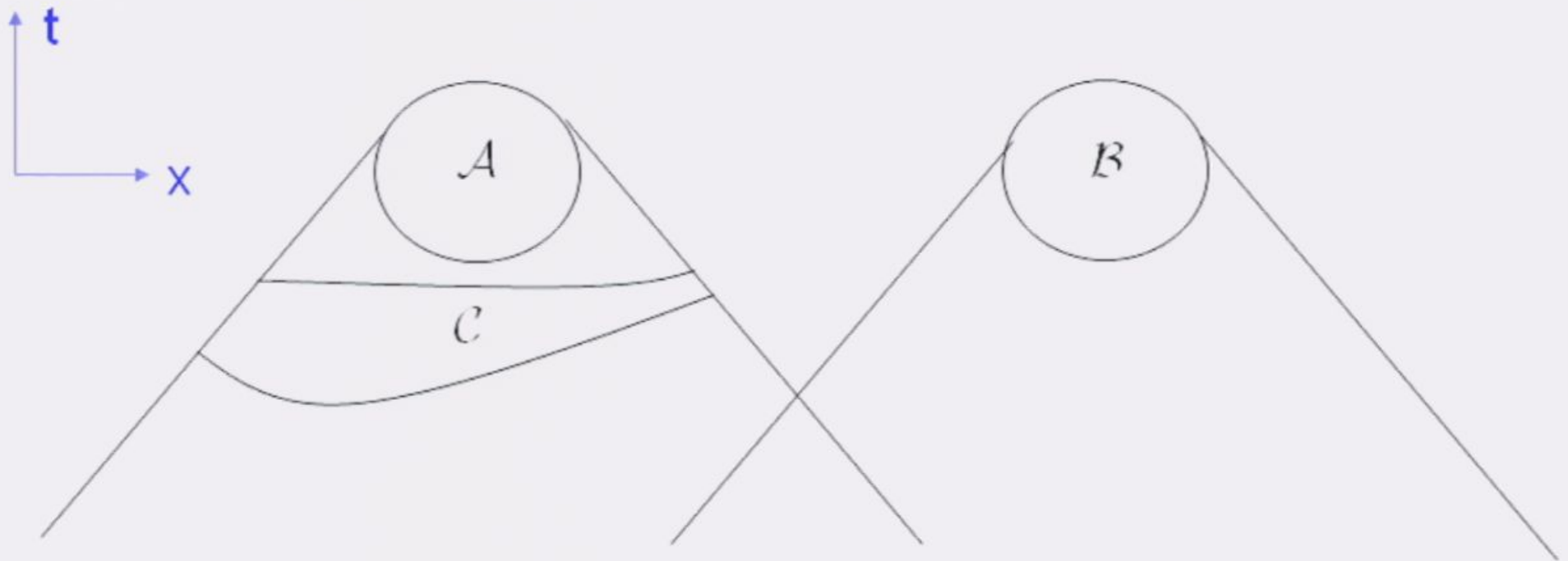


Title: Foundations of Quantum Mech. (PHYS 639) - Lecture 9

Date: Dec 10, 2009 11:00 AM

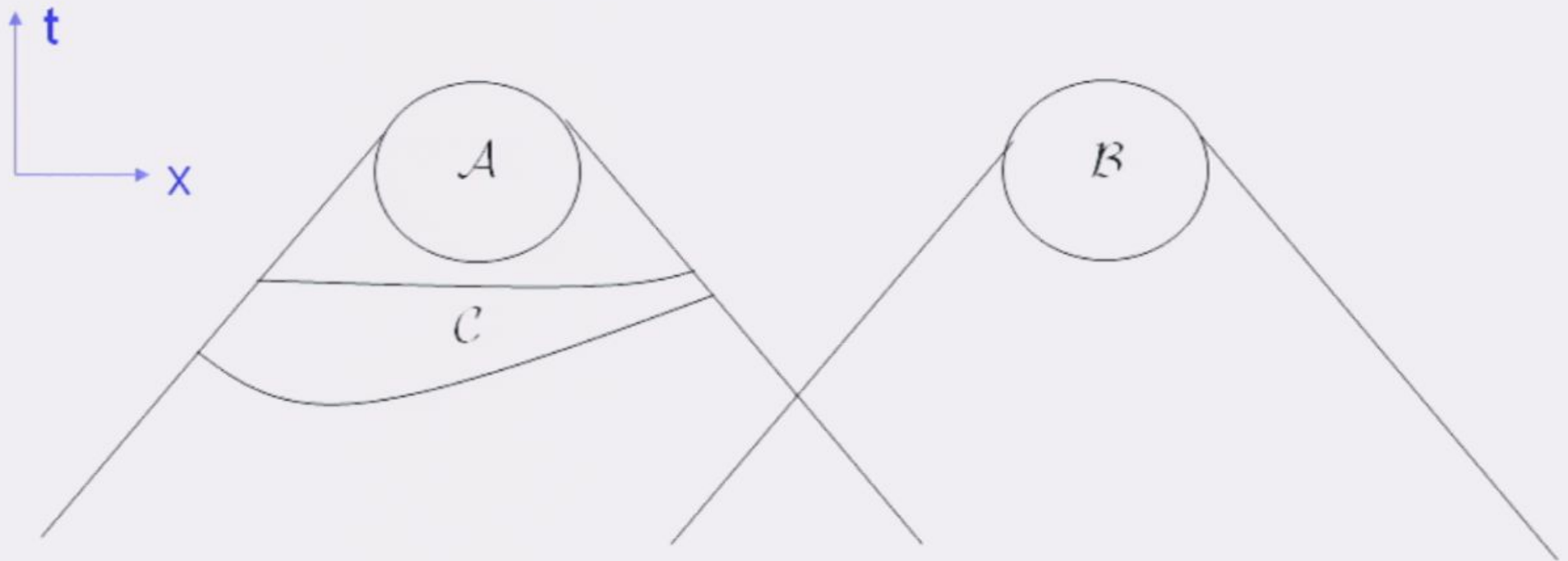
URL: <http://pirsa.org/09120073>

Abstract:



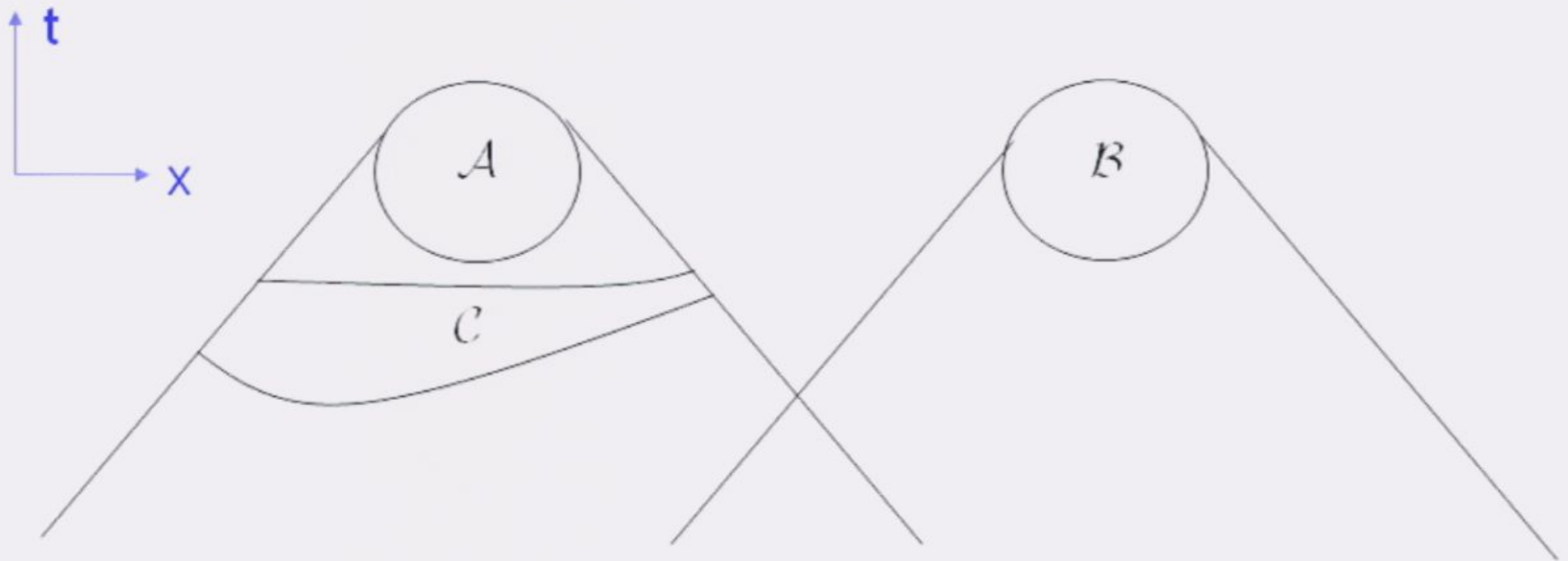
“A theory will be said to be **locally causal** if the probabilities for the values of local beables in a space-time region  $A$  are unaltered by specification of values of local beables in a space-time region  $B$ , when what happens in the backward light cone of  $A$  is already sufficiently specified, for example by a full specification of local beables in a space-time region  $C$ .”

-- J. S. Bell



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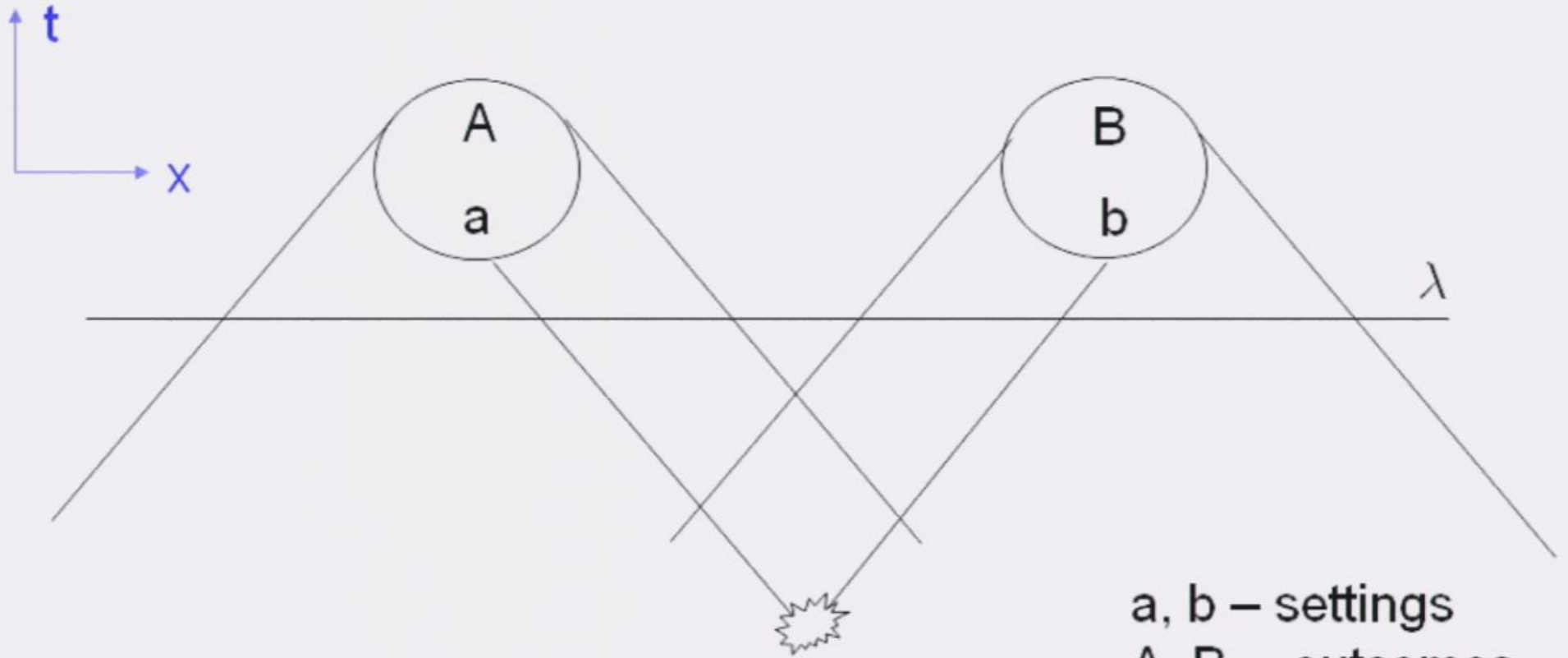


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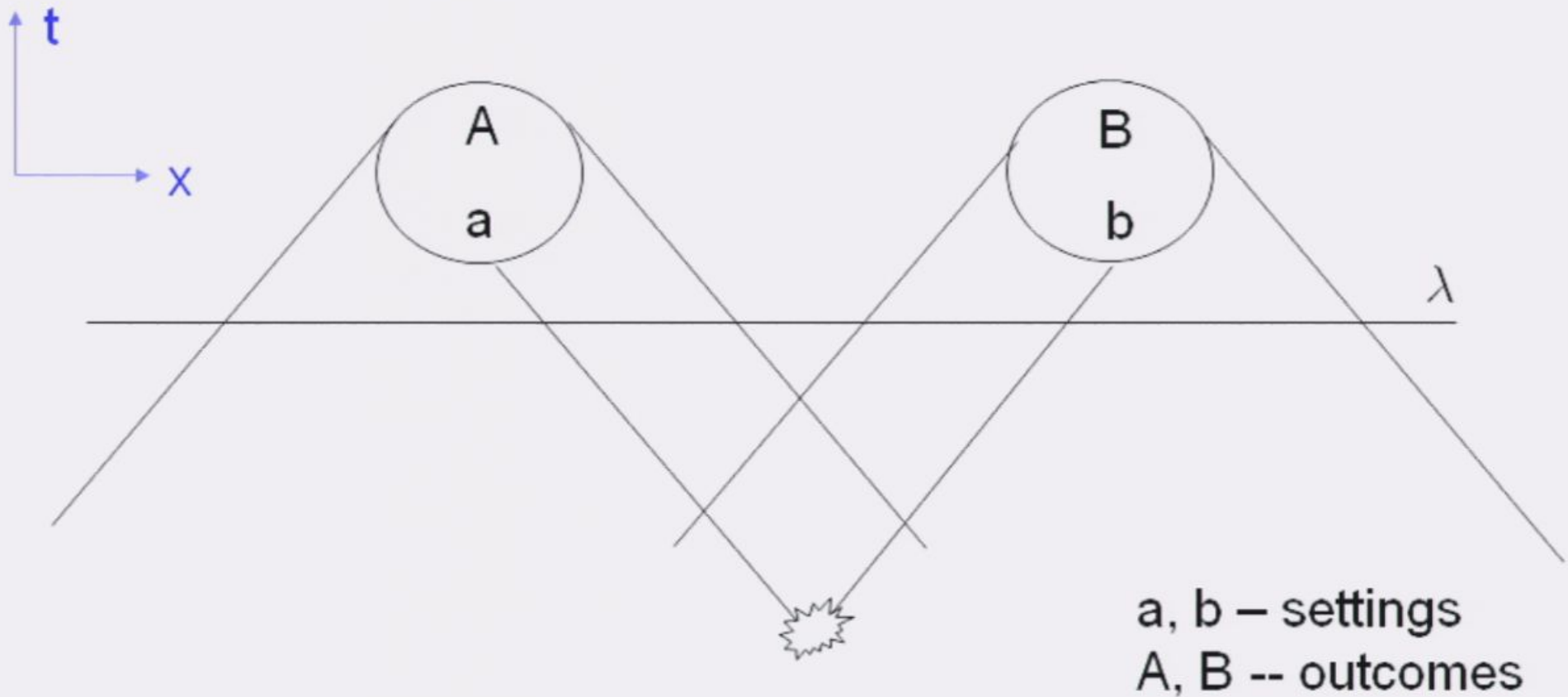
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## Local causality

$$p(X_A | X_B, \lambda_C) \equiv p(X_A | \lambda_C)$$

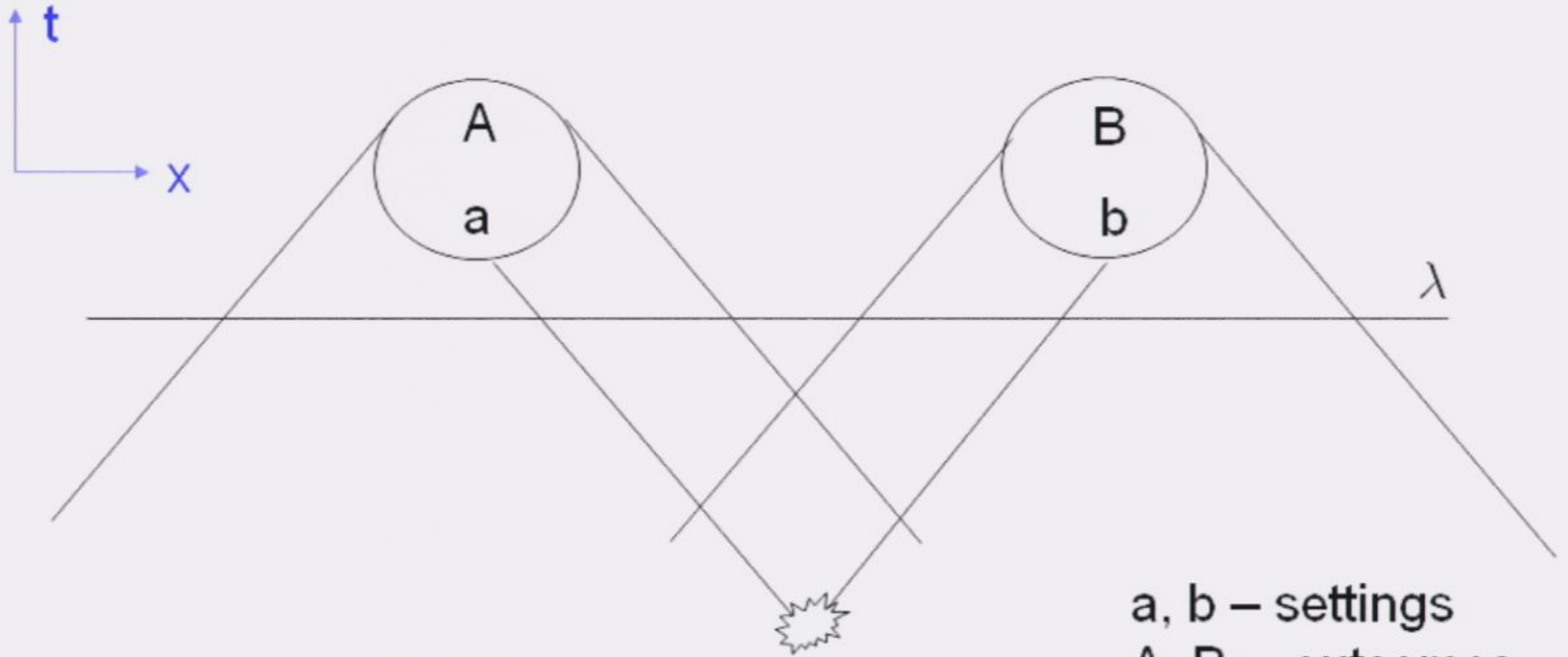


a, b – settings  
A, B -- outcomes



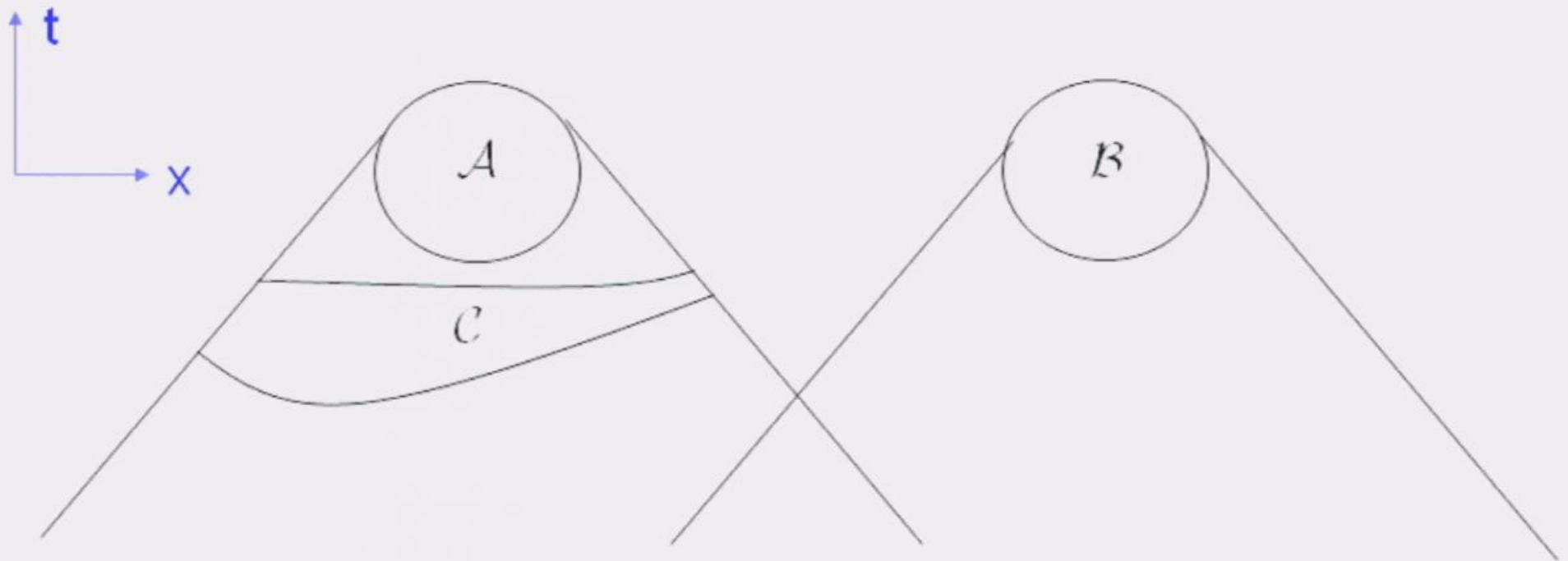
Locality causality implies

$$p(A|a, b, B, \lambda) = p(A|a, \lambda)$$



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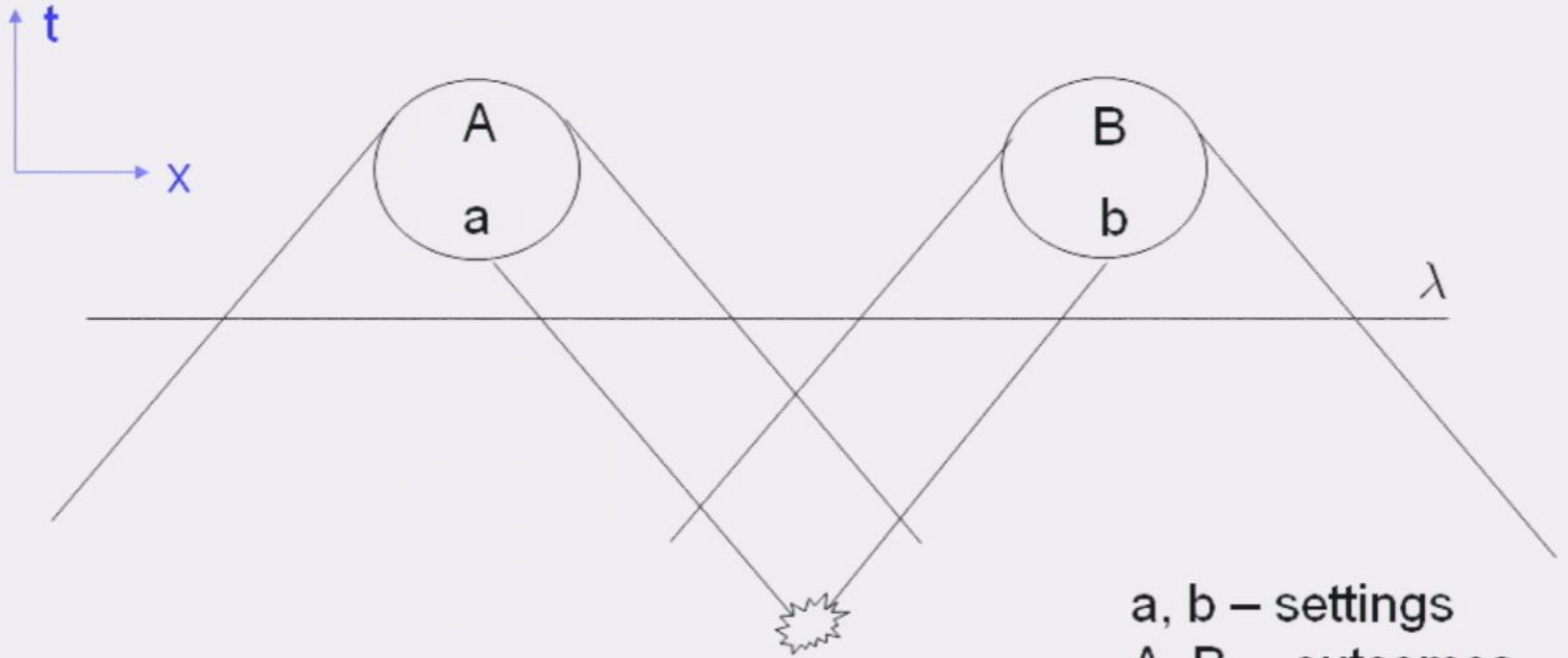
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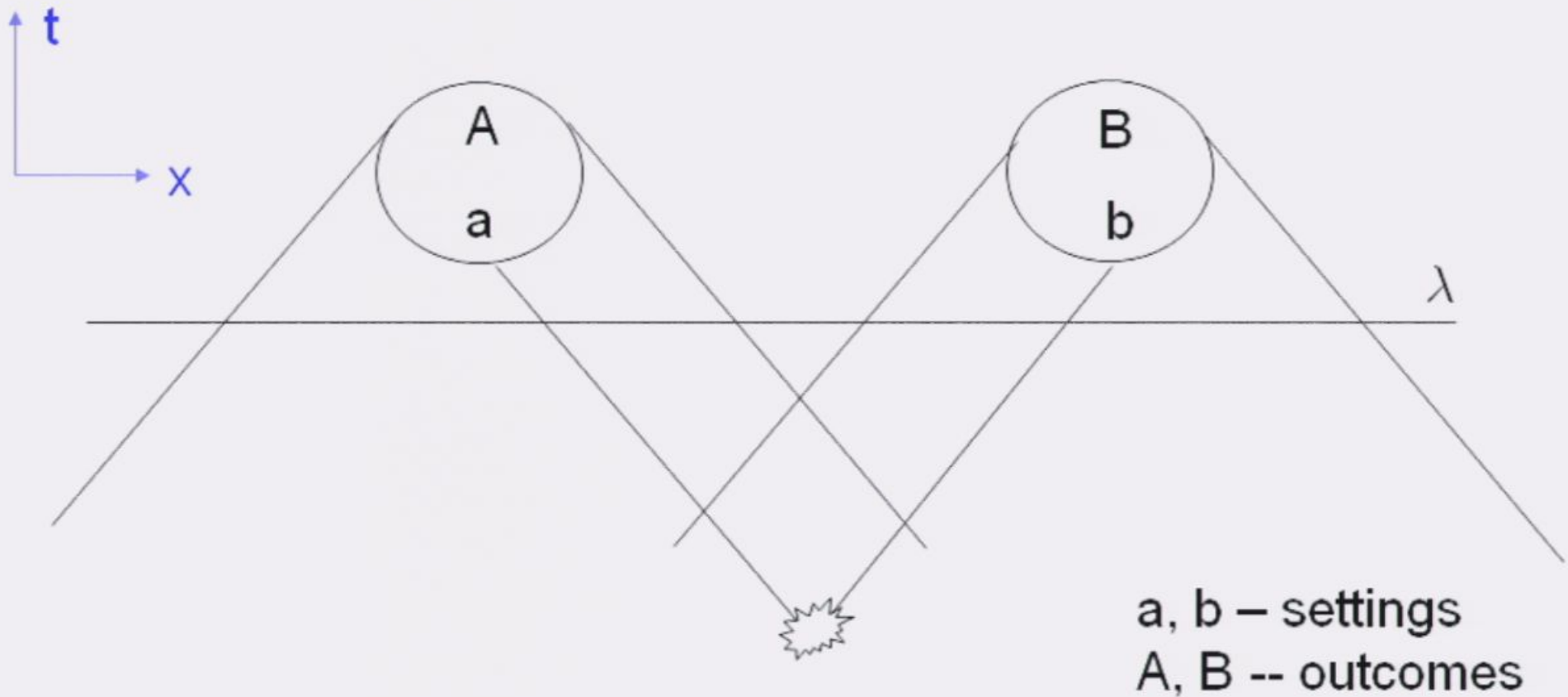
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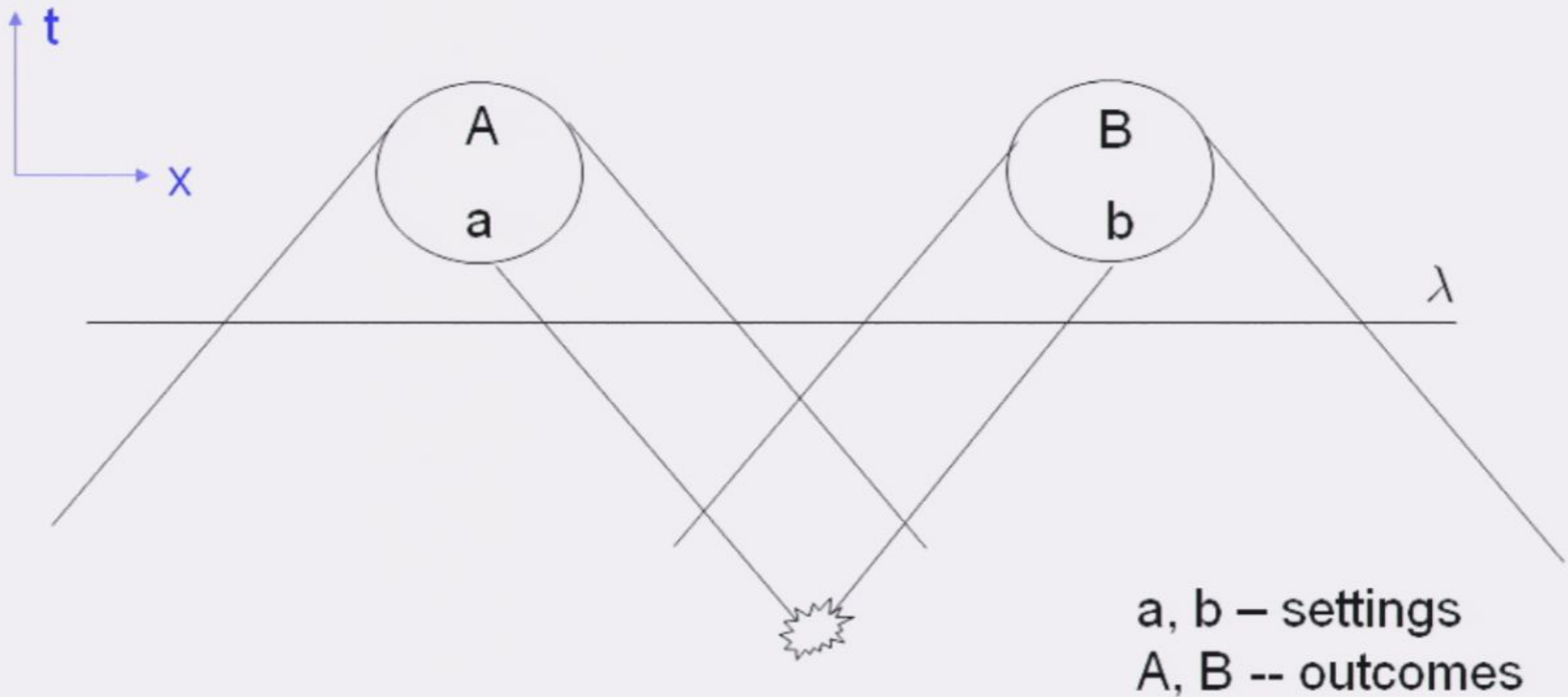


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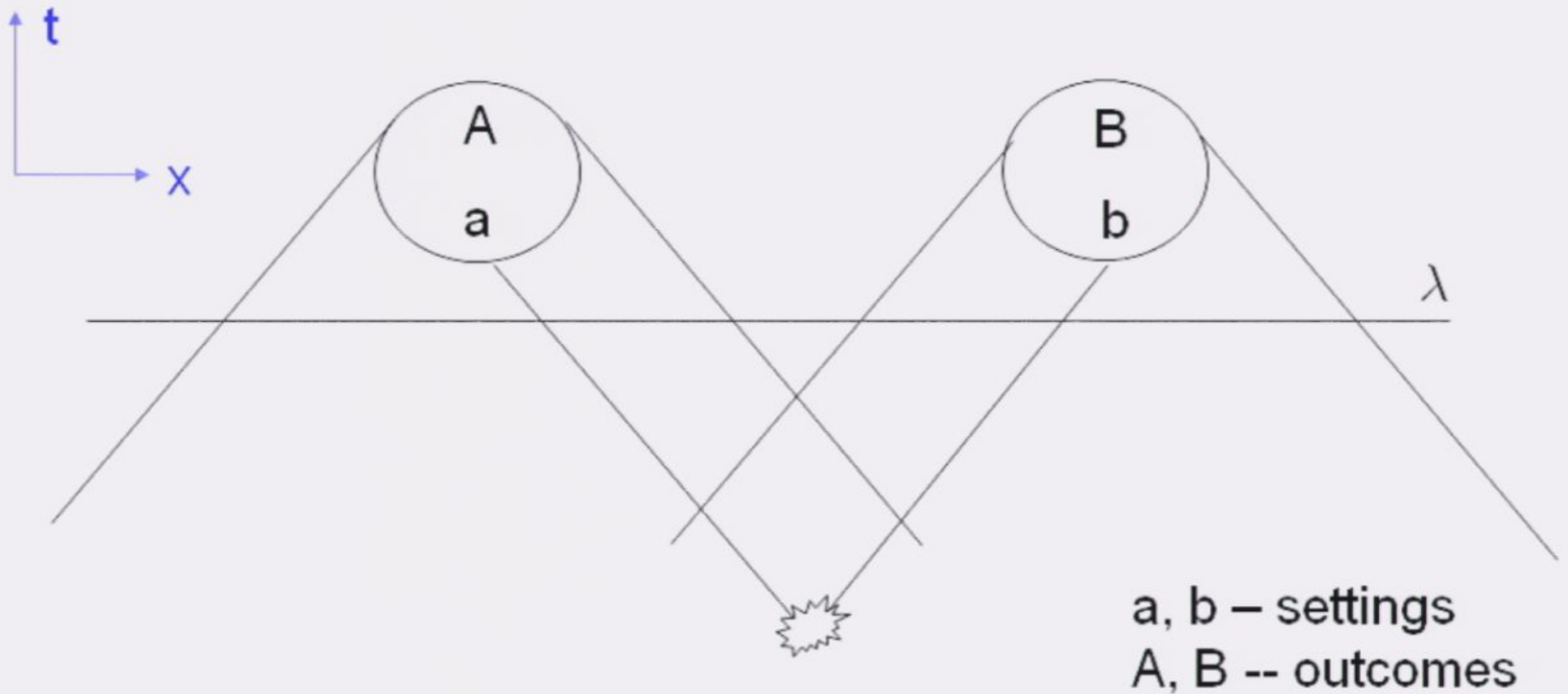
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$$p(A|a, b, B, \lambda) = p(A|a, \lambda)$$

$$p(B|a, b, A, \lambda) = p(B|b, \lambda)$$

and implies *factorizability*

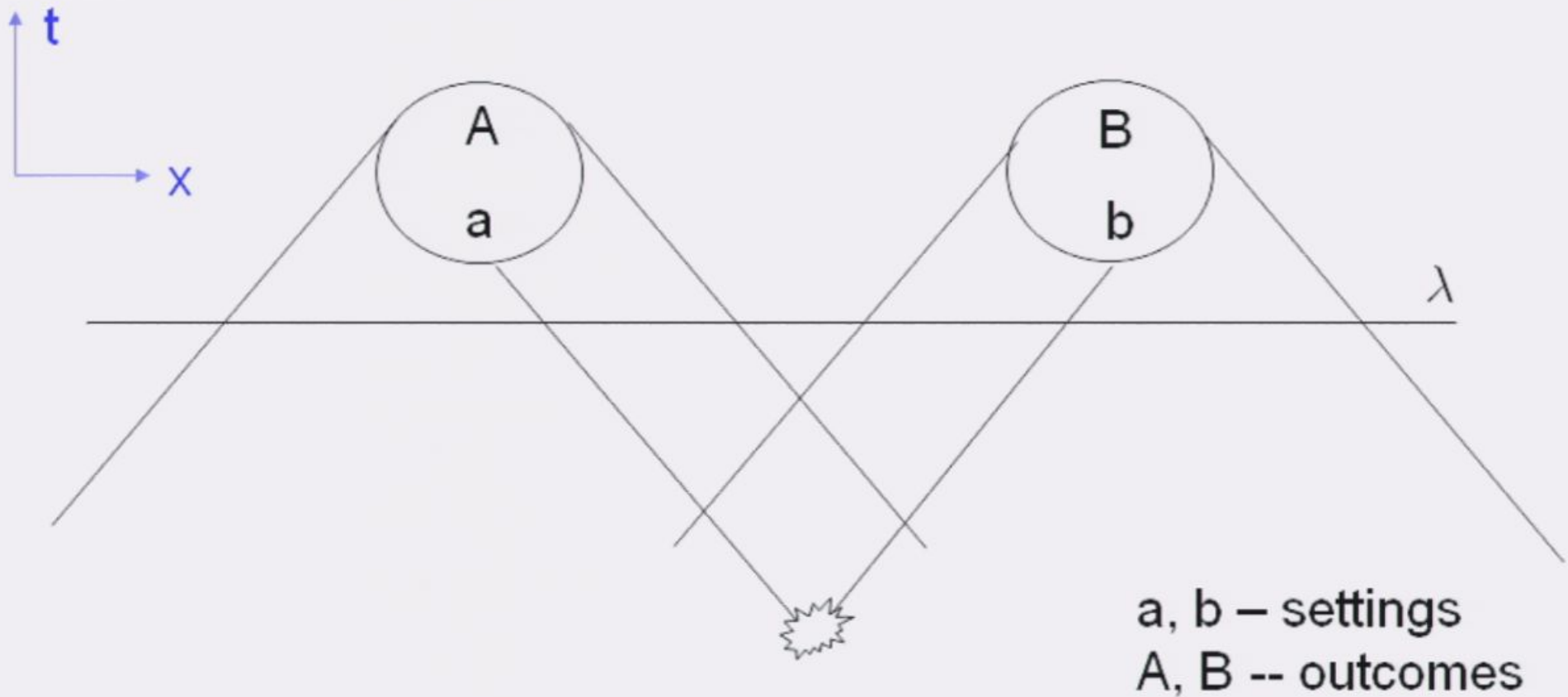
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Factorizability from local causality

Recall Bayes' rule

$$p(A, B) = p(A|B)p(B)$$



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$$\frac{1}{4} [ p(\text{agree}|ab) + p(\text{agree}|ab') + p(\text{agree}|a'b) + p(\text{disagree}|a'b') ] \leq 3/4$$

Evidence for Atoms

How Big Is A Molecule?

$$p(\text{agree} | a, b) = p(+, + | a, b) + p(-, - | a, b)$$

$$p(+, + | a, b) = \int d\lambda p(\lambda) p(+, + | a, b, \lambda)$$

$\uparrow$   
 $p(+ | a, \lambda) p(+ | b, \lambda)$


 $\frac{\pi}{2}$ 

 $\pi \rightarrow \mu \nu$ 
 $\bar{\pi} \rightarrow \bar{\mu} \bar{\nu}$ 


← 1 mile →







$$\frac{1}{4} [ p(\text{agree}|ab) + p(\text{agree}|ab') + p(\text{agree}|a'b) + p(\text{disagree}|a'b') ] \leq 3/4$$

Define  $C(a, b) = (+1)p(\text{agree}|ab) + (-1)p(\text{disagree}|ab)$

$$|C(a, b) + C(a', b) + C(a, b') - C(a', b')| \leq 2$$

The Clauser-Horn-Shimony-Holt (CHSH) inequality



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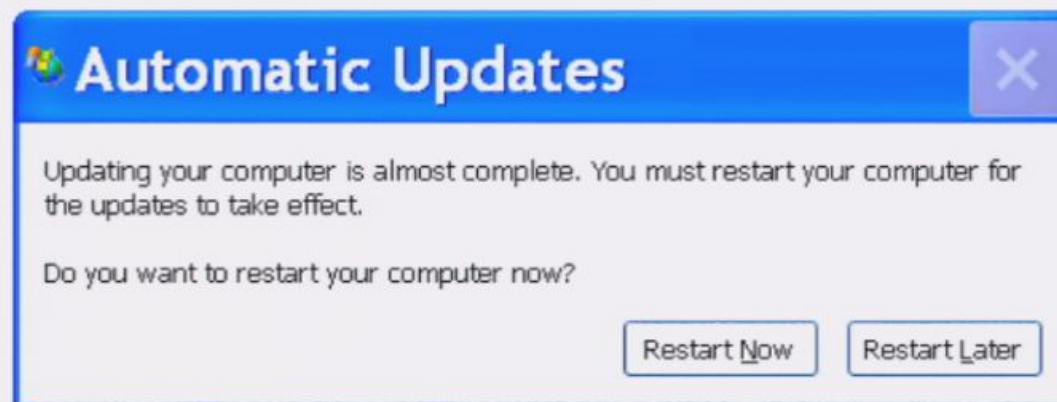
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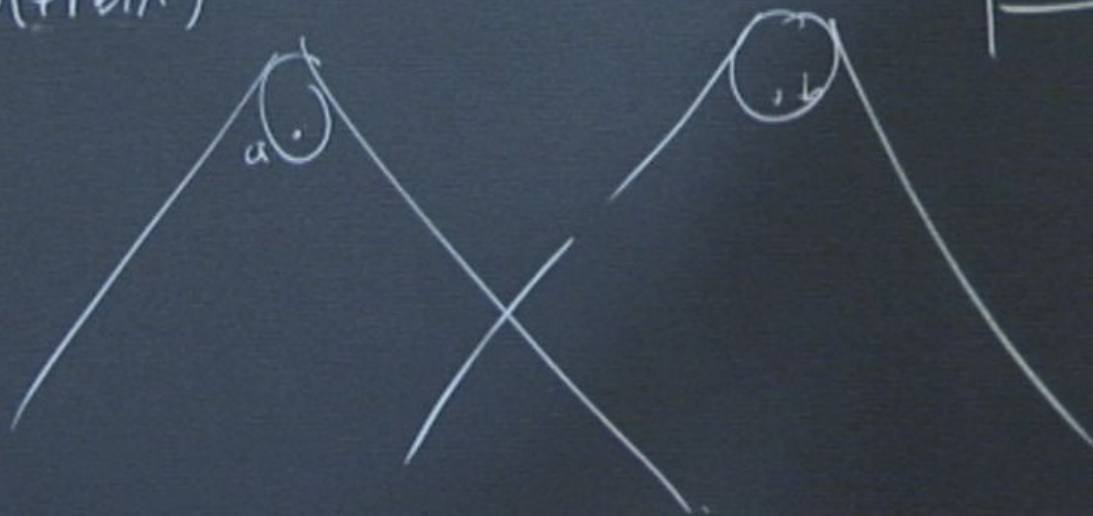
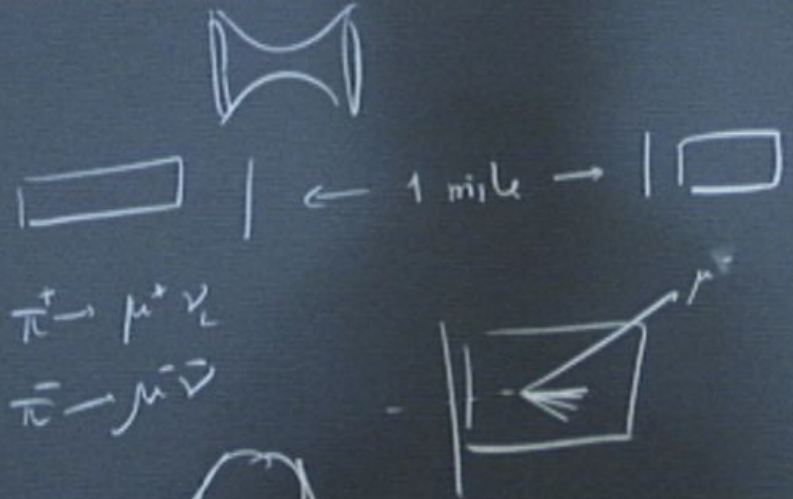
# Responses to Bell's theorem

- Experimental loopholes
- Superdeterminism  $p(a, b, \lambda) \neq p(a)p(b)p(\lambda)$ 
  - free will
  - complexity
  - causal connection in the past

$$= p(+,+|a,b) + p(-,-|a,b)$$

$$p(b) = \int d\lambda \mu(\lambda) p(+,+|a,b,\lambda)$$

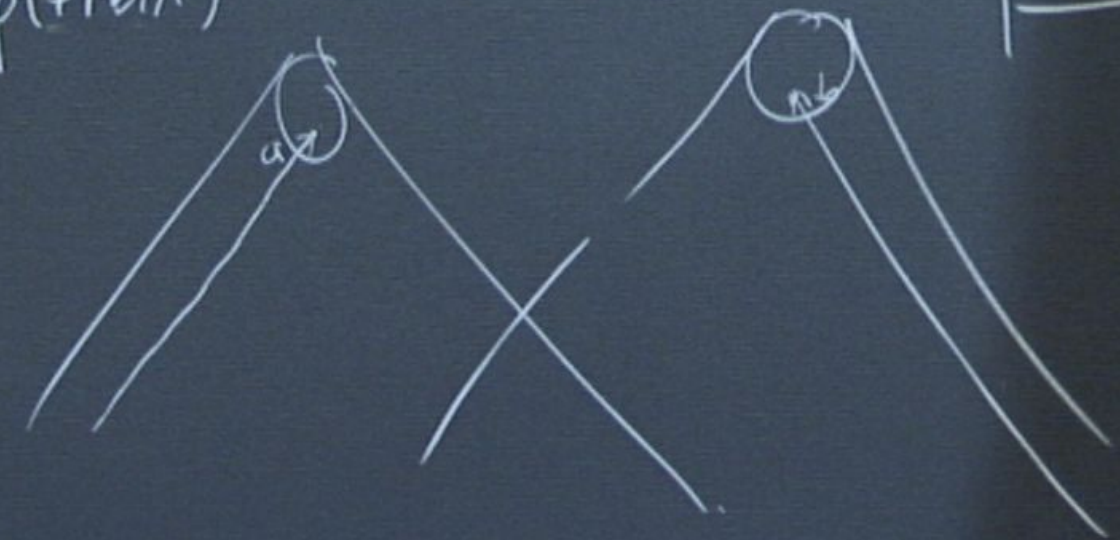
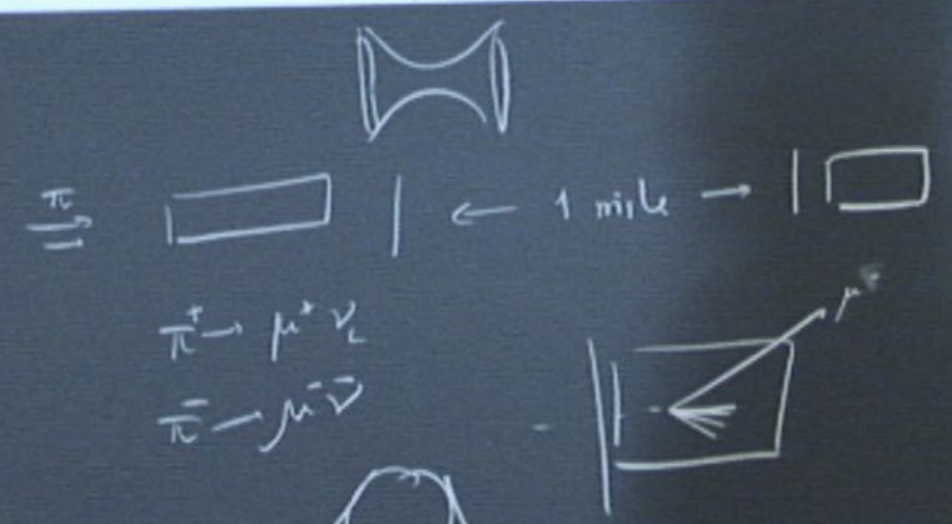
$\uparrow$   
 $p(+|a,\lambda) p(+|b,\lambda)$


 $\rightleftharpoons$ 


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$\underbrace{\hspace{10em}}_{p(+|a,\lambda) p(+|b|\lambda)}$





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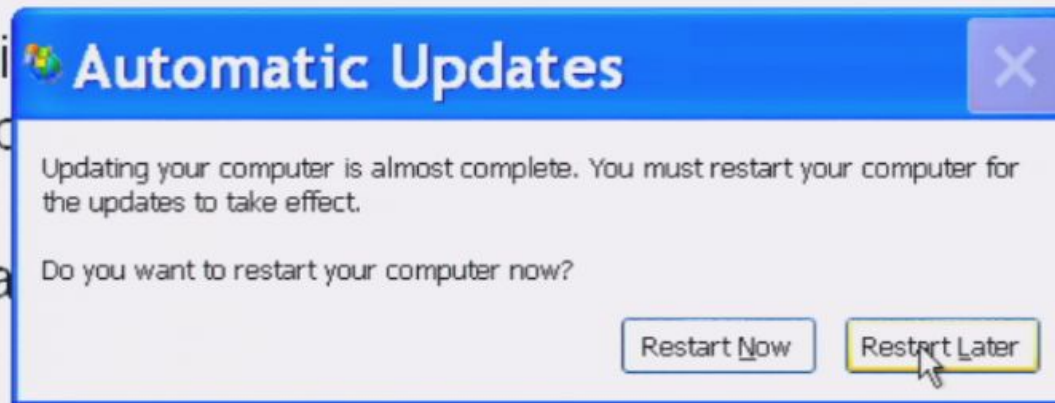
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- Failure of separability
  - how to recover *macroscopic* separability?

# History of nonlocality in realist theories

Einstein's Solvay argument, 1927

Einstein, Podolsky and Rosen, 1935

$\psi$ -completeness + Q Statistics + local causality  $\rightarrow$  contradiction

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So locality can't be achieved for *any* of the alternatives!

Locality offers no verdict on  $\psi$ -complete vs.  $\psi$ -incomplete



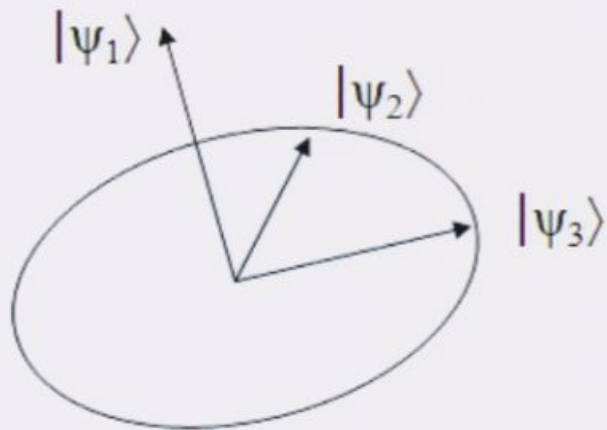
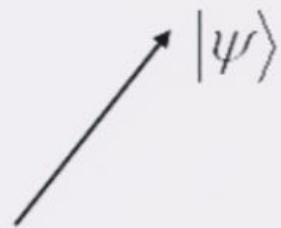
When seeking a realist explanation of Bell's theorem, there is a tension between:

- 1) No superluminal signalling (independence of statistics at one wing on choice of measurement at the other)
- 2) The necessity of superluminal causation (dependence of particular outcomes at one wing on choice of measurement at the other)

# Contextuality

# The traditional notion

# Deterministic hidden variable models

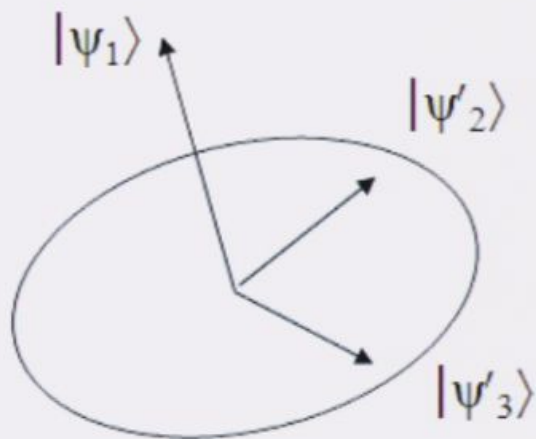
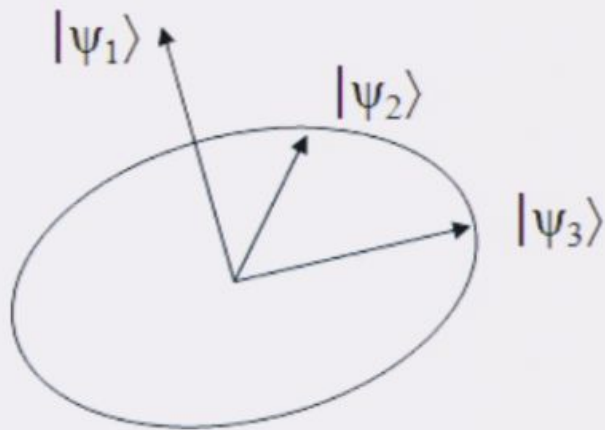


It is assumed that the outcomes are deterministic given.

$$|\langle \psi | \chi_k \rangle|^2 = \int d\lambda \mu(\lambda) \chi_k(\lambda)$$

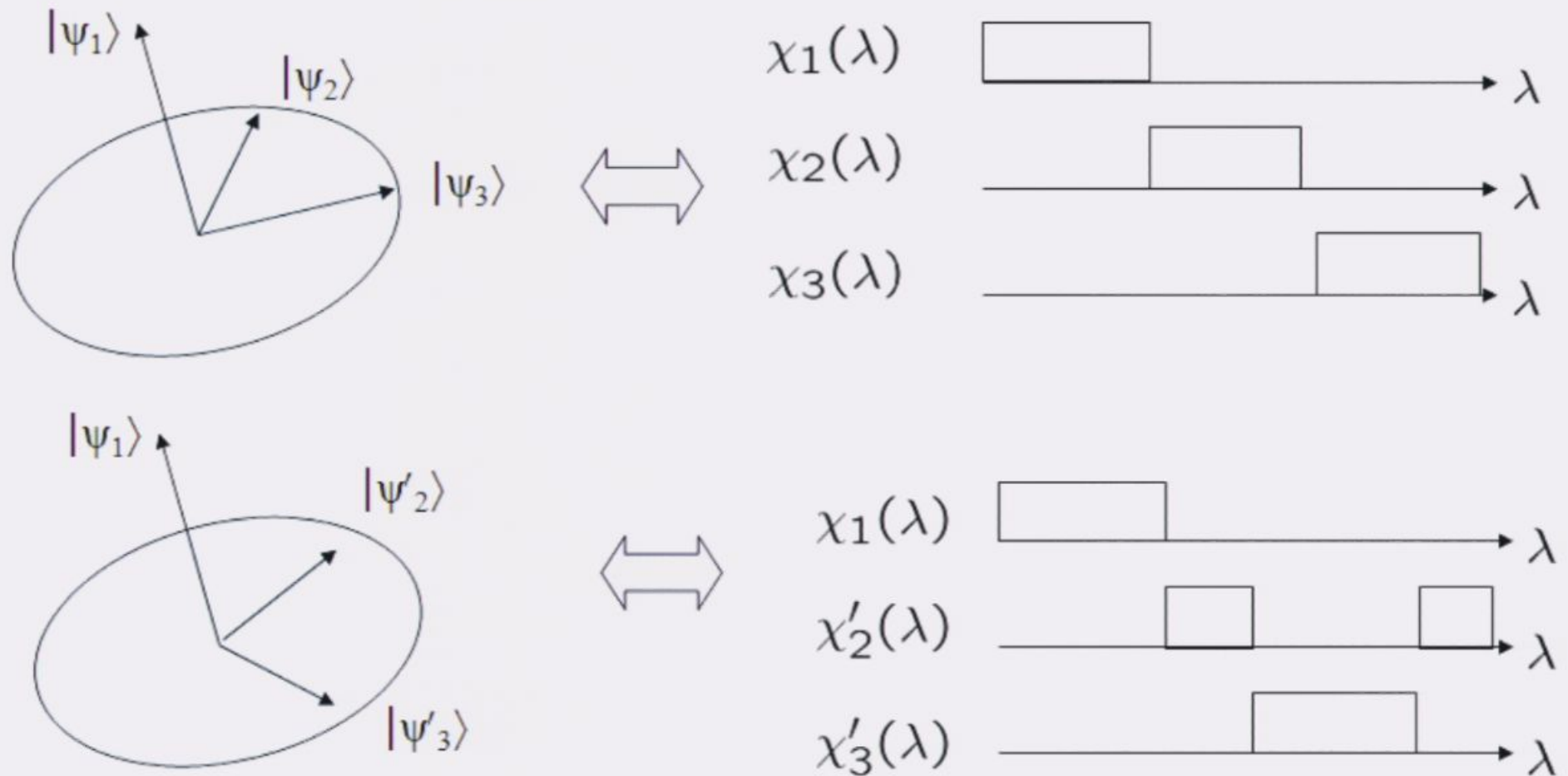
## Traditional notion of noncontextuality

A given projector may appear in many different measurements



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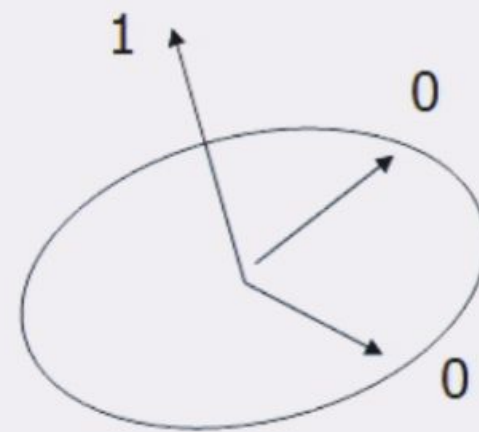
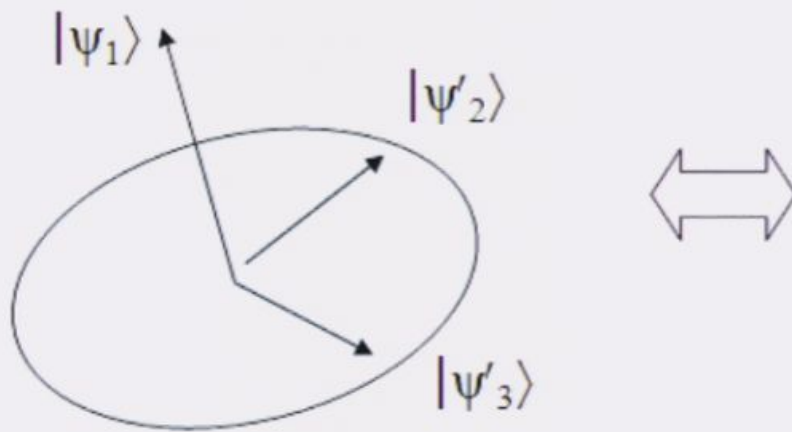
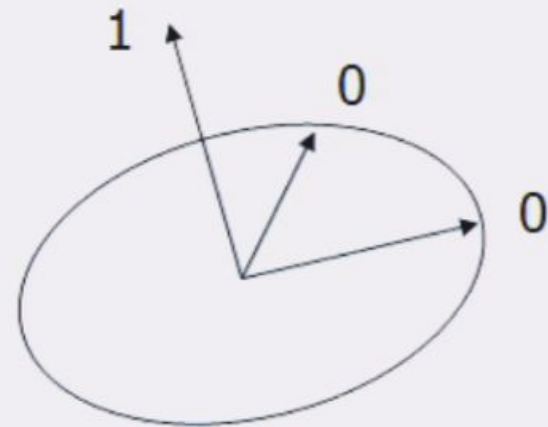
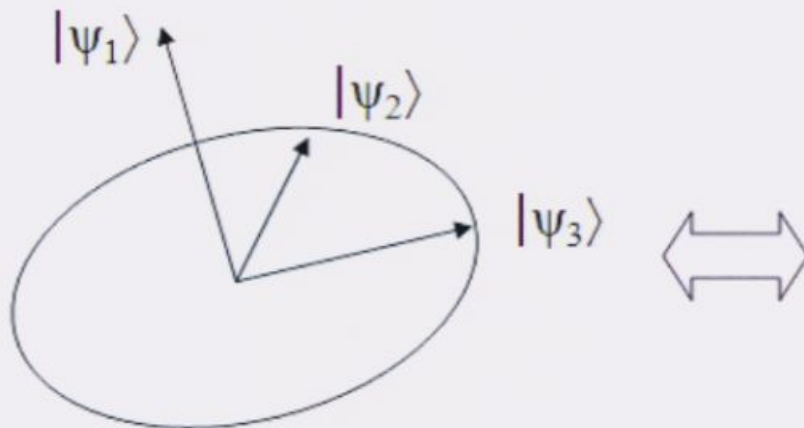
Every projector  $P$  is associated with the same  $\chi(\lambda)$



Alternatively,

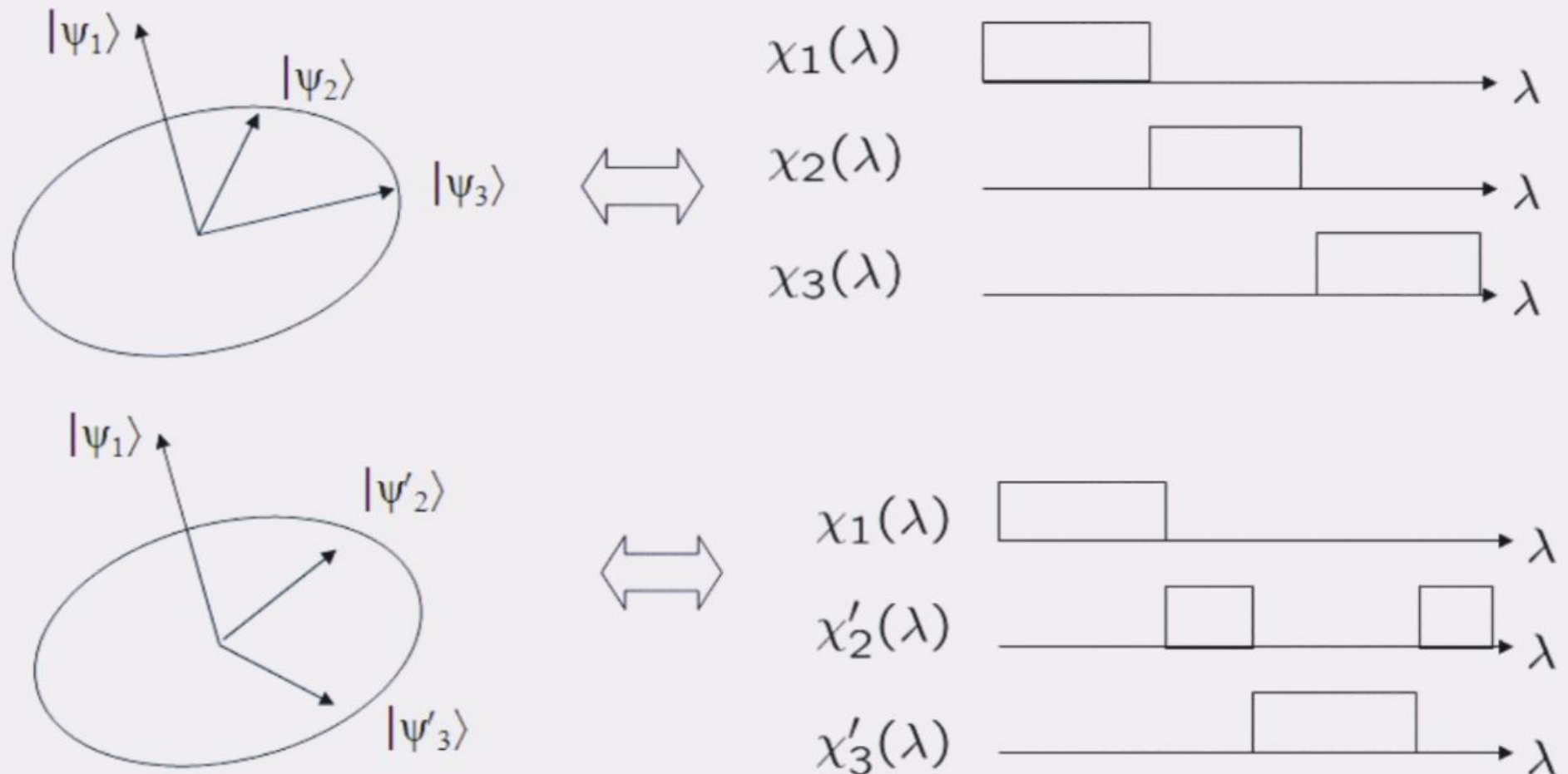
$$\chi_{\psi}(\lambda) = 0 \text{ or } 1$$

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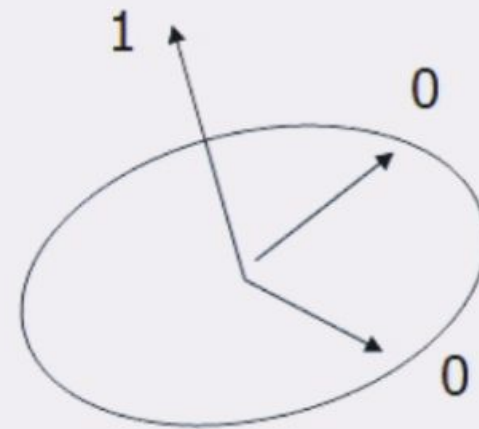
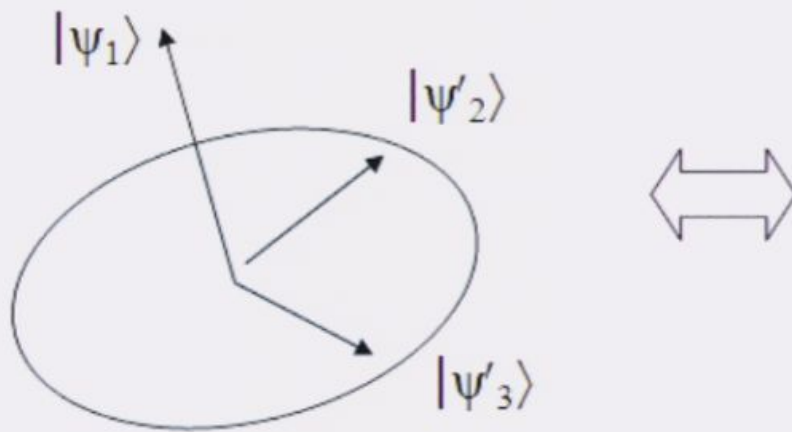
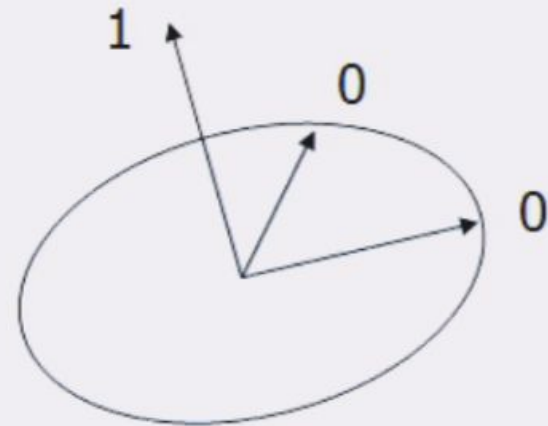
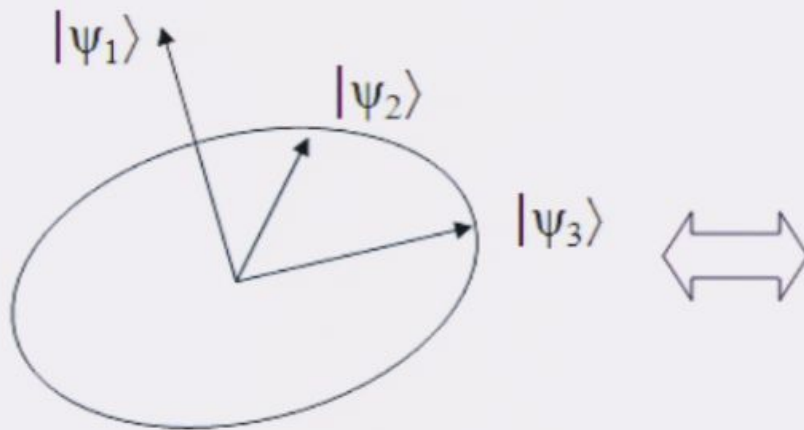
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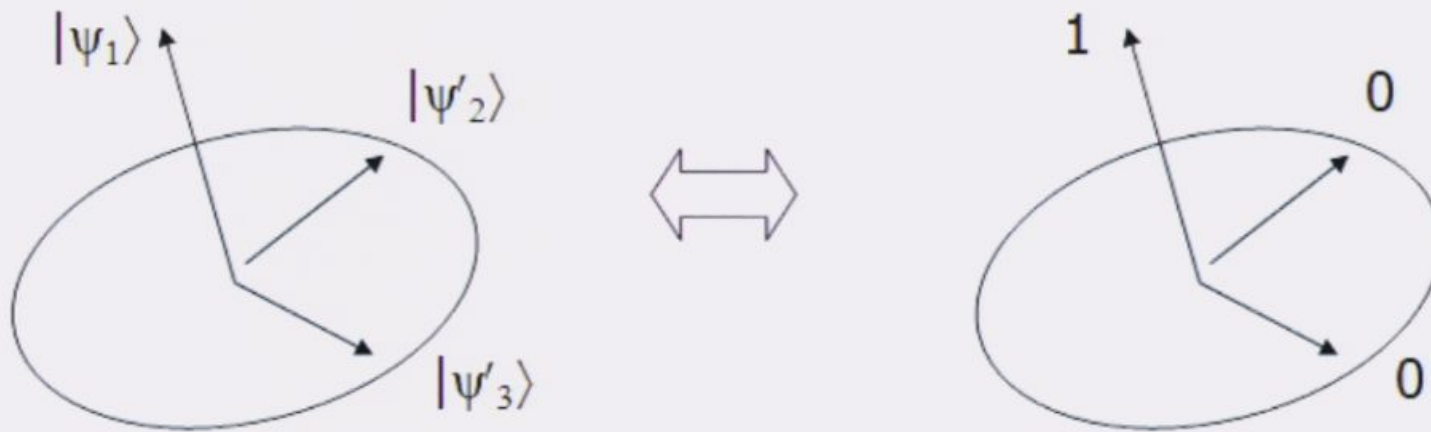
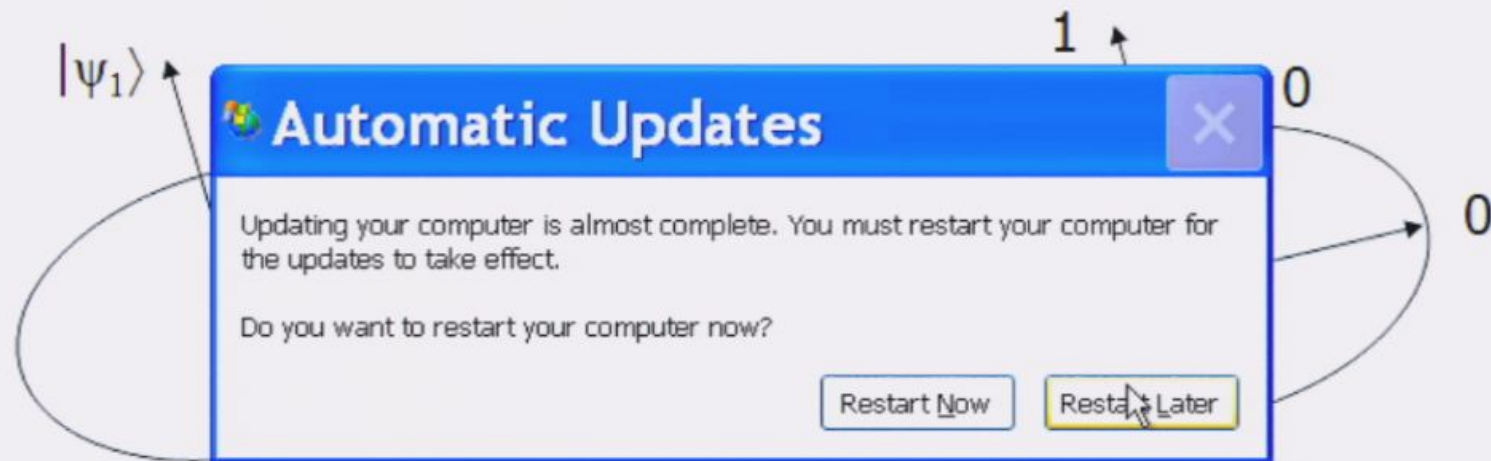
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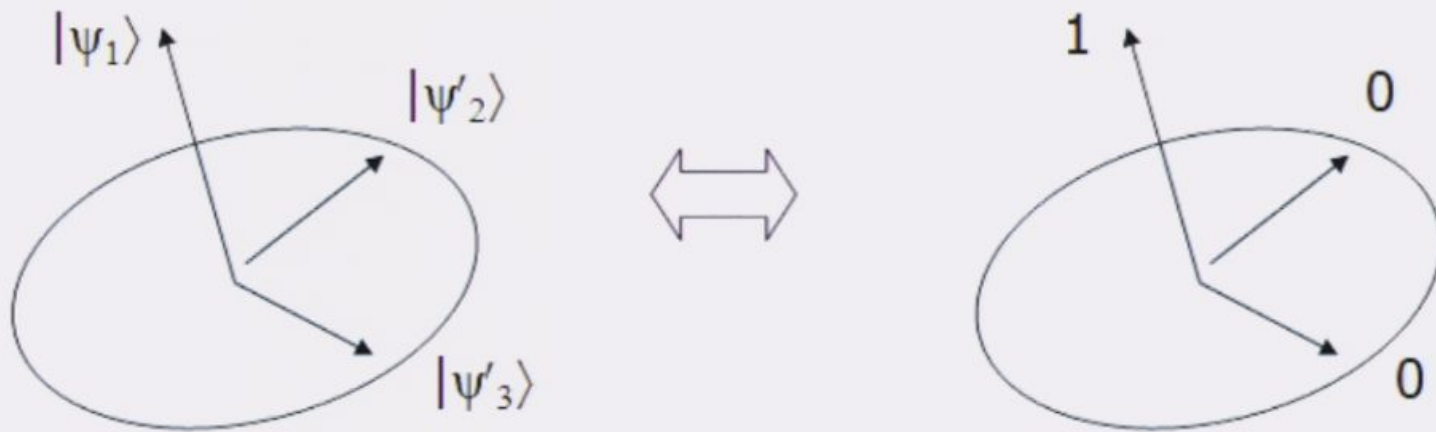
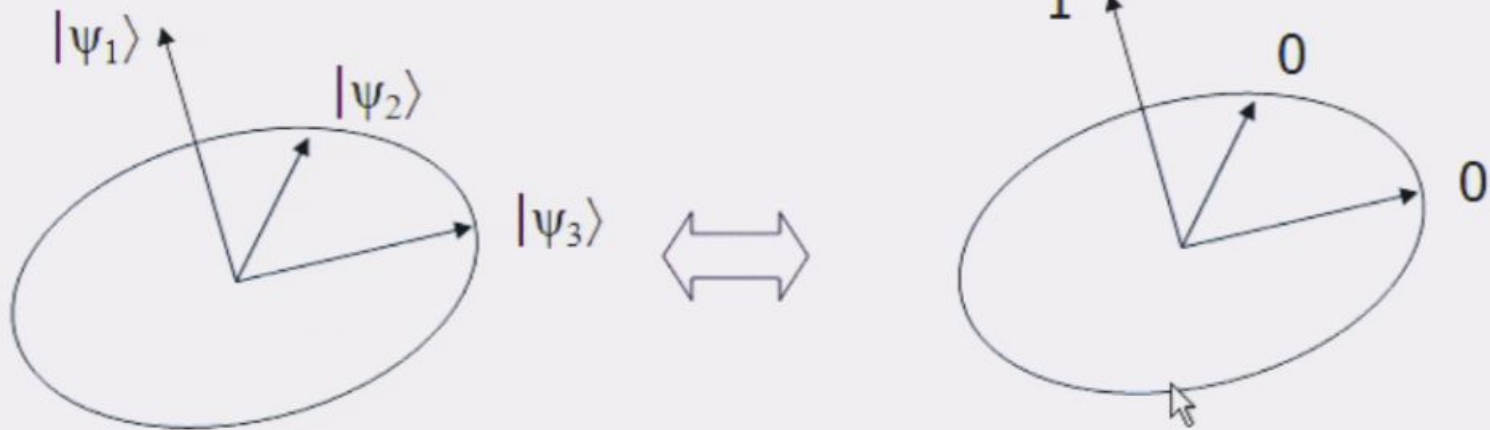
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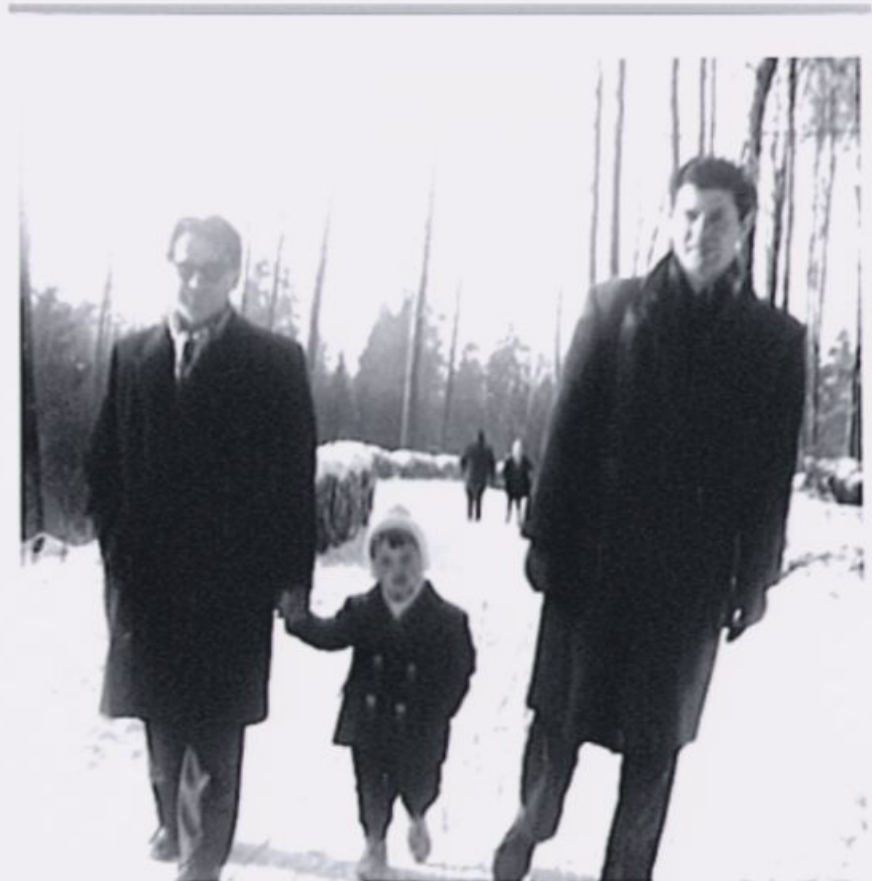
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John S. Bell



Ernst Specker (with son) and  
Simon Kochen

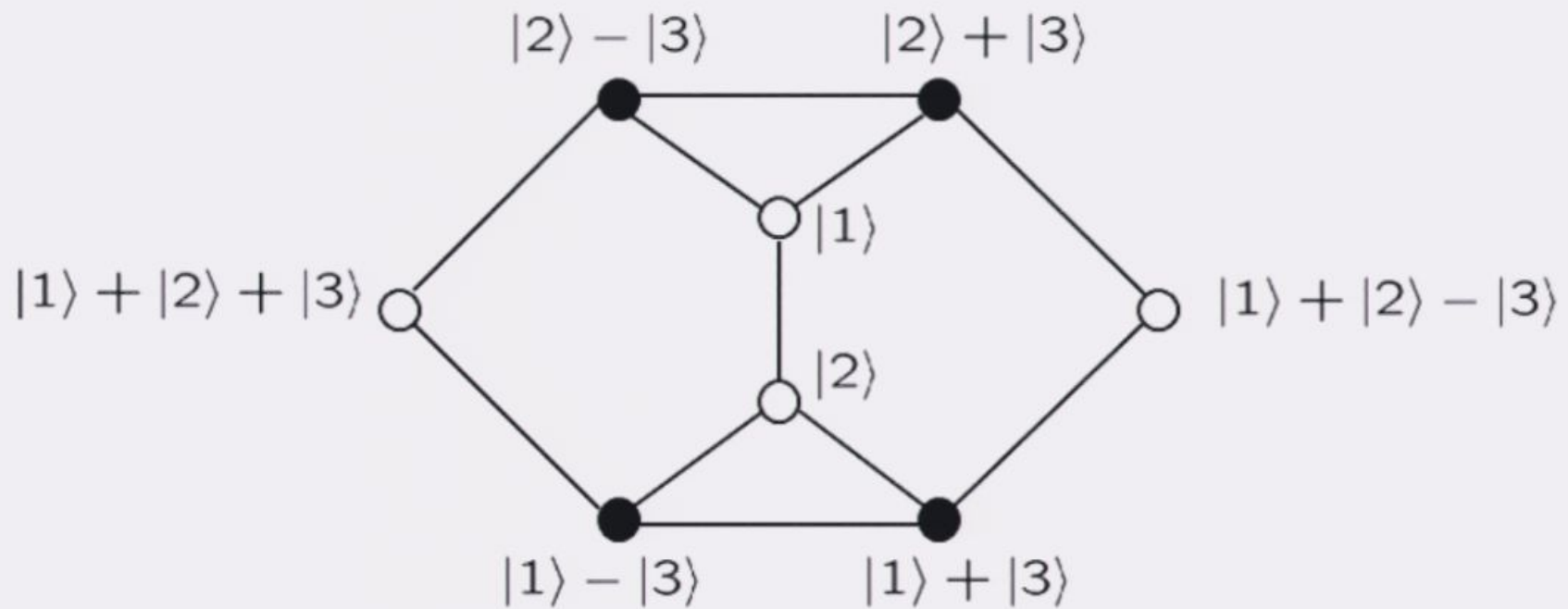
**Bell-Kochen-Specker theorem:** A noncontextual hidden variable model of quantum theory for Hilbert spaces of dimension 3 or greater is **impossible**.



## Example (Clifton's statistical 8 ray proof in 3d):

$$|\psi\rangle \circ \longrightarrow \chi_{|\psi\rangle}(\lambda) = 1$$

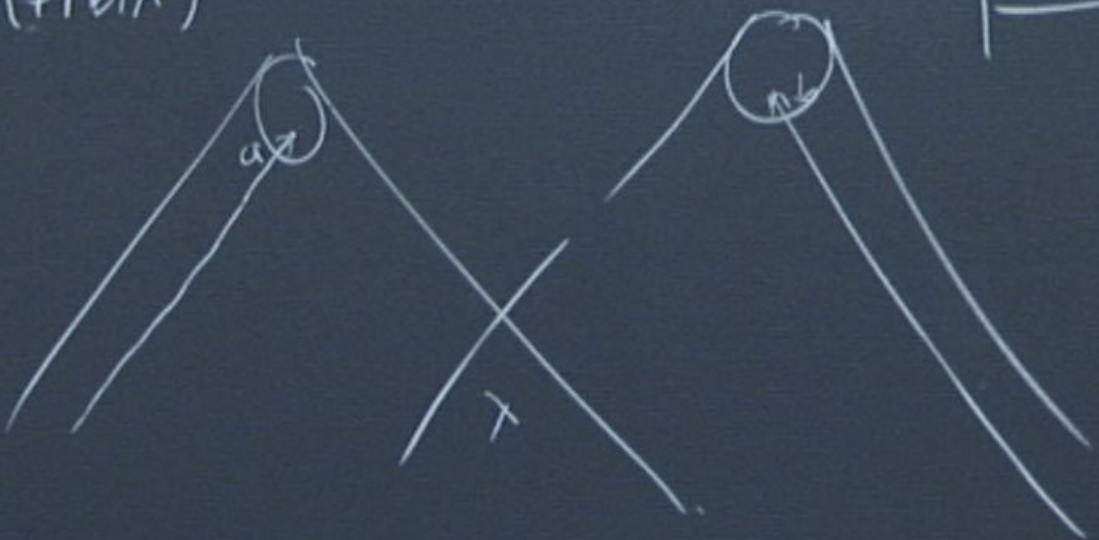
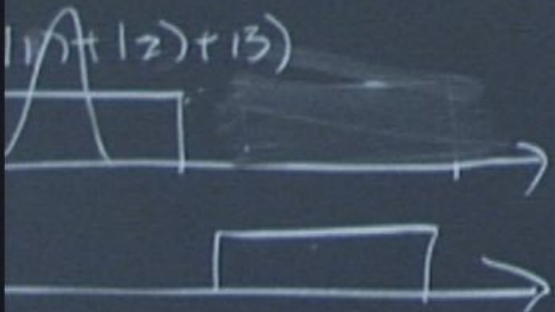
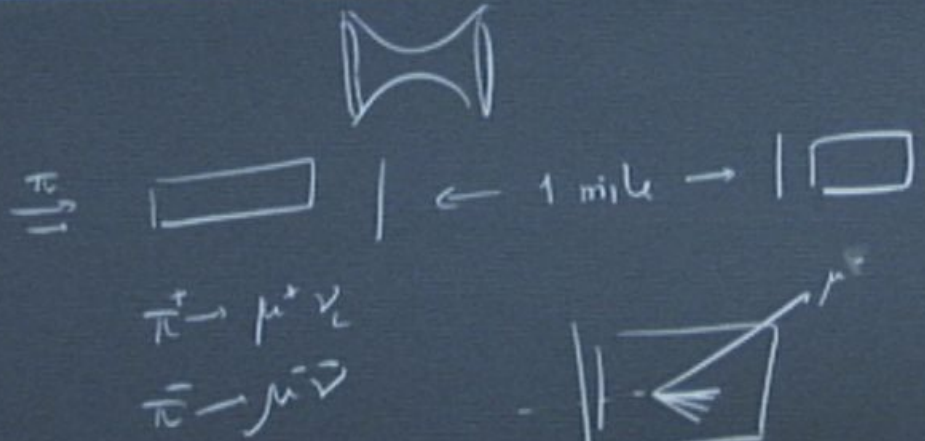
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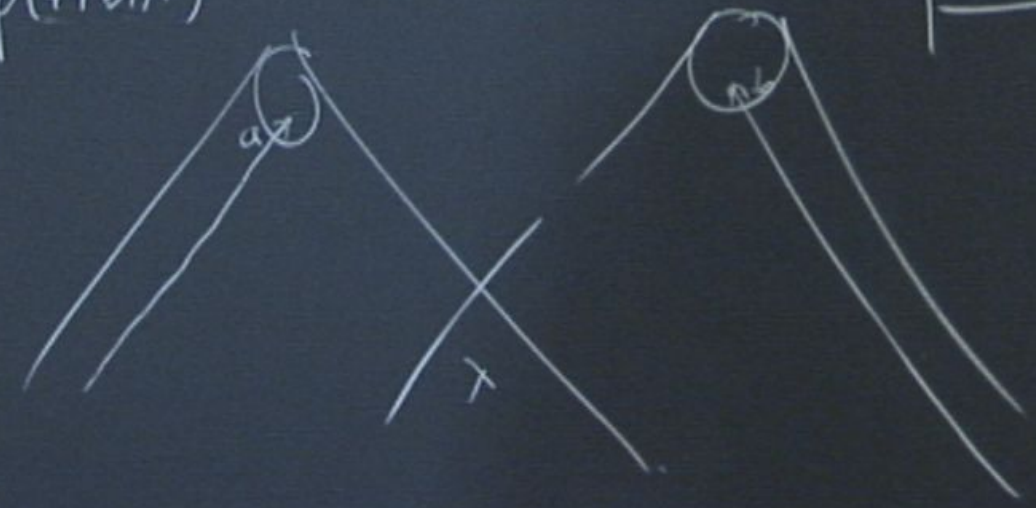
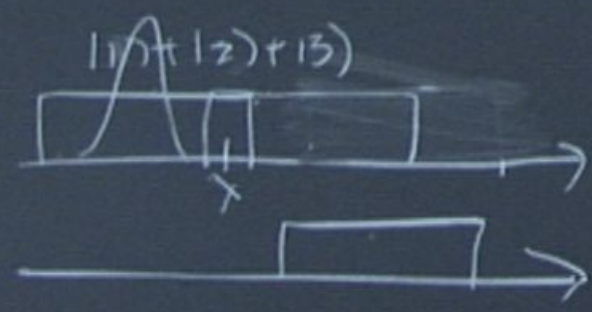
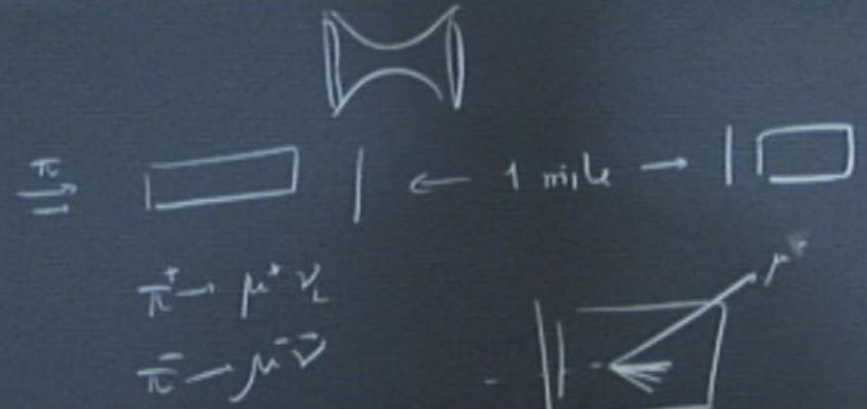
$$p(b) = \int d\lambda \rho(\lambda) p(+,+|a,b,\lambda) \xrightarrow{\dagger} \boxed{\text{shaded}} \xrightarrow{\pi} \boxed{\text{empty}}$$

$p(+|a,\lambda) p(+|b,\lambda)$



agree  $|a, b\rangle = p(+, + | a, b) + p(-, - | a, b)$

$$p(+, + | a, b) = \int d\lambda \mu(\lambda) \underbrace{p(+, + | a, b, \lambda)}_{p(+ | a, \lambda) p(+ | b, \lambda)}$$

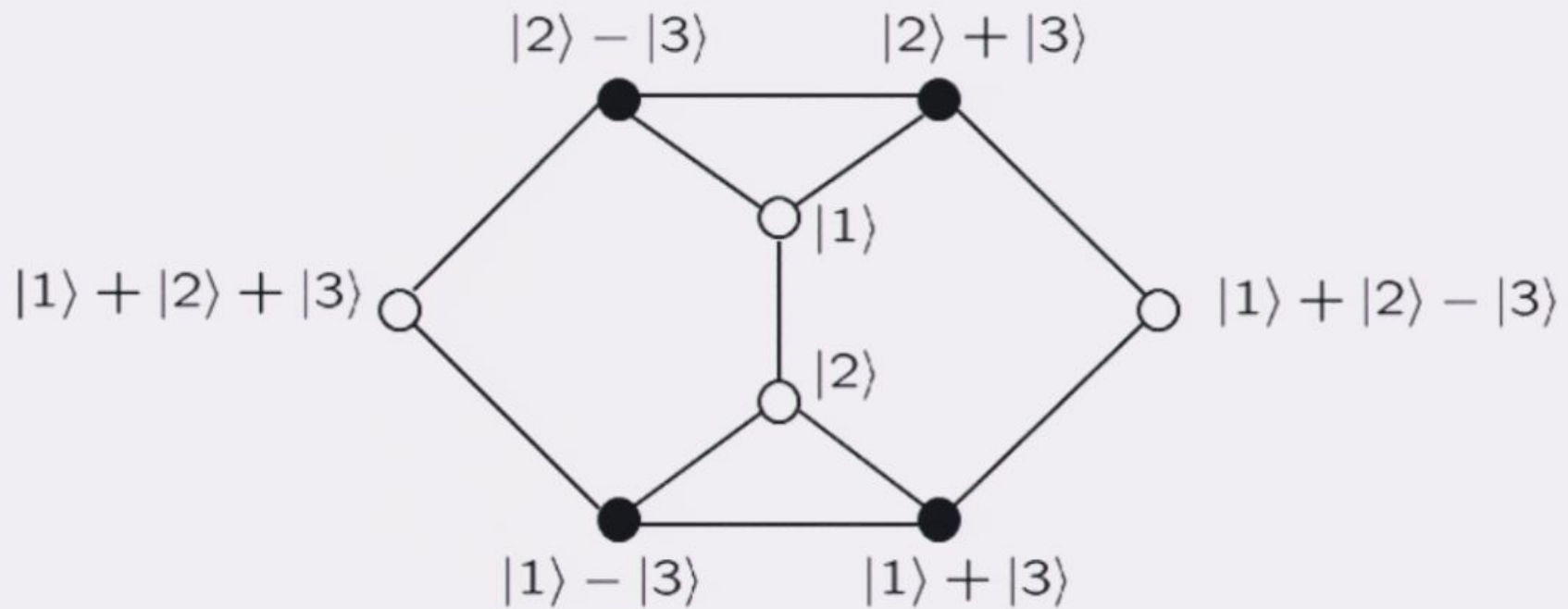




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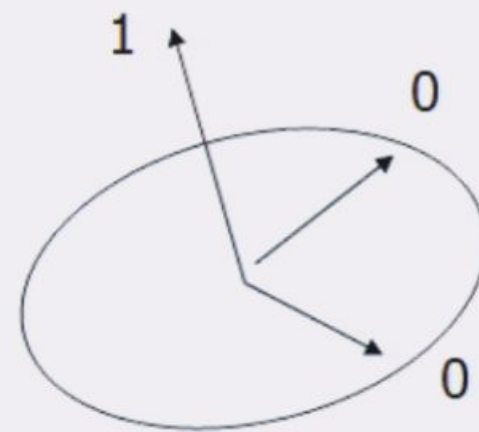
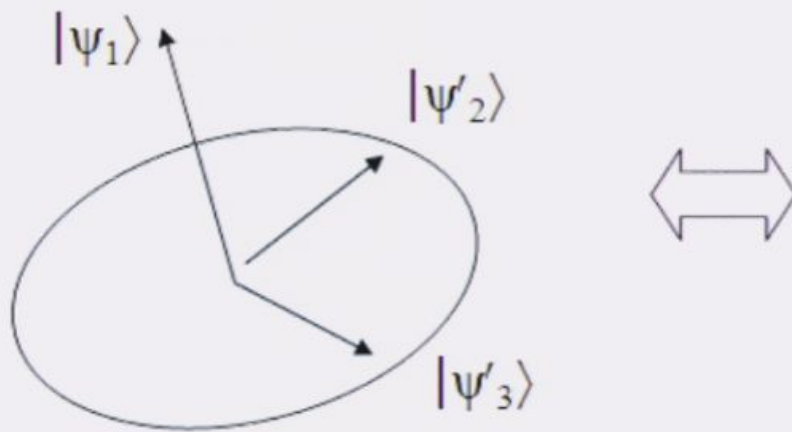
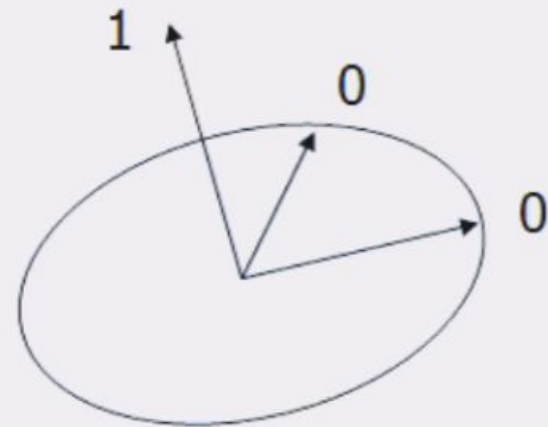
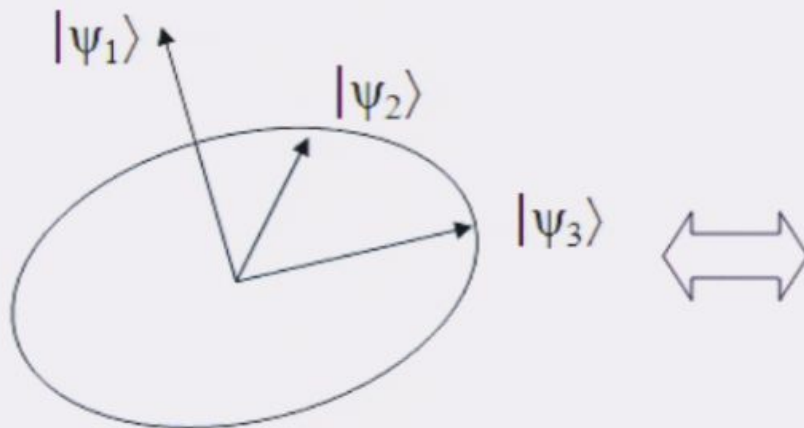
$$|\psi\rangle \bullet \rightarrow \chi_{|\psi\rangle}(\lambda) = 0$$



Alternatively,

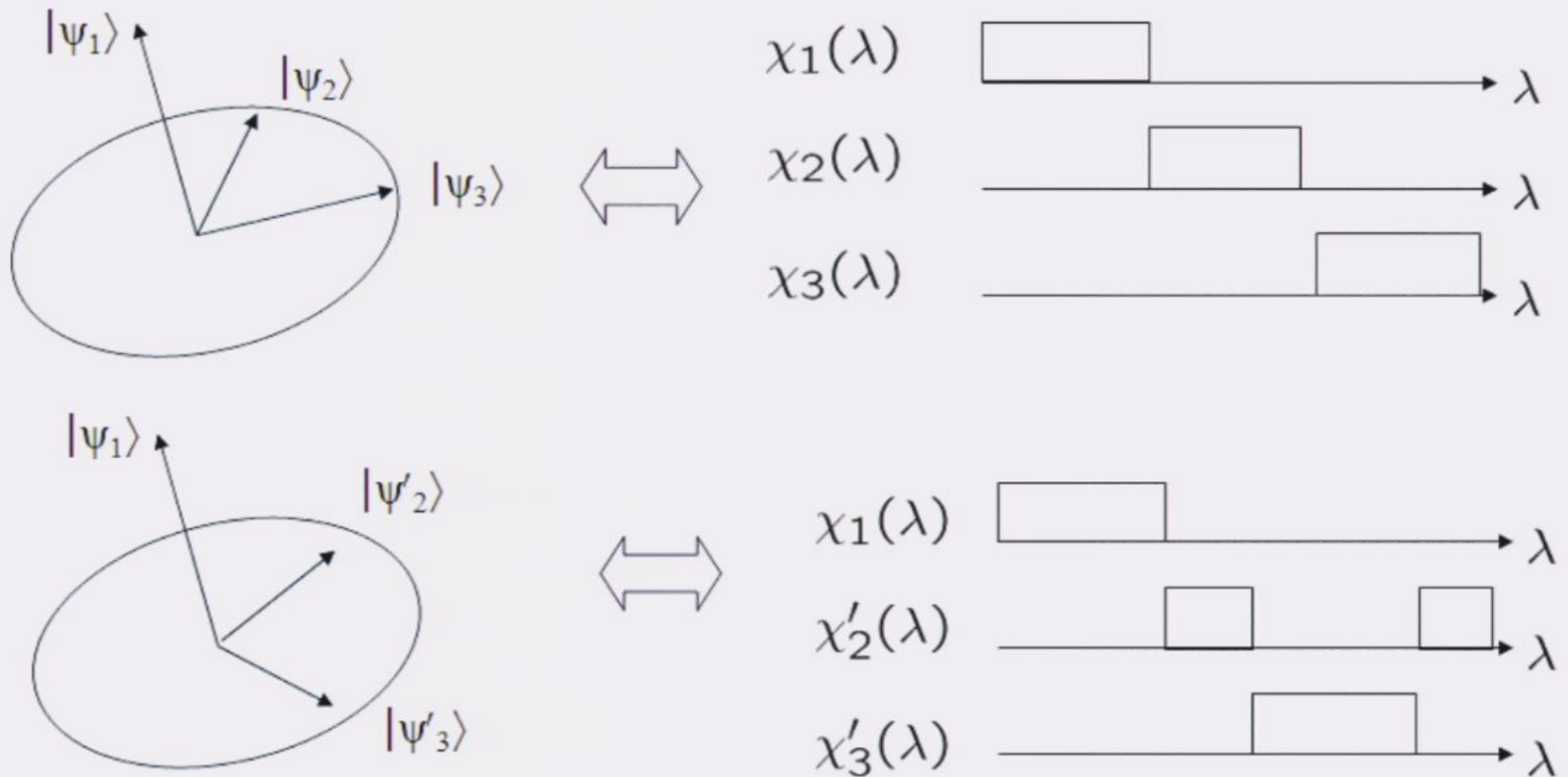
$$\chi_{\psi}(\lambda) = 0 \text{ or } 1$$

$$\sum_k \chi_{\psi_k}(\lambda) = 1$$



## Traditional notion of noncontextuality

A given projector may appear in many different measurements



The traditional notion of noncontextuality:

Every projector  $P$  is associated with the same  $\chi(\lambda)$