

Title: Foundations of Quantum Mech. (PHYS 639) - Lecture 2

Date: Dec 01, 2009 11:00 AM

URL: <http://pirsa.org/09120066>

Abstract:

Towards a purely operational formulation of quantum theory

“Orthodox” postulates of quantum theory

Representational completeness of ψ . The rays of Hilbert space correspond one-to-one with the **physical states** of the system.

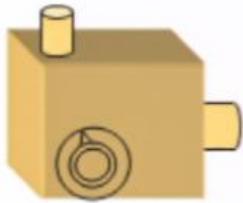
Measurement. If the Hermitian operator A with spectral projectors $\{P_k\}$ is measured, the probability of outcome k is $\langle \psi | P_k | \psi \rangle$. These **probabilities are objective -- indeterminism.**

Evolution of isolated systems. It is unitary, $|\psi\rangle \rightarrow U|\psi\rangle = e^{-\frac{i}{\hbar}Ht}|\psi\rangle$ therefore **deterministic and continuous.**

Evolution of systems undergoing measurement. If Hermitian operator A with spectral projectors $\{P_k\}$ is measured and outcome k is obtained, the physical state of the system **changes discontinuously,**

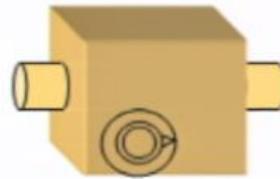
$$|\psi\rangle \rightarrow |\psi_k\rangle = \frac{P_k|\psi\rangle}{\sqrt{\langle \psi | P_k | \psi \rangle}}$$

Operational Quantum Mechanics



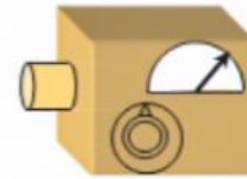
Preparation
 P

Vector
 $|\psi\rangle$



Transformation
 T

Unitary map
 U

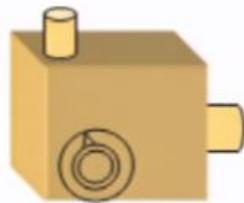


Measurement
 M

Hermitian operator
 A
 $A = \sum_k a_k P_k$

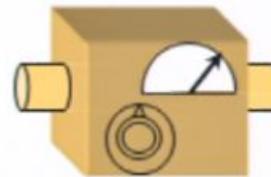
$$Pr(k|P, T, M) = \langle \psi | U^\dagger P_k U | \psi \rangle$$

Operational Quantum Mechanics



Preparation

P



Measurement

M

Effective preparation

P_k

Update map

$$|\psi\rangle \rightarrow |\psi_k\rangle = \frac{P_k|\psi\rangle}{\sqrt{\langle\psi|P_k|\psi\rangle}}$$

Some puzzles

How does one represent a preparation that corresponds to one element of a pair of entangled systems?

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|+\hat{n}\rangle|-\hat{n}\rangle - |-\hat{n}\rangle|+\hat{n}\rangle)$$

Not by a Hilbert space vector!

How does one represent irreversible dynamics? Cooling a quantum system to its ground state for example?

Not by a unitary!

When a photon is detected at position r , the quantum state of the EM field is updated as follows

$$|\psi\rangle \rightarrow \hat{\Psi}(r)|\psi\rangle$$

But $\hat{\Psi}(r)$ is **not a projector!**

Photodetection theory shows that the probability of detecting k photons is

$$\langle \psi | E_k | \psi \rangle$$

$$E_k = : \frac{(\eta a^\dagger a)^k e^{-\eta a^\dagger a}}{k!} :$$

$$= \sum_n \lambda_n^k |n\rangle \langle n|$$

$$\lambda_n^k = \binom{n}{k} \eta^k (1 - \eta)^{n-k} \quad \text{for } k \leq n$$
$$= 0 \quad \text{otherwise}$$

But the operator in question is **not a projector!**

Consider a system and two pointers

The coupling

$$\gamma_A \hat{A} \otimes \hat{P} \otimes \hat{I}$$

achieves a measurement of A.

The coupling

$$\gamma_B \hat{B} \otimes \hat{I} \otimes \hat{P}$$

achieves a measurement of B.

What about

$$\gamma_A \hat{A} \otimes \hat{P} \otimes \hat{I} + \gamma_B \hat{B} \otimes \hat{I} \otimes \hat{P}$$

If A and B don't commute, this **can't be modeled by a Hermitian operator!**

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Generalized preparations

A density operator

$$\rho \in \mathcal{L}(\mathbb{C}_d)$$

Positive definite operator: $\langle \psi | \rho | \psi \rangle > 0, \forall \psi \neq 0$

Unit trace: $\text{Tr}(\rho) = 1$

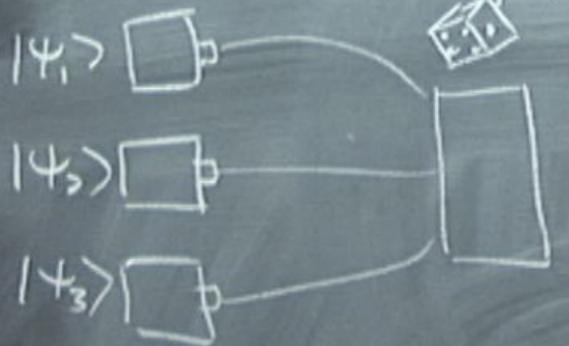
$$\begin{aligned} \rho = |\psi\rangle\langle\psi| &\leftrightarrow \text{Pure preparation} \\ \rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| &\leftrightarrow \text{Mixed preparation} \end{aligned}$$

Generalized Preparations

Mixtures of pure states

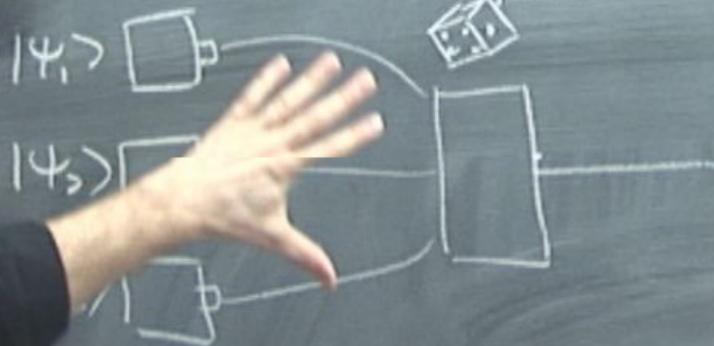
Generalized Preparations

Mixtures of pure states



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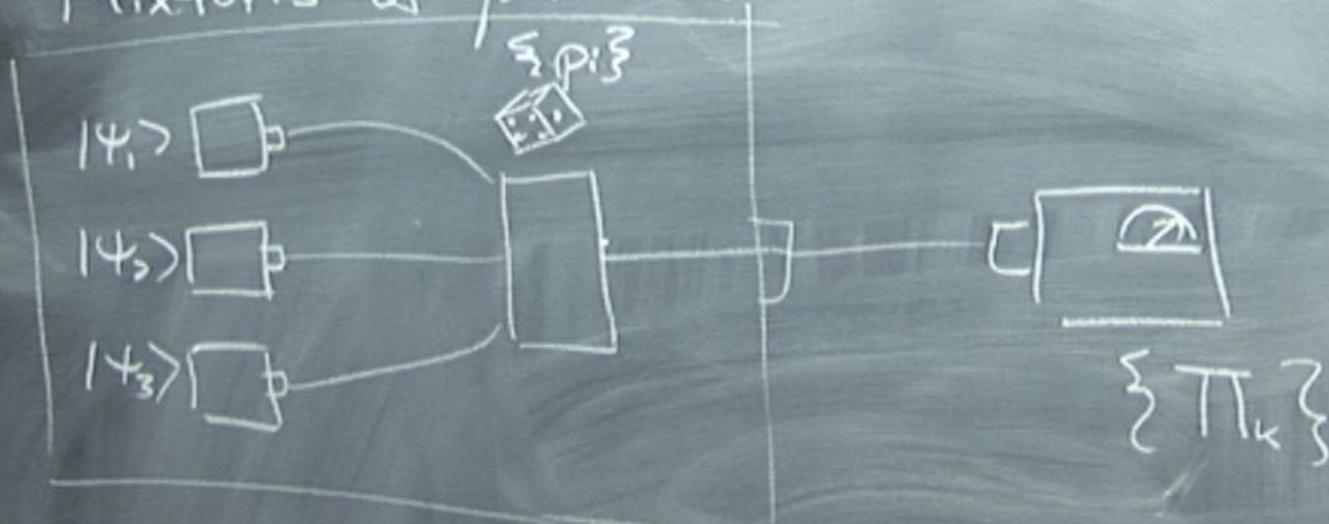
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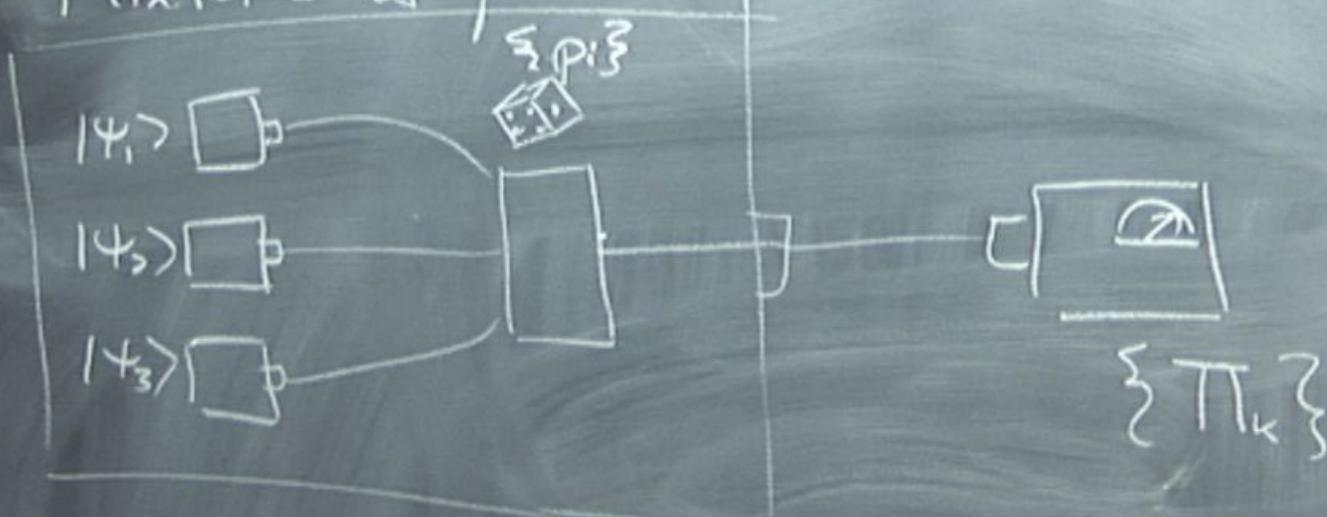
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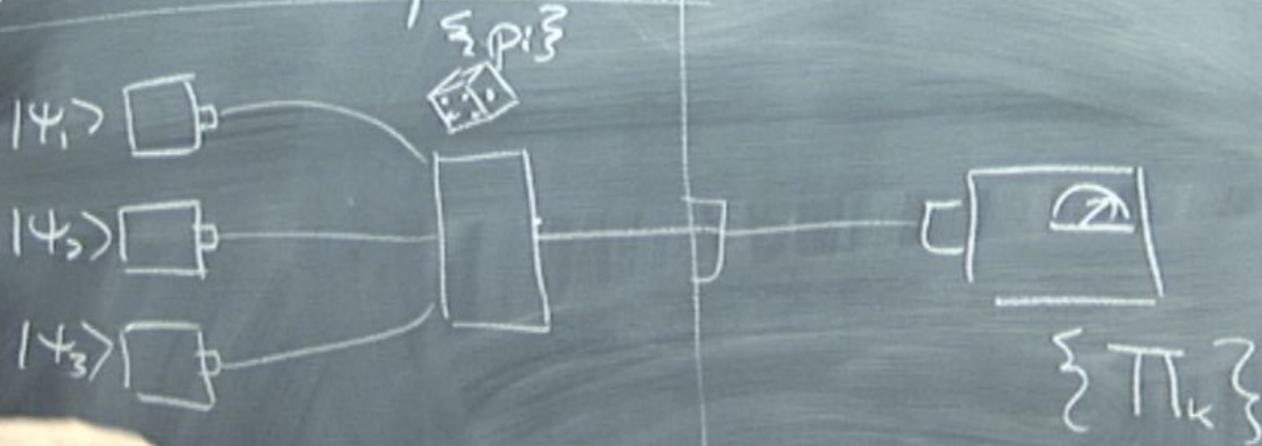
Mixtures of pure states



$$P(k) =$$

Generalized Preparations

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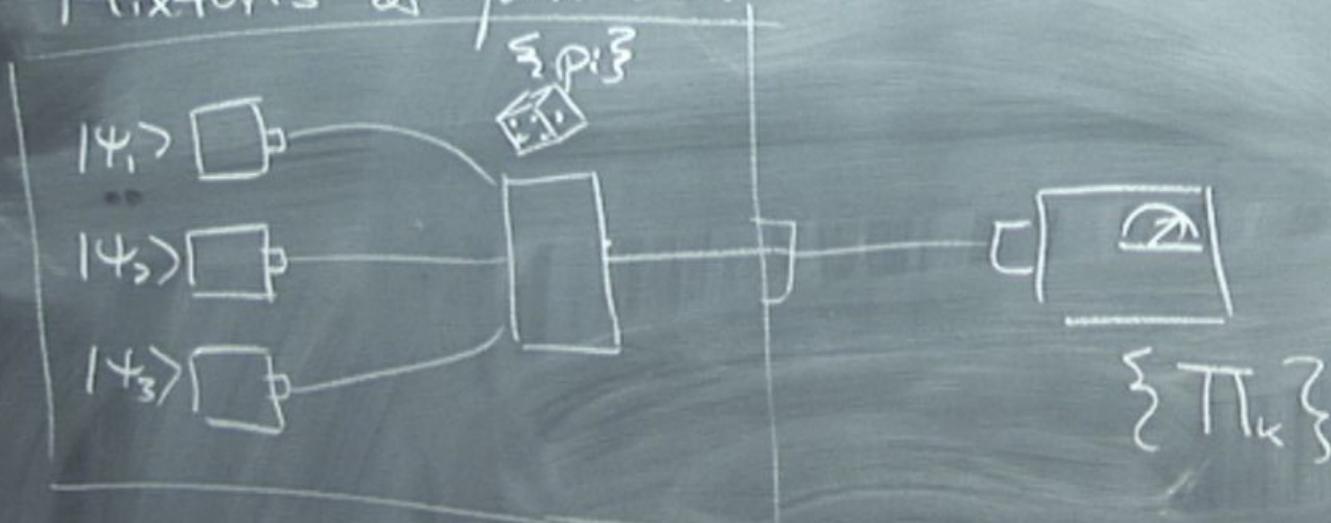


$$P(k) = \sum_i P(k|i) p(i)$$

Realized Preparations

$$\langle \psi | A | \psi \rangle = \text{Tr}(A | \psi \rangle \langle \psi |)$$

Mixtures of pure states



$$\begin{aligned} p(k) &= \sum_i p(k|i) p(i) \\ &= \sum_i \langle \psi_i | \pi_k | \psi_i \rangle p_i \\ &= \sum_i \end{aligned}$$

Realized Preparations

$$\langle \psi | A | \psi \rangle = \text{Tr}(A | \psi \rangle \langle \psi |) \\ = \sum_k \langle k | A | \psi \rangle \langle \psi | k \rangle$$

Mixtures of pure states



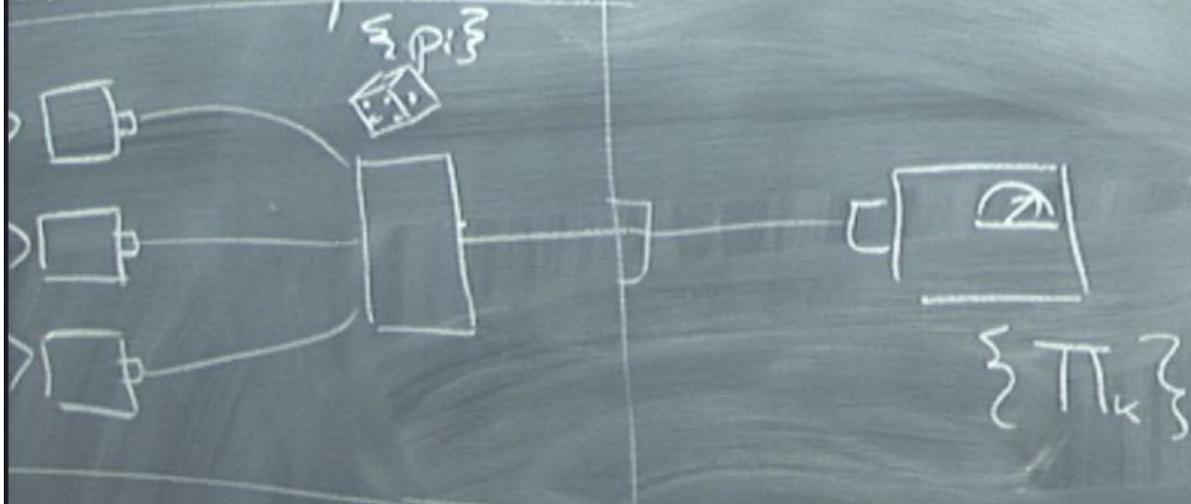
$$P(k) = \sum_i$$

$$P(i)$$

$$|\psi_i\rangle p_i$$

Preparations

Measurements of pure states

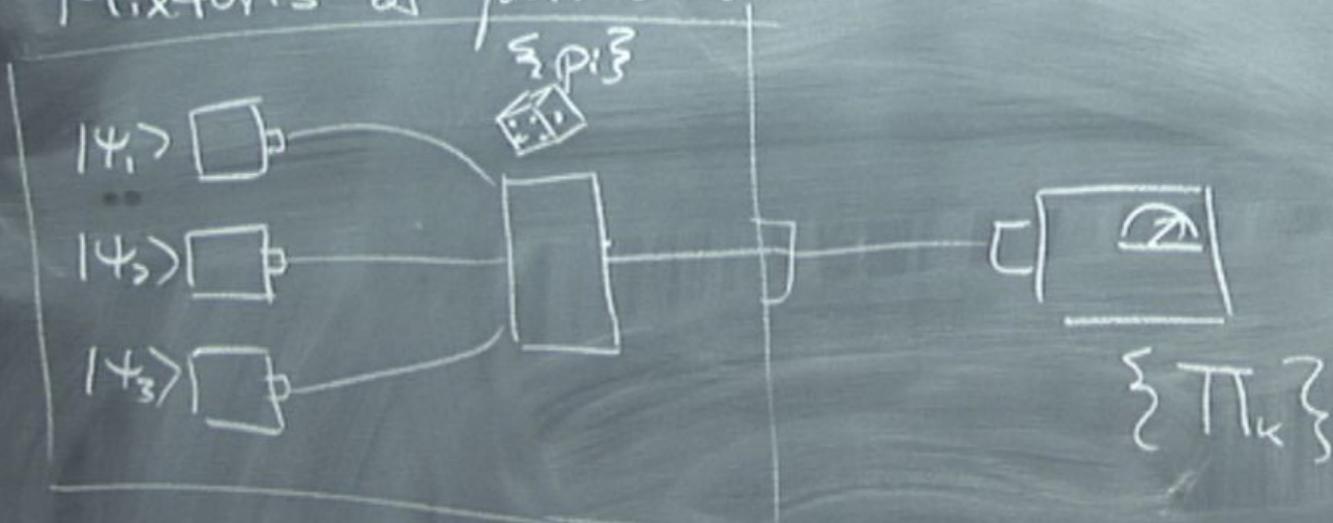


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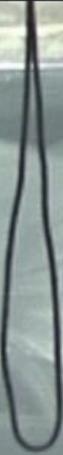
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$$\begin{aligned} p(k) &= \sum_i p(k|i) p(i) \\ &= \sum_i \langle \psi_i | \Pi_k | \psi_i \rangle p_i \\ &= \sum_i \text{Tr}(\Pi_k |\psi_i\rangle \langle \psi_i|) p_i \\ &= \text{Tr}(\Pi_k \underbrace{\sum_i p_i |\psi_i\rangle \langle \psi_i|}_{\rho}) \end{aligned}$$

Positive definite



Positive sem. definite: $\langle \psi | \rho | \psi \rangle \geq 0 \quad \forall |\psi\rangle$ w/ equality only if $|\psi\rangle = 0$

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proof: $\langle \psi | \sum_i p_i |\psi_i\rangle \langle \psi_i | \psi \rangle$
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 ≥ 0

Unit trace: $\text{Tr}(\rho)$

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Note: $e^{i\phi} |\psi\rangle$



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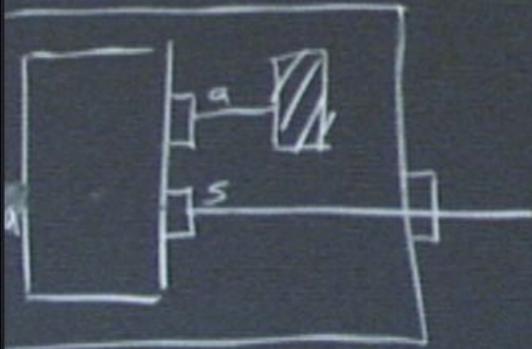
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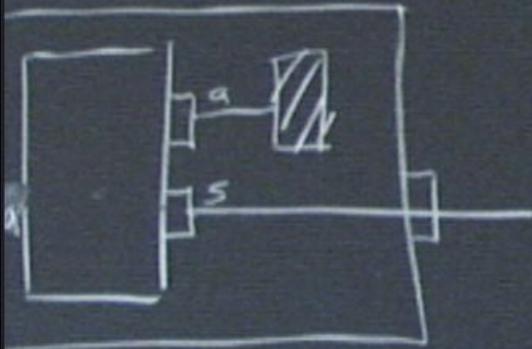
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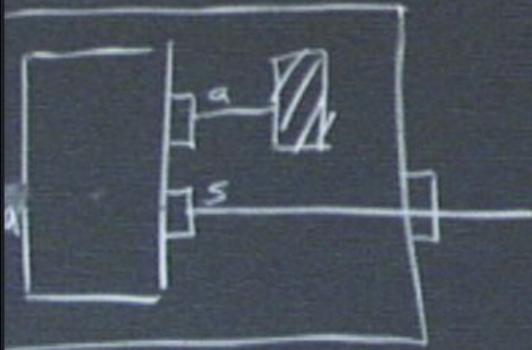
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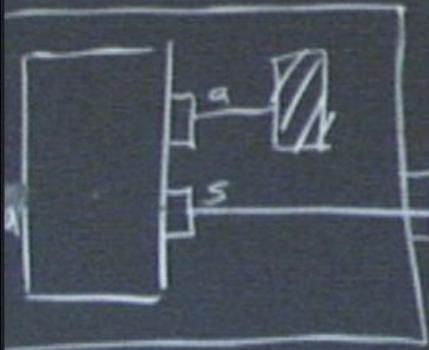
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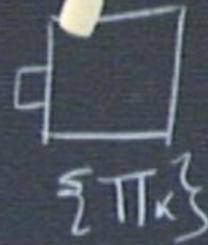
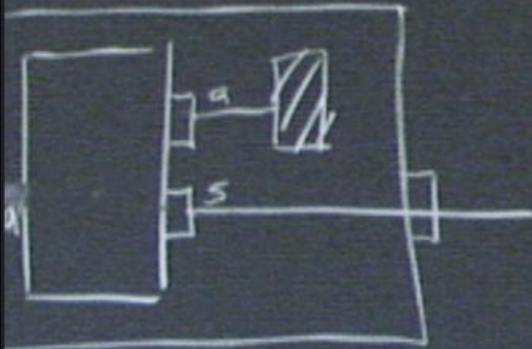






$$\rho(k) = \text{Tr}_{S_A} \left(\left(\Pi_k^{(S)} \otimes \Pi^{(A)} \right) |\psi\rangle_{S_A} \langle \psi| \right)$$

$$|\psi\rangle_{S_A} \in \mathcal{H}_S \otimes \mathcal{H}_A$$



$$|\psi\rangle_{sa} \in \mathcal{H}_s \otimes \mathcal{H}_a$$

$$\rho(k) = \text{Tr}_{sa} \left(\left(\Pi_k^{(s)} \otimes \mathbb{I}^{(a)} \right) |\psi\rangle_{sa} \langle \psi| \right)$$



H_q

$$\rho(k) = \text{Tr}_a \left(\left(\Pi_k^{(s)} \otimes \mathbb{I}^{(a)} \right) |\psi\rangle_{sa} \langle \psi| \right)$$

$$= \sum_{j,k} \langle j|_s \langle k|_a \left(\dots \right) |j\rangle_s |k\rangle_a$$

$$= \text{Tr}_s \left(\Pi_k^{(s)} \text{Tr}_a (|\psi\rangle_{sa} \langle \psi|) \right)$$



H_a

$$\rho(k) = \text{Tr}_a \left(\left(\Pi_k^{(s)} \otimes \mathbb{I}^{(a)} \right) |\Psi\rangle_{sa} \langle\Psi| \right)$$

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$$= \text{Tr}_s \left(\Pi_k^{(s)} \underbrace{\text{Tr}_a (|\Psi\rangle_{sa} \langle\Psi|)}_{\rho_s} \right)$$

ρ_s
reduced density operator
of $|\Psi_{sa}\rangle$ on S

Generalized Preparations

$$|4\rangle_{sa} = \frac{1}{\sqrt{2}} (|0\rangle_s |0\rangle_a + |1\rangle_s |1\rangle_a)$$

Generalized Preparations

$$|\psi\rangle_{sa} = \frac{1}{\sqrt{2}} (|0\rangle_s |0\rangle_a + |1\rangle_s |1\rangle_a)$$

$$\begin{aligned} \rho_s &= \text{Tr}_a (|\psi\rangle_{sa} \langle\psi|_{sa}) \\ &= \text{Tr}_a \left(\frac{1}{2} (|0\rangle_s \langle 0|_s |0\rangle_a \langle 0|_a + |0\rangle_s \langle 1|_s |0\rangle_a \langle 1|_a \right. \\ &\quad \left. + |1\rangle_s \langle 0|_s |1\rangle_a \langle 0|_a + |1\rangle_s \langle 1|_s |1\rangle_a \langle 1|_a) \right) \end{aligned}$$

Normalized Preparations

$$|4\rangle_{sa} = \frac{1}{\sqrt{2}} (|0\rangle_s |0\rangle_a + |1\rangle_s |1\rangle_a)$$

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Positive

prod

~~Unit tr~~

Normalized Preparations

$$|4\rangle_{sa} \in \mathcal{H}_s \otimes \mathcal{H}_a$$

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Generalized Preparations

$$|\psi\rangle_{sa} \in \mathcal{H}_s \otimes \mathcal{H}_a$$

$$|\psi\rangle_{sa} = |\phi\rangle_s \otimes |\chi\rangle_a$$

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Local Preparations

$$|\psi\rangle_{sa} \in \mathcal{H}_s \otimes \mathcal{H}_a$$

$\otimes |X\rangle_a$

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$$\text{Tr}(|X\rangle\langle X|) = \langle X|X\rangle^{\text{prod}}$$

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Unit tr

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$$= \frac{1}{2}(|0\rangle_s\langle 0| + |1\rangle_s\langle 1|)$$

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$$+ |1\rangle_s\langle 0|_s |1\rangle_a\langle 0|_a + |1\rangle_s\langle 1|_s |1\rangle_a\langle 1|_a)$$

$$= \frac{1}{2}(|0\rangle_s\langle 0|_s + |1\rangle_s\langle 1|_s)$$

$$= \frac{1}{2} \mathbb{1}_s$$

Positive

Unit tr

realized Preparations

$$|\psi\rangle_{sa} \in \mathbb{H}_s \otimes \mathbb{H}_a$$

$b\rangle_s \otimes |X\rangle_a$

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$$\text{Tr}(|X\rangle\langle X|) = \langle X|X\rangle$$

$$\rho_s = \text{Tr}_a(|\psi\rangle_{sa}\langle\psi|)$$

$$= \text{Tr}_a\left(\frac{1}{2}(|0\rangle_s\langle 0| |0\rangle_a\langle 0| + |0\rangle_s\langle 1| |0\rangle_a\langle 1| + |1\rangle_s\langle 0| |1\rangle_a\langle 0| + |1\rangle_s\langle 1| |1\rangle_a\langle 1|)\right)$$

$$= \frac{1}{2}|0\rangle_s\langle 0| + \frac{1}{2}|1\rangle_s\langle 1|$$

$$= \frac{1}{2}\mathbb{1}_s$$

Position

Unit

Realized Preparations

$$|\psi\rangle_{sa} \in \mathbb{H}_s \otimes \mathbb{H}_a$$

$b\rangle_s \otimes |X\rangle_a$

$$|\psi\rangle_{sa} = \frac{1}{\sqrt{2}} (|0\rangle_s |0\rangle_a + |1\rangle_s |1\rangle_a)$$

$\text{Tr}(|X\rangle\langle X|) = \langle X|X\rangle = 1$

$$\rho_s = \text{Tr}_a (|\psi\rangle_{sa} \langle\psi|_{sa})$$

$$= \text{Tr}_a \left(\frac{1}{2} (|0\rangle_s \langle 0|_s |0\rangle_a \langle 0|_a + |0\rangle_s \langle 1|_s |0\rangle_a \langle 1|_a + |1\rangle_s \langle 0|_s |1\rangle_a \langle 0|_a + |1\rangle_s \langle 1|_s |1\rangle_a \langle 1|_a) \right)$$

$$= \frac{1}{2} (|0\rangle_s \langle 0|_s + |1\rangle_s \langle 1|_s)$$

$$= \frac{1}{2} \mathbb{1}_s$$

$$= \frac{1}{2} \mathbb{1}_s$$

Measure $\{ |+\eta\rangle\langle +\eta|, |-\eta\rangle\langle -\eta| \}$

Position

Unit

Realized Preparations

$$|\psi\rangle_{sa} \in \mathcal{H}_s \otimes \mathcal{H}_a$$

$$|b\rangle_s \otimes |X\rangle_a \quad |\psi\rangle_{sa} = \frac{1}{\sqrt{2}} (|0\rangle_s |0\rangle_a + |1\rangle_s |1\rangle_a)$$

$$\text{Tr}(|X\rangle\langle X|) = \langle X|X\rangle = 1$$

$$\rho_s = \text{Tr}_a(|\psi\rangle_{sa}\langle\psi|)$$

$$= \text{Tr}_a\left(\frac{1}{2}(|0\rangle_s\langle 0|_s |0\rangle_a\langle 0|_a + |0\rangle_s\langle 1|_s |0\rangle_a\langle 1|_a\right.$$

$$+ |1\rangle_s\langle 0|_s |1\rangle_a\langle 0|_a + |1\rangle_s\langle 1|_s |1\rangle_a\langle 1|_a)$$

$$= \frac{1}{2}(|0\rangle_s\langle 0|_s + |1\rangle_s\langle 1|_s)$$

$$= \frac{1}{2} \mathbb{1}_s$$

Measure $\{ |+\eta\rangle\langle +\eta|, |-\eta\rangle\langle -\eta| \}$

$$p(\pm) = \text{Tr}(\rho_s | \pm \eta \rangle \langle \pm \eta |)$$

$$= \frac{1}{2}$$

Realized Preparations

$$|\psi\rangle_{sa} \in \mathbb{H}_s \otimes \mathbb{H}_a$$

$b\rangle_s \otimes |X\rangle_a$

$$|\psi\rangle_{sa} = \frac{1}{\sqrt{2}} (|0\rangle_s |0\rangle_a + |1\rangle_s |1\rangle_a)$$

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$$+ |1\rangle_s\langle 0|_s |1\rangle_a\langle 0|_a + |1\rangle_s\langle 1|_s |1\rangle_a\langle 1|_a)$$

$$= \frac{1}{2}(|0\rangle_s\langle 0|_s + |1\rangle_s\langle 1|_s)$$

$$= \frac{1}{2} \mathbb{1}_s$$

Measure $\{|+\eta\rangle\langle +\eta|, |-\eta\rangle\langle -\eta|\}$

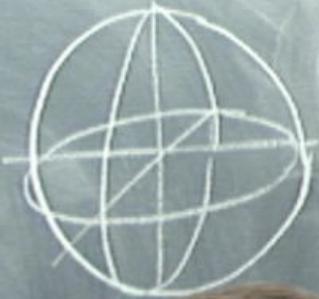
$$p(\pm) = \text{Tr}(\rho_s | \pm \eta \rangle \langle \pm \eta |)$$

$$= \frac{1}{2}$$

Blach phase rep'n



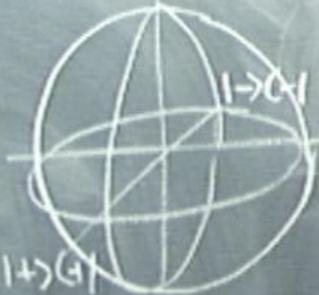
Bloch sphere rep'n



Bloch sphere rep'n

$|0\rangle\langle 0|$

$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



$|z\rangle\langle z|$

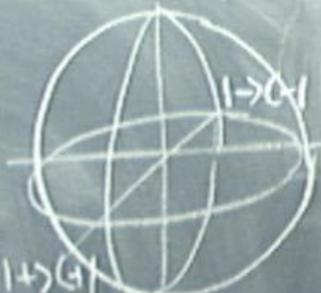
$|1\rangle\langle 1|$



Bloch sphere rep'n

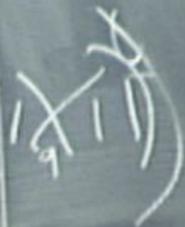
$|0\rangle$ col

$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



$|z\rangle$

$|1\rangle$

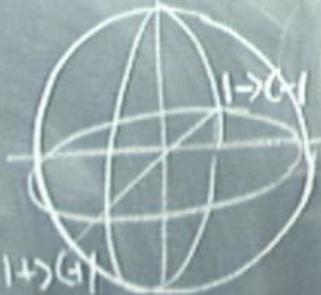


Hermitian op's on Hilbert space of dim d
form a real Hilbert space of dim d^2

Bloch sphere rep'n

$|0\rangle$ col

$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



$|z\rangle$



Hermitian ops on Hilbert space of dim. d
form a real Hilbert space of dim. d^2

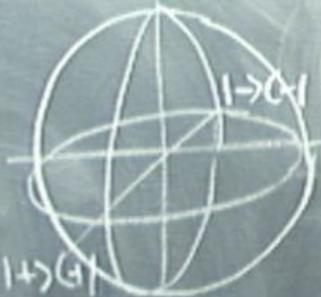
Suppose A, B Hermitian

$$rA \quad r \in \mathbb{R}$$

Bloch sphere rep'n

$|0\rangle$ col

$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



$|1/2\rangle$

$|1/2\rangle$



Hermitian op's on Hilbert space of dim. d
form a real Hilbert space of dim. d^2

Suppose A, B Hermitian

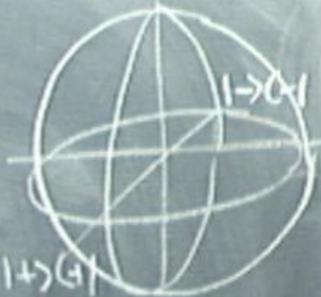
$$rA \quad r \in \mathbb{R}$$

$$A + B$$

Bloch sphere rep'n

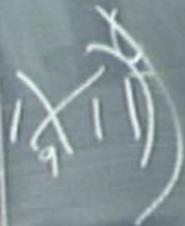
$|0\rangle$ col

$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



$|1\rangle$ bot

$|X\rangle$



Hermitian op's on Hilbert space of dim. d
form a real Hilbert space of dim. d^2

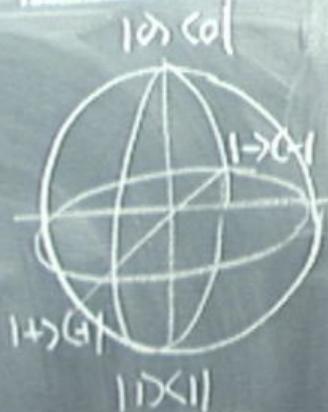
Suppose A, B Hermitian

$$rA \quad r \in \mathbb{R}$$

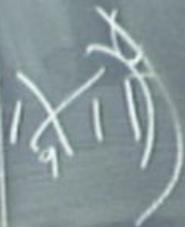
$$A + B$$

inner product $(A, B) = \text{Tr}(AB)$

Bloch sphere rep'n



$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$



Hermitian op's on Hilbert space of dim. d
form a real Hilbert space of dim. d^2

Suppose A, B Hermitian

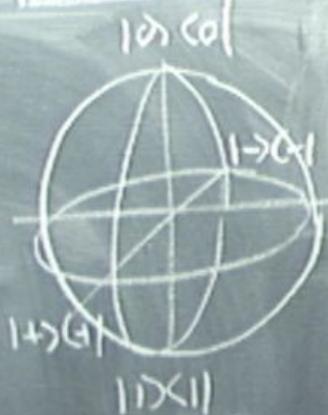
$$rA \quad r \in \mathbb{R}$$

$$A + B$$

inner product $(A, B) = \text{Tr}(AB)$

(i) linearity $(A, rB) = r(A, B)$

Bloch sphere rep'n



$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

Hermitian ops on Hilbert space of dim d
form a real Hilbert space of dim d^2

Suppose A, B Hermitian

$$rA \quad r \in \mathbb{R}$$

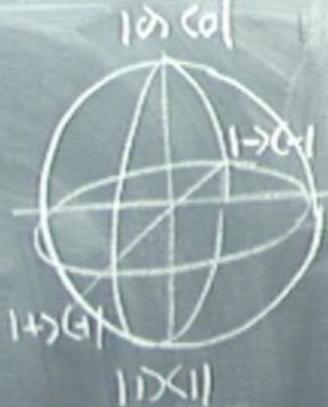
$$A + B$$

inner product $(A, B) = \text{Tr}(AB)$

(1) linearity $(A, rB) = r(A, B)$

(2) symmetric $(A, B) = (B, A)$

Bloch sphere rep'n



$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

Hermitian ops on Hilbert space of dim d
form a real Hilbert space of dim d^2

Suppose A, B Hermitian

$$rA \quad r \in \mathbb{R}$$

$$A + B$$

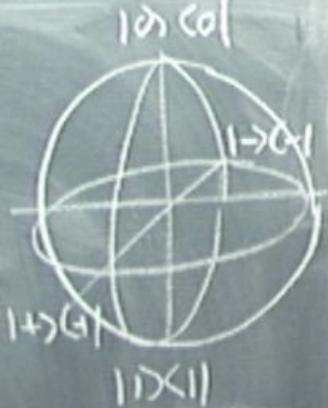
inner product $(A, B) = \text{Tr}(AB)$

(1) linearity $(A, rB) = r(A, B)$

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(3) $(A, A) \geq 0$

Bloch sphere rep'n



$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Hermitian ops on Hilbert space of dim d
form a real Hilbert space of dim d^2

Suppose A, B Hermitian

$$rA \quad r \in \mathbb{R}$$

$$A + B$$

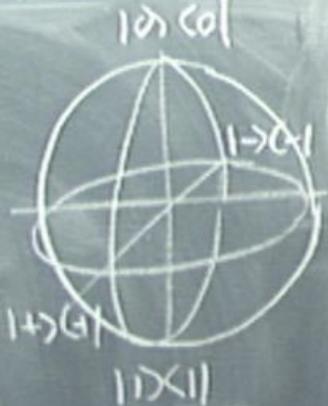
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Bloch sphere rep'n



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Hermitian ops on Hilbert space of dim d
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Suppose A, B Hermitian

$$rA \quad r \in \mathbb{R}$$

$$A + B$$

An orthogonal basis

$$\sigma_0 = \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

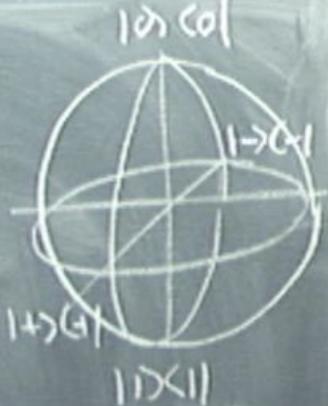
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Bloch sphere rep'n



$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

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$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z$$

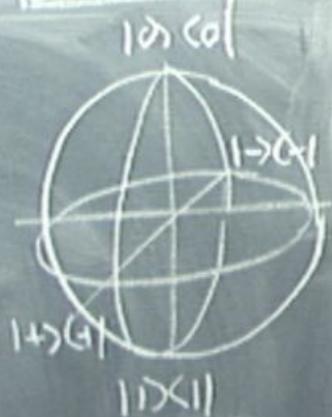
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(1) linearity $(A, rB) = r(A, B)$

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$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

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$$\sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

inner product $(A, B) = \text{Tr}(AB)$

(1) linearity $(A, rB) = r(A, B)$

(2) symmetric $(A, B) = (B, A)$

(3) $(A, A) \geq 0$

$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z$$

$$\rho = \frac{1}{2} (\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

where $|\vec{r}| \leq 1$

Generalized Preparations

$$\begin{aligned}\text{Tr}(\rho\rho') &= \text{Tr}\left[\frac{1}{2}(1+\vec{r}\cdot\vec{\sigma})\frac{1}{2}(1+\vec{s}\cdot\vec{\sigma})\right] \\ &= \frac{1}{2}(1+\vec{r}\cdot\vec{s})\end{aligned}$$

Generalized Preparations

$$\text{Tr}(\rho\rho') = \text{Tr}\left[\frac{1}{2}(1 + \vec{r}\cdot\vec{\sigma})\frac{1}{2}(1 + \vec{s}\cdot\vec{\sigma})\right]$$

$$= \frac{1}{2}(1 + \vec{r}\cdot\vec{s})$$

$$\text{Tr}(\rho\rho') = 0$$

$$|\psi\rangle\langle\psi| |\psi'\rangle\langle\psi'|$$

$$\vec{r} = -\vec{s}$$

Generalized Preparations

$$\text{Tr}(\rho\rho') = \text{Tr}\left[\frac{1}{2}(1 + \vec{r} \cdot \vec{\sigma}) \frac{1}{2}(1 + \vec{s} \cdot \vec{\sigma})\right]$$

$$= \frac{1}{2}(1 + \vec{r} \cdot \vec{s})$$

$$\text{Tr}(\rho\rho') = 0$$

$$\langle \psi | \langle \psi | \psi \rangle \langle \psi | \rangle = 0$$

$$\vec{r} = -\vec{s}$$

Ambiguity of mixtures

Generalized Preparations

$$\text{Tr}(\rho\rho') = \text{Tr}\left[\frac{1}{2}(1 + \vec{r}\cdot\vec{\sigma})\frac{1}{2}(1 + \vec{s}\cdot\vec{\sigma})\right]$$

$$= \frac{1}{2}(1 + \vec{r}\cdot\vec{s})$$

$$\text{Tr}(\rho\rho') = 0$$

$$\vec{r} = -\vec{s}$$

$$\langle\psi|\psi\rangle\langle\psi|\psi\rangle = 0$$

Ambiguity of mixtures

$$\begin{aligned}\frac{1}{2} &= \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \\ &= \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|\end{aligned}$$

Generalized Preparations

$$\begin{aligned}\text{Tr}(\rho\rho') &= \text{Tr}\left[\frac{1}{2}(1+\vec{r}\cdot\vec{\sigma})\frac{1}{2}(1+\vec{s}\cdot\vec{\sigma})\right] \\ &= \frac{1}{2}(1+\vec{r}\cdot\vec{s})\end{aligned}$$

$$\text{Tr}(\rho\rho') = 0$$

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Ambiguity of mixtures

$$\begin{aligned}\frac{1}{2} &= \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \\ &= \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -| \\ &= \frac{1}{2}\frac{|0\rangle+|1\rangle}{\sqrt{2}}\frac{\langle 0|+\langle 1|}{\sqrt{2}} + \frac{1}{2}\frac{|0\rangle-|1\rangle}{\sqrt{2}}\frac{\langle 0|-\langle 1|}{\sqrt{2}}\end{aligned}$$

Generalized Preparations

$$\rho = \frac{3}{4} |a\rangle\langle a| + \frac{1}{4} |b\rangle\langle b|$$
$$= \frac{1}{2} |a\rangle\langle a| + \frac{1}{2} |b\rangle\langle b|$$

$$|a\rangle = \sqrt{\frac{3}{4}} |0\rangle + \sqrt{\frac{1}{4}} |1\rangle$$

A mixture

$$|a\rangle = \frac{\sqrt{3}}{2} |0\rangle + \frac{1}{2} |1\rangle$$

$$\frac{\sqrt{3}}{2} \frac{|0\rangle + |1\rangle}{\sqrt{2}} + \frac{1}{2} \frac{|0\rangle - |1\rangle}{\sqrt{2}} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Generalized Preparations

$$\rho = \frac{3}{4} |0\rangle\langle 0| + \frac{1}{4} |1\rangle\langle 1|$$

$$= \frac{1}{2} |a\rangle\langle a| + \frac{1}{2} |b\rangle\langle b|$$

$$|a\rangle = \sqrt{\frac{3}{4}} |0\rangle + \sqrt{\frac{1}{4}} |1\rangle$$

$$|b\rangle = \sqrt{\frac{3}{4}} |0\rangle - \sqrt{\frac{1}{4}} |1\rangle$$

Ambiguity of mixtures

$$\frac{1}{2} = \frac{1}{2} |0\rangle\langle 0| + \frac{1}{2} |1\rangle\langle 1|$$

$$= \frac{1}{2} |+\rangle\langle +| + \frac{1}{2} |-\rangle\langle -|$$

$$\frac{|0\rangle+|1\rangle}{\sqrt{2}} \langle 0|+\langle 1| + \frac{1}{2} \frac{|0\rangle-|1\rangle}{\sqrt{2}} \langle 0|-\langle 1|$$

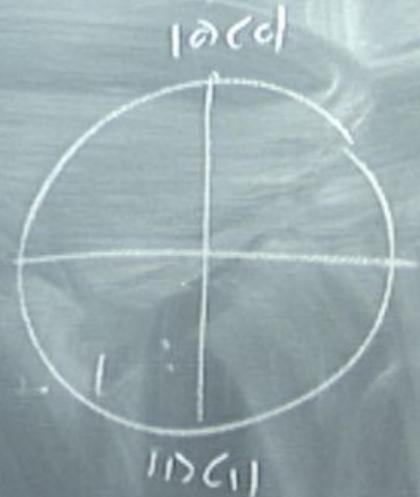
Generalized Preparations

$$\rho = \frac{3}{4} |0\rangle\langle 0| + \frac{1}{4} |1\rangle\langle 1|$$

$$= \frac{1}{2} |a\rangle\langle a| + \frac{1}{2} |b\rangle\langle b|$$

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$$|b\rangle = \sqrt{\frac{3}{4}} |0\rangle - \sqrt{\frac{1}{4}} |1\rangle$$



4 mixtures

$$= \frac{1}{2} |0\rangle\langle 0| + \frac{1}{2} |1\rangle\langle 1|$$

$$= \frac{1}{2} |+\rangle\langle +| + \frac{1}{2} |-\rangle\langle -|$$

$$= \frac{1}{2} \frac{|0\rangle + |1\rangle}{\sqrt{2}} \langle 0| + \langle 1| + \frac{1}{2} \frac{|0\rangle - |1\rangle}{\sqrt{2}} \langle 0| - \langle 1|$$

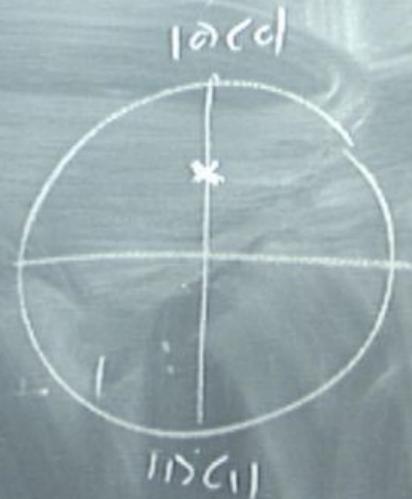
Generalized Preparations

$$\rho = \frac{3}{4} |0\rangle\langle 0| + \frac{1}{4} |1\rangle\langle 1|$$

$$= \frac{1}{2} |a\rangle\langle a| + \frac{1}{2} |b\rangle\langle b|$$

$$|a\rangle = \sqrt{\frac{3}{4}} |0\rangle + \sqrt{\frac{1}{4}} |1\rangle$$

$$|b\rangle = \sqrt{\frac{3}{4}} |0\rangle - \sqrt{\frac{1}{4}} |1\rangle$$



Justy's mixture

$$\frac{1}{2} = \frac{1}{2} |0\rangle\langle 0| + \frac{1}{2} |1\rangle\langle 1|$$

$$= \frac{1}{2} |+\rangle\langle +| + \frac{1}{2} |-\rangle\langle -|$$

$$\frac{1}{2} \frac{|0\rangle + |1\rangle}{\sqrt{2}} \frac{\langle 0| + \langle 1|}{\sqrt{2}} + \frac{1}{2} \frac{|0\rangle - |1\rangle}{\sqrt{2}} \frac{\langle 0| - \langle 1|}{\sqrt{2}}$$

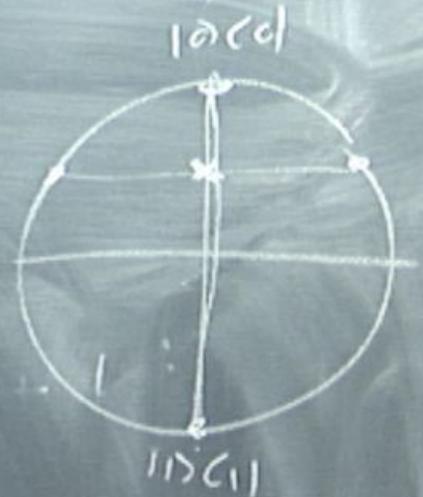
Generalized Preparations

$$\rho = \frac{3}{4} |0\rangle\langle 0| + \frac{1}{4} |1\rangle\langle 1|$$

$$= \frac{1}{2} |a\rangle\langle a| + \frac{1}{2} |b\rangle\langle b|$$

$$|a\rangle = \sqrt{\frac{3}{4}} |0\rangle + \sqrt{\frac{1}{4}} |1\rangle$$

$$|b\rangle = \sqrt{\frac{3}{4}} |0\rangle - \sqrt{\frac{1}{4}} |1\rangle$$



density as mixture

$$= \frac{1}{2} |0\rangle\langle 0| + \frac{1}{2} |1\rangle\langle 1|$$

$$= \frac{1}{2} |+\rangle\langle +| + \frac{1}{2} |-\rangle\langle -|$$

$$= \frac{1}{2} \frac{|0\rangle + |1\rangle}{\sqrt{2}} \frac{\langle 0| + \langle 1|}{\sqrt{2}} + \frac{1}{2} \frac{|0\rangle - |1\rangle}{\sqrt{2}} \frac{\langle 0| - \langle 1|}{\sqrt{2}}$$

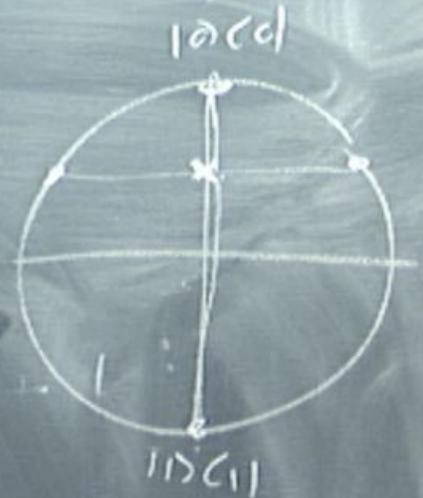
Generalized Preparations

$$\rho = \frac{3}{4} |0\rangle\langle 0| + \frac{1}{4} |1\rangle\langle 1|$$

$$= \frac{1}{2} |a\rangle\langle a| + \frac{1}{2} |b\rangle\langle b|$$

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Ambiguity of mixtures

$$\frac{1}{2} = \frac{1}{2} |0\rangle\langle 0| + \frac{1}{2} |1\rangle\langle 1|$$

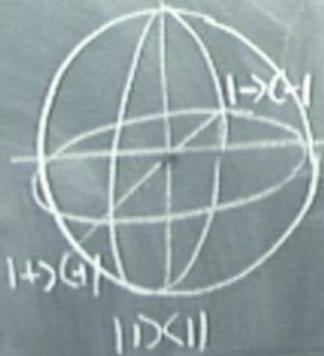
$$= \frac{1}{2} |+\rangle\langle +| + \frac{1}{2} |-\rangle\langle -|$$

$$= \frac{1}{2} \frac{|0\rangle + |1\rangle}{\sqrt{2}} \frac{\langle 0| + \langle 1|}{\sqrt{2}} + \frac{1}{2} \frac{|0\rangle - |1\rangle}{\sqrt{2}} \frac{\langle 0| - \langle 1|}{\sqrt{2}}$$

Bloch sphere rep'n

$|0\rangle|0\rangle$

$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



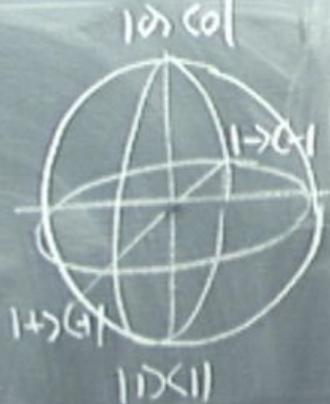
$$\rho = \sum_{k=1}^n p_k \rho_k$$

$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z \quad (3)$$

$$\rho = \frac{1}{2} (\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

where $|\vec{r}| \leq 1$

Bloch sphere rep'n



$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$\rho = \sum_{k=1}^n p_k \rho_k$$

$$\frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma}) = \sum_{k=1}^n p_k \frac{1}{2}(\mathbb{1} + \vec{r}_k \cdot \vec{\sigma})$$

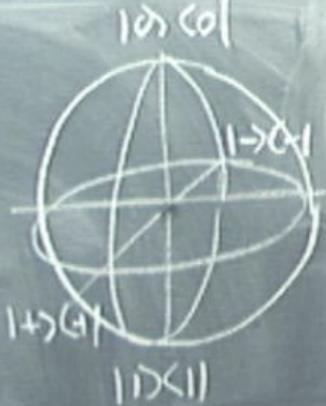
$$\vec{r} = \sum_{k=1}^n p_k \vec{r}_k$$

$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z \quad (3) \quad 11$$

$$\rho = \frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

where $|\vec{r}| \leq 1$

Bloch sphere rep'n



$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$\rho = \sum_{k=1}^n p_k \rho_k$$

$$\frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma}) = \sum_{k=1}^n p_k \frac{1}{2}(\mathbb{1} + \vec{r}_k \cdot \vec{\sigma})$$

$$\vec{r} = \sum_{k=1}^n p_k \vec{r}_k$$

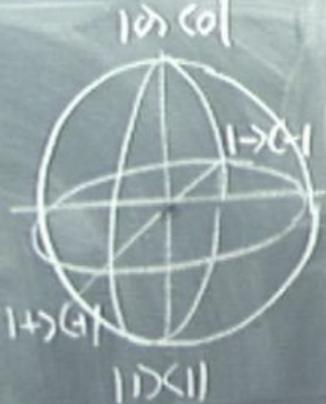
$$\underline{n=2} \quad \vec{r} = p \vec{r}_1 + (1-p) \vec{r}_2$$

$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z \quad (3) \quad (1)$$

$$\rho = \frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

where $|\vec{r}| \leq 1$

Bloch sphere rep'n



$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

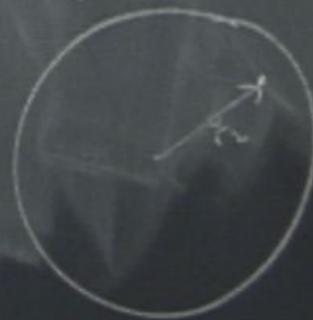
$$\rho = \sum_{k=1}^n p_k \rho_k$$

$$\frac{1}{2}(1 + \vec{r} \cdot \vec{\sigma}) = \sum_{k=1}^n p_k \frac{1}{2}(1 + \vec{r}_k \cdot \vec{\sigma})$$

$$\vec{r} = \sum_{k=1}^n p_k \vec{r}_k$$

$n=2$

$$\begin{aligned} \vec{r} &= p \vec{r}_1 + (1-p) \vec{r}_2 \\ &= \vec{r}_2 + p(\vec{r}_1 - \vec{r}_2) \end{aligned}$$

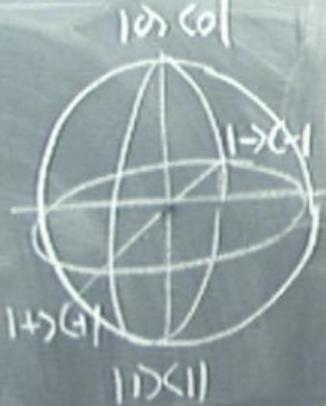


$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z \quad (3)$$

$$\rho = \frac{1}{2}(1 + \vec{r} \cdot \vec{\sigma})$$

where $|\vec{r}| \leq 1$

Bloch sphere rep'n



$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

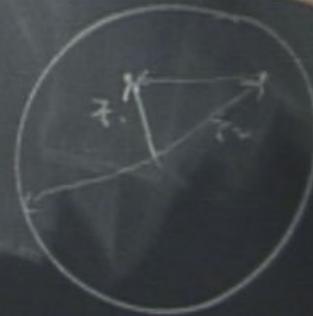
$$\rho = \sum_{k=1}^n p_k \rho_k$$

$$\frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma}) = \sum_{k=1}^n p_k \frac{1}{2}(\mathbb{1} + \vec{r}_k \cdot \vec{\sigma})$$

$$\vec{r} = \sum_{k=1}^n p_k \vec{r}_k$$

$n=2$

$$\begin{aligned} \vec{r} &= p \vec{r}_1 + (1-p) \vec{r}_2 \\ &= \vec{r}_2 + p(\vec{r}_1 - \vec{r}_2) \end{aligned}$$

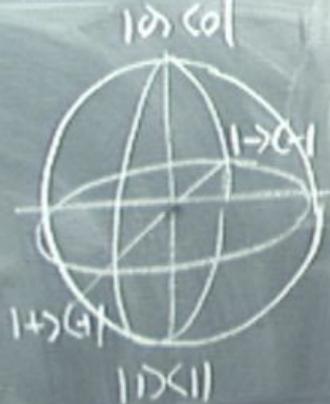


$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z \quad (3)$$

$$\rho = \frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

where $|\vec{r}| \leq 1$

Bloch sphere rep'n



$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

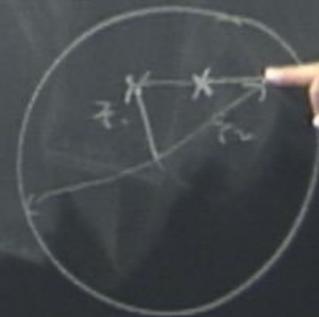
$$\rho = \sum_{k=1}^n p_k \rho_k$$

$$\frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma}) = \sum_{k=1}^n p_k \frac{1}{2}(\mathbb{1} + \vec{r}_k \cdot \vec{\sigma})$$

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$$\begin{aligned} \vec{r} &= p \vec{r}_1 + (1-p) \vec{r}_2 \\ &= \vec{r}_2 + p(\vec{r}_1 - \vec{r}_2) \end{aligned}$$

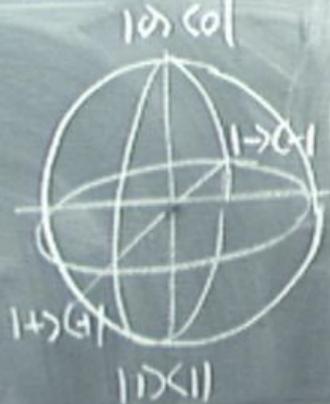


$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z \quad (3)$$

$$\rho = \frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

where $|\vec{r}| \leq 1$

Bloch sphere rep'n



$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

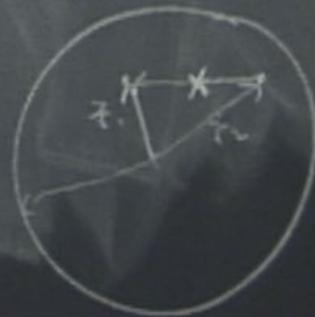
$$\rho = \sum_{k=1}^n p_k \rho_k$$

$$\frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma}) = \sum_{k=1}^n p_k \frac{1}{2}(\mathbb{1} + \vec{r}_k \cdot \vec{\sigma})$$

$$\vec{r} = \sum_{k=1}^n p_k \vec{r}_k$$

n=2

$$\begin{aligned} \vec{r} &= p \vec{r}_1 + (1-p) \vec{r}_2 \\ &= \vec{r}_2 + p(\vec{r}_1 - \vec{r}_2) \end{aligned}$$



$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z \quad (3)$$

$$\rho = \frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

where $|\vec{r}| \leq 1$

Bloch sphere rep'n



$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$\rho = \sum_{k=1}^n p_k \rho_k$$

$$\frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma}) = \sum_{k=1}^n p_k \frac{1}{2}(\mathbb{1} + \vec{r}_k \cdot \vec{\sigma})$$

$$\vec{r} = \sum_{k=1}^n p_k \vec{r}_k$$

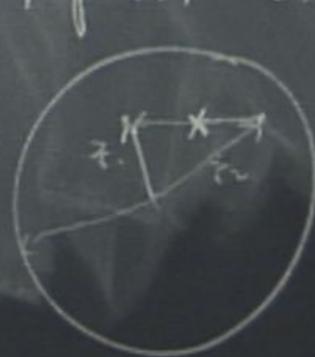
$n=2$

$$\begin{aligned} \vec{r} &= p \vec{r}_1 + (1-p) \vec{r}_2 \\ &= \vec{r}_2 + p(\vec{r}_1 - \vec{r}_2) \end{aligned}$$

$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z \quad (3)$$

$$\rho = \frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

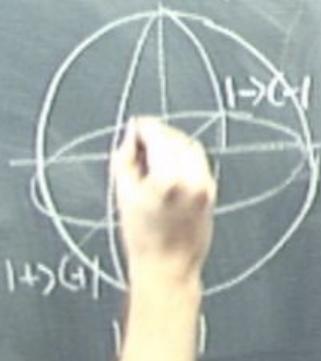
where $|\vec{r}| \leq 1$



Bloch sphere rep'n

$|0\rangle$ col

$$|±\rangle = \frac{|0\rangle ± |1\rangle}{\sqrt{2}}$$



$$\rho = \sum_{k=1}^n p_k \rho_k$$

$$\frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma}) = \sum_{k=1}^n p_k \frac{1}{2}(\mathbb{1} + \vec{r}_k \cdot \vec{\sigma})$$

$$\vec{r} = \sum_{k=1}^n p_k \vec{r}_k$$

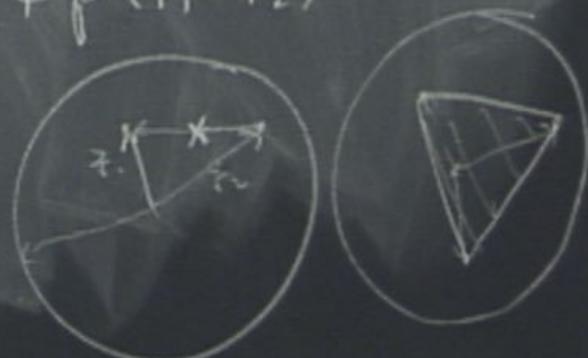
$n=2$

$$\begin{aligned} \vec{r} &= p \vec{r}_1 + (1-p) \vec{r}_2 \\ &= \vec{r}_2 + p(\vec{r}_1 - \vec{r}_2) \end{aligned}$$

$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z \quad (3)$$

$$\rho = \frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

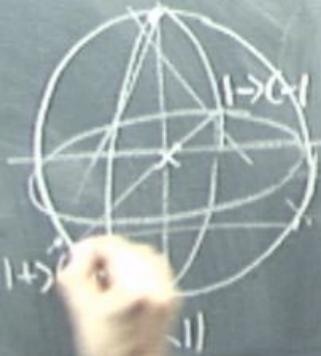
where $|\vec{r}| \leq 1$



Bloch sphere rep'n

101 col

$$|z\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



$$\rho = \sum_{k=1}^n p_k \rho_k$$

$$\frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma}) = \sum_{k=1}^n p_k \frac{1}{2}(\mathbb{1} + \vec{r}_k \cdot \vec{\sigma})$$

$$\vec{r} = \sum_{k=1}^n p_k \vec{r}_k$$

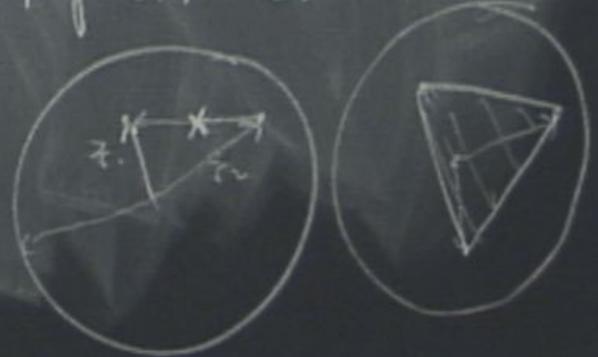
n=2

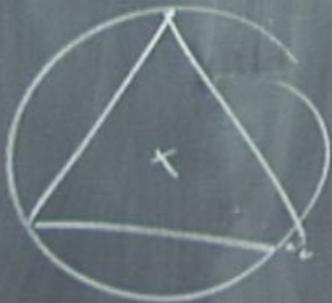
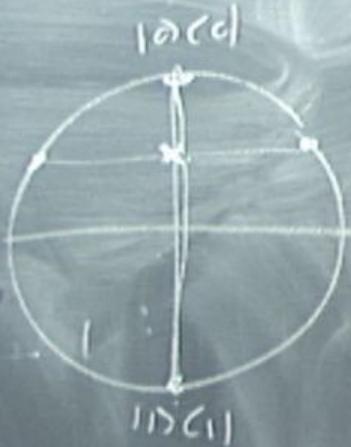
$$\begin{aligned} \vec{r} &= p \vec{r}_1 + (1-p) \vec{r}_2 \\ &= \vec{r}_2 + p(\vec{r}_1 - \vec{r}_2) \end{aligned}$$

$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z \quad (3)$$

$$\rho = \frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

where $|\vec{r}| \leq 1$



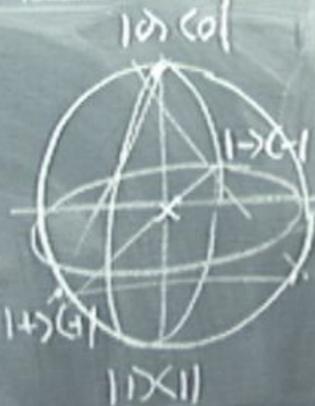


110

111

$$\frac{|0\rangle - |1\rangle}{\sqrt{2}} \quad \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Bloch sphere rep'n



$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

$$\rho = \sum_{k=1}^n p_k$$

$$\frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

$$\vec{r} =$$

$$n=2$$

$$\vec{r} = \rho$$

$$= \frac{\vec{\sigma}}{2}$$

$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z \quad (3)$$

$$\rho = \frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

where $|\vec{r}| \leq 1$

Bloch sphere rep'n

$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

$$\rho = \sum_{k=1}^n p_k \rho_k$$

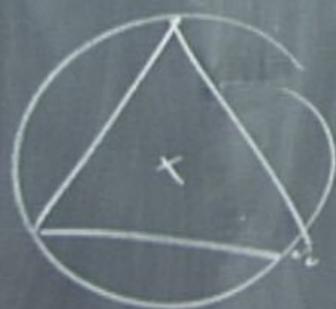
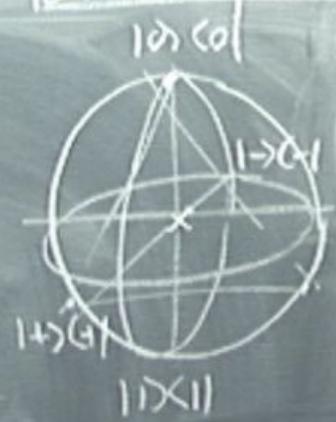
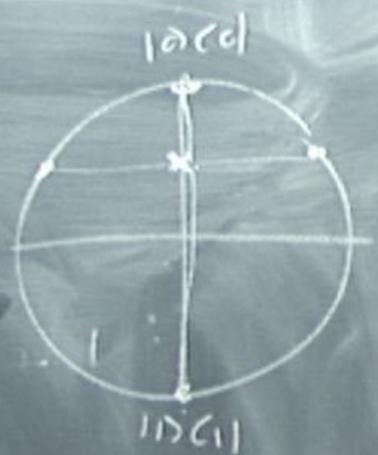
$$\frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

$$\vec{r} = \rho$$

$$n=2$$

$$\vec{r} = \rho$$

$$= \vec{r}_2$$



$$A = c_0 \mathbb{1} + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z \quad (3)$$

$$\rho = \frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma})$$

where $|\vec{r}| \leq 1$

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$$\frac{|0\rangle - |1\rangle}{\sqrt{2}} \quad \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$