

Title: Cosmology - Review (PHYS 621) - Lecture 13

Date: Dec 16, 2009 10:00 AM

URL: <http://pirsa.org/09120095>

Abstract:

$$N = 68 + \frac{1}{4} \ln \frac{V_{Hor}}{M_p^4}$$



$$N = 68 + \frac{1}{4} \ln \frac{V_{Hor}}{M_p^4}$$

$$\frac{a_{eq} H_{eq}}{a_0 H_0} \approx 219 \Omega_m^{(0)} h$$

x.
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$$N = 68 + \frac{1}{4} \ln \frac{V_{\text{Hor}}}{M_{\text{p}}^4}$$

$$\frac{a_{\text{eq}} H_{\text{eq}}}{a_0 H_0} \simeq 219 \Omega_{\text{m}(0)} h$$

$$x. \quad \Omega_{\text{r}(0)} = 4.15 \times 10^{-5} h^{-2}$$

$$1 + z_{\text{eq}} = 2.41 \Omega_{\text{m}(0)} h^2 \times 10^4$$

$$N = 68 + \frac{1}{4} \ln \frac{V_{Hor}}{M_p^4}$$

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x. $\Omega_{r(0)} = 4.15 \times 10^{-5} h^{-2}$

$$z_{eq} = 2.41 \Omega_{m(0)} h^2 \times 10^4$$

$$\frac{H_{eq}^2}{H_0^2} = \frac{8\pi}{3m_p^2} \left(\rho_{m(z_{eq})} + \rho_{r(z_{eq})} \right)$$

$$\rho_m(z_{eq}) = \rho_r(z_{eq})$$

$$N = 68 + \frac{1}{4} \ln \frac{V_{Hor}}{M_p^4}$$

$$\frac{a_{eq} H_{eq}}{a_0 H_0} \approx 219 \Omega_{m(0)} h$$

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$$\frac{H_{eq}}{H_0} ; H_{eq}^2 = \frac{8\pi}{3m_p^2} (\rho_m + \rho_r) = \frac{4\pi}{3m_p^2} \rho_m(z_{eq}) = \frac{4\pi}{3m_p^2} \rho_{m,0} (1+z_{eq})^3$$

$$\rho_m(z_{eq}) = \rho_r(z_{eq})$$

$$q = \sqrt{2} \Omega_{m0} (1+z_{eq})^{3/2}$$

$$\frac{H_{eq}}{H_0} = \sqrt{2} \Omega_{m0} (1+z_{eq})^{1/2}$$

$$\rho(z_{eq}) = \frac{4\pi}{3m_p^3} \rho_{m0} (1+z_{eq})^3 = \frac{4\pi}{3m_p^3} \frac{3m_p^2}{8\pi} \Omega_{m0} H_0^2 (1+z_{eq})^3$$

$$\frac{H_{eq}}{H_0} = \sqrt{2} \Omega_{m(0)} (1+z_{eq})^{3/2}$$

$$\frac{a_{eq} H_{eq}}{a_0 H_0} = \sqrt{2} \Omega_{m(0)} (1+z_{eq})^{1/2}$$

$$= \frac{4\pi}{3m_p^2} \rho_m(z_{eq}) = \frac{4\pi}{3m_p^2} \rho_{m,0} (1+z_{eq})^3 = \frac{4\pi}{3m_p^2} \frac{3m_p^2}{8\pi} \Omega_{m(0)} H_0^2 (1+z_{eq})^3$$

$$N = 68 + \frac{1}{4} \ln \frac{V_{\text{Hor}}}{M_{\text{P}}^4} \rightarrow \epsilon$$

$$\rightarrow 63 + \frac{1}{4} \ln \epsilon$$

219 $\Omega_{\text{m}0}$

$$\frac{C_{\text{eq}} H_{\text{eq}}}{\alpha_0 H_0} = \sqrt{2 \Omega_{\text{m}0}} \quad (1)$$

$$N = 68 + \frac{1}{4} \ln \frac{V_{\text{Hor}}}{M_{\text{Pl}}^4} \rightarrow \epsilon$$

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old Inflation (Guth 1981)

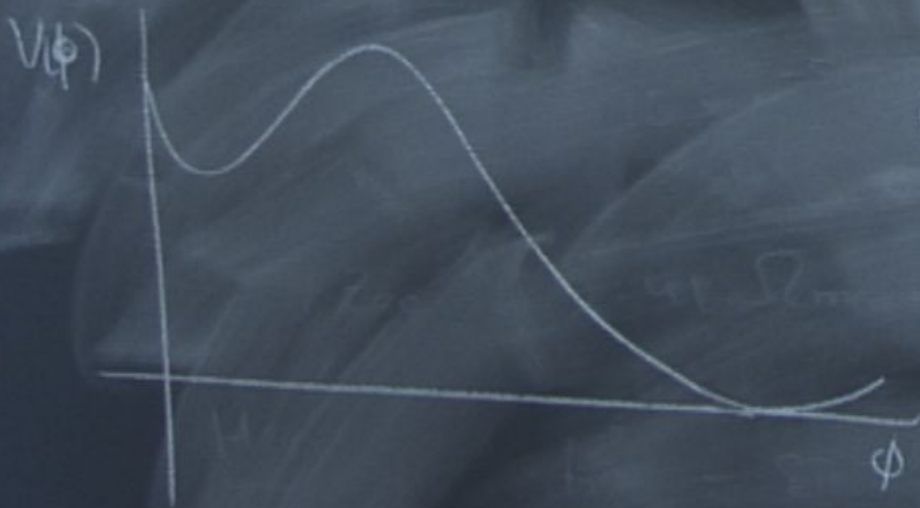


$$\frac{c_{\text{eq}} H_{\text{eq}}}{a_0 H_0} = \sqrt{2\Omega_{\text{m}0}} \quad (1)$$

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Old Inflation (Guth 1981)



$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi)$$

$$p = \frac{\dot{\phi}^2}{2} - V(\phi)$$

$$\frac{c_{\text{eq}} H_{\text{eq}}}{a_0 H_0} = \sqrt{2\Omega_{\text{m}0}}$$

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Old Inflation (Guth 1981)

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$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi)$$

$$p = \frac{\dot{\phi}^2}{2} - V(\phi)$$

$$w = \frac{p}{\rho} < -\frac{1}{3}$$



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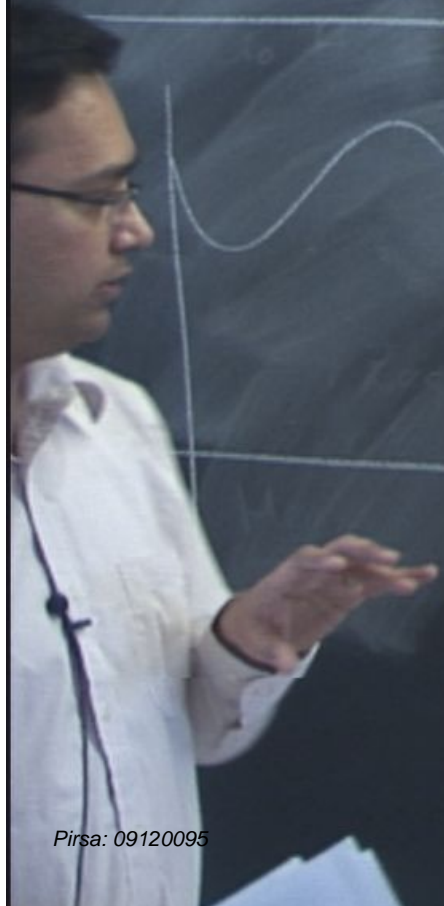
$$\frac{a_{\text{eq}} H_{\text{eq}}}{a_0 H_0} = \sqrt{2 \Omega_{\text{m}0}}$$

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Old Inflation (Guth 1981)



$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi)$$

$$p = \frac{\dot{\phi}^2}{2} - V(\phi)$$

$$\omega = \frac{p}{\rho} < -\frac{1}{3}$$

$$\omega \approx -1$$

$$\frac{a_{\text{eq}} H_{\text{eq}}}{a_0 H_0} = \sqrt{2\Omega_{\text{m}0}}$$

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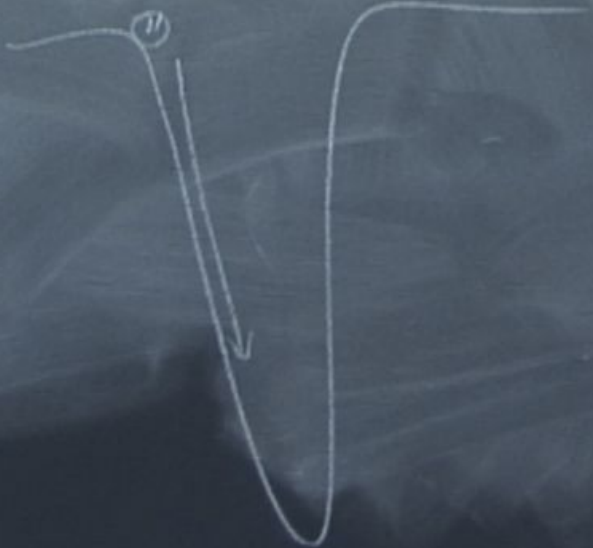
$$a \propto e^{\sqrt{\lambda} t}$$

$$\frac{a_{\text{eq}} H_{\text{eq}}}{a_0 H_0} = \sqrt{2 \Omega_{\text{m,eq}}}$$

Heg

$$\sqrt{2S_{m10}}$$

$$(1+Z_{eq})^{1/2}$$



$$N = 68 + \frac{1}{4} \ln \frac{V_{\text{Hor}}}{M_{\text{Pl}}^4} \rightarrow \epsilon$$

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Old Inflation (Guth 1981)



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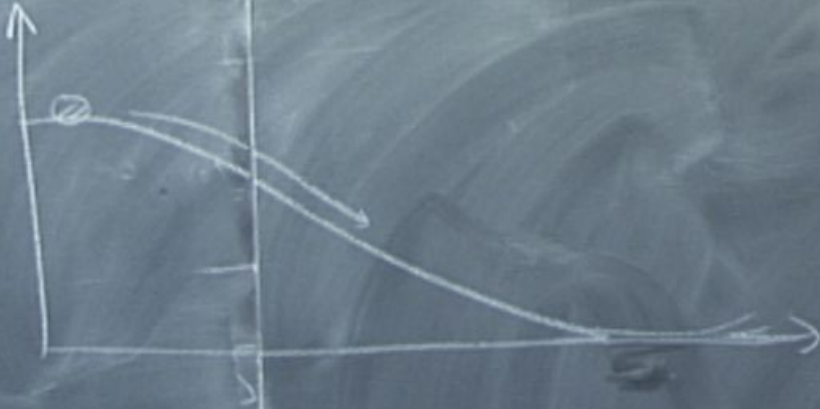
$$a \propto e^{-\sqrt{\lambda} t}$$

$$\frac{a_{\text{eq}} H_{\text{eq}}}{a_0 H_0} = \sqrt{2 \Omega_{\text{max}}}$$

New Inflation ; Linde ; Albrecht & Steinhardt (1992)

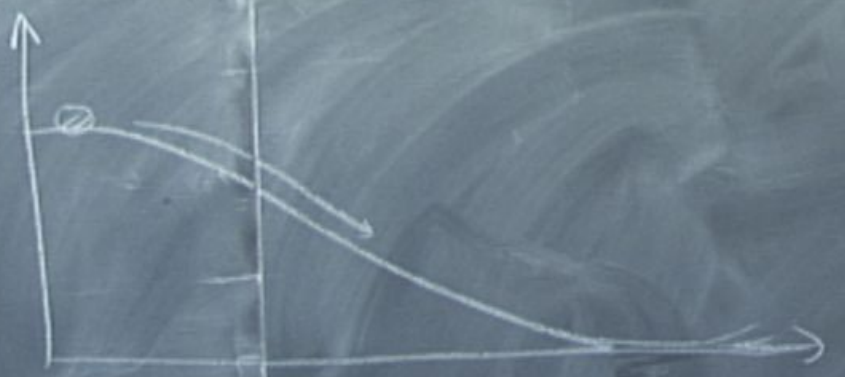


New Inflation ; Linde ; Albrecht & Steinhardt. (1992)

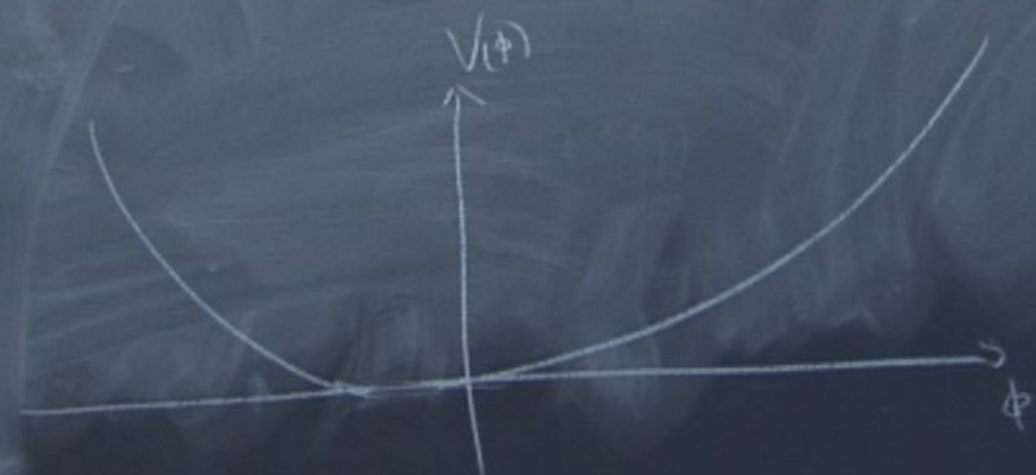


⊛ Chaotic Inflation

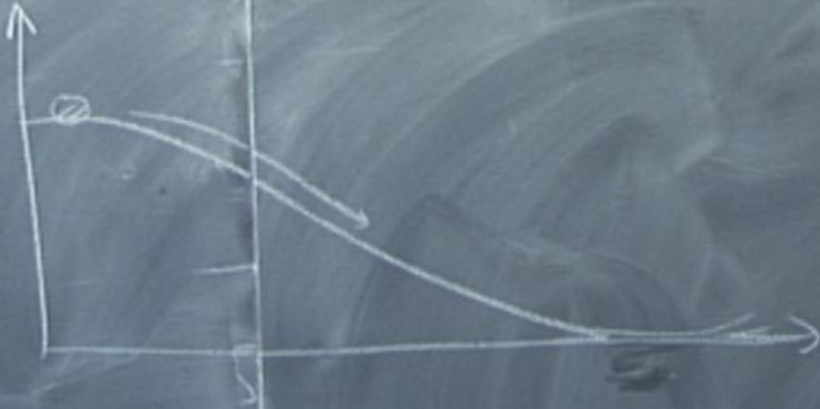
New Inflation ; Linde ; Albrecht & Steinhardt. (1992)



* Chaotic Inflation



New Inflation ; Linde ; Albrecht & Steinhardt. (1992)



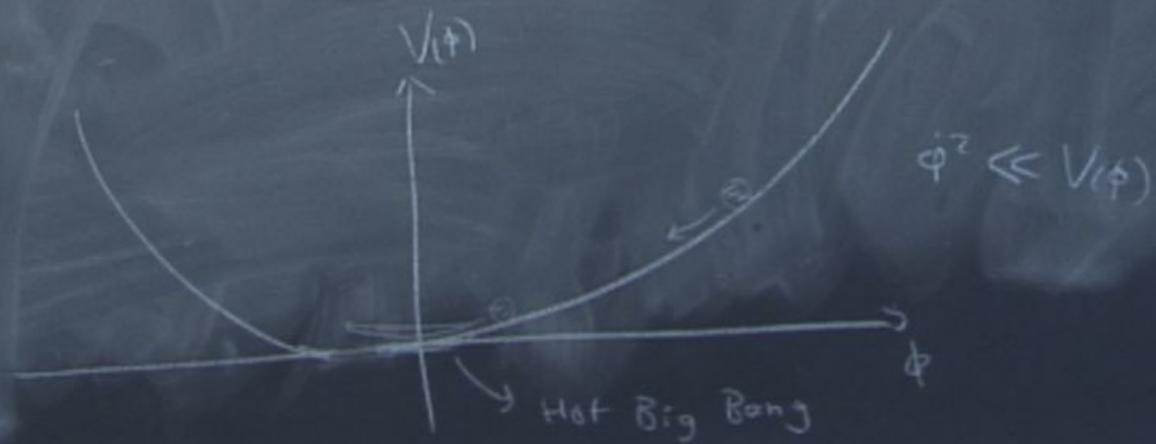
⊛ Chaotic Inflation



New Inflation ; Linde ; Albrecht & Steinhardt. (1992)



* Chaotic Inflation



Inflation ; Linde ; Albrecht & Steinhardt (1992)



Chaotic Inflation

$V(\phi)$

$$-\dot{\phi}^2 + V > 0$$

$$\dot{\phi}^2 \ll V(\phi)$$



Hot Big Bang

$$N = 68 + \frac{1}{4} \ln \frac{V_{\text{Hor}}}{M_{\text{p}}^4} \rightarrow \epsilon$$

$$\rightarrow 63 + \frac{1}{4} \ln \epsilon$$

$$H^2 = \frac{8\pi G}{3} \left(\frac{\dot{\phi}^2}{2} + V \right)$$

New Inf



6)

⊗ Chaot

$$N = 68 + \frac{1}{4} \ln \frac{V_{\text{Hor}}}{M_{\text{p}}^4} \rightarrow \epsilon$$

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New Inf

$$H^2 = \frac{8\pi G}{3} \left(\frac{\dot{\phi}^2}{2} + V \right)$$

$$P = \frac{\dot{\phi}^2}{2} + V$$

$$P = \frac{\dot{\phi}^2}{2} - V$$

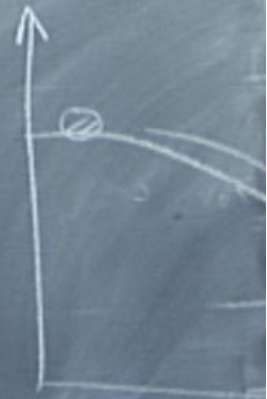
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (P + 3P) = \frac{8\pi G}{3} (V - \dot{\phi}^2)$$

$$\dot{\phi} + 3H(P + P) = 0$$

$$\rightarrow \ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

Klein-Gordon Eqn

⊗ Chaotic



$$\ddot{H} = \frac{\ddot{a}}{a} - H^2$$

$$\dot{H} = -4\pi G \dot{\phi}^2$$

Linde ; Albrecht & Steinhardt.

$V(\phi)$

Hot Big Bang

$$\ddot{H} = \frac{\ddot{a}}{a} - H^2$$

$$\dot{H} = -4\pi G \dot{\phi}^2$$

; Linde ; Albrecht & Steinhardt.

Slow roll

(i) $\dot{\phi}^2 \ll V(\phi)$

(ii) $\dot{\phi}$ should not change in one Hubble time

$$\frac{8\pi G}{3} V(\phi)$$

$$\ddot{H} = \frac{\ddot{a}}{a} - H^2$$

; Linde ; Albrecht & Ste

$$\dot{H} = -4\pi G \dot{\phi}^2$$

$$\frac{1}{2}\dot{\phi}^2 + V$$
$$= \frac{1}{2}\dot{\phi}^2 - V$$

Slow roll

(i) $\dot{\phi}^2 \ll V(\phi)$

(ii) $\dot{\phi}$ should not change in one Hubble time

(*) $H^2 \simeq \frac{8\pi G}{3} V(\phi)$

$$\dot{H} = -4\pi G \dot{\phi}^2$$

$$|\dot{H}| \ll H^2$$

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$$\frac{8\pi G}{3} V(\phi)$$

$$-4\pi G \dot{\phi}^2$$

$$\frac{d}{dt} \left(\frac{3H^2}{8\pi G} \right) \sim \frac{d}{dt} V$$

Linde ; Albrecht & Ste

$$\frac{\dot{\phi}^2}{2} + V$$
$$= \frac{\dot{\phi}^2}{2} - V$$
$$\dot{\phi}^2$$

$$\ddot{H} = \frac{\ddot{a}}{a} - H^2$$

; Linde ; Albrecht & Ste

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$$\dot{H} = -4\pi G \dot{\phi}^2$$

$$|\dot{H}| \ll H^2$$

$$\frac{d}{dt} \left(\frac{3H^2}{8\pi G} \right) \simeq \frac{d}{dt} V = V_{,\phi} \dot{\phi}$$

$$\Rightarrow -3H\dot{\phi} \simeq V_{,\phi}$$

$$\ddot{H} = \frac{\ddot{a}}{a} - H^2$$

; Linde ; Albrecht & Ste

$$\dot{H} = -4\pi G \dot{\phi}^2$$

$$\frac{\dot{\phi}^2}{2} + V$$

$$= \frac{\dot{\phi}^2}{2} - V$$

$$\dot{\phi}^2$$

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$$\Rightarrow -3H\dot{\phi} \simeq V_{,\phi}$$

$$|\dot{\phi}| \ll 3H\dot{\phi}$$

; Linde ; Albrecht & Steinhardt (1992)

$$x. \dot{\phi} \ll v$$

$$\Rightarrow \left(\frac{v_{14}}{v}\right)^2 \ll 24\pi G.$$

change in one Hubble #

$$\left(\frac{3H^2}{8\pi G}\right) \approx \frac{dV}{dt} = v$$

$$\Rightarrow -3H\dot{\phi} \approx v$$

$$|\dot{\phi}| \approx \frac{v}{3H}$$

; Linde ; Albrecht & Steinhardt (1992)

$$x. \dot{\varphi} \ll v \Rightarrow \left(\frac{v_{1\varphi}}{v} \right)^2 \ll 24\pi G.$$

$$\dot{\varphi} \approx -\frac{v_{1\varphi}}{3H}, \text{ use } H^2 \approx \frac{8\pi G}{3} V$$

$$\Rightarrow \dot{\varphi}^2 \approx \frac{v_{1\varphi}^2}{9H^2} \approx \frac{v_{1\varphi}^2}{24\pi G V}$$

change in one time

$$\left(\frac{3H^2}{8\pi} \right)$$

; Linde ; Albrecht & Steinhardt (1992)

$$x. \dot{\phi} \ll V \Rightarrow \left(\frac{V_{,\phi}}{V}\right)^2 \ll 24\pi G.$$

$$\dot{\phi} \approx -\frac{V_{,\phi}}{3H}, \text{ use } H^2 \approx \frac{8\pi G}{3} V$$

$$\Rightarrow \dot{\phi}^2 \approx \frac{V_{,\phi}^2}{9H^2} \approx \frac{V_{,\phi}^2}{24\pi G V}$$

$$\left(\frac{V_{,\phi}}{V}\right)^2 \ll 24\pi G.$$

change in one Hubble time

$$\left(\frac{3H^2}{8\pi G}\right) \approx \frac{dV}{dt} = V_{,\phi} \dot{\phi}$$

$$\Rightarrow -3H\dot{\phi} \approx V_{,\phi}$$

$$|\dot{\phi}| \ll 3H\dot{\phi}$$

; Linde ; Albrecht & Steinhardt (1992)

$$x. \dot{\varphi} \ll V \Rightarrow \left(\frac{V_{,\varphi}}{V}\right)^2 \ll 24\pi G.$$

$$\dot{\varphi} \simeq -\frac{V_{,\varphi}}{3H}, \text{ use } H^2 \simeq \frac{8\pi G}{3} V$$

$$\Rightarrow \dot{\varphi}^2 \simeq \frac{V_{,\varphi}^2}{9H^2} \simeq \frac{V_{,\varphi}^2}{24\pi G V}$$

$$\left(\frac{V_{,\varphi}}{V}\right)^2 \ll 24\pi G.$$

change in one Hubble time

$$\left(\frac{3H^2}{8\pi G}\right) \simeq \frac{dV}{dt} = V_{,\varphi} \dot{\varphi}$$

$$\Rightarrow -3H\dot{\varphi} \simeq V_{,\varphi}$$

$$|\dot{\varphi}| \ll 3H\dot{\varphi}$$

$$N = 68 + \frac{1}{4} \ln \frac{V_{\text{Hor}}}{M_{\text{p}}^4} \rightarrow \epsilon$$

$$3H\dot{\phi} \simeq -V_{,\phi}$$

$$\ddot{\phi} \simeq -\frac{d}{dt} \left(\frac{V_{,\phi}}{3H} \right)$$

\simeq

$$N = 68 + \frac{1}{4} \ln \frac{V_{\text{Hor}}}{M_{\text{p}}^4} \rightarrow \epsilon$$

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$$\ddot{\varphi} \simeq -\frac{d}{dt} \left(\frac{V_{,\varphi}}{3H} \right)$$

$$\simeq -\frac{V_{,\varphi\varphi} \dot{\varphi}}{3H} + \underbrace{\frac{V_{,\varphi} \dot{H}}{3H^2}}_{-\frac{H\dot{\varphi}}{H}}$$

$$N = 68 + \frac{1}{4} \ln \frac{V_{\text{Hor}}}{M_{\text{p}}^4} \rightarrow \epsilon$$

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$$\simeq -\frac{V_{,\varphi\varphi} \dot{\varphi}}{3H} - \frac{H\dot{\varphi}}{H}$$

$$\simeq -\frac{V_{,\varphi\varphi} V_{,\varphi}}{9H^2}$$

$$N = 68 + \frac{1}{4} \ln \frac{V_{\text{Hor}}}{M_{\text{P}}^4} \rightarrow \epsilon$$

$$* \quad \frac{V_{,\phi}}{V} \ll \sqrt{\epsilon}$$

$$3H\dot{\phi} \approx -V_{,\phi}$$

$$\ddot{\phi} \approx -\frac{d}{dt} \left(\frac{V_{,\phi}}{3H} \right)$$

$$\approx -\frac{V_{,\phi\phi}\dot{\phi}}{3H} + \underbrace{\frac{V_{,\phi}}{3H^2} \dot{H}}_{-\frac{\dot{H}\dot{\phi}}{H}}$$

$$\approx -\frac{V_{,\phi\phi}\dot{\phi}}{3H}$$

$$\approx \frac{V_{,\phi\phi}\dot{\phi} V_{,\phi}}{9H^2} \approx \frac{V_{,\phi\phi} V_{,\phi}}{9.81 \times 10^{16}}$$

$$\ddot{\phi} \ll 3H\dot{\phi}$$

$V(\phi)$
2

$$N = 68 + \frac{1}{4} \ln \frac{V_{\text{Hor}}}{M_{\text{p}}^4} \rightarrow \epsilon$$

$$* \quad \frac{V_{,\varphi}}{V} \ll \sqrt{24\pi G} ;$$

$$3H\dot{\varphi} \approx -V_{,\varphi}$$

$$\ddot{\varphi} \approx -\frac{d}{dt} \left(\frac{V_{,\varphi}}{3H} \right)$$

$$\approx -\frac{V_{,\varphi\varphi}\dot{\varphi}}{3H} + \underbrace{\frac{V_{,\varphi}}{3H^2} \dot{H}}_{-\frac{H\dot{\varphi}}{H}}$$

$$\approx -\frac{V_{,\varphi\varphi}\dot{\varphi}}{3H}$$

$$\approx \frac{V_{,\varphi\varphi} V_{,\varphi}}{9H^2} \approx \frac{V_{,\varphi\varphi} V_{,\varphi}}{\frac{9.8116}{3} V} \Rightarrow$$

$$\ddot{\varphi} \ll 3H\dot{\varphi}$$

*. $\frac{v, \varphi}{v} \ll \sqrt{2\pi\lambda G}$; $\frac{v, \varphi}{v} \ll 2\pi\lambda G$ Linde ; Albrecht & Ste

$$\epsilon := \frac{1}{16\pi G} \left(\frac{v, \varphi}{v} \right)^2$$

$$\eta :=$$

$$\frac{3}{2} V(\phi)$$

$$\frac{d}{dt} \left(\frac{3H^2}{8\pi G} \right) \approx \frac{d}{dt} V = v, \varphi \dot{\varphi}$$

$$\Rightarrow -3H\dot{\phi} \approx v, \varphi$$

$$|\dot{\phi}| \ll 3H\phi$$

$$\boxed{|v, \varphi \dot{\varphi}| \ll 2\pi\lambda G}$$

x. $\dot{\varphi}$
 $\dot{\varphi}$
 \Rightarrow
 (V)

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$$\epsilon := \frac{1}{16\pi G} \left(\frac{v, \varphi}{v} \right)^2$$

$$\eta := \frac{1}{8\pi G} \left(\frac{v, \varphi}{v} \right)$$

$$\frac{3}{2} V(\phi)$$

$$\frac{d}{dt} \left(\frac{3H^2}{8\pi G} \right) \approx \frac{d}{dt} V = v, \varphi \dot{\varphi}$$

$$\Rightarrow -3H\dot{\varphi} \approx v, \varphi$$

$$|\dot{\varphi}| \ll 3H\varphi$$

$$|v, \varphi| \ll 2\pi\lambda G$$

x. $\dot{\varphi}$
 $\dot{\varphi}$
 \Rightarrow
 (v, φ)

*. $\frac{V, \varphi}{v} \ll \sqrt{24\pi G}$; $\frac{V_{14\varphi}}{v} \ll 24\pi G$ Linde ; Albrecht & Ste

$$\epsilon := \frac{1}{16\pi G} \left(\frac{V, \varphi}{v} \right)^2$$

$$\eta := \frac{1}{8\pi G} \left(\frac{V_{14\varphi}}{v} \right)$$

$$\epsilon \ll 1$$

$$\eta \ll 1$$

3
2
V(φ)

$$\frac{d}{dt} \left(\frac{3H^2}{8\pi G} \right) \approx \frac{d}{dt} V = V, \varphi \dot{\varphi}$$

$$\Rightarrow -3H\dot{\varphi} \approx V, \varphi$$

$$|\dot{\varphi}| \ll 3H\varphi$$

$$|V, \varphi \dot{\varphi}| \ll 24\pi G$$

x. φ
φ
⇒

x. $\frac{v_{1\varphi}}{v} \ll \sqrt{2\pi\pi G}$; $\frac{v_{1\varphi\varphi}}{v} \ll 2\pi\pi G$ Linde ; Albrecht & Ste

$$\epsilon := \frac{1}{16\pi G} \left(\frac{v_{1\varphi}}{v} \right)^2$$

$$\eta := \frac{1}{8\pi G} \left(\frac{v_{1\varphi\varphi}}{v} \right)$$

$$\epsilon \ll 1$$

$$\eta \ll 1$$

$$\omega =$$

$$\frac{d}{dt} \left(\frac{3H^2}{8\pi G} \right) \approx \frac{d}{dt} V = v_{1\varphi} \dot{\varphi}$$

$$\Rightarrow -3H\dot{\varphi} \approx v_{1\varphi}$$

$$|\dot{\varphi}| \ll 3H\varphi$$

*. $\frac{V, \varphi}{v} \ll \sqrt{24\pi G}$; $\frac{V, \varphi \varphi}{v} \ll 24\pi G$ Linde ; Albrecht & Ste

$$\epsilon := \frac{1}{16\pi G} \left(\frac{V, \varphi}{v} \right)^2$$

$$\eta := \frac{1}{8\pi G} \left(\frac{V, \varphi \varphi}{v} \right)$$

$$\epsilon \ll 1$$

$$\eta \ll 1.$$

$$\omega = -1 + \frac{2}{3}\epsilon$$

3
2
V(φ)

$$\frac{d}{dt} \left(\frac{3H^2}{8\pi G} \right) \approx \frac{d}{dt} V = V, \varphi \dot{\varphi}$$

$$\Rightarrow -3H\dot{\varphi} \approx V, \varphi$$

$$|\dot{\varphi}| \ll 3H\varphi$$

$$|V, \varphi \varphi| \ll 24\pi G$$

x. φ
φ
⇒

Linde ; Albrecht & Steinhardt (1992)

$$\ll 1$$

$$\ll 1$$

$$0 = -1 + \frac{2}{3} \epsilon$$

$$\epsilon = 1$$

$$\omega = -1/3$$

$$x. \dot{\phi} \ll V \Rightarrow \left(\frac{V_{1/4}}{V}\right)^2 \ll 24\pi G.$$

$$\dot{\phi} \approx -\frac{V_{1/4}}{3H}, \text{ use } H^2 \approx \frac{8\pi G}{3} V$$

$$\Rightarrow \dot{\phi}^2 \approx \frac{V_{1/4}^2}{9H^2} \approx \frac{V_{1/4}^2}{24\pi G V}$$

$$\left(\frac{V_{1/4}}{V}\right)^2 \ll 24\pi G.$$



$$N = 68 + \frac{1}{4} \ln \frac{V_{Hor}}{M_p^4} \rightarrow \epsilon$$

$$* \quad \frac{V_{1,\varphi}}{V} \ll \sqrt{\epsilon}$$

(*) In the slow-roll approx

$$P_s = \frac{8V}{3m_p^4} \frac{1}{\epsilon}$$

$$\epsilon := \frac{2}{M_{pl}^2} \left(\frac{V_{1,\varphi}}{V} \right)^2$$

$$\eta := \frac{1}{M_{pl}^2} \left(\frac{V_{2,\varphi}}{V} \right)$$

$$\sim - \frac{V_{1,\varphi\varphi} \varphi}{3H}$$

$$\sim \frac{V_{1,\varphi\varphi} V_{1,\varphi}}{9H^2} \sim \frac{V_{1,\varphi\varphi} V_{1,\varphi}}{9.8116 \frac{V}{3}}$$

$$\ddot{\varphi} \ll 3H\dot{\varphi}$$

$$\Rightarrow \left| \frac{V_{1,\varphi\varphi} \varphi}{V} \right| \ll 24\pi^2 \epsilon$$

$$N = 68 + \frac{1}{4} \ln \frac{V_{Hor}}{M_p^4} \rightarrow \epsilon$$

$$* \quad \frac{V_{1,4}}{V} \ll \sqrt{\alpha}$$

(*) In the slow-roll approx

$$P_s = \frac{8V}{3m_p^4} \epsilon \approx 2.6 \times 10^{-9}$$

$$\epsilon := \frac{21}{16\pi G} \left(\frac{V_{1,4}}{V} \right)^2$$

$$\frac{V}{m_p^4} \approx 10^{-9} \epsilon$$

$$\eta := \frac{1}{8\pi G} \left(\frac{V_{1,4}}{V} \right)$$

$$\approx - \frac{V_{1,4\phi} \phi}{3H}$$

$$\approx \frac{V_{1\phi\phi} V_{1,4}}{9H^2} \approx \frac{V_{1,4\phi} V_{1,4}}{9.8116 \frac{V}{3}}$$

$$\ddot{\phi} \ll 3H\dot{\phi}$$

$$\Rightarrow \left| \frac{V_{1,4\phi} \phi}{V} \right| \ll 24\pi G$$

$$N = 68 + \frac{1}{4} \ln \frac{V_{Hor}}{M_p^4} \rightarrow \epsilon$$

$$* \quad \frac{V_{,\varphi}}{V} \ll \sqrt{\epsilon}$$

(*) In the slow-roll approx

$$P_s = \frac{8V}{3m_{pl}^4} \frac{1}{\epsilon} \approx 2.6 \times 10^{-9}$$

$$\epsilon := \frac{2}{16\pi G} \left(\frac{V_{,\varphi}}{V} \right)^2$$

$$\frac{V}{m_{pl}^4} \approx 10^{-9} \epsilon$$

$$\eta := \frac{1}{8\pi G} \left(\frac{V_{,\varphi\varphi}}{V} \right)$$

$$N \approx 63 + \frac{1}{4} \ln \epsilon$$

$$\approx - \frac{V_{,\varphi\varphi} \varphi}{3H}$$

$$\approx \frac{V_{,\varphi\varphi} V_{,\varphi}}{9H^2} \approx \frac{V_{,\varphi\varphi} V_{,\varphi}}{9.8116 \frac{V}{3}}$$

$$\varphi \ll 3H\varphi$$

$$\Rightarrow \left| \frac{V_{,\varphi\varphi}}{V} \right| \ll 24\pi G$$

$$N = 68 + \frac{1}{4} \ln \frac{V_{Hor}}{M_p^4} \quad \epsilon$$

$$* \quad \frac{V_{,\varphi}}{V} \ll \sqrt{\epsilon}$$

(*) In the slow-roll approx

$$P_s = \frac{8V}{3m_p^4} \frac{1}{\epsilon} \approx 2.6 \times 10^{-9}$$

$$\epsilon := \frac{1}{16\pi G} \left(\frac{V_{,\varphi}}{V} \right)^2$$

$$\frac{V}{m_p^4} \approx 10^{-9} \epsilon$$

$$\eta := \frac{1}{8\pi G} \left(\frac{V_{,\varphi\varphi}}{V} \right)$$

$$N \approx 63 + \frac{1}{4} \ln \epsilon$$

$$\approx - \frac{V_{,\varphi\varphi} \varphi}{3H}$$

$$\approx \frac{V_{,\varphi\varphi} V_{,\varphi}}{9H^2} \approx \frac{V_{,\varphi\varphi} V_{,\varphi}}{9.8116 \frac{V}{3}}$$

$$\varphi \ll 3H\varphi$$

$$\Rightarrow \left| \frac{V_{,\varphi\varphi} \varphi}{V} \right| \ll 24\pi G$$

$$N = 68 + \frac{1}{4} \ln \frac{V_{\text{Hor}}}{M_{\text{p}}^4} \rightarrow \epsilon$$

$$* \quad \frac{V_{,4}}{V} \ll \sqrt{\epsilon}$$

(*) In the slow-roll approx

$$P_s = \frac{8V}{3m_{\text{p}}^4} \frac{1}{\epsilon} \approx 2.6 \times 10^{-9}$$

$$\epsilon := \frac{2}{16\pi G} \left(\frac{V_{,4}}{V} \right)^2$$

$$\frac{V}{m_{\text{p}}^4} \approx 10^{-9} \epsilon$$

$$\eta := \frac{1}{8\pi G} \left(\frac{V_{,4}}{V} \right)$$

$$N \approx 63 + \frac{1}{4} \ln \epsilon$$

$$\approx - \frac{V_{,4} \varphi}{3H}$$

$$\approx \frac{V_{,4} \varphi V_{,4}}{9H^2} \approx \frac{V_{,4} \varphi V_{,4}}{9 \frac{8\pi G}{3} V}$$

$$\varphi \ll 3H\varphi$$

$$\Rightarrow \left| \frac{V_{,4} \varphi}{V} \right| \ll 24\pi G$$

$\frac{V_{1\phi} \ll 24\pi G}{v}$ Linde ; Albrecht & Steinhardt (1982)

$$\epsilon \ll 1$$

$$\eta \ll 1$$

$$\omega = -1 + \frac{2}{3}\epsilon$$

if $\epsilon = 1$

$$\omega = -1/3$$

x. $\dot{\phi} \ll v \Rightarrow \left(\frac{V_{1\phi}}{v}\right)^2 \ll$

$$\dot{\phi} \approx -\frac{V_{1\phi}}{3H}$$

, use $H^2 \approx 8\pi G V$

$$\Rightarrow \dot{\phi}^2 \approx \frac{V_{1\phi}^2}{9H^2} \approx \frac{V_{1\phi}^2}{24\pi G V}$$

$$\left(\frac{V_{1\phi}}{v}\right)^2 \ll 24\pi G$$

