

Title: Primordial nongaussianity and large-scale structure

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Abstract: Standard inflationary theory predicts that primordial fluctuations in the universe were nearly Gaussian random. Therefore, searches for, and limits on, primordial nongaussianity are some of the most fundamental tests of inflation and the early universe in general. I first briefly review the history of its measurements from the cosmic microwave background anisotropies and large-scale structure in the universe. I then present results from recent work where effects of primordial nongaussianity on the distribution of largest virialized objects was studied numerically and analytically. We found that the bias of dark matter halos takes strong scale dependence in nongaussian cosmological models. Therefore, measurements of scale dependence of the bias, using various tracers of large-scale structure, can - and do - constrain primordial nongaussianity more than an order of magnitude better than previously thought.

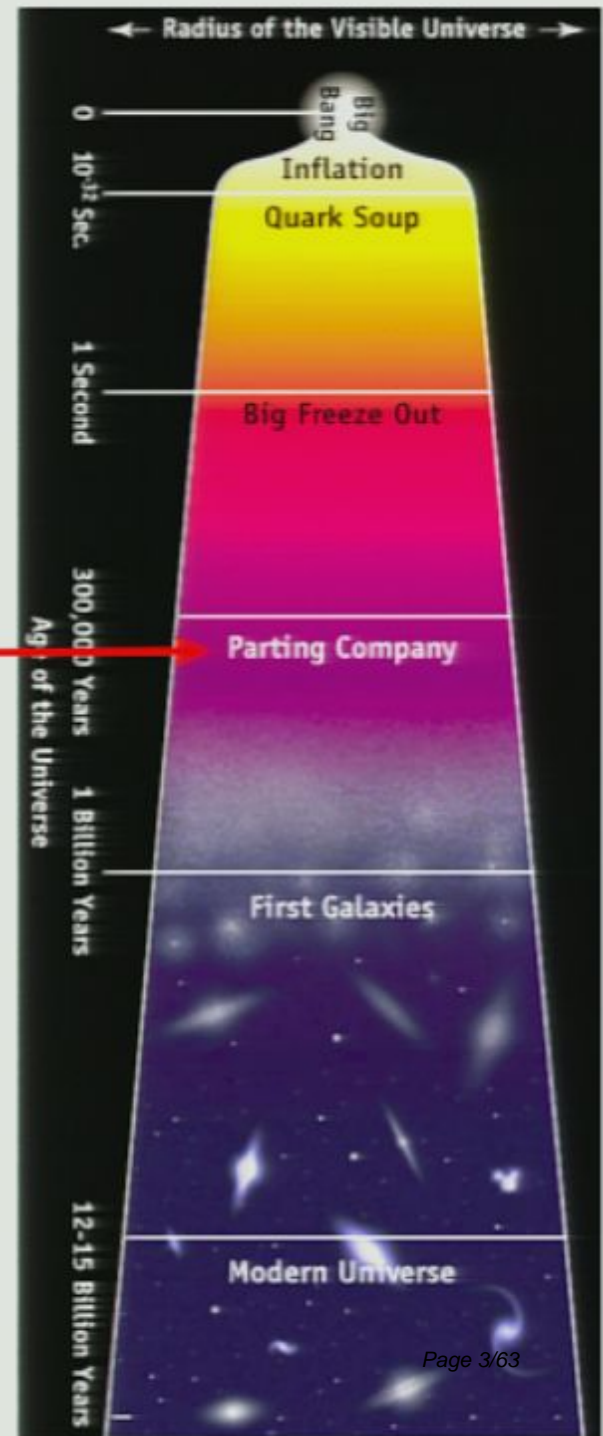
Primordial Nongaussianity and Large-scale Structure

Dragan Huterer
(University of Michigan)

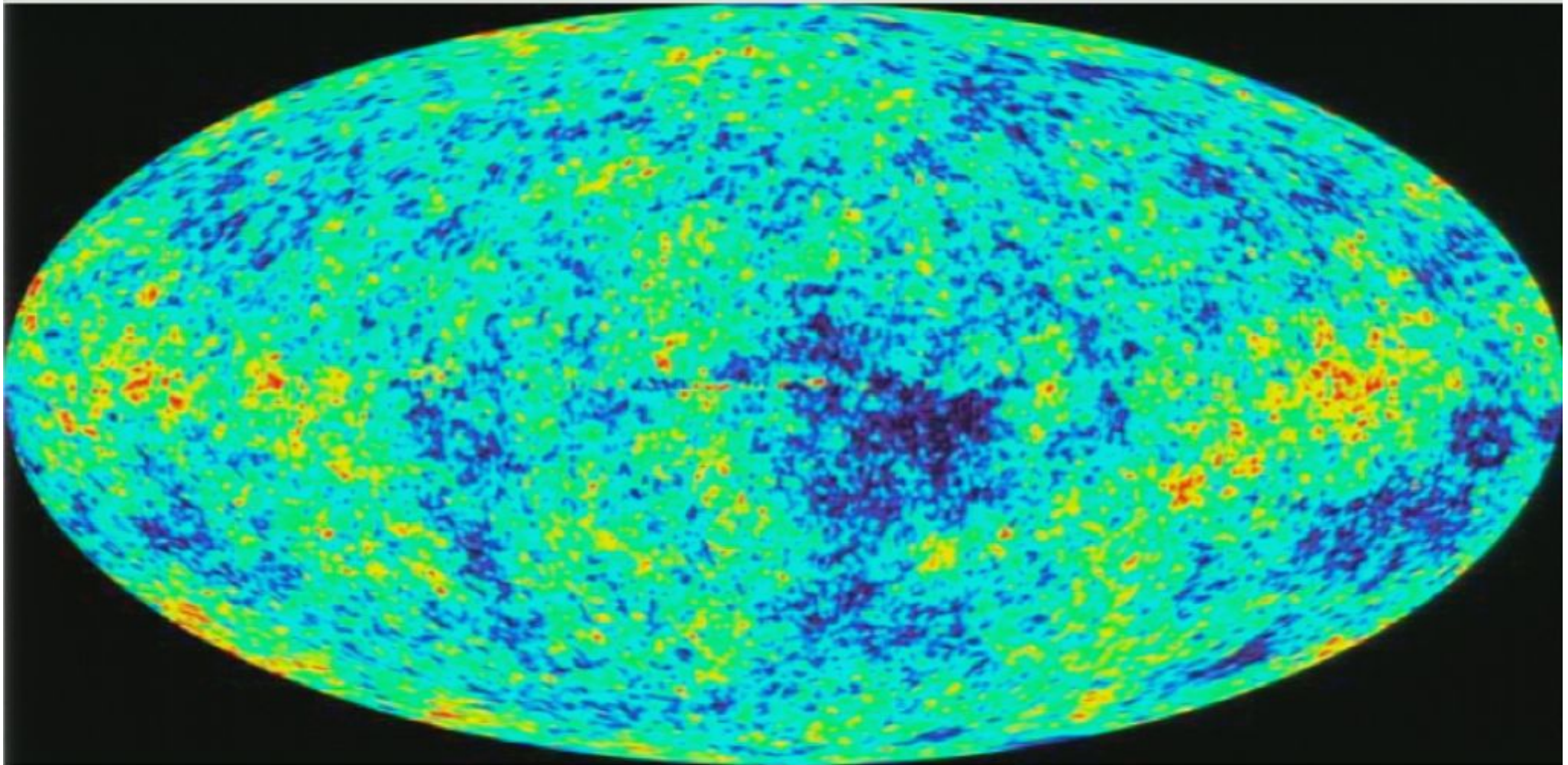
with Neal Dalal (CITA), Olivier Doré (JPL),
Carlos Cunha (Michigan), and others

Universe becomes transparent ($t=380,000$ yrs)

- Radiation finally free to propagate - universe has become cool enough for atoms to form
- The **Cosmic Microwave Background** radiation we observe has been released at this time
- Temp = 3000 Kelvin (2.725 Kelvin today)
- Uniform to one part in 100,000



Fluctuations 1 part in 100,000 (of 2.725 Kelvin)



The cosmic Rosetta Stone

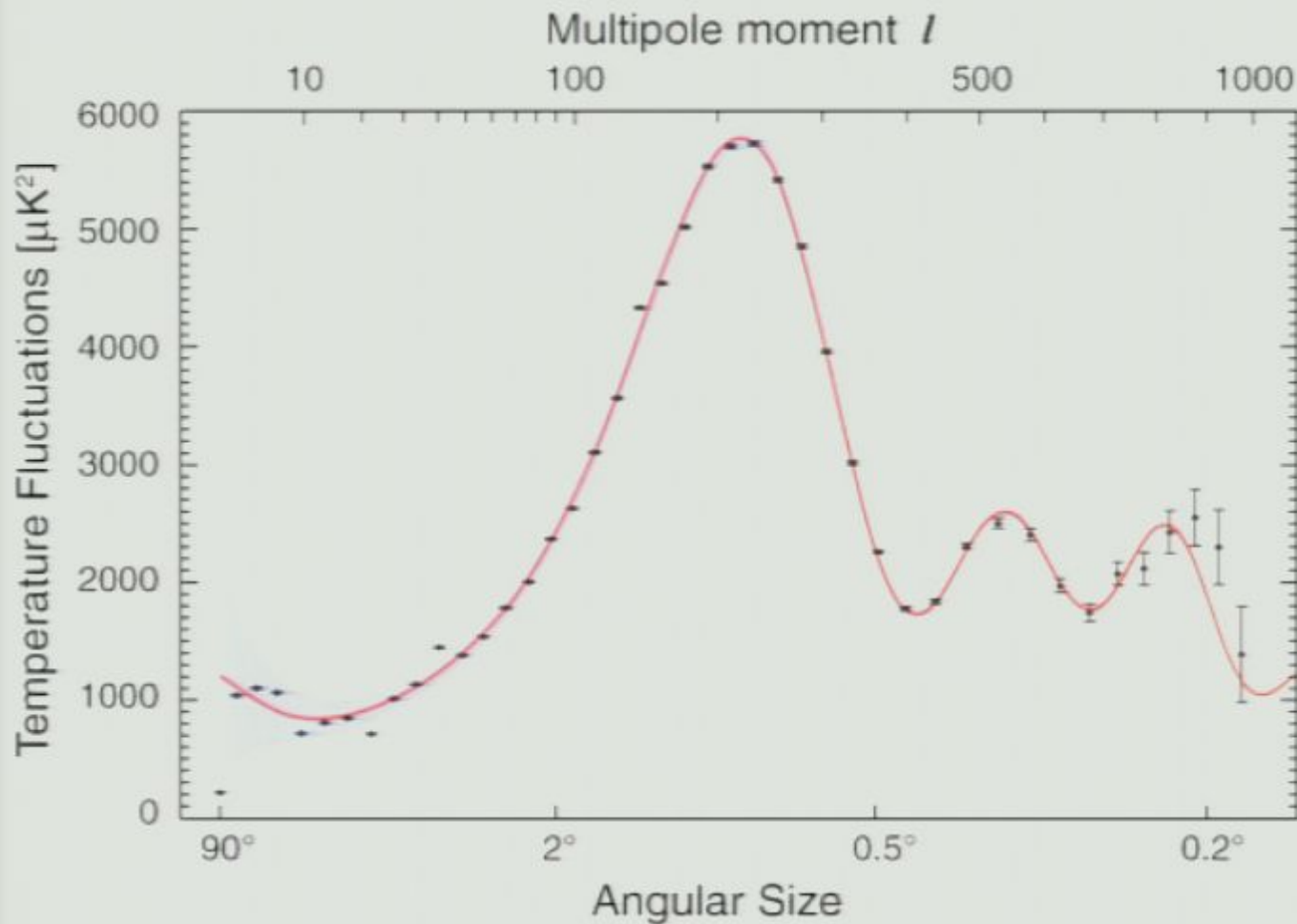


Image From <http://map.gsfc.nasa.gov>

Pirsa: 10010002

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{l, m} a_{lm} Y_{lm}(\theta, \phi), \quad C_\ell \equiv \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

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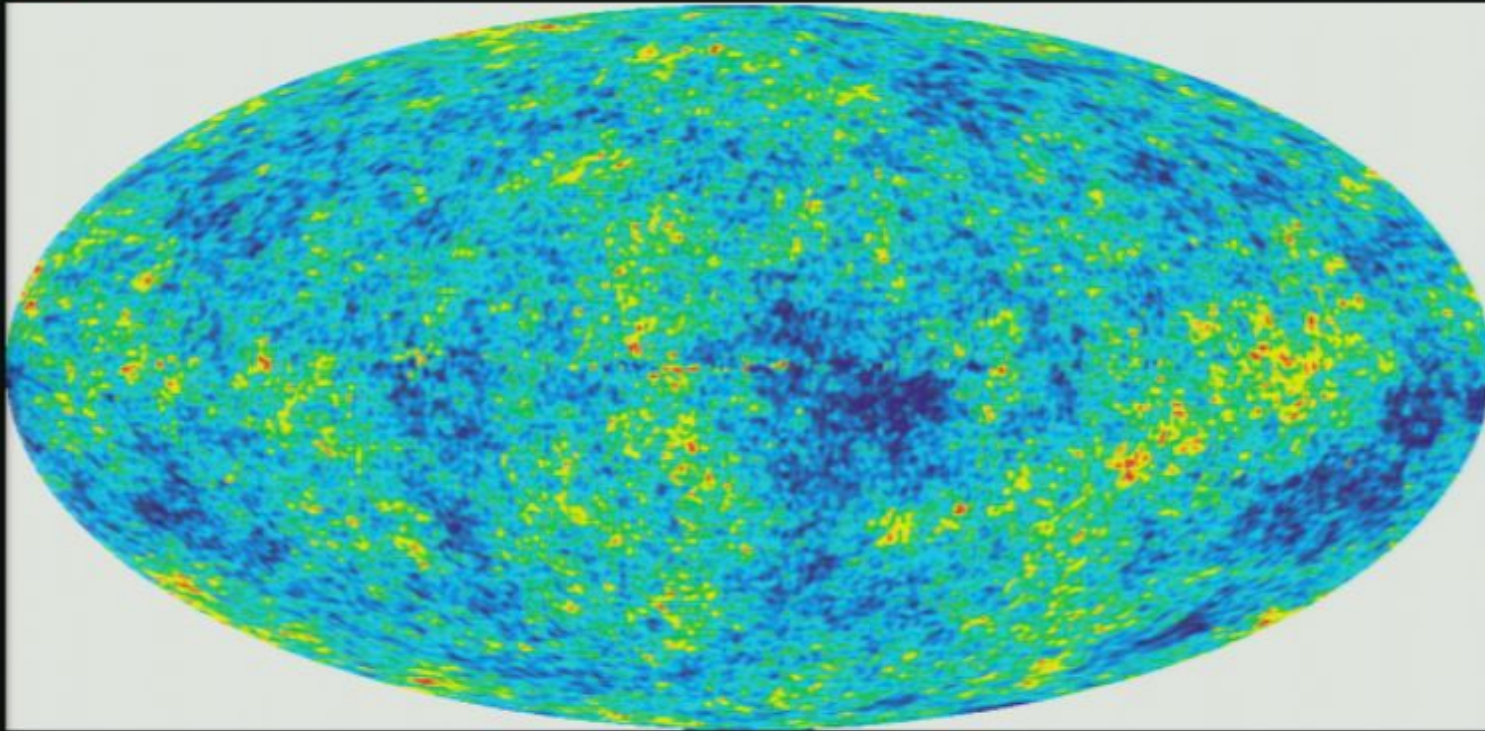
Class	Parameter	WMAP 5-year ML ^a	WMAP+BAO+SN ML	WMAP 5-year Mean ^b	WMAP+BAO+SN Mean
Primary	$100\Omega_b h^2$	2.268	2.263	2.273 ± 0.062	2.265 ± 0.059
	$\Omega_c h^2$	0.1081	0.1136	0.1099 ± 0.0062	0.1143 ± 0.0034
	Ω_Λ	0.751	0.724	0.742 ± 0.030	0.721 ± 0.015
	n_s	0.961	0.961	$0.963^{+0.014}_{-0.015}$	$0.960^{+0.014}_{-0.013}$
	τ	0.089	0.080	0.087 ± 0.017	0.084 ± 0.016
	$\Delta_{\mathcal{R}}^2 (k_0^{-2})$	2.41×10^{-9}	2.42×10^{-9}	$(2.41 \pm 0.11) \times 10^{-9}$	$(2.457^{+0.092}_{-0.093}) \times 10^{-9}$
Derived	σ_8	0.787	0.811	0.796 ± 0.036	0.817 ± 0.026
	H_0	72.4 km/s/Mpc	70.3 km/s/Mpc	$71.9^{+2.6}_{-2.7}$ km/s/Mpc	70.1 ± 1.3 km/s/Mpc
	Ω_b	0.0432	0.0458	0.0441 ± 0.0030	0.0462 ± 0.0015
	Ω_c	0.206	0.230	0.214 ± 0.027	0.233 ± 0.013
	$\Omega_m h^2$	0.1308	0.1363	0.1326 ± 0.0063	0.1369 ± 0.0037
	z_{reion}^f	11.2	10.5	11.0 ± 1.4	10.8 ± 1.4
	t_0^g	13.69 Gyr	13.72 Gyr	13.69 ± 0.13 Gyr	13.73 ± 0.12 Gyr

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Section	Name	Type	WMAP 5-year	WMAP+BAO+SN
§ 3.2	Gravitational Wave ^a	No Running Ind.	$r < 0.43^b$	$r < 0.20$
§ 3.1.3	Running Index	No Grav. Wave	$-0.090 < dn_s/d \ln k < 0.019^c$	$-0.0728 < dn_s/d \ln k < 0.0087$
§ 3.4	Curvature ^d		$-0.063 < \Omega_k < 0.017^e$	$-0.0175 < \Omega_k < 0.0085^f$
	Curvature Radius ^g	Positive Curv.	$R_{\text{curv}} > 12 h^{-1} \text{Gpc}$	$R_{\text{curv}} > 23 h^{-1} \text{Gpc}$
		Negative Curv.	$R_{\text{curv}} > 23 h^{-1} \text{Gpc}$	$R_{\text{curv}} > 33 h^{-1} \text{Gpc}$
§ 3.5	Gaussianity	Local	$-9 < f_{\text{NL}}^{\text{local}} < 111^h$	N/A
		Equilateral	$-151 < f_{\text{NL}}^{\text{equil}} < 253^i$	N/A
§ 3.6	Adiabaticity	Axion	$\alpha_0 < 0.16^j$	$\alpha_0 < 0.067^k$
		Curvaton	$\alpha_{-1} < 0.011^l$	$\alpha_{-1} < 0.0037^m$
§ 4	Parity Violation	Chern-Simons ⁿ	$-5.9^o < \Delta\alpha < 2.4^o$	N/A
§ 5	Dark Energy	Constant w^p	$-1.37 < 1 - w < 0.32^p$	$-0.11 < 1 - w < 0.14$
		Evolving $w(z)^q$	N/A	$-0.38 < 1 + w_0 < 0.14^r$
§ 6.1	Neutrino Mass ^s		$\sum m_\nu < 1.3 \text{ eV}^t$	$\sum m_\nu < 0.61 \text{ eV}^u$
§ 6.2	Neutrino Species		$N_{\text{eff}} > 2.3^v$	$N_{\text{eff}} = 4.4 \pm 1.5^w$ (68%)

Initial conditions in our universe

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

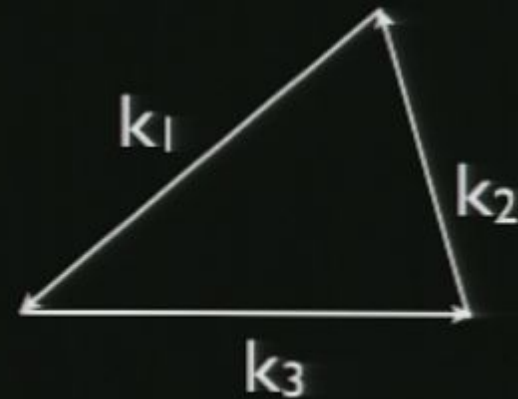
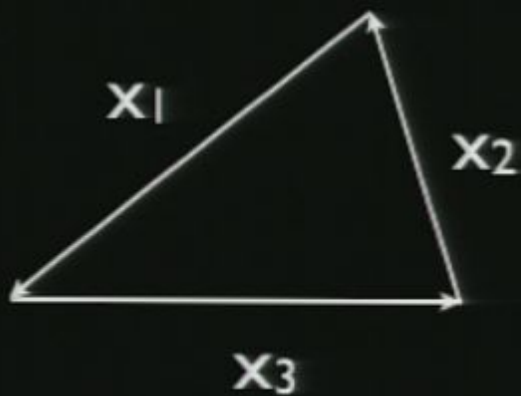


Generic inflationary predictions:

- Nearly scale-invariant spectrum of density perturbations
- Background of gravity waves
- (Very nearly) gaussian initial conditions:

3-pt function as a measure of cosmological NonGaussianity (NG)

- Principal measure of NG: three-pt correlation function



“local”



“equilateral”



Inflation generically predicts (very nearly) gaussian random fluctuations

- Nongaussianity is proportional to slow-roll parameters, V'/V and V''/V
- Reasonable and commonly used approximation: the “local” model of primordial nongaussianity

$$\Phi = \Phi_G + f_{\text{NL}} (\Phi_G^2 - \langle \Phi_G^2 \rangle)$$

- Inflation predicts $f_{\text{NL}} \sim O(0.1)$, which is basically extremely small
- More exotic inflationary models can produce observable NG, however

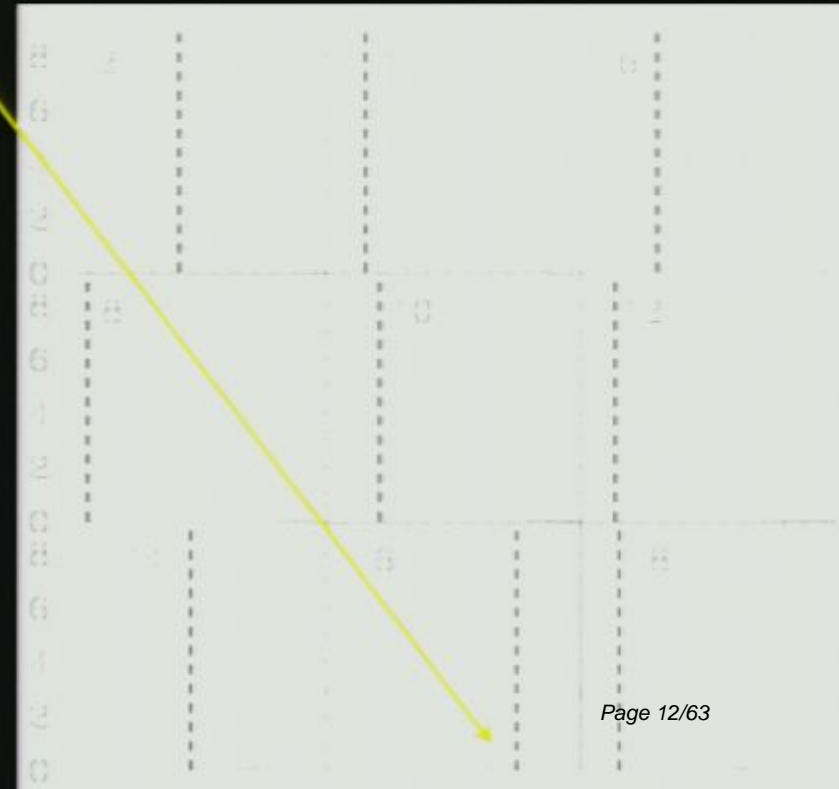
Brief history of NG measurements: 1990's

Early 1990s; COBE: Gaussian CMB sky (Kogut et al 1996)

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1998; COBE: claim of NG at $l=16$ equilateral bispectrum (Ferreira, Magueijo & Gorski 1998)



Brief history of NG measurements: 2000's

Pre-WMAP CMB: all is gaussian (e.g. MAXIMA; Wu et al 2001)

WMAP pre-2008: all is gaussian

(Komatsu et al. 2003; Creminelli, Senatore, Zaldarriaga & Tegmark 2007)

$$-36 < f_{NL} < 100 \quad (95\% \text{ CL})$$

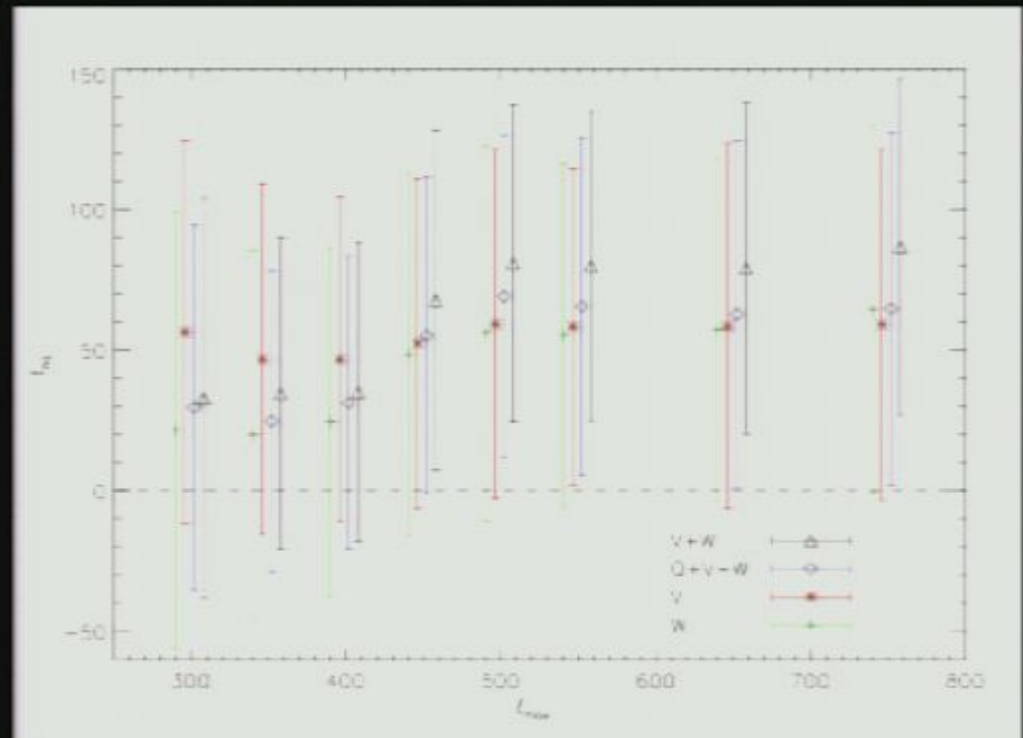
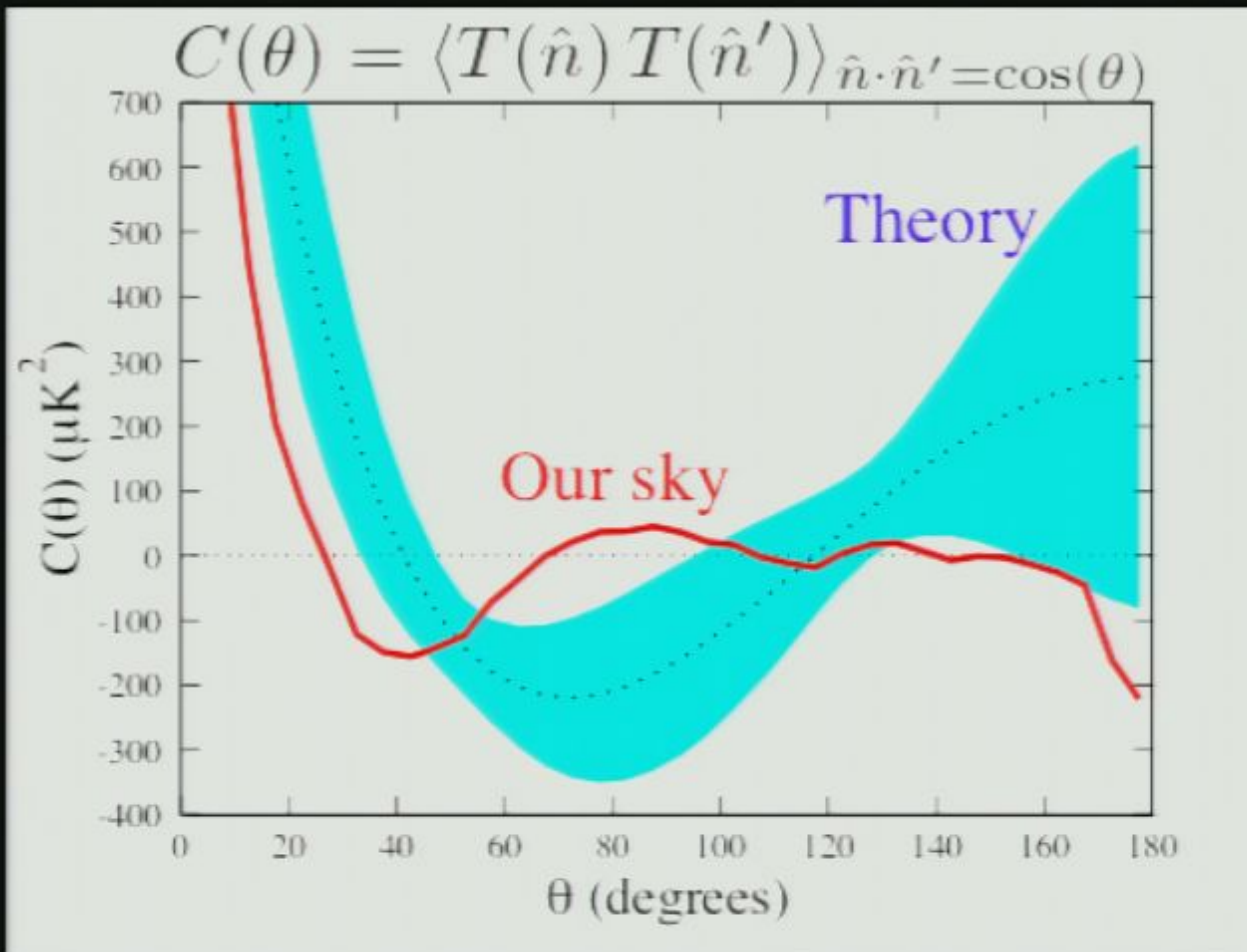


TABLE 6
 NULL TESTS, FREQUENCY DEPENDENCE, AND
 RAW-MAP ESTIMATES OF THE LOCAL FORM OF
 PRIMORDIAL NON-GAUSSIANITY, f_{NL}^{local} , FOR
 $l_{\text{max}} = 500$

Band	Foreground	Mask	f_{NL}^{local}
Q-W	Raw	<i>KQ75</i>	-0.53 ± 0.22
V-W	Raw	<i>KQ75</i>	-0.31 ± 0.23
Q-W	Clean	<i>KQ75</i>	0.10 ± 0.22
V-W	Clean	<i>KQ75</i>	0.06 ± 0.23
Q	Raw	<i>KQ75p1^a</i>	-42 ± 45
V	Raw	<i>KQ75p1</i>	38 ± 34
W	Raw	<i>KQ75p1</i>	43 ± 33
Q	Raw	<i>KQ75</i>	-42 ± 48
V	Raw	<i>KQ75</i>	41 ± 35
W	Raw	<i>KQ75</i>	46 ± 35
Q	Clean	<i>KQ75p1</i>	9 ± 45
V	Clean	<i>KQ75p1</i>	47 ± 34
W	Clean	<i>KQ75p1</i>	60 ± 33
Q	Clean	<i>KQ75</i>	10 ± 48
V	Clean	<i>KQ75</i>	50 ± 35
W	Clean	<i>KQ75</i>	62 ± 35
V+W	Raw	<i>KQ85</i>	9 ± 26
V+W	Raw	<i>Kp0</i>	48 ± 26
V+W	Raw	<i>KQ75p1</i>	41 ± 28
V+W	Raw	<i>KQ75</i>	43 ± 30

^aThis mask replaces the point-source mask in *KQ75* with the one that does not mask the sources identified in the WMAP K-band data

... and also “large-scale anomalies”



lack of power
at >60 deg;
significant at
99.97%

Hinshaw et al. 1996

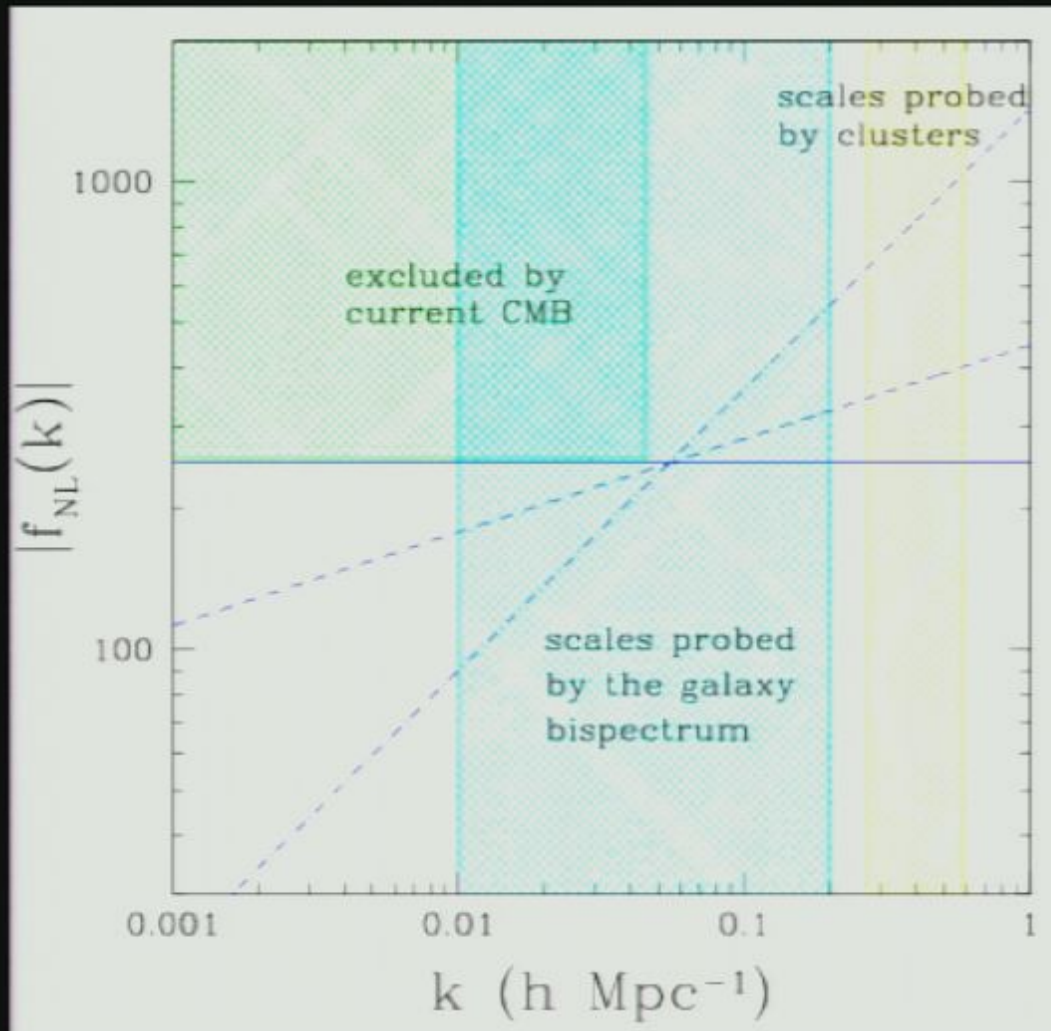
(COBE)

Spergel et al. 2003

(WMAP 1)

Copi, Huterer, Schwarz & Starkman 2007, 08 (WMAP 3, 5)

Constraints from future LSS surveys



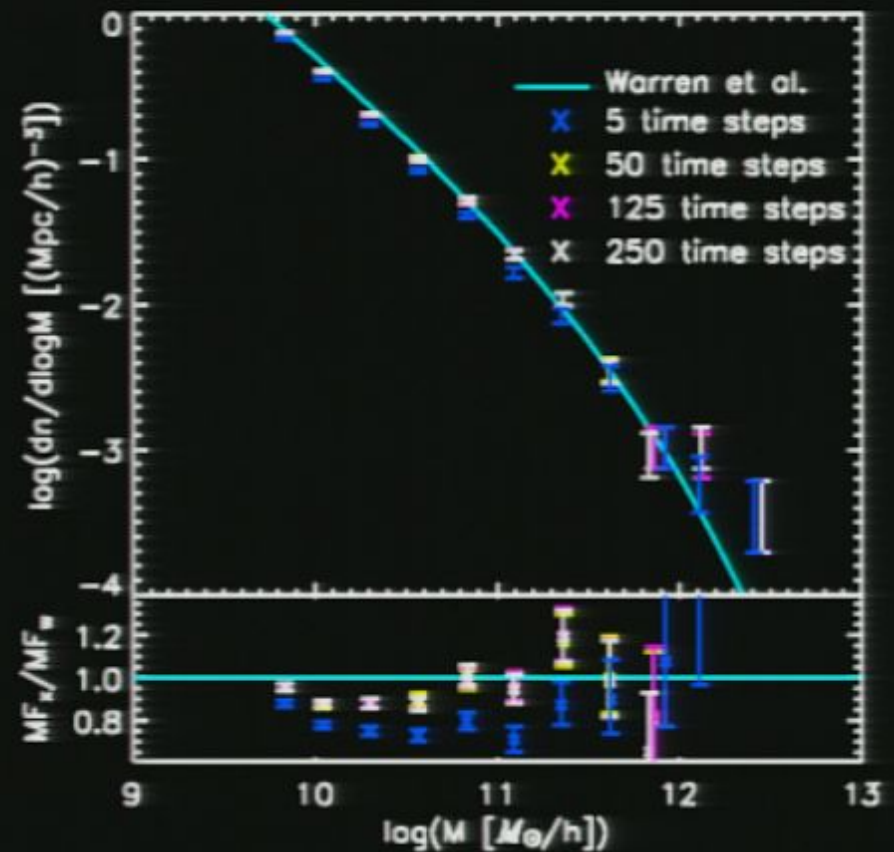
Abundance of halos: the mass function

Lots of interest in using halo counts as a cosmological probe.

- Mass function can be computed precisely (~5%) and robustly for standard cosmology (Jenkins et al. 01, Warren et al. 03)

- dN/dM appears universal — i.e. $f(\sigma)$ — for standard cosmologies

$$\sigma^2(M, z) = \frac{1}{2\pi^2} \int_0^\infty k^2 P(k) W^2(k, M) dk$$



Mass function, usual analytic approach

Press & Schechter 1974:

$$\frac{dn}{dM} dM = \frac{\rho_M}{M} \left| \frac{dF}{dM} \right| dM \quad F(> M) = 2 \int_{\delta_c/\sigma(M)}^{\infty} P_G(\nu) d\nu$$

therefore

$$\left(\frac{dn}{d \ln M} \right)_{\text{PS}} = 2 \frac{\rho_M}{M} \frac{\delta_c}{\sigma} \left| \frac{d \ln \sigma}{d \ln M} \right| P_G(\delta/\sigma)$$

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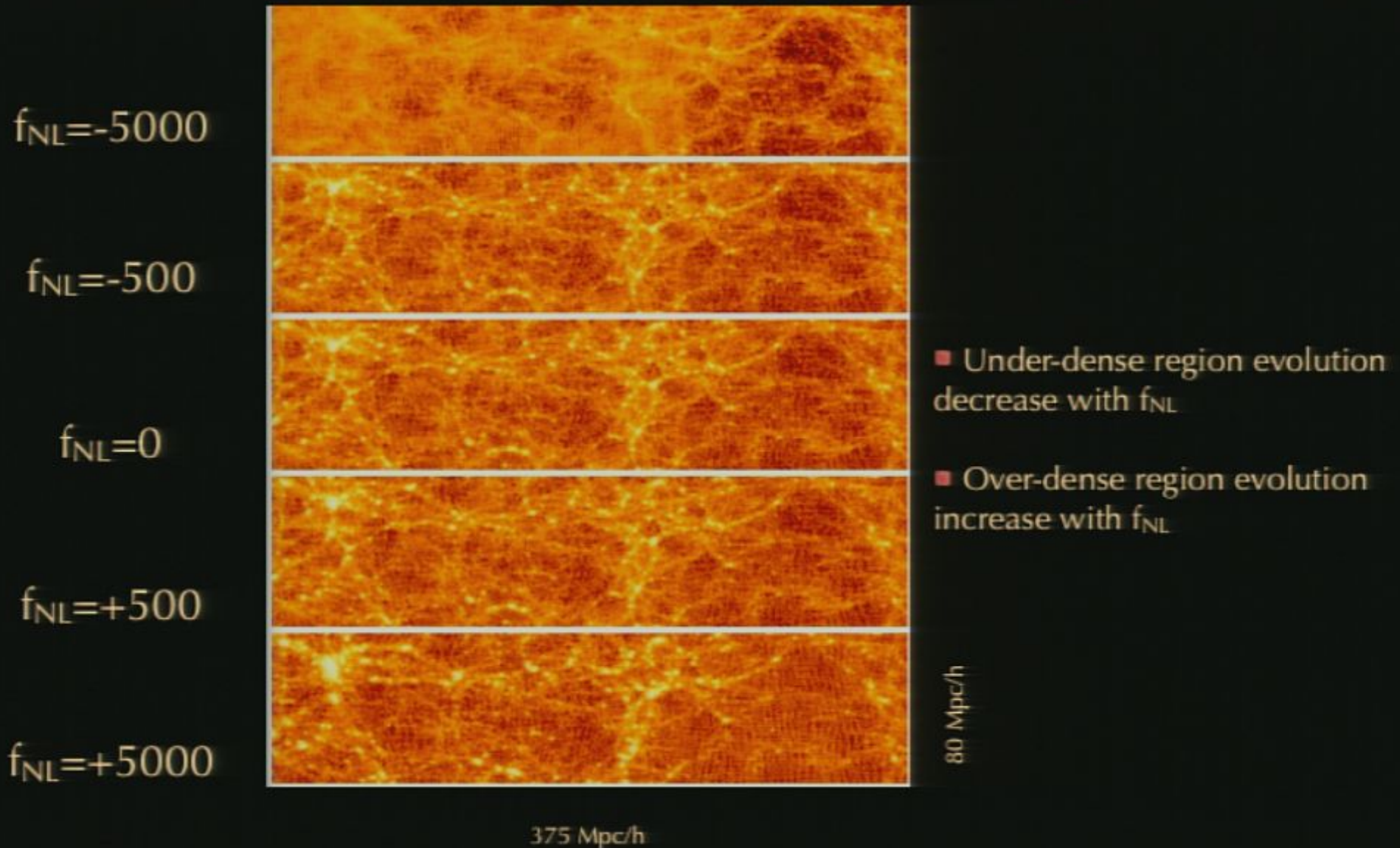
therefore
$$\left(\frac{dn}{d \ln M} \right)_{\text{PS}} = 2 \frac{\rho_M}{M} \frac{\delta_c}{\sigma} \left| \frac{d \ln \sigma}{d \ln M} \right| P_G(\delta/\sigma)$$

“Extended Press-Schechter” (EPS): $P_G(\nu) \rightarrow P_{NG}(\nu)$

Matarrese, Verde & Jimenez (2000; MVJ):
follow EPS, then expand P_{NG} in terms of skewness, do the integral
(also LoVerde, Miller, Shandera & Verde 2008)

However, no convincing reason why either should work!
Need to check these formulae with simulations

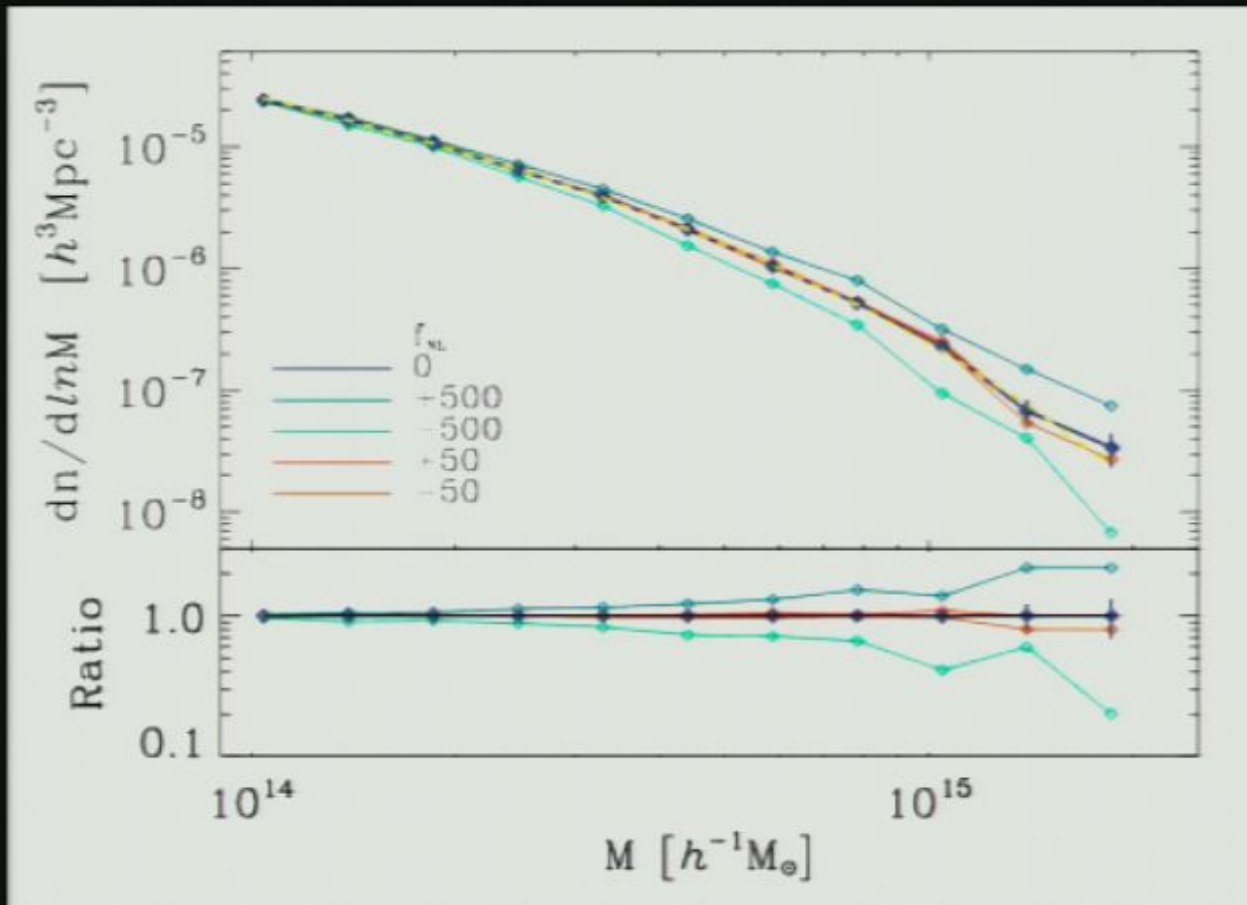
Simulations with nongaussianity (f_{NL})



■ Same initial conditions, different f_{NL}

■ Slice through a box in a simulation $N_{\text{part}}=512^3$, $L=800$ Mpc/h

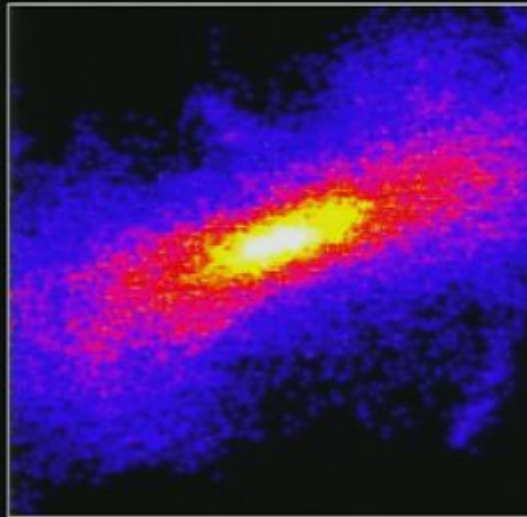
The measured halo mass function



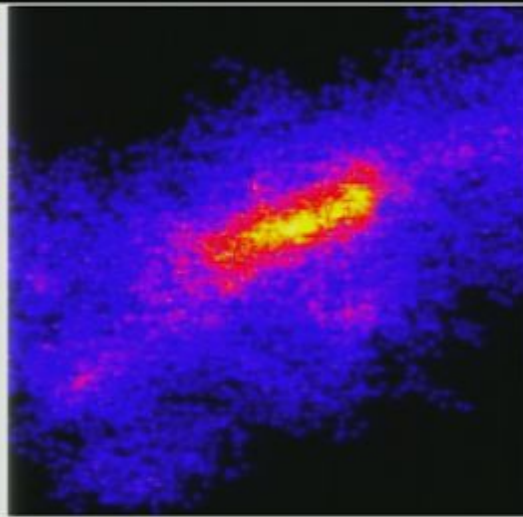
- 512^3 (1024^3) particle simulations with box size 800 (1600) Mpc/h
- Gracos code (www.gracos.com); add quadratic Φ term in real space; apply transfer function in Fourier space

Looking at one individual cluster

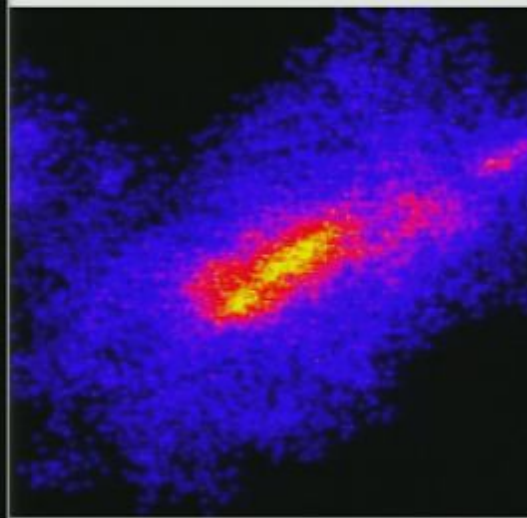
$f_{\text{NL}}=+5000$
 $M=1.2 \cdot 10^{16} M_{\odot}$



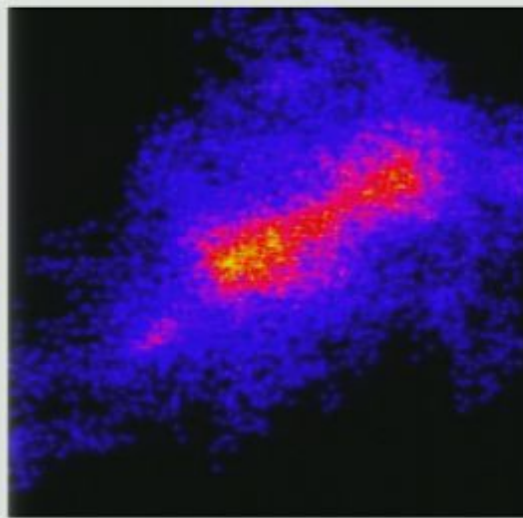
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$f_{\text{NL}}=0$
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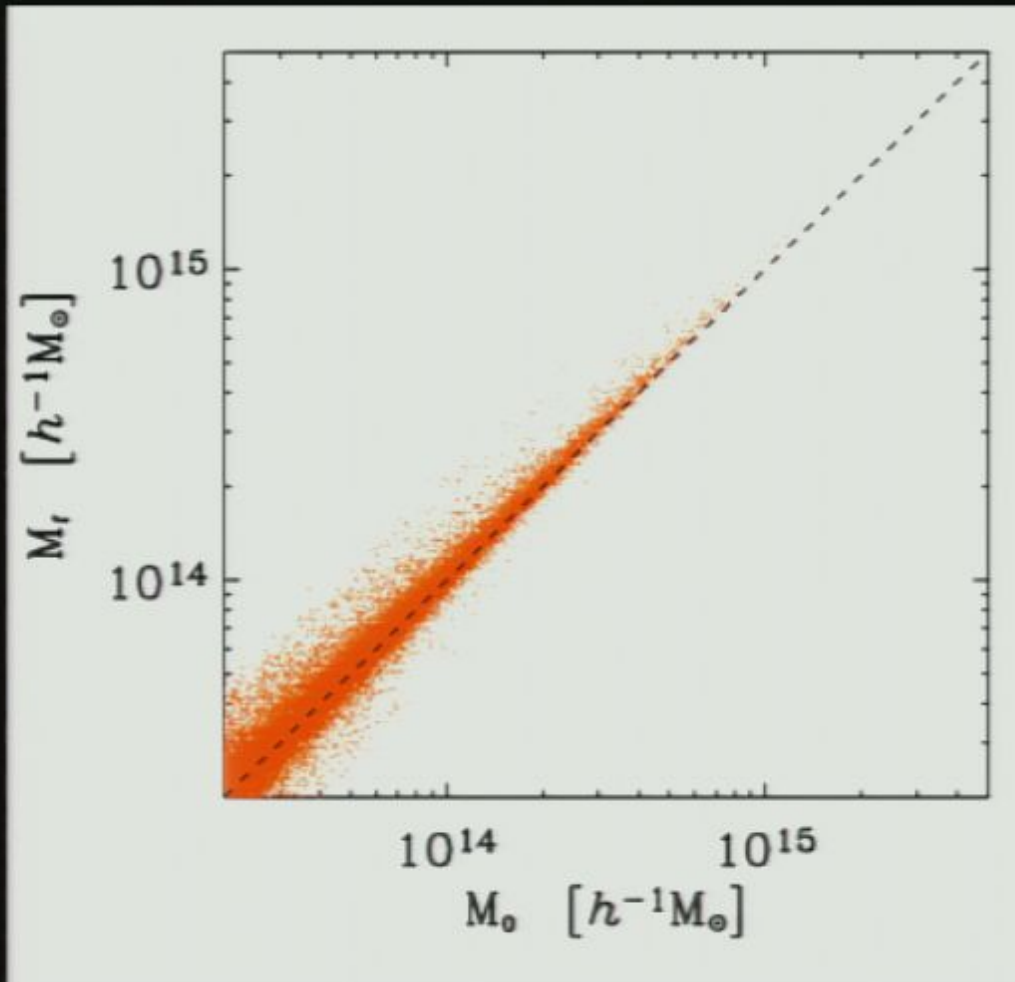


$f_{\text{NL}}=-500$
 $M=4.3 \cdot 10^{15} M_{\odot}$



- Most massive cluster in our simulation
- For small enough f_{NL} , same peaks arise, with different heights (implying different masses)
- Can we extend to any cluster?

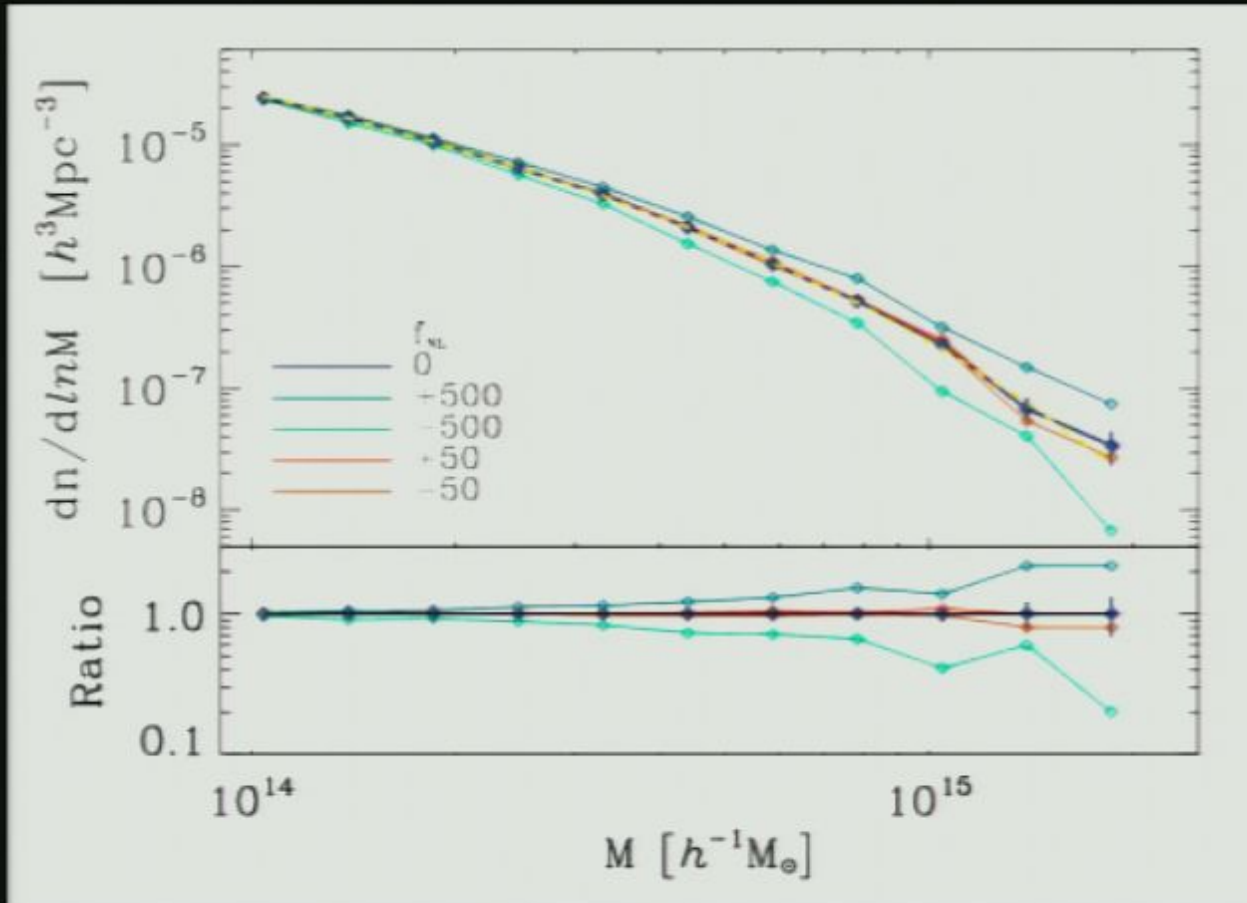
Building the $P(M_f|M_0)$ distribution



$f_{NL} = 500$

- Idea: identify the *same* cluster for different f_{NL} , keep track how its mass changed!
- Significantly saves computational expenses

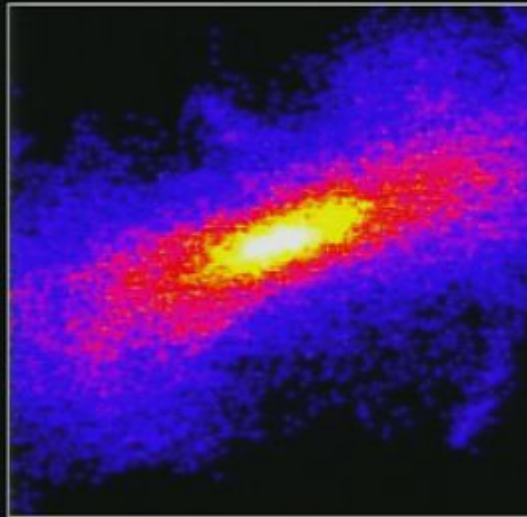
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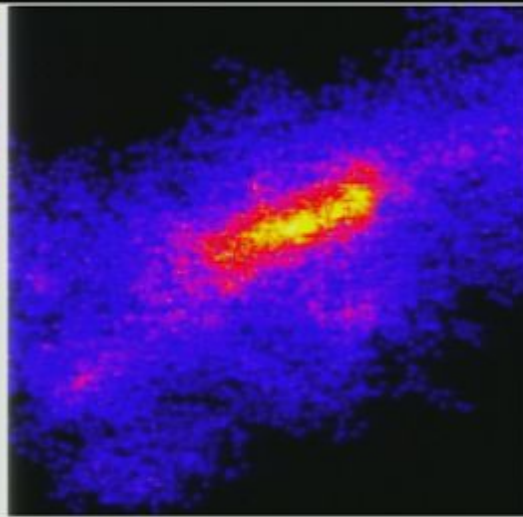
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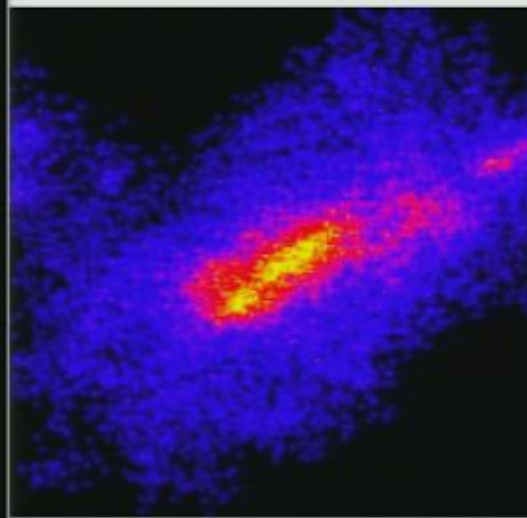
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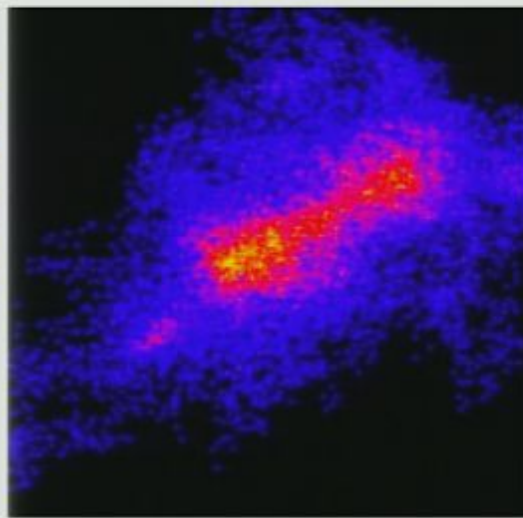
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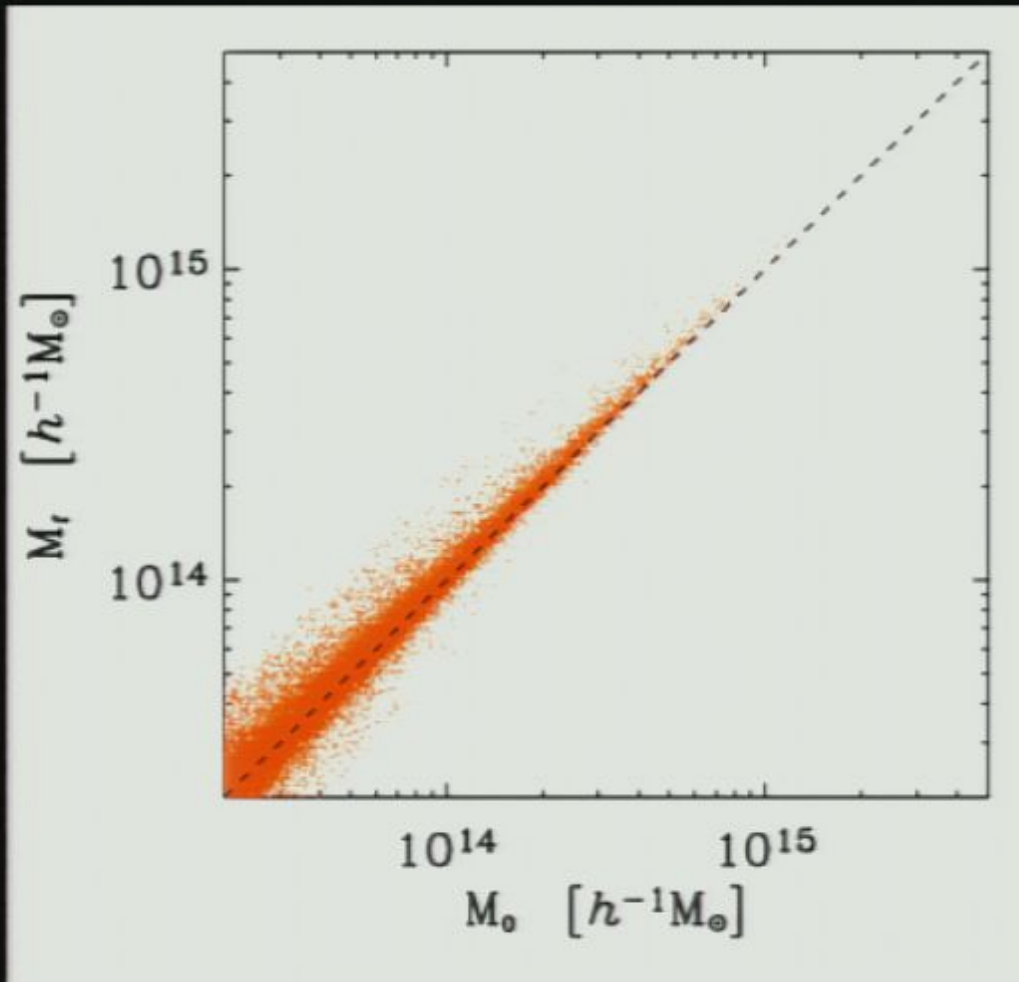


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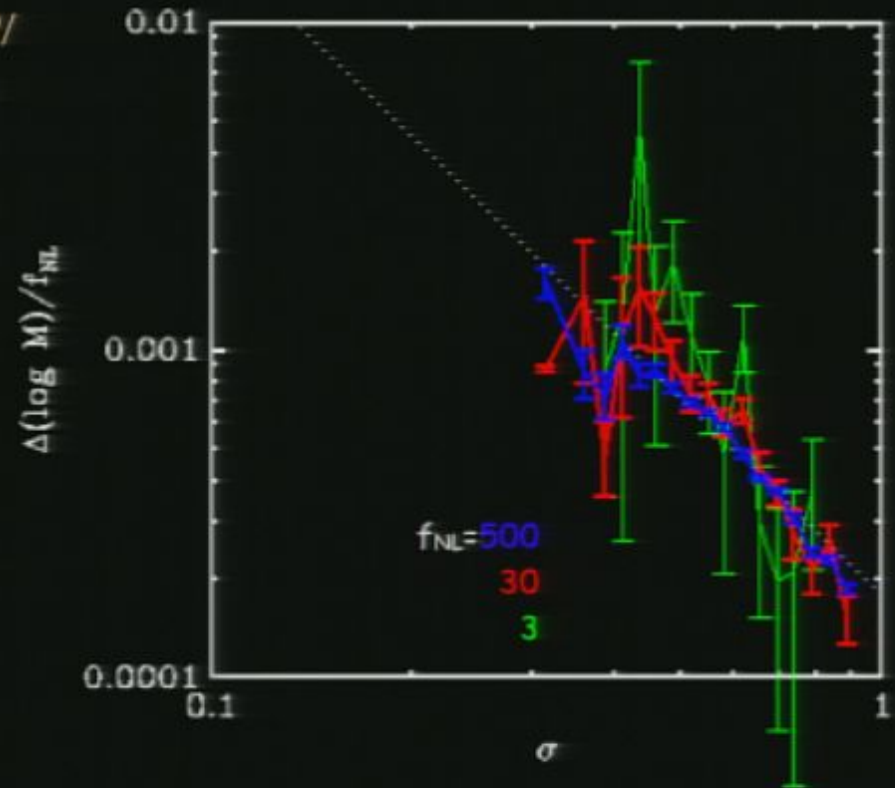
Towards a fitting function

- If the mapping $M_0 \rightarrow M_f$ is described by a PDF $dP/dM_f(M_0)$, then the non-gaussian mass function is a convolution over the (known) gaussian mass function

$$\frac{dN}{dM} = \int \frac{dP(M_f|M_0)}{dM_f} \frac{dN}{dM_0} dM_0$$

usual Gaussian mass function
(e.g. Jenkins et al)

non-Gaussian mass function

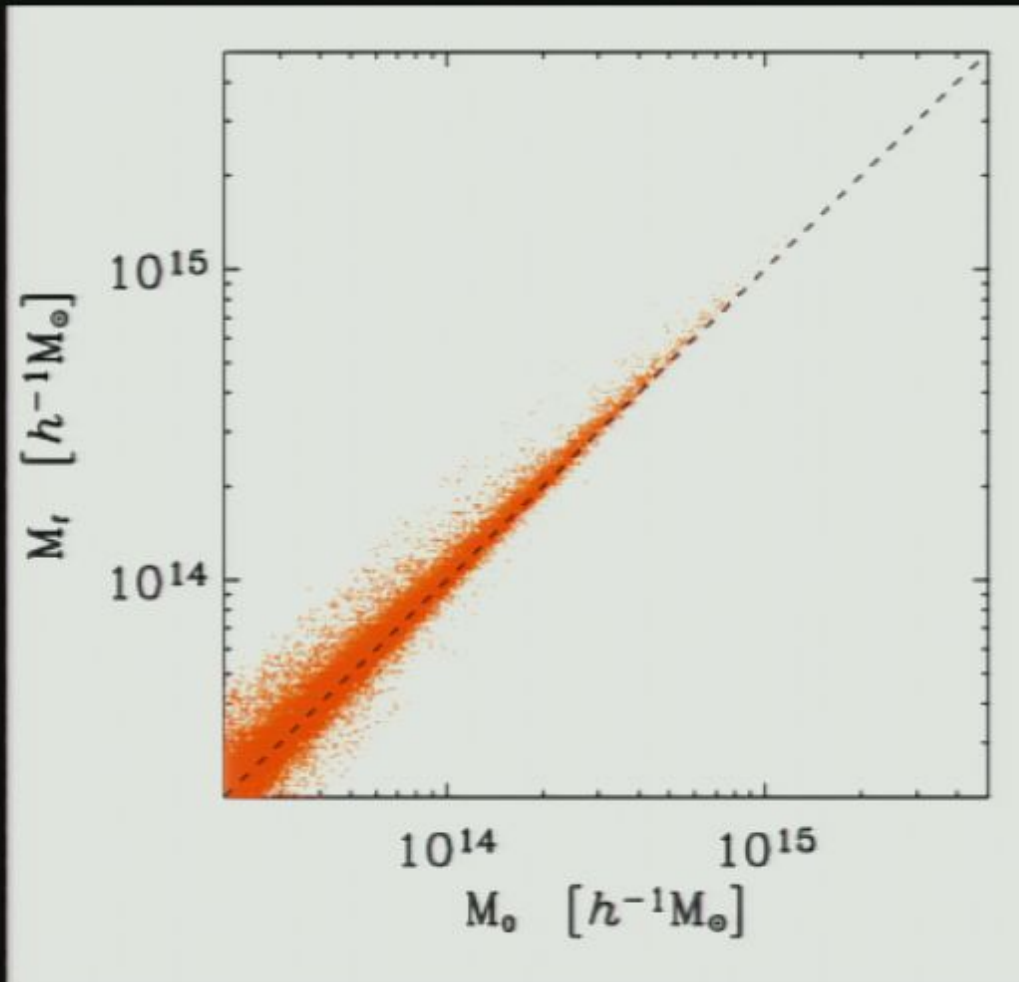


Mean and variance of $P(M_f/M_0)$ are well fit by:

$$\left[\frac{\bar{M}_f}{M_0} \right] - 1 = 6 \cdot 10^{-5} f_{NL} \sigma_8 \sigma(M_0, z)^{-2}$$

$$\sigma \left(\left[\frac{\bar{M}_f}{M_0} \right] - 1 \right) = 0.012 (f_{NL} \sigma_8)^{0.4} \sigma(M_0, z)^{-0.5}$$

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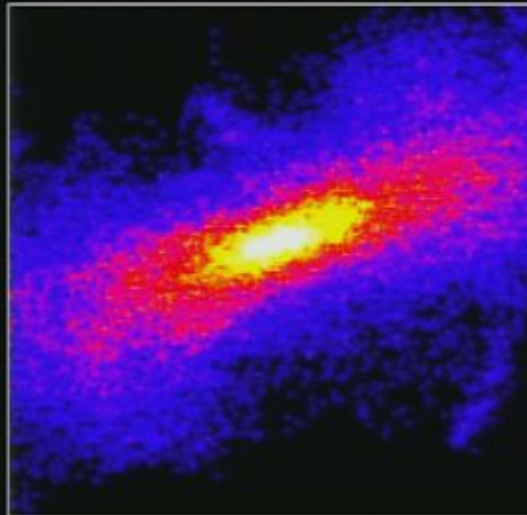


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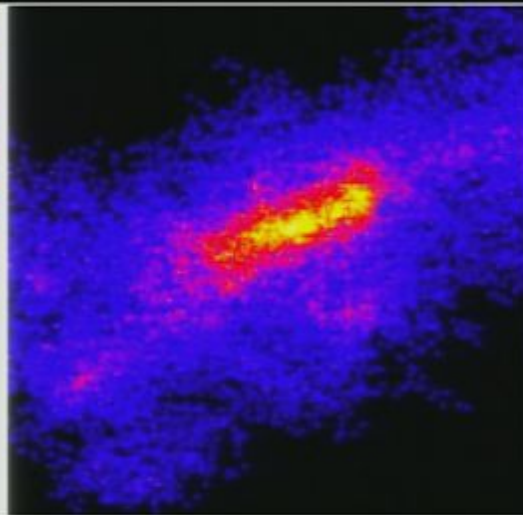
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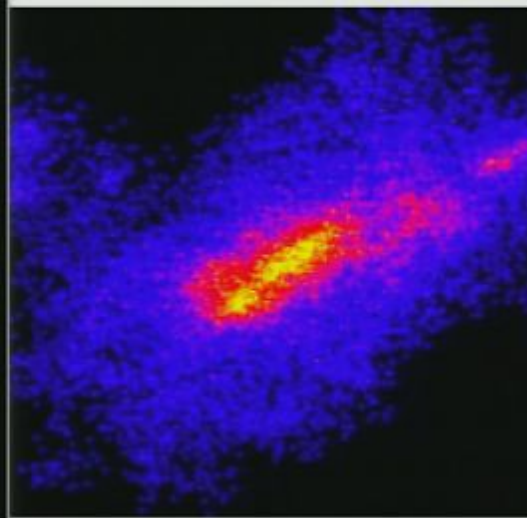
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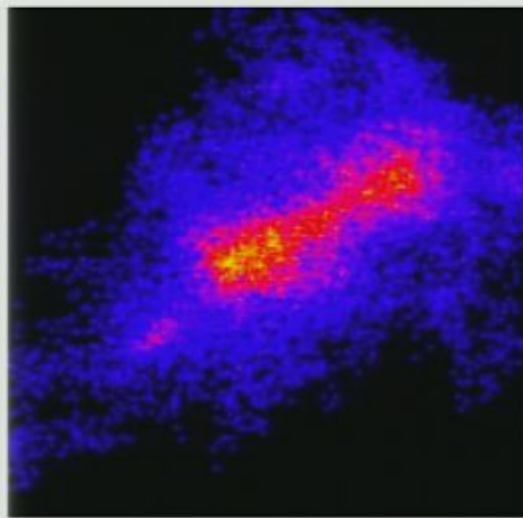
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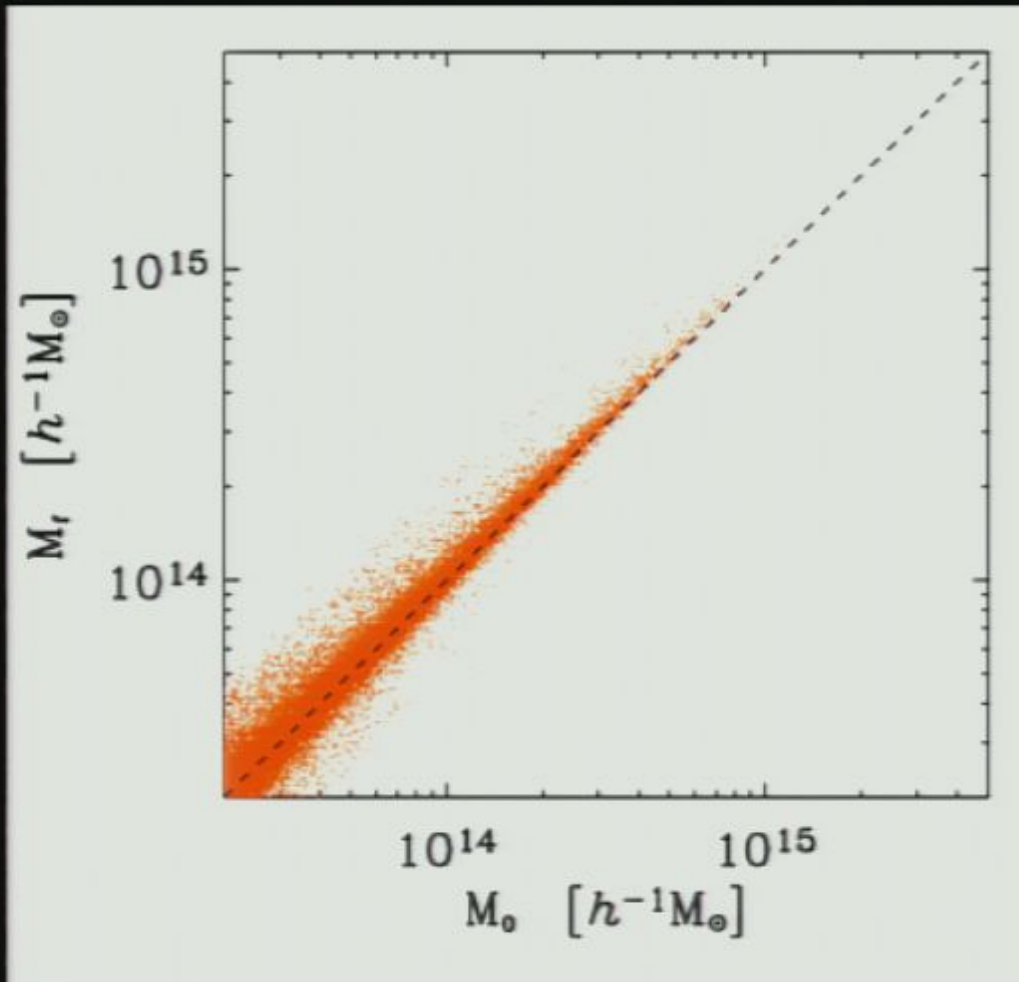


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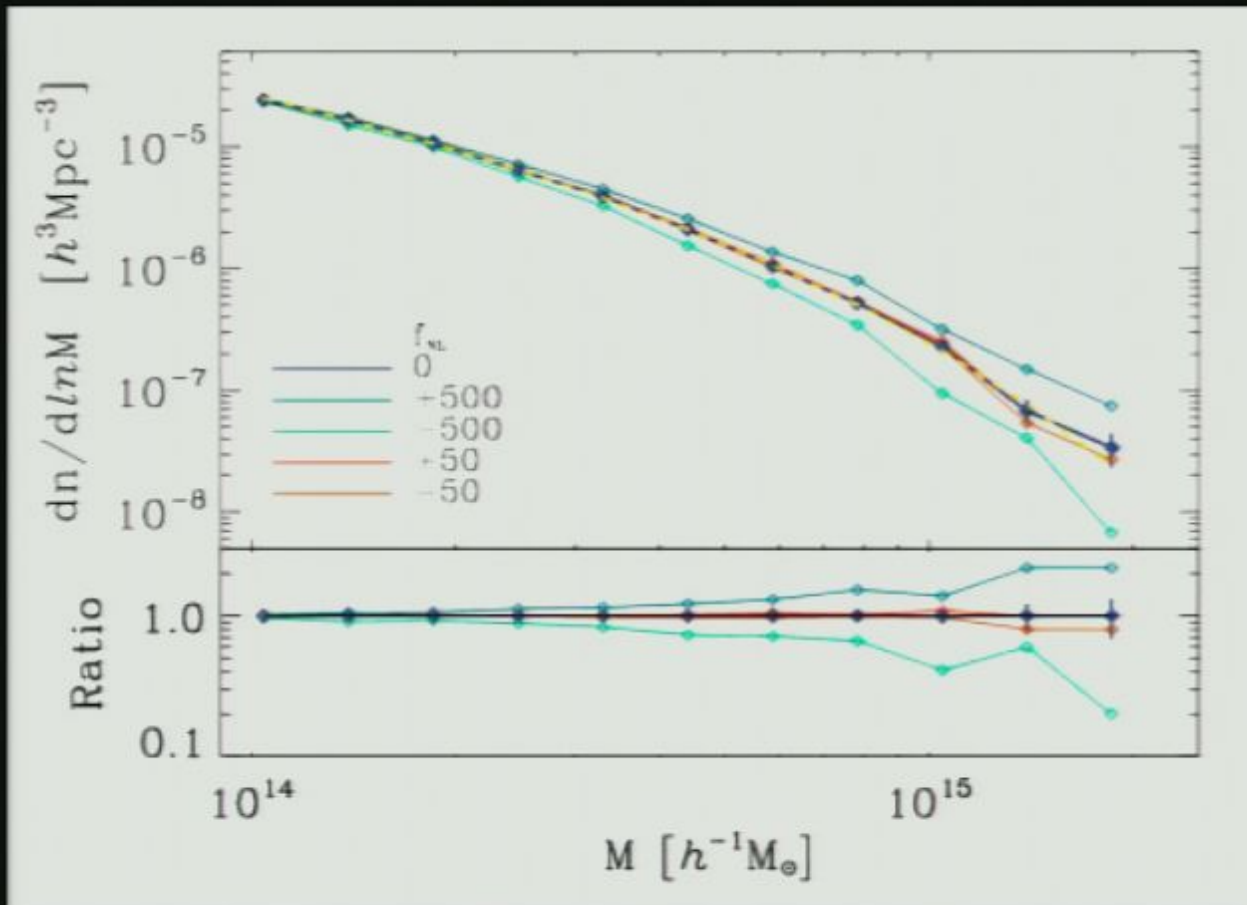
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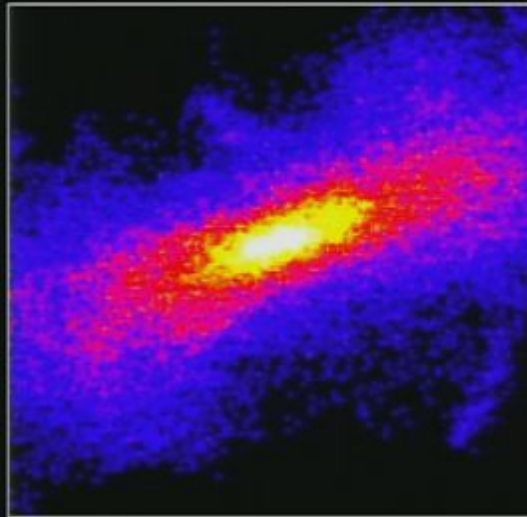
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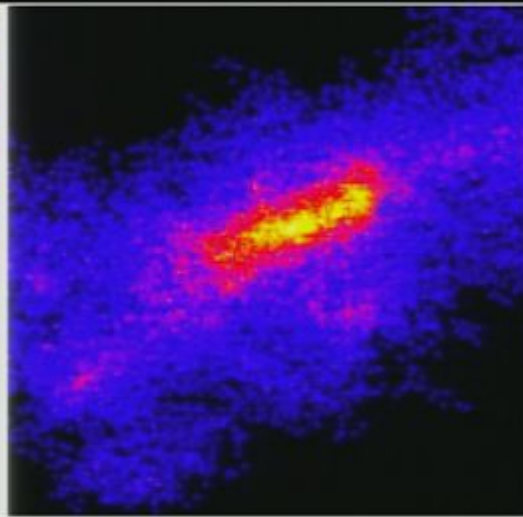
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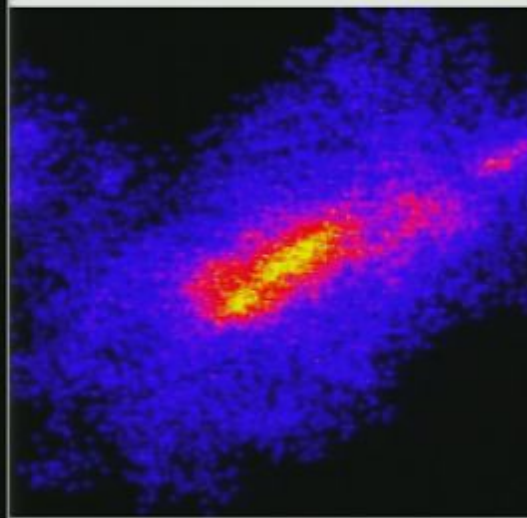
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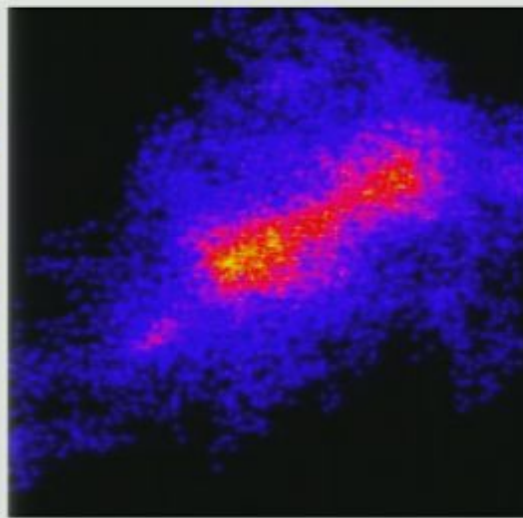
$f_{\text{NL}}=+500$
 $M=5.9 \cdot 10^{15} M_{\odot}$



$f_{\text{NL}}=0$
 $M=5.1 \cdot 10^{15} M_{\odot}$

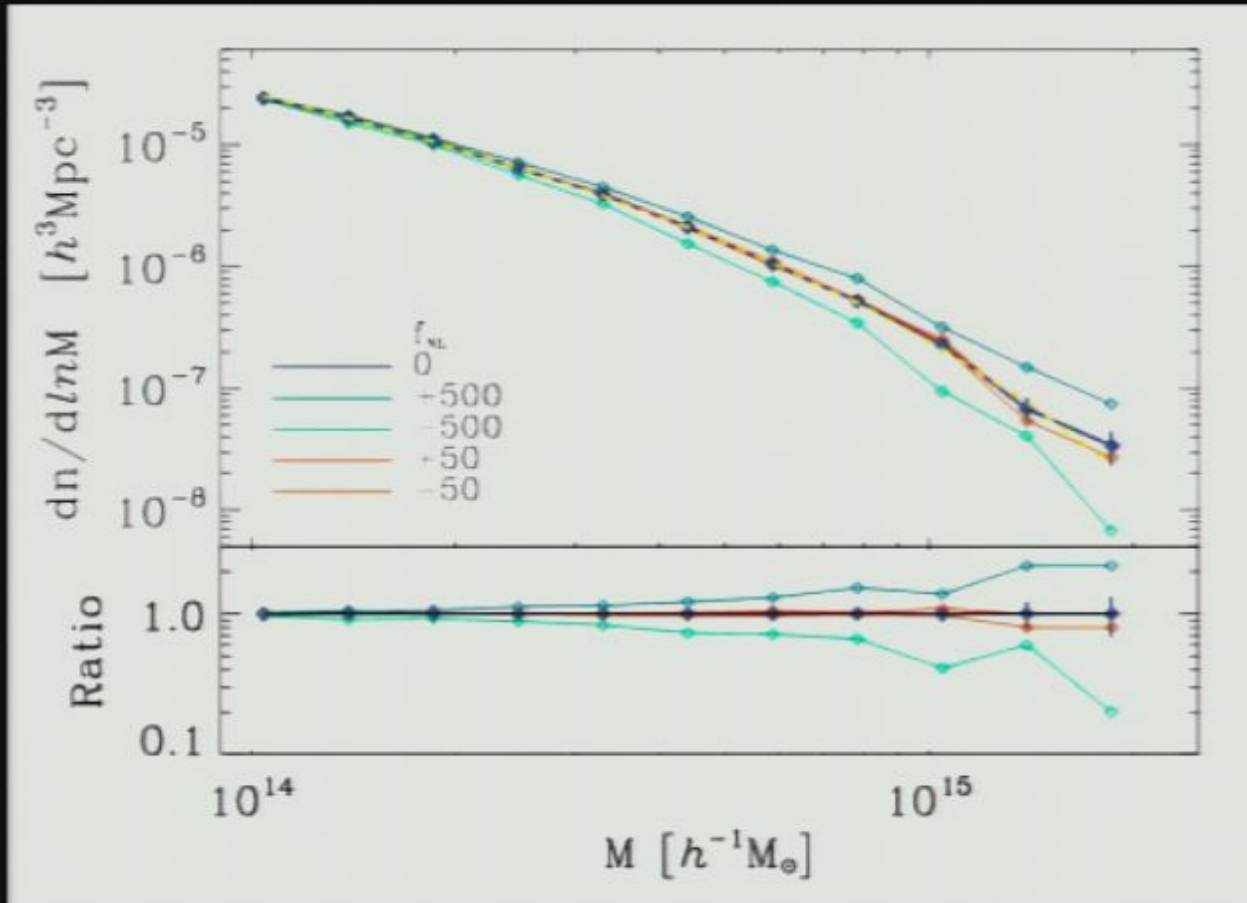


$f_{\text{NL}}=-500$
 $M=4.3 \cdot 10^{15} M_{\odot}$



- Most massive cluster in our simulation
- For small enough f_{NL} , same peaks arise, with different heights (implying different masses)
- Can we extend to any cluster?

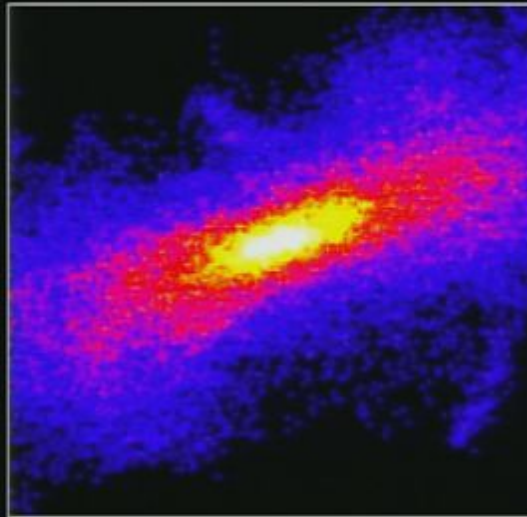
The measured halo mass function



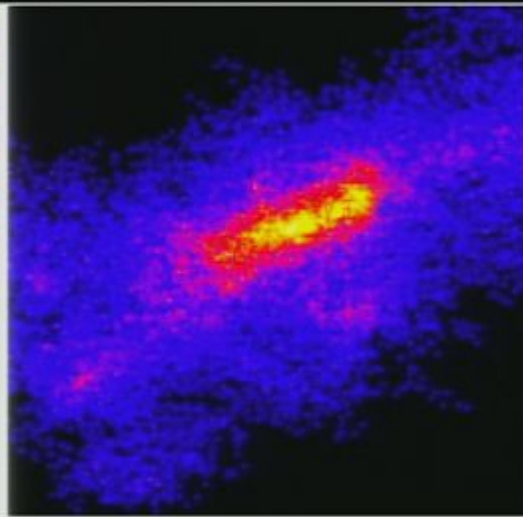
- 512^3 (1024^3) particle simulations with box size 800 (1600) Mpc/h
- Gracos code (www.gracos.com); add quadratic Φ term in real space; apply transfer function in Fourier space

Looking at one individual cluster

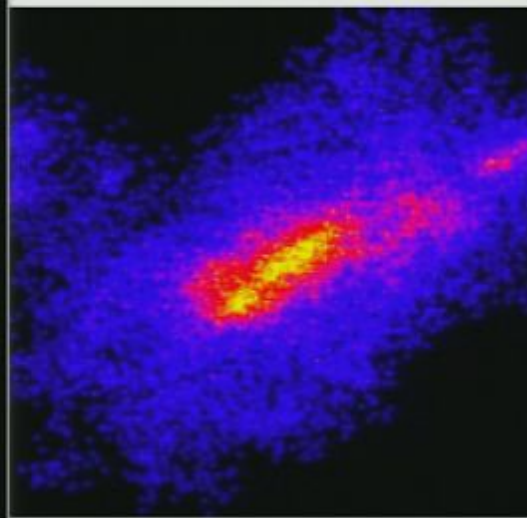
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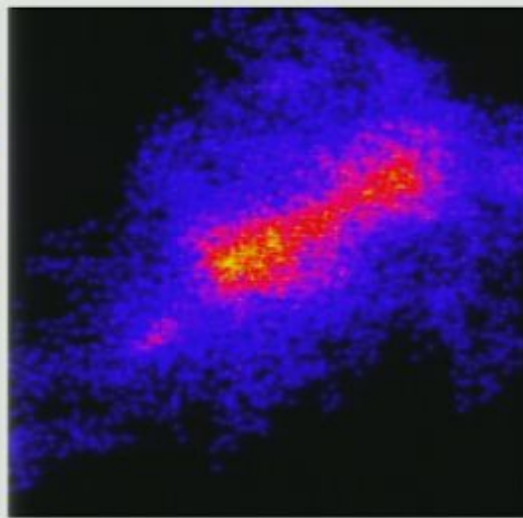
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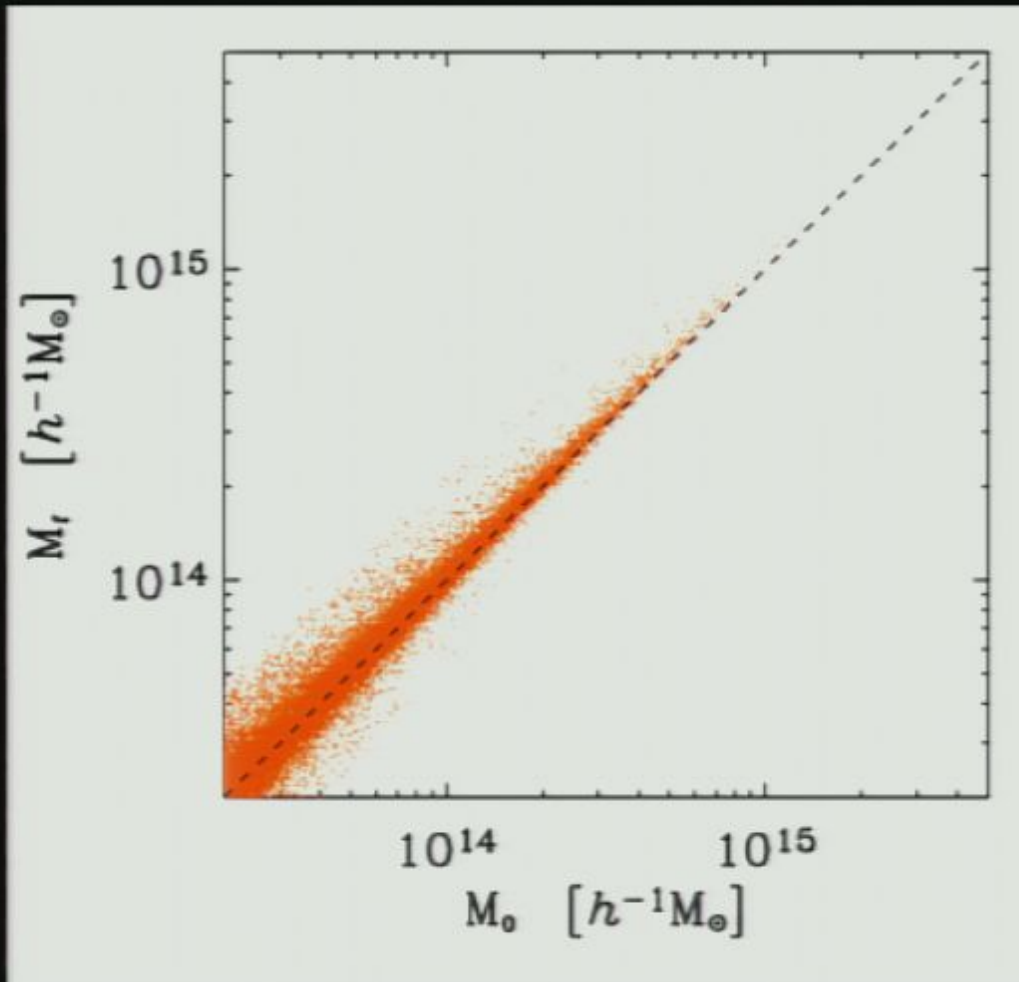


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Building the $P(M_f|M_0)$ distribution



$f_{NL} = 500$

- Idea: identify the *same* cluster for different f_{NL} , keep track how its mass changed!
- Significantly saves computational expenses

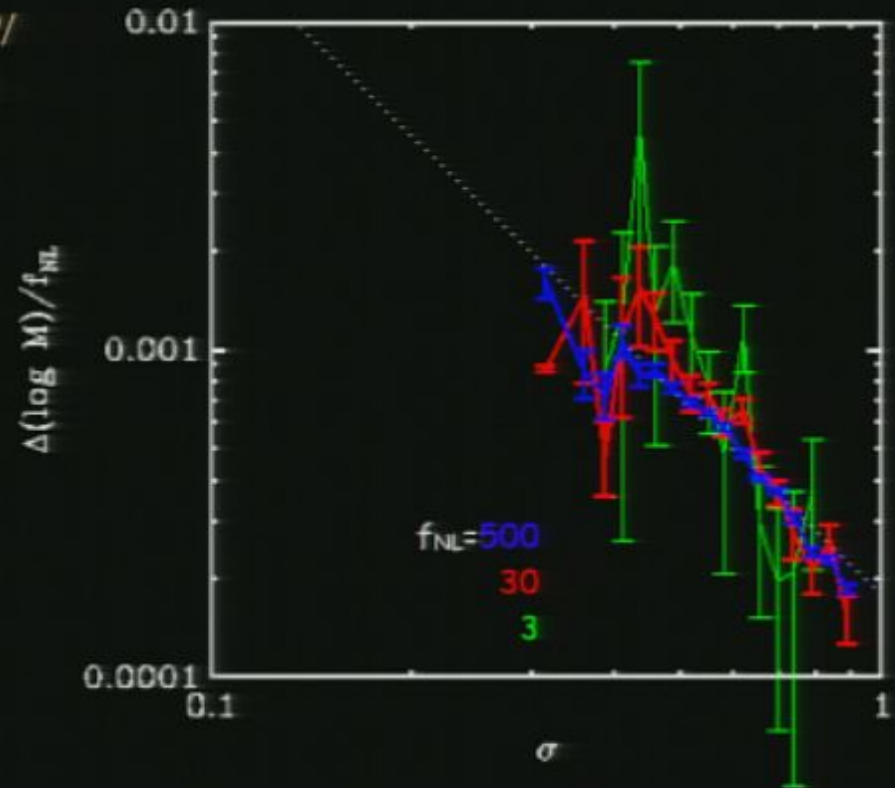
Towards a fitting function

- If the mapping $M_0 \rightarrow M_f$ is described by a PDF $dP/dM_f(M_0)$, then the non-gaussian mass function is a convolution over the (known) gaussian mass function

$$\frac{dN}{dM} = \int \frac{dP(M_f|M_0)}{dM_f} \frac{dN}{dM_0} dM_0$$

usual Gaussian mass function
(e.g. Jenkins et al)

non-Gaussian mass function

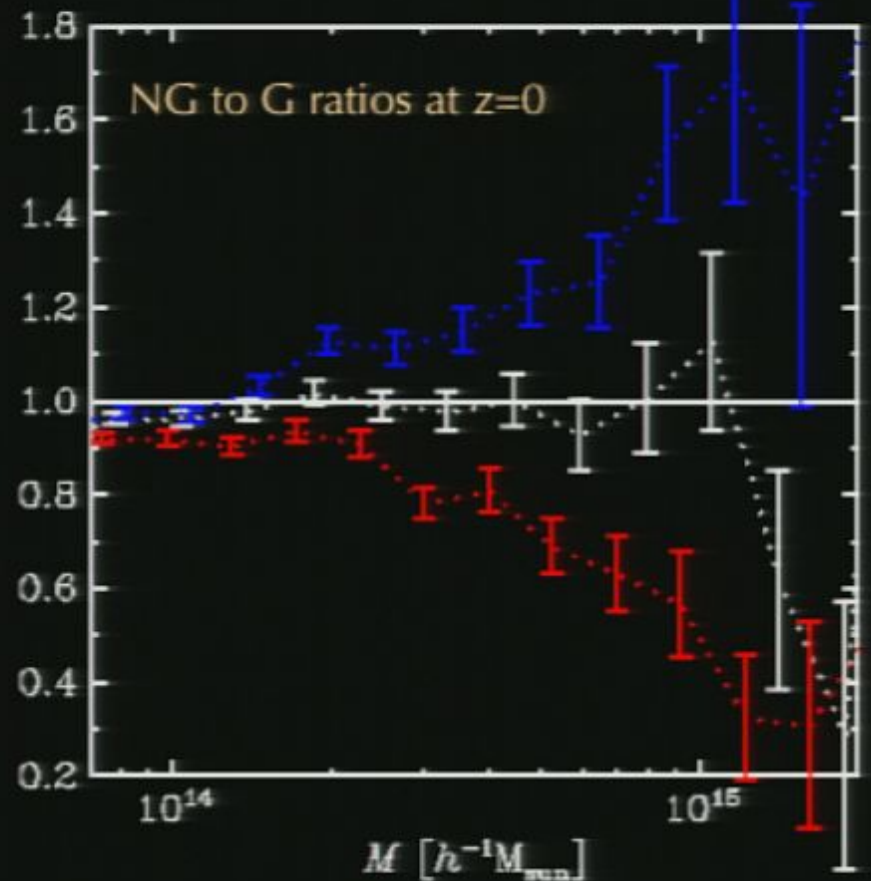
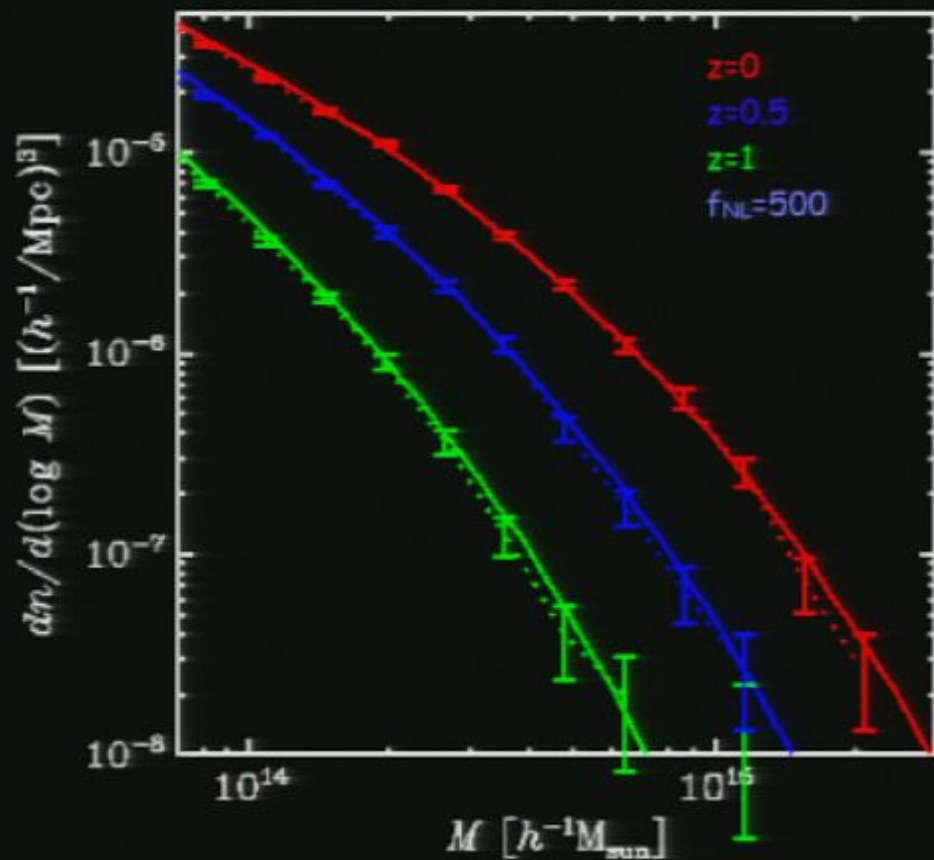


Mean and variance of $P(M_f/M_0)$ are well fit by:

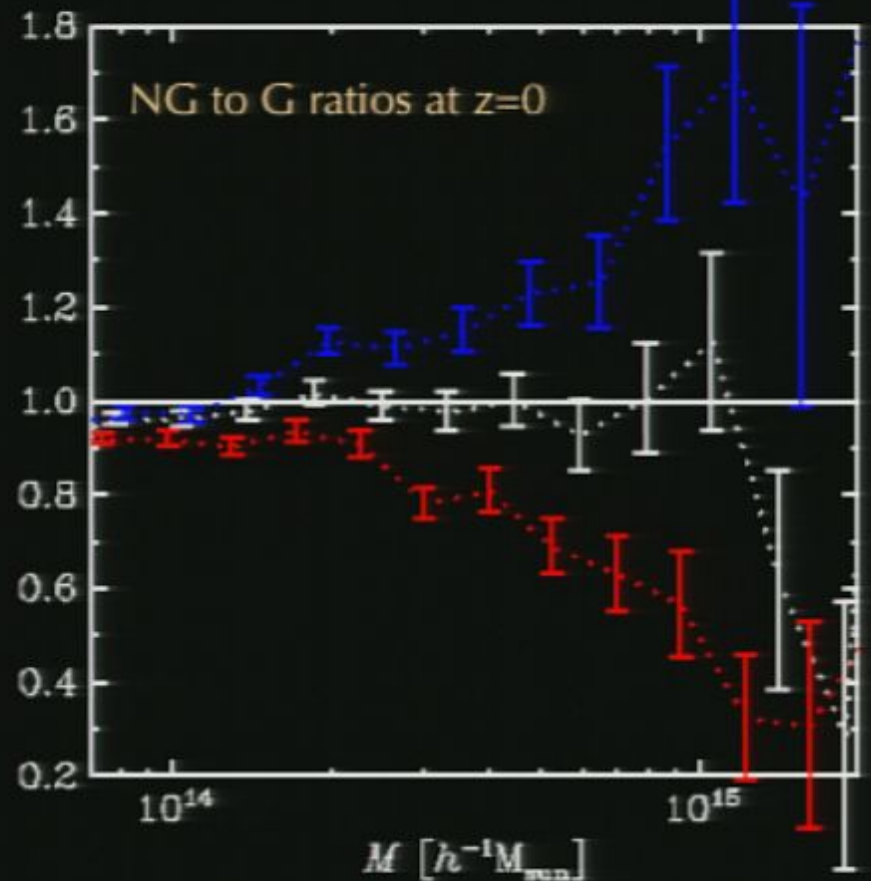
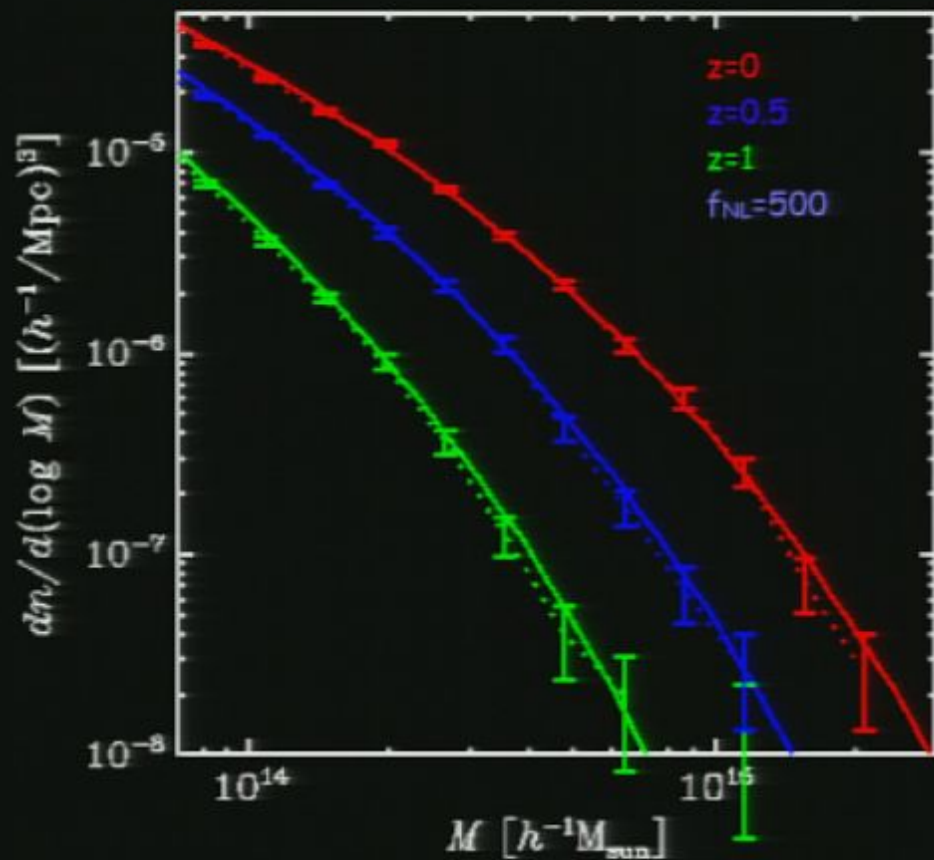
$$\left[\frac{\bar{M}_f}{M_0} \right] - 1 = 6 \cdot 10^{-5} f_{NL} \sigma_8 \sigma(M_0, z)^{-2}$$

$$\sigma \left(\left[\frac{\bar{M}_f}{M_0} \right] - 1 \right) = 0.012 (f_{NL} \sigma_8)^{0.4} \sigma(M_0, z)^{-0.5}$$

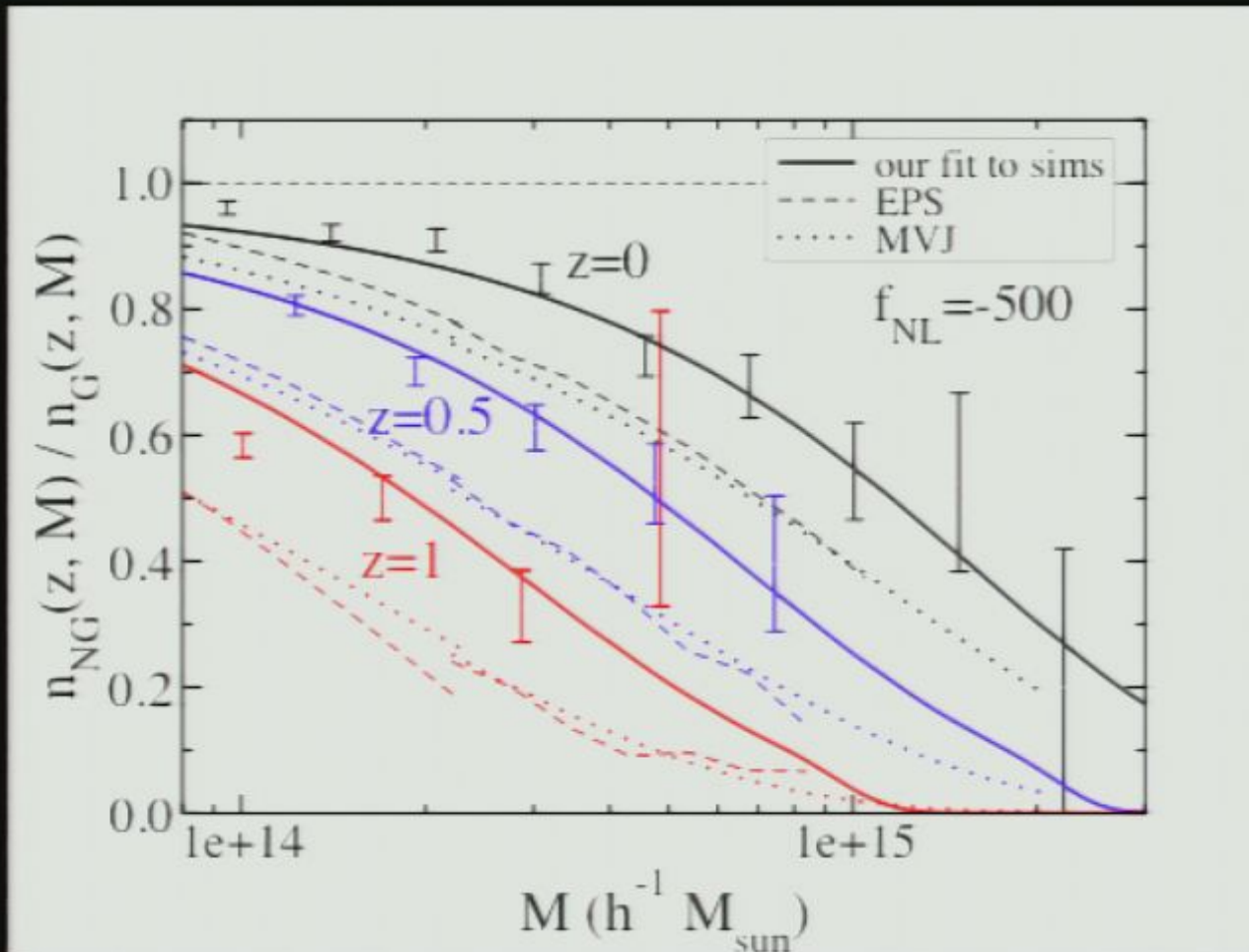
Mass function from N-body simulation and our fitting formula



Mass function from N-body simulation and our fitting formula

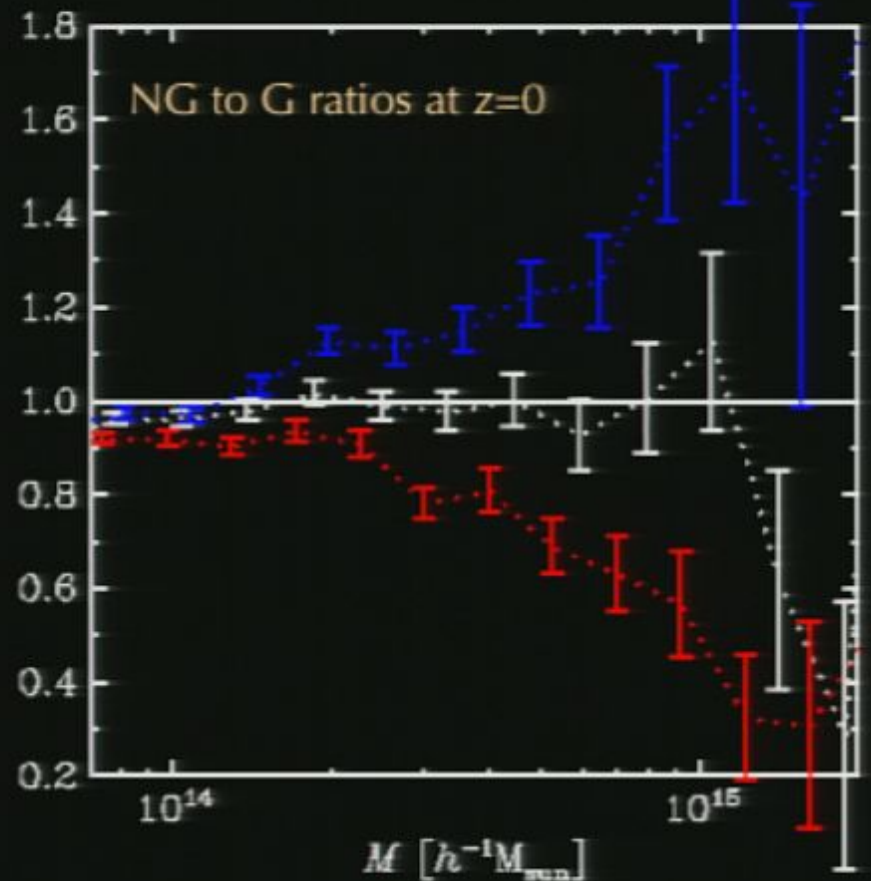
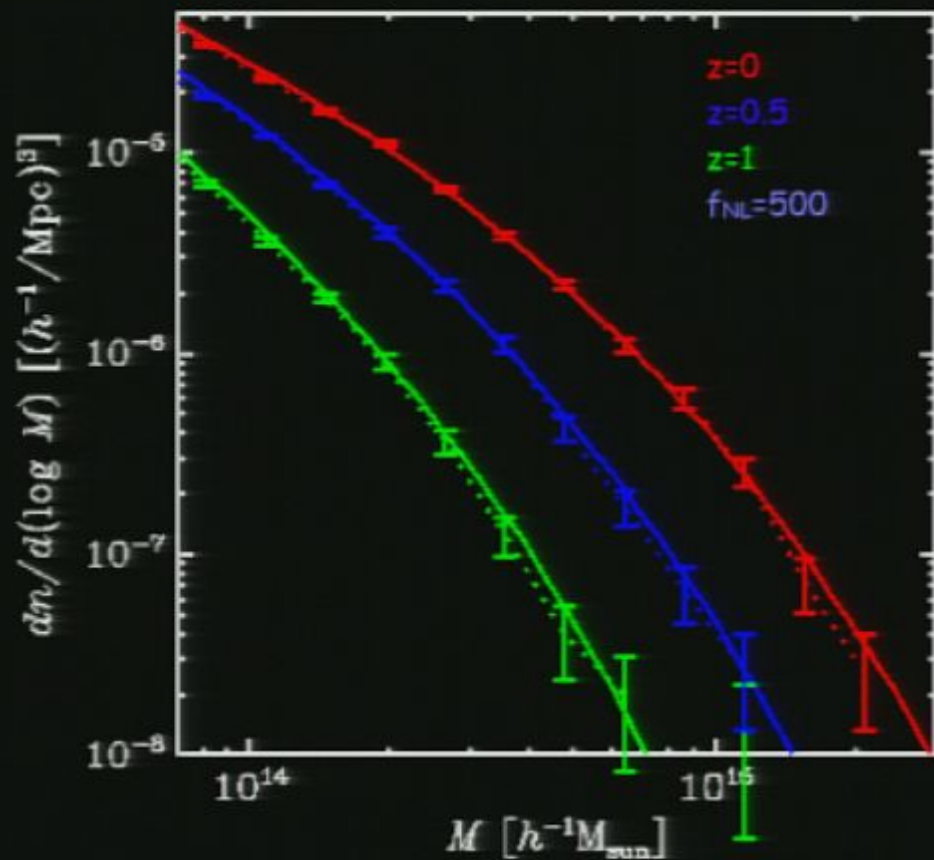


Old fitting functions are discrepant;
off by O(100%) wrt truth

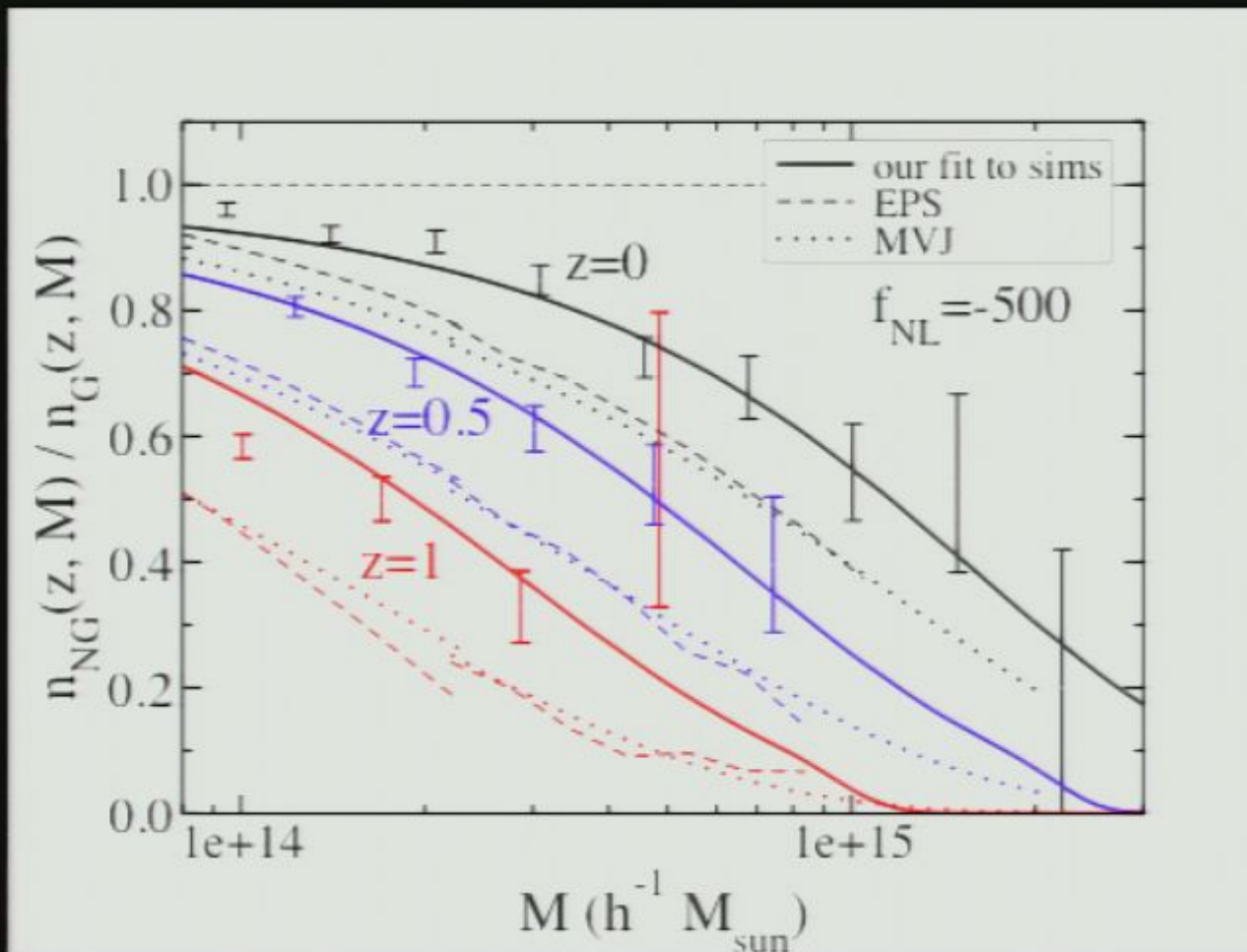


Moreover, it is not much harder to run a simulation
than evaluate Extended Press-Schechter $n(M)$

Mass function from N-body simulation and our fitting formula

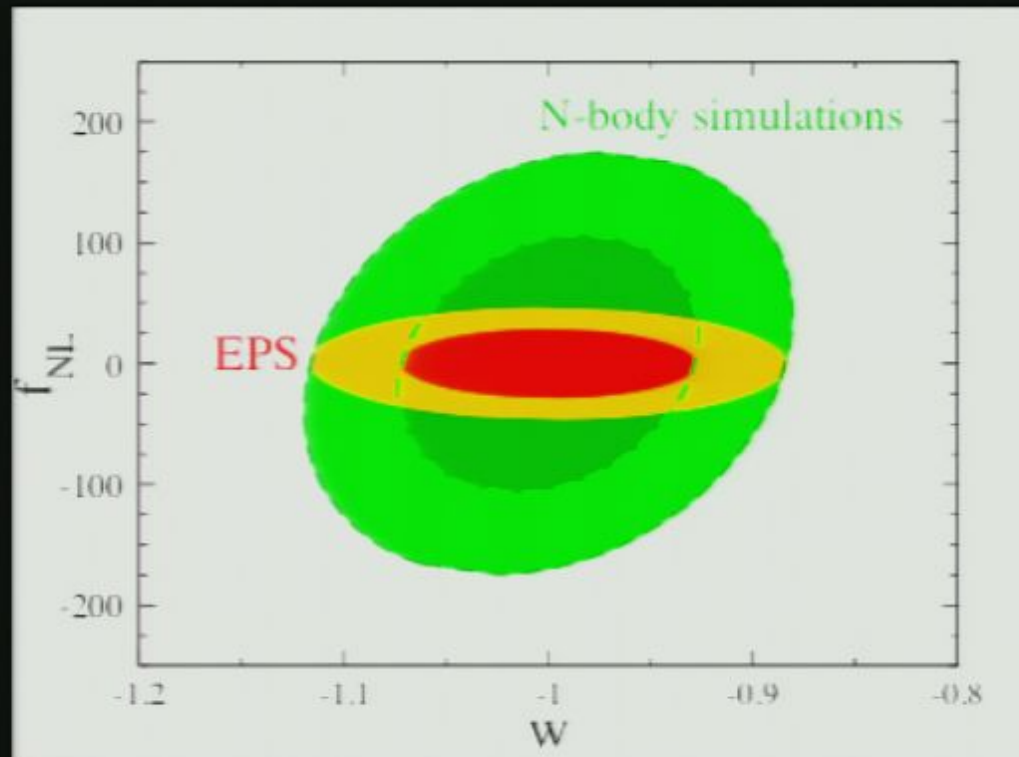


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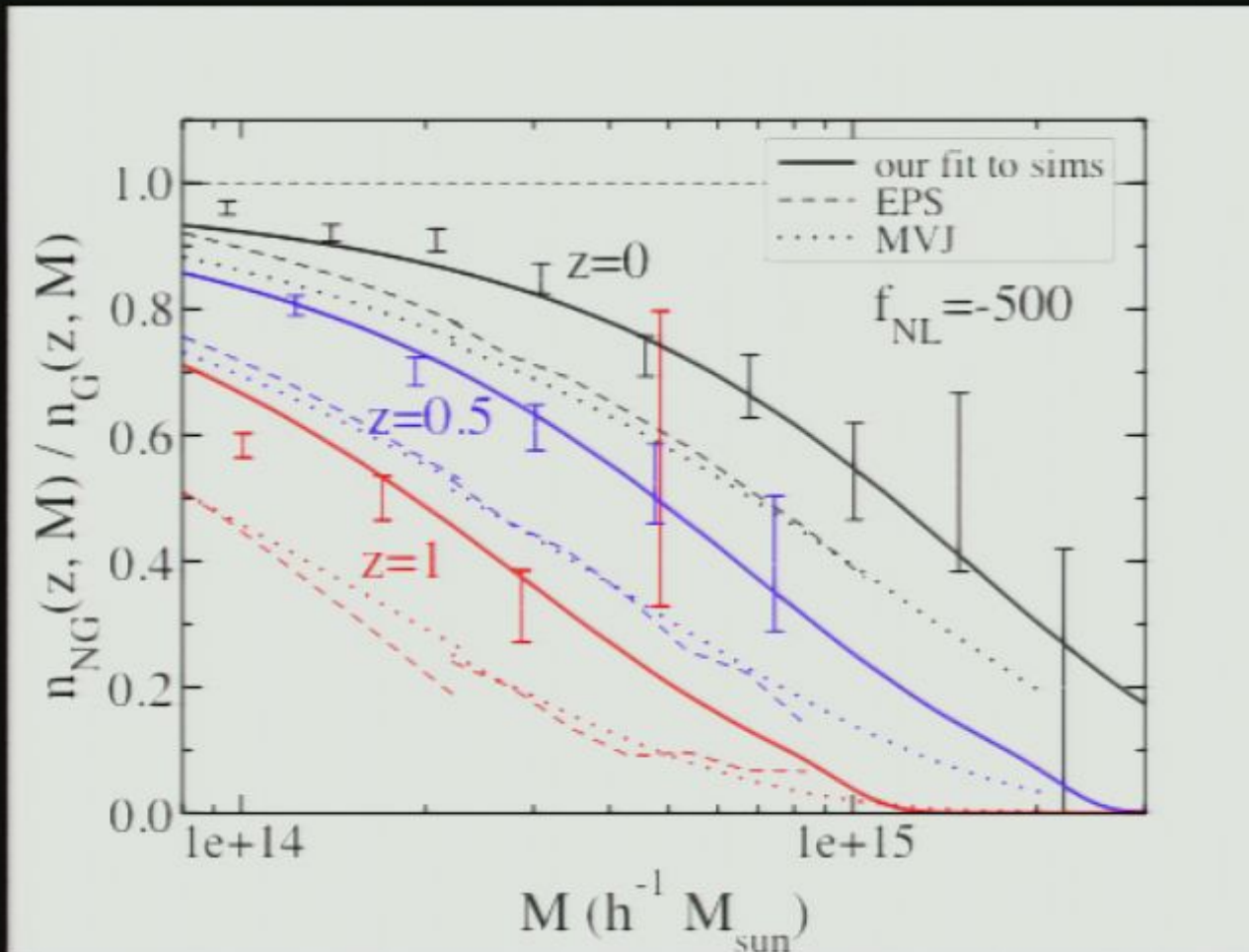
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Cosmological constraints - dark energy and NG



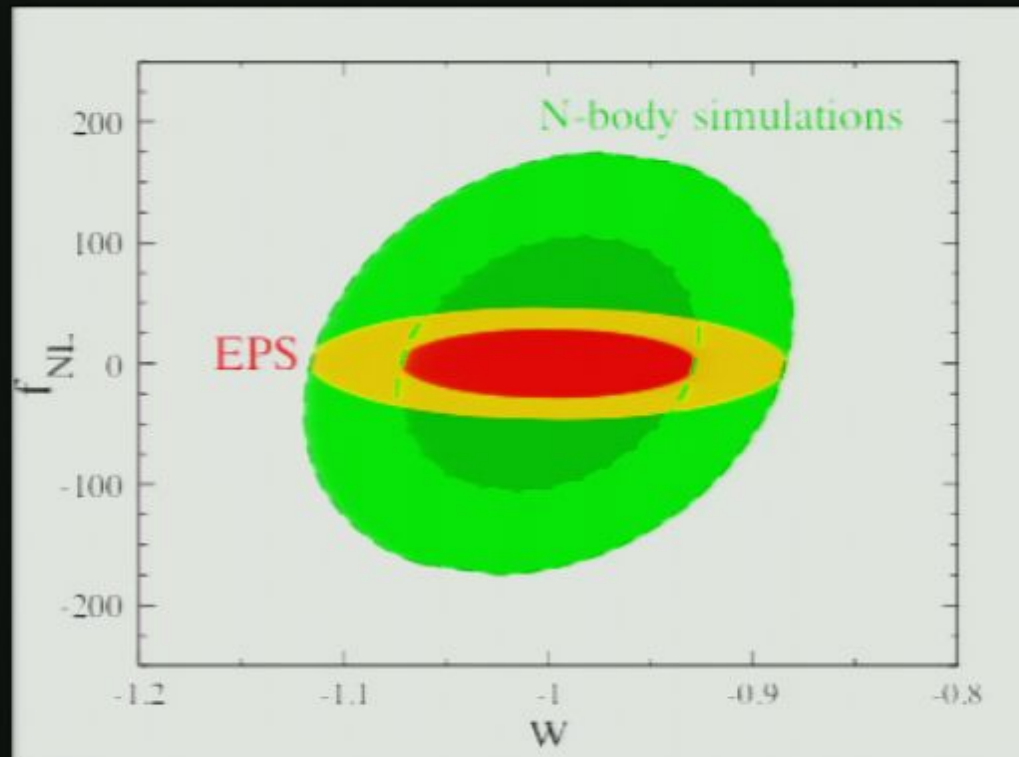
SPT-type survey, $\sim 7,000$ clusters, 4000 sq.deg., $0.1 < z < 1.5$
Planck prior

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Cosmological constraints - dark energy and NG



SPT-type survey, $\sim 7,000$ clusters, 4000 sq.deg., $0.1 < z < 1.5$
Planck prior

**Recent exciting developments (2007-present):
how to constrain primordial NG
much more accurately
from the LSS**

We looked at the galaxy bias

usually nuisance parameter(s) ←

$$\text{bias} \equiv \frac{\text{clustering of galaxies}}{\text{clustering of dark matter}} = \frac{\left(\frac{\delta\rho}{\rho}\right)_{\text{halos}}}{\left(\frac{\delta\rho}{\rho}\right)_{\text{DM}}}$$

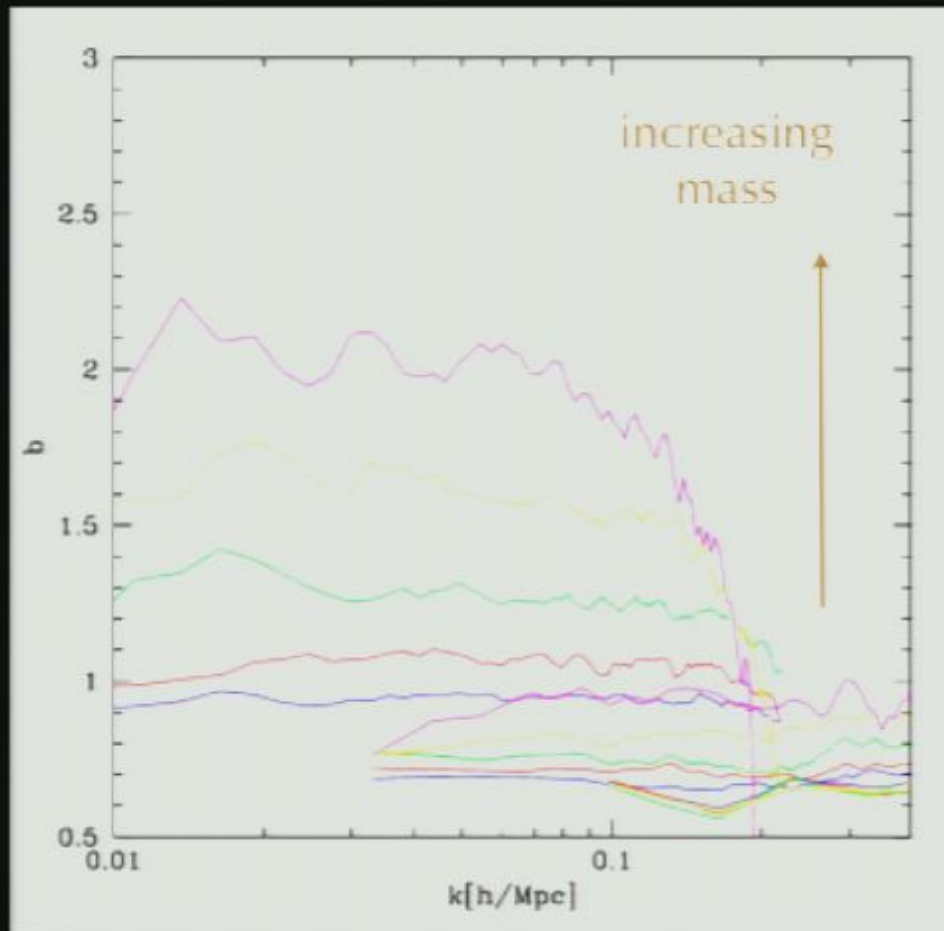
→ cosmologists measure

→ theory predicts

Simulations and theory both say:
large-scale bias is scale-independent

Bias of dark matter halos - Gaussian case

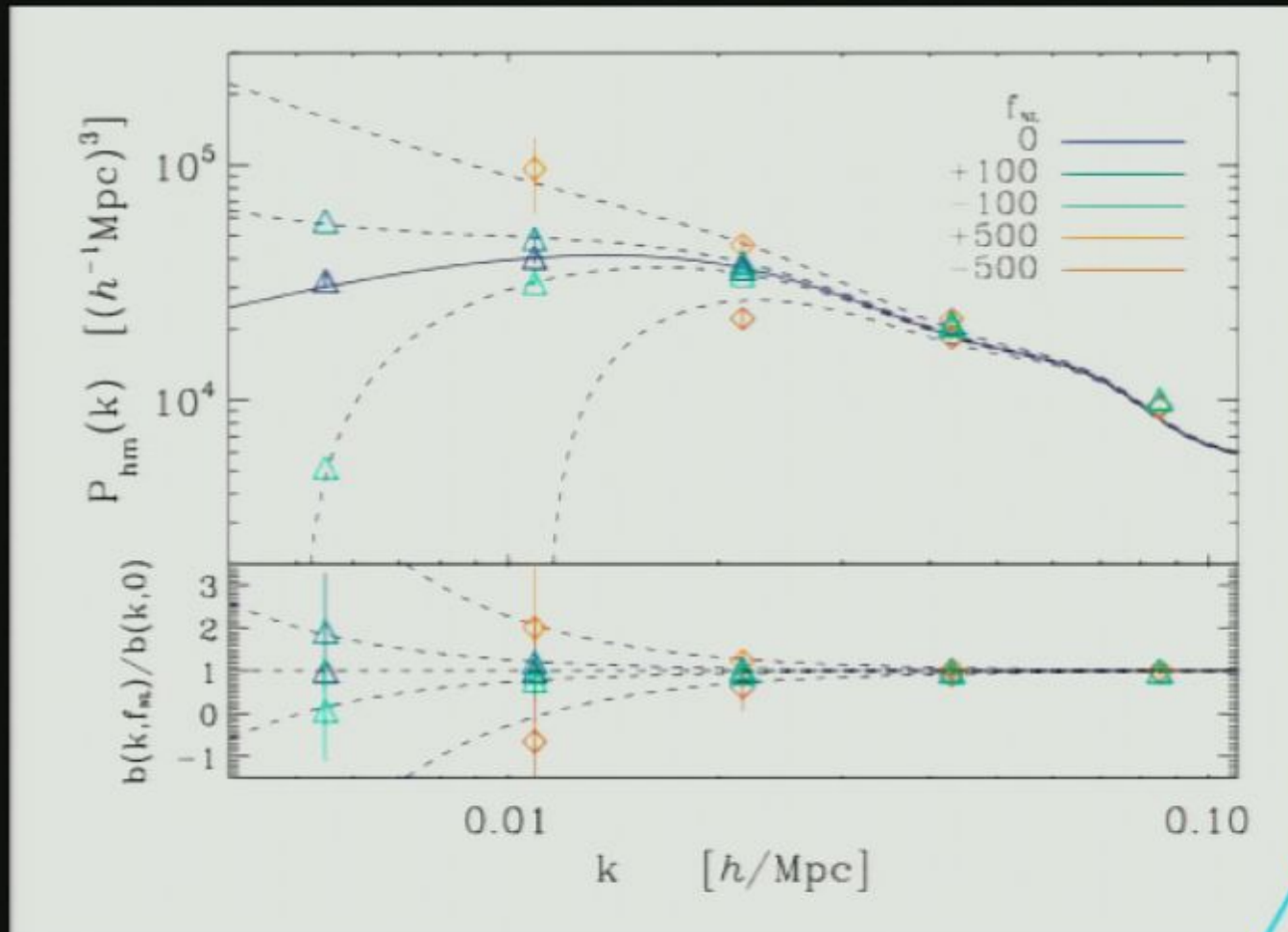
$$b \equiv \delta_h / \delta_{\text{DM}}$$



Seljak & Warren 2004

Simulations and theory both say: large-scale bias is scale-independent
(theorem if halo abundance is function of **local** density)

Scale dependence of NG halo bias!



$$b(k) = b_G + f_{\text{NL}} \frac{\text{const}}{k^2}$$

Halo clustering with NG: Analytic confirmation

Rigorous derivations exist, but here's back-of-envelope:

$$\Phi_{\text{NG}} = \phi + f_{\text{NL}}(\phi^2 - \langle \phi^2 \rangle)$$

Then, near the peaks of the potential

$$\begin{aligned}\nabla^2 \Phi_{\text{NG}} &= \nabla^2 \phi + 2f_{\text{NL}}(\phi \nabla^2 \phi + |\nabla \phi|^2) \\ &\approx \nabla^2 \phi (1 + 2f_{\text{NL}}\phi)\end{aligned}$$

And in particular

$$\delta_{\text{NG}} = \delta(1 + 2f_{\text{NL}}\phi)$$

Halo clustering with NG: Analytic confirmation

Definition of bias: $\delta_h = b_L \delta$

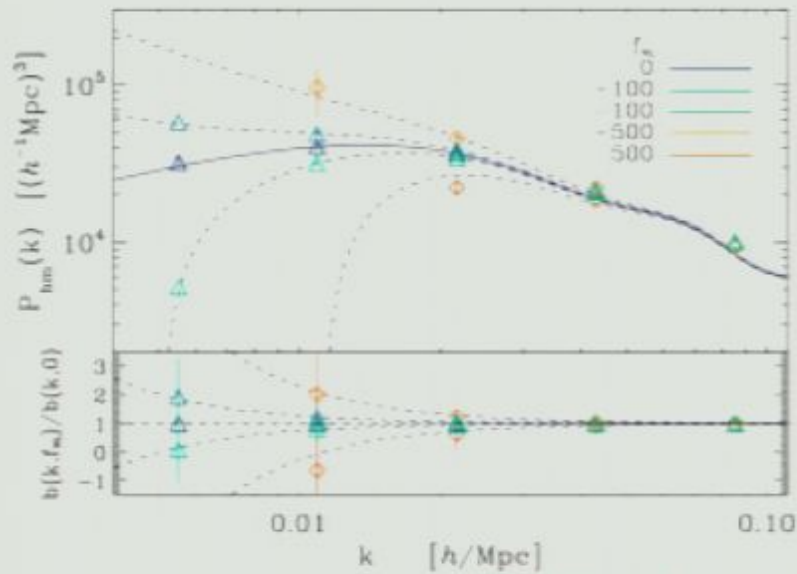
With NG, for peaks: $\delta \rightarrow \delta + 2f_{\text{NL}}\phi\delta$

Reinterpreting as change in bias $\delta_h = b_L (\delta + 2f_{\text{NL}}\phi\delta) = (b_L + \Delta b) \delta$

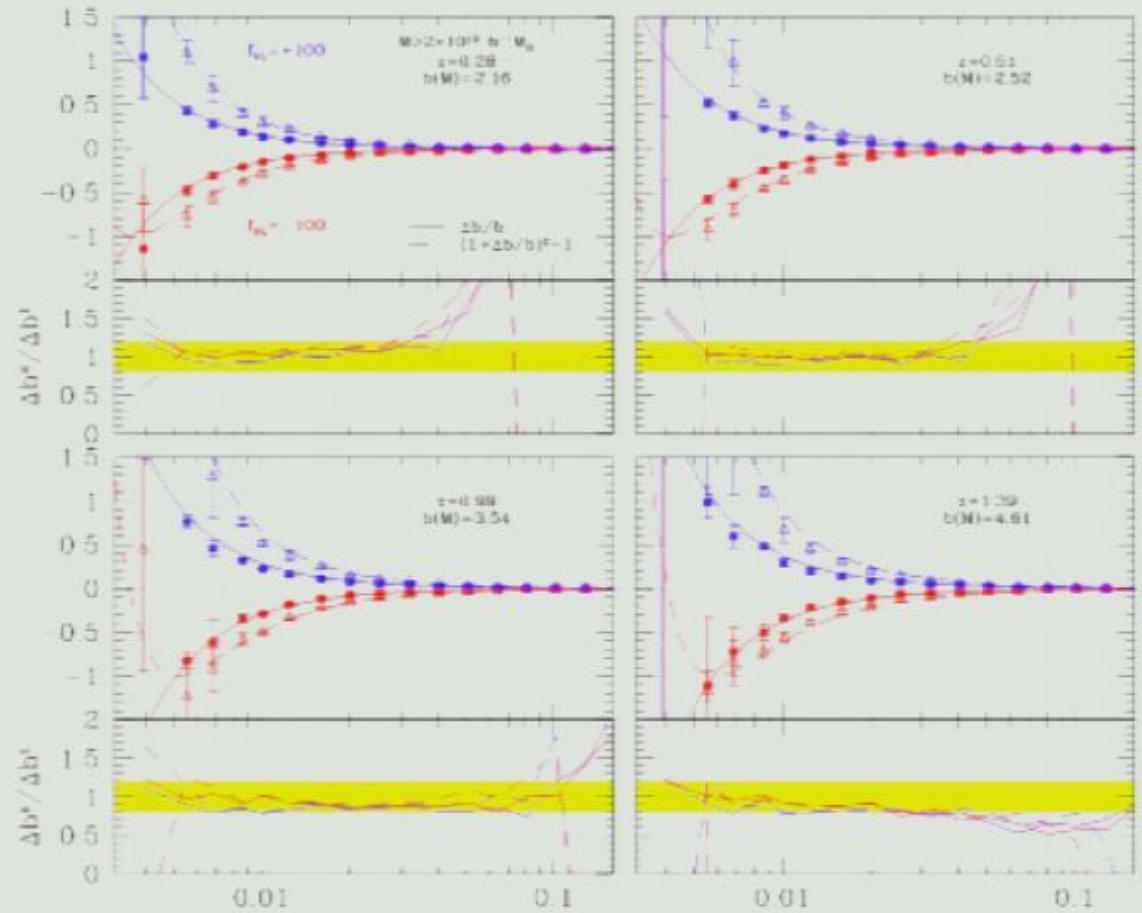
Using Poisson Eq. (to replace ϕ with δ) and including late-time perturbation evolution, you immediately get

$$\Delta b(k) = 2 b_L f_{\text{NL}} \delta_c \frac{3}{2} \frac{\Omega_M H_0^2}{T(k) D(a) k^2}$$

Analytic and numerical results agree



Dalal et al 2008

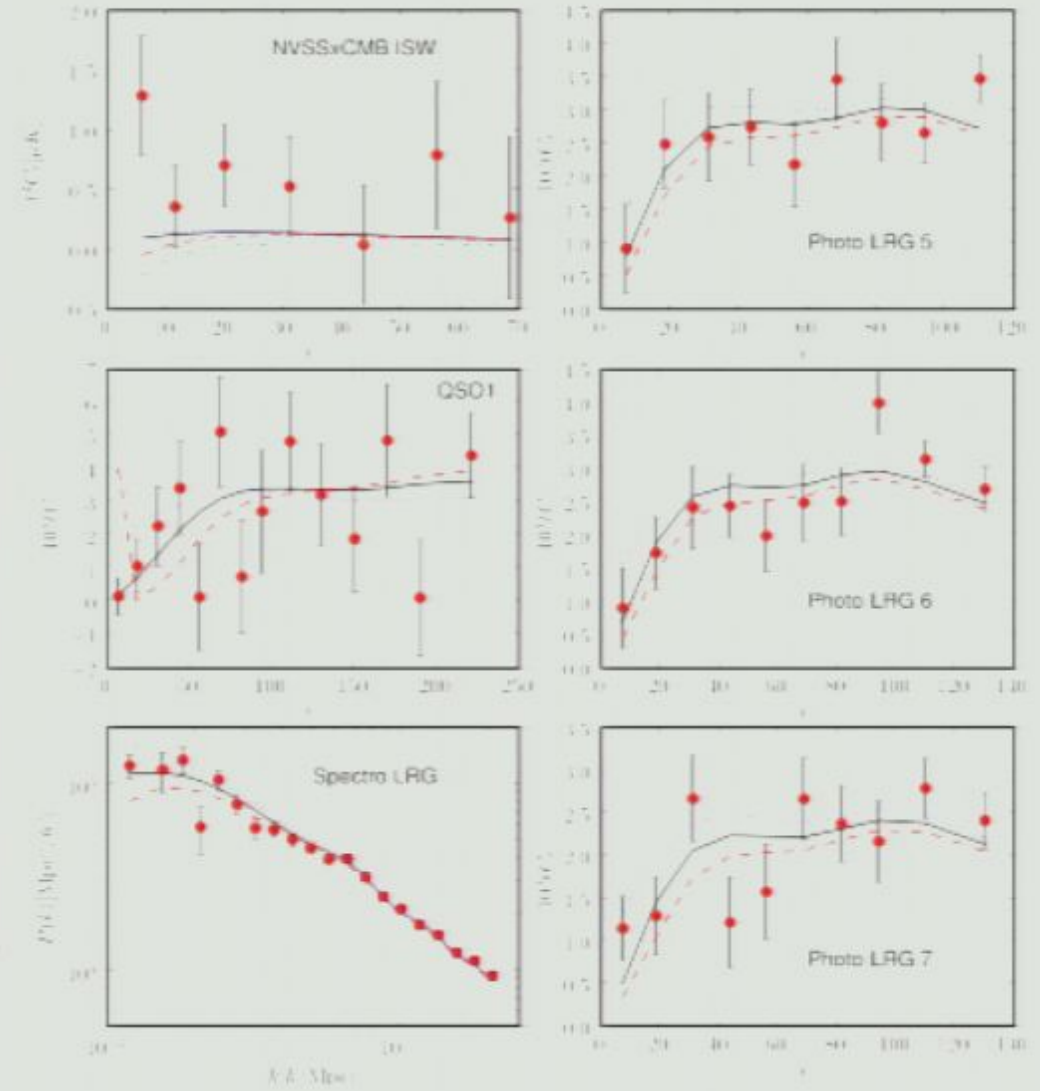
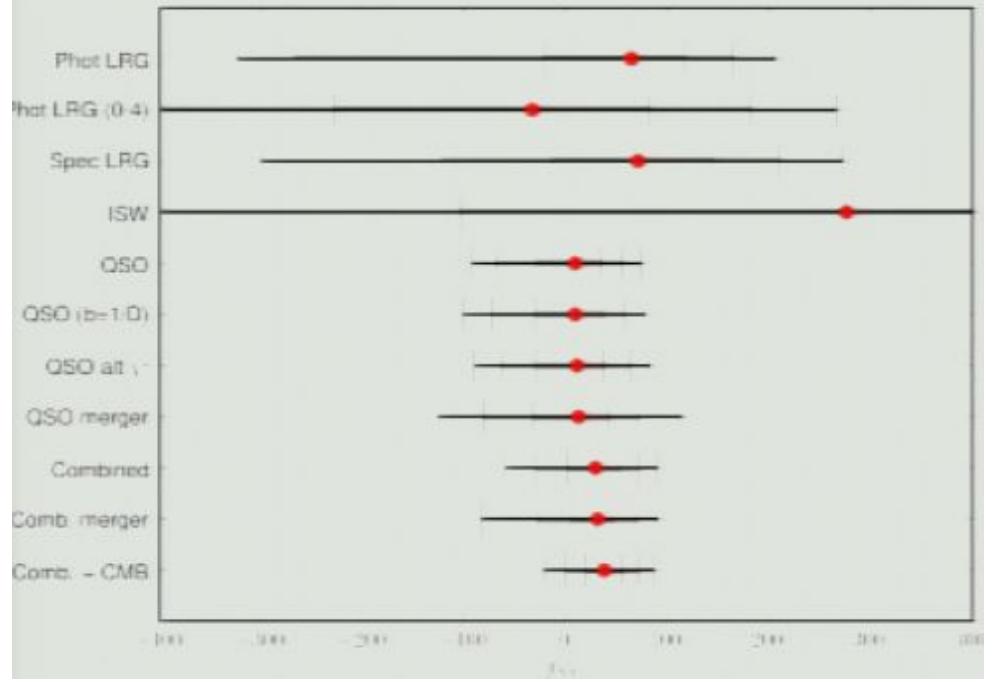


Desjacques, Seljak & Iliev 2009

$$\Delta b(k) = 2 b_L f_{\text{NL}} \delta_c \frac{3}{2} \frac{\Omega_M H_0^2}{T(k) D(a) k^2}$$

Constraints from the bias of DM halos

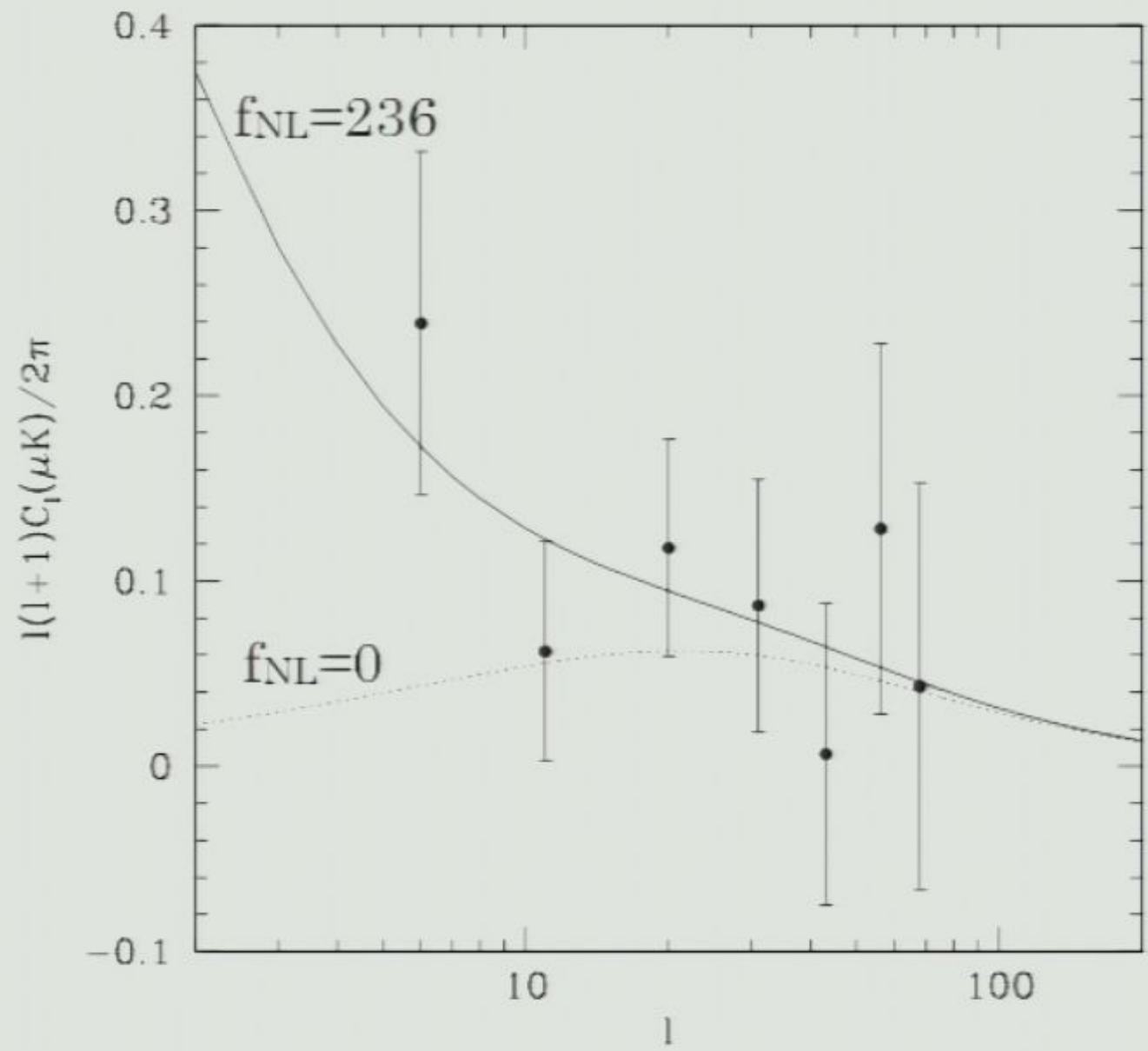
Constraints from *current* data - west coast team



$f_{NL} = 8 \pm 30$ (68%, QSO)

Pirsa: 10010002 $f_{NL} = 23 \pm 23$ (68%, all)

Constraints from *current* data - Canada team



Future NG from measurements of $b(k)$

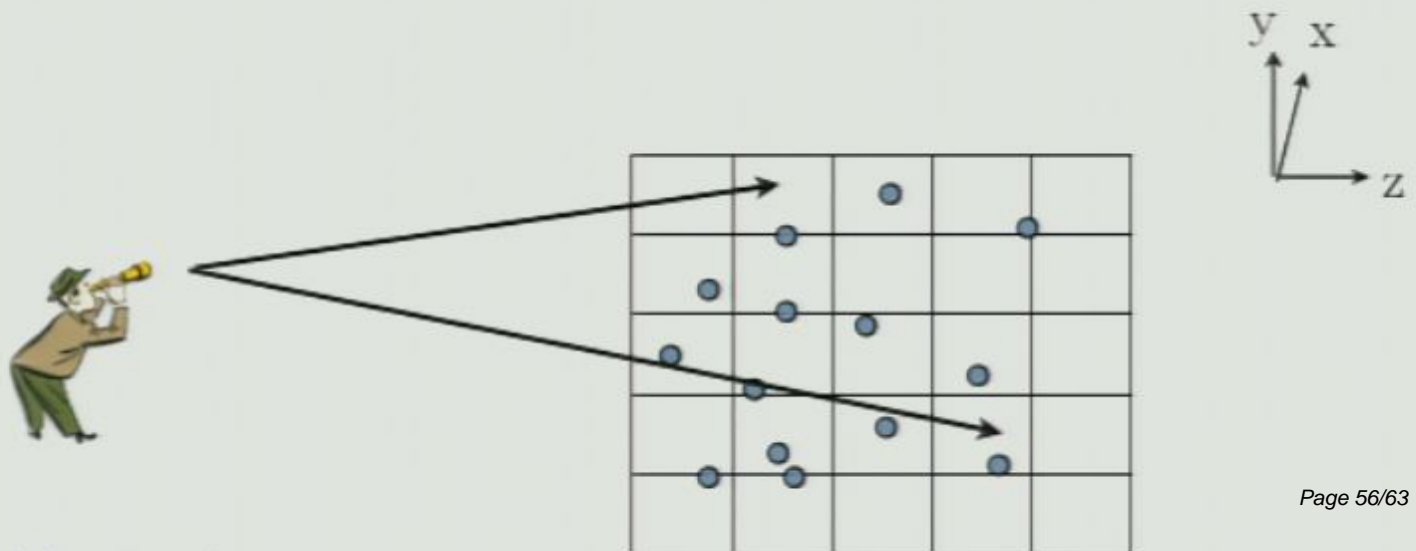
- Numerous cosmological probes, such as the baryon acoustic oscillations (BAO) or probes of Integrated Sachs-Wolfe effect (galaxy-CMB cross-corr) can be used to measure $b(k)$
- The effect (going as k^{-2}) provides a fairly unique signature and a clear target; **almost no degeneracy with other cosmological parameters**
- Expect accuracy of order $\sigma(f_{\text{NL}}) < 10$ or even ~ 1 in the future

TABLE 1
GALAXY SURVEYS CONSIDERED

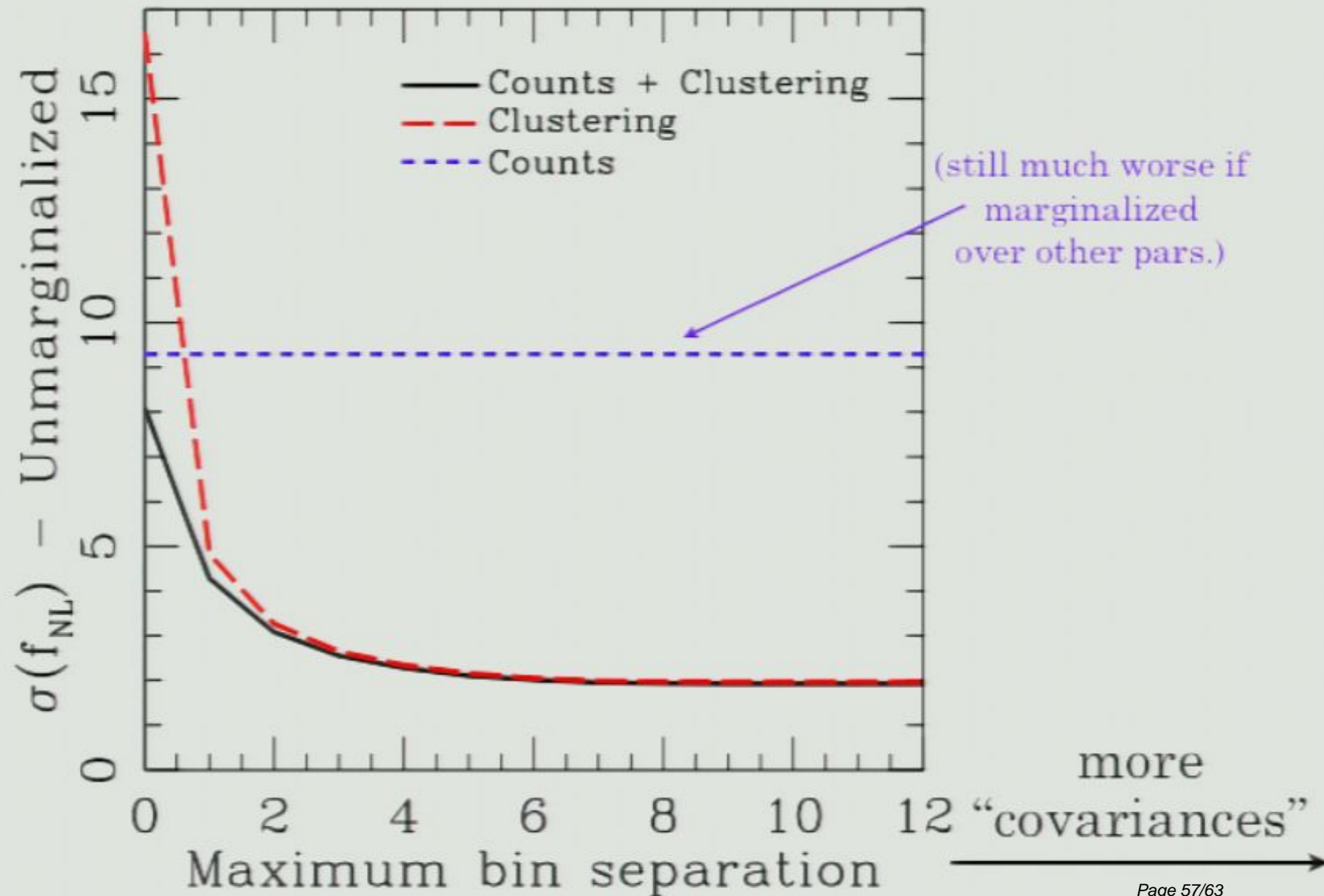
survey	z range	sq deg	mean galaxy density (h/Mpc) ³	$\Delta f_{\text{NL}}/q'$ LSS
SDSS LRG's	$0.16 < z < 0.47$	7.6×10^3	1.36×10^{-4}	40
BOSS	$0 < z < 0.7$	10^4	2.66×10^{-4}	18
WFMOS low z	$0.5 < z < 1.3$	2×10^3	4.88×10^{-4}	15
WFMOS high z	$2.3 < z < 3.3$	3×10^2	4.55×10^{-4}	17
ADEPT	$1 < z < 2$	2.8×10^4	9.37×10^{-4}	1.5
EUCLID	$0 < z < 2$	2×10^4	1.56×10^{-3}	1.7
DES	$0.2 < z < 1.3$	5×10^3	1.85×10^{-3}	8
PanSTARRS	$0 < z < 1.2$	3×10^4	1.72×10^{-3}	3.5
LSST	$0.3 < z < 3.6$	3×10^4	2.77×10^{-3}	0.7

Nongaussianity form galaxy clusters

- For dark energy/dark matter constraints, **counts** of clusters are most important; probe the volume and distance; $d^2N/dz d\Omega = r^2(z)/H(z)$
- For nongaussianity, **variance** of cluster counts improves constraints on NG by **an order of magnitude**, since it is proportional to (integral over) power spectrum $P_{\text{cluster}}(k) = b^2(k)P(k)$.
- However, we found that **covariance** of cluster counts improves those constraints by nearly **another order of magnitude**, since basically covariance you get correlation across large distances (or small k); recall, the effect goes as $b^2(k) \propto k^{-4}$

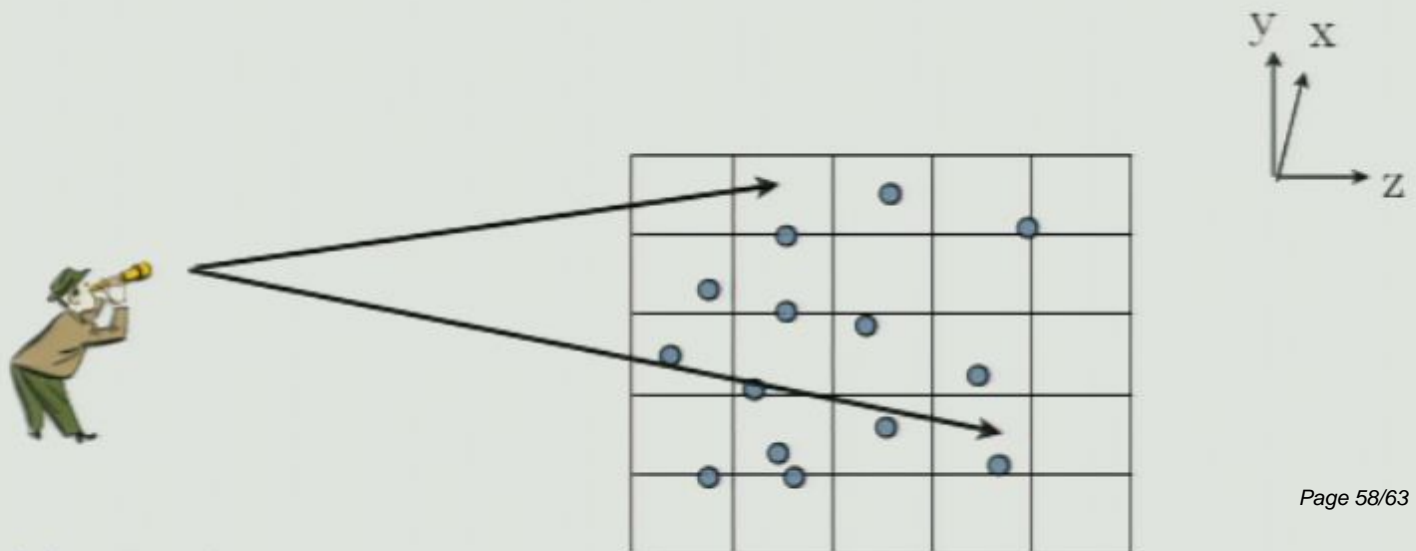


NG form **covariance** of galaxy clusters

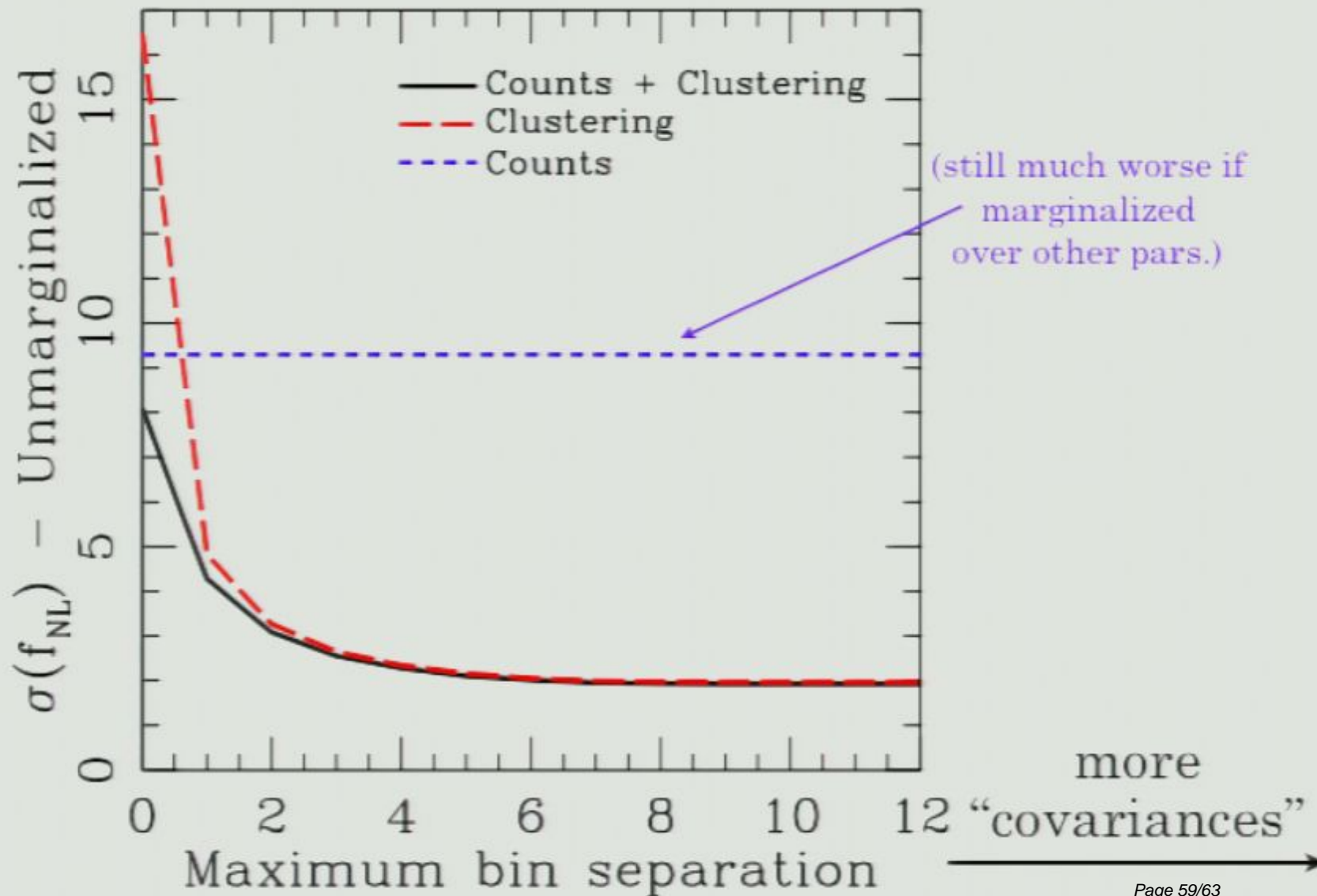


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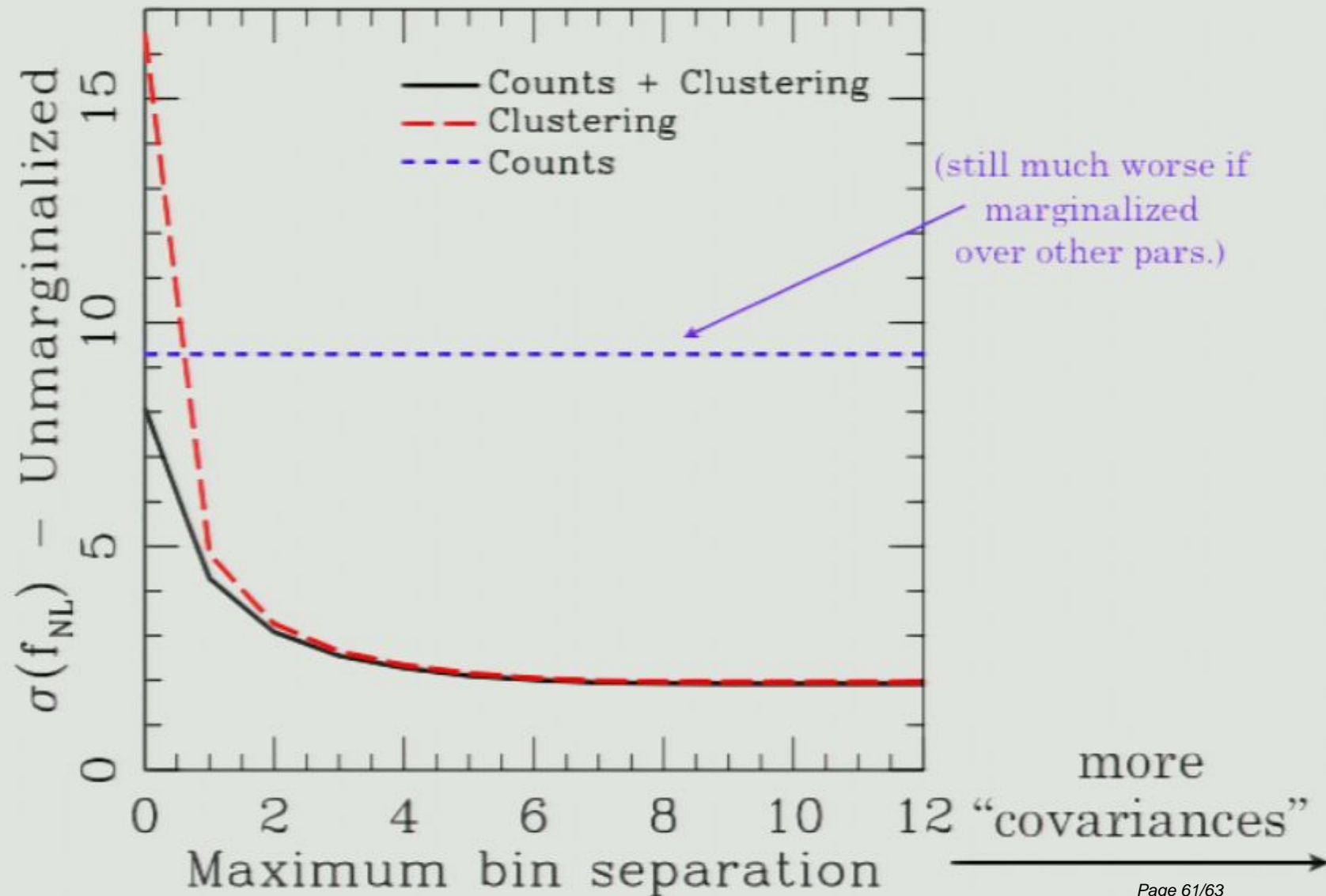
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NG form **covariance** of galaxy clusters



Open questions \Rightarrow current research

- What is the effect on large-scale structure (and bias) of **other models of nongaussianity**? (problem: NG = “non-dog”; could be anything)
- What **systematic** error control (e.g. atmospheric blurring; pixel response; astrophysical effects) is required **not to degrade** the projected future constraints?
- How best to design a survey to measure primordial nongaussianity?
- What galaxies and clusters are best to use (mass, color, etc)?
- Do you still get the same theoretical $b(k)$ result as $k^{-1} \rightarrow H_0^{-1}$?
- What about the f_{NL} model where **f_{NL} depends on scale**, $f_{NL}=f_{NL}(k)$?

Conclusions

- Searching for primordial nongaussianity is one of the most fundamental tests of the early universe cosmology
- CMB bispectrum traditionally most promising tool; current results favor $f_{\text{NL}} > 0$ but only at 1-2 sigma
- Mass function of cluster counts is in principle sensitive to NG, but not competitive with the CMB
- **Cosmological models with (local) primordial NG lead to significant scale dependence of halo bias; theory and simulations appear to be in remarkable agreement on this**
- Therefore, LSS probes (baryon oscillations, galaxy-CMB cross-correlations, etc) are likely to lead to constraints on NG **nearly 2 orders of magnitude stronger** than previously thought
- **$\sigma(f_{\text{NL}})$ ~ few expected from future LSS surveys (DES, LSST, JDEM etc)**