Title: Negative Energy and theGeneralized Second Law

Date: Dec 10, 2009 03:00 PM

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Abstract: In quantum field theory it is possible to create negative local energy densities. This would violate the Generalized Second Law (GSL) unless there is some sort of energy condition requiring the negative energy to be counterbalanced by positive energy. TO explore what this energy condition is, I will assume that the GSL holds in semiclassical gravity for all future causal horizons. From CPT symmetry it follows that the time-reverse of the GSL, properly understood, holds for all past causal horizons. These two conditions together then imply that the Averaged Null Energy Condition (ANEC) holds on any null line, i.e. a complete achronal lightlike null geodesic. In curved spacetimes, the ANEC can be violated on general geodesics. But even if the ANEC only holds on null lines, theorems by Sorkin, Penrose and Woolgar, and by Graham and Olum imply that semiclassical gravity should satisfy positivity of energy, topological censorship, and should not admit closed timelike curves. These results can thus be seen as consequences of the GSL. However, these theorems don't apply when gravitational fluctuations are taken into account. In that case, the GSL argument suggests a modification to the ANEC which may make these theorems applicable to perturbative quantum gravity.

Negative Energy

and

The Generalized Second Law

by Aron Wall

based on:

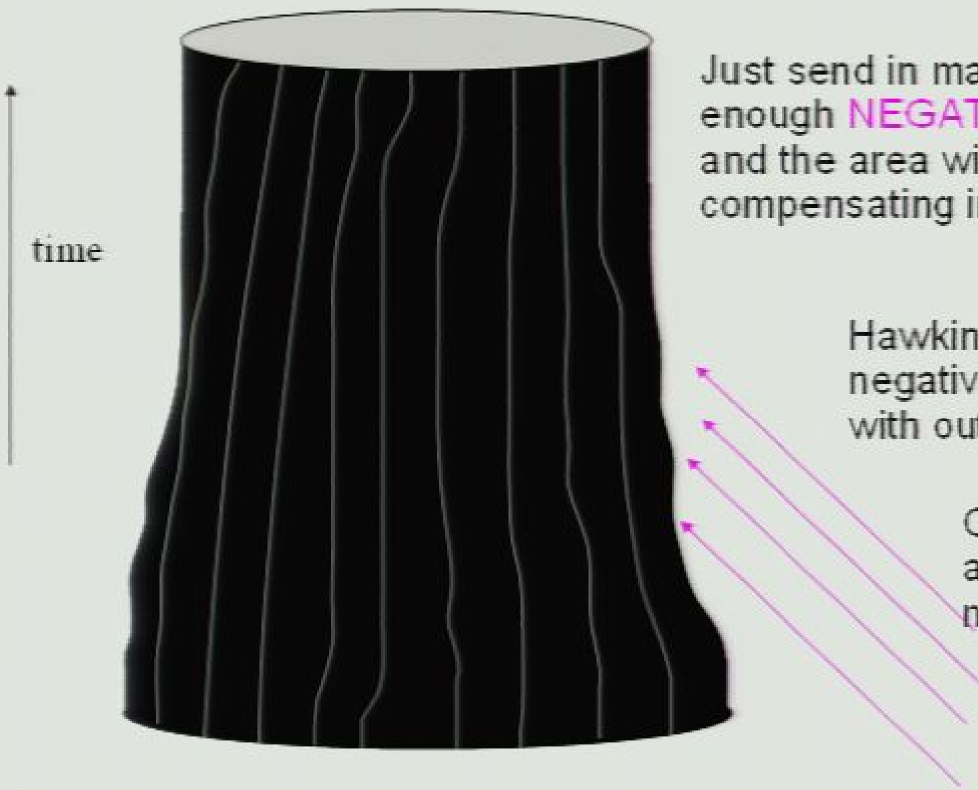
"Proving the Achronal Averaged Null Energy Condition from the Generalized Second Law", arXiv:0910.5751

Outline of Talk

1. Motivating the achronal Averaged Null Energy Condition (ANEC)
2. Defining the Generalized Second Law (GSL)
3. Proving the achronal ANEC from the GSL
4. Taking gravitational fluctuations into account

Why we would like to believe in the Average Null Energy Condition

Suppose we have access to arbitrary negative energies.
Then violating the Generalized Second Law (GSL) is easy.



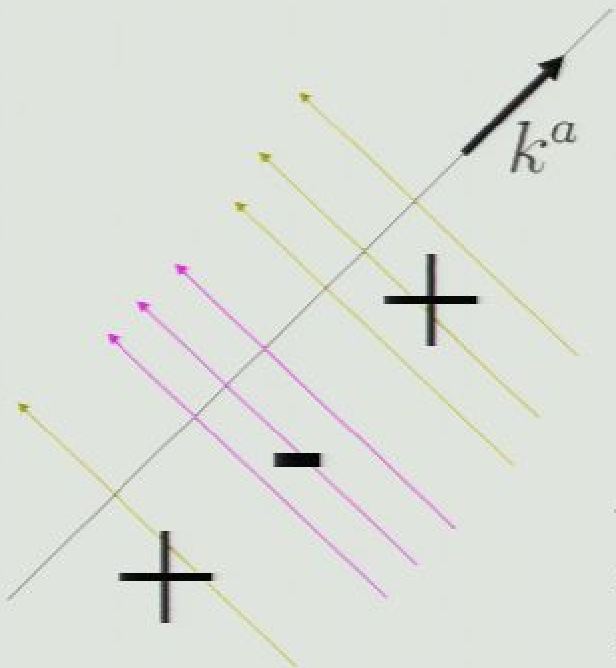
Just send in matter which contains enough NEGATIVE ENERGY and the area will shrink without any compensating increase of entropy outside

Hawking radiation also has negative energy but it is associated with outgoing entropy.

Other forms of negative energy also exist in QFT—e.g. moving mirrors/squeezed states.

What energy condition is required for black hole thermodynamics?

The Averaged Null Energy Condition (ANEC)



$$\int_{-\infty}^{\infty} T_{ab} k^a k^b d\lambda \geq 0$$

k^a —a covariantly constant tangent vector

λ —an affine parameter

The integral of the null-null energy over an entire null geodesic must be nonnegative.

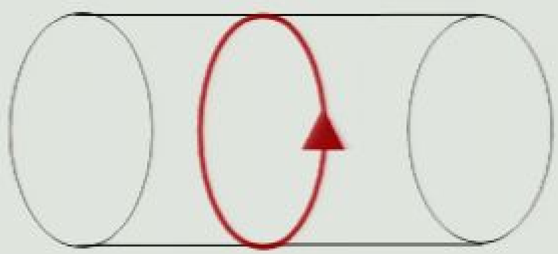
In Minkowski space, proven for:

1) free scalars in n dimensions by Klinkhammer (1991)
2) free electromagnetism in 4 dimensions by Folacci (1992)
3) all theories with a mass gap in 2 dimensions by Verch (2000)

But the ANEC does NOT hold generally in curved spacetimes.

Two counterexamples:

1. Compactify a dimension



Casimir effect can make the null energy in the compact direction negative.

Null ray goes round and round.

shown by G. Klinkhammer (1991)

Boulware state of black hole, obtained by removing all Hawking quanta

Every null geodesic on this spacetime violates the ANEC

State not regular on horizon, but shouldn't matter for null geodesics outside.

shown by M. Visser (1996)

Both these examples have in common that the null geodesic is *chronal*

Null Lines

A *chronal* null geodesic is one which has a timelike curve connecting two of its points.

On a typical curved spacetime, most geodesics will be chronal.

An *achronal* null geodesic is one that is not chronal, i.e. it goes “faster” than any timelike curve.
A null line is a *complete* achronal null geodesic.

Graham and Olum (2007) proposed that the ANEC should hold on null lines for any self-consistent semiclassical state.

I will show this is true perturbatively about a dozen slides from now.

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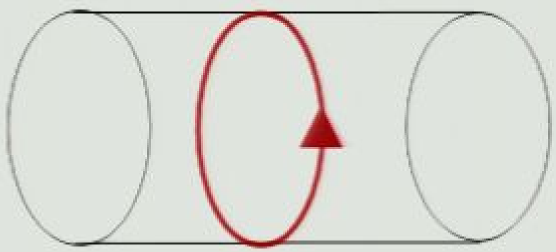

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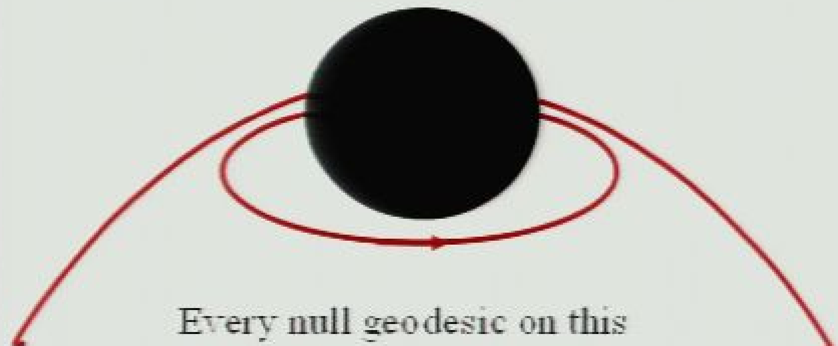

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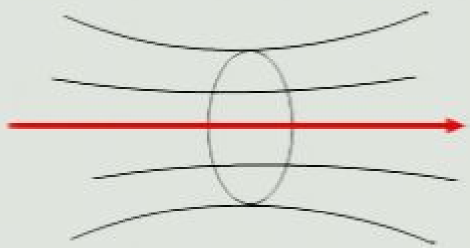
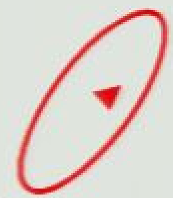
Several pathologies require generic null lines to exist

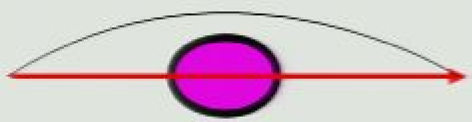
Assume that spacetime is asymptotically flat and null geodesically complete in the relevant regions. Then spacetimes with:



Traversable wormholes,

Closed timelike curves,





or negative ADM mass

all generically have a "fastest possible" light curve which goes through the wormhole/timelike loop/negative energy & which is therefore a null line.

BUT, by the focusing thorem of Borde (1987), any null geodesic satisfying the ANEC and the "generic condition" must have conjugate points, and thus is NOT a fastest possible light curve and NOT a null line.

So the "achronal ANEC" rules these out.

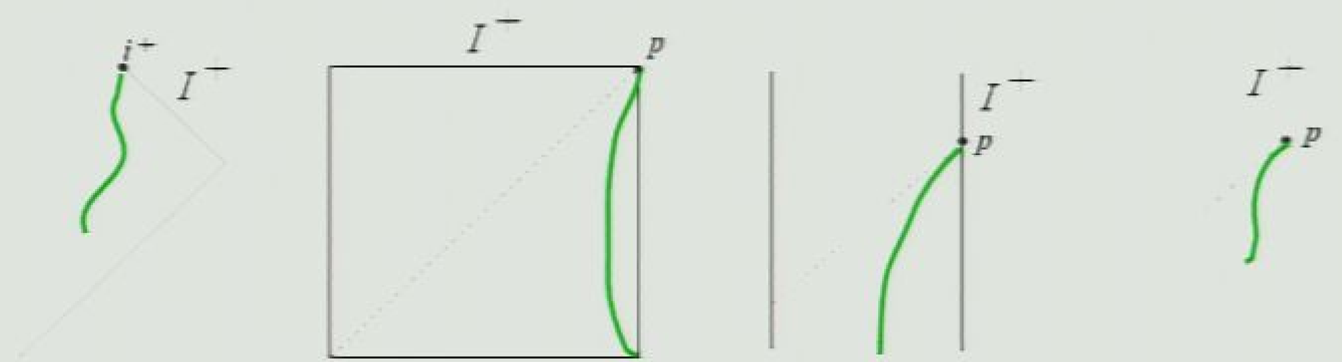
Graham and Olum (2007)
Penrose, Sorkin, and Woolgar (1993)

What does the GSL really say?

CAUSAL HORIZONS

$$\frac{\partial}{\partial t}\left(\frac{A}{4G\hbar} + S_{\text{out}}\right)\Big|_{horizon} \geq 0$$

A future "causal horizon" means the boundary of the past of any future-infinite worldline (shown in **GREEN**); i.e. an "observer".



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"Black hole thermodynamics" isn't just about black holes!

See Jacobson (1999) for review and discussion.

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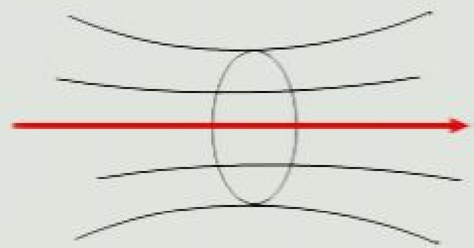

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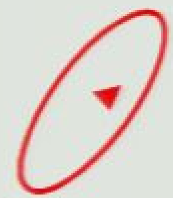

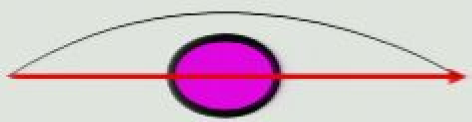

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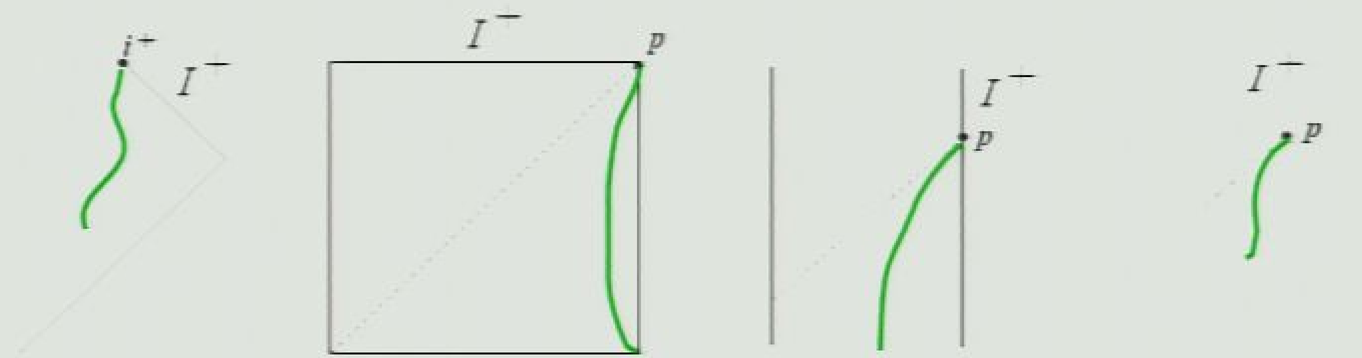

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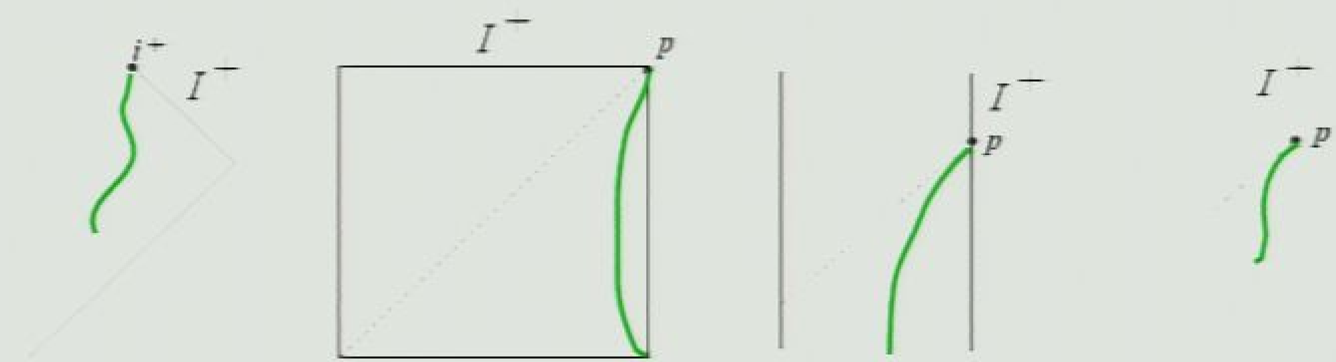

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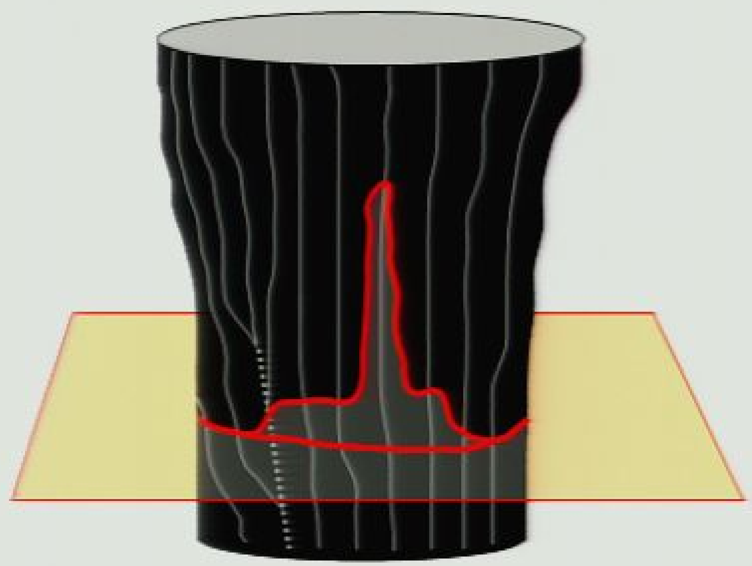
TIME EVOLUTION

$$\frac{\partial}{\partial t}\left(\frac{A}{4G\hbar} + S_{\text{out}}\right)\Big|_{horizon} \geq 0$$

By time evolution I mean an arbitrarily wiggly way of pushing a time slice forward in time along the horizon.



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By entropy “outside” the horizon I mean the von Neumann entropy of the spatial slice restricted to the observer's side of the horizon:

$$S_{\text{out}} = -\text{tr}(\rho \ln \rho)$$

EXCEPT that this quantity is actually ill-defined due to the divergent ultraviolet entanglement entropy due to quantum fields.

outside

inside

For now let's just pretend we have a well-defined renormalization scheme; I'll come back to this later.

THE AREA TERM

$$\frac{\partial}{\partial t}\left(\frac{A}{4G\hbar} + S_{\text{out}}\right)\Big|_{horizon} \geq 0$$

Von Neumann entropy S_{out} is a c-number,
area should also be a c-number.

Should use the expectation value of the area $\langle A \rangle$
as argued by Sorkin & Sudarsky (1999).

Advantages:

1. Don't have to worry about area fluctuations,
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   $8\pi G\langle T_{ab}\rangle = \langle R_{ab} - (1/2)g_{ab}R\rangle$

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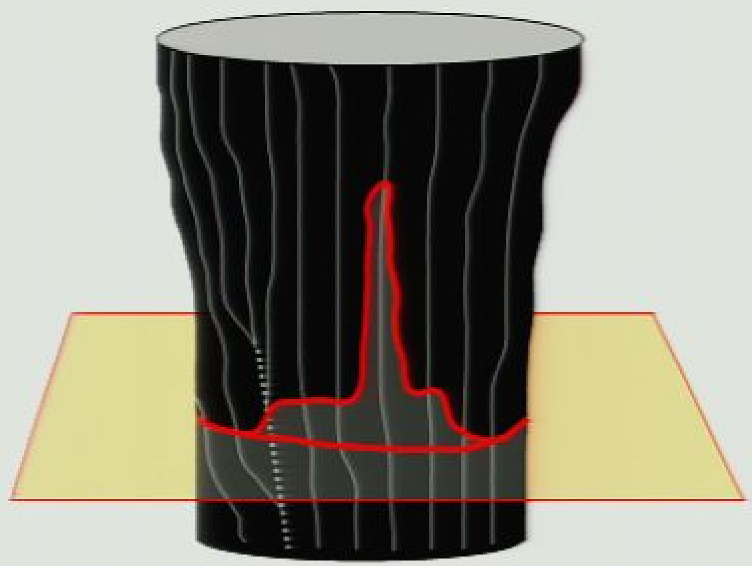

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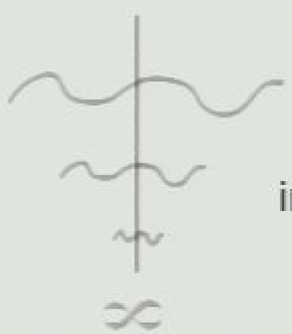

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The Anti-GSL

* Generalized entropy of *future* causal horizons cannot *decrease*,

* CPT symmetry (C and P are irrelevant)

* Generalized entropy of *past* causal horizons cannot *increase*.
(e.g. white holes)

? Huh? Isn't the whole point of the Second Law of Thermodynamics that it only holds in one direction?

Unlike the ordinary second law, the GSL is a time asymmetric statement.
The only form of coarse graining is the restriction to outside the horizon.
With no horizons, the fine-grained entropy neither increases nor decreases.

"Objective" coarse graining argued for in Sorkin (2005).

Thus it is not a contradiction to assume the anti-GSL.
If the GSL is always true, so is the anti-GSL.

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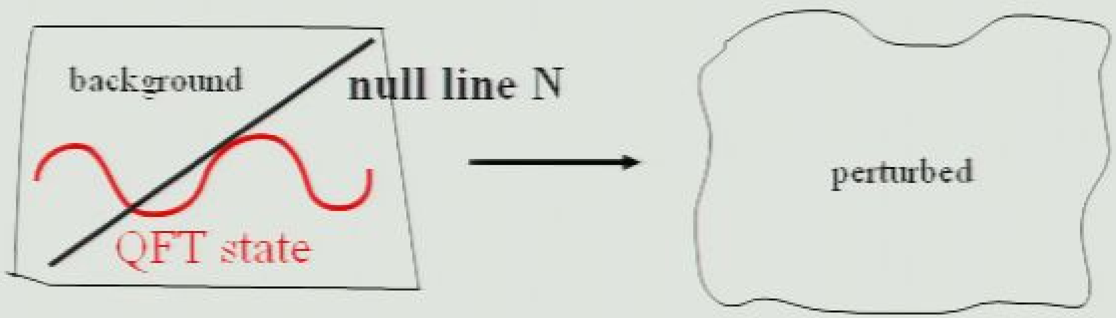
The GSL and anti-GSL
imply the Achronal ANEC
(semiclassically)

Gravitational Perturbation Theory

Do an expansion in $\hbar$. Set G = c = 1.

Expand metric as $g_{ab} = \overset{0}{g}_{ab} + \overset{1/2}{g}_{ab} + \overset{1}{g}_{ab} + \mathcal{O}(\hbar^{3/2})$
& impose the Einstein equation order by order.



* Zeroth order metric is some classical background metric with a null line.
Assume the background obeys the null curvature condition $\overset{0}{R}_{ab} k^a k^b = 0$.

* Half order metric perturbation comes from graviton fluctuations.
Ignored semiclassically. Justified only with large number of species.

* First order metric represents gravitational effects of quantum fields on the classical background. Assume this is a small perturbation.

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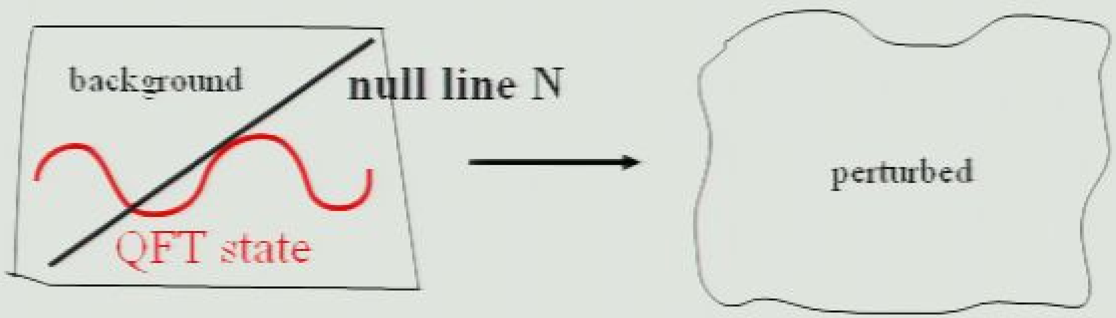

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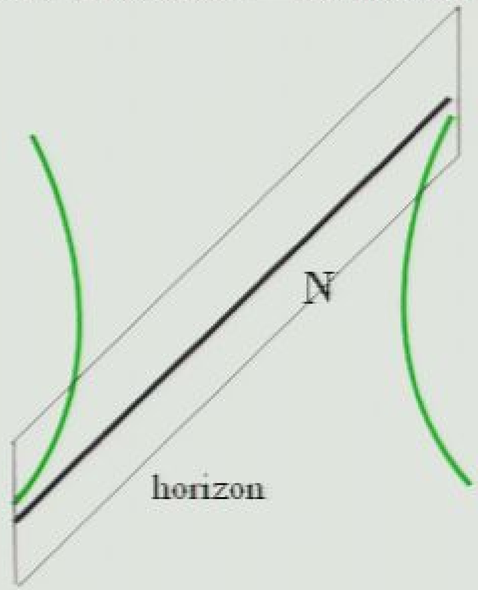
The Classical Background

Every null geodesic generates a past and a future horizon by thinking of the null geodesic itself as an "observer"—same horizon seen by accelerating observers.

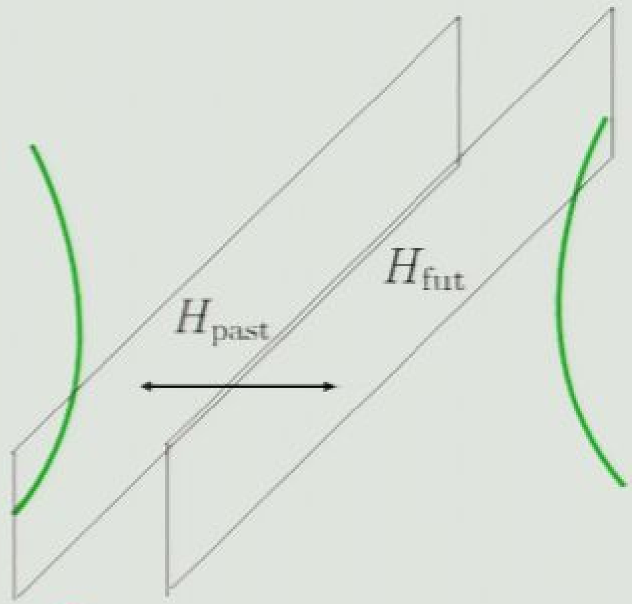


N is achronal so it lies on its past and future horizons

The past and future horizons coincide and are stationary
(i.e. no expansion or shear)
on any background satisfying the null energy condition
& a form of cosmic censorship (Galloway 2000)

Perturbed Spacetime

Past and future horizons split at first order in $\hbar$



asymptotic observers unaffected by sufficiently local perturbation

$$\frac{A}{4\hbar} + S_{\text{out}} \longrightarrow T_{ab}$$

$\hbar^1$ perturbed metric

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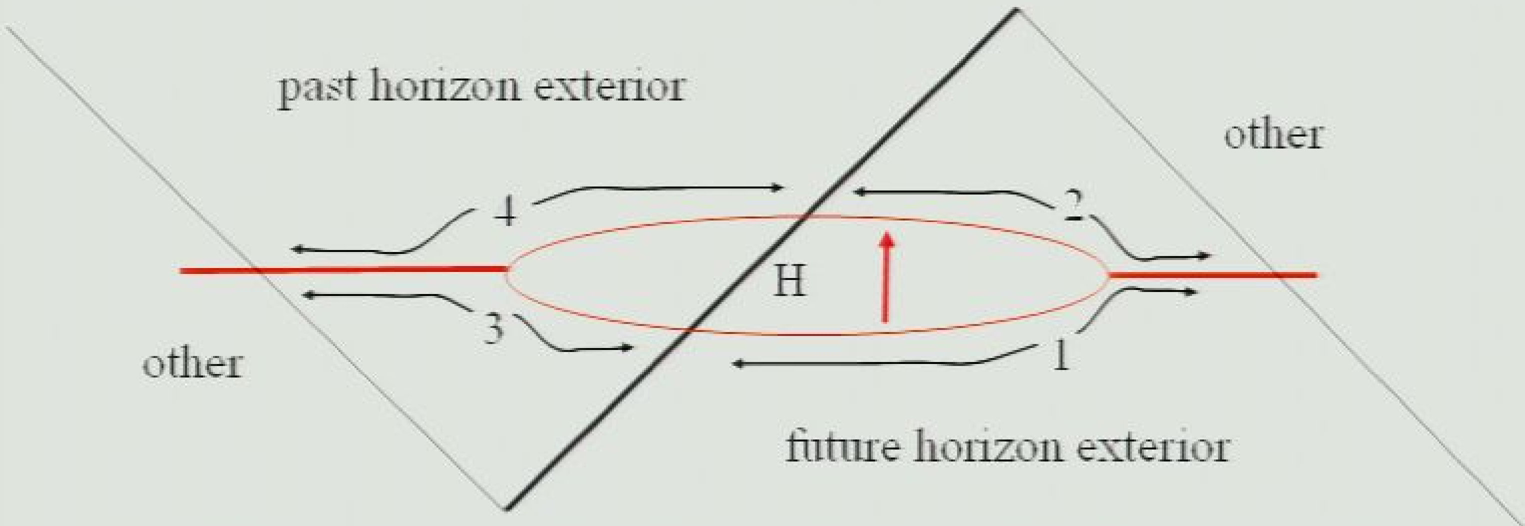
properties of quantum fields on background

metric perturbation only affects Bekenstein-Hawking term due to the $\hbar$ in the denominator

causal diagram of the proof



diffeomorphism ambiguities from identifying perturbed/background manifolds are higher order in $\hbar$

GSL: $S_2 + A_2/4\hbar \geq S_1 + A_1/4\hbar$

anti-GSL: $-S_4 - A_4/4\hbar \geq -S_3 - A_3/4\hbar$

weak monotonicity: $S_1 + S_4 \geq S_2 + S_3$ (follows from strong subadditivity)

Therefore $A_2 - A_1 \geq A_4 - A_3$, which implies $\theta_{\rm fut} \geq \theta_{\rm past}$

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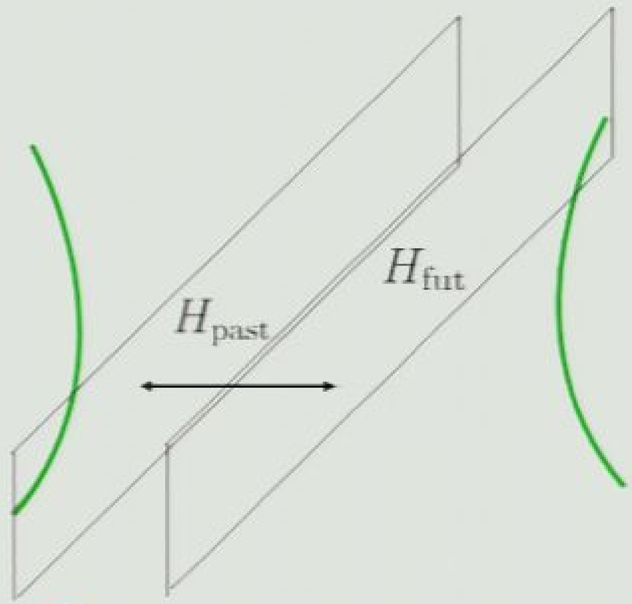

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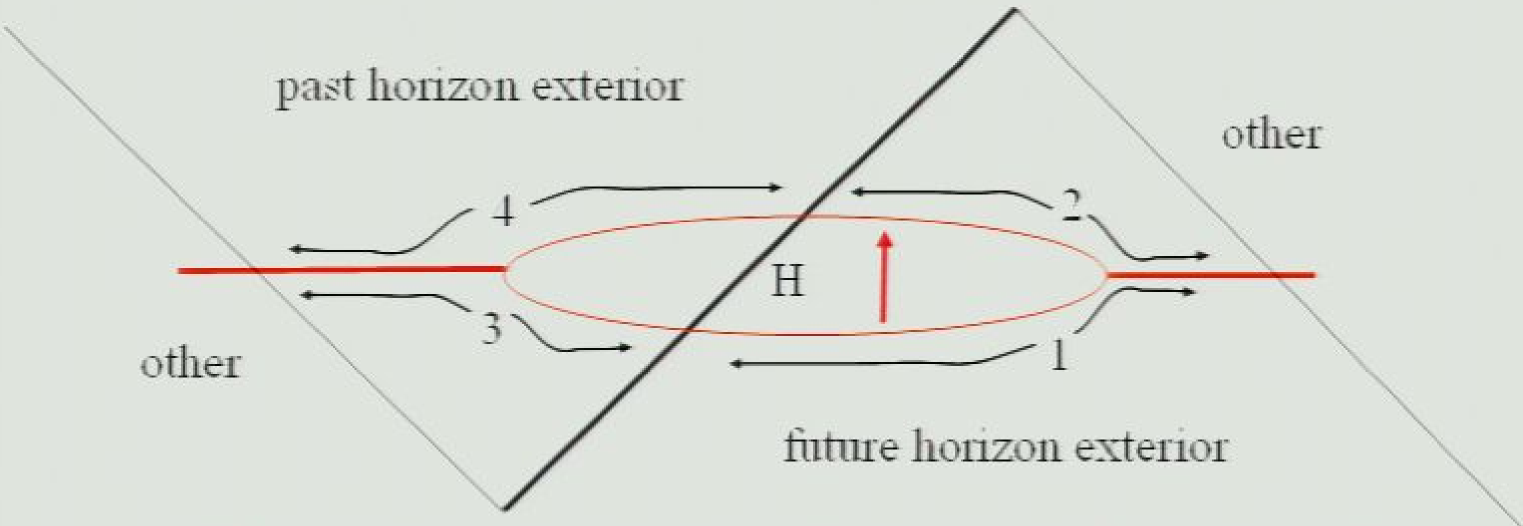

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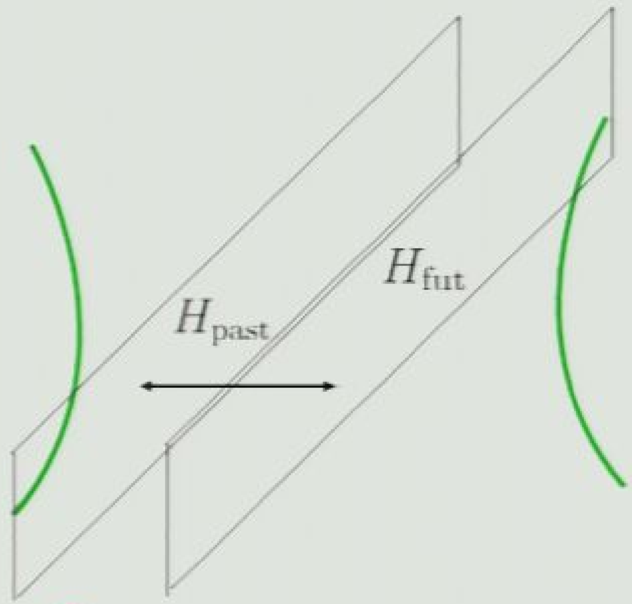

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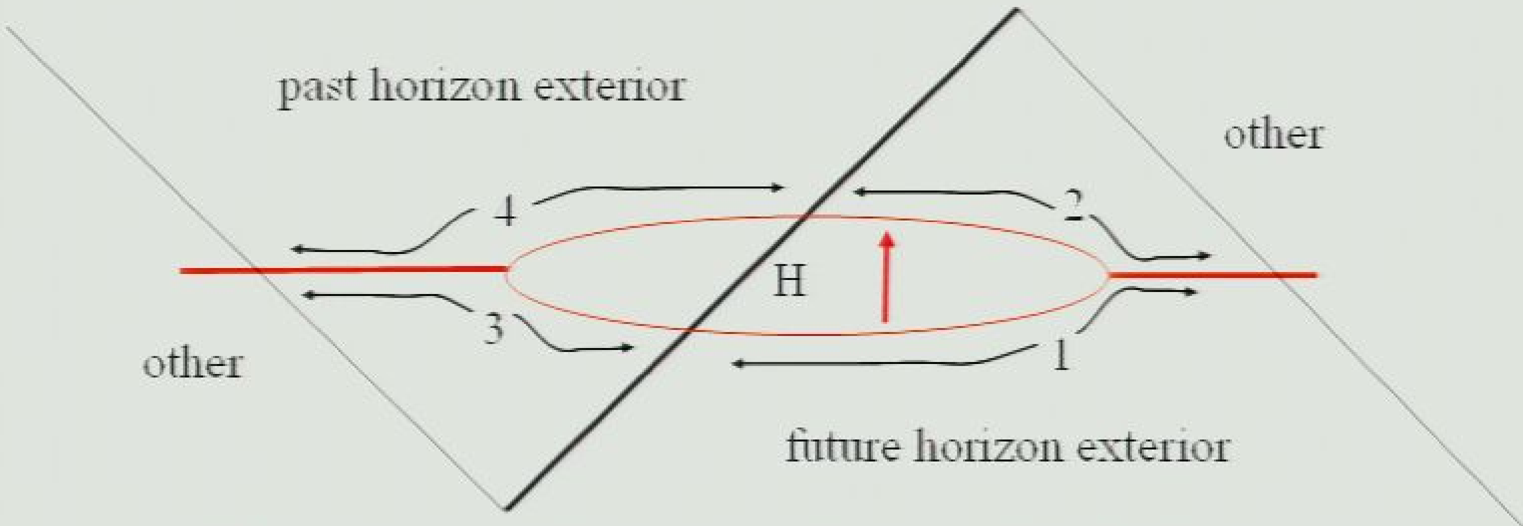

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So at any point X on the null line,
the first order perturbation to the metric satisfies:

$$\theta_{\rm fut} \geq \theta_{\rm past} \qquad\qquad \theta = \frac{1}{A}\frac{dA}{d\lambda}$$

Integrate the linearized Raychaudhuri equation

$$\frac{d\theta}{d\lambda} = -8\pi T_{ab}k^a k^b$$

using appropriate boundary conditions for the horizons:

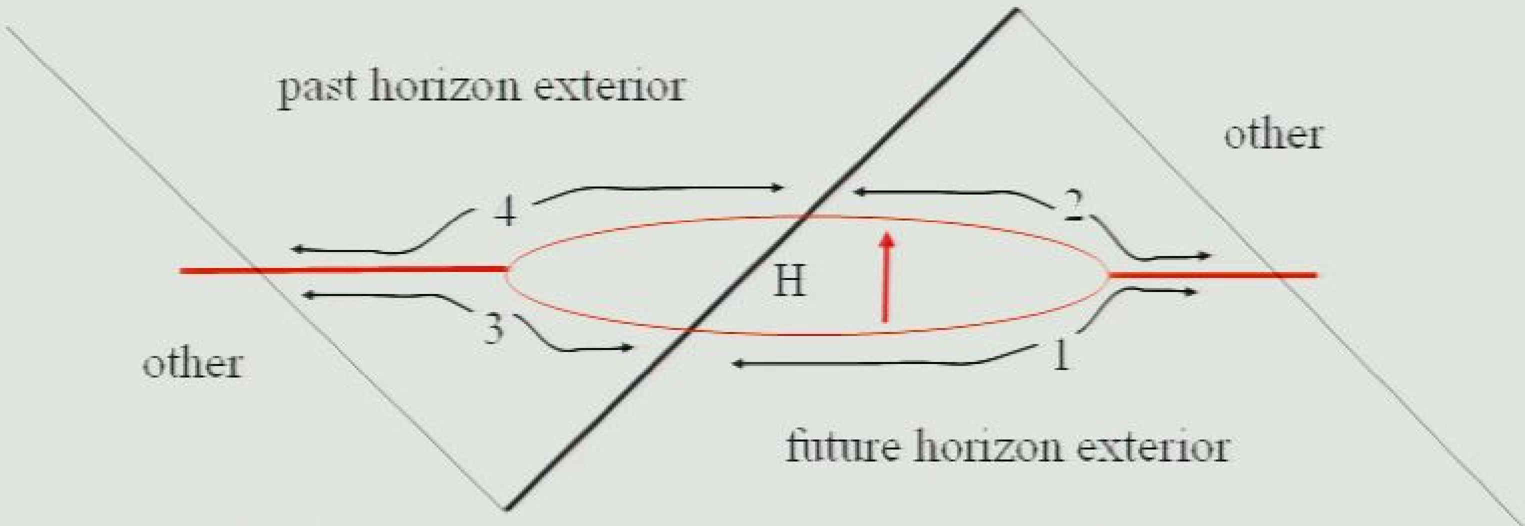
$$\theta_{\rm fut}|_{\infty} = \theta_{\rm past}|_{-\infty} = 0$$

and obtain

$$\theta_{\rm fut} = \int_X^{\infty} T_{ab}k^a k^b d\lambda \quad \& \quad \theta_{\rm past} = -\int_{-\infty}^{X} T_{ab}k^a k^b d\lambda$$

proving the ANEC holds on the null line.

causal diagram of the proof



diffeomorphism ambiguities from identifying perturbed/background manifolds are higher order in $\hbar$

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Therefore $A_2 - A_1 \geq A_4 - A_3$, which implies $\theta_{\text{fut}} \geq \theta_{\text{past}}$

So at any point X on the null line,
the first order perturbation to the metric satisfies:

$$\theta_{\text{fut}} \geq \theta_{\text{past}} \qquad\qquad \theta = \frac{1}{A}\frac{dA}{d\lambda}$$

Integrate the linearized Raychaudhuri equation

$$\frac{d\theta}{d\lambda} = -8\pi T_{ab}k^a k^b$$

using appropriate boundary conditions for the horizons:

$$\theta_{\text{fut}}|_{\infty} = \theta_{\text{past}}|_{-\infty} = 0$$

and obtain

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Gravitational Fluctuations

Limitations of the preceding result

1. Renormalization of $S_{\rm out}$ has been ignored. Renormalization OK if
A) regulated states satisfy weak monotonicity, and
B) the divergences must be associated with connected components of region boundaries, equal on both sides of the boundary.

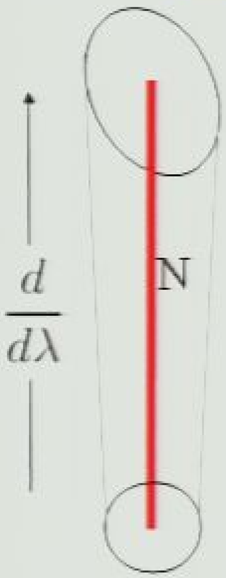
2. Requires minimal coupling to Einstein gravity, or extra terms in
a) Einstein's equation,
b) the horizon entropy, & possibly
c) the ANEC integral itself.

3. Gravitational fluctuations neglected. But these can be important if e.g. a black hole Hawking radiates gravitons.

What happens when gravitons are included?

Raychaudhuri equation:

$$\frac{d\theta}{d\lambda} = -\tfrac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} - R_{ab}k^a k^b$$



σ_{ab} shear tensor gives rate at which circle deforms into ellipse with respect to first order change in λ

When the gravitons are quantized, σ_{ab} is order $\hbar^{1/2}$, so $\sigma_{ab}\sigma^{ab}$ is order $\hbar$. (θ^2 is still negligible).

Renormalized $\sigma_{ab}\sigma^{ab}$ can be *negative*
Candelas & Sciama (1977).

Borde's focussing result doesn't apply! To use focusing to prove theorems, need

$$\int_{-\infty}^{\infty} (T_{ab}k^a k^b + \frac{1}{8\pi G}\sigma_{ab}\sigma^{ab})\, d\lambda > 0$$ in generic states.

Can this shear-inclusive ANEC be proven by the same method?

Same argument gives $\theta_{\rm fut} \geq \theta_{\rm past}$,
but now integrating Raychaudhuri gives

$$\theta_{\rm fut} = \int_X^{\infty} (8\pi G T_{ab} k^a k^b + \sigma_{ab}\sigma^{ab}|_{\rm fut})\, d\lambda$$

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To get the shear-inclusive ANEC, need to assume $\sigma_{ab}|_{past} = \sigma_{ab}|_{fut}$ holds at $\hbar^{1/2}$ order.

True for nonextremal black holes/pp-wave spacetimes. General proof?

This gives $\displaystyle\int_{-\infty}^{\infty} (T_{ab} k^a k^b + \frac{1}{8\pi G}\sigma_{ab}\sigma^{ab})\, d\lambda \geq 0$ shear-inclusive ANEC

Inequality should only be saturated when GSL itself is saturated—not generic.

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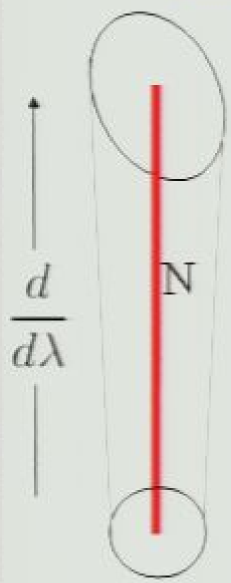

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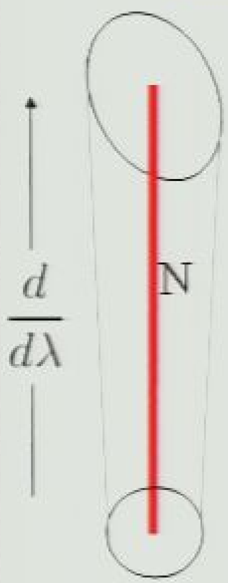

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Conclusions

1. If the GSL is true (and CPT, cosmic censorship, etc., then the ANEC holds on null lines perturbatively.

2. Thus the GSL prohibits negative mass objects, traversable wormholes and closed timelike curves.

3. In order to be useful as an energy condition, a shear-squared term must be added to the ANEC when gravitational fluctuations are considered.

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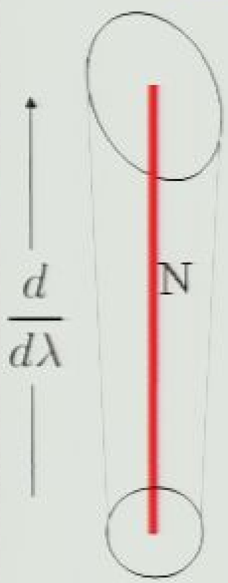

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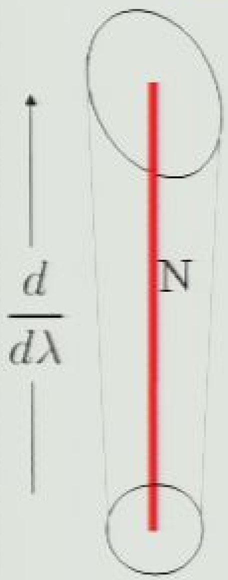

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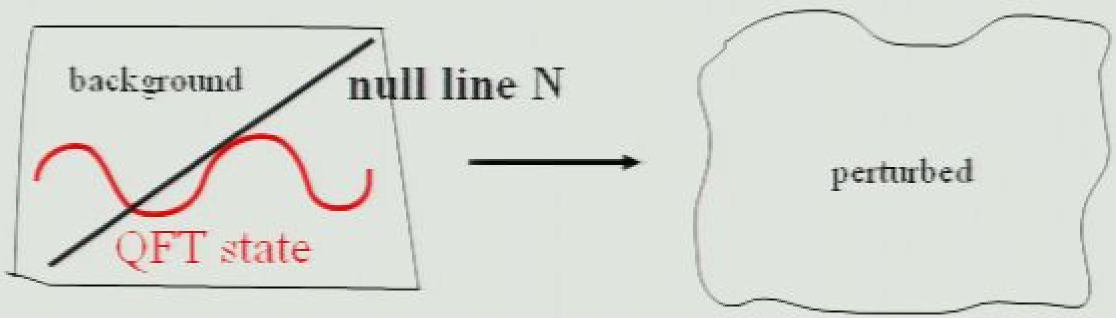
Gravitational Perturbation Theory

Do an expansion in $\hbar$.   Set G = c = 1.

Expand metric as $g_{ab} = \overset{0}{g}_{ab} + \overset{1/2}{g}_{ab} + \overset{1}{g}_{ab} + \mathcal{O}(\hbar^{3/2})$
& impose the Einstein equation order by order.



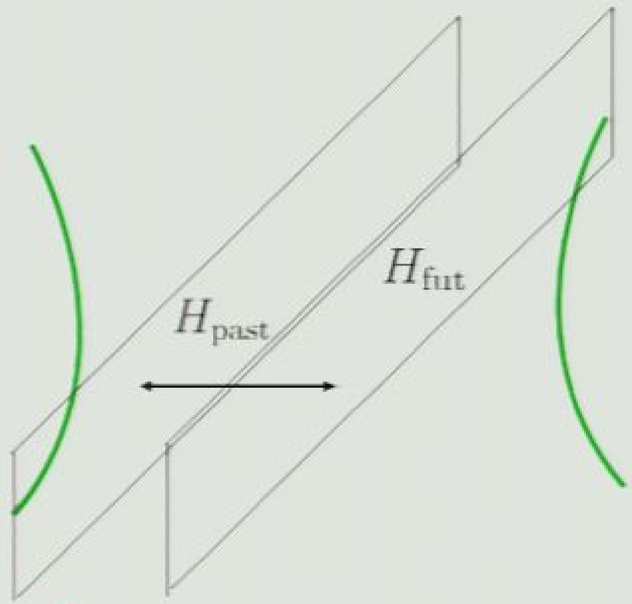
* Zeroth order metric is some classical background metric with a null line.
Assume the background obeys the null curvature condition $\overset{0}{R}_{ab} k^a k^b = 0$.

* Half order metric perturbation comes from graviton fluctuations.
Ignored semiclassically. Justified only with large number of species.

* First order metric represents gravitational effects of quantum fields
on the classical background. Assume this is a small perturbation.

Perturbed Spacetime

Past and future horizons split at first order in $\hbar$



asymptotic observers unaffected by sufficiently local perturbation

$\hbar^1$ perturbed metric

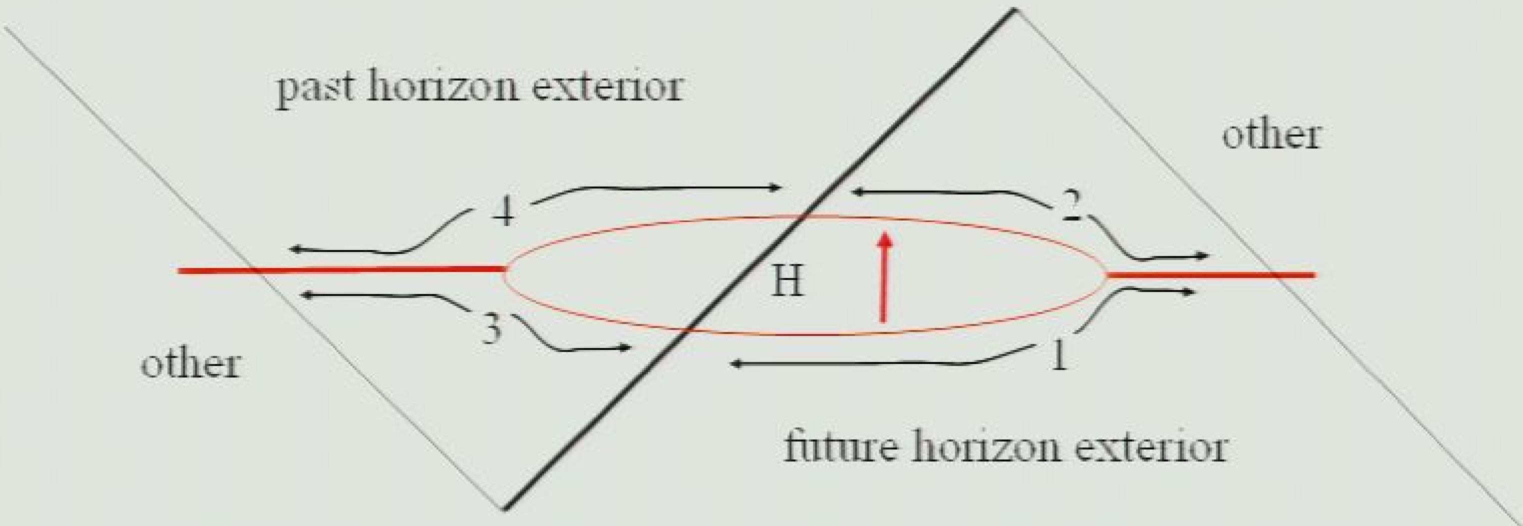
$$\frac{A}{4\hbar} + S_{\rm out} \longrightarrow T_{ab}$$

$\hbar^0$ $\qquad$ $\hbar^1$

properties of quantum fields on background

metric perturbation only affects Bekenstein-Hawking term due to the $\hbar$ in the denominator

causal diagram of the proof



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