Title: Foundations of Quantum Mech. (PHYS 639) - Lecture 4

Date: Dec 03, 2009 11:00 AM

URL: http://pirsa.org/09120068

Abstract:

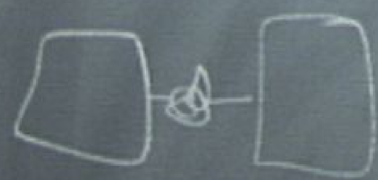
Why?

Why?

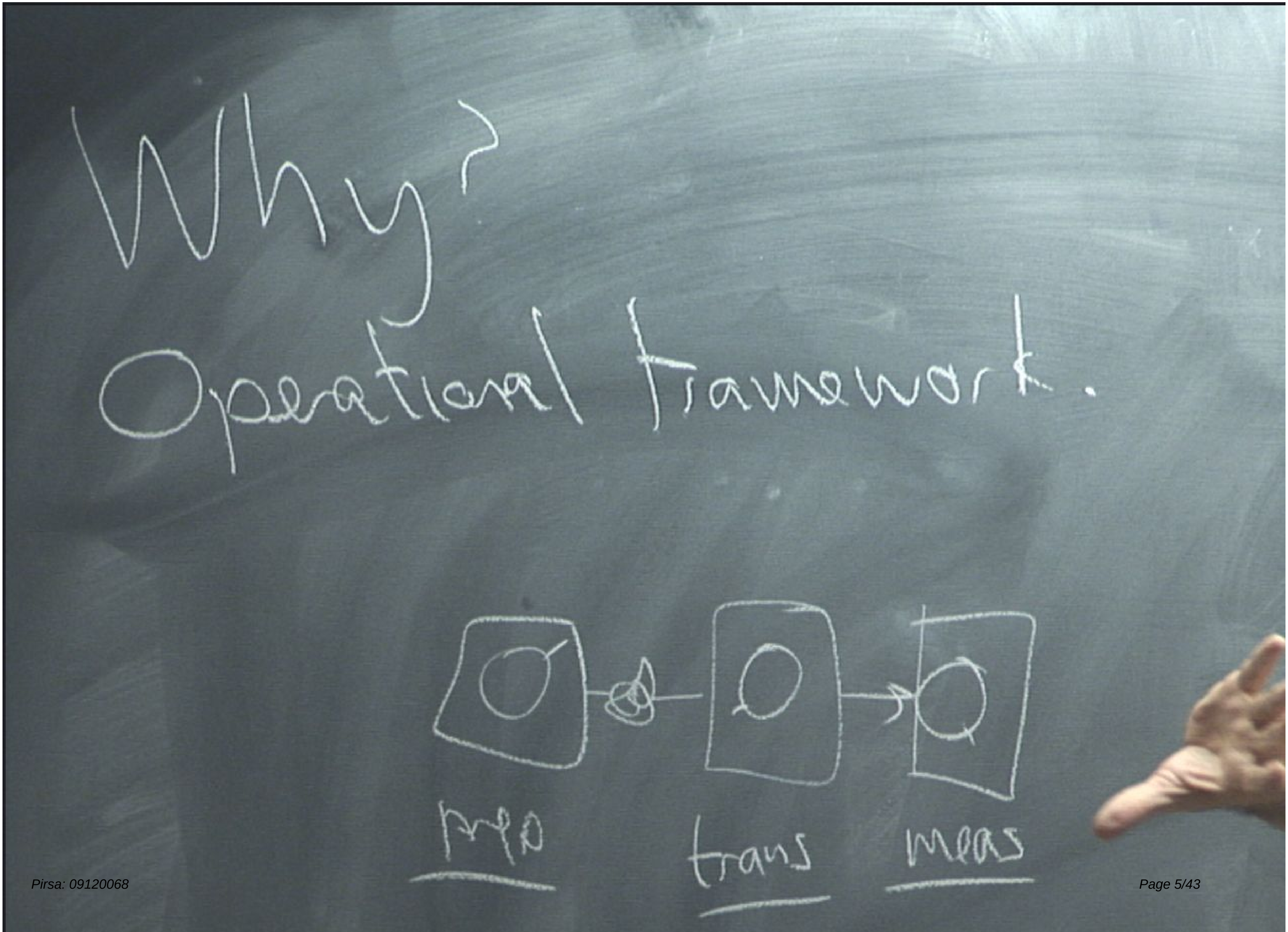
Operational framework.

Why?

Operational framework.

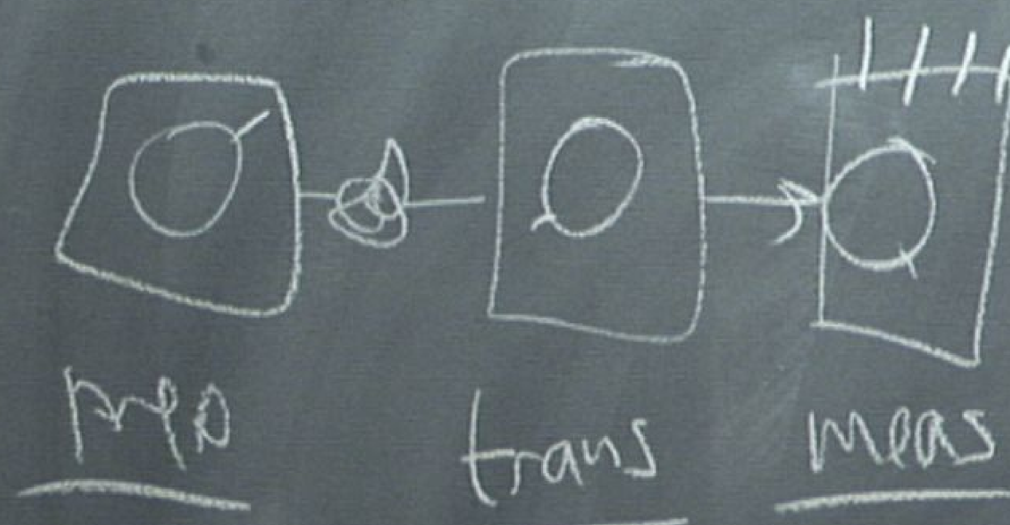


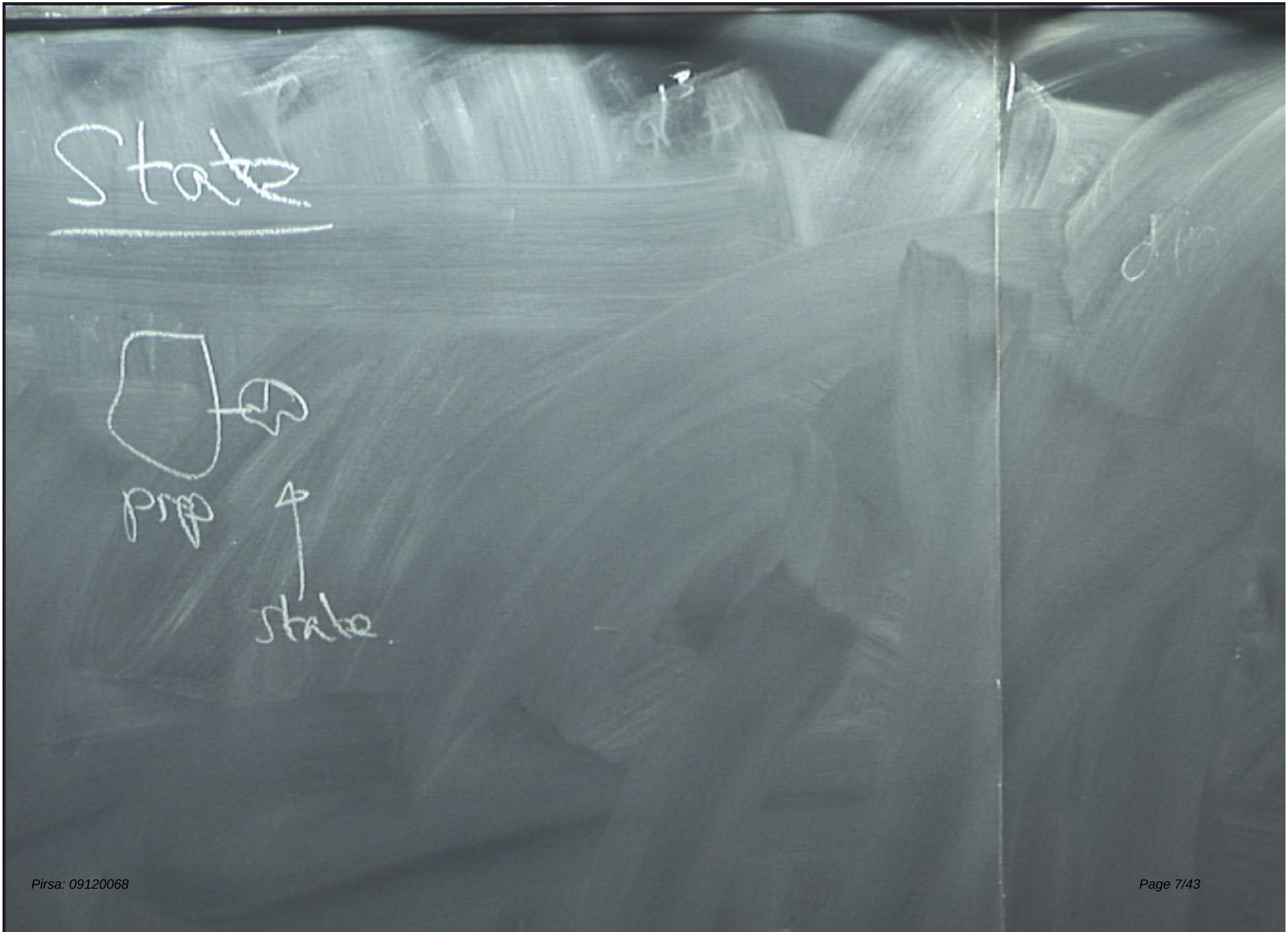
prep

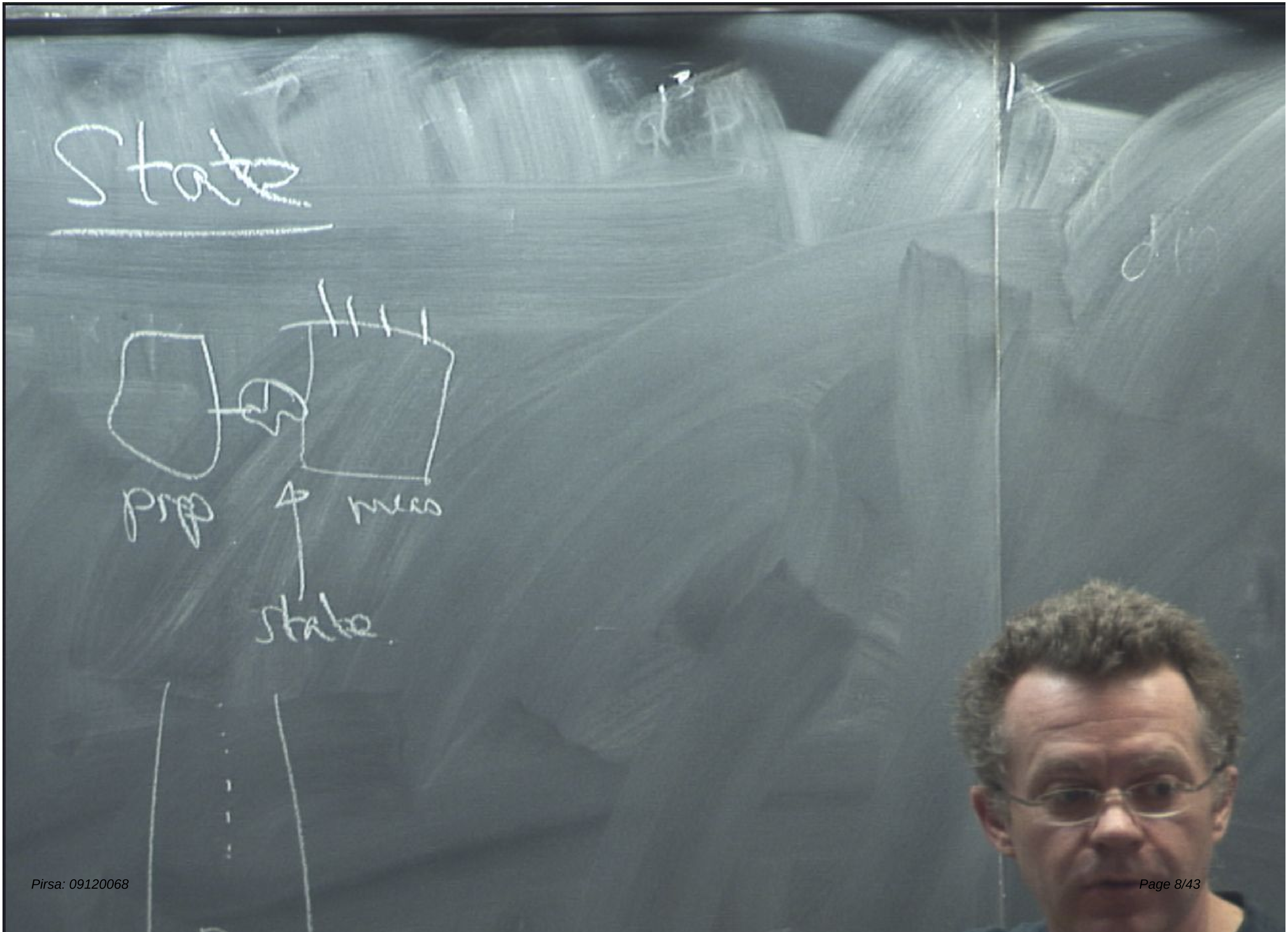

Why?
Operational framework.
prep
trans
meas

Why?

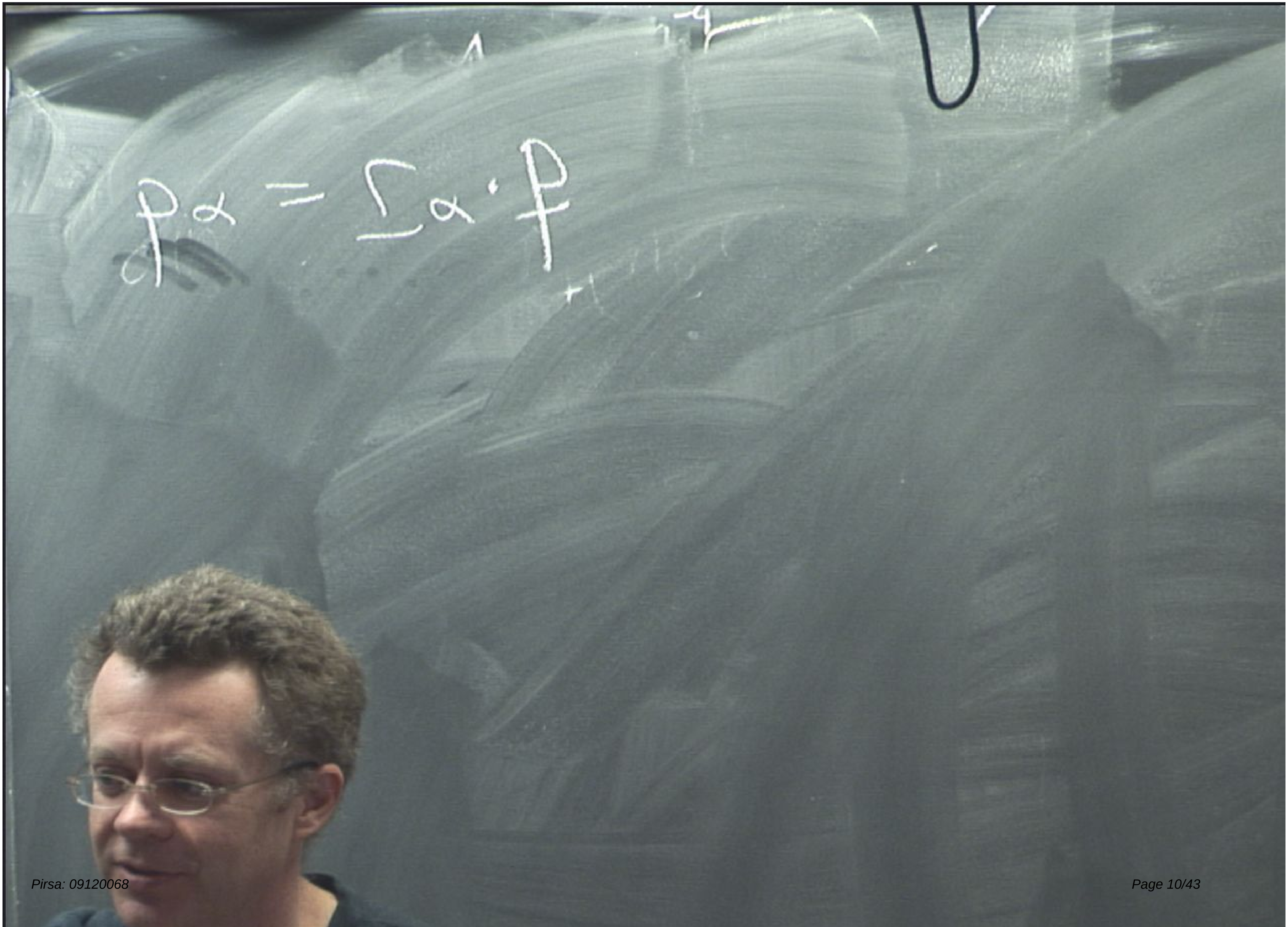
Operational framework.




State
prp
state


State
prep
meas
state

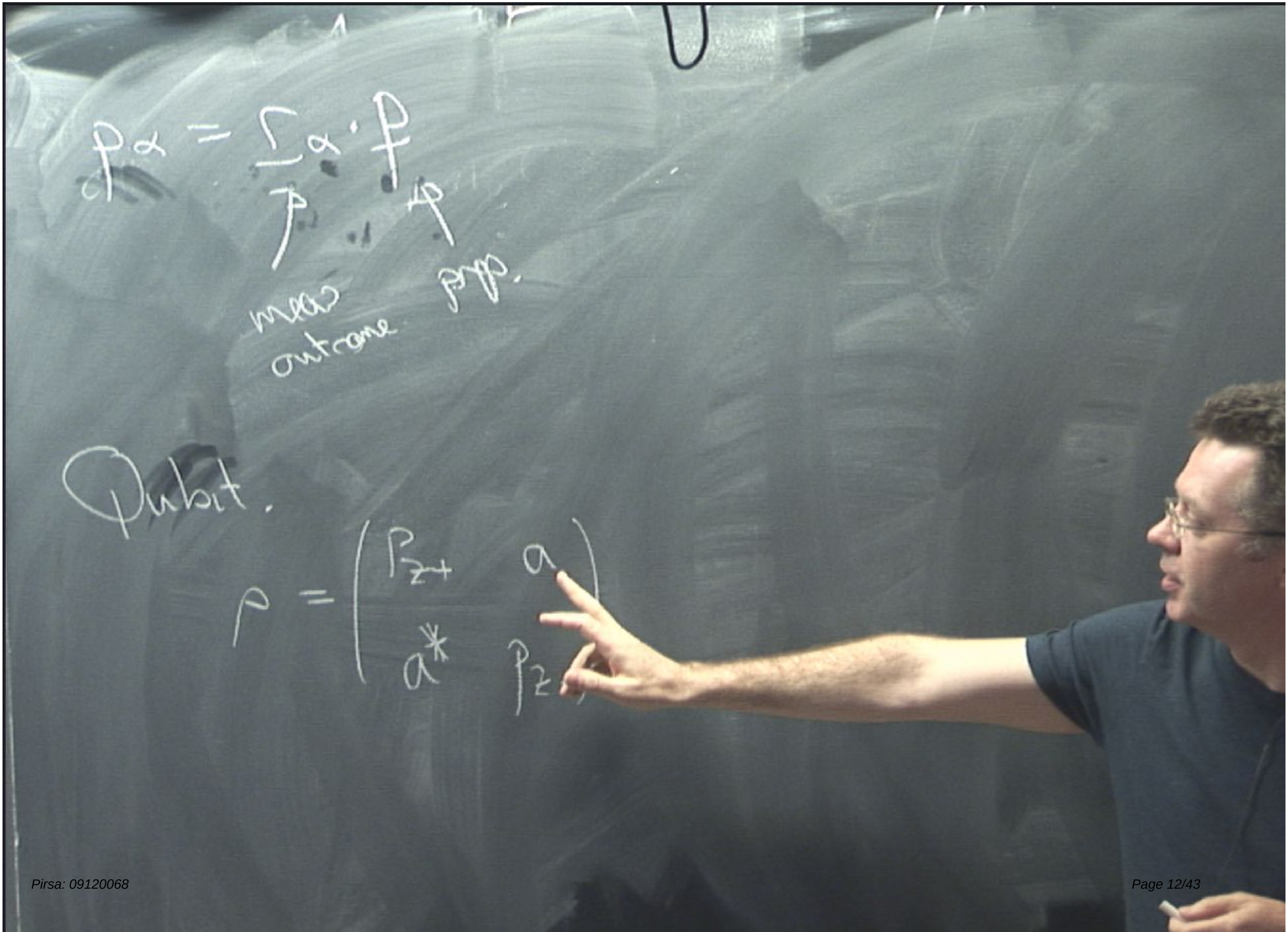
$= \sum \alpha \cdot \rho$

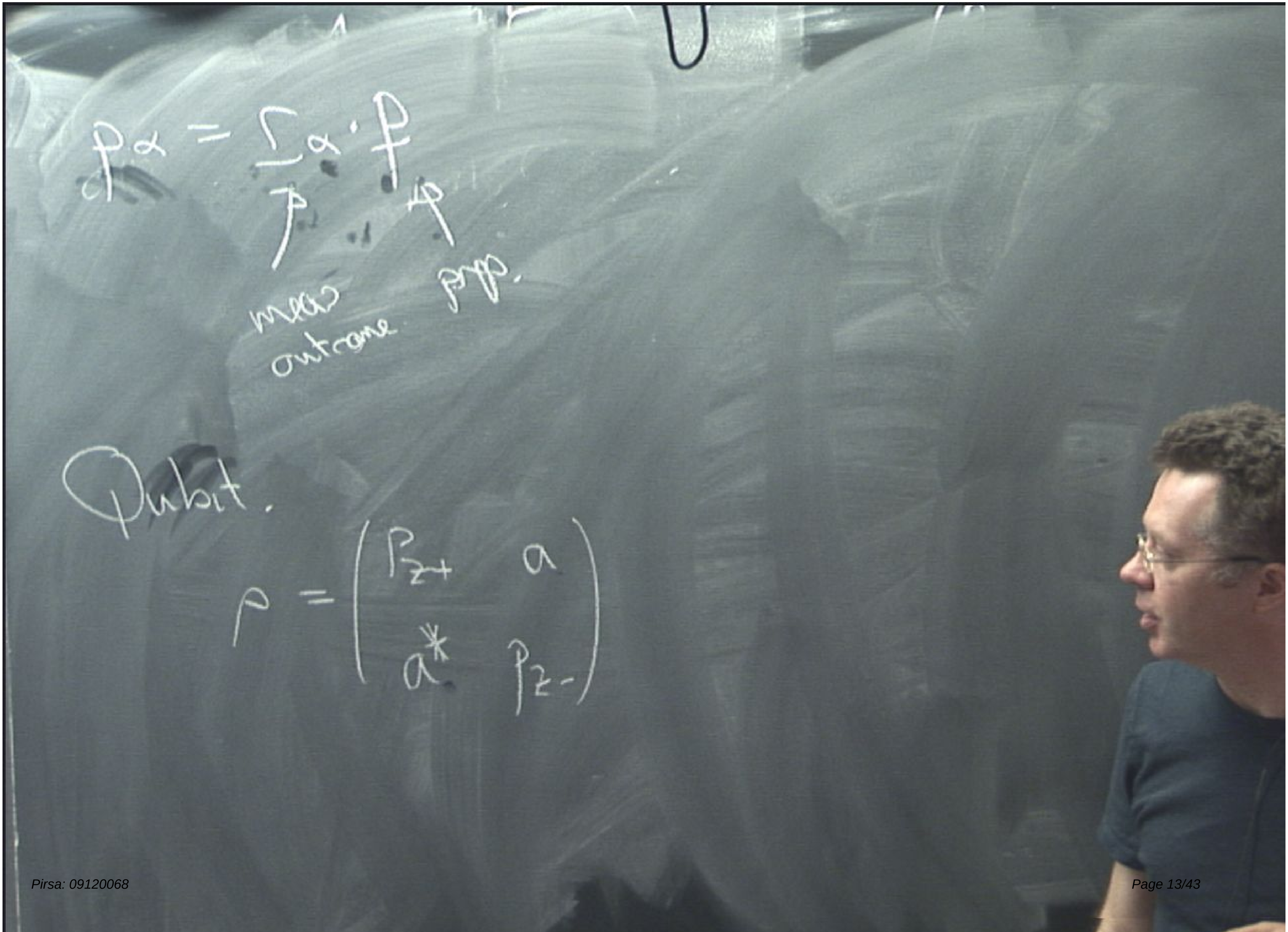

$\rho_\alpha = \Sigma_\alpha \cdot \rho$

$$p_\alpha = \frac{\Gamma_\alpha \cdot p}{p \cdot A}$$

meas outcome

prep.


$p_\alpha = \Gamma_\alpha \cdot p$
meas outcome
prep.
Qubit.
$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix}$


$p_\alpha = \sum_\beta \alpha \cdot p_\beta$
meas outcome
prop.
Qubit.
$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix}$

$$p_\alpha = \frac{\Sigma_\alpha \cdot p}{\uparrow \quad \uparrow}$$

meas outcome prop.

Qubit.

$$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix}$$

$$a = p_{x+} - i p_{y+} - \frac{1}{2}(1-i)\left(p_{z+} + p_{z-}\right)$$

$$p_\alpha = \frac{\Gamma_\alpha \cdot p}{\text{meas outcome} \quad \text{prep.}}$$

Qubit.

$$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \iff p = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

ex

$$a = p_{x+} - i p_{y+} - \frac{1}{2}(1-i)\left(p_{z+} + p_{z-}\right)$$

$$p_\alpha = \vec{r}_\alpha \cdot \vec{p}$$

meas outcome

prep.

$$\vec{p} = \begin{pmatrix} p_H \\ p_T \end{pmatrix}$$

Qubit.

$$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \Longleftrightarrow \vec{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

ex

$$a = p_{x+} - i p_{y+} - \frac{1}{2}(1-i)\left(p_{z+} + p_{z-}\right)$$

$$p_\alpha = \Gamma_\alpha \cdot \vec{p}$$

meas. outcome $\;\uparrow\;$ $\;\;\uparrow\;$ prep.

$$\vec{p} = \begin{pmatrix} p_H \\ p_T \end{pmatrix} \qquad \vec{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix}$$

Qubit.

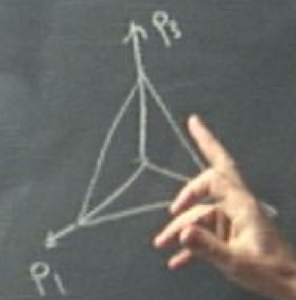
$$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \iff \vec{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

ex

$$a = p_{x+} - i\,p_{y+} - \tfrac{1}{2}(1-i)\left(p_{z+} + p_{z-}\right)$$

$$p_\alpha = \underbrace{\underline{\Sigma}_\alpha}_{\text{meas outcome}} \cdot \underbrace{\underline{p}}_{\text{prep.}}$$

$$\underline{p} = \begin{pmatrix} p_H \\ p_T \end{pmatrix} \qquad \underline{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix}$$



Qubit.

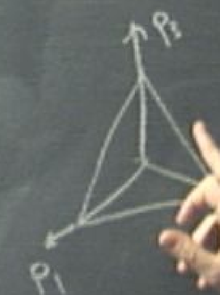
$$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \Longleftrightarrow \underline{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

ex

$$a = p_{x+} - i p_{y+} - \frac{1}{2}(1-i)\left(p_{z+} + p_{z-}\right)$$

$$p_\alpha = \underbrace{\alpha}_{\text{meas outcome}} \cdot \underbrace{\vec{p}}_{\text{prep.}}$$

$$\vec{p} = \begin{pmatrix} p_H \\ p_T \end{pmatrix} \qquad \vec{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix}$$



Qubit.

$$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \iff \vec{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

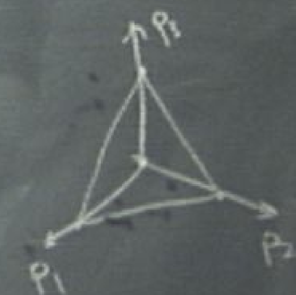
ex

$$a = p_{x+} - i\, p_{y+} - \frac{1}{2}(1-i)\left(p_{z+} + p_{z-}\right)$$

$$p_\alpha = \underline{\Omega}_\alpha \cdot \vec{p}$$

meas outcome prop.

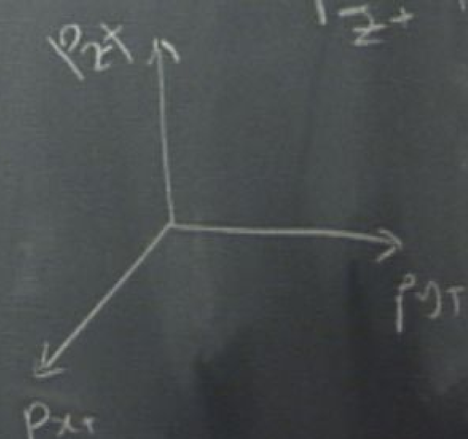
$$\vec{p} = \begin{pmatrix} p_H \\ p_T \end{pmatrix} \qquad \vec{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix}$$



$$p_{z+} + p_{z-} = 1$$

Qubit.

$$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \Longleftrightarrow \vec{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$



ex

$$a = p_{x+} - i\, p_{y+} - \frac{1}{2}(1-i)\left(p_{z+} + p_{z-}\right)$$

$$p_\alpha = \underset{\text{meas outcome}}{r_\alpha} \cdot \underset{\text{prep.}}{\vec{p}}$$

$$\vec{p} = \begin{pmatrix} p_H \\ p_T \end{pmatrix} \qquad \vec{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix}$$

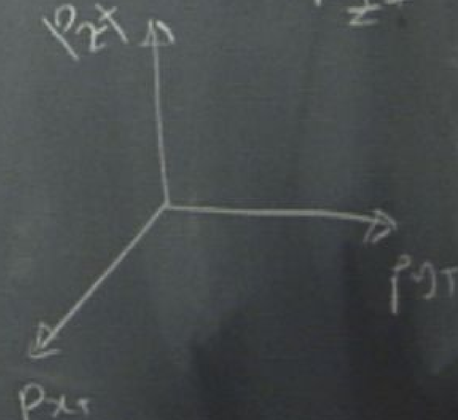
$$p_{z+} + p_{z-} = 1$$

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \cdot \vec{p}$$

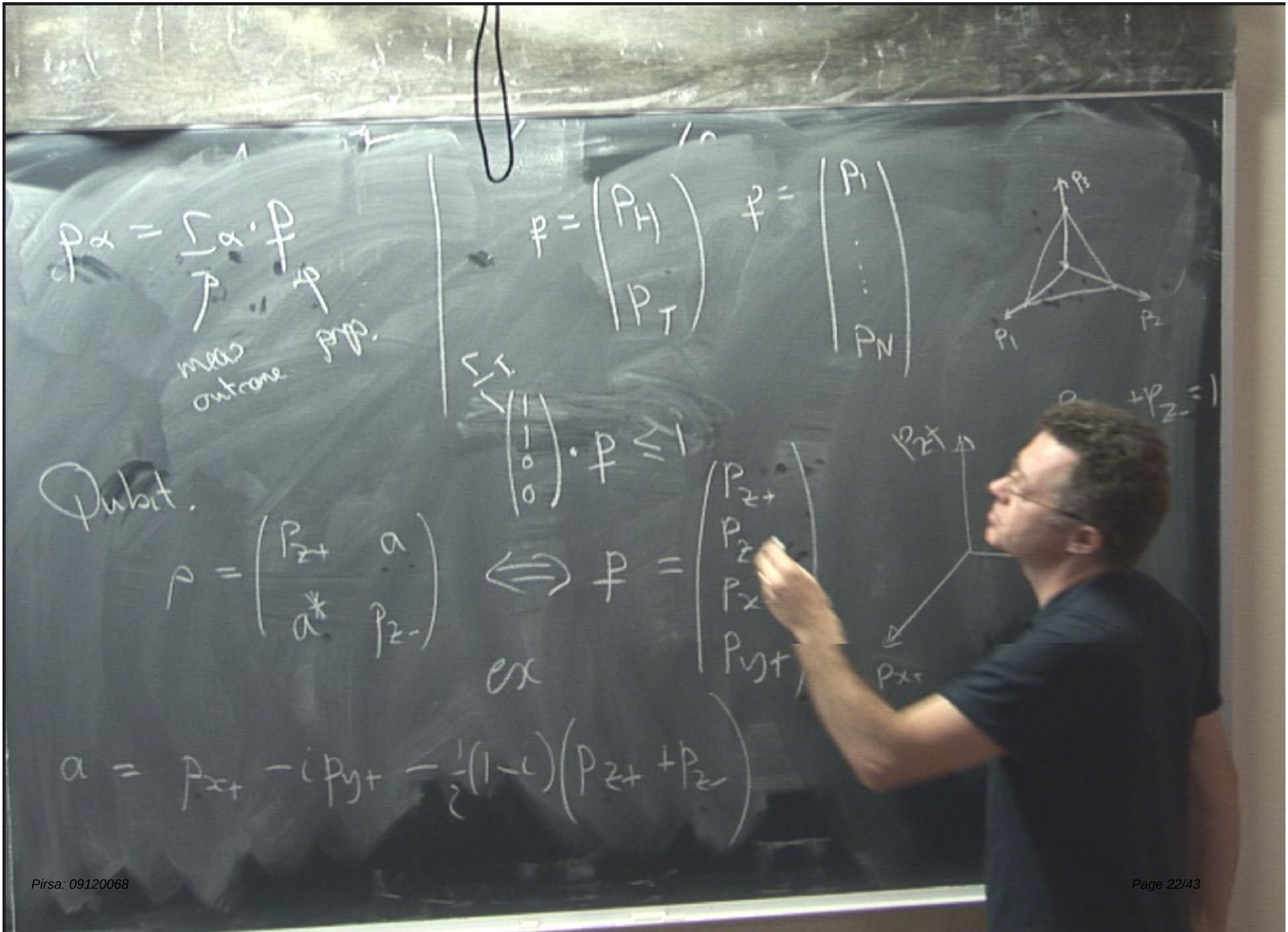
Qubit.

$$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \Longleftrightarrow \vec{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

ex



$$a = p_{x+} - i p_{y+} - \frac{1}{2}(1-i)(p_{z+} + p_{z-})$$

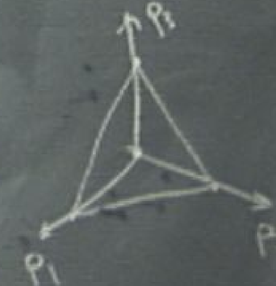


meas outcome
prob.
Qubit.
ex

$$p_\alpha = \underline{\Sigma}_\alpha \cdot \underline{p}$$

meas outcome prep.

$$\underline{p} = \begin{pmatrix} p_H \\ p_T \end{pmatrix} \qquad \underline{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix}$$



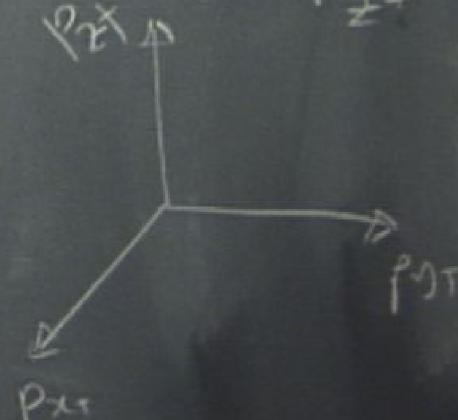
$$\underline{\Sigma}_I = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \cdot \underline{p} \le 1$$

$$p_{z+} + p_{z-} = 1$$

Qubit.

$$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \Longleftrightarrow \underline{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

ex



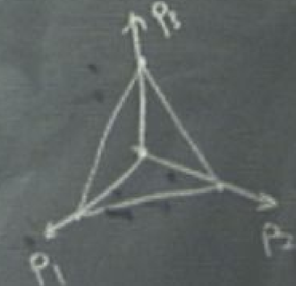
$$a = p_{x+} - i p_{y+} - \frac{1}{2}(1-i)\left(p_{z+} + p_{z-}\right)$$

$p_\alpha = \underline{\Sigma}_\alpha \cdot \underline{p}$

prop.

meas outcome

$\underline{p} = \begin{pmatrix} p_H \\ p_T \end{pmatrix}$ $\quad \underline{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix}$

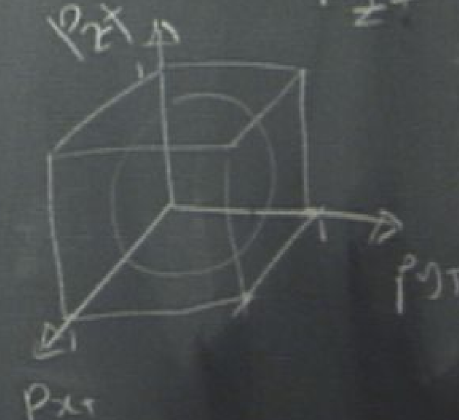


$\underline{\Sigma}_I = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \cdot \underline{p} \le 1$

$p_{Z+} + p_{Z-} = 1$

Qubit.

$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \Longleftrightarrow \underline{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$



ex

$a = p_{x+} - i p_{y+} - \frac{1}{2}(1-i)\left(p_{z+} + p_{z-}\right)$

State

K min # probs

$p_\alpha = \underset{\text{meas outcome}}{\underline{r}_\alpha} \cdot \underset{\text{prep}}{p}$

$$p = \begin{pmatrix} p_1 \\ \vdots \\ p_K \end{pmatrix}$$

Qubit.

$$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \Longleftrightarrow p$$

ex

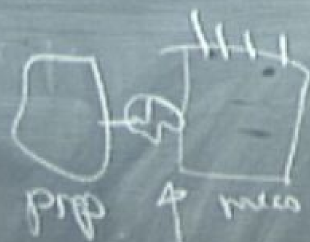
$a = p_{x+} - i p_{y+} - \frac{1}{2}(1-i)(p_{z+}$

$p = \begin{pmatrix} p \\ p \end{pmatrix}$

$\underline{r}_I$

$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \cdot p$

State



K min # probs

N

p_α

$$\mathbf{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_K \end{pmatrix}$$

$$p_\alpha = \underline{r}_\alpha \cdot \mathbf{p}$$

meas outcome

prep.

Qubit.

$$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix}$$

$$a = p_{x+} - i\, p_{y+} - \tfrac{1}{2}(1-i)\big(p_{z+}$$

$$\mathbf{p} = \begin{pmatrix} p \\ p \end{pmatrix}$$

$$\underline{r}_I = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \cdot \mathbf{p}$$

$\Longleftrightarrow \mathbf{p}$

ex

hy?

erational framework.

State

prep

meas

state

K min # probs

N max # dis state

p_α

$$\mathbf{p} = \begin{pmatrix} p_1 \\ \vdots \\ \vdots \\ p_K \end{pmatrix}$$

Qu

a =

...hy?

...erational framework.

State

prep ↑ meas

state

K min # probs

N max # dis states

$$p_\alpha$$

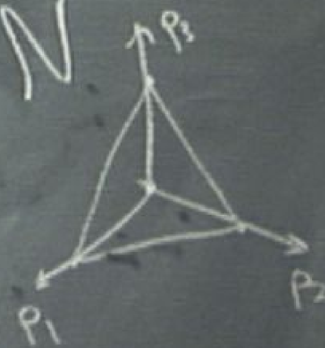
$$\mathbf{p} = \begin{pmatrix} p_1 \\ \vdots \\ \vdots \\ p_K \end{pmatrix}$$

$a =$

$\vec{\alpha}\cdot\vec{p}$

meas outcome   prop.

$\vec{p} = \begin{pmatrix} p_H \\ p_T \end{pmatrix}$ $\vec{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix}$ $K = N$



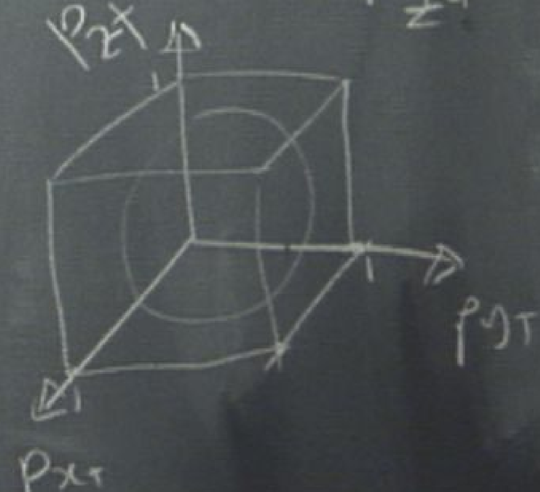
$\vec{\Sigma}_I$

$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \cdot \vec{p} \le 1$

$p_{z+} + p_{z-} = 1$

$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \Longleftrightarrow \vec{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$

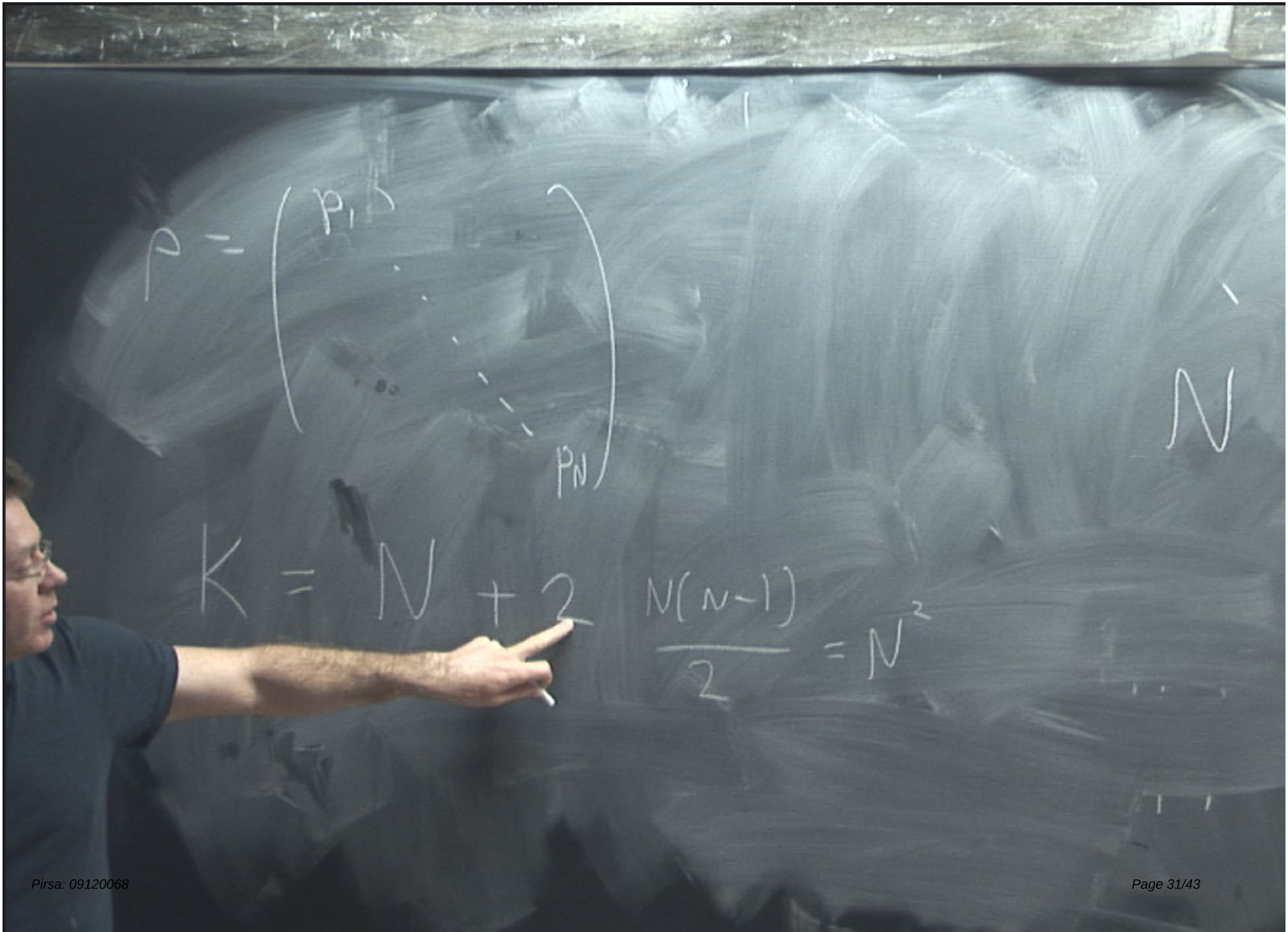
ex



$p_{x+} - i p_{y+} - \frac{1}{2}(1-i)(p_{z+} + p_{z-})$ $K = N^2$

$$\rho = \begin{pmatrix} p_1 & & \\ & \ddots & \\ & & p_N \end{pmatrix}$$

N


$\rho = \begin{pmatrix} p_1 & & \\ & \ddots & \\ & & p_N \end{pmatrix}$
N
$K = N + 2\,\frac{N(N-1)}{2} = N^2$

$$\rho = \begin{pmatrix} p_1 & & \\ & \ddots & \\ & & p_N \end{pmatrix}$$

N

$$K = N + 2\,\frac{N(N-1)}{2} = N^2$$

Postulate 3

There exists a continuous reversible transformation between any pair of pure states.

$\rho \to \sum p$

min # probs

N max # dis state

$p_\alpha = \underset{\text{meas outcome}}{\Gamma_\alpha} \cdot \underset{\text{prep.}}{p}$

$\frac{(N-1)}{2} = N^2$

Qubit.

$$\rho = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix}$$

$a = p_{x+} - i p_{y+} - \frac{1}{2}$

Postulate 3 There exists a continuous reversible transformation between any pair of pure states.

Postulate 2

$$N_{AB} = N_A N_B$$

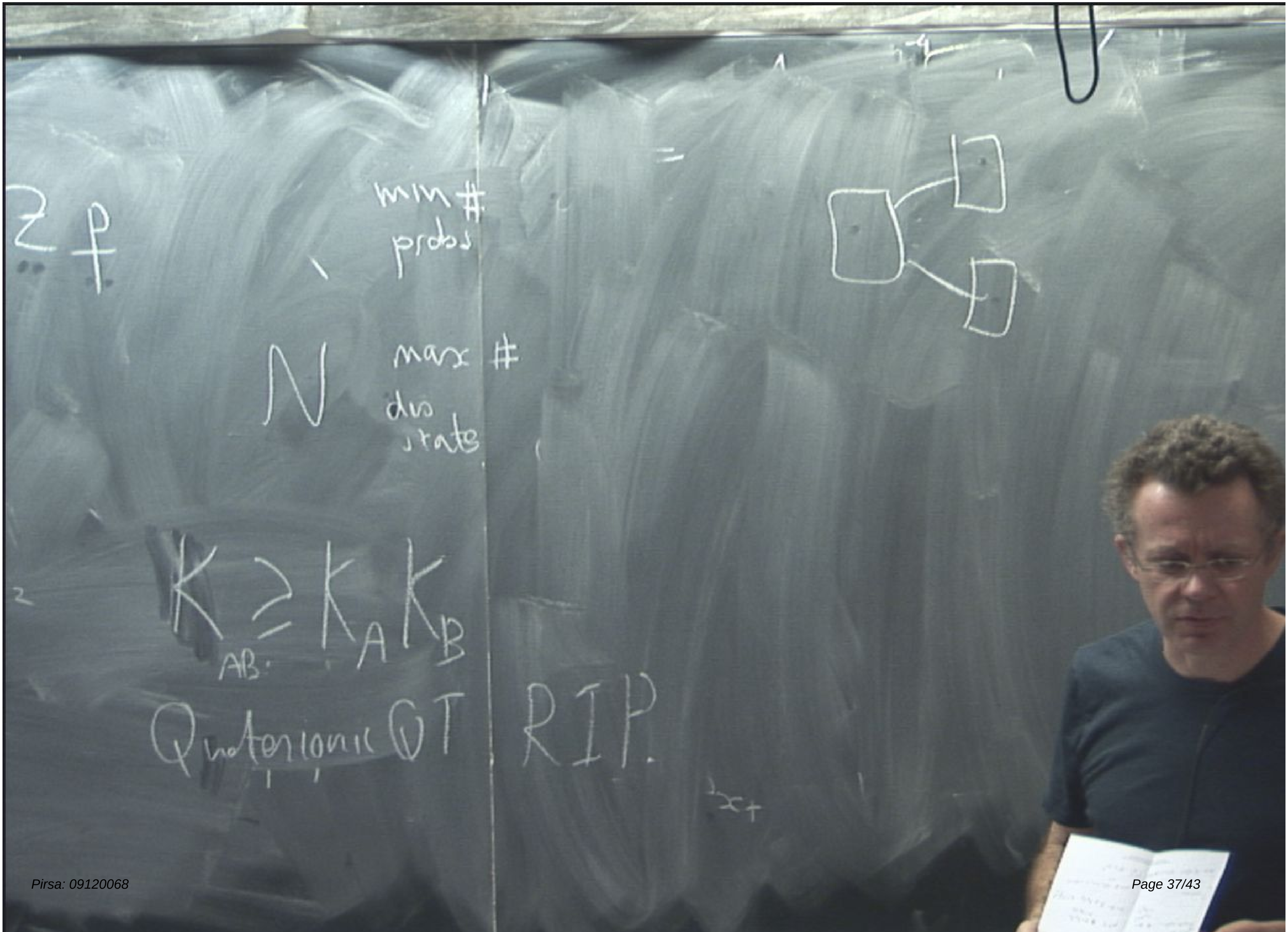
$$K_{AB} = K_A K_B$$

Z p

min # probs

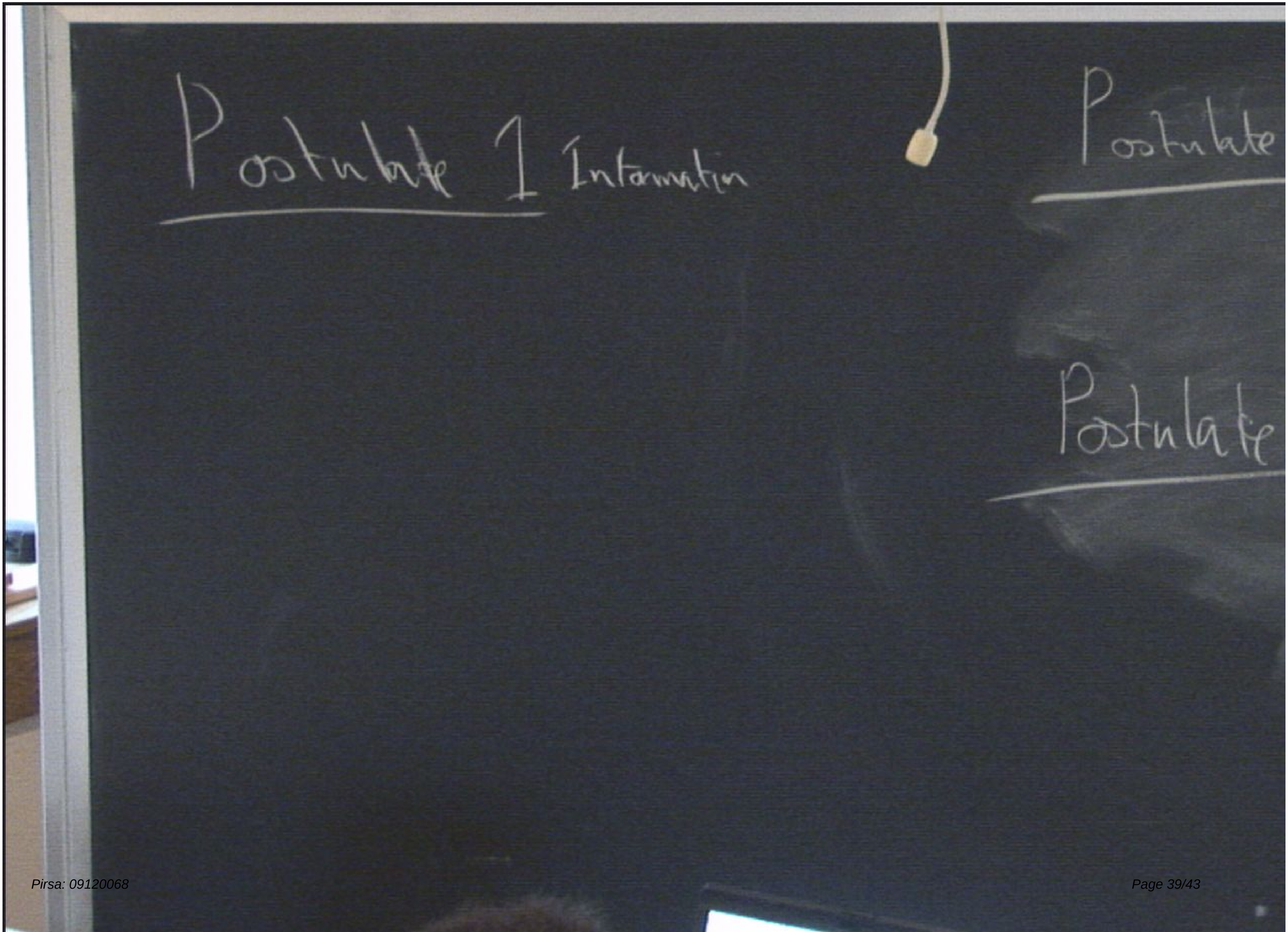
N max # dis states

$K_{AB} \geq K_A K_B$


min #
probs
N
max #
dis
state
K_AB ≥ K_A K_B
Quaternionic QT
R.I.P.

$K(N)$ $\quad K(N+1) > K(N)$

$K = N^{\,}$


Postulate 1 Information
Postulate
Postulate

Postulate 1 Information

Systems having or constrained to have a given information carrying capacity have the same properties.

Postulate

Postulate

$K = K(N) \qquad K(N+1) > K(N)$

$K = N^r . \quad r = 1, 2, 3, \ldots$

Postulate 1 Information

Systems having or constrained to have a given information carrying capacity have the same properties.

Postulate simplicity.

Post

Postu

$K = K(N) \quad K(N+1) > K(N)$

$K = N^r, \quad r = 1, 2, 3, \ldots$