Title: Effective Field Theory Approach to the Post-Newtonian Physics

Date: Dec 03, 2009 03:00 PM

URL: http://pirsa.org/09120099

Abstract:

Effective Field Theory approach to the Post-Newtonian physics

Misha Smolkin

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Outline

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1. Einstein-Infeld-Hoffman Lagrangian (Improved derivation)

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2. Unrealistic scalar toy model
   - *redundant operators*
   - *classical RG flow*

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1. **Einstein-Infeld-Hoffman Lagrangian (Improved derivation)**

2. **Unrealistic scalar toy model**
   - *redundant operators*
   - *classical RG flow*

3. **Finite size effects** *(work in progress)*

4. **Results and open questions**

Post-Newtonian approximation

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$$r_s << R$$

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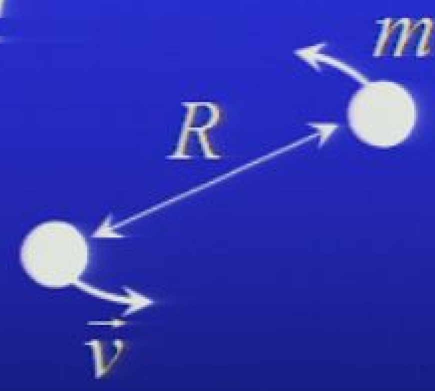
$$r_s << R$$

Post-Newtonian approximation

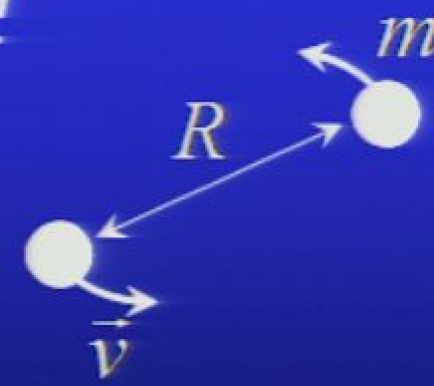
Einstein-Infeld-Hoffman Lagrangian derivation:



Post-Newtonian approximation

Einstein-Infeld-Hoffman Lagrangian derivation:

(W.Goldberger and I.Rothstein)



Virial theorem: $\vec{v}^2 \sim \frac{Gm}{R} \sim \frac{r_s}{R} = 1PN$

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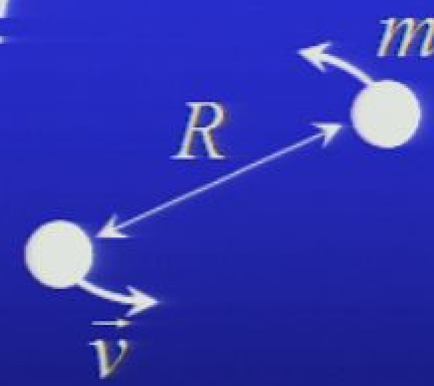

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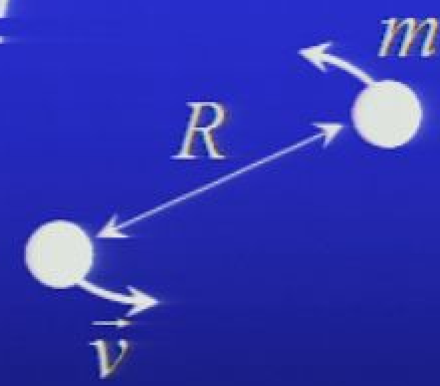
Typical lengths:

Post-Newtonian approximation



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Typical lengths:

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potential length R

radiation length $\frac{R}{v}$

Corresponding fields:

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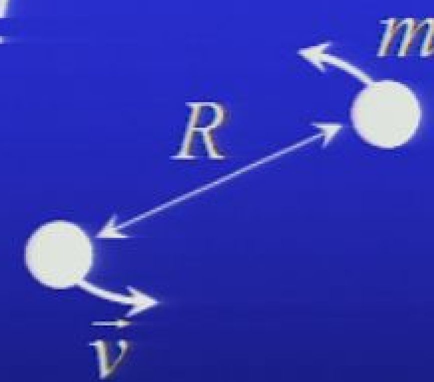
Post-Newtonian approximation



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radiation length $\frac{R}{v}$

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strong field $g_{\mu\nu}$

potential field $H_{\mu\nu}$

radiation field $h_{\mu\nu}$

$$\frac{\partial H_{\mu\nu}}{\partial x^i} \sim \frac{1}{R} H_{\mu\nu} \ , \ \frac{\partial H_{\mu\nu}}{\partial t} \sim \frac{v}{R} H_{\mu\nu}$$

$$\frac{\partial h_{\mu\nu}}{\partial x^i} \sim \frac{v}{R} h_{\mu\nu} \ , \ \frac{\partial h_{\mu\nu}}{\partial t} \sim \frac{v}{R} h_{\mu\nu}$$

Post-Newtonian approximation

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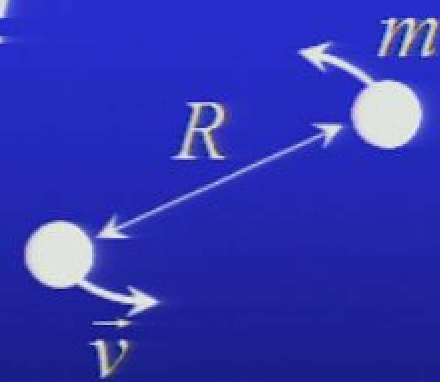
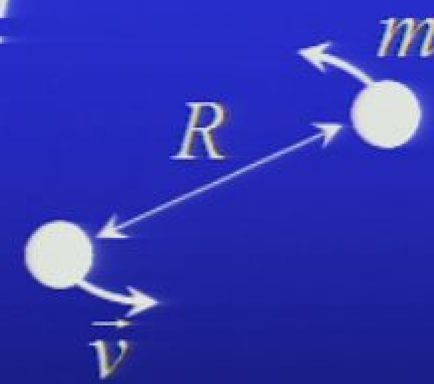


Step 1

Post-Newtonian approximation

Einstein-Infeld-Hoffman Lagrangian derivation:



Step 1

$$\text{"Integrate out } r_s\text{"}$$

$$S = -\frac{1}{16\pi G}\int\sqrt{-\bar{g}}\, d^4x\, R[\bar{g}] - \sum_{a=1}^{2} m_a \int d\tau_a + \ldots$$

$$\text{where } \bar{g}_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu} + h_{\mu\nu} \text{ and } d\tau_a^2 = \bar{g}_{\mu\nu} dx_a^\mu dx_a^\nu$$

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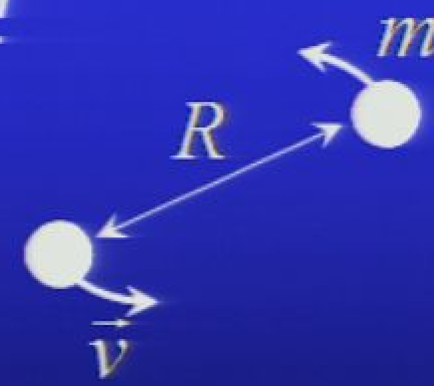

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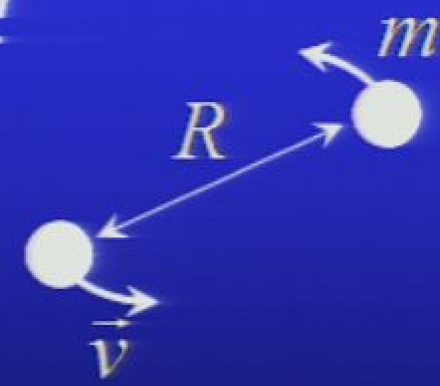
Step 2

Apply temporal dimensional reduction ansatz:

$$ds^2 = e^{2\phi}\left(dt - A_i dx^i\right)^2 - e^{-2\phi}\gamma_{ij} dx^i dx^j$$

This defines a change of variables $\bar{g}_{\mu\nu} \rightarrow \left(\gamma_{ij}, A_i, \phi\right)$

Post-Newtonian approximation



Einstein-Infeld-Hoffman Lagrangian derivation:

As a result we get

$$S_{pp} = -\sum_{a=1}^{2} m_a \int dt\, e^{\phi} \sqrt{\left(1 - \vec{A}\cdot\vec{v}_a\right)^2 - e^{-4\phi}\gamma_{ij} v_a^i v_a^j} + \dots$$

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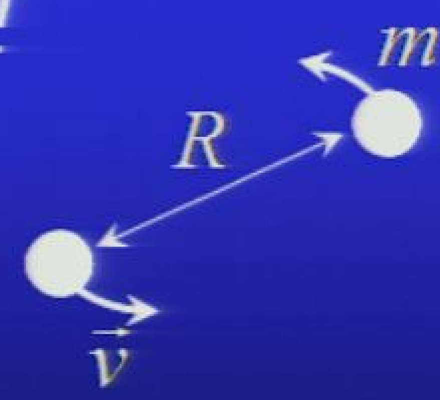

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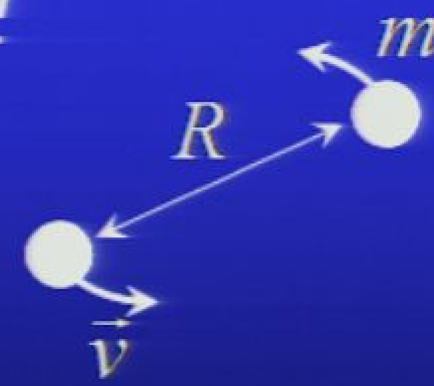

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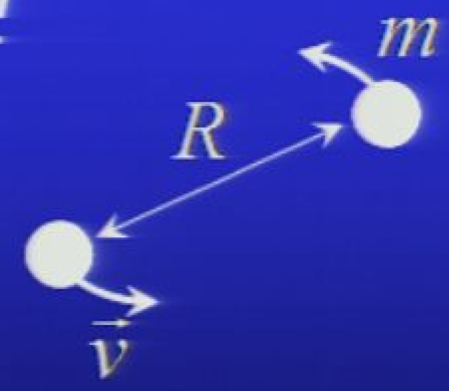

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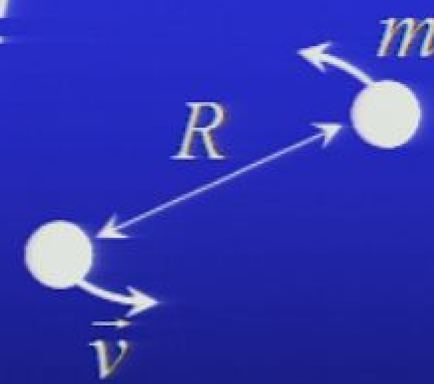

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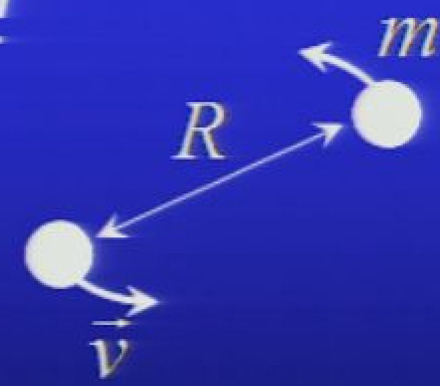

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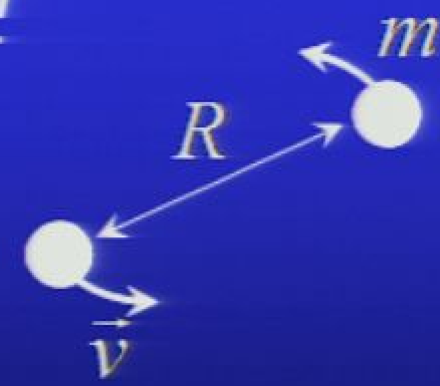
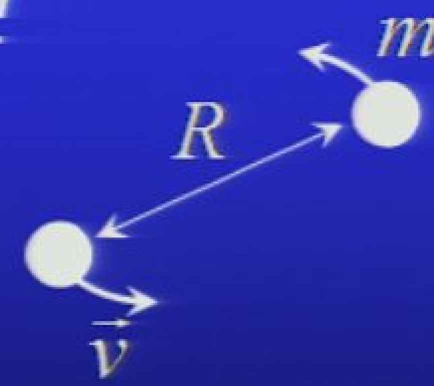


Step 3

Post-Newtonian approximation

Einstein-Infeld-Hoffman Lagrangian derivation:



Step 3

Decompose all the fields into potential and radiation components

$$\left(\gamma_{ij}, A_i, \phi\right) \rightarrow \left(\gamma_{ij} + \bar{\gamma}_{ij},\ A_i + \bar{a}_i,\ \phi + \bar{\phi}\right)$$

and integrate out the potential modes in order to obtain an action for two interacting massive particles which depends on their velocities as well as their locations

Post-Newtonian approximation

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$$S_{bulk} = -\frac{1}{8\pi G}\int d^4x \left[(\nabla\phi)^2 - \frac{1}{8}F_{ij}F^{ij} - \dot{\phi}^2 \right] + \ldots$$

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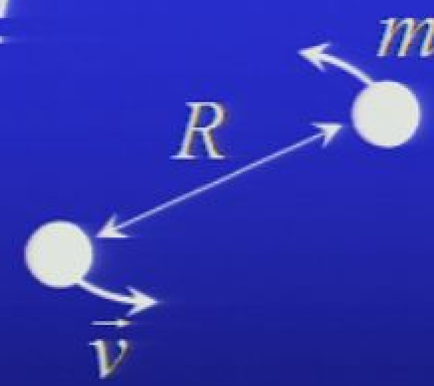

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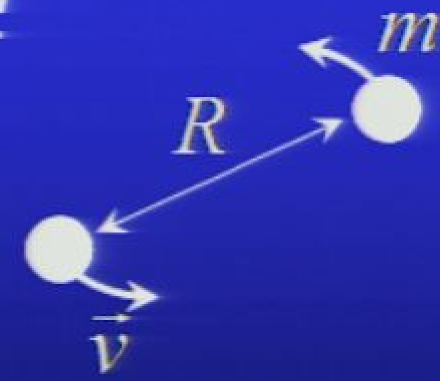

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Integrating out the potential modes leads to

Post-Newtonian approximation

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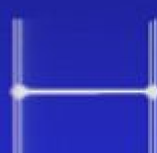
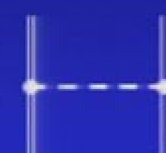
Integrating out the potential modes leads to

$$S_{eff}\left(\vec{x}_a, \vec{v}_a\right) = \sum_{a=1}^{2} \int dt \, \frac{m_a \vec{v}_a^2}{2} +$$



$$+ \sum_{a=1}^{2} \int dt \, \frac{m_a \left(\vec{v}_a^2\right)^2}{8} +$$



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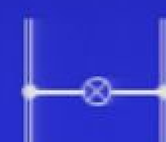

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$$S_{bulk} = -\frac{1}{8\pi G}\int d^4x\left[(\nabla\phi)^2 - \frac{1}{8}F_{ij}F^{ij} - \dot{\phi}^2\right] + \ldots$$

$$\underline{\phi \qquad \phi} \quad = 4\pi G\,\delta(t_1 - t_2)\int \frac{d^3k}{(2\pi)^3}\frac{e^{i\vec{k}(\vec{x}_1 - \vec{x}_2)}}{\vec{k}^2}$$

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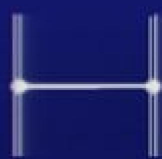

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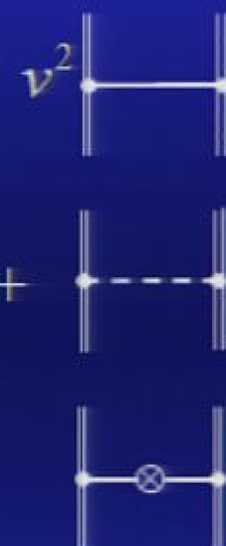
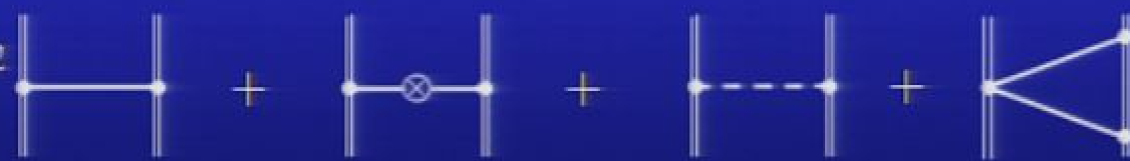


$$+$$

Post-Newtonian approximation

Einstein-Infeld-Hoffman Lagrangian derivation:

$$L_{EIH}\left(\vec{x}_a, \vec{v}_a\right) = \frac{1}{8}\sum_{a=1}^{2} m_a\left(\vec{v}_a^2\right)^2 + v^2$$



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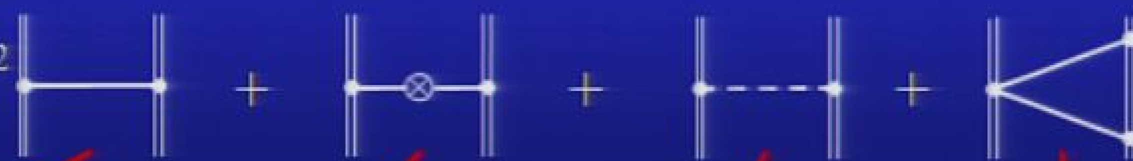

$$\frac{3Gm_1m_2}{2r}\left(\vec{v}_1^2 + \vec{v}_2^2\right)$$

$$\frac{Gm_1m_2}{2r}\vec{v}_{1\perp}\cdot\vec{v}_{2\perp}$$

$$-4\frac{Gm_1m_2}{r}\vec{v}_1\cdot\vec{v}_2$$

$$-\frac{G^2m_1m_2\left(m_1+m_2\right)}{2r^2}$$

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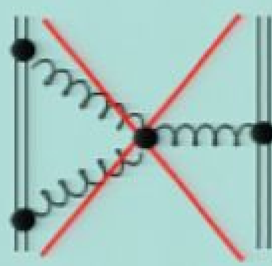
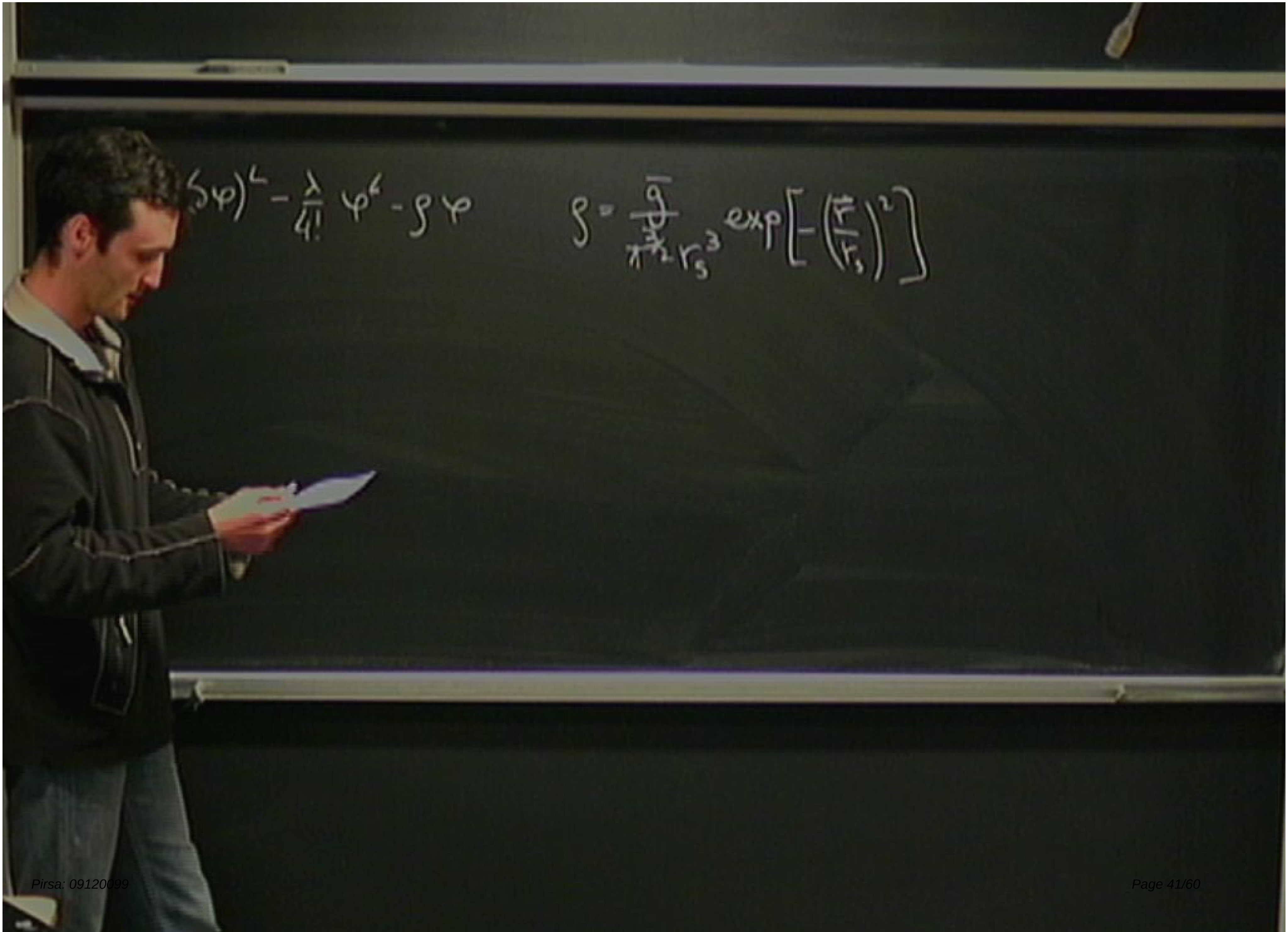
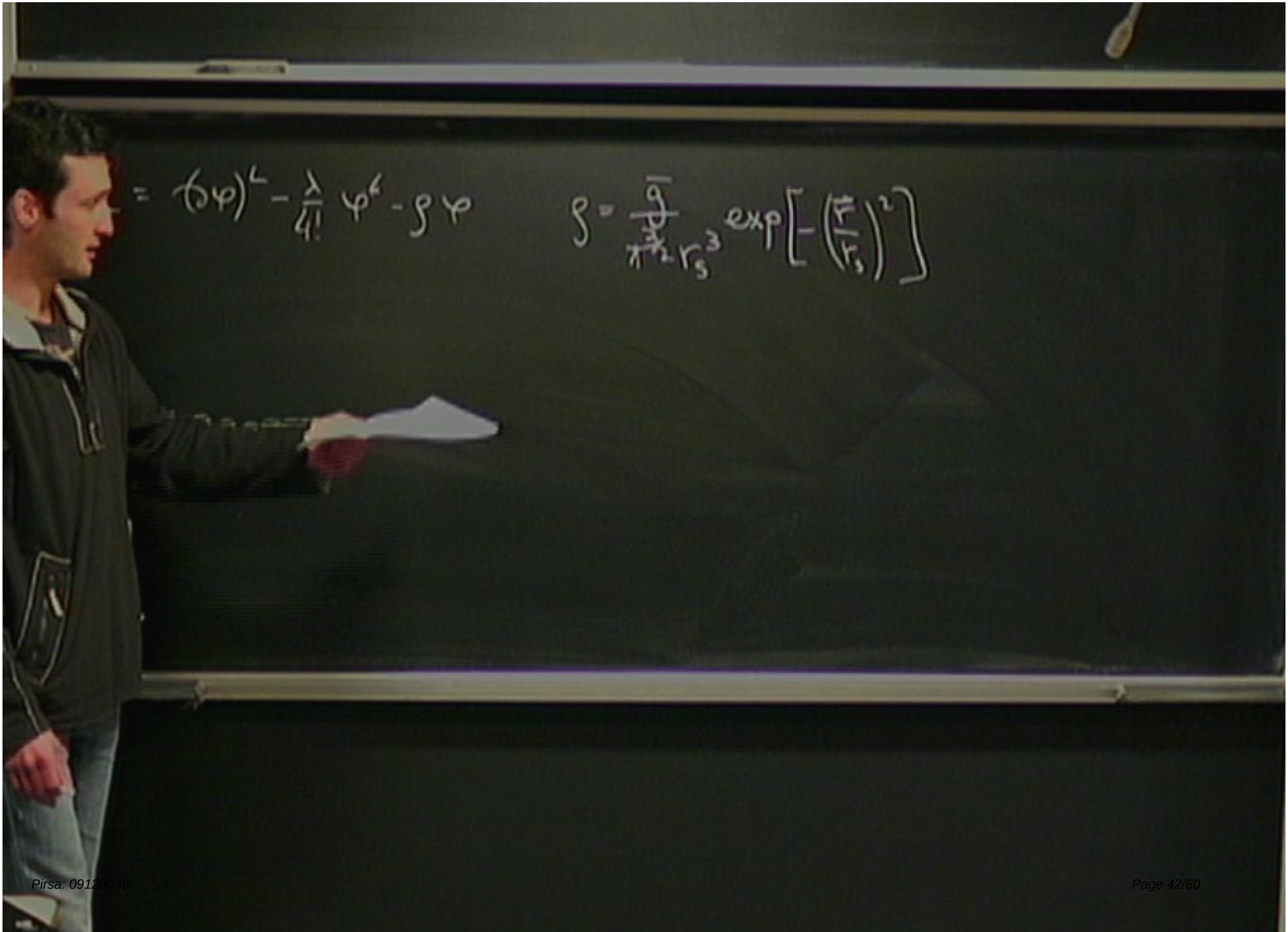
Improvement: *The pay-off of dimensional reduction* $\bar{g}_{\mu\nu} \rightarrow \left(\gamma_{ij}, A_i, \phi\right)$ *is that the triple vertex diagram gets eliminated and there is no need to cope with a loop diagram containing voluminous 3-graviton vertex.*




$(\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4 - \rho\varphi$
$\rho = \frac{g}{\pi^{3/2} r_s^3} \exp\left[-\left(\frac{r}{r_s}\right)^2\right]$


$= (\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4 - \rho\varphi$
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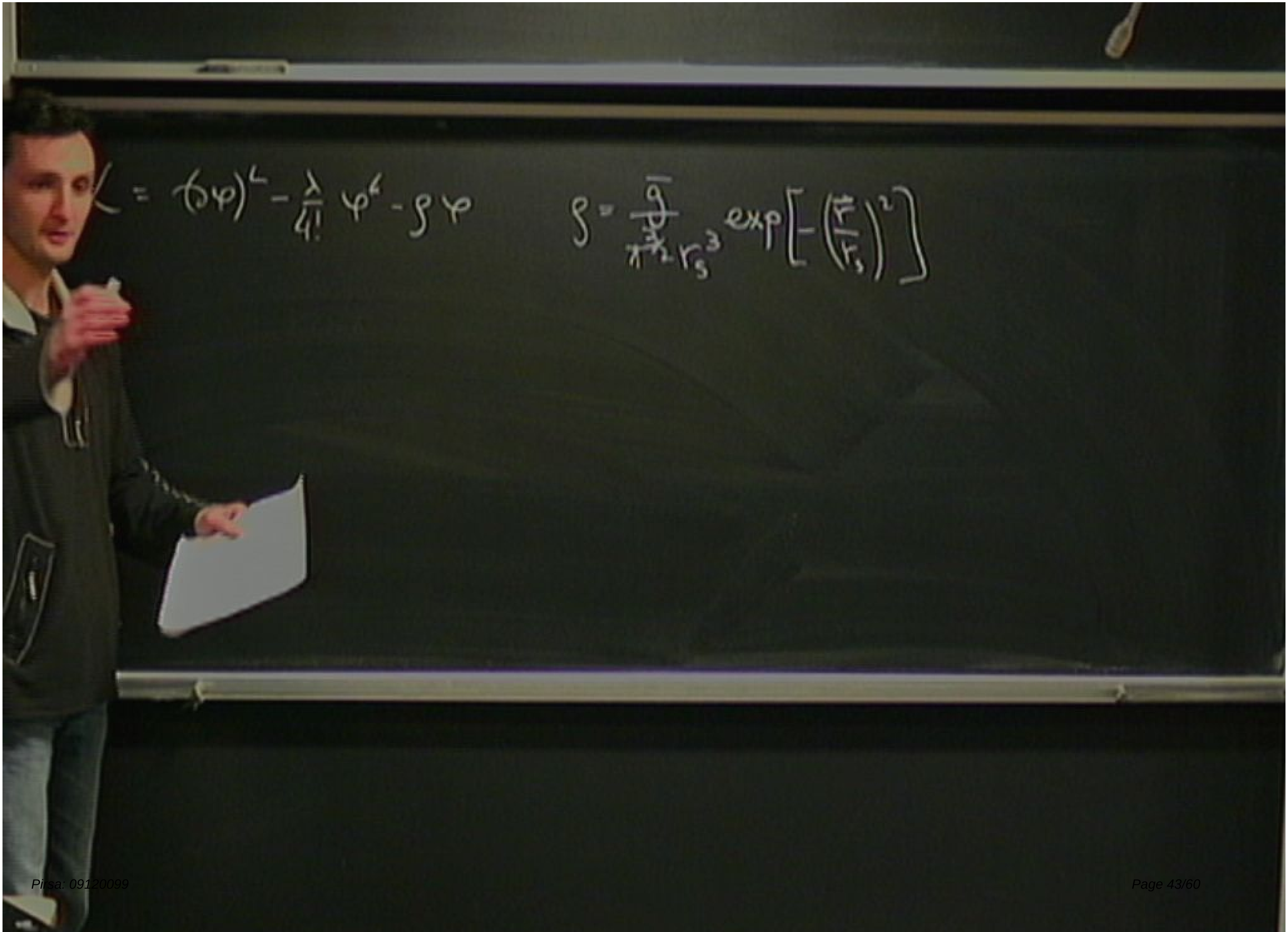
$\mathcal{L} = (\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4 - \rho\varphi$
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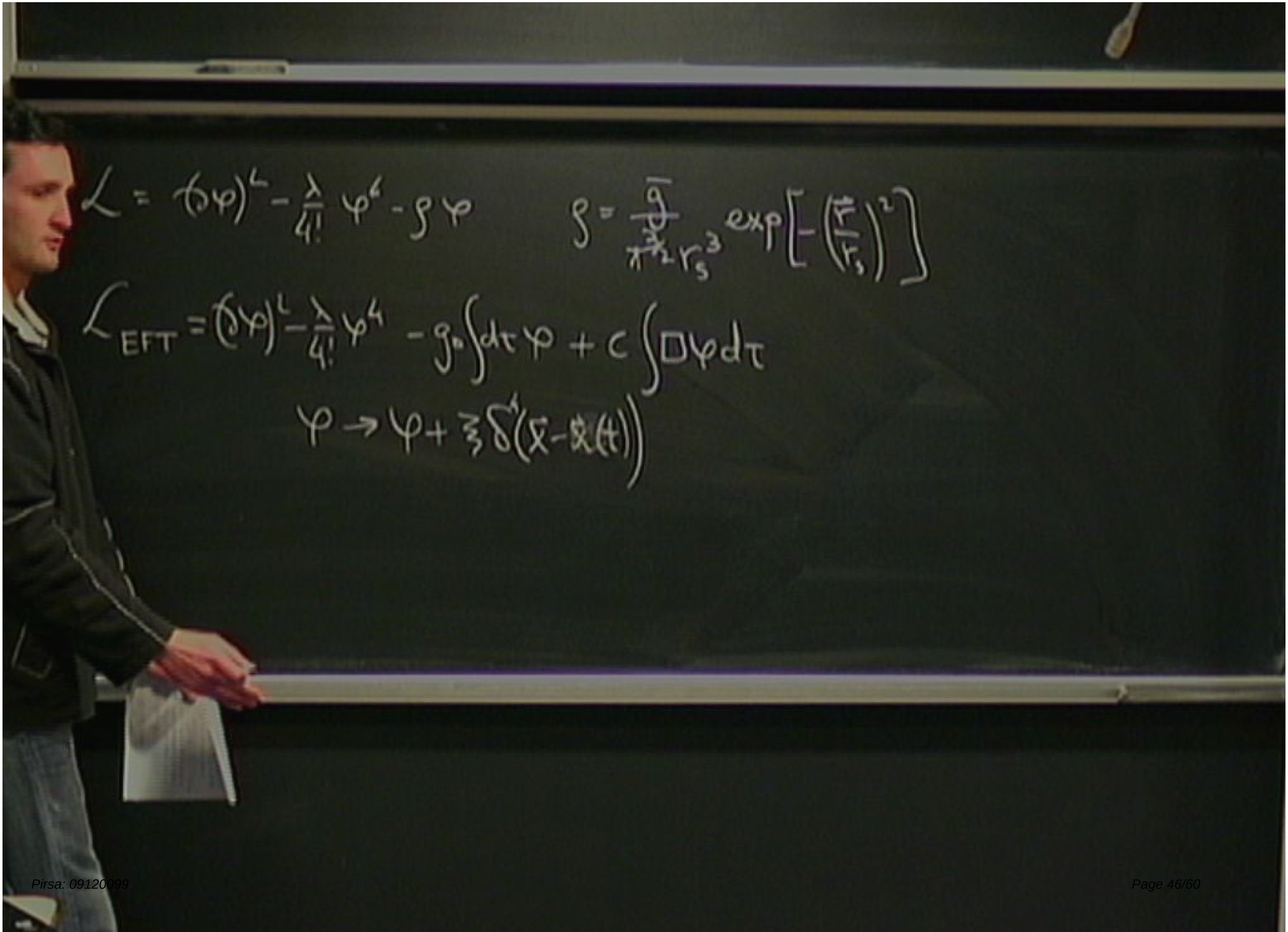
$$\mathcal{L} = (\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4 - \rho\varphi \qquad \rho = \frac{g}{\pi^{3/2} r_s^3}\exp\left[-\left(\frac{\vec{r}}{r_s}\right)^2\right]$$

$$\mathcal{L}_{EFT} = (\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4 - g_0\int d\tau\,\varphi + c\int \Box\varphi\, d\tau$$



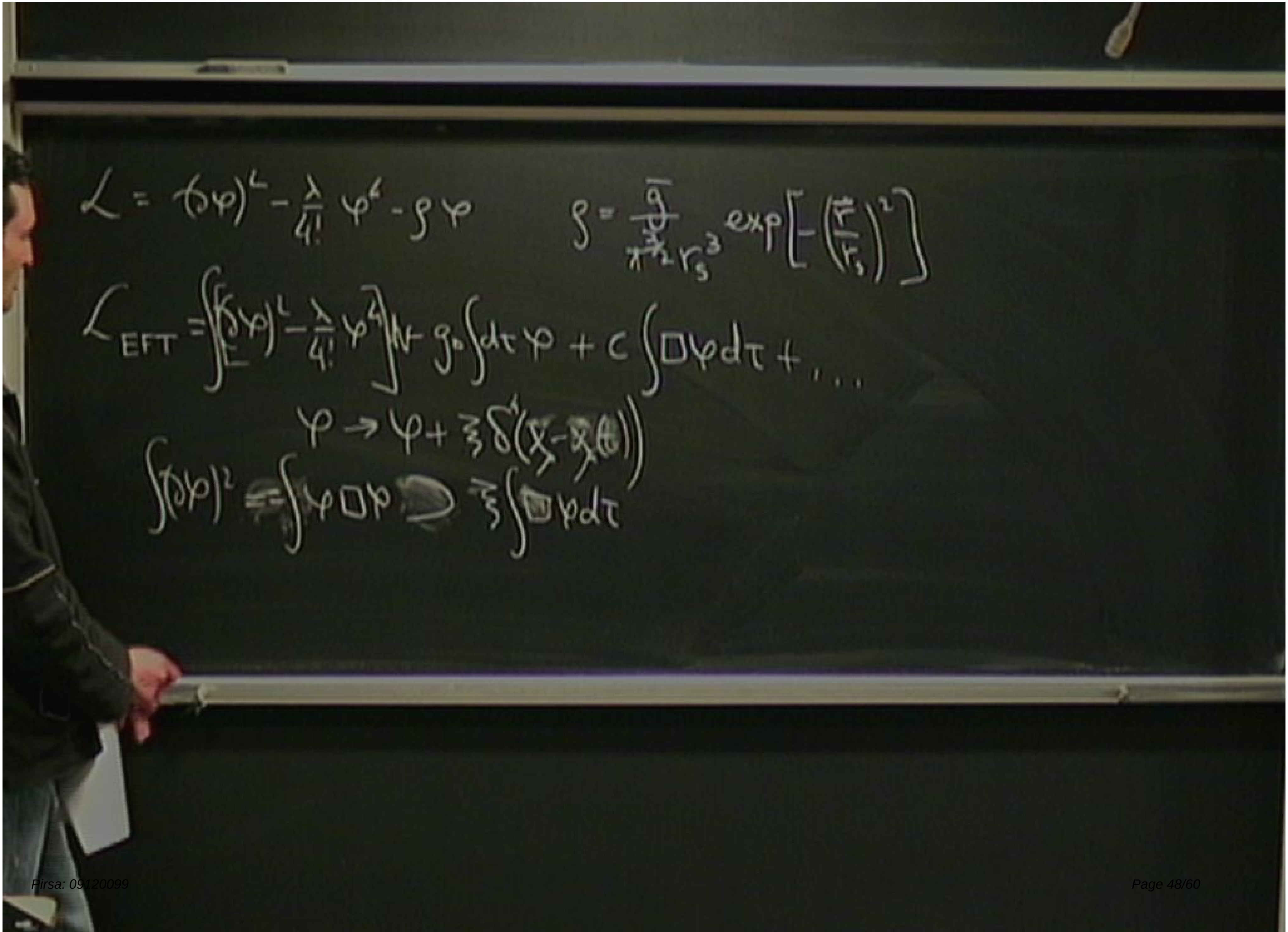
$\mathcal{L} = (\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4 - \rho\varphi$
$\rho = \frac{g}{\pi^{3/2} r_s^3} \exp\left[-\left(\frac{\vec{r}}{r_s}\right)^2\right]$
$\mathcal{L}_{EFT} = (\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4 - g_0\int d\tau\,\varphi + c\int \Box\varphi\, d\tau$
$\varphi \to \varphi + \xi\,\delta^4(\vec{x} - \vec{x}(t))$

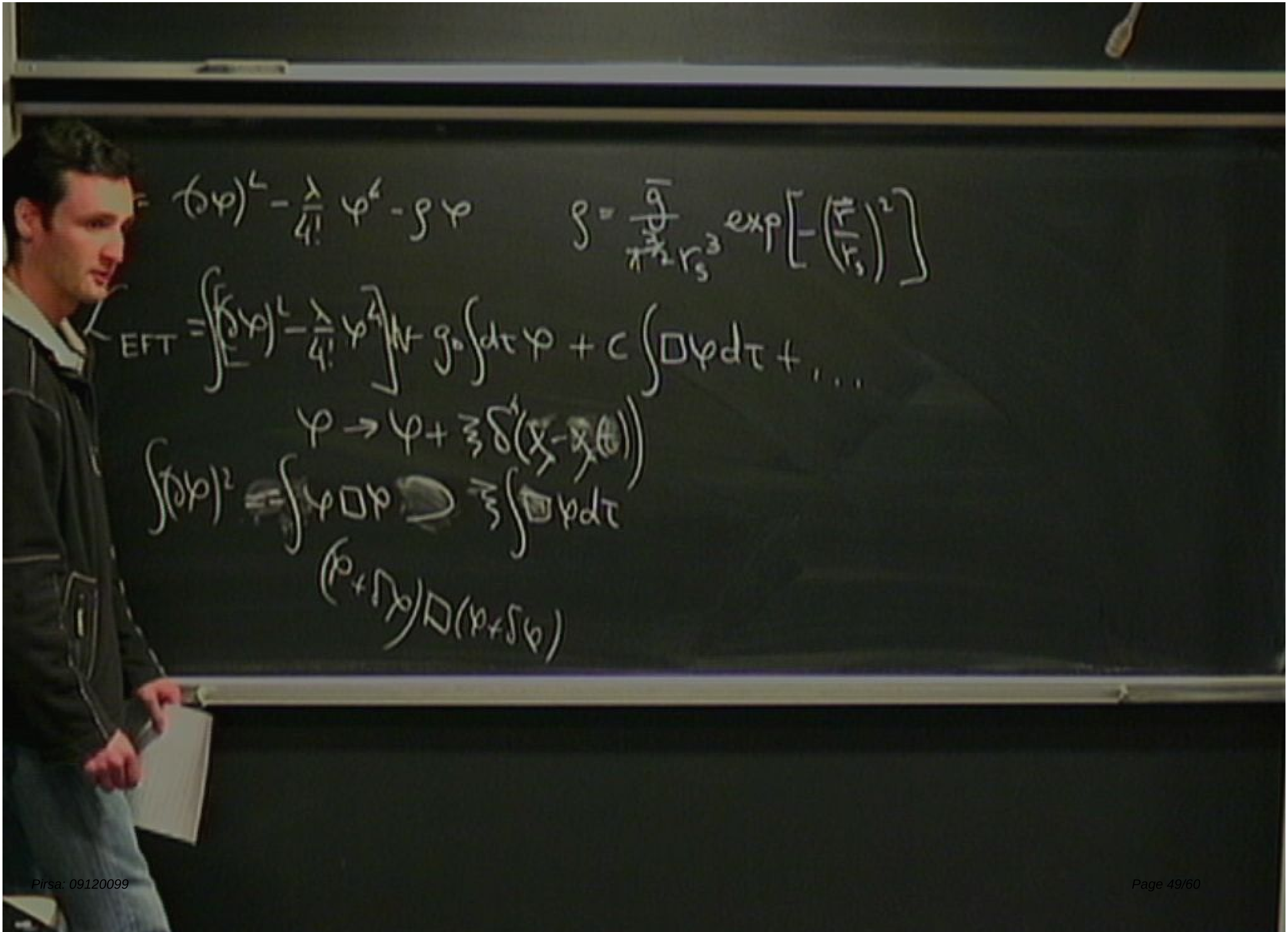
$$\mathcal{L} = \tfrac{1}{2}(\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4 - \rho\varphi \qquad \rho = \frac{g}{\pi^{3/2} r_s^3}\exp\left[-\left(\frac{r}{r_s}\right)^2\right]$$

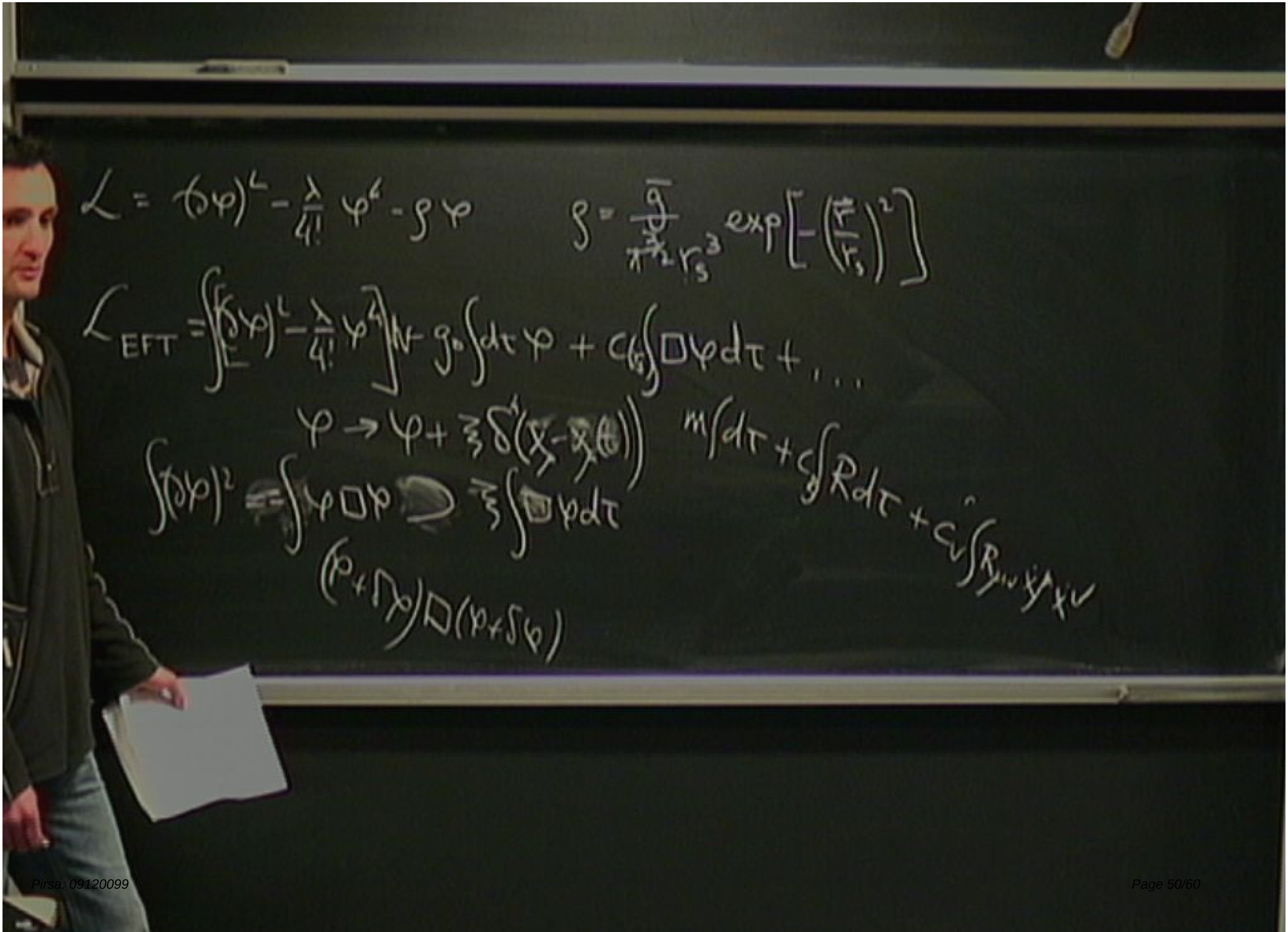
$$\mathcal{L}_{EFT} = \int\left[\tfrac{1}{2}(\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4\right]d^4x - g_0\int d\tau\,\varphi + c\int \Box\varphi\, d\tau$$

$$\varphi \to \varphi + \xi\,\delta^4(x - x(\tau))$$

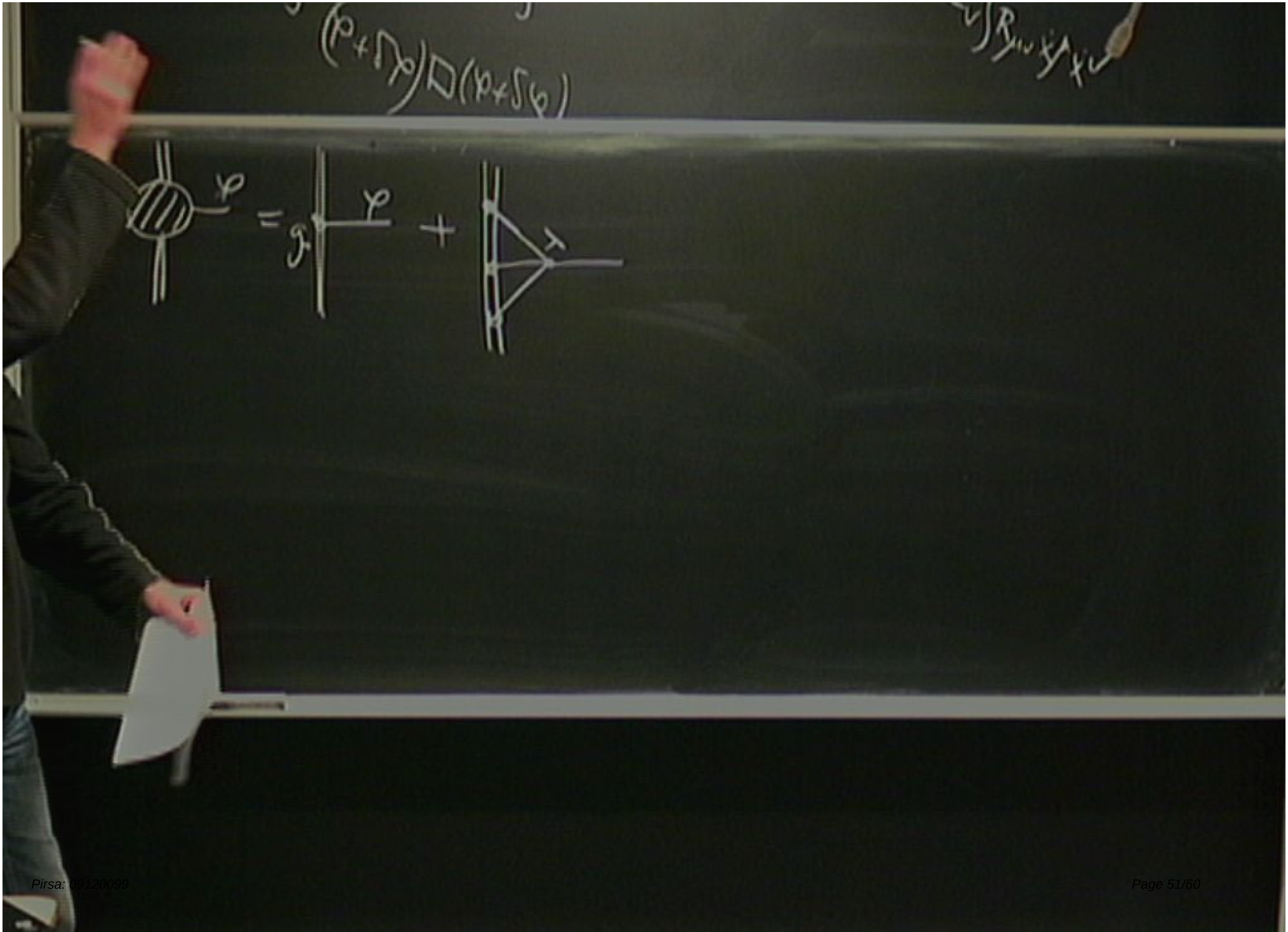
$$\int(\partial\varphi)^2 = -\int\varphi\Box\varphi \supset$$

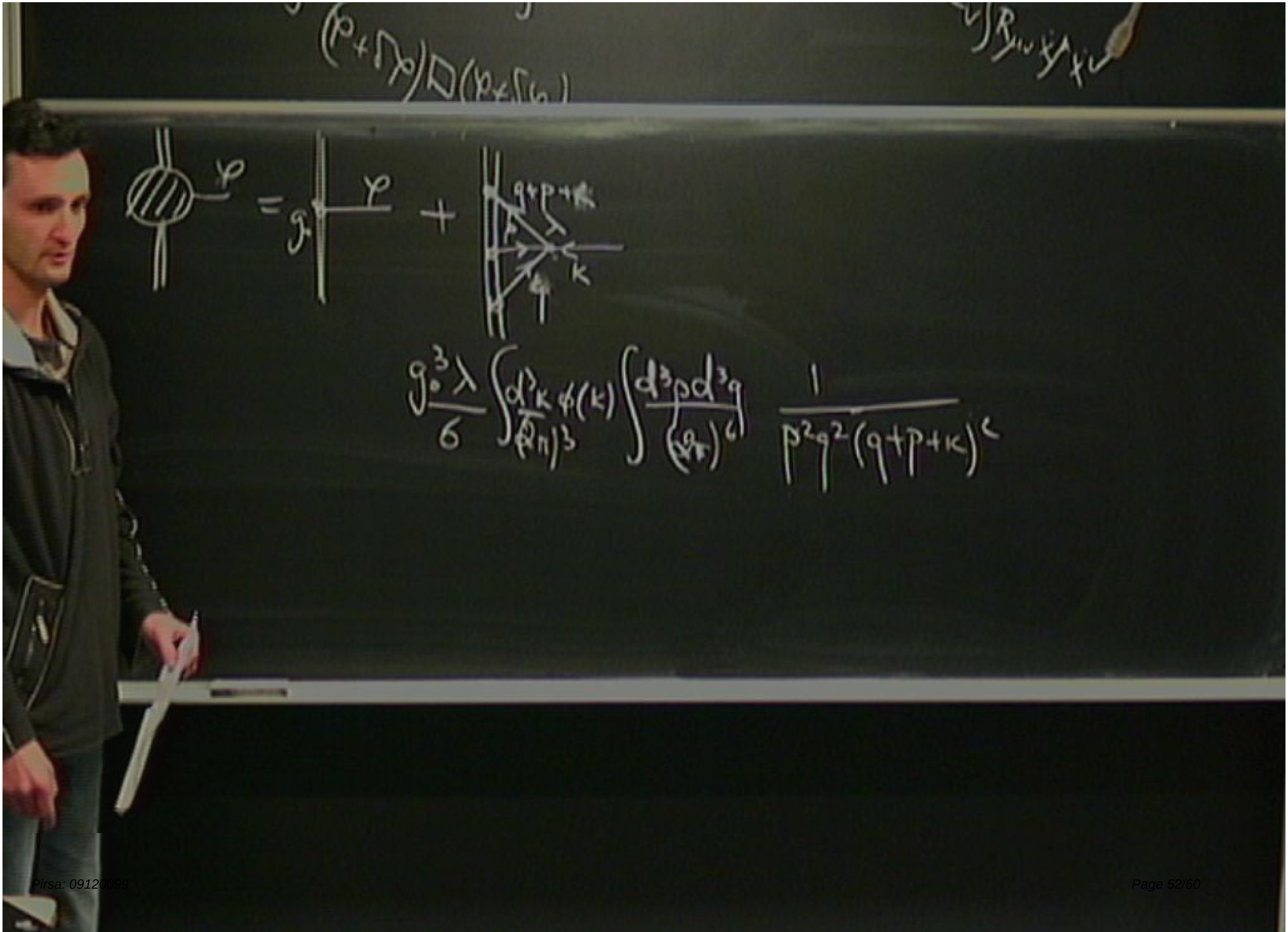

$\mathcal{L} = (\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4 - \rho\varphi$
$\rho = \frac{g}{\pi^{3/2} r_s^3} \exp\left[-\left(\frac{\vec{r}}{r_s}\right)^2\right]$
$\mathcal{L}_{EFT} = \int\left[(\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4\right] d^4x - g_0\int d\tau\,\varphi + c\int \Box\varphi\, d\tau + \ldots$
$\varphi \to \varphi + \xi\,\delta^4(x - x(\tau))$
$\int(\partial\varphi)^2 = -\int\varphi\Box\varphi \supset \xi\int\Box\varphi\, d\tau$

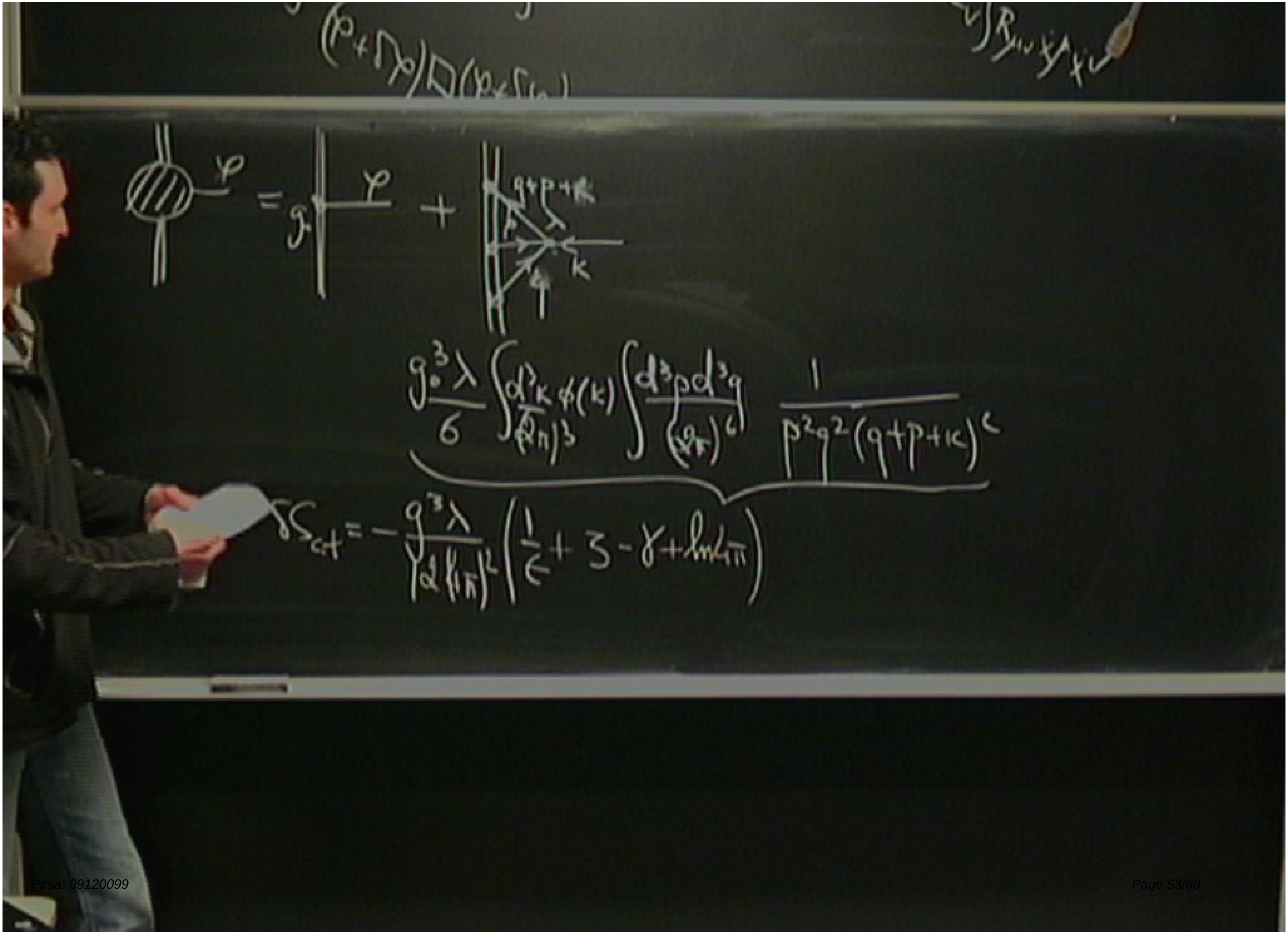

$= (\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4 - \rho\varphi$
$\rho = \frac{g}{\pi^{3/2} r_s^3} \exp\left[-\left(\frac{r}{r_s}\right)^2\right]$
$\mathcal{L}_{EFT} = \int\left[(\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4\right]d^4x - g_0\int d\tau\,\varphi + c\int \Box\varphi\, d\tau + \ldots$
$\varphi \to \varphi + \xi\delta^3(x - x(t))$
$\int(\partial\varphi)^2 = -\int\varphi\Box\varphi \quad \xi\int\Box\varphi\, d\tau$
$(\varphi+\delta\varphi)\Box(\varphi+\delta\varphi)$

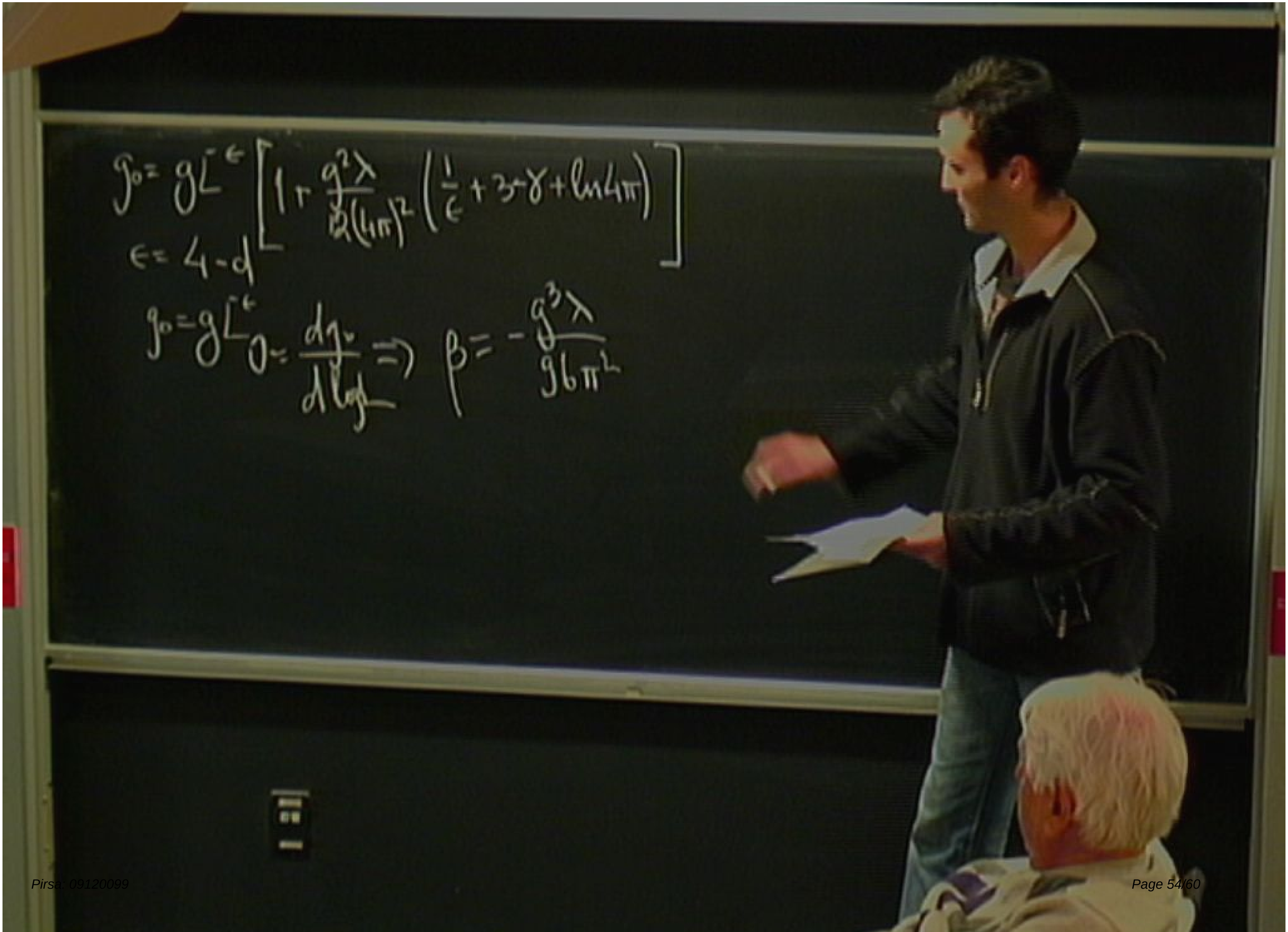


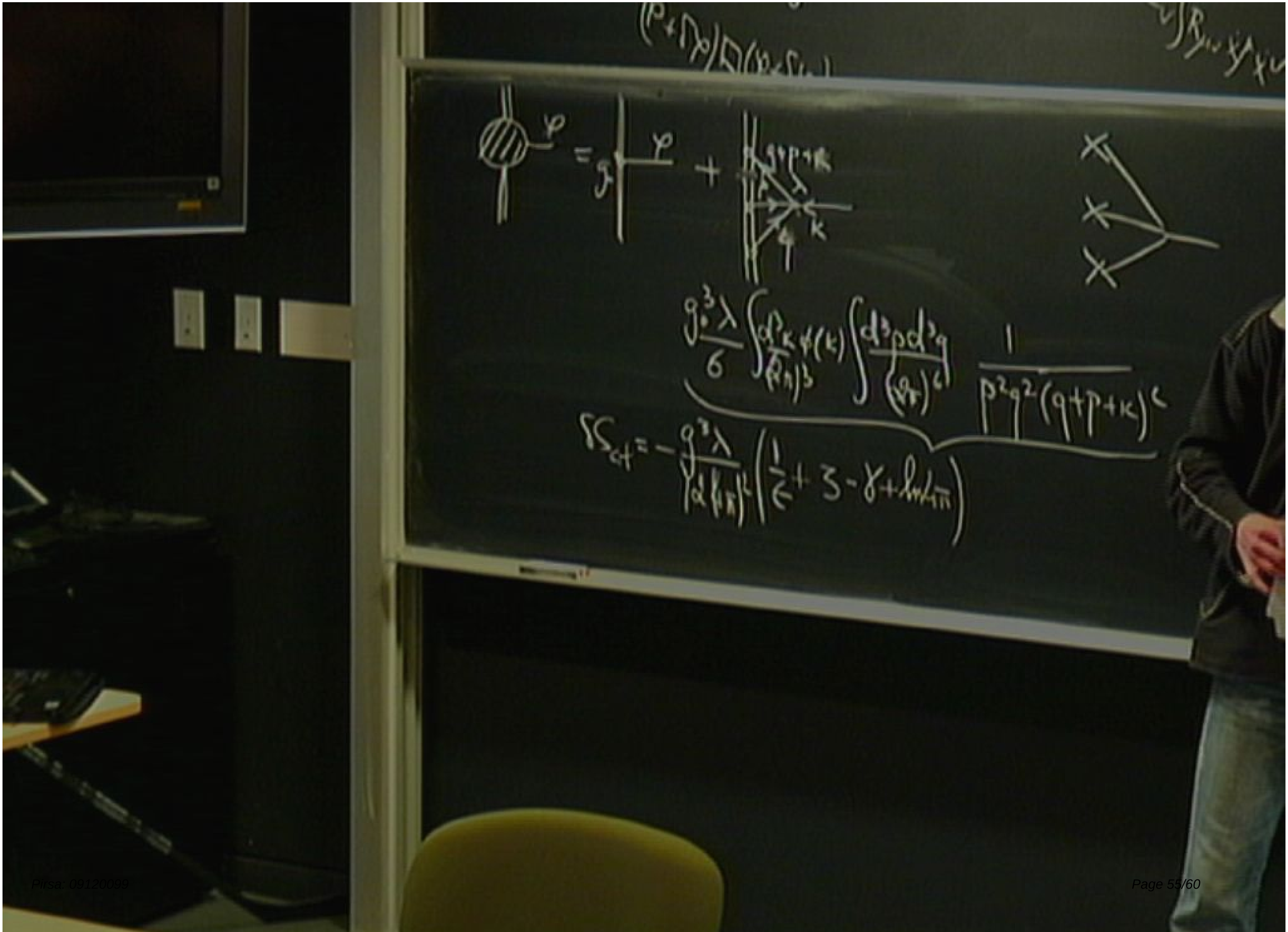
$\mathcal{L} = (\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4 - \rho\varphi$
$\rho = \frac{g}{\pi^{3/2} r_s^3} \exp\left[-\left(\frac{r}{r_s}\right)^2\right]$
$\mathcal{L}_{EFT} = \int\left[\frac{1}{2}(\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4\right]d^4x - g_0\int d\tau\,\varphi + c(r_s)\int \Box\varphi\, d\tau + \ldots$
$\varphi \to \varphi + \xi\,\delta^4(x - x(\tau))$
$m\int d\tau + c_R\int R\, d\tau + c_V\int R_{\mu\nu}\dot{x}^\mu \dot{x}^\nu$
$\int(\partial\varphi)^2 = -\int\varphi\Box\varphi \quad \xi\int\Box\varphi\, d\tau$
$(\varphi+\delta\varphi)\Box(\varphi+\delta\varphi)$


(φ+δφ)□(φ+δφ)
φ
=
g
φ
+
T


φ
q+p+k
k
g₀³λ/6 ∫ d³k/(2π)³ φ(k) ∫ d³p d³q/(2π)⁶ 1/(p²q²(q+p+k)²)


$\frac{g_0^3\lambda}{6}\int\frac{d^3k}{(2\pi)^3}\phi(k)\int\frac{d^3p\,d^3q}{(2\pi)^6}\frac{1}{p^2q^2(q+p+k)^2}$
$\delta S_{ct} = -\frac{g^3\lambda}{2(4\pi)^2}\left(\frac{1}{\epsilon}+3-\gamma+\ln 4\pi\right)$


$g_0 = g L^{-\epsilon} \left[1 + \frac{g^2 \lambda}{2(4\pi)^2} \left(\frac{1}{\epsilon} + 3 - \gamma + \ln 4\pi \right) \right]$
$\epsilon = 4 - d$
$g_0 = g L^{-\epsilon}$
$0 = \frac{d g_0}{d \log L} \Rightarrow \beta = -\frac{g^3 \lambda}{96\pi^2}$

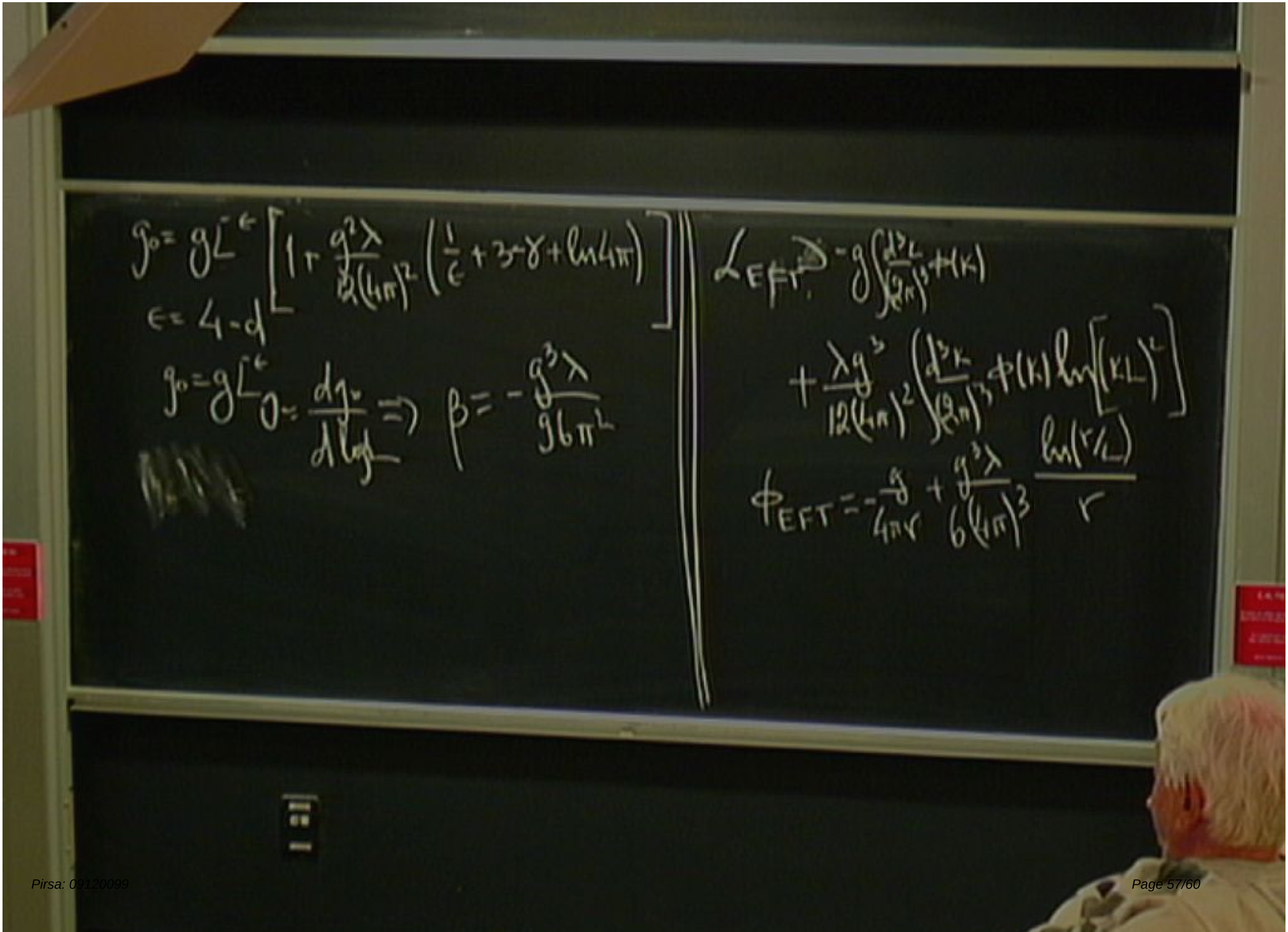


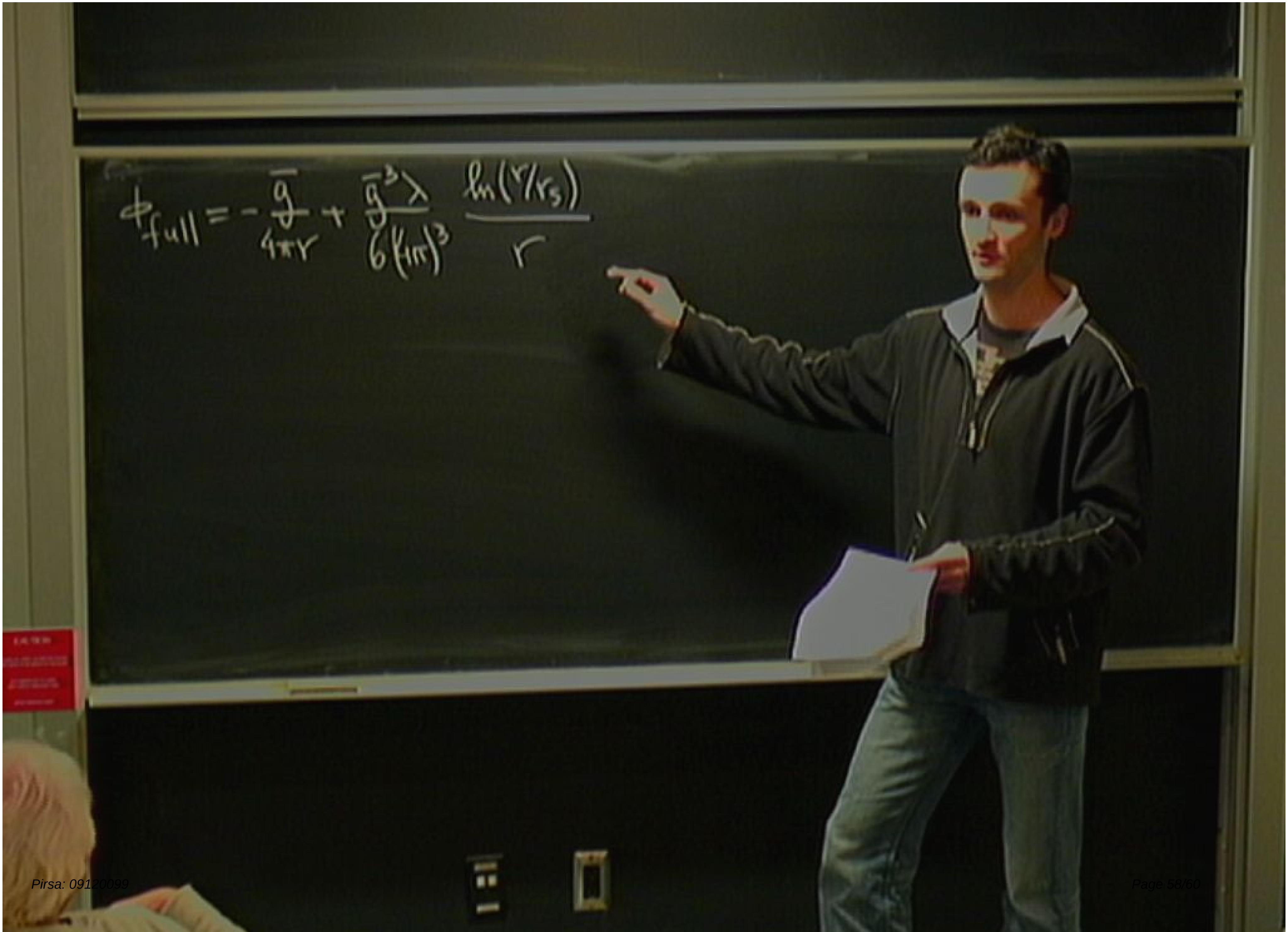
$$g_0 = g L^{-\epsilon}\left[1 + \frac{g^2\lambda}{12(4\pi)^2}\left(\frac{1}{\epsilon} + 3 - \gamma + \ln 4\pi\right)\right]$$

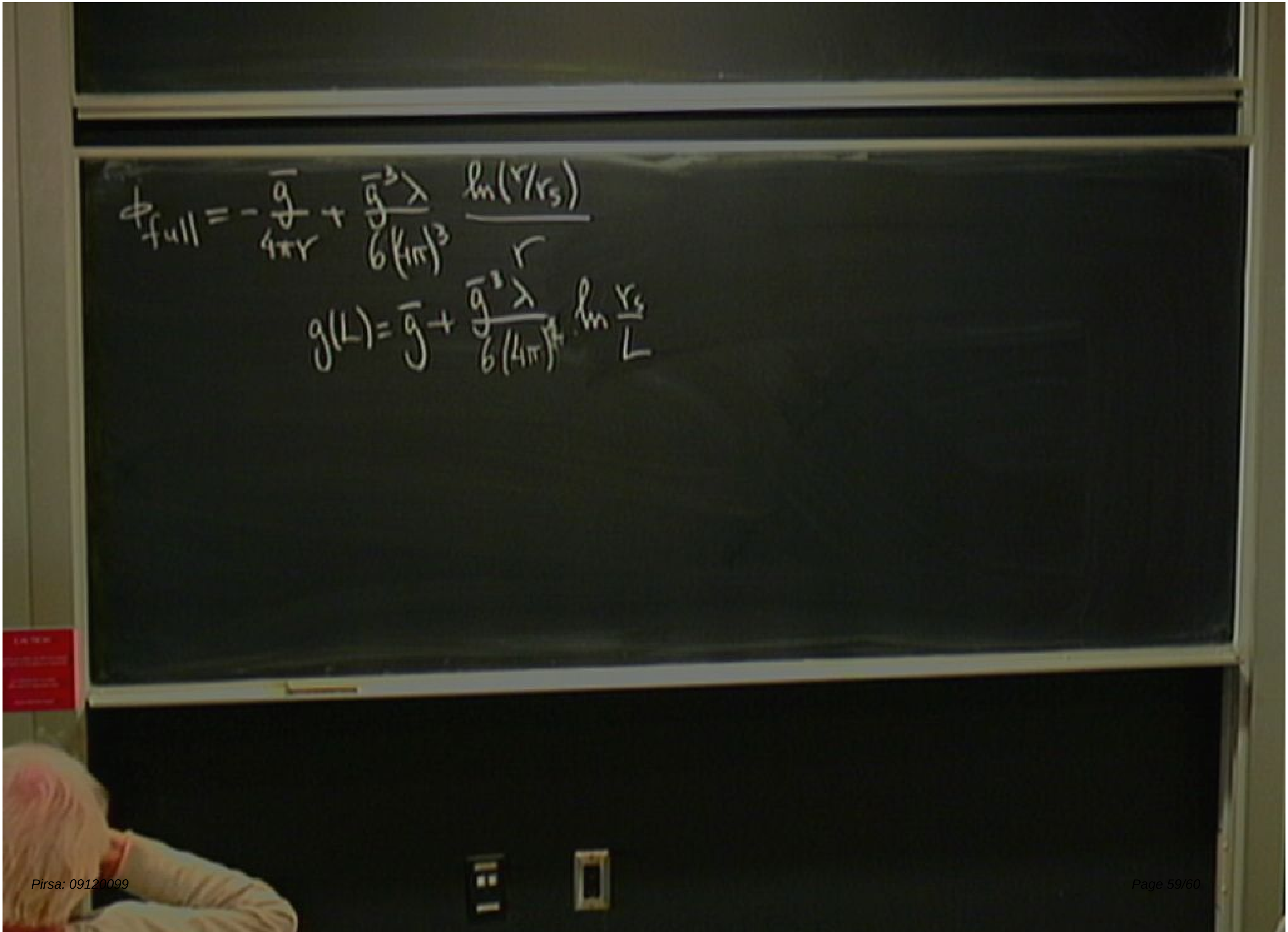
$$\epsilon = 4 - d$$

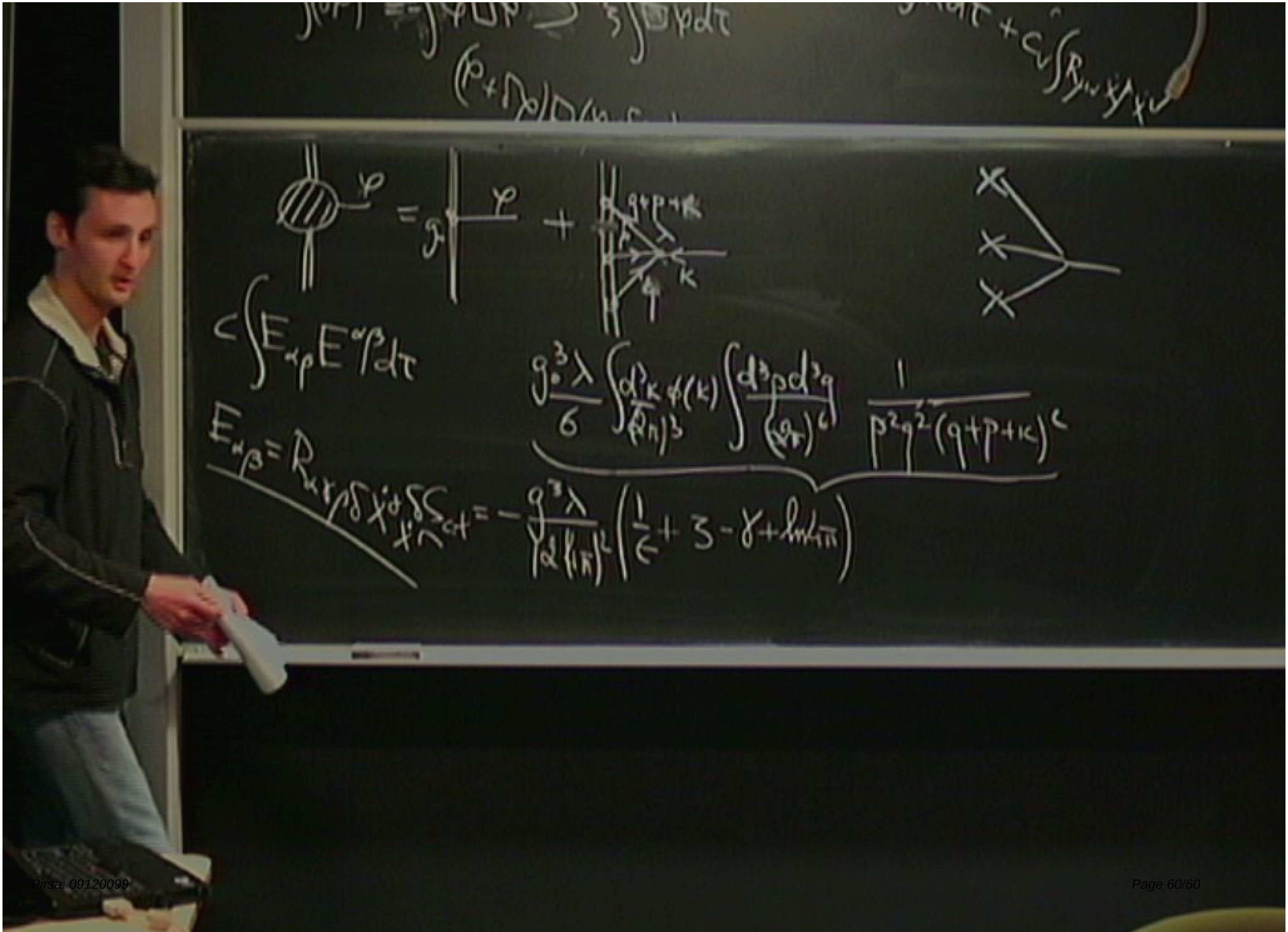
$$g_0 = g L^{-\epsilon} \quad 0 = \frac{dg_0}{d\log L} \Rightarrow \beta = -\frac{g^3\lambda}{96\pi^2}$$

$$\mathcal{L}_{EFT} \ni -g\int\frac{d^3k}{(2\pi)^3}\phi(k) + \frac{\lambda g^3}{12(4\pi)^2}\int\frac{d^3k}{(2\pi)^3}\phi(k)\ln\left[(kL)^2\right]$$


$g_0 = g L^{-\epsilon}\left[1 + \frac{g^2\lambda}{12(4\pi)^2}\left(\frac{1}{\epsilon} + 3 - \gamma + \ln 4\pi\right)\right]$
$\epsilon = 4 - d$
$g_0 = g L^{\epsilon}$ $0 = \frac{dg_0}{d\log L} \Rightarrow \beta = -\frac{g^3\lambda}{96\pi^2}$
$\mathcal{L}_{EFT} \ni -g\int\frac{d^3k}{(2\pi)^3}\phi(k)$
$+ \frac{\lambda g^3}{12(4\pi)^2}\int\frac{d^3k}{(2\pi)^3}\phi(k)\ln\left[(kL)^2\right]$
$\phi_{EFT} = -\frac{g}{4\pi r} + \frac{g^3\lambda}{6(4\pi)^3}\frac{\ln(r/L)}{r}$


$\phi_{full} = -\frac{\bar{g}}{4\pi r} + \frac{\bar{g}^3 \lambda}{6(4\pi)^3} \frac{\ln(r/r_s)}{r}$


$\phi_{full} = -\frac{\bar{g}}{4\pi r} + \frac{\bar{g}^3 \lambda}{6(4\pi)^3} \frac{\ln(r/r_s)}{r}$
$g(L) = \bar{g} + \frac{\bar{g}^3 \lambda}{6(4\pi)^2} \ln \frac{r_s}{L}$


c∫E_αρ E^αρ dτ
g₀³λ/6 ∫d³k/(2π)³ φ(k) ∫d³p d³q/(2π)⁶ 1/(p²q²(q+p+k)²)
E_αβ = R_αγβδ ẋ^γ ẋ^δ
1/ε + 3 − γ + ln 4π