Title: Foundations of Quantum Mech. (PHYS 639) - Lecture 8

Date: Dec 09, 2009 11:00 AM

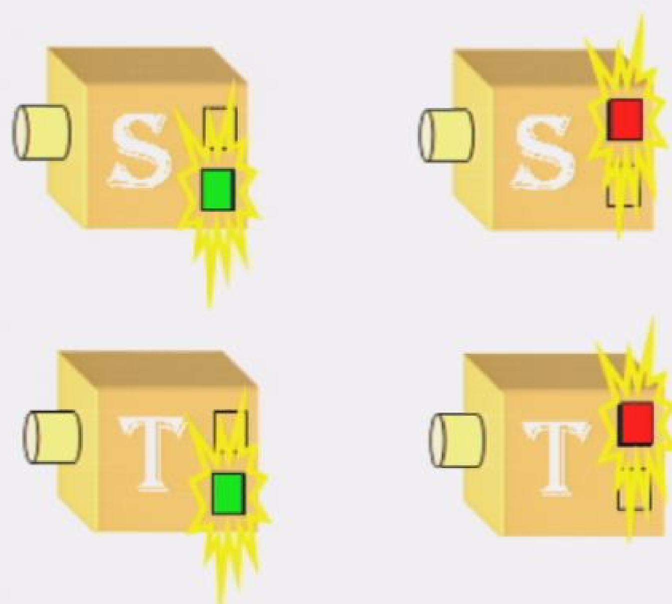
URL: http://pirsa.org/09120072

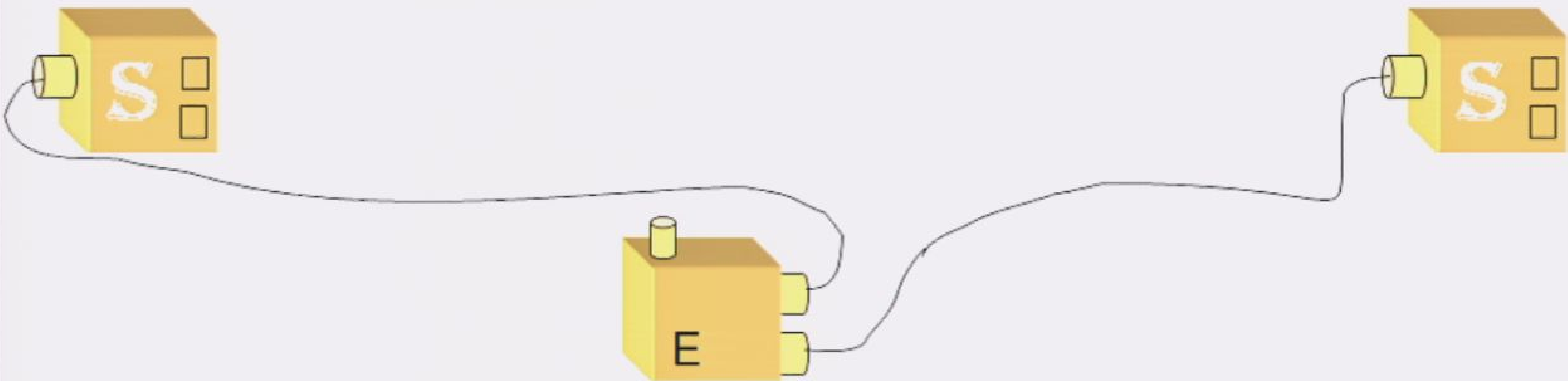
Abstract:

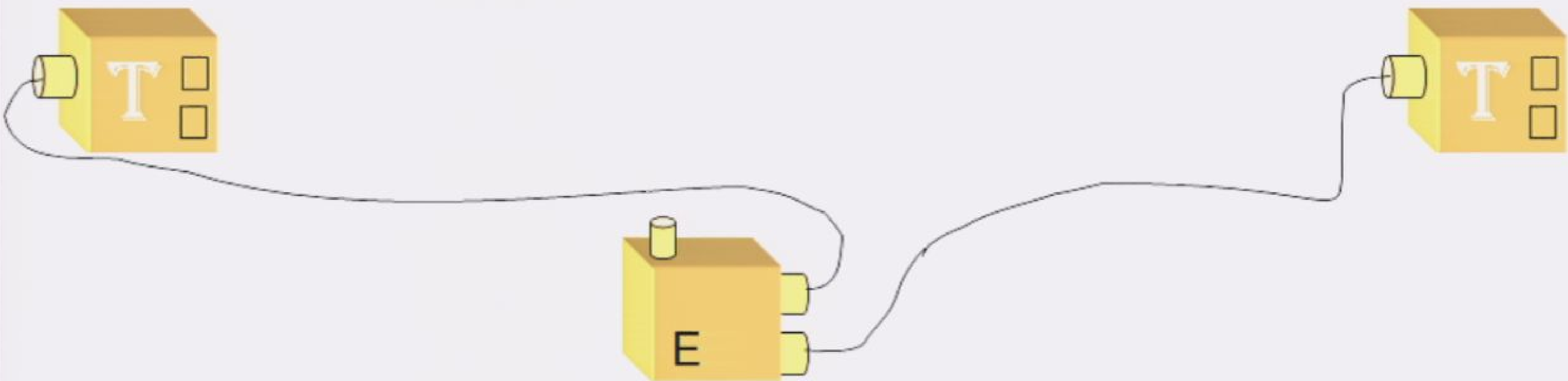
Bell's theorem

John S. Bell

A pair of two-outcome measurements




S
E
S


T
E
T

There are two possible measurements, S and T, with two outcomes each: green or red

Suppose which of S or T occurs at each wing is chosen at random

Scenario 1

1. Whenever the same measurement is made on A and B, the outcomes always agree — S and S or T and T

2. Whenever different measurements are made on A and B, the outcomes always disagree — S and T or T and S


S
S
outcomes agree
outcomes disagree
outcomes disagree
outcomes agree
T
T

There are two possible measurements, S and T, with two outcomes each: green or red

Suppose which of S or T occurs at each wing is chosen at random

Scenario 2

1. Whenever the same measurement is made on A and B, the outcomes always disagree	S and S or T and T
2. Whenever different measurements are made on A and B, the outcomes always agree	S and T or T and S


S
S
T
T
outcomes disagree
outcomes agree
outcomes agree
outcomes disagree

There are two possible "measurements", S and T, with two outcomes each: green or red

Suppose which of S or T occurs at each wing is chosen at random

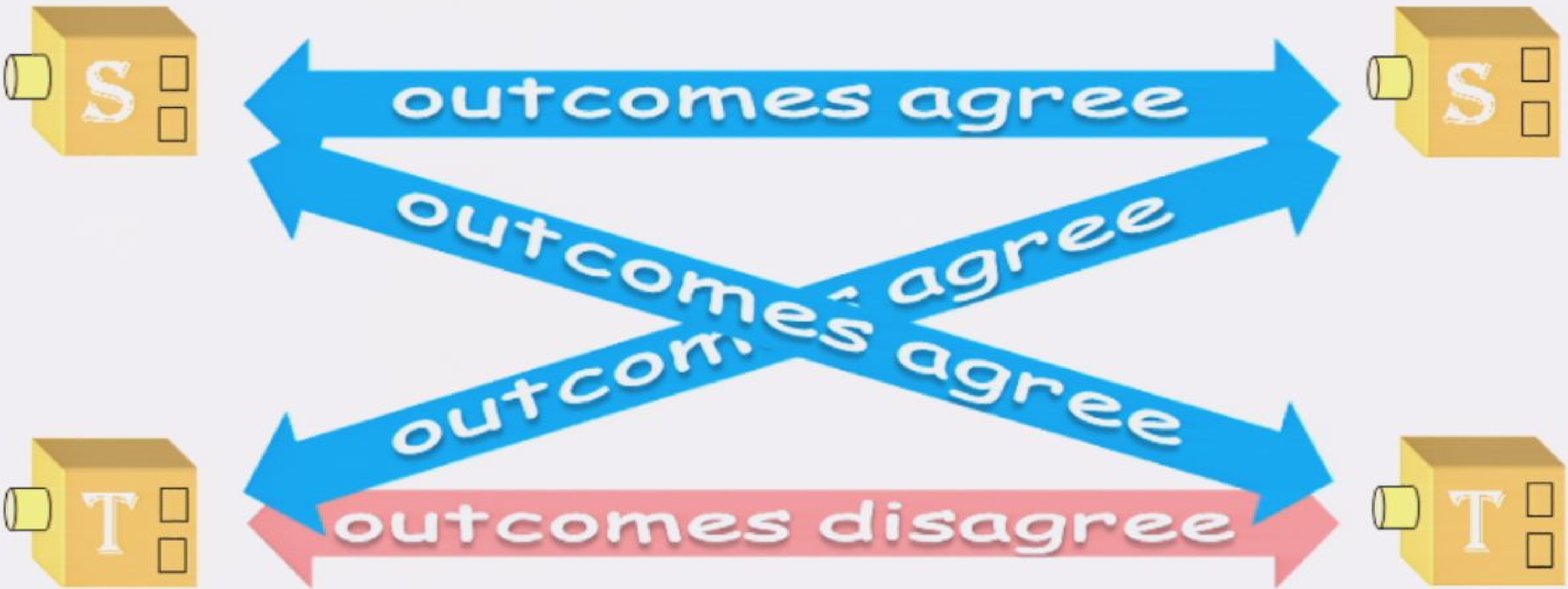
Scenario 3

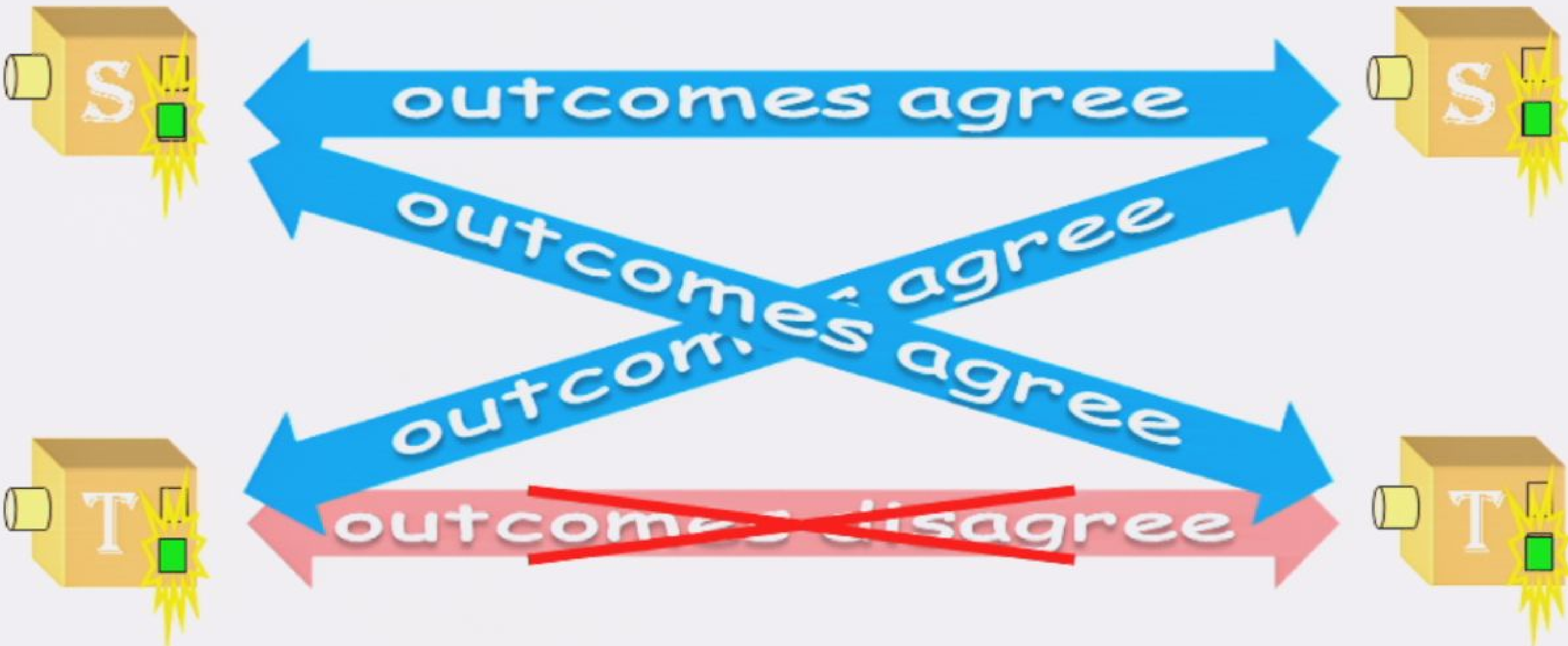
1. Whenever the measurement T is made on both A and B, the outcomes always disagree

   T and T

2. Otherwise, the outcomes always agree

   S and S
   or
   S and T
   or
   T and S


S
S
T
T
outcomes agree
outcomes agree
outcomes agree
outcomes disagree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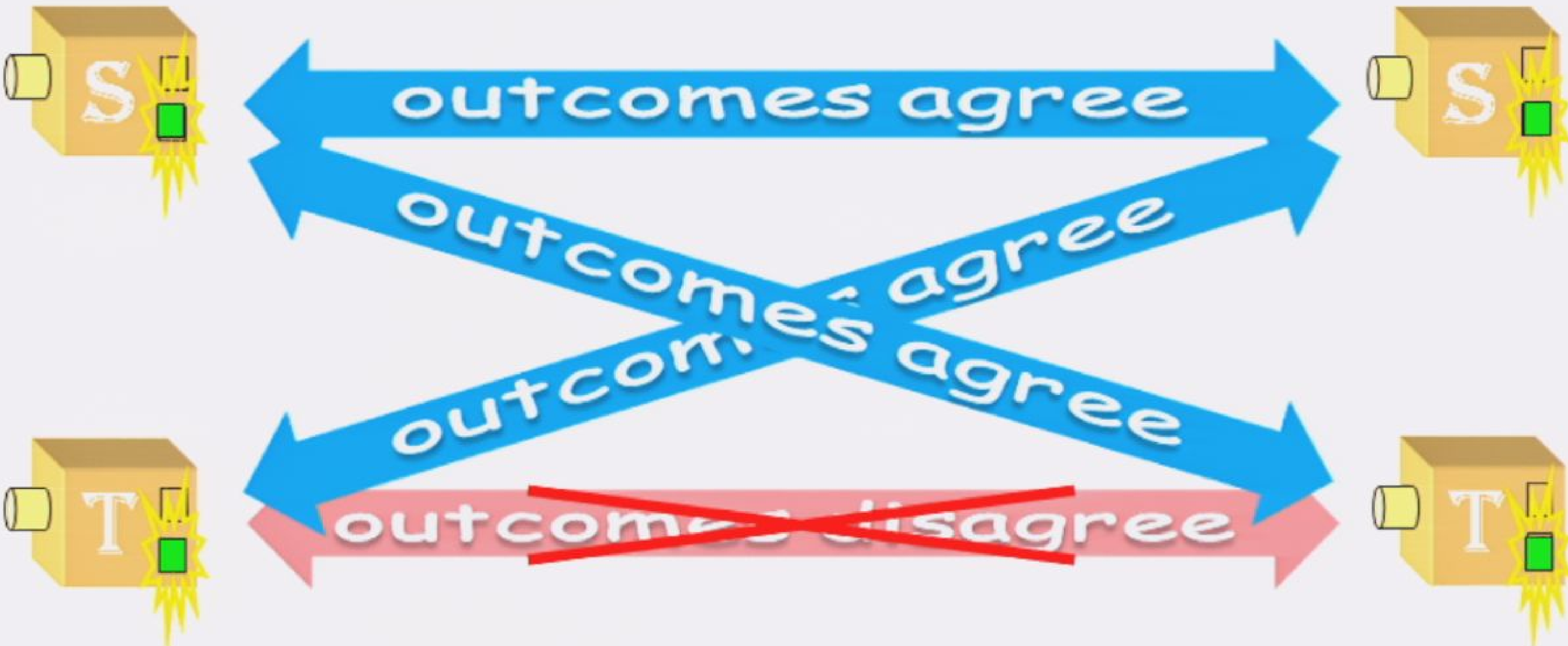
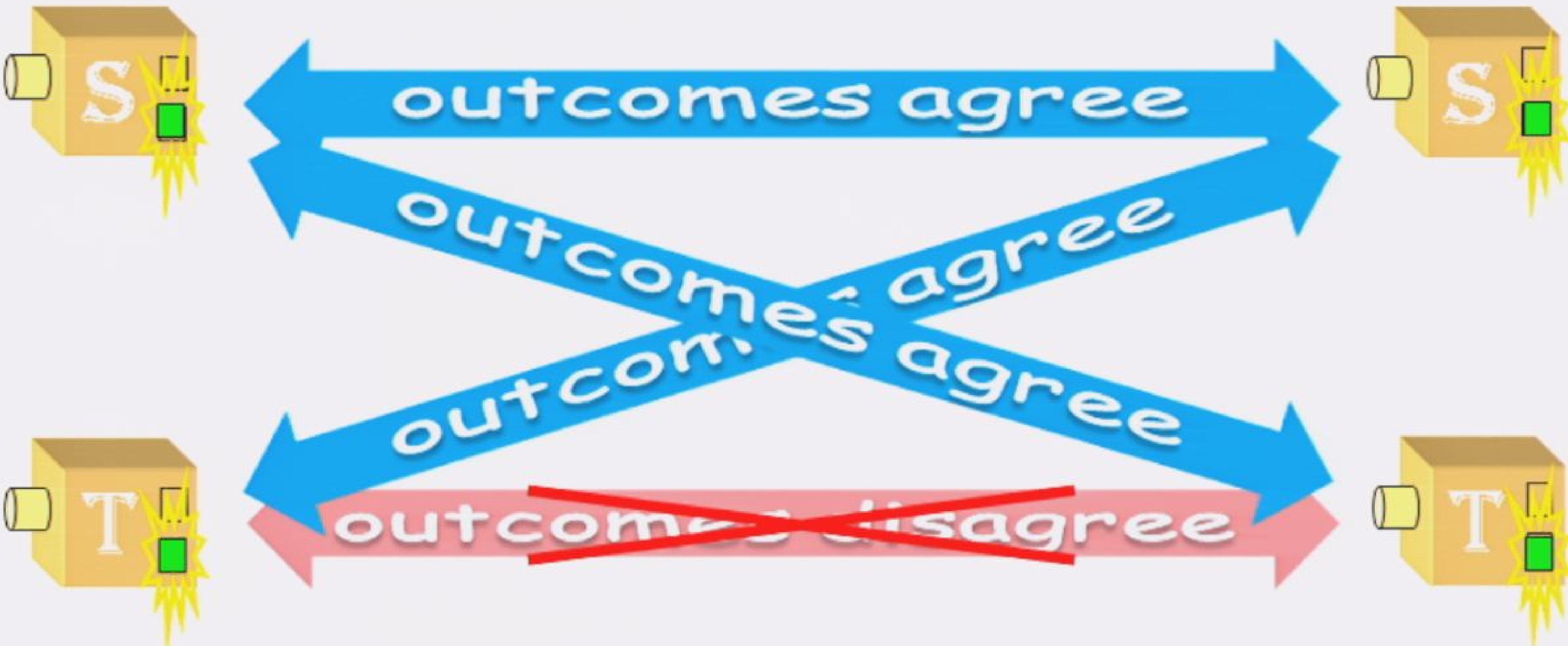


The game can be won at most 75% of the time by local strategies



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Using quantum theory, it can be won 85% of the time!

Q: How could you cheat and win the game all the time?

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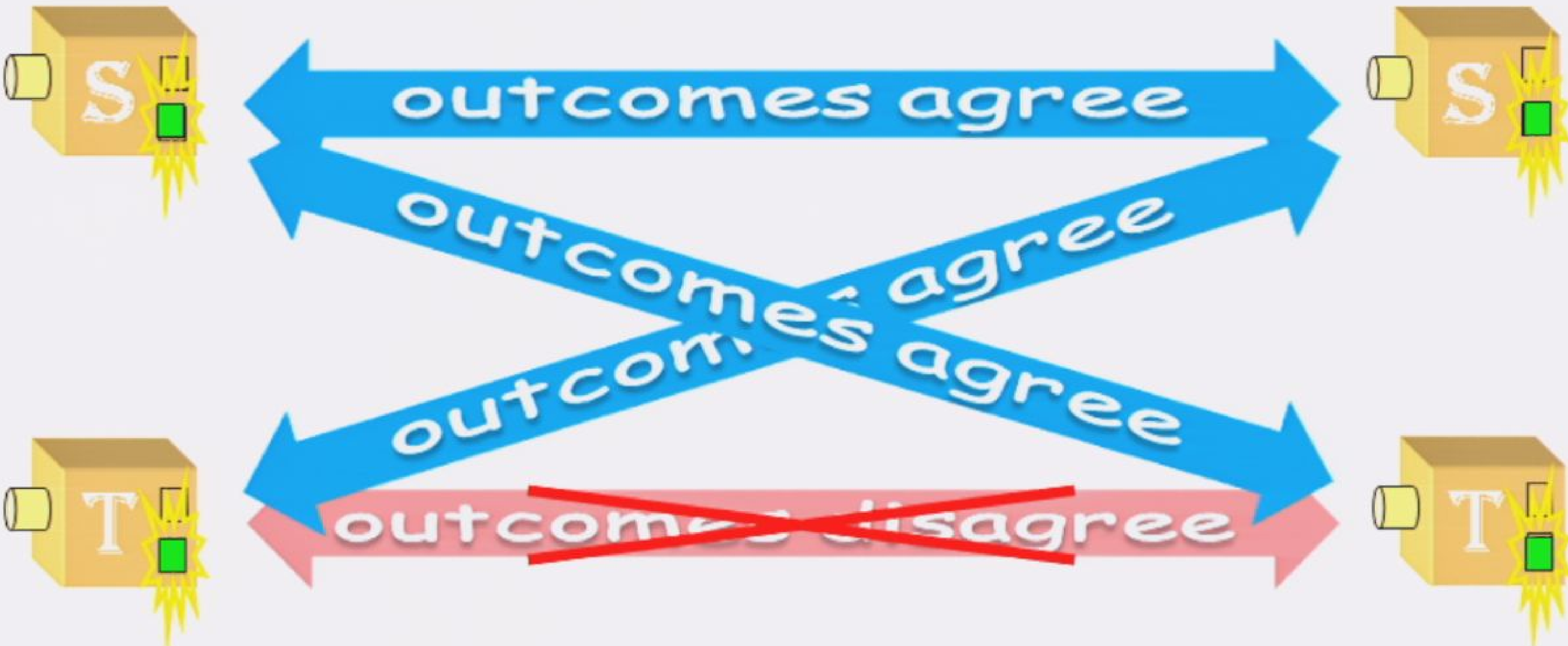
But there's a problem...



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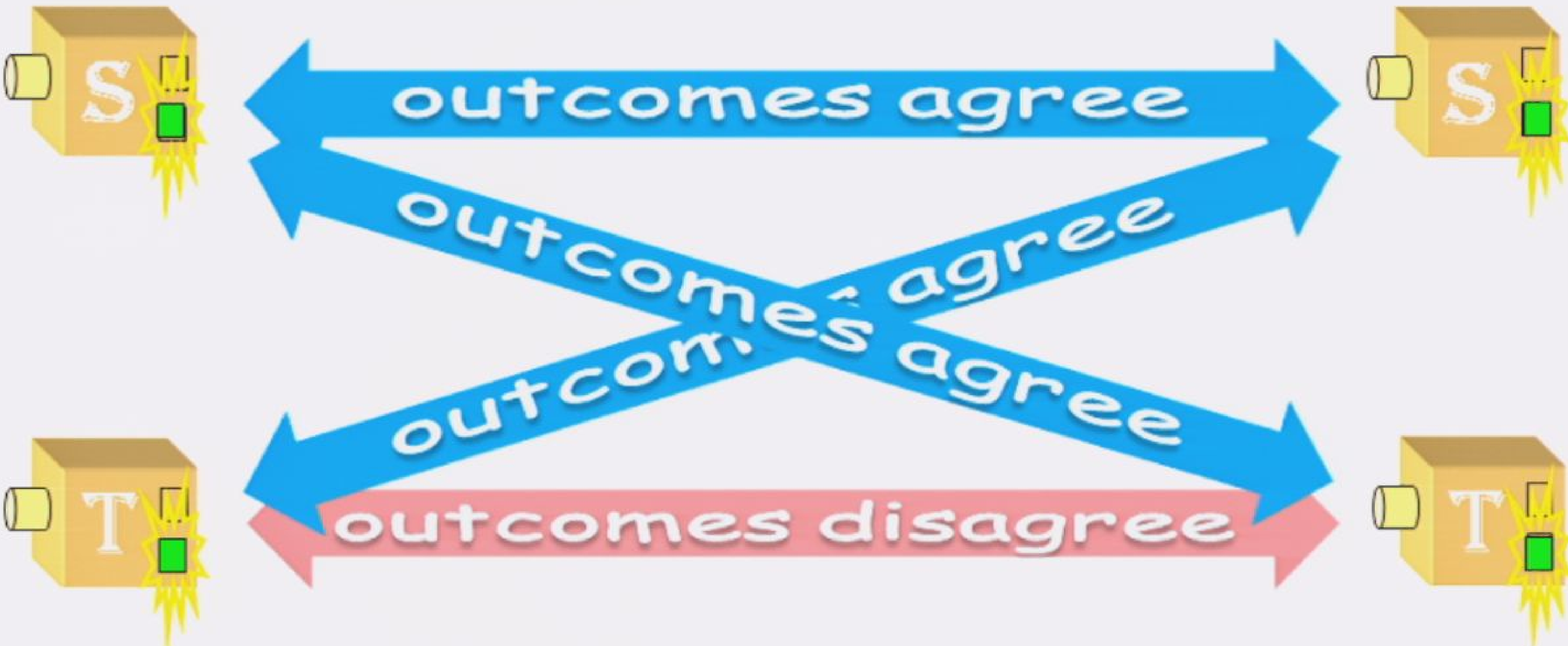
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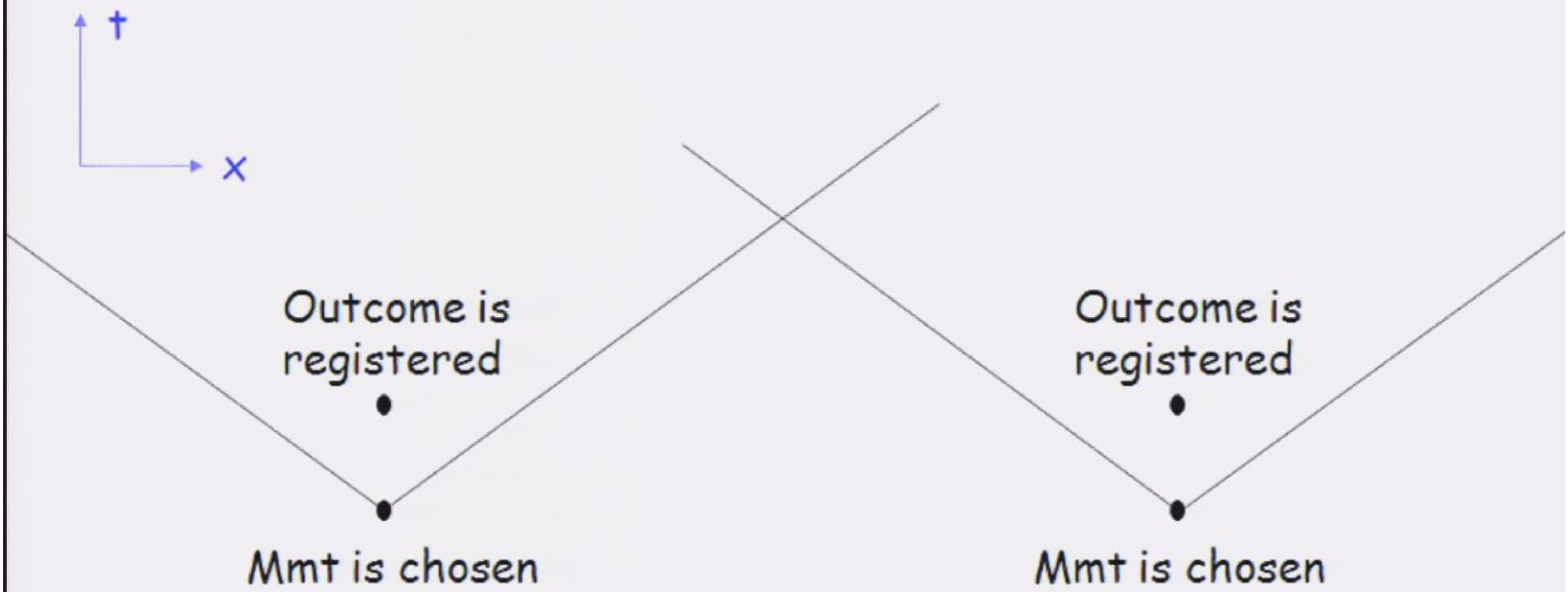
Tension with the theory of relativity



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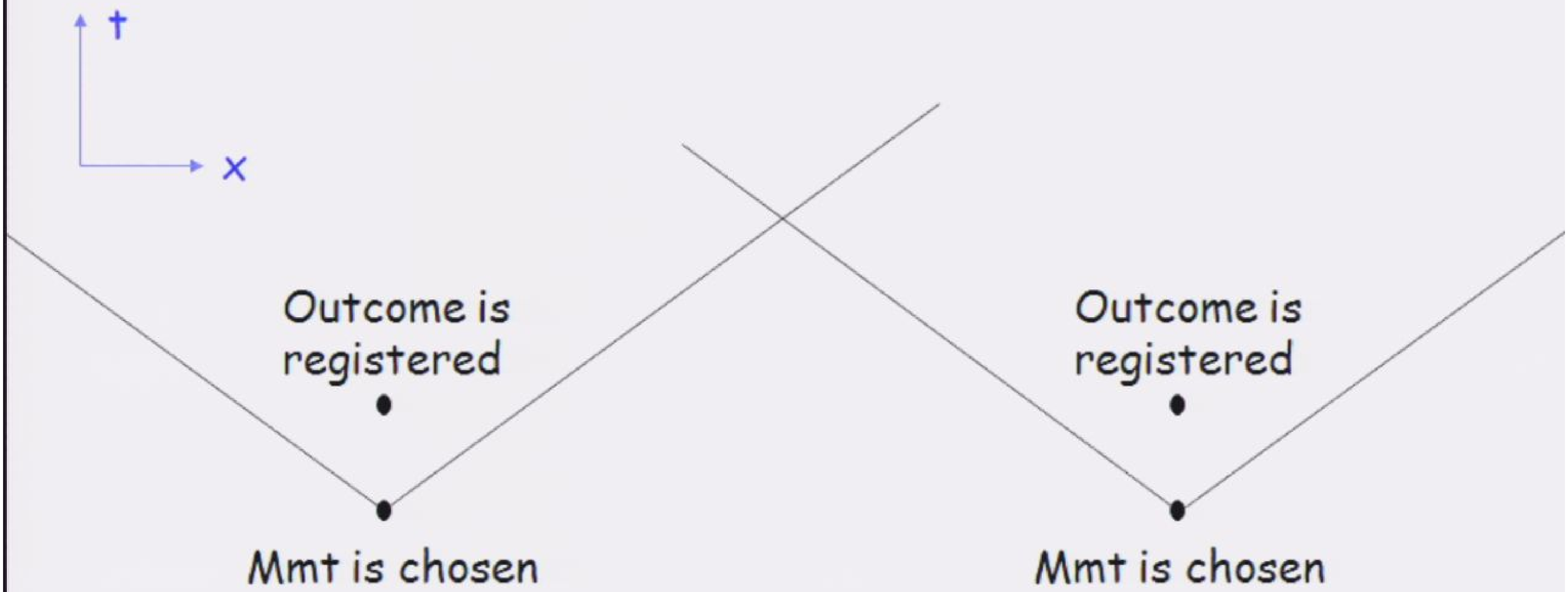
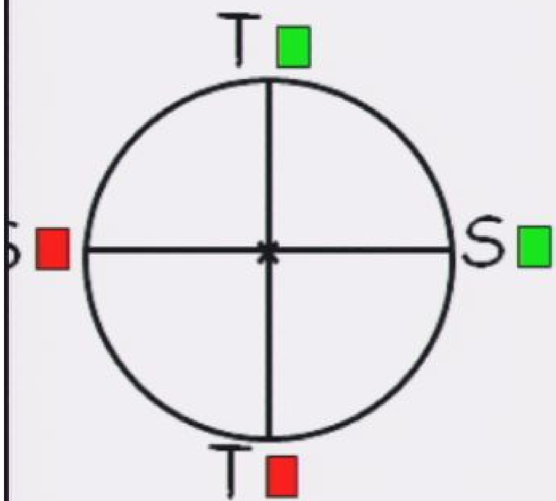
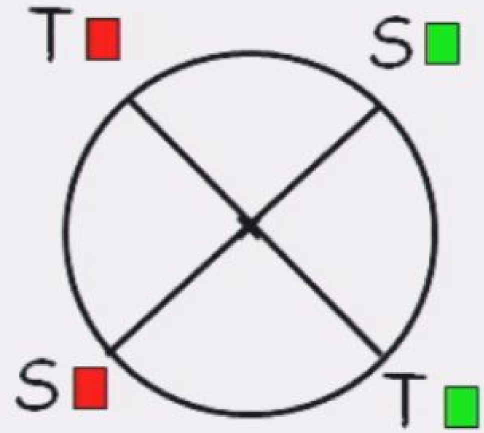


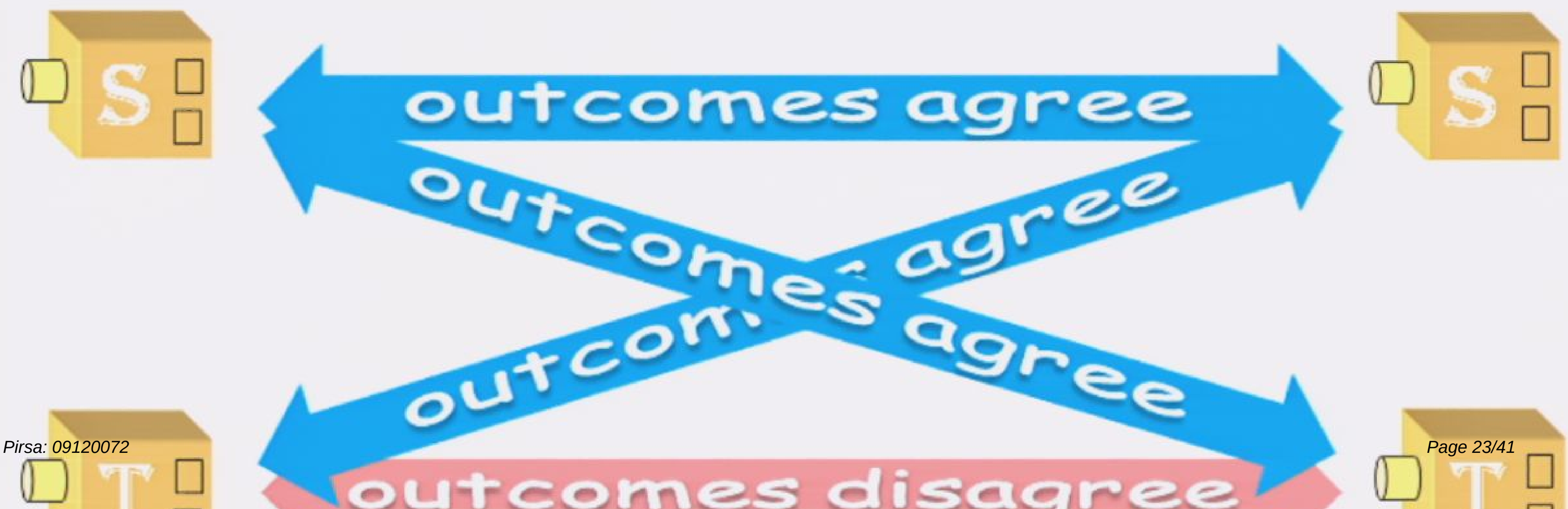
$$p(\text{success}) = p(\text{agree}|SS)p(SS) + p(\text{agree}|ST)p(ST) + p(\text{agree}|TS)p(TS) + p(\text{disagree}|TT)p(TT)$$

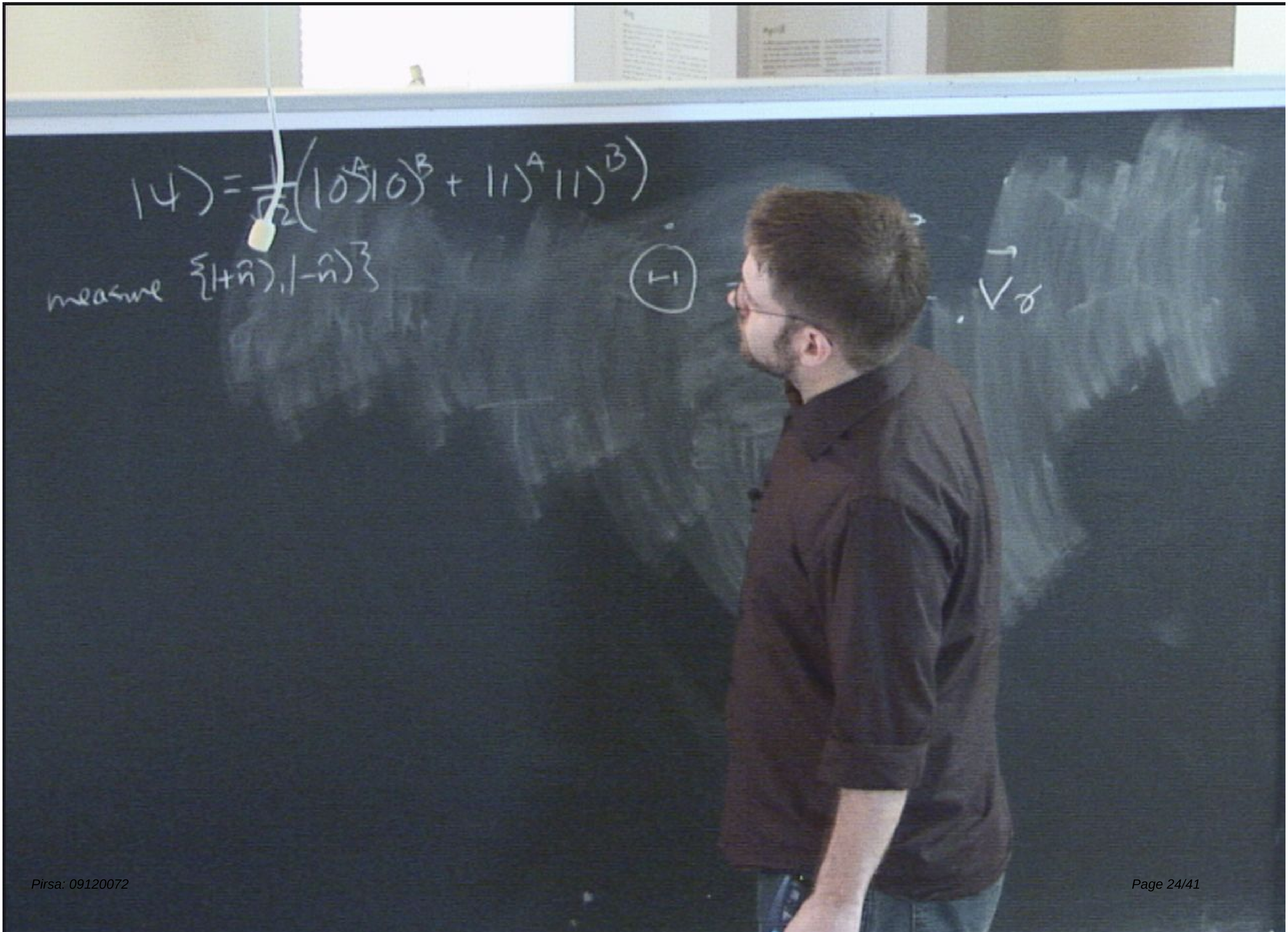
The quantum correlations

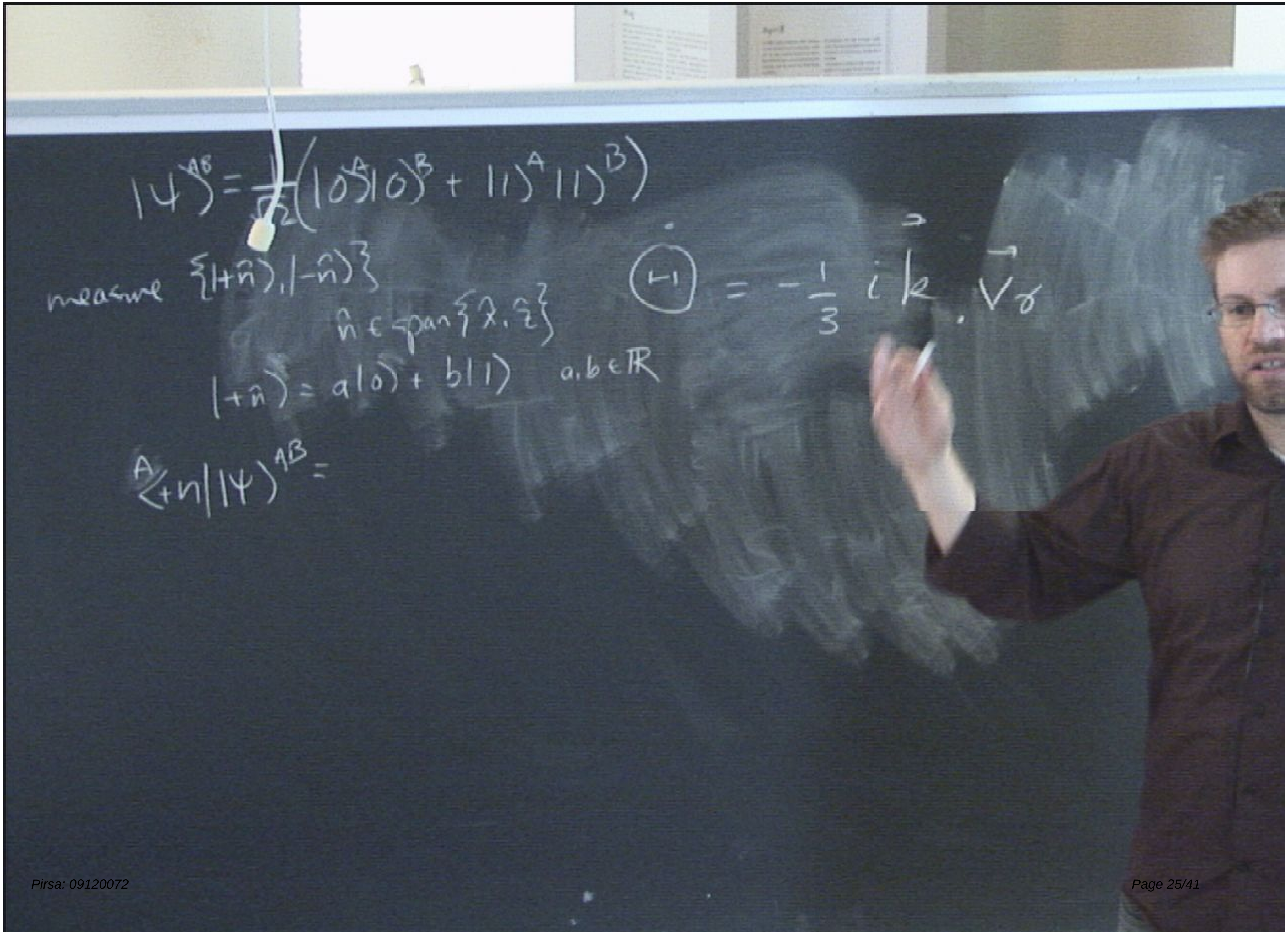


$$cos^2(\pi/8) = \frac{1}{2} + \frac{1}{2\sqrt{2}}$$
$$\simeq 0.85$$






|ψ⟩ = 1/√2 (|0⟩^A |0⟩^B + |1⟩^A |1⟩^B)
measure {|+n̂⟩, |−n̂⟩}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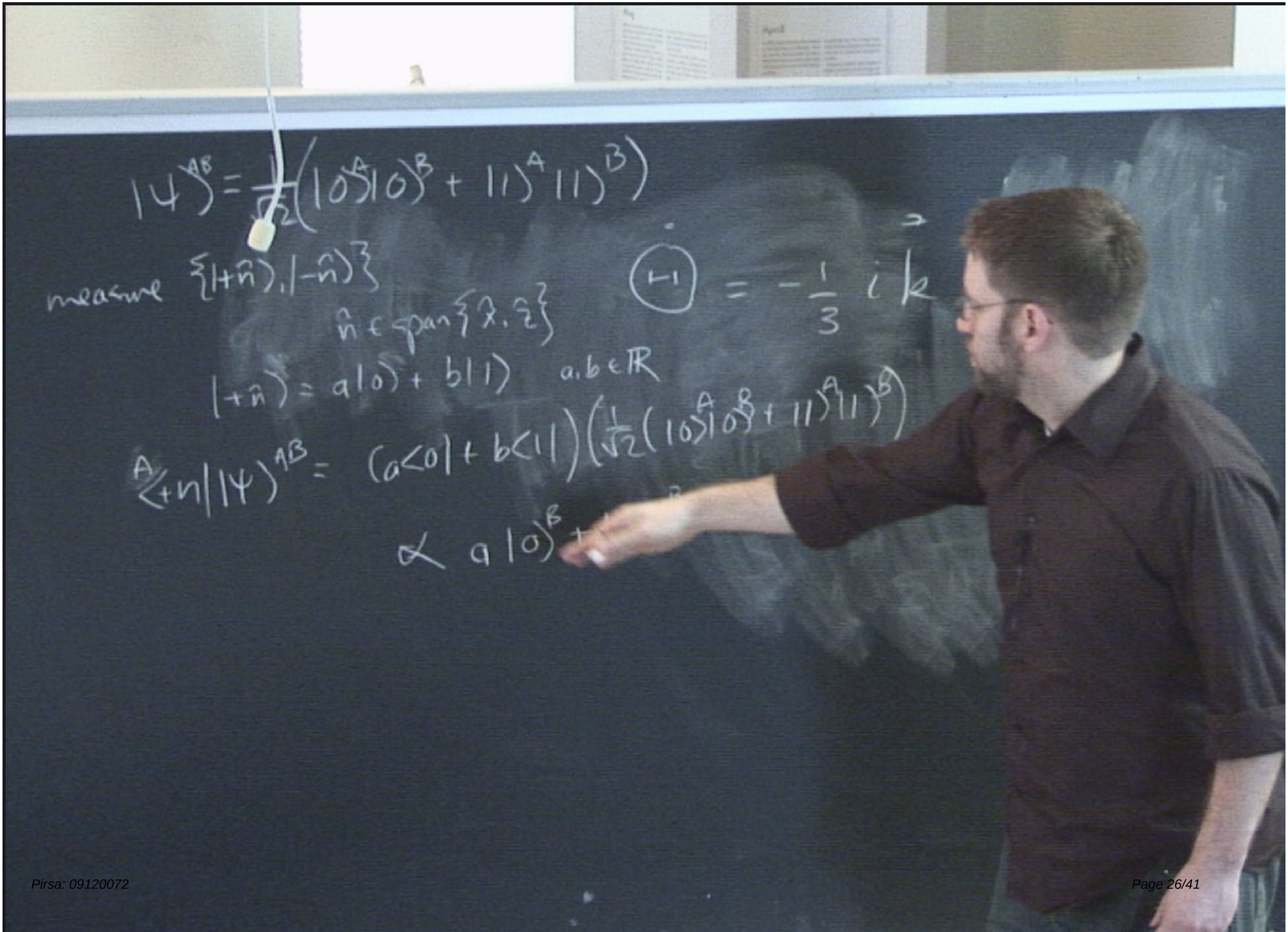
measure {|+n̂⟩, |−n̂⟩}
n̂ ∈ span{x̂, ẑ}
|+n̂⟩ = a|0⟩ + b|1⟩   a, b ∈ ℝ
^A⟨+n|ψ⟩^AB =
= −1/3 i k⃗ · V⃗γ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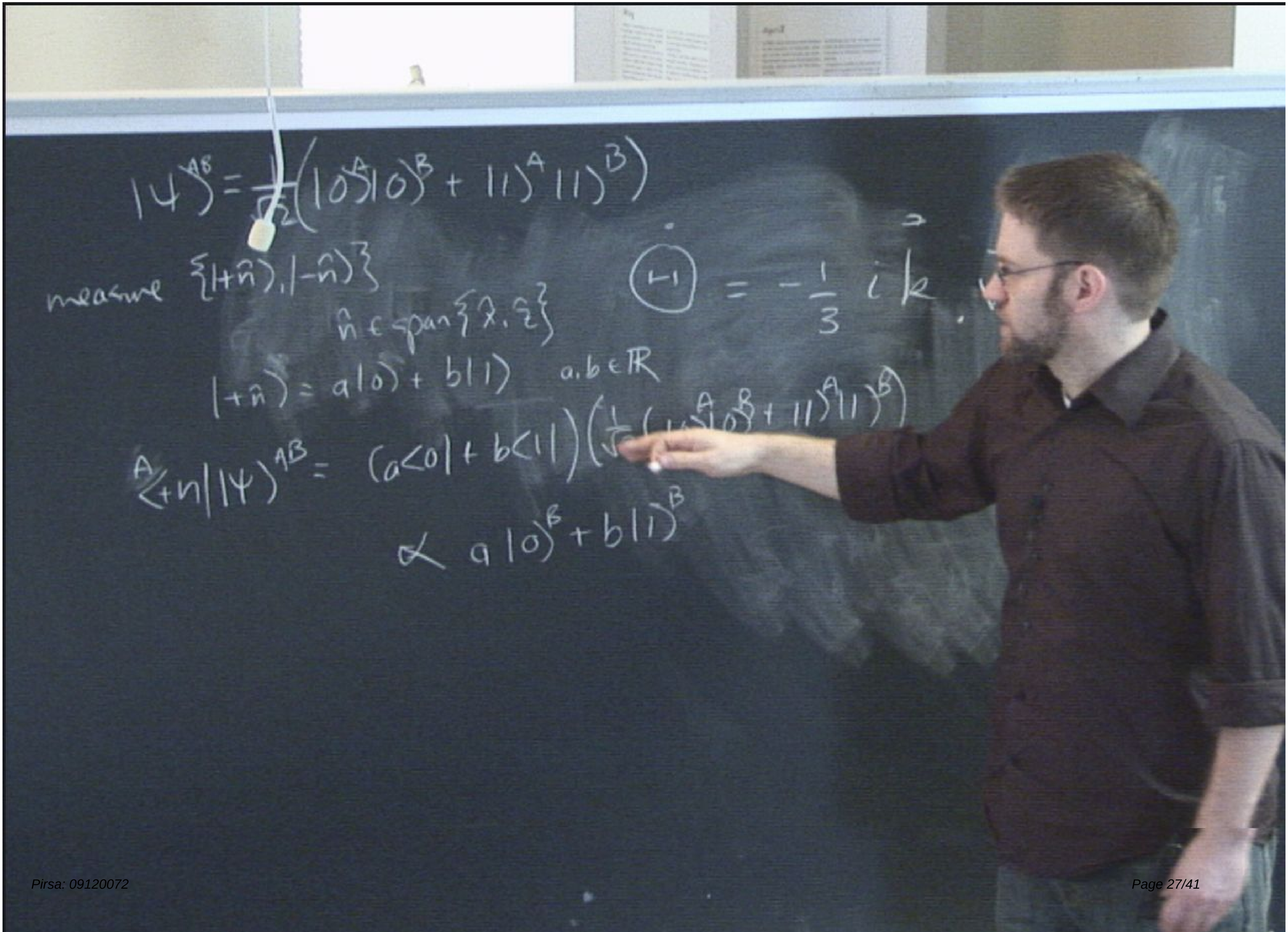
∝ a|0⟩^B +
= −1/3 i k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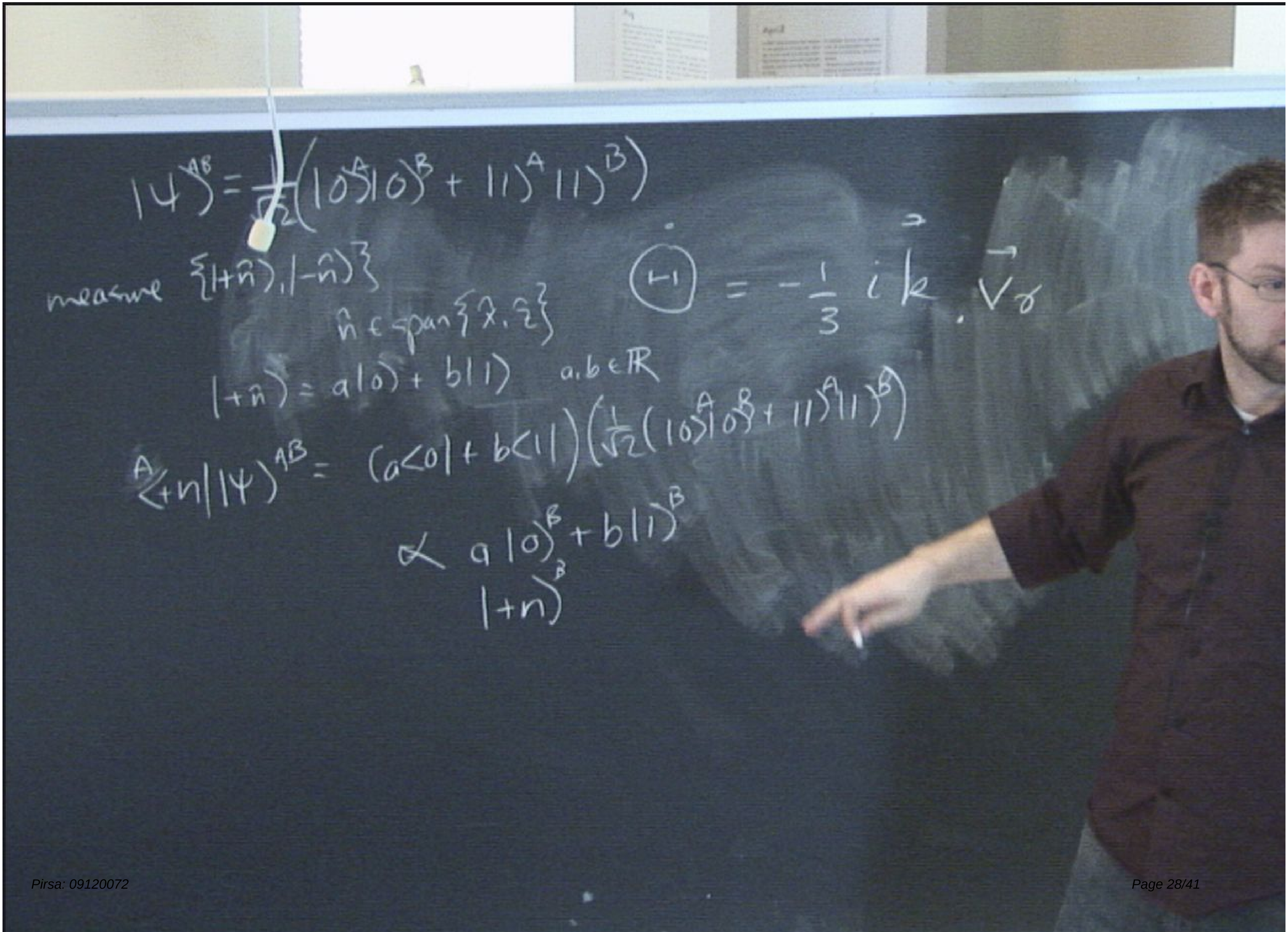
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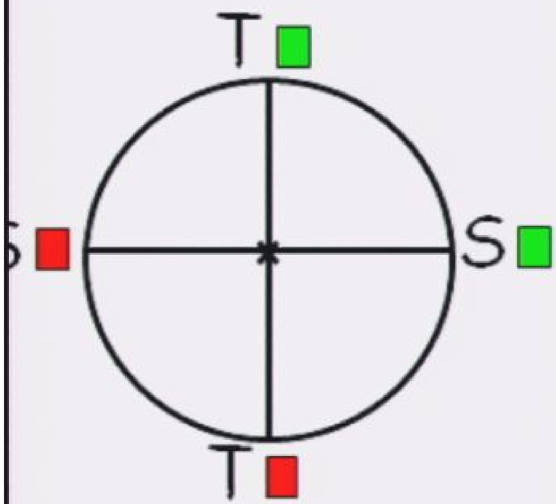
∝ a|0⟩^B + b|1⟩^B
|+n⟩^B
(−1) = −1/3 i k⃗ · V⃗ γ

The quantum correlations



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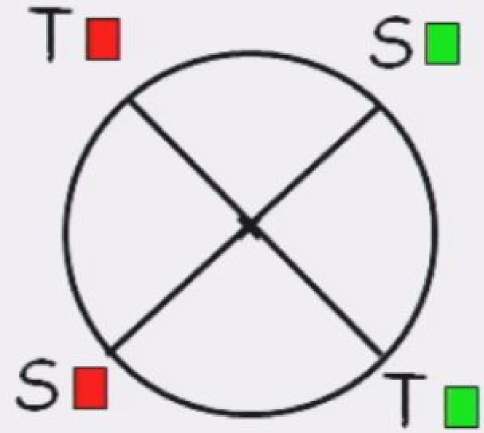

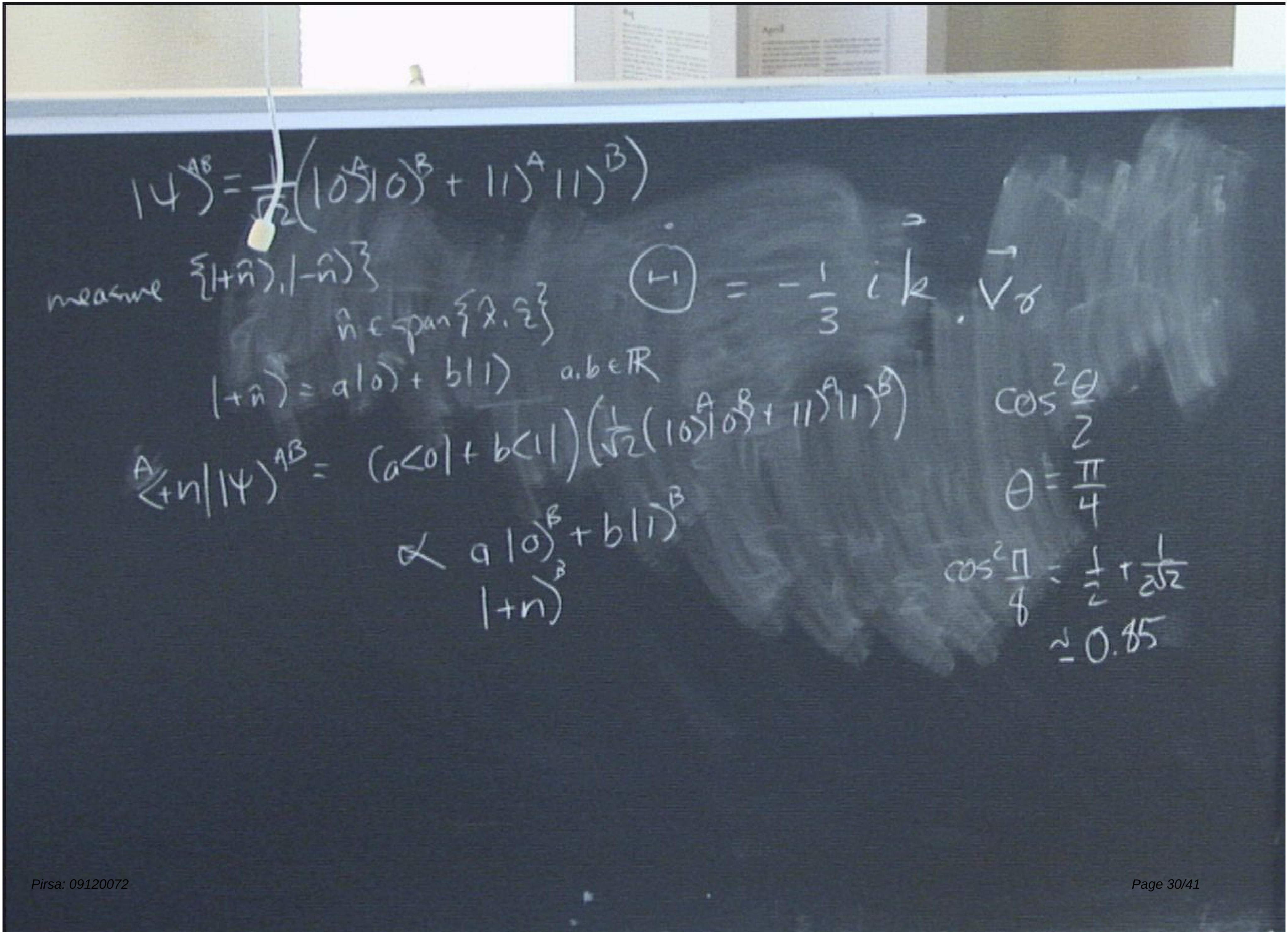
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cos² θ/2
θ = π/4
cos² π/8 = 1/2 + 1/(2√2)
≃ 0.85

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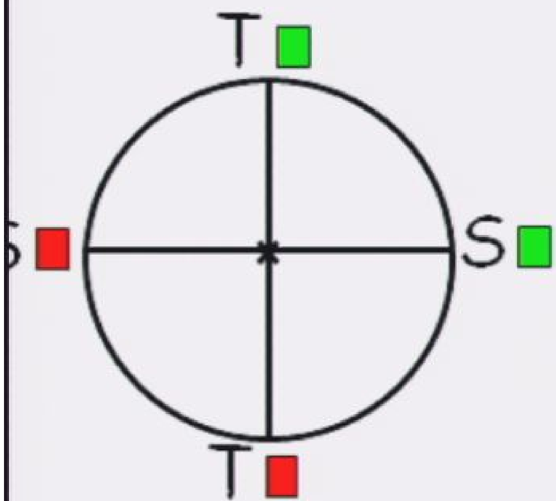

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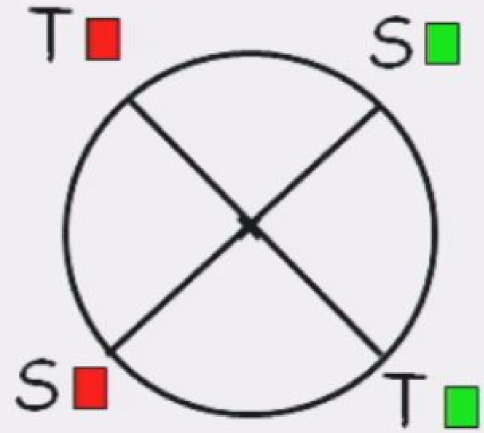
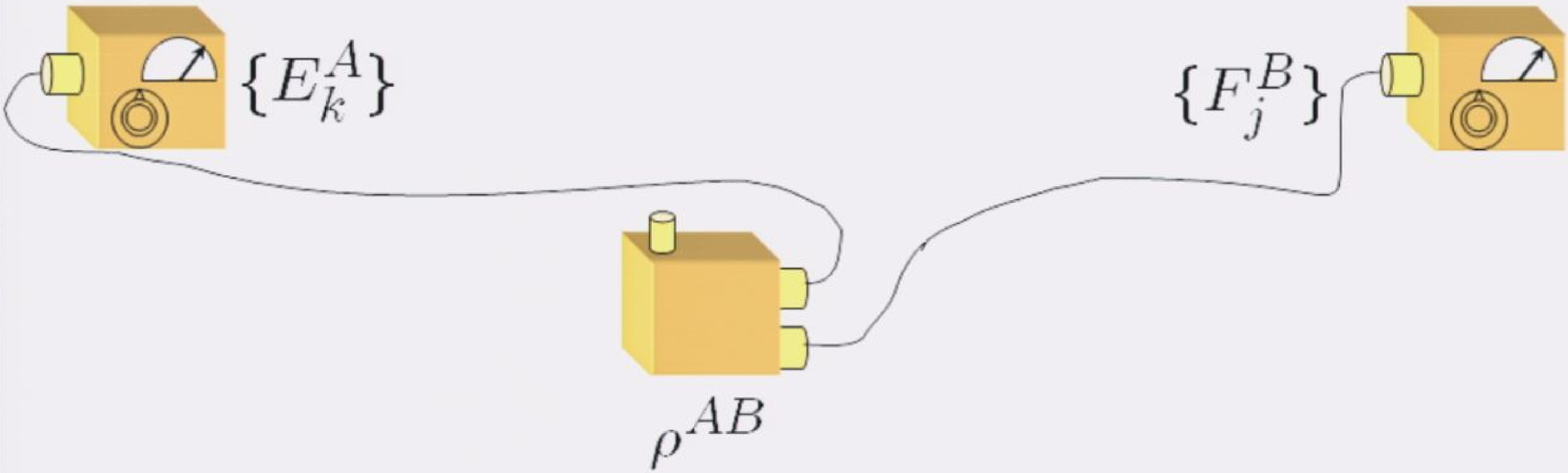


No signalling in quantum theory

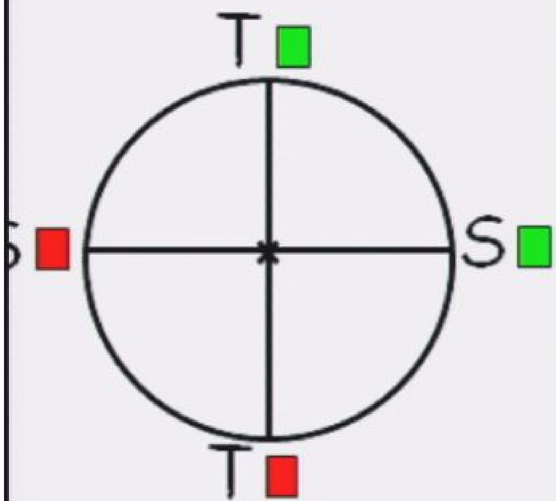


$$\begin{aligned} p(j) &= \sum_k p(k,j) \\ &= \sum_k \mathrm{Tr}_{AB}[\,(E_k^A \otimes F_j^B)\,\rho^{AB}] \\ &= \mathrm{Tr}_{AB}[\,(I^A \otimes F_j^B)\,\rho^{AB}] \end{aligned}$$

Independent of choice of measurement at A

Note that $[E_k^A, F_j^B] = 0$ for A and B space-like separated

The quantum correlations



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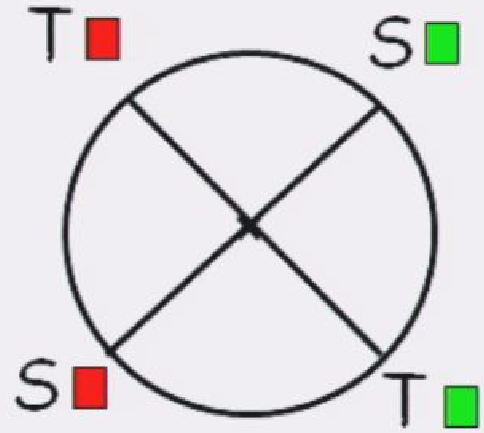

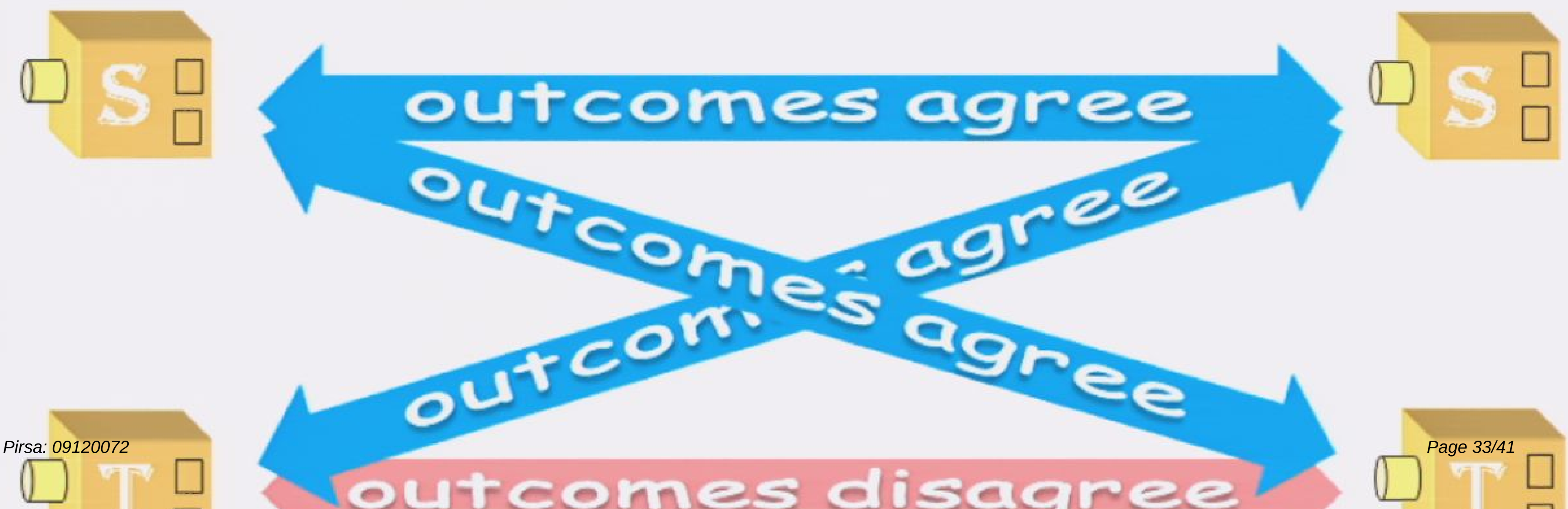

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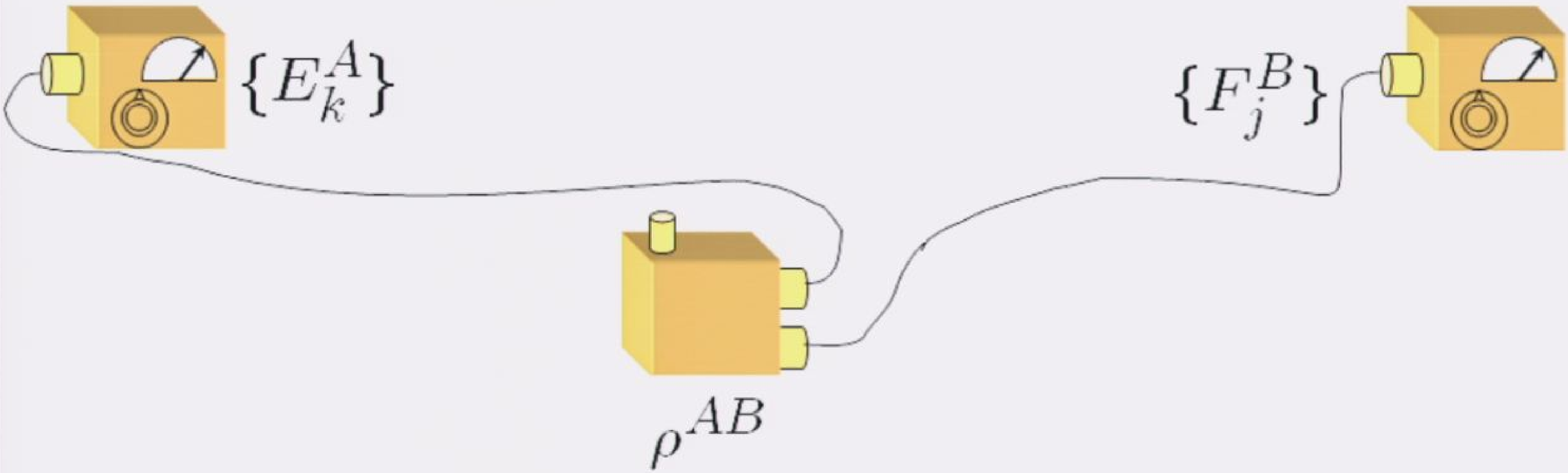

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