Title: Foundations of Quantum Mech. (PHYS 639) - Lecture 13

Date: Dec 16, 2009 11:00 AM

URL: http://pirsa.org/09120090

Abstract:

"QBism" – the quantum Bayesian program of

C. M. Caves

R. Schack

D. M. Appleby

myself

See arXiv.org.

See also:

C. G. Timpson, "Quantum Bayesianism: A Study"

and pirsa.org/09080010

My Favorite Convex Set

(My Favorite Shape)

Christopher Fuchs
P̂I – Perimeter Inst.

Work with:

Marcus Appleby
Åsa Ericsson
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Jim Hartle 1968 *(Section IV)* Interpretation of Quantum Mechanics *(suitably modified)*

Am. J. Phys. 36, 704–712 (1968)

A quantum state, being a summary of the observers' information about an individual physical system, changes both by dynamical laws and whenever the observer acquires new information about the system through the process of measurement. The existence of two laws for the evolution of the state vector becomes problematical only if it is believed that the state vector is an objective property of the system. If the state of a system is defined as a list of [*experimental*] propositions together with [*their probabilities of occurrence*], it is not surprising that after a measurement the state must be changed to be in accord with the new information. The "reduction of the wave packet" does take place in the consciousness of the observer, not because of any unique physical process which takes place there, but only because the state is a construct of the observer and not an objective property of the physical system.

The hypothesis that there is an external world, not dependent on human minds, made of something, is so obviously useful and so strongly confirmed by experience down through the ages that we can say without exaggerating that it is better confirmed than any other empirical hypothesis.

— Martin Gardner

A Single-User Theory

- probability theory
- quantum theory

"The Bayesian, subjectivist, or coherent, paradigm is egocentric. It is a tale of one person contemplating the world and not wishing to be stupid (technically incoherent). He realizes that to do this his statements of uncertainties must be probabilistic."

— D. V. Lindley

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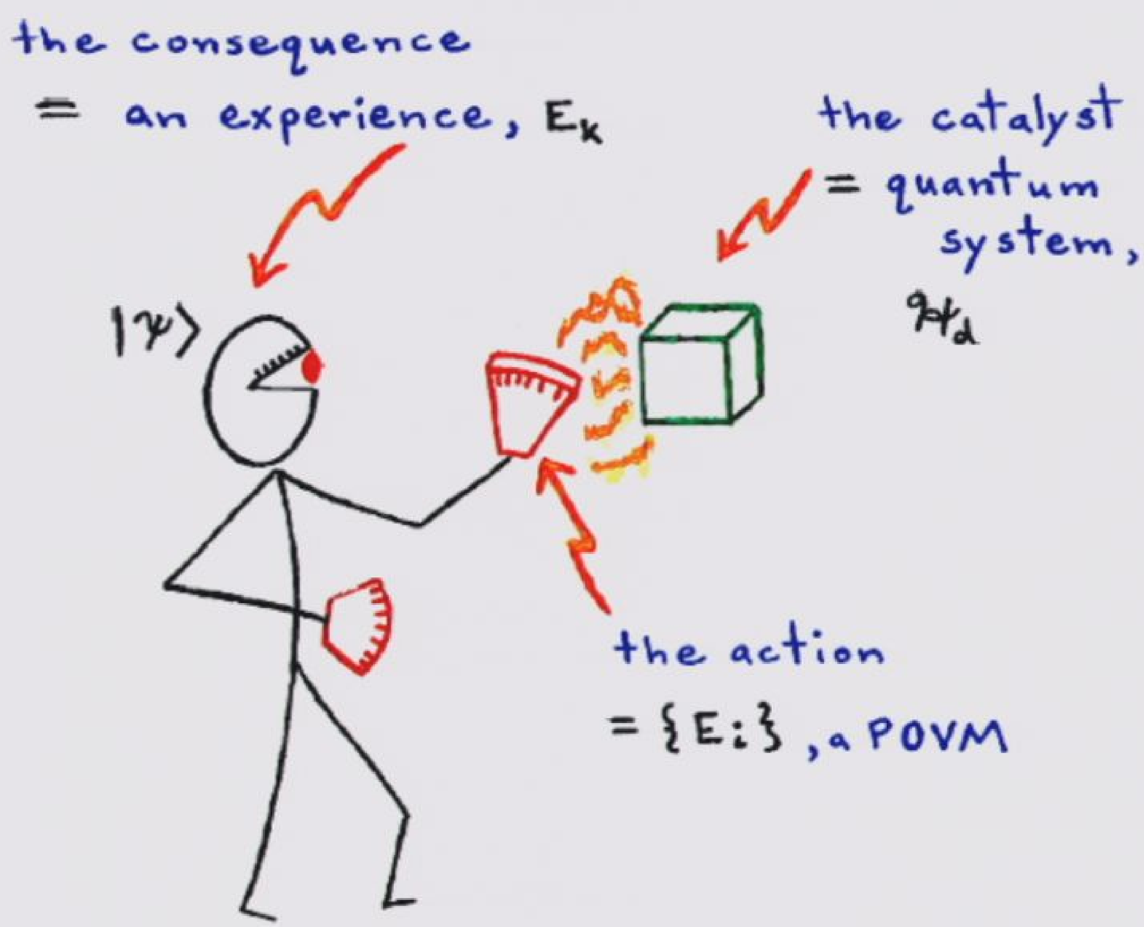
the consequence
= an experience, E_k
$|\psi\rangle$
the catalyst
= quantum
system,
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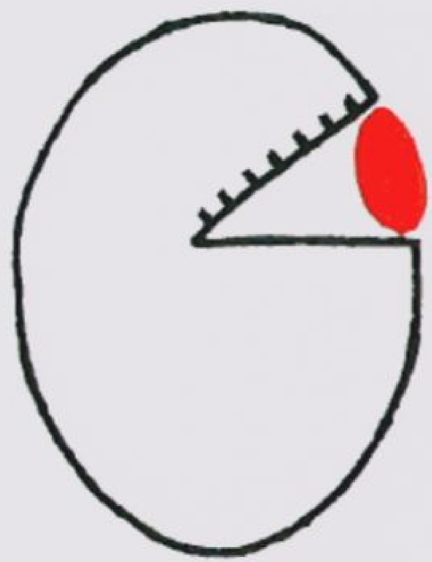
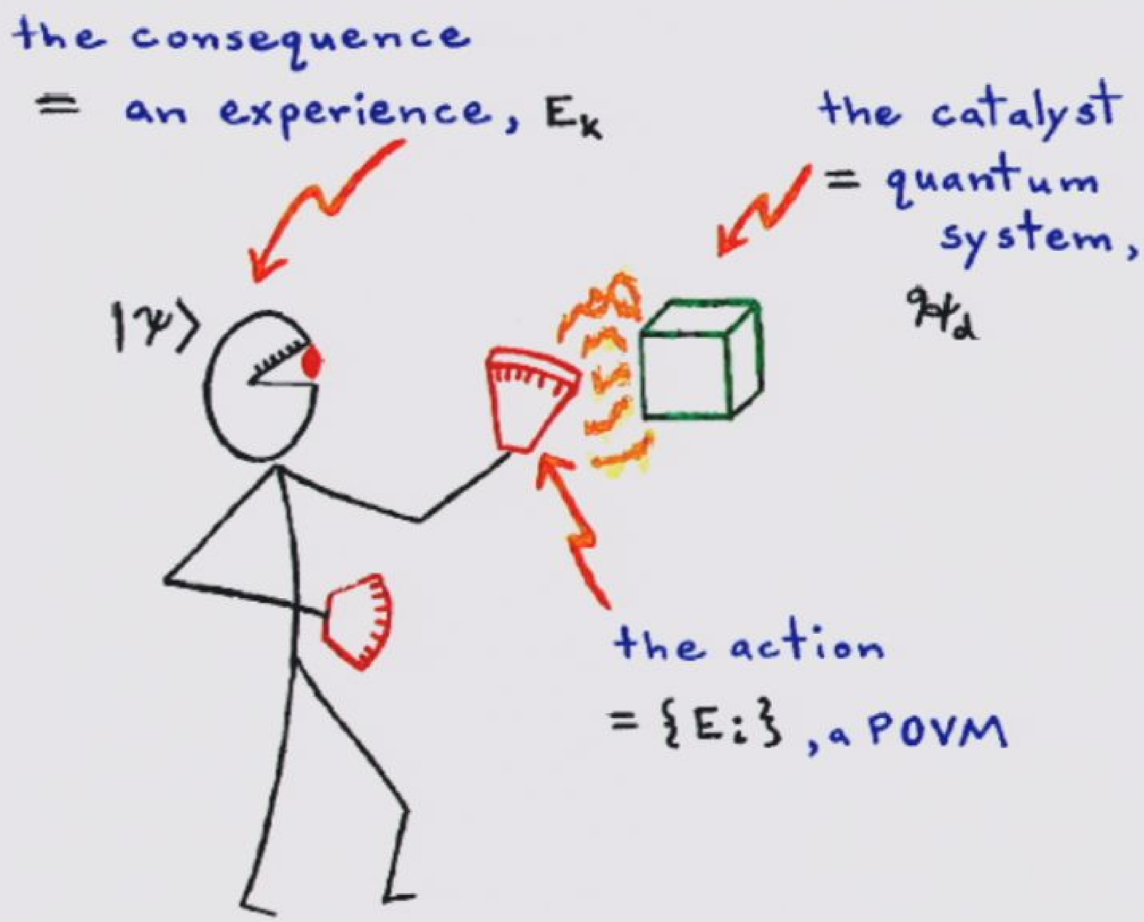
the action
= $\{E_i\}$, a POVM

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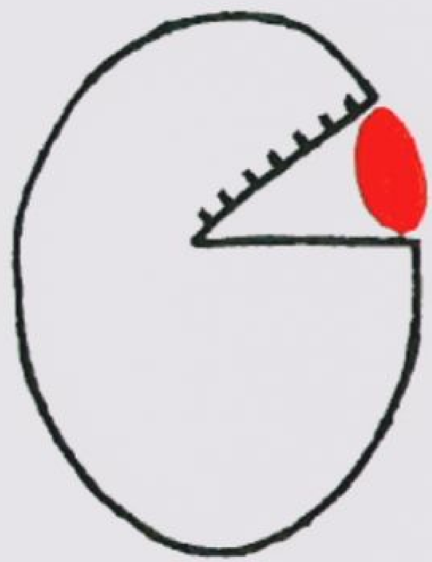

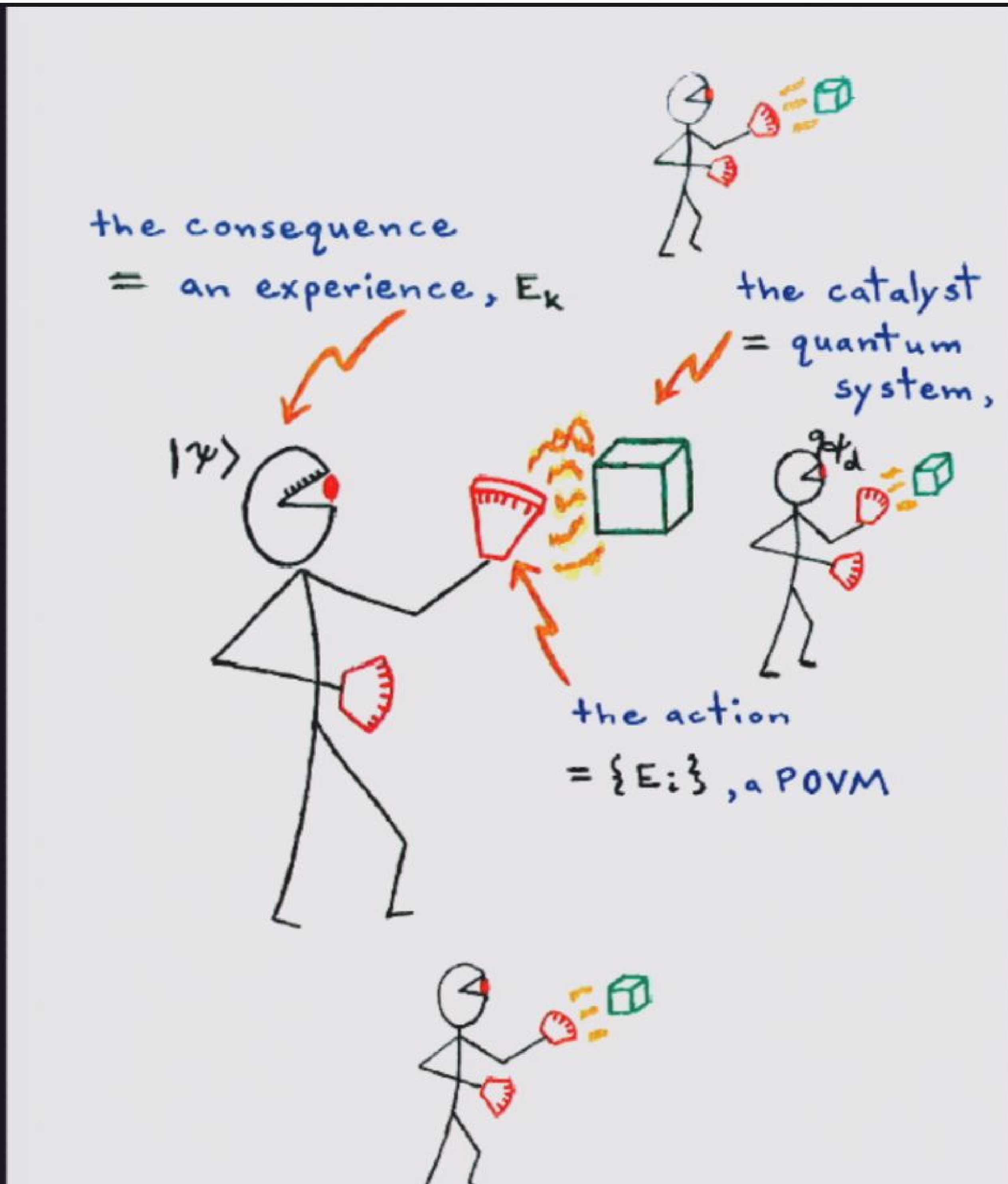
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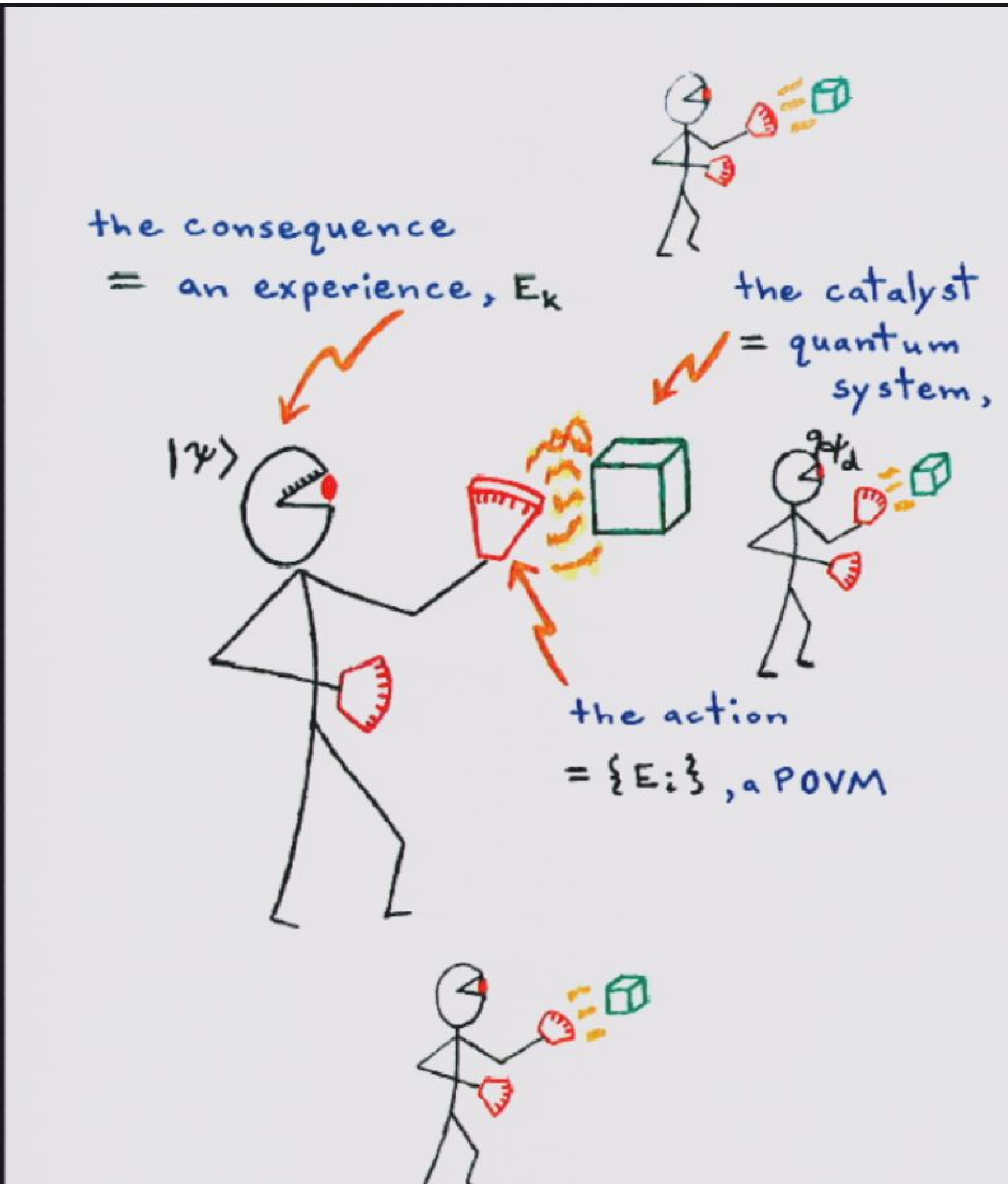
Calculus 1 → Character 1

Calculus 2 → Character 2

Calculus 3 → Character 3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Calculus 1 $\Rightarrow$ Character 1

Calculus 2 $\Rightarrow$ Character 2

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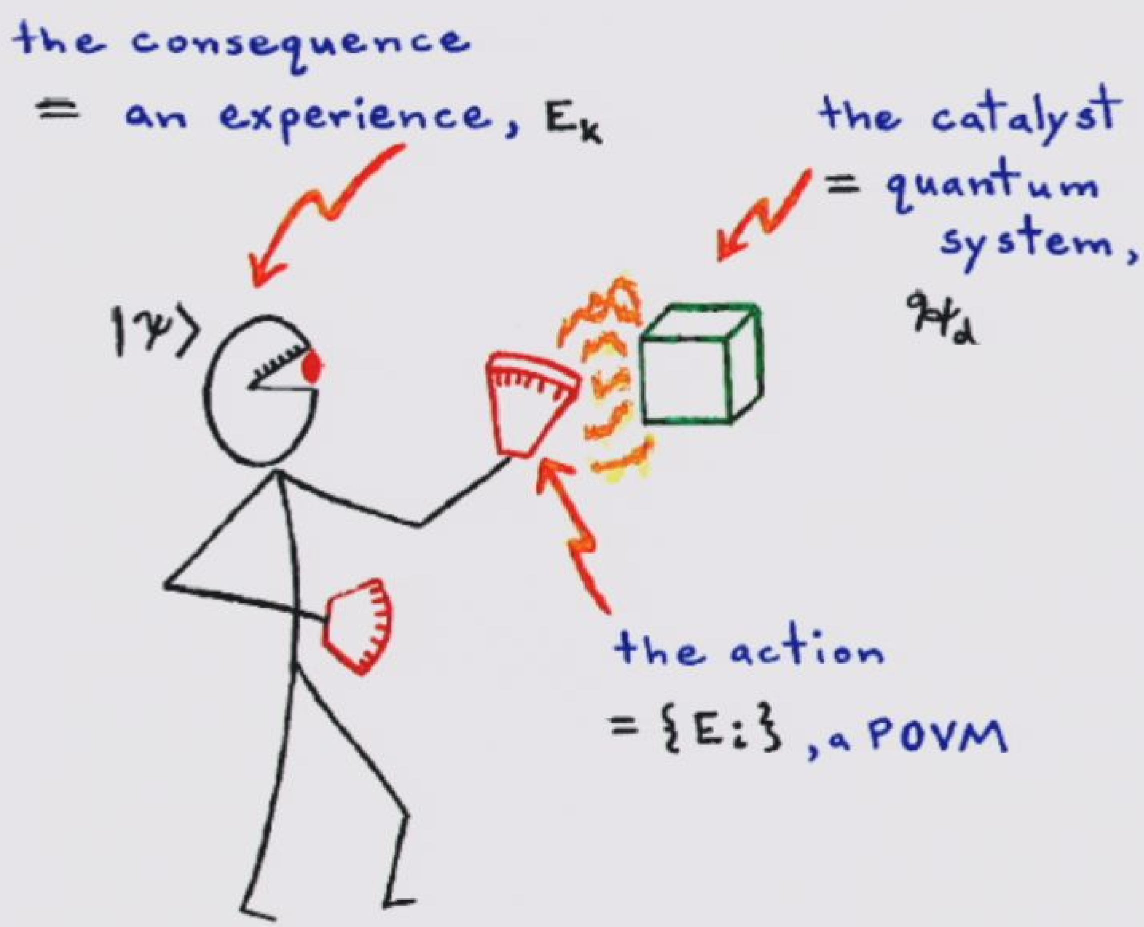
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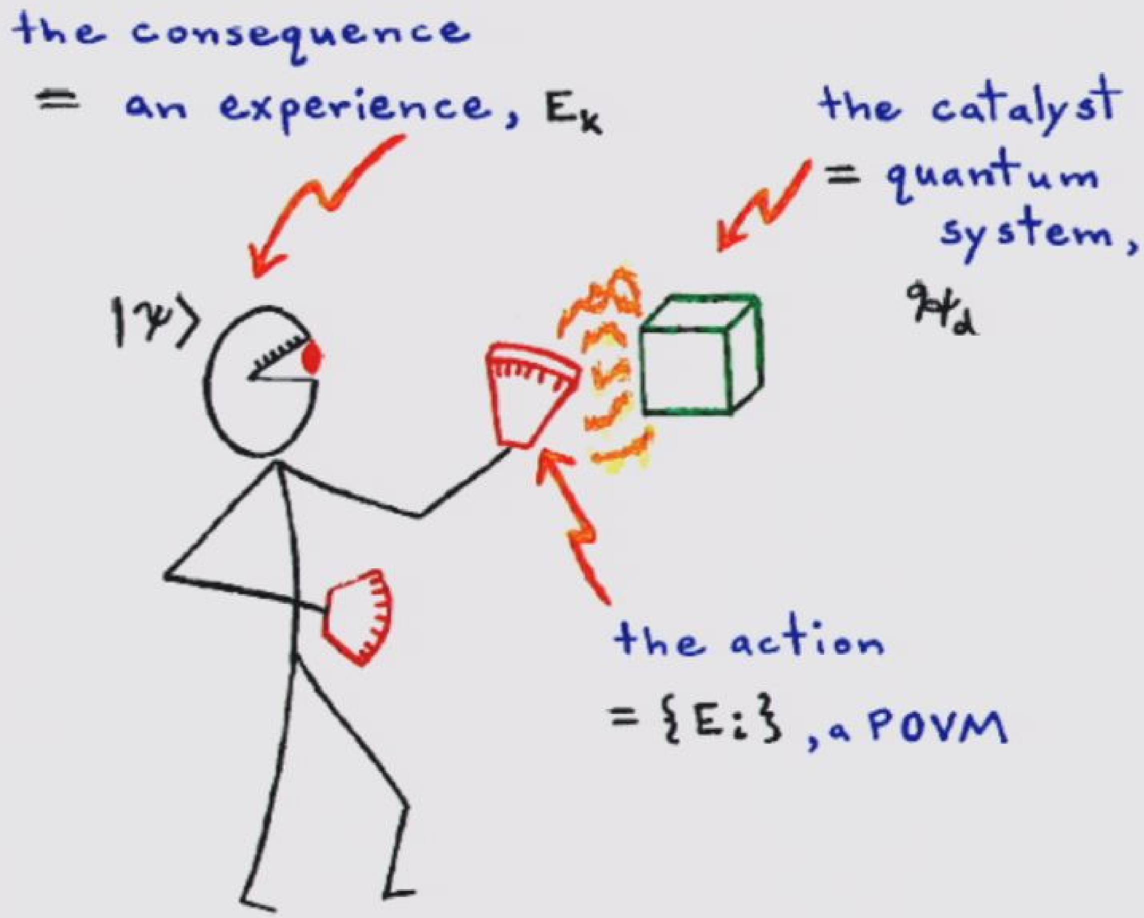
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A satisfactory statement about the actual (objective) characteristics of the quantum world should contain no $|\psi\rangle$'s at all.

Really. <u>None</u>!

Density Operators

linear operators

$$\rho \in \mathcal{L}(\mathcal{H}_d)$$

complex vector space

catalog of uncertainties

1) $\rho^\dagger = \rho$

2) $\mathrm{tr}\,\rho = 1$

convex hull of the set $\{ |\psi\rangle\langle\psi| : |\psi\rangle \in \mathcal{H}_d \}$

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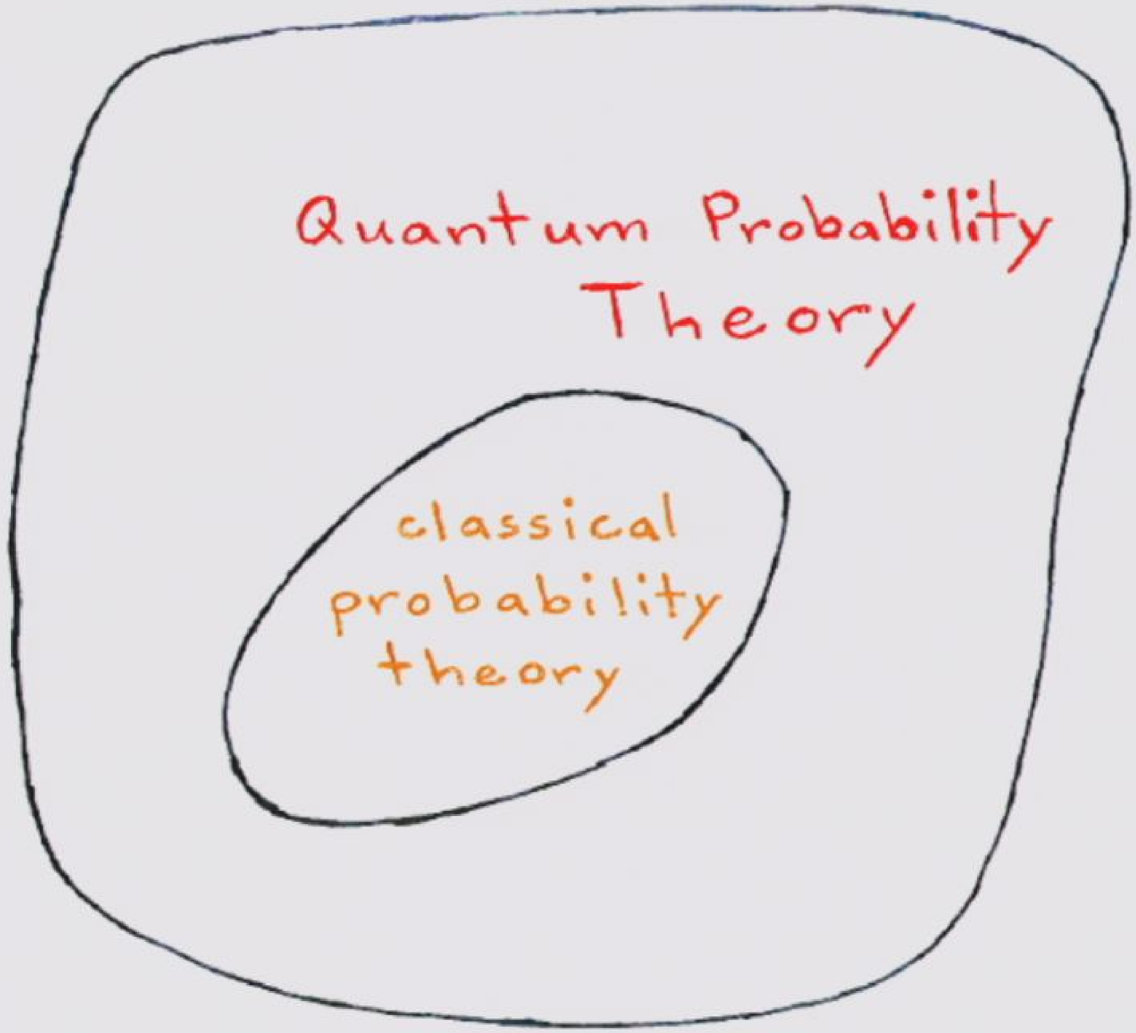
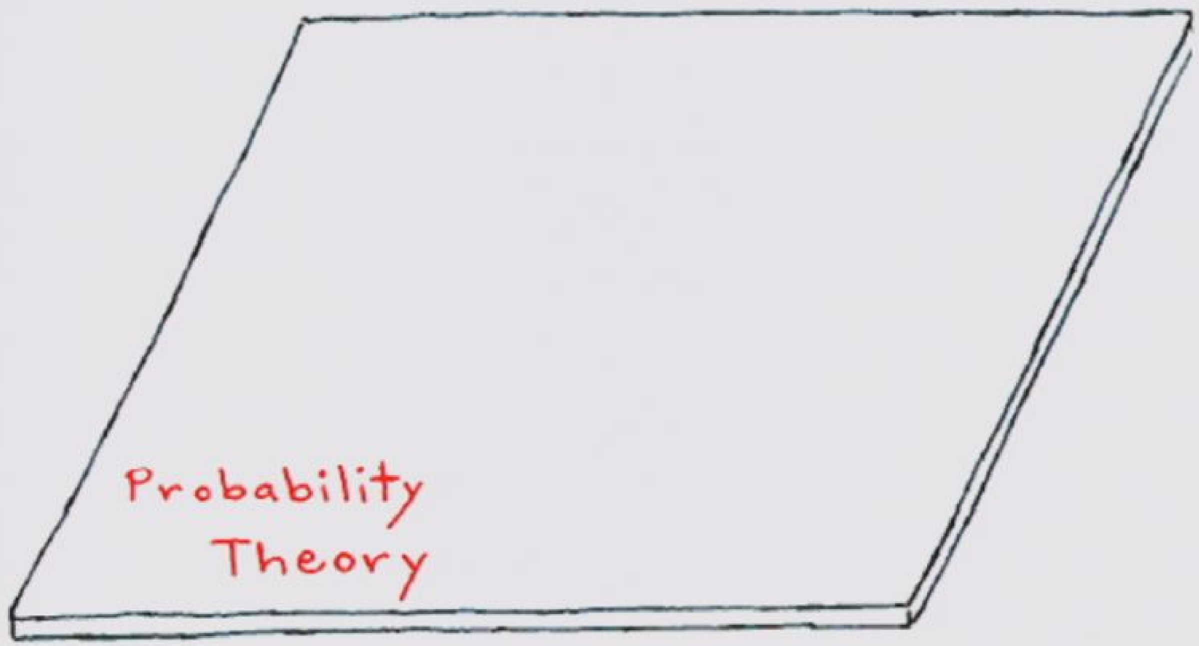
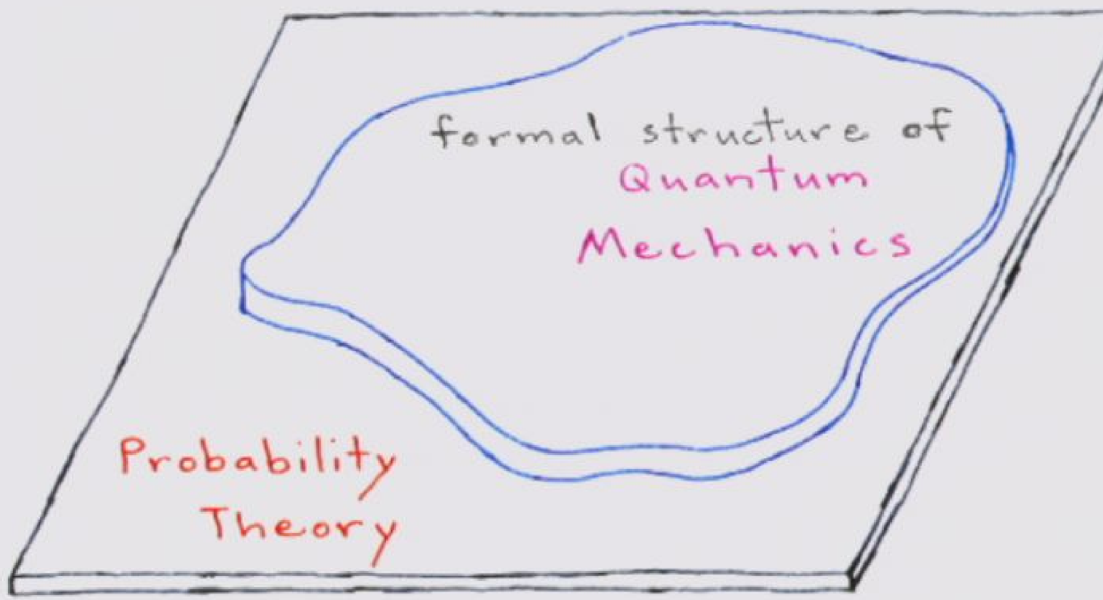


Classical probability is "just" the commutative case.


Probability
Theory


formal structure of
Quantum
Mechanics
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The Born Rule

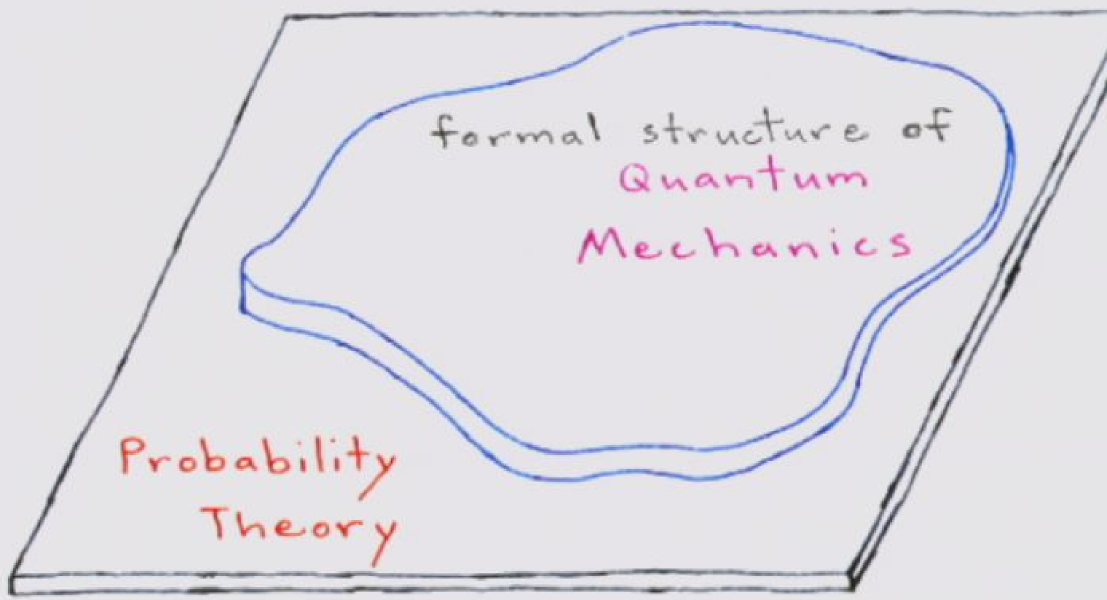
Given ρ and $\{E_i\}$,

quantum state

POVM measurement

$$\boxed{p(i) = \mathrm{tr}\, \rho E_i}$$

"The Born Rule"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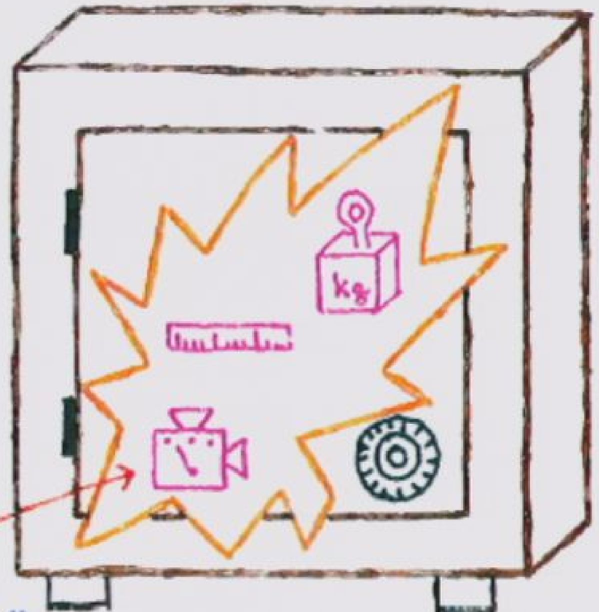
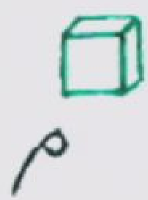
"The Born Rule"

$$\rho \longleftrightarrow p(h)$$

Bureau of Standards



the
"standard"
quantum measurement



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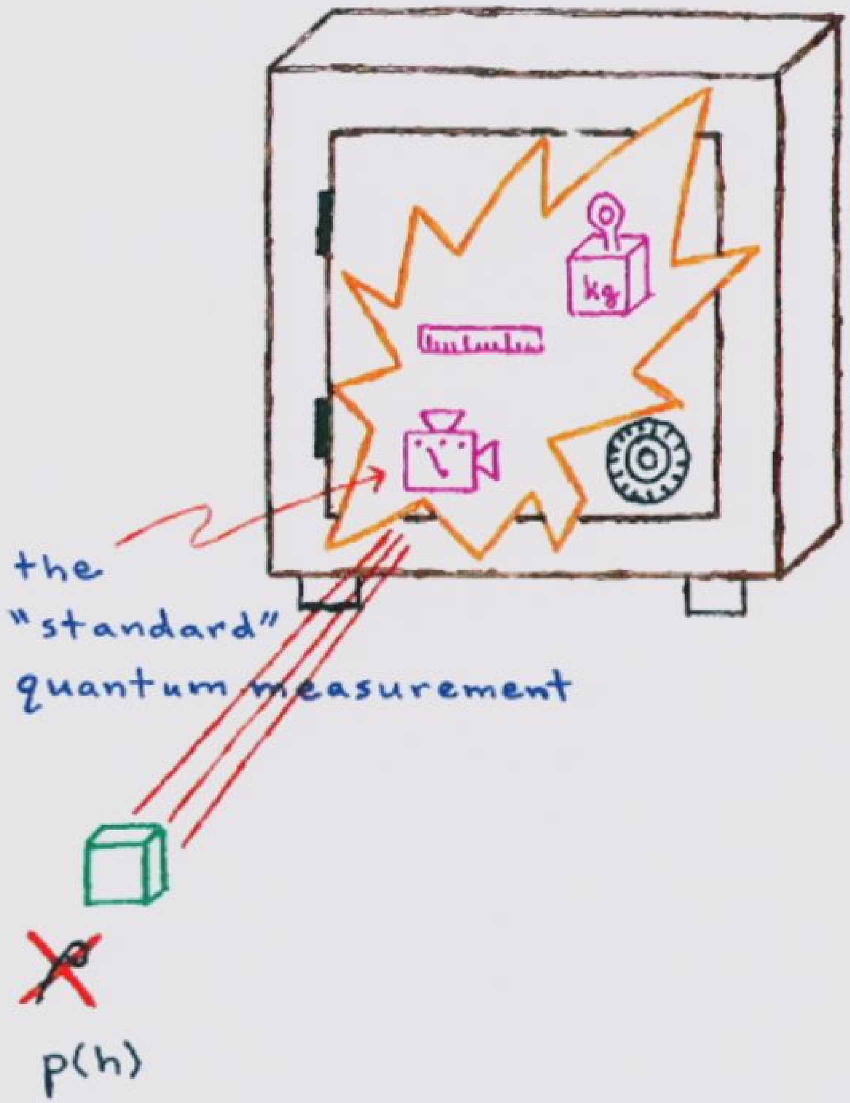

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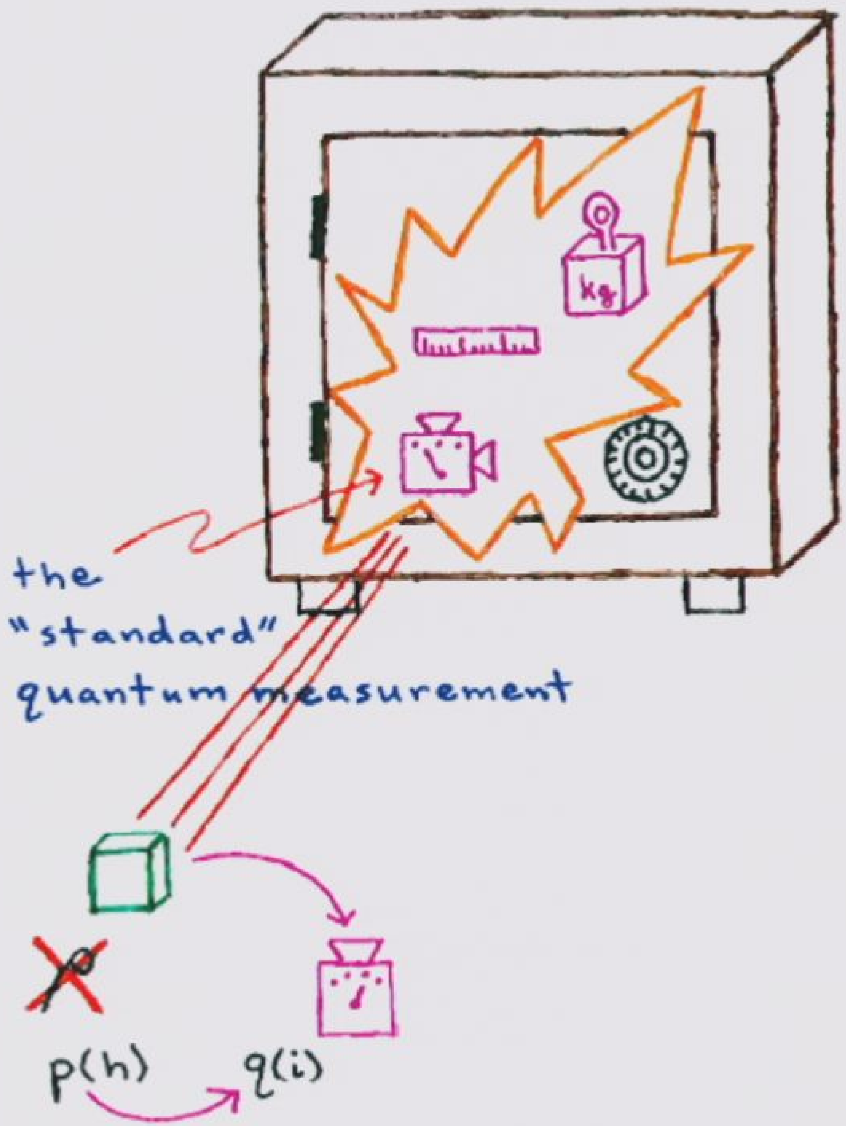

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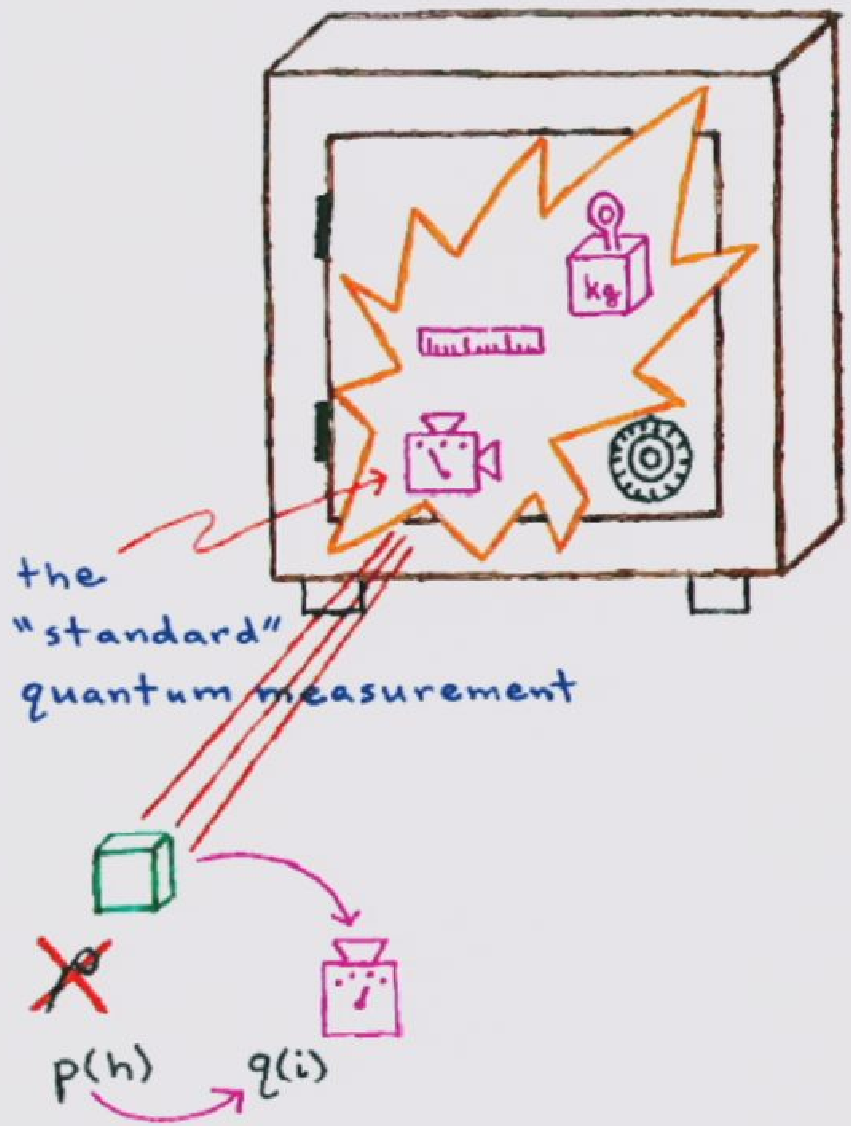


von Neumann

~~Standard~~ measurements not good enough for the bureau.

$$H = \sum_i \alpha_i \Pi_i \quad , \quad \Pi_i = |i\rangle\langle i|$$

$$p(i) = \mathrm{tr}\,\rho\,\Pi_i = \langle i|\rho|i\rangle$$

$$\Rightarrow \begin{pmatrix} \rho_{11} & & \\ & \rho_{22} & \\ & & \ddots \end{pmatrix}$$

Standard Measurements	Generalized Measurements
$\{\Pi_i\}$	$\{E_b\}$
$\langle\psi\|\Pi_i\|\psi\rangle \geq 0, \forall \|\psi\rangle$	$\langle\psi\|E_b\|\psi\rangle \geq 0, \forall \|\psi\rangle$
$\sum_i \Pi_i = I$	$\sum_b E_b = I$
$p(i) = \mathrm{tr}\,\rho\Pi_i$	$p(b) = \mathrm{tr}\,\rho E_b$
$\Pi_i \Pi_j = \delta_{ij}\Pi_i$	—

Informational Completeness

quantum states

$\rho \in \mathcal{L}(\mathcal{H}_D)$ — D^2-dimensional vector space

Choose POVM $\{E_h\}$, $h = 1, \ldots, D^2$, with E_h all linearly independent.

(Can be done.)

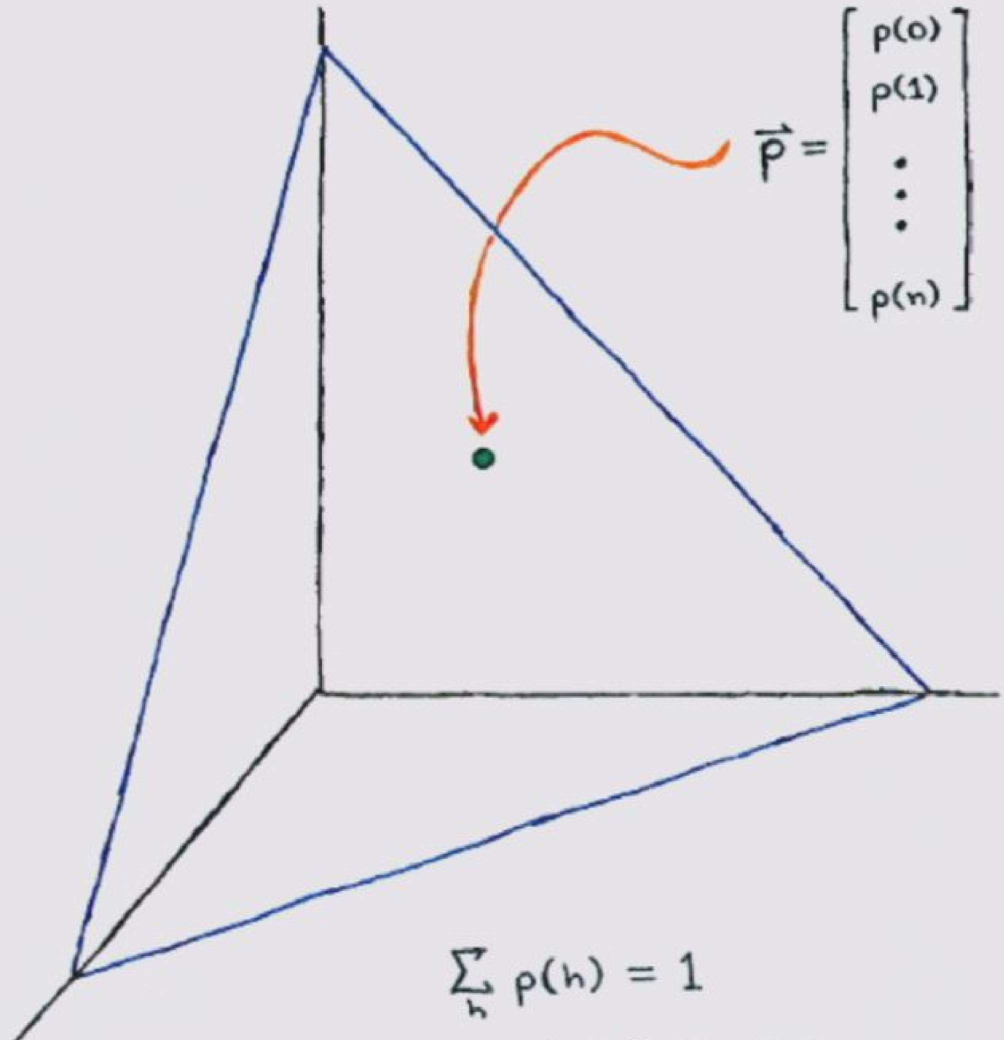
D^2 numbers $p(h) = \mathrm{tr}\,\rho E_h$ determine ρ.

Because $(A,B) = \mathrm{tr}\,A^\dagger B$ is an inner product.

projection of ρ onto E_h

Any such $\{E_h\}$ can be the standard quantum measurement.

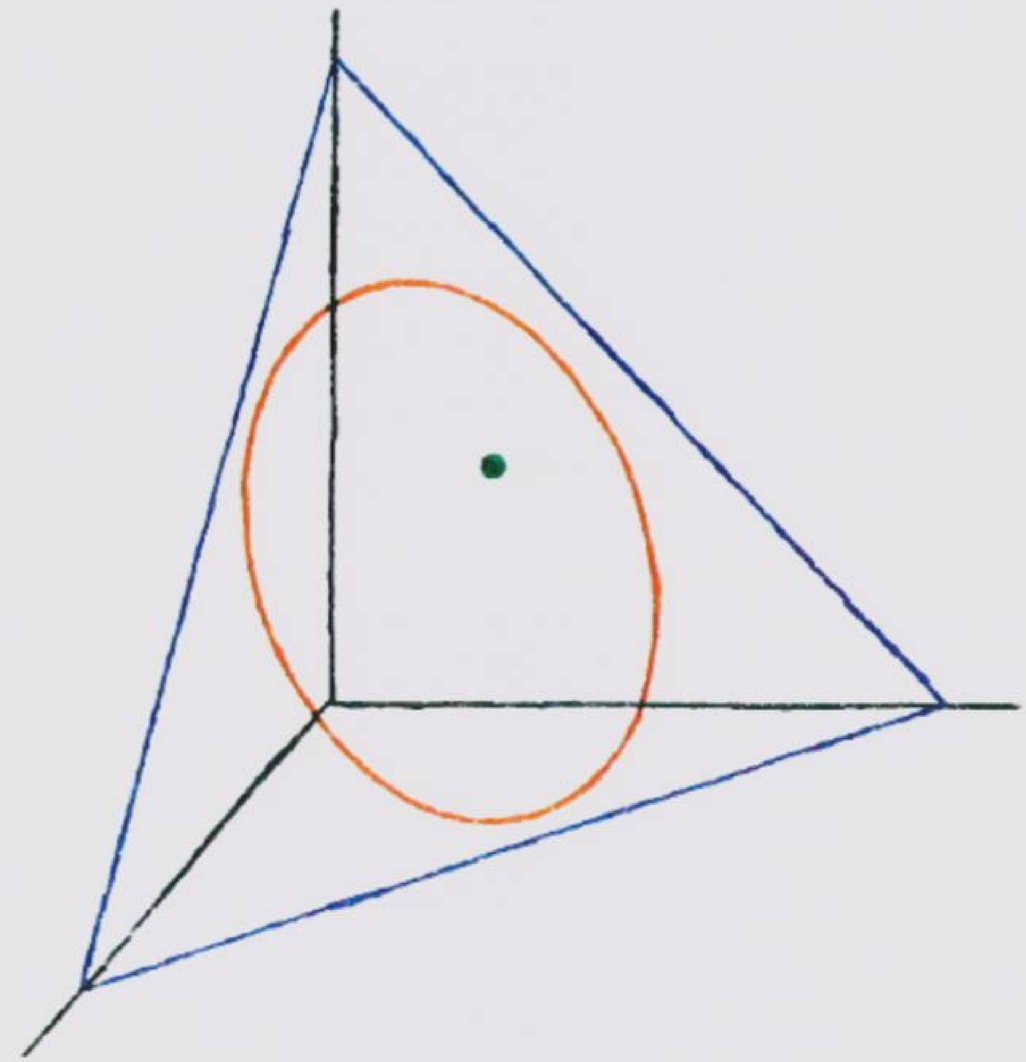
Probability Simplex



$$\vec{p} = \begin{bmatrix} p(0) \\ p(1) \\ \vdots \\ p(n) \end{bmatrix}$$

$$\sum_h p(h) = 1$$

$$p(h) \geq 0 \quad \forall h$$



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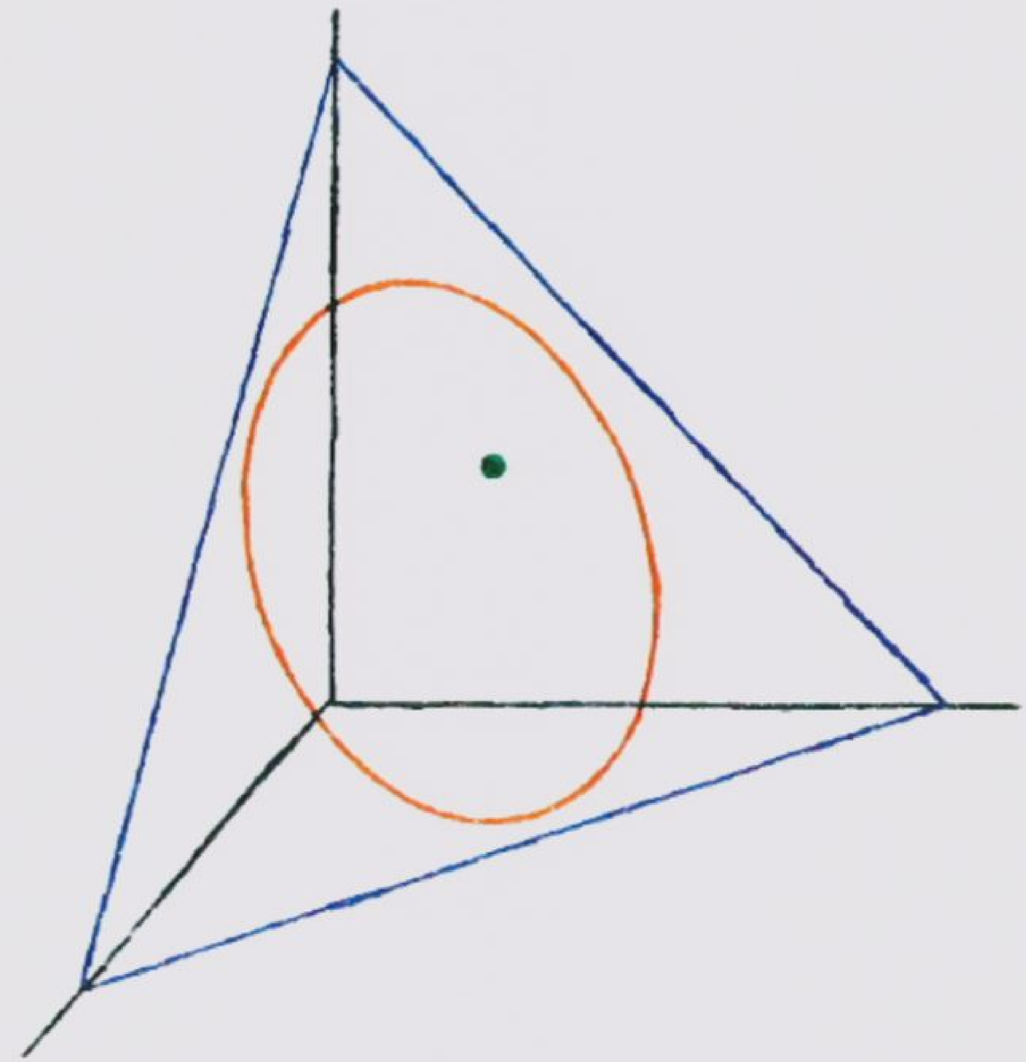


A Very Fundamental Mmt ?

Caves, 1999
Zauner

Suppose d^2 projectors $\Pi_i = |\psi_i\rangle\langle\psi_i|$ satisfying

$$\mathrm{tr}\,\Pi_i\Pi_j = \frac{1}{d+1}\ , \quad i \neq j$$

exist.

Can prove:

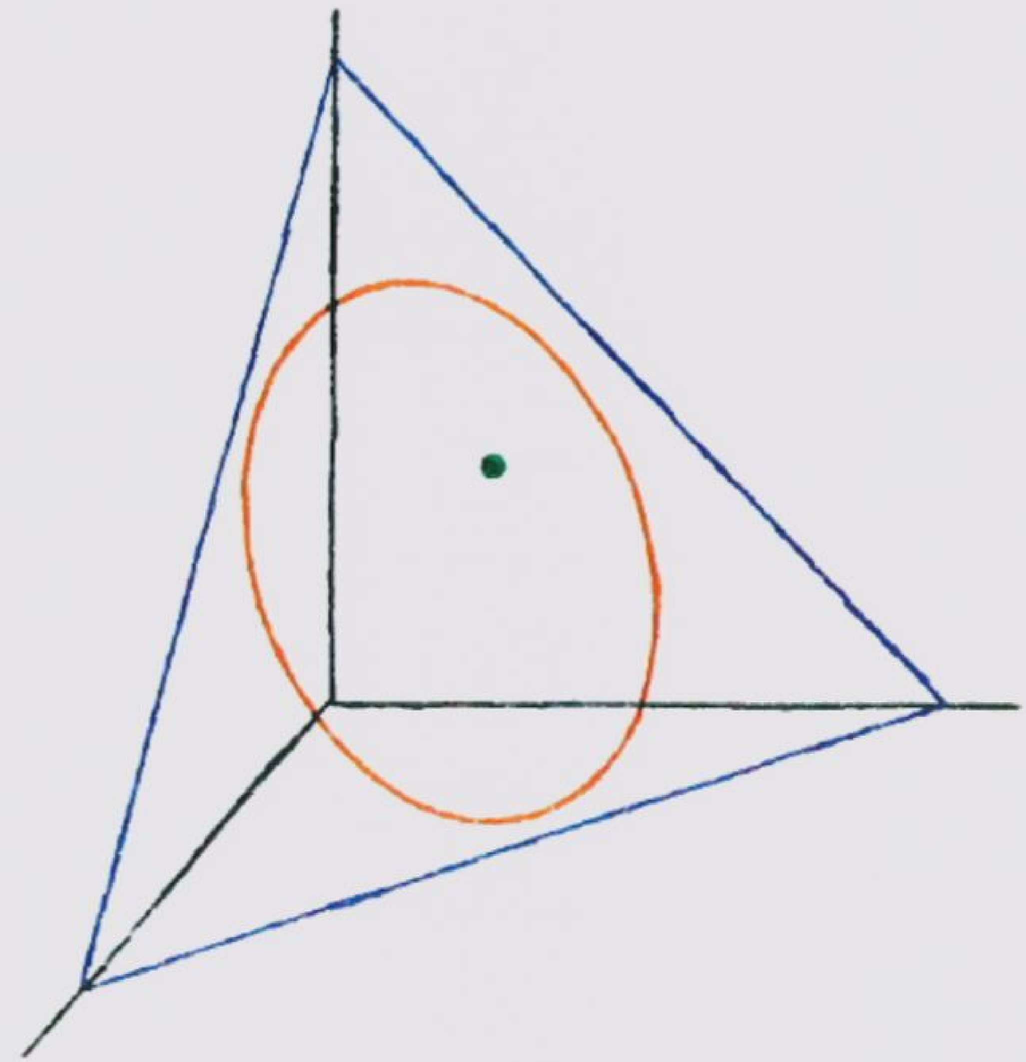
1) the Π_i linearly independent

2) $\sum_i \frac{1}{d}\Pi_i = I$

So good for Bureau of Standards.

Also
$$p(i) = \frac{1}{d}\,\mathrm{tr}\,\rho\Pi_i$$

$$\boxed{\rho = \sum_i \left[(d+1)p(i) - \frac{1}{d}\right]\Pi_i}$$



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But do such sets of states

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Inequivalent SIC Sets

Let $d = 3$, $\omega = e^{\frac{2\pi i}{3}}$.

$$\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -\omega \\ \bar{\omega} \end{bmatrix} \quad \begin{bmatrix} 0 \\ -\bar{\omega} \\ \omega \end{bmatrix}$$

Set 1

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ \bar{\omega} \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ -\bar{\omega} \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} -1 \\ \omega \\ 0 \end{bmatrix} \quad \begin{bmatrix} -1 \\ \bar{\omega} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} -2 \\ \omega \\ \bar{\omega} \end{bmatrix} \quad \begin{bmatrix} -2 \\ \bar{\omega} \\ \omega \end{bmatrix}$$

Set 2

$$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -2\omega \\ \bar{\omega} \end{bmatrix} \quad \begin{bmatrix} 1 \\ -2\bar{\omega} \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ \text{[illegible]} \end{bmatrix} \quad \begin{bmatrix} 1 \\ \omega \\ \text{[illegible]} \end{bmatrix} \quad \begin{bmatrix} 1 \\ \bar{\omega} \\ \text{[illegible]} \end{bmatrix}$$

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$$

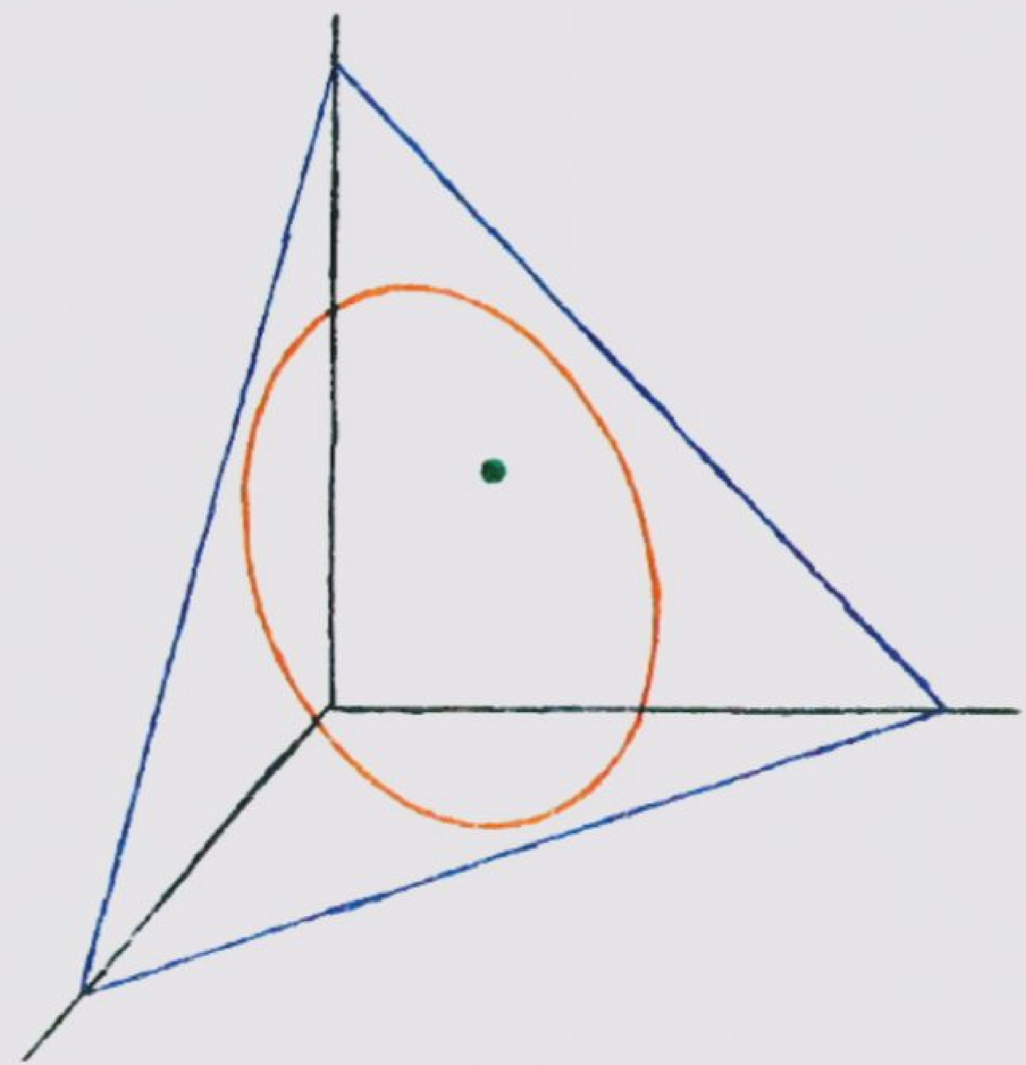
Evidence for Existence

Analytical Constructions

$$d = 2 - 13,\ 14,\ 15,\ 19$$

Numerical ($\epsilon \leq \cancel{10^{-11}}$) 10^{-38}!

$$d = 2 - \cancel{47}\ 67$$



Remarkable Theorem

Jones & Linden, PRA 71 (2005)
Flammia, (unpub, 2004)

Let A be Hermitian, $A^{\dagger} = A$.

Then, $A = |\psi\rangle\langle\psi|$ if and only if

$$\mathrm{tr}\, A^2 = \mathrm{tr}\, A^3 = 1 .$$

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Pure States in SIC Language

Conditions

$$\rho^\dagger = \rho \quad , \quad \mathrm{tr}\,\rho^2 = \mathrm{tr}\,\rho^3 = 1$$

translate to

$$\sum_i p(i)^2 = \frac{2}{d(d+1)}$$

and

$$\sum_{jkl} c_{jkl}\, p(j)\, p(k)\, p(l) = \frac{d+7}{(d+1)^3}$$

where

$$c_{jkl} = \mathrm{Re}\ \mathrm{tr}\, \Pi_j \Pi_k \Pi_l$$

Could these be independently motivatable physical constants?

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Proof:

a_i — eigenvalues of A

$$\mathrm{tr}\,A^2 = \sum_i a_i^2 = 1 \quad \Rightarrow \quad |a_i| \leq 1$$

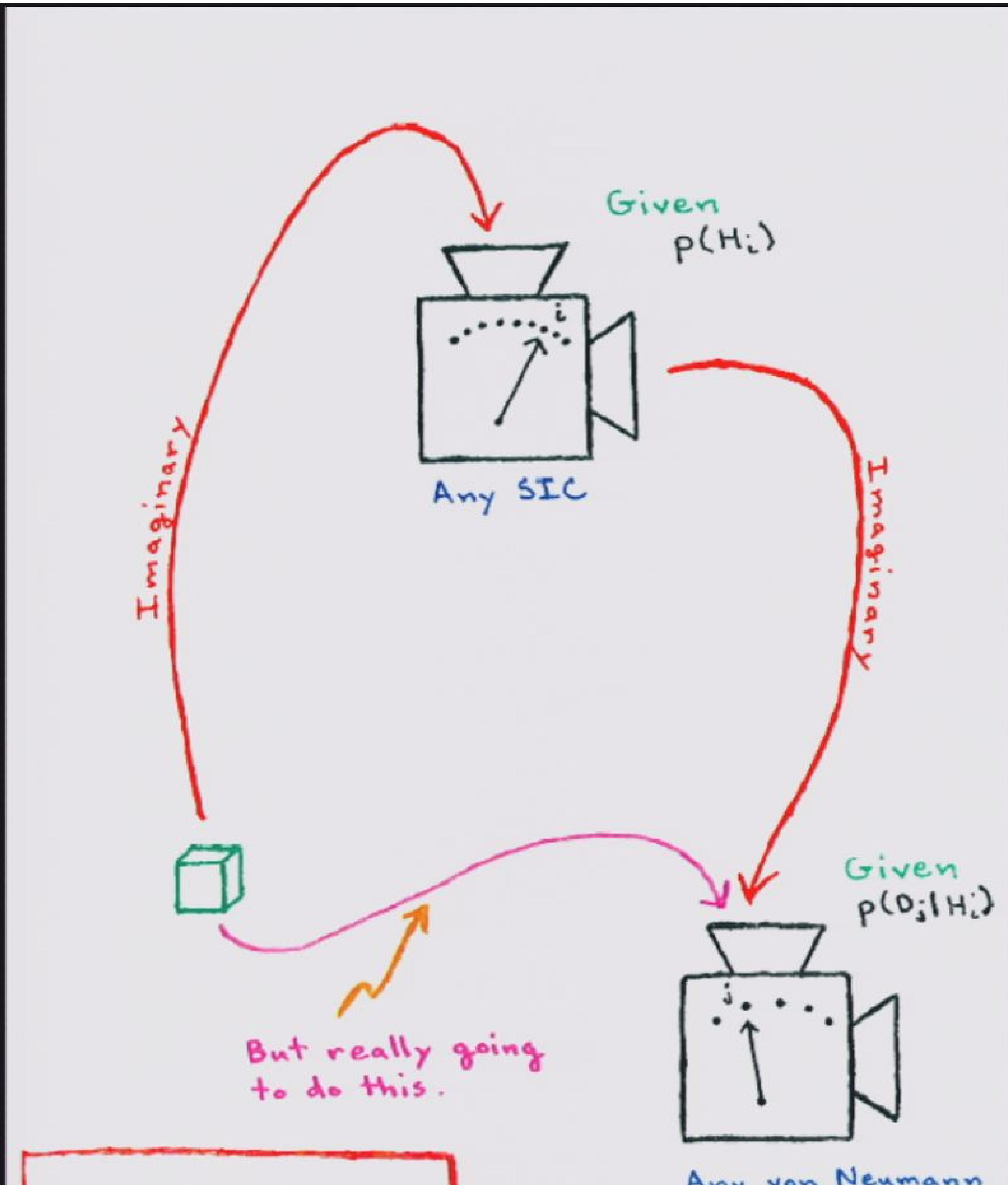
$$1 - a_i \geq 0$$

$$0 = \mathrm{tr}\,A^2 - \mathrm{tr}\,A^3 = \sum_i a_i^2 (1 - a_i)$$

$$\Rightarrow \quad a_i = 0 \text{ or } 1 - a_i = 0$$

$\mathrm{tr}\,A^2 = 1 \quad \Rightarrow \quad a_i = 1$ for one and only one i.

QED


Given
p(H_i)
i
Any SIC
Imaginary
Imaginary
Given
p(D_j|H_i)
j
But really going
to do this.

Laws of Probability

H_i – various hypotheses one might have

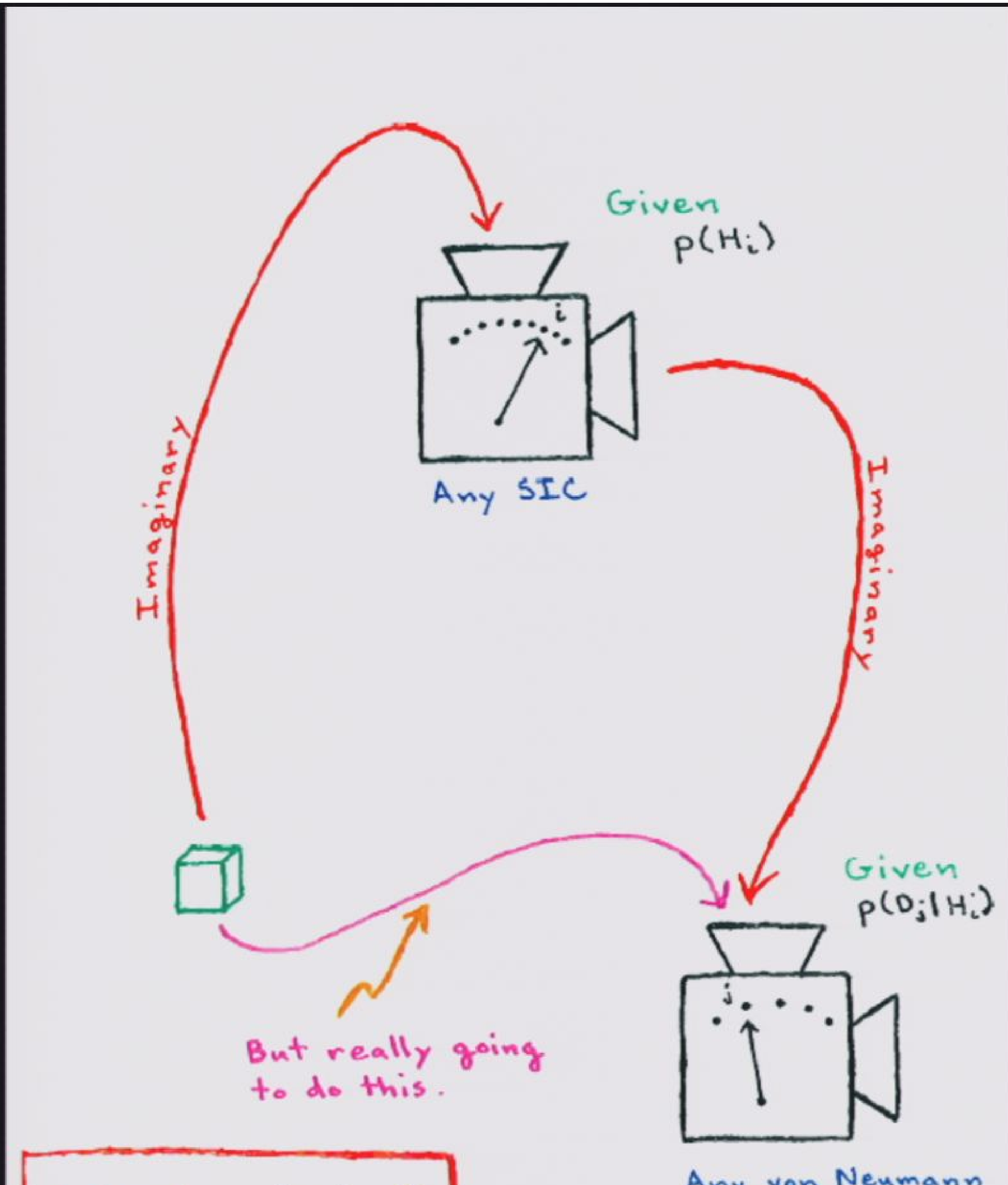
D_j – data values one might gather

Given: $p(D_j | H_i)$ ← expectations for data given hypothesis

$p(H_i)$ ← expectations for hypotheses themselves

Question: What expectations should one have for the D_j?

Answer: $P(D_j) = \sum_i p(H_i)\, p(D_j | H_i)$



Given
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Any von Neumann

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Answer: $P(D_j) = \sum_i p(H_i)\, p(D_j|H_i)$

In this case,

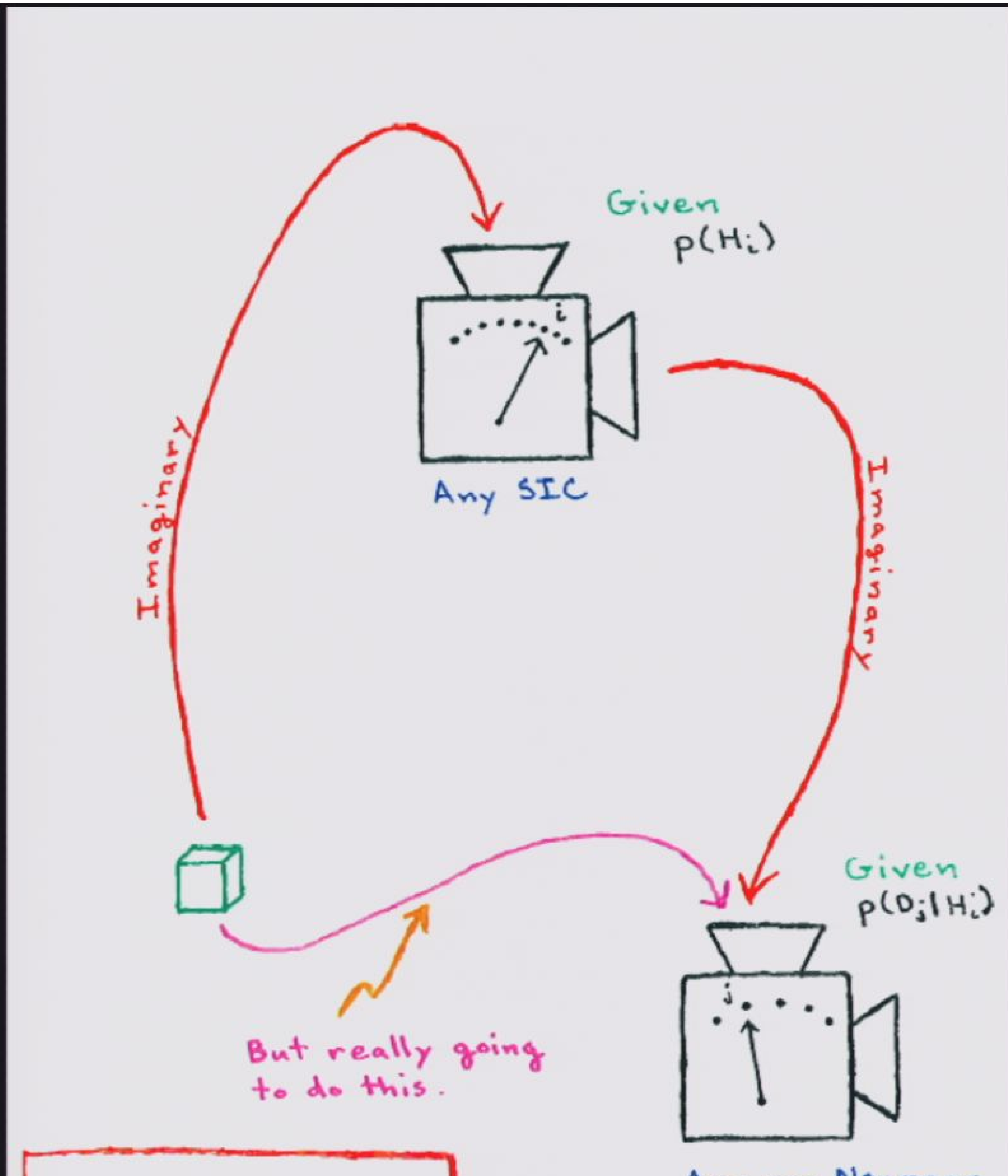
$$p(D_j) \neq \sum_i p(H_i)\, p(D_j | H_i) .$$

As Ballentine (1986) points out, there are hidden conditionals

$p(D_j)$ really $p(D_j | C_1)$

$p(H_i)$ really $p(H_i | C_2)$

$p(D_j | H_i)$ really $p(D_j | H_i, C_2)$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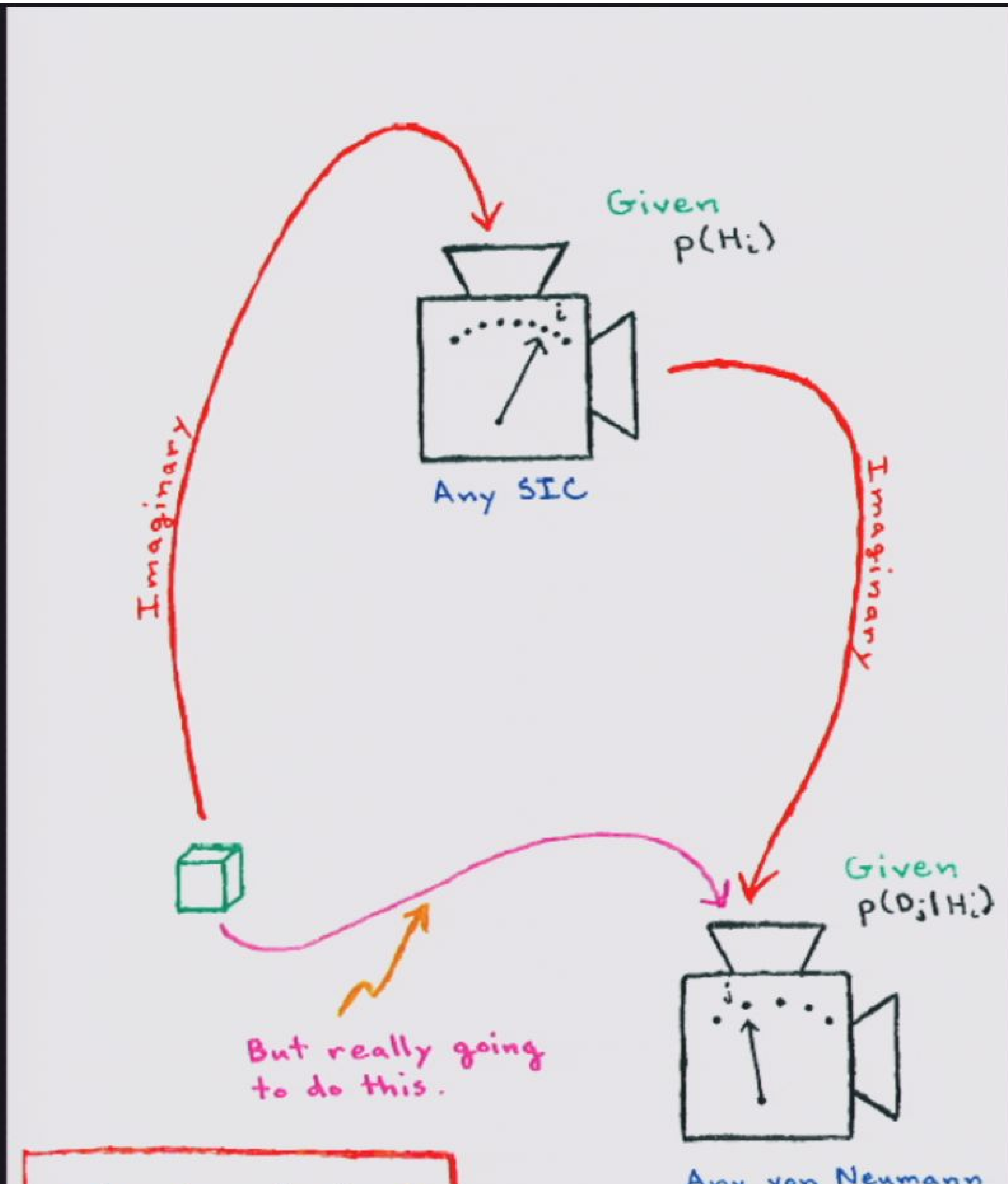
$p(H_i)$ really $p(H_i | C_2)$

$p(D_j | H_i)$ really $p(D_j | H_i, C_2)$


Given
p(H_i)
i
Any SIC
Imaginary
Imaginary
Given
p(D_j|H_i)
j
But really going
to do this.

Laws of Probability

H_i – various hypotheses one might have

D_j – data values one might gather

Given: $p(D_j | H_i)$ ← expectations for data given hypothesis

$p(H_i)$ ← expectations for hypotheses themselves

Question: What expectations should one have for the D_j?

Answer: $P(D_j) = \sum_i p(H_i)\, p(D_j | H_i)$

In this case,

$$p(D_j) \neq \sum_i p(H_i)\, p(D_j | H_i) .$$

As Ballentine (1986) points out, there are hidden conditionals

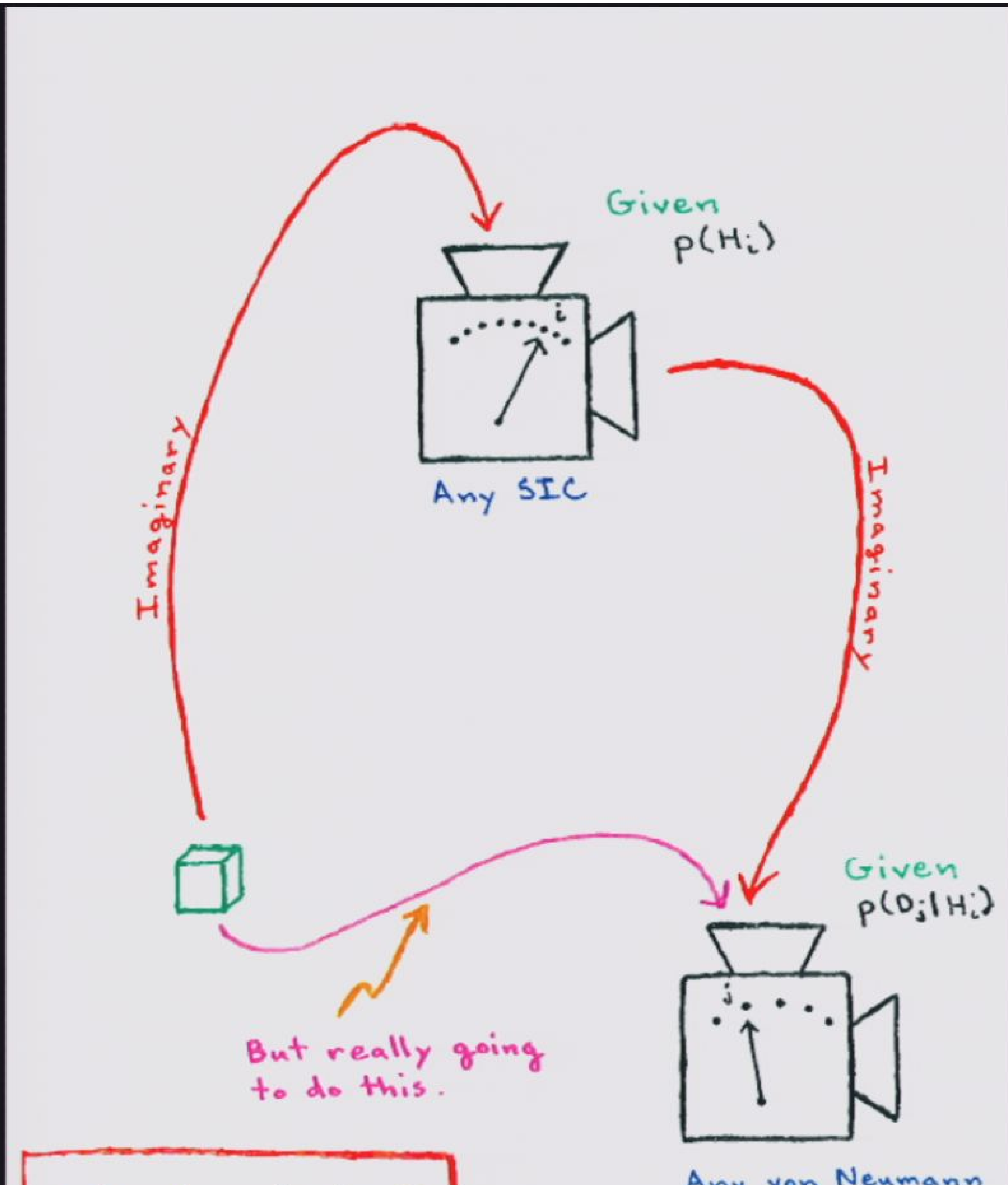
$p(D_j)$ really $p(D_j | C_1)$

$p(H_i)$ really $p(H_i | C_2)$

$p(D_j | H_i)$ really $p(D_j | H_i, C_2)$

$$\underbrace{p(D_j)}_{\text{Quantum}} = (d+1) \underbrace{\sum_i p(H_i)\, p(D_j|H_i)}_{\text{(Usual) Bayesian}} - 1$$

Magic!


Given
p(H_i)
Imaginary
Any SIC
Imaginary
Given
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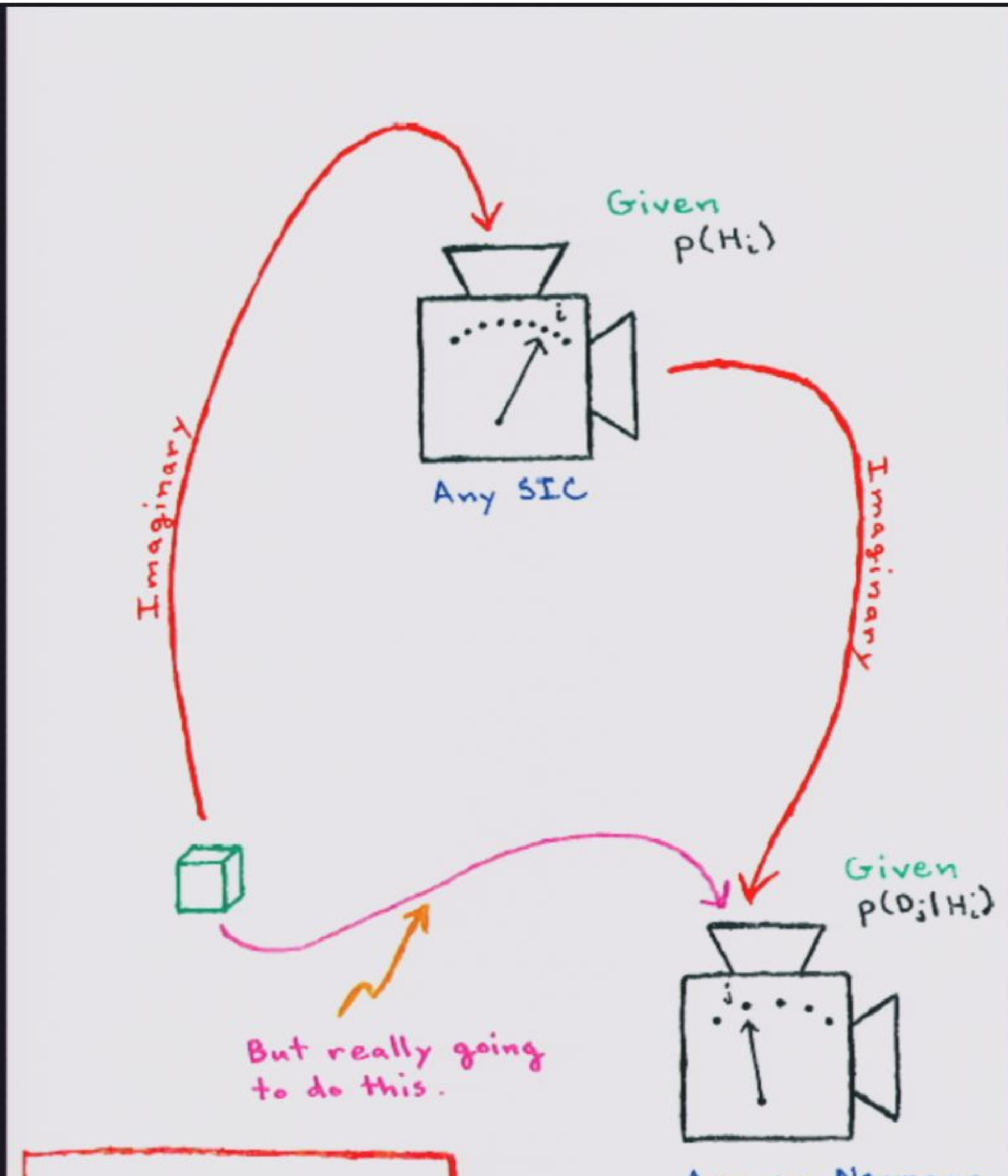
Given
p(H_i)
Imaginary
Any SIC
Imaginary
Given
p(D_j|H_i)
But really going
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Any von Neumann

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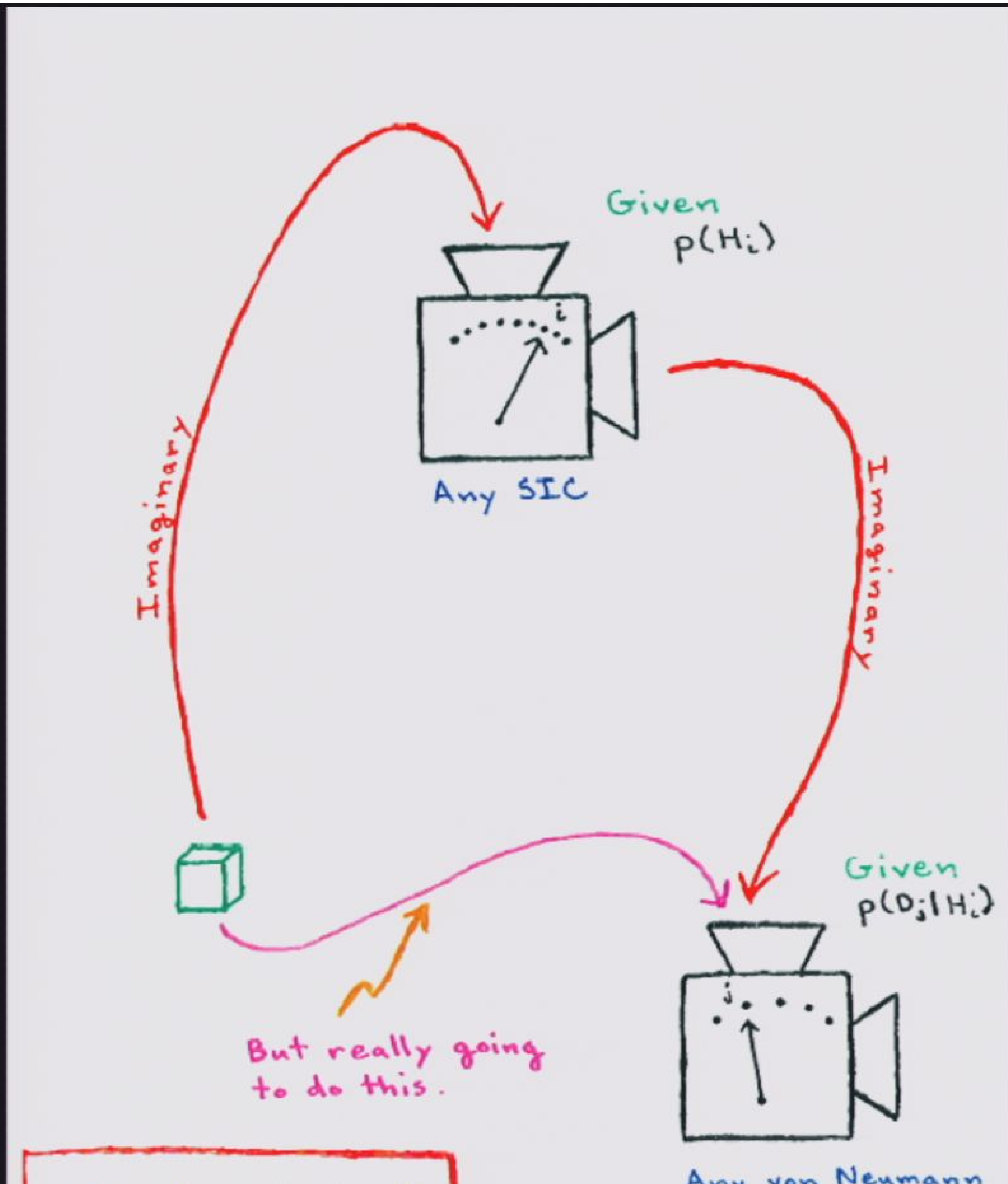
Magic!


Given
p(Hi)
Any SIC
Imaginary
Imaginary
Given
p(Dj|Hi)
But really going
to do this.
Any von Neumann

$$\underbrace{p(D_j)}_{\text{Quantum}} = (d+1)\underbrace{\sum_i p(H_i)\,p(D_j|H_i)}_{\text{(Usual) Bayesian}} - 1$$

Magic!

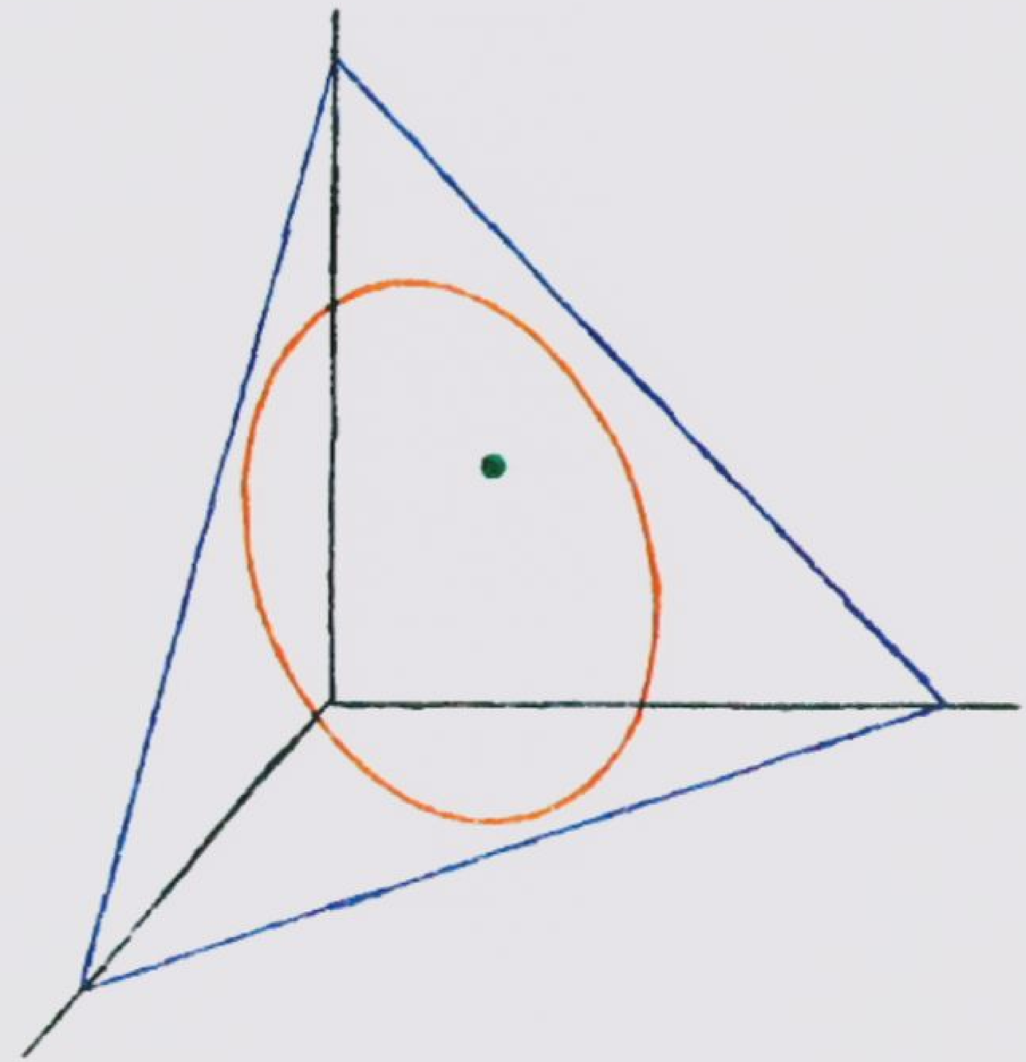
Remarkable Theorem

Jones & Linden, PRA 71 (2005)
Flammia, (unpub, 2004)

Let A be Hermitian, $A^{+} = A$.

Then, $A = |\psi\rangle\langle\psi|$ if and only if

$$\mathrm{tr}\, A^2 = \mathrm{tr}\, A^3 = 1 .$$



Laws of Probability

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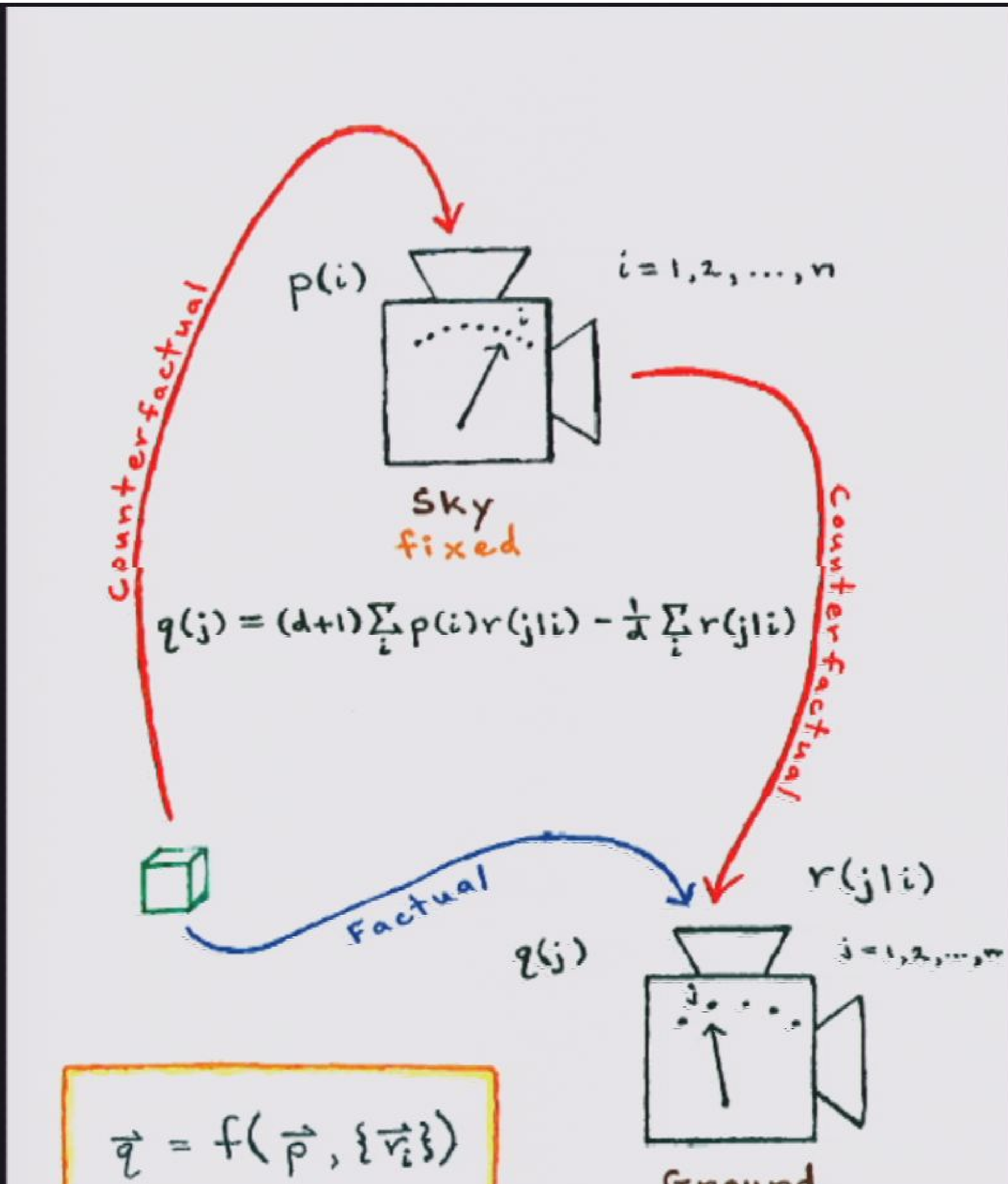
p(i)
i = 1, 2, ..., n
Sky
fixed
Counterfactual
Counterfactual
q(j) = (d+1) Σ_i p(i) r(j|i) − 1/d Σ_i r(j|i)
Factual
r(j|i)
q(j)
j = 1, 2, ..., m
q⃗ = f(p⃗ , {r⃗_i})
Ground

Examples

1) Take $\vec{q} = \vec{p}$. Consequently must have

$$\vec{p} \cdot \vec{p} \leq \frac{2}{d(d+1)}$$

Same as quantum.

2) Consider a subset $\{\vec{p}_k\} \subseteq \mathcal{S}$ with $k = 1, \ldots, m$ such that

$$\vec{p}_k \cdot \vec{p}_k = \frac{2}{d(d+1)}$$

$$\vec{p}_k \cdot \vec{p}_\ell = \frac{1}{d(d+1)} \qquad k \neq \ell.$$

How large can m be?

Answer: d, same as quantum

Think SIC thoughts!

... and maybe by way of it we'll come to understand quantum mechanics a little better.

Bayesian Perspective

No logical reason why situation with conditional lotteries should be commensurate with situation without conditional lotteries.

$$p(D_j) \neq \sum_i p(H_i)\, p(D_j | H_i)$$

(Need better notation, though.)

Quantum Perspective

Nonetheless, there may be

$$\boxed{p(D_j) = (d+1) \sum_i p(H_i)\, p(D_j | H_i) - \frac{1}{d} \sum_i p(D_j | H_i)}$$

Remarkable Theorem

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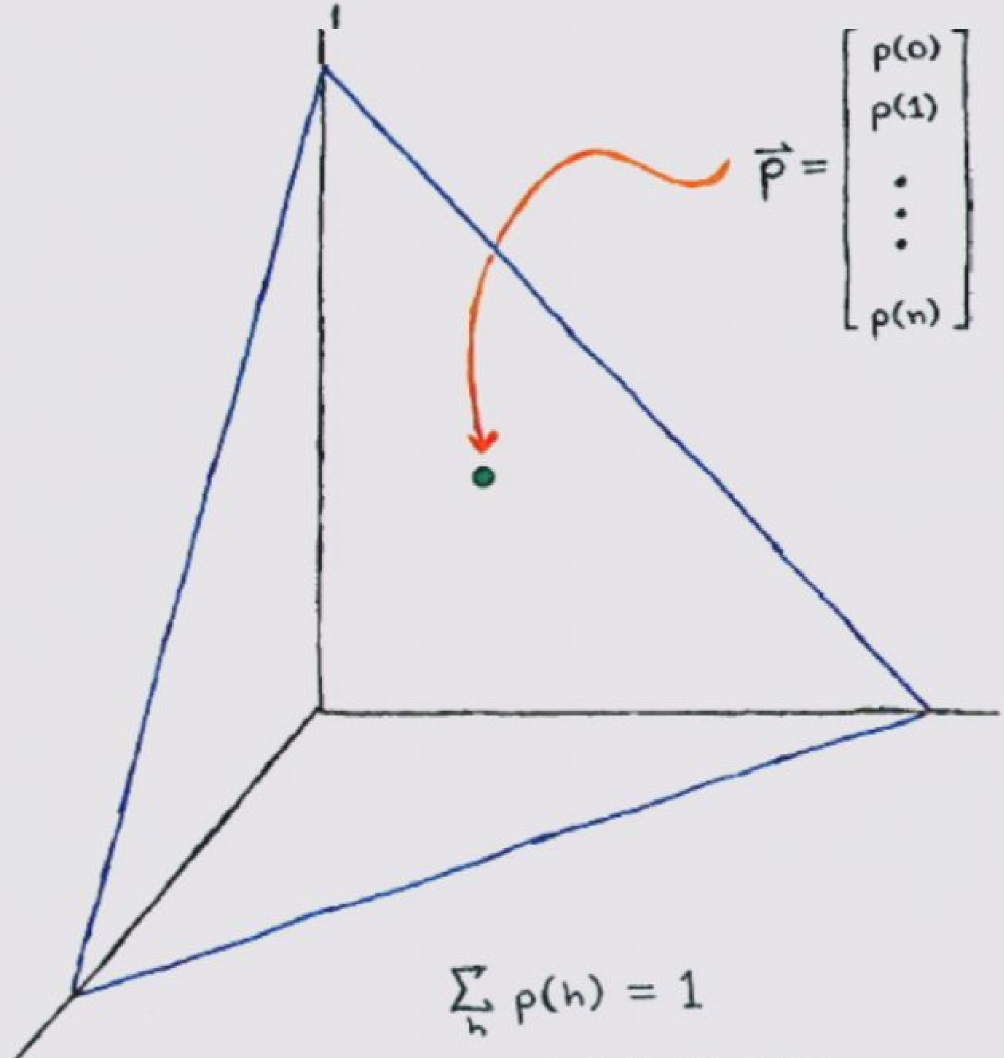
Flammia, (unpub, 2004)

Let A be Hermitian, $A^{\dagger} = A$.

Then, $A = |\Psi\rangle\langle\Psi|$ if and only if

$$\mathrm{tr}\, A^2 = \mathrm{tr}\, A^3 = 1 .$$

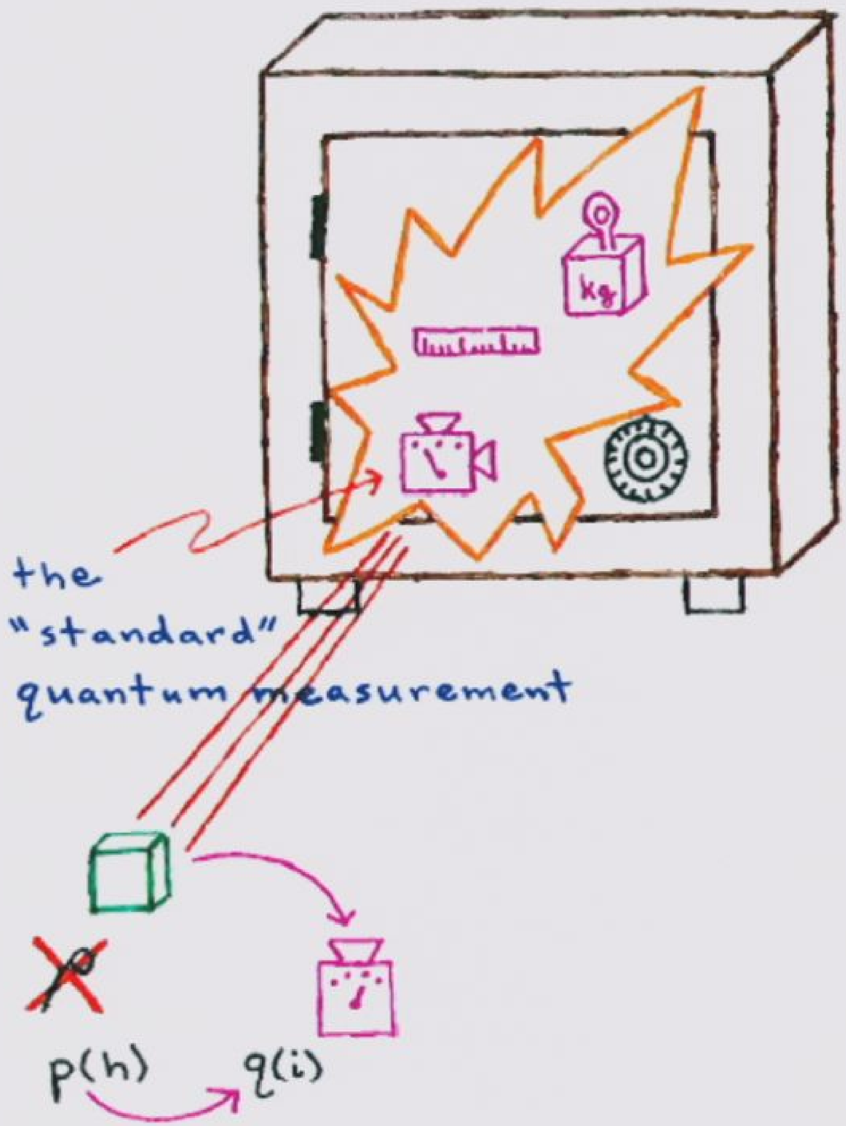


$$\vec{p} = \begin{bmatrix} p(0) \\ p(1) \\ \vdots \\ p(n) \end{bmatrix}$$

$$\sum_h p(h) = 1$$

$$p(h) \geq 0 \quad \forall h$$

Bureau of Standards



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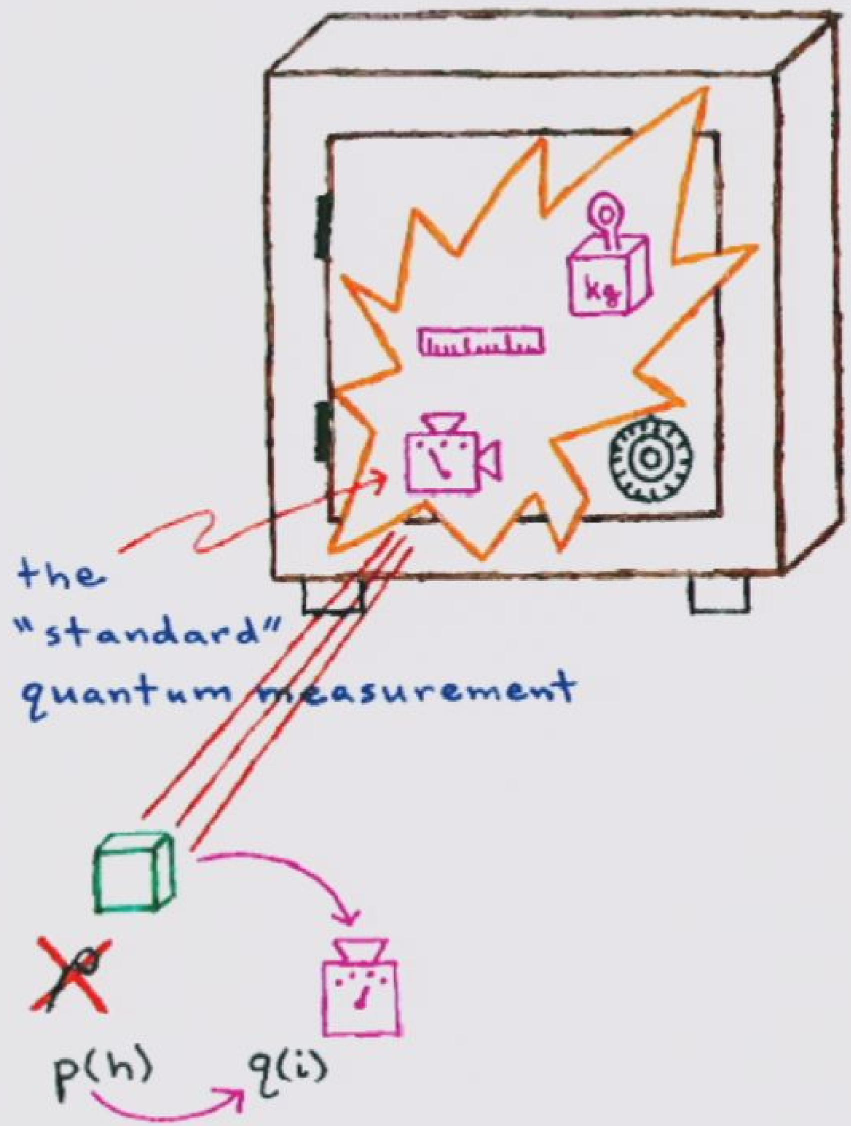

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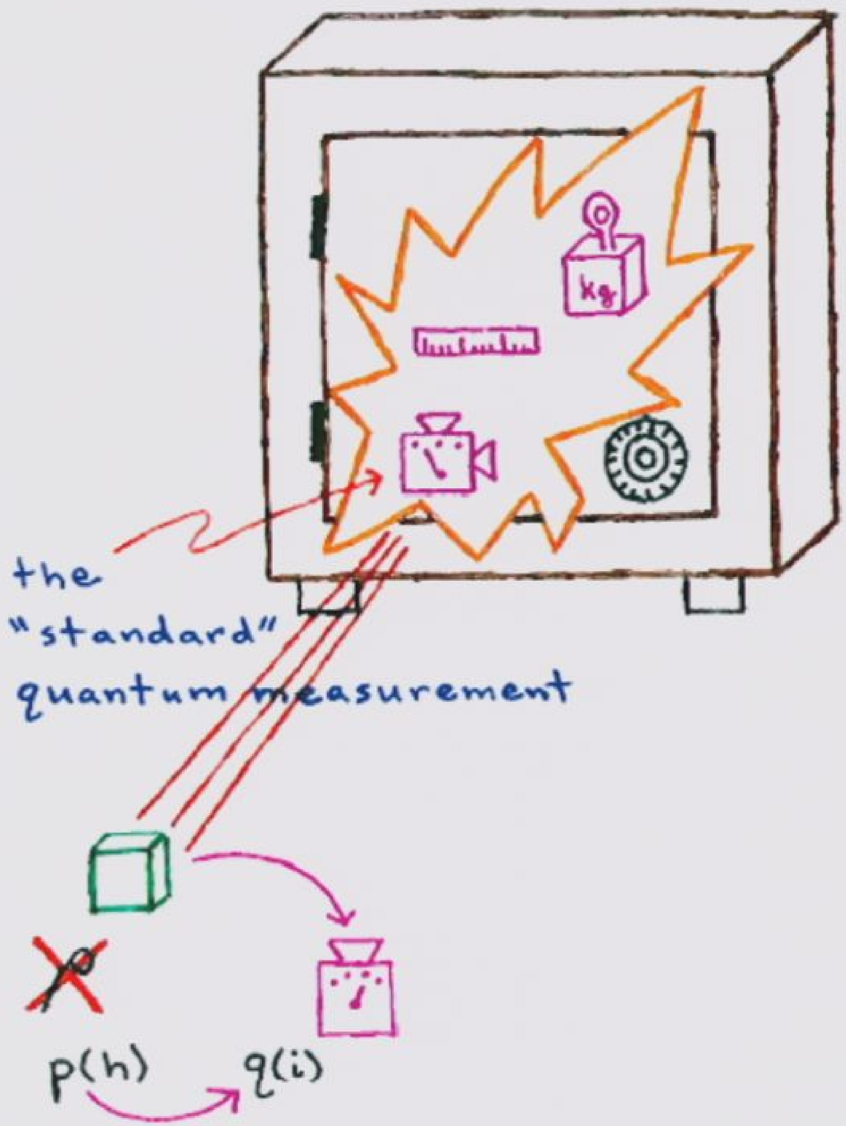

Density Operators

linear operators

$$\rho \in \mathcal{L}(\mathcal{H}_d)$$

complex vector space

catalog of uncertainties

1) $\rho^{\dagger} = \rho$

A Single-User Theory

- probability theory
- quantum theory

"The Bayesian, subjectivist, or coherent, paradigm is egocentric. It is a tale of one person contemplating the world and not wishing to be stupid (technically incoherent). He realizes that to do this his statements of uncertainties must be probabilistic."

— D. V. Lindley

The hypothesis that there is an external world, not dependent on human minds, made of something, is so obviously useful and so strongly confirmed by experience down through the ages that we can say without exaggerating that it is better confirmed than any other empirical hypothesis.

— Martin Gardner

Jim Hartle 1968 *(Section IV)* Interpretation of Quantum Mechanics *(suitably modified)*

Am. J. Phys. 36, 704–712 (1968)

A quantum state, being a summary of the observers' information about an individual physical system, changes both by dynamical laws and whenever the observer acquires new information about the system through the process of measurement. The existence of two laws for the evolution of the state vector becomes problematical only if it is believed that the state vector is an objective property of the system. If the state of a system is defined as a list of [*experimental*] propositions together with [*their probabilities of occurrence*], it is not surprising that after a measurement the state must be changed to be in accord with the new information. The "reduction of the wave packet" does take place in the consciousness of the observer, not because of any unique physical process which takes place there, but only because the state is a construct of the observer and not an objective property of the physical system.

My Favorite Convex Set

(My Favorite Shape)

Christopher Fuchs
$\hat{\text{PI}}$ - Perimeter Inst.

Work with:

Marcus Appleby
Åsa Ericsson
Rüdiger Schack

Jim Hartle 1968 *(Section IV)* Interpretation of Quantum Mechanics *(suitably modified)*

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