Title: Foundations of Quantum Mech. (PHYS 639) - Lecture 6

Date: Dec 07, 2009 11:00 AM

URL: http://pirsa.org/09120070

Abstract:

ψ-complete model:

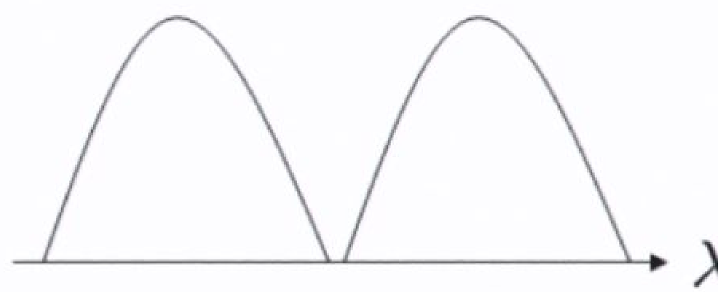
Space of physical states = space of rays in Hilbert space

$$\lambda = \psi$$

ψ-ontic model:

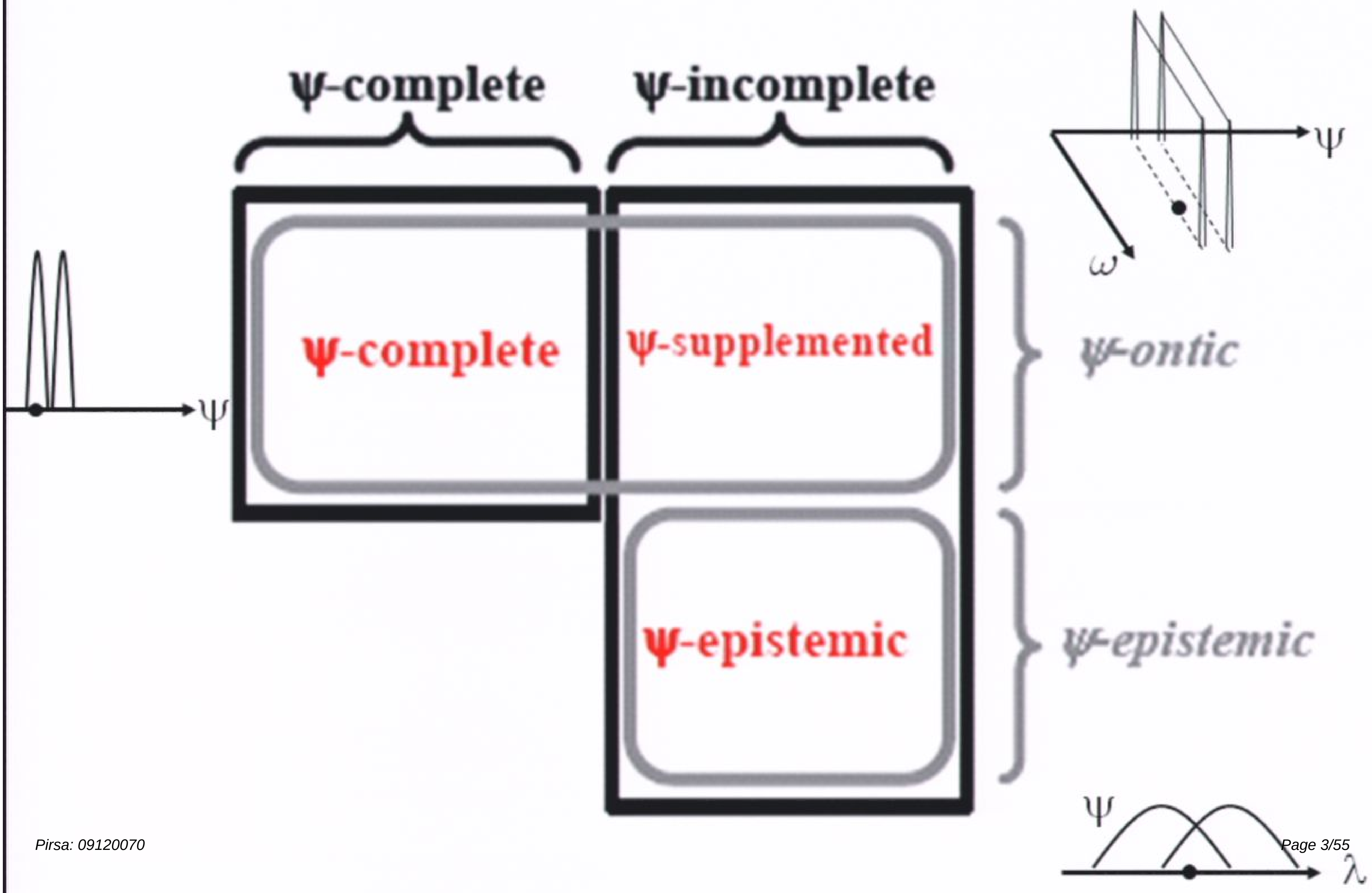
For any preparation procedures $P_{|\psi_1\rangle}$, $P_{|\psi_2\rangle}$ with $|\psi_1\rangle \neq |\psi_2$

$\mu(\lambda|P_{|\psi_1\rangle})\mu(\lambda|P_{|\psi_2\rangle}) = 0$ for all λ



ψ-epistemic model: $\exists\ |\psi_1\rangle \neq |\psi_2\rangle$

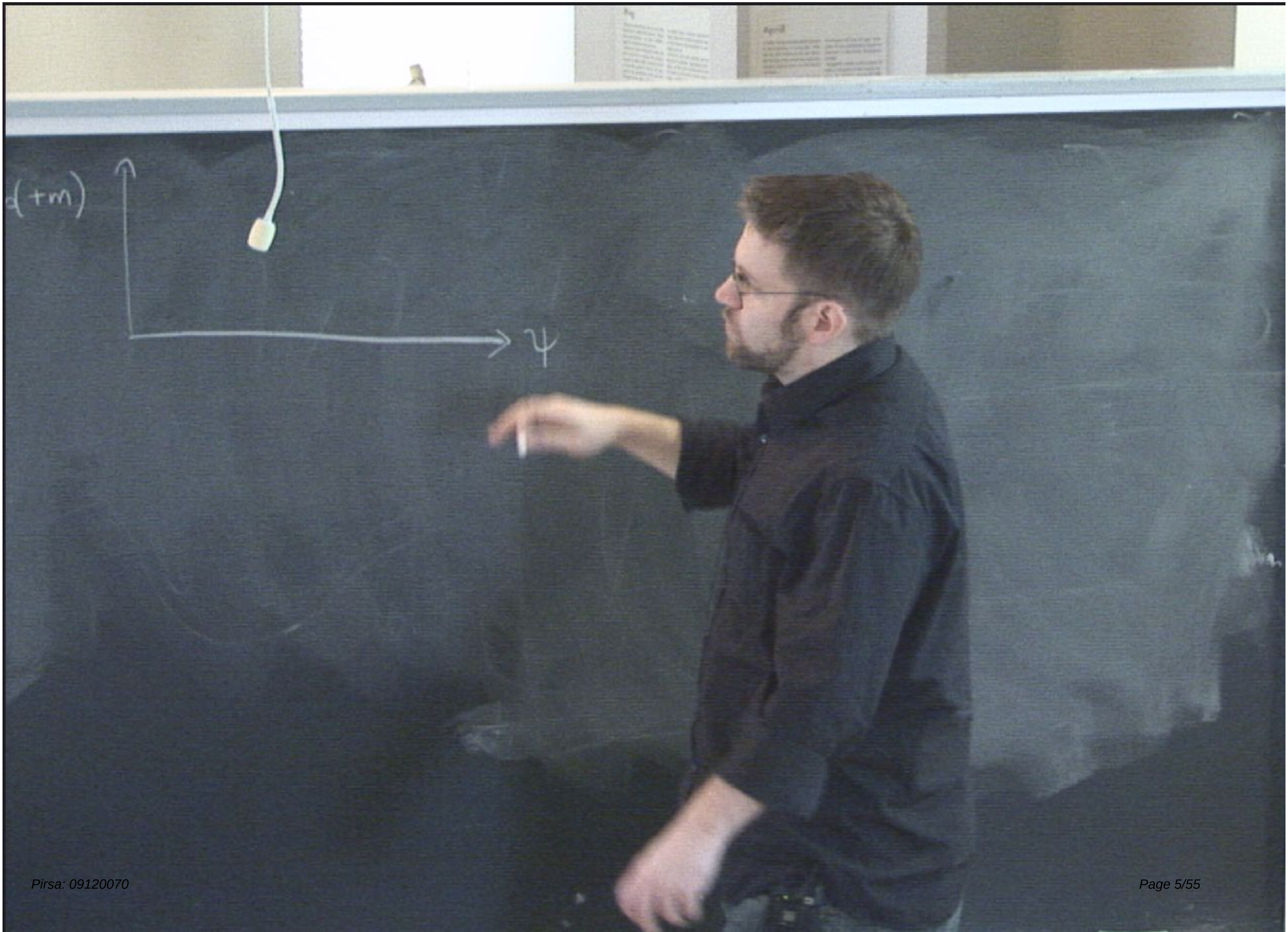
$\mu(\lambda|P_{|\psi_1\rangle})\mu(\lambda|P_{|\psi_2\rangle}) \neq 0$ for some λ

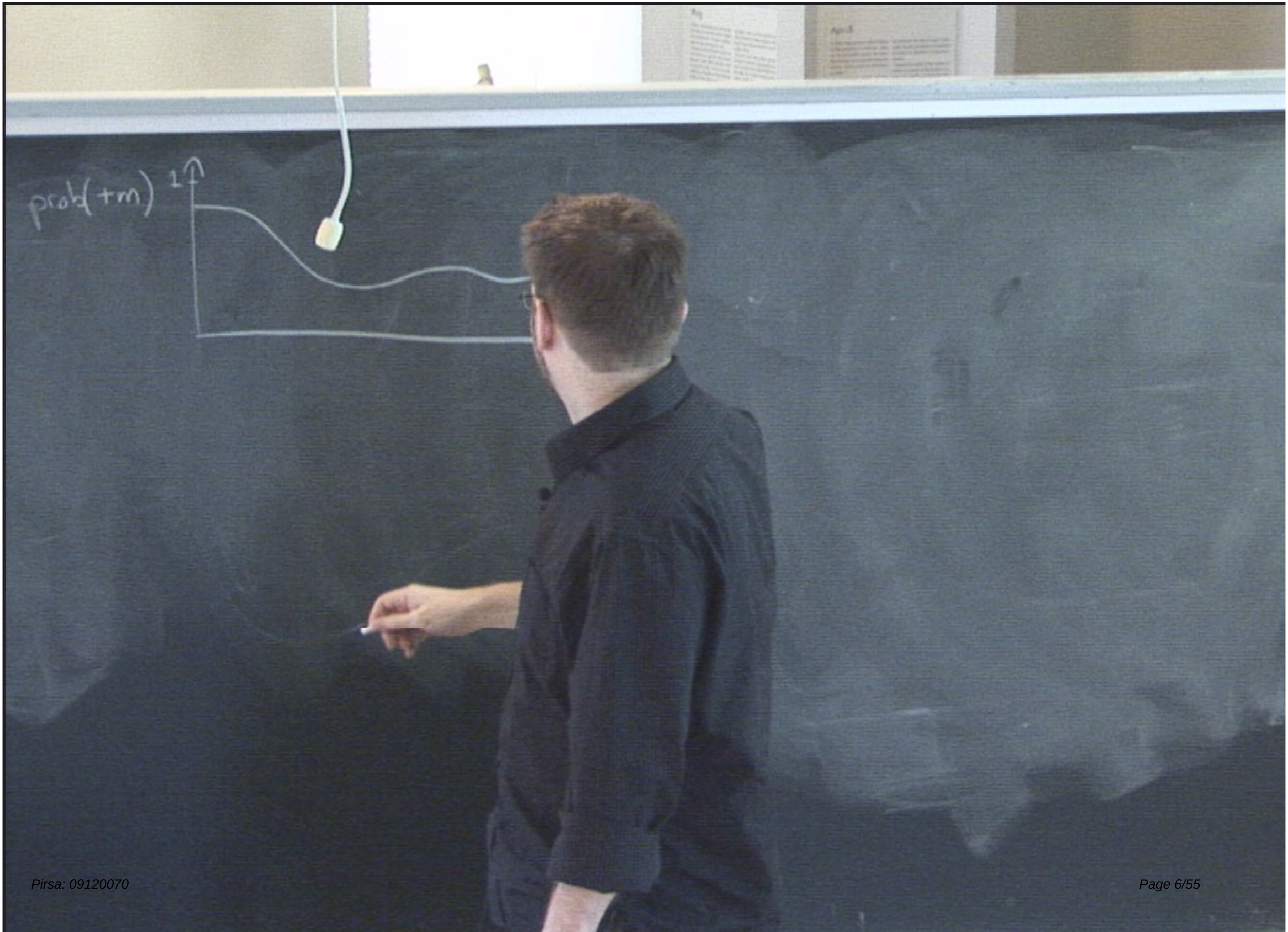

ψ-complete
ψ-incomplete
ψ-complete
ψ-supplemented
ψ-epistemic
ψ-ontic
ψ-epistemic
Ψ
Ψ
ω
Ψ
λ

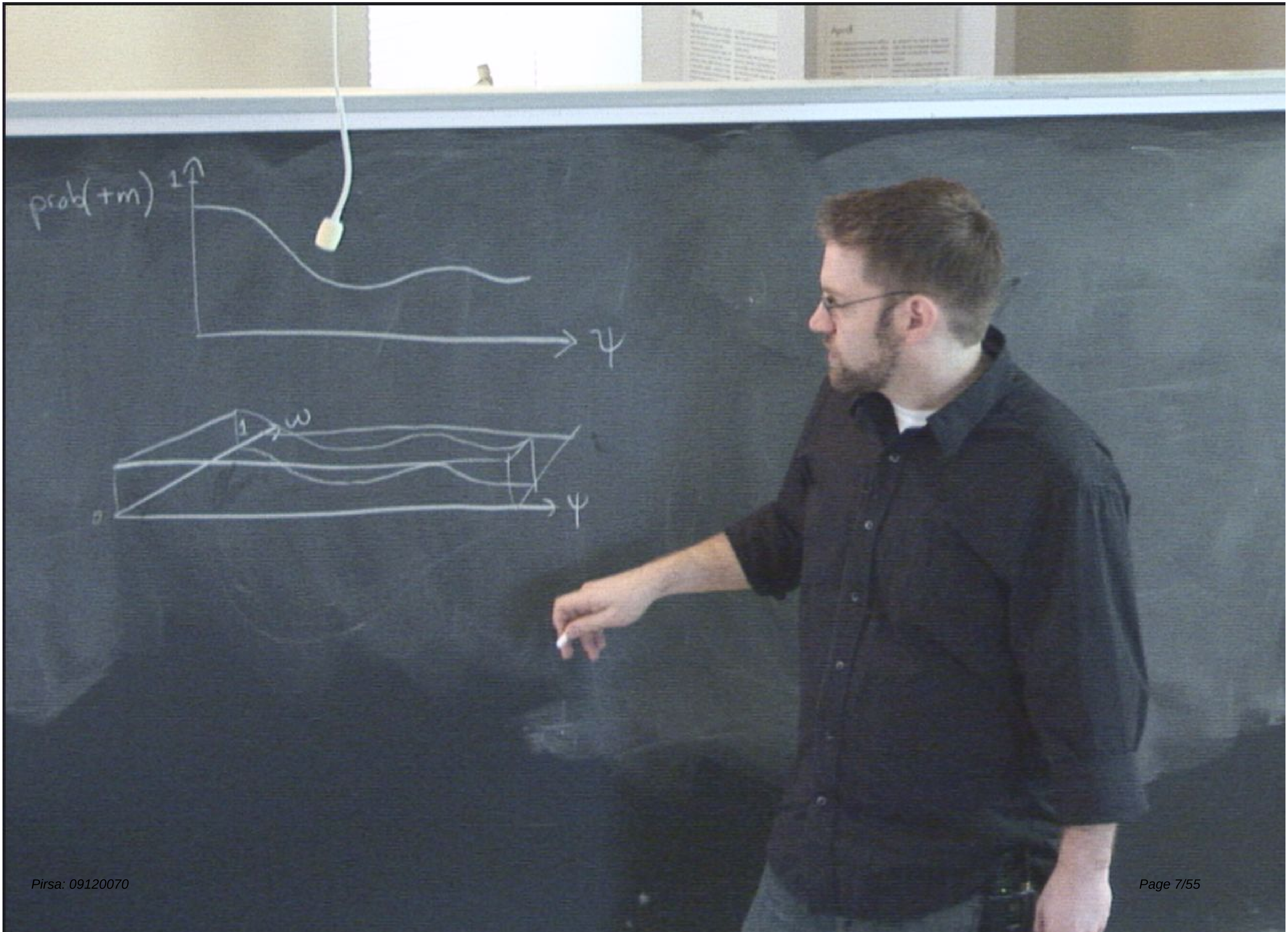
ψ-complete

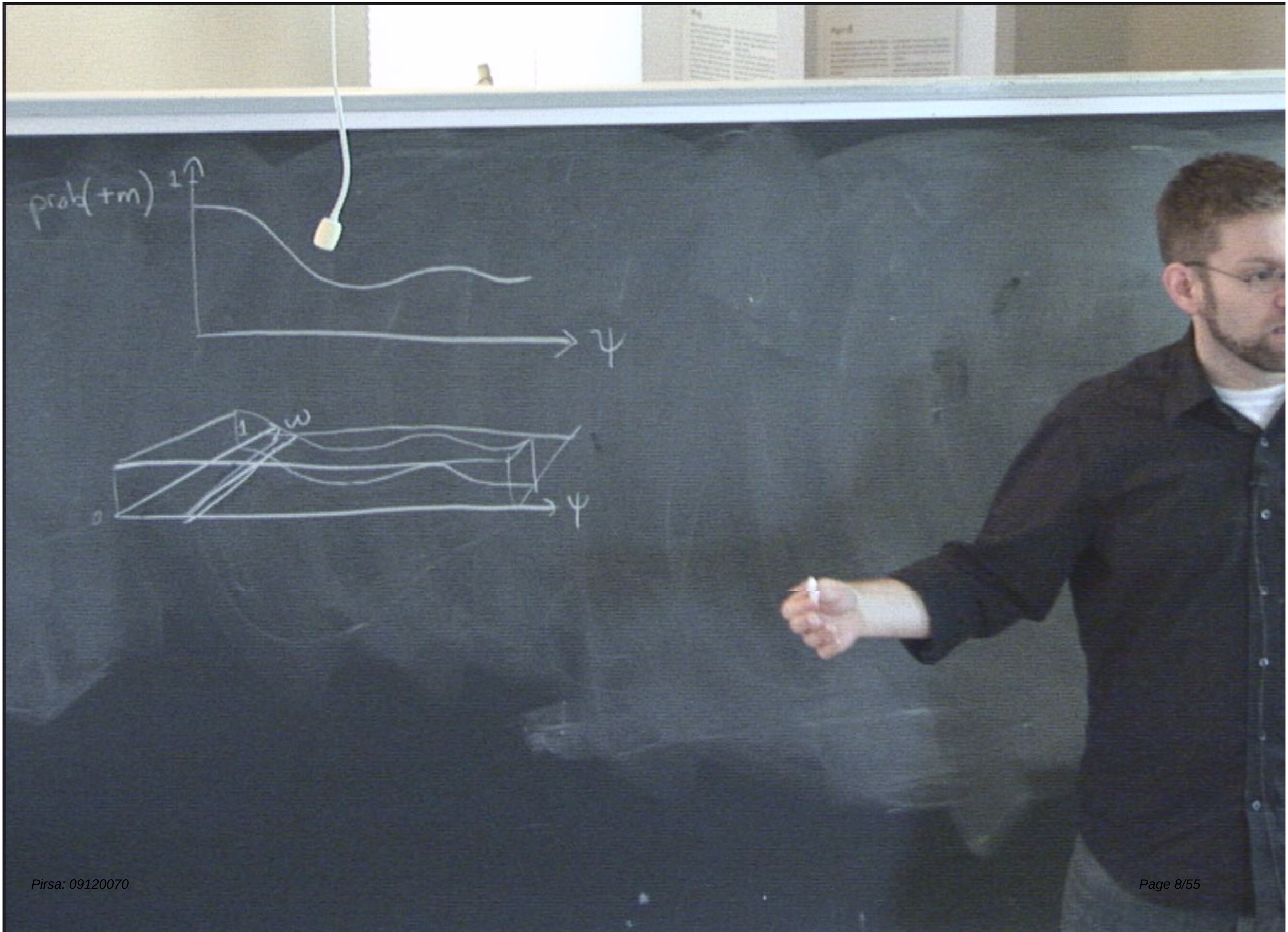
$\big\downarrow$ Eliminate indeterminism

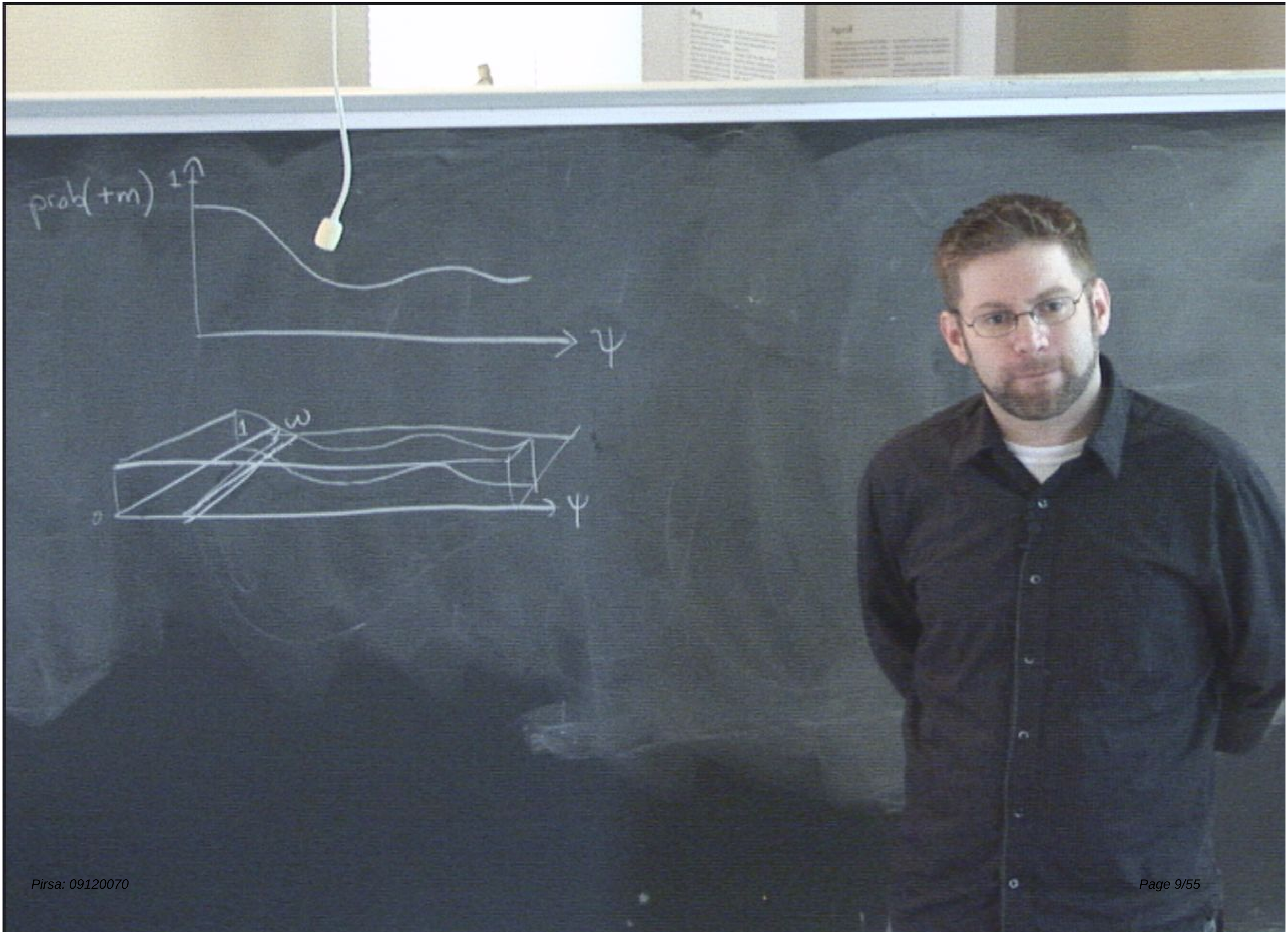
ψ-supplemented

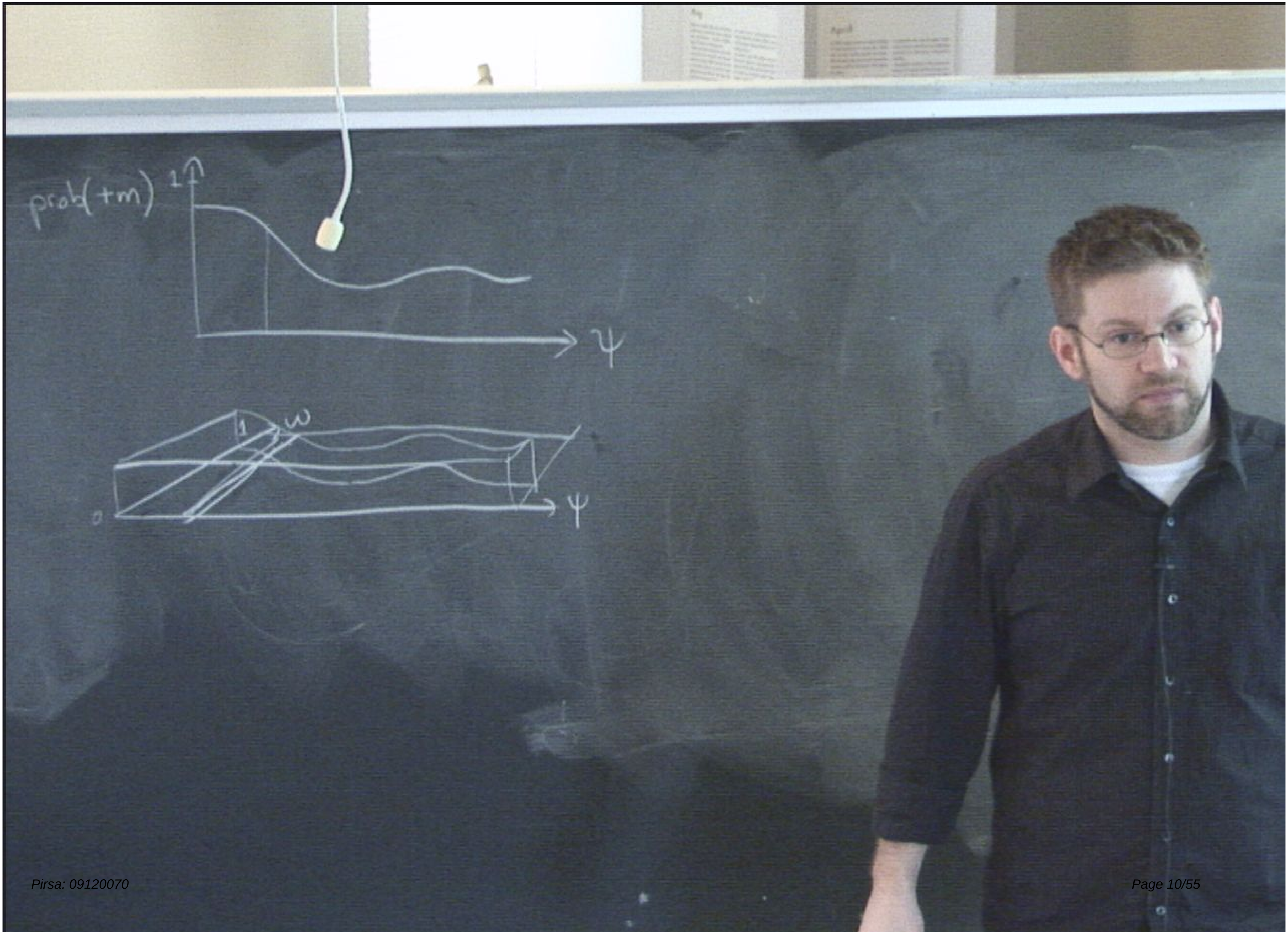

d(+m)
ψ


prob(+m)
1


prob(+m)
1
ψ
1
ω
0
ψ

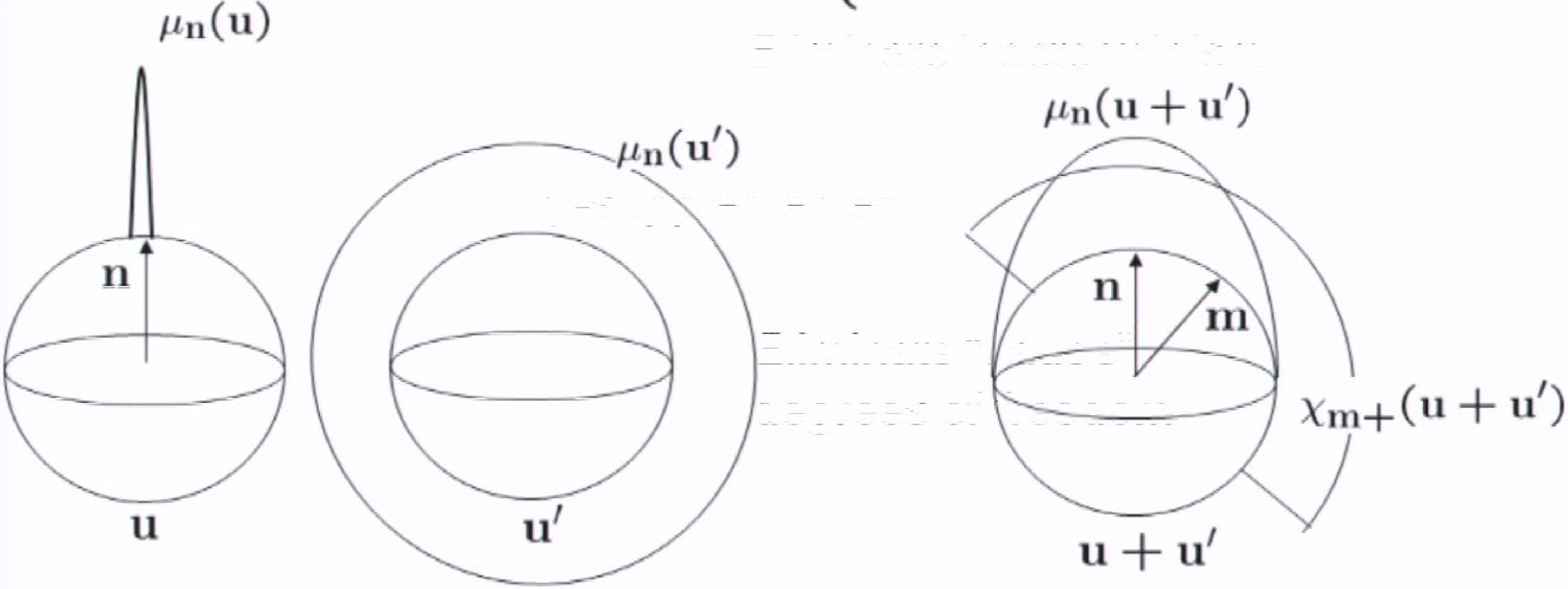

prob(+m)
1
ψ
ω
1
0
ψ


prob(+m)
1
Ψ
ω
1
0
Ψ


prob(+m)
1
ψ
1
ω
0
ψ

ψ-complete

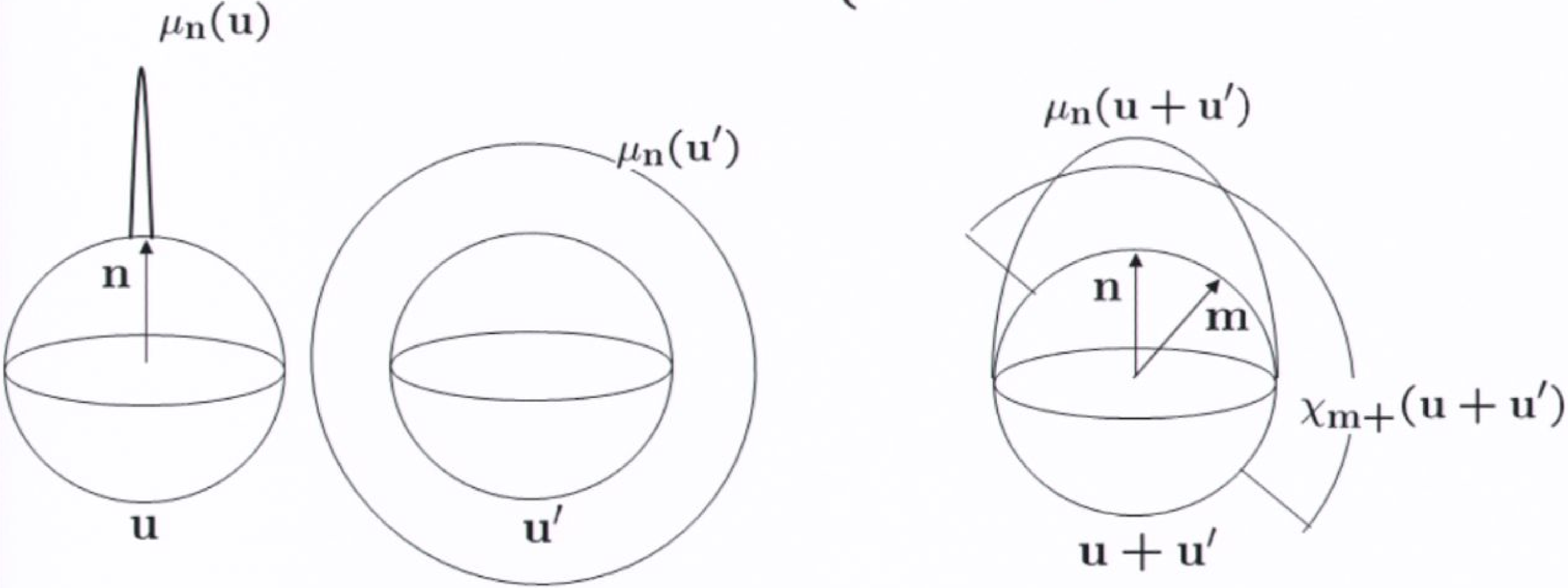
0 otherwise.



The Bell-Mermin model

$$|+\mathbf{n}\rangle \leftrightarrow \quad \mu_{\mathbf{n}}(\mathbf{u},\mathbf{u}') = \frac{1}{4\pi}\delta(\mathbf{u}-\mathbf{n})$$

$$|+\mathbf{m}\rangle \leftrightarrow \quad \chi_{\mathbf{m}+}(\mathbf{u},\mathbf{u}') = \begin{cases} 1 & \text{for } \mathbf{m}\cdot(\mathbf{u}+\mathbf{u}') > 0 \\ 0 & \text{otherwise.} \end{cases}$$



The Kochen-Specker model

$$|+\mathbf{n}\rangle \leftrightarrow \quad \mu_{\mathbf{n}}(\mathbf{u}) = \begin{cases} \frac{1}{\pi}\mathbf{n}\cdot\mathbf{u} & \text{for } \mathbf{n}\cdot\mathbf{u} > 0 \\ 0 & \text{otherwise.} \end{cases}$$

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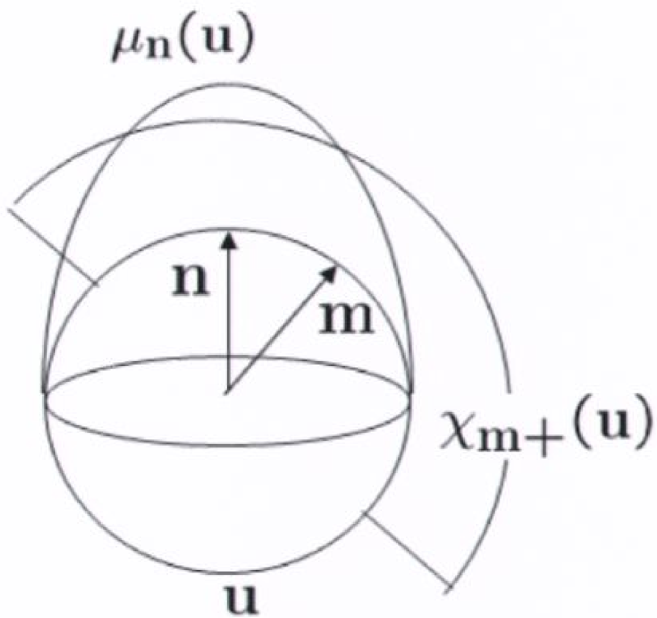


Locality in realist models: Early Work

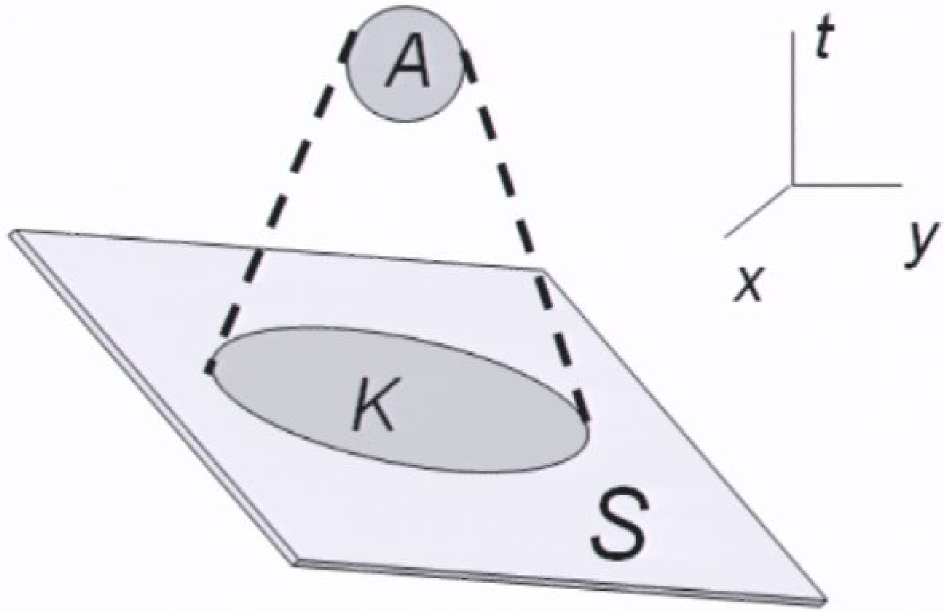
Suppose a region R can be divided into local regions R_1, R_2, ...,R_n. A realist theory is said to be ***separable*** only if the ontic state space Λ_R of region R is the Cartesian product of the ontic state spaces Λ_{R_i} of the regions R_i,

$$\Lambda_R = \Lambda_{R_1} \times \Lambda_{R_2} \times \cdot \quad \cdot \quad \cdot \times \Lambda_{R_n}.$$

This is the paradigm of field theories

A separable realist theory is ***locally deterministic*** if the ontic state of entities within a given region of space-time is determined completely by the ontic state, at any earlier time, of entities lying inside the backward light cone of that region.

"*No action at a distance*"



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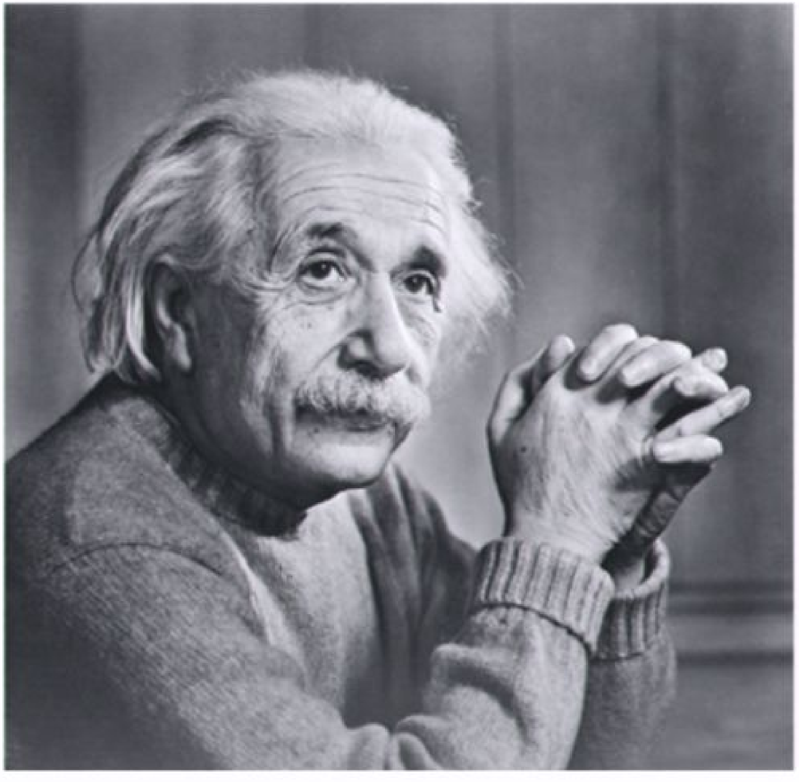
A realist theory is said to be ***local*** if and only if it is separable and

Einstein's arguments concerning the locality of quantum theory



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ψ-complete model: $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$

$$\mathcal{PH}_{AB} \neq \mathcal{PH}_A \times \mathcal{PH}_B$$

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$$\mathcal{P}\mathcal{H}_{AB} \neq \mathcal{P}\mathcal{H}_A \times \mathcal{P}\mathcal{H}_B$$ not separable

ψ-ontic model:

$$\lambda_{AB} = (\psi_{AB}, \omega_{AB})$$
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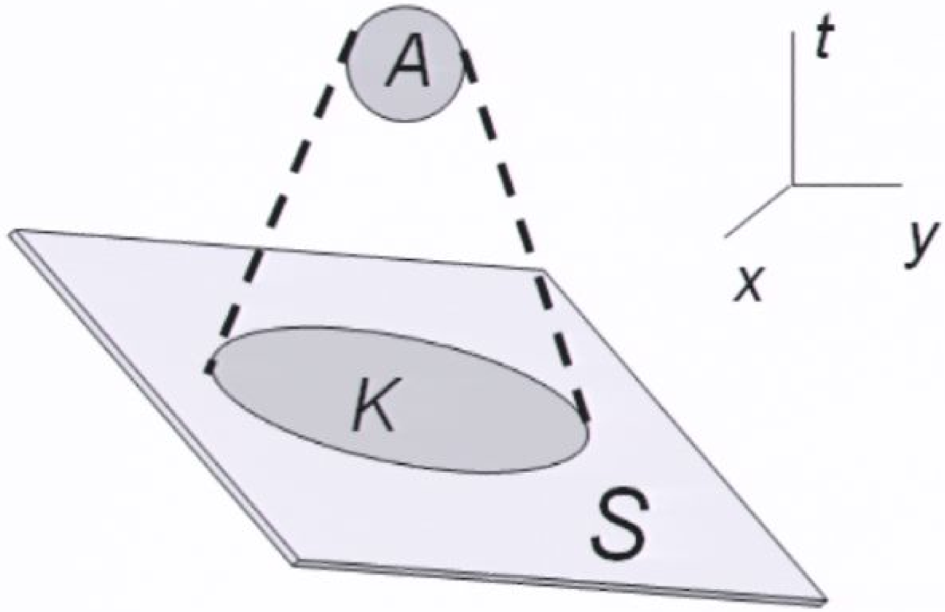
$\lambda_{AB} = (\lambda_A, \lambda_B)?$ not separable (with a caveat)

Einstein: "The field in a many-dimensional coordinate space does not smell like something real"

"If only the undulatory fields introduced there could be transplanted

A separable realist theory is ***locally deterministic*** if the ontic state of entities within a given region of space-time is determined completely by the ontic state, at any earlier time, of entities lying inside the backward light cone of that region.

"*No action at a distance*"



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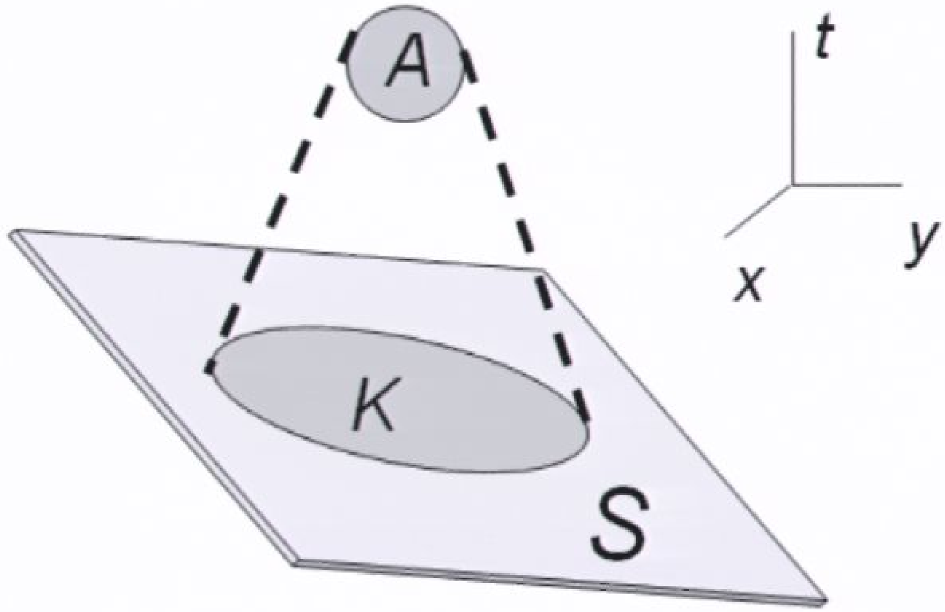

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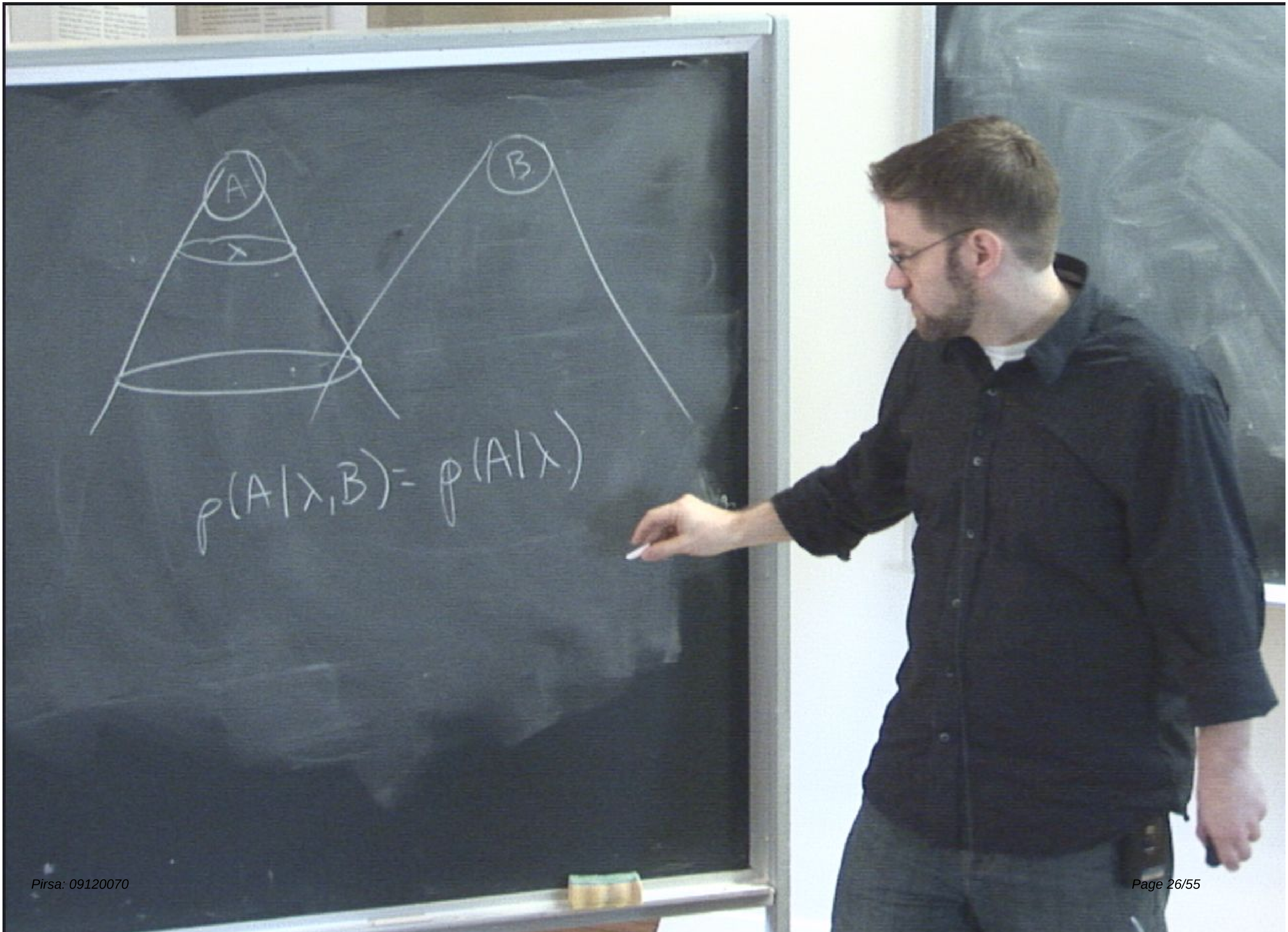
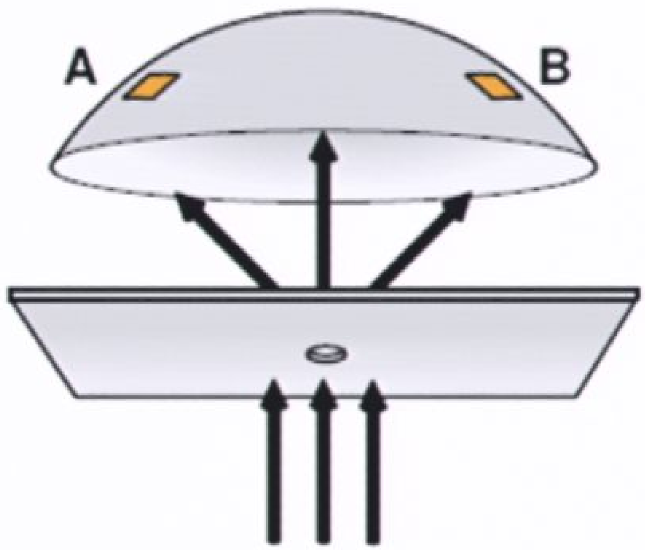

A
λ
B
p(A|λ,B) = p(A|λ)

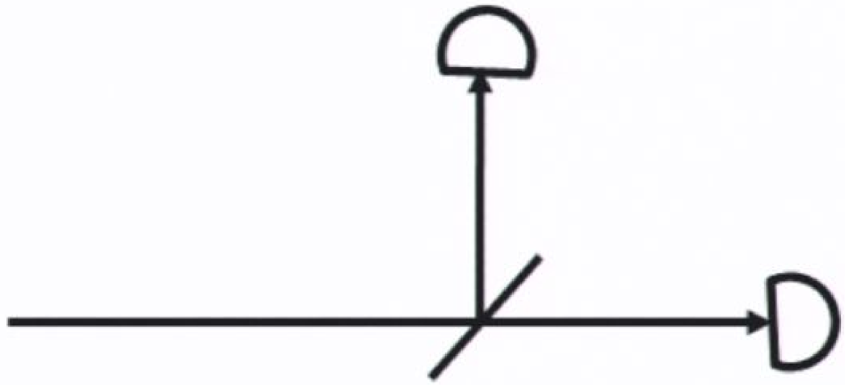
MR EINSTEIN.[a] — Despite being conscious of the fact that I have not entered deeply enough into the essence of quantum mechanics, nevertheless I want to present here some general remarks.[b]

Einstein's 1927 Solvay argument



f $|\psi|^2$ were simply regarded as the probability that at a certain point a given article is found at a given time, it could happen that *the same* elementary rocess produces an action *in two or several* places on the screen. But the nterpretation, according to which $|\psi|^2$ expresses the probability that *this* article is found at a given point, assumes an entirely peculiar mechanism f action at a distance, which prevents the wave continuously distributed in pace from producing an action in *two* places on the screen.

Einstein's 1927 Solvay argument



$$|\psi\rangle = \sqrt{2}^{-1}(|A\rangle + |B\rangle)$$

"Now, if nothing is observed, this negative result will signify that the particle is not in box [A] and it is thus in box [B] in Paris. But this can reasonably signify only one thing: the particle was already in Paris in box [B] prior to the drainage experiment made in Tokyo in box [A]. Every other interpretation is absurd. How can we imagine that the simple fact of having observed nothing in Tokyo has been able to promote the localization of the particle at a distance of many thousands of miles away?"

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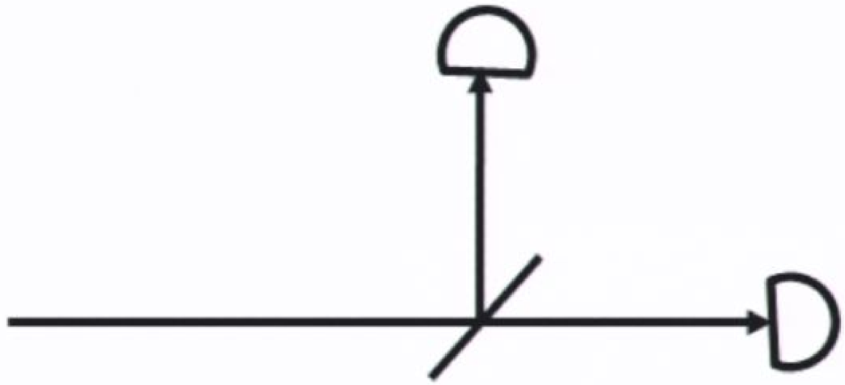

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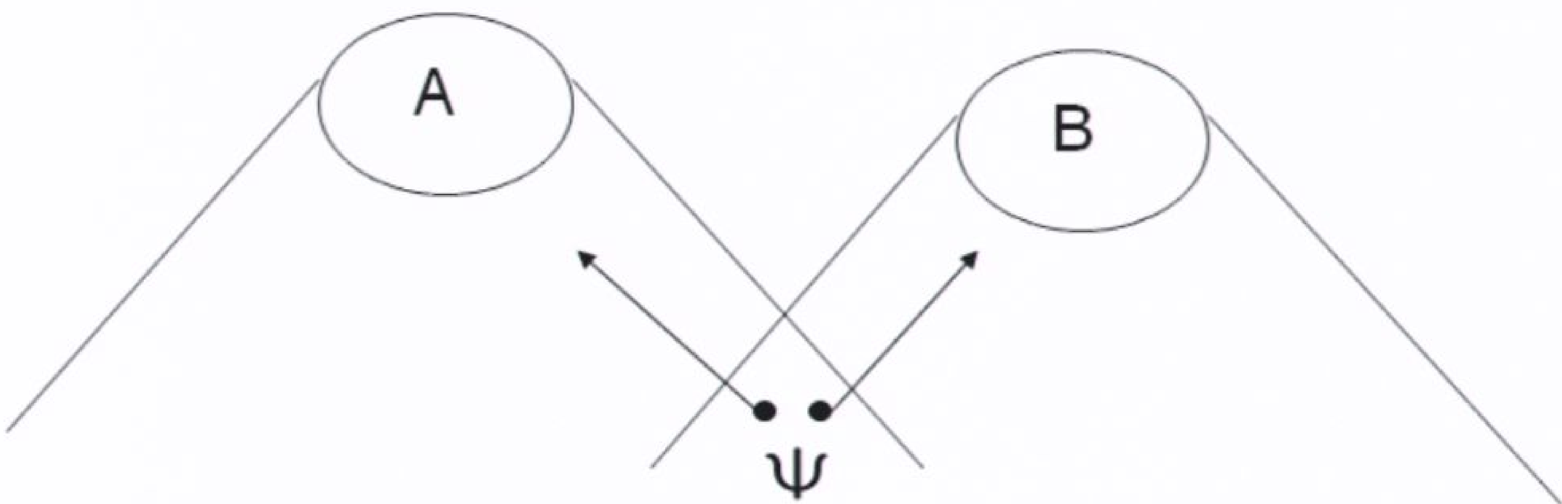
Einstein's 1927 Solvay argument

Adapted to spin

Suppose

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}\left(|+z\rangle|+z\rangle + |-z\rangle|-z\rangle\right)$$

At A one measures $\{|+z\rangle, |-z\rangle\}$



Theorem: Quantum statistics + local causality → ψ-incompleteness

Proof by contradiction: (see Norsen, 2005):

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|+\mathbf{z}\rangle|+\mathbf{z}\rangle + |-\mathbf{z}\rangle|-\mathbf{z}\rangle)$$

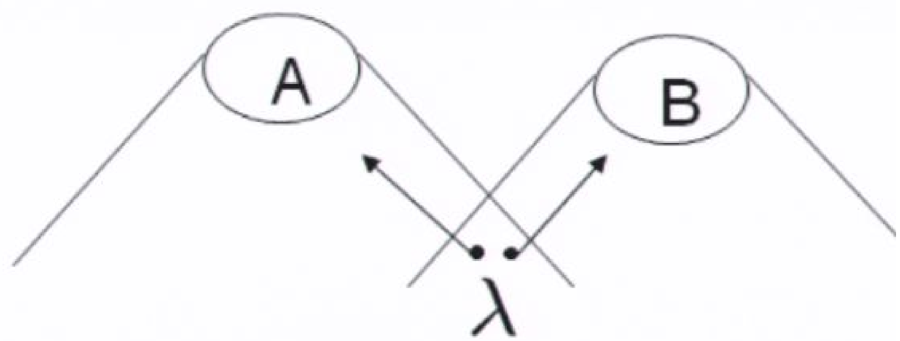
Probability theory guarantees that

$$p(+\mathbf{z}_A, +\mathbf{z}_B|\Psi) = p(+\mathbf{z}_A|\Psi)p(+\mathbf{z}_B|\Psi, +\mathbf{z}_A)$$

By local causality

$$p(E_B|\lambda, E_A) = p(E_B|\lambda)$$

where λ is a complete description



ψ-completeness → $\lambda = \Psi$

$$p(+\mathbf{z}_B|\Psi, +\mathbf{z}_A) = p(+\mathbf{z}_B|\Psi)$$

Thus $p(+\mathbf{z}_A, +\mathbf{z}_B|\Psi) = p(+\mathbf{z}_A|\Psi)p(+\mathbf{z}_B|\Psi) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

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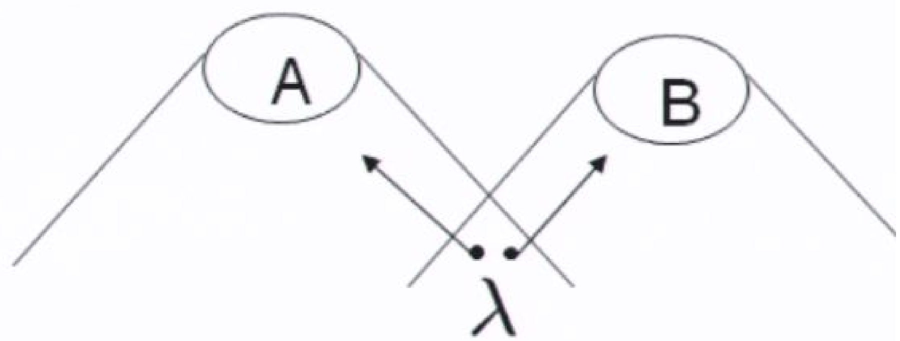

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MR BOHR.[a] — I feel myself in a very difficult position because I don't understand what precisely is the point which Einstein wants to [make]. No doubt it is my fault.


P. Ehrenfest, PD-OLD


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MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

The EPR argument

EPR completeness condition:

Every element of the physical reality must have a counterpart in the physical theory

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EPR criterion of reality:

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity.

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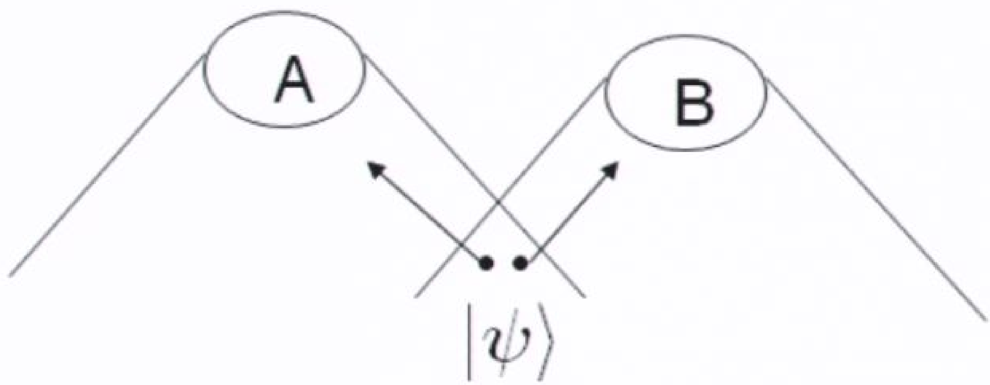
If , without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the outcome of a measurement procedure, then there exists an attribute associated with that measurement.

The EPR argument



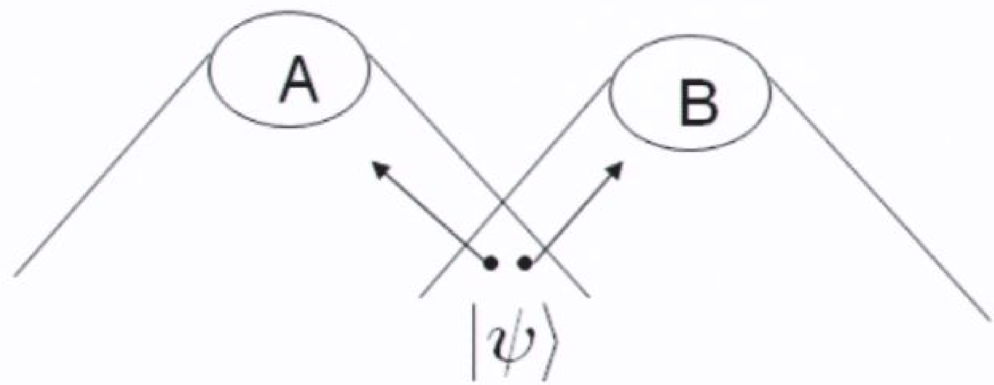
$$|\psi\rangle = \int dx_1\, dx_2\, \delta(x_1 - x_2)|x_1\rangle|x_2\rangle$$
$$= \int dp_1\, dp_2\, \delta(p_1 + p_2)|p_1\rangle|p_2\rangle$$

On particle 1, measure either X or P

If a measurement of X is made on 1, then an attribute X exists for 2

By locality, the existence of the attribute for 2 cannot be dependent

The EPR argument



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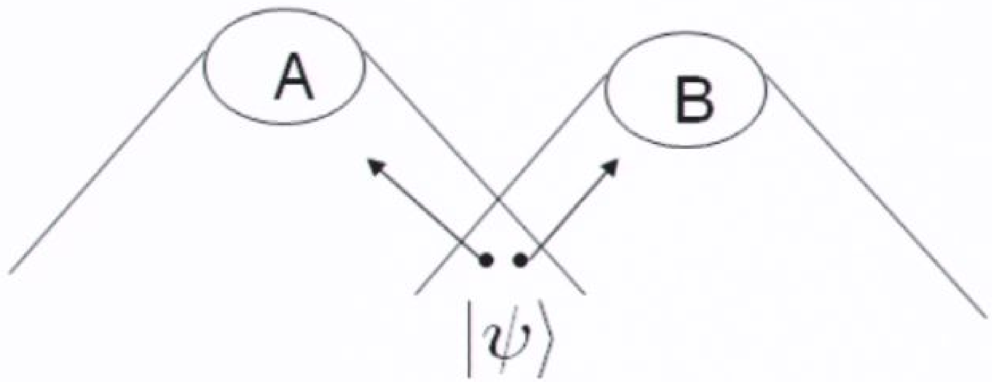

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}\left(|+z\rangle|+z\rangle + |-z\rangle|-z\rangle\right)$$
$$= \tfrac{1}{\sqrt{2}}\left(|+x\rangle|+x\rangle + |-x\rangle|-x\rangle\right)$$

On particle 1, measure either $\{|+z\rangle, |-z\rangle\}$ or $\{|+x\rangle, |-x\rangle\}$

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

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OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

"… an ambiguity as regards the meaning of the expression 'without in any way disturbing a system'. Of course, there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage ***there is essentially the question of an influence on the very conditions which define the possible types of predictions regarding the future behaviour of the system***."

"…there can be no question of any unambiguous interpretation of the symbols of quantum mechanics other than that embodied in the well-known rules which allow to predict the results obtained by a given experimental arrangement described in a totally classical way."

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Why complicate the argument?
-- Redhead, Fine, Maudlin, Norsen, …

Maudlin suggests that it was to also beat the uncertainty principle by establishing the real existence of particle 2's momentum. He refers to this as "an unnecessary bit of grandstanding (probably due to Podolsky)" which "plunged the previous simple EPR argument into the muddy waters of [modal logic and counterfactual definiteness]."

Why complicate the argument?
-- Redhead, Fine, Maudlin, Norsen, …

Maudlin suggests that it was to also beat the uncertainty principle by establishing the real existence of particle 2's momentum. He refers to this as "an unnecessary bit of grandstanding (probably due to Podolsky)" which "plunged the previous simple EPR argument into the muddy waters of [modal logic and counterfactual definiteness]."

Einstein's attitude to EPR

"For reasons of language, this was written by Podolsky after many discussions. But still it has not come out as well as I really wanted; on the contrary, the main point was, so to speak, buried by the erudition."
--Einstein, letter to Schroedinger, June 1935 (trans. by D. Howard)

Einstein's own 1935 argument

in correspondence with Schroedinger, June 1935

Einstein's notion of completeness (distinct from EPR):
"… ψ is correlated one-to-one with the real state of the real system … If this works, then I speak of a compete description of reality by the theory." → ψ-completeness

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Recall: ψ-ontic view: different ψ imply different physical states

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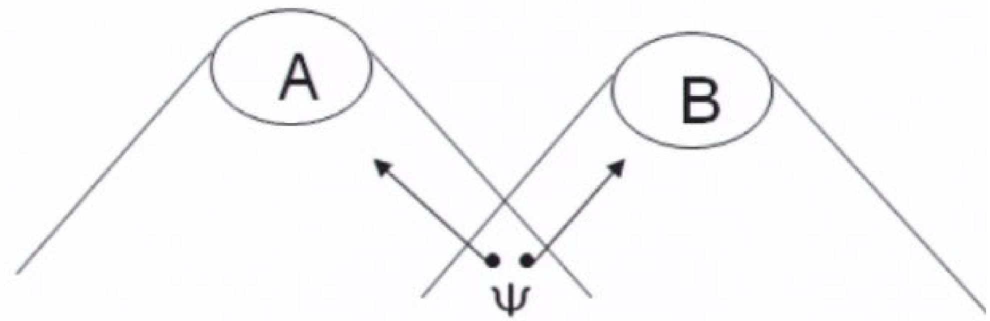
Einstein's own 1935 argument

Suppose A and B share



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|+z\rangle|+z\rangle + |-z\rangle|-z\rangle\right)$$

If A measures $\{|+z\rangle, |-z\rangle\}$

B's state becomes $|+z\rangle$ with probability 1/2
$|-z\rangle$ with probability 1/2

If A measures $\{|+x\rangle, |-x\rangle\}$

B's state becomes $|+x\rangle$ with probability 1/2
$|-x\rangle$ with probability 1/2

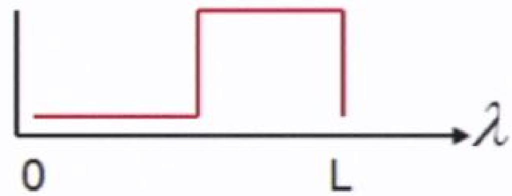
"Steering"

$$p(\lambda', \lambda) = \frac{1}{L}\delta(\lambda - \lambda') \qquad \lambda, \lambda' \in [0, L]$$

If A measures whether $\lambda' \in [0, \frac{L}{2}]$ or not

Her knowledge of B is updated to

with prob. 1/2

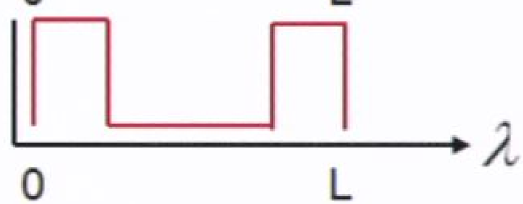


with prob. 1/2

If A measures whether $\lambda' \in [\frac{L}{4}, \frac{3L}{4}]$ or not

Her knowledge of B is updated to

with prob. 1/2



with prob. 1/2

This requires a model that is not just ψ-incomplete, but ψ-epistemic