

Title: Primordial power spectra and cosmological observations

Date: Dec 08, 2009 11:50 AM

URL: <http://pirsa.org/09120107>

Abstract:

## Outline

- Introduction: Concordance Model and Primordial Power Spectra
- Cosmological Observations: Current Data and Forecasts
  - Single-Field Slow-Roll Inflation
  - Releasing the Slow-Roll Assumptions
  - Summary and Conclusions

# Outline

Introduction: Concordance Model and Primordial Power Spectra

Cosmological Observations: Current Data and Forecasts

Single-Field Slow-Roll Inflation

Releasing the Slow-Roll Assumptions

Phenomenological Parametrization

Consistency Treatment

Summary and Conclusions

# 6-parameter Concordance Model

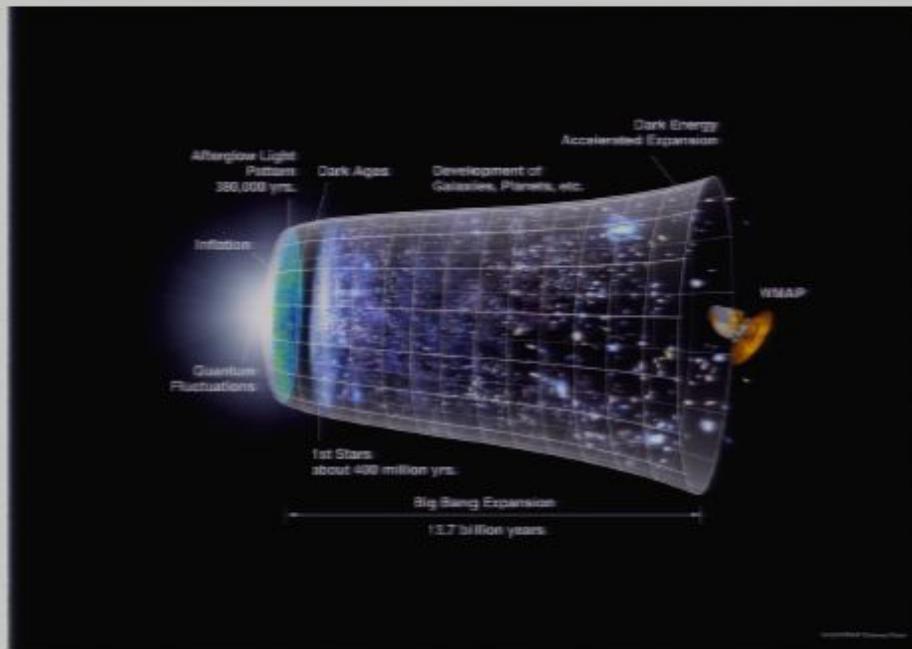


Figure from LAMBDA website

$\tau$ : reionization optical depth

$\Omega_b h^2$ : today's physical density of baryon

$\Omega_c h^2$ : today's physical density of cold dark matter

$\theta$ : the angle subtended by sound horizon on CMB sky

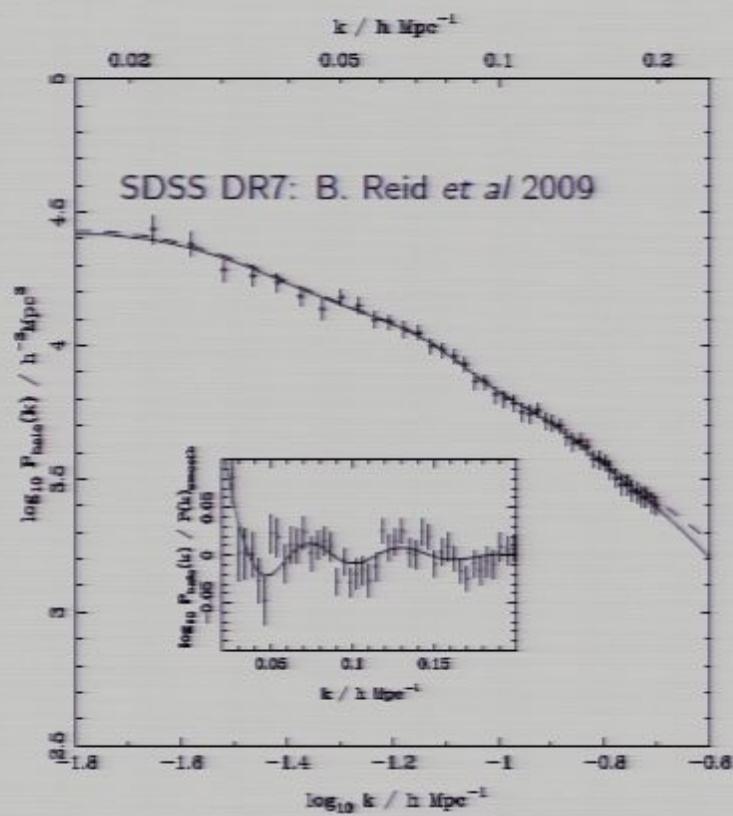
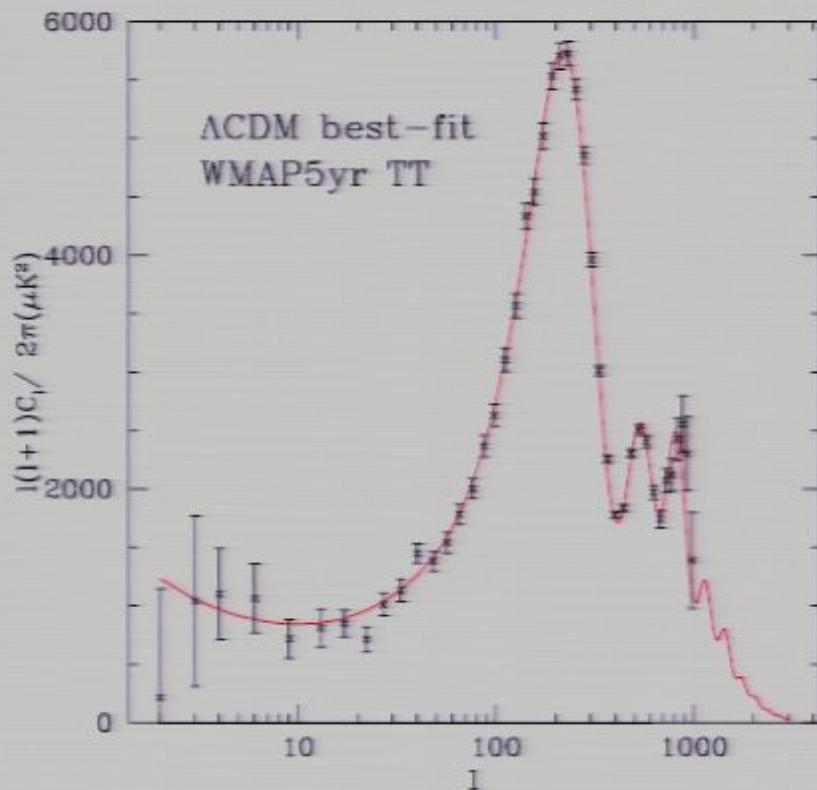
$A_s$ : the amplitude of primordial scalar metric perturbation

$n_s$ : the spectral index of primordial scalar metric perturbation

## Outline

- Introduction: Concordance Model and Primordial Power Spectra
- Cosmological Observations: Current Data and Forecasts
- Single-Field Slow-Roll Inflation
- Releasing the Slow-Roll Assumptions
- Summary and Conclusions

# Observational Confirmation of the Concordance Model



## Beyond 6 parameters:

- ▶ Late universe:
  - dark energy ( $w$ ): quintessence,  $f(R)$ , ...
  - dark matter: annihilation, decay, ...
  - reionization history
  - neutrinos
  - ...
- ▶ Early universe:
  - inflation & (p)reheating { primordial power spectra
    - non-Gaussianity( $f_{nl}$ )
  - phase transition
  - topological defect
  - CMB physics
  - ...

## Outline

Introduction: Concordance Model and Primordial Power Spectra

Cosmological Observations: Current Data and Forecasts

Single-Field Slow-Roll Inflation

Releasing the Slow-Roll Assumptions

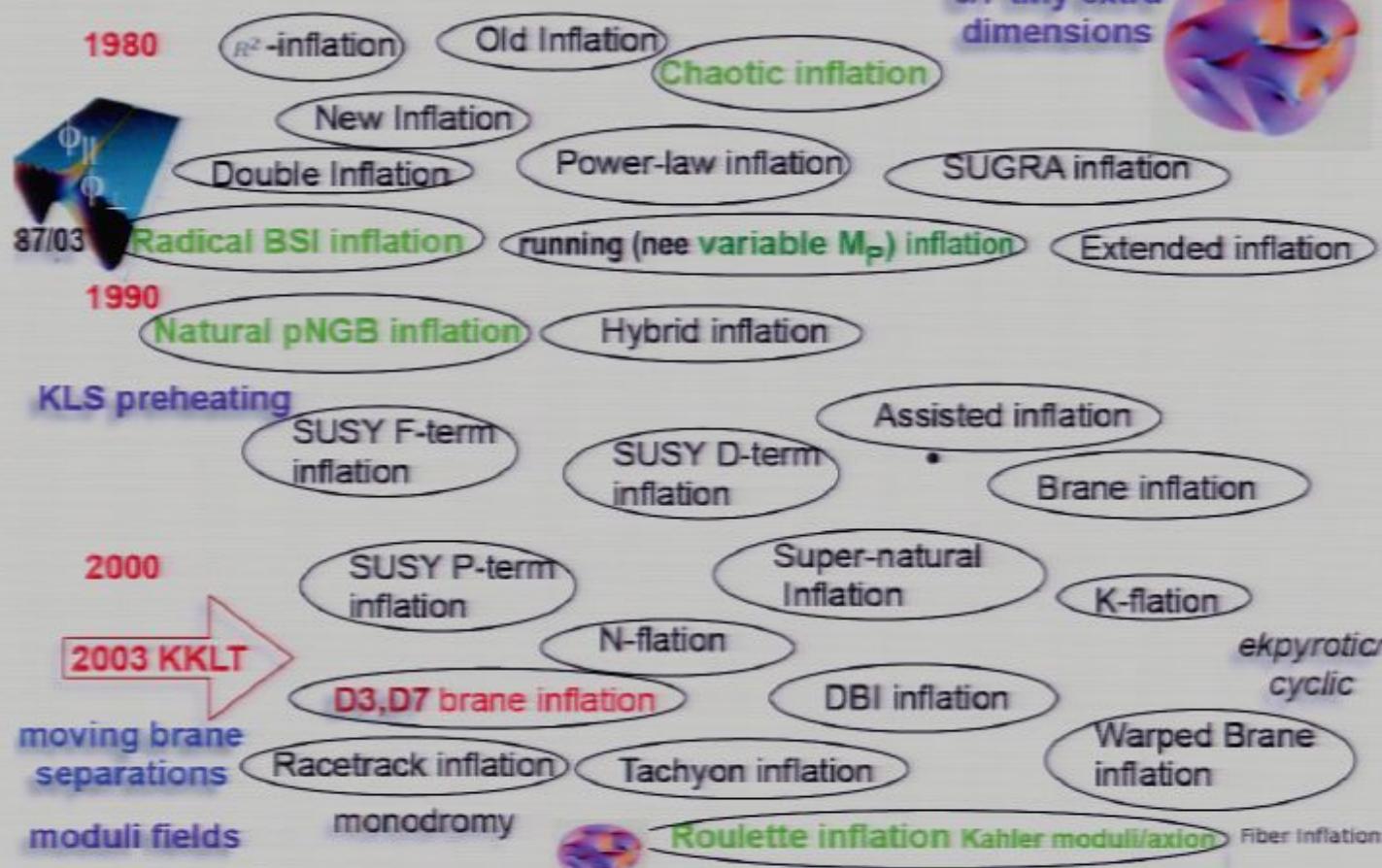
Summary and Conclusions

# Inflation & (p)reheating: the zoology of theories

Old view: Theory prior = delta function of THE correct one and only theory

New: Theory prior = probability distribution of late-flows on an energy LANDSCAPE

6/7 tiny extra dimensions



## Primordial Power Spectra: Standard Extensions $n_{\text{run}}$ , $r$ (and $n_t$ ).

$n_{\text{run}}$ : the running of primordial scalar power spectrum

$r$ : the tensor-to-scalar ratio of primordial metric perturbations

$n_t$ : the spectral index of primordial tensor power spectrum

$$\mathcal{P}_s(k) = A_s \left( \frac{k}{k_{\text{pivot}}} \right)^{n_s - 1 + \frac{1}{2} n_{\text{run}} \ln(\frac{k}{k_{\text{pivot}}})}$$

$$\mathcal{P}_t(k) = A_t \left( \frac{k}{k_{\text{pivot}}} \right)^{n_t},$$

where  $A_t \equiv rA_s$ , and often chosen  $k_{\text{pivot}}$ :  $0.002 \text{Mpc}^{-1}$  or  $0.05 \text{Mpc}^{-1}$ .

## Cosmological data sets

- ▶ Cosmic Microwave Background (**CMB**): WMAP5yr (09), Acbar (09), QUAD (09), BICEP (09), CBI (08), Boomerang (06), DASI (05), VSA (04), MAXIMA (00)
- ▶ Type Ia Supernova (**SN**): LOWZ + SDSS + ESSENCE + SNLS1yr + HST (Kessler et al 09) (soon will + SNLS3yr)
- ▶ Weak Lensing (**WL**): COSMOS + CFHTLS-wide + RCS + VIRIMOS + GaBoDS (Massey et al 07, Lesgourgues et al 07, Benjamin et al 07)
- ▶ Large Scale Structure (**LSS**): SDSS-DR7 LRG (Reid et al 09)
- ▶ Ly $\alpha$  Forest (**Lya**): SDSS Lya (McDonald et al 05, 06)
- ▶ Others: HST constraint on Hubble parameter (Riess et al 09); Cluster x-ray gas mass fraction (Allen et al 08)

## Tool: Modified CosmoMC

- ▶ Arbitrary Primordial Power spectra functions  $P_s(k)$  and  $P_t(k)$ .
- ▶ An integrator to calculate  $P_s(k)$  and  $P_t(k)$  from arbitrary single-field inflation model
- ▶ Automatic adjusting  $l, k$  interpolation for more oscillatory primordial power spectra.
- ▶ Dark energy equation of state arbitrary function  $w(z)$ , with an analytic quintessence/phantom motivated parametrization built-in.
- ▶ Decaying dark matter
- ▶ CMB, WL, SN, BAO mock data simulator, add an action  $= -1$  for running simulations
- ▶ Self-written GetDist to do more statistics & visualizations

## Outline

Introduction: Concordance Model and Primordial Power Spectra

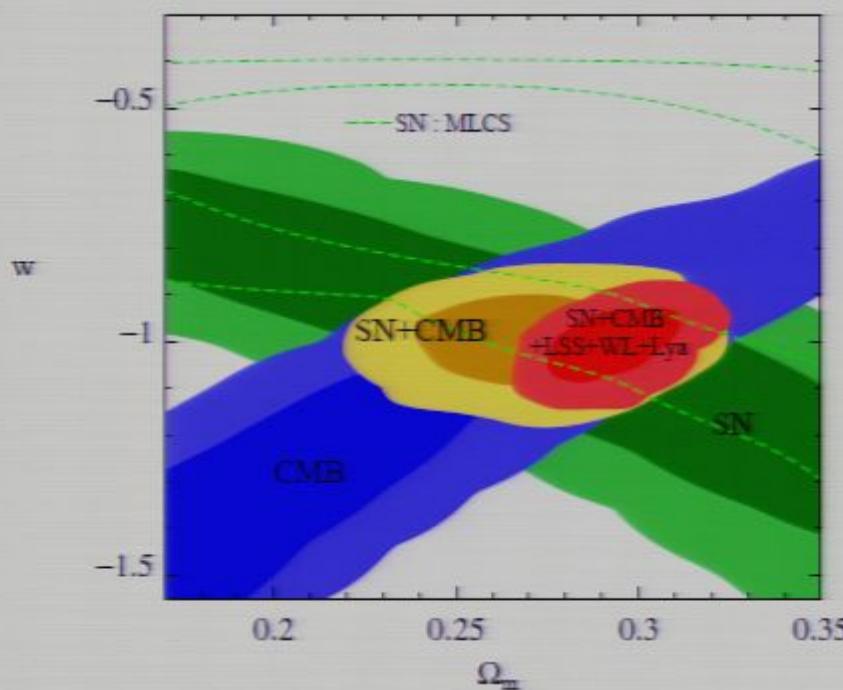
Cosmological Observations: Current Data and Forecasts

Single-Field Slow-Roll Inflation

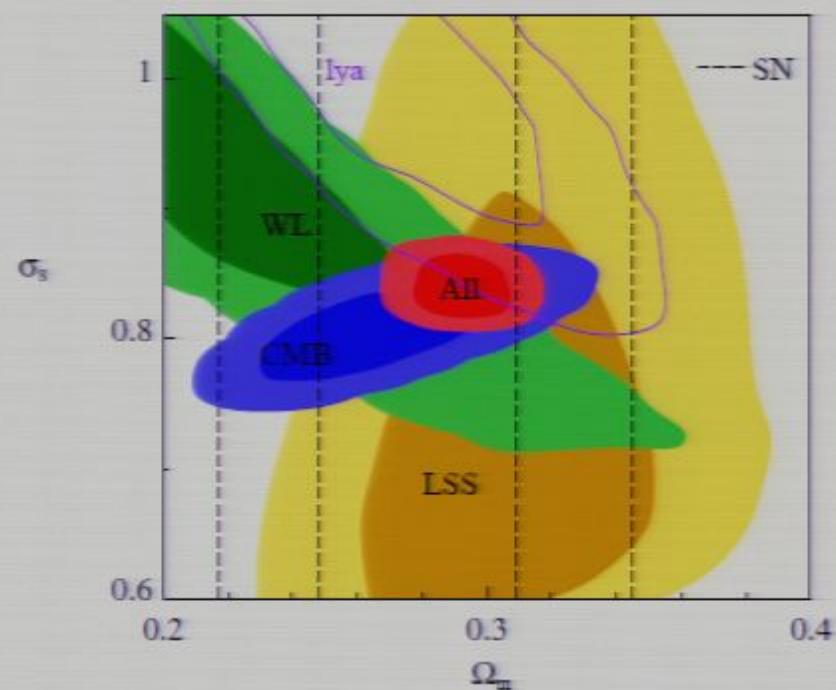
Releasing the Slow-Roll Assumptions

Summary and Conclusions

# The Consistency between Cosmological Data Sets



default SN filter: SALT II



# Primordial Power Spectra: Standard Extensions $n_{\text{run}}$ , $r$ (and $n_t$ ).

**$n_{\text{run}}$ :** the running of primordial scalar power spectrum

**$r$ :** the tensor-to-scalar ratio of primordial metric perturbations

**$n_t$ :** the spectral index of primordial tensor power spectrum

$$\mathcal{P}_s(k) = A_s \left( \frac{k}{k_{\text{pivot}}} \right)^{n_s - 1 + \frac{1}{2} n_{\text{run}} \ln(\frac{k}{k_{\text{pivot}}})}$$

$$\mathcal{P}_t(k) = A_t \left( \frac{k}{k_{\text{pivot}}} \right)^{n_t},$$

where  $A_t \equiv rA_s$ , and often chosen  $k_{\text{pivot}}: 0.002 \text{Mpc}^{-1}$  or  $0.05 \text{Mpc}^{-1}$ .

## Cosmological data sets

- ▶ Cosmic Microwave Background (**CMB**): WMAP5yr (09), Acbar (09), QUAD (09), BICEP (09), CBI (08), Boomerang (06), DASI (05), VSA (04), MAXIMA (00)
- ▶ Type Ia Supernova (**SN**): LOWZ + SDSS + ESSENCE + SNLS1yr + HST (Kessler et al 09) (soon will + SNLS3yr)
- ▶ Weak Lensing (**WL**): COSMOS + CFHTLS-wide + RCS + VIRMOS + GaBoDS (Massey et al 07, Lesgourgues et al 07, Benjamin et al 07)
- ▶ Large Scale Structure (**LSS**): SDSS-DR7 LRG (Reid et al 09)
- ▶ Ly $\alpha$  Forest (**Lya**): SDSS Lya (McDonald et al 05, 06)
- ▶ Others: HST constraint on Hubble parameter (Riess et al 09); Cluster x-ray gas mass fraction (Allen et al 08)

# Primordial Power Spectra: Standard Extensions $n_{\text{run}}$ , $r$ (and $n_t$ ).

$n_{\text{run}}$ : the running of primordial scalar power spectrum

$r$ : the tensor-to-scalar ratio of primordial metric perturbations

$n_t$ : the spectral index of primordial tensor power spectrum

$$\mathcal{P}_s(k) = A_s \left( \frac{k}{k_{\text{pivot}}} \right)^{n_s - 1 + \frac{1}{2} n_{\text{run}} \ln(\frac{k}{k_{\text{pivot}}})}$$

$$\mathcal{P}_t(k) = A_t \left( \frac{k}{k_{\text{pivot}}} \right)^{n_t},$$

where  $A_t \equiv rA_s$ , and often chosen  $k_{\text{pivot}}$ :  $0.002 \text{Mpc}^{-1}$  or  $0.05 \text{Mpc}^{-1}$ .

## Outline

Introduction: Concordance Model and Primordial Power Spectra

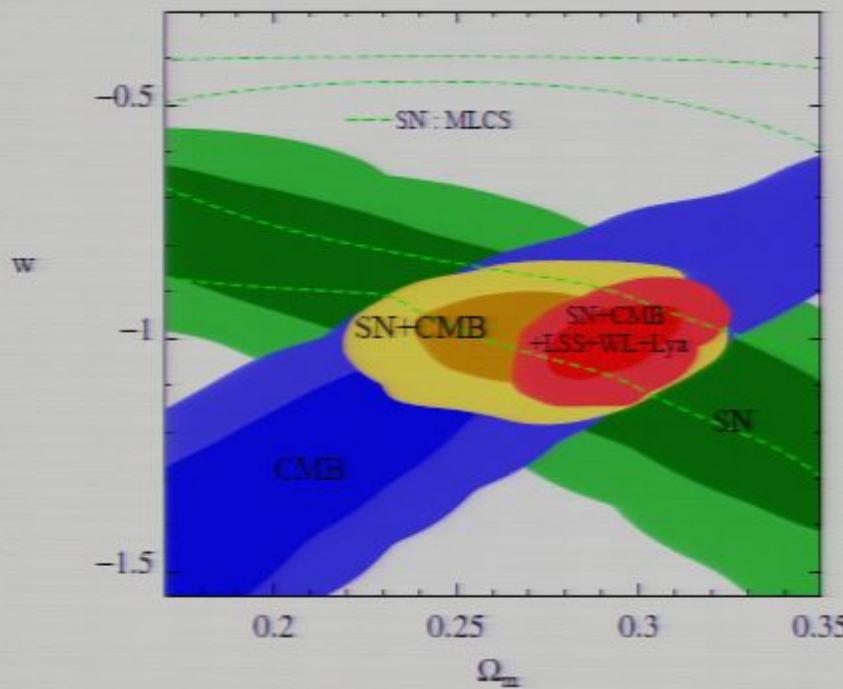
Cosmological Observations: Current Data and Forecasts

Single-Field Slow-Roll Inflation

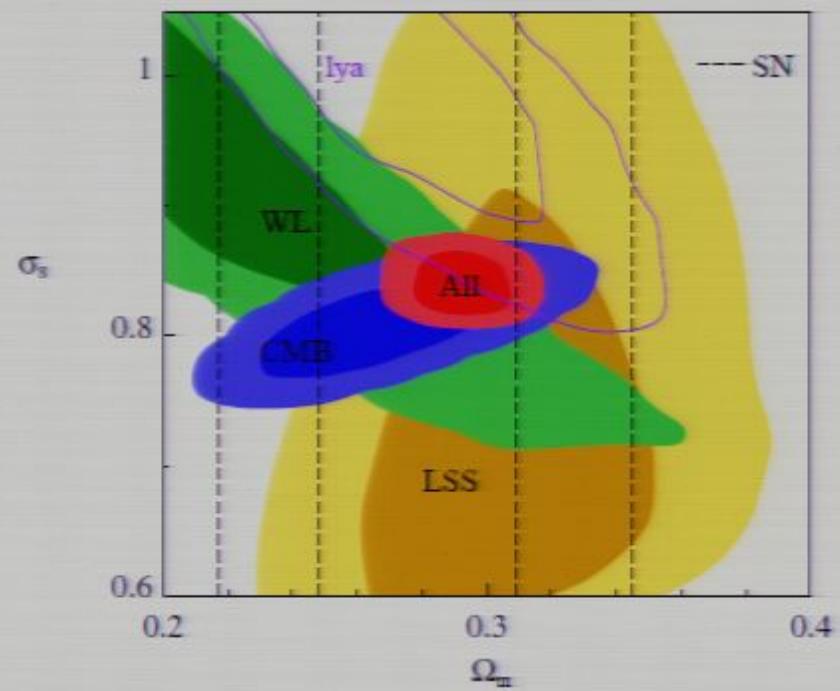
Releasing the Slow-Roll Assumptions

Summary and Conclusions

# The Consistency between Cosmological Data Sets



default SN filter: SALT II



## Forecasts

- ▶ **CMB:** Planck2.5yr, using 3 channels (70GHz, 100GHz, 143GHz), assuming 5% foreground residual (synchrotron + dust),  $f_{\text{sky}} = 3/4$ ,  $l_{\text{max}} = 2500$ .  
other future polarization experiments: SPIDER, EBEX, QUIET, KECK, CMBPol ...
- ▶ **WL:** DUNE-like weak lensing tomography, 20000 degree<sup>2</sup>, depth  $z \sim 1$ , 35 galaxies/arcmin<sup>2</sup>, two redshift bins,  $l_{\text{max}} = 1500$ .  
the other proposed deep and wide WL surveys: JDEM, LSST, ...
- ▶ **SN:** JDEM, 500 LOWZ ( $z < 0.03$ ) + 2500 HIGHZ ( $0.03 < z < 1.7$ )  
other ongoing/future SN surveys: SNLS, SDSS, LSST ...
- ▶ **BAO:** JDEM, 10000 degree<sup>2</sup>,  $0.5 < z < 2$ , 10 redshift bins  
other ongoing/future BAO surveys: CHIME, WIGGLEZ, BOSS, LSST, ...

# Prediction from Single-Field Slow-Roll Inflation

## Slow-Roll Approximation:

$$n_s - 1 \approx 2\eta_V - 6\epsilon_V$$

$$r \approx -8n_t \approx 16\epsilon_V$$

$$n_{\text{run}} \approx -16\epsilon_V \eta_V + 24\epsilon_V^2 + 2\zeta_V^2 \approx 0,$$

where

$$\epsilon_V \equiv \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2$$

$$\eta_V \equiv M_p^2 \left( \frac{V''}{V} \right)$$

$$\zeta_V^2 \equiv M_p^4 \left( \frac{V' V'''}{V^2} \right)$$

## Classification in $\epsilon_V$ - $\eta_V$ space (or $r$ - $n_s$ space):

- ▶ Small Field Inflation:

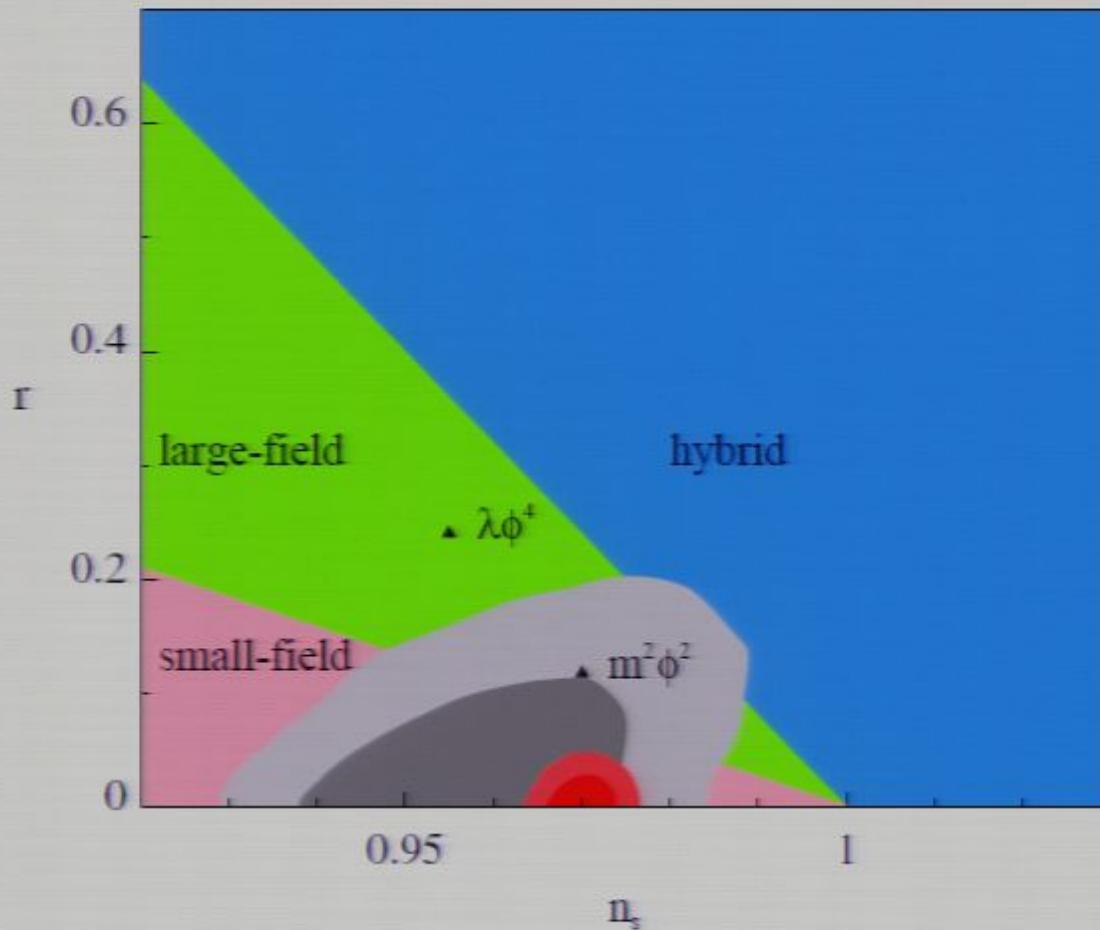
$$\eta_V < 0$$

- ▶ Large Field Inflation:

$$0 < \eta_V < 2\epsilon_V$$

- ▶ Hybrid Inflation:  $\eta_V > 2\epsilon_V$

# Constraining Single-Field Slow-Roll Inflation



68.3% and 95.4%  
CL constraints, current  
data

Forecast (fiducial  
 $n_s = 0.97, r = 0$ )

current  $r < 0.16$  (95%  
CL)

forecast  $r < 0.037$  (95%  
CL)

# Prediction from Single-Field Slow-Roll Inflation

## Slow-Roll Approximation:

$$\begin{aligned} n_s - 1 &\approx 2\eta_V - 6\epsilon_V \\ r &\approx -8n_t \approx 16\epsilon_V \\ n_{\text{run}} &\approx -16\epsilon_V \eta_V + 24\epsilon_V^2 + 2\zeta_V^2 \approx 0, \end{aligned}$$

where

$$\epsilon_V \equiv \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2$$

$$\eta_V \equiv M_p^2 \left( \frac{V''}{V} \right)$$

$$\zeta_V^2 \equiv M_p^4 \left( \frac{V' V'''}{V^2} \right)$$

## Classification in $\epsilon_V$ - $\eta_V$ space (or $r$ - $n_s$ space):

- ▶ Small Field Inflation:

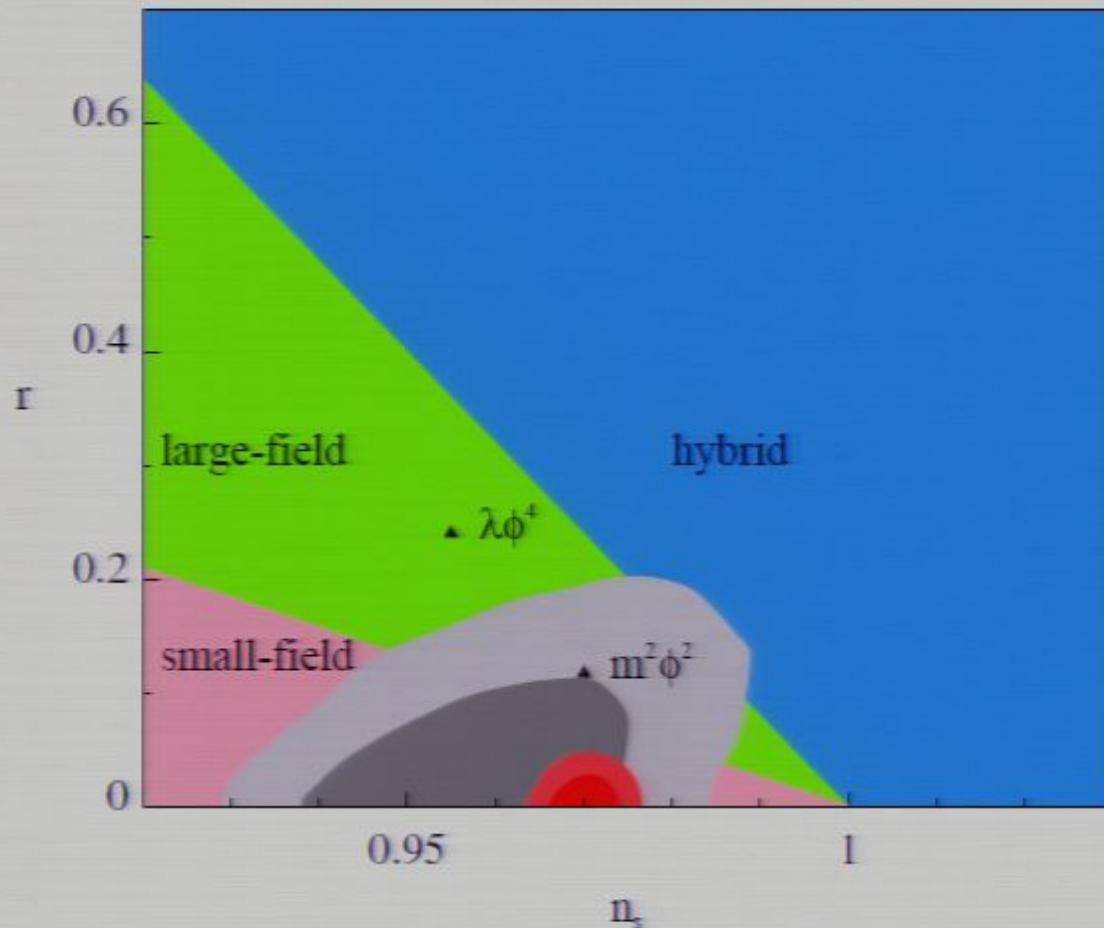
$$\eta_V < 0$$

- ▶ Large Field Inflation:

$$0 < \eta_V < 2\epsilon_V$$

- ▶ Hybrid Inflation:  $\eta_V > 2\epsilon_V$

# Constraining Single-Field Slow-Roll Inflation



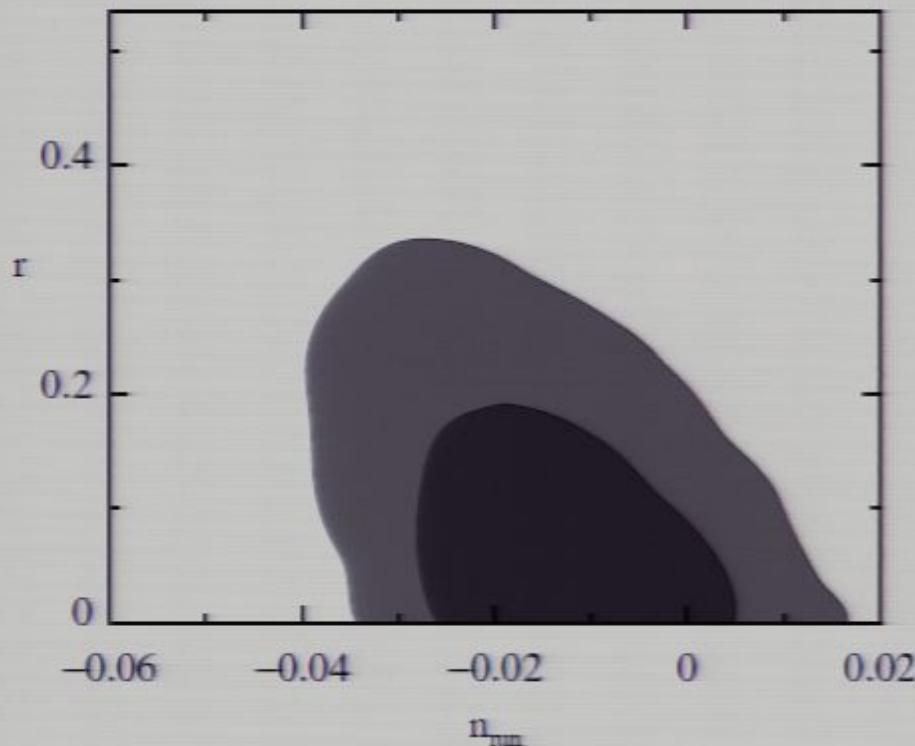
68.3% and 95.4%  
CL constraints, current  
data

Forecast (fiducial  
 $n_s = 0.97, r = 0$ )

current  $r < 0.16$  (95%  
CL)

forecast  $r < 0.037$  (95%  
CL)

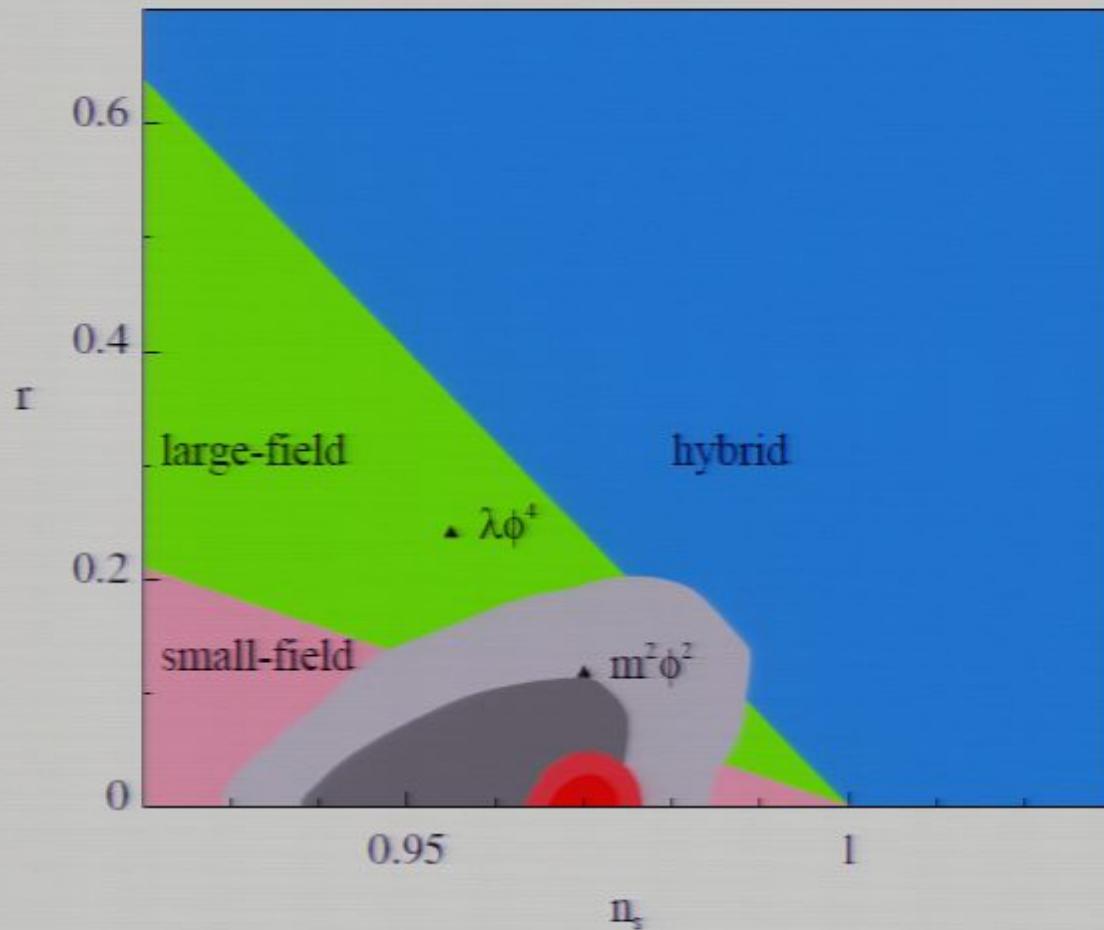
# The Degeneracy between Scalar and Tensor Spectrum



constraint on  $r$  (95%CL):

- ▶  $r < 0.16$  (no running,  
data sets: all)
- ▶  $r < 0.32$  (no running,  
data sets: CMB all)
- ▶  $r < 0.27$  (with running,  
data sets: all)

# Constraining Single-Field Slow-Roll Inflation



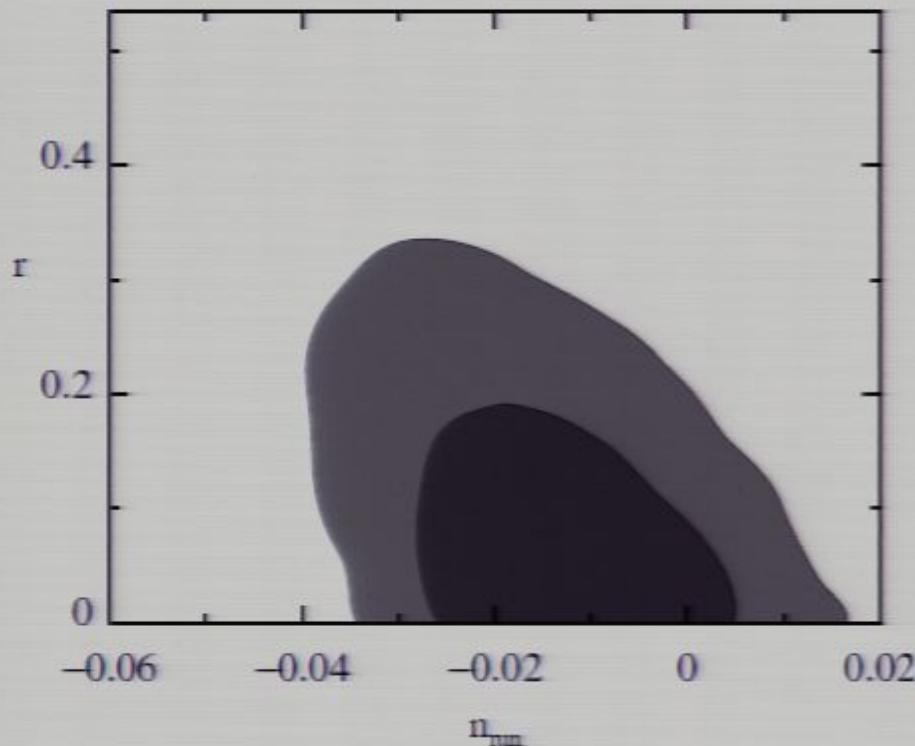
68.3% and 95.4%  
CL constraints, current  
data

Forecast (fiducial  
 $n_s = 0.97, r = 0$ )

current  $r < 0.16$  (95%  
CL)

forecast  $r < 0.037$  (95%  
CL)

# The Degeneracy between Scalar and Tensor Spectrum



constraint on  $r$  (95%CL):

- ▶  $r < 0.16$  (no running, data sets: all)
- ▶  $r < 0.32$  (no running, data sets: CMB all)
- ▶  $r < 0.27$  (with running, data sets: all)

## Releasing the Slow-Roll Assumptions:

If allow  $V''$  to be big,  $\zeta_V$  not necessarily small  $\rightarrow$  nontrivial  $n_{\text{run}}$ .

Single field models are not limited in the  $r - n_s$  plane.

Two approaches to extend the parametrization of primordial power spectra:

- ▶ **Phenomenologically** take  $P_s(k)$  and  $P_t(k)$  as arbitrary functions.
- ▶ **Consistently** solve  $P_s(k)$  and  $P_t(k)$  for all possible expansion histories.

Previous works on this topic:

- ▶ Simple binning techniques: Briddle et al 03; Hannestad 04; Bridges et al 06, 07; Spergel et al 07;
- ▶ Direct inversion: Shafieloo et al 04, 08; Kogo et al 04; Tocchini-Valentini et al 05 06; Nagata et al 08; Nicholson et al 09a, 09b;
- ▶ Basis function expansion: Mukherjee 05; Leach 06;
- ▶ Cubic spline interpolation: Sealfon et al 05; Peris et al 08, 09;
- ▶ Slow-roll reconstruction (flow equations): Peris et al 03, 06a, 06b; Easther 06; Adshead et al 09;

## Phenomenological Parametrization

conventional “3 + 2”  $\Rightarrow$  “ $n + 2$ ”.

- ▶ For the well constrained scalar power spectrum, take  $\ln P_s(k)$  at  $n$  fixed knots  $k_1, k_2, \dots, k_n$  in the observable range, interpolate in between.  
 $n$  parameters
- ▶ For the poorly constrained tensor power spectrum, use simple

$$P_t(k) = A_t \left( \frac{k}{k_{\text{pivot}}} \right)^{n_t}, \text{ with prior } -0.1 < n_t < 0$$

2 parameters

(optional: do similar expansion as  $P_s \Rightarrow$  “ $n + m$ ” parametrization )

Previous works on this topic:

- ▶ Simple binning techniques: Briddle et al 03; Hannestad 04; Bridges et al 06, 07; Spergel et al 07;
- ▶ Direct inversion: Shafieloo et al 04, 08; Kogo et al 04; Tocchini-Valentini et al 05 06; Nagata et al 08; Nicholson et al 09a, 09b;
- ▶ Basis function expansion: Mukherjee 05; Leach 06;
- ▶ Cubic spline interpolation: Sealfon et al 05; Peris et al 08, 09;
- ▶ Slow-roll reconstruction (flow equations): Peris et al 03, 06a, 06b; Easther 06; Adshead et al 09;

## Phenomenological Parametrization

conventional “3 + 2”  $\Rightarrow$  “ $n + 2$ ”.

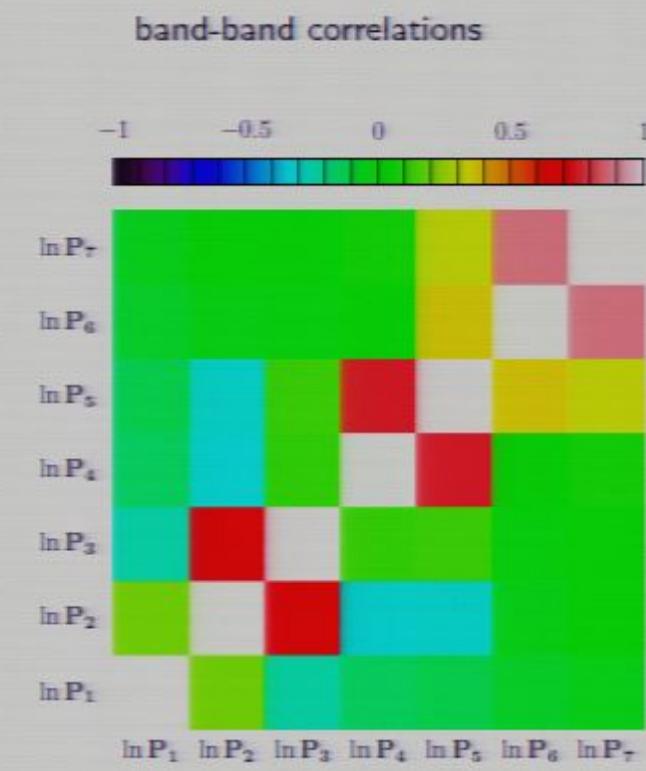
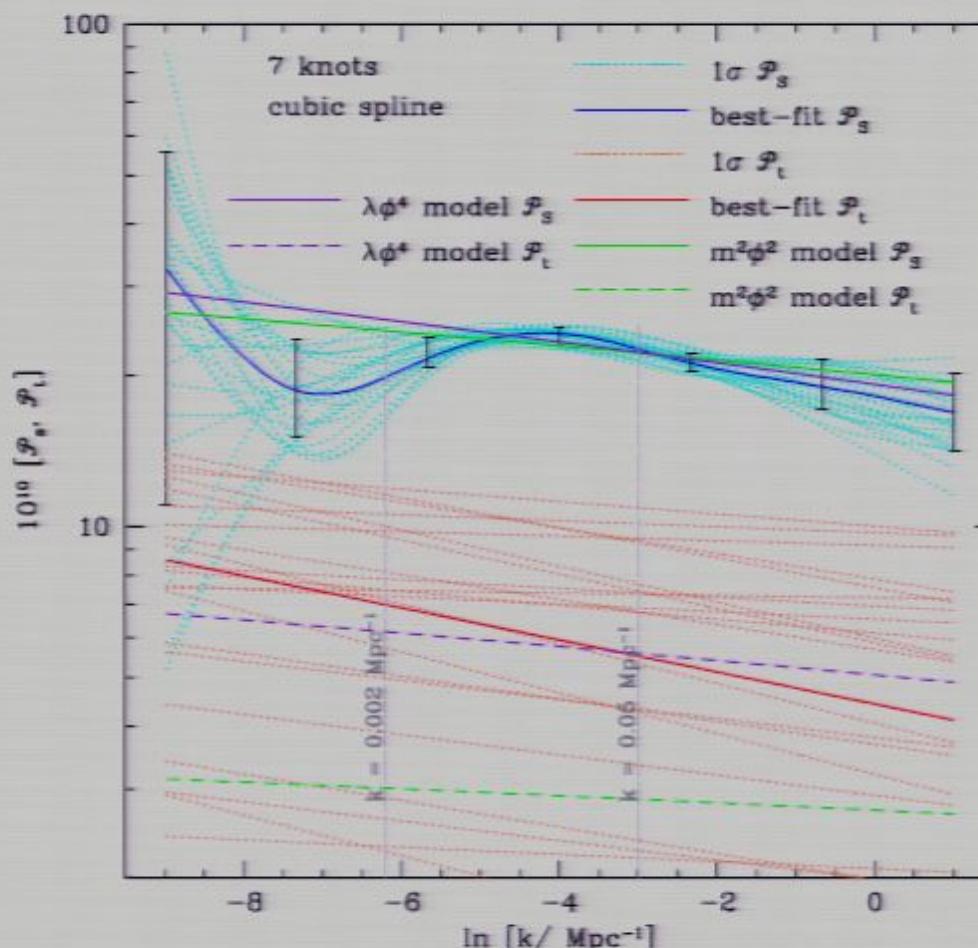
- ▶ For the well constrained scalar power spectrum, take  $\ln P_s(k)$  at  $n$  fixed knots  $k_1, k_2, \dots, k_n$  in the observable range, interpolate in between.  
 $n$  parameters
- ▶ For the poorly constrained tensor power spectrum, use simple

$$P_t(k) = A_t \left( \frac{k}{k_{\text{pivot}}} \right)^{n_t}, \text{ with prior } -0.1 < n_t < 0$$

2 parameters

(optional: do similar expansion as  $P_s \Rightarrow$  “ $n + m$ ” parametrization )

## Reconstructed Primordial Power Spectra



$$r(0.002 \text{Mpc}^{-1}) < 0.89 \text{ (95% CL)}$$

## Phenomenological Parametrization

conventional “3 + 2”  $\Rightarrow$  “ $n + 2$ ”.

- ▶ For the well constrained scalar power spectrum, take  $\ln P_s(k)$  at  $n$  fixed knots  $k_1, k_2, \dots, k_n$  in the observable range, interpolate in between.  
 $n$  parameters
- ▶ For the poorly constrained tensor power spectrum, use simple

$$P_t(k) = A_t \left( \frac{k}{k_{\text{pivot}}} \right)^{n_t}, \text{ with prior } -0.1 < n_t < 0$$

2 parameters

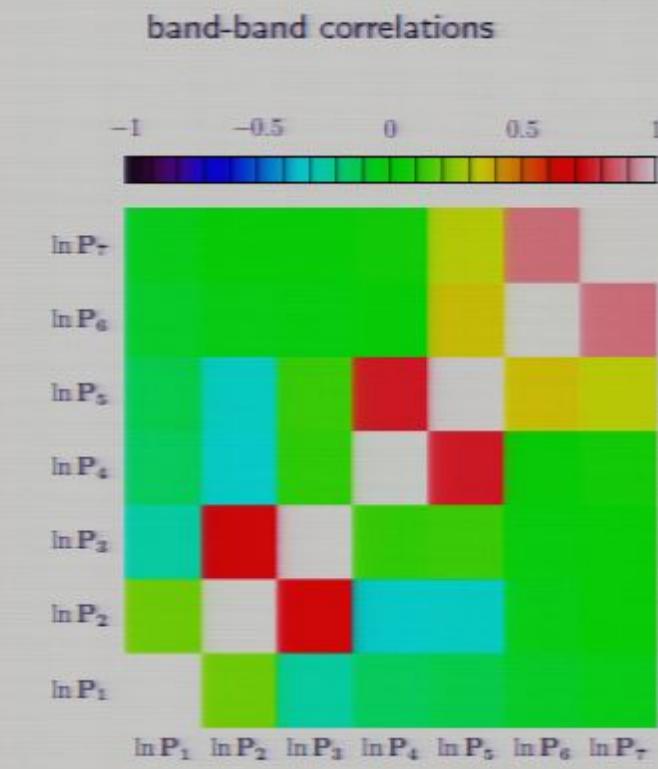
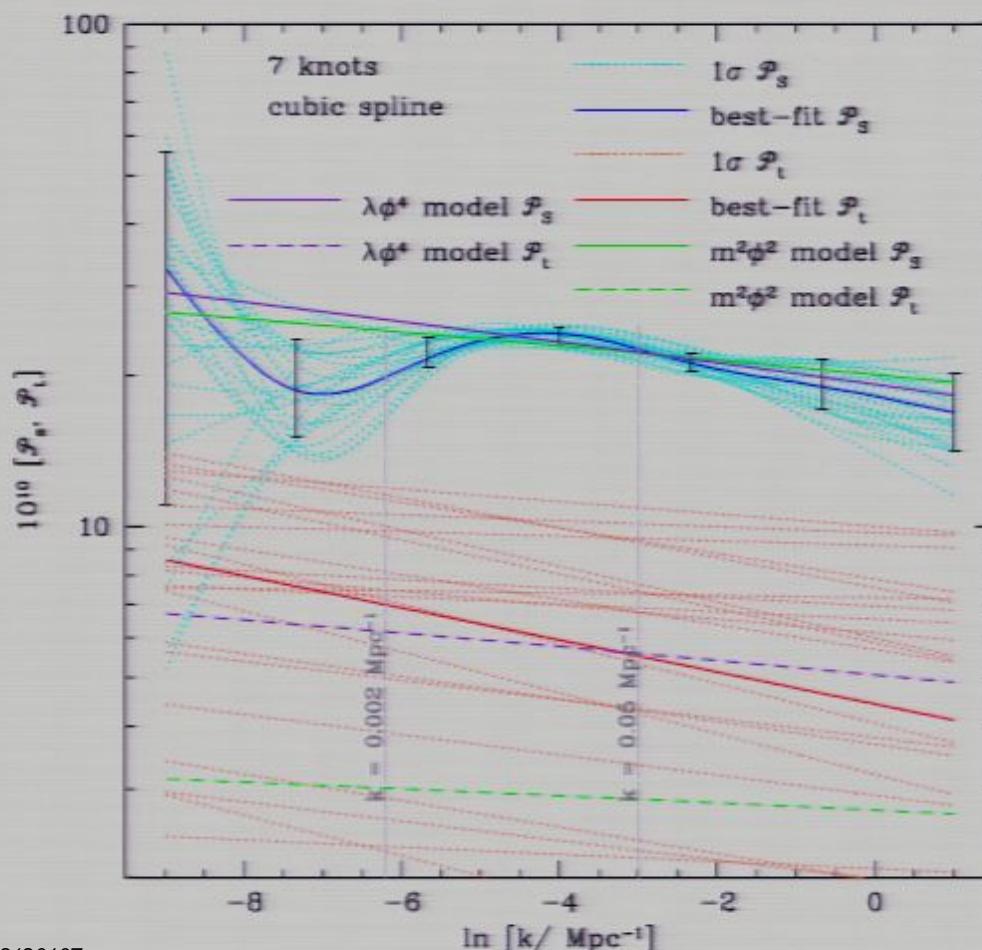
(optional: do similar expansion as  $P_s \Rightarrow$  “ $n + m$ ” parametrization )

## Outline

- Introduction: Concordance Model and Primordial Power Spectra
- Cosmological Observations: Current Data and Forecasts
- Single-Field Slow-Roll Inflation
- Releasing the Slow-Roll Assumptions**
- Summary and Conclusions

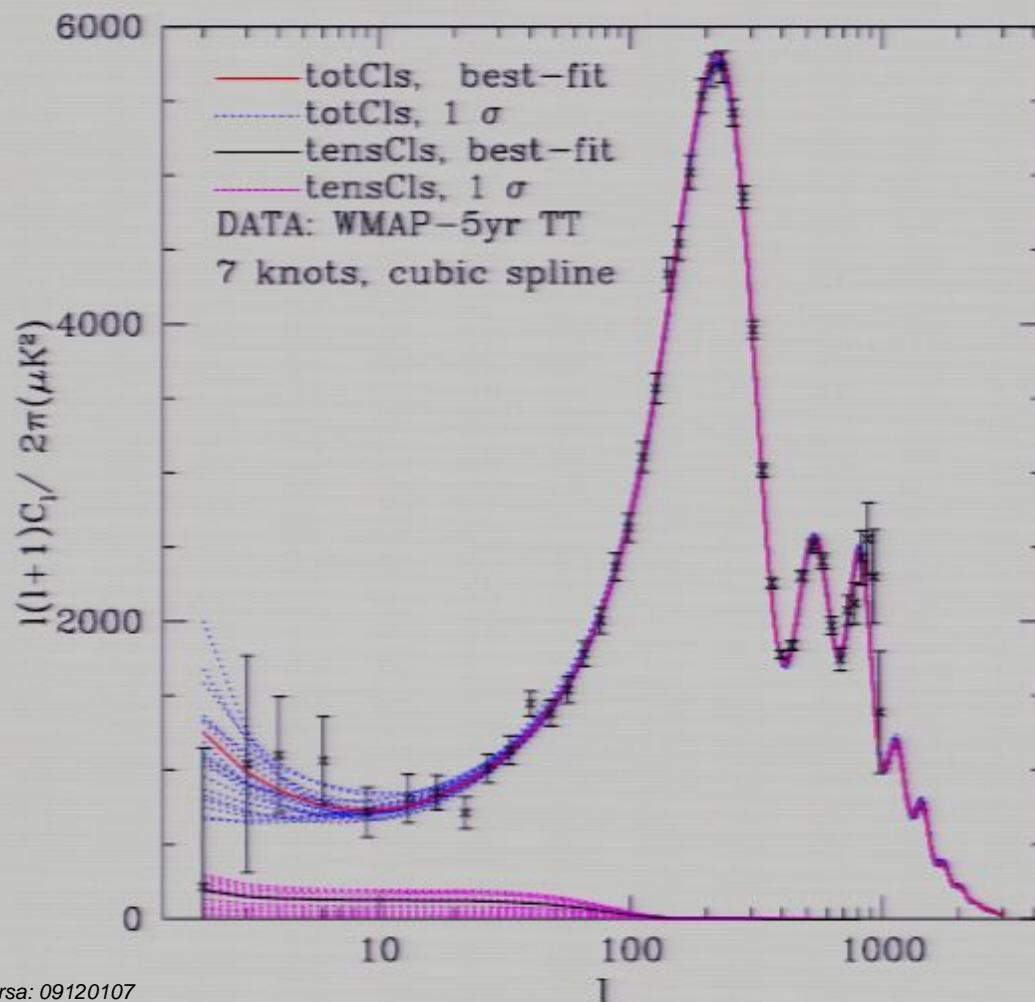
## Phenomenological Parametrization Consistency Treatment

# Reconstructed Primordial Power Spectra

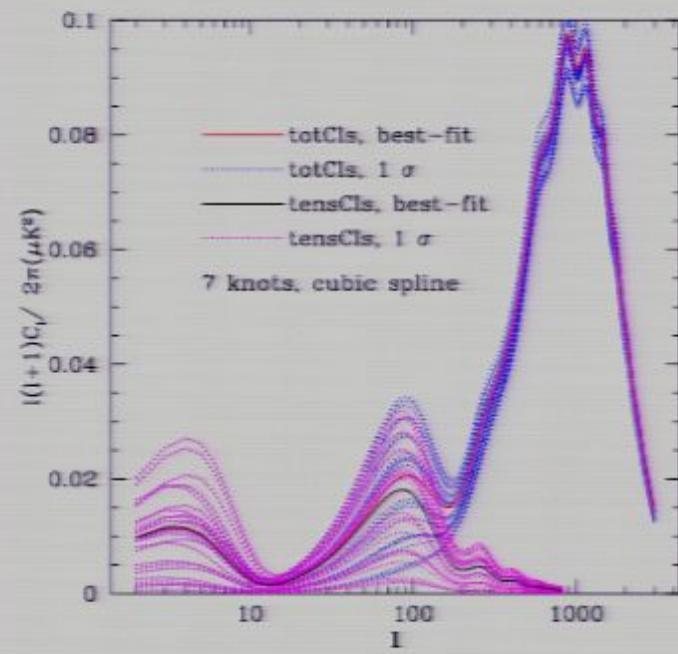


$$r(0.002 \text{ Mpc}^{-1}) < 0.89 \text{ (95% CL)}$$

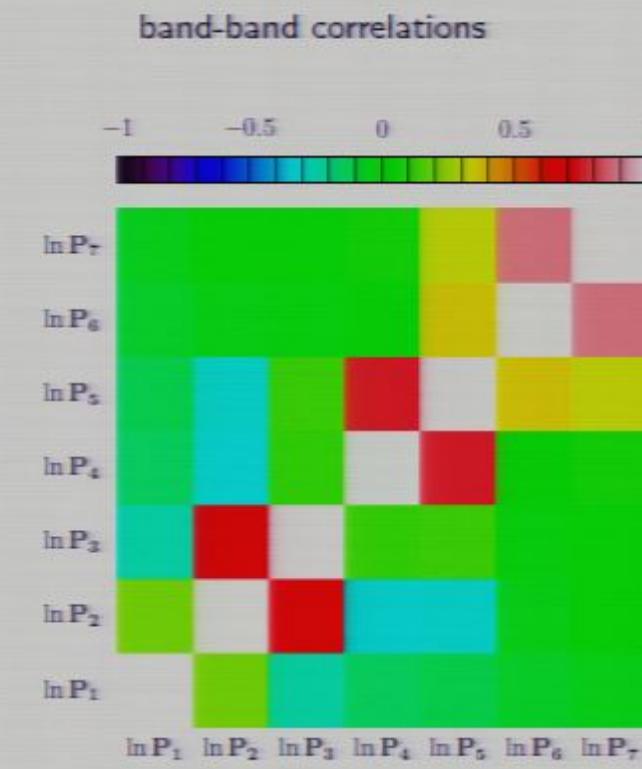
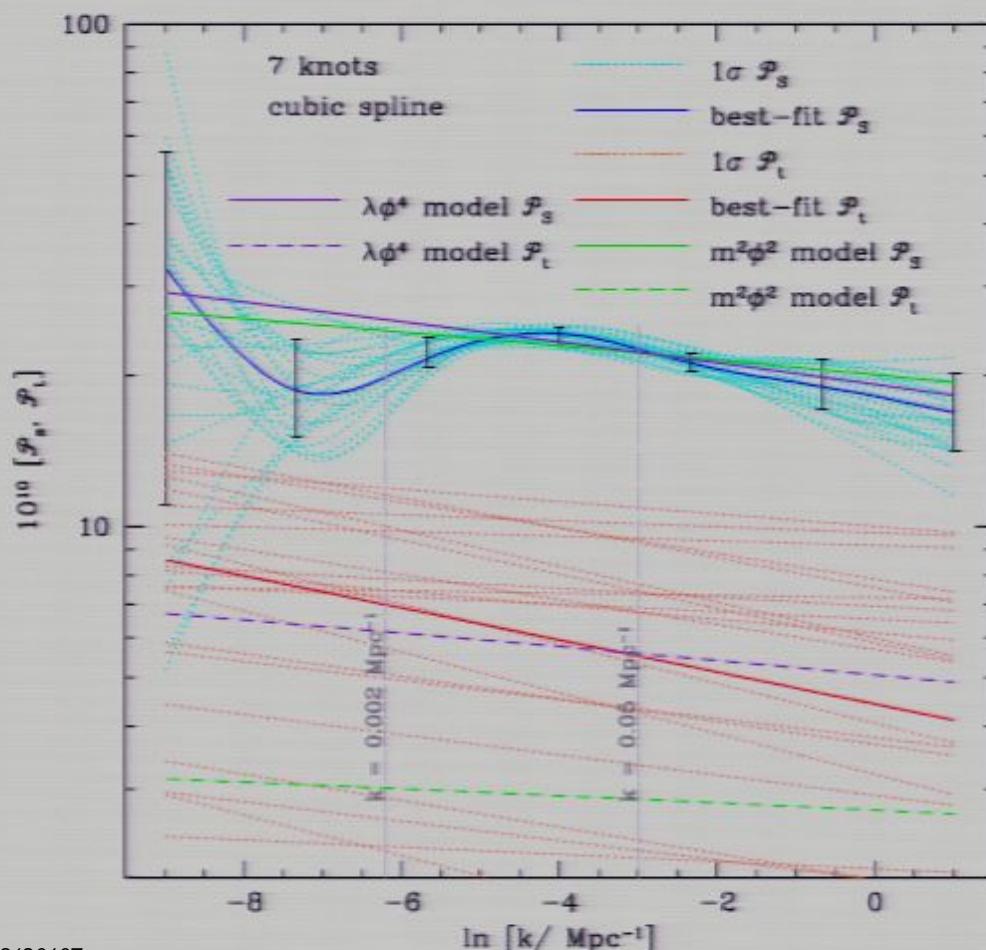
# Reconstructed Trajectories in $C_l$ Space



BB spectrum

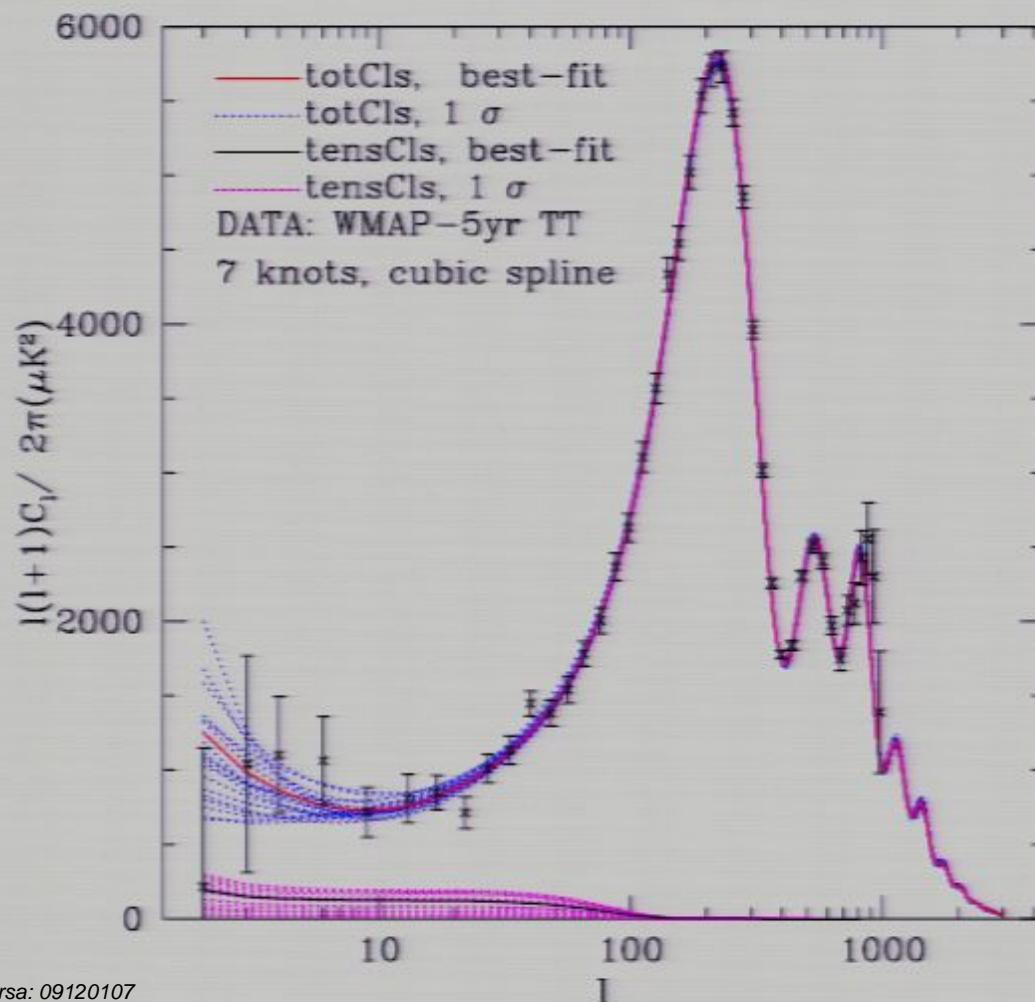


# Reconstructed Primordial Power Spectra

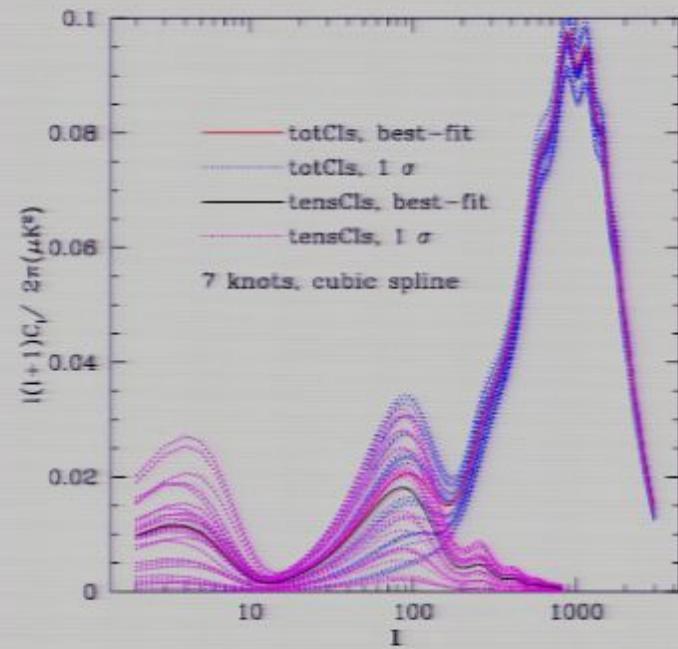


$$r(0.002 \text{Mpc}^{-1}) < 0.89 \text{ (95% CL)}$$

## Reconstructed Trajectories in $C_l$ Space



BB spectrum

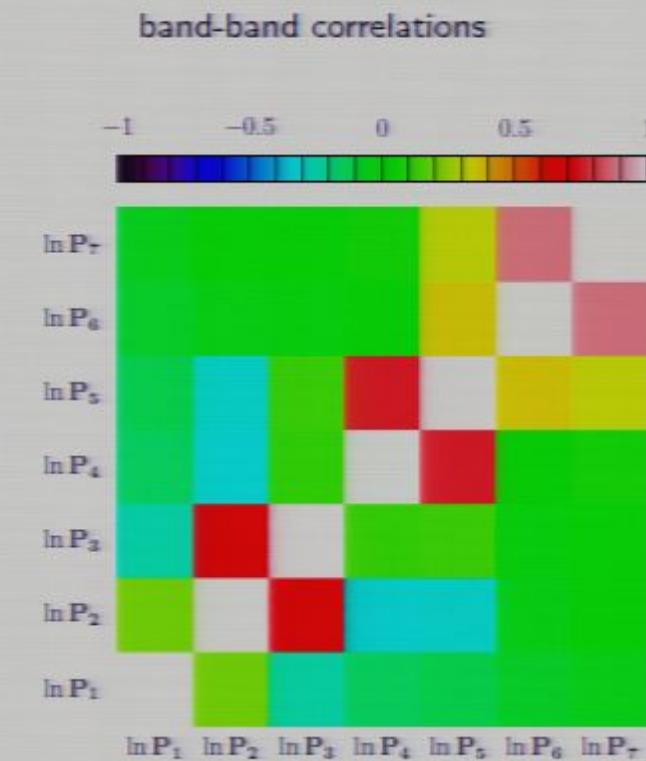
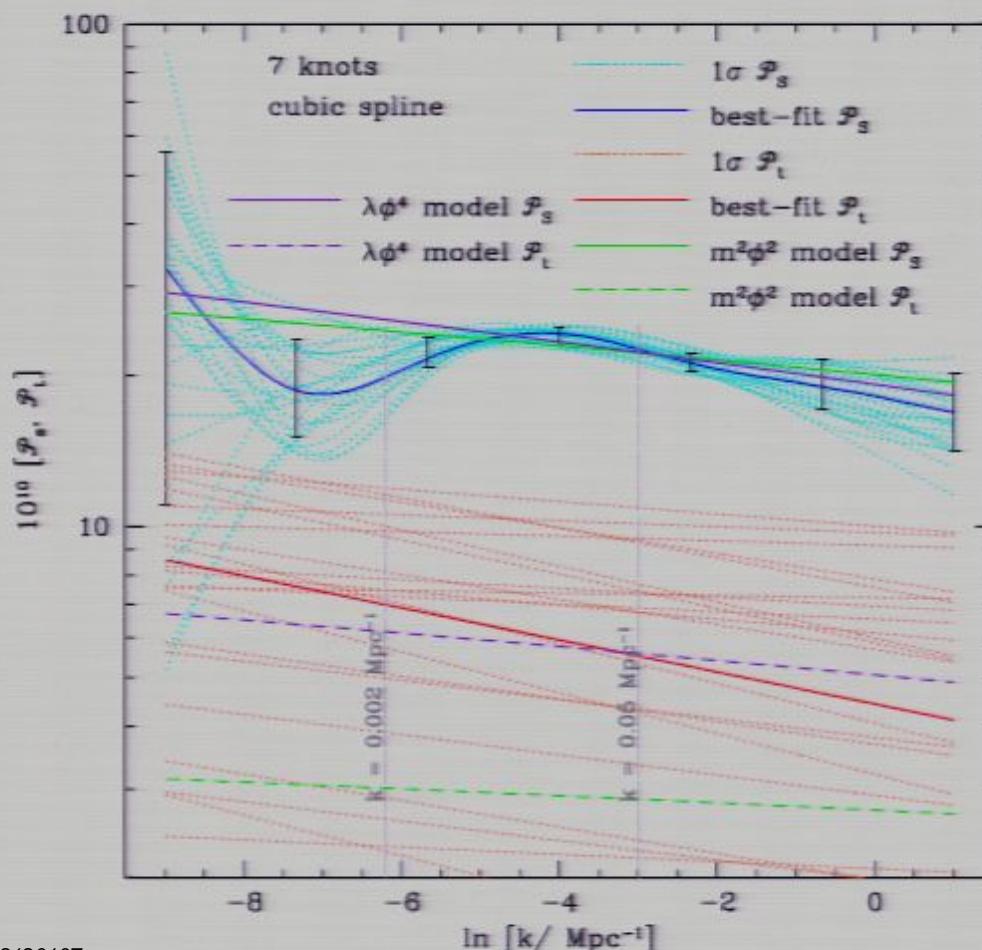


## Outline

- Introduction: Concordance Model and Primordial Power Spectra
- Cosmological Observations: Current Data and Forecasts
- Single-Field Slow-Roll Inflation
- Releasing the Slow-Roll Assumptions
- Summary and Conclusions

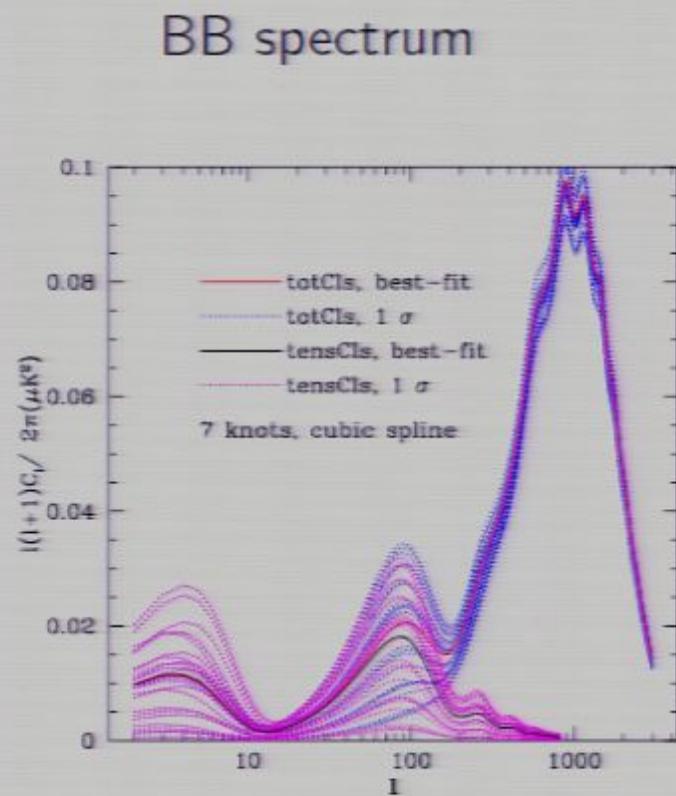
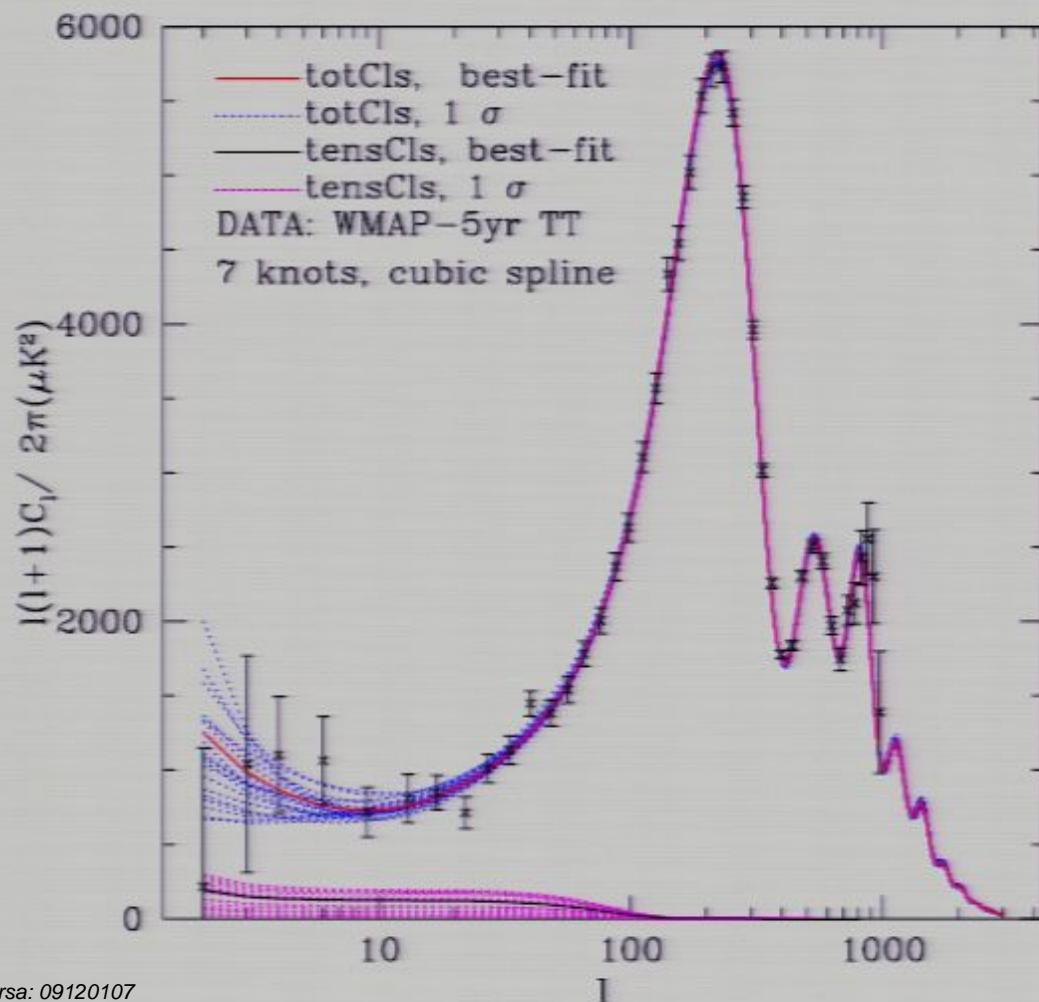
## Phenomenological Parametrization Consistency Treatment

# Reconstructed Primordial Power Spectra

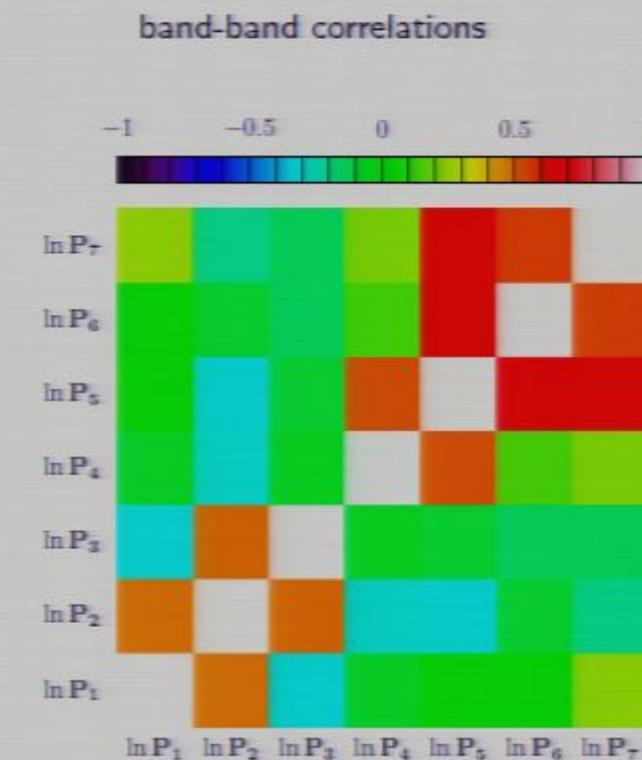
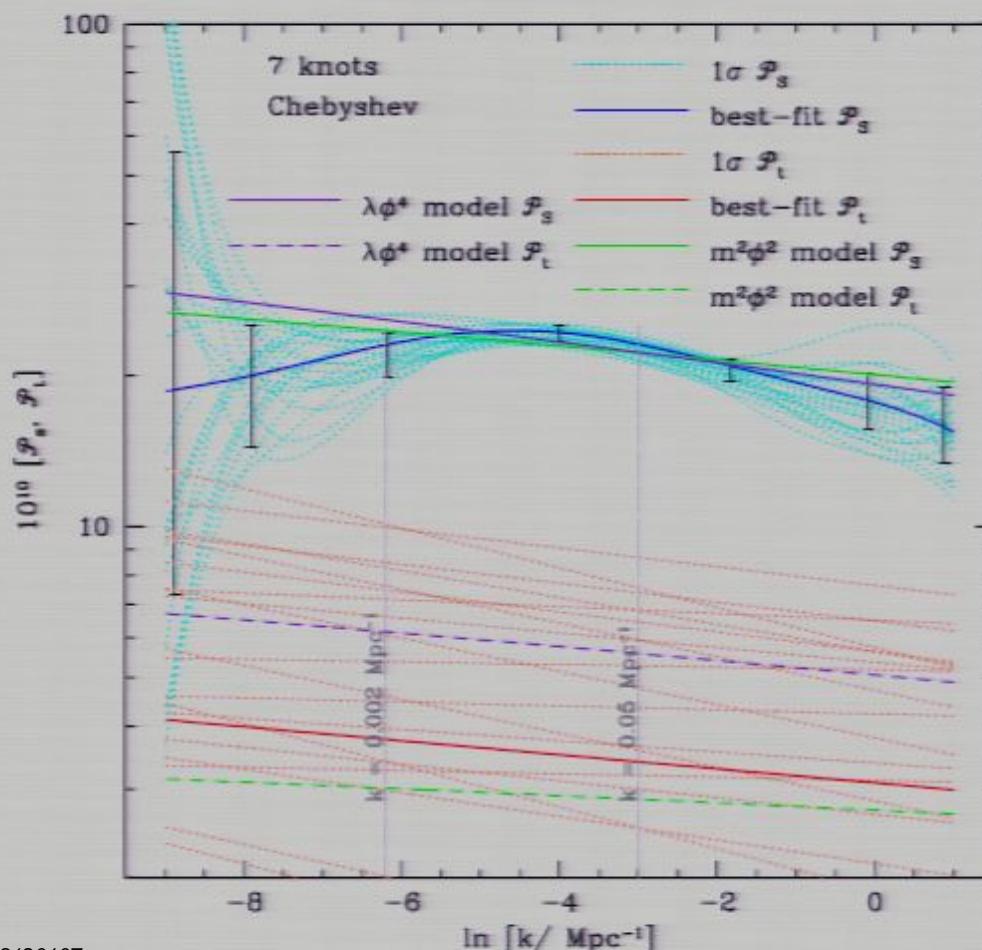


$$r(0.002 \text{Mpc}^{-1}) < 0.89 \text{ (95% CL)}$$

## Reconstructed Trajectories in $C_l$ Space



## Vary the Interpolation Method: Chebyshev Interpolation



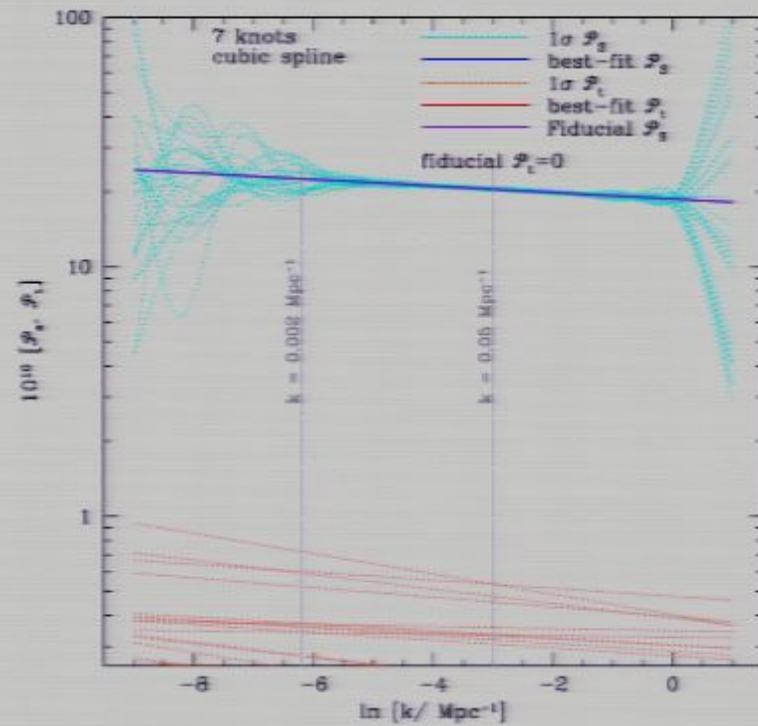
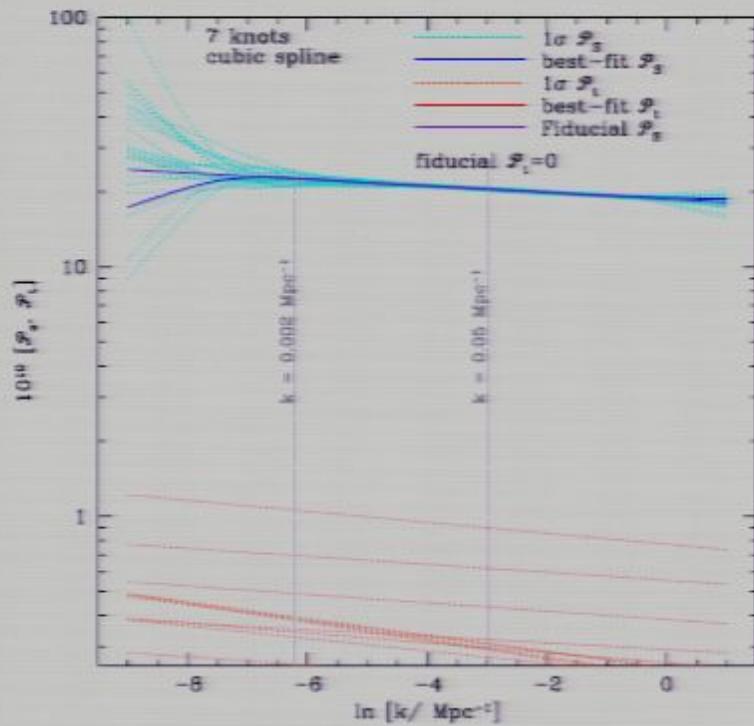
$r(0.002 \text{Mpc}^{-1}) < 0.68 \text{ (95\% CL)}$

## Outline

- Introduction: Concordance Model and Primordial Power Spectra
- Cosmological Observations: Current Data and Forecasts
- Single-Field Slow-Roll Inflation
- Releasing the Slow-Roll Assumptions
- Summary and Conclusions

## Phenomenological Parametrization Consistency Treatment

# Forecast: 7 knots and 13 knots



$$r(0.002 \text{ Mpc}^{-1}) < 0.041 \text{ (95\% CL)}$$

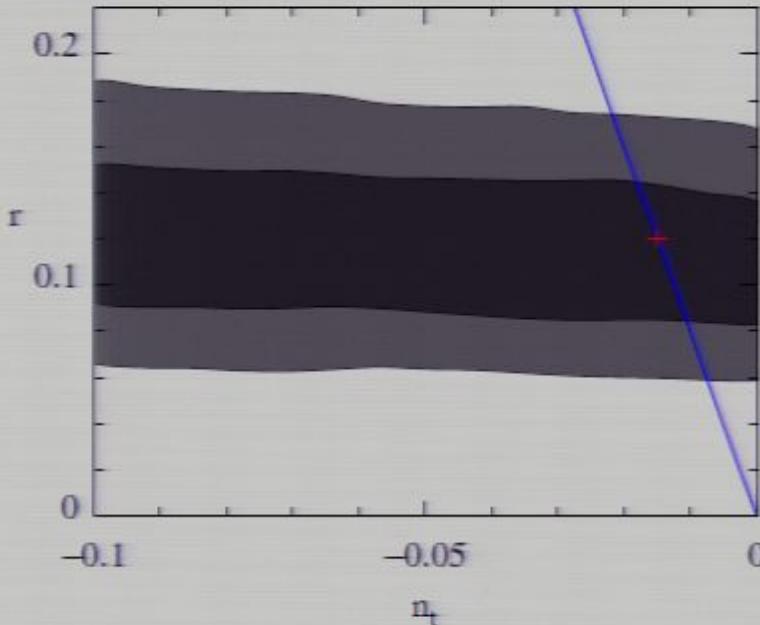
Pirsa: 09120107

$$r(0.002 \text{ Mpc}^{-1}) < 0.044 \text{ (95\% CL)}$$

Page 37/63

## What is Consistency and Why Consistency

- ▶ In the context of single-field inflation:  
 uniform acceleration  $\epsilon \approx \text{const}$ :  $n_t \approx -2\epsilon \approx -\frac{r}{8}$   
 generic  $\epsilon(\ln k)$ : more complicated functional relationship .
- ▶ Even the next generation data can not measure the consistency  
 $\Rightarrow$  need to be treated as a theoretical prior.



## Consistency Treatment

Expansion history  $H(\ln a) \Rightarrow P_s(k)$  and  $P_t(k)$ .

Method #1: slow-roll approximation

$$P_s(k) \approx \frac{1}{8\pi^2 M_p^2} \frac{H^2}{\epsilon} \Big|_{k=aH},$$

$$P_t(k) \approx \frac{2}{\pi^2 M_p^2} H^2 \Big|_{k=aH},$$

where

$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$

Inflation  $\Leftrightarrow 0 < \epsilon < 1$ .

Method #2: numerical exact solution

$$P_s(k) \propto |\mathcal{R}_k|^2,$$

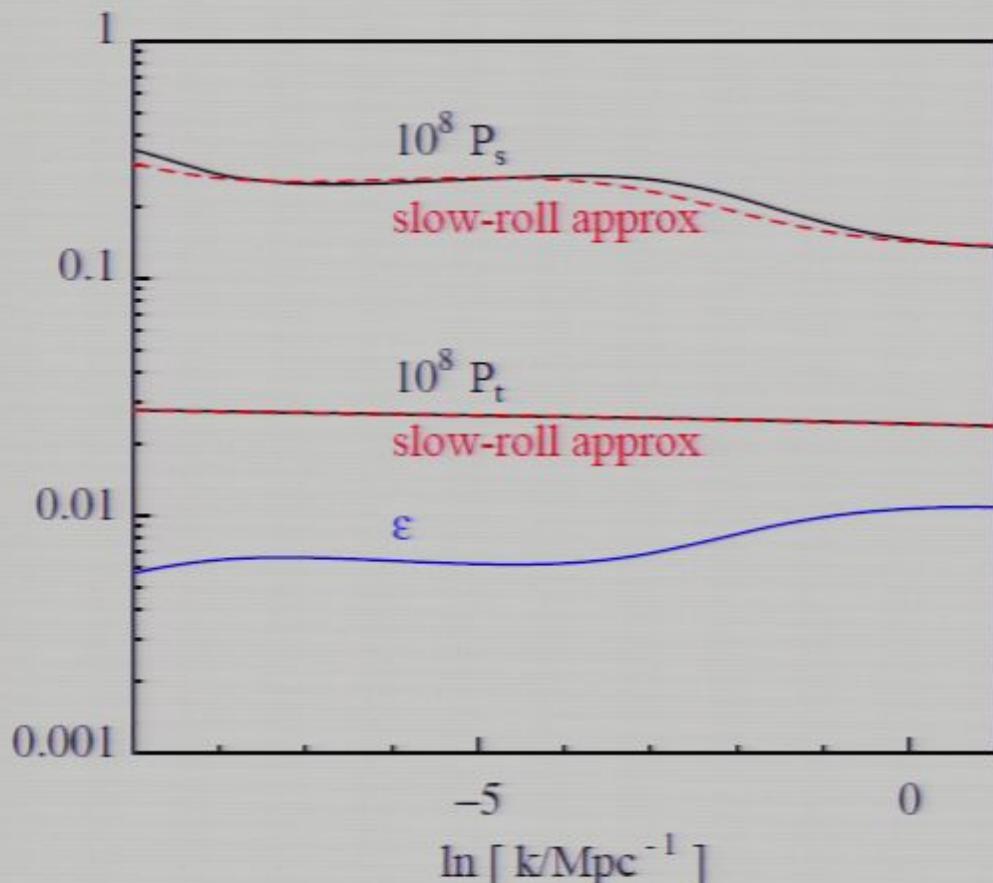
$$P_t(k) \propto |h_k|^2,$$

where  $\mathcal{R}_k$  and  $h_k$  can be solved from:

$$\mathcal{R}_k'' + 2 \frac{(\sqrt{\epsilon}a)'}{\sqrt{\epsilon}a} \mathcal{R}_k + k^2 \mathcal{R}_k = 0,$$

$$h_k'' + 2 \frac{a'}{a} h_k + k^2 h_k = 0.$$

## How Good Is the Slow-Roll Approximation?



If  $| \frac{d \ln \epsilon}{d \ln k} | \lesssim 0.1$ ,

- ▶  $P_s$ : a few percents error
- ▶  $P_t$ : sufficiently good probably ok for current data

For future precision cosmology better use exact power spectra.

## Consistency Treatment

Expansion history  $H(\ln a) \Rightarrow P_s(k)$  and  $P_t(k)$ .

Method #1: slow-roll approximation

Method #2: numerical exact solution

$$P_s(k) \approx \frac{1}{8\pi^2 M_p^2} \frac{H^2}{\epsilon} \Big|_{k=aH},$$

$$P_t(k) \approx \frac{2}{\pi^2 M_p^2} H^2 \Big|_{k=aH},$$

where

$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$

Inflation  $\Leftrightarrow 0 < \epsilon < 1$ .

$$P_s(k) \propto |\mathcal{R}_k|^2,$$

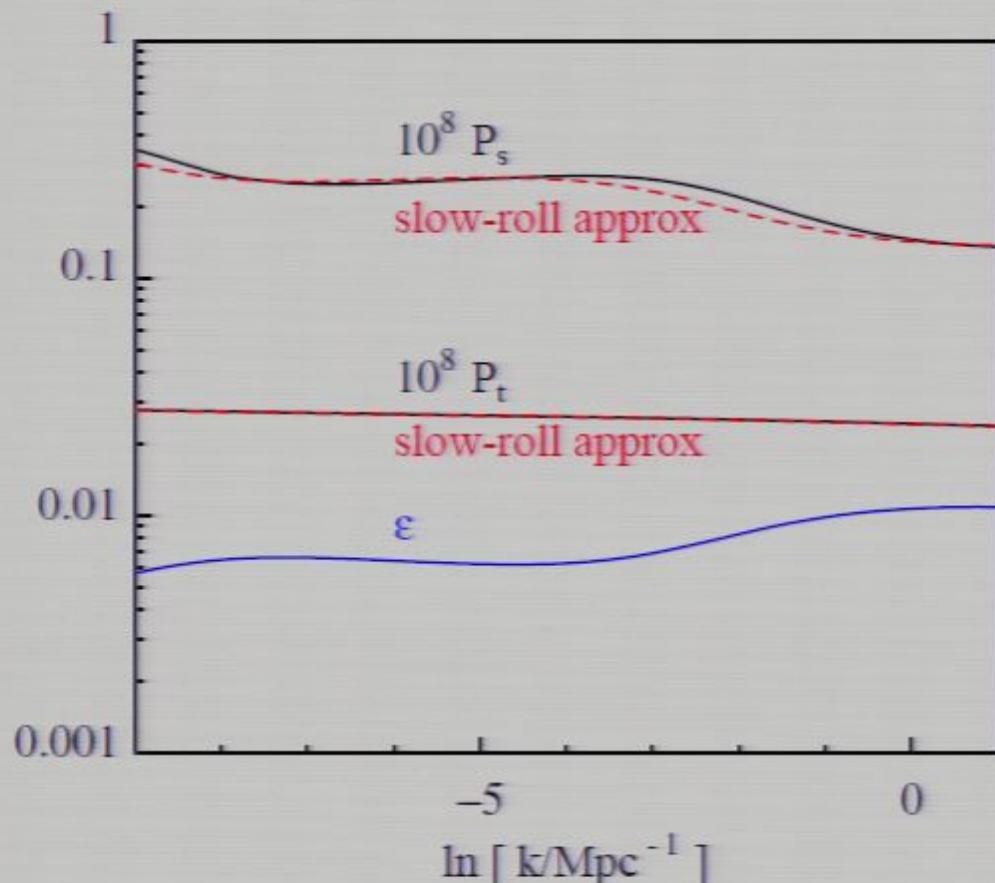
$$P_t(k) \propto |h_k|^2,$$

where  $\mathcal{R}_k$  and  $h_k$  can be solved from:

$$\mathcal{R}_k'' + 2 \frac{(\sqrt{\epsilon}a)'}{\sqrt{\epsilon}a} \mathcal{R}_k + k^2 \mathcal{R}_k = 0,$$

$$h_k'' + 2 \frac{a'}{a} h_k + k^2 h_k = 0.$$

## How Good Is the Slow-Roll Approximation?



If  $\left| \frac{d \ln \epsilon}{d \ln k} \right| \lesssim 0.1$ ,

- ▶  $P_s$ : a few percents error
- ▶  $P_t$ : sufficiently good probably ok for current data

For future precision cosmology better use exact power spectra.

## Consistency Treatment

Expansion history  $H(\ln a) \Rightarrow P_s(k)$  and  $P_t(k)$ .

Method #1: slow-roll approximation

$$P_s(k) \approx \frac{1}{8\pi^2 M_p^2} \frac{H^2}{\epsilon} \Big|_{k=aH},$$

$$P_t(k) \approx \frac{2}{\pi^2 M_p^2} H^2 \Big|_{k=aH},$$

where

$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$

Inflation  $\Leftrightarrow 0 < \epsilon < 1$ .

Method #2: numerical exact solution

$$P_s(k) \propto |\mathcal{R}_k|^2,$$

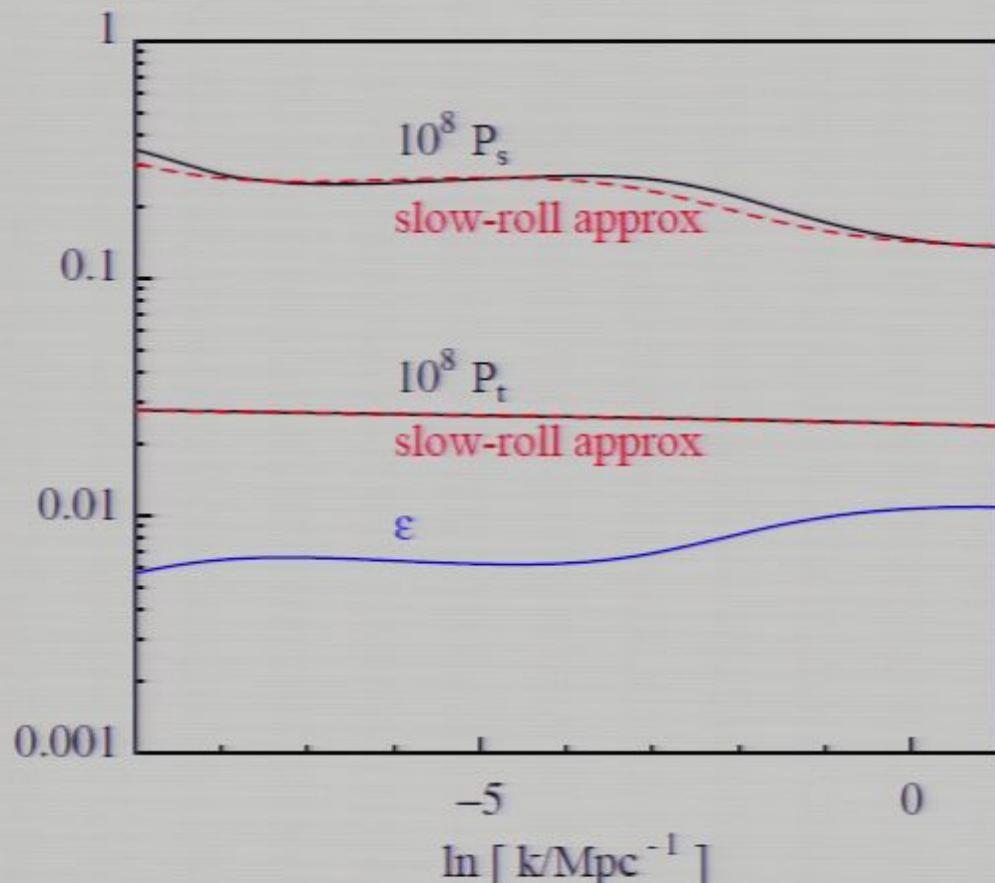
$$P_t(k) \propto |h_k|^2,$$

where  $\mathcal{R}_k$  and  $h_k$  can be solved from:

$$\mathcal{R}_k'' + 2 \frac{(\sqrt{\epsilon}a)'}{\sqrt{\epsilon}a} \mathcal{R}_k + k^2 \mathcal{R}_k = 0,$$

$$h_k'' + 2 \frac{a'}{a} h_k + k^2 h_k = 0.$$

## How Good Is the Slow-Roll Approximation?



If  $\left| \frac{d \ln \epsilon}{d \ln k} \right| \lesssim 0.1$ ,

- ▶  $P_s$ : a few percents error
- ▶  $P_t$ : sufficiently good probably ok for current data

For future precision cosmology better use exact power spectra.

## Consistency Treatment

Expansion history  $H(\ln a) \Rightarrow P_s(k)$  and  $P_t(k)$ .

Method #1: slow-roll approximation

Method #2: numerical exact solution

$$P_s(k) \approx \frac{1}{8\pi^2 M_p^2} \frac{H^2}{\epsilon} \Big|_{k=aH},$$

$$P_t(k) \approx \frac{2}{\pi^2 M_p^2} H^2 \Big|_{k=aH},$$

where

$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$

Inflation  $\Leftrightarrow 0 < \epsilon < 1$ .

$$P_s(k) \propto |\mathcal{R}_k|^2,$$

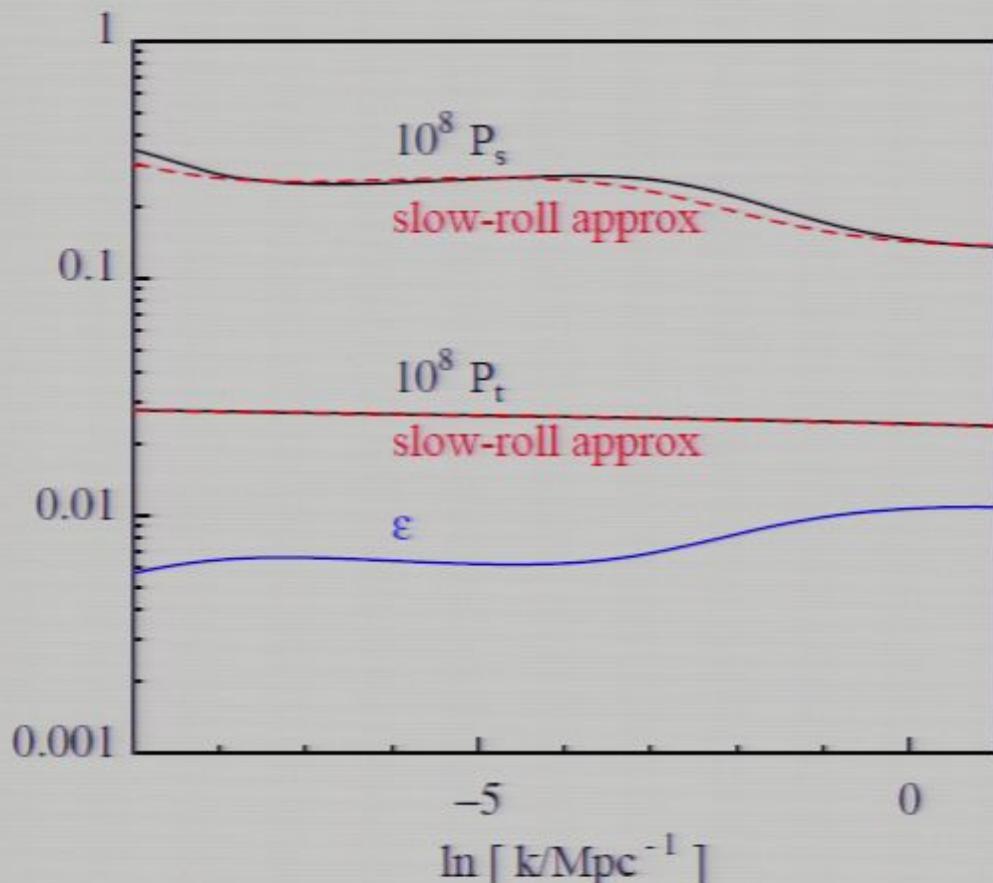
$$P_t(k) \propto |h_k|^2,$$

where  $\mathcal{R}_k$  and  $h_k$  can be solved from:

$$\mathcal{R}_k'' + 2\frac{(\sqrt{\epsilon}a)'}{\sqrt{\epsilon}a} \mathcal{R}_k + k^2 \mathcal{R}_k = 0,$$

$$h_k'' + 2\frac{a'}{a} h_k + k^2 h_k = 0.$$

## How Good Is the Slow-Roll Approximation?

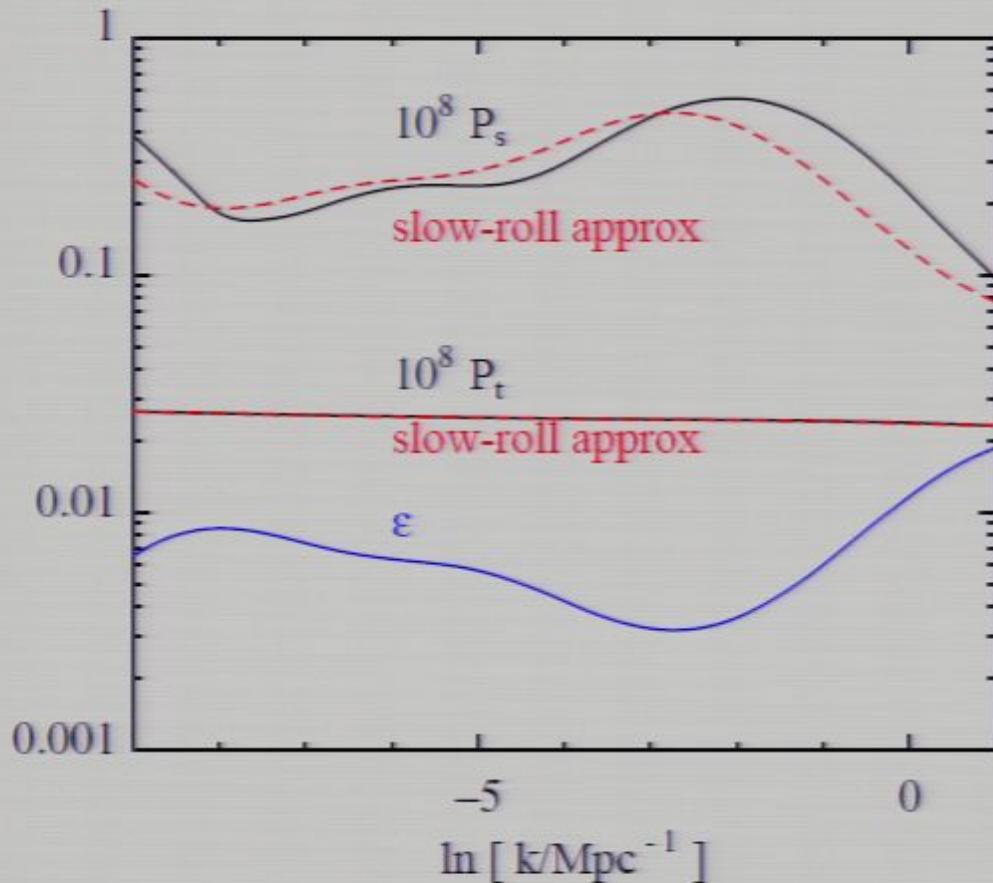


If  $\left| \frac{d \ln \epsilon}{d \ln k} \right| \lesssim 0.1$ ,

- ▶  $P_s$ : a few percents error
- ▶  $P_t$ : sufficiently good probably ok for current data

For future precision cosmology better use exact power spectra.

## A wilder example.



When  $|\frac{d \ln \epsilon}{d \ln k}| \sim 1$ ,

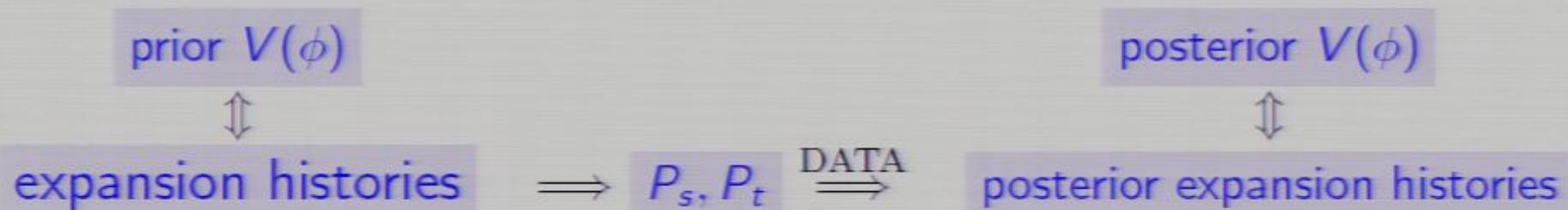
- ▶  $P_s$ : errors of order 1.
- ▶  $P_t$ : still not bad (this is generic given small  $\epsilon$ ).

## Constraining Expansion Histories

“ $2 + n$ ” parametrization:

- ▶  $\ln A_p \equiv \ln\left(\frac{H_{\text{pivot}}^2}{\epsilon_{\text{pivot}}}\right)$
- ▶  $\epsilon_{\text{pivot}}$
- ▶ Take  $\frac{d \ln \epsilon}{d \ln k}$  at  $n$  fixed knots  $k_1, k_2, \dots, k_n$ , interpolate in between (at knots prior  $-1 < \frac{d \ln \epsilon}{d \ln k} < 1$ ).

Each MCMC step numerically solve  $P_s(k)$  and  $P_t(k)$  (CPU time  $\sim$  a few WMAP likelihood evaluations).

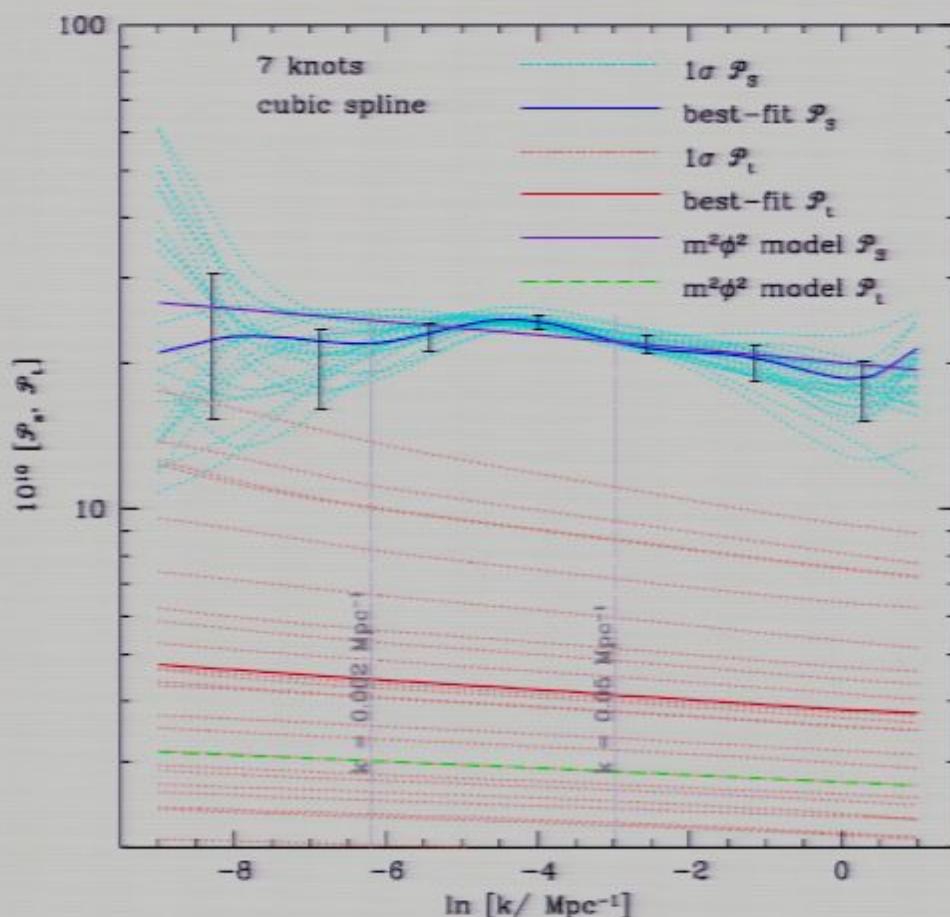


## Outline

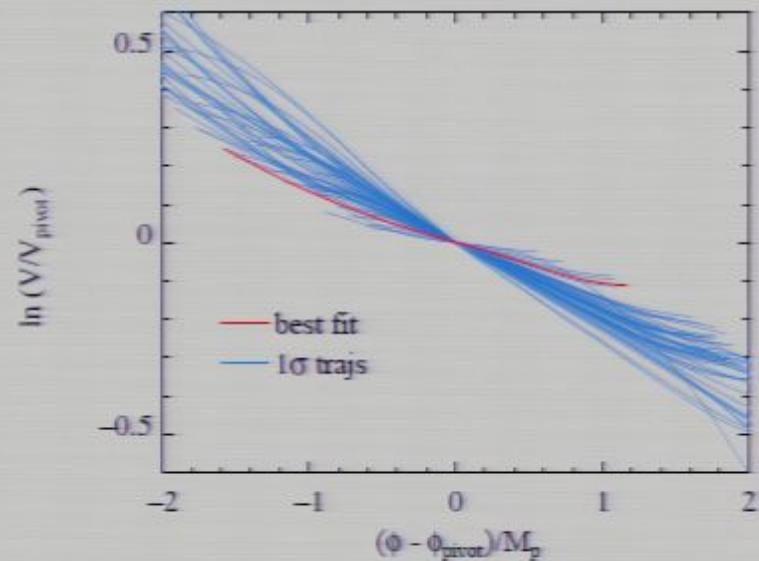
- Introduction: Concordance Model and Primordial Power Spectra
- Cosmological Observations: Current Data and Forecasts
- Single-Field Slow-Roll Inflation
- Releasing the Slow-Roll Assumptions**
- Summary and Conclusions

## Phenomenological Parametrization Consistency Treatment

# Reconstructed Power Spectra and Potentials



reconstructed potentials



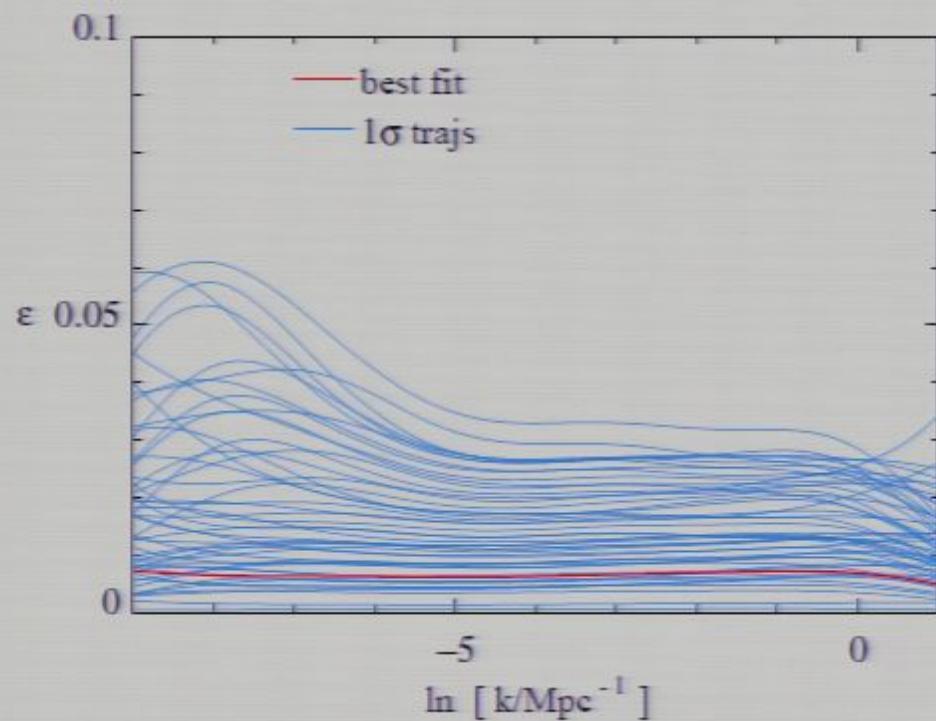
$$r(0.002 \text{ Mpc}^{-1}) < 0.66 \text{ (95% CL)}$$

## Outline

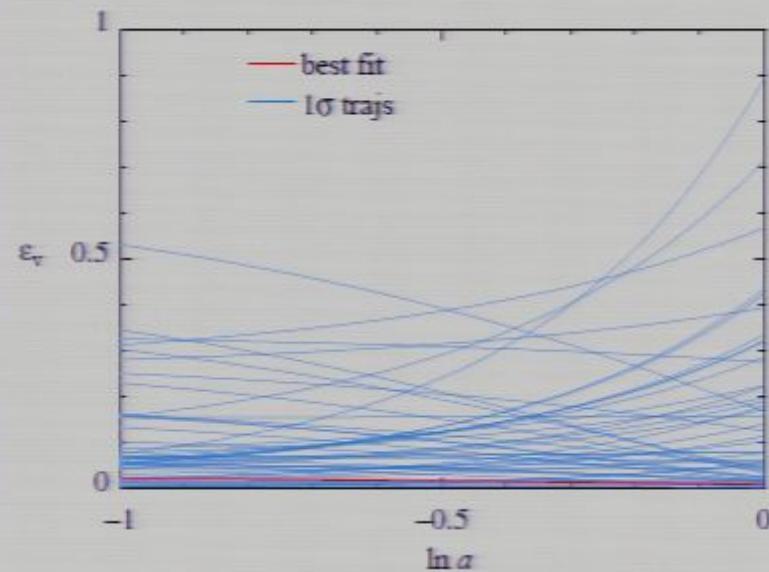
- Introduction: Concordance Model and Primordial Power Spectra
- Cosmological Observations: Current Data and Forecasts
- Single-Field Slow-Roll Inflation
- Releasing the Slow-Roll Assumptions**
- Summary and Conclusions

## Phenomenological Parametrization Consistency Treatment

# Reconstructed $\epsilon$ trajectories



comparing to dark energy  $\epsilon_v$  (just for fun)



# How Good Is the Slow-Roll Approximation?

Expansion theory ( $\ln(a) \rightarrow s(\eta)$  and  $t(\eta)$ )

Method #1: slow-roll approximation

$$P_s(k) \approx \frac{1}{8\pi^2 M_p^2} \frac{H^2}{\epsilon} \Big|_{k=aH},$$

$$P_t(k) \approx \frac{2}{\pi^2 M_p^2} H^2 \Big|_{k=aH},$$

where

$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$

Inflation  $\Leftrightarrow 0 < \epsilon < 1$ .

Method #2: numerical exact solution

$$P_s(k) \propto |\mathcal{R}_k|^2,$$

$$P_t(k) \propto |h_k|^2,$$

where  $\mathcal{R}_k$  and  $h_k$  can be solved from:

$$\mathcal{R}_k'' + 2\frac{(\sqrt{\epsilon}a)'}{\sqrt{\epsilon}a}\mathcal{R}_k + k^2\mathcal{R}_k = 0,$$

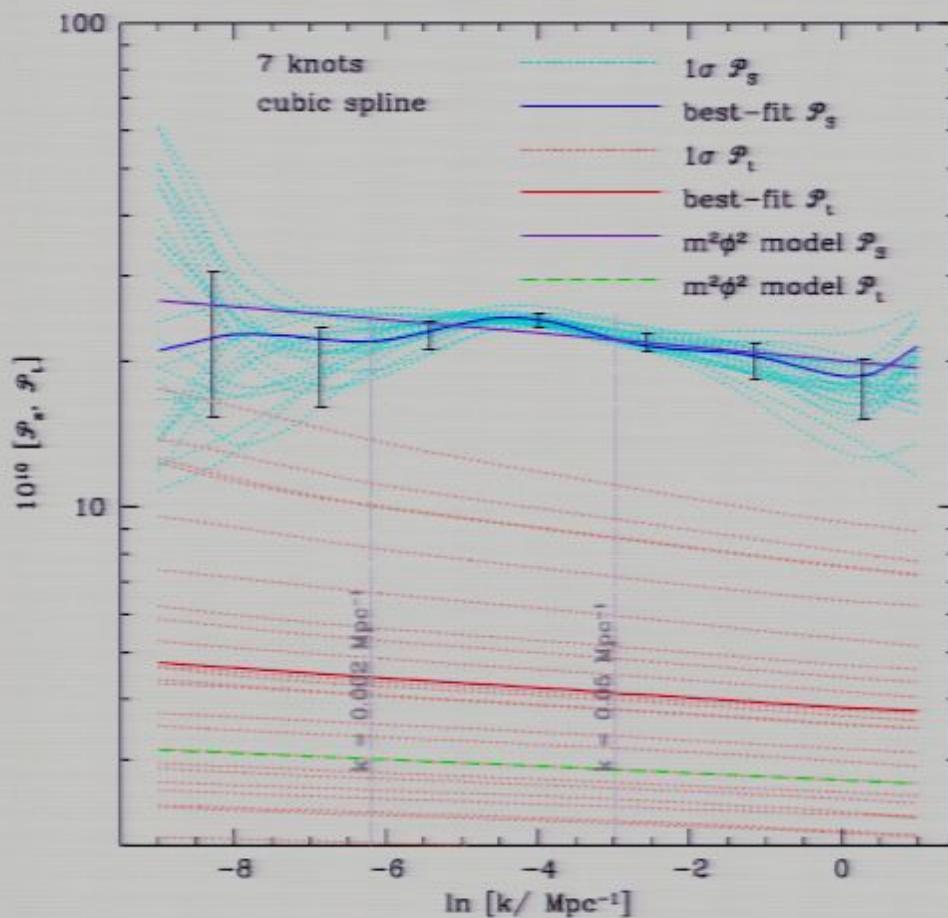
$$h_k'' + 2\frac{a'}{a}h_k + k^2h_k = 0.$$

## Outline

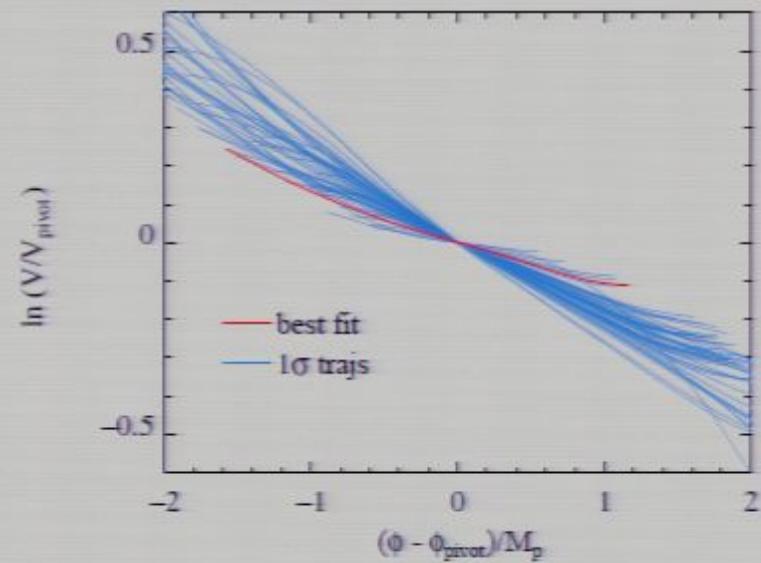
- Introduction: Concordance Model and Primordial Power Spectra
- Cosmological Observations: Current Data and Forecasts
- Single-Field Slow-Roll Inflation
- Releasing the Slow-Roll Assumptions
- Summary and Conclusions

## Phenomenological Parametrization Consistency Treatment

# Reconstructed Power Spectra and Potentials



reconstructed potentials



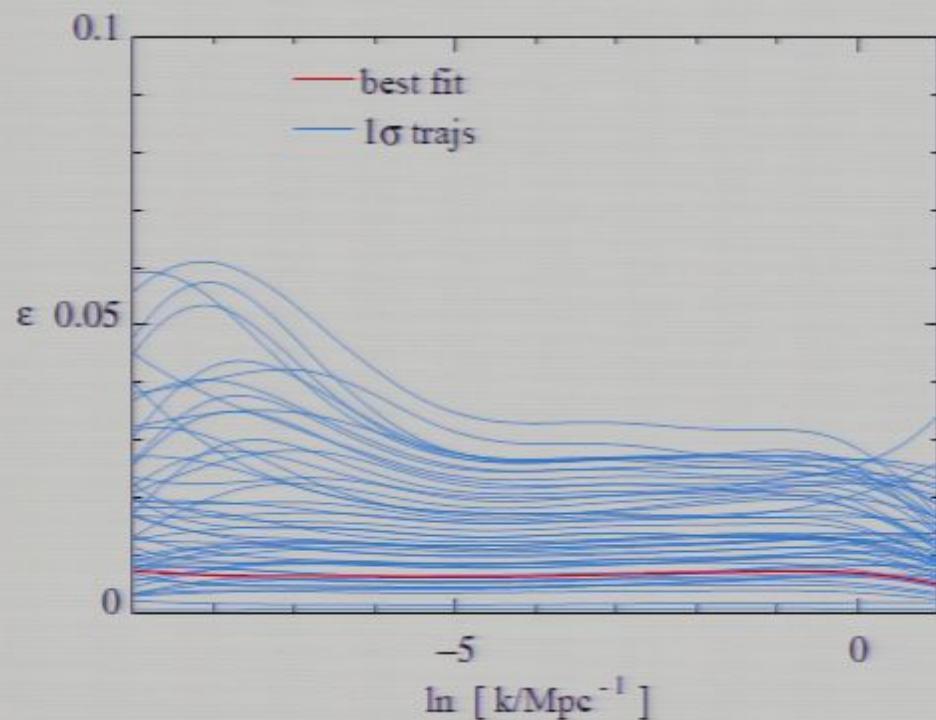
$$r(0.002 \text{ Mpc}^{-1}) < 0.66 \text{ (95% CL)}$$

## Outline

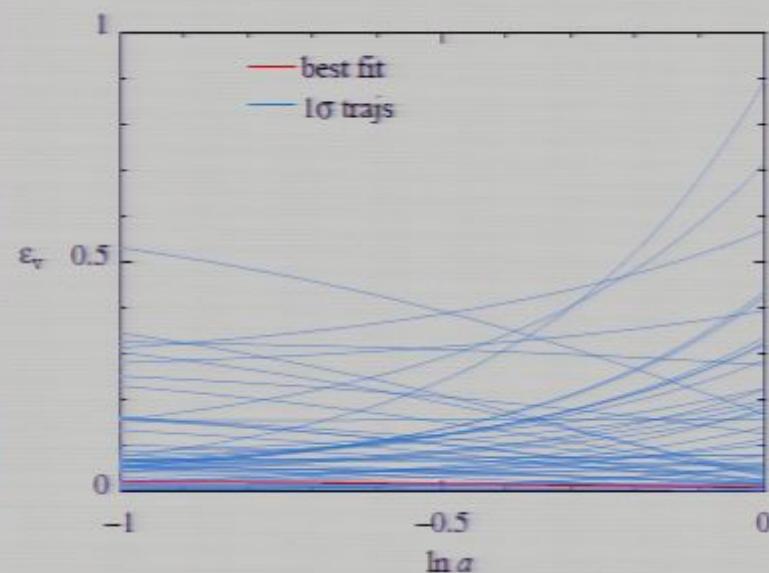
- Introduction: Concordance Model and Primordial Power Spectra
- Cosmological Observations: Current Data and Forecasts
- Single-Field Slow-Roll Inflation
- Releasing the Slow-Roll Assumptions**
- Summary and Conclusions

## Phenomenological Parametrization Consistency Treatment

# Reconstructed $\epsilon$ trajectories



comparing to dark energy  $\epsilon_V$  (just for fun)

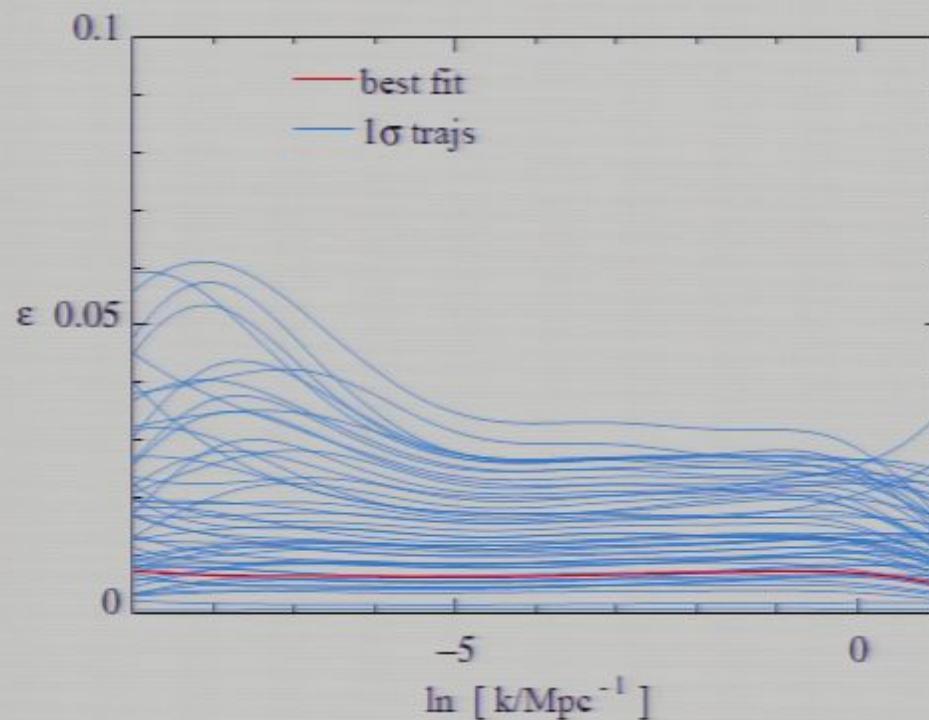


## Outline

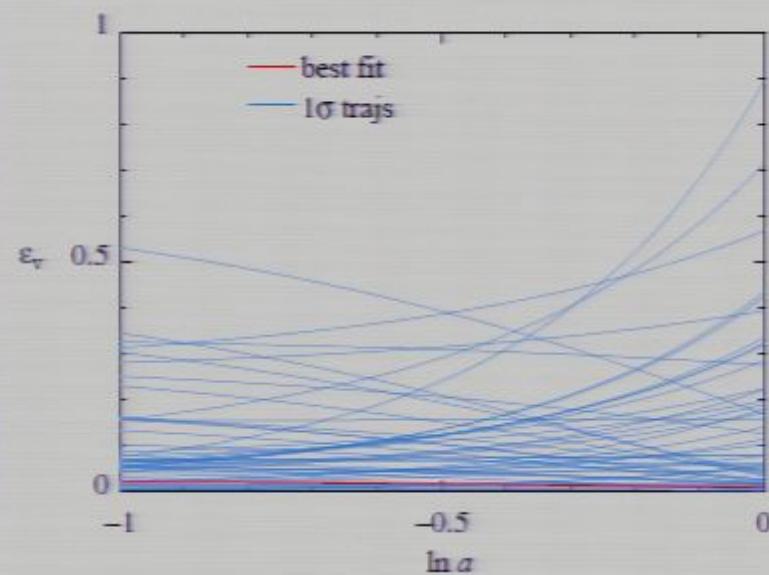
- Introduction: Concordance Model and Primordial Power Spectra
- Cosmological Observations: Current Data and Forecasts
- Single-Field Slow-Roll Inflation
- Releasing the Slow-Roll Assumptions
- Summary and Conclusions

## Phenomenological Parametrization Consistency Treatment

# Reconstructed $\epsilon$ trajectories



comparing to dark energy  $\epsilon_V$  (just for fun)



## Outline

Introduction: Concordance Model and Primordial Power Spectra

Cosmological Observations: Current Data and Forecasts

Single-Field Slow-Roll Inflation

Releasing the Slow-Roll Assumptions

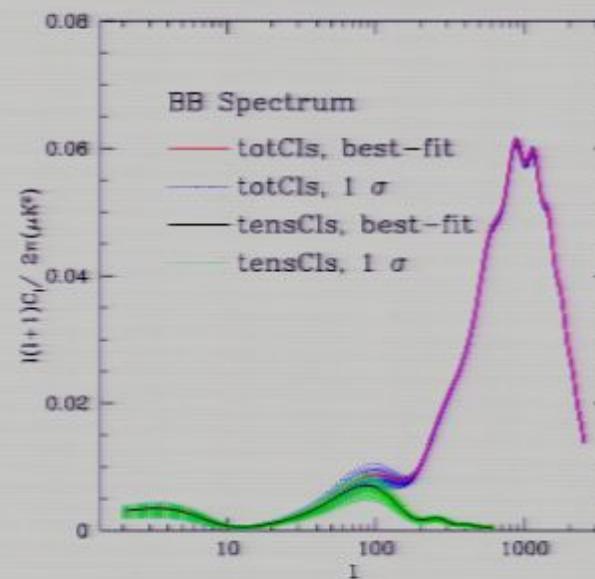
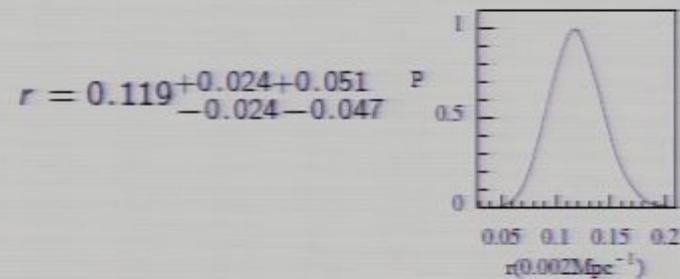
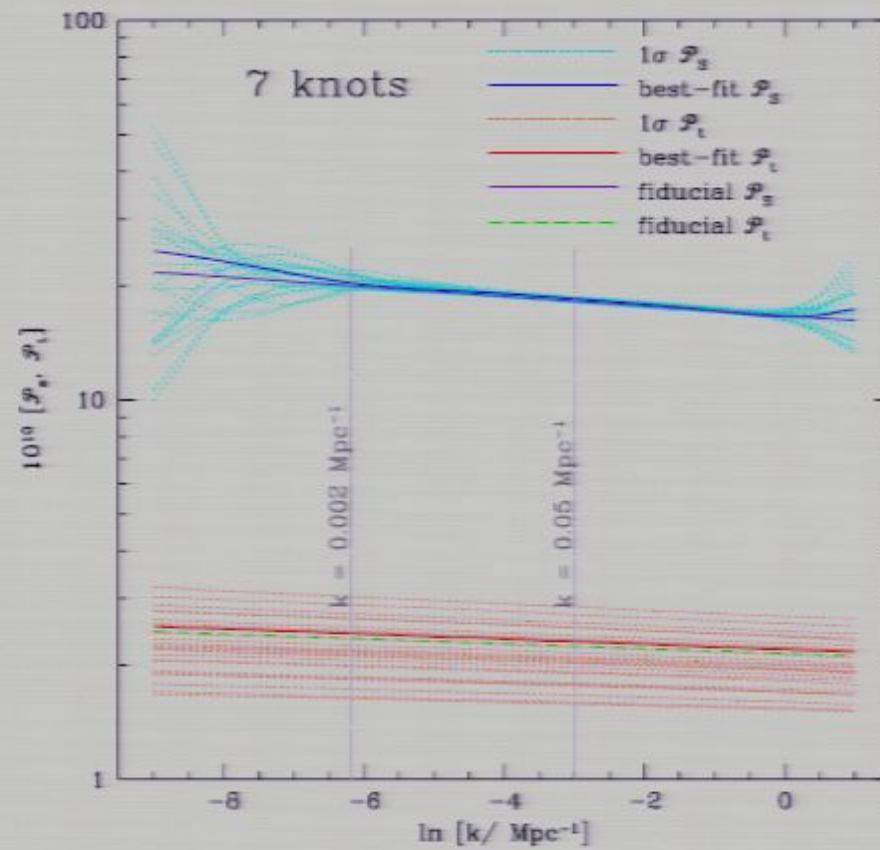
Summary and Conclusions

Phenomenological Parametrization

Consistency Treatment

# Reconstructing $m^2\phi^2$ model with mock data

Fiducial model:  $V(\phi) = \frac{1}{2}m^2\phi^2$ ,  $m = 5.5 \times 10^{-6} M_p$ ,  $\phi|_{0.002 \text{ Mpc}^{-1}} = 16.5 M_p$



# Directly Reconstruct $V(\phi)$

Parametrization:

$$V(\phi) = V_0 \alpha^2 \left( 1 + \alpha \Delta\phi + \frac{\beta}{2} \Delta\phi^2 + \frac{\gamma}{3!} \Delta\phi^3 + \frac{\omega}{4!} \Delta\phi^4 \right),$$

where

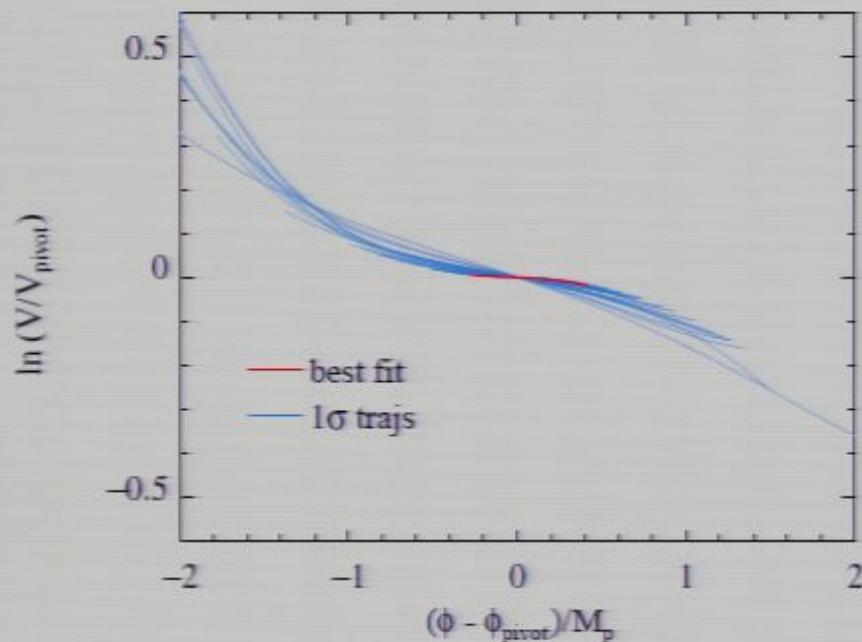
$$\Delta\phi \equiv (\phi - \phi_{\text{pivot}})/M_p$$

Priors:

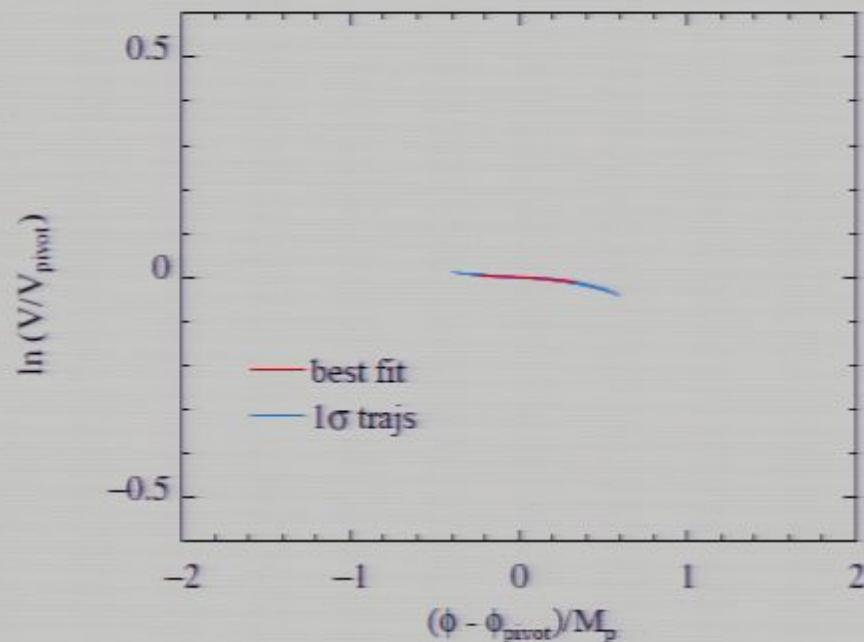
- ▶  $V(\phi)$  positive and monotonic (WLOG assume  $dV/d\phi < 0$ ).
- ▶  $0 < \epsilon < 1$
- ▶ (optional) small-field assumption  $\phi_{\max} - \phi_{\min} < M_p$

## Reconstructed potentials

without small-field assumption



with small-field assumption



## Directly Reconstruct $V(\phi)$

Parametrization:

$$V(\phi) = V_0 \alpha^2 \left( 1 + \alpha \Delta\phi + \frac{\beta}{2} \Delta\phi^2 + \frac{\gamma}{3!} \Delta\phi^3 + \frac{\omega}{4!} \Delta\phi^4 \right),$$

where

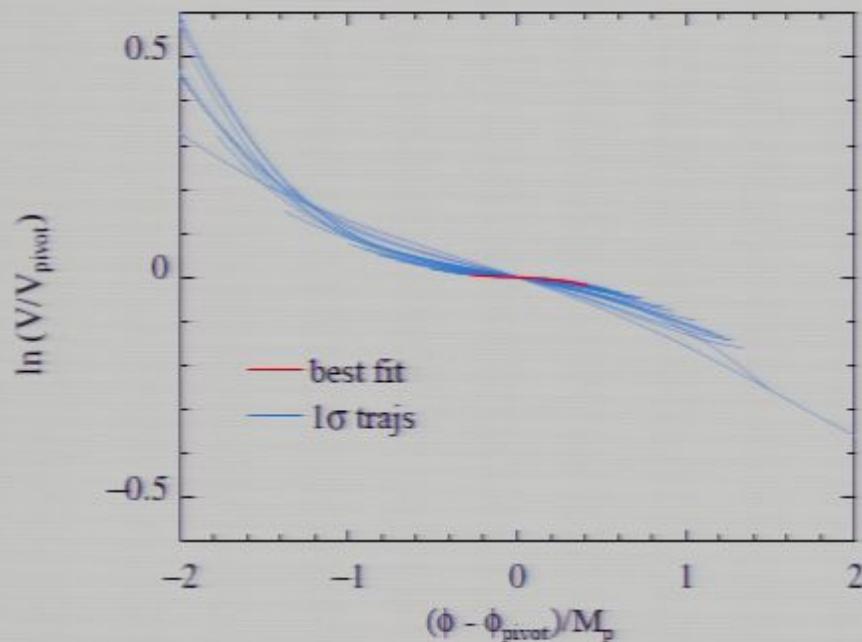
$$\Delta\phi \equiv (\phi - \phi_{\text{pivot}})/M_p$$

Priors:

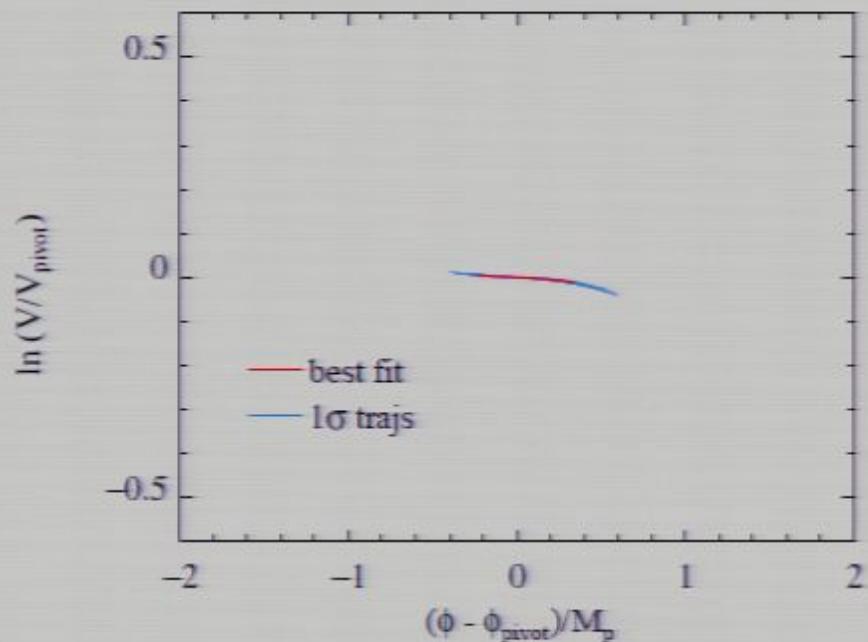
- ▶  $V(\phi)$  positive and monotonic (WLOG assume  $dV/d\phi < 0$ ).
- ▶  $0 < \epsilon < 1$
- ▶ (optional) small-field assumption  $\phi_{\text{max}} - \phi_{\text{min}} < M_p$

## Reconstructed potentials

without small-field assumption



with small-field assumption



## Summary and Conclusions

- ▶ Current constraint  $r < 0.16$  strongly depends on the uniform acceleration assumption.
- ▶ Completely releasing slow-roll assumption  $\Rightarrow$  degeneracy between  $P_s$  and  $P_t \Rightarrow r \lesssim 0.7$
- ▶ The next generation cosmological observations can break the degeneracy between  $P_s$  and  $P_t$  at a much better level (constrain  $r < 0.04$  without assuming slow-roll).

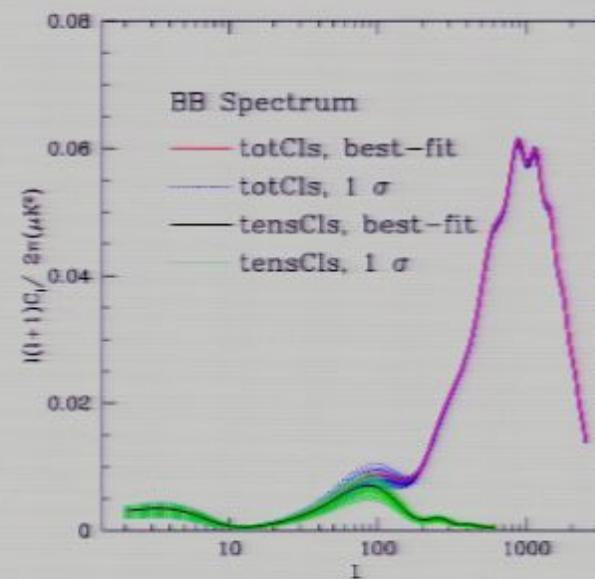
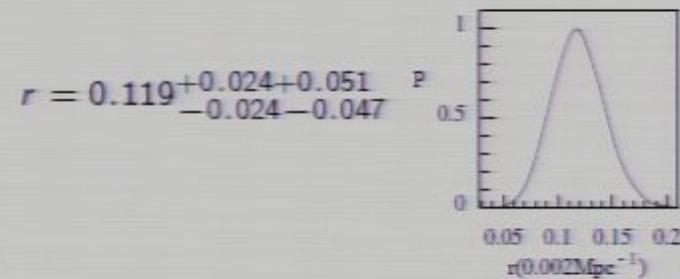
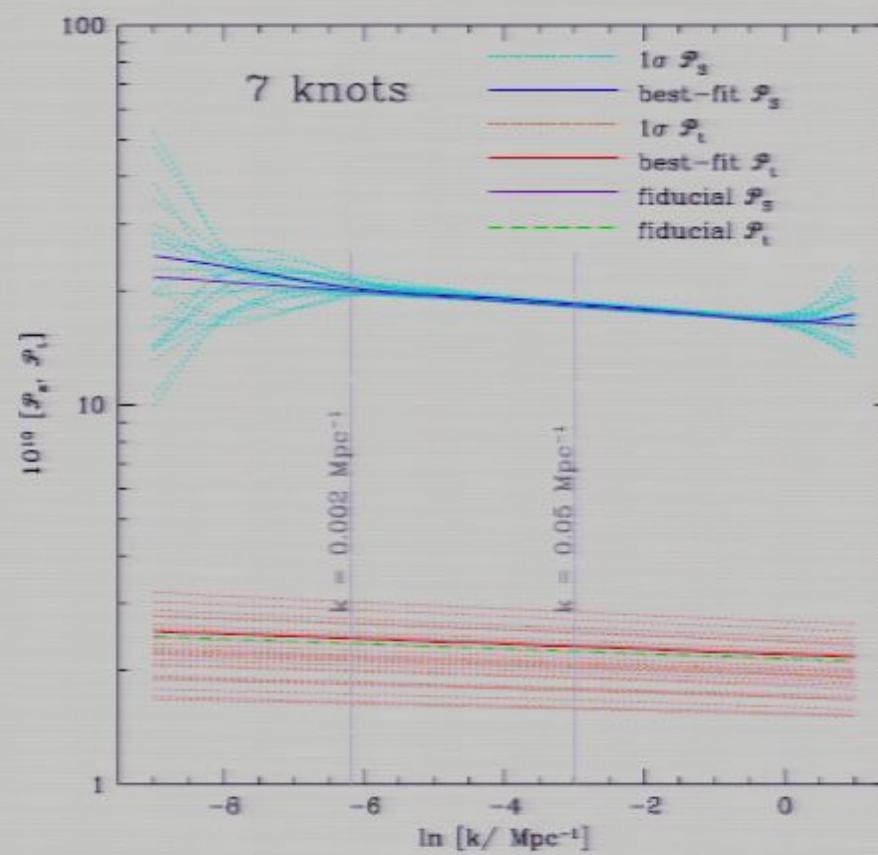
## Outline

- Introduction: Concordance Model and Primordial Power Spectra
- Cosmological Observations: Current Data and Forecasts
- Single-Field Slow-Roll Inflation
- Releasing the Slow-Roll Assumptions**
- Summary and Conclusions

## Phenomenological Parametrization Consistency Treatment

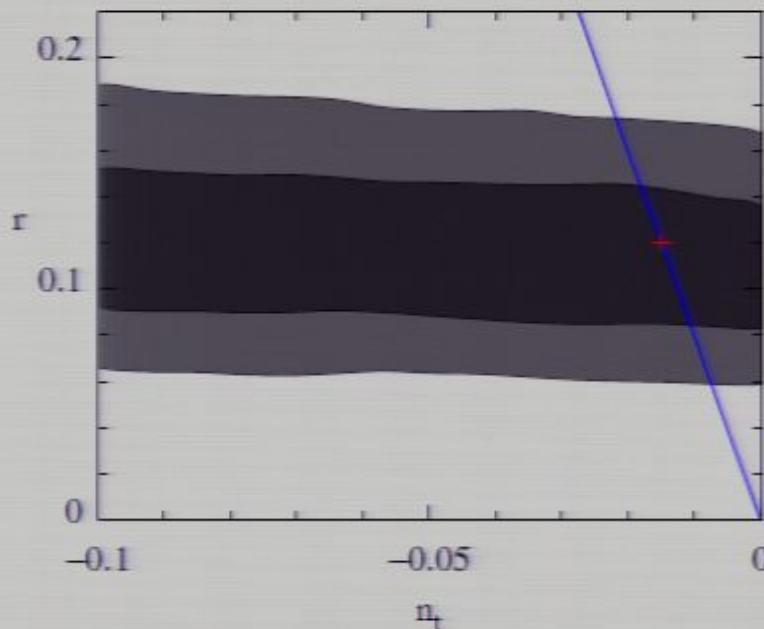
# Reconstructing $m^2\phi^2$ model with mock data

Fiducial model:  $V(\phi) = \frac{1}{2}m^2\phi^2$ ,  $m = 5.5 \times 10^{-6} M_p$ ,  $\phi|_{0.002 \text{Mpc}^{-1}} = 16.5 M_p$



## What is Consistency and Why Consistency

- ▶ In the context of single-field inflation:  
 uniform acceleration  $\epsilon \approx \text{const}$ :  $n_t \approx -2\epsilon \approx -\frac{r}{8}$   
 generic  $\epsilon(\ln k)$ : more complicated functional relationship .
- ▶ Even the next generation data can not measure the consistency  
 $\Rightarrow$  need to be treated as a theoretical prior.

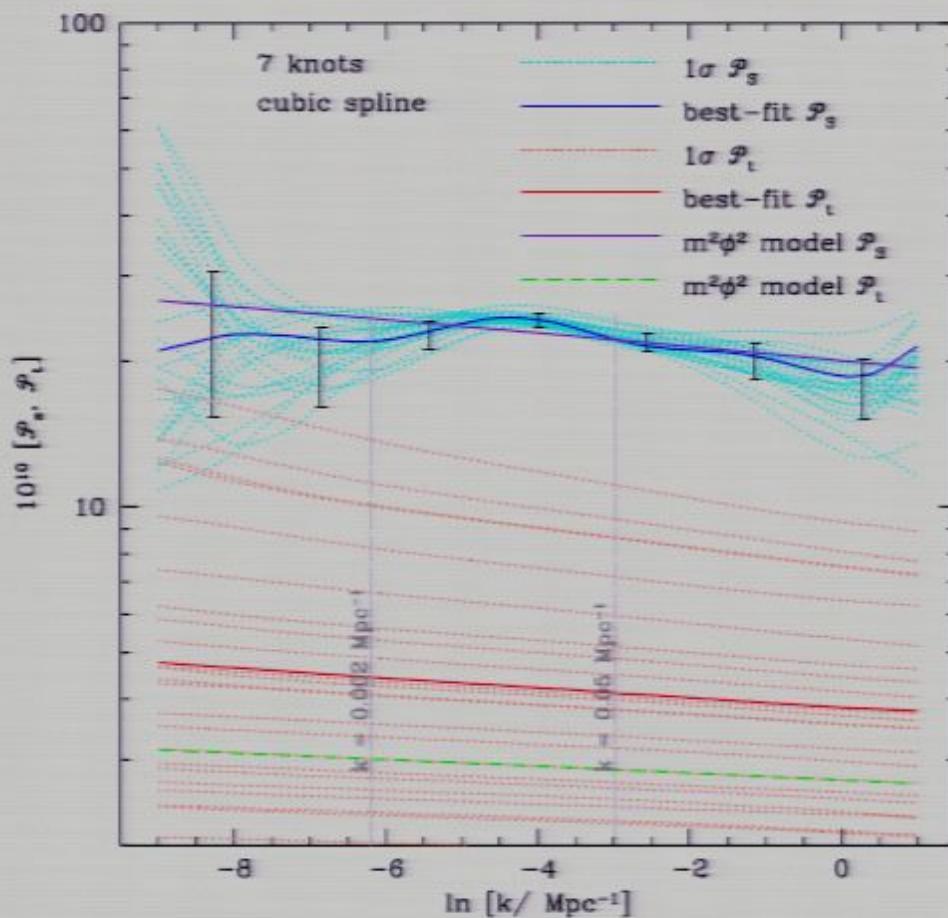


## Outline

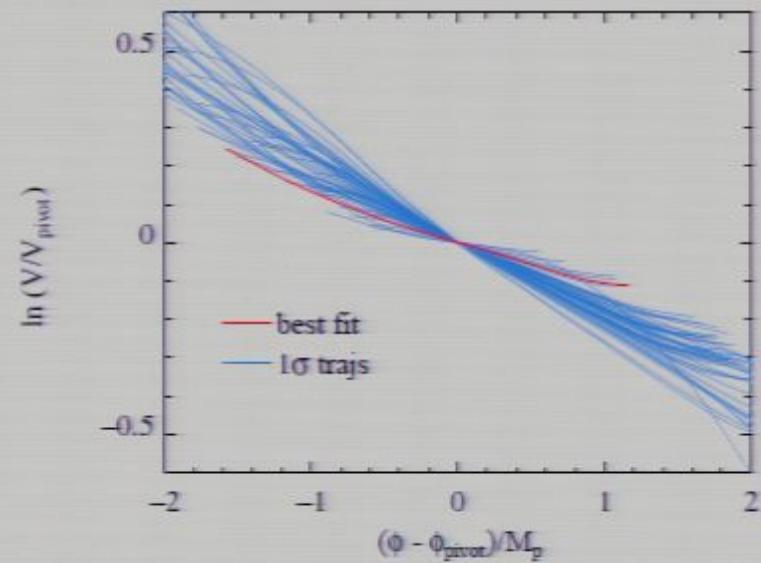
- Introduction: Concordance Model and Primordial Power Spectra
- Cosmological Observations: Current Data and Forecasts
- Single-Field Slow-Roll Inflation
- Releasing the Slow-Roll Assumptions**
- Summary and Conclusions

## Phenomenological Parametrization Consistency Treatment

# Reconstructed Power Spectra and Potentials



reconstructed potentials



$$r(0.002 \text{ Mpc}^{-1}) < 0.66 \text{ (95% CL)}$$