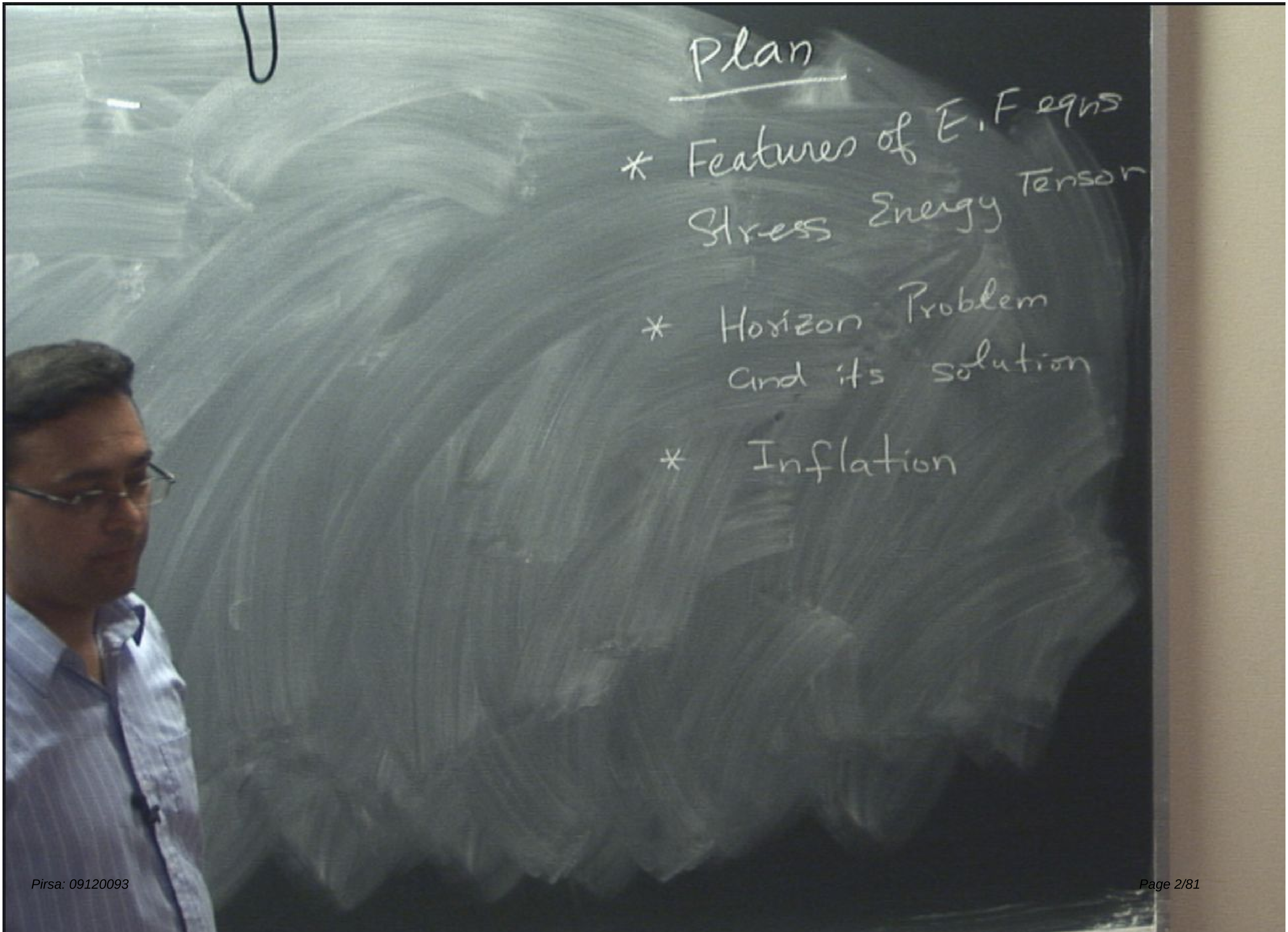
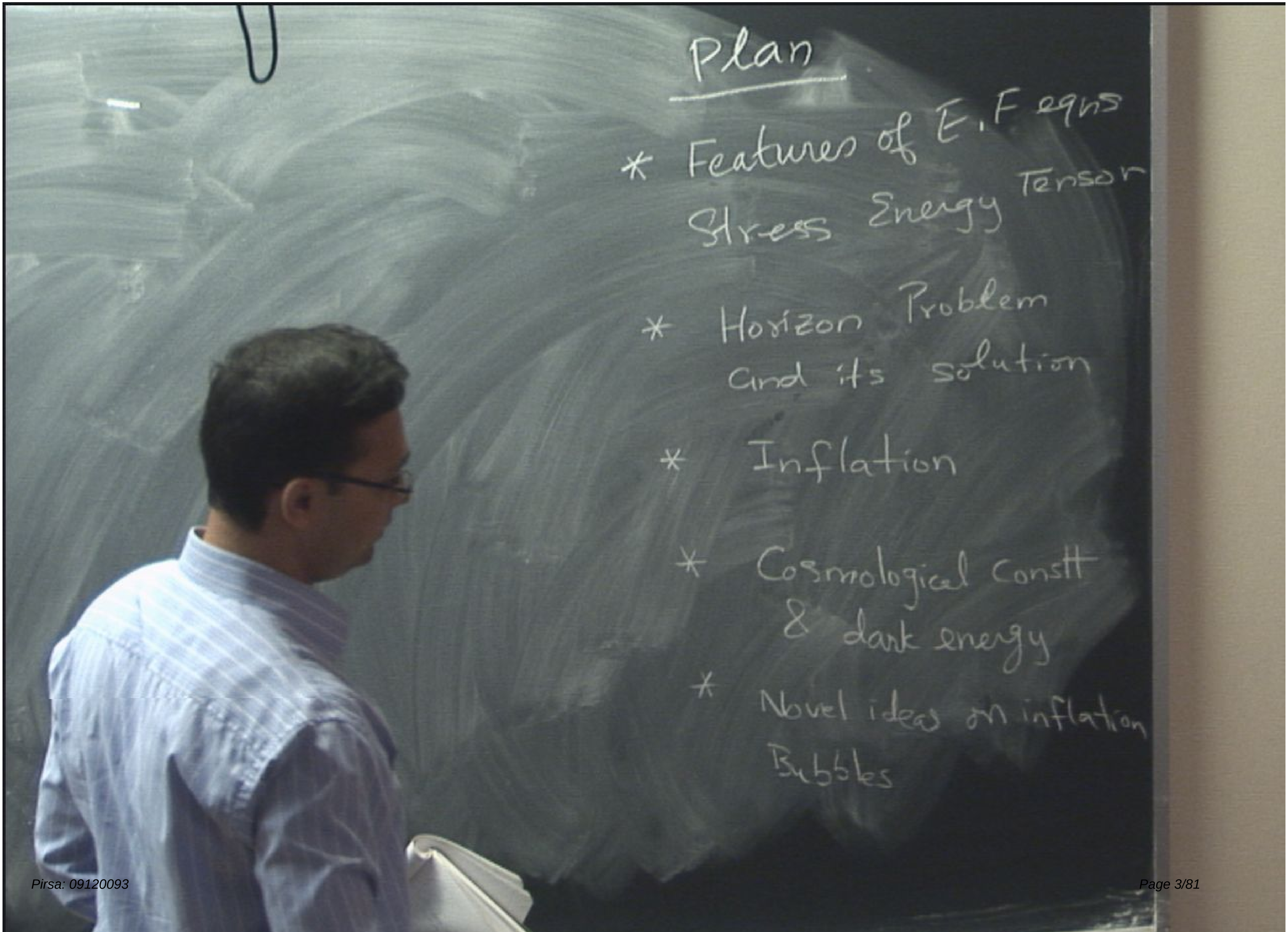
Title: Cosmology - Review (PHYS 621) - Lecture 11

Date: Dec 14, 2009 10:00 AM

URL: http://pirsa.org/09120093

Abstract:


Plan
* Features of E.F eqns
Stress Energy Tensor
* Horizon Problem
and its solution
* Inflation


Plan
* Features of E.F eqns
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* Inflation
* Cosmological Constt
& dark energy
* Novel ideas on inflation
Bubbles

Plan

* Features of E.F eqns
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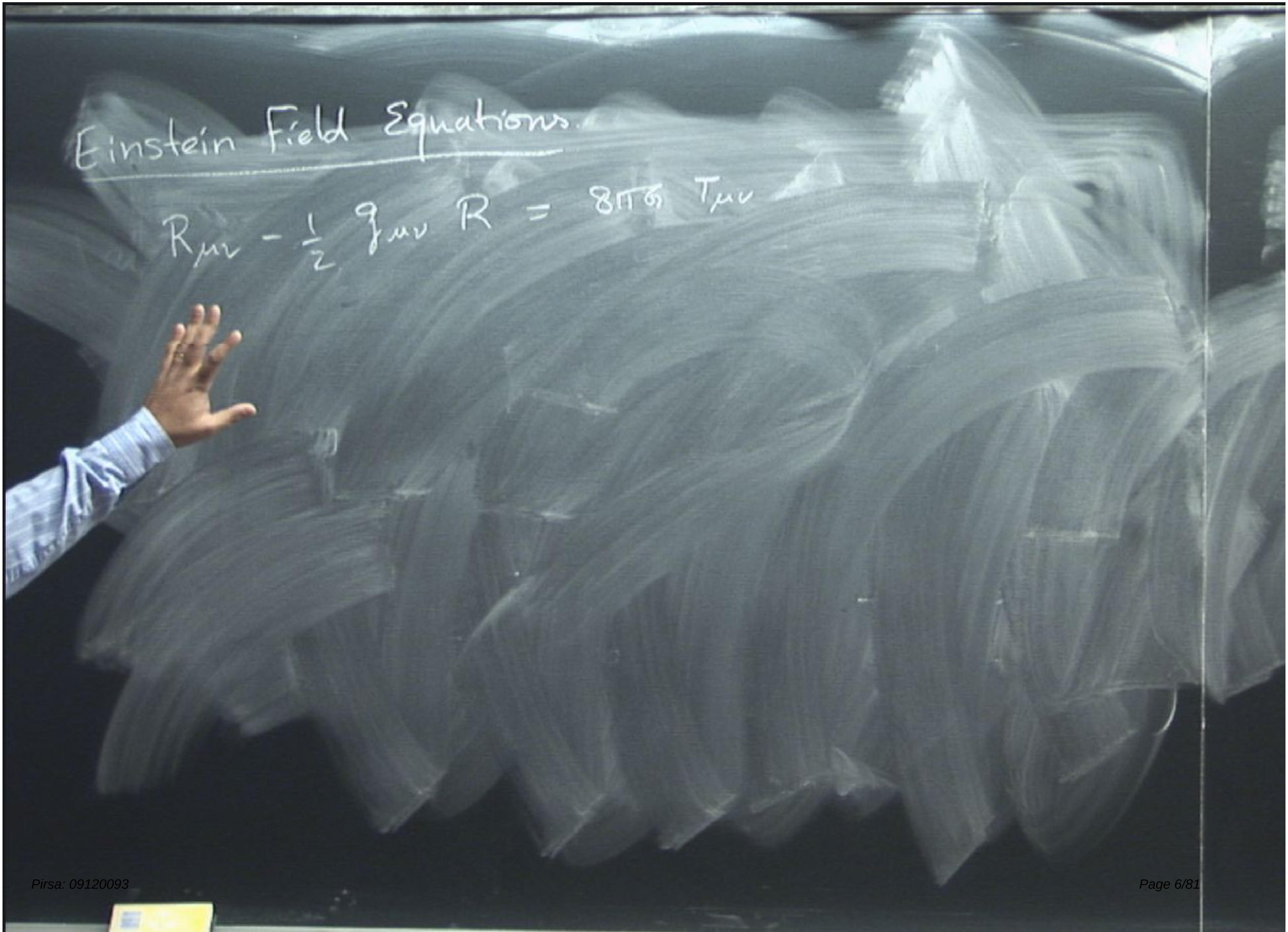
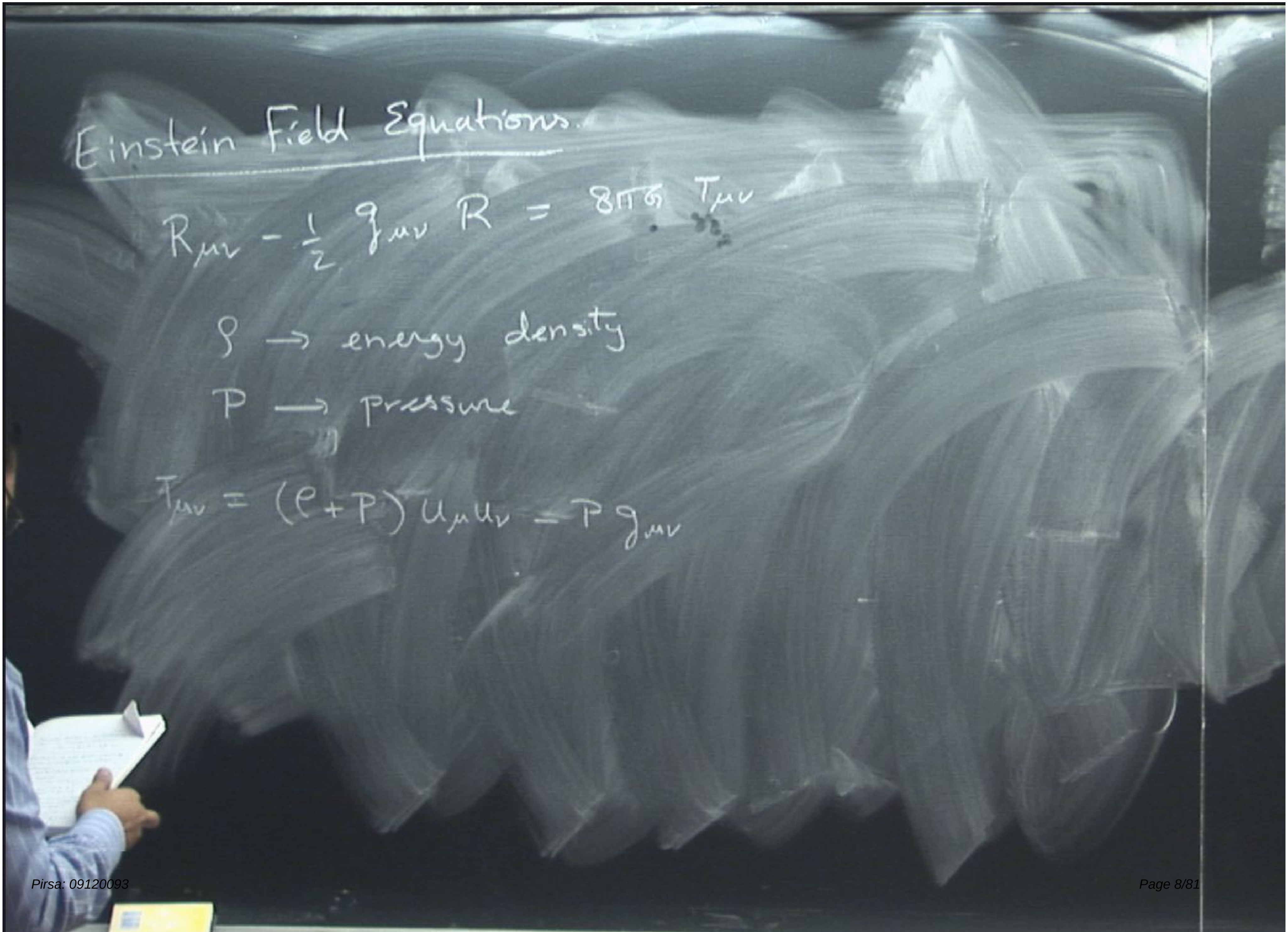
  Bubbles


Einstein Field Equations.


Einstein Field Equations.
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$

Einstein Field Equations.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \, T_{\mu\nu}$$


Einstein Field Equations.
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \, T_{\mu\nu}$
ρ → energy density
P → pressure
$T_{\mu\nu} = (\rho + P) u_\mu u_\nu - P g_{\mu\nu}$

Einstein Field Equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

$\rho \rightarrow$ energy density

$P \longrightarrow$ pressure

$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu - P\, g_{\mu\nu}$$

$u_\mu \rightarrow$ 4-volocity of the fluid

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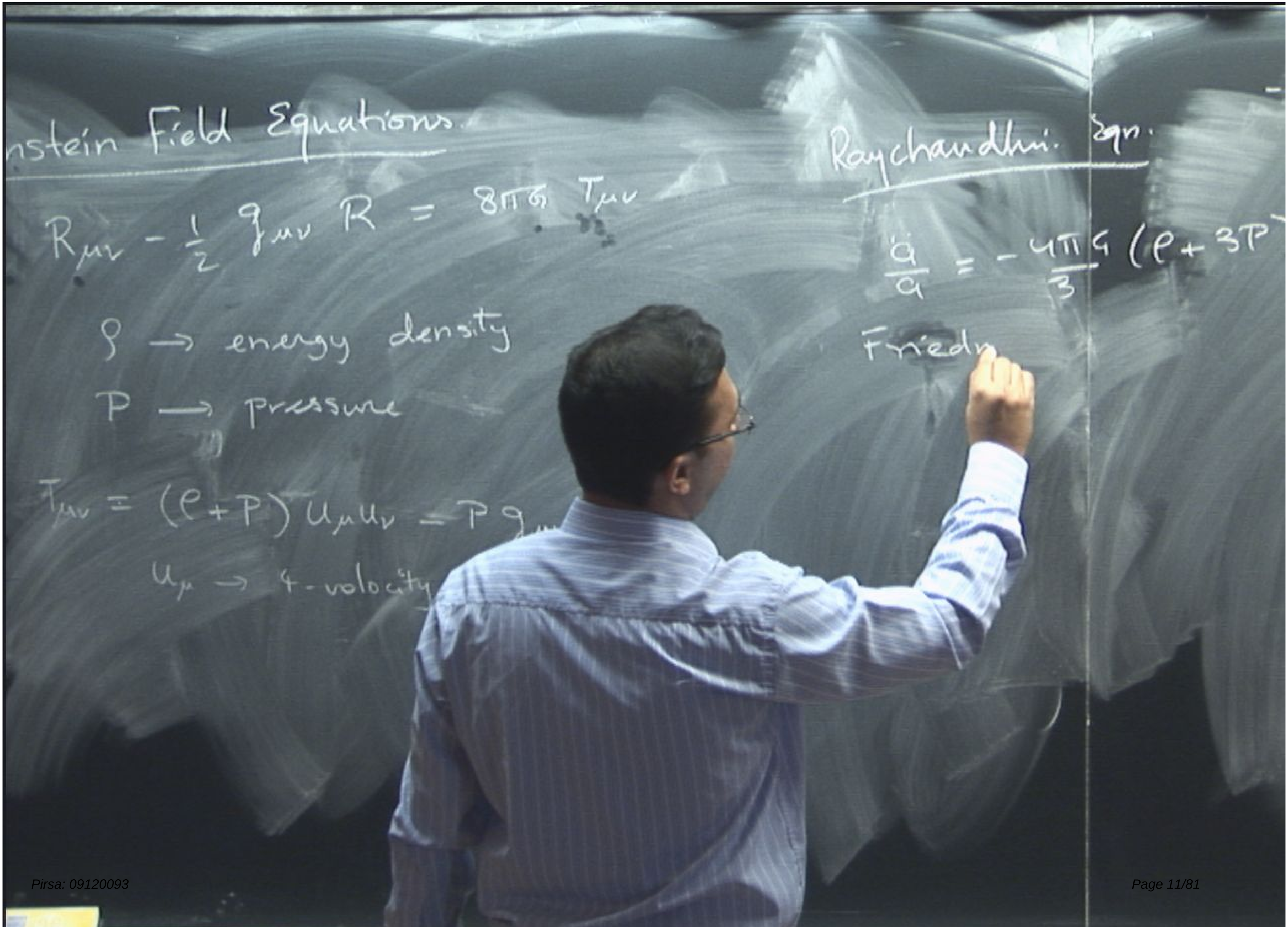
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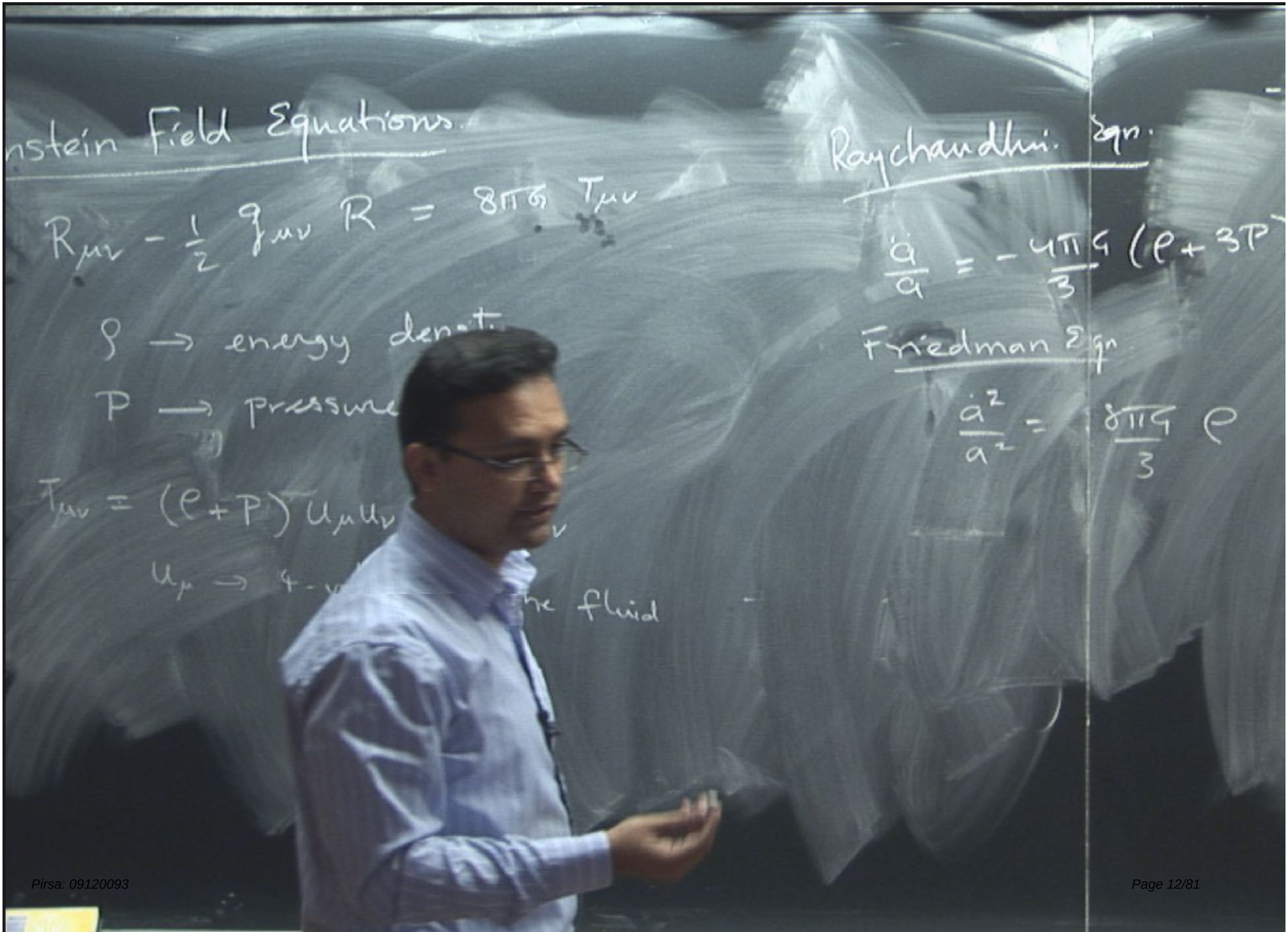
$u_\mu \rightarrow$ 4-volocity
Raychaudhuri Eqn.
$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$
Friedm


nstein Field Equations.
Rμν - 1/2 gμν R = 8πG Tμν
ρ → energy dens
P → pressure
Tμν = (ρ+P) uμuν
uμ → 4-v
he fluid
Raychaudhuri Eqn.
ä/a = - 4πG/3 (ρ + 3P
Friedman Eqn.
ȧ²/a² = 8πG/3 ρ

...nstein Field Equations.

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Friedman Eqn

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho$$

...ions.

$$\ldots = 8\pi G\, T_{\mu\nu}$$

density

...re

$$\ldots u_\nu - P\, g_{\mu\nu}$$

...ocity of the fluid

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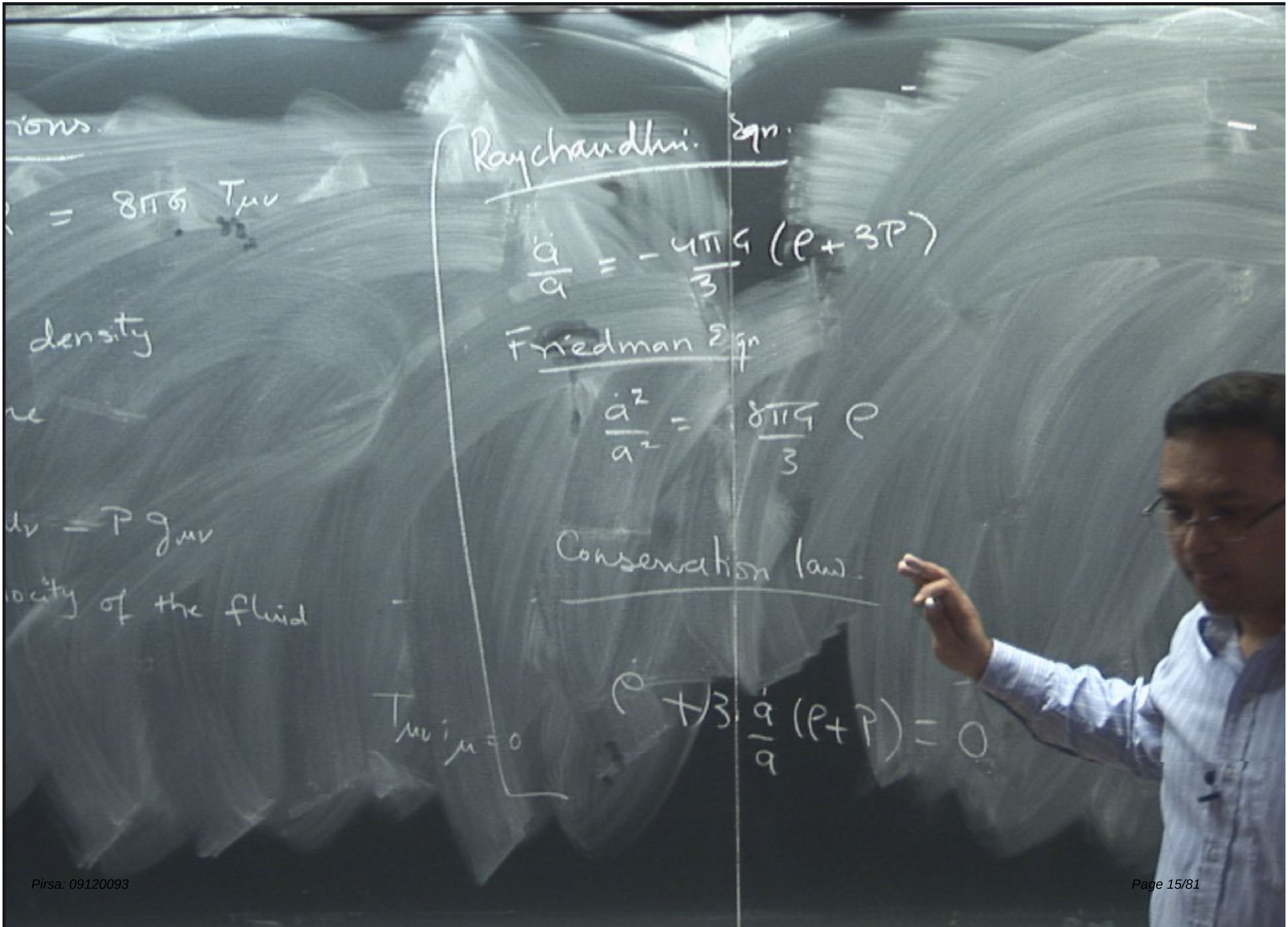
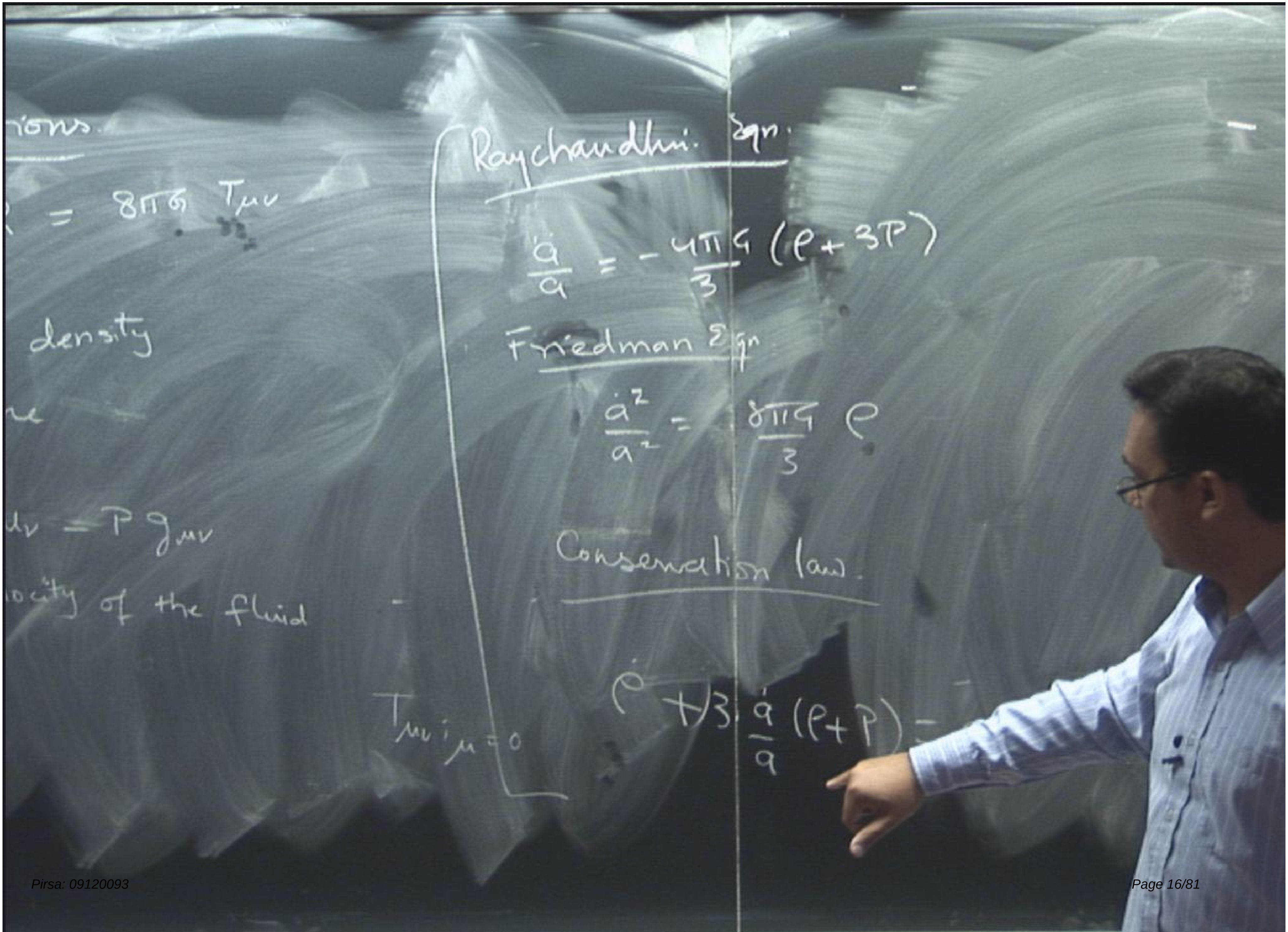
Conservation law.

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$$



Raychaudhuri Eqn.
$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$
Friedman Eqn.
$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho$
Conservation law.
$T_{\mu\nu;\mu} = 0$
$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$
$= 8\pi G\ T_{\mu\nu}$
density
$P g_{\mu\nu}$
of the fluid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...ions.

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density

...re

$\ldots_{\mu\nu} - P\, g_{\mu\nu}$

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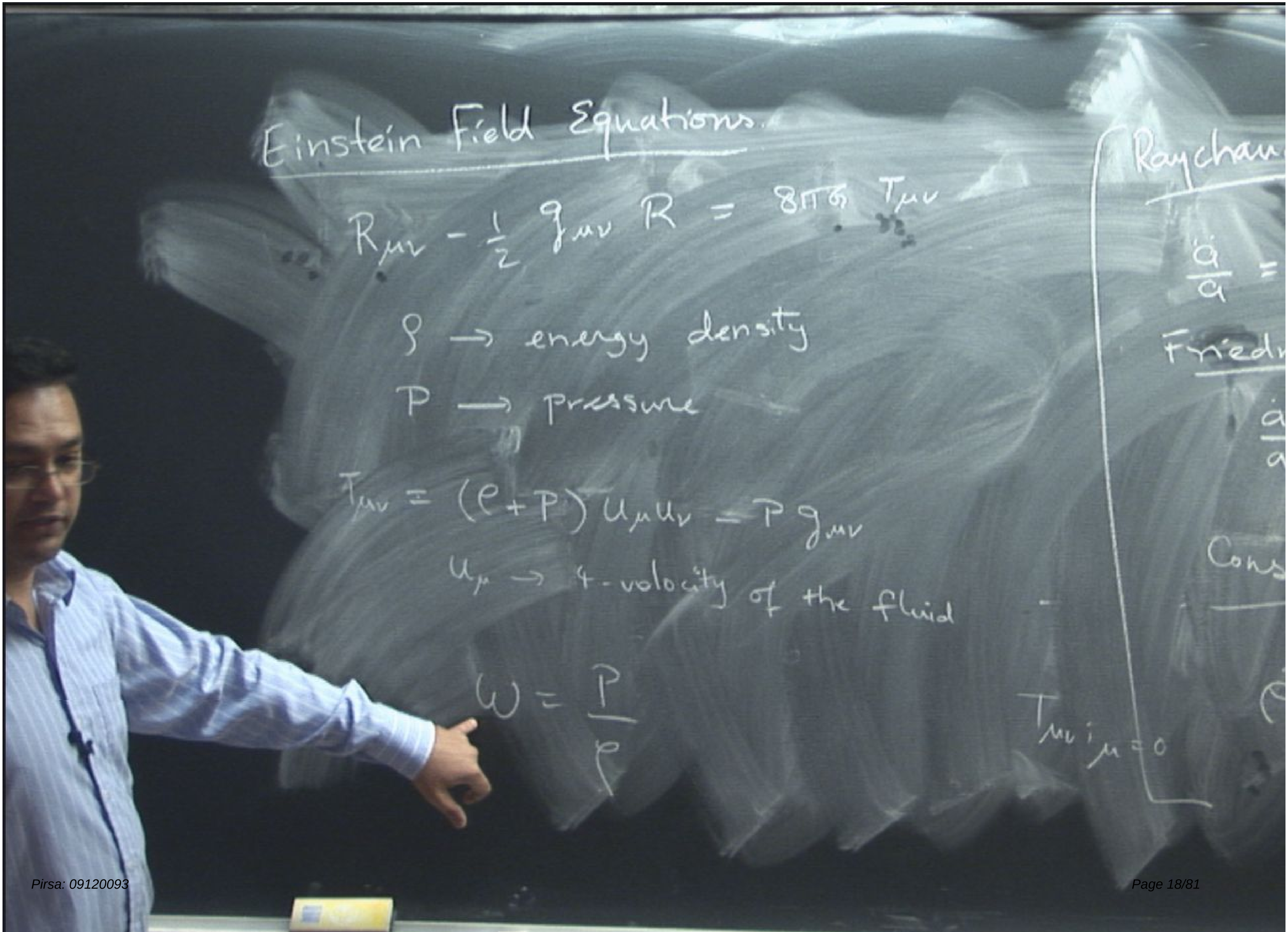
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$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$
$\rho \rightarrow$ energy density
$P \rightarrow$ pressure
$T_{\mu\nu} = (\rho + P) u_\mu u_\nu - P g_{\mu\nu}$
$u_\mu \rightarrow$ 4-volocity of the fluid
$\omega = \frac{P}{\rho}$
$T_{\mu\nu ;\mu} = 0$

Einstein Field Equations

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Raychau

$\frac{\ddot{a}}{a} =$

Friedm

$\frac{\dot{a}}{a}$

Cons

$\dot{\rho}$

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luid

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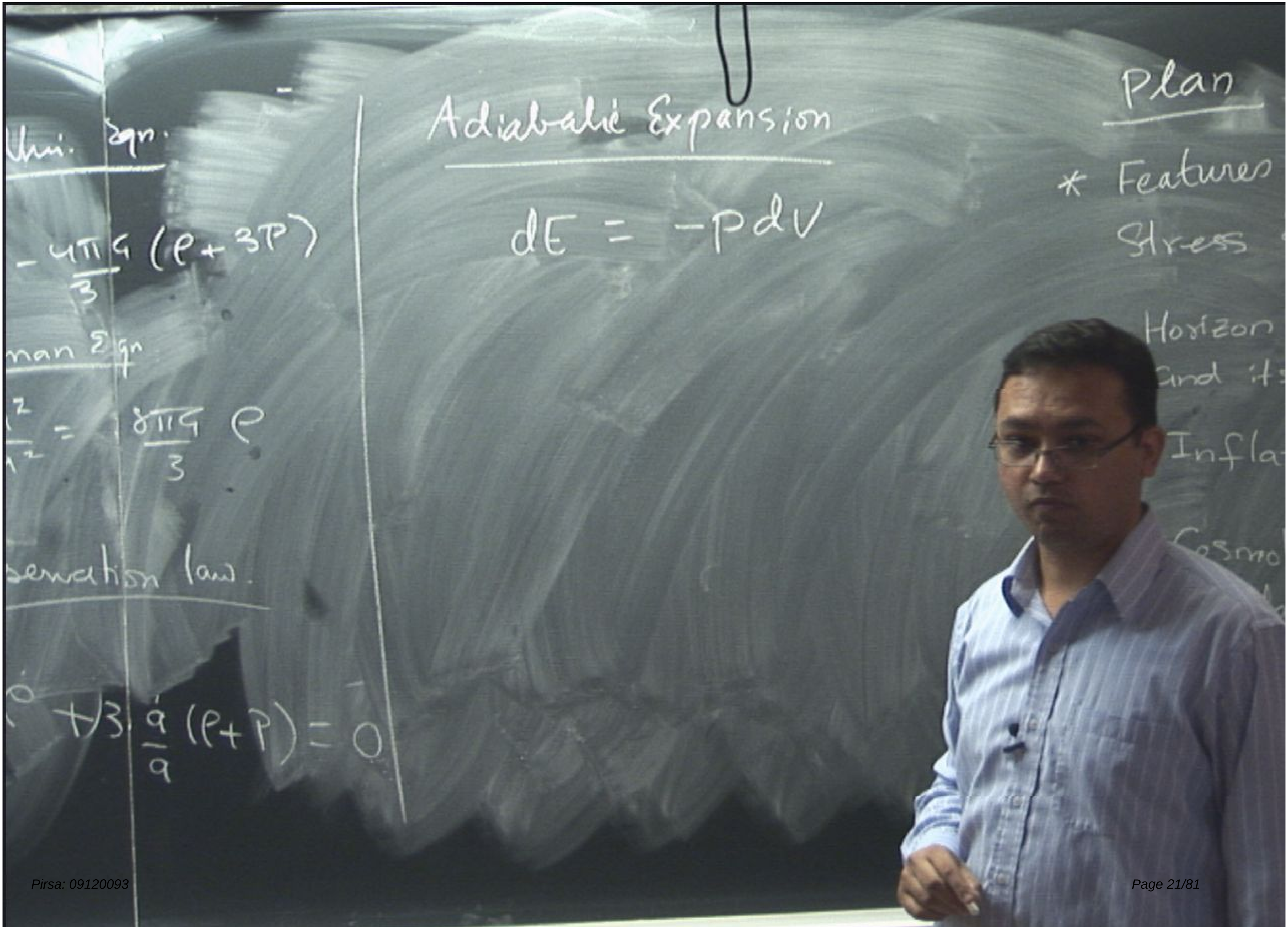
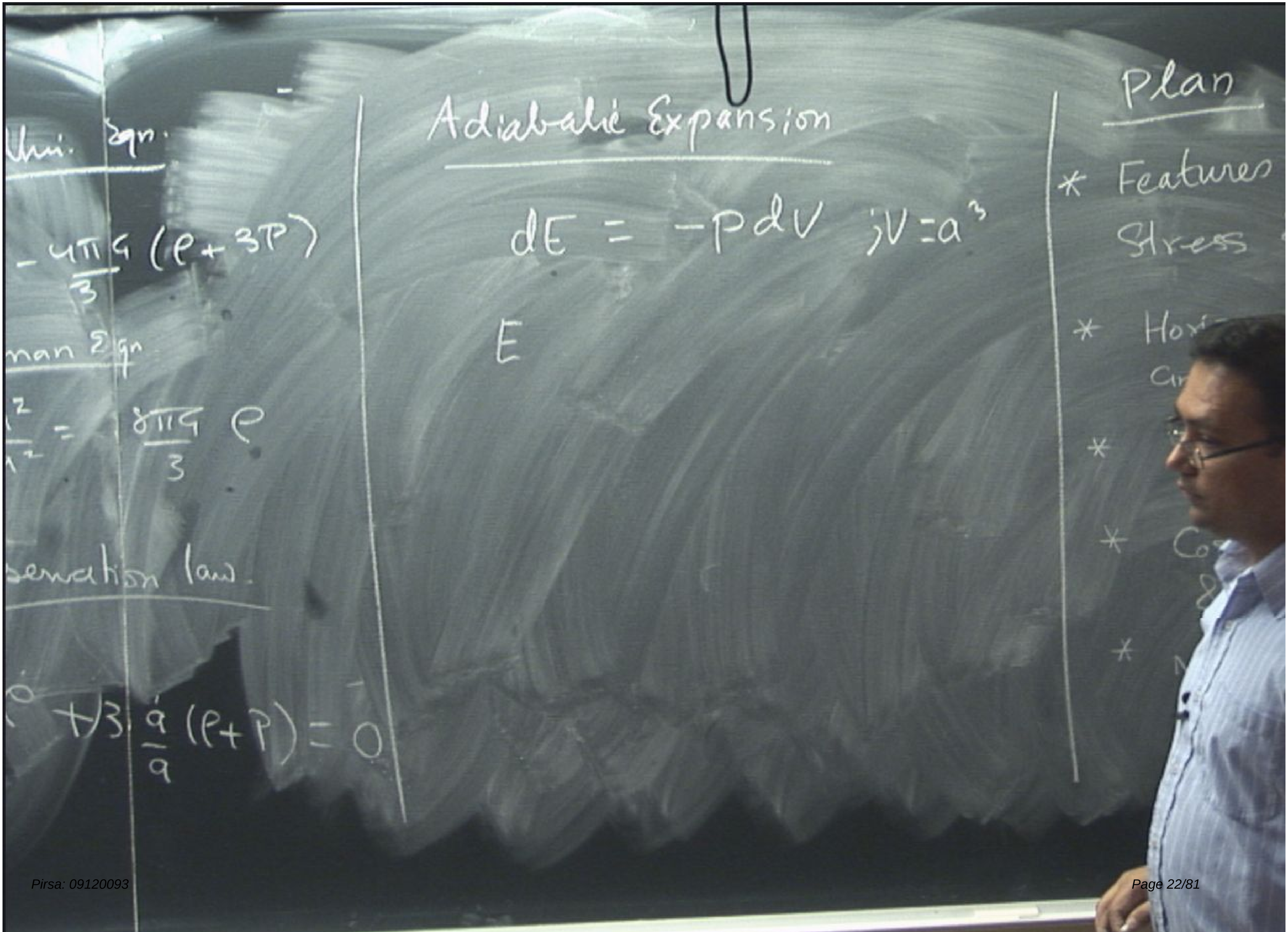
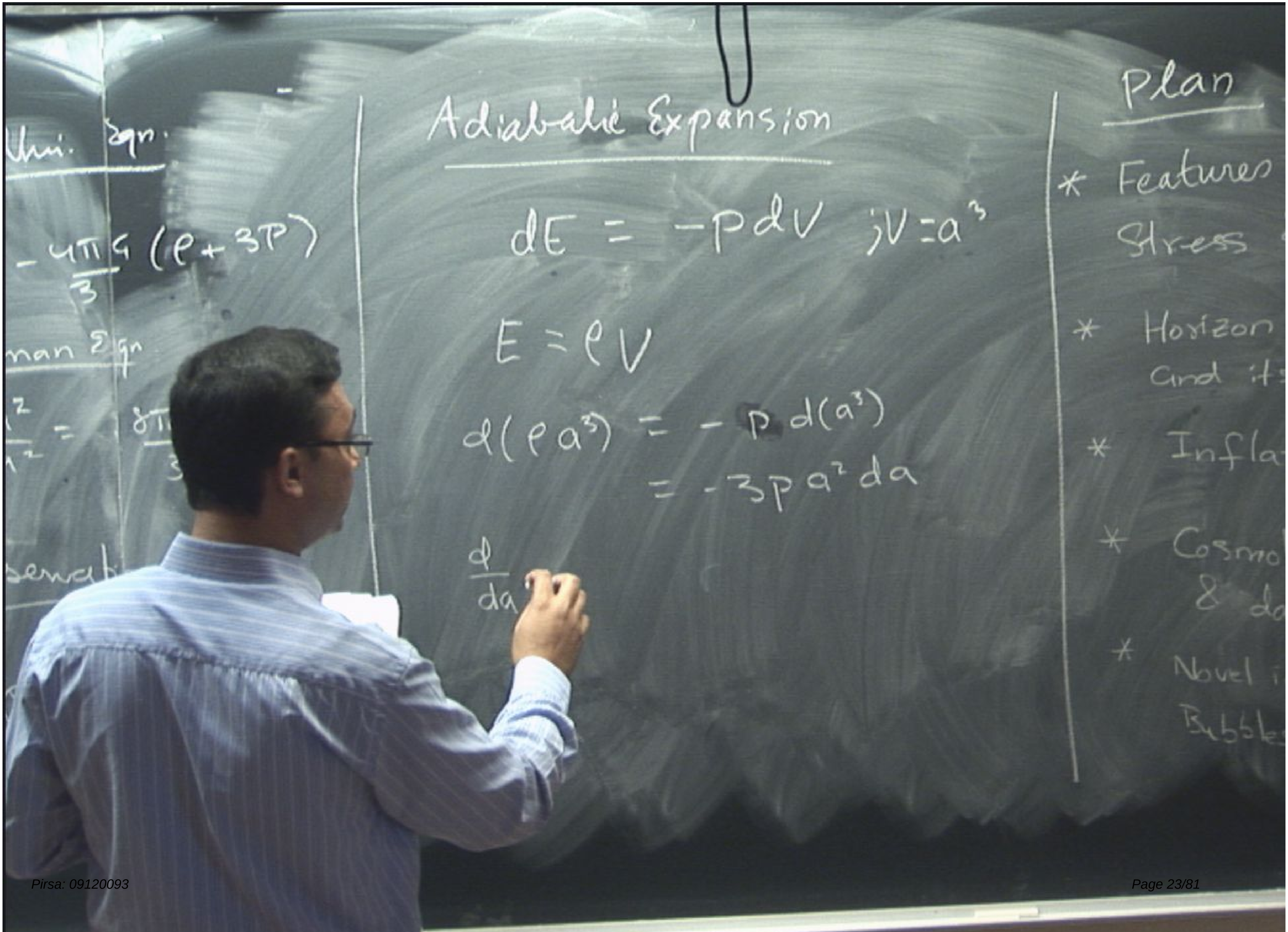
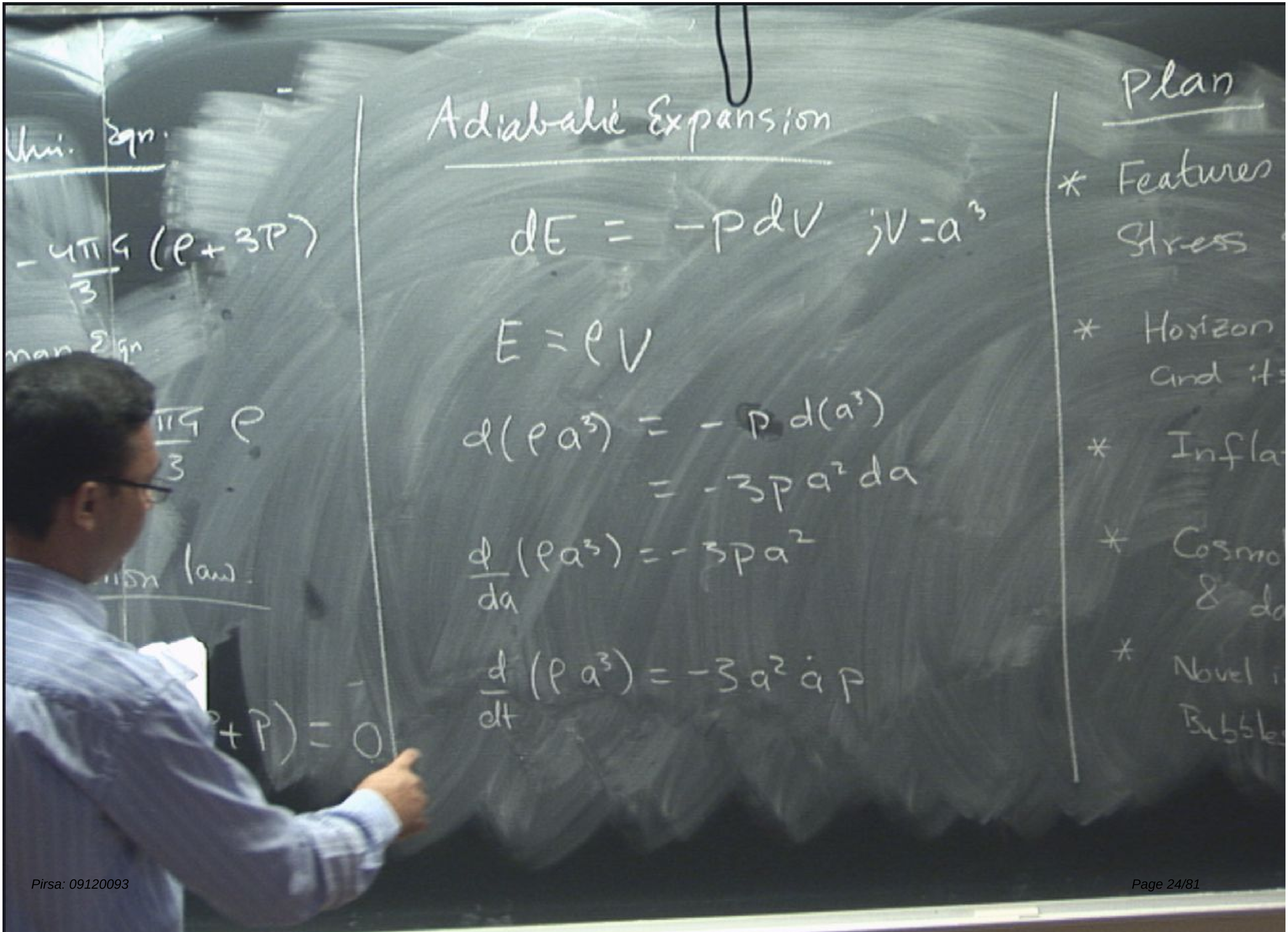

Adiabatic Expansion
dE = -pdV
Plan
* Features
Stress
Horizon
and it
Infla
Cosmo
man Eqn
servation law


Adiabatic Expansion
dE = -pdV ; V = a³
E
Plan
* Features
Stress
man Eqn
servation law


Adiabatic Expansion
dE = -p dV ; V = a³
E = ρV
d(ρa³) = - p d(a³)
= -3p a² da
d/da
Plan
* Features
Stress
* Horizon
and its
* Infla
* Cosmo
& da
* Novel
Bubble
Univ. Eqn.
-4πG/3 (ρ + 3P)
man Eqn


Adiabatic Expansion
dE = -p dV ; V = a^3
E = ρV
d(ρa^3) = - p d(a^3)
= -3p a^2 da
d/da (ρa^3) = -3pa^2
d/dt (ρa^3) = -3a^2 ȧ p
Plan
* Features
Stress
* Horizon
and it
* Infla
* Cosmo
& da
* Novel
Bubble
Uni. Eqn.
-4πG/3 (ρ+3P)
law.
+P) = 0

Uni. Eqn.

$$-\frac{4\pi G}{3}(\rho + 3P)$$

...man Eqn

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho$$

...servation law.

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$$

Adiabatic Expansion

$$dE = -P\,dV \;;\; V = a^3$$

$$E = \rho V$$

$$d(\rho a^3) = -P\,d(a^3)$$
$$= -3P a^2\,da$$

$$\frac{d}{da}(\rho a^3) = -3Pa^2$$

$$\frac{d}{dt}(\rho a^3) = -3a^2\dot{a}P$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + P)$$

Plan

* Features ...
Stress ...
* Horizon ...
and it...
* Infla...
* Cosmo...
& da...
* Novel i...
Bubble...

Uni. Eqn.

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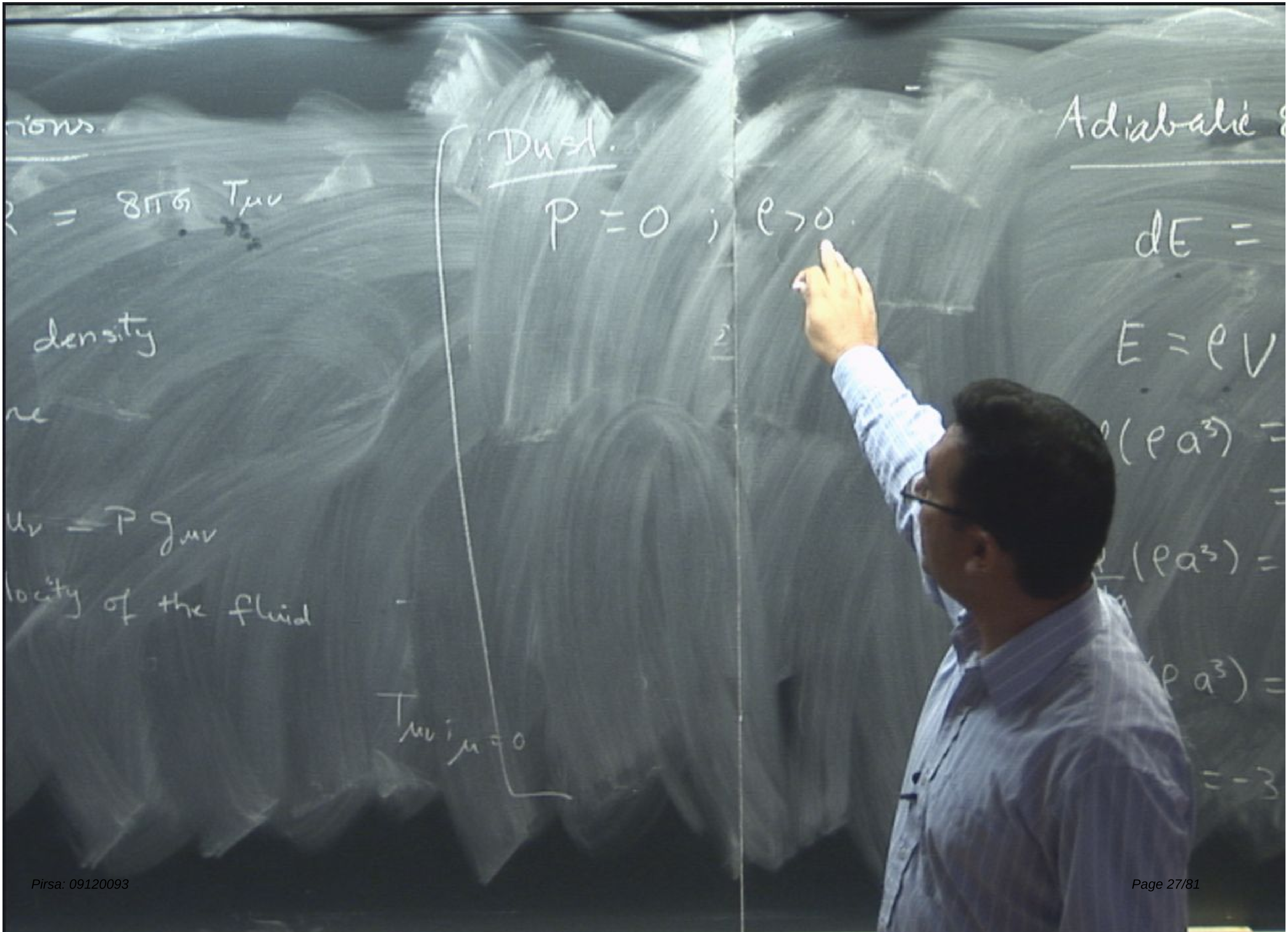
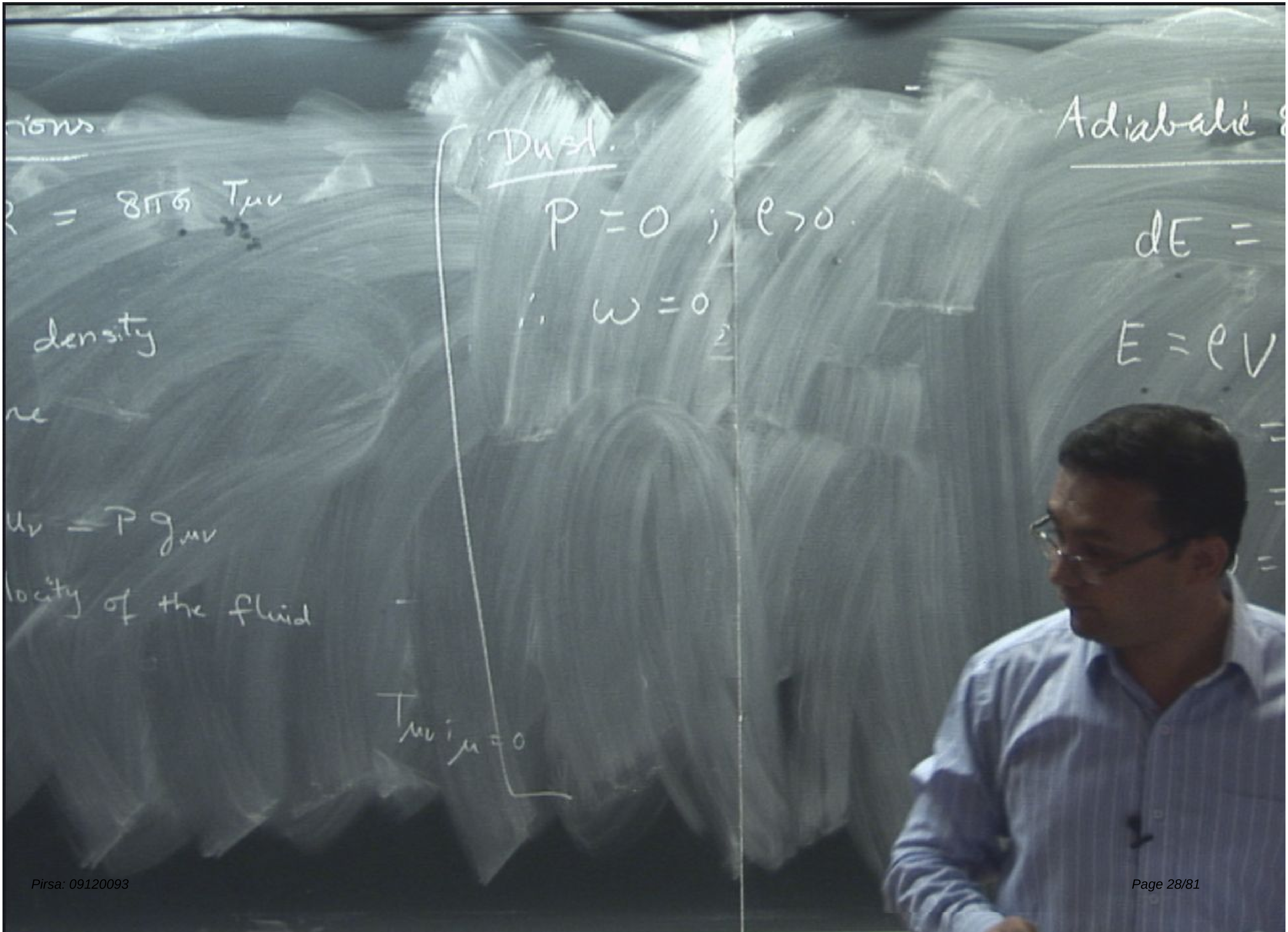
$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)$$

Plan

* Features
  Stress ...
* Horizon
  and it...
* Infla...
* Cosmo...
  & ...
* N...
  B...



Dust.
$P = 0 \; ; \; \rho > 0$
density
velocity of the fluid
$T_{\mu\nu;\mu} = 0$
Adiabatic
$dE =$
$E = \rho V$


ions.
R = 8πG Tμν
density
ne
uν − P gμν
locity of the fluid
Dust.
P = 0 ; ρ > 0
∴ ω = 0
Tμν ; μ = 0
Adiabatic
dE =
E = ρV

...ions.

$R = 8\pi G\, T_{\mu\nu}$

density

...re

$u_\nu - P g_{\mu\nu}$

...locity of the fluid

$T_{\mu\nu\,;\,\mu} = 0$

Dust.

$P = 0 \;;\; \rho > 0$

$\therefore\ w = 0$

Radiation

Adiabatic ...

$dE = $

$E = \rho V$

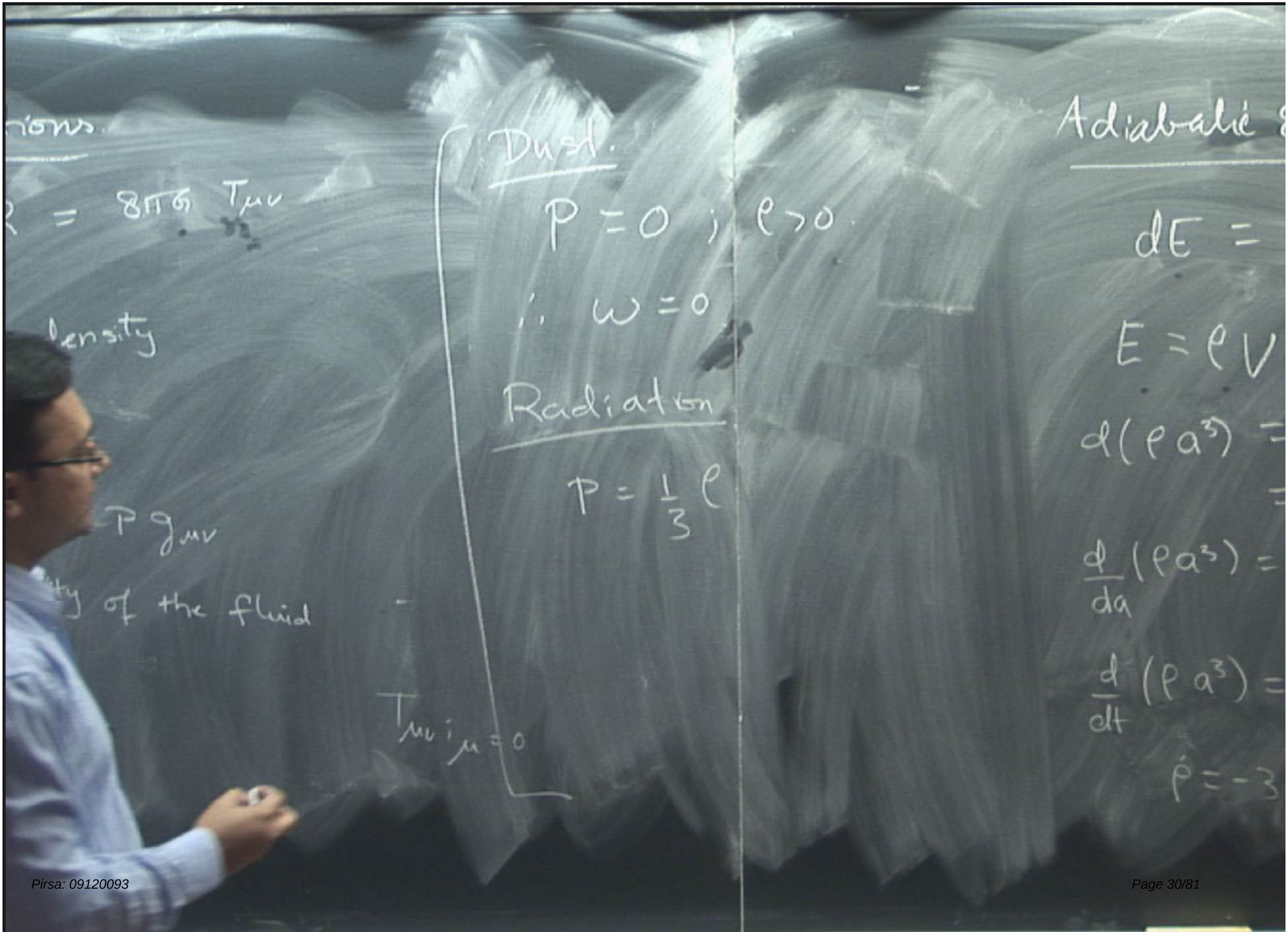
$d(\rho a^3) = $

$=$

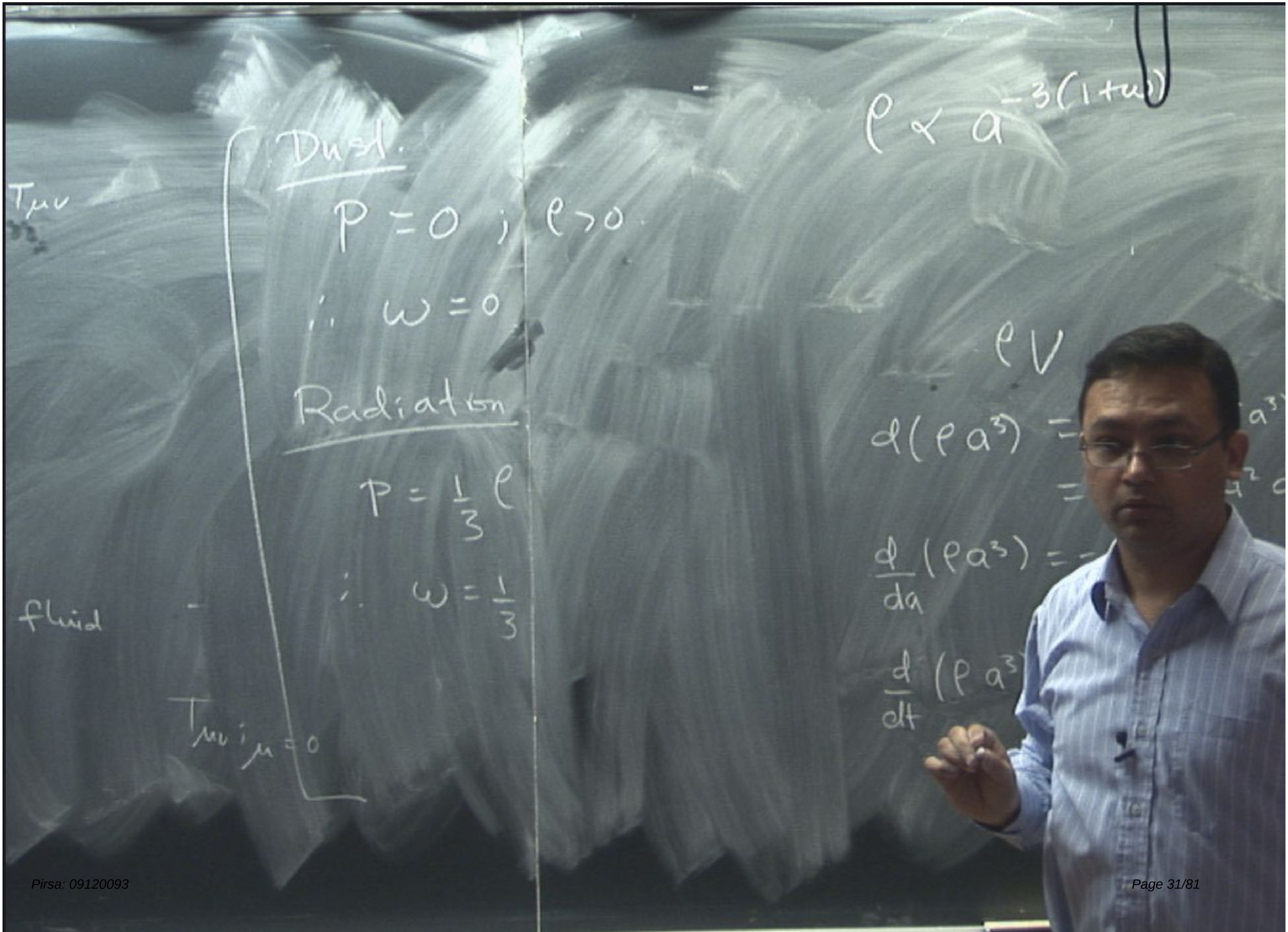
$\frac{d}{da}(\rho a^3) = $

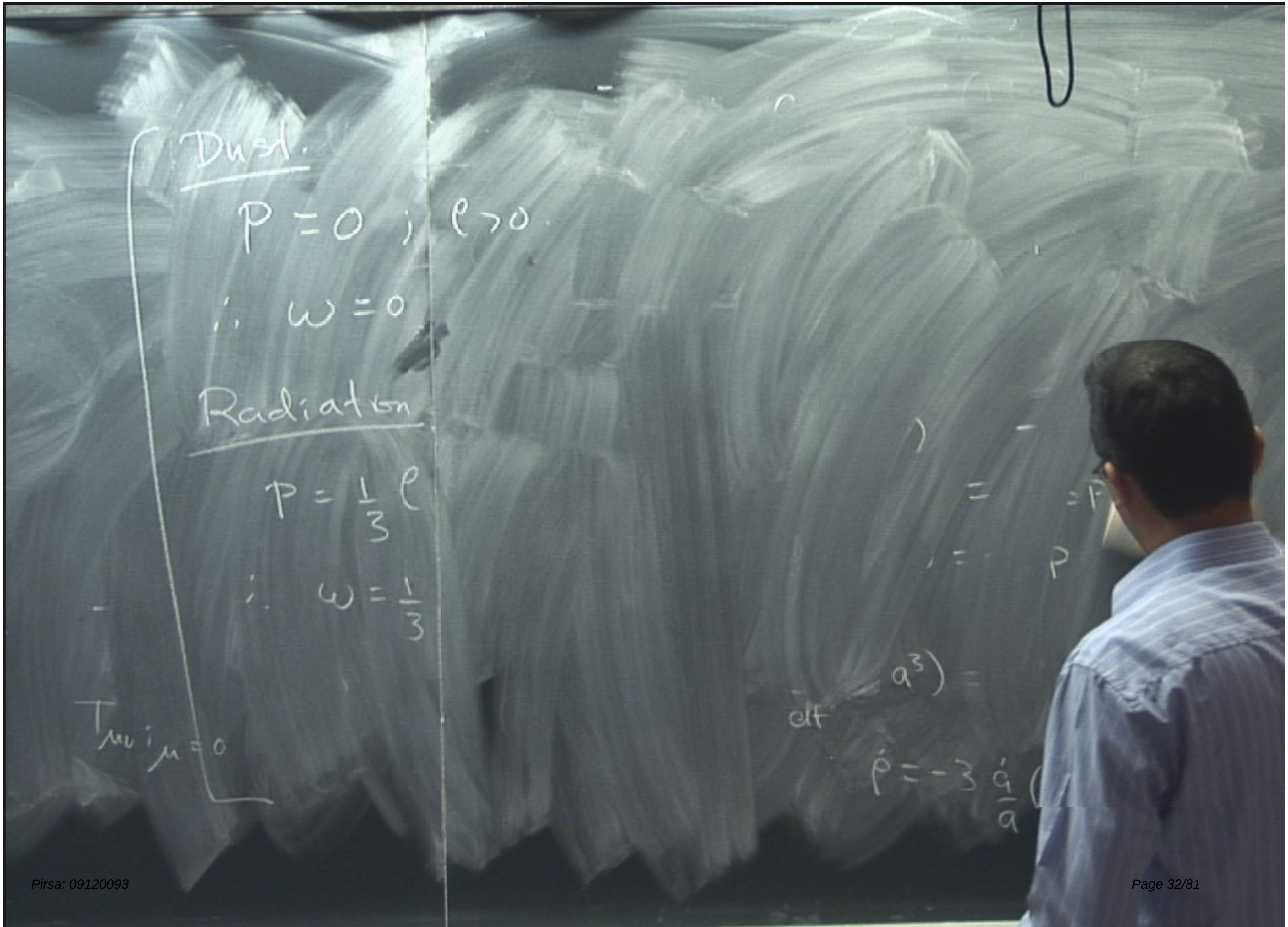
$\frac{d}{dt}(\rho a^3) = $

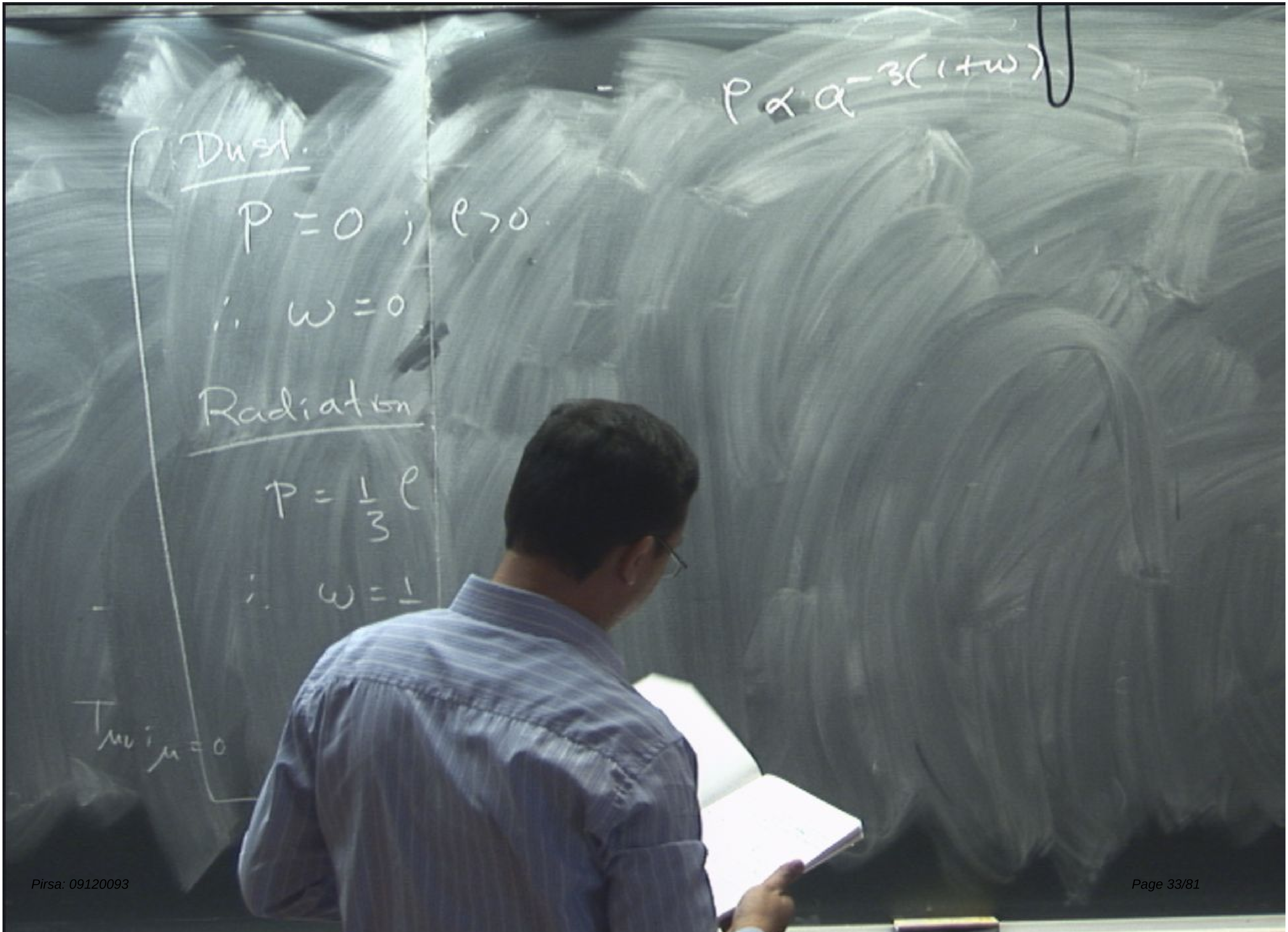
$\dot{\rho} = -3$



tions.
R = 8πG Tμν
density
P gμν
ty of the fluid
Tμν;μ = 0
Dust.
P = 0 ; ρ>0
∴ ω = 0
Radiation
P = 1/3 ρ
Adiabatic
dE =
E = ρV
d(ρa³) =
d/da (ρa³) =
d/dt (ρa³) =
ρ̇ = -3


Dust.
P = 0 ; ρ > 0
∴ ω = 0
Radiation
P = 1/3 ρ
∴ ω = 1/3
ρ ∝ a^{-3(1+ω)}
ρV
d(ρa³) =
d/da (ρa³) =
d/dt (ρ a³)
Tμν
fluid
Tμν;μ = 0


Dust.
p = 0 ; ρ > 0
∴ ω = 0
Radiation
p = 1/3 ρ
∴ ω = 1/3
T_μν;μ = 0
a³) =
dt
ρ̇ = −3 ȧ/a (


$\rho \propto a^{-3(1+w)}$
Dust.
$p = 0 \; ; \; \rho > 0$
$\therefore \; w = 0$
Radiation
$p = \frac{1}{3}\rho$
$\therefore \; w = 1$
$T_{\mu\nu;\mu} = 0$

$\rho \propto a^{-3(1+w)}$

Dust.

$p = 0 \; ; \; \rho > 0 \longrightarrow \rho \propto a^{-3}$

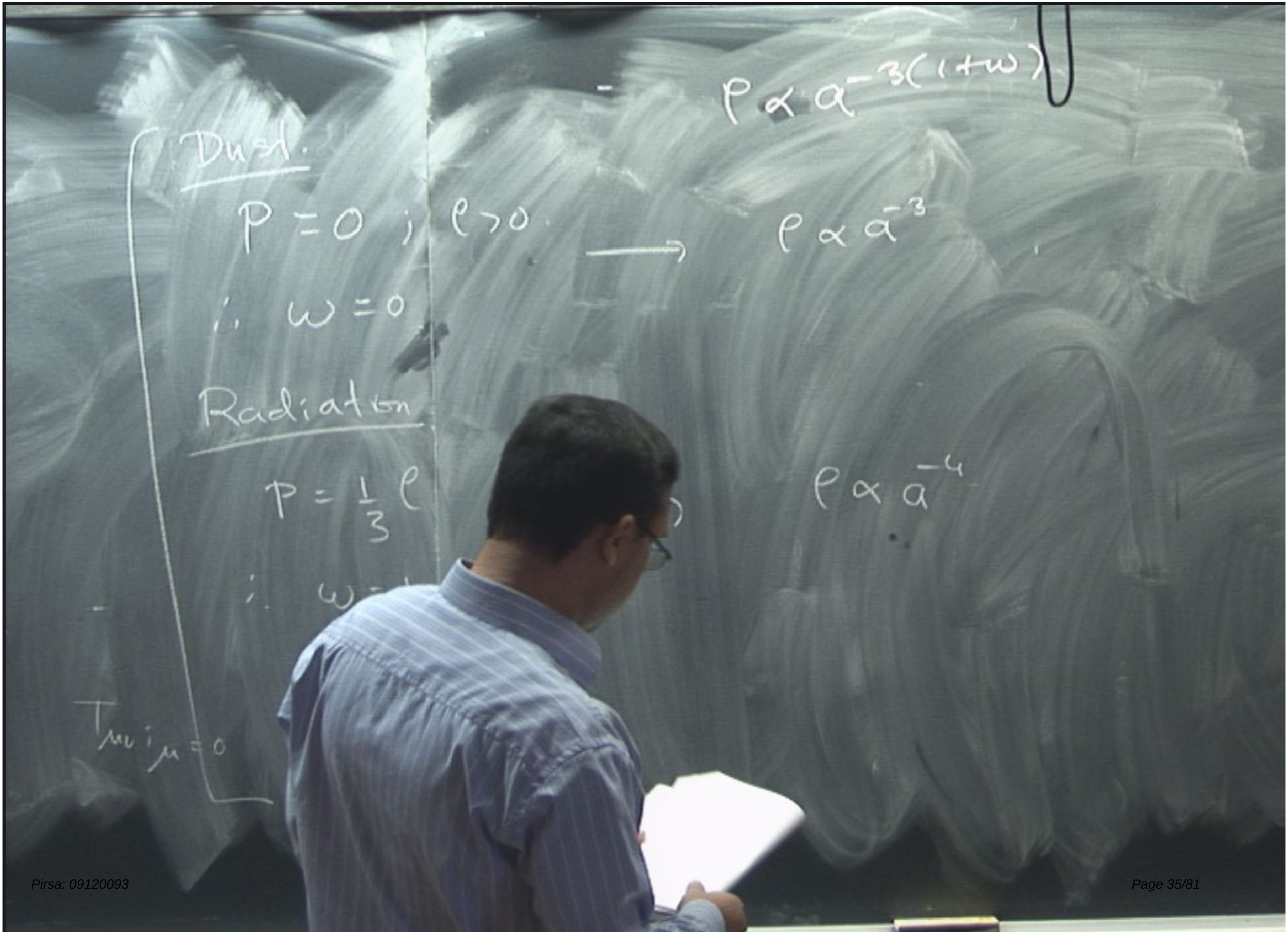
$\therefore \; w = 0$

Radiation

$p = \frac{1}{3}\rho \longrightarrow \rho \propto a^{-4}$

$\therefore \; w = \frac{1}{3}$

$T_{\mu\nu;\mu} = 0$


ρ ∝ a^-3(1+ω)
Dust.
p = 0 ; ρ > 0
ρ ∝ a^-3
∴ ω = 0
Radiation
p = 1/3 ρ
ρ ∝ a^-4
T_μν;μ = 0

$\rho \propto a^{-3(1+w)}$

Dust:

$p = 0 \; ; \; \rho > 0 \longrightarrow \rho \propto a^{-3} \propto (1+z)^3$

$\therefore \; w = 0$

Radiation

$p = \frac{1}{3}\rho \longrightarrow \rho \propto a^{-4} \propto (1+z)^4$

$\therefore \; w = \frac{1}{3}$

$T_{\mu\nu;\mu} = 0$

If $a \longrightarrow 0$ $\quad \rho_{dust} \longrightarrow \infty$

$\rho_{rad} \longrightarrow \infty$

$$\rho \propto a^{-3(1+w)}$$

Dust:

$p = 0$; $\rho > 0$ $\longrightarrow$ $\rho \propto a^{-3} \propto (1+z)^3$

$\therefore\ w = 0$

Radiation

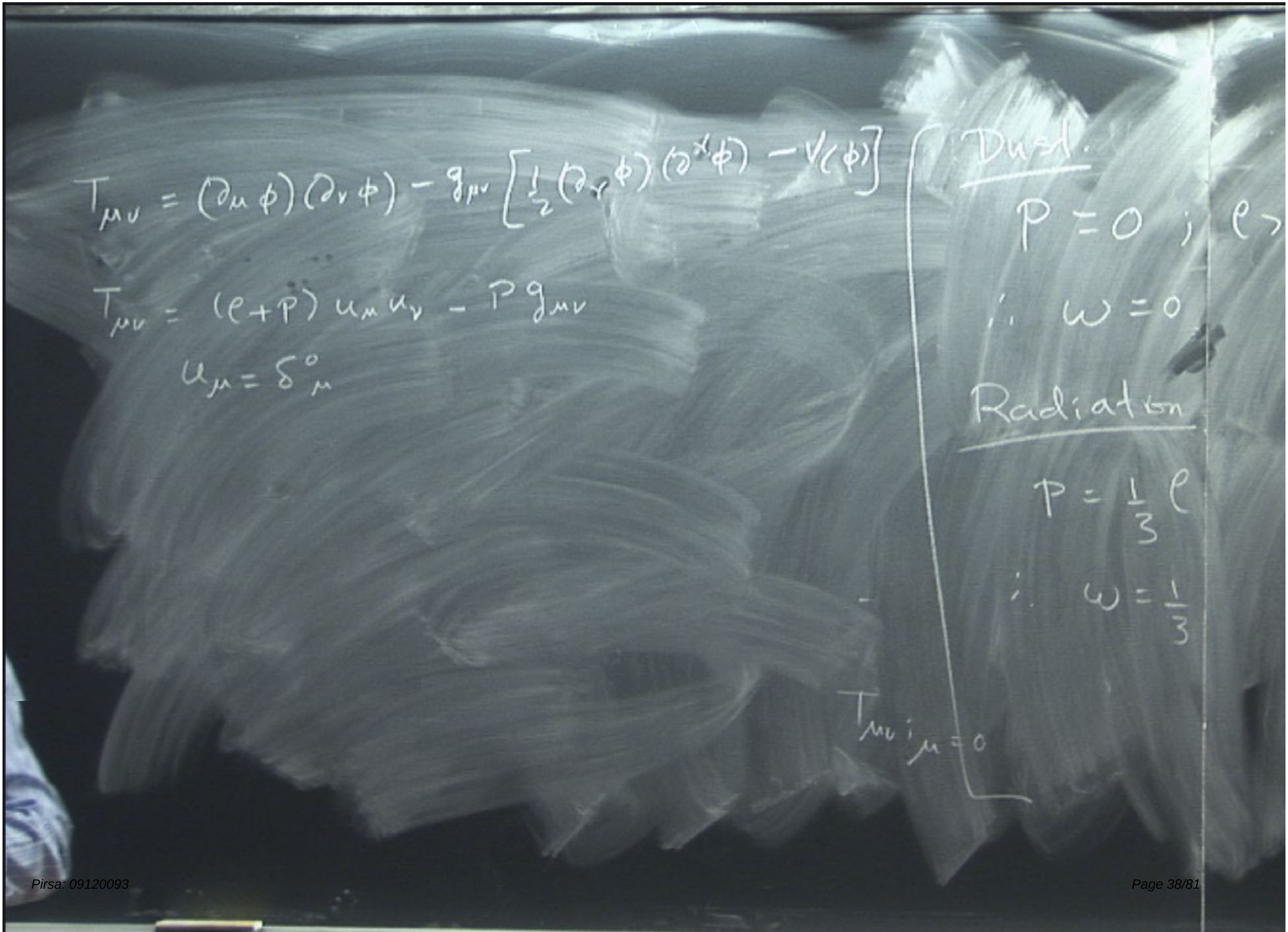
$p = \frac{1}{3}\rho$ $\longrightarrow$ $\rho \propto a^{-4} \propto (1+z)^4$

$\therefore\ w = \frac{1}{3}$

If $a \to 0$ $\quad \rho_{dust} \to \infty$

$\rho_{rad} \to \infty$

$T_{\mu\nu;\mu} = 0$


$T_{\mu\nu} = (\partial_\mu \phi)(\partial_\nu \phi) - g_{\mu\nu}\left[\frac{1}{2}(\partial_\alpha \phi)(\partial^\alpha \phi) - V(\phi)\right]$
$T_{\mu\nu} = (\rho + P) u_\mu u_\nu - P g_{\mu\nu}$
$u_\mu = \delta^0_\mu$
Dust.
$P = 0 \; ; \; \rho >$
$\therefore \; \omega = 0$
Radiation
$P = \frac{1}{3}\rho$
$\therefore \; \omega = \frac{1}{3}$
$T_{\mu\nu;\mu} = 0$

$$T_{\mu\nu} = (\partial_\mu \phi)(\partial_\nu \phi) - g_{\mu\nu}\left[\frac{1}{2}(\partial_\gamma \phi)(\partial^\gamma \phi) - V(\phi)\right]$$

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Dust.

$P = 0$; $\rho >$

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Radiation

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$$u_\mu = \delta^0_\mu$$

$$T_{\mu\nu} = (\rho + p)\, \delta^0_\mu \delta^0_\nu - p\, g_{\mu\nu}$$

Dust.

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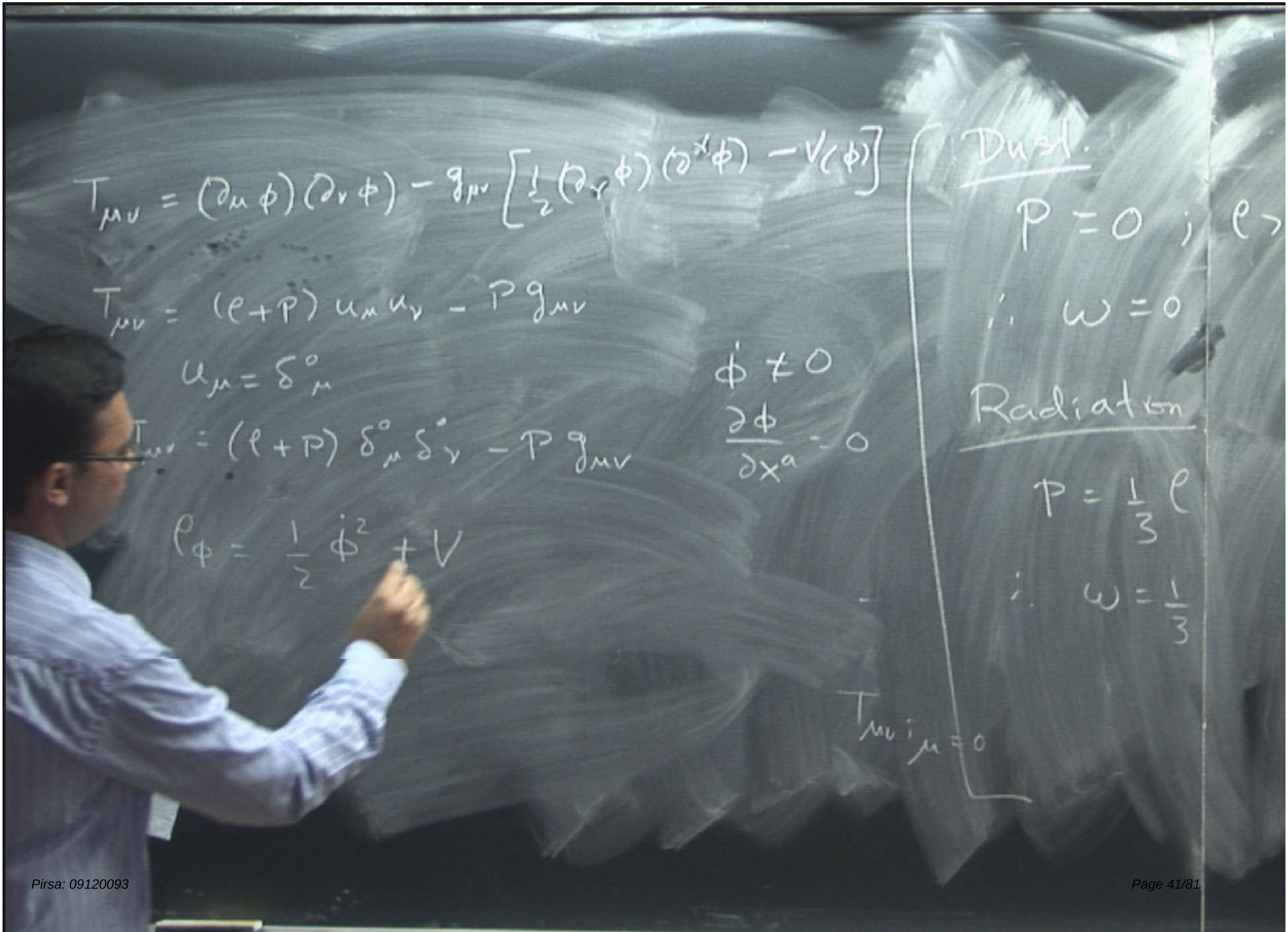
$\therefore \; \omega = 0$

Radiation

$p = \frac{1}{3}\rho$

$\therefore \; \omega = \frac{1}{3}$

$T_{\mu\nu ; \mu} = 0$



$T_{\mu\nu} = (\partial_\mu \phi)(\partial_\nu \phi) - g_{\mu\nu}\left[\frac{1}{2}(\partial_\gamma \phi)(\partial^\gamma \phi) - V(\phi)\right]$
$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu - P g_{\mu\nu}$
$u_\mu = \delta^0_\mu$
$T_{\mu\nu} = (\rho + P)\, \delta^0_\mu \delta^0_\nu - P g_{\mu\nu}$
$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V$
$\dot{\phi} \neq 0$
$\frac{\partial \phi}{\partial x^a} = 0$
$T_{\mu\nu;\mu} = 0$
Dust.
$P = 0$; $\rho >$
$\therefore\ \omega = 0$
Radiation
$P = \frac{1}{3}\rho$
$\therefore\ \omega = \frac{1}{3}$

$$T_{\mu\nu} = (\partial_\mu \phi)(\partial_\nu \phi) - g_{\mu\nu}\left[\frac{1}{2}(\partial_\alpha \phi)(\partial^\alpha \phi) - V(\phi)\right]$$

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p\, g_{\mu\nu}$$

$$u_\mu = \delta^0_\mu$$

$$T_{\mu\nu} = (\rho + p)\, \delta^0_\mu \delta^0_\nu - p\, g_{\mu\nu}$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\dot{\phi} \neq 0$$

$$\frac{\partial \phi}{\partial x^a} = 0$$

$$T_{\mu\nu;\mu} = 0$$

Dust.

$p = 0$; $\rho >$

$\therefore\ \omega = 0$

Radiation

$p = \frac{1}{3}\rho$

$\therefore\ \omega = \frac{1}{3}$

$$T_{\mu\nu} = (\partial_\mu \phi)(\partial_\nu \phi) - g_{\mu\nu}\left[\frac{1}{2}(\partial_\alpha \phi)(\partial^\alpha \phi) - V(\phi)\right]$$

$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu - P g_{\mu\nu}$$

$$u_\mu = \delta^0_\mu$$

$$T_{\mu\nu} = (\rho + P)\, \delta^0_\mu \delta^0_\nu - P g_{\mu\nu}$$

$$\dot\phi \neq 0$$

$$\frac{\partial \phi}{\partial x^a} = 0$$

$$\rho_\phi = \frac{1}{2}\dot\phi^2 + V(\phi)$$

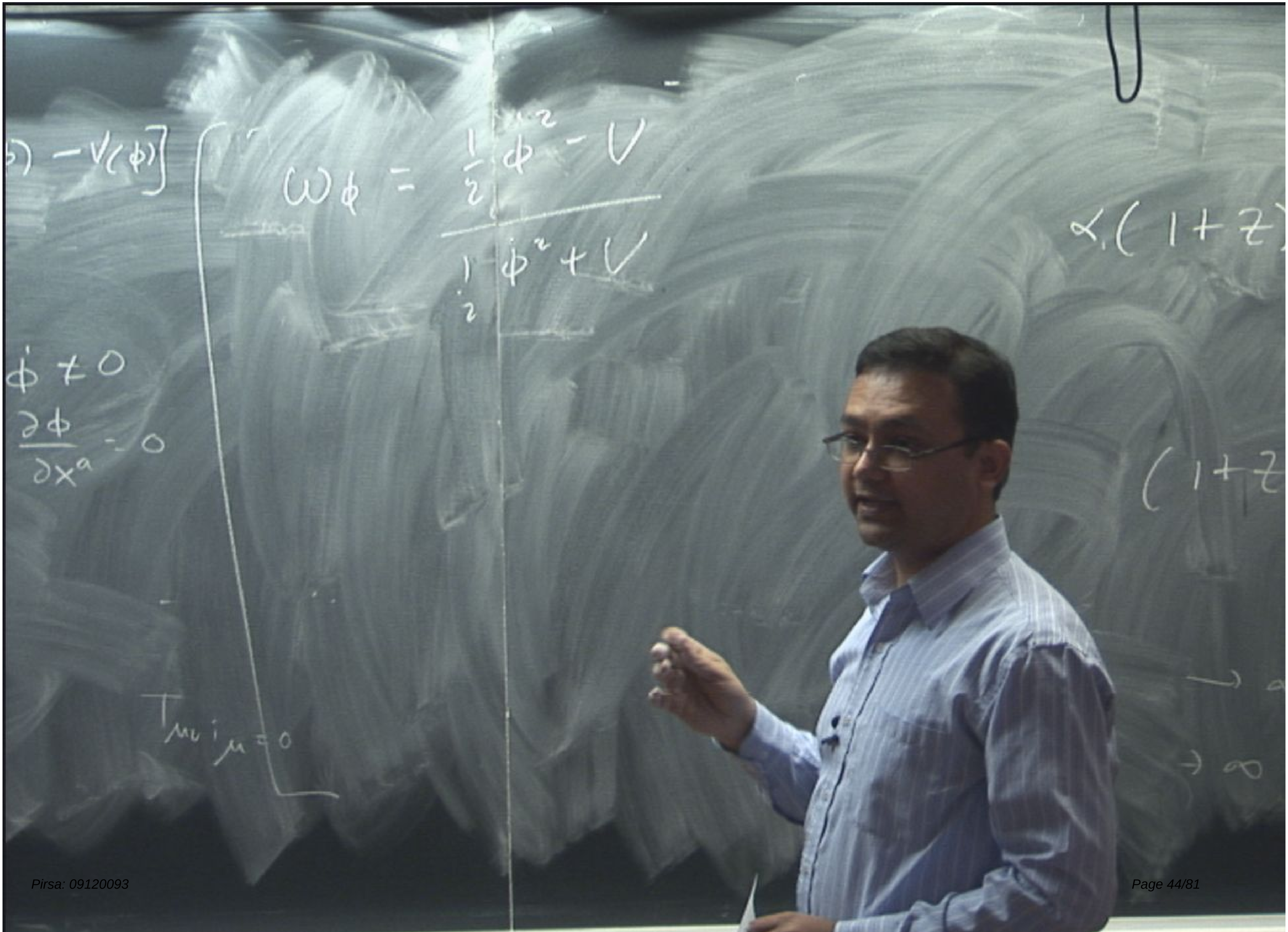
$$P_\phi = \frac{1}{2}\dot\phi^2 - V(\phi)$$

Dust:

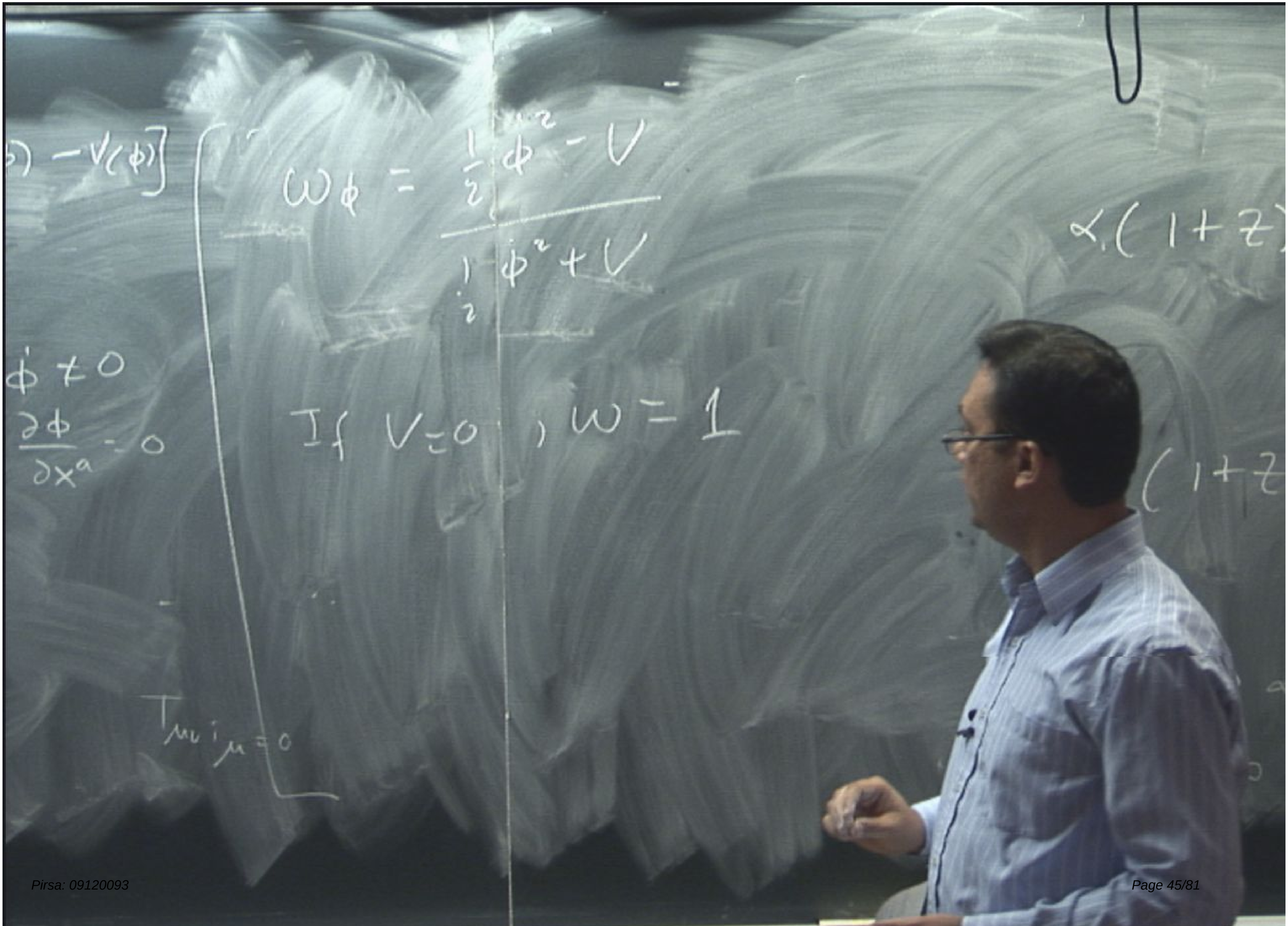
$P = 0 \; ; \; \rho >$

$= 0$

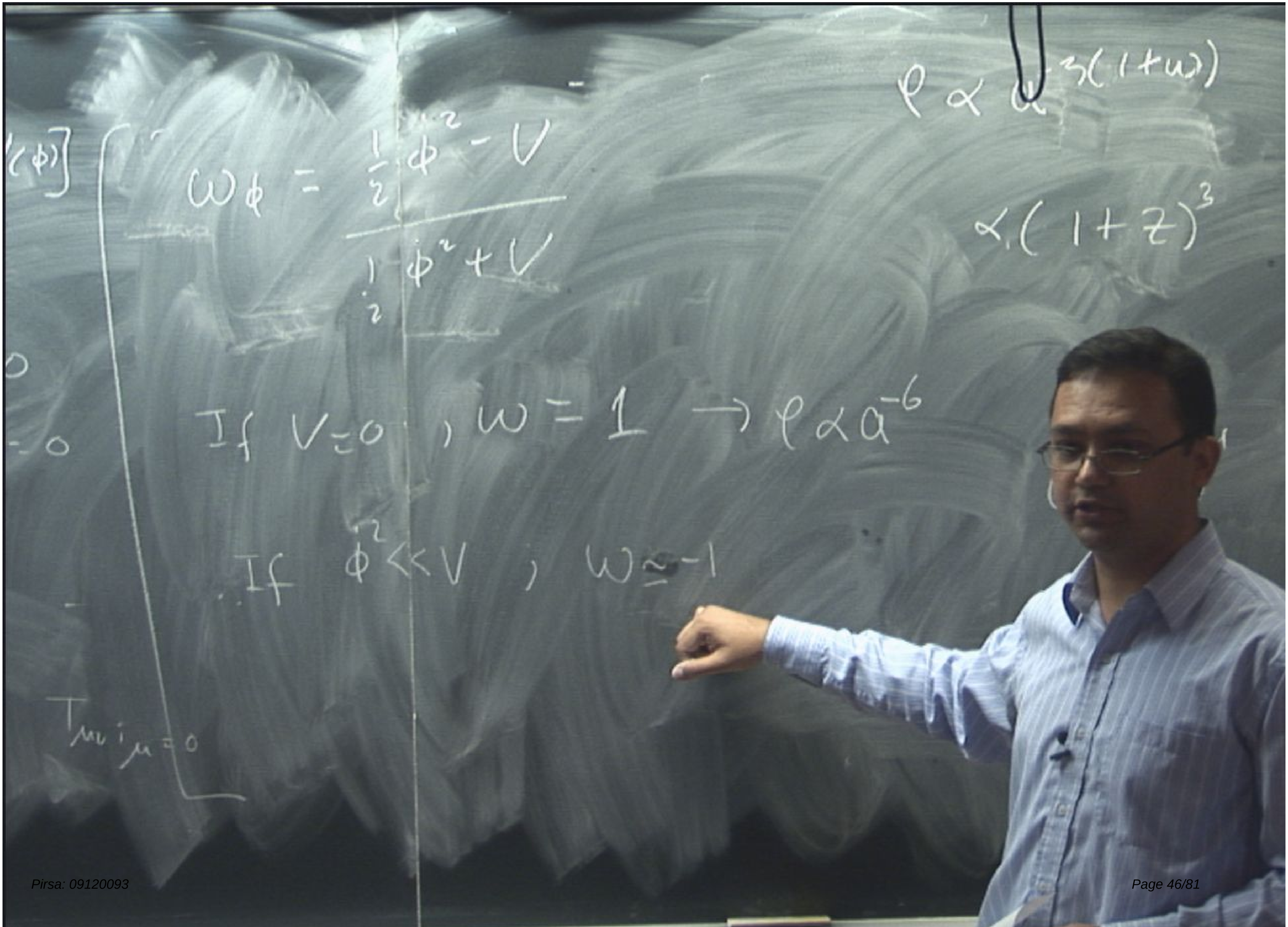
$$T_{\mu\nu\,;\,\mu} = 0$$



$\omega_\phi = \frac{\frac{1}{2}\dot\phi^2 - V}{\frac{1}{2}\dot\phi^2 + V}$
$-V(\phi)]$
$\dot\phi \neq 0$
$\frac{\partial\phi}{\partial x^a} = 0$
$T_{\mu\nu} i_\mu = 0$
$\propto (1+z$
$(1+z$
$\rightarrow \infty$



$-V(\phi)]$
$\omega_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}$
$\propto (1+z)$
$\dot{\phi} \neq 0$
$\frac{\partial \phi}{\partial x^a} = 0$
If $V = 0$, $\omega = 1$
$(1+z$
$T_{\mu\nu} i_\mu = 0$


ρ ∝ a^-3(1+w)
ω_φ = (1/2 φ̇² − V) / (1/2 φ̇² + V)
∝ (1+z)^3
If V=0 , ω = 1 → ρ ∝ a^-6
If φ̇² << V ; ω ≃ -1
T_μν ; μ = 0

$V(\phi)]$

$$\rho \propto a^{-3(1+w)}$$

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}$$

$$\propto (1+z)^3$$

$= 0$

If $V = 0$, $w = 1 \to \rho \propto a^{-6}$

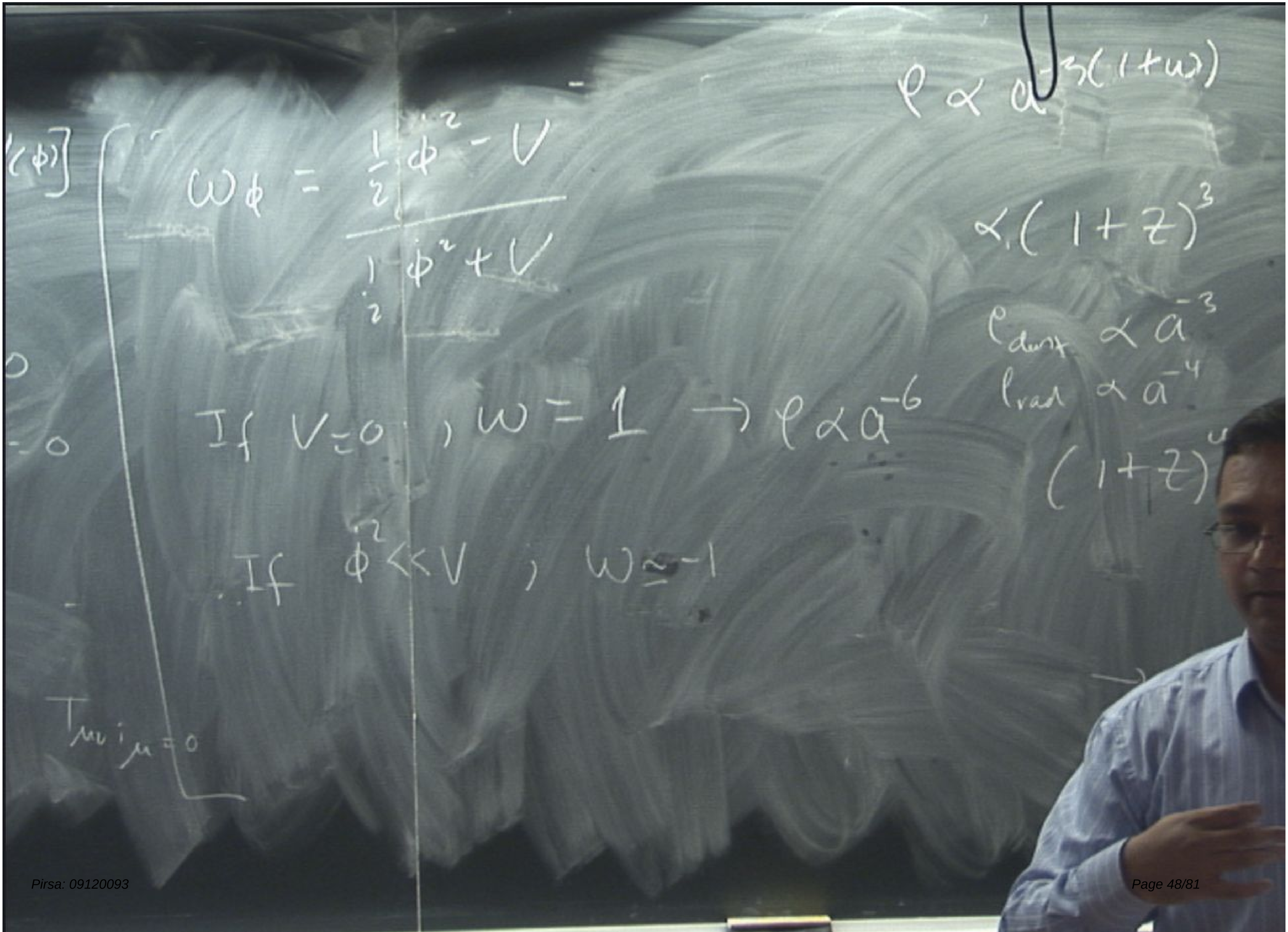
$$(1+z)^4$$

If $\dot{\phi}^2 \ll V$; $w \simeq -1$

$\to \infty$

$\to \infty$

$T_{\mu\nu;\mu} = 0$


ρ ∝ a^{-3(1+w)}
ω_φ = (½ φ̇² − V) / (½ φ̇² + V)
∝ (1+z)³
ρ_dust ∝ a^{-3}
ρ_rad ∝ a^{-4}
(1+z)
If V=0 , ω = 1 → ρ ∝ a^{-6}
If φ̇² << V ; ω ≃ −1
T_{μν ; μ} = 0

$$\rho \propto a^{-3(1+w)}$$

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}$$

$$\propto (1+z)^3$$

$$\rho_{dust} \propto a^{-3}$$

$$\rho_{rad} \propto a^{-4}$$

$$(1+z)^4$$

If $V=0$, $w = 1 \rightarrow \rho \propto a^{-6}$

If $\dot{\phi}^2 \ll V$; $w \simeq -1$

$\rightarrow \infty$

$\rightarrow \infty$

$T_{\mu\nu;\mu} = 0$

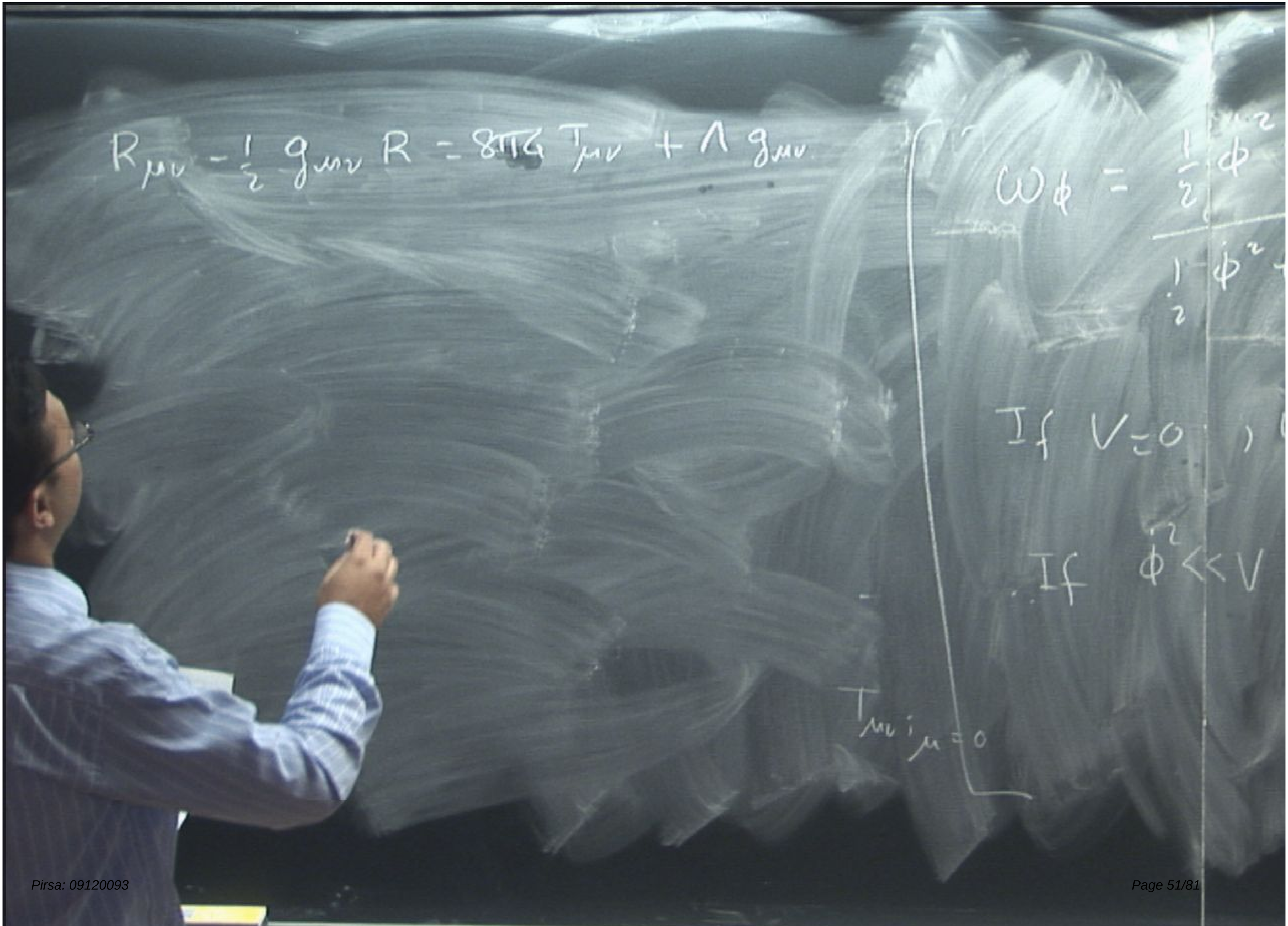
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

$$T_{\mu\nu ; \mu} = 0$$

$$\omega_\phi = \frac{\frac{1}{2}\dot{\phi}^2 \ldots}{\frac{1}{2}\dot{\phi}^2 \ldots}$$

If $V = 0$,

If $\dot{\phi}^2 \ll V$


$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$
$\omega_\phi = \frac{\frac{1}{2}\dot\phi^2}{\frac{1}{2}\dot\phi^2}$
If $V=0$,
If $\dot\phi^2 \ll V$
$T_{\mu\nu;\mu} = 0$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu} + \underbrace{\Lambda g_{\mu\nu}}_{T^{(v)}_{\mu\nu}}$$

$$T_{\mu\nu ; \mu} = 0$$

$$\omega_\phi = \frac{\frac{1}{2}\dot\phi^2 \ldots}{\frac{1}{2}\dot\phi^2 \ldots}$$

If $V = 0$, ...

If $\dot\phi^2 \ll V$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu} + \underbrace{\Lambda g_{\mu\nu}}_{T^{(\Lambda)}_{\mu\nu}}$$

$$T^{(\Lambda)}_{\mu\nu} \equiv \frac{\Lambda}{8\pi G} g_{\mu\nu}$$

$T_{\mu\nu;\mu} = 0$

$$\omega_\phi = \frac{\frac{1}{2}\dot\phi^2 \ldots}{\frac{1}{2}\dot\phi^2 + \ldots}$$

If $V = 0$, ...

If $\dot\phi^2 \ll V$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$$\underbrace{\Lambda g_{\mu\nu}}_{T^{(\Lambda)}_{\mu\nu}}$$

$$\boxed{T^{(\Lambda)}_{\mu\nu} \equiv \frac{\Lambda}{8\pi G} g_{\mu\nu}}$$

$$T_{\mu\nu} = (\rho + P)\,\delta^0_\mu \delta^0_\nu - P\, g_{\mu\nu}$$

$$P = -\rho \quad ; \; \omega = -1$$

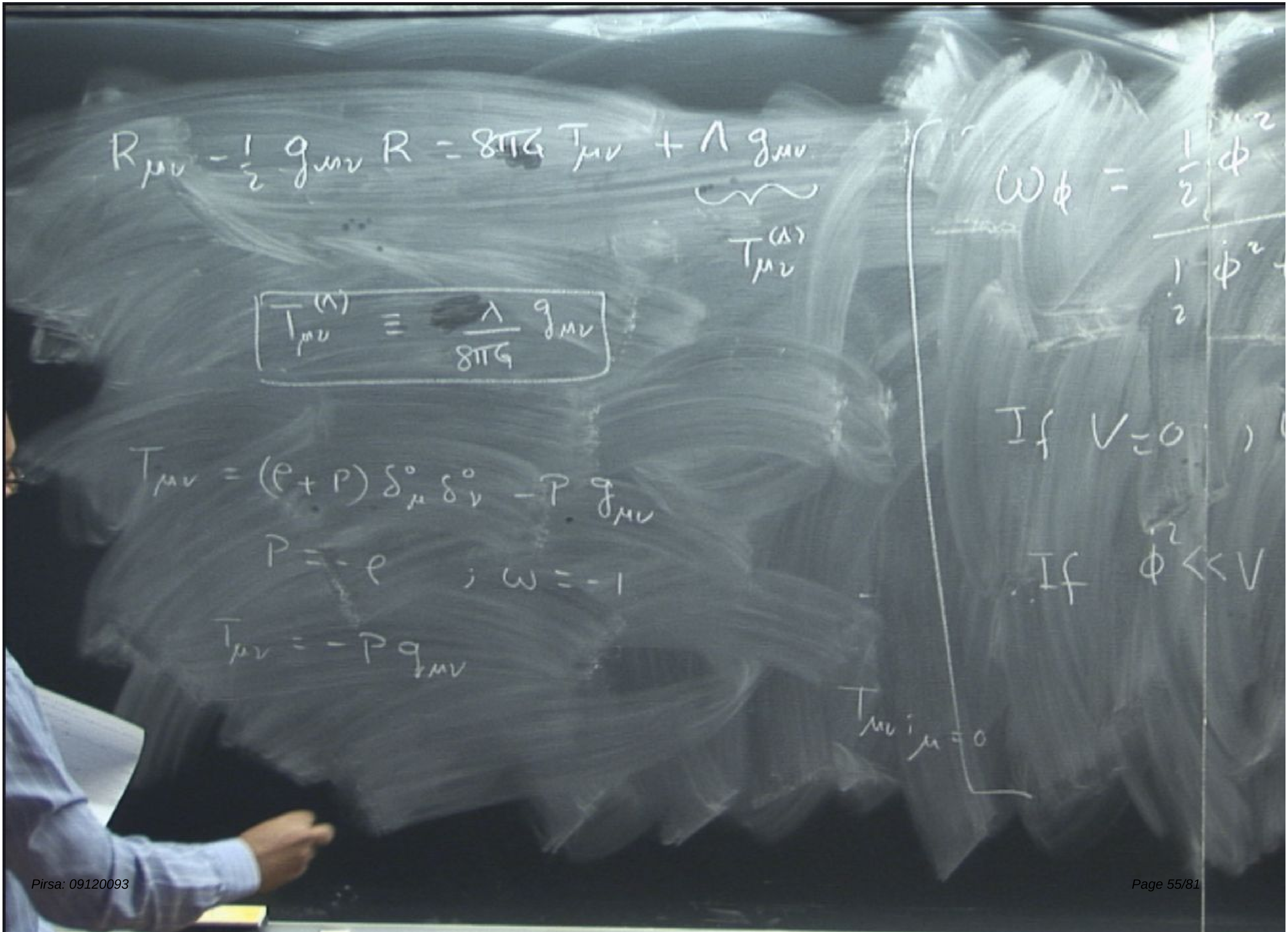
$$T_{\mu\nu} = -P\, g_{\mu\nu}$$

$$T_{\mu\nu\,;\,\mu} = 0$$

$$\omega_\phi = \frac{\frac{1}{2}\dot{\phi}^2 \ldots}{\frac{1}{2}\dot{\phi}^2 + \ldots}$$

If $V = 0$,

If $\dot{\phi}^2 << V$


$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu} + \Lambda g_{\mu\nu}$
$T^{(\Lambda)}_{\mu\nu}$
$T^{(\Lambda)}_{\mu\nu} \equiv \frac{\Lambda}{8\pi G} g_{\mu\nu}$
$T_{\mu\nu} = (\rho + P)\, \delta^0_\mu \delta^0_\nu - P g_{\mu\nu}$
$P = -\rho \; ; \; \omega = -1$
$T_{\mu\nu} = -P g_{\mu\nu}$
$T_{\mu\nu\,;\mu} = 0$
$\omega_\phi = \frac{1}{2}\dot{\phi}^2$
$\frac{1}{2}\dot{\phi}^2$
If $V = 0$,
If $\dot{\phi}^2 << V$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu} + \underbrace{\Lambda g_{\mu\nu}}_{T^{(\Lambda)}_{\mu\nu}}$$

$$\boxed{T^{(\Lambda)}_{\mu\nu} \equiv \frac{\Lambda}{8\pi G} g_{\mu\nu}}$$

$$T_{\mu\nu} = (\rho + p)\,\delta^0_\mu \delta^0_\nu - p\, g_{\mu\nu}$$

$$p = -\rho \quad ; \quad \omega = -1$$

$$T_{\mu\nu} = -p\, g_{\mu\nu} = \rho\, g_{\mu\nu}$$

$$T_{\mu\nu\,;\mu} = 0$$

$$\omega_\phi = \frac{\frac{1}{2}\dot\phi^2 \ldots}{\frac{1}{2}\dot\phi^2 \ldots}$$

If $V = 0$,

If $\dot\phi^2 \ll V$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu} + \underbrace{\Lambda g_{\mu\nu}}_{T^{(\Lambda)}_{\mu\nu}}$$

$$\boxed{T^{(\Lambda)}_{\mu\nu} \equiv \frac{\Lambda}{8\pi G} g_{\mu\nu}}$$

$$T_{\mu\nu} = (\rho + p)\,\delta^0_\mu \delta^0_\nu - p\, g_{\mu\nu}$$

$$p = -\rho \quad ; \quad w = -1$$

$$T_{\mu\nu} = -p\, g_{\mu\nu} = \rho\, g_{\mu\nu}$$

$$T_{\mu\nu\,;\,\mu} = 0$$

$$w_\phi = \frac{\frac{1}{2}\dot\phi^2 \ldots}{\frac{1}{2}\dot\phi^2 + \ldots}$$

If $V = 0$, …

If $\dot\phi^2 \ll V$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} + \underbrace{\Lambda g_{\mu\nu}}_{T^{(\Lambda)}_{\mu\nu}}$$

$$\boxed{T^{(\Lambda)}_{\mu\nu} \equiv \frac{\Lambda}{8\pi G} g_{\mu\nu}}$$

$$T_{\mu\nu} = (\rho + P)\delta^0_\mu \delta^0_\nu - P g_{\mu\nu}$$

$$P = -\rho \quad ; \omega = -1$$

$$T_{\mu\nu} = -P g_{\mu\nu} = \rho g_{\mu\nu}$$

$$\boxed{\rho_\Lambda = \frac{\Lambda}{8\pi G}}$$

$$T_{\mu\nu;\mu} = 0$$

$$\omega_\phi = \frac{\frac{1}{2}\dot\phi^2 \ldots}{\frac{1}{2}\dot\phi^2 + \ldots}$$

If $V = 0$, ...

If $\dot\phi^2 \ll V$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu} + \underbrace{\Lambda g_{\mu\nu}}_{T^{(\Lambda)}_{\mu\nu}}$$

$$\boxed{T^{(\Lambda)}_{\mu\nu} \equiv \frac{\Lambda}{8\pi G} g_{\mu\nu}}$$

$$T_{\mu\nu} = (\rho + P)\,\delta^0_\mu \delta^0_\nu - P g_{\mu\nu}$$

$$P = -\rho \quad ; \quad \omega = -1$$

$$T_{\mu\nu} = -P g_{\mu\nu} = \rho\, g_{\mu\nu}$$

$$\boxed{\rho_\Lambda = \frac{\Lambda}{8\pi G}}$$

$$T_{\mu\nu\,;\mu} = 0$$

$$\omega_\phi = \frac{\frac{1}{2}\dot\phi^2 \ldots}{\frac{1}{2}\dot\phi^2 + \ldots}$$

If $V = 0$, ...

If $\dot\phi^2 \ll V$

Energy Conditions.

$u^i{}_\mu = 0$

Energy Conditions.

$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu - P\, g_{\mu\nu}.$$

i) Strong Energy

$u^i{}_\mu = 0$

*

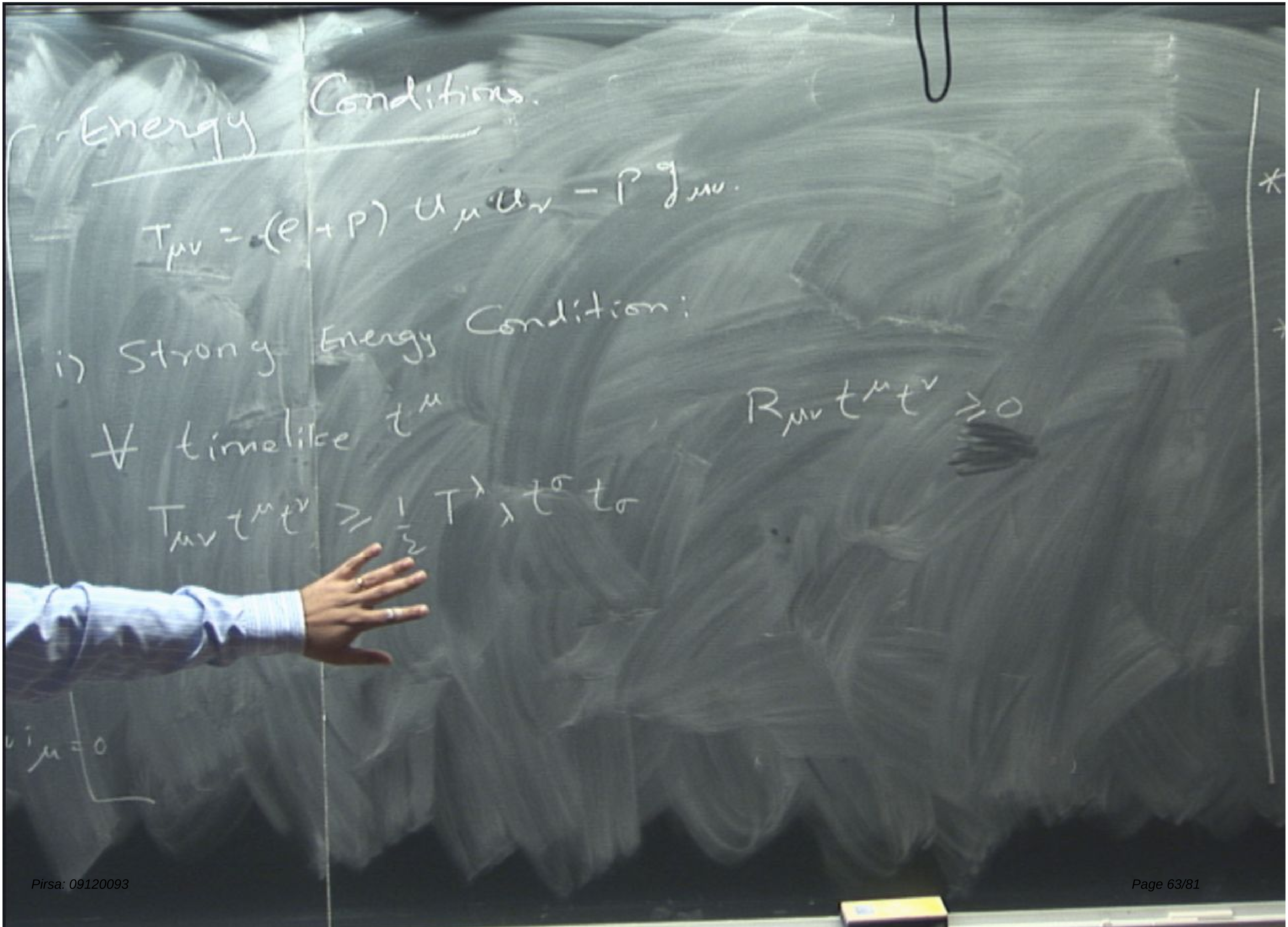
*

Energy Conditions.

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p\, g_{\mu\nu}.$$

i) Strong Energy Condition:

$u^i{}_\mu = 0$


Energy Conditions.
Tμν = (ρ + p) uμ uν − p gμν.
i) Strong Energy Condition:
∀ timelike t^μ
Tμν t^μ t^ν ≥ 1/2 T^λ_λ t^σ tσ
Rμν t^μ t^ν ≥ 0

Energy Conditions.

$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu - P\, g_{\mu\nu}.$$

Energy Condition:

i) Strong Energy Condition:

$\forall$ timelike t^μ, $\quad R_{\mu\nu} t^\mu t^\nu \geq 0$

$$T_{\mu\nu} t^\mu t^\nu \geq \frac{1}{2} T^\lambda{}_\lambda t^\sigma t_\sigma$$

$$\Rightarrow \boxed{\rho + P \geq 0 \,, \quad \rho + 3P \geq 0}$$

Energy Conditions.

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p\, g_{\mu\nu}.$$

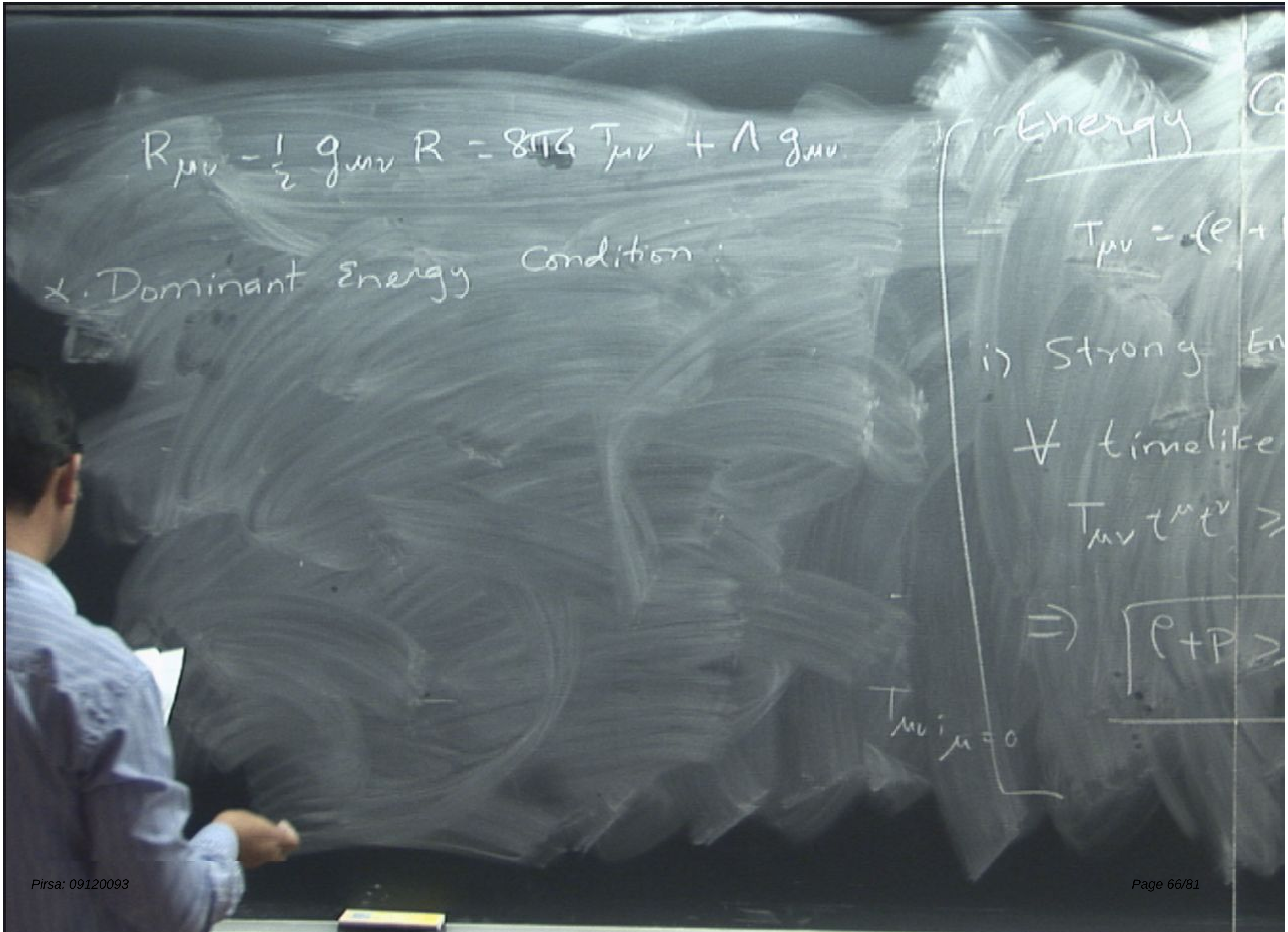
i) Strong Energy Condition:

$\forall$ timelike t^μ $\qquad R_{\mu\nu} t^\mu t^\nu \geq 0$

$$T_{\mu\nu} t^\mu t^\nu \geq \frac{1}{2} T^\lambda{}_\lambda t^\sigma t_\sigma$$

$$\Rightarrow \boxed{\rho + P \geq 0 \;, \; \rho + 3P \geq 0}$$

$u^i{}_\mu = 0$


$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$
x. Dominant Energy Condition:
Energy
$T_{\mu\nu} = (\rho +$
i) Strong
∀ timelike
$T_{\mu\nu} t^\mu t^\nu \geq$
$\Rightarrow \rho + P \geq$
$T_{\mu\nu\,;\mu} = 0$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

x. Dominant Energy Condition:

$\forall\, t^\mu$; $T_{\mu\nu} t^\mu t^\nu \geq 0$ & $T_{\mu\nu} T^\nu{}_\lambda t^\mu t^\lambda \leq 0$

$$\boxed{\rho + p \geq 0\ ,\ \rho - p \geq 0}$$

$T_{\mu\nu;\mu} = 0$

Energy C...

$T_{\mu\nu} = (\rho + \ldots$

i) Strong En...

$\forall$ timelike

$T_{\mu\nu} t^\mu t^\nu \geq \ldots$

$\Rightarrow$ $\boxed{\rho + p \geq \ldots}$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu} + \Lambda g_{\mu\nu}$$

* Dominant Energy Condition:

$\forall\, t^\mu$; $T_{\mu\nu} t^\mu t^\nu \geq 0$ & $T_{\mu\nu} T^\nu{}_\lambda t^\mu t^\lambda \leq 0$

$\rho + p \geq 0 \;,\; \rho - p \geq 0$

* Weak energy condition

$T_{\mu\nu} t^\mu t^\nu \geq 0 \Rightarrow \rho \geq 0 \,,\, \rho + p \geq 0$

$T_{\mu\nu;\mu} = 0$

* Null Energy condition

$T_{\mu\nu} \ell^\mu \ell^\nu \geq 0 \quad \forall\, \ell_\mu \rightarrow \rho + p \geq 0$

Energy

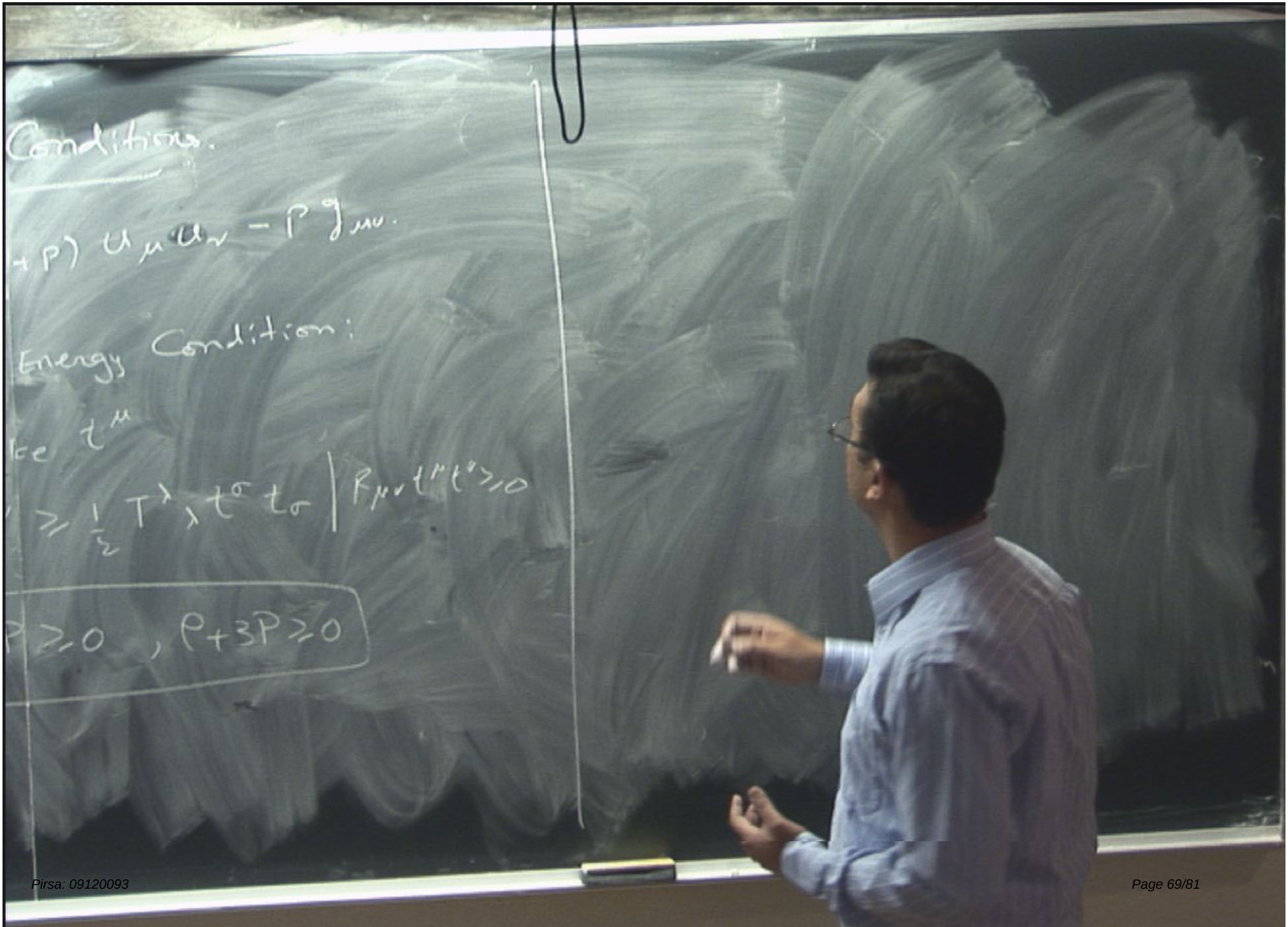
$T_{\mu\nu} = (\rho$

i) Strong

$\forall$ timeli[ke]

$T_{\mu\nu} t^\mu t^\nu$

$\Rightarrow \rho + p$


Conditions.
+p) u_μ u_ν − p g_μν.
Energy Condition:
ke t^μ
≥ ½ T^λ_λ t^σ t_σ
R_μν t^μ t^ν ≥ 0
P ≥ 0 , ρ+3P ≥ 0

Conditions:

$(\rho + P)\, u_\mu u_\nu - P\, g_{\mu\nu}$.

Energy Condition:

...ke t^μ

$\geq \frac{1}{2} T^\lambda{}_\lambda t^\sigma t_\sigma \quad | \quad R_{\mu\nu} t^\mu t^\nu \geq 0$

$P \geq 0 \quad , \quad \rho + 3P \geq 0$

Dust:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3P)$$

Conditions:

$(\rho + P)\, u_\mu u_\nu - P\, g_{\mu\nu}$.

Energy Condition:

...ke t^μ

$\ldots \geq \frac{1}{2} T^\lambda{}_\lambda t^\sigma t_\sigma \;\Big|\; R_{\mu\nu} t^\mu t^\nu \geq 0$

$P \geq 0 \;,\; \rho + 3P \geq 0$

Dust:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

If SEC is satisfied then $\ddot{a} < 0$

Conditions.

$(\rho + p)\, u_\mu u_\nu - p\, g_{\mu\nu}$.

Energy Condition:

ke t^μ

$\geq \frac{1}{2} T^\lambda{}_\lambda t^\sigma t_\sigma \;\Big|\; R_{\mu\nu} t^\mu t^\nu \geq 0$

$p \geq 0 \;, \; \rho + 3p \geq 0$

Dust:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p)$$

If SEC is satisfied then $\ddot{a} < 0$

$w = 0 \;, \; p = 0 \;, \; \rho > 0$

$\rho + 3p > 0$

Conditions:

$+p) u_\mu u_\nu - p\, g_{\mu\nu}.$

Energy Condition:

ke t^μ

$\geq \frac{1}{2} T^\lambda{}_\lambda t^\sigma t_\sigma \quad \Big| \quad R_{\mu\nu} t^\mu t^\nu \geq 0$

$p \geq 0 \quad , \quad \rho + 3P \geq 0$

Dust:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

If SEC is satisfied then $\ddot{a} < 0$

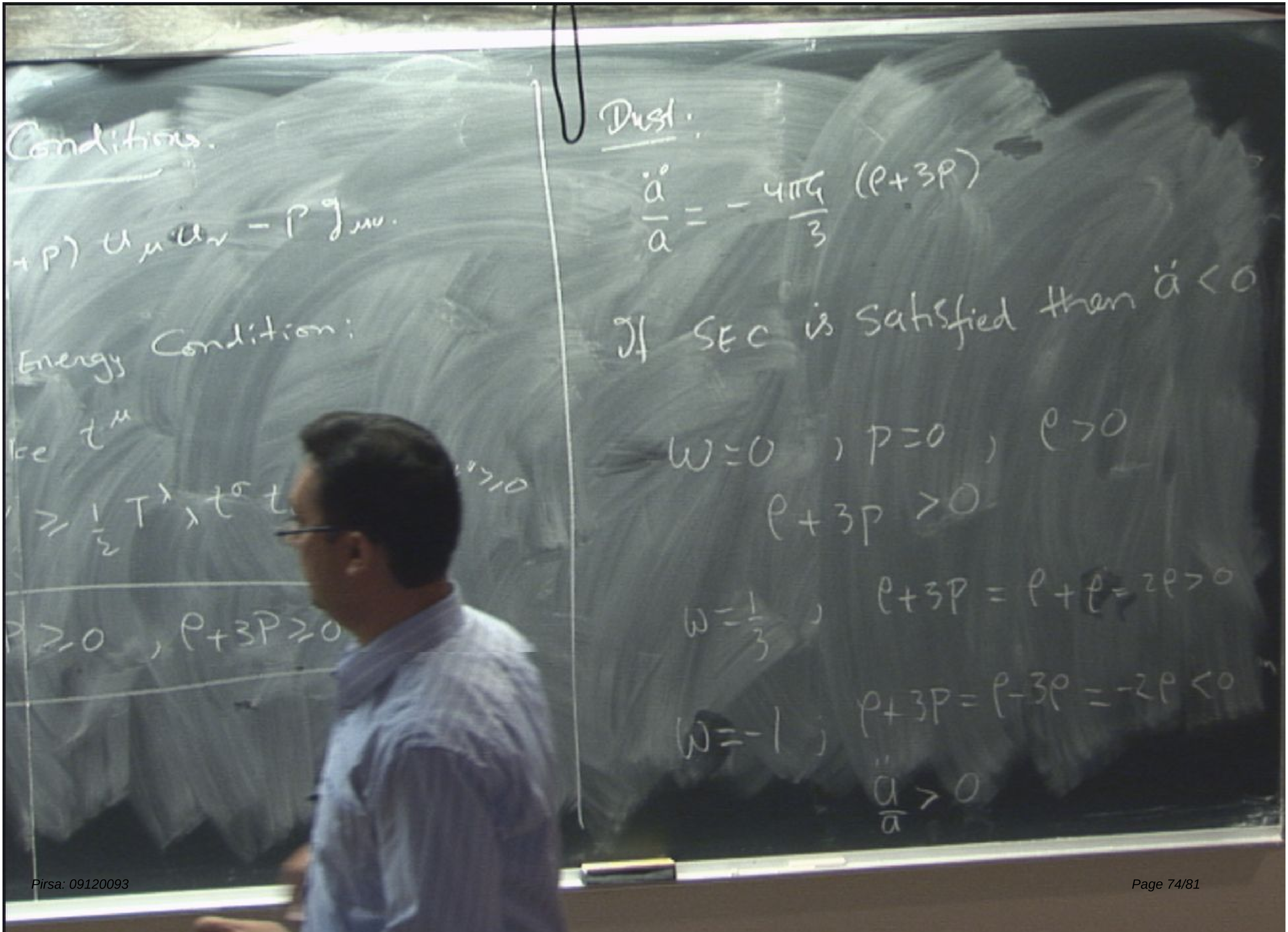
$w = 0 \ , \ P = 0 \ , \ \rho > 0$

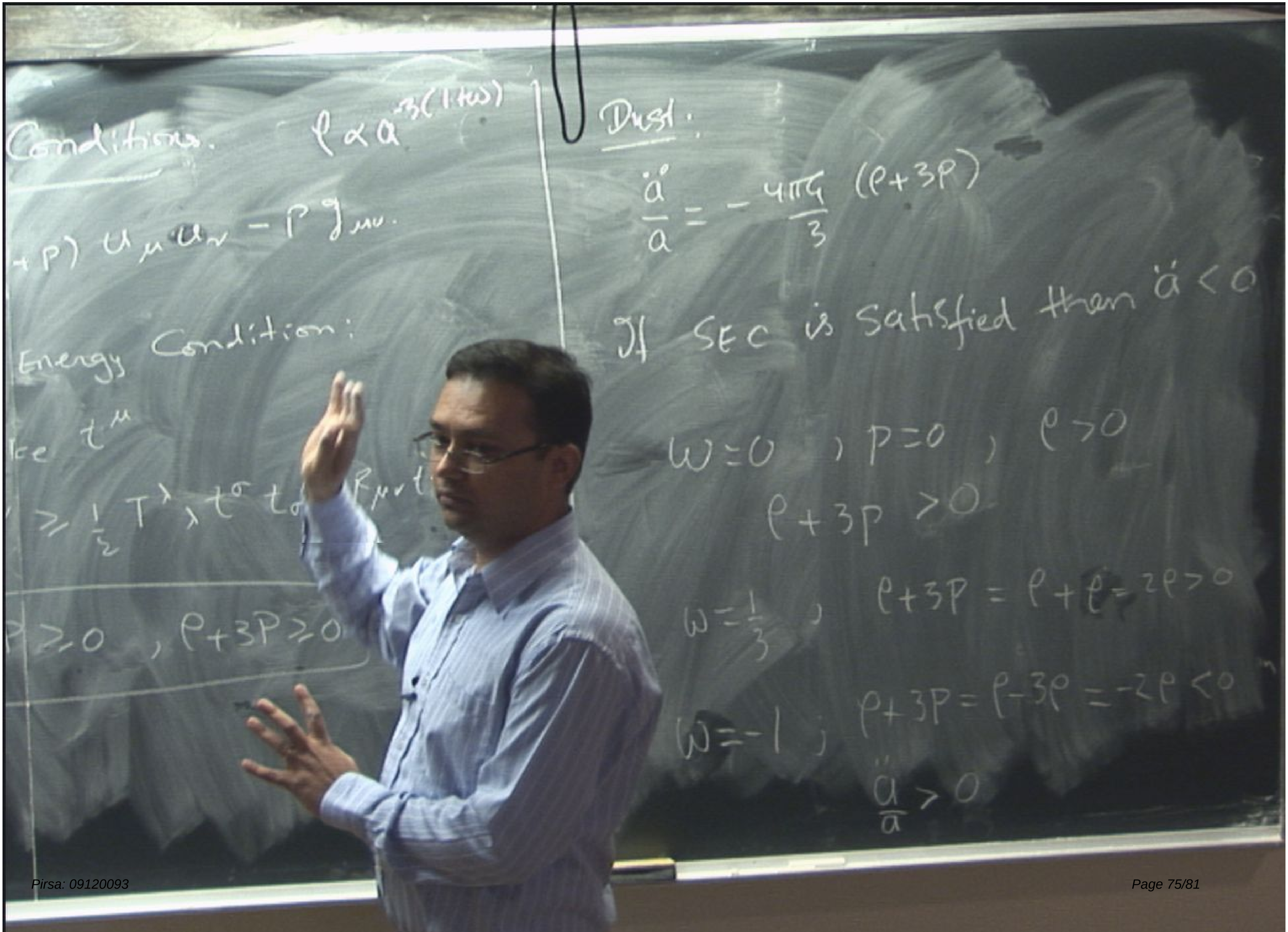
$\rho + 3P > 0$

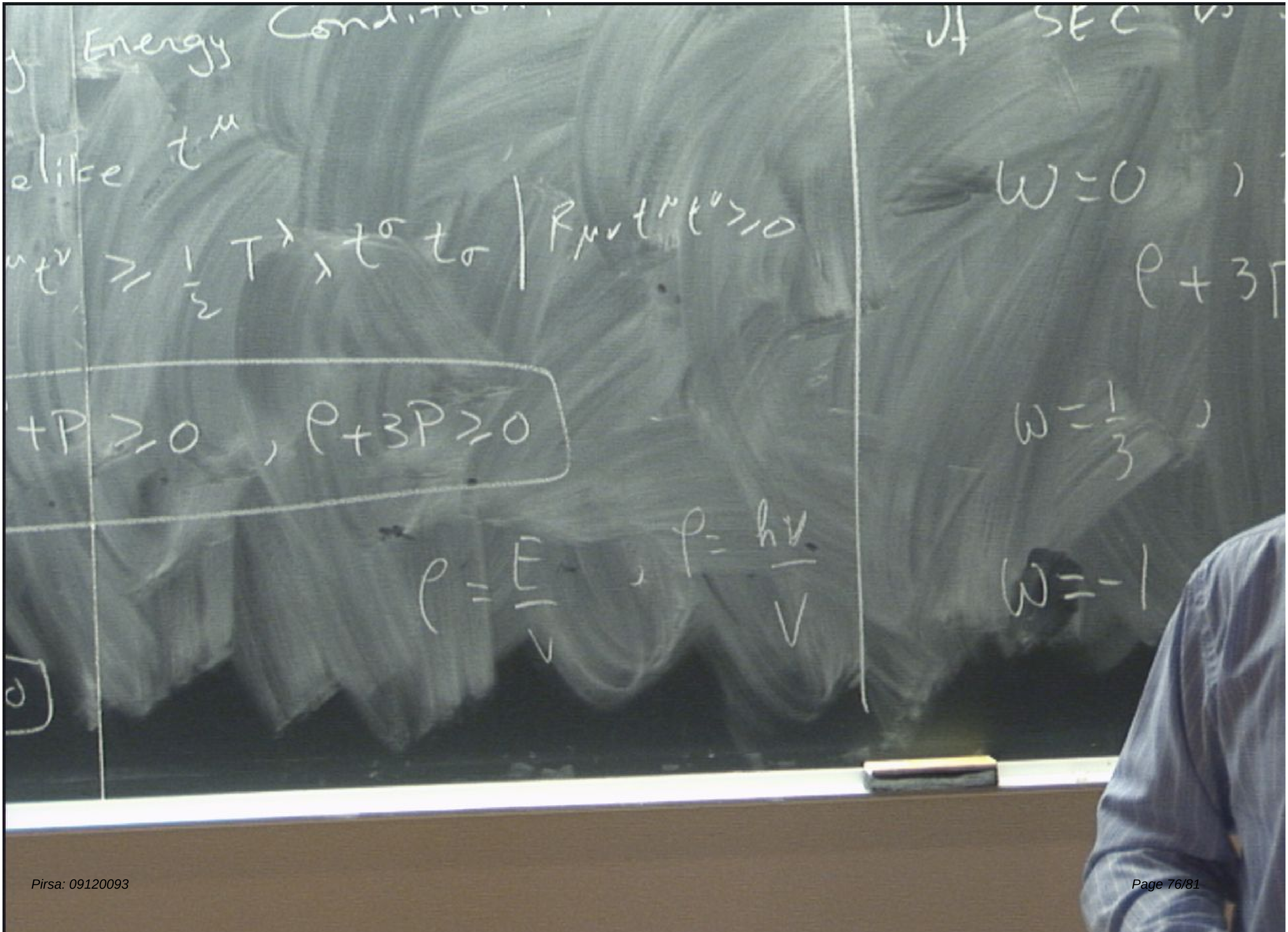
$w = \frac{1}{3} \ , \quad \rho + 3P = \rho + \rho = 2\rho > 0$

$w = -1 \ ; \ \rho + 3P = \rho - 3\rho = -2\rho < 0$

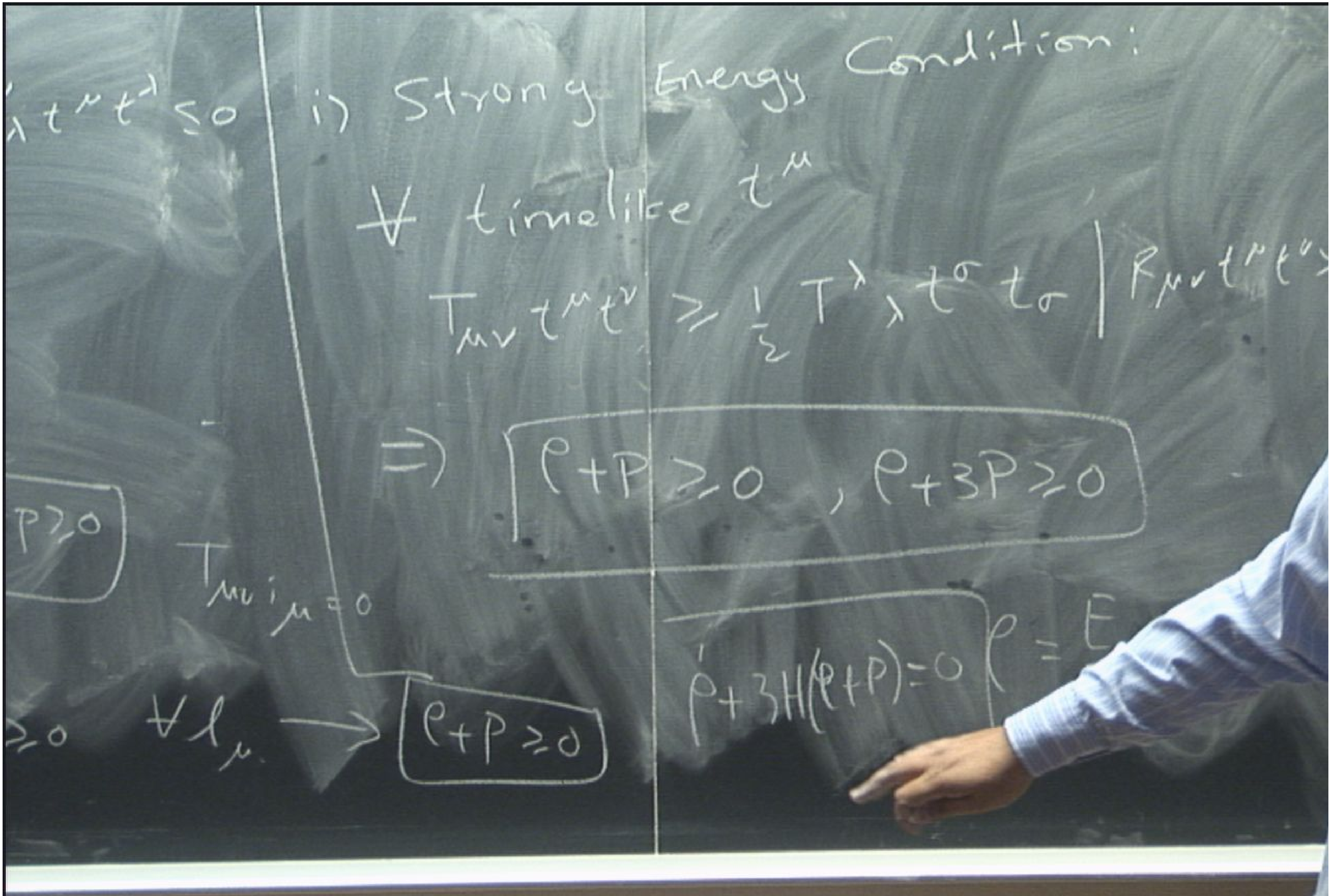
$\frac{\ddot{a}}{a} > 0$

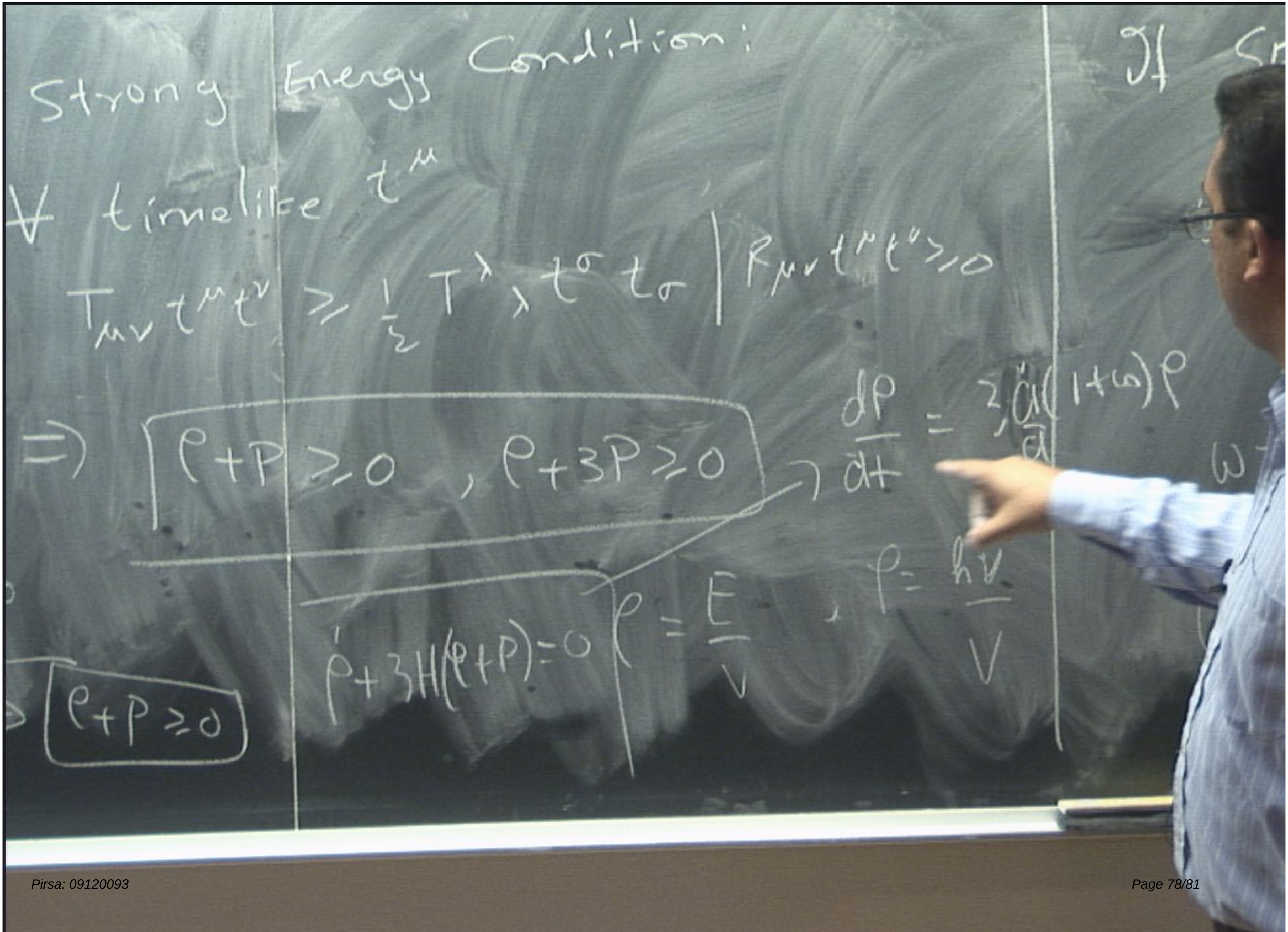

Conditions.
+ p) u_μ u_ν − p g_μν.
Energy Condition:
ke t^μ
≥ ½ T^λ_λ t^σ t
p ≥ 0 , ρ + 3P ≥ 0
Dust:
ä/a = − 4πG/3 (ρ + 3P)
If SEC is satisfied then ä < 0
w = 0 , P = 0 , ρ > 0
ρ + 3P > 0
w = 1/3 , ρ + 3P = ρ + ρ = 2ρ > 0
w = −1 , ρ + 3P = ρ − 3ρ = −2ρ < 0
ä/a > 0

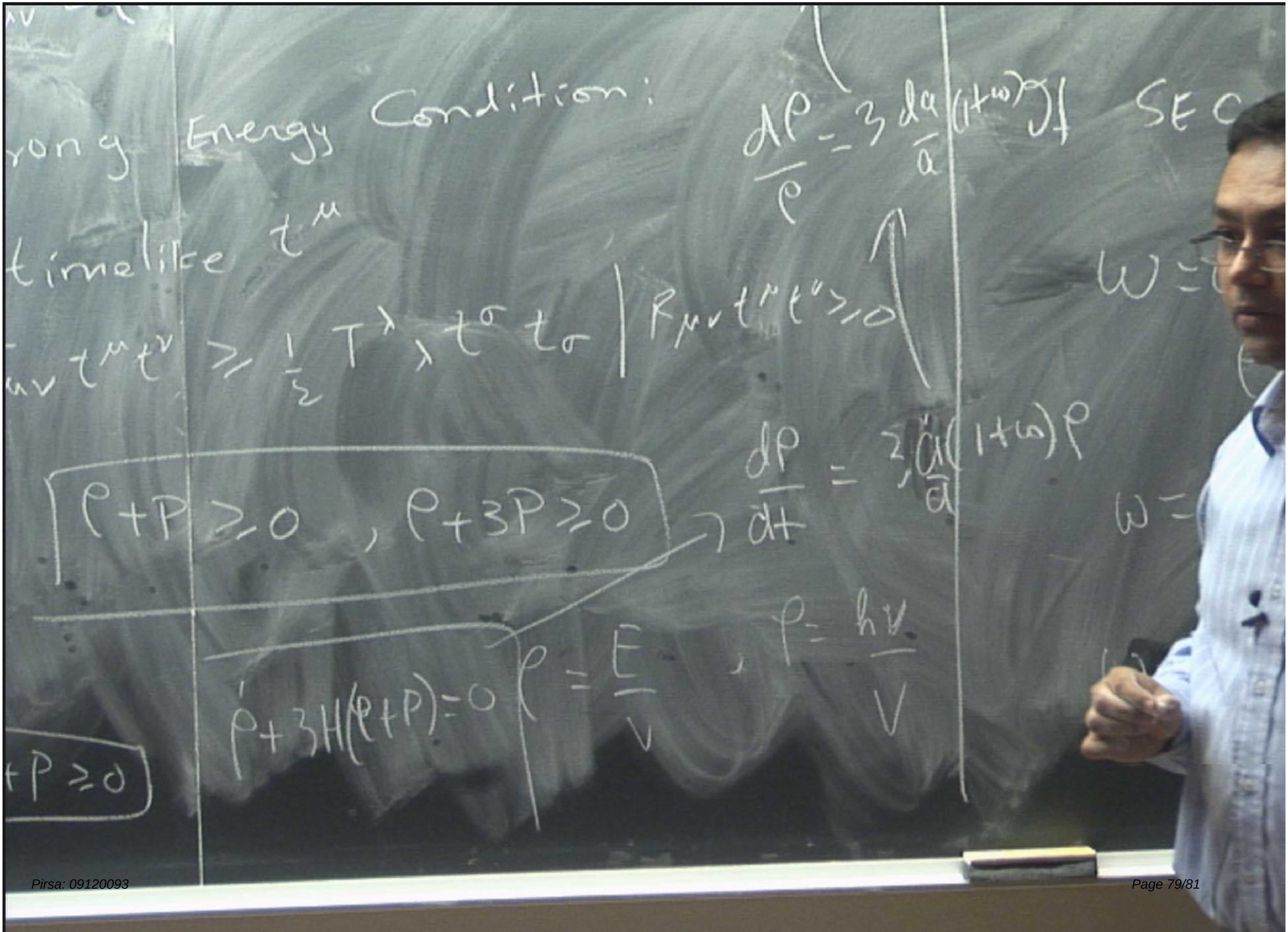

Conditions.
$\rho \propto a^{-3(1+\omega)}$
$+p) u_\mu u_\nu - p g_{\mu\nu}$.
Energy Condition:
t^μ
$\geq \frac{1}{2} T^\lambda_\lambda t^\sigma t_\sigma$
≥ 0 , $\rho + 3P \geq 0$
Dust:
$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$
If SEC is satisfied then $\ddot{a} < 0$
$\omega = 0$, $P = 0$, $\rho > 0$
$\rho + 3P > 0$
$\omega = \frac{1}{3}$, $\rho + 3P = \rho + \rho = 2\rho > 0$
$\omega = -1$; $\rho + 3P = \rho - 3\rho = -2\rho < 0$
$\frac{\ddot{a}}{a} > 0$

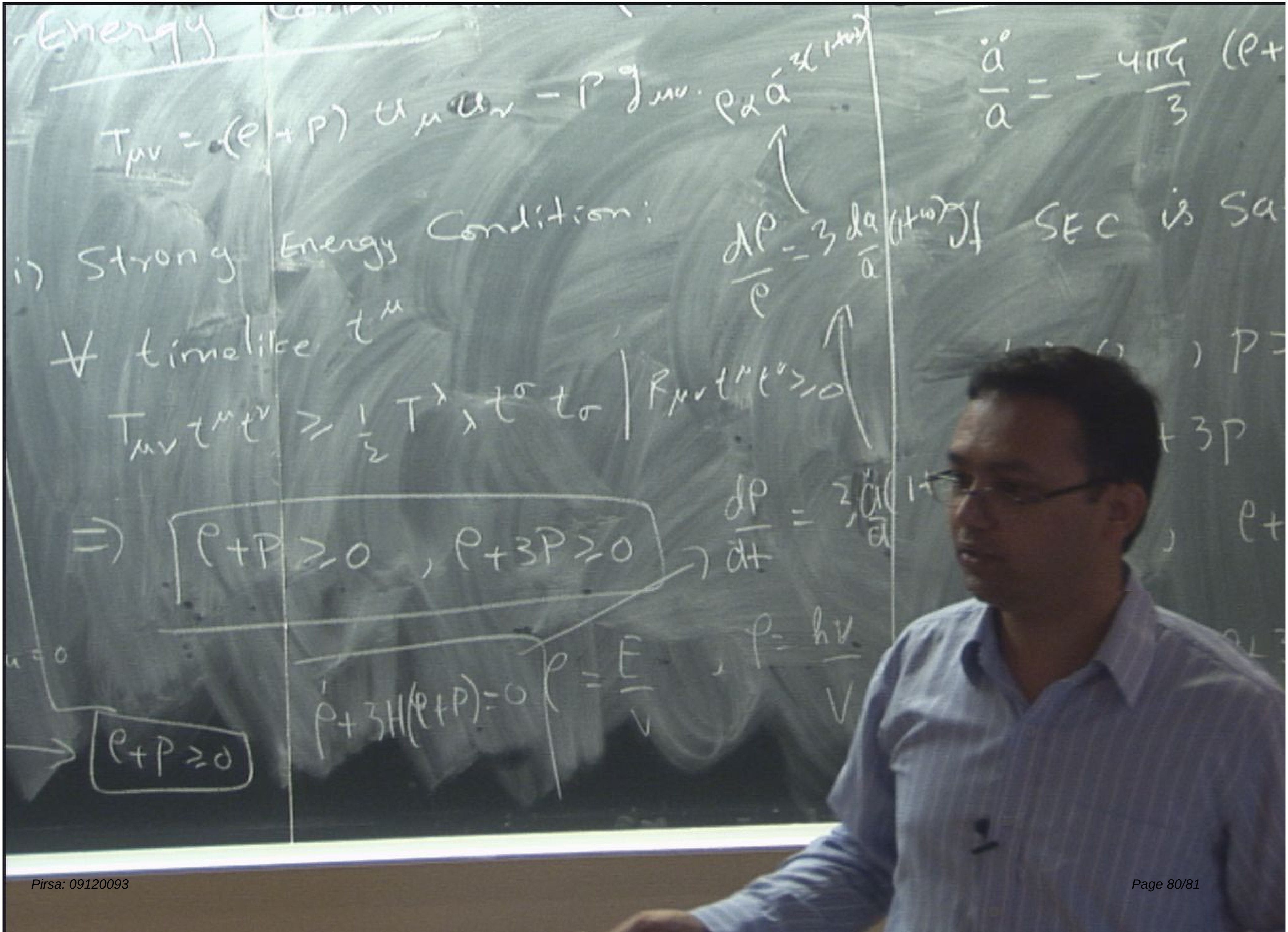


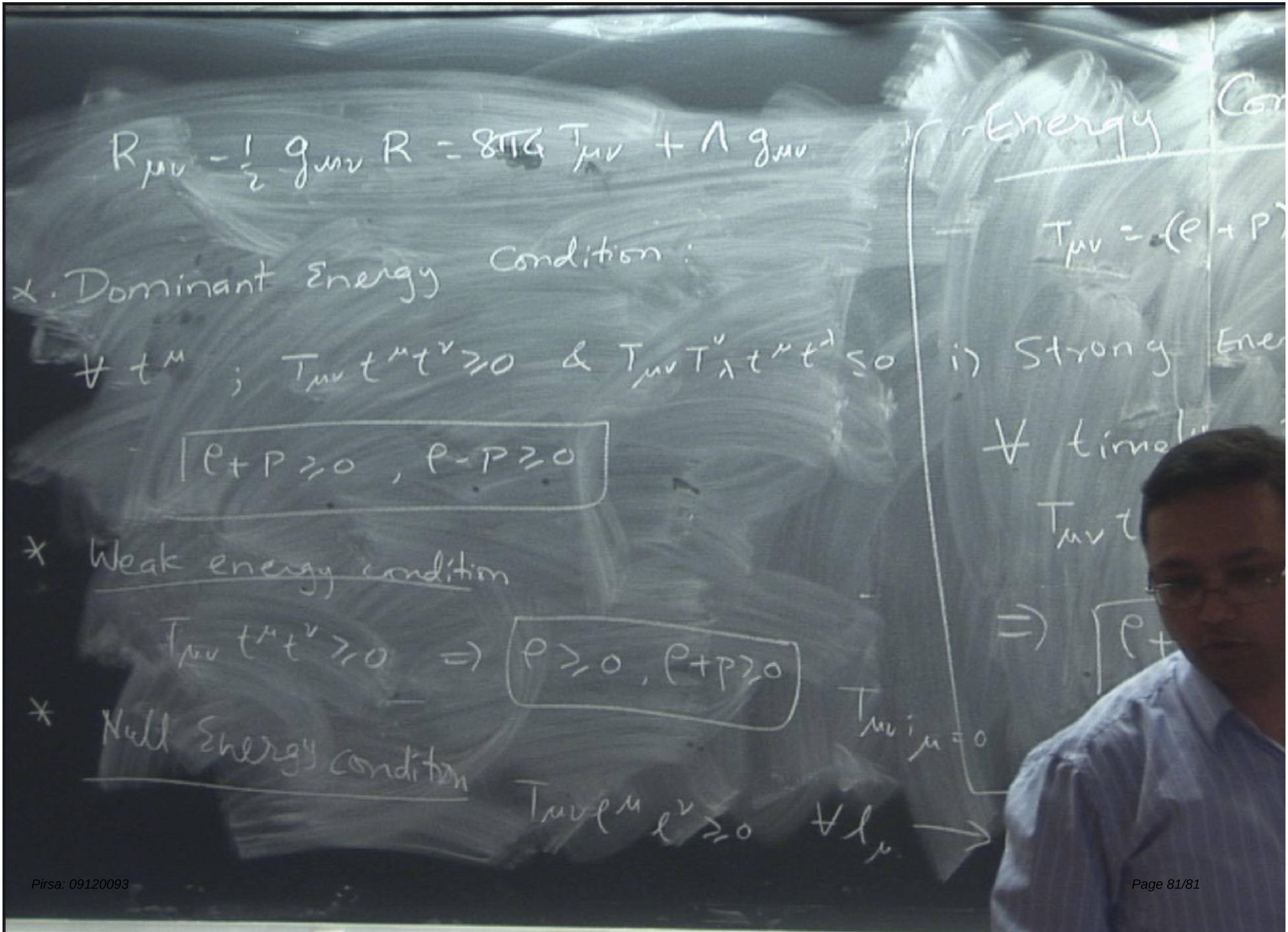
Energy Condition
elike t^μ
$t^\nu \geq \frac{1}{2} T^\lambda{}_\lambda t^\sigma t_\sigma$
$R_{\mu\nu} t^\mu t^\nu \geq 0$
$+P \geq 0 \quad , \rho + 3P \geq 0$
$\rho = \frac{E}{V} \quad , \rho = \frac{h\nu}{V}$
SEC
$\omega = 0$
$\rho + 3P$
$\omega = \frac{1}{3}$
$\omega = -1$


is Strong Energy Condition:
∀ timelike t^μ
$T_{\mu\nu} t^\mu t^\nu \geq \frac{1}{2} T^\lambda{}_\lambda t^\sigma t_\sigma$
$\Rightarrow \rho + P \geq 0 \ , \ \rho + 3P \geq 0$
$\dot{\rho} + 3H(\rho + P) = 0$
$\rho + P \geq 0$
$\forall l_\mu$


Strong Energy Condition:
∀ timelike t^μ
T_μν t^μ t^ν ≥ 1/2 T^λ_λ t^σ t_σ
R_μν t^μ t^ν ≥ 0
⇒ ρ+P ≥ 0 , ρ+3P ≥ 0
dρ/dt
ρ̇+3H(ρ+P)=0
ρ = E/V
ρ = hν/V
ρ+P ≥ 0


Energy Condition:
timelike t^μ
$T_{\mu\nu} t^\mu t^\nu \geq \frac{1}{2} T^\lambda{}_\lambda t^\sigma t_\sigma$
$R_{\mu\nu} t^\mu t^\nu \geq 0$
$\rho + P \geq 0$, $\rho + 3P \geq 0$
$\dot{\rho} + 3H(\rho + P) = 0$
$\rho = \frac{E}{V}$, $\rho = \frac{h\nu}{V}$
SEC


Energy
Strong Energy Condition:
∀ timelike
SEC is sa


R_μν - ½ g_μν R = 8πG T_μν + Λ g_μν
* Dominant Energy Condition :
∀ t^μ ; T_μν t^μ t^ν ≥ 0 & T_μν T^ν_λ t^μ t^λ ≤ 0
ρ + p ≥ 0 , ρ - p ≥ 0
* Weak energy condition
T_μν t^μ t^ν ≥ 0 ⇒ ρ ≥ 0 , ρ + p ≥ 0
* Null Energy condition
T_μν ℓ^μ ℓ^ν ≥ 0 ∀ ℓ_μ
T_μν ;_μ = 0
Energy Con
T_μν = (ρ + p)
ii) Strong Ener
∀ timel
T_μν t
⇒ ρ +