

Title: Foundations of Quantum Mech. (PHYS 639) - Lecture 5

Date: Dec 04, 2009 11:00 AM

URL: <http://pirsa.org/09120069>

Abstract:

Forays into Realism

Realist talk

Entities:

toy blocks,...

Attributes:

color, shape, taste,...

Instances of the attribute “color”:

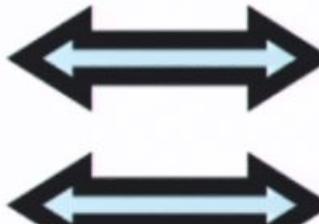
red, blue, green,....

A naïve realist



In mathematical theories,

attributes



variables

instances of attributes

values of variables

Possible features of realist theories:

A realist theory is **atomic** if macroscopic entities can be thought of as composed of fundamental entities.

A realist theory is **reductionist** if the state of a composite system can be specified by specifying the state of all the fundamental entities of which it is composed.

More realist talk

Elements:

Fundamental entities

Attributes

Instances of attribute

Set of independent attributes

Specification of the instances of a
set of independent attributes
= state $\leftrightarrow \lambda$

State space $\rightarrow \Lambda$

In Newtonian physics:

Particles

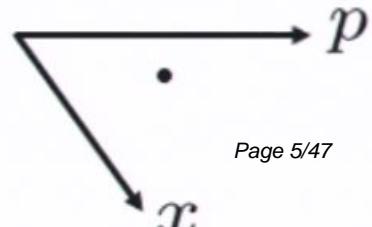
Position, momentum, energy

$x=0, x=1, x=2$

$\{x, p\}$

$\lambda = (x=3, p=5)$

$\Lambda = \mathbb{R}^2$



More realist talk

Elements:

Fundamental entities

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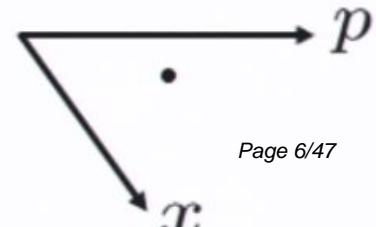
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Possible features of realist theories:

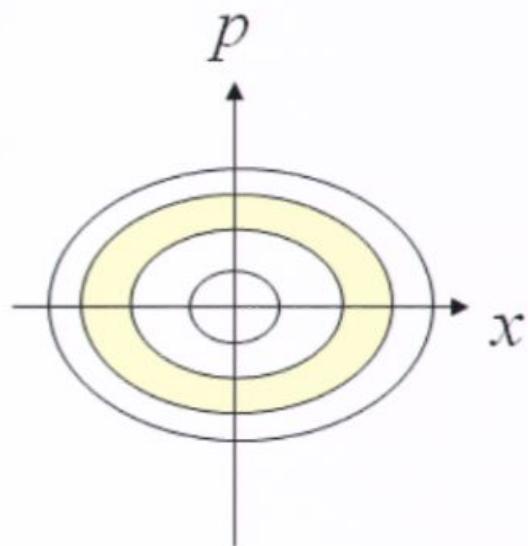
A realist theory is **deterministic** if the law of state evolution is such that for each state that is possible at a given time, there is a unique state possible at every future and earlier time.



An intelligent being who knew for a given instant all the forces by which nature is animated and possessed complete information on the state of matter of which nature consists – providing his mind were powerful enough to analyze these date – could express in the same equation the motion of the largest bodies of the universe and the motion of the smallest atoms. Nothing would be uncertain for him, and he would see the future as well as the past at one glance.

Pierre-Simon Laplace, 1820

Properties: attributes that have only two instances
A property either holds or fails to hold.
They are associated with regions of the state space



$$E(x, p) = \frac{p^2}{2m} + \frac{kx^2}{2}$$

$$E_1 \leq E \leq E_2$$

Consider $\alpha(x, p) = a_1$ if $x < x_1$,
 $= a_2$ if $x_1 \leq x \leq x_2$,
 $= a_3$ if $x > x_2$.

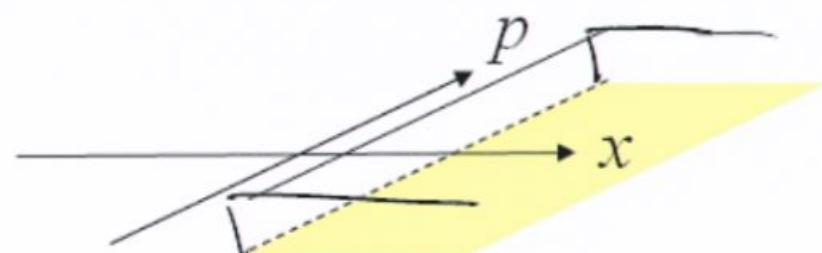
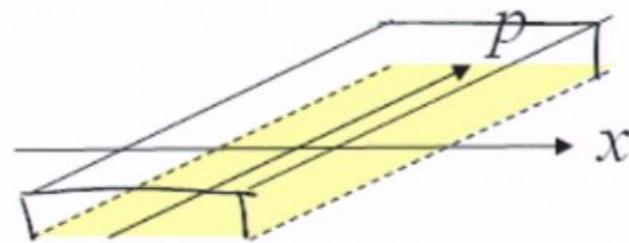
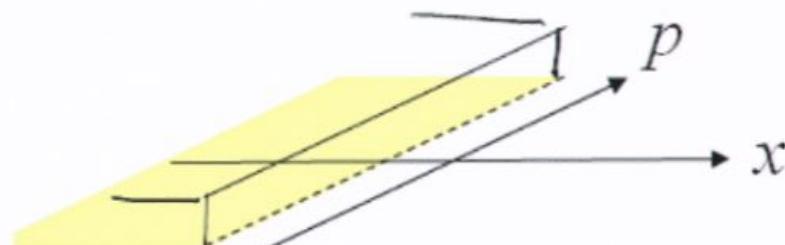
Equivalently, $\alpha(x, p) = \sum_k a_k \chi_k(x, p)$

where

$$\begin{aligned}\chi_1(x, p) &= 1 \text{ if } x < x_1 \\ &= 0 \text{ otherwise,}\end{aligned}$$

$$\begin{aligned}\chi_2(x, p) &= 1 \text{ if } x_1 \leq x \leq x_2 \\ &= 0 \text{ otherwise,}\end{aligned}$$

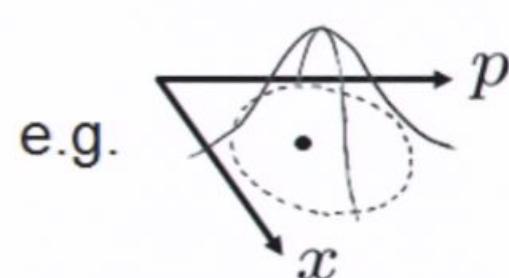
$$\begin{aligned}\chi_3(x, p) &= 1 \text{ if } x > x_2 \\ &= 0 \text{ otherwise,}\end{aligned}$$



Ontic states vs. Epistemic states

Ontic state = real state of affairs

Epistemic state = state of knowledge



Classical case:

Points in phase space are ontic states

Probability distributions over phase space are epistemic states

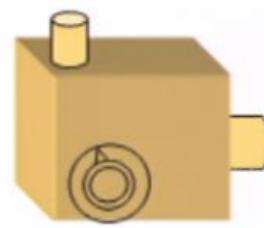


“But our present QM formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature --- all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble. Yet we think that the unscrambling is a prerequisite for any further advance in basic physical theory. For, if we cannot separate the subjective and objective aspects of the formalism, we cannot know what we are talking about; it is just that simple.”

--E.T. Jaynes

Realism meets operationalism

Operational Quantum Mechanics

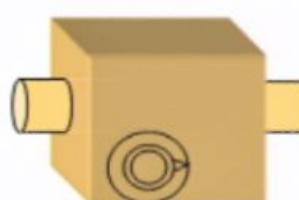


Preparation

P

Density operator

ρ

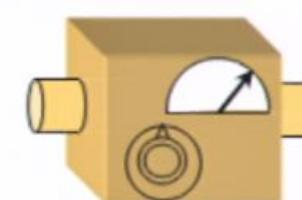


Transformation

T

Trace-preserving
completely positive
linear map (CP map)

\mathcal{T}



Measurement

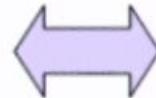
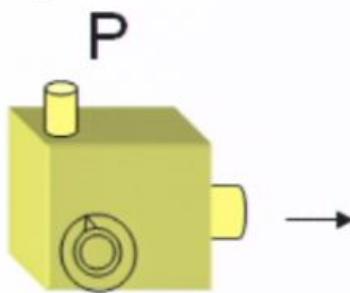
M

Positive operator-valued
measure (POVM)

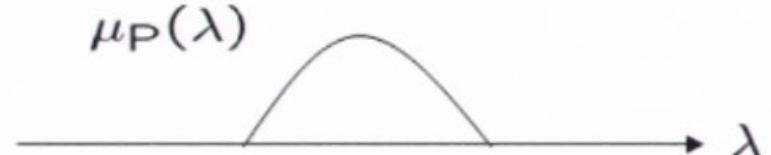
$\{E_k\}$

A realist model of an operational theory
assumes primitives of systems, properties and states

Preparation

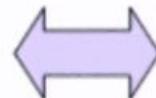
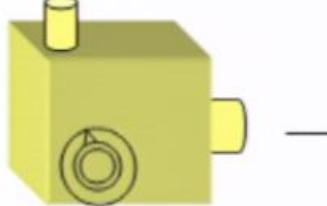


$$\int \mu_P(\lambda) d\lambda = 1$$



A realist model of an operational theory
assumes primitives of systems, properties and states

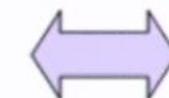
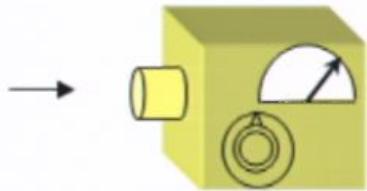
Preparation P



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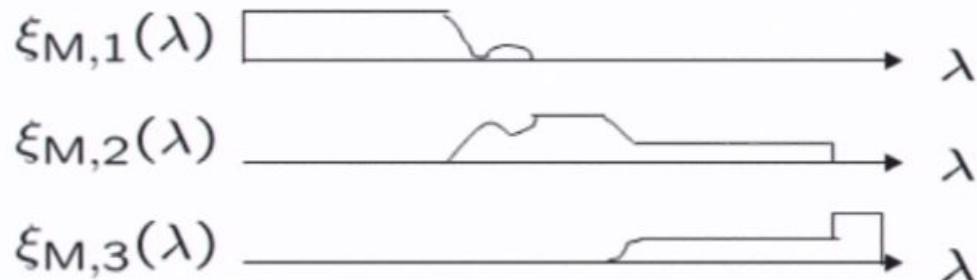


Measurement M



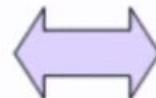
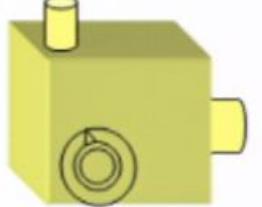
$$0 \leq \xi_{M,k} \leq 1$$

$$\sum_k \xi_{M,k}(\lambda) = 1 \text{ for all } \lambda$$

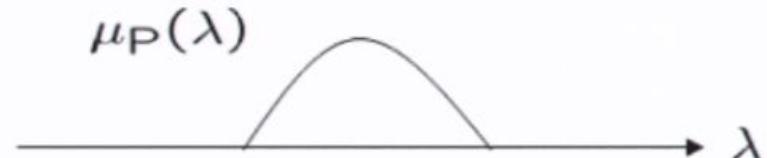


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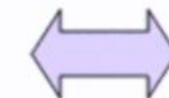
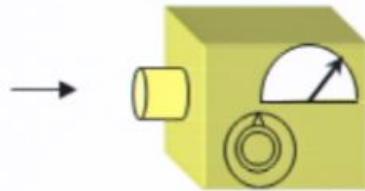
Preparation P



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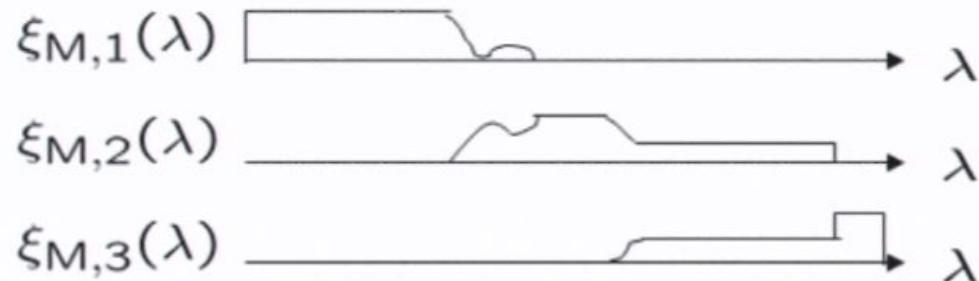


Measurement M

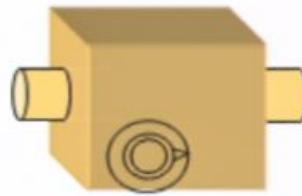


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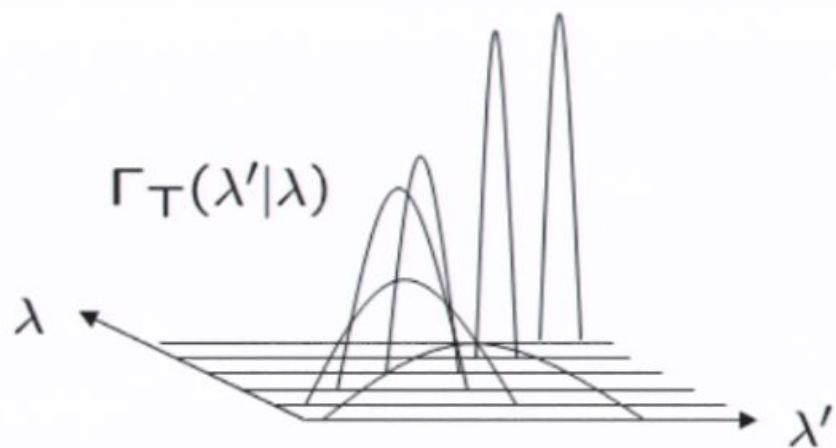


$$p(k|P, M) = \int d\lambda \xi_{M,k}(\lambda) \mu_P(\lambda)$$

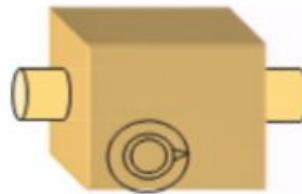


Transformation

T

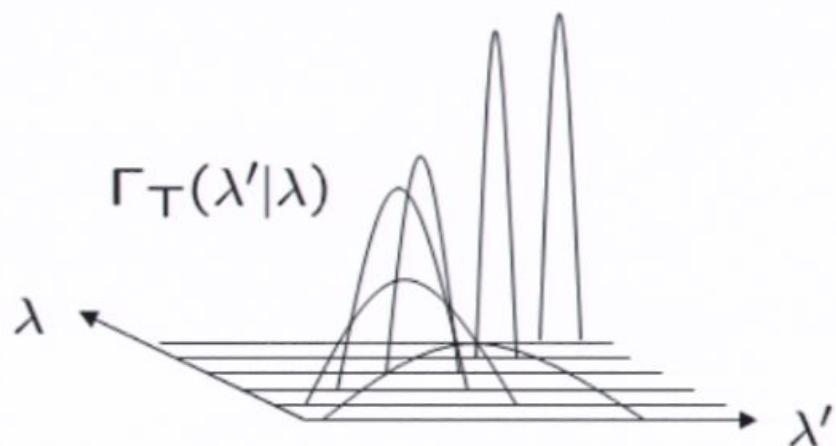


$$\int \Gamma_T(\lambda'|\lambda) d\lambda' = 1 \text{ for all } \lambda$$



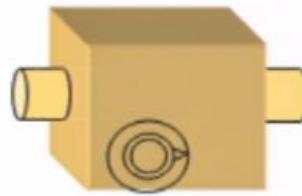
Transformation

T



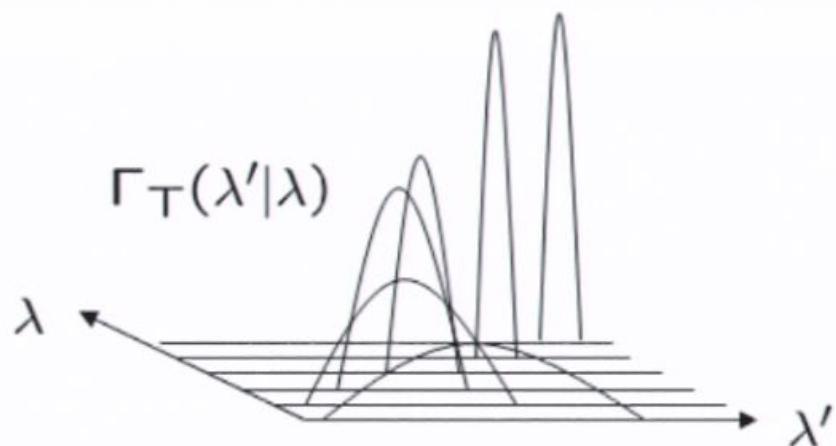
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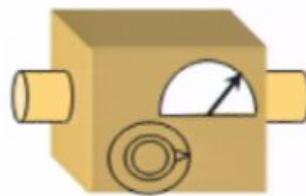
Transformation

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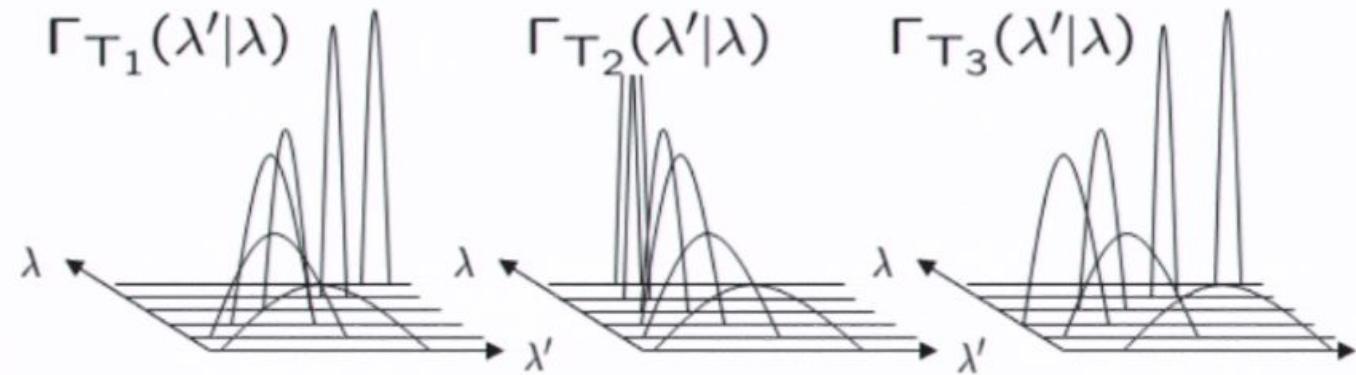
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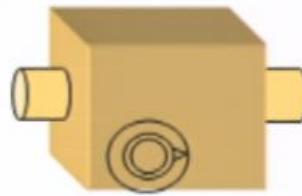


Measurement-induced
transformations

$\{\mathsf{T}_k\}$

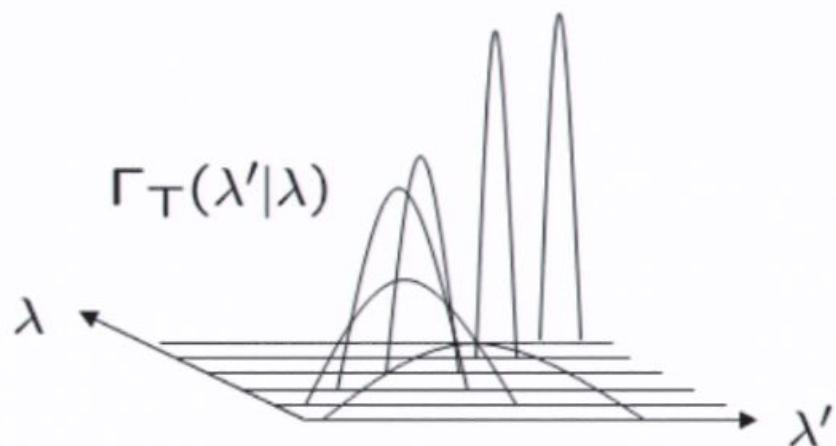


$$\int \Gamma_{T_k}(\lambda'|\lambda) d\lambda' \leq 1 \text{ for all } \lambda, k$$



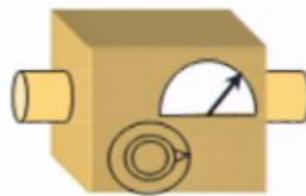
Transformation

T



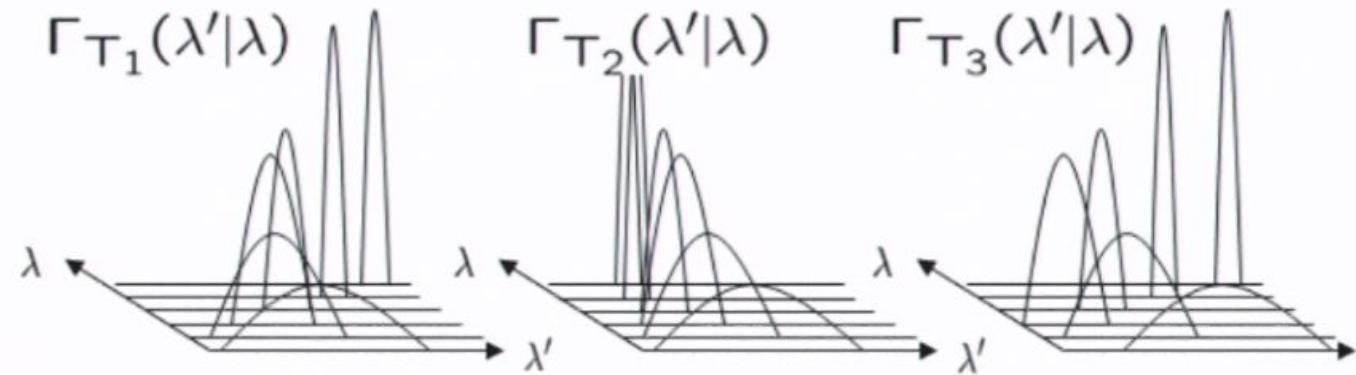
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$$p(k|P, T, M) = \int d\lambda' \xi_{M,k}(\lambda) \int d\lambda \Gamma_T(\lambda'|\lambda) \mu_P(\lambda)$$



Measurement-induced
transformations

$\{\mathsf{T}_k\}$



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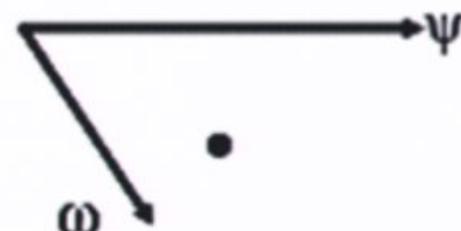
A useful classification of realist models

a) ψ -complete



Complete state is ψ

b) ψ -supplemented

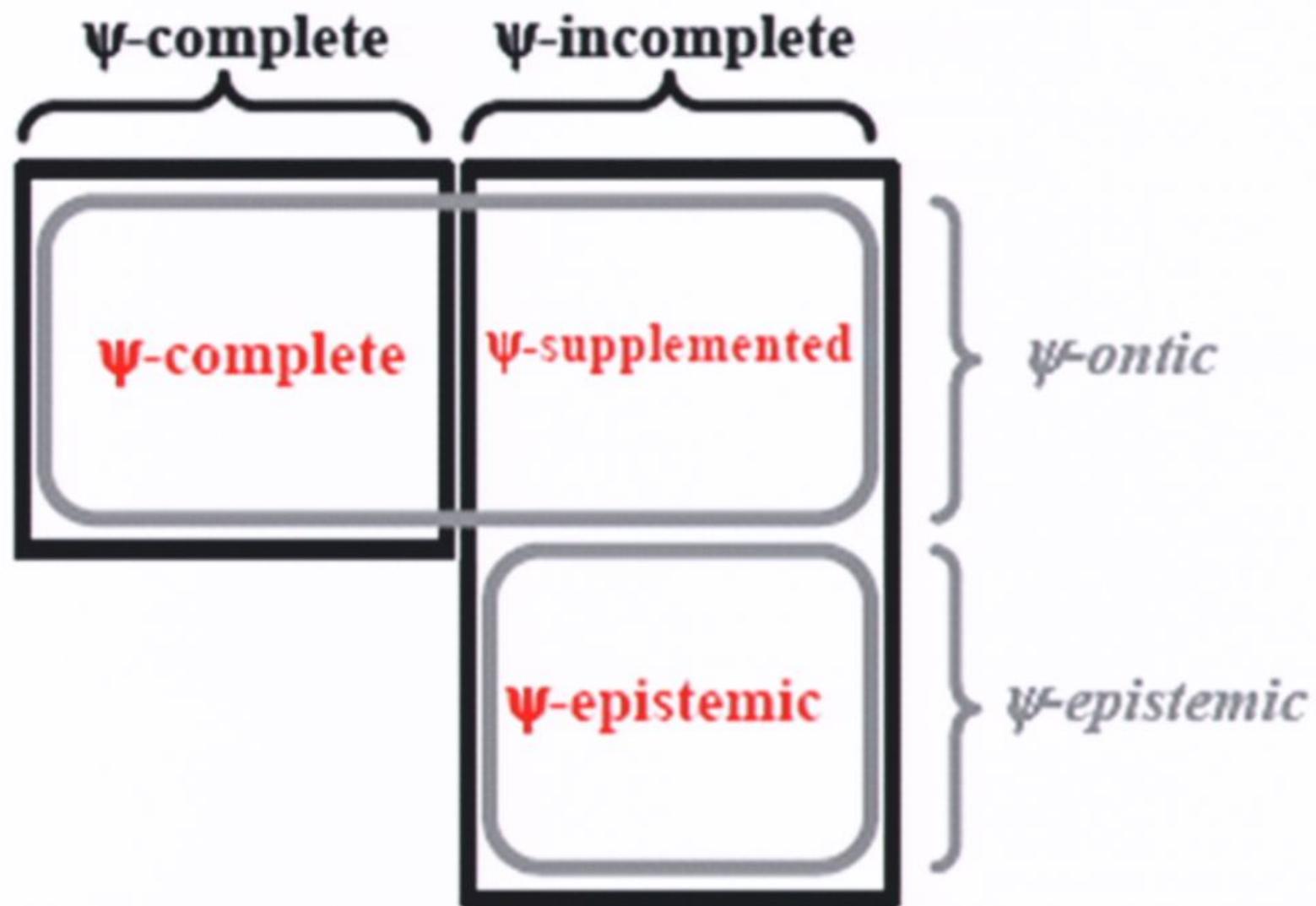


Complete state is $\lambda = (\psi, \omega)$

c) ψ -epistemic



Complete state is λ



ψ -complete model:

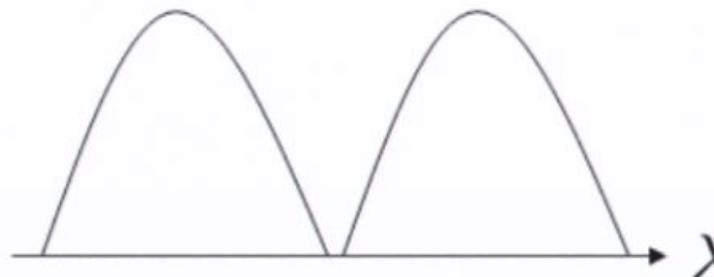
Space of physical states = space of rays in Hilbert space

$$\lambda = \psi$$

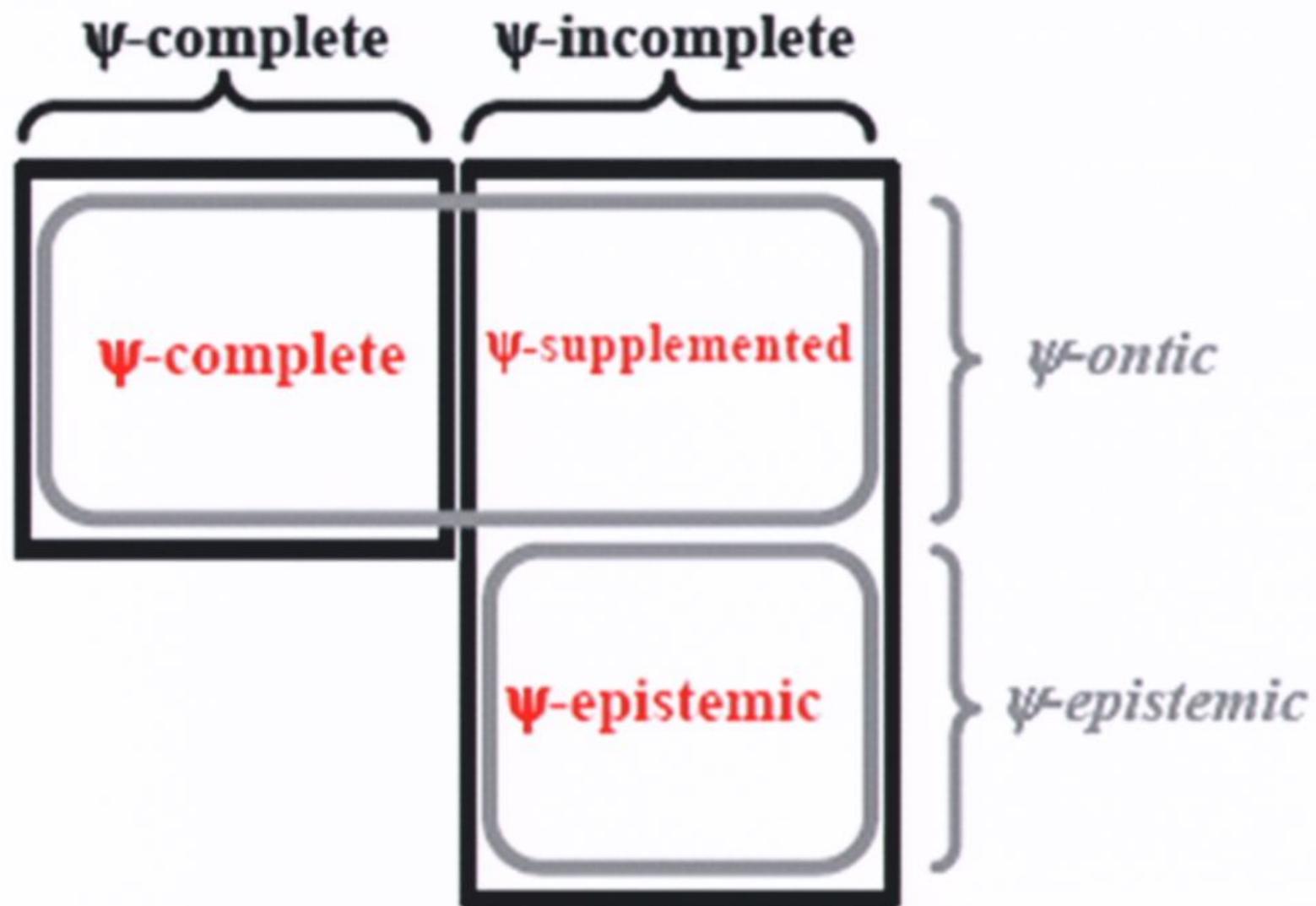
ψ -ontic model:

For any preparation procedures $P_{|\psi_1\rangle}$, $P_{|\psi_2\rangle}$ with $|\psi_1\rangle \neq |\psi_2\rangle$

$$\mu(\lambda|P_{|\psi_1\rangle})\mu(\lambda|P_{|\psi_2\rangle}) = 0 \text{ for all } \lambda$$



ψ -epistemic model: $\exists |\psi_1\rangle \neq |\psi_2\rangle$



ψ -complete



ψ

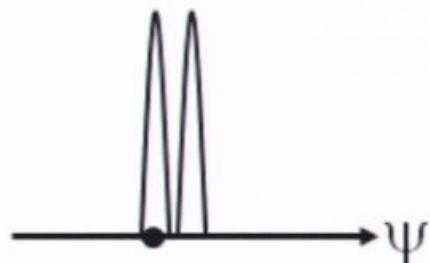
Complete state is ψ

ψ -complete



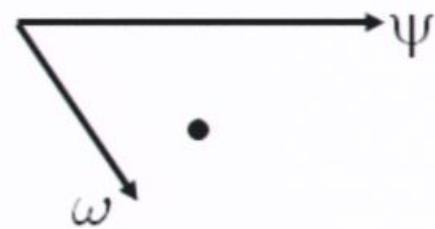
Complete state is ψ

ψ -complete $\rightarrow \psi$ -ontic



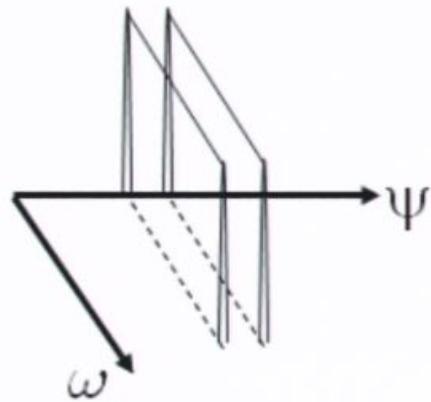
Complete state is ψ

ψ -supplemented



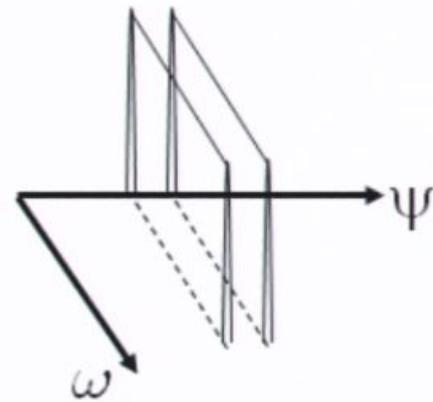
Complete state is (ψ, ω)

ψ -supplemented \rightarrow ψ -ontic



Complete state is (ψ, ω)

ψ -supplemented \rightarrow ψ -ontic



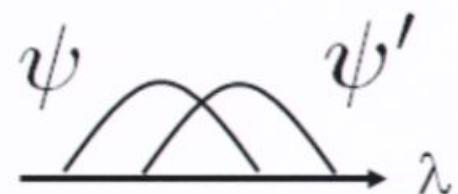
Complete state is (ψ, ω)

ψ -epistemic



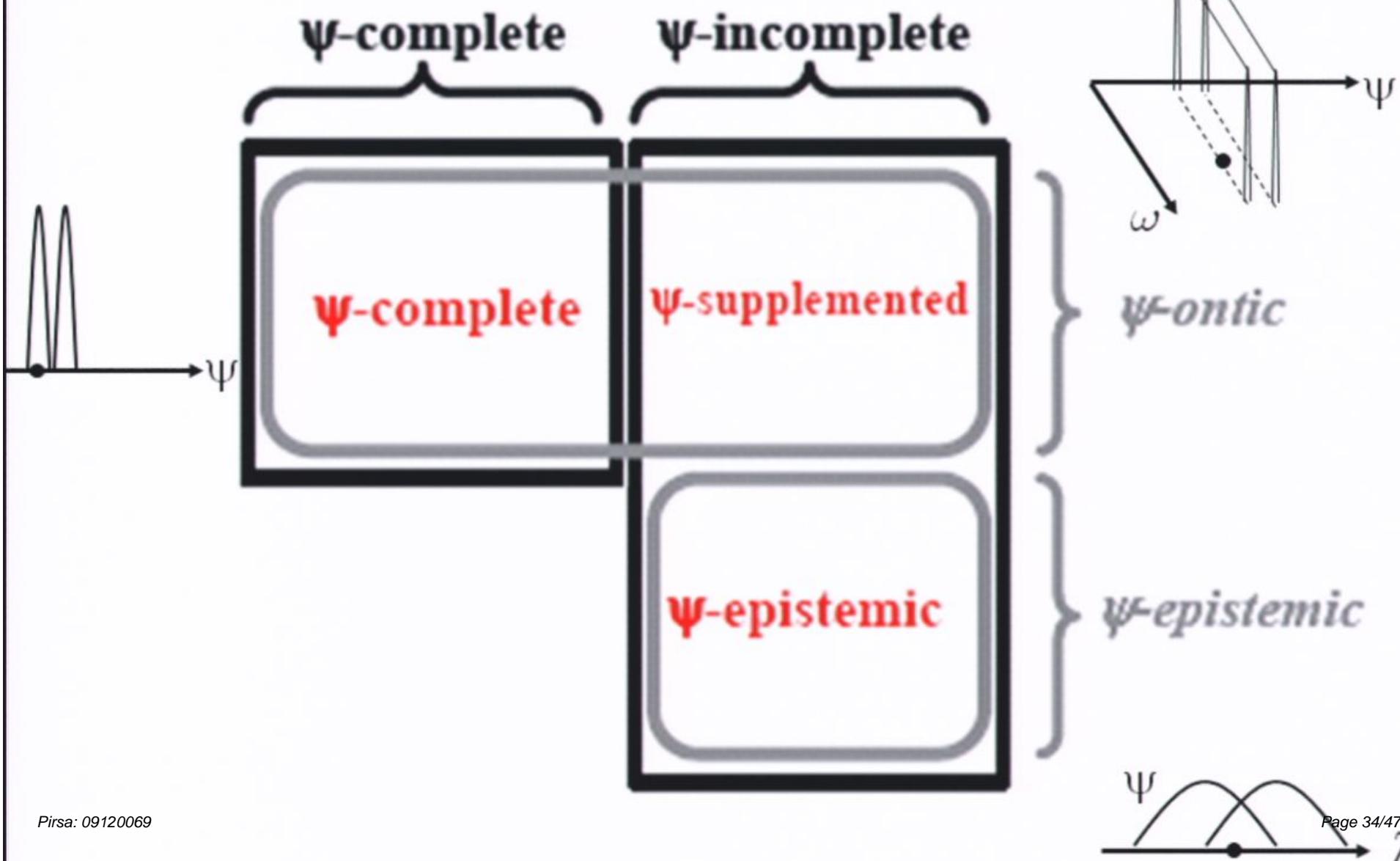
Complete state is λ

ψ -epistemic



Complete state is λ

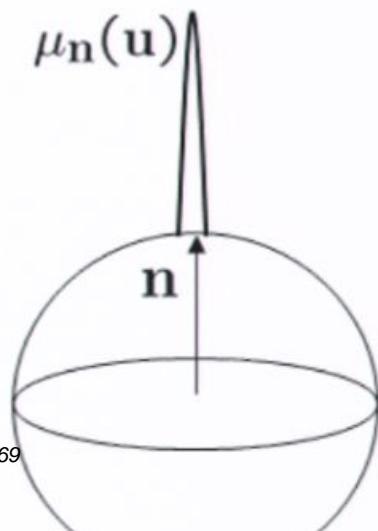
Hidden variable models



Some simple models for pure states and sharp measurements in a 2D Hilbert space

The orthodox model (Beltrametti-Bugajski)

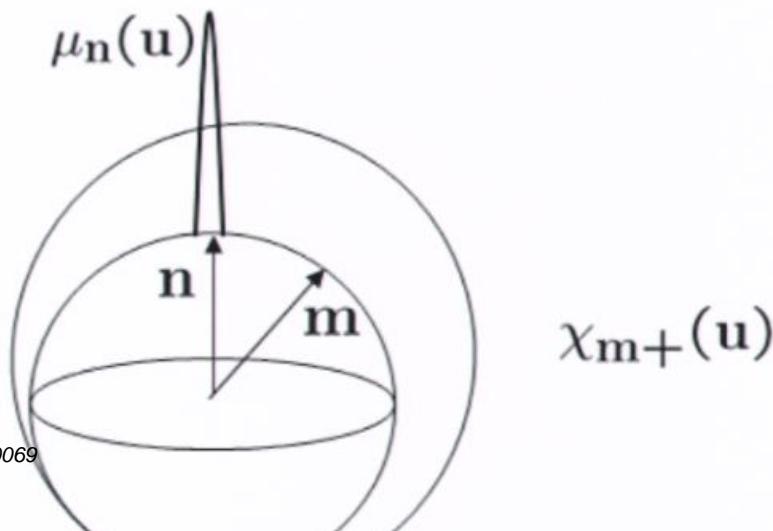
$$| +n \rangle \leftrightarrow \mu_n(u) = \delta(u - n)$$



The orthodox model (Beltrametti-Bugajski)

$$| +n \rangle \leftrightarrow \mu_n(u) = \delta(u - n)$$

$$| +m \rangle \leftrightarrow \chi_{m+}(u) = \frac{1}{2} (1 + m \cdot u)$$

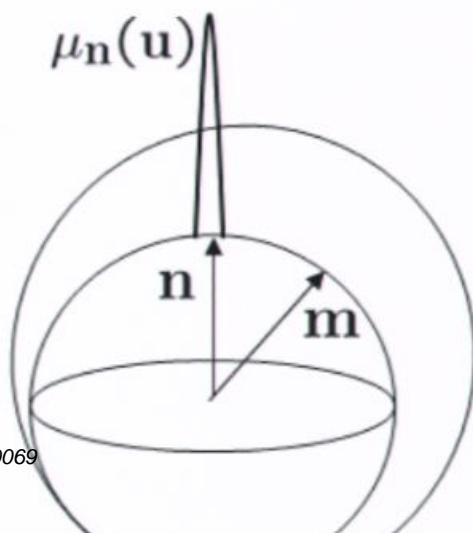


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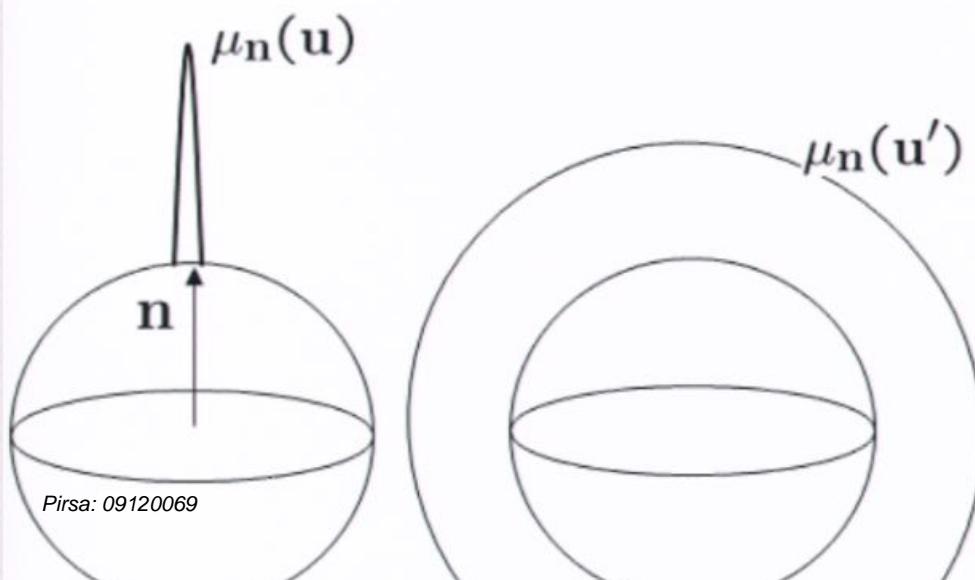
$$\begin{aligned}\int \mu_n(u) \chi_{m+}(u) du &= \int \delta(u - n) \frac{1}{2} (1 + m \cdot u) du \\ &= \frac{1}{2} (1 + m \cdot n) \\ &= |\langle +m | +n \rangle|^2\end{aligned}$$



$\chi_{m+}(u)$

The Bell-Mermin model

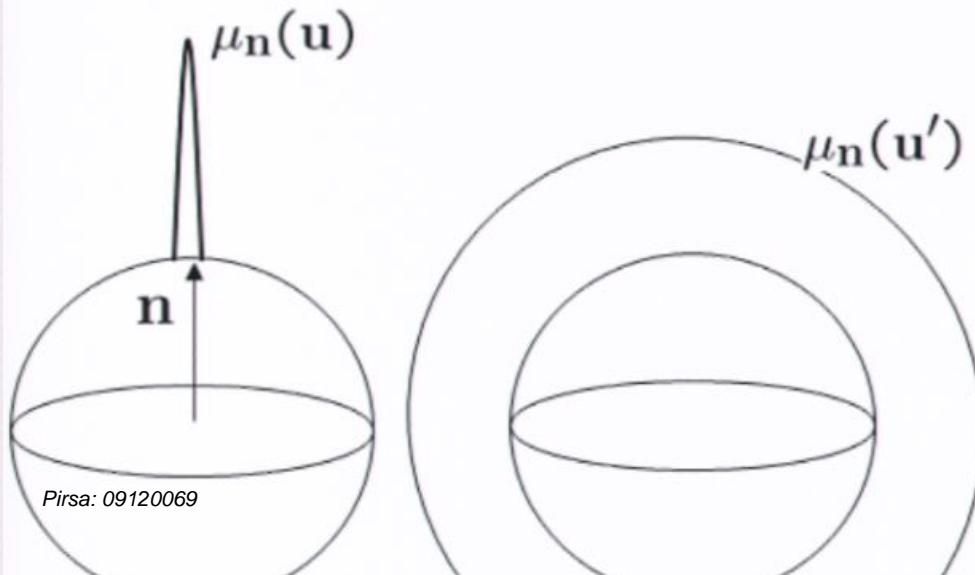
$$| +\mathbf{n} \rangle \leftrightarrow \mu_{\mathbf{n}}(\mathbf{u}, \mathbf{u}') = \frac{1}{4\pi} \delta(\mathbf{u} - \mathbf{n})$$



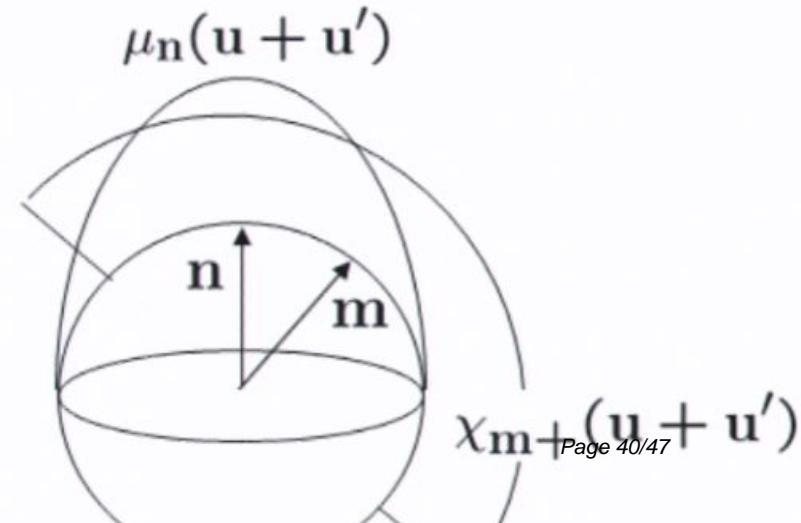
The Bell-Mermin model

$$| + \mathbf{n} \rangle \leftrightarrow \mu_{\mathbf{n}}(\mathbf{u}, \mathbf{u}') = \frac{1}{4\pi} \delta(\mathbf{u} - \mathbf{n})$$

$$| + \mathbf{m} \rangle \leftrightarrow \chi_{\mathbf{m}+}(\mathbf{u}, \mathbf{u}') = \begin{cases} 1 & \text{for } \mathbf{m} \cdot (\mathbf{u} + \mathbf{u}') > 0 \\ 0 & \text{otherwise.} \end{cases}$$



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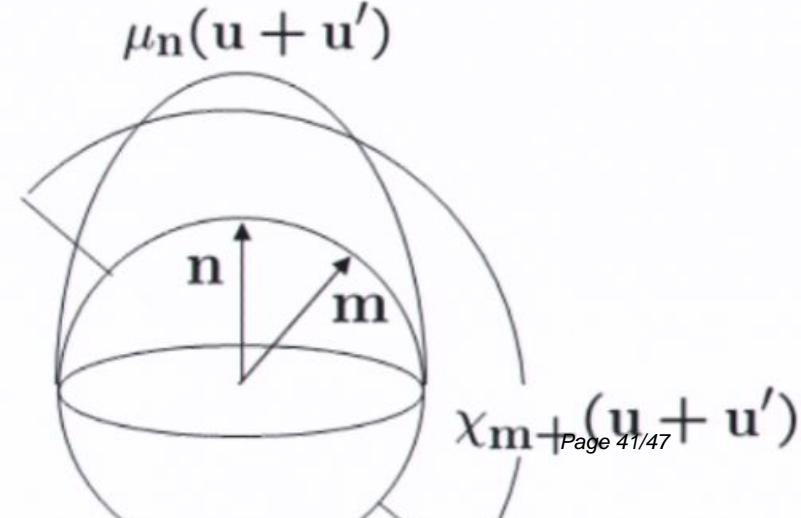
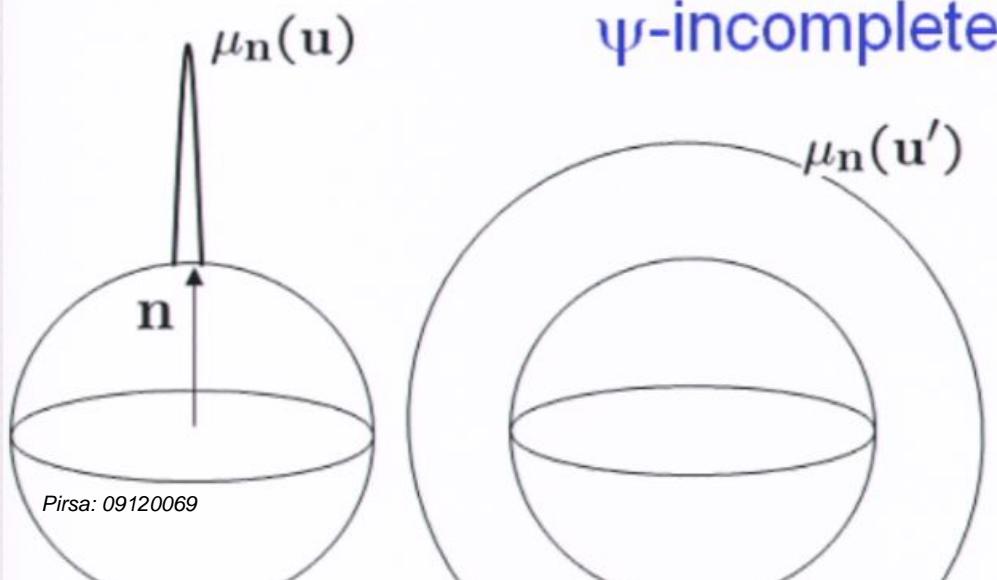
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The Bell-Mermin model

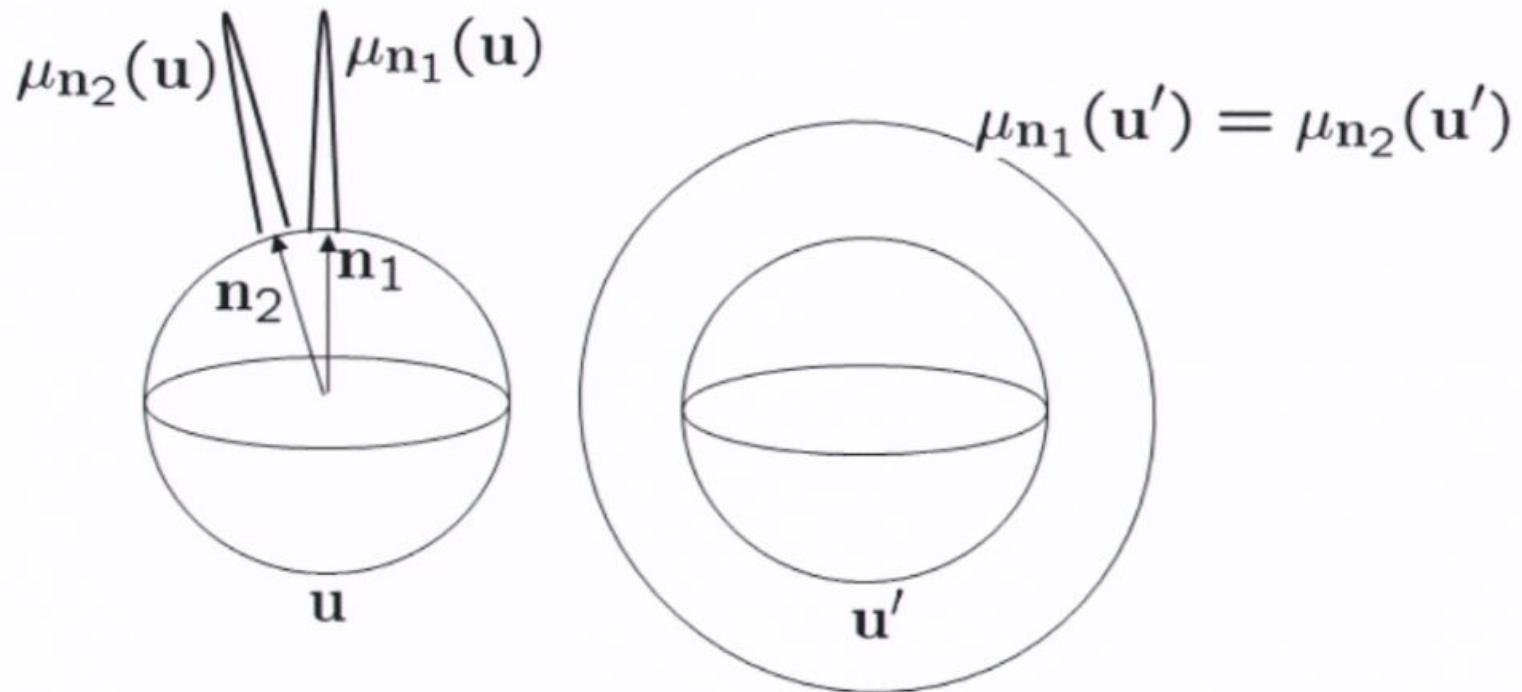
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$$\begin{aligned} \int \mu_{\mathbf{n}}(\mathbf{u}, \mathbf{u}') \chi_{\mathbf{m}+}(\mathbf{u}, \mathbf{u}') d\mathbf{u} d\mathbf{u}' &= \frac{1}{2} (1 + \mathbf{m} \cdot \mathbf{n}) \\ &= |\langle +\mathbf{m} | + \mathbf{n} \rangle|^2 \end{aligned}$$



The Bell-Mermin model



$$\mu_{n_1}(u, u') \mu_{n_2}(u, u') = \frac{1}{(4\pi)^2} \delta(u - n_1) \delta(u - n_2) = 0$$

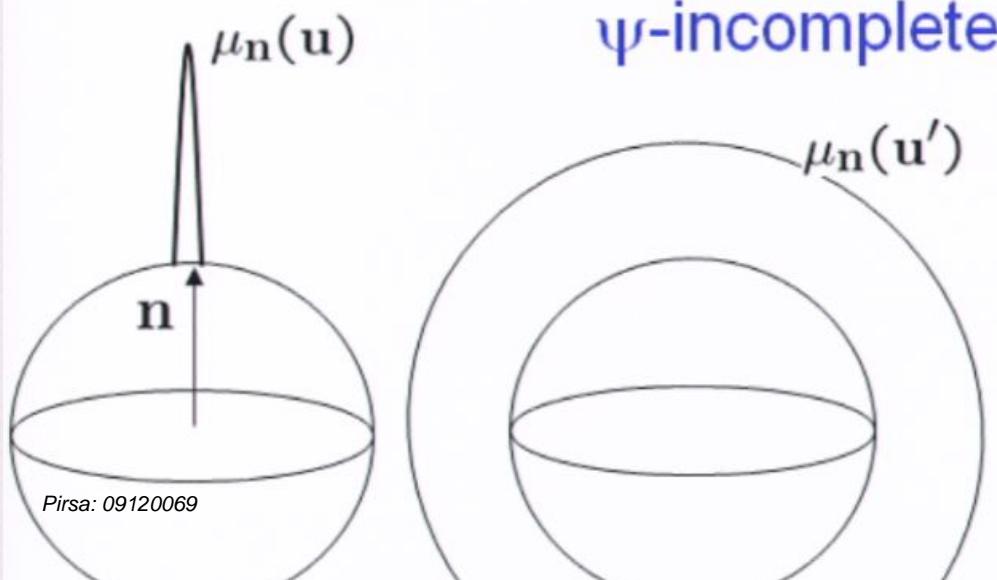
for all u, u'

The Bell-Mermin model

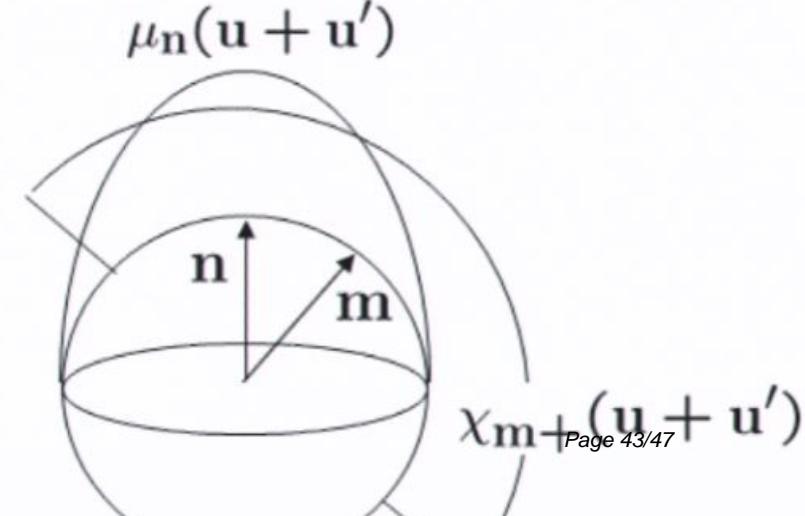
$$| + \mathbf{n} \rangle \leftrightarrow \mu_{\mathbf{n}}(\mathbf{u}, \mathbf{u}') = \frac{1}{4\pi} \delta(\mathbf{u} - \mathbf{n})$$

$$| + \mathbf{m} \rangle \leftrightarrow \chi_{\mathbf{m}+}(\mathbf{u}, \mathbf{u}') = \begin{cases} 1 & \text{for } \mathbf{m} \cdot (\mathbf{u} + \mathbf{u}') > 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$\begin{aligned} \int \mu_{\mathbf{n}}(\mathbf{u}, \mathbf{u}') \chi_{\mathbf{m}+}(\mathbf{u}, \mathbf{u}') d\mathbf{u} d\mathbf{u}' &= \frac{1}{2} (1 + \mathbf{m} \cdot \mathbf{n}) \\ &= |\langle +\mathbf{m} | + \mathbf{n} \rangle|^2 \end{aligned}$$

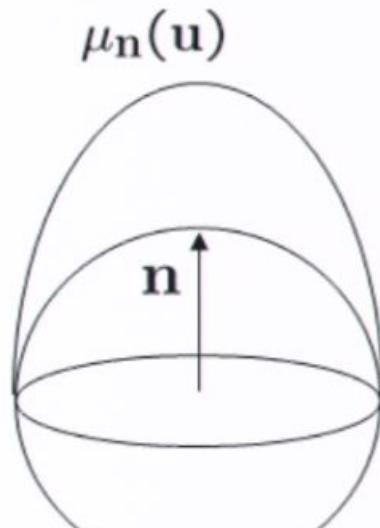


ψ -incomplete



The Kochen-Specker model

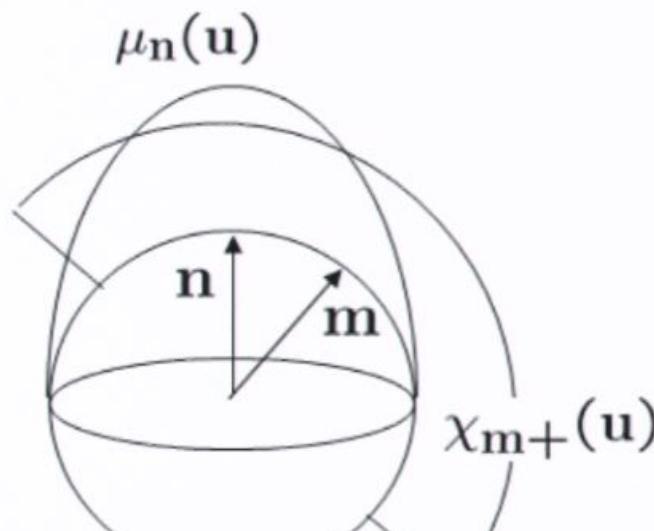
$$| +n \rangle \leftrightarrow \mu_n(u) = \begin{cases} \frac{1}{\pi} n \cdot u & \text{for } n \cdot u > 0 \\ 0 & \text{otherwise.} \end{cases}$$



The Kochen-Specker model

$$| +n \rangle \leftrightarrow \mu_n(u) = \begin{cases} \frac{1}{\pi} n \cdot u & \text{for } n \cdot u > 0 \\ 0 & \text{otherwise.} \end{cases}$$

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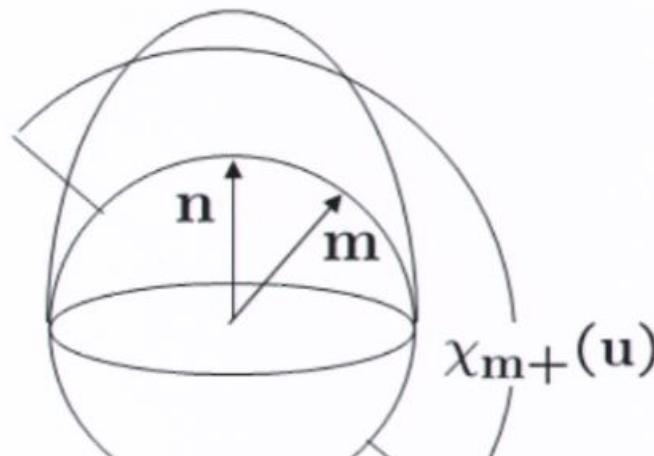


The Kochen-Specker model

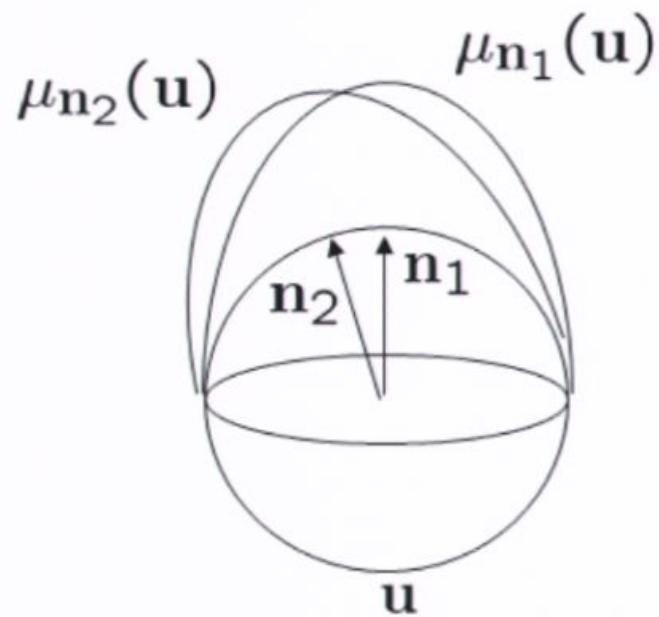
$$| +n \rangle \leftrightarrow \mu_n(u) = \begin{cases} \frac{1}{\pi} n \cdot u & \text{for } n \cdot u > 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$| +m \rangle \leftrightarrow \chi_{m+}(u) = \begin{cases} 1 & \text{for } m \cdot u > 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$\int_{\mu_n(u)} \mu_n(u) \chi_{m+}(u) du = \frac{1}{2} (1 + m \cdot n)$$
$$= |\langle +m | +n \rangle|^2$$



The Kochen-Specker model



unless $n_1 = -n_2$

there exist u such that $\mu_{n_1}(u)\mu_{n_2}(u) \neq 0$