

Title: Gravity dual of Polchinski's theorem: forbidden landscape

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Abstract:

# Gravity dual of Polchinski's theorem: forbidden landscape

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[arxiv:0909.4297](https://arxiv.org/abs/0909.4297)

[arxiv:0907.0227](https://arxiv.org/abs/0907.0227)

## My belief

**Holography** is one of the fundamental properties of **quantum gravity**

It is a good strategy to constrain candidates of quantum gravity from holography

## Objective

- Show gravity dual of Polchinski's theorem in string/M-theory
- Discuss consistency/constraint of quantum gravity from holography
- String Landscape/ Swampland
  - Bad examples: spontaneous Lorentz symmetry breaking, ghost condensation

Theorem (Polchinski): the scale invariant theory is conformal (in 1+1 dimension)



Gravity dual

Scale inv field configuration  $\rightarrow$   
automatically conformal inv (AdS isometry)

# Gravy dual of Polchinski's theorem

Theorem (Polchinski, 1988):  
the scale invariant theory is  
conformal (in 1+1 dimension)

IF

1. The theory is unitary
2. The theory is Poincare invariant
3. The spectrum is discrete

## Scale inv vs conformal inv

- Scale invariance

$$x^\mu \rightarrow \lambda x^\mu$$

→ trace of EM tensor is total derivative

$$T^\mu{}_\mu = \partial^\mu A_\mu$$

- Special conformal invariance

$$x^\mu \rightarrow \frac{x^\mu + a^\mu x^2}{1 + 2a^\mu x_\mu + a^2 x^2}$$

→ EM tensor is traceless (or A is total derivative)

$$T^\mu{}_\mu = 0, A_\mu = \partial_\mu \lambda$$



Theorem (Polchinski):  
the scale invariant theory is  
conformal (in 1+1 dimension)

Why 2D?

1. Tensor structure is simple
2. C-theorem
3. No counterexample in higher dim

## Field theory proof

$$\begin{aligned} F(x^2) &= z^2 \langle T(x)T(0) \rangle & T &\equiv T_{zz} \ , \ \Theta \equiv T^\mu{}_\mu \\ G(x^2) &= z^3 \bar{z} \langle \Theta(x)T(0) \rangle & & \\ H(x^2) &= z^2 \bar{z}^2 \langle \Theta(x)\Theta(0) \rangle \ . & \bar{\partial}T + 4\partial\Theta &= 0 \end{aligned}$$

Following Zamolodchikov, we define

$$\begin{aligned} C &= 2\left(F - \frac{1}{2}G - \frac{3}{16}H\right) \\ \dot{C} &\equiv x^2 \frac{d}{dx^2} C = -\frac{3}{4}H \leq 0 && \leftarrow \text{C-theorem!} \end{aligned}$$

At fixed point  $\dot{C} = 0$  so  $H = 0$

$$\rightarrow \langle \Theta(x)\Theta(0) \rangle = 0 \iff \Theta = 0$$

## Gravity counterpart

Scale inv geometry  $\rightarrow$  conformal inv (AdS)

Consider string/M-theory compactification

$$S_M = -\frac{1}{2\kappa^2} \int d^{11}x \sqrt{-g} \left( R - \frac{1}{2 \cdot 4!} |F|^2 \right) - \frac{1}{6} \int C_3 \wedge F_4 \wedge F_4$$

The following discussion applies to string theory as well....

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# Conjecture

Most general scale inv configuration:

$$ds_{11}^2 = f(\xi) \frac{dz^2}{z^2} + g(\xi) \frac{-dt^2 + dx^2}{z^2} + 2h_i(\xi) \frac{dz d\xi^i}{z} + ds_8^2(\xi)$$

$$F_4 = A + \frac{dz}{z} \wedge B + \frac{dt \wedge dx}{z^2} \wedge C + \frac{dz \wedge dx \wedge dt}{z^3} \wedge D$$

$$z \rightarrow \lambda z, \quad t \rightarrow \lambda t, \quad x \rightarrow \lambda x$$

Claim:  $f(\xi) = g(\xi)$ ,  $h_i(\xi) = B = C = 0$

so that it is invariant under

$$\delta x_a = 2(\epsilon^a x_b) x_a - (z^2 + x^b x_b) \epsilon_a, \quad \delta z = 2(\epsilon^b x_b) z$$

## Proof

We impose gauge condition  $h_i(\xi) = f(\xi)\partial_i\Lambda(\xi)$

$$z^2(f^2(\xi)R_{zz} + g^2(\xi)R_{tt}) = D^i J_i(\xi)$$

Null energy (weaker energy) condition

$$\rightarrow f^2(\xi)R_{zz} + g^2(\xi)R_{tt} \geq 0$$

Integrate over  $X_g$

$$\rightarrow \int f^2(\xi)R_{zz} + g^2(\xi)R_{tt} = \int |B|^2 + |C|^2 = 0$$

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## More generally

Nontrivial  $h$  gives additional terms:

$$z^2(R_{tt} + R_{zz}) = \tilde{h}_i \tilde{h}^i + (\partial_i \tilde{h}_j - \partial_j \tilde{h}_i)^2 - D^i j_i$$

Null energy condition gives positive definite in LHS...but non-trivial  $h$  can compensate.

Effective violation of null-energy condition?

→ Most probably EOM for flux + other Einstein equations makes it vanish...

Should be true at least in (1+2) d.

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- Similar evidence can be presented in string theory (or any gravity theory with null/weaker energy condition)
- Does not depend on the dimension  
→ Polchinski's theorem was only proved in  $1+1$  d.  
(but no counterexamples...)
- Suggests higher dim generalization?

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# Forbidden Landscape

So far so good

Question: is effective field theory consistent as quantum gravity?

Bad examples

- Spontaneous Lorentz breaking
- Ghost condensation

## Spontaneous Lorentz Symmetry breaking model

$$\mathcal{L} = F_{\mu\nu}F^{\mu\nu} - \sum_n \frac{g_n}{2n} (A_\mu A^\mu)^n$$

- EOM is solved by  $\text{AdS}_3$  and the vector condensation

$$A = A^\mu dx_\mu = a \frac{dz}{z}$$

- EM tensor is proportional to  $g_{\mu\nu}$

This solution is bad...

$$A = A^\mu dx_\mu = a \frac{dz}{z}$$

- Scale invariant but not conformal invariant

$$\begin{aligned}\delta x_a &= 2x_a(\epsilon^b x_b) - (z^2 + x^2)\epsilon_a \\ \delta z &= 2z(\epsilon^a x_a)\end{aligned}$$



So,  
spontaneous Lorentz symmetry  
breaking based on the action  
is forbidden  
in any consistent quantum theories  
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## Ghost condensation model

$$\mathcal{L} = \sum \frac{h_n}{2n} (\partial^\mu \phi \partial_\mu \phi)^n$$

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$$\phi = c \log z$$

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- $\phi(x) \sim \phi(x) + \Lambda$  so that scale inv is OK
- These scale inv but non-conformal field configurations are forbidden in quantum gravity (in 1+2D)

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## Higher dimension

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