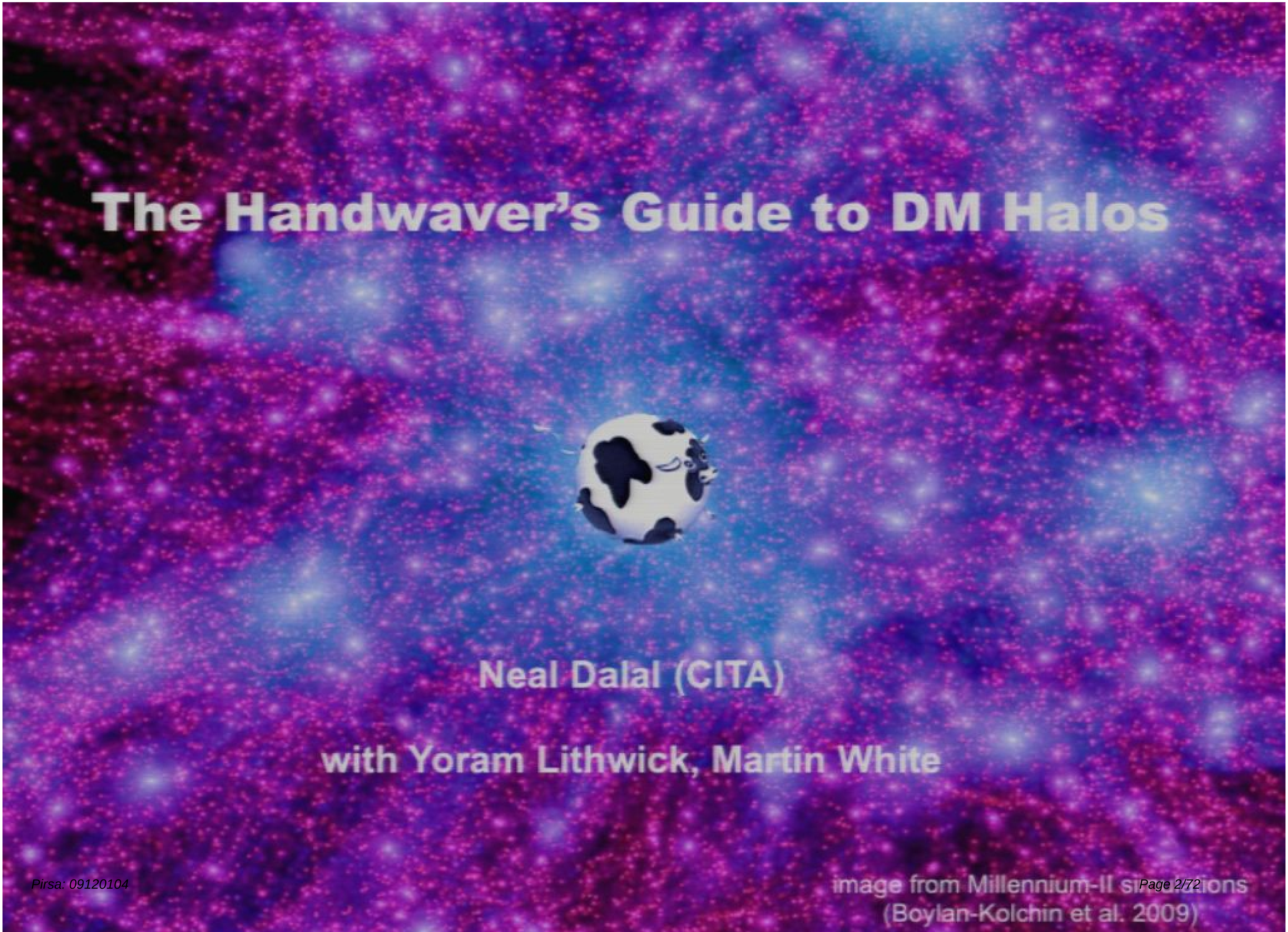
Title: Dark Matter Halos

Date: Dec 08, 2009 02:00 PM

URL: http://pirsa.org/09120104

Abstract:


The Handwaver's Guide to DM Halos
Neal Dalal (CITA)
with Yoram Lithwick, Martin White
image from Millennium-II simulations
(Boylan-Kolchin et al. 2009)

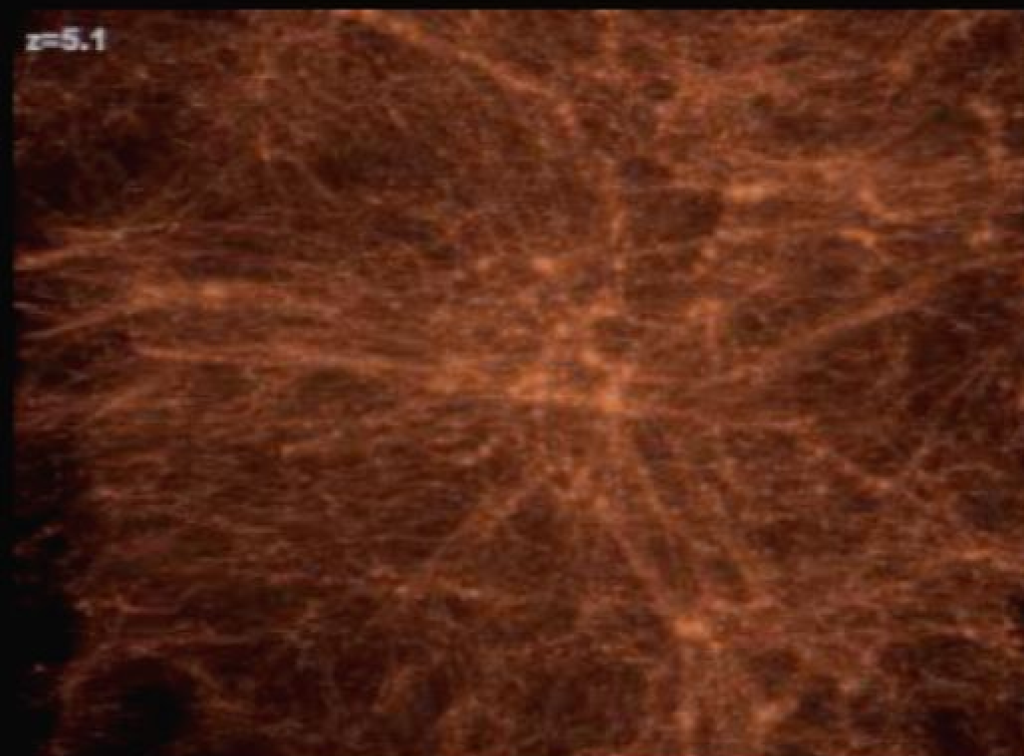
Halos

- N-body sims show regularity in halo properties:
  1. profile (NFW-ish)
  2. abundance (dn/dM)
  3. clustering (bias)
- I'll try to give a simple way to understand where these come from
- Then I'll discuss variations, e.g. what changes for cosmologies different than ΛCDM

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions



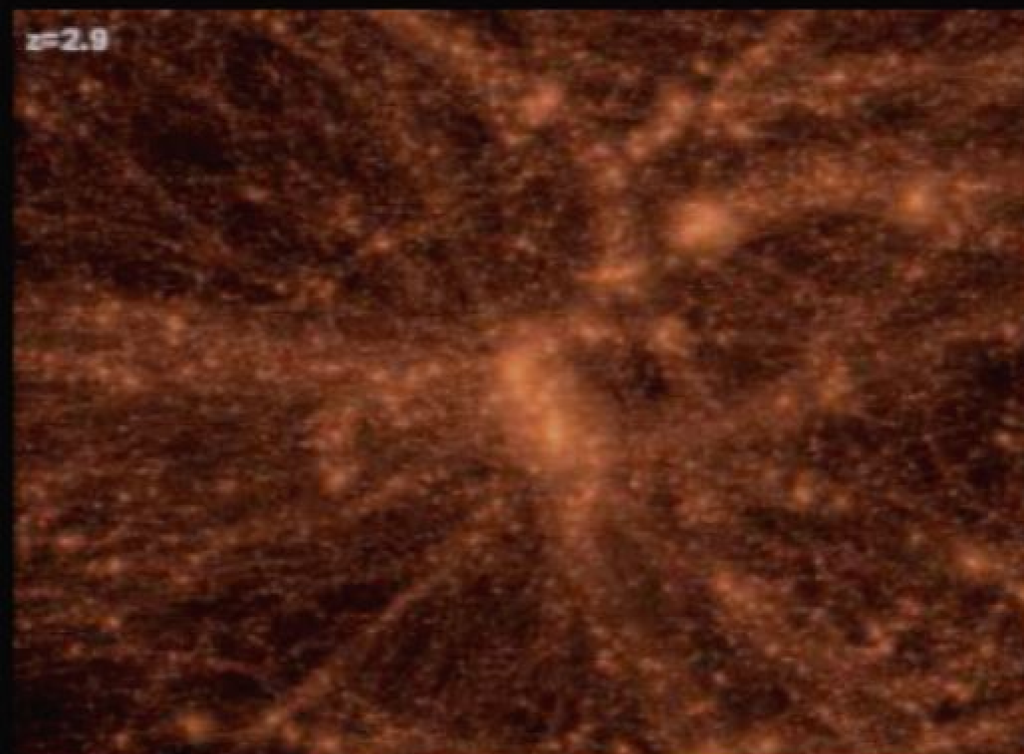
"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define,
but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions



“Via Lactea”
Diemand et al. 2006

“[A halo] is hard to define, but I know it when I see it.”
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define,
but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define,
but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

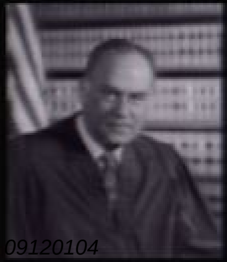
"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006



"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

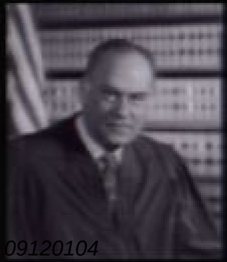
"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006



"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

What are halos?

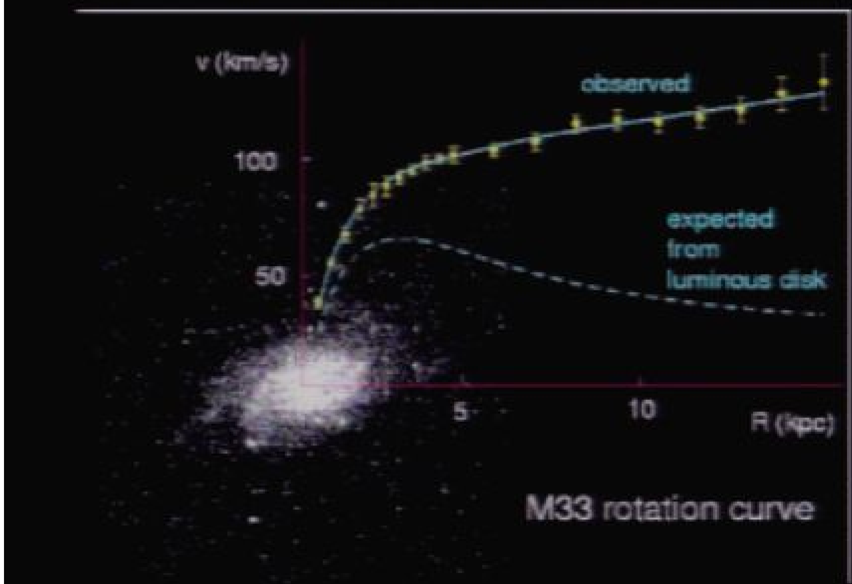
To theorists, halos are cosmological objects that are:

- gravitationally self-bound,
- virialized, and
- collapsed in all 3 dimensions

"Via Lactea"
Diemand et al. 2006

"[A halo] is hard to define, but I know it when I see it."
-- Potter Stewart, 1964

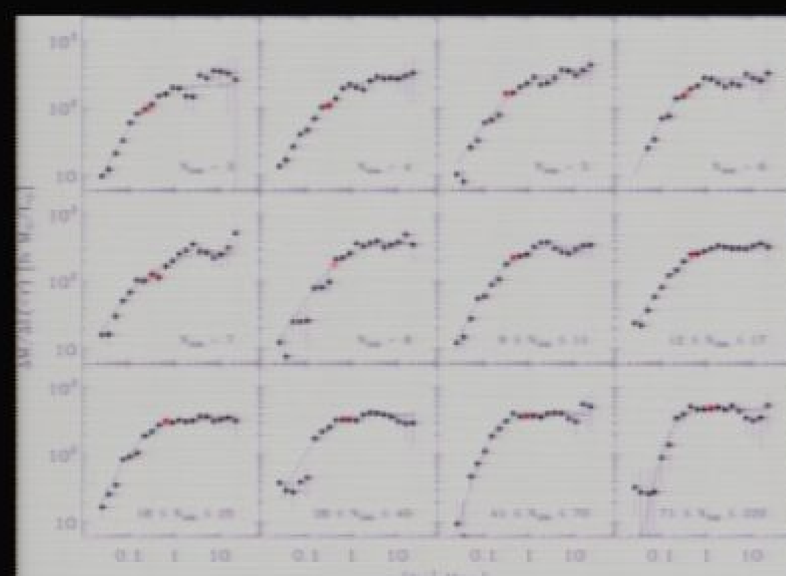
halos observed



M33 rotation curve


Sgr tidal stream


bullet cluster


gal-gal lensing

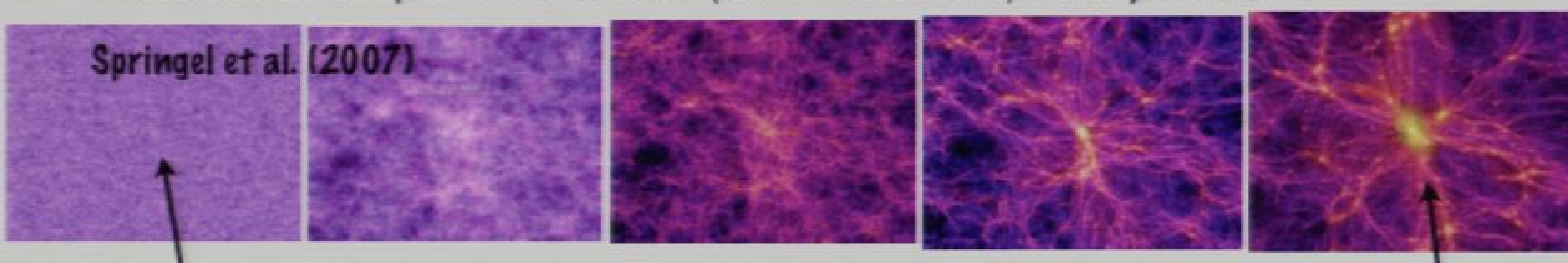
Who cares?

Halo properties are important for a huge range of topics in astrophysics & cosmology, e.g.

- sites of galaxy & star formation
- determines galaxy properties
- DM annihilation signal
- cluster abundance
- large-scale structure
- etc...

So we'd like to understand where our predictions for halo properties come from, in some simple robust way. The approach I'll take:

Halos come from peaks of the initial (Gaussian random) density field, so...



start with this

end with this

so properties of initial peaks ⇒ final halo properties

Self-similar calculations

To figure out what's going on, we'll examine a particular example in great detail:

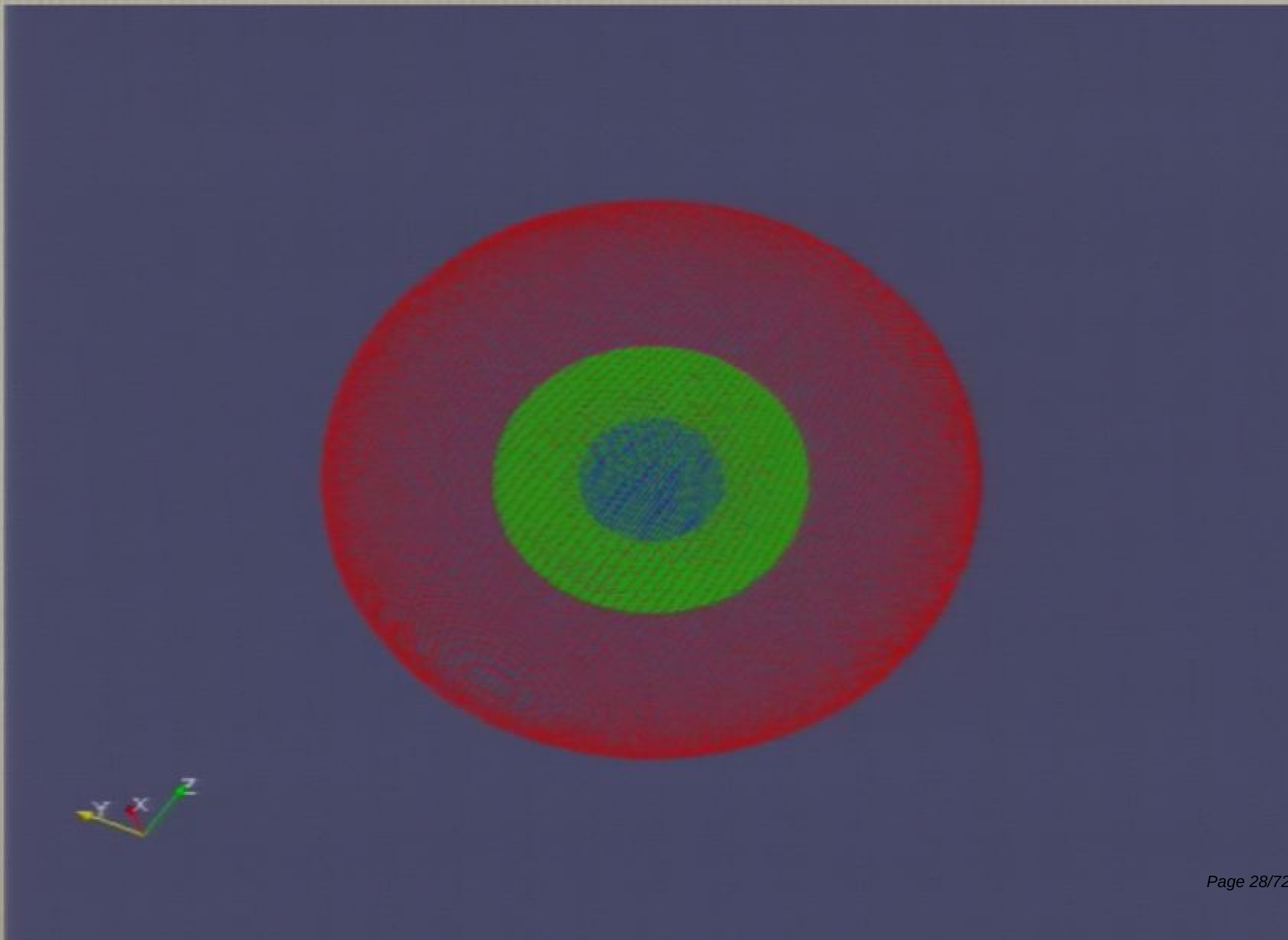
Self-similar calculations

To figure out what's going on, we'll examine a particular example in great detail:

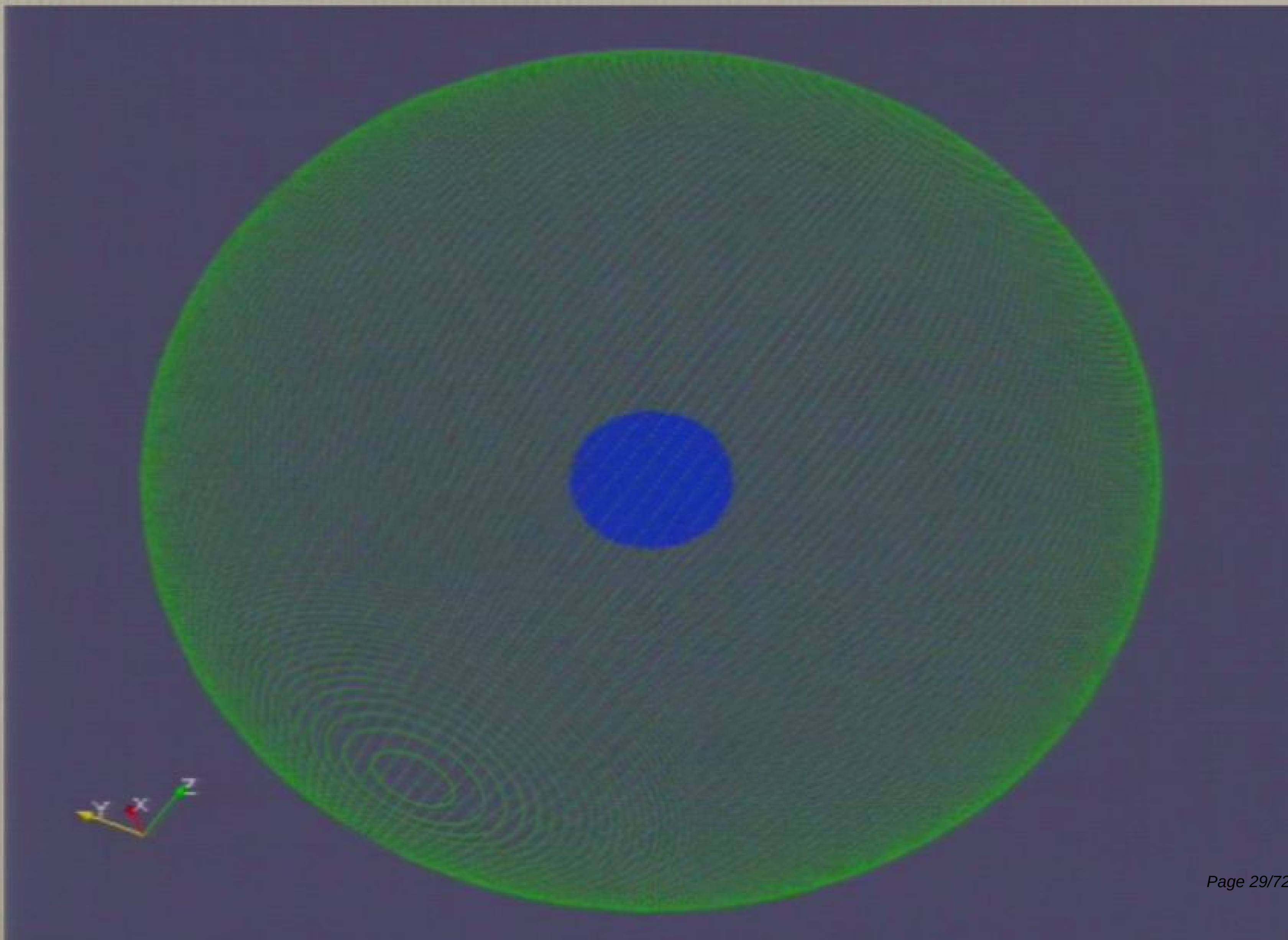
collapse of a scale-free, nonspherical profile $\delta\rho \propto r^{-\gamma} f(\theta,\varphi)$

Because this initial profile is scale-free (and gravity is scale-free), the problem admits a self-similar solution. Self-similarity allows us to achieve high spatial resolution just by integrating for a longer time. Our calculations typically have a spatial dynamic range of $\gtrsim 10^{10}$, with run-times many orders of magnitude faster than usual N-body simulations.

Spherical Self-Similar Solution
(Fillmore & Goldreich 1984, Bertschinger 1985)

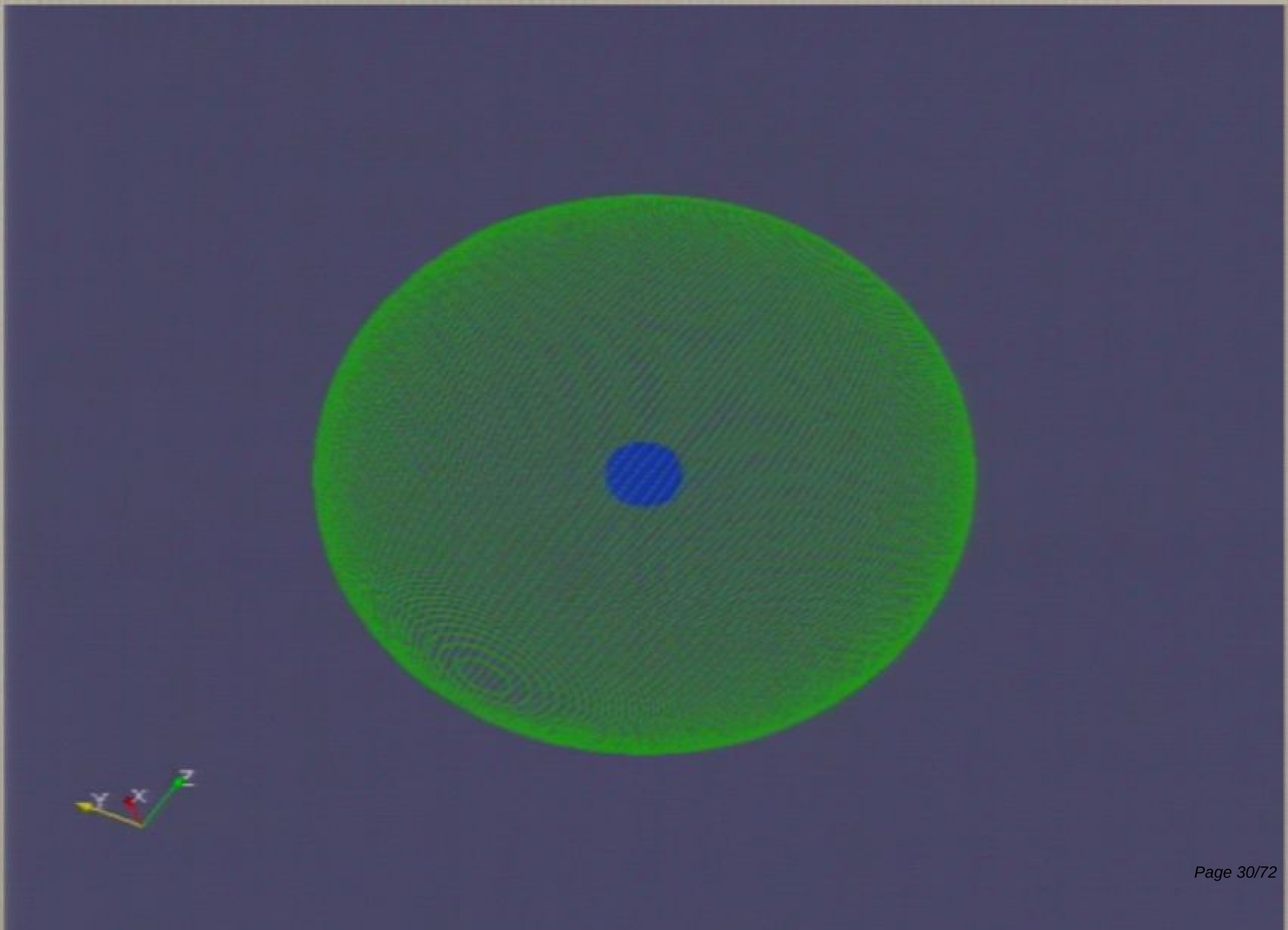


Spherical Self-Similar Solution
(Fillmore & Goldreich 1984, Bertschinger 1985)



Spherical Self-Similar Solution

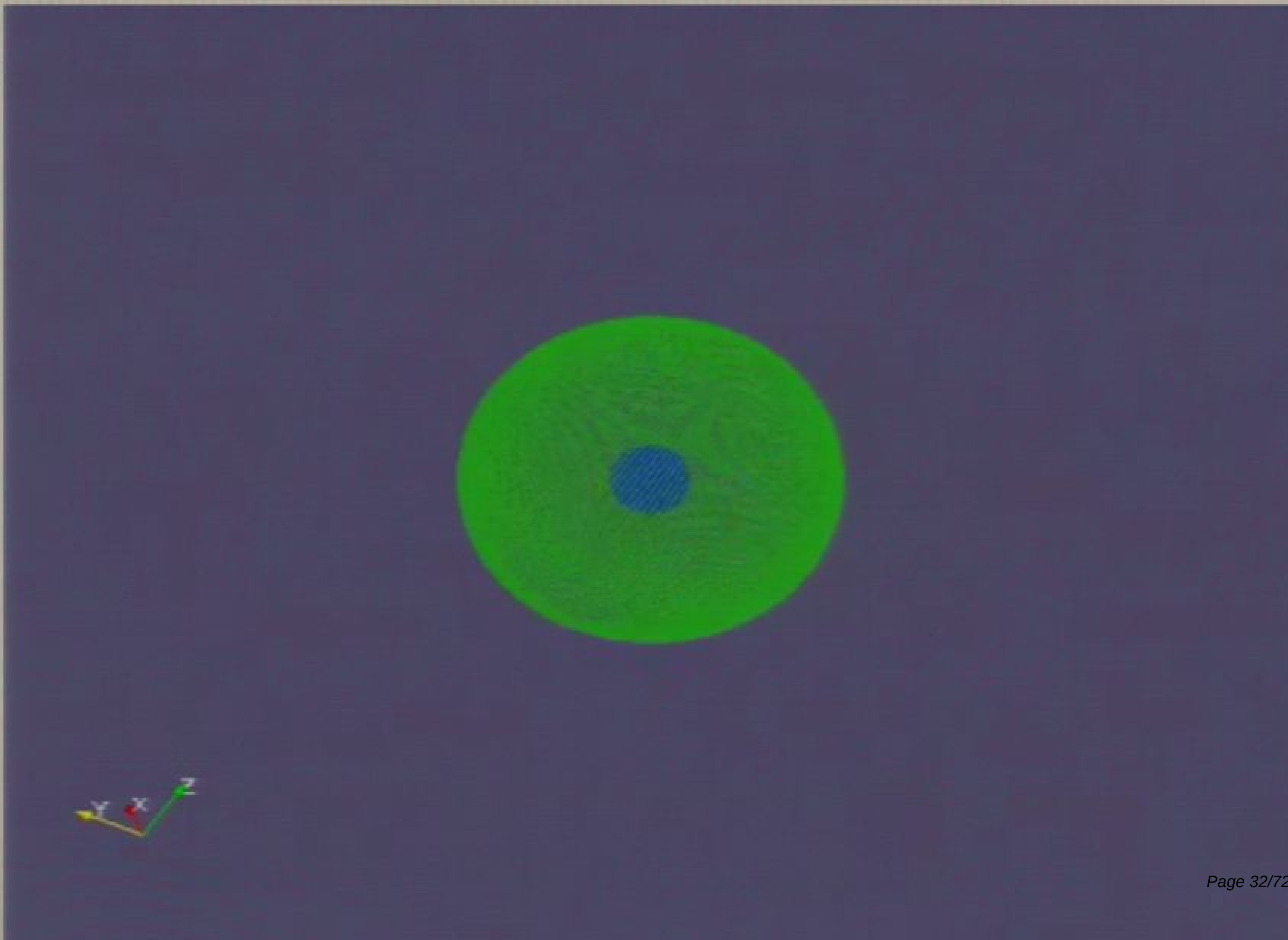
(Fillmore & Goldreich 1984, Bertschinger 1985)



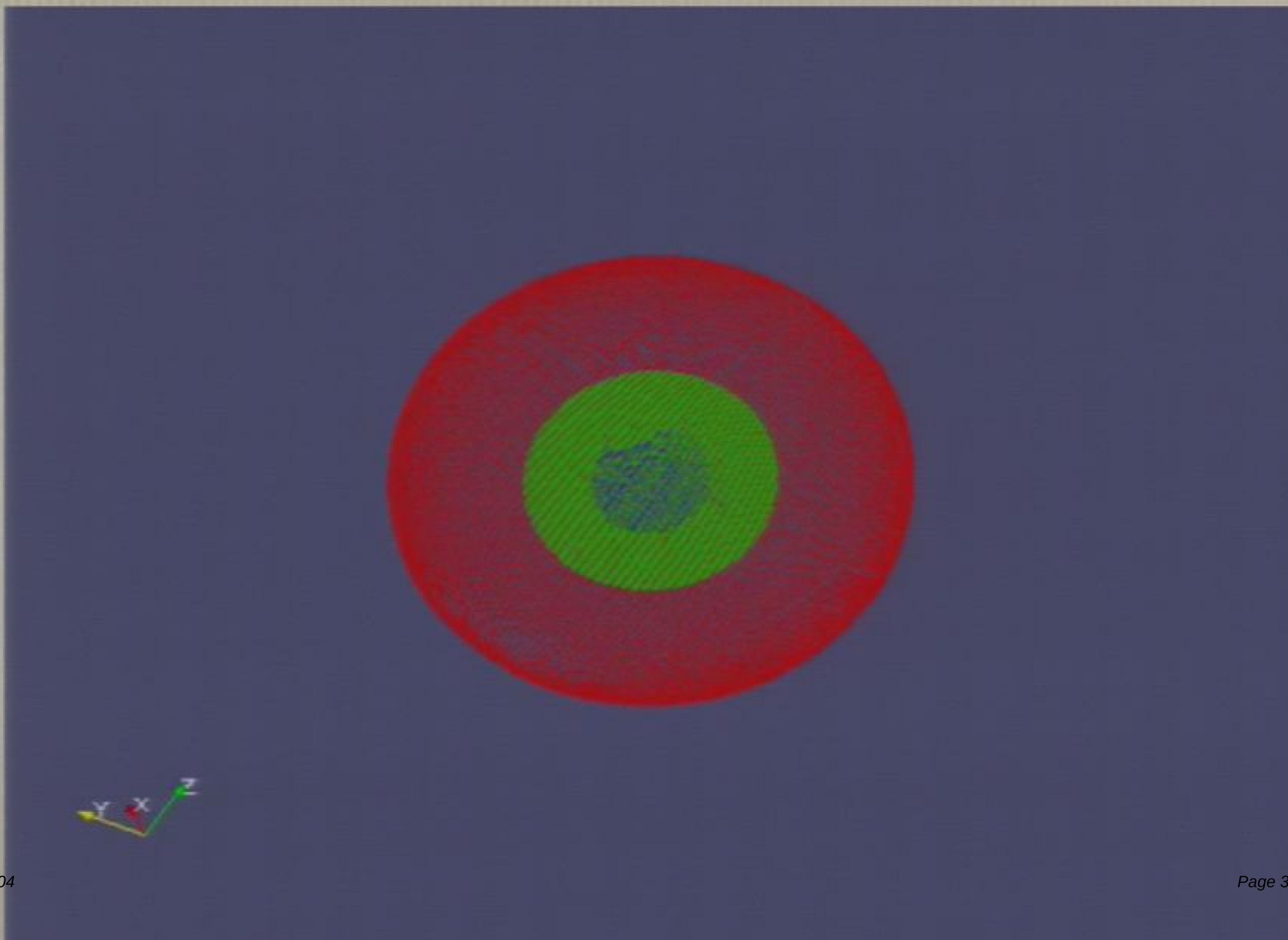
Spherical Self-Similar Solution

(Fillmore & Goldreich 1984, Bertschinger 1985)

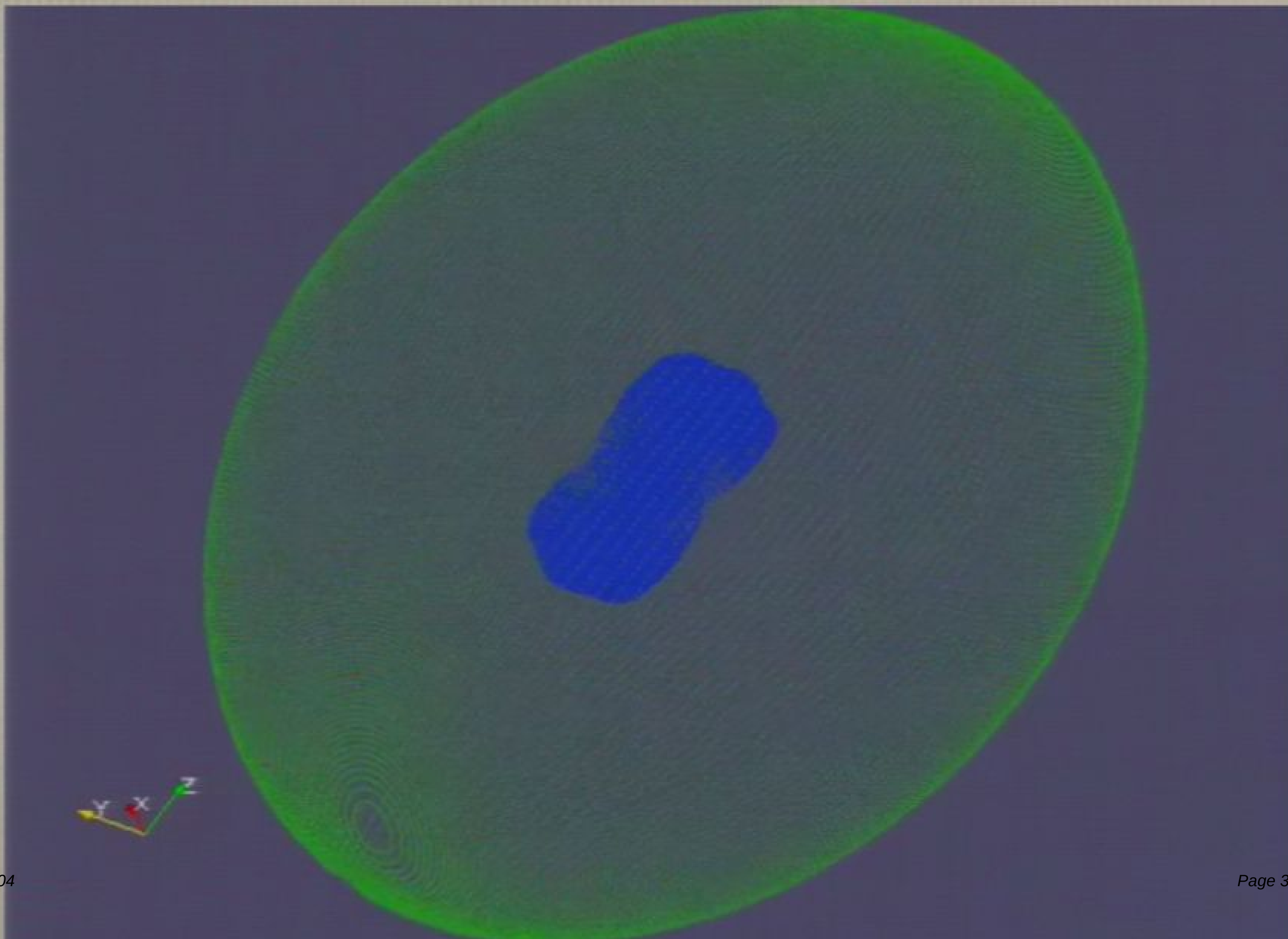
Spherical Self-Similar Solution
(Fillmore & Goldreich 1984, Bertschinger 1985)



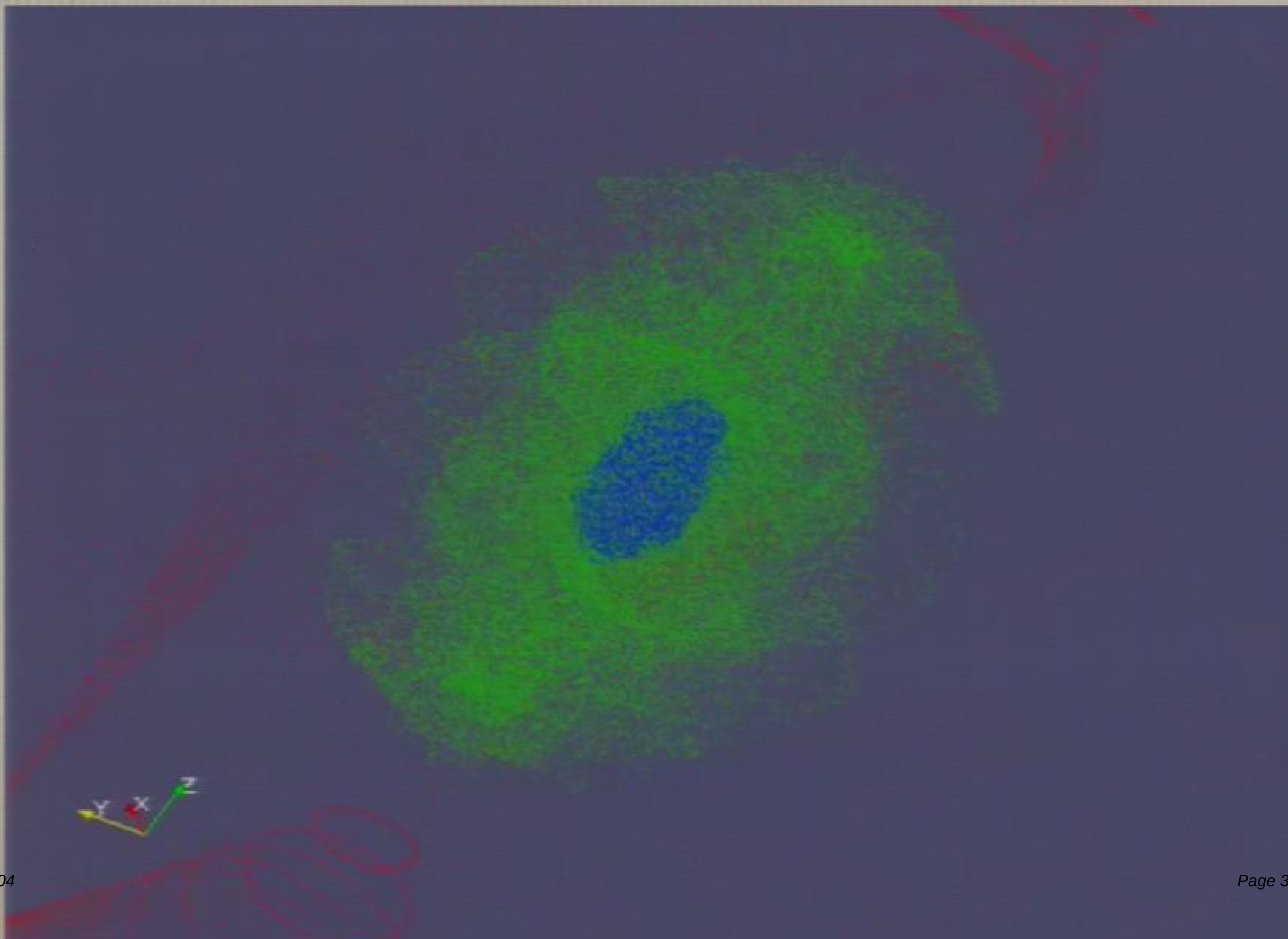
Nonspherical Self-Similar Solution



Nonspherical Self-Similar Solution



Nonspherical Self-Similar Solution



Halo statistics

Halo mass function

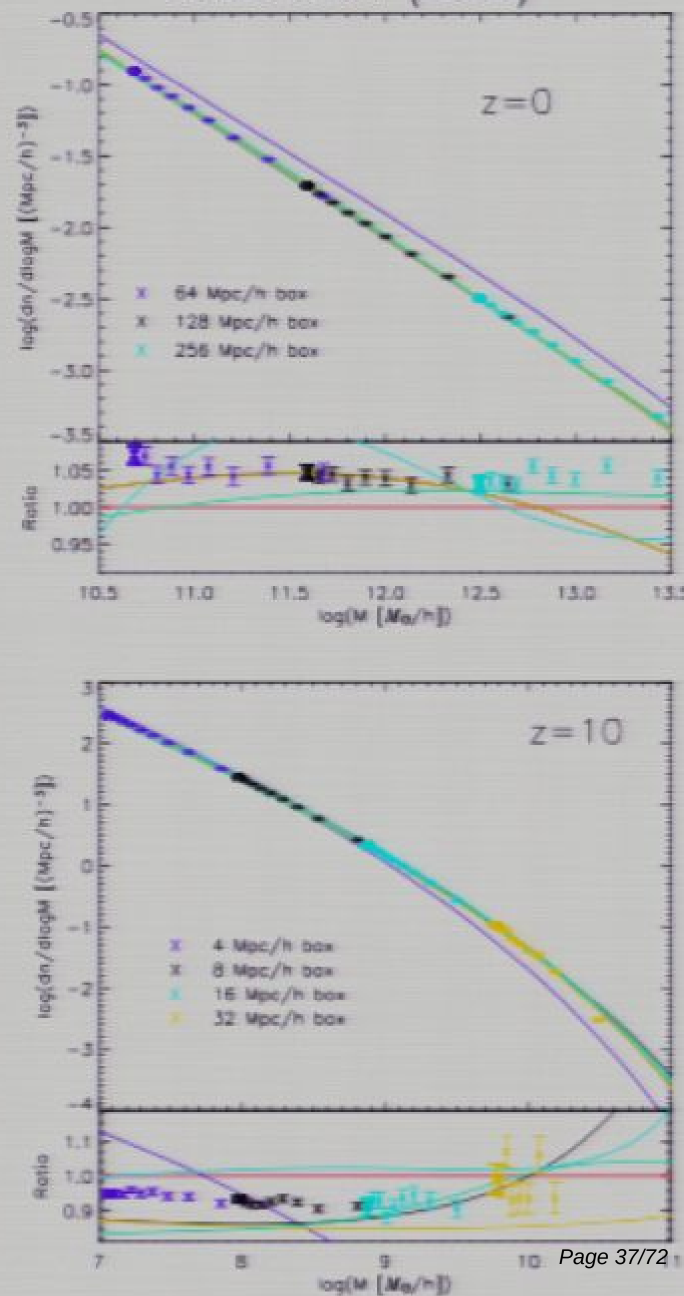
- theoretical literature is largely based on Press & Schechter (1974), which doesn't work so well:
  - too low at high M
  - too high at low M

- nowadays we just use fitting functions, which generally assume 'universality':

$$\frac{dn}{dM} = \frac{\rho_m}{M^2}\frac{d\log\sigma}{d\log M}f(\sigma)$$

i.e. the shape of the mass function is assumed to be independent of the shape of the matter power spectrum

Lukic et al. (2008)



Press-Schechter:

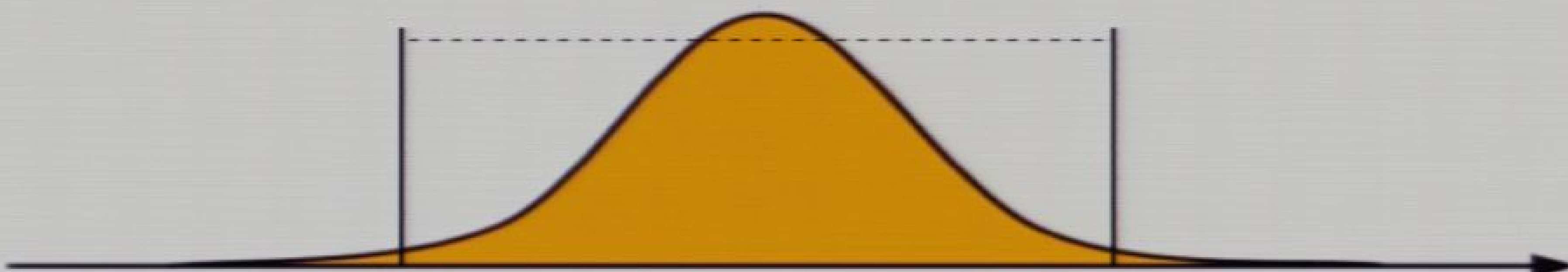
the hidden menace

- PS model: $f_c(>M) = f\,(\delta>\delta_c) \quad \times 2$

Press-Schechter:

the hidden menace

- PS model: $f_c(>M) = f(\delta>\delta_c) \quad \times 2$
- grossly under-predicts mass function at high M
- issues: cloud-in-cloud, sharp-k filter, ellipsoidal collapse, no big deal
- bigger problem for high peaks:



halo statistics

- Basic idea: Assuming that peaks form halos, we determine which peaks make which halos using our collapse calculation. We compute collapse thresholds δ_c and relation between smoothing scale R and halo mass M, as a function of peak parameters γ, e, p.

- Then in combination with with **known** peak statistics, we derive halo statistics -- mass function dn/dM, 2-point function, etc.

- This is NOT <u>Press-Schechter</u>

- e.g. the mass function is dn/dM, where n is :

$$n = \int de\, dp \ldots \int_{\nu_c}^{\infty} \mathcal{N}(\nu, e, p, \ldots) d\nu$$

this is all of the work
(self-similar calc)

peak statistics (Gaussian)

- First step is to count the number of peaks of the linear density δ (smoothed on some scale R) as a function of height ν, curvature x, triaxiality e,p, etc..... which was already worked out by BBKS (1986), e.g.:

$$\mathcal{N}_{\rm pk}(\nu) \approx \frac{(\sigma^2_{\delta\nabla^2\delta}/3\sigma^2_\delta)^{3/2}}{2\pi^2}(\nu^3 - 3\nu)e^{-\nu^2/2}, \qquad \nu \to \infty$$

halo statistics

- Next, we use a collapse model to find a correspondence between initial peak parameters and the final halo properties, e.g. $M(t)$.
- In practice, the model should provide a collapse threshold δ_c, and a relation between smoothing scale R and halo mass M, e.g.:

$$n = \int de\, dp \ldots \int_{\nu_c}^{\infty} \mathcal{N}(\nu, e, p, \ldots) d\nu$$

halo statistics

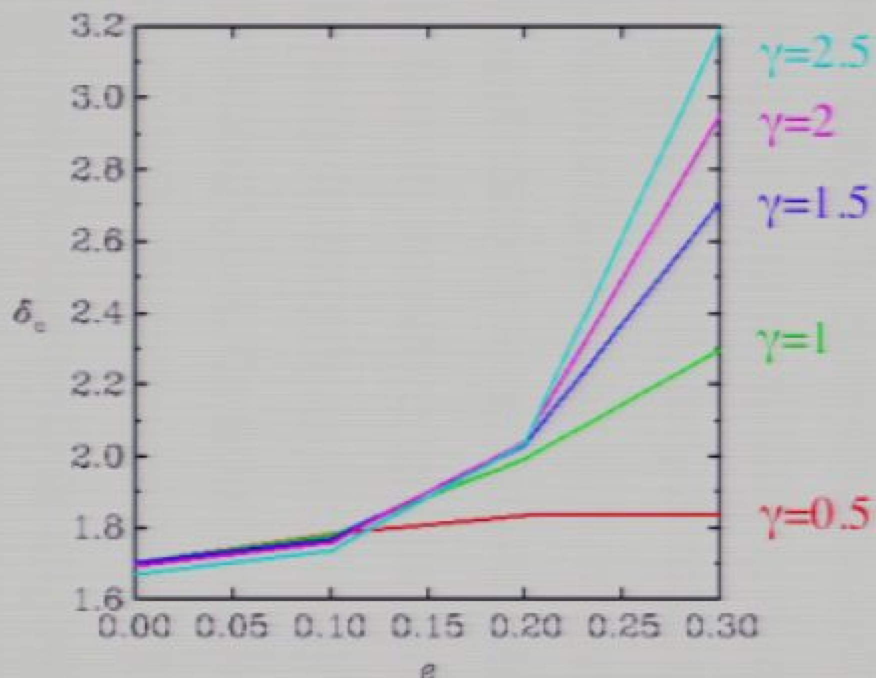
- Next, we use a collapse model to find a correspondence between initial peak parameters and the final halo properties, e.g. $M(t)$.
- In practice, the model should provide a collapse threshold δ_c, and a relation between smoothing scale R and halo mass M, e.g.:

$$n = \int de\, dp \ldots \int_{\nu_c}^{\infty} \mathcal{N}(\nu, e, p, \ldots) d\nu$$

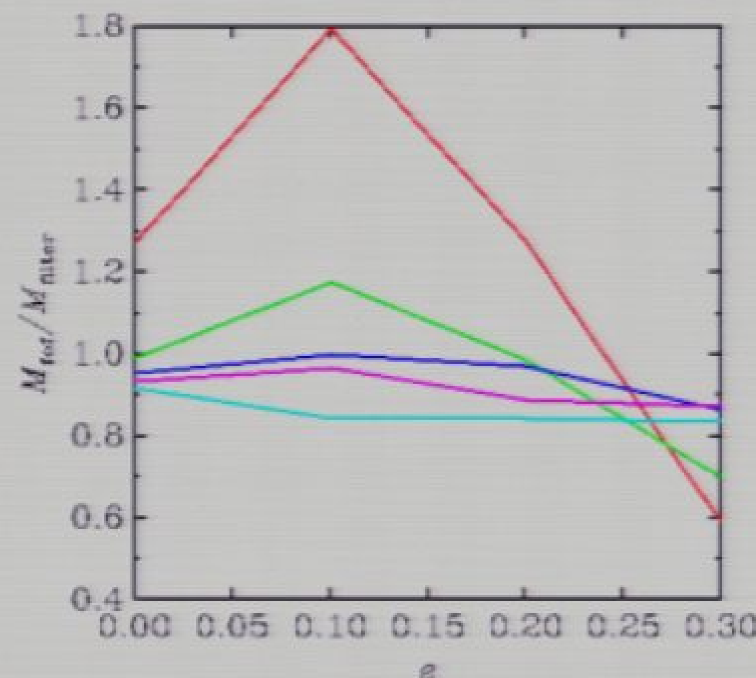
- For example, the spherical collapse model predicts δ_c=1.686, for all peaks
- Of course, peaks are more complicated than this. We assume that just a few peak properties are important:
  - radial slope γ

Trends

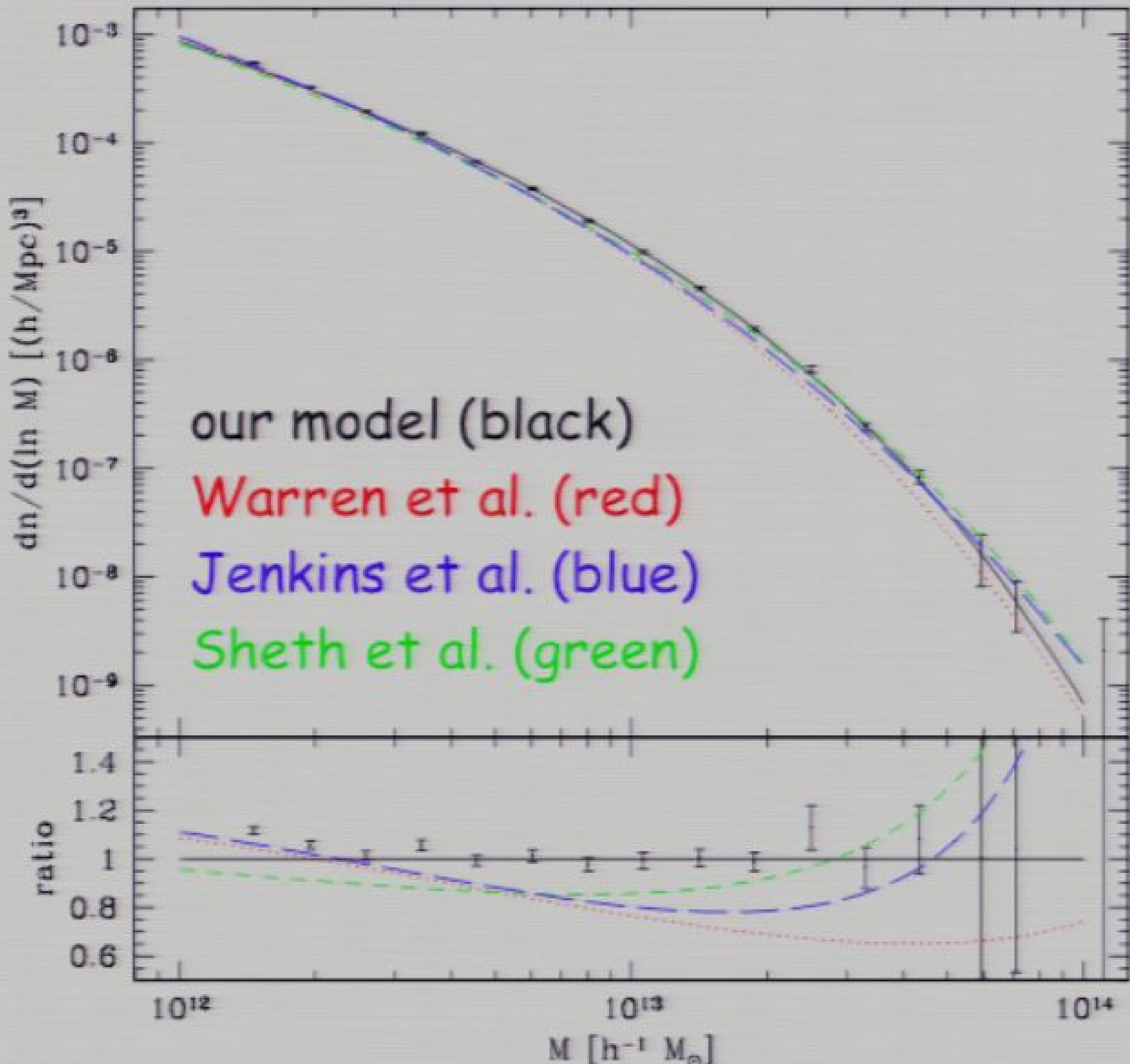
triaxiality
delays collapse



steeper slopes
⇒ smaller mass



z=3 mass function



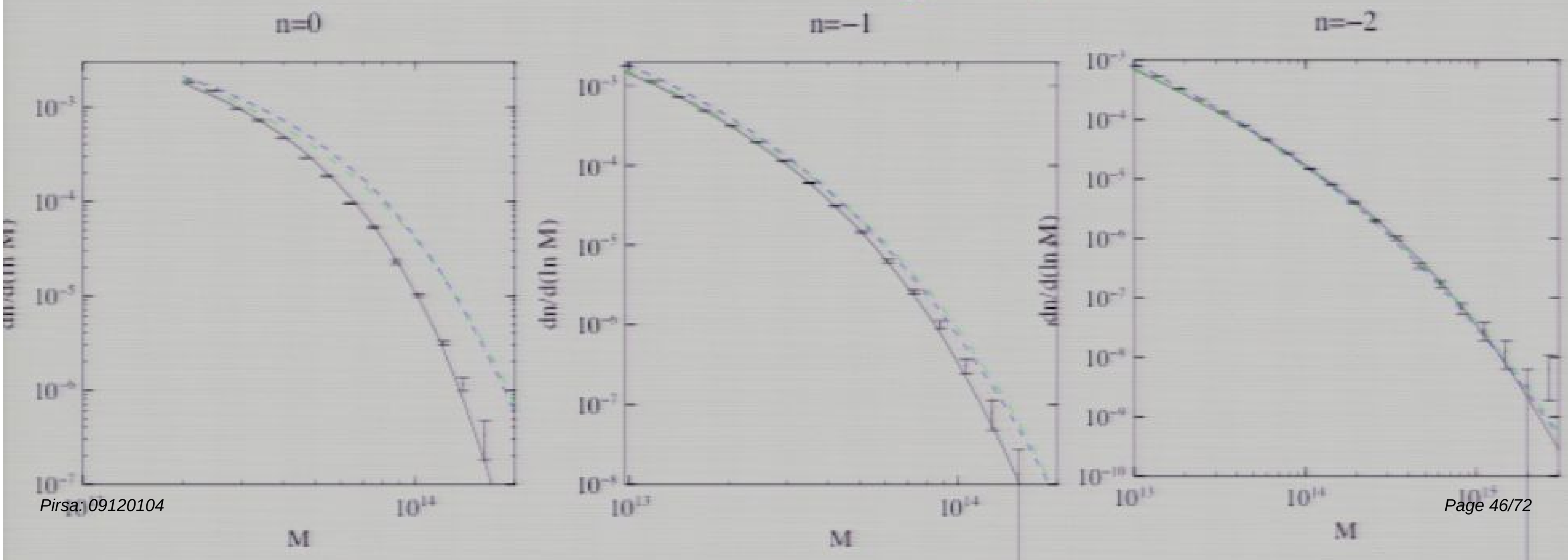
halo mass function

Example: dn/dM for scale-free cosmologies with $\Omega_m=1$, $P(k)\propto k^n$.

Below, colors denote: our model (black)

Warren et al. (red)
Jenkins et al. (blue)
Sheth et al. (green)

} fitting functions



halo clustering

* we can predict halo clustering from the clustering of peaks of the linear density field
* clustering is usually measured by the "**bias**" b=d(log n)/dδ

halo clustering

* we can predict halo clustering from the clustering of peaks of the linear density field
* clustering is usually measured by the "**bias**" $b = \mathrm{d}(\log n)/\mathrm{d}\delta$
* for large scales where $\delta \ll 1$, this gives $\delta_h = b\ \delta$, $\xi_h = b^2\ \xi$, etc.
* example: Press-Schechter

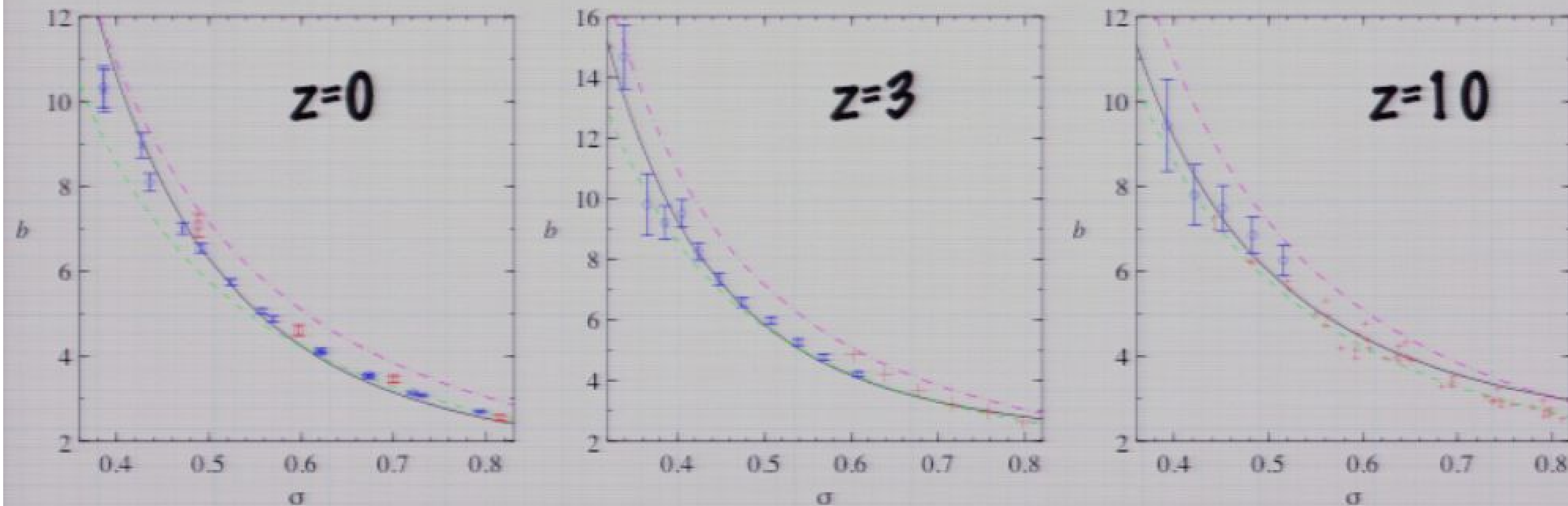
$$n_{\rm PS} \propto \delta_c \exp[-½(\delta_c/\sigma)^2]$$

$$\Rightarrow b_{\rm PS} = \delta_c/\sigma^2 - \delta_c^{-1}$$

* more careful peaks calculation: $$b_{\rm L} = \sigma^{-1}\frac{\nu - \nu_\star}{1 - \Gamma_\nu^2}$$

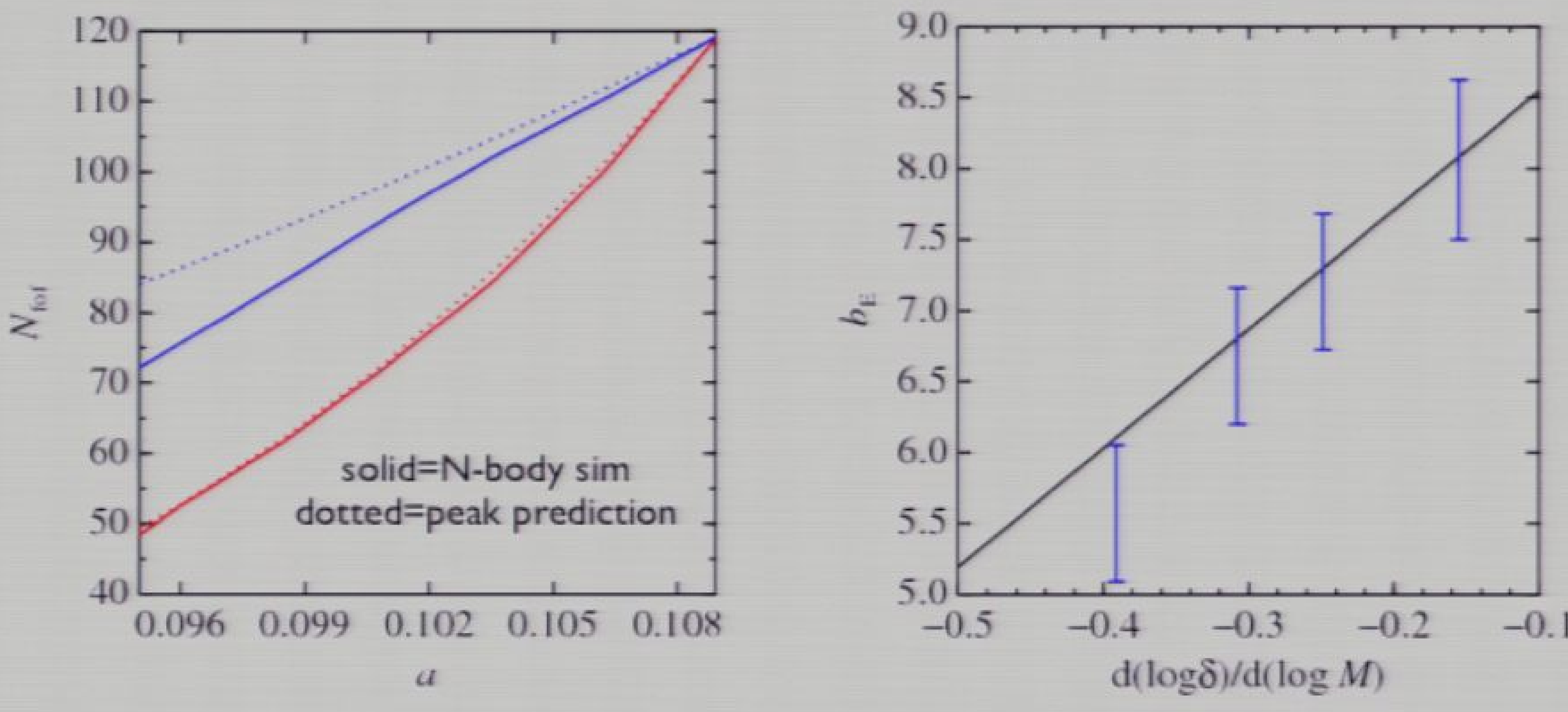
conditional mean & variance of ν

Halo bias



curves: our model (black)
Mo & White (magenta)
Sheth et al. (green)

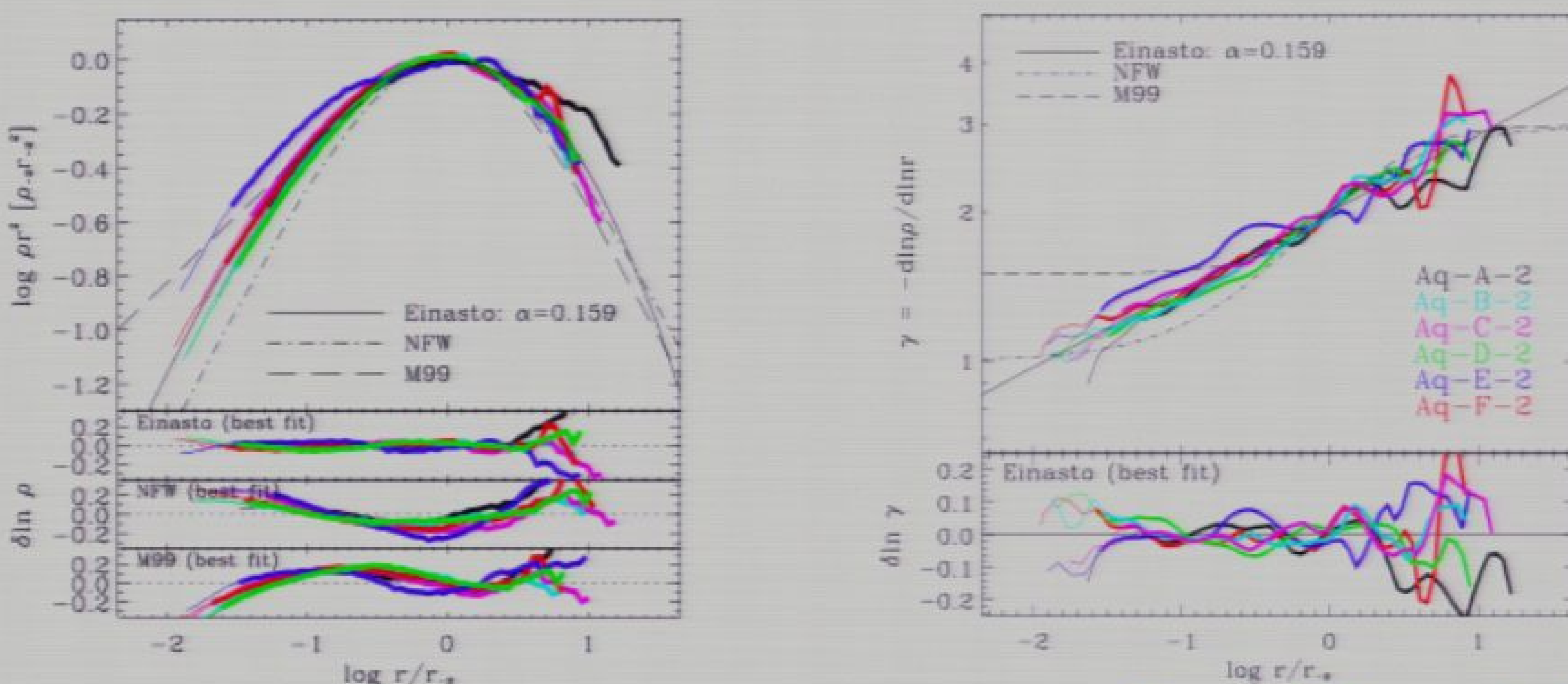
assembly bias



Halo profile

Halo Profile

Slope is steep at large radii, and becomes more shallow at small *r*. The rollover is very gradual, occurring over many decades in *r*.



"Aquarius" (Navarro et al. 2008)

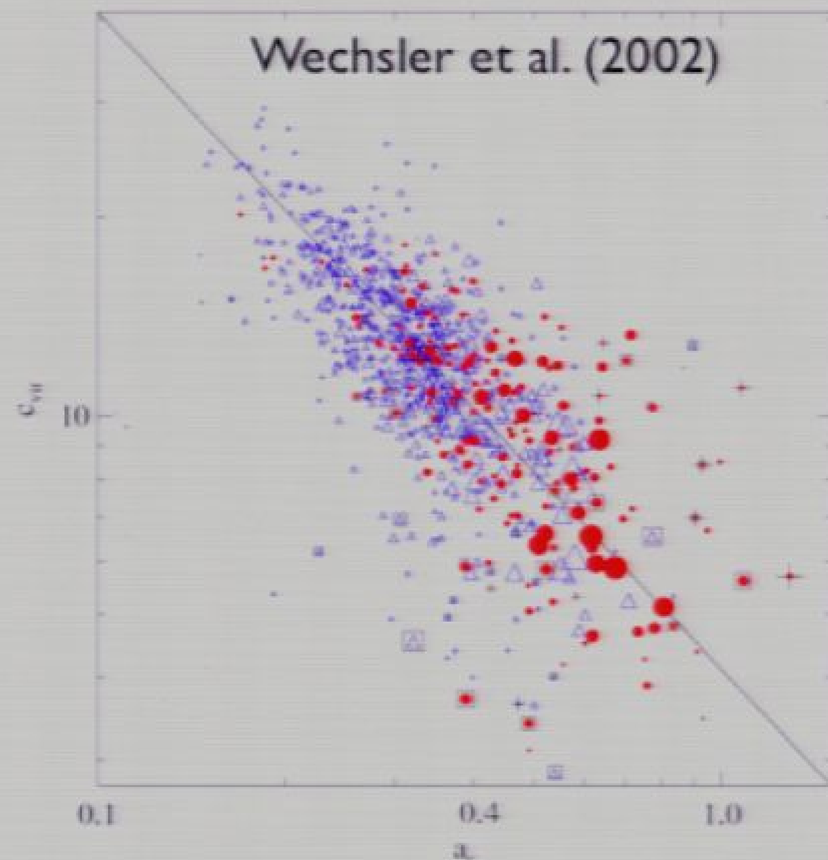
The vast majority of simulated halos behave this way; exceptions tend to be recent mergers or bridged halos.

concentrations

$c_{vir} = r_{vir}/r_{-2}$ measures the extent of the outer, steep portion of the profile.

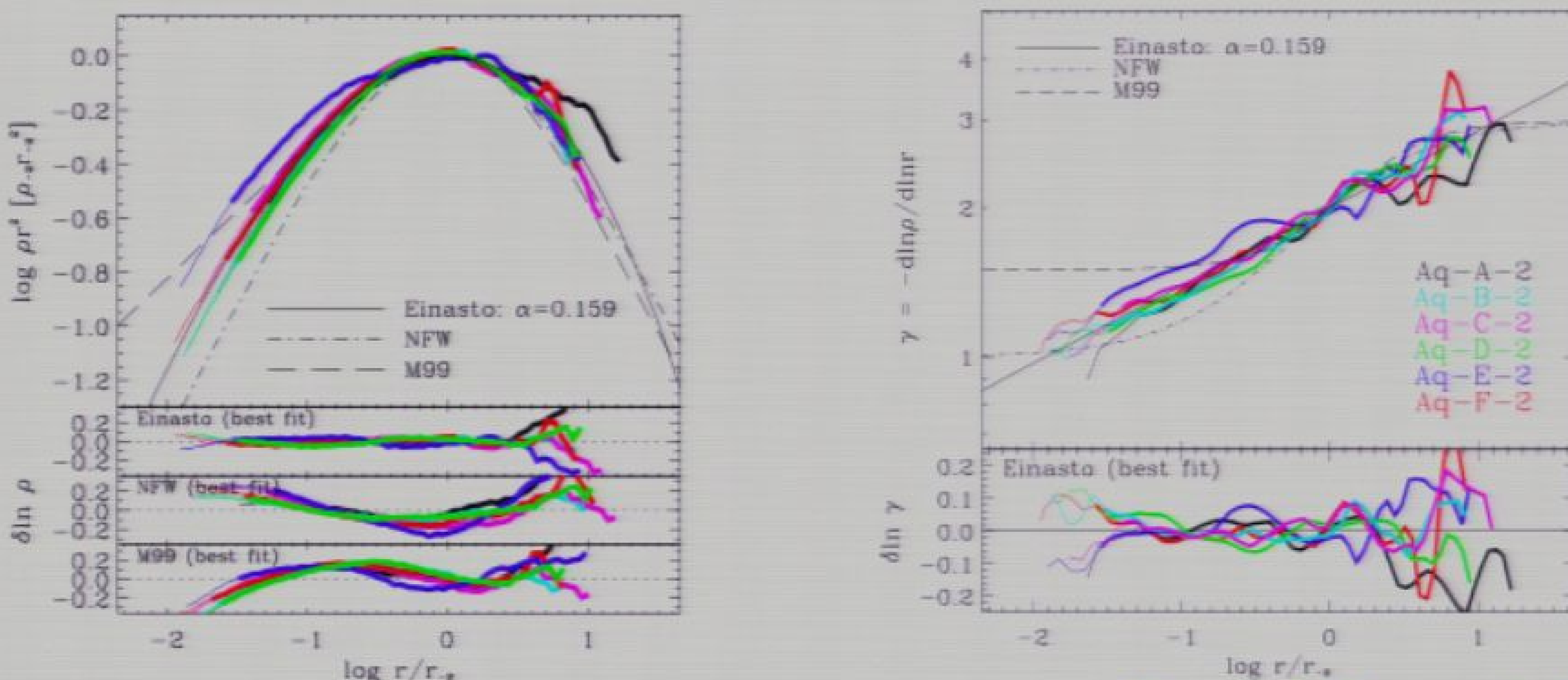
correlates with other parameters, in the sense that

- old, low mass ⇒ high c_{vir}
- young, high mass ⇒ low c_{vir}



Halo Profile

Slope is steep at large radii, and becomes more shallow at small r.
The rollover is very gradual, occurring over many decades in r.



"Aquarius" (Navarro et al. 2008)

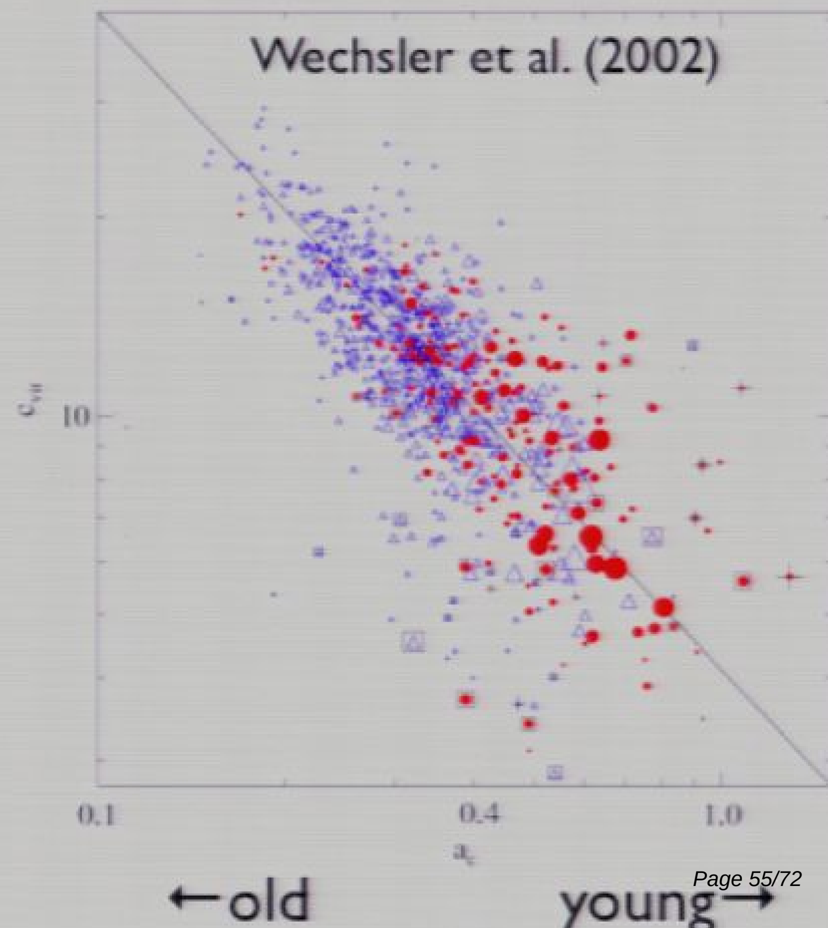
The vast majority of simulated halos behave this way; exceptions tend to be recent mergers or bridged halos.

concentrations

$c_{vir}=r_{vir}/r_{-2}$ measures the extent of the outer, steep portion of the profile.

correlates with other parameters, in the sense that

- old, low mass ⇒ high c_{vir}
- young, high mass ⇒ low c_{vir}



What is responsible for this generic profile?

- many suggestions, including shape of power spectrum (e.g. Nusser & Sheth 1999), or substructure (e.g. Dekel et al. 2003)

- but NFW-ish profile is found for essentially any P(k) shape, including models with a cutoff, like HDM (Wang & White 2009)

- so hierarchical structure formation does not seem important

Are mergers responsible for universal halo properties? 5

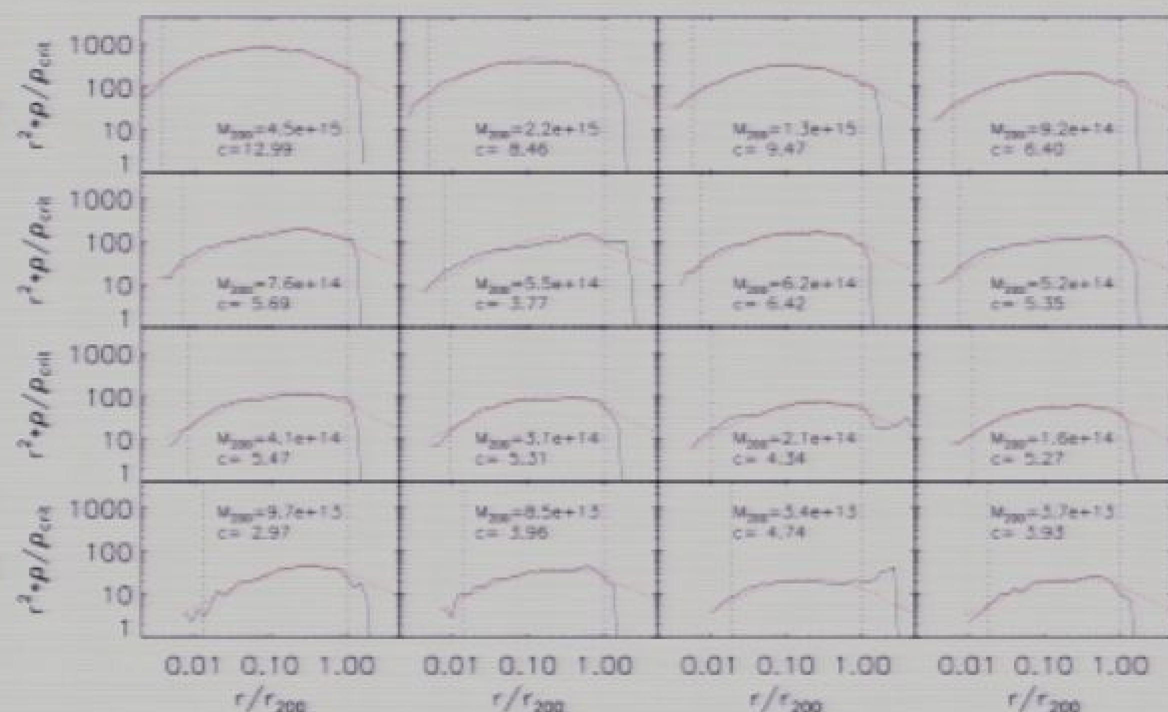


Figure 4. Density profiles for 16 haloes with particle number within r_{200} (N_{200}) greater than 10,000. These 16 haloes cover two orders of magnitude in mass. The density profiles are normalized by r^2/ρ_{crit} and the radius are normalized by r_{200}. In each panel, the red dotted curve is the NFW fit to the numerical measurement (black solid curve). Two vertical dotted lines show the softening length and r_{200}. The corresponding M_{200} (in unit of $h^{-1}M_\odot$) and the concentration parameter c are listed in each panel.

Why is this happening?

we'll use self-similar collapse as an example

What is responsible for this generic profile?

- many suggestions, including shape of power spectrum (e.g. Nusser & Sheth 1999), or substructure (e.g. Dekel et al. 2003)

- but NFW-ish profile is found for essentially any P(k) shape, including models with a cutoff, like HDM (Wang & White 2009)

- so hierarchical structure formation does not seem important

Are mergers responsible for universal halo properties? 5

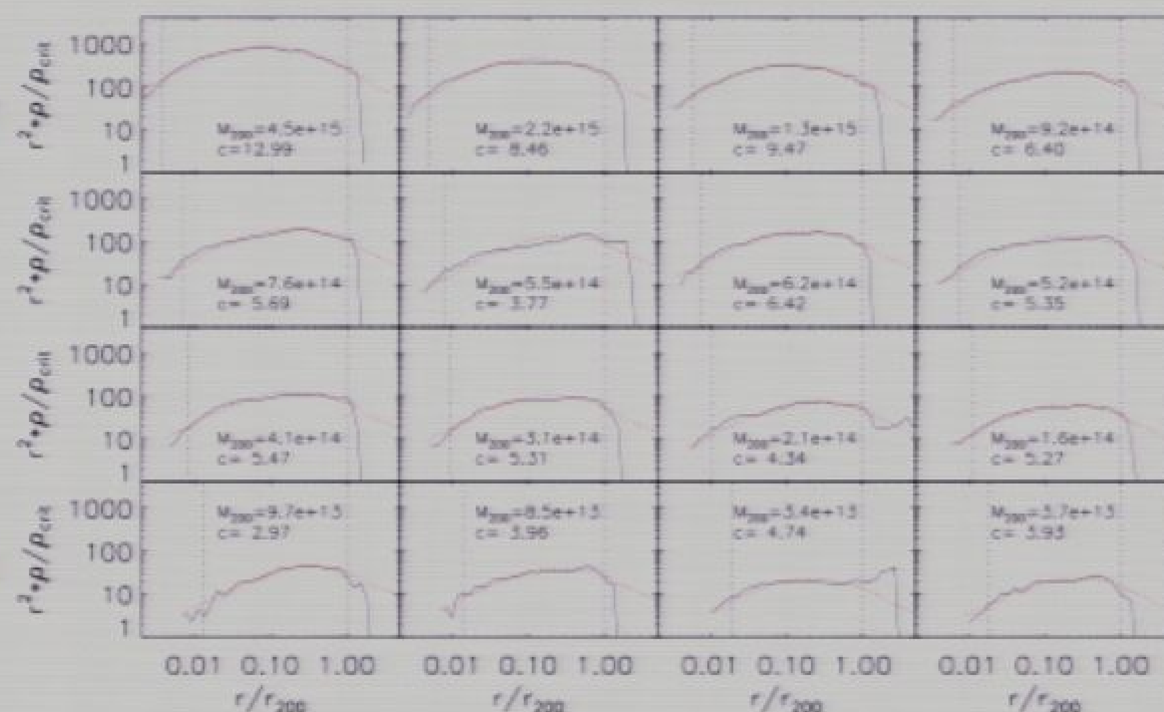


Figure 4. Density profiles for 16 haloes with particle number within r_{200} (N_{200}) greater than 10,000. These 16 haloes cover two orders of magnitude in mass. The density profiles are normalized by r^2/ρ_{crit} and the radius are normalized by r_{200}. In each panel, the red dotted curve is the NFW fit to the numerical measurement (black solid curve). Two vertical dotted lines show the softening length and r_{200}. The corresponding M_{200} (in unit of $h^{-1}M_\odot$) and the concentration parameter c are listed in each panel.

Why is this happening?

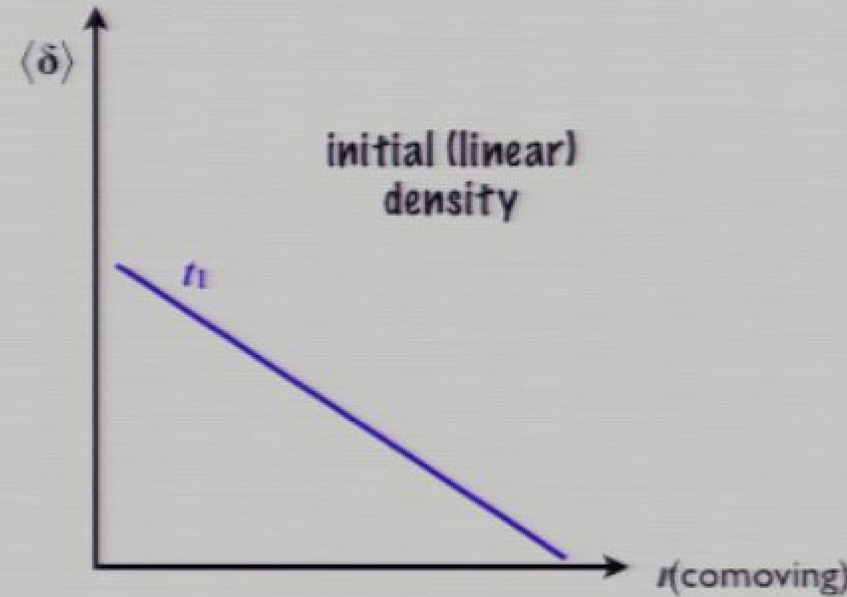
we'll use self-similar collapse as an example

Density profile

see Fillmore & Goldreich (1984)

Suppose linear density profile has local slope γ, so that

$$\delta(r,a) \propto a\, r^{-\gamma}$$



Density profile

see Fillmore & Goldreich (1984)

Suppose linear density profile has local slope γ, so that

$$\delta(r,a) \propto a\, r^{-\gamma}$$

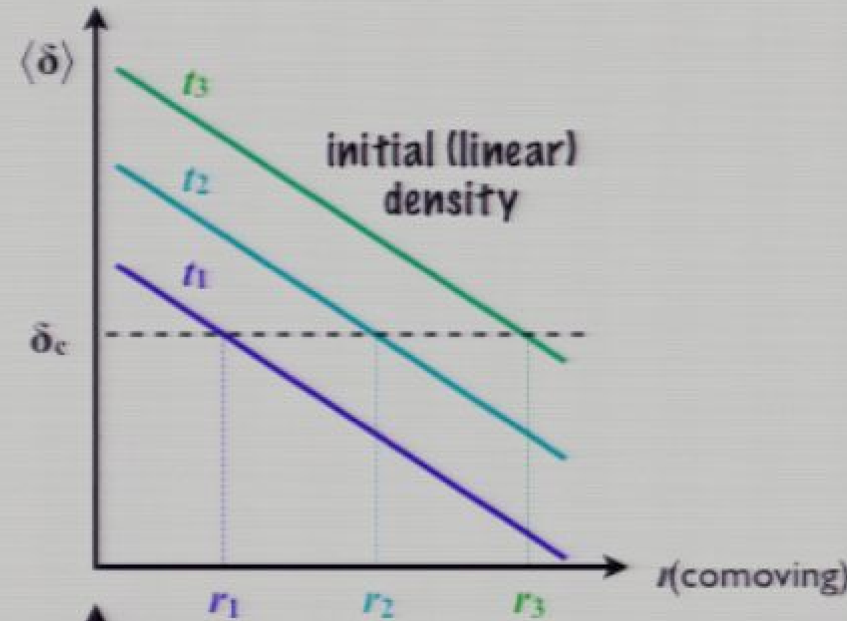
Turnaround occurs when $\delta \sim 1$, so

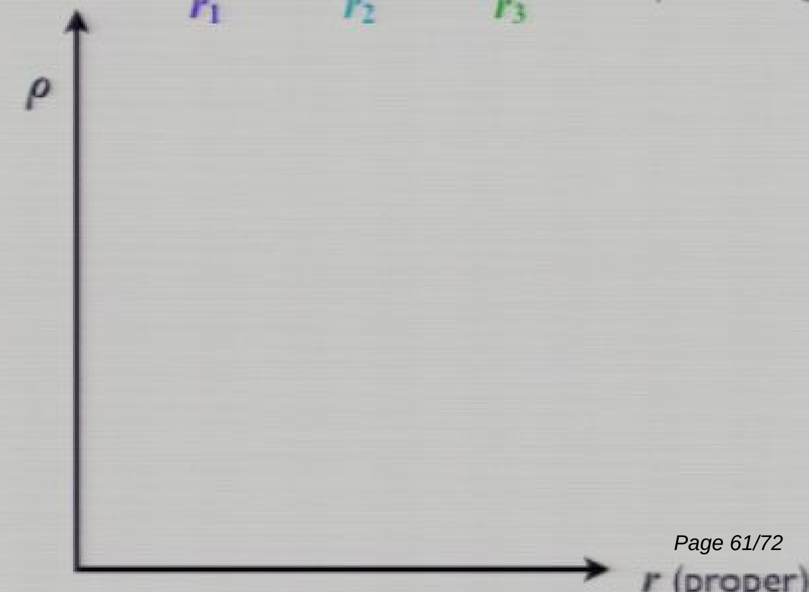
$r_{\rm ta} \propto a^{1/\gamma}$ (comoving)

$r_{\rm ta} \propto a^{(1+\gamma)/\gamma}$ (proper)

Suppose (for now) that the annulus collapsing at $r_{\rm ta}$ remains thereafter at the same radius.

Background $\rho \propto a^{-3}$, and $a_{\rm ta} \propto r_{\rm ta}^{\gamma/(1+\gamma)}$, so the slope of the density is





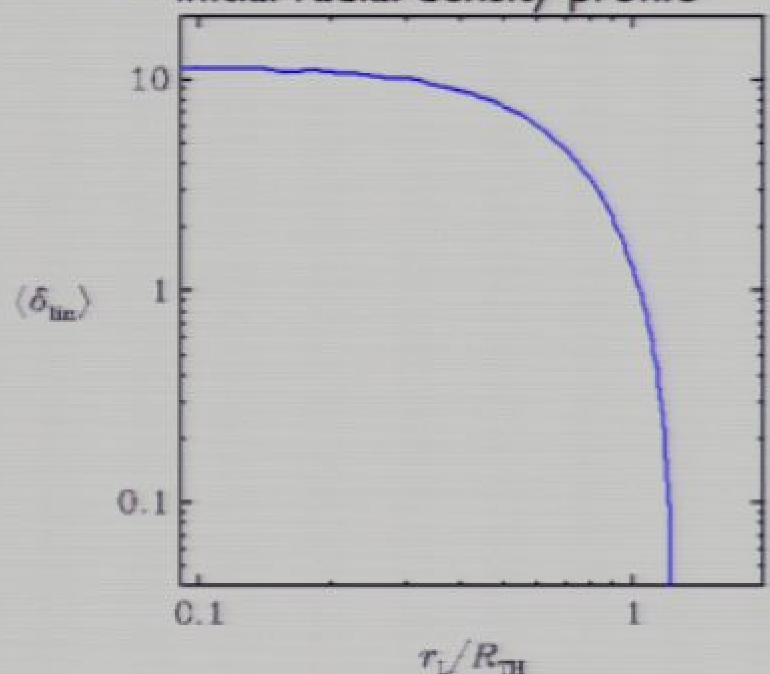
Density profile

The preceding argument ($\rho \propto d^3 r_L / d^3 r$) can be used to predict the halo profile given the initial peak profile:

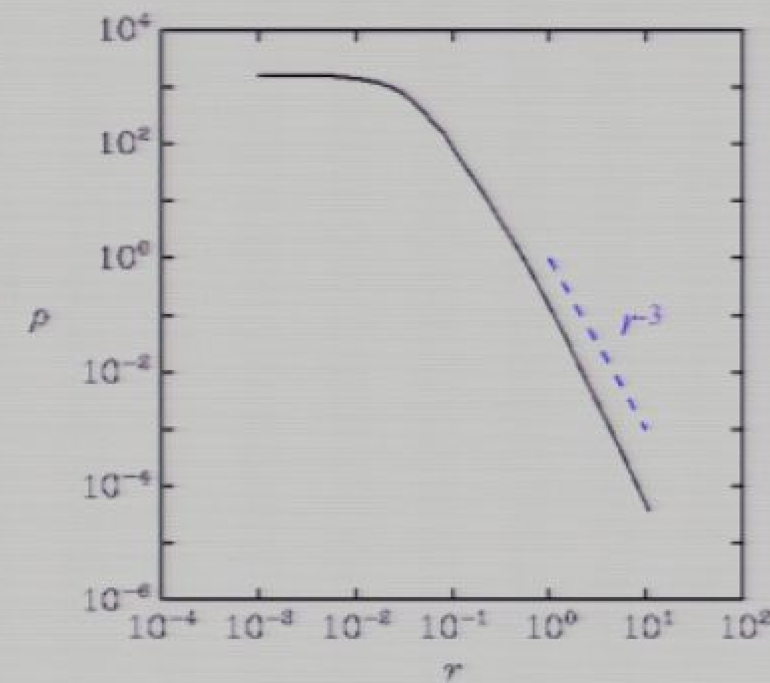
Density profile

The preceding argument ($\rho \propto d^3 r_L / d^3 r$) can be used to predict the halo profile given the initial peak profile:

- initial radial density profile



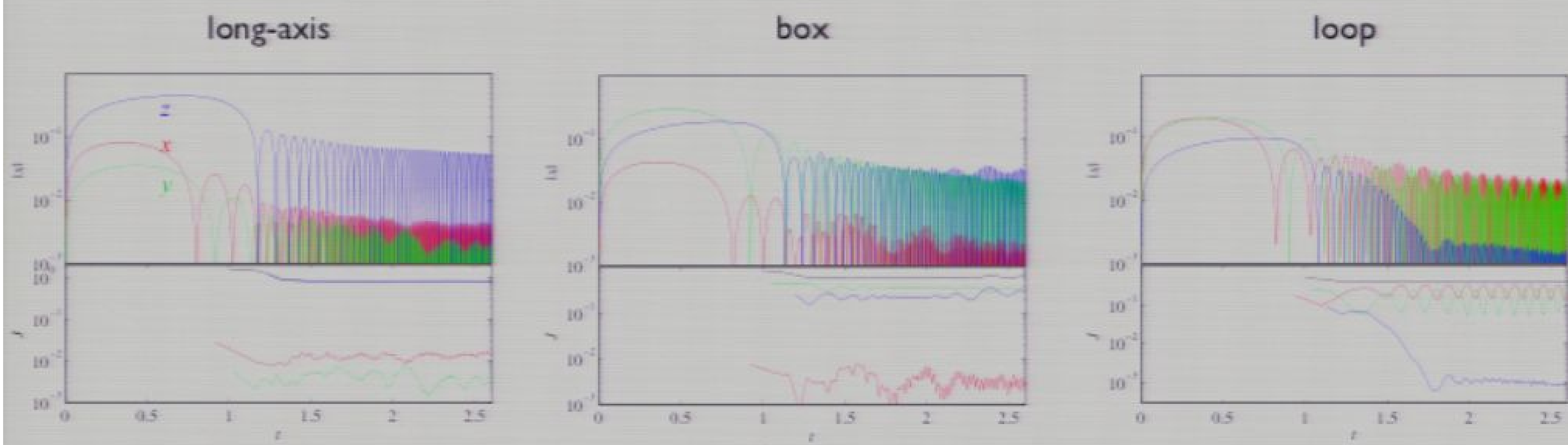
➔



recall slope $\alpha \approx 3\gamma/(1+\gamma)$

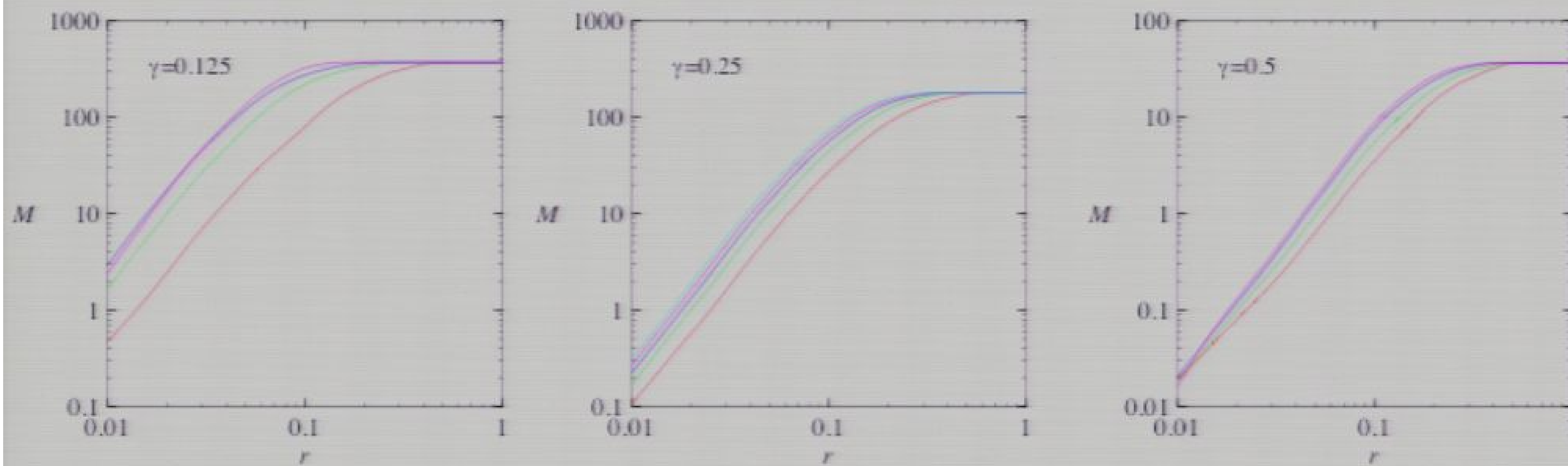
see also Gunn & Ryden (1988), Ascasibar et al. (2004), Lu et al. (2006)

individual orbits

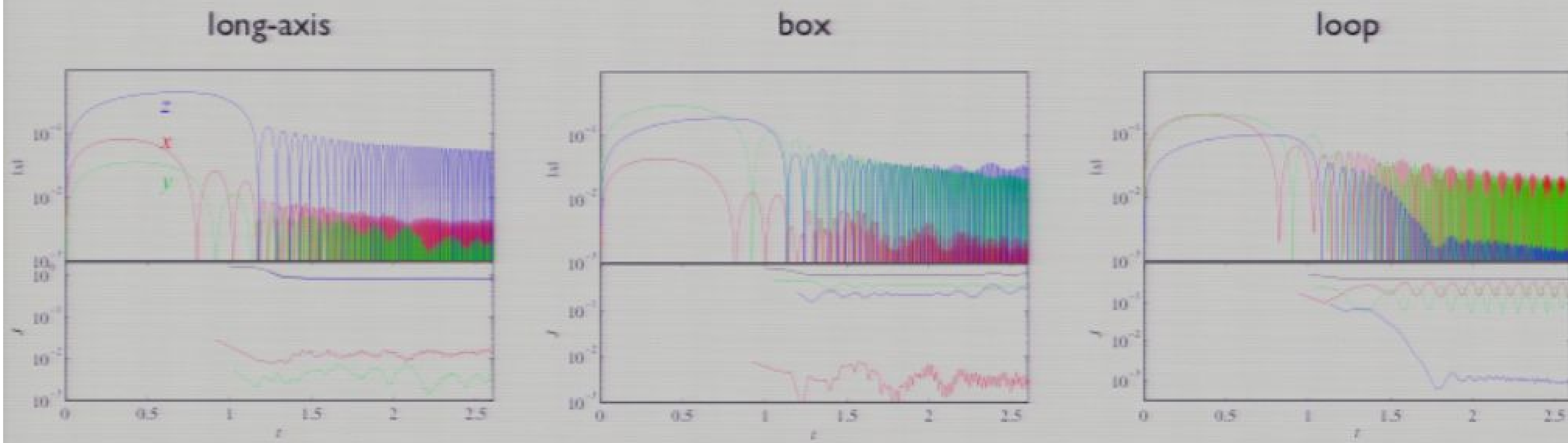


All of the orbits contract over time as the potential deepens. However, the contraction is mostly adiabatic: the action $J_i = \int v_i \, dx_i$ is (roughly) conserved (usually).

contraction

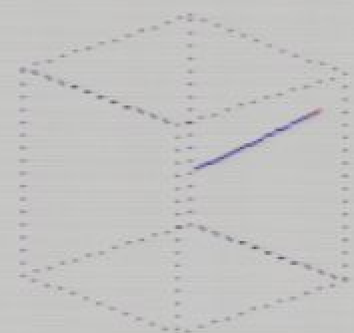


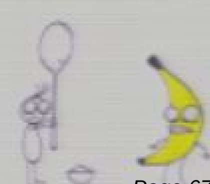
individual orbits



All of the orbits contract over time as the potential deepens. However, the contraction is mostly adiabatic: the action $J_i = \int v_i \, dx_i$ is (roughly) conserved (usually).

orbit transformation

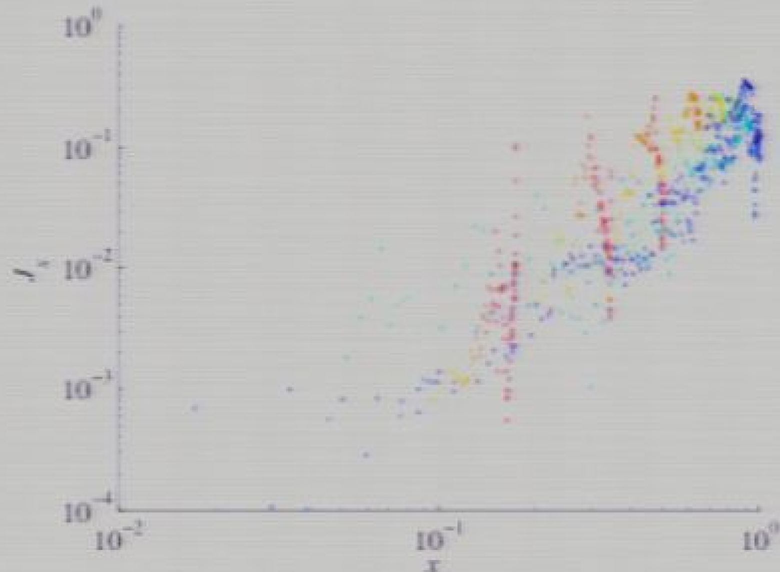


more examples: 

adiabatic compression

- the long tails of outer shells extending to small radius deepen the potential at small r, causing contraction
- the effect is described well by assuming that the action $J_i = \int v_i \, dx_i \sim x_i \, v_i \sim (x_i^3 \, \partial_i \Phi)^{1/2}$ is an adiabatic invariant. [NB – this is not the same as conservation of angular momentum!]
- the action can be predicted from the linear density profile

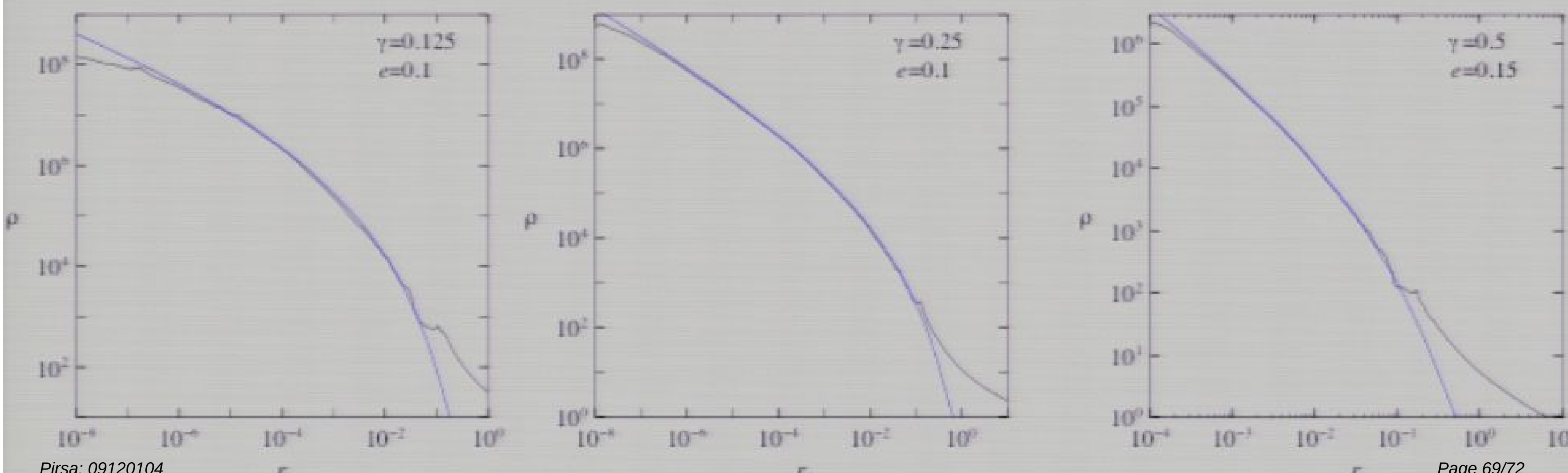
γ=0.125
e=0.1



toy model

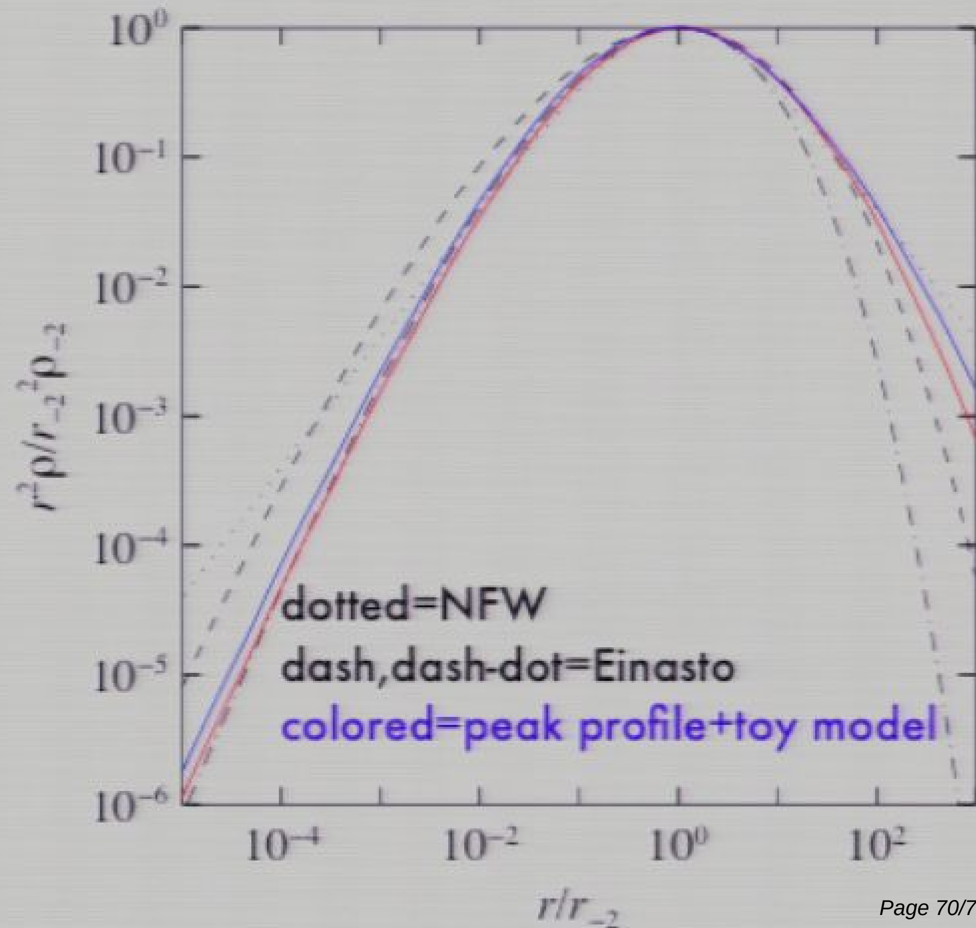
using these observations, we can write down a simple model for profile:

- use linear profile to determine actions J_i
- given J_i's and potential Φ, estimate x,y,z apo's
- assume orbits deposit uniform density inside ellipsoid bounded by the x,y,z apo's
- add up all the orbits to get total ρ and Φ



putting it all together

- combining everything gives profiles qualitatively similar to NFW
- however, we find no reason for r^{-1} cusp to extend to small r
- important physics: triaxiality & adiabatic contraction



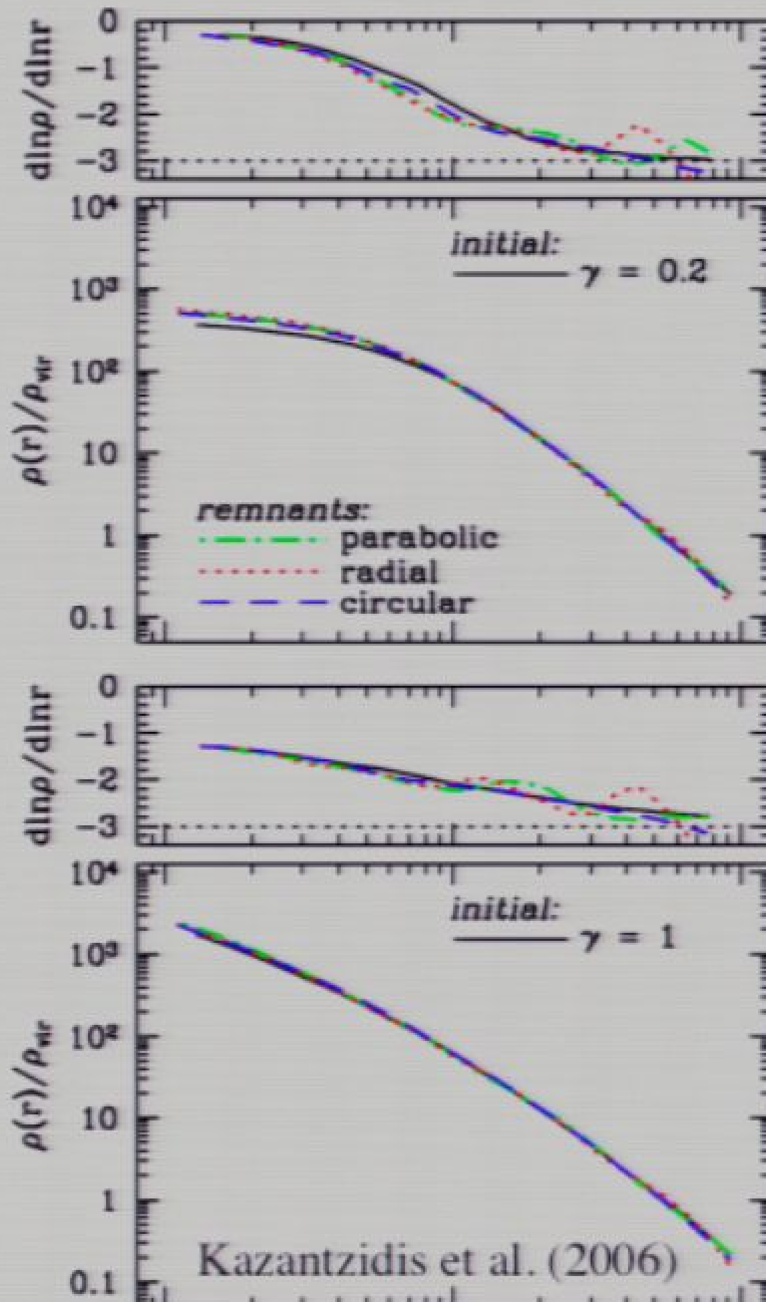
Major mergers?

Our model relies upon the (rough) conservation of the orbital actions.

If halos violently relax (e.g. in major mergers) then there's no reason for them to retain their profiles

But halos have NFW profiles even following major (1:1) mergers!

Explanation: halos do not violently relax in major mergers, but instead retain memory of their profiles prior to merger. (Only a fraction of particles get kicked onto very different orbits.)



Variations