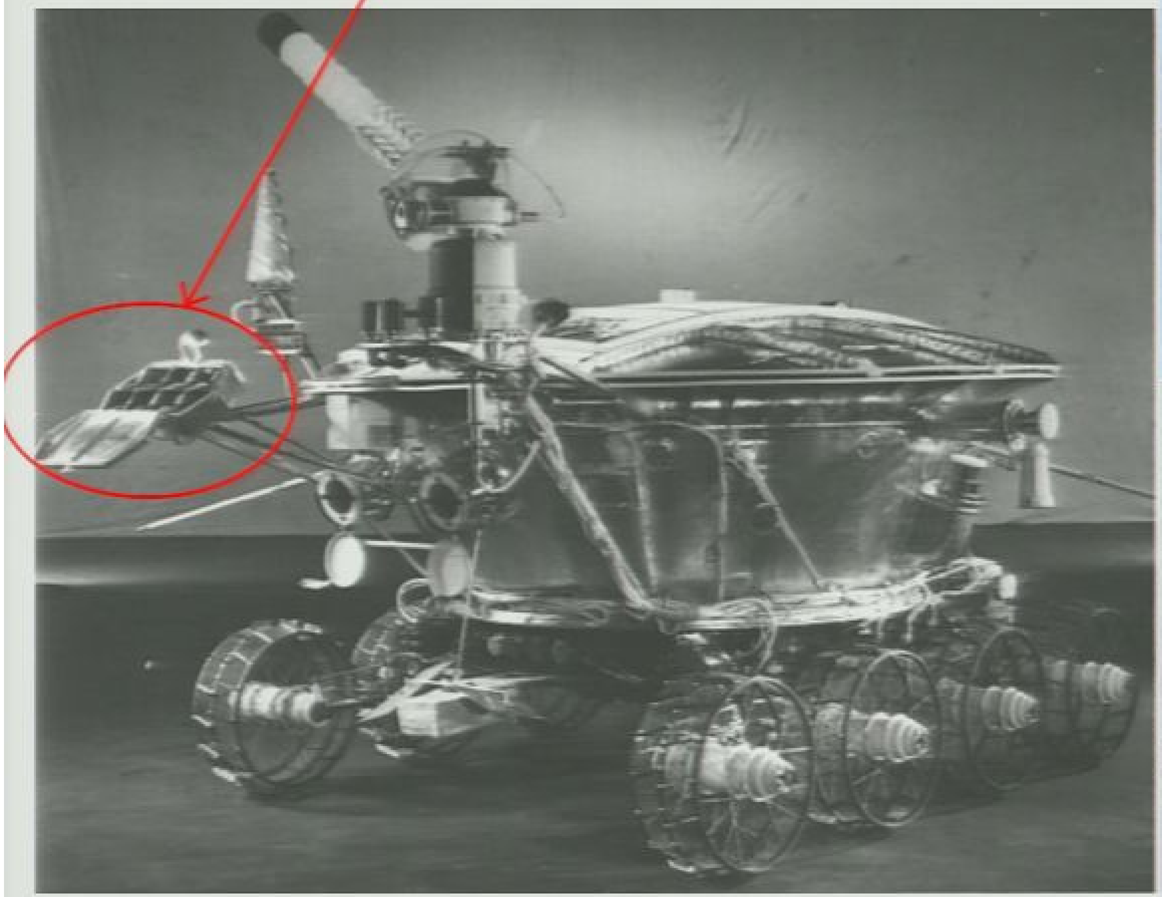


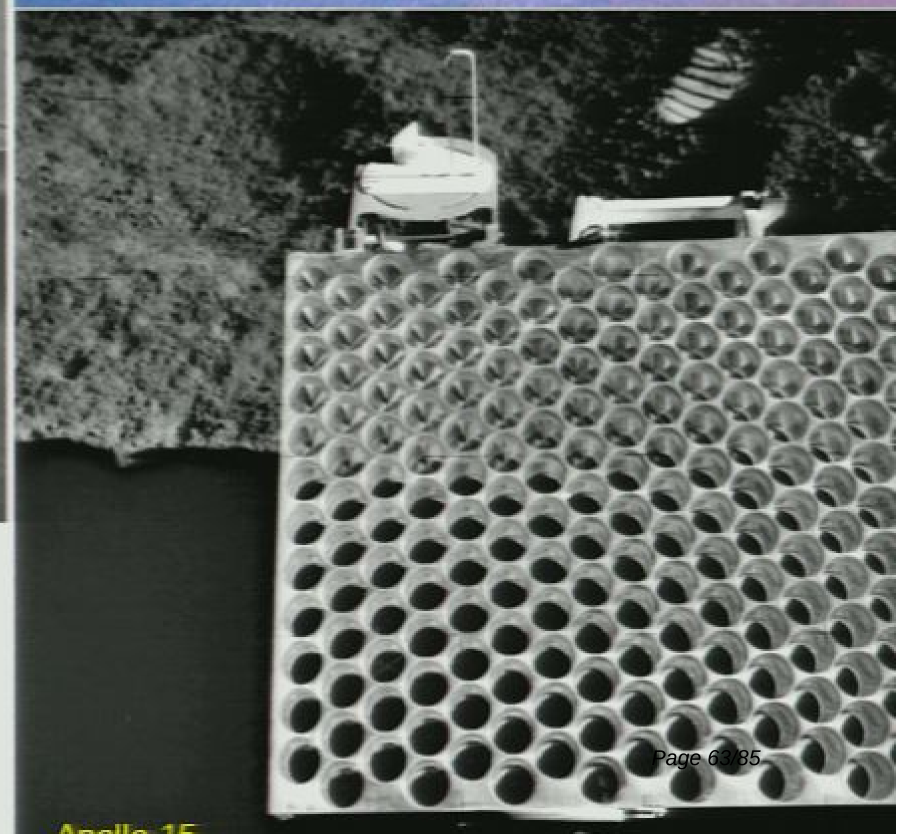
LUNAR LASER RANGING SCEINCE

Lunar Retroreflectors

French-built retroreflector array



Lunokhod Rover (USSR, 1972)



Beginning of the laser ranging technology.

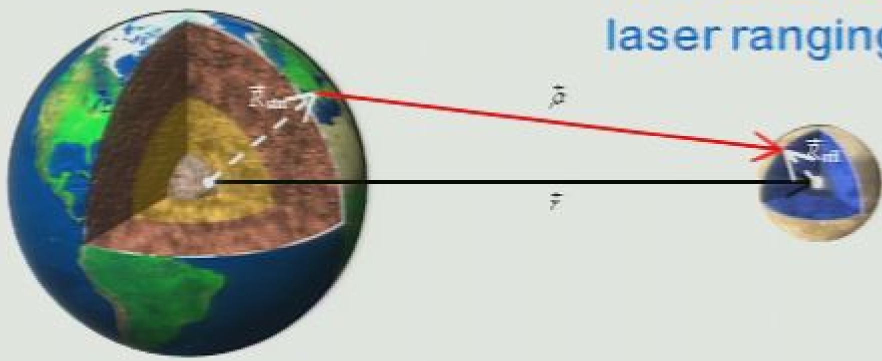
Today, laser ranging has many applications:

- Satellite laser ranging, communication systems, metrology, 3-D scanning, altimetry, etc.

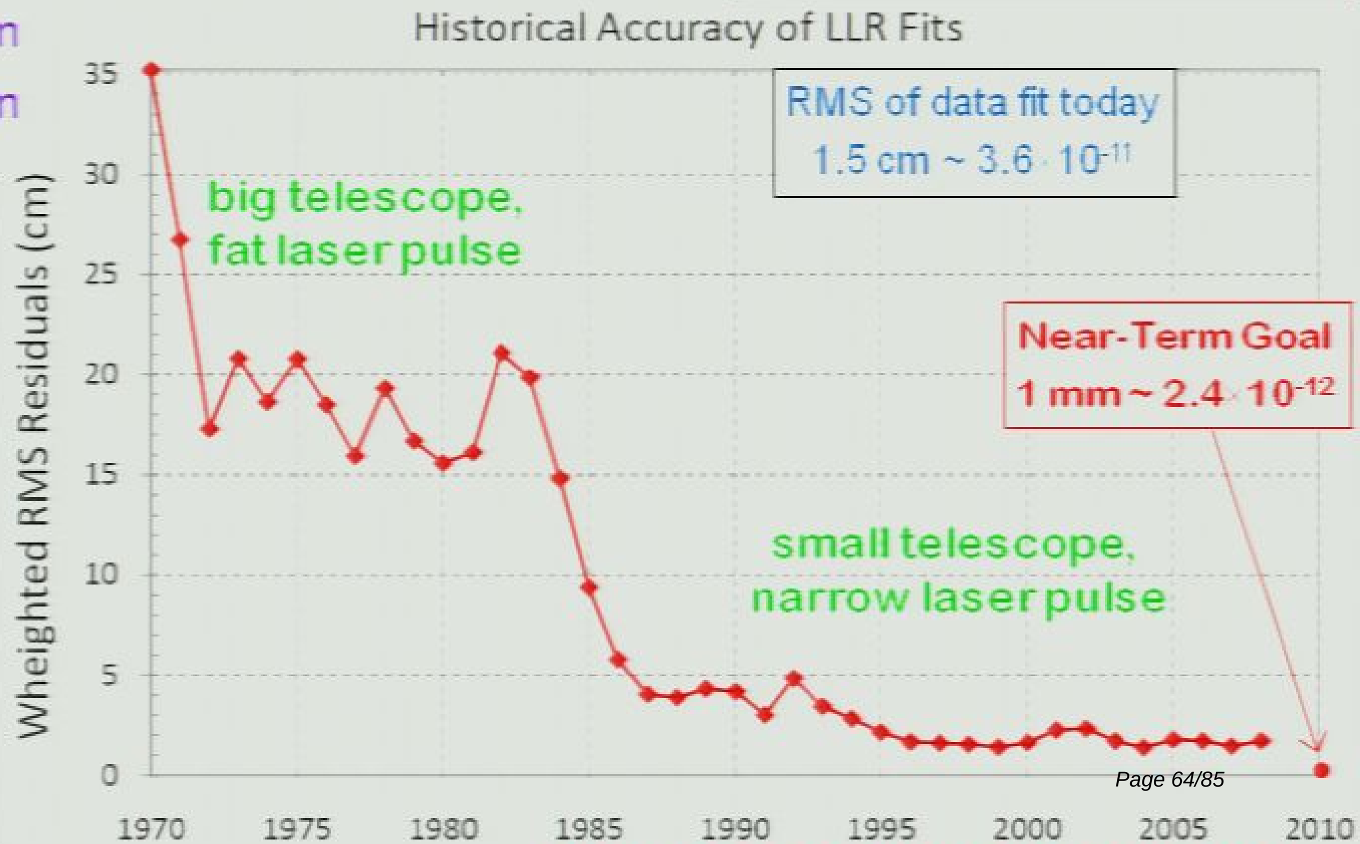
LUNAR LASER RANGING SCEINCE

Historical Accuracy of LLR

Schematics of the lunar laser ranging experiment



- Raw ranges vary by ~1,000s km
- Present range accuracy ~1.5cm



Solution parameters include:

- Dissipation: tidal & solid / fluid core mantle boundary (CMB);
- Dissipation related coefficients for rotation & orientation terms;
- Love numbers k_2, h_2, l_2;
- Correction to tilt of equator to the ecliptic – approximates influence of CMB flattening;
- Number of relativity parameters.

LUNAR LASER RANGING SCEINCE

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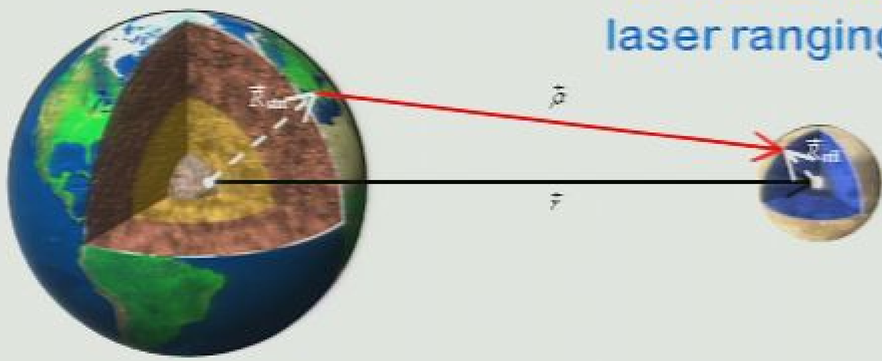
Schematics of the lunar laser ranging experiment



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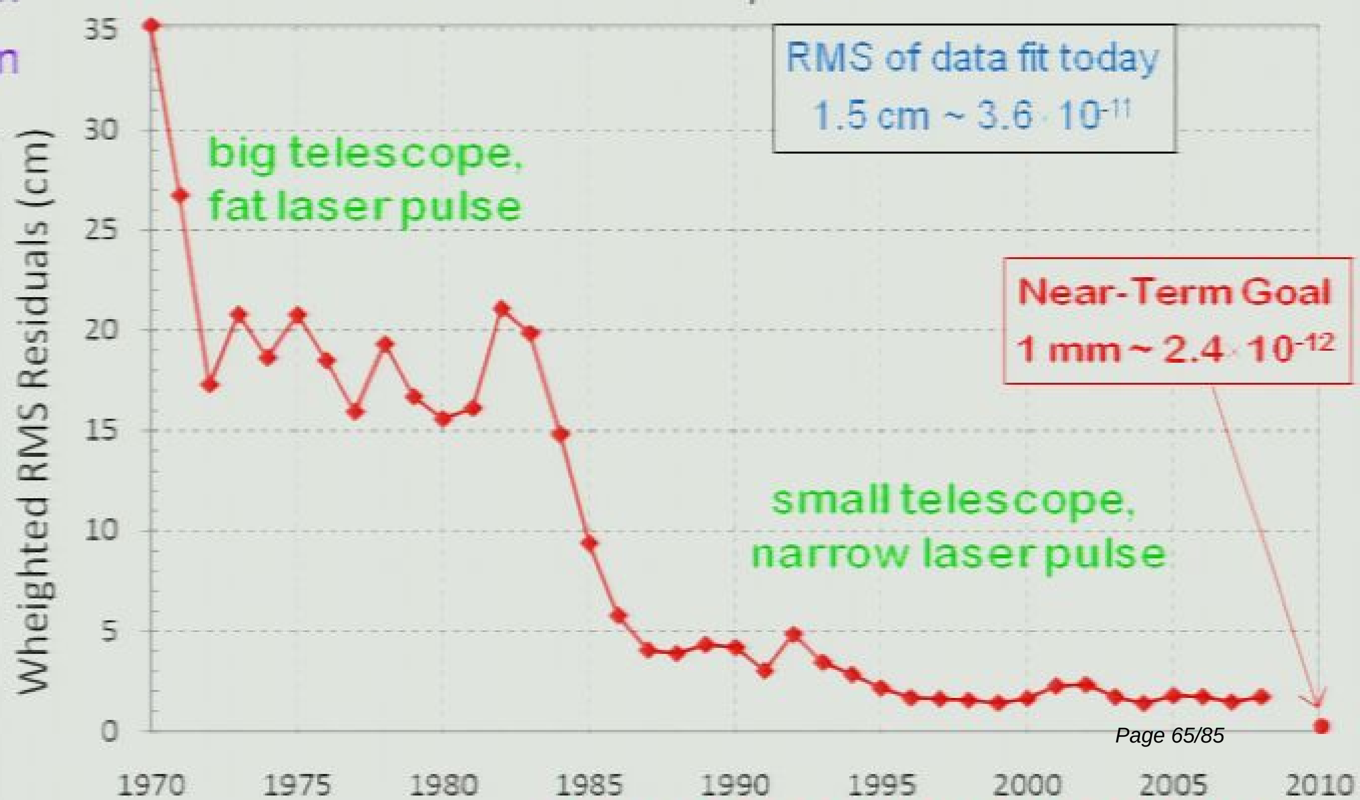
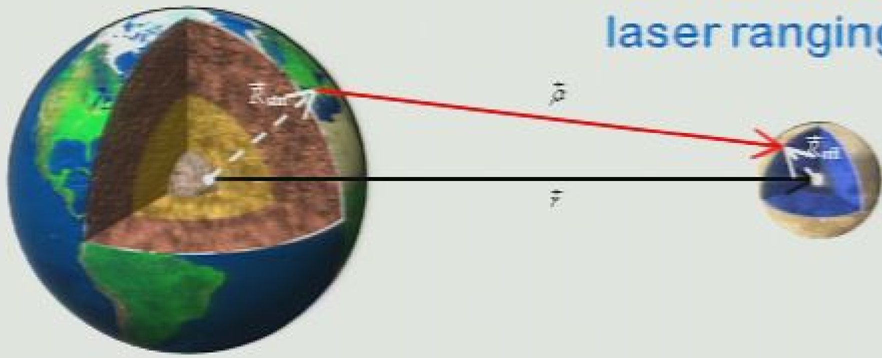
Historical Accuracy of LLR Fits



LUNAR LASER RANGING SCEINCE

Historical Accuracy of LLR

Schematics of the lunar laser ranging experiment



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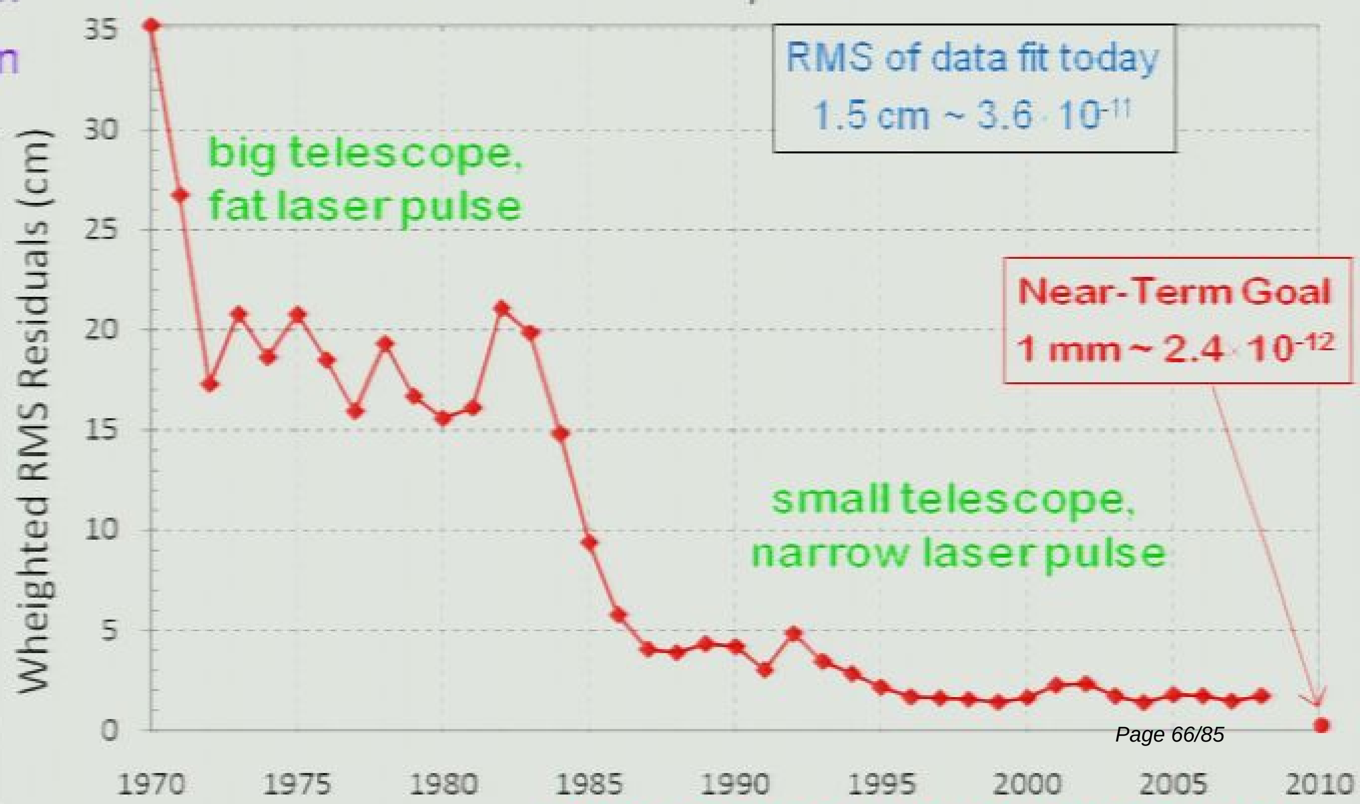
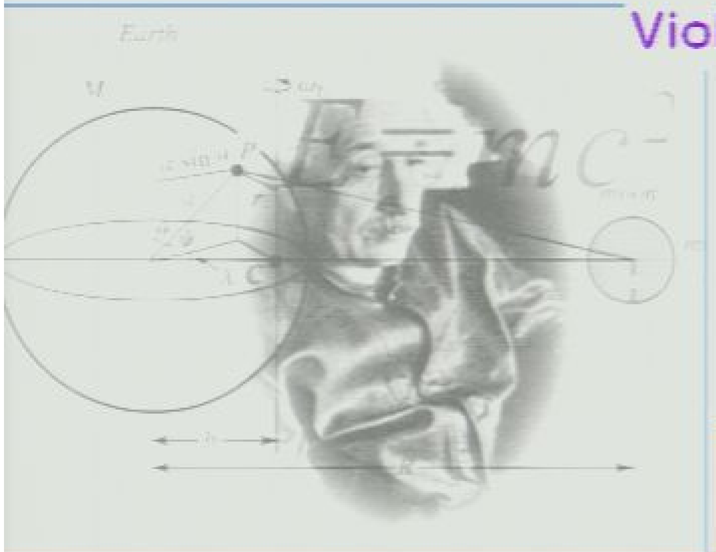


Testing General Relativity with LLR



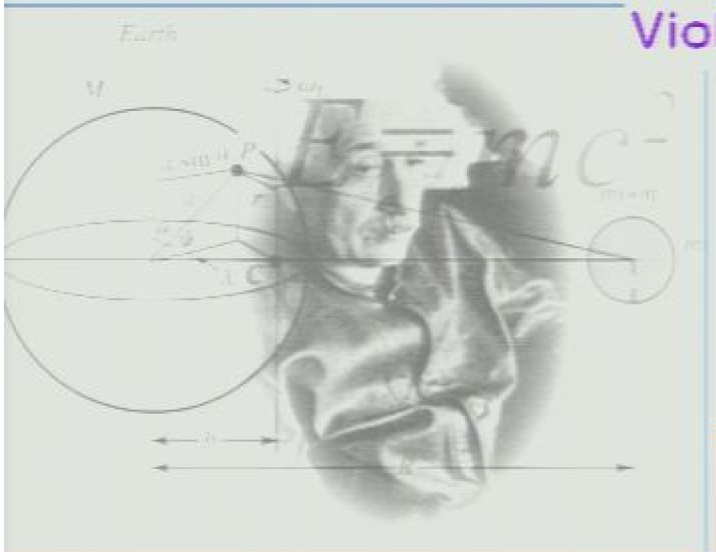
Violation of the Equivalence Principle in PPN formalism:

$$\frac{\Delta a}{a} \equiv \frac{2(a_1 - a_2)}{(a_1 + a_2)} = \left(\frac{m_G}{m_I}\right)_1 - \left(\frac{m_G}{m_I}\right)_2 . \qquad \frac{m_G}{m_I} = 1 + (4\beta - \gamma - 3)\frac{\Omega}{mc^2}$$

$$\frac{\Delta a}{a} = \eta \cdot \left(\frac{\Omega_e}{m_e c^2} - \frac{\Omega_m}{m_m c^2}\right) = -\eta \cdot 4.45 \times 10^{-10}. \qquad \eta \equiv 4\beta - \gamma - 3.$$

If η =1, this would produce a 13 m displacement of lunar orbit.
By 2007, range accuracy is ~1.5 cm, the effect was not seen.

Testing General Relativity with LLR



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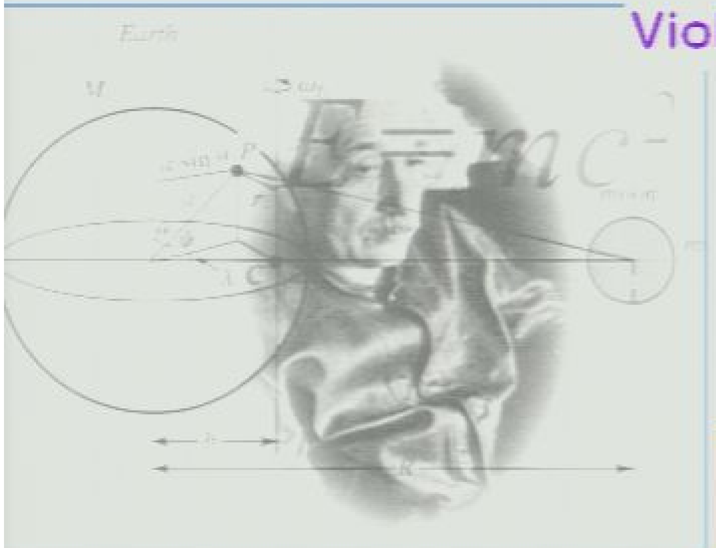

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If η =1, this would produce a 13 m displacement of lunar orbit. By 2007, range accuracy is ~1.5 cm, the effect was not seen.

LLR results (April 2008):

16,471 normal points through May 29, 2007, including 147 APOLLO points plus MLRS, OCA, and HALA

$\Delta\left(\frac{m_G}{m_I}\right) = (-0.95 \pm 1.30) \times 10^{-13}$ – corrected for solar radiation pressure from Vokrouhlicky (1997).

$\frac{\Delta a}{a} = (-1.95 \pm 1.91) \times 10^{-13}$ – test of the Strong Equivalence Principle with Adelberger (2001) results for WEP $\quad \eta = 4\beta - \gamma - 3 = (4.4 \pm 4.3) \times 10^{-4}$

Using Cassini '03 result $\quad \gamma - 1 = (2.1 \pm 2.3) \times 10^{-5} \quad \Rightarrow \quad \beta - 1 = (1.2 \pm 1.1) \times 10^{-4}$

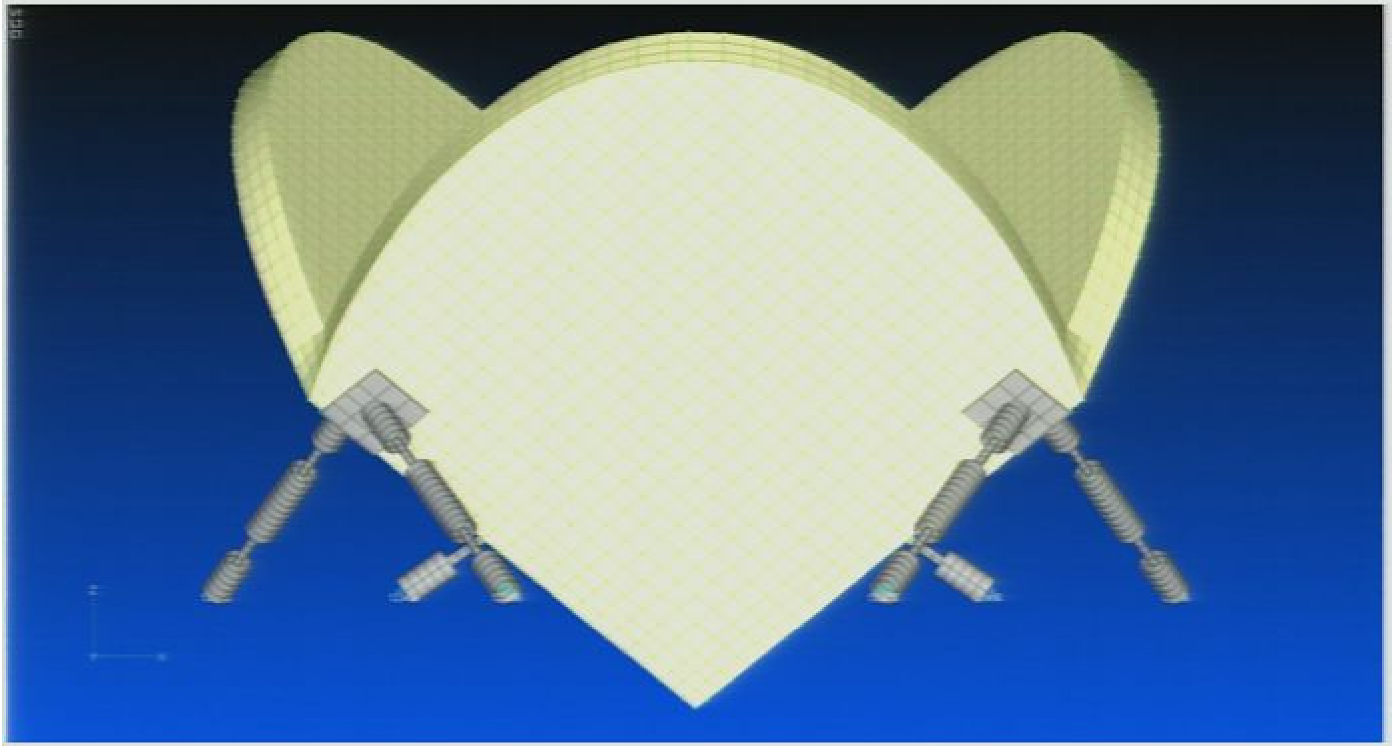
$K_{GP} = -0.0007 \pm 0.0047$ – Geodetic / de Sitter-Fokker precession

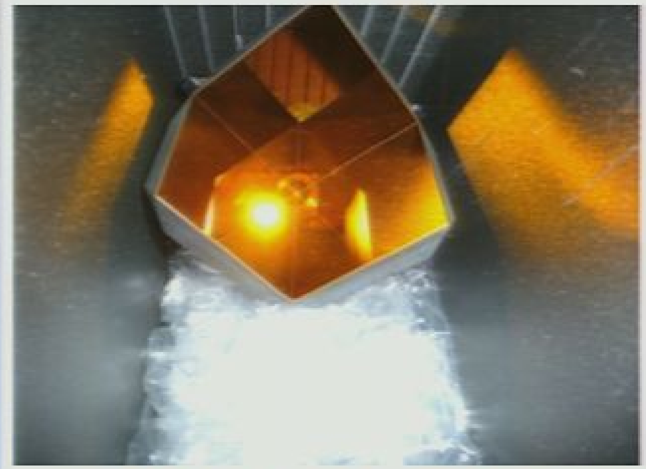
$\dot{G}/G = (4.9 \pm 5.7) \times 10^{-13} \ \text{yr}^{-1}$

ADVANCED LUNAR LASER RANGING EXPERIMENT

Large Hollow Corner Cube for LLR

- 170 mm diameter Silica Cylindrical Corner Cube
  10 mm thick – 104 mm internal edge length





Prototype of advanced corner cube for LLR: a SIM precision CC (~0.5 urad), 10 cm side length

- A 170mm diameter, hollow CC, retro reflector system has been modeled using a combination of thermal, mechanical, and optical diffraction physics models.
- The optical power returned by one 170mm hollow CC is approximately the same as the power returned by 100 Apollo solid corner cubes.

Advanced LLR: anticipated results

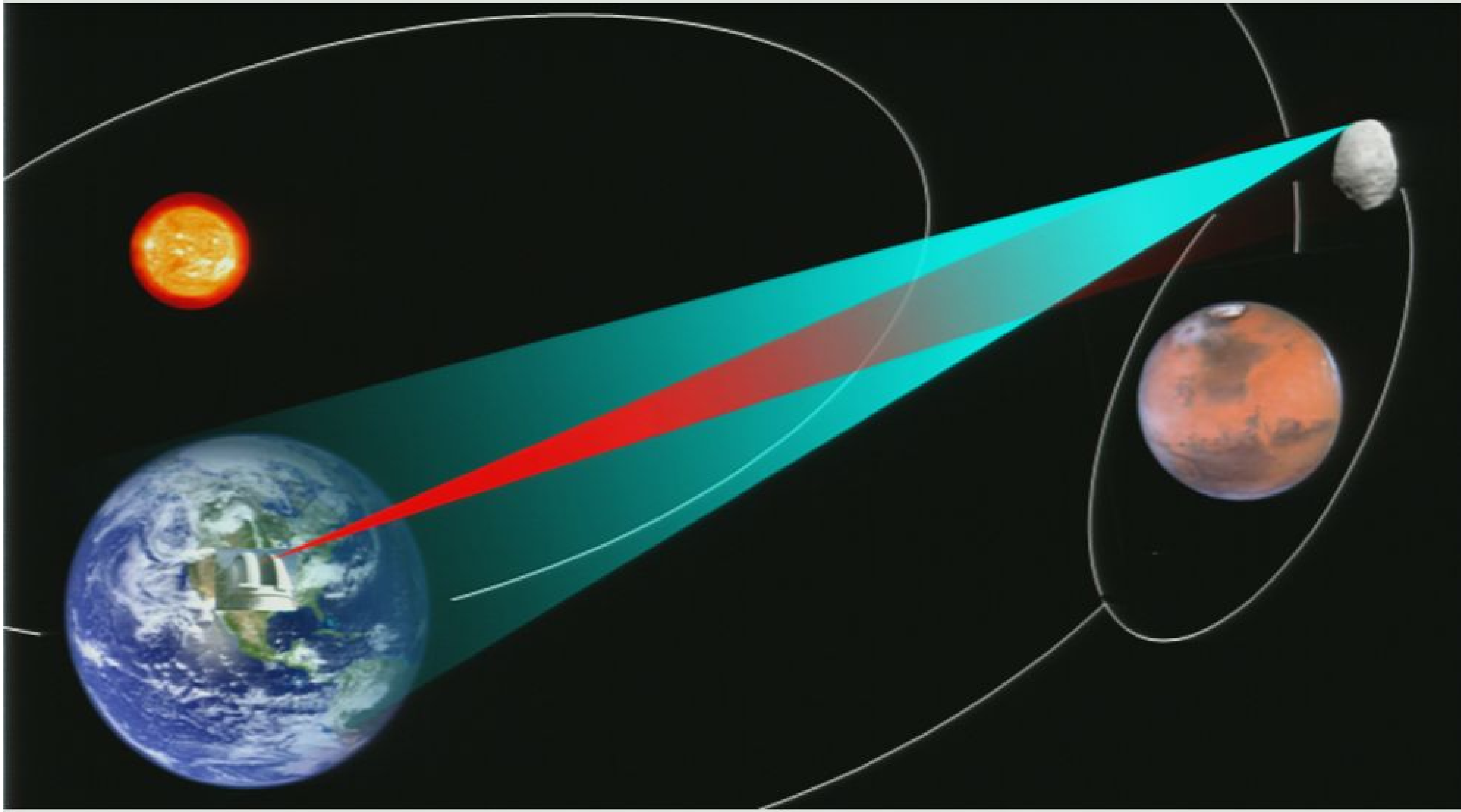
Tests of GR

Science	Timescale	Current (cm)	1 mm	0.1 mm
Weak Equivalence Principle	Few years	$\lvert \Delta a/a \rvert < 1.3\times10^{-13}$	10^{-14}	10^{-15}
Strong Equivalence Principle	Few years	$\lvert \eta \rvert < 4.3\times10^{-4}$	3×10^{-5}	3×10^{-6}
PPN parameter β	Few years	$\lvert \beta-1 \rvert < 1.1\times10^{-4}$	10^{-5}	10^{-6}
Time variation of G	~10 years	$5.7\times10^{-13}\ \mathrm{yr}^{-1}$	5×10^{-14}	5×10^{-15}
Inverse Square Law	~10 years	$\lvert \alpha \rvert < 3\times10^{-11}$	10^{-12}	10^{-13}

Lunar science

Effect	Current	Future Goals
Positions on Moon	yes	More locations
Low-degree gravity field	yes	Distinguish mantle from inner core for gravity and moments
3 free libration mantle modes	yes	Seek stimulating events
Solid-body tides	yes	Improve Love number accuracies
Tidal dissipation	yes	Improve tidal Q vs frequency
Core/mantle boundary dissipation	yes	Improve uncertainty, used to limit fluid core size
Core/mantle boundary flattening	yes	Improve uncertainty
Fluid core moment of inertia	no	Detect and determine
Fluid core free precession mode	no	Detect mode, determine amplitude & period
Inner solid core	no	Detect inner core, determine gravity
3 inner core free libration modes	no	Detect modes, determine amplitudes & periods
Inner core boundary dissipation	no	Limit inner core size

Phobos Laser Ranging Architecture



Next Step – Interplanetary Laser Ranging

Interplanetary laser ranging trade space

- **Mercury**
  - Surface
    - *Dark crater* — *Expensive, conjunction communication problem*
    - *Plains* — *Expensive, thermal difficulties*
  - Orbit — *Expensive, orbital & thermal difficulties*
- **Mars**
  - Surface — ***Promising**, but problems with dust & atmosphere*
  - Orbit
    - *Free orbit* — *Orbit precision a challenge*
    - *Phobos* — ***Promising***
    - *Deimos* — *Precession too long*
- **Asteroid** — *No suitable candidate with enough conjunctions*
- **Free-flyer** — *Expensive, as a single spacecraft architecture would need a drag-free platform; Insufficient science with a time-of-flight transponder (need for coherent detection).*

Simulated: laser ranging over 1-6 years of operation based on daily 1 mm range points.

Estimated parameters (total up to 230) include orbital elements (60), up to 67 individual asteroid GMs, asteroid class densities (3), spacecraft biases (8), solar corona corrections (8), planetary features (Mars, Mercury, Phobos, etc.) and others.

TESTS OF GENERAL RELATIVITY WITH INTERPLANETARY LASER RANGING

Interplanetary laser ranging: Mercury vs Mars

1mm laser ranging to Mars:

Estimated uncertainties for parameters of interest as a function of Mars lander mission duration, with 1 mm laser ranging once per day with 2° SEP cut-off and 67 asteroid mass parameters estimated.

Relativistic Effect to be studied by PLR	Mission duration, months		
	18	36	72
The Eddington parameter γ	3.1×10^{-7}	1.4×10^{-7}	7.8×10^{-8}
The Eddington parameter β	4.3×10^{-4}	1.7×10^{-4}	8.6×10^{-5}
Strong Equivalence Principle, η	8.8×10^{-4}	3.1×10^{-4}	8.5×10^{-5}
Solar oblateness, J_2	6.9×10^{-8}	3.2×10^{-8}	2.1×10^{-8}
Parameter $\dot{G}/G$, yr^{-1}	1.7×10^{-14}	2.8×10^{-15}	1.0×10^{-15}
Measure $\dot{M}_\odot/M_\odot$, yr^{-1}	4.7×10^{-14}	1.8×10^{-14}	9.4×10^{-15}

Estimated uncertainties for parameters of interest as a function of number of estimated asteroid mass parameters, for a 36 month Mars lander mission with 1 mm laser ranging once per day with 2° SEP cut-off.

Relativistic Effect to be studied by PLR	Number of asteroid GMs		
	11	36	67
The Eddington parameter γ	7.8×10^{-8}	1.1×10^{-7}	1.4×10^{-7}
The Eddington parameter β	6.9×10^{-5}	9.7×10^{-5}	1.7×10^{-4}
Strong Equivalence Principle, η	4.3×10^{-5}	8.8×10^{-5}	3.1×10^{-4}
Solar oblateness, J_2	1.6×10^{-8}	2.5×10^{-8}	3.2×10^{-8}
Parameter $\dot{G}/G$, yr^{-1}	2.6×10^{-15}	2.6×10^{-15}	2.8×10^{-15}
Measure $\dot{M}_\odot/M_\odot$, yr^{-1}	4.1×10^{-15}	9.9×10^{-15}	1.8×10^{-14}

Practical issues:

dust (life time), laser communication through atmosphere (conjunctions are worst case), etc.

1mm laser ranging to Mercury:

Estimated uncertainties for parameters of interest as a function of number of Mercury lander mission duration, with 1 mm laser ranging once per day with 2° SEP cut-off and 67 asteroid mass parameters estimated.

Relativistic Effect to be studied by PLR	Mission duration, months		
	18	36	72
The Eddington parameter γ	9.8×10^{-8}	4.6×10^{-8}	2.7×10^{-8}
The Eddington parameter β	1.5×10^{-5}	6.6×10^{-6}	7.5×10^{-7}
Strong Equivalence Principle, η	1.5×10^{-4}	3.4×10^{-5}	6.5×10^{-6}
Solar oblateness, J_2	1.7×10^{-9}	6.9×10^{-10}	7.4×10^{-11}
Parameter $\dot{G}/G$, yr^{-1}	1.7×10^{-14}	2.8×10^{-15}	9.1×10^{-16}
Measure $\dot{M}_\odot/M_\odot$, yr^{-1}	2.1×10^{-14}	2.8×10^{-15}	9.1×10^{-16}

Estimated uncertainties for parameters of interest as a function of number of estimated asteroid mass parameters, for a 18 month Mercury lander mission w/ 1 mm laser ranging once per day w/ 2° SEP cut-off.

Relativistic Effect to be studied by PLR	Number of asteroid GMs		
	11	36	67
The Eddington parameter γ	6.7×10^{-8}	8.0×10^{-8}	9.8×10^{-8}
The Eddington parameter β	4.2×10^{-6}	7.3×10^{-6}	1.5×10^{-5}
Strong Equivalence Principle, η	2.6×10^{-5}	5.1×10^{-5}	1.5×10^{-4}
Solar oblateness, J_2	4.4×10^{-10}	7.9×10^{-10}	1.7×10^{-9}
Parameter $\dot{G}/G$, yr^{-1}	1.2×10^{-14}	1.4×10^{-14}	1.7×10^{-14}
Measure $\dot{M}_\odot/M_\odot$, yr^{-1}	1.2×10^{-14}	1.6×10^{-14}	2.1×10^{-14}

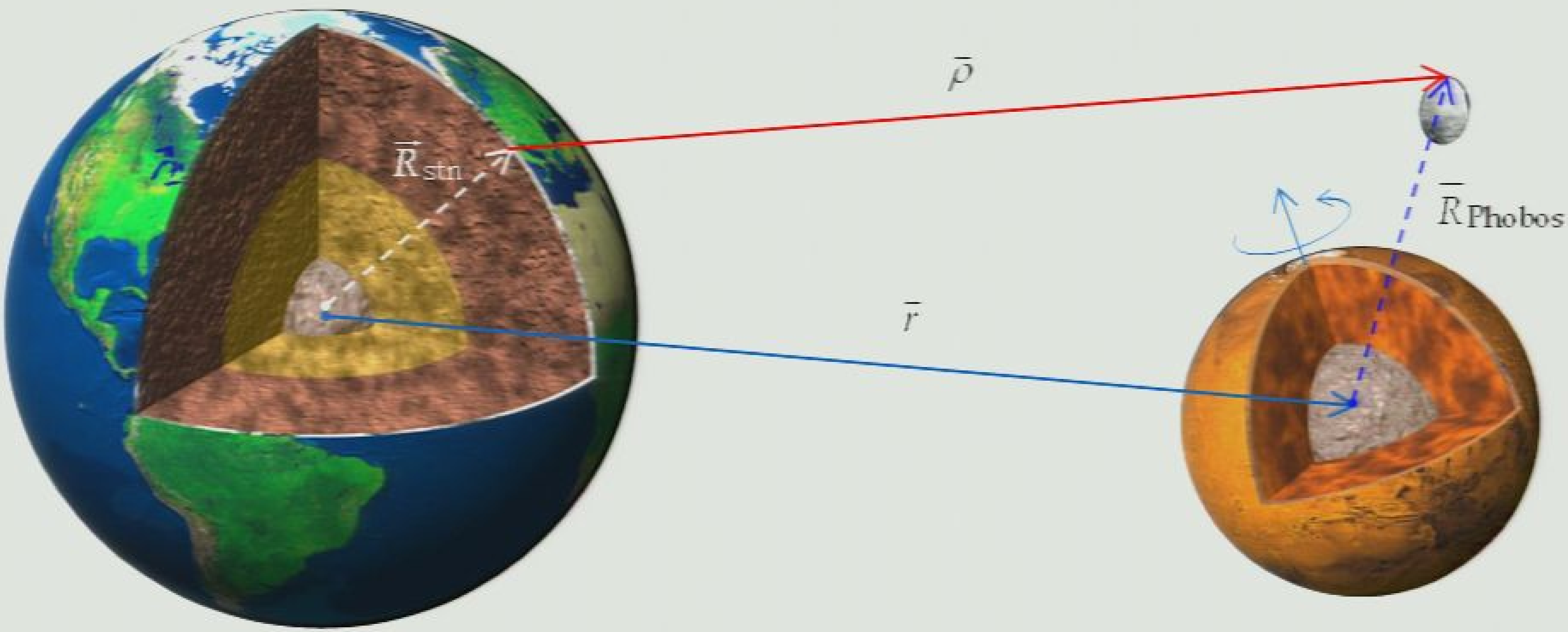
Practical issues:

challenging thermal conditions (life time), communication through conjunctions, nuclear power, overall cost ($1.5B+), etc.


Typical atmosphere on Mars...

Phobos Laser Ranging: Principle

1 mm range accuracy with PLR is possible



Impact on:

- Test of general relativity
- The science of Phobos, especially its interior

@ $550M (FY 2009 $)

Gravity Tests with PLR vs Experiment Duration

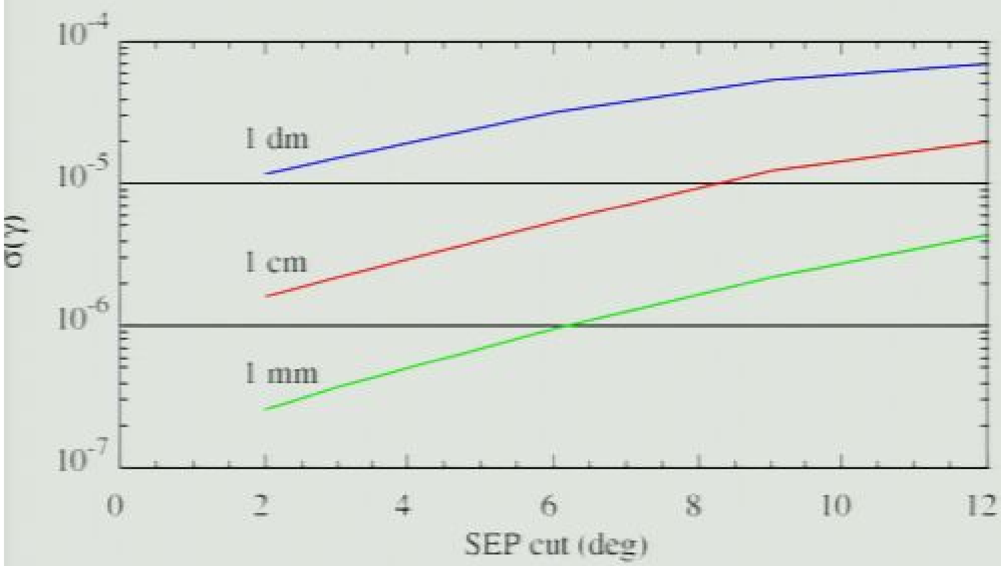
Relativistic Effect	Current best	Mission duration / N of conjunctions		
		1 yr / 1 cnj	3 yr / 2 cnj	6 yr / 3 cnj
PPN parameter γ	2.3×10^{-5}	3.1×10^{-7}	1.4×10^{-7}	7.9×10^{-8}
PPN parameter β	1.1×10^{-4}	4.3×10^{-4}	1.6×10^{-4}	9.4×10^{-5}
Test of Strong Equiv. Principle, η	4.3×10^{-4}	1.5×10^{-3}	2.8×10^{-4}	8.8×10^{-5}
Solar oblateness, J_2	2.0×10^{-7}	6.9×10^{-8}	3.2×10^{-8}	2.3×10^{-8}
Search for time variation in the grav. constant G, $dG/dt/G$, yr^{-1}	7×10^{-13}	1.7×10^{-14}	2.8×10^{-15}	1.0×10^{-15}
Gravitational inverse square law	2×10^{-9} @ 1.5 AU	4×10^{-11} @ 1.5 AU	2×10^{-11} @ 1.5 AU	1×10^{-11} @ 1.5 AU

Estimated uncertainties for parameters of interest as a function of Phobos lander mission duration, with 1 mm laser ranging once per day with 2° SEP cut-off and 67 asteroid mass parameters estimated.

Simulations by W.M. Folkner, JPL

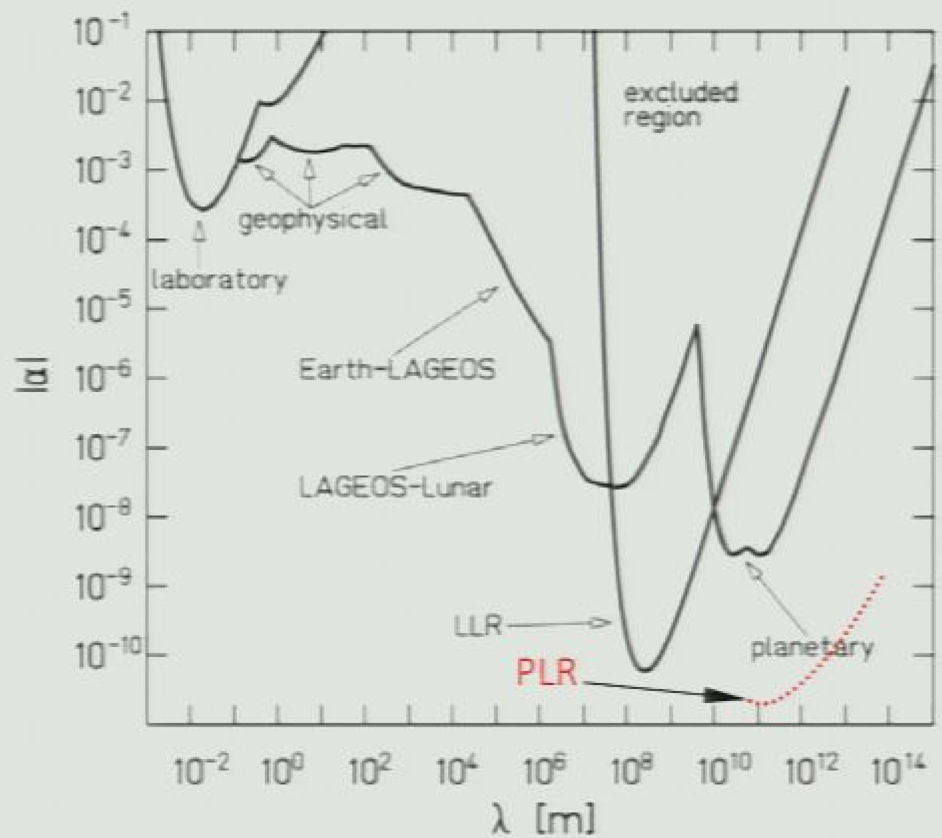
Gravity Tests with PLR: PPN γ and the ISL

Estimated uncertainty in PPN γ



Estimated uncertainty in PPN γ as a function of data accuracy and data cut-off with angular separation from the Sun as viewed from Earth.

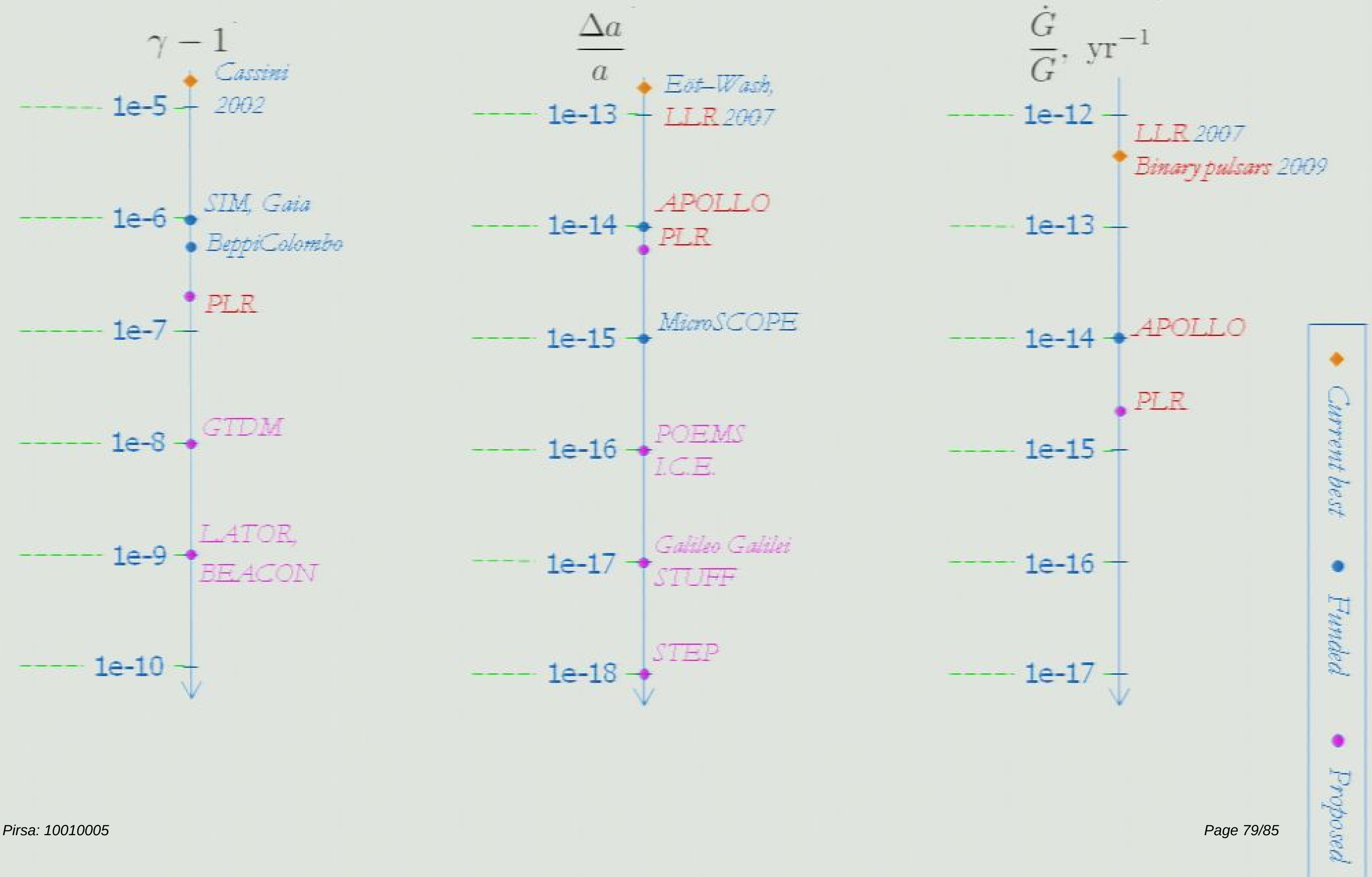
Limits on the ISL violations



Simulations by W.M. Folkner;
background graphics from (Adelberger et al., 2003)

Theoretical Landscape of the 21th Century:

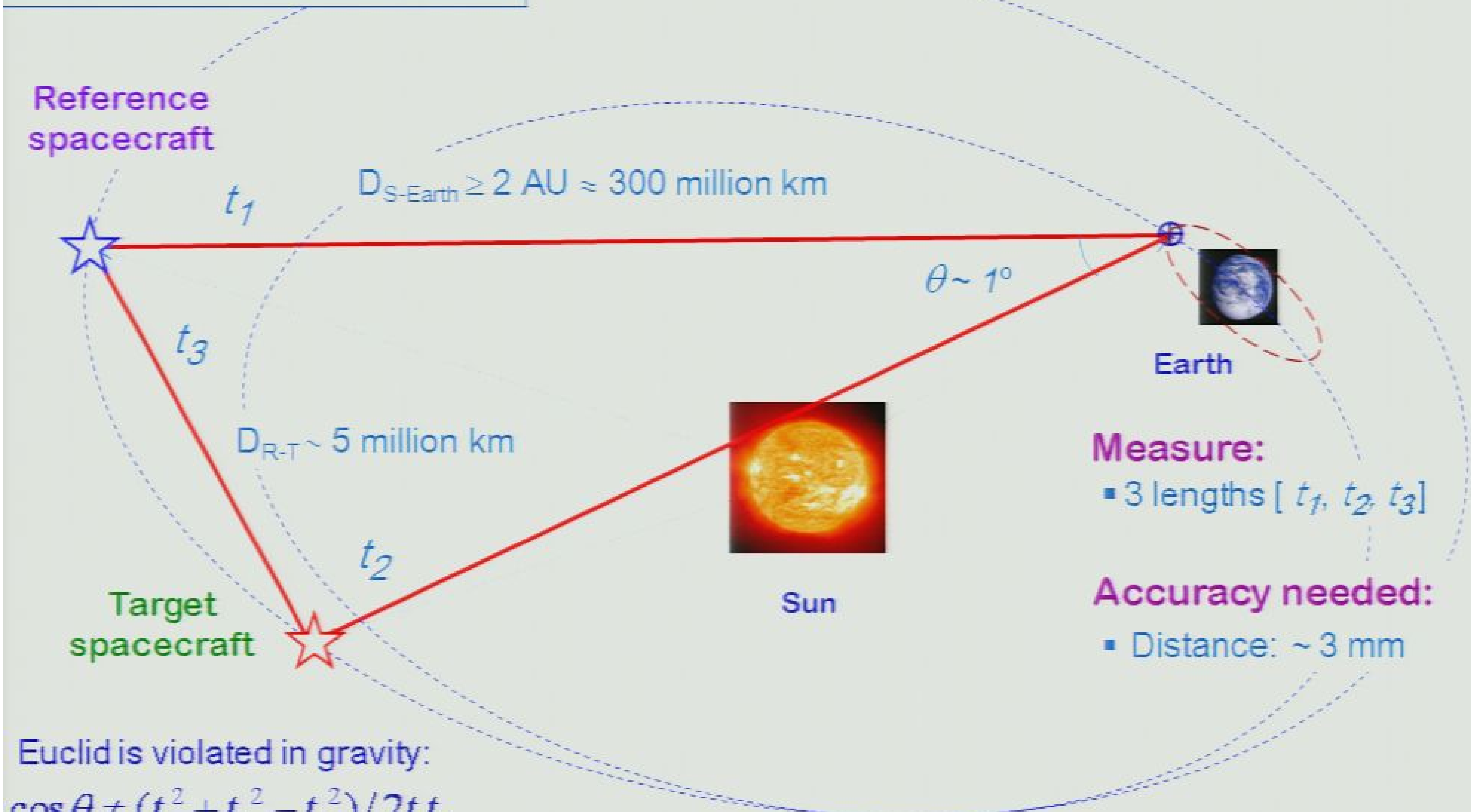
Confrontation Between Theory and Experiment



LASER ASTROMETRIC TEST OF RELATIVITY

LATOR Mission Concept

CQG 21 (2004) 2773-2799, gr-qc/0311020



Measure:

- 3 lengths [t_1, t_2, t_3]

Accuracy needed:

- Distance: ~ 3 mm

Euclid is violated in gravity:

$$\cos\theta \neq (t_1^2 + t_2^2 - t_3^2)/2t_1t_2$$

@ $630M (FY 2009 $)

JPL

LASER ASTROMETRIC TEST OF RELATIVITY

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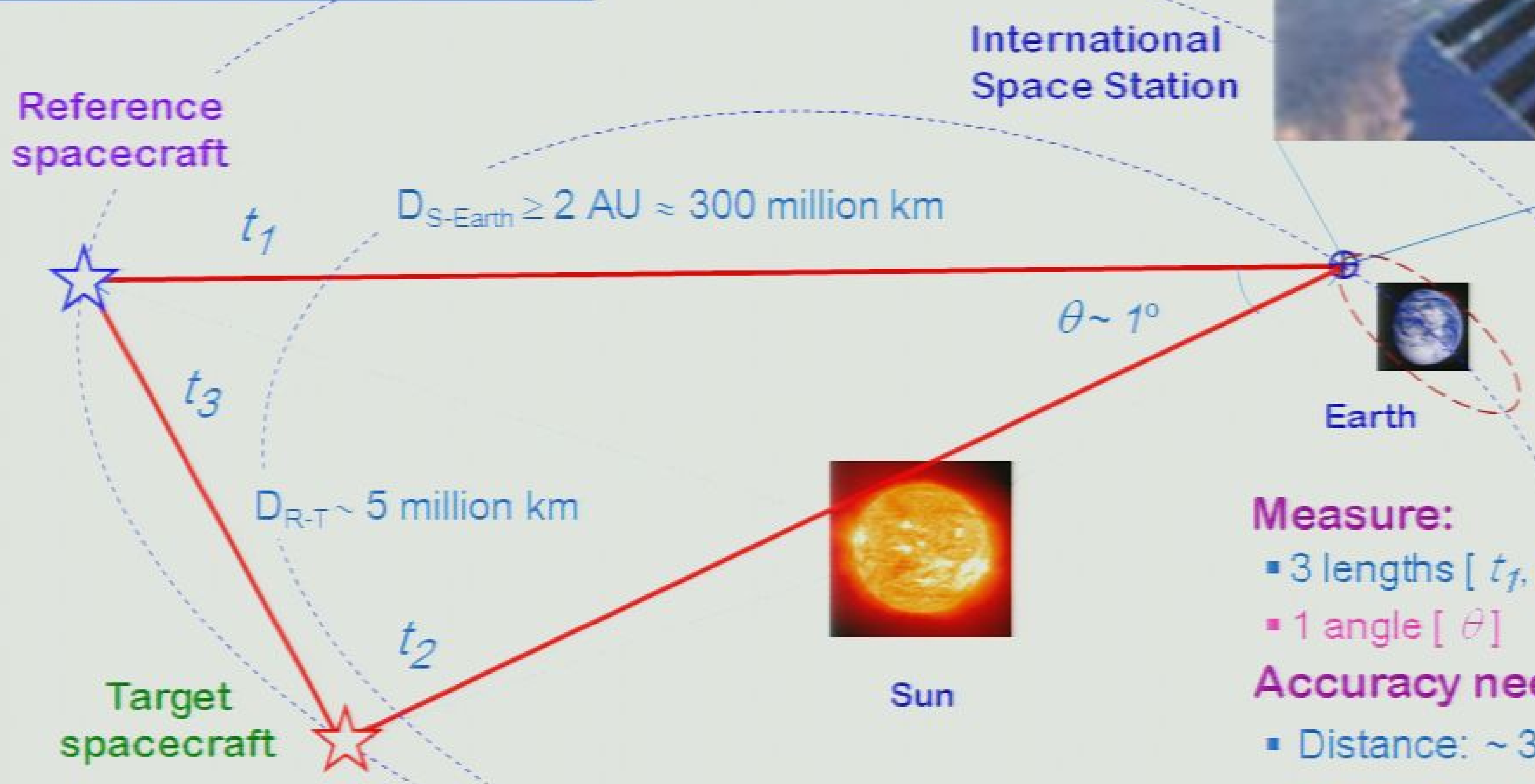


Measure:

- 3 lengths [t_1, t_2, t_3]
- 1 angle [θ]

Accuracy needed:

- Distance: ~ 3 mm
- Angle: 0.01 picorad

Euclid is violated in gravity:

$$\cos\theta \neq (t_1^2 + t_2^2 - t_3^2)/2t_1t_2$$

Geometric redundancy enables a very accurate measurement of curvature of the solar gravity field

Test of gravitational deflection of light: PPN γ to 1 part in 10^9 @ $630M (FY 2009 $)

Eddington Experiment of the 21st Century

Optical vs. Microwave:

- Solar plasma effects decrease as λ^2: from 10cm (3GHz) to 1 μm 300 THz is a 10^{10} reduction in solar plasma optical path fluctuations

Orbit Determination (OD):

- No need for drag-free environment for LATOR spacecraft
- Redundant optical truss – alternative to ultra-precise OD

A Low Cost Experiment:

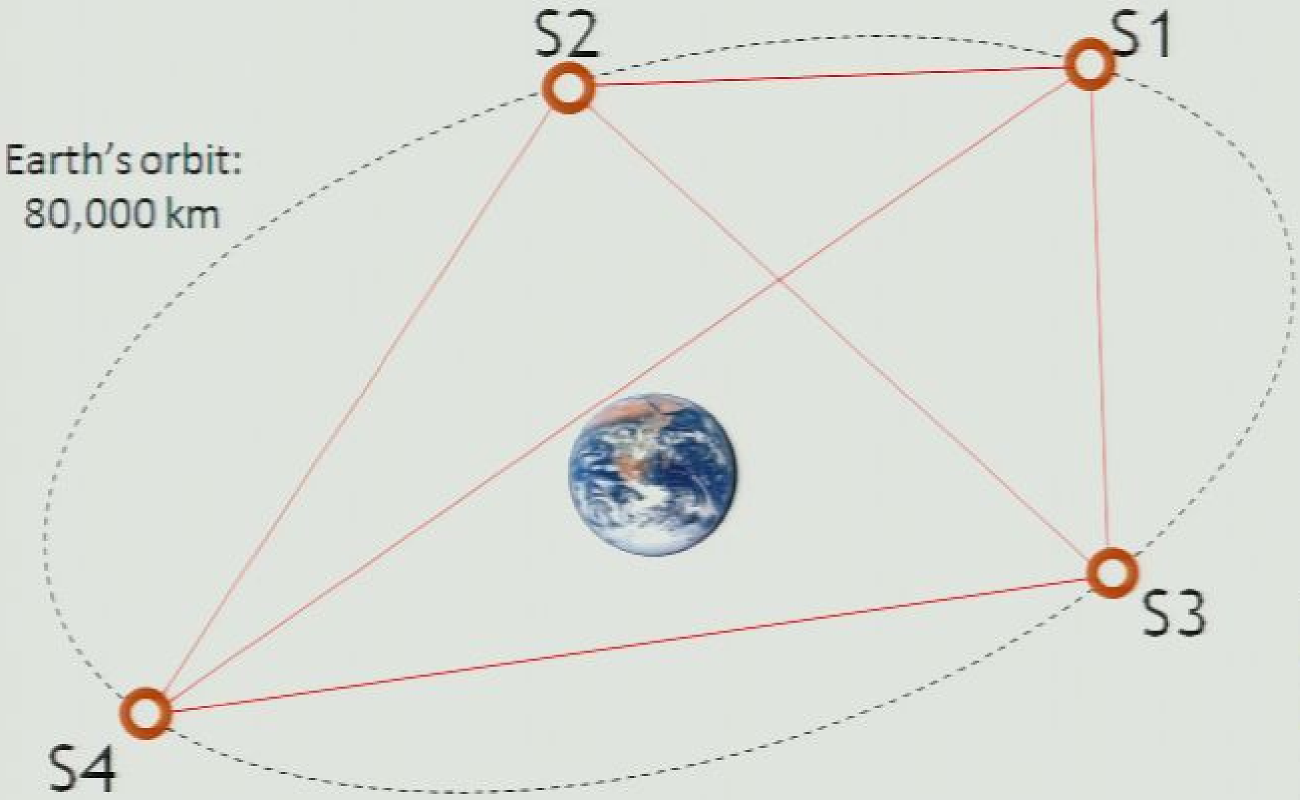
- Optical apertures ~15-25 cm – sufficient; high SNR ~1700
- Options exist for NO motorized moving parts
- Many technologies exist: laser components and spacecraft
- Possibilities for further improvements: clocks, accelerometers, etc.

Toward Centennial of General Relativity (2015):

- 1919: Light deflection during solar eclipse: $|1-\gamma| \leq 10^{-1}$
- 1980: Viking – Shapiro Time Delay: $|1-\gamma| \leq 2 \times 10^{-3}$
- 2003: Cassini – Doppler [d(Time Delay)/dt]: $|1-\gamma| \leq 2.3 \times 10^{-5}$
- 2016: LATOR – Astrometric Interferometry: $|1-\gamma| \rightarrow 10^{-8}$–$10^{-9}$

TESTS OF RELATIVISTIC GRAVITY IN SPACE

Beyond Einstein Advanced Coherent Optical Network (BEACON)



Schematic of the BEACON experiment

Measure:

- 4 lengths [t_1, t_2, t_3, t_4]

Accuracy needed:

- Distance: ~ 0.1 nm

Geometric redundancy:

- Enables a very accurate measurement of curvature of the Earth's gravity field
- Reduces the need for drag-free spacecraft

Accurate test of gravitational time delay (Shapiro effect): PPN γ to 1 part in 10^9

Emerging Technologies

- Frequency Standards:
  - *μ-wave* atomic clocks:
    - NIST F-1 Cs fountain μ-wave clock: 5×10^{-16} accuracy (1 day)
  - *μ-wave* atomic clocks on a chip:
    - NIST Rb chip-scale μ-wave clock: 6×10^{-12} accuracy (1 sec)
  - *Optical* atomic clocks
    - NIST Hg+ clock now $< 3 \times 10^{-17}$ (10^4 sec)
- Matter-wave interferometers:
  - Gravity gradiometer:
    - Demonstrated differential acceleration sensitivity: 3×10^{-9} g/Hz$^{1/2}$
  - Gyroscope
    - Achieved stability 2×10^{-6} deg/hr$^{1/2}$ ARW (angle random walk)
  - Accelerometer
    - Demonstrated performance 2.3×10^{-9} g/Hz$^{1/2}$

TESTS OF RELATIVISTIC GRAVITY IN SPACE

Conclusions

- Recent technological progress: **arXiv:0902.3004 [gr-qc]**
  - Resulted in new instruments with unique performance
  - Could lead to major improvements in the tests of relativistic gravity
  - Already led to a number of recently proposed gravitational experiments
- Challenges for solar system tests of gravity:
  - Dedicated space-based experiments are very expensive – the science must worth the cost… – *EP, G-dot and PPN γ tests are most relevant.*
  - Motivation for the tests in a weak gravity field is a challenge: there is no strong expectation to see deviations from GR in the solar system (we are looking for anomalies…) – *access to strong(er) gravity regime is needed!*
  - GR is very hard to modify, embed, extend or augment (whatever your favorite verb is…) – *thus, perhaps, those anomalies are important…*
  - PPN formalism becomes less relevant for modern gravity research…
  - Looking to Cosmos for help? There is none: Little or no correspondence between cosmological tests and physical principles in the foundation of tests of PPN gravity – *EP, LLI, LPI, energy-momentum conservation, etc…*