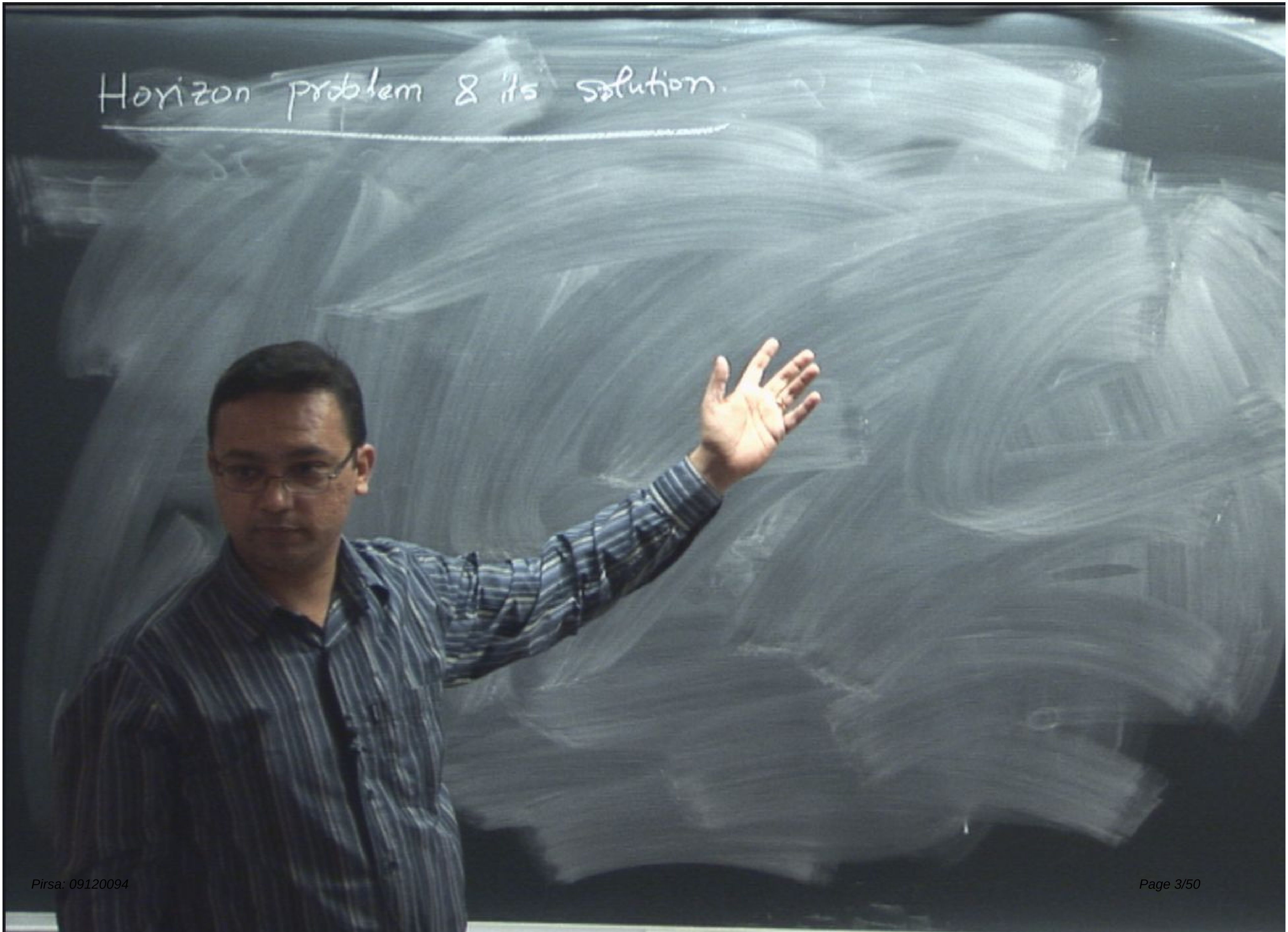
Title: Cosmology - Review (PHYS 621) - Lecture 12

Date: Dec 15, 2009 10:00 AM

URL: http://pirsa.org/09120094

Abstract:

Horizon problem & its solution.


Horizon problem & its solution.

Horizon problem & its solution.

* Comoving

Horizon problem & its solution.

* Comoving Horizon

$(\vec{x}_1, \vec{x}_2)$ $\quad d = [(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2]$

$$ds^2 = -dt^2 + a^2 d\vec{x}^2$$

comoving distance traveled by light rays

$$dx = \frac{dt}{a(t)}$$

Horizon problem & its solution.

* Comoving Horizon

$(\vec{x_1}, \vec{x_2})$ $\quad \delta = [(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2]$

$$ds^2 = -dt^2 + a^2 d\vec{x}^2$$

comoving distance traveled by light rays

$$dx = \frac{dt}{a(t)}$$

Total comoving distance : $\eta = \int_0^t \frac{dt'}{a(t')}$

Behavior of η.

1) Satisfies SEC.

dust. $\omega = 0$; $a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+\omega)}}$

$a \propto t^{\frac{2}{3}} \Rightarrow \eta \propto a^{1/2}$

Behavior of η.

1) Satisfies SEC.

$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$

dust. $w = 0$;

$a \propto t^{\frac{2}{3}} \Rightarrow \eta \propto a^{1/2}$

radiation $w = 1/3$; $\eta \propto a$

Behavior of η.

1) Satisfies SEC.

$\omega > -1/3$

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+\omega)}}$$

dust. $\omega = 0$; $a \propto t^{\frac{2}{3}} \Rightarrow \eta \propto a^{1/2}$

radiation $\omega = 1/3$; $\eta \propto a$

Behavior of η.

$w \; -1/3$

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

i) Satisfies SEC.

dust. $w = 0$;

$a \propto t^{\frac{2}{3}} \Rightarrow \eta \propto a^{1/2}$

radiation $w = 1/3$; $\eta \propto a$

ii) violation of SEC.

$w = -2/3$

Behavior of η.

1) Satisfies SEC.

$\omega > -1/3$,

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+\omega)}}$$

dust. $\omega = 0$;

$a \propto t^{\frac{2}{3}} \Rightarrow \eta \propto a^{1/2}$

radiation $\omega = 1/3$; $\eta \propto a$

ii) violation of SEC.

$\omega = -2/3$ $\qquad$ $\omega < -1/3$

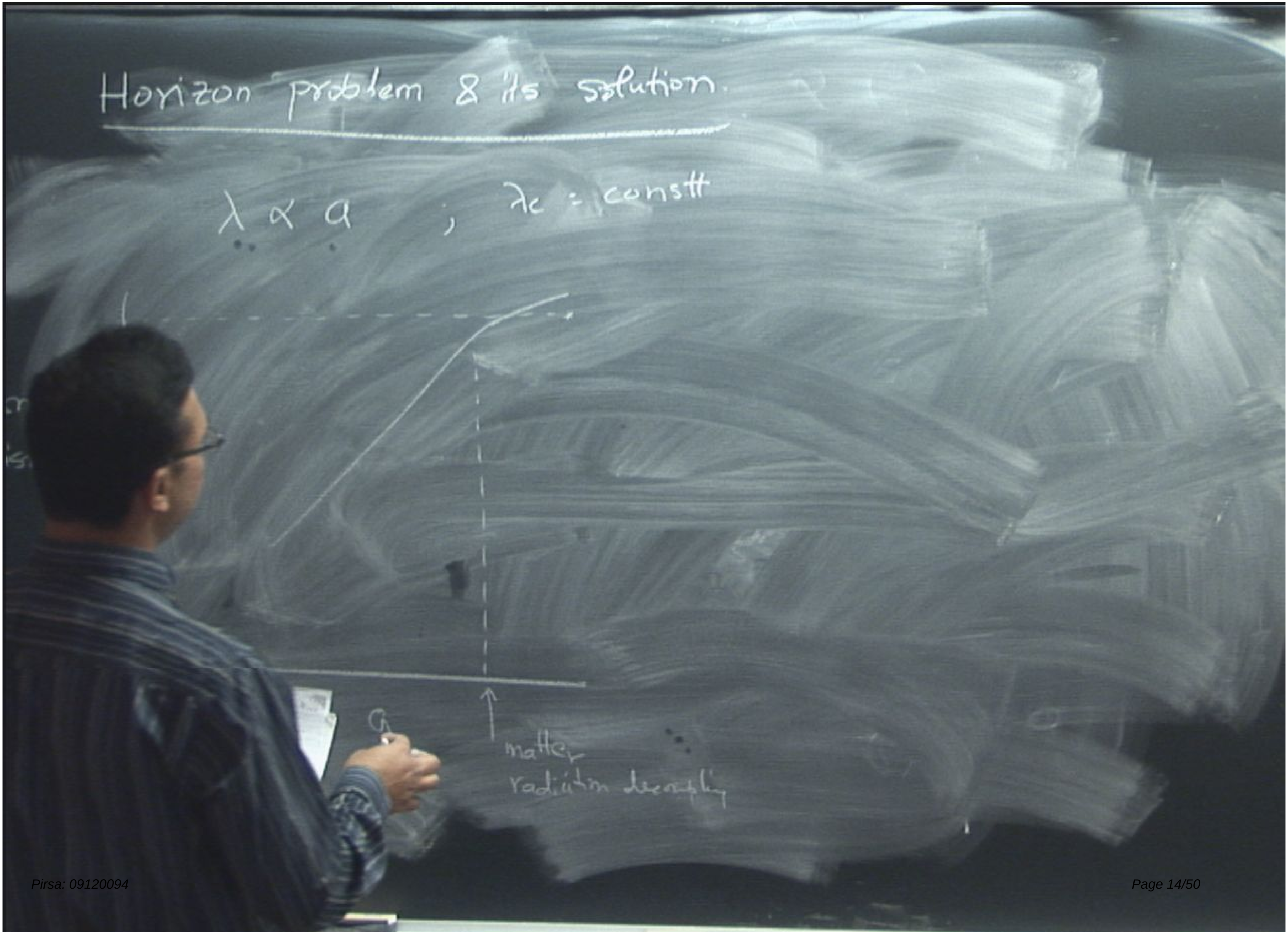
$a \propto t^2$, $\eta \propto a^{-1/2}$

Horizon problem & its solution.

$\lambda \propto a$; λ_c = constt

Horizon problem & its solution.

$\lambda \propto a \quad ; \quad \lambda_c = \text{constt}$


Horizon problem & its solution.
λ ∝ a ; λc = constt
a
matter
radiation decoupling

Horizon problem & its solution.

$\lambda \propto a \quad ; \quad \lambda_c = \text{constt}$

solution

= constt

$$\eta = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{a'}\left(\frac{1}{a' H(a')}\right)$$

$(aH)^{-1}$

...tter

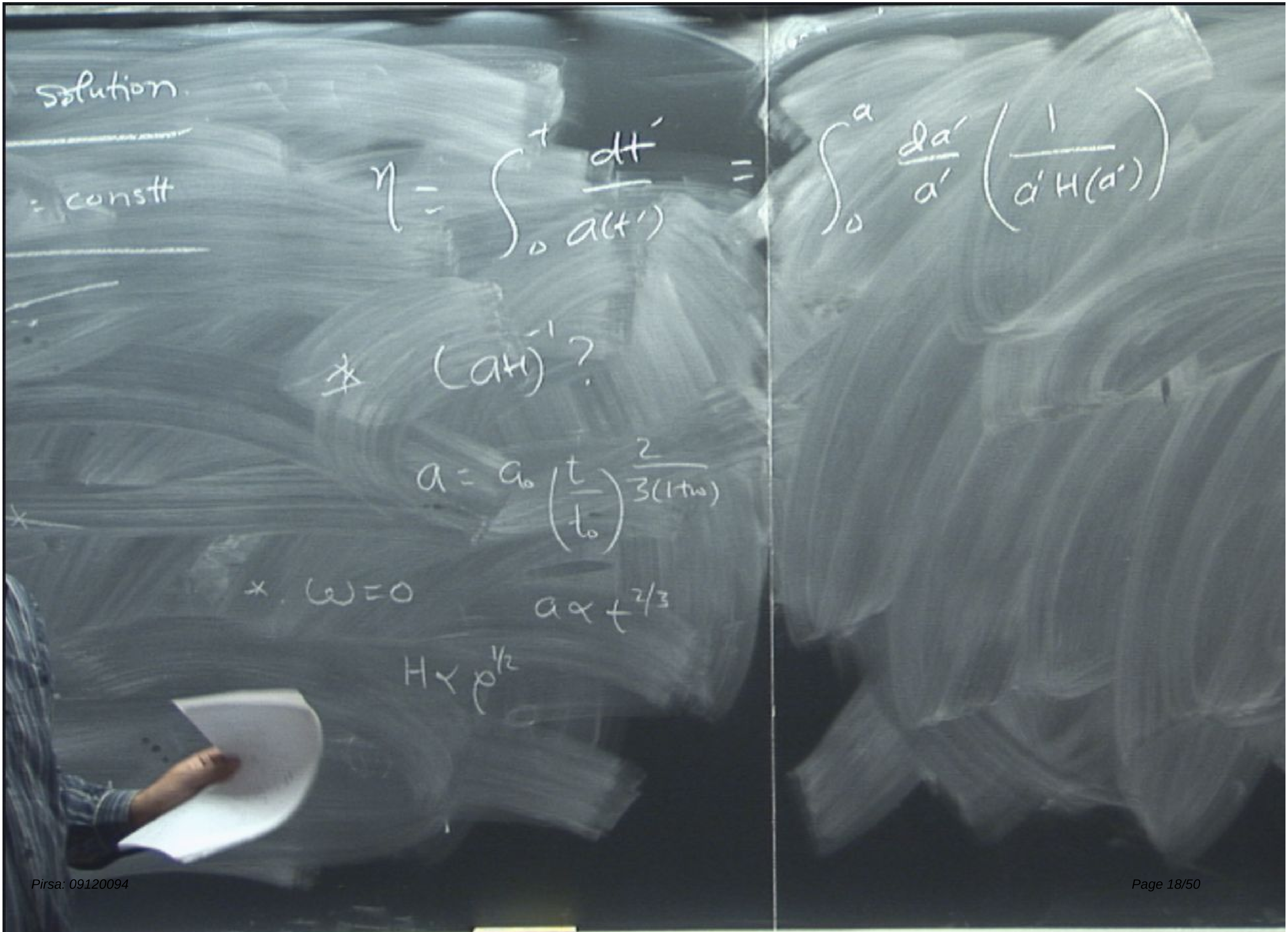
...diation decoupling

solution

: constt

$$\eta = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{a'}\left(\frac{1}{a' H(a')}\right)$$

tter
diation decoupling


solution.
= const
$\eta = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{a'} \left(\frac{1}{a' H(a')} \right)$
* $(aH)^{-1}$?
$a = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$
* $w = 0$
$a \propto t^{2/3}$
$H \propto \rho^{1/2}$

solution

= constt

$$\eta = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{a'}\left(\frac{1}{a' H(a')}\right)$$

$* \quad (aH)^{-1}?$

$$a = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

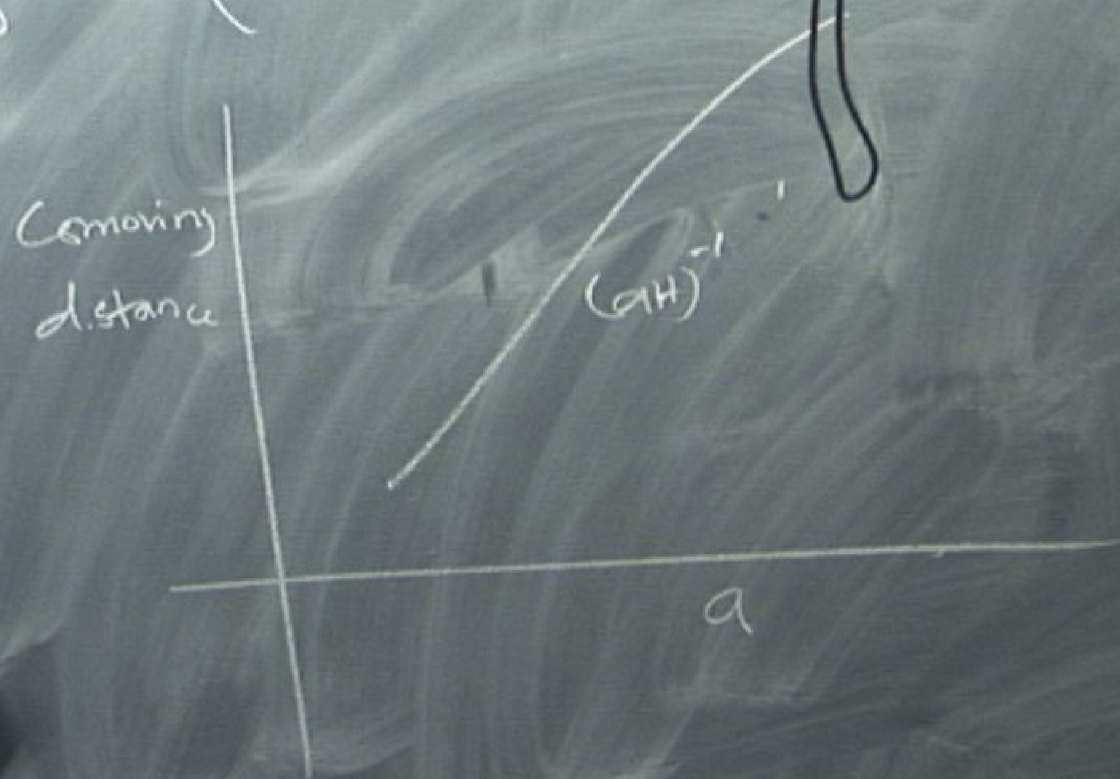
$* \quad \omega = 0 \qquad a \propto t^{2/3}$

$$H \propto \rho^{1/2} \propto a^{-3/2}$$

$$aH \propto a^{-1/2} \quad ; \quad (aH)^{-1} \propto a^{1/2}$$

tter
diation decoupling

$$\eta = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{a'}\left(\frac{1}{a' H(a')}\right)$$



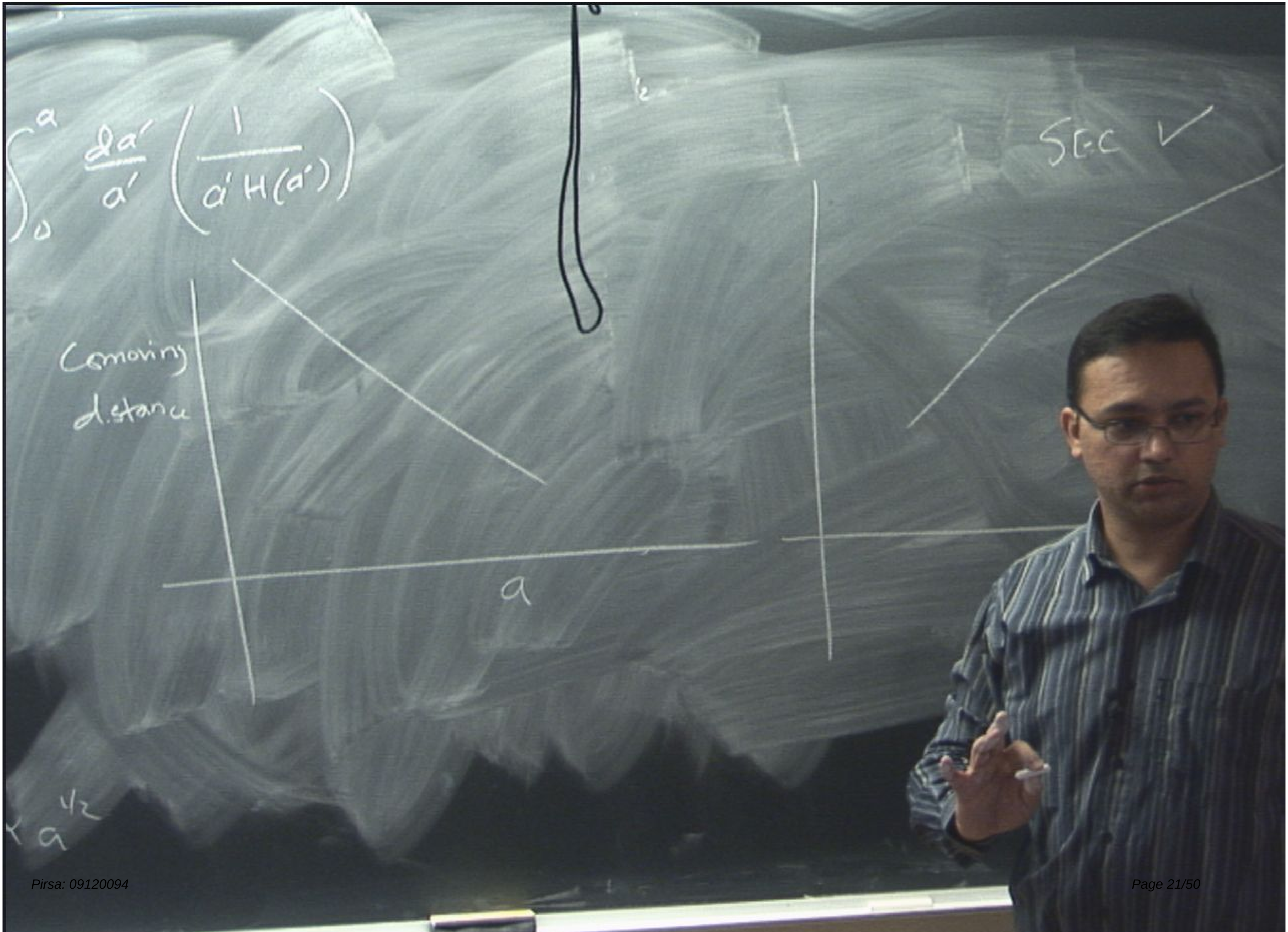
$*$ $(aH)^{-1}$?

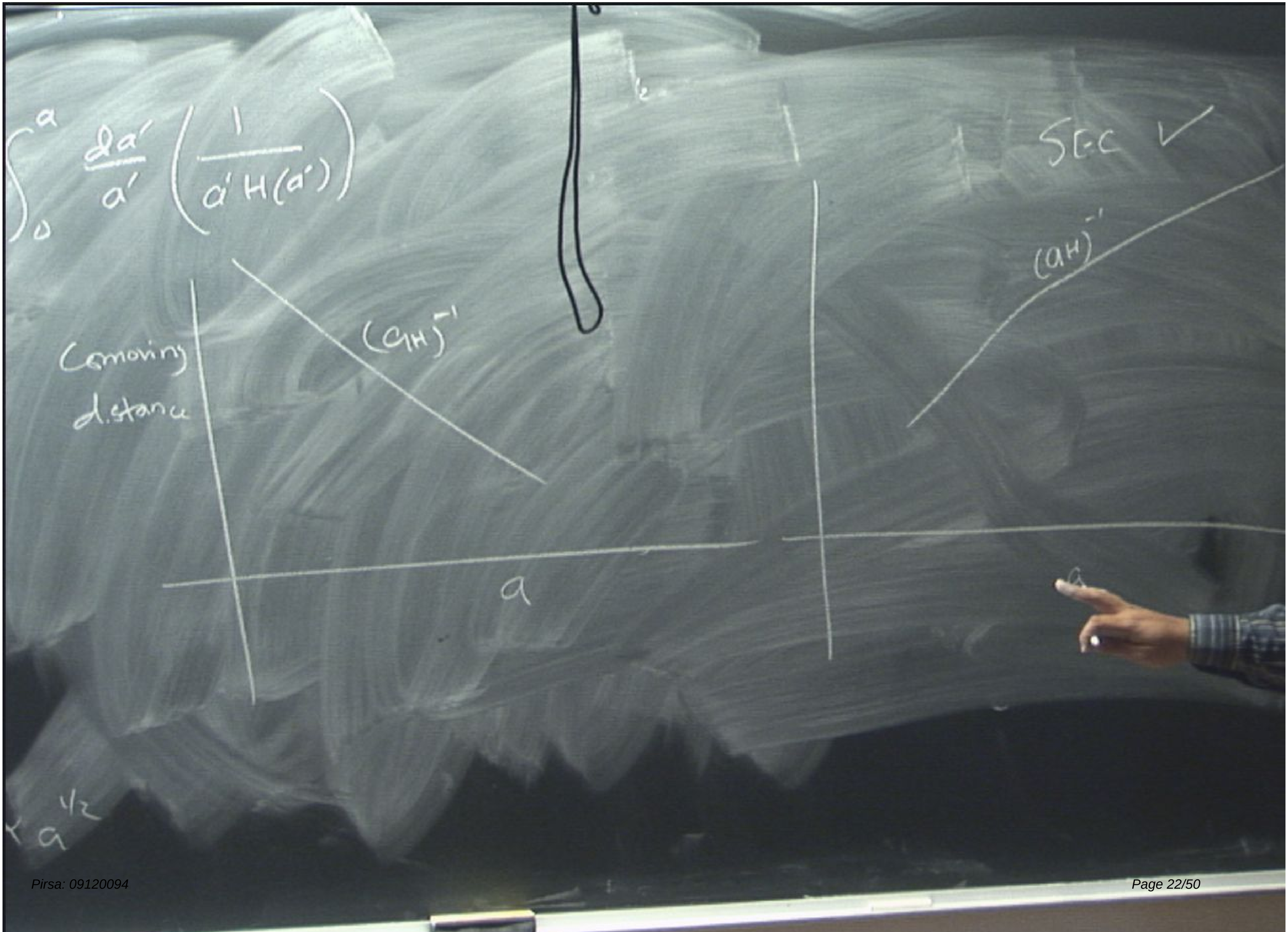
$$a = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

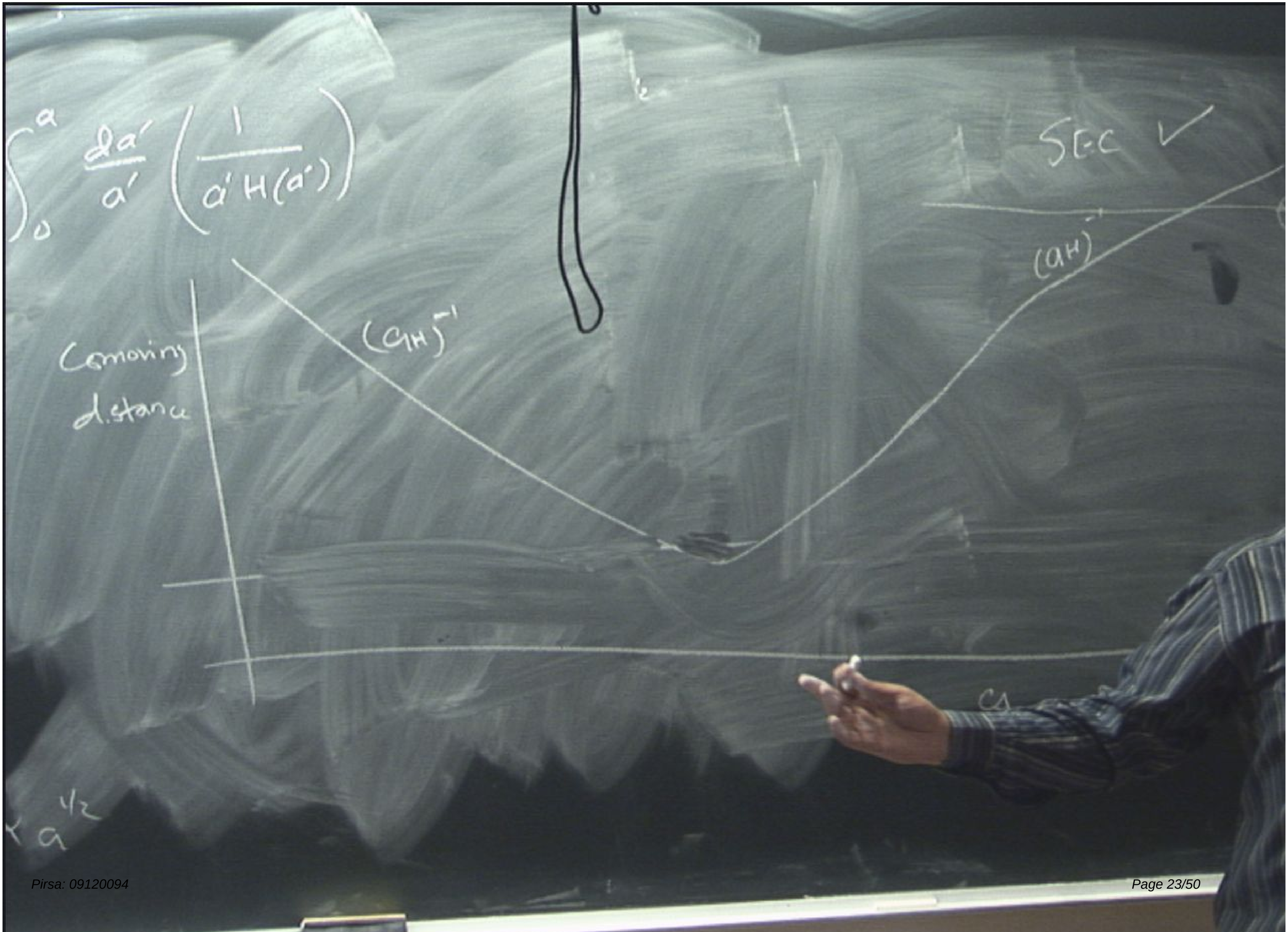
$w = 0$ $\quad a \propto t^{2/3}$

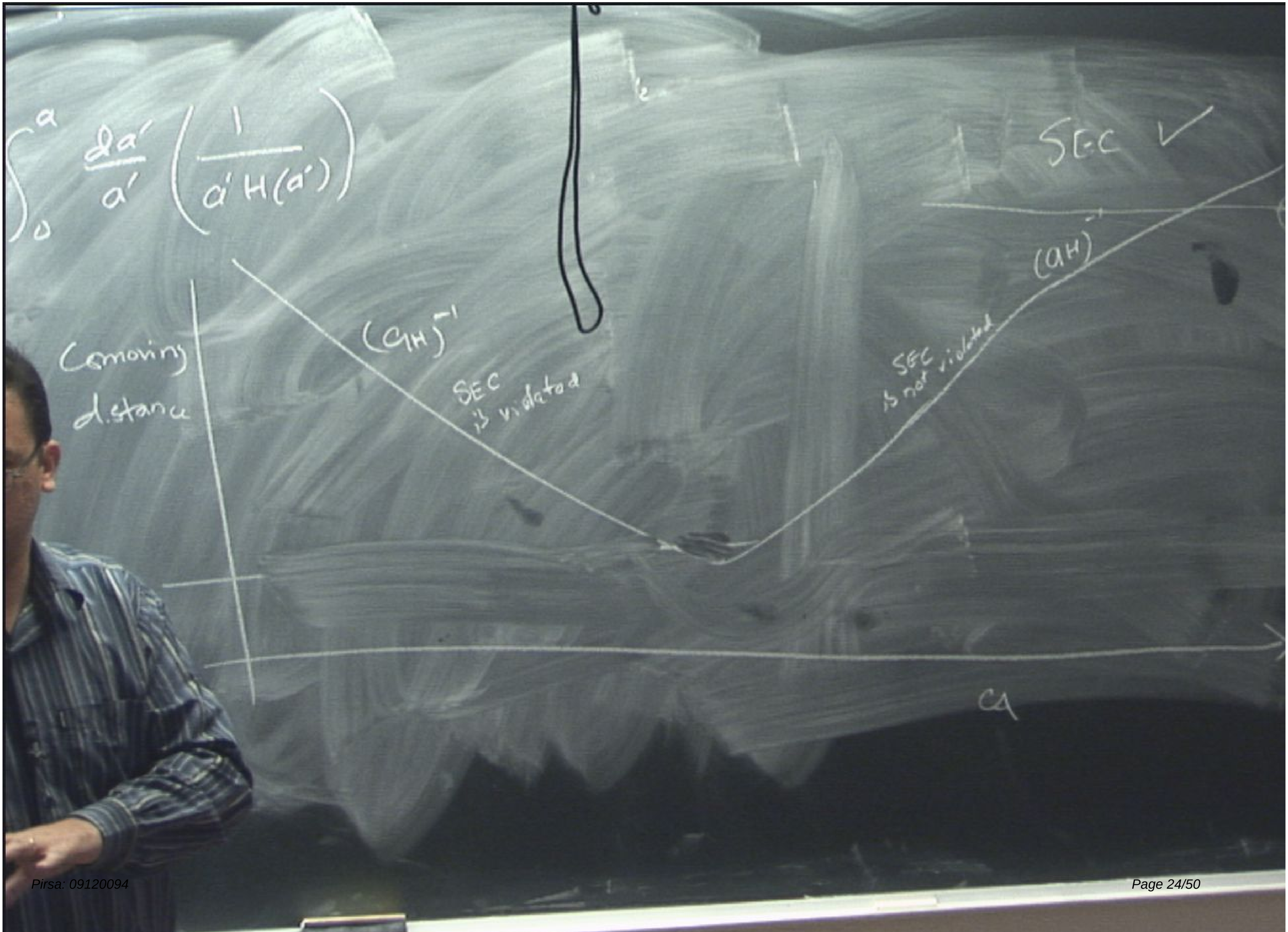
$H \propto \rho^{1/2} \propto a^{-3/2}$

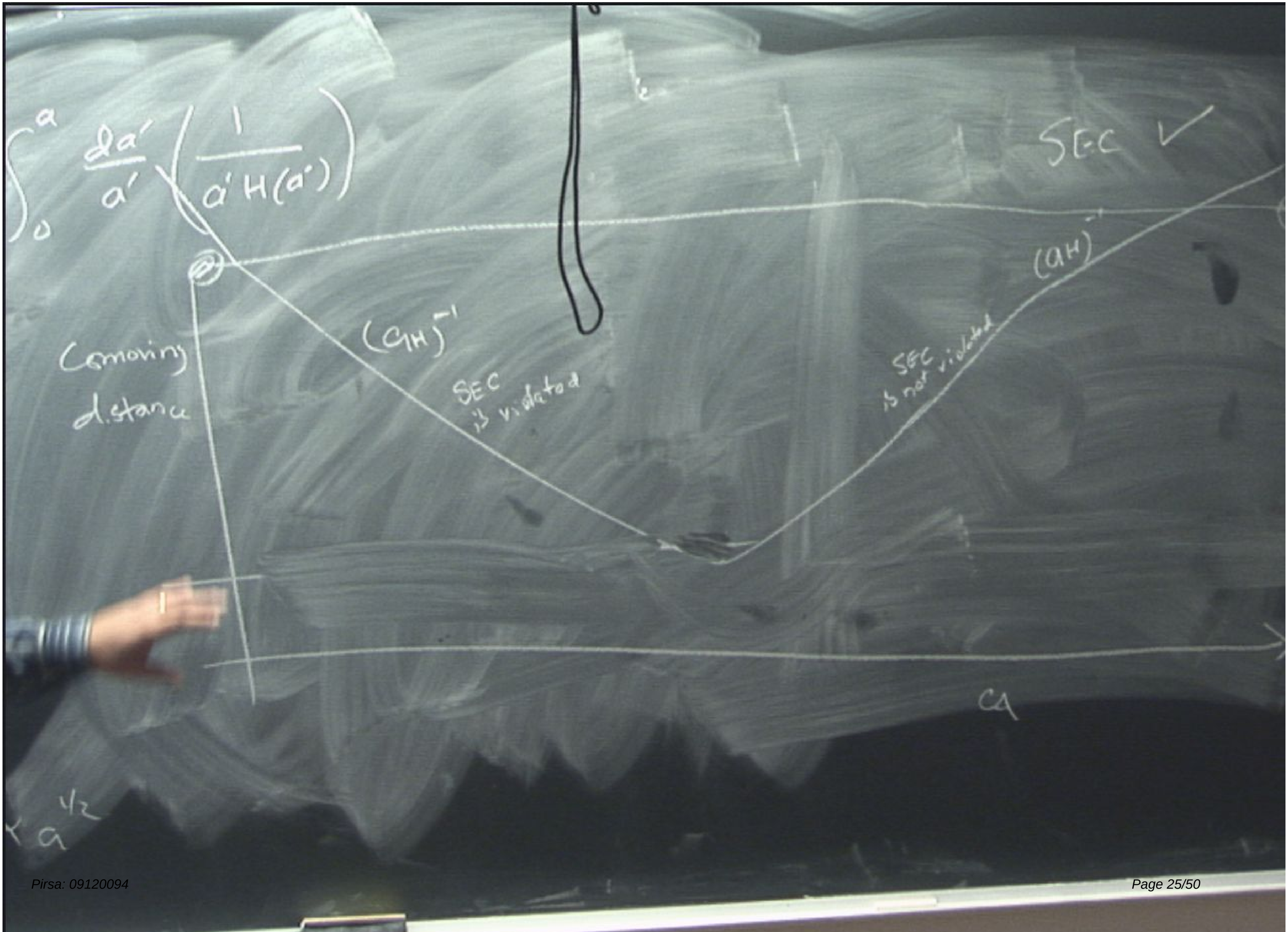
$aH \propto a^{-1/2}$; $(aH)^{-1} \propto a^{1/2}$

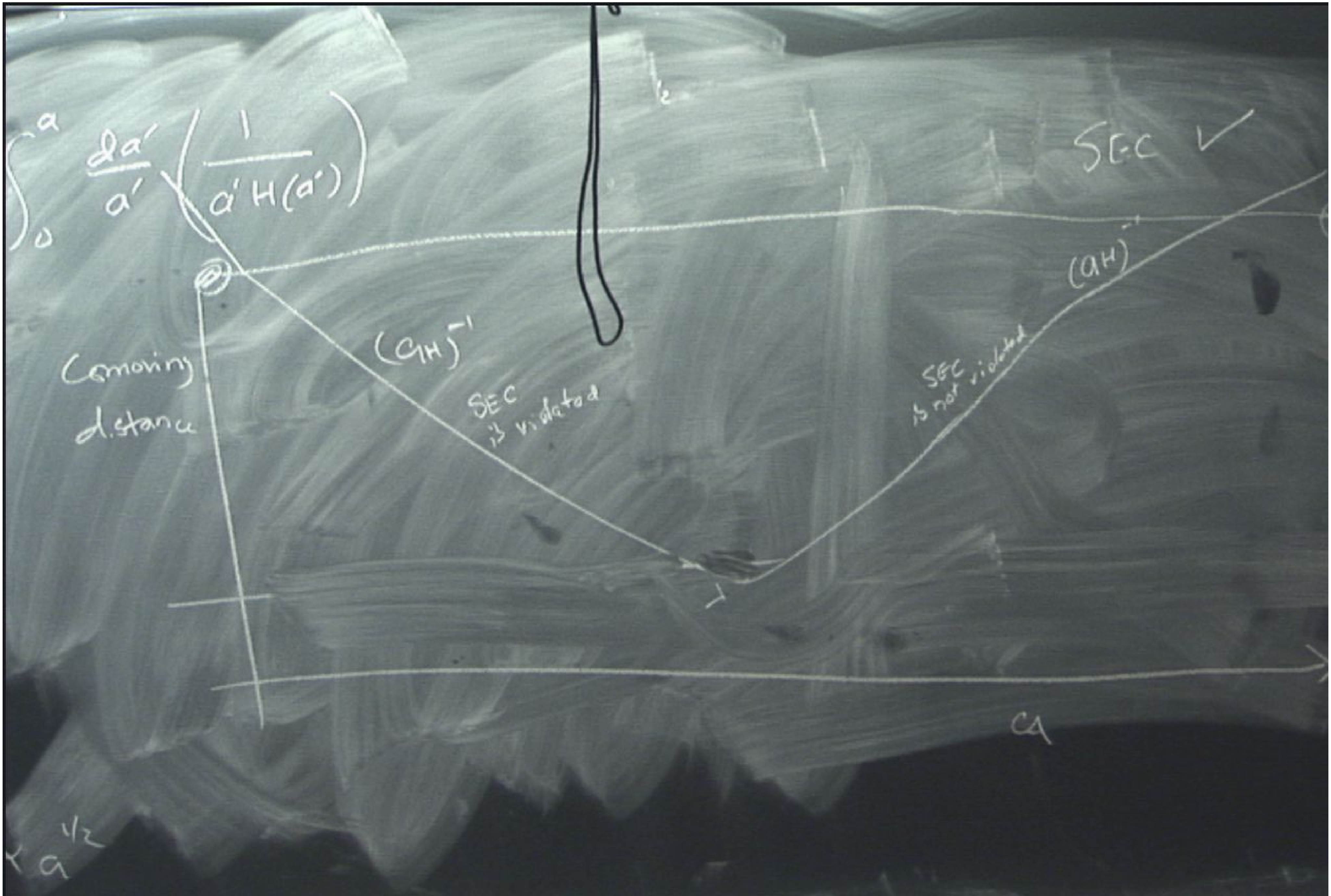

$\int_0^a \frac{da'}{a'} \left(\frac{1}{a' H(a')} \right)$
Comoving distance
a
SEC ✓
$a^{1/2}$


$\int_0^a \frac{da'}{a'}\left(\frac{1}{a'H(a')}\right)$
Comoving distance
$(aH)^{-1}$
a
SEC ✓
$(aH)^{-1}$
a
$a^{1/2}$

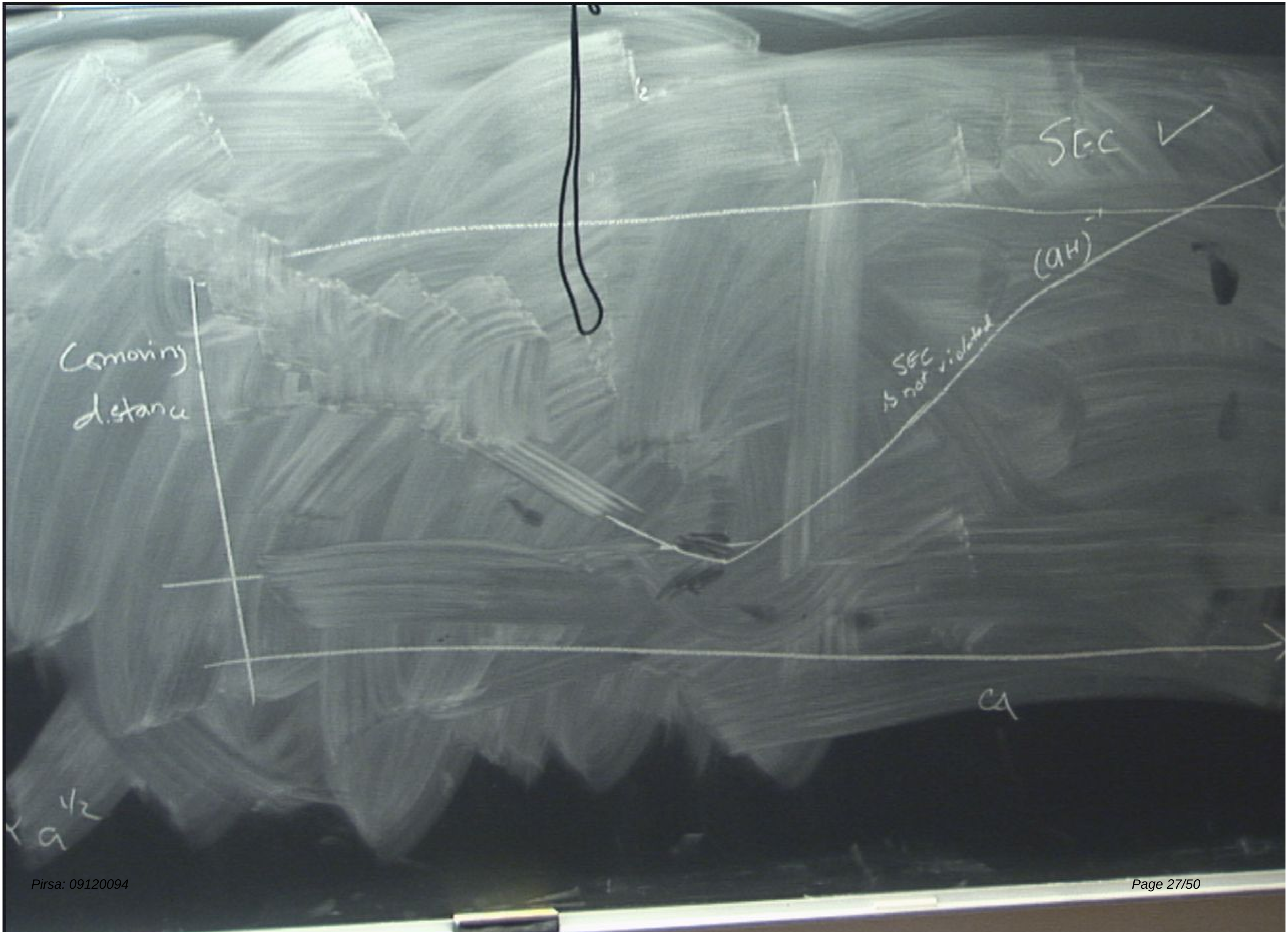

$\int_0^a \frac{da'}{a'} \left(\frac{1}{a' H(a')} \right)$
SEC ✓
$(aH)^{-1}$
$(aH)^{-1}$
Comoving distance
$a^{1/2}$

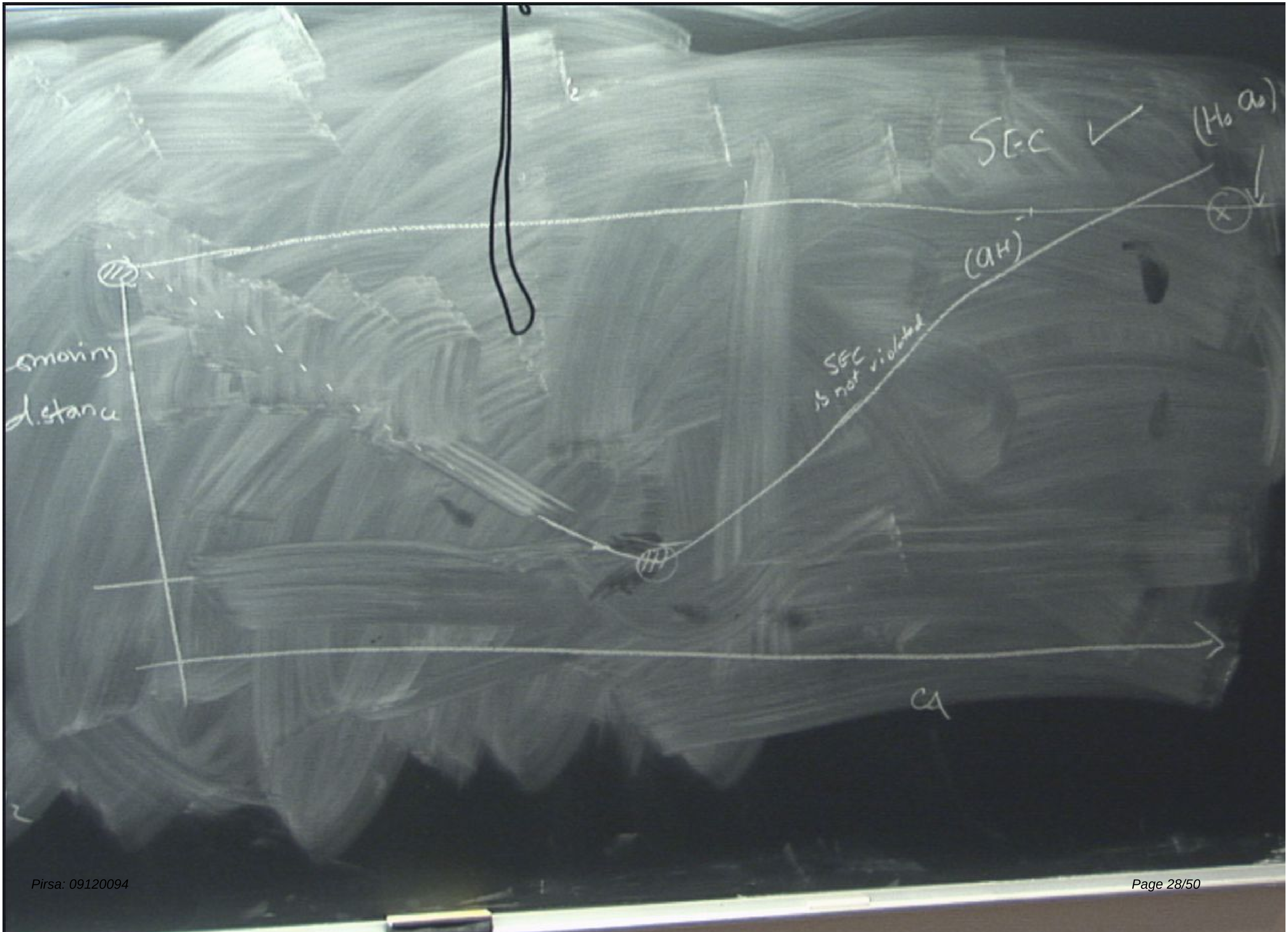

$\int_0^a \frac{da'}{a'}\left(\frac{1}{a'H(a')}\right)$
Comoving distance
$(aH)^{-1}$
SEC is violated
SEC is not violated
$(aH)^{-1}$
SEC ✓
a

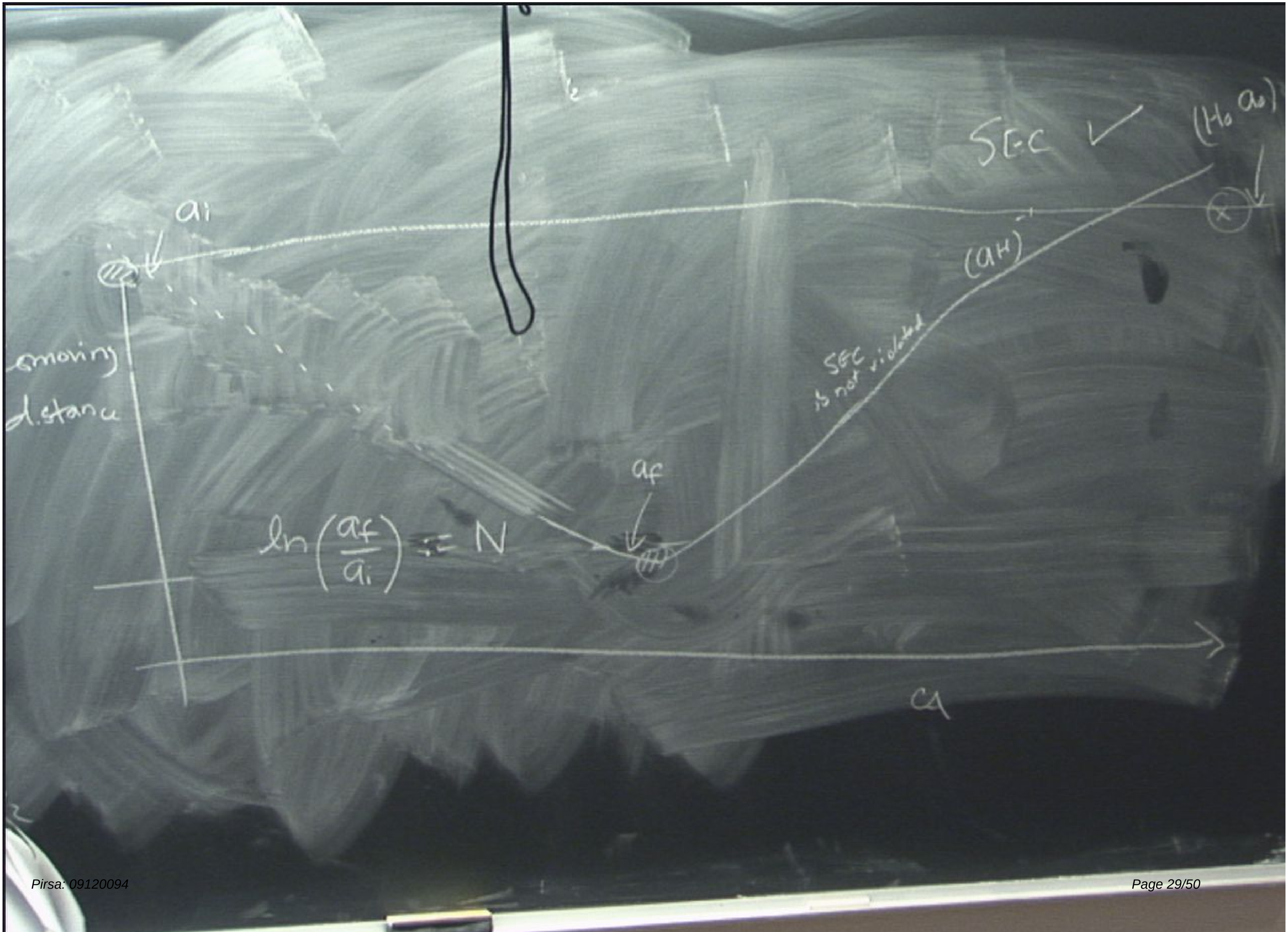

$\int_0^a \frac{da'}{a'} \left(\frac{1}{a' H(a')} \right)$
SEC ✓
$(aH)^{-1}$
comoving distance
SEC is violated
SEC is not violated
$a^{1/2}$

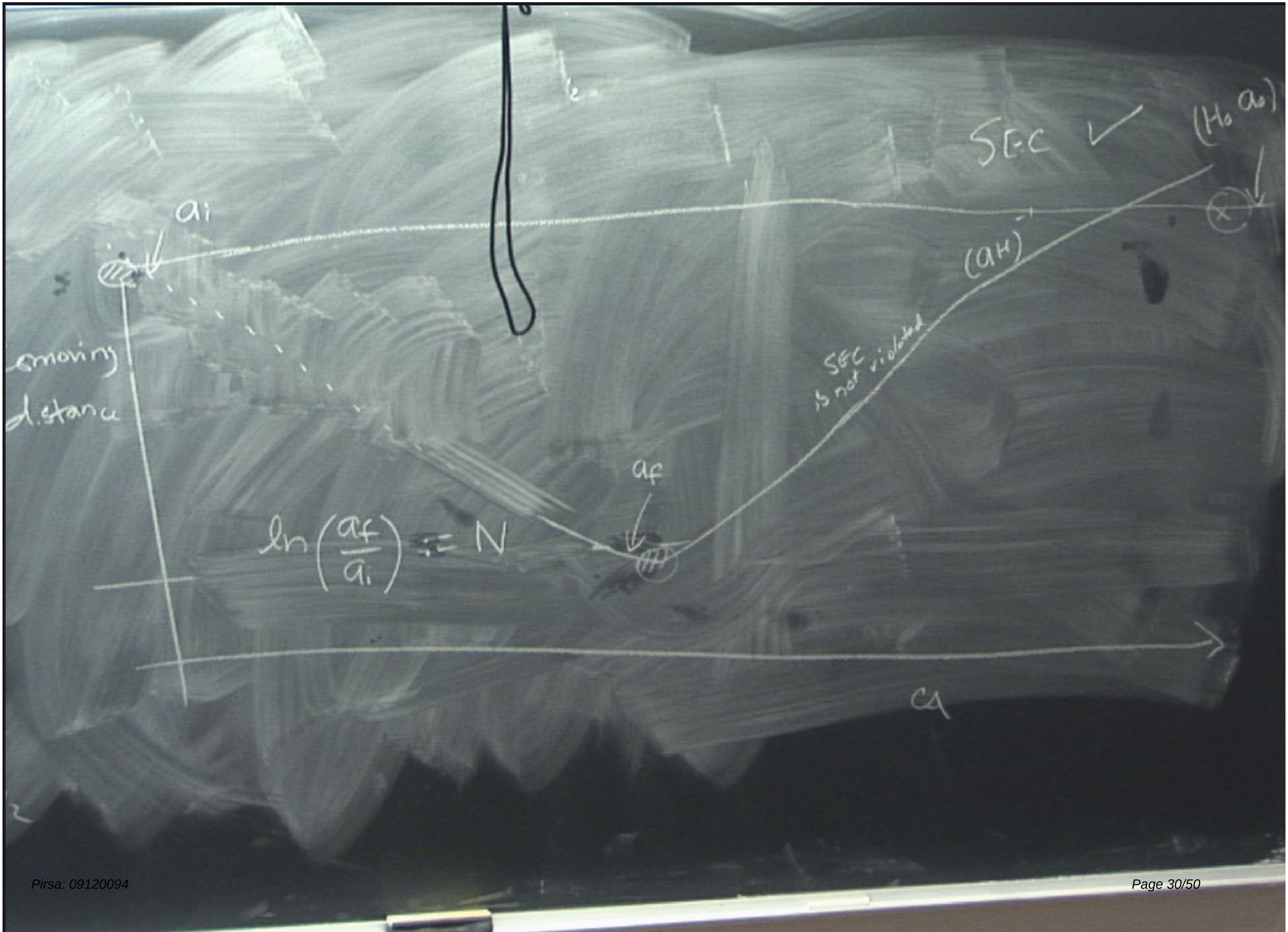


$\int_0^a \frac{da'}{a'}\left(\frac{1}{a'H(a')}\right)$
SEC ✓
$(aH)^{-1}$
comoving distance
$(aH)^{-1}$
SEC is violated
SEC is not violated
$a^{1/2}$

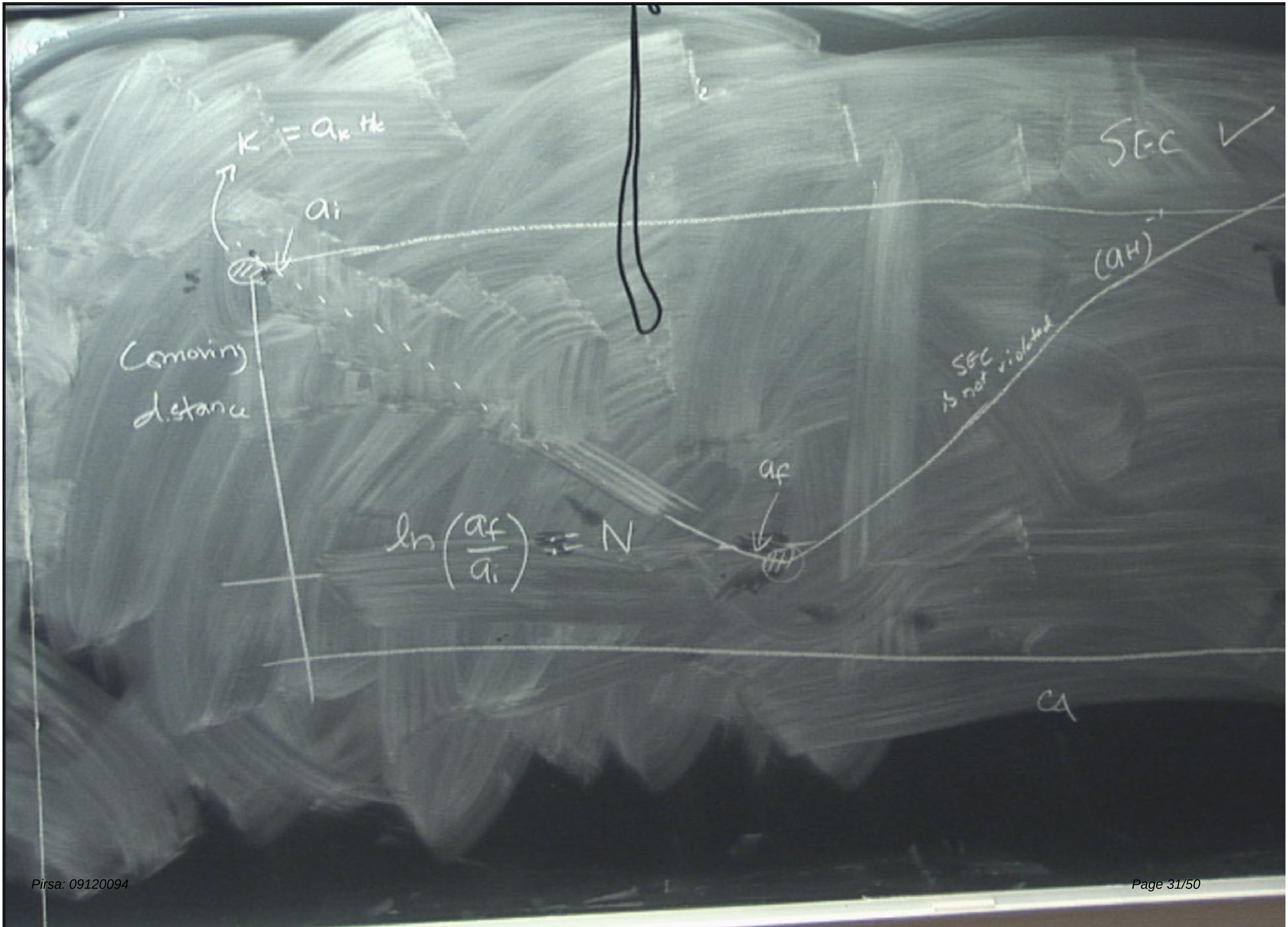

SEC ✓
(aH)⁻¹
SEC is not violated
Comoving distance
a


SEC
(H0 a0)
(aH)-1
SEC is not violated
comoving
distance
CA


SEC ✓
$(H_0\, a_0)$
a_i
comoving
distance
$(aH)^{-1}$
SEC is not violated
a_f
$\ln\left(\frac{a_f}{a_i}\right) = N$



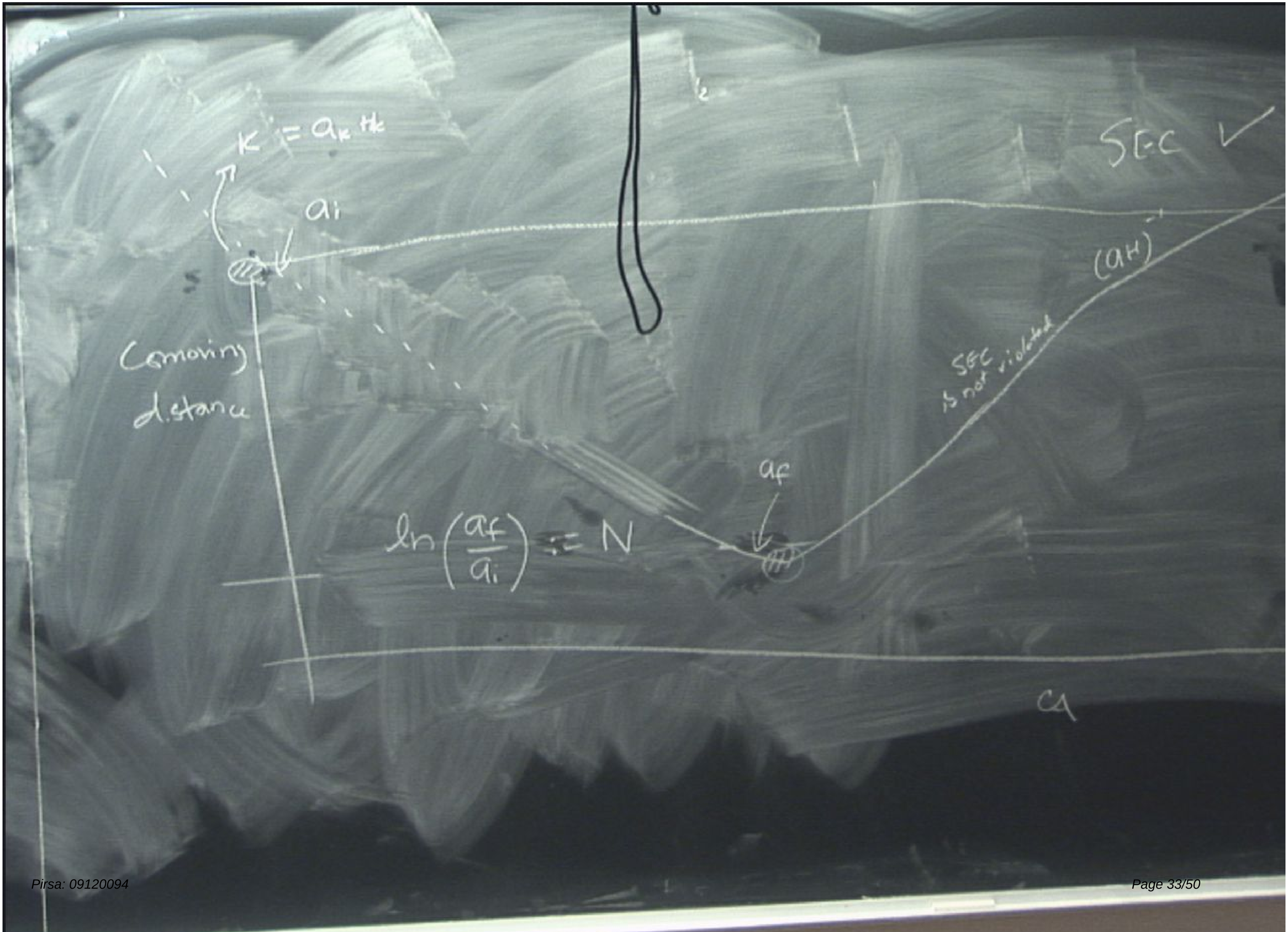
SEC ✓
(H0 a0)
ai
(aH)
comoving
distance
SEC
is not violated
af
ln(af/ai) = N



K = a_k H_k
a_i
comoving distance
SEC ✓
(aH)^{-1}
SEC is not violated
a_f
ln(a_f / a_i) = N
CA

Horizon problem & its solution.

$$\frac{K}{a_0 H_0}$$


$k = a_k H_k$
a_i
Comoving
distance
SEC ✓
$(aH)^{-1}$
SEC is not violated
a_f
$\ln\left(\frac{a_f}{a_i}\right) = N$
a

Horizon problem & its solution.

$$e^{N(k)} := \frac{a_{end}}{a_k}$$

$$\frac{k}{a_0 H_0}$$

Horizon problem & its solution.

$$e^{N(k)} := \frac{a_{end}}{a_k}$$

$$\frac{k}{a_0 H_0} = \frac{a_k H_k}{a_0 H_0} = e^{-N} \frac{a_{end}}{a_{eq}} \frac{a_{eq}}{a_0} \frac{H_k H_{eq}}{H_{eq} H_0}$$

Horizon problem & its solution.

$$e^{N(k)} := \frac{a_{end}}{a_k}$$

$$\frac{k}{a_0 H_0} = \frac{a_k H_k}{a_0 H_0} = e^{-N} \frac{a_{end}}{a_{eq}} \frac{a_{eq}}{a_0} \frac{H_k H_{eq}}{H_{eq} H_0}$$

*. $$\boxed{\frac{a_{eq} H_{eq}}{a_0 H_0} = 219\, \Omega_{m(0)}\, h}$$

Horizon problem & its solution.

$$e^{N(k)} := \frac{a_{end}}{a_k}$$

$$\frac{k}{a_0 H_0} = \frac{a_k H_k}{a_0 H_0} = e^{-N} \frac{a_{end}}{a_{eq}} \frac{a_{eq}}{a_0} \frac{H_k H_{eq}}{H_{eq} H_0}$$

*. $$\boxed{\frac{a_{eq} H_{eq}}{a_0 H_0} = 219\, \Omega_{m(0)}\, h}$$

$$H_{eq} = 5.25 \times 10^{6}\, h^{3}\, \Omega_{m(0)}^{2}\, H_0$$

$$H_0 = 1.75 \times 10^{-61}\, h\, m_{pl}$$

Horizon problem & its solution.

$$e^{N(k)} := \frac{a_{end}}{a_k}$$

$$\frac{k}{a_0 H_0} = \frac{a_k H_k}{a_0 H_0} = e^{-N} \frac{a_{end}}{a_{eq}} \frac{a_{eq}}{a_0} \frac{H_k H_{eq}}{H_{eq} H_0}$$

$$\ast. \quad \frac{a_{eq} H_{eq}}{a_0 H_0} = 219\, \Omega_m \ldots$$

$$H_{eq} = 5.25 \times 10^{6}\, h^3 \ldots H_0$$

$$H_0 = 1.75 \times 10^{-61} \ldots$$

$$H_k^2 \simeq \frac{8\pi}{3 m_p^2} V \ldots$$

$$H_k^2 = \frac{8\pi G}{3} \rho$$

$$= \frac{8\pi G}{3} \left(\frac{\dot{\varphi}_k^2}{2} + V_k \right)$$

$$\simeq \frac{8\pi G}{3} V_k = \frac{8\pi}{3 m_p^2} V_k$$

Horizon problem & its solution.

$$e^{N(k)} := \frac{a_{end}}{a_k}$$

$$\frac{k}{a_0 H_0} = \frac{a_k H_k}{a_0 H_0} = e^{-N} \frac{a_{end}}{a_{eq}} \frac{a_{eq}}{a_0} \frac{H_k H_{eq}}{H_{eq} H_0}$$

$$*. \quad \boxed{\frac{a_{eq} H_{eq}}{a_0 H_0} = 219\, \Omega_{m(0)}\, h}$$

$$\frac{k}{a_0 H_0} = e^{-N} \frac{a_{end}}{a_{eq}} \left(\frac{a_{eq} H_{eq}}{a_0 H_0}\right) \frac{H_k}{H_{eq}}$$

$$H_{eq} = 5.25 \times 10^{6}\, h^3\, \Omega_{m(0)}^2\, H_0$$

$$H_0 = 1.75 \times 10^{-61}\, h\, m_{pl}$$

$$\boxed{H_k^2 \simeq \frac{8\pi}{3 m_p^2} V_k}$$

Horizon problem & its solution.

$$e^{N(k)} := \frac{a_{end}}{a_k}$$

$$\frac{k}{a_0 H_0} = \frac{a_k H_k}{a_0 H_0} = e^{-N} \frac{a_{end}}{a_{eq}} \frac{a_{eq}}{a_0} \frac{H_k H_{eq}}{H_{eq} H_0}$$

$$* \quad \boxed{\frac{a_{eq} H_{eq}}{a_0 H_0} = 219\, \Omega_{m(0)}\, h}$$

$$H_{eq} = 5.25 \times 10^{6}\, h^3\, \Omega^2_{m(0)}\, H_0$$

$$H_0 = 1.75 \times 10^{-61}\, h\, m_{pl}$$

$$\boxed{H_k^2 \simeq \frac{8\pi}{3 m_p^2} V_k}$$

$$\frac{k}{a_0 H_0} = e^{-N} \underbrace{\frac{a_{end}}{a_{eq}}}_{\rho \propto a^{-4}} \left(\frac{a_{eq} H_{eq}}{a_0 H_0}\right) \frac{H_k}{H_{eq}}$$

$$N = -\ln \frac{k}{a_0 H_0} + \frac{1}{4} \ln \frac{\rho_{eq}}{\rho_{end}} + \ln(219\, \Omega_0 h) + \ln \sqrt{\frac{8\pi V_k}{3 m_p^2}}$$

Horizon problem & its solution.

$$e^{N(k)} := \frac{a_{end}}{a_k}$$

$$\frac{k}{a_0 H_0} = \frac{a_k H_k}{a_0 H_0} = e^{-N} \frac{a_{end}}{a_{eq}} \frac{a_{eq}}{a_0} \frac{H_k H_{eq}}{H_{eq} H_0}$$

$$* \quad \boxed{\frac{a_{eq} H_{eq}}{a_0 H_0} = 219\, \Omega_{m(0)}\, h}$$

$$H_{eq} = 5.25 \times 10^{6}\, h^3\, \Omega^2_{m(0)}\, H_0$$

$$H_0 = 1.75 \times 10^{-61}\, h\, m_{pe}$$

$$\boxed{H_k^2 \simeq \frac{8\pi}{3 m_p^2} V_k}$$

$$\frac{k}{a_0 H_0} = e^{-N} \underbrace{\frac{a_{end}}{a_{eq}}}_{\rho \propto a^{-4}} \left(\frac{a_{eq} H_{eq}}{a_0 H_0}\right) \frac{H_k}{H_{eq}}$$

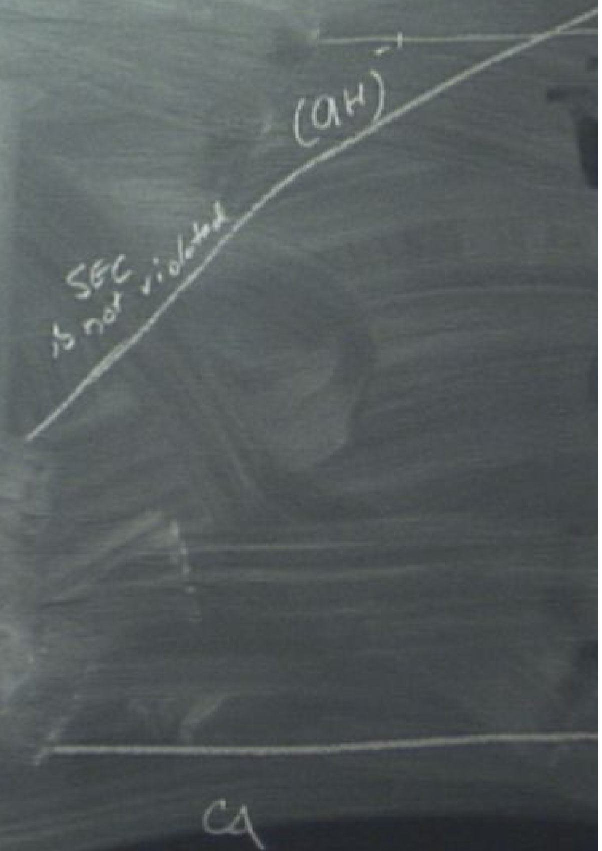
$$N = -\ln \frac{k}{a_0 H_0} + \frac{1}{4} \ln \frac{\rho_{eq}}{\rho_{end}} + \ln(219\, \Omega_0 h) + \ln \sqrt{\frac{8\pi V_k}{3 m_p^2}}$$

$$N = \frac{1}{4}\ln\frac{\rho_{eq}}{V_{hor}} + \ln\sqrt{\frac{8\pi V_{hor}}{3m_p^2}}\,\frac{1}{H_{eq}} + 3.83$$

$h = 0.7$

$\Omega_{m0} = 0.3$

SEC ✓



$V_k \equiv V_{hor}$

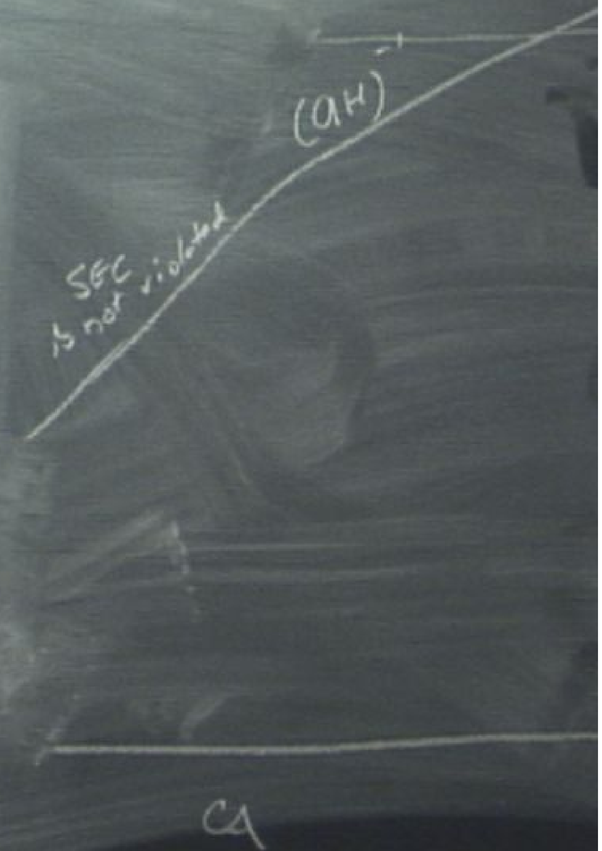
$\frac{1}{H_{eq}}$

$h = 0.7$

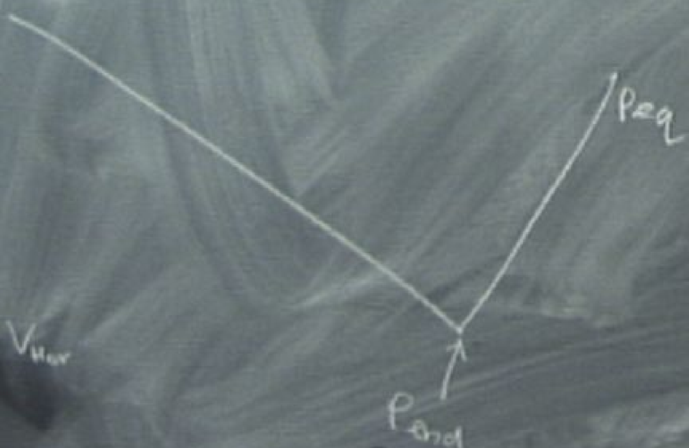
$\Omega_{mat} = 0.3$

$$N = \frac{1}{4}\ln\frac{\rho_{eq}}{V_{hor}} + \ln\sqrt{\frac{8\pi V_{hor}}{3M_p^2}}\,\frac{1}{H_{eq}} + 3.83$$

SEC ✓



$V_k \equiv V_{hor}$



$$H^2 = \frac{8\pi}{3m_p^2}\rho$$

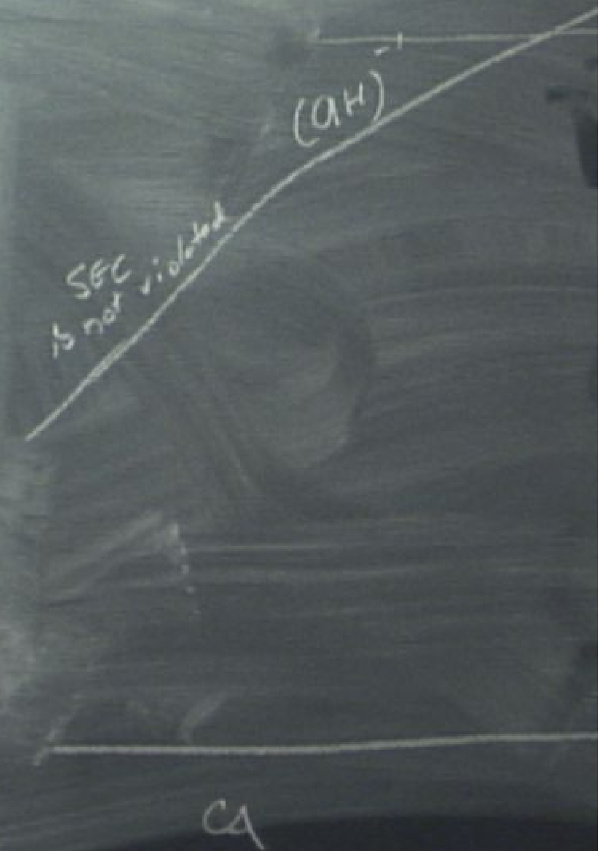
$$N = \frac{1}{4}\ln\frac{\rho_{eq}}{V_{Hor}} + \ln\sqrt{\frac{8\pi V_{Hor}}{3m_p^2}}\,\frac{1}{H_{eq}} + 3.83$$

$h = 0.7$

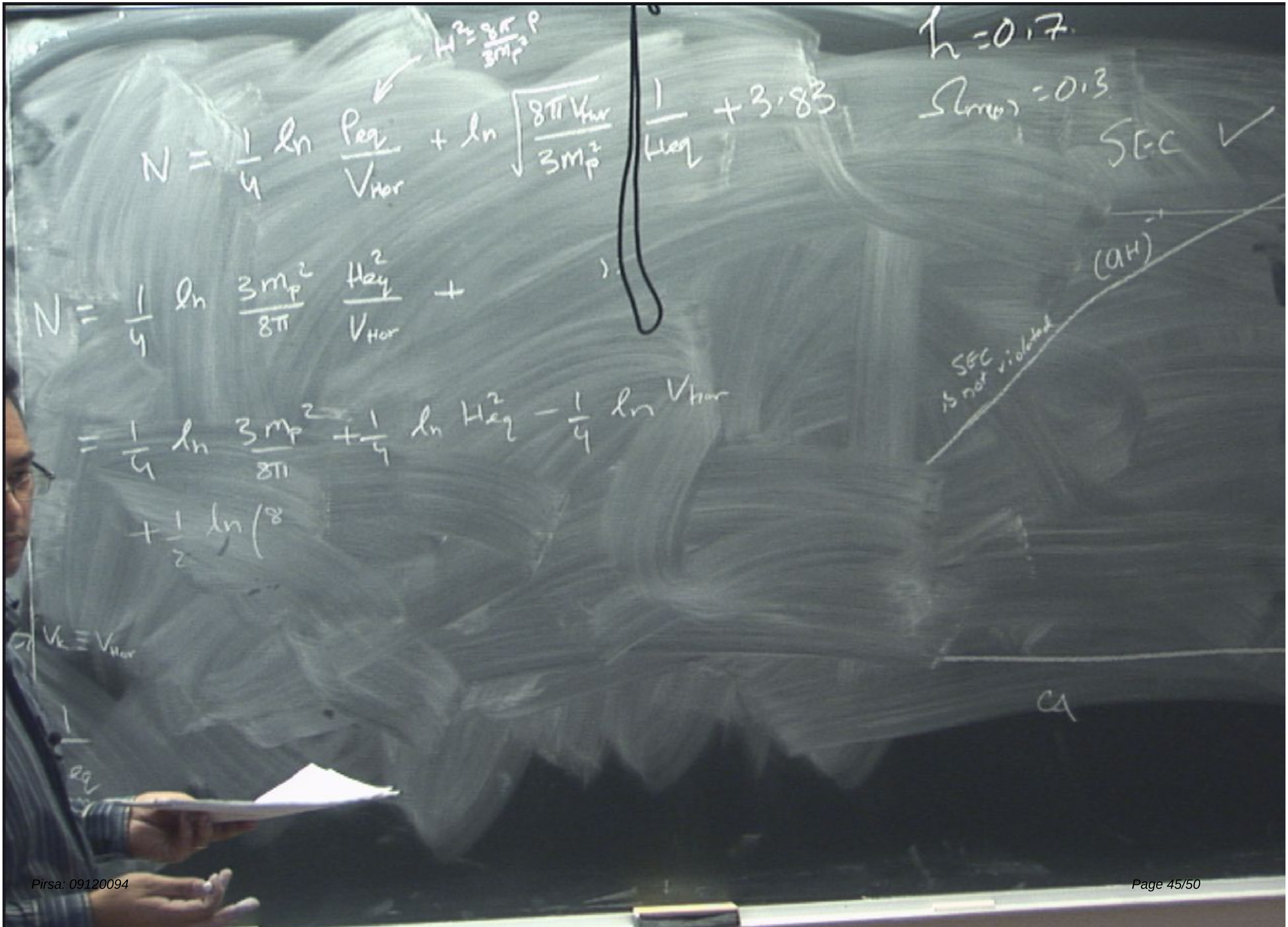
$\Omega_{matter} = 0.3$

SEC ✓

$$N = \frac{1}{4}\ln\frac{3m_p^2}{8\pi}\,\frac{H_{eq}^2}{V_{Hor}} +$$



$V_k \equiv V_{Hor}$


$H^2 = \frac{8\pi}{3m_p^2}\rho$
$N = \frac{1}{4} \ln \frac{\rho_{eq}}{V_{Hor}} + \ln \sqrt{\frac{8\pi V_{Hor}}{3m_p^2}} \frac{1}{H_{eq}} + 3.83$
$h = 0.7$
$\Omega_{matter} = 0.3$
SEC ✓
(aH)$^{-1}$
SEC is not violated
$N = \frac{1}{4} \ln \frac{3m_p^2}{8\pi} \frac{H_{eq}^2}{V_{Hor}} +$
$= \frac{1}{4} \ln \frac{3m_p^2}{8\pi} + \frac{1}{4} \ln H_{eq}^2 - \frac{1}{4} \ln V_{Hor}$
$+ \frac{1}{2} \ln (8$
$V_k \equiv V_{Hor}$
CA

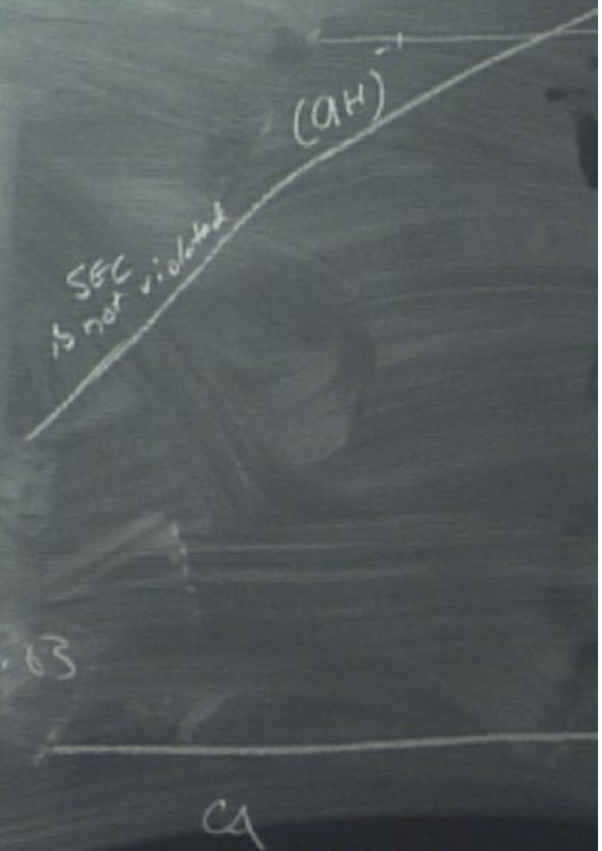
$$H^2 = \frac{8\pi}{3m_p^2}\rho$$

$h = 0.7$

$\Omega_{matter} = 0.3$

$$N = \frac{1}{4}\ln\frac{\rho_{eq}}{V_{hor}} + \ln\sqrt{\frac{8\pi V_{hor}}{3m_p^2}}\,\frac{1}{H_{eq}} + 3.83$$

SEC ✓



$$N = \frac{1}{4}\ln\frac{3m_p^2}{8\pi}\,\frac{H_{eq}^2}{V_{hor}} +$$

$$= \frac{1}{4}\ln\frac{3m_p^2}{8\pi} + \frac{1}{4}\ln H_{eq}^2 - \frac{1}{4}\ln V_{hor}$$

$$+ \frac{1}{2}\ln\left(\frac{8\pi}{3m_p^2}\right) + \frac{1}{2}\ln V_{hor} - \ln H_{eq} + 3.63$$

$V_k \equiv V_{hor}$

$$= \frac{1}{4}\ln\left(\frac{8\pi}{3m_p^2}\right) + \frac{1}{4}\ln V_{hor} - \frac{1}{2}\ln H_{eq} + 3.63$$

$\frac{1}{H_{eq}}$

$$H^2 = \frac{8\pi}{3m_p^2}\rho$$

$$N = \frac{1}{4}\ln\frac{\rho_{eq}}{V_{hor}} + \ln\sqrt{\frac{8\pi V_{hor}}{3m_p^2}}\,\frac{1}{H_{eq}} + 3.83$$

$h = 0.7$

$\Omega_{matter} = 0.3$

SEC ✓

$$N = \frac{1}{4}\ln\frac{3m_p^2}{8\pi}\frac{H_{eq}^2}{V_{hor}} +$$

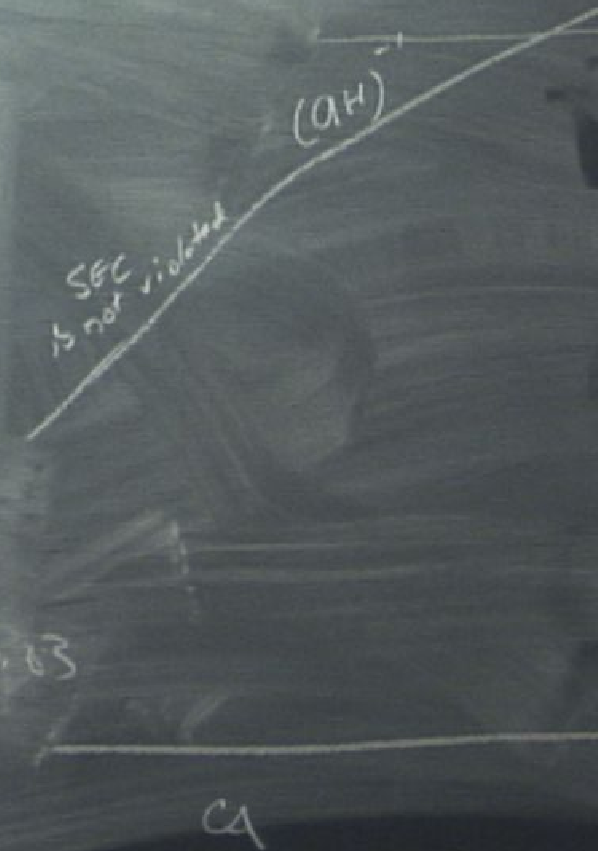
$$= \frac{1}{4}\ln\frac{3m_p^2}{8\pi} + \frac{1}{4}\ln H_{eq}^2 - \frac{1}{4}\ln V_{hor}$$

$$+ \frac{1}{2}\ln\left(\frac{8\pi}{3m_p^2}\right) + \frac{1}{2}\ln V_{hor} - \ln H_{eq} + 3.83$$

$V_k \equiv V_{hor}$

$$= \frac{1}{4}\ln\left(\frac{8\pi}{3m_p^2}\right) + \frac{1}{4}\ln V_{hor} - \frac{1}{2}\ln H_{eq} + 3.83$$

$\frac{1}{H_{eq}}$



$H^2 = \frac{8\pi}{3m_p^2}\rho$

$h = 0.7$

$\Omega_{mat} = 0.3$

SEC ✓

$$N = \frac{1}{4}\ln\frac{\rho_{eq}}{V_{Hor}} + \ln\sqrt{\frac{8\pi V_{Hor}}{3m_p^2}}\,\frac{1}{H_{eq}} + 3.83$$

$$N = \frac{1}{4}\ln\frac{3m_p^2}{8\pi}\frac{H_{eq}^2}{V_{Hor}} +$$

$$= \frac{1}{4}\ln\frac{3m_p^2}{8\pi} + \frac{1}{4}\ln H_{eq}^2 - \frac{1}{4}\ln V_{Hor}$$

$$+ \frac{1}{2}\ln\left(\frac{8\pi}{3m_p^2}\right) + \frac{1}{2}\ln V_{Hor} - \ln H_{eq} + 3.63$$

$V_k \equiv V_{Hor}$

$$= \frac{1}{4}\ln\left(\frac{8\pi}{3m_p^2}\right) + \frac{1}{4}\ln V_{Hor} - \frac{1}{2}\ln H_{eq} + 3.63$$

$\frac{1}{H_{eq}}$

$$N = \frac{1}{4}\ln\left(\frac{8\pi}{3}\right) + 63.3 + 3.8 + \frac{1}{4}\ln\frac{V_{Hor}}{m_p^4}$$

(aH)$^{-1}$

SEC is not violated

CA

$H^2 = \frac{8\pi}{3m_p^2}\rho$

$h = 0.7$

$\Omega_{matter} = 0.3$

SEC ✓

$$N = \frac{1}{4}\ln\frac{\rho_{eq}}{V_{Hor}} + \ln\sqrt{\frac{8\pi V_{Hor}}{3m_p^2}}\frac{1}{H_{eq}} + 3.83$$

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$\frac{1}{H_{eq}}$

$$N = \frac{1}{4}\ln\left(\frac{8\pi}{3}\right) + 63.3 + 3.8 + \frac{1}{4}\ln\frac{V_{Hor}}{m_p^4} \approx 68 + \frac{1}{4}\ln\frac{V_{Hor}}{m_p^4}$$

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$$\ln\frac{3m_p^2}{8\pi}\,\frac{H_{eq}^2}{V_{Hor}} + \ldots$$

$$\ln\frac{3m_p^2}{8\pi} + \frac{1}{4}\ln H_{eq}^2 - \frac{1}{4}\ln V_{Hor}$$

$$\frac{1}{2}\ln\left(\frac{8\pi}{3m_p^2}\right) + \frac{1}{2}\ln V_{Hor} - \ln H_{eq} + 3.83$$

$$= \frac{1}{4}\ln\left(\frac{8\pi}{3m_p^2}\right) + \frac{1}{4}\ln V_{Hor} - \frac{1}{2}\ln H_{eq} + 3.83$$

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SEC ✓