

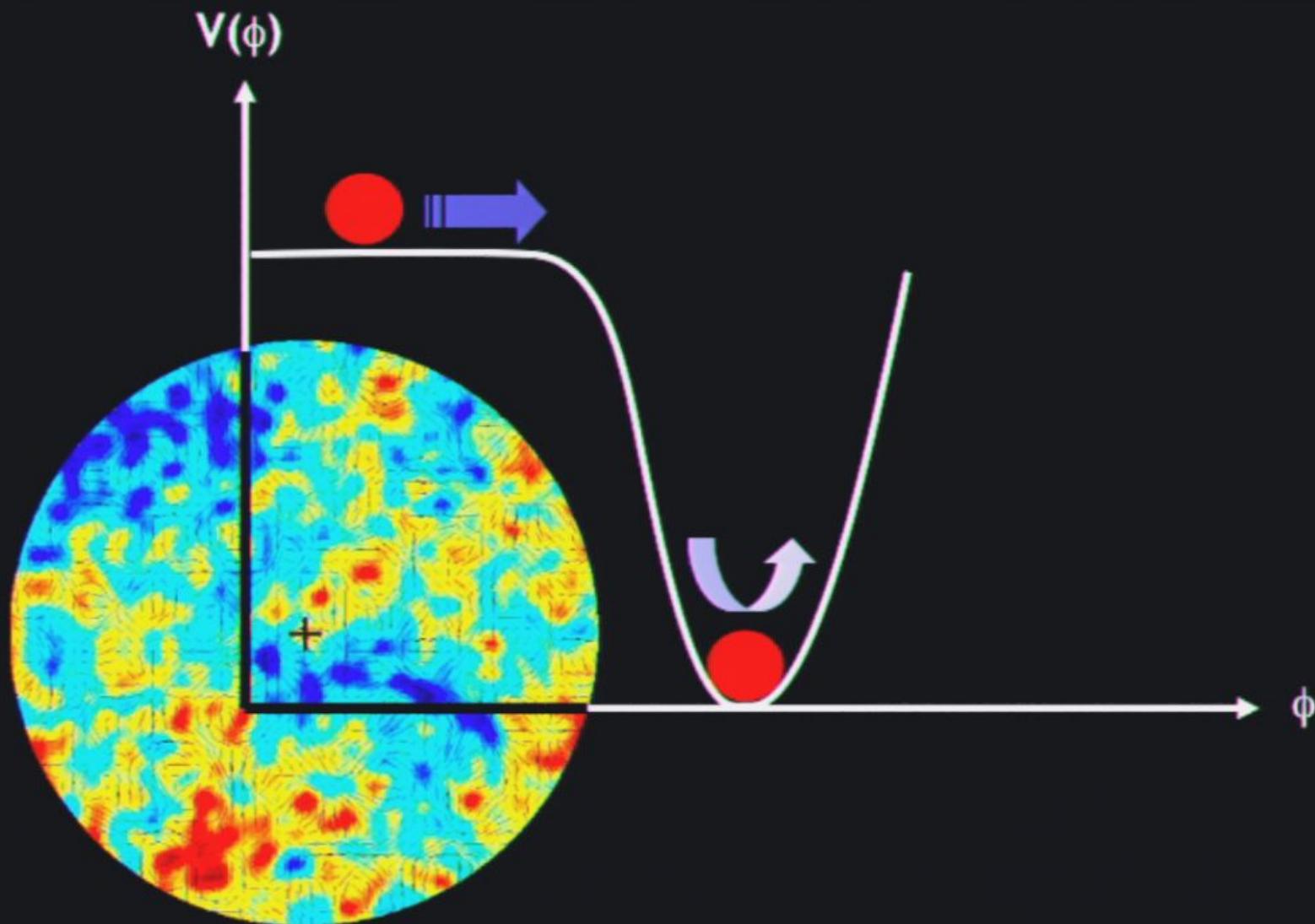
Title: Cosmology - Review (PHYS 621) - Lecture 10

Date: Dec 11, 2009 10:00 AM

URL: <http://www.pirsa.org/09120064>

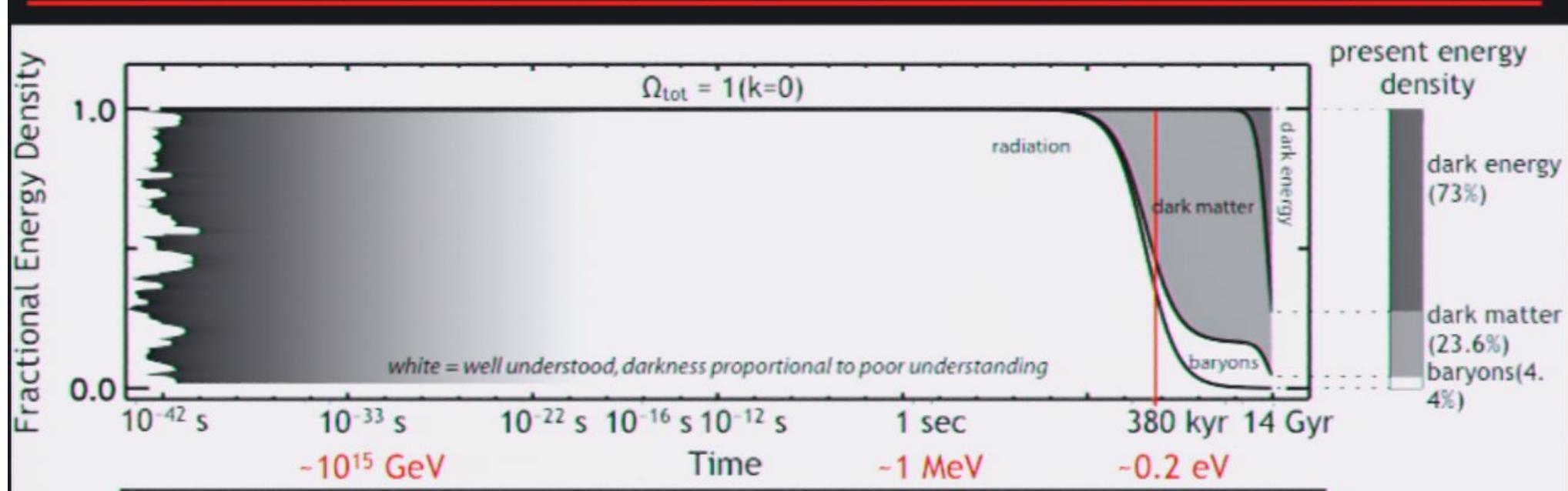
Abstract:

Prospects for CMB observations



Hiranya Peiris

Cosmic History / Cosmic Mystery



Planck Energy GUT symmetry $T=100 \text{ TeV}$ (ILC X 100) nucleosynthesis

Generation of primordial perturbations

Cosmic Microwave Background Emitted carries signature of acoustic oscillations and potentially primordial gravitational waves

non-linear growth of perturbations: signature on CMB through weak gravitational lensing

Λ CDM: The “Standard Model” of Cosmology

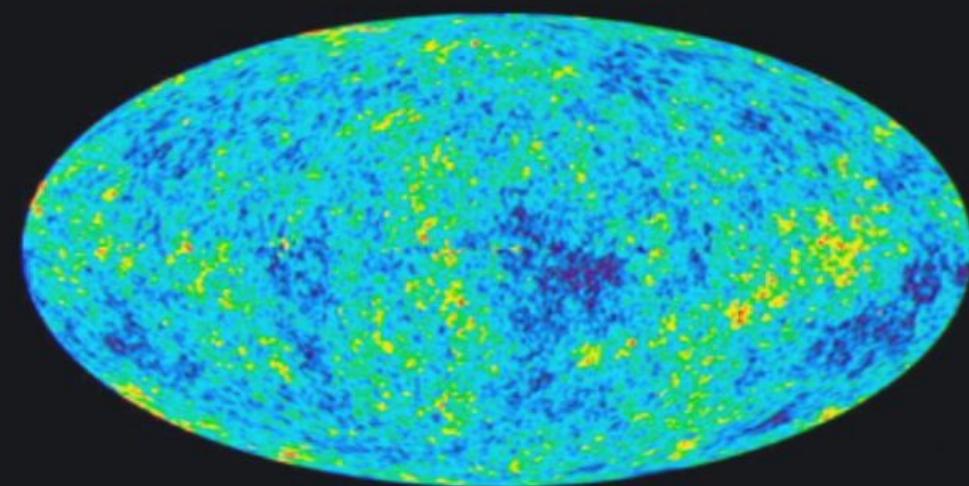
Homogeneous background



$\Omega_b, \Omega_c, \Omega_\Lambda, H_0, \tau$

- atoms 4%
- cold dark matter 23%
- dark energy 73%

Perturbations



A_s, n_s, r

- nearly scale-invariant
- adiabatic
- Gaussian

History of CMB temperature measurements

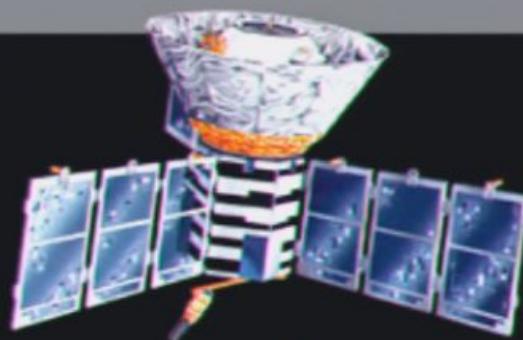
1965



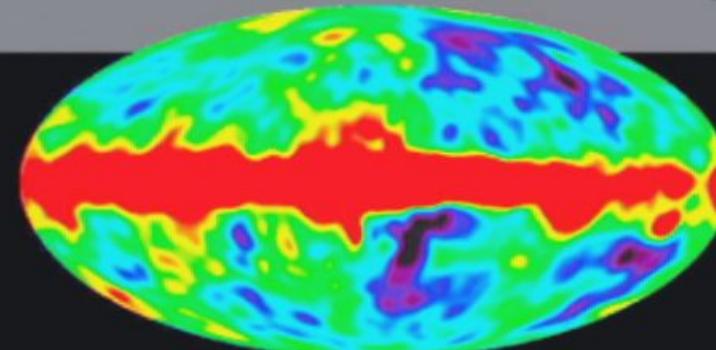
Penzias and
Wilson

2.725 K

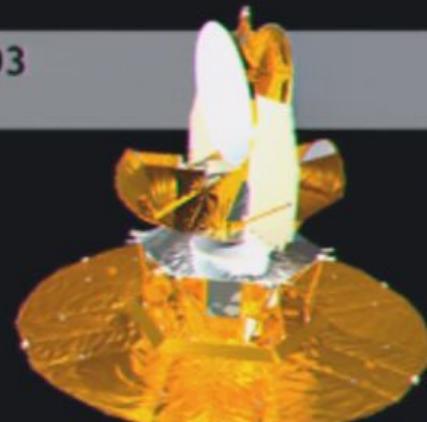
1992



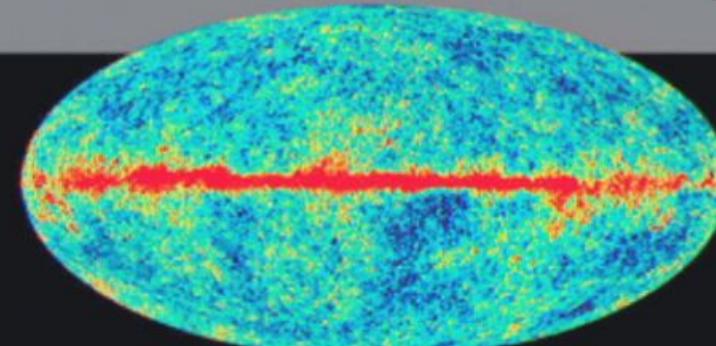
COBE



2003



WMAP



History of CMB temperature measurements

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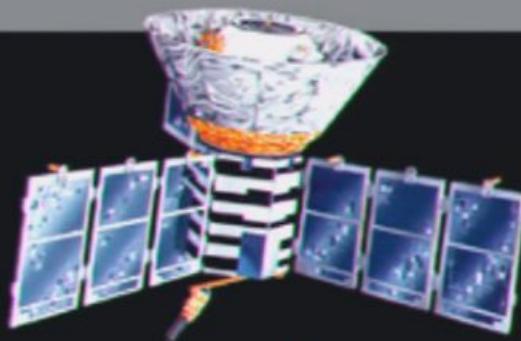


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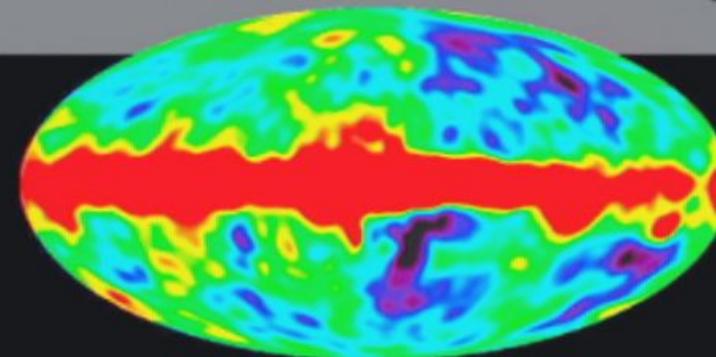
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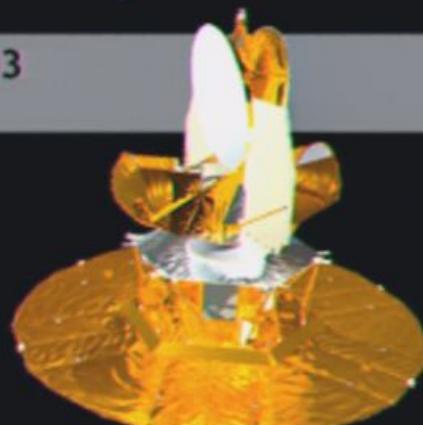
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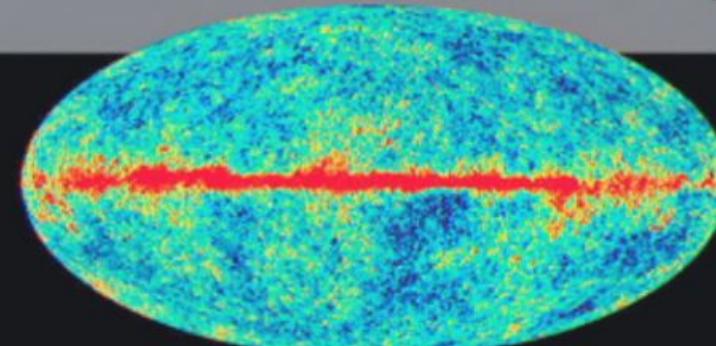
COBE



2003



WMAP



Compress the CMB map to study cosmology

Express sky as:

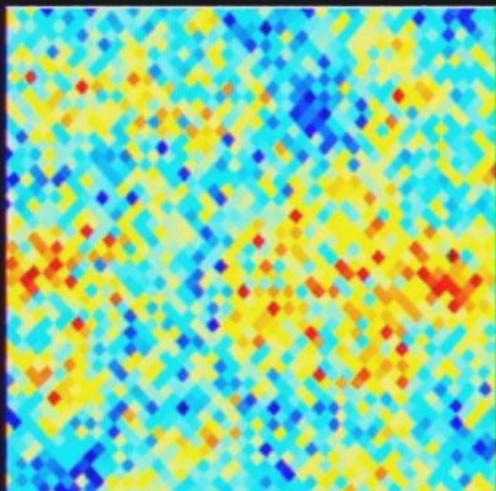
$$\delta T(\theta, \phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi)$$

If the anisotropy is a Gaussian random field

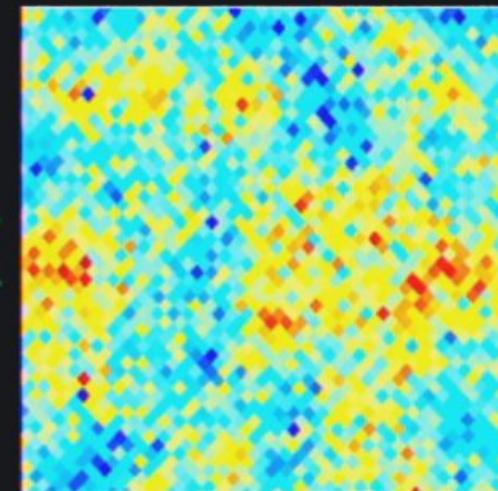
real and imaginary parts of each a_{lm} independent normal deviates, not correlated)

All the statistical information is contained in the angular power spectrum.

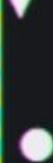
0.06% of map



5 deg



1 deg



$$C_l = \frac{1}{2\ell + 1} \sum_m |a_{lm}|^2$$

ANGULAR POWER SPECTRUM

A simplified CMB likelihood function

$$-2 \ln \mathcal{L} = \sum_{\ell} (2\ell + 1) \left[\ln \left(\frac{C_{\ell}^{\text{th}} + N_{\ell}}{\hat{C}_{\ell}} \right) + \frac{\hat{C}_{\ell}}{C_{\ell}^{\text{th}} + N_{\ell}} - 1 \right]$$

The diagram illustrates the components of the likelihood function. Three boxes are shown: 'theory' with an arrow pointing to C_{ℓ}^{th} , 'noise bias' with an arrow pointing to N_{ℓ} , and 'estimator for sky Cls' with an arrow pointing to \hat{C}_{ℓ} .

- Logarithmic at large scales; more likely to scatter low.
- Approaches Gaussian at small scales.

Cosmic variance: even ideal experiment can only measure $(2\ell + 1)$ modes

A simplified CMB likelihood function

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The diagram illustrates the components of the likelihood function. It shows three boxes with labels: 'theory' (green), 'noise bias' (blue), and 'estimator for sky Cls' (orange). Arrows point from each box to its corresponding term in the equation:

- An arrow from the 'theory' box points to the term $\ln \left(\frac{C_{\ell}^{\text{th}} + N_{\ell}}{\hat{C}_{\ell}} \right)$.
- An arrow from the 'noise bias' box points to the term $\frac{\hat{C}_{\ell}}{C_{\ell}^{\text{th}} + N_{\ell}}$.
- An arrow from the 'estimator for sky Cls' box points to the term \hat{C}_{ℓ} .

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Radical data compression

Time-ordered data (e.g. WMAP 5 years 60-100 GB)



mostly experimental
characteristics

map (12-50 million pixels)



physically motivated statistics

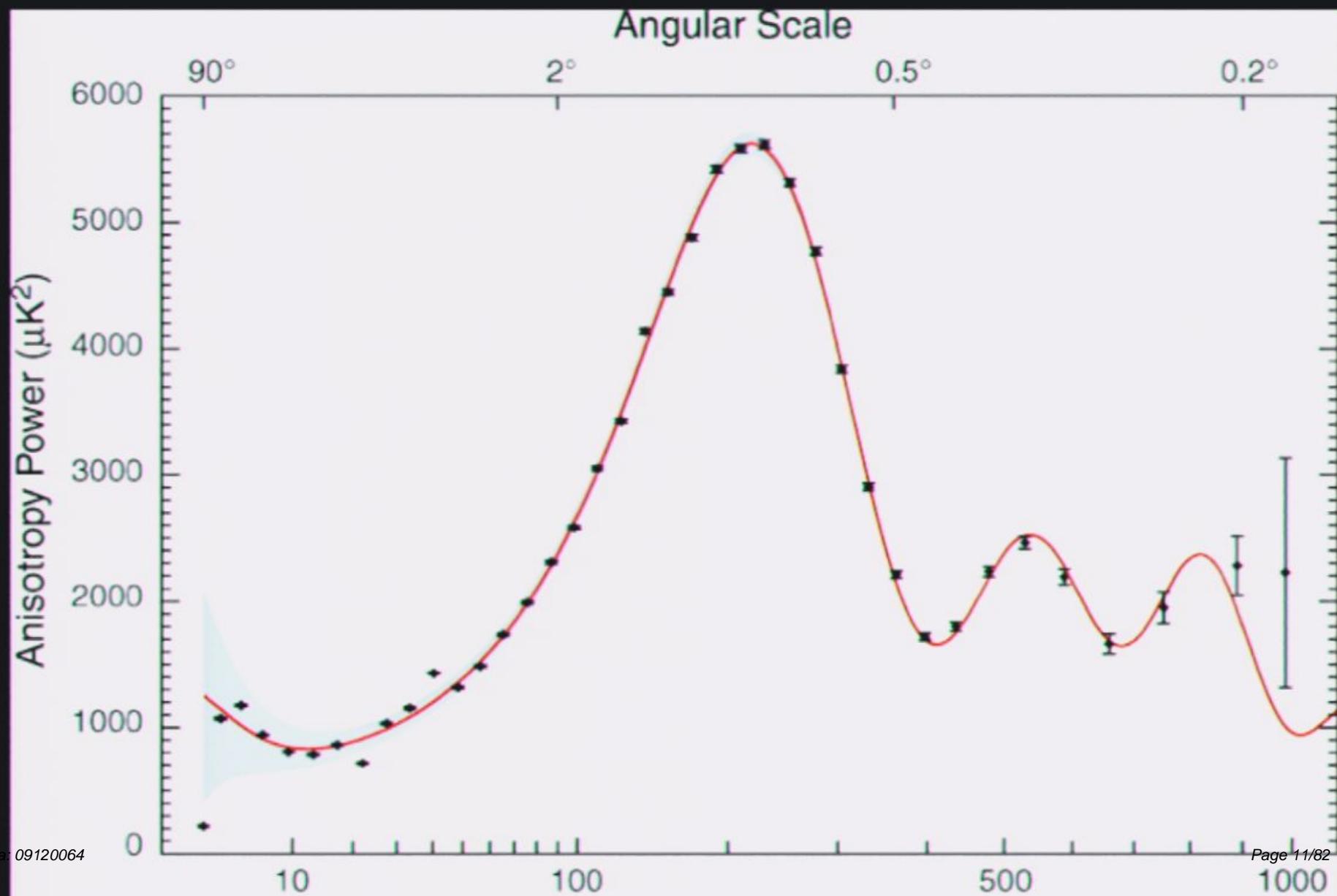
angular power spectrum - $O(1000)$ to $O(10000)$ numbers



experiment, physics, statistics

model - $O(10)$ parameters

WMAP temperature power spectrum



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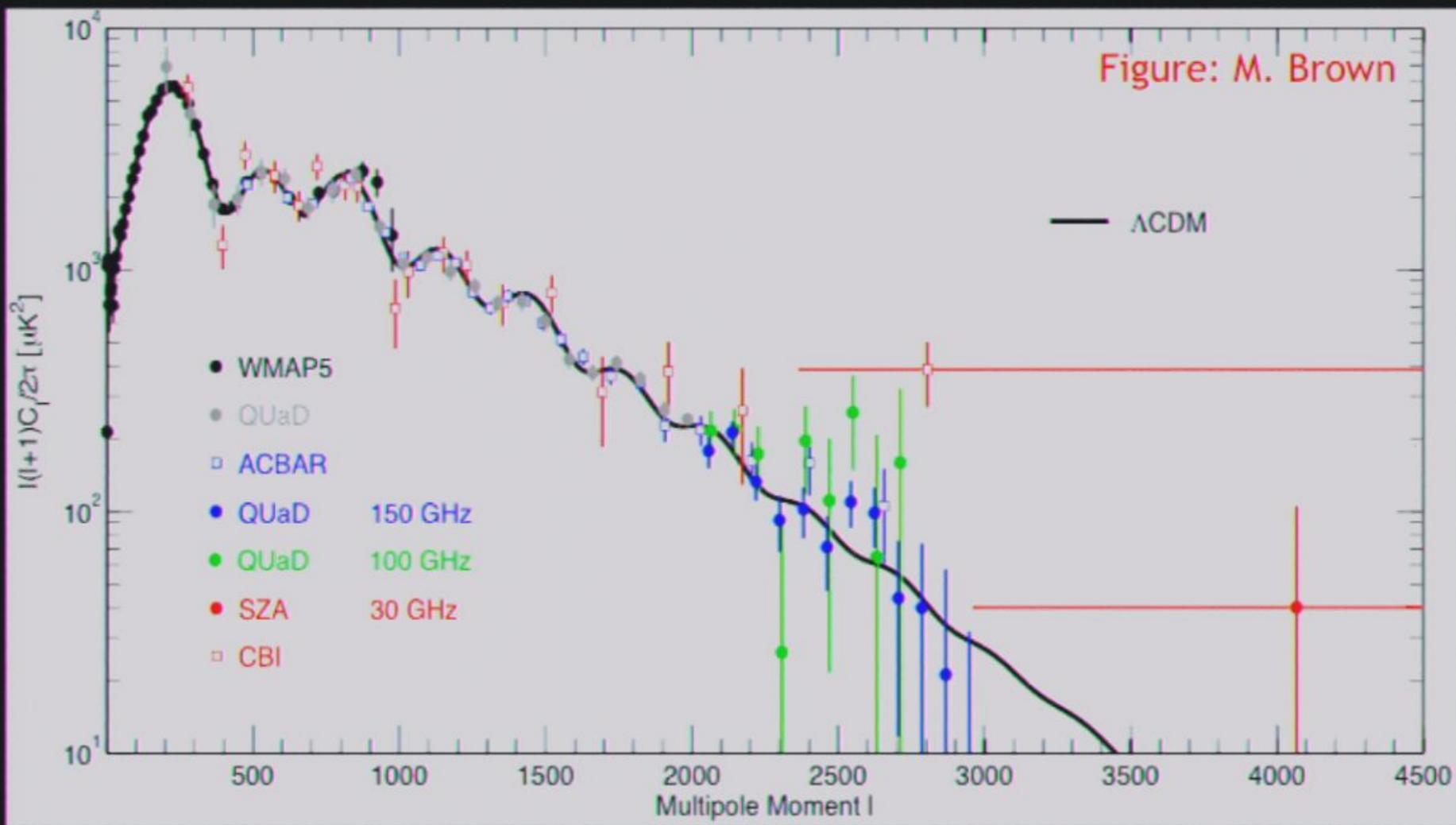
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State of the art: temperature

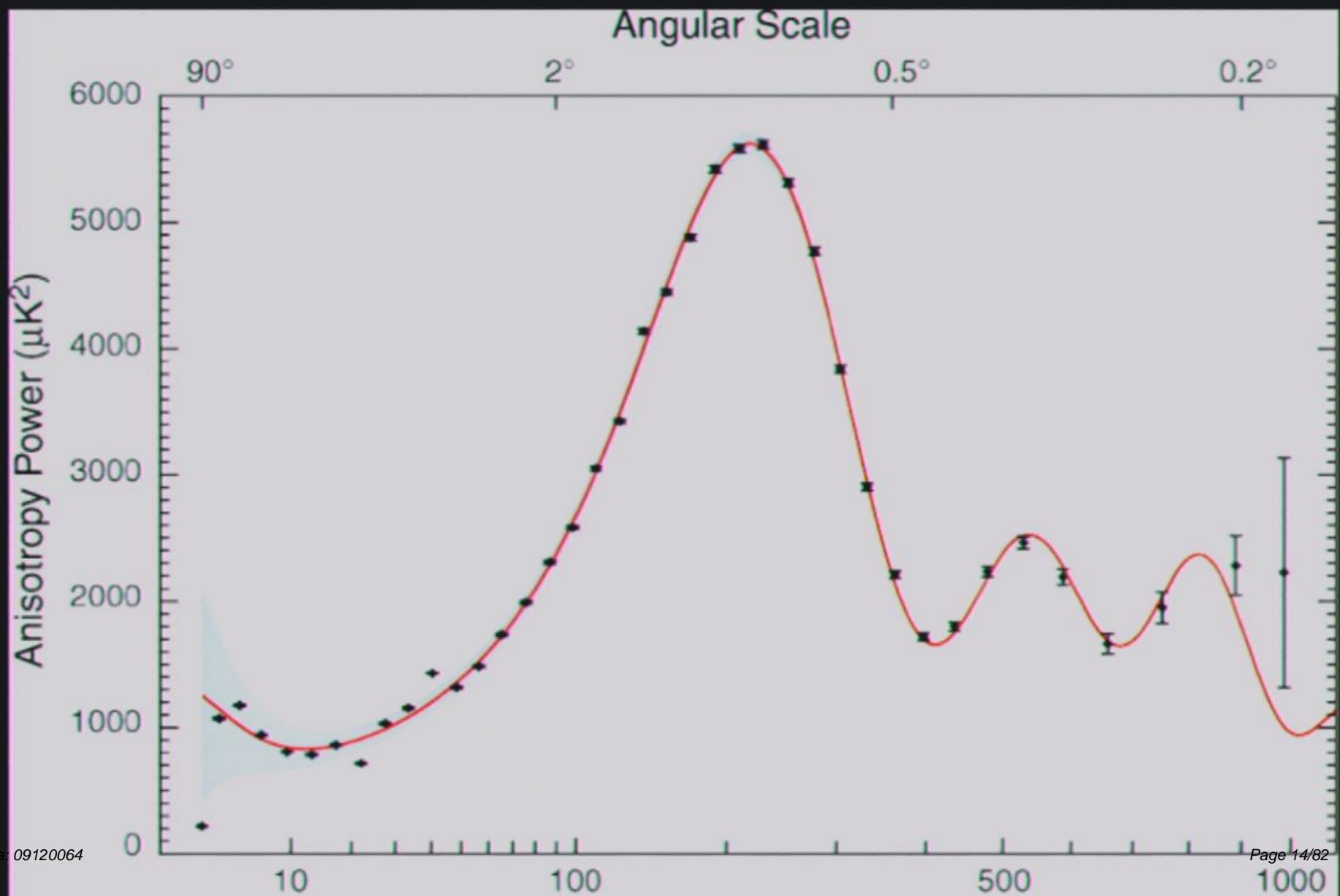


► Sachs-Wolfe plateau and the late time Integrated Sachs-Wolfe effect

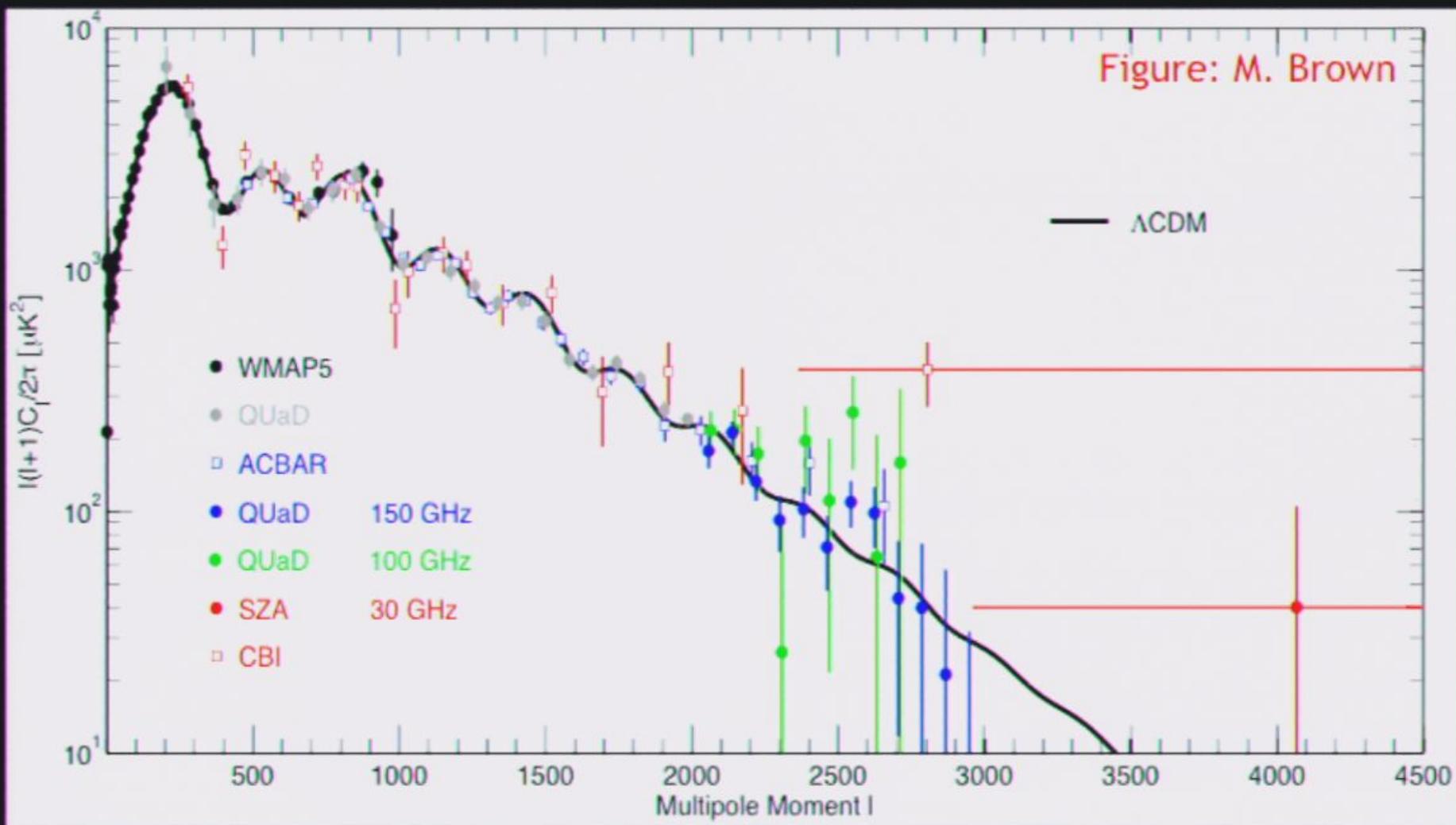
► Acoustic peaks at “adiabatic” locations

► Damping tail and photon diffusion

WMAP temperature power spectrum



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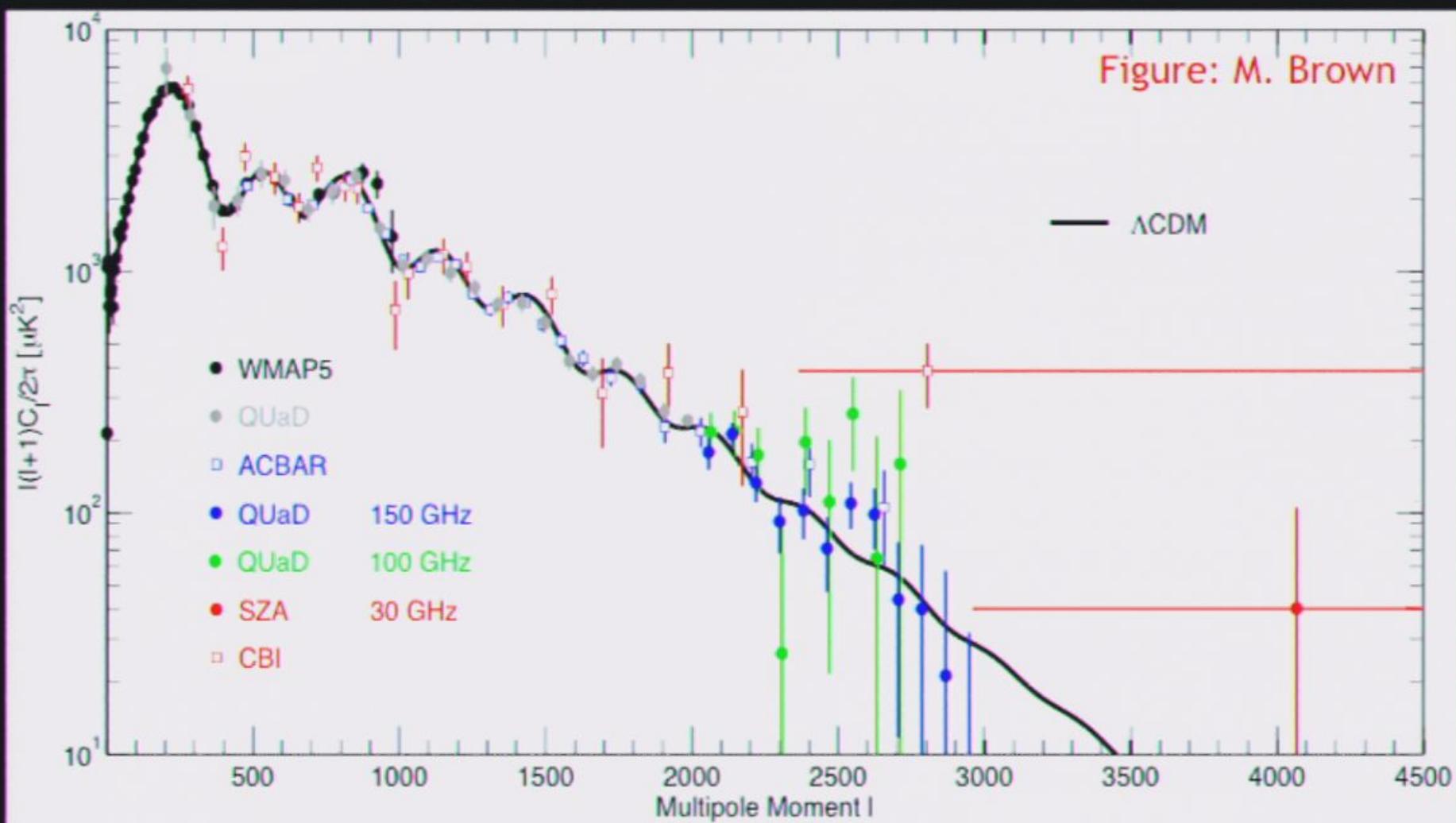


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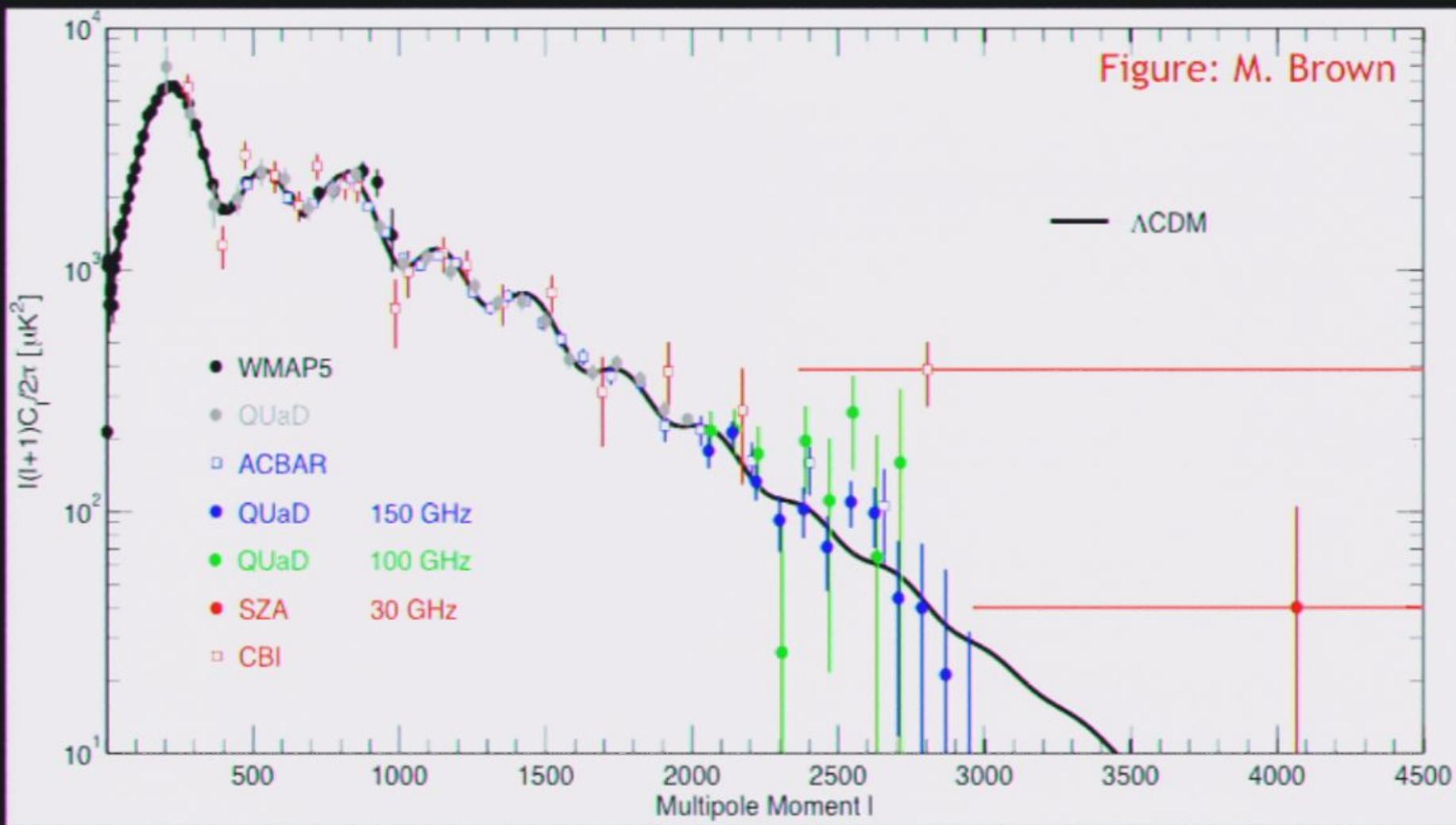


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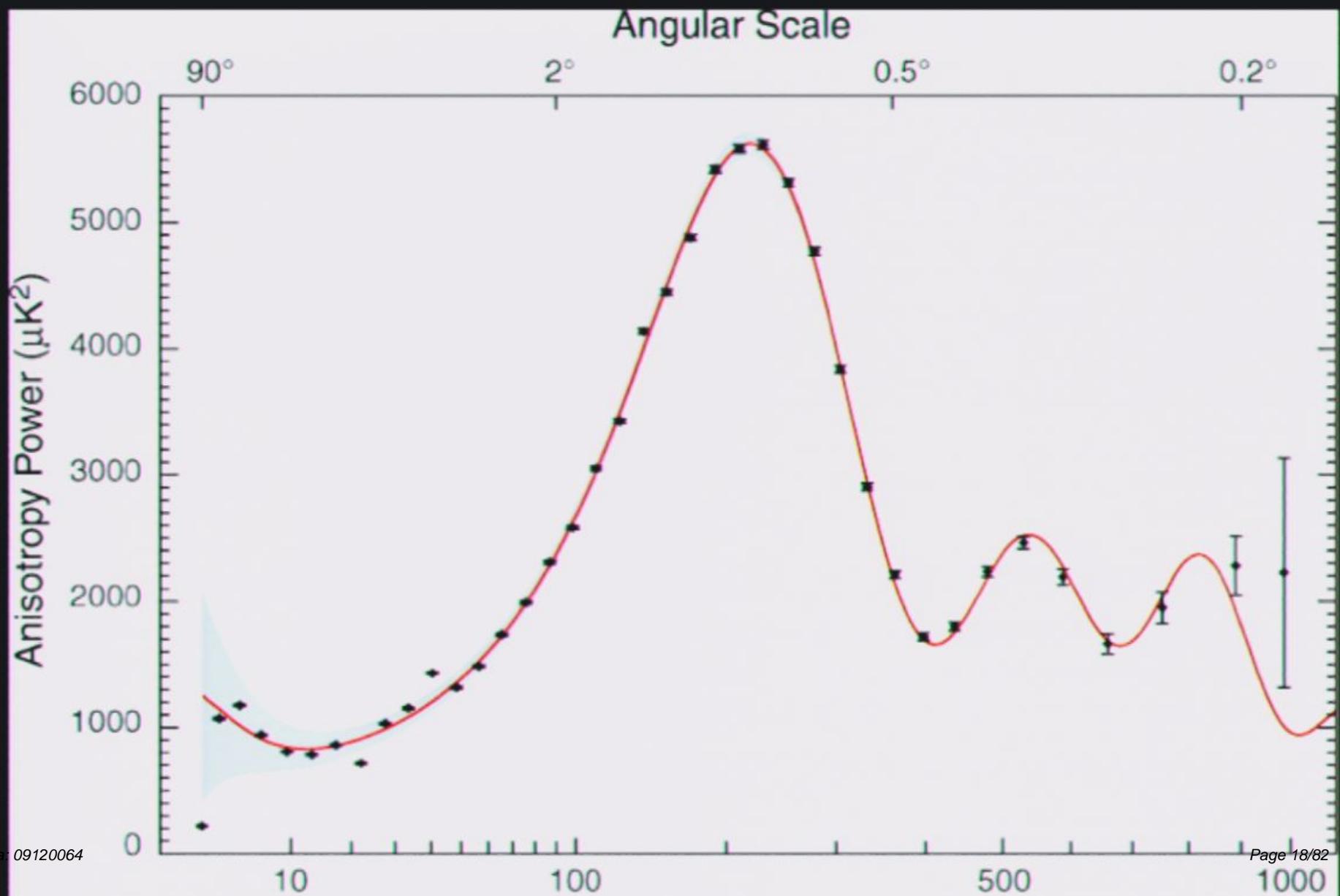


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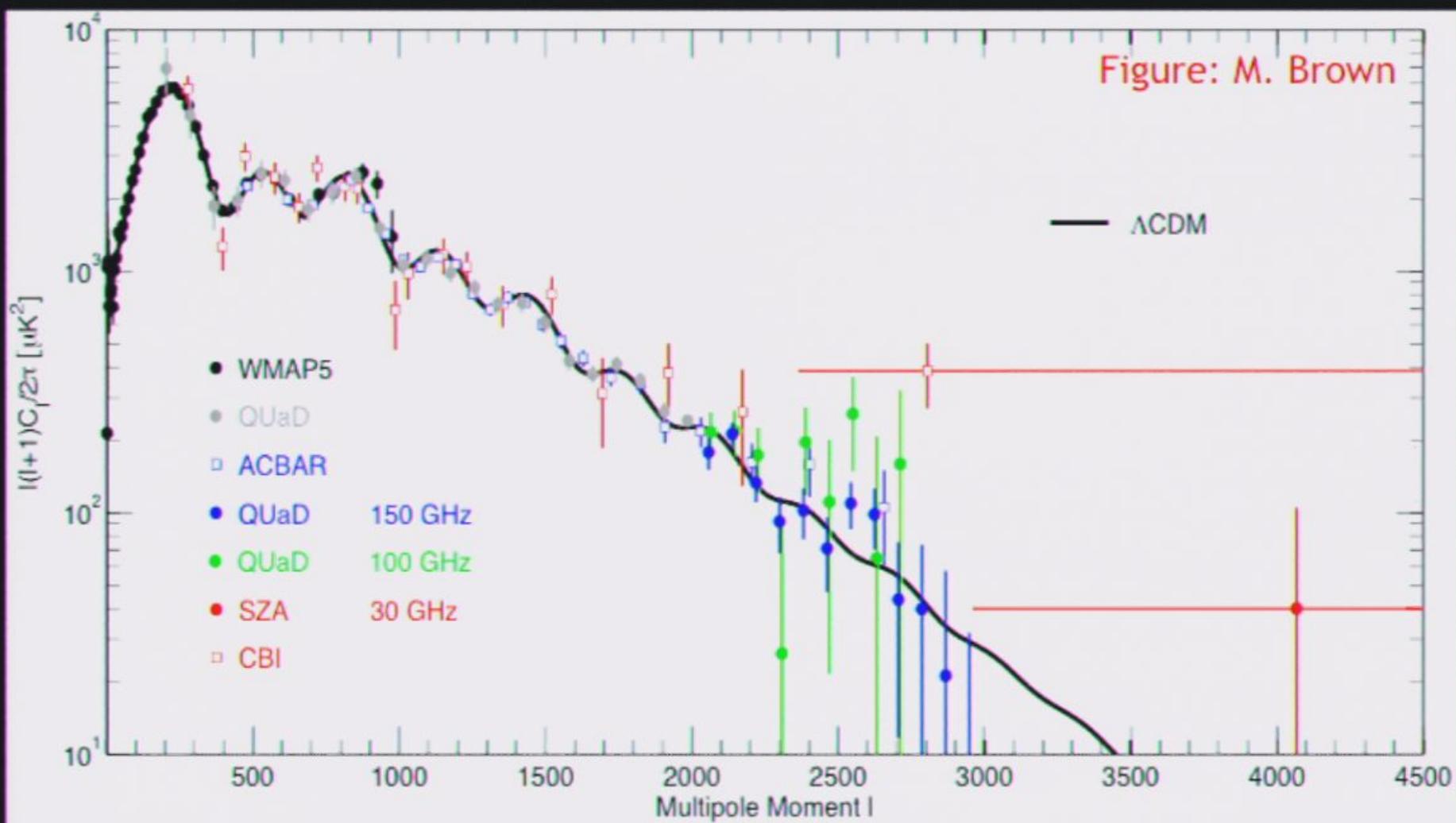
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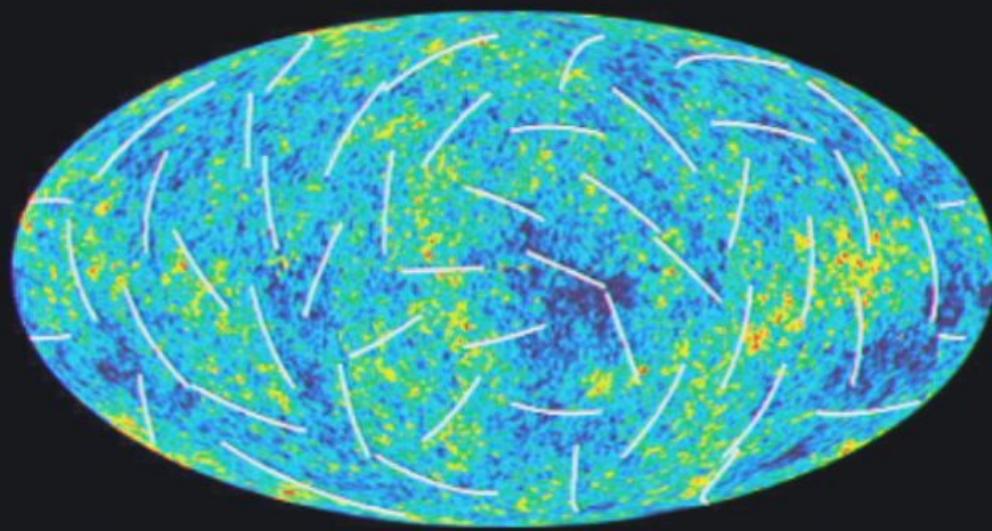


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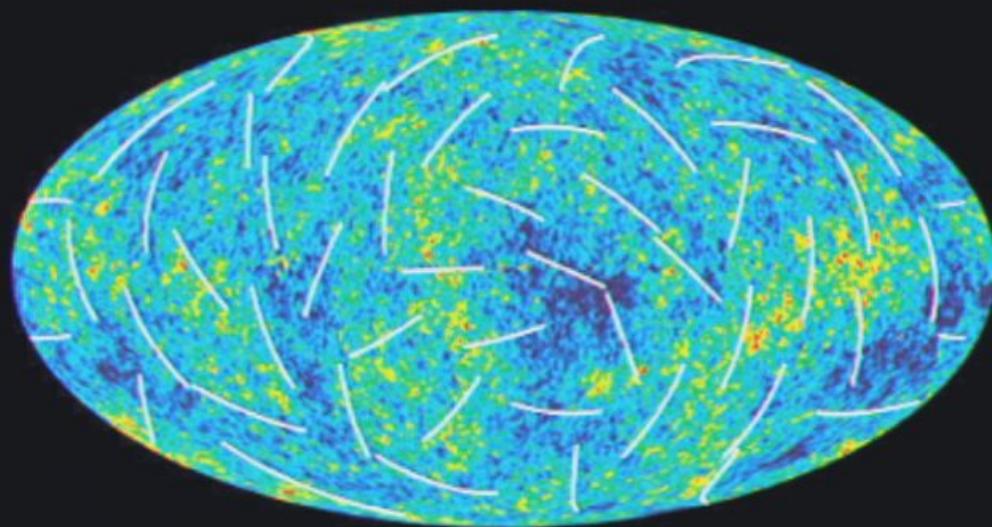
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History of CMB polarization measurements



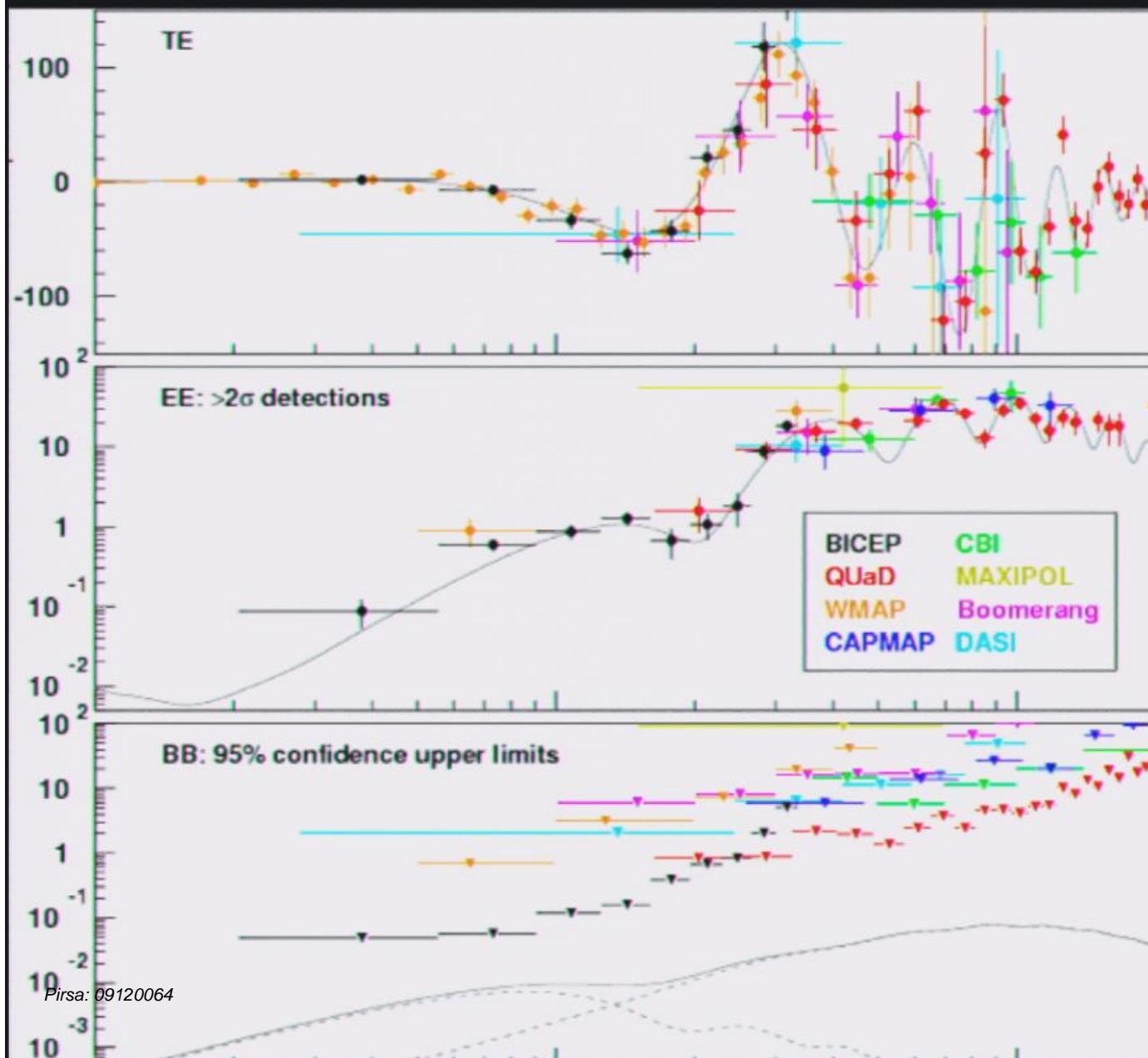
E-mode first detected by DASI in 2002

History of CMB polarization measurements



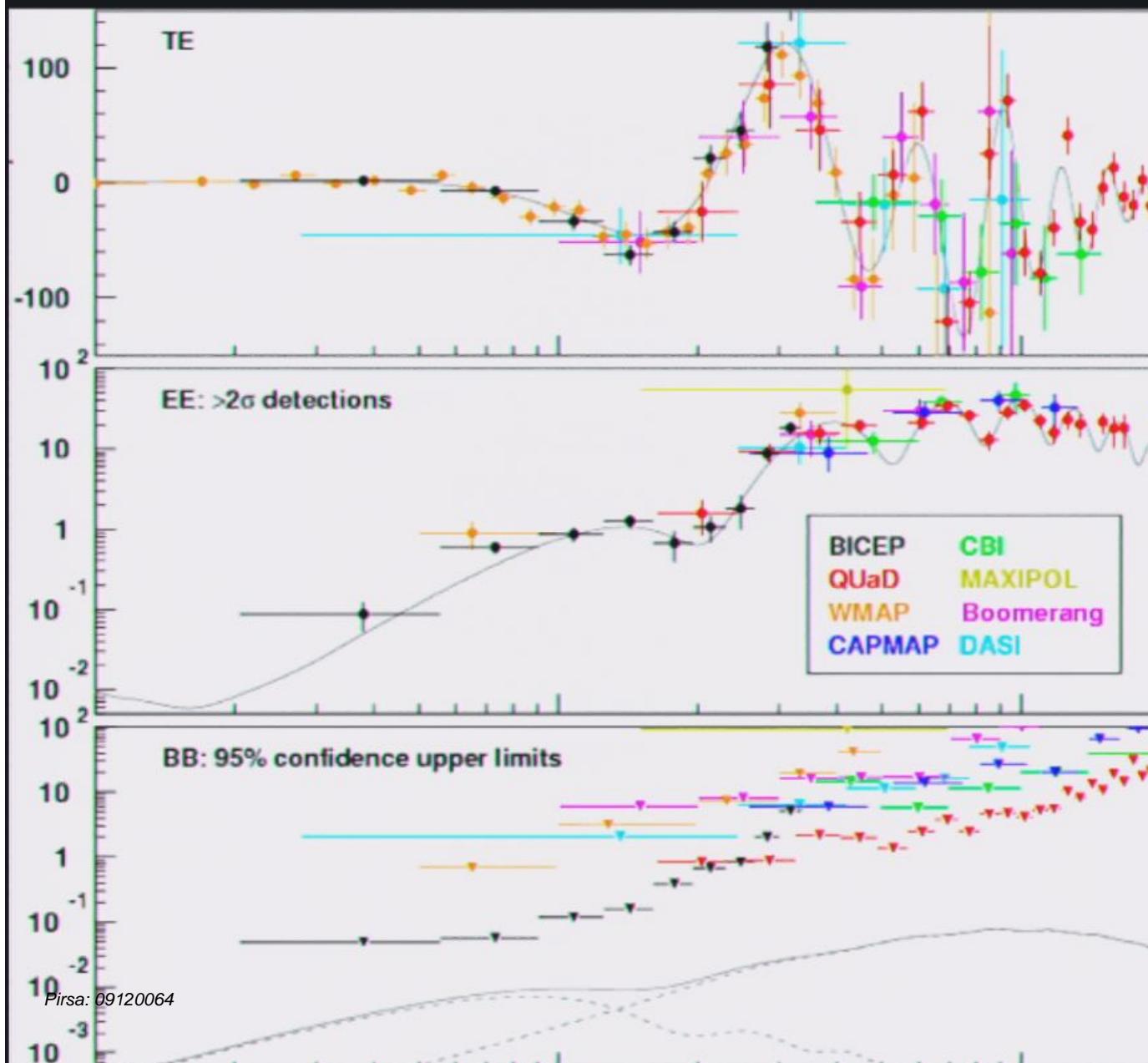
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State of the art: polarization



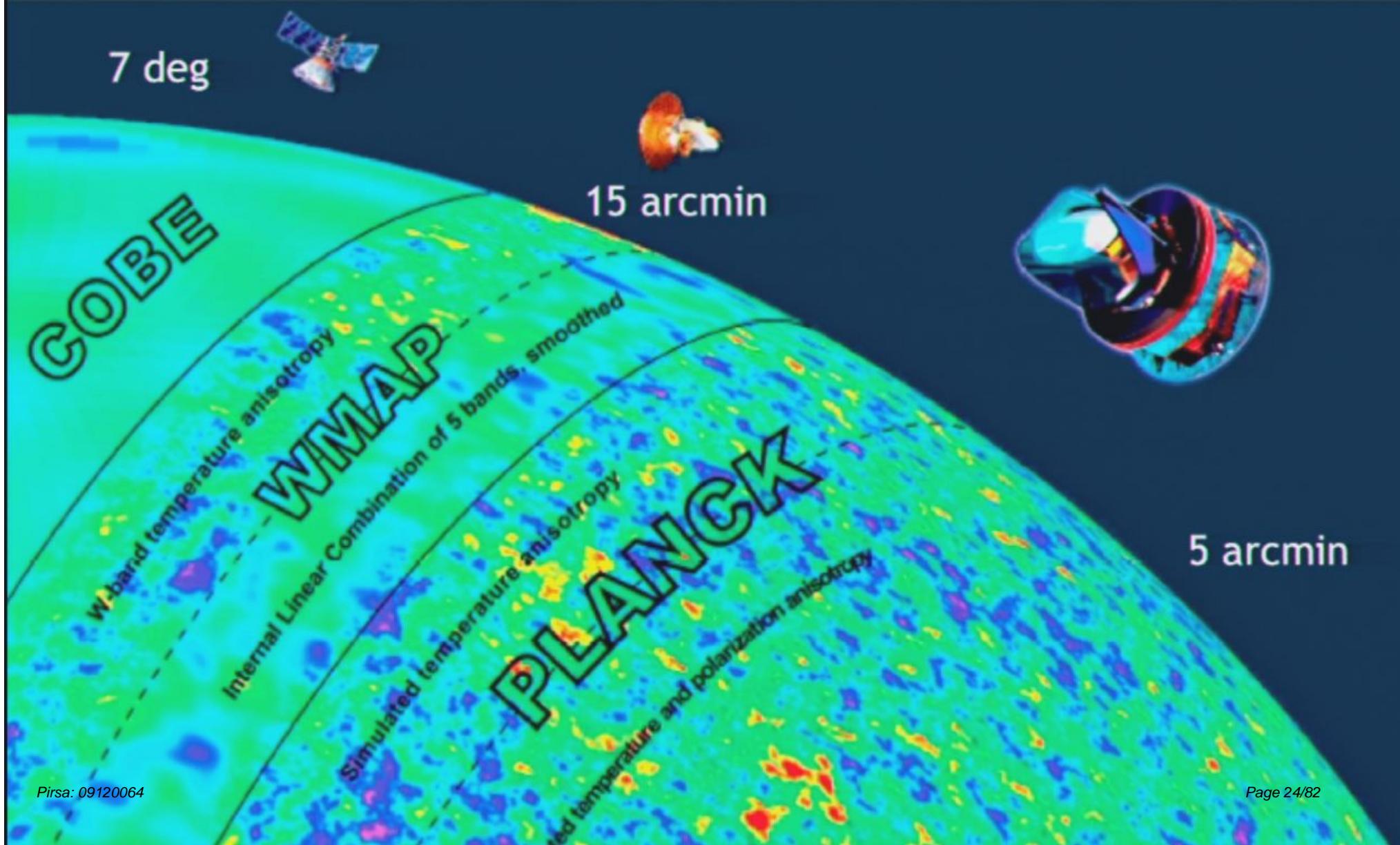
- ▶ Acoustic peaks at “adiabatic” locations
- ▶ E-mode polarization and cross-correlation with T
- ▶ Large angle polarization from reionization
- ▶ BICEP limit from BB-alone: $T/S < 0.73$ (95% CI)

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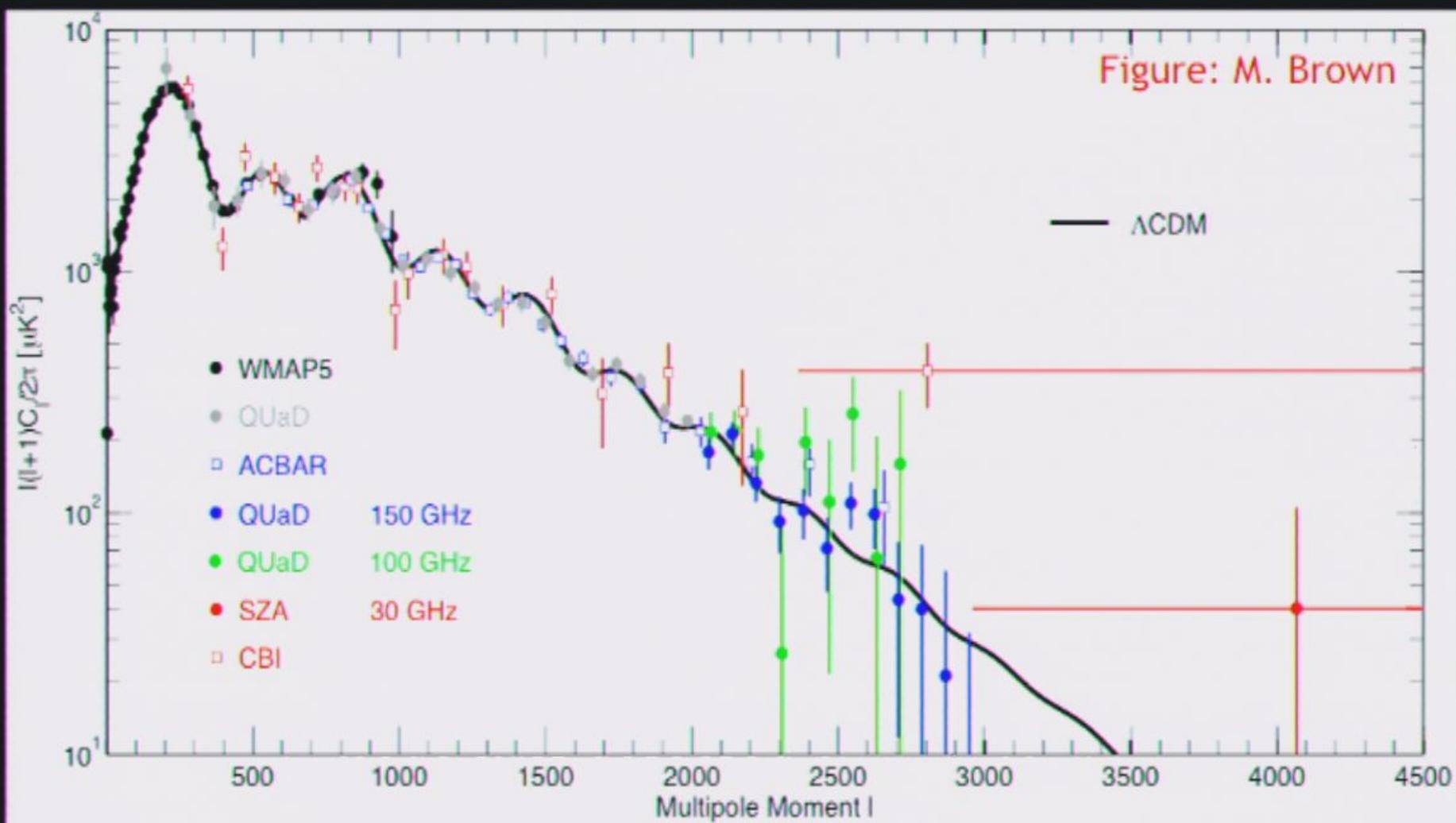


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Planck: *THE NEXT GENERATION*



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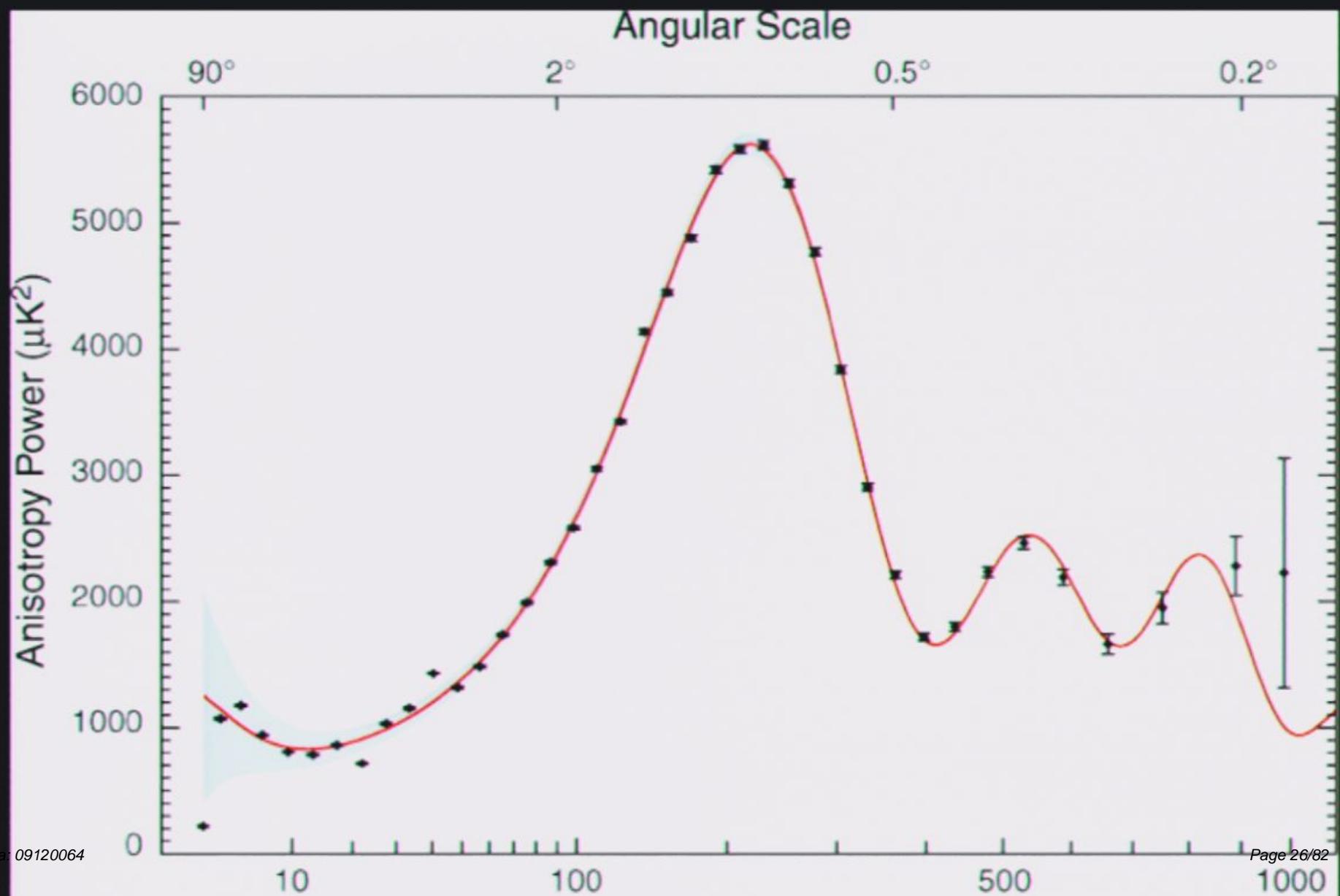


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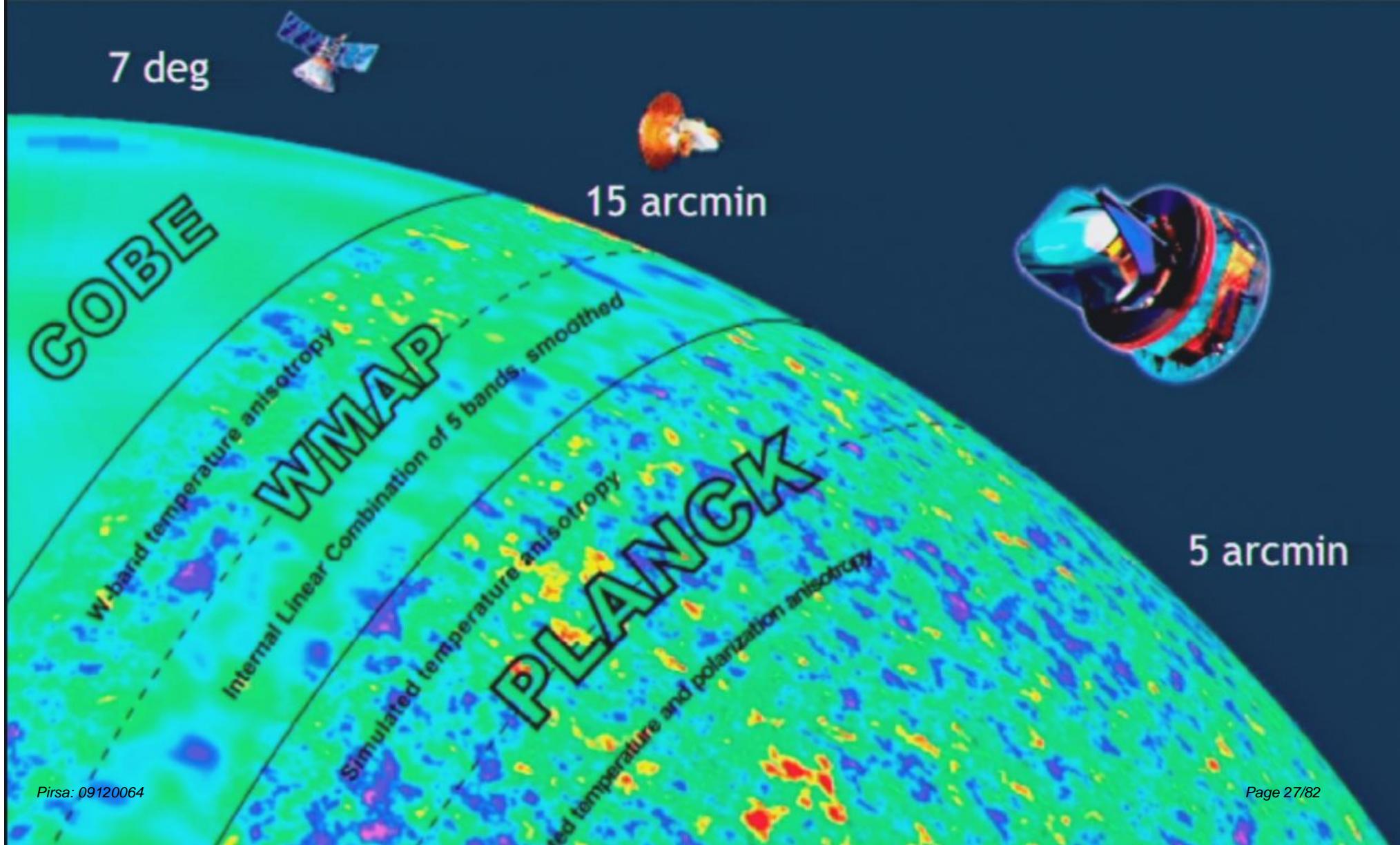
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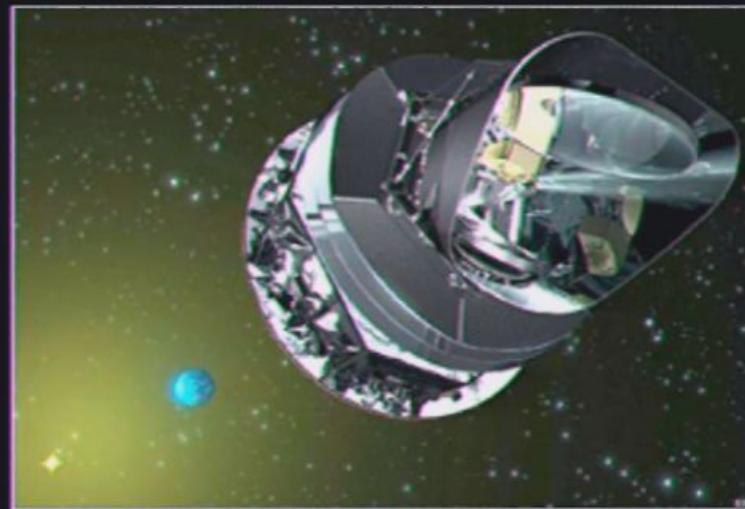
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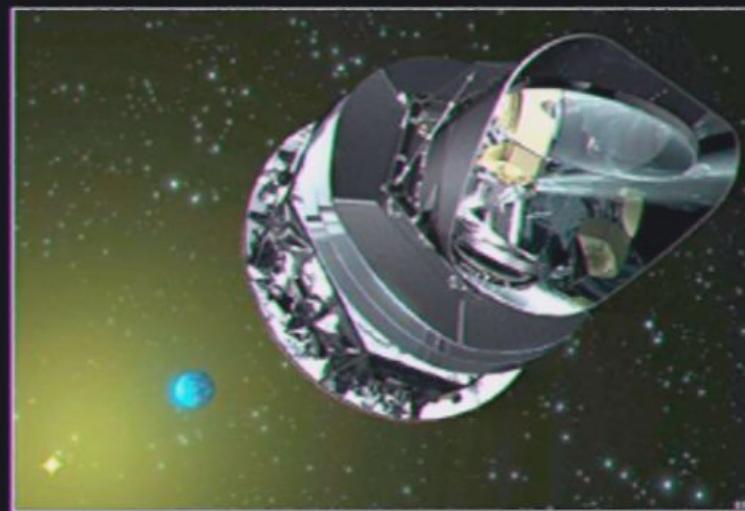
Planck



ESA

Extract essentially all information in primary CMB temperature anisotropy; big advance in polarization measurements.

Planck

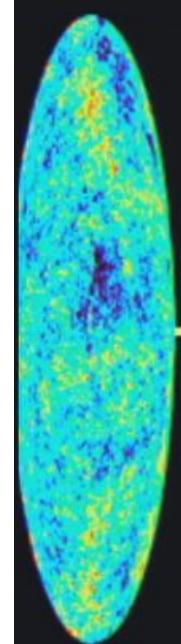


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Next frontier: secondary anisotropies

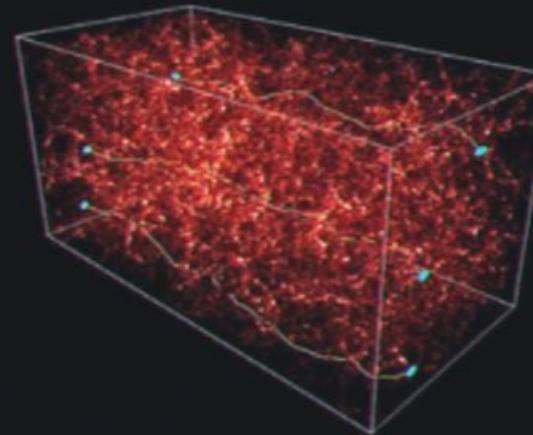
Use the CMB as a backlight to illuminate the growth of cosmological structure.



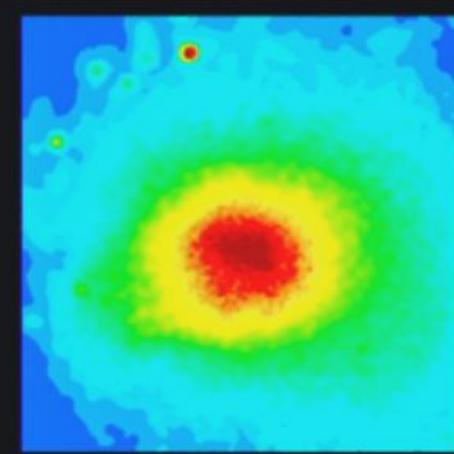
- First galaxies
- Universe is reionized
- Ostriker-Vishniac/kSZ



- weak lensing

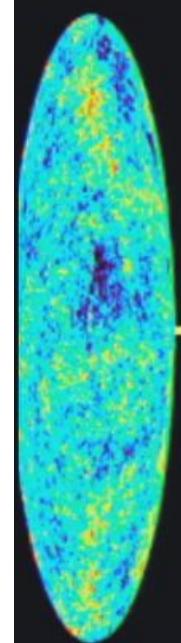


- Sunyaev-Zel'dovich (SZ) clusters
- Diffuse thermal SZ
- Kinetic SZ
- Rees-Sciama/ISW



Next frontier: secondary anisotropies

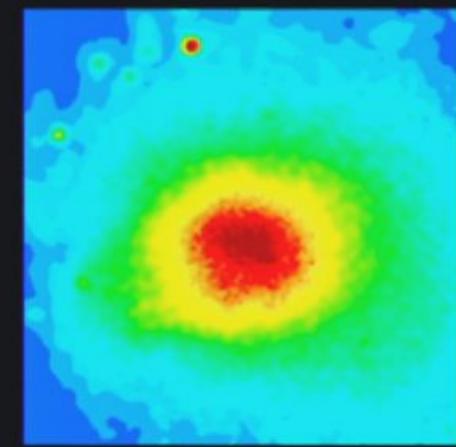
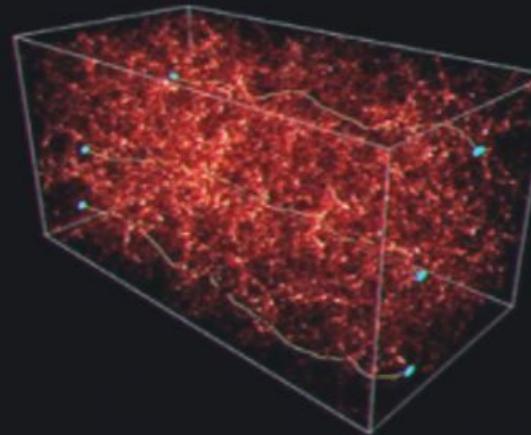
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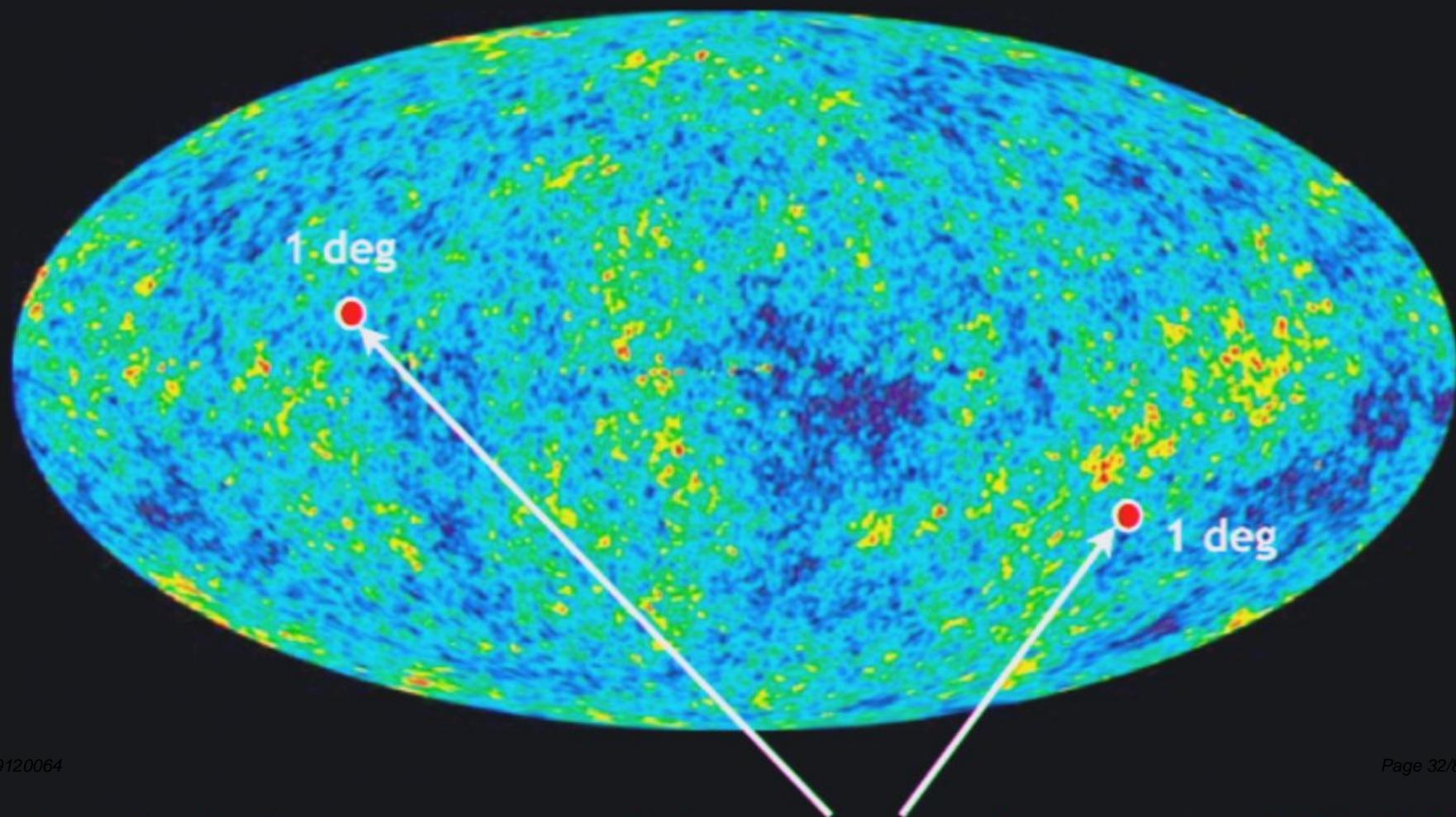
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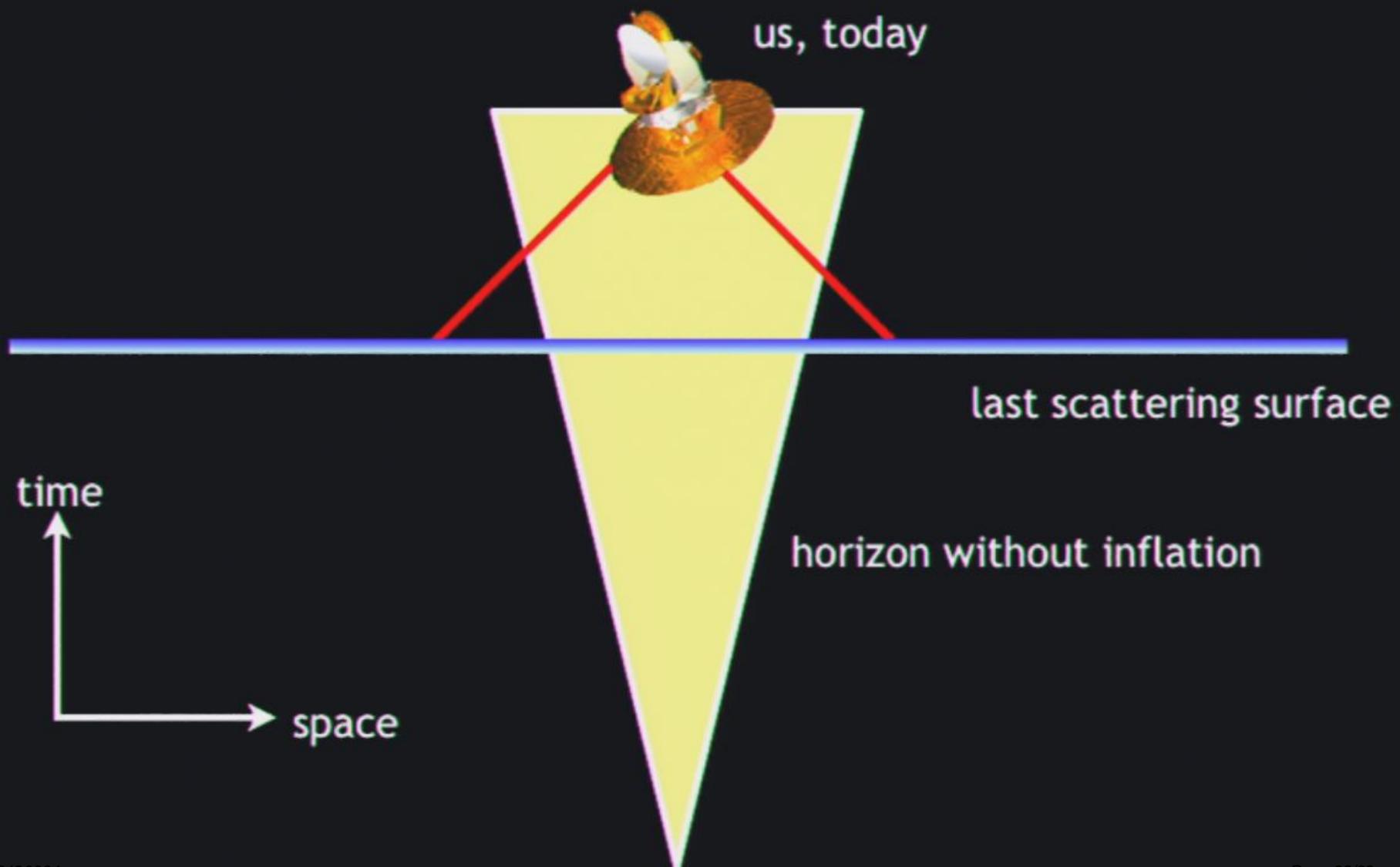


The Horizon Problem

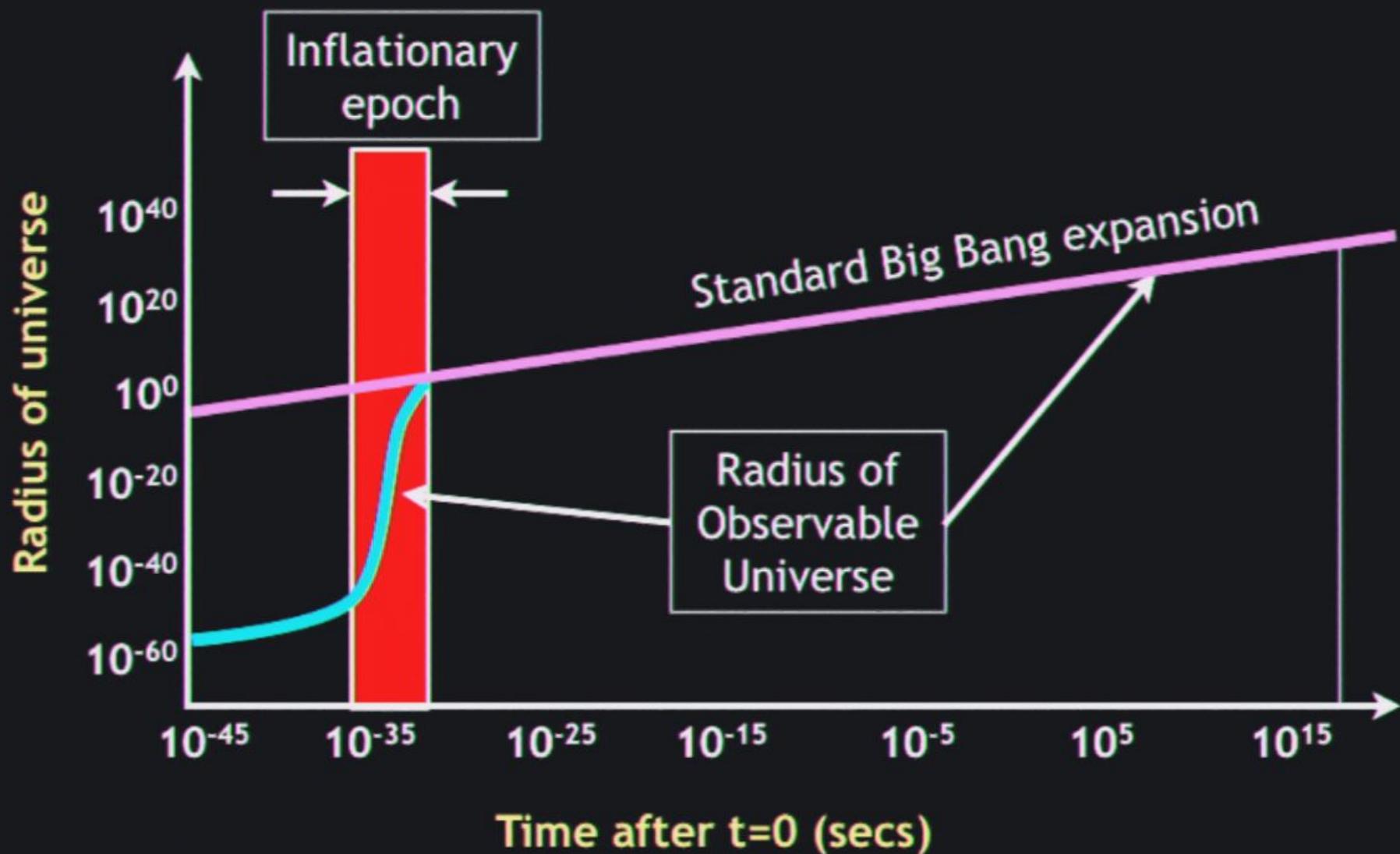
In Standard Big Bang Model, horizon scale at CMB release subtends ~ 1 deg
Regions separated by more than 1 deg could not have interacted previously



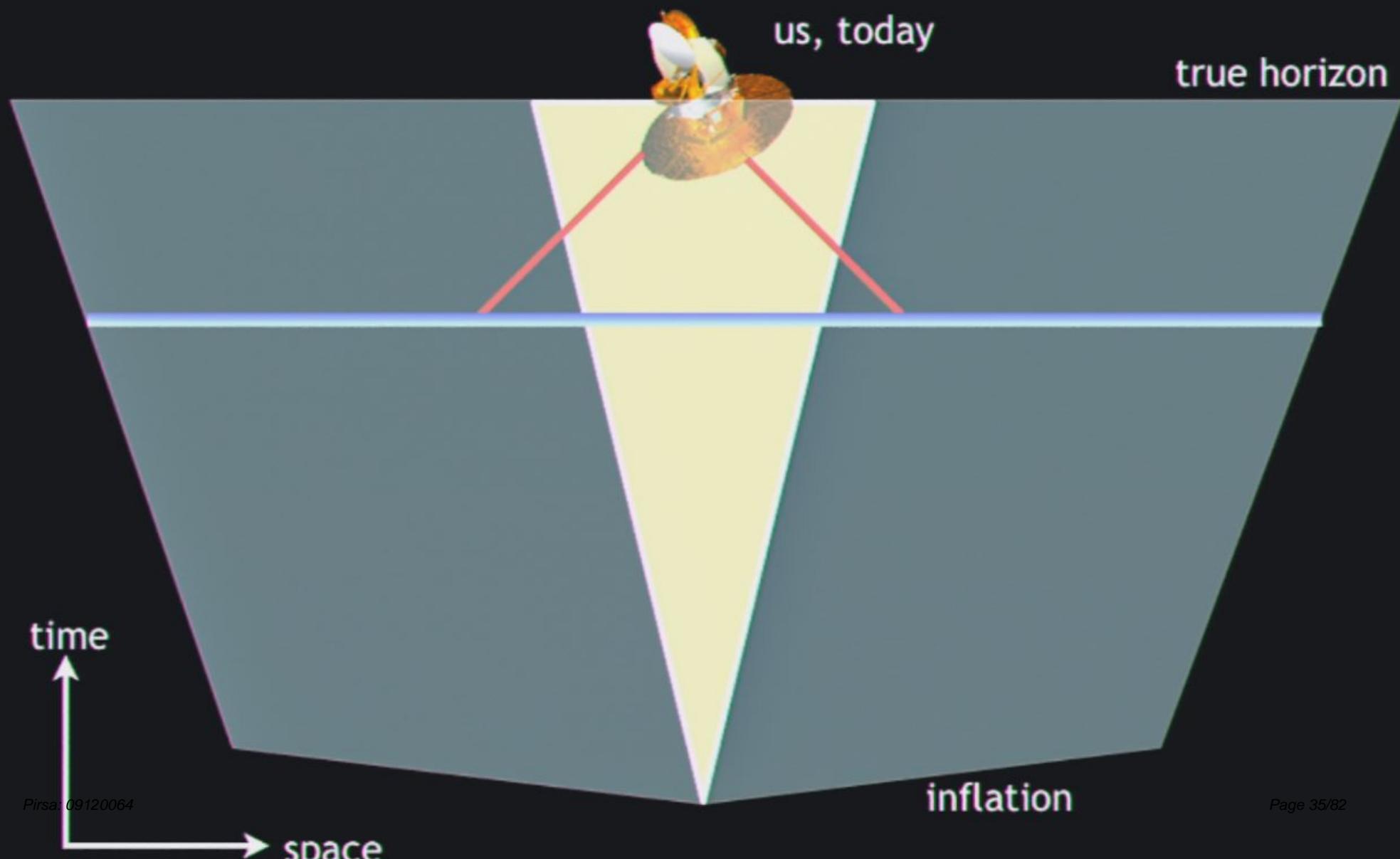
Horizon problem



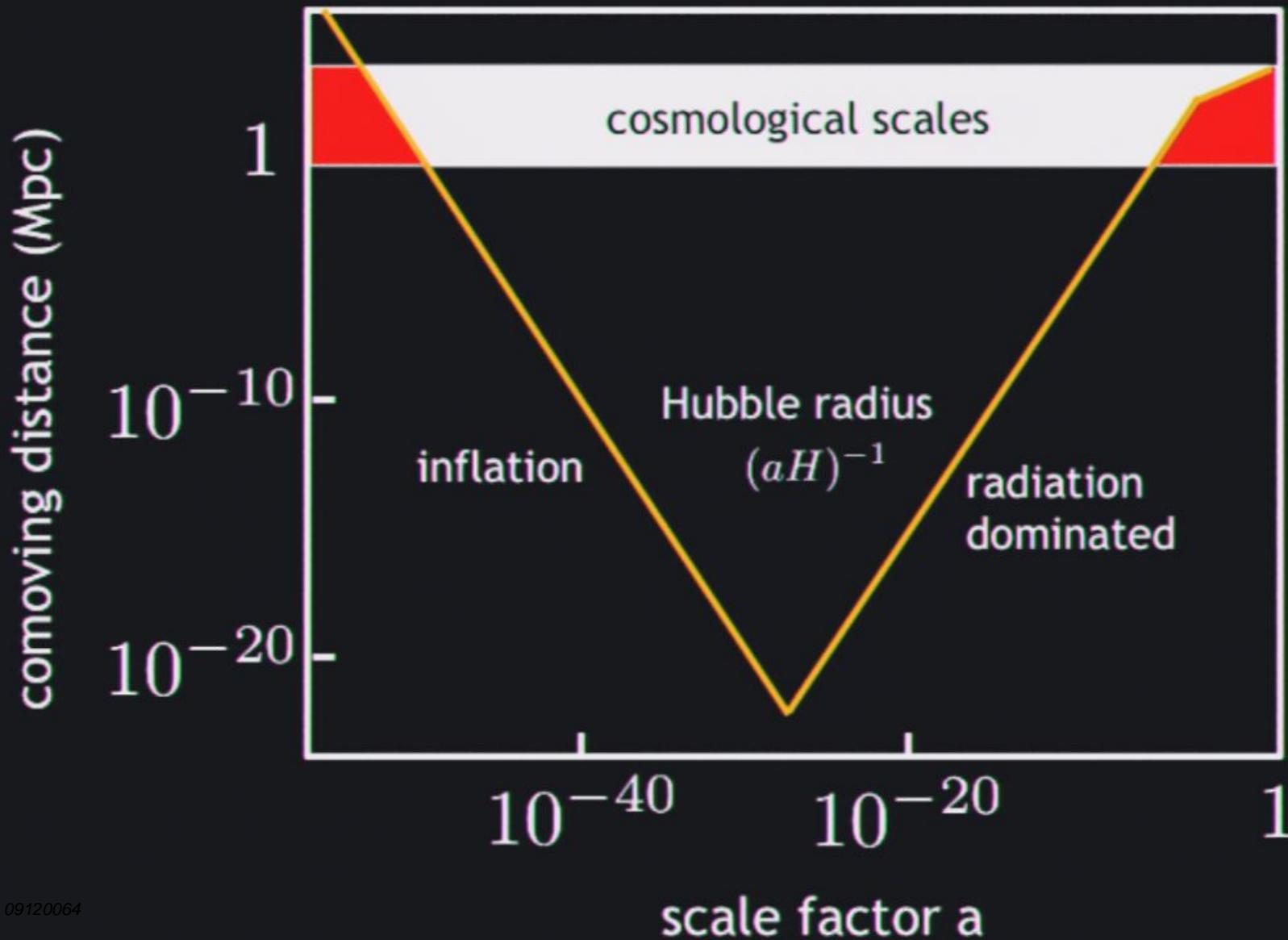
Inflation: accelerated super-expansion



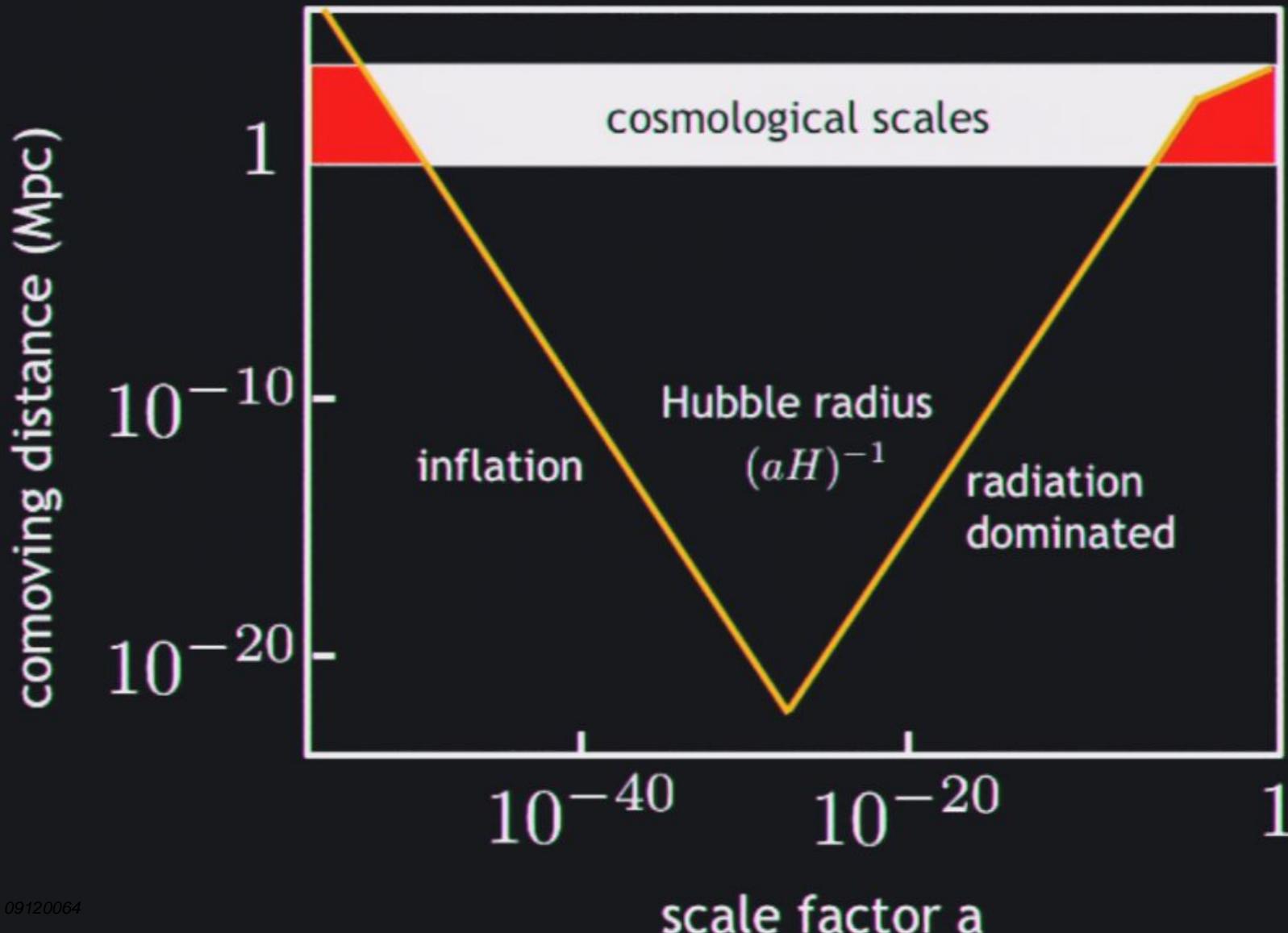
Inflationary resolution of horizon problem



Comoving Hubble radius during inflation



Comoving Hubble radius during inflation



Inflation

A period of accelerated expansion

$$ds^2 = -dt^2 + e^{2Ht}dx^2 \quad H \simeq \text{const}$$

- Solves:
 - ▶ horizon problem
 - ▶ flatness problem
 - ▶ monopole problemi.e. explains why the Universe is so large, so flat, and so empty
- Predicts:
 - ▶ scalar fluctuations in the CMB temperature
 - ✓ nearly scale-invariant
 - ✓ approximately Gaussian (?)

Inflation

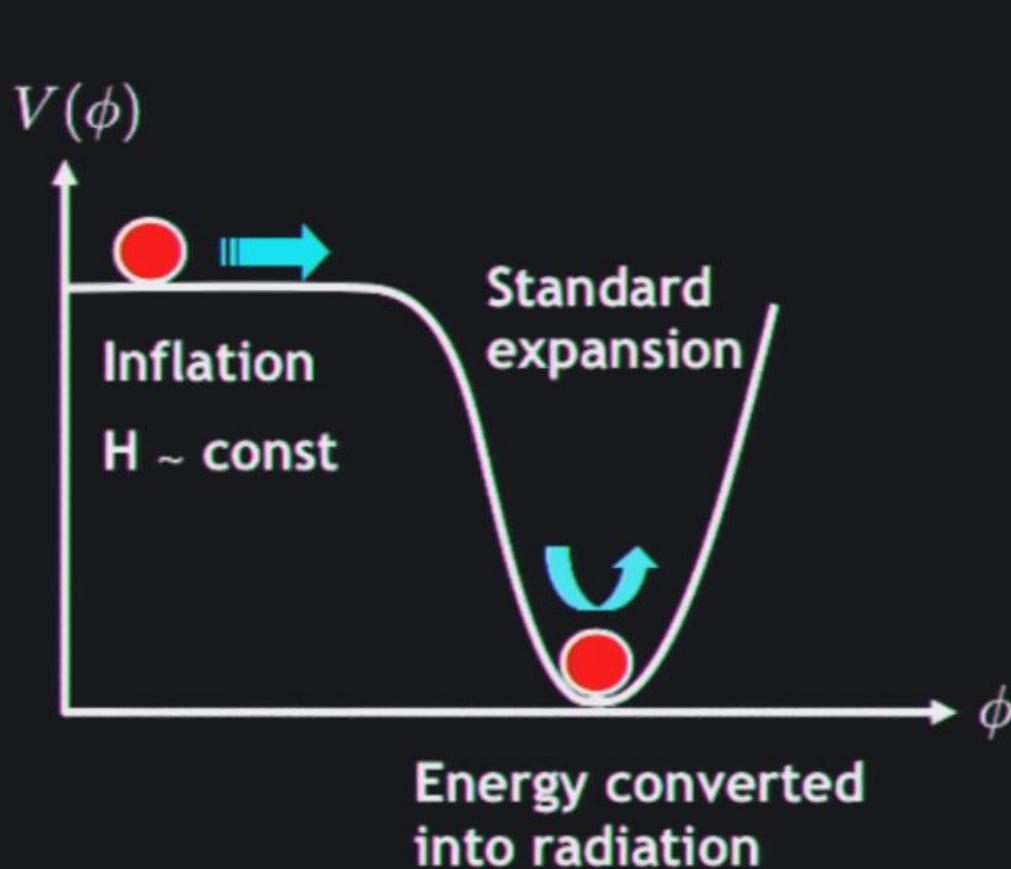
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Inflation

Implemented as a slowly-rolling scalar field evolving in a potential:



$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 \quad \xleftarrow{\text{expansion rate}}$$
$$= \frac{8\pi G}{3} \left[\frac{1}{2} \dot{\phi}^2 + V(\phi) \right]$$

$$\uparrow$$

density

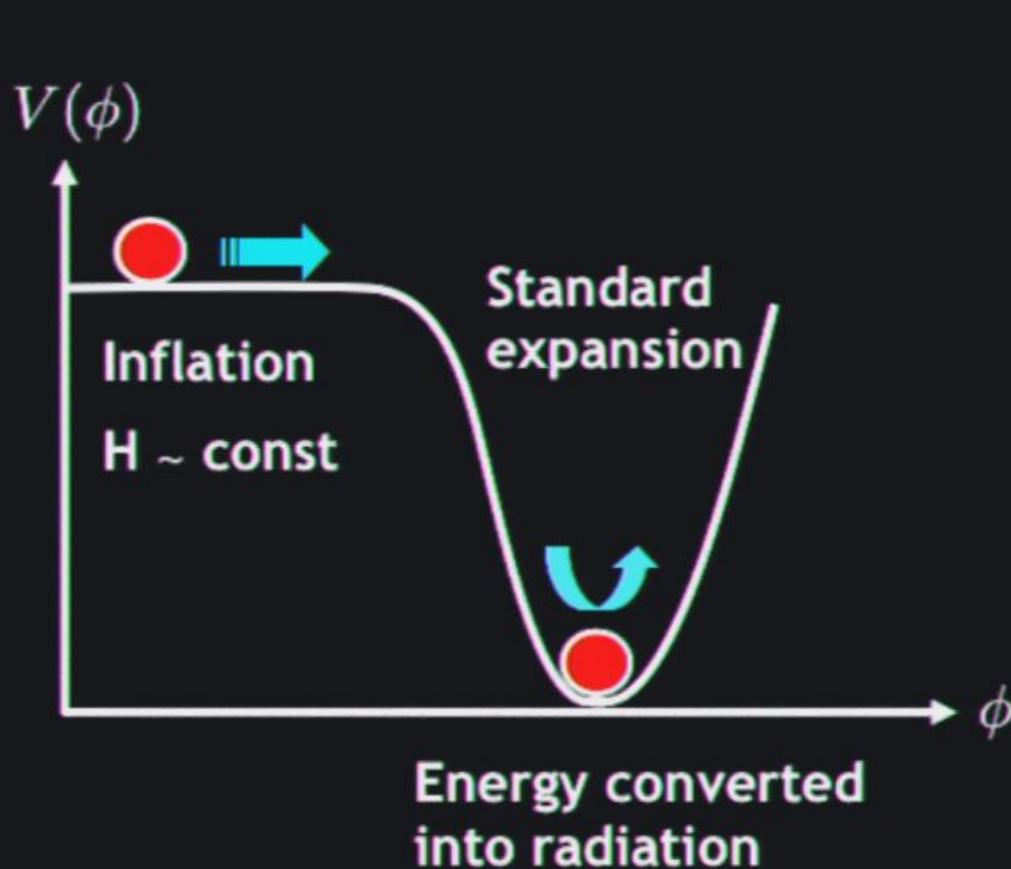
$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

$$\uparrow$$

friction

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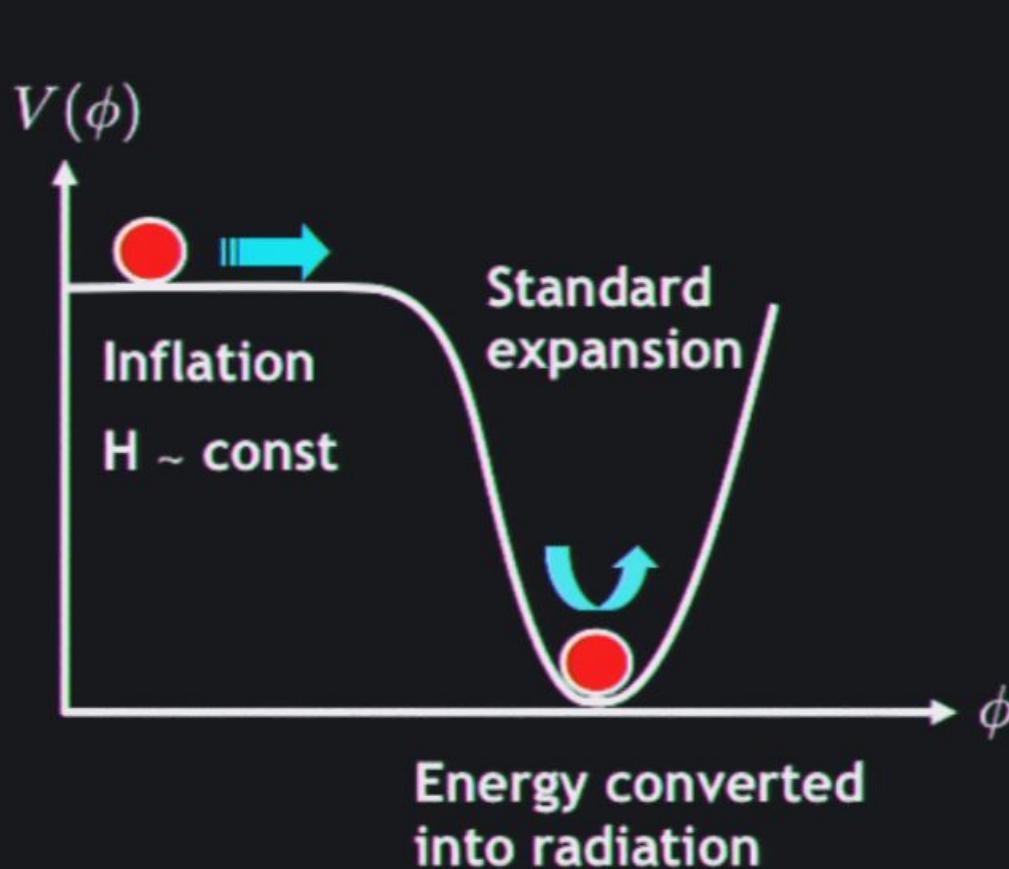
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friction

Perturbations from inflation

Cosmological perturbations arise from quantum fluctuations, evolve classically.



Perturbations from inflation

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Inflation

- Solves the flatness/horizon problems if the early universe inflates by factor $\sim 10^{30}$.
- Cosmological perturbations arise from quantum fluctuations, evolve classically.

$$P_\phi(k) \simeq \hbar \left(\frac{H}{2\pi} \right)^2$$
$$P_R \simeq \frac{\hbar}{4\pi^2} \left(\frac{H^4}{\dot{\phi}^2} \right)_{k=aH}^2 \quad \text{scalar}$$
$$P_h \simeq \frac{2\hbar}{\pi^2} \left(\frac{H}{m_{\text{Pl}}} \right)_{k=aH}^2 \quad \text{tensor}$$

Inflation

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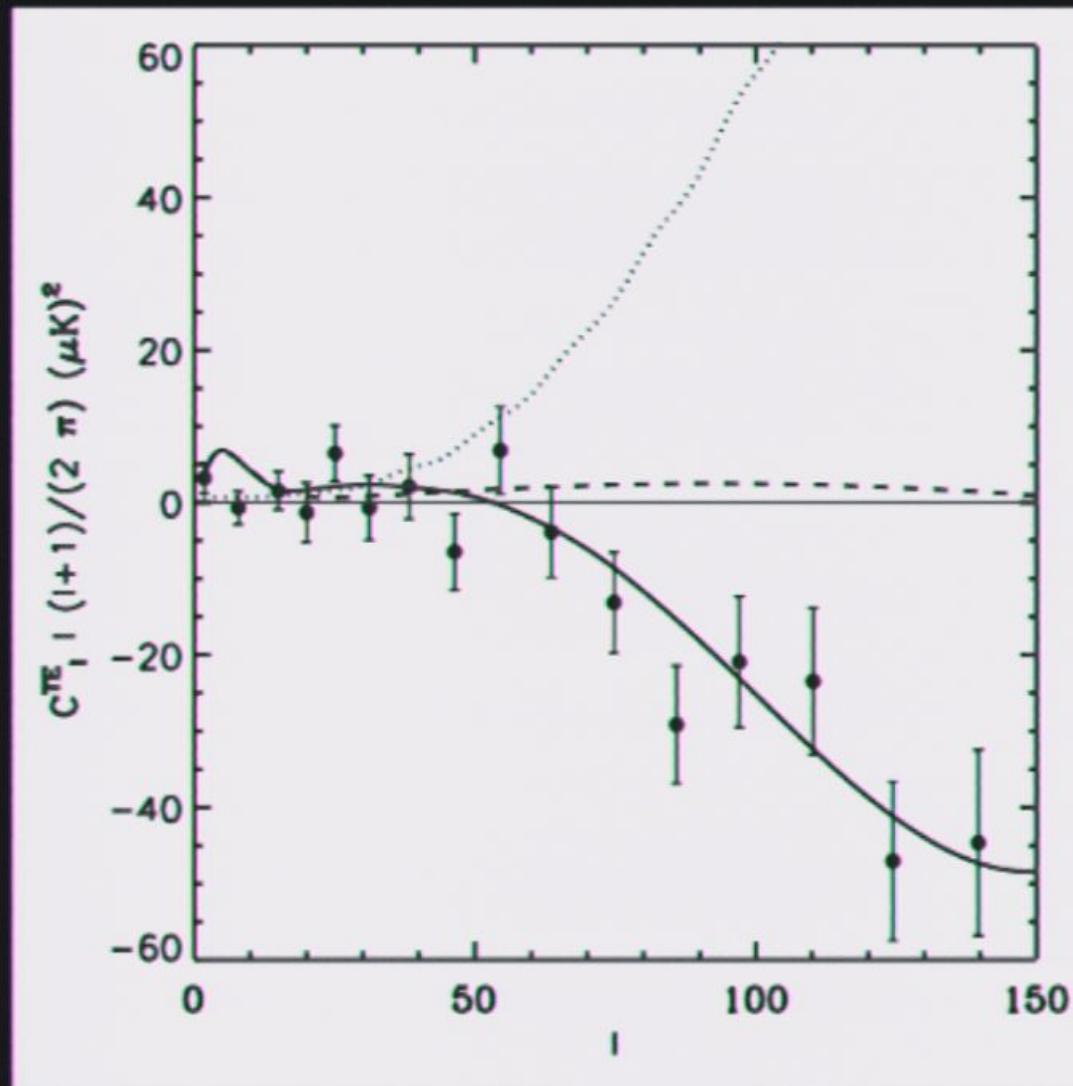
$$P_h \simeq \frac{2\hbar}{\pi^2} \left(\frac{H}{m_{\text{Pl}}} \right)_{k=aH}^2$$

tensor

- Don't know the dynamics of inflation: parameterize weakly scale-dependent functions with a few numbers to pin down observationally.

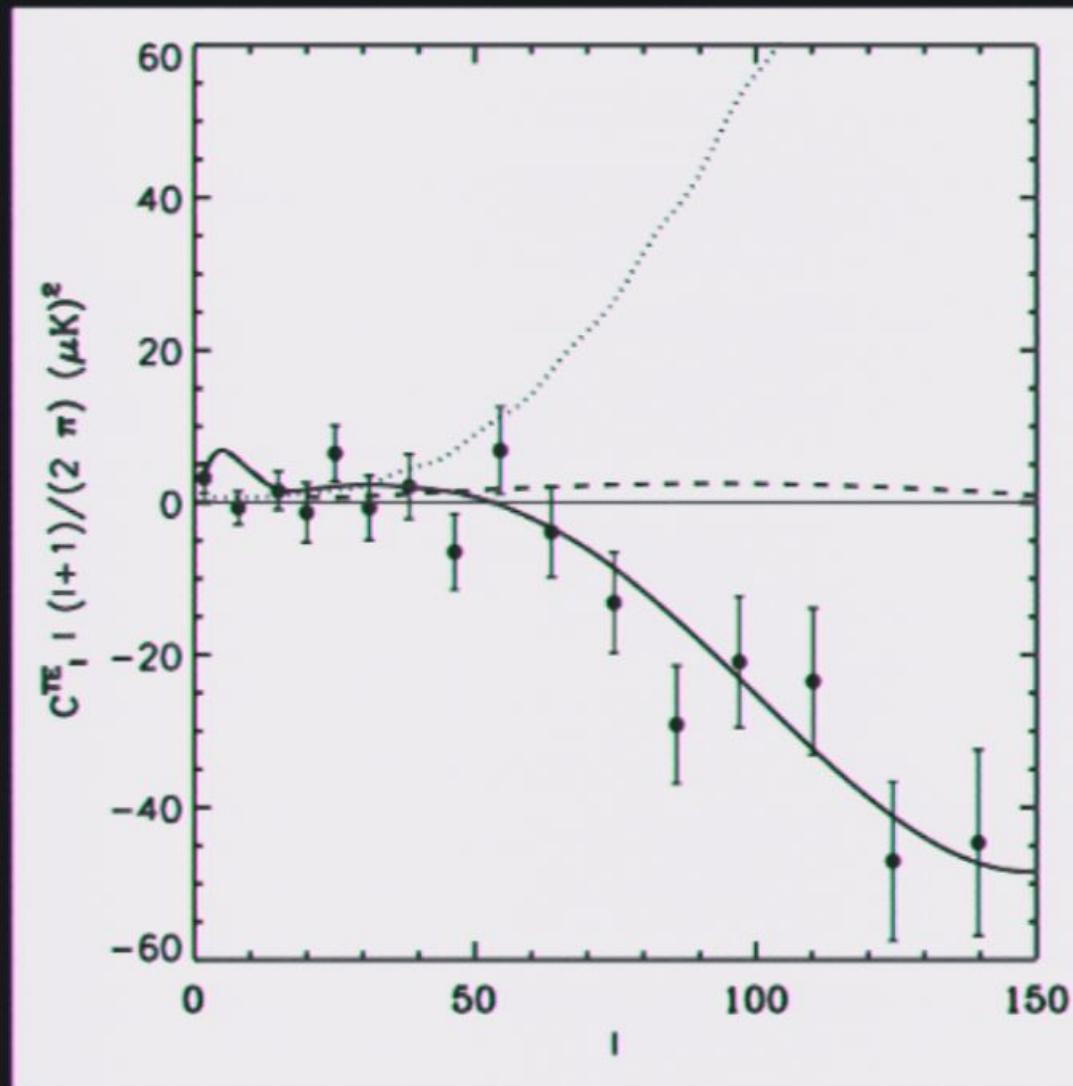
Slow roll inflation consistent with WMAP+

- ▶ Superhorizon, adiabatic fluctuations
 - T and E anticorrelated at superhorizon scales
- ▶ Flatness tested to 1%.
- ▶ Gaussianity tested to 0.1%.
- ▶ nearly scale-invariant fluctuations
 - red tilt indicated at $\sim 2.5 \sigma$



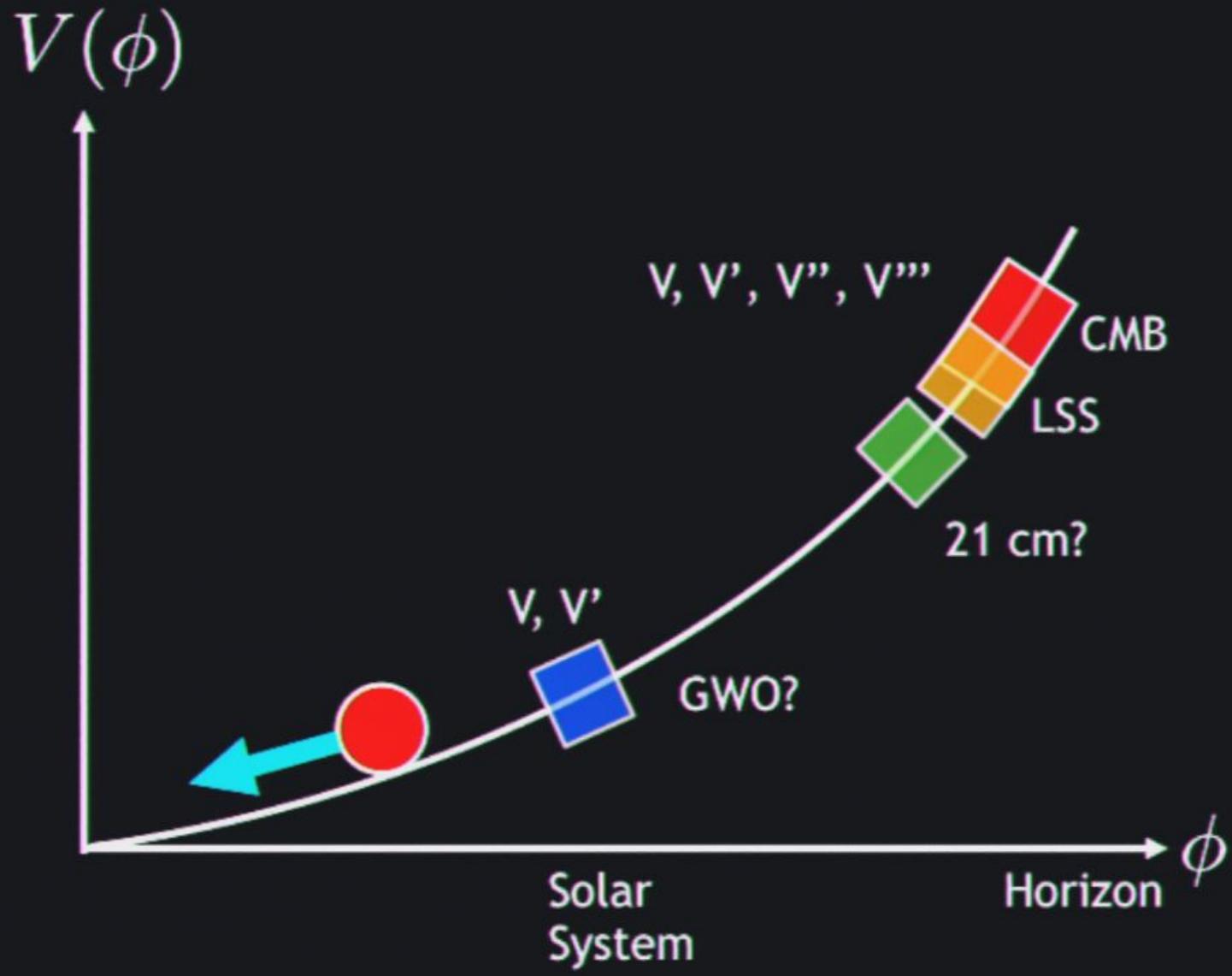
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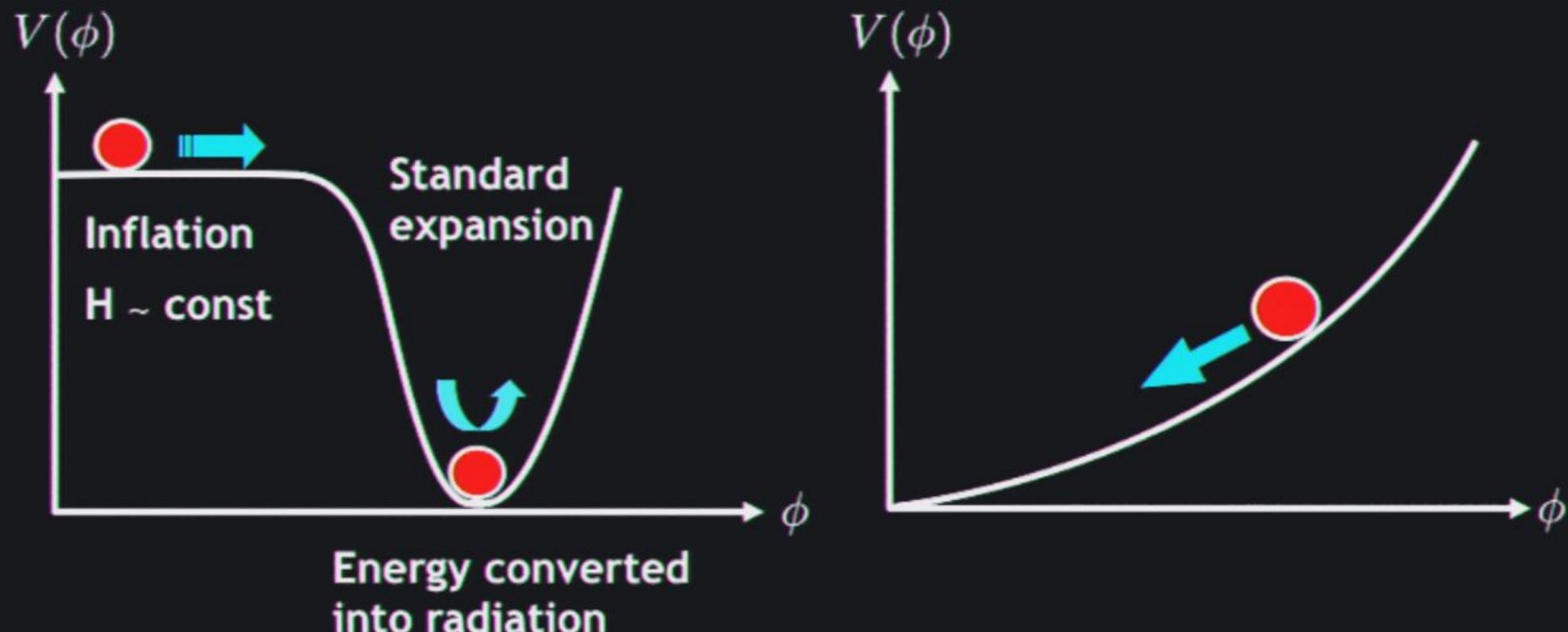
- ▶ Still testing basic aspects of inflationary mechanism rather than specific implementation.

Fingerprints of the early universe

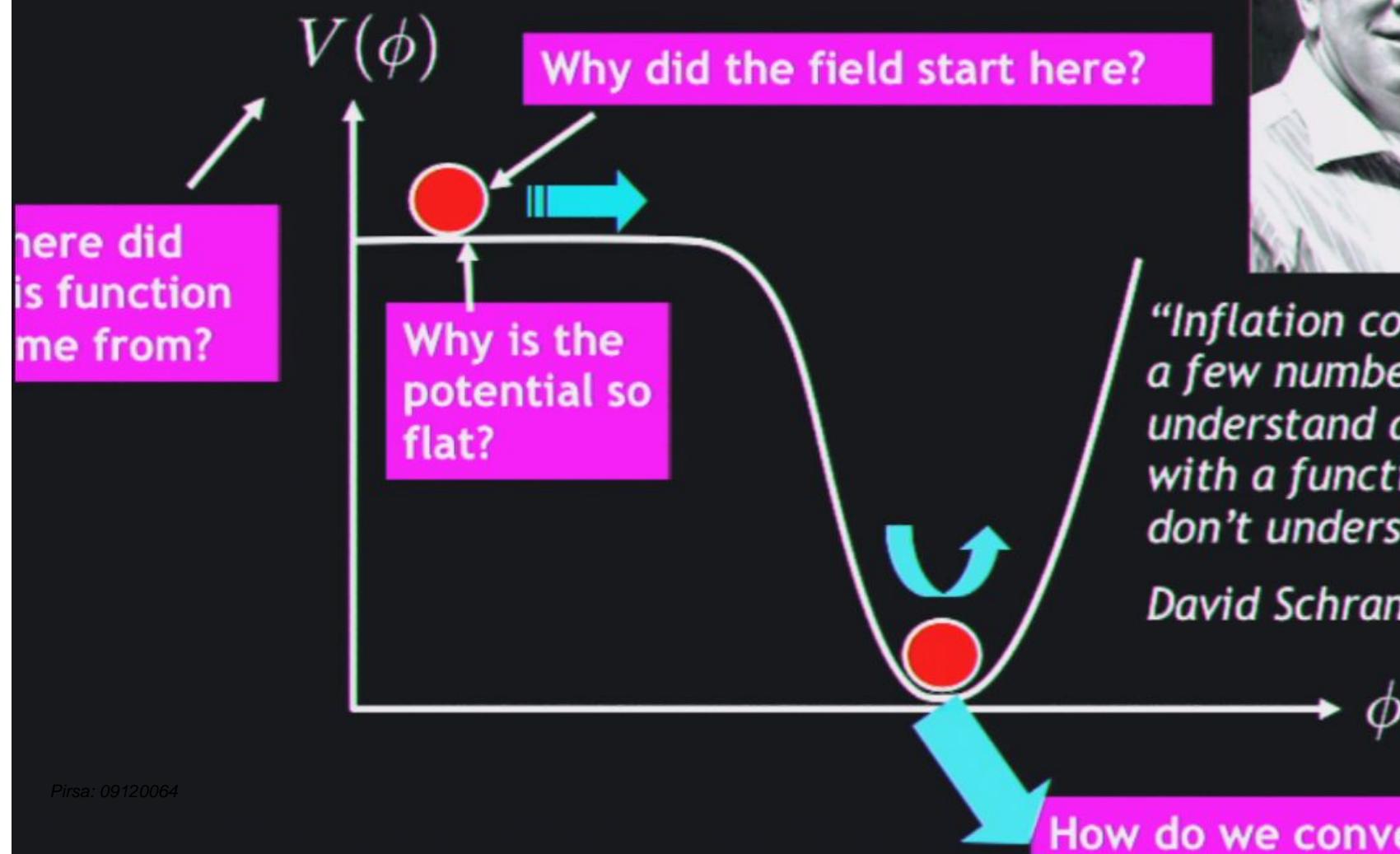


Inflation

Modelled as a scalar field (inflaton) evolving in a potential



What is the physics of inflation?



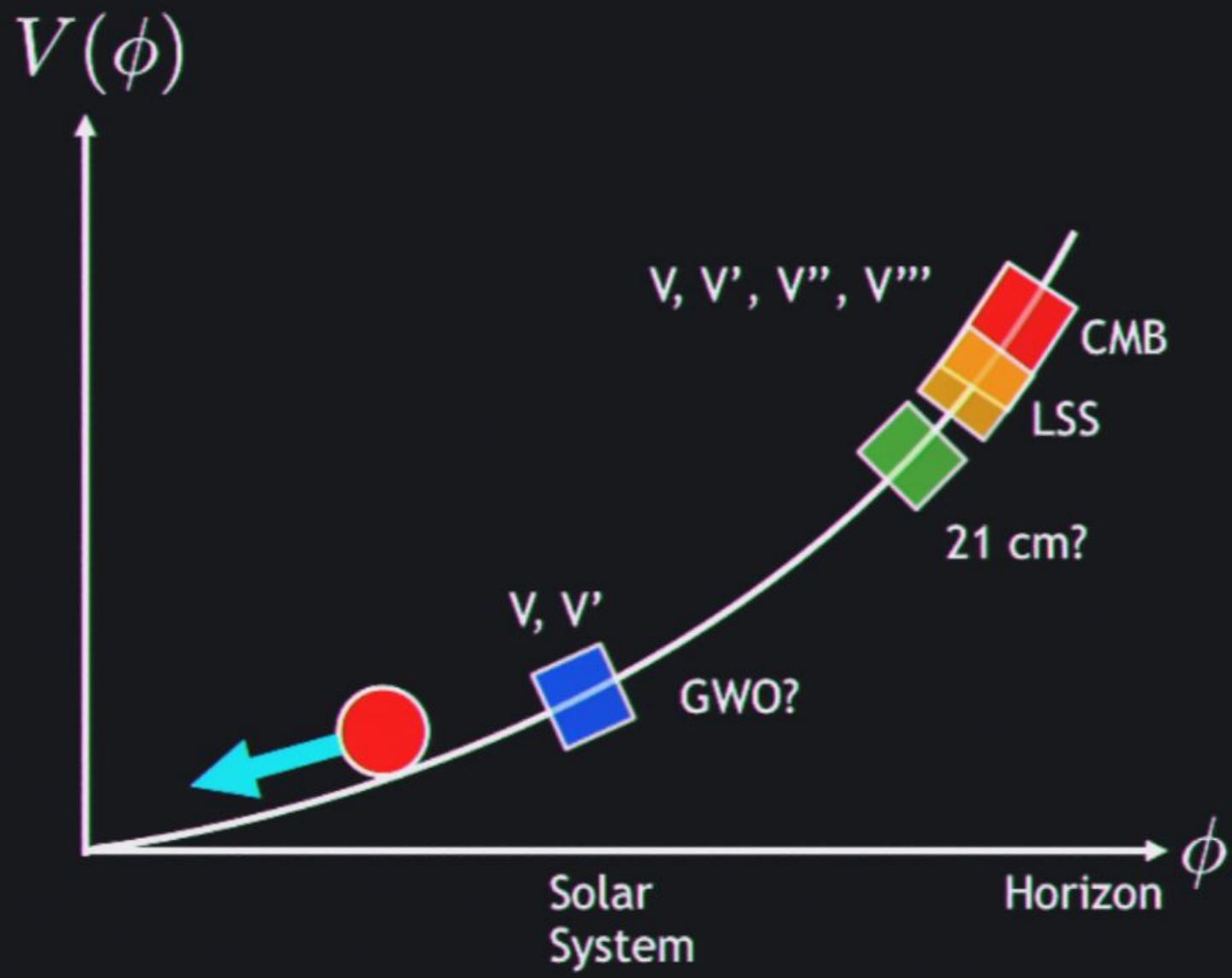
"Inflation consists of taking a few numbers that we don't understand and replacing it with a function that we don't understand"

David Schramm 1945 - 1997

Primordial gravitational waves: a smoking gun

- ▶ Temperature only sensitive to scalars. Polarization can differentiate between scalars (density) and tensors (gravitational waves).
- ▶ Current limit $r_{\text{CMB}} < 0.2$. “Realistically” observable: $r_{\text{CMB}} \geq 0.01$
- ▶ Measurement gives two critical pieces of info:
 - energy scale of inflation: $V^{1/4} \sim \left(\frac{r_{\text{CMB}}}{0.01}\right)^{1/4} 10^{16} \text{ GeV}$
 - super-Planckian field variation: $\frac{\Delta\phi}{M_{\text{Pl}}} > \mathcal{O}(1) \left(\frac{r_{\text{CMB}}}{0.01}\right)^{1/2}$

Fingerprints of the early universe



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 - super-Planckian field variation: $\frac{\Delta\phi}{M_{\text{Pl}}} > \mathcal{O}(1) \left(\frac{r_{\text{CMB}}}{0.01}\right)^{1/2}$

Lyth Bound

In a de Sitter spacetime,

$$\text{tensors: } P_h \propto \frac{H^2}{M_{\text{Pl}}^2} \quad \text{scalars: } P_s \propto H^2 \left(\frac{H}{\dot{\phi}} \right)^2$$

$$\text{tensor to scalar ratio: } r \equiv \frac{P_h}{P_s} = 8 \left(\frac{1}{M_{\text{Pl}}} \frac{d\phi}{dN_e} \right)^2$$

$$\text{where } dN_e \equiv d \ln a = H dt = \left(\frac{H}{\dot{\phi}} \right) d\phi$$

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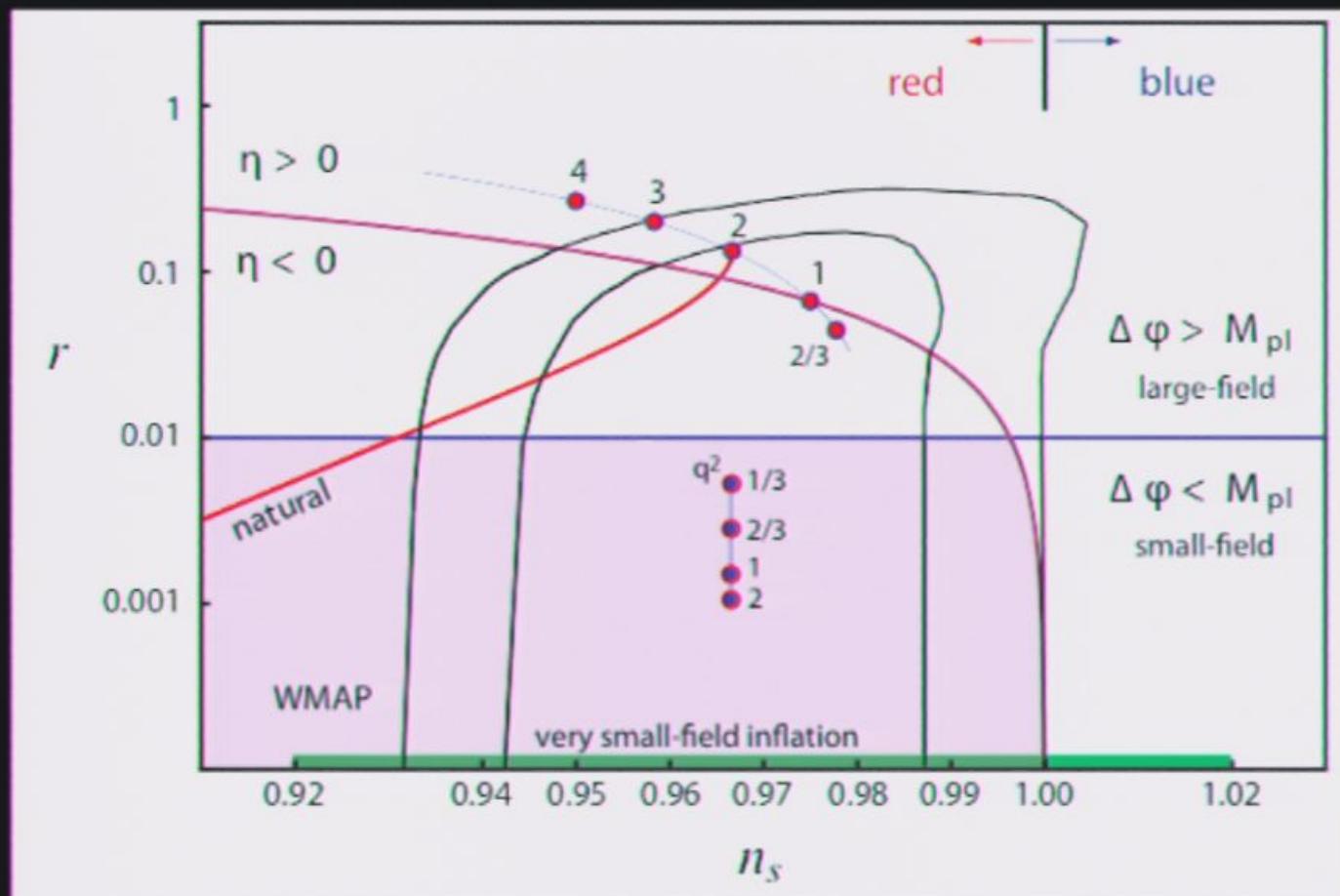
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Primordial gravitational waves: the challenges

*Gravity's waves are
Traceless; which does not mean they
Can never be found.*

Haiku by Peter Coles

Challenge I: what is the amplitude?



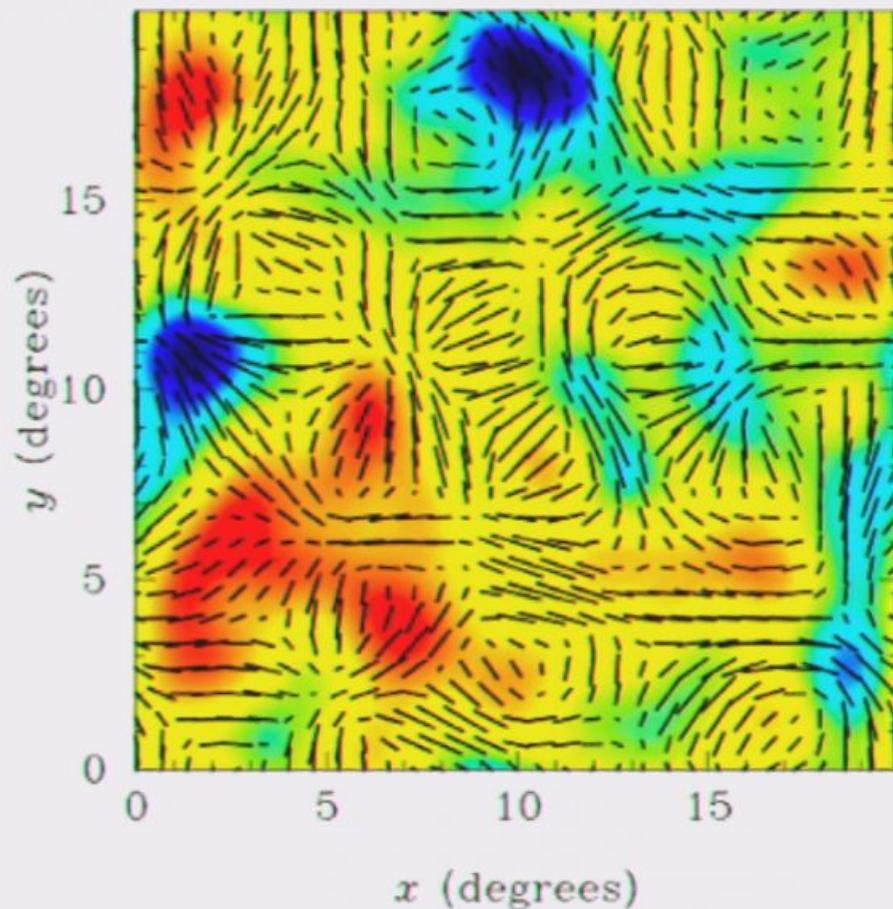
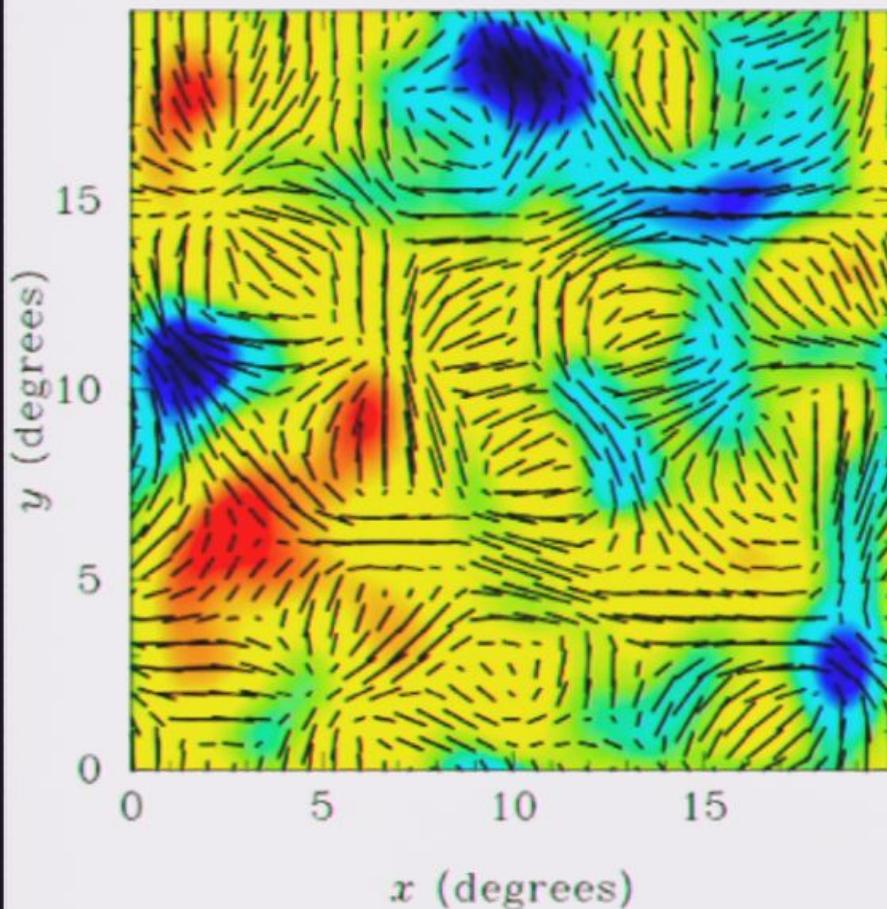
► r determines whether model is large or small field.

► n_s determines whether spectrum is red or blue.

► a combination of n_s and r determines the curvature of the potential η .

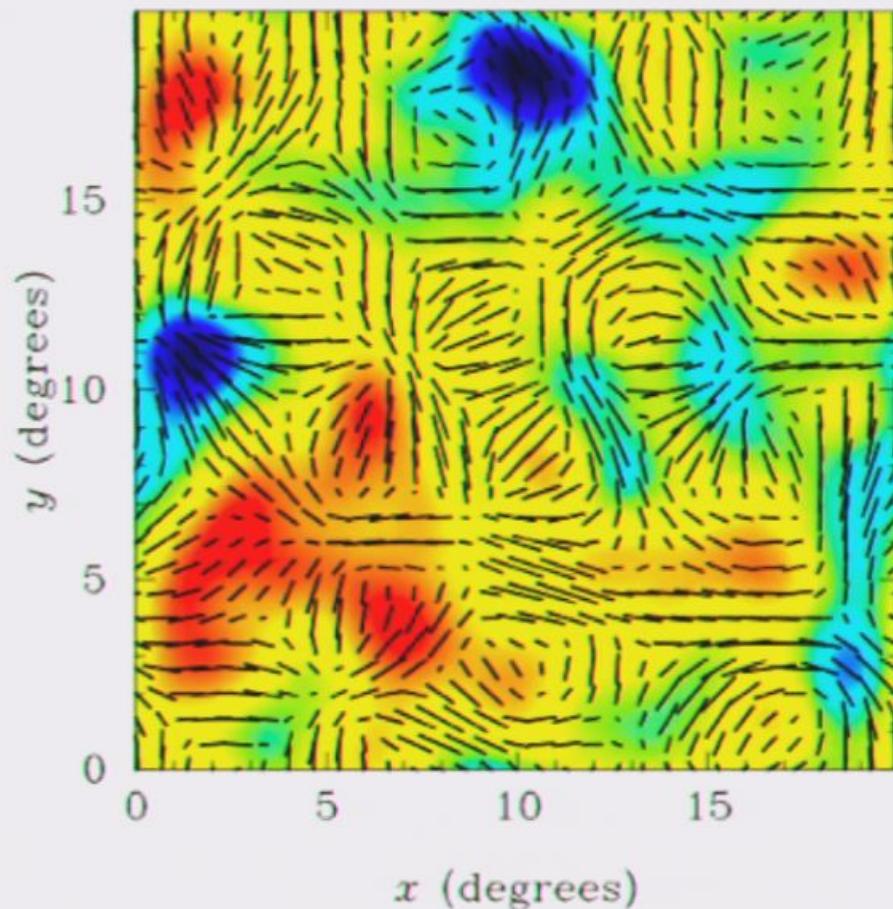
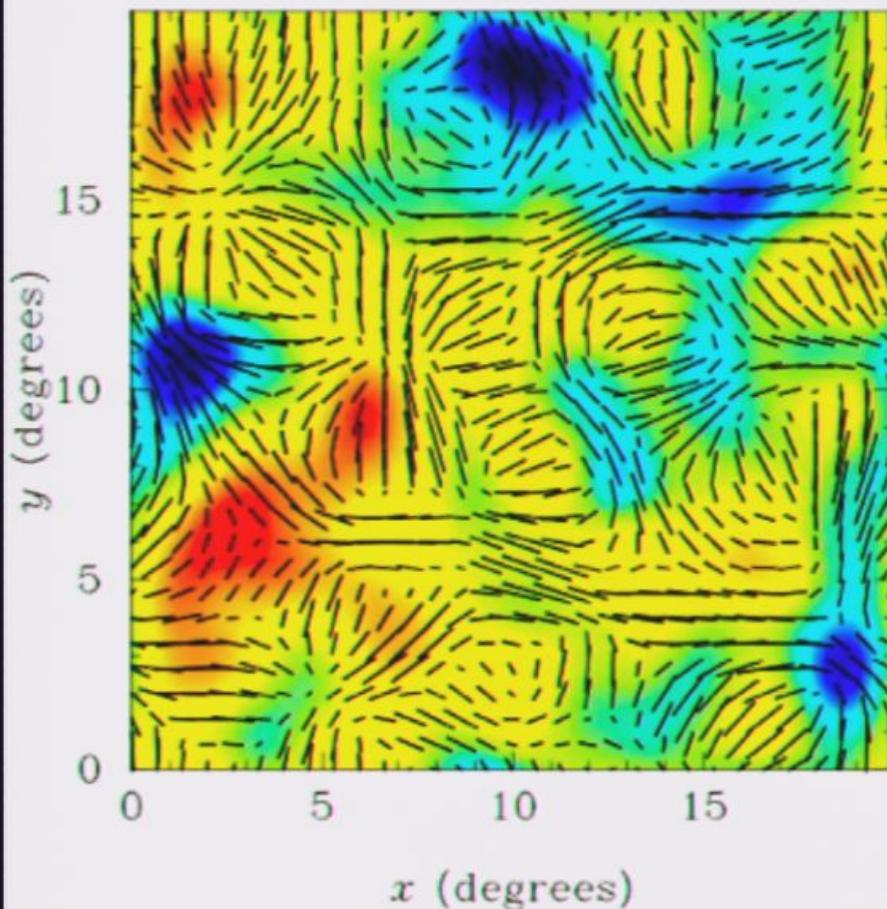
Tensors: B-mode contribution is small!

- R.m.s. B -mode signal from gravity waves $< 200 \text{ nK}$

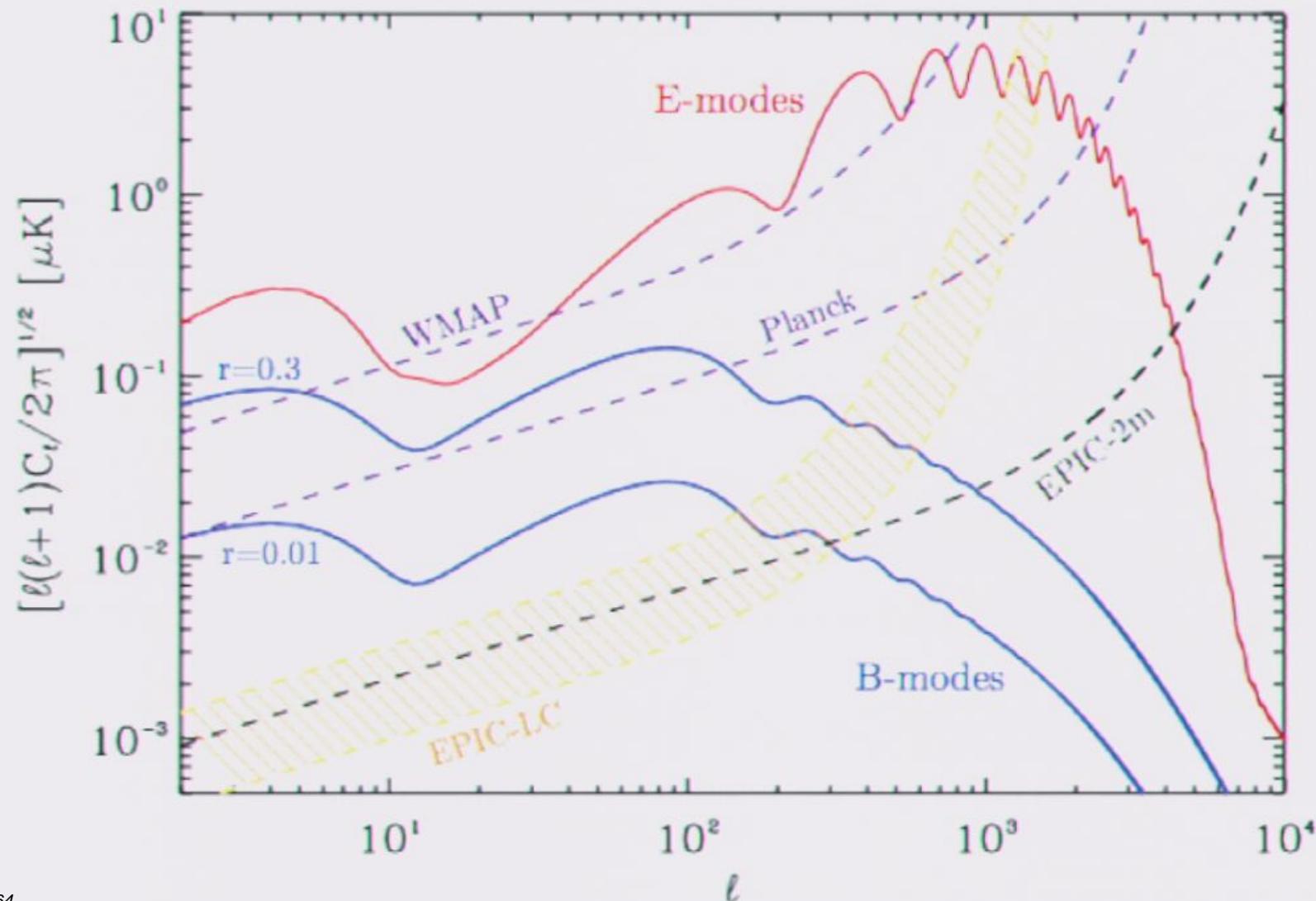


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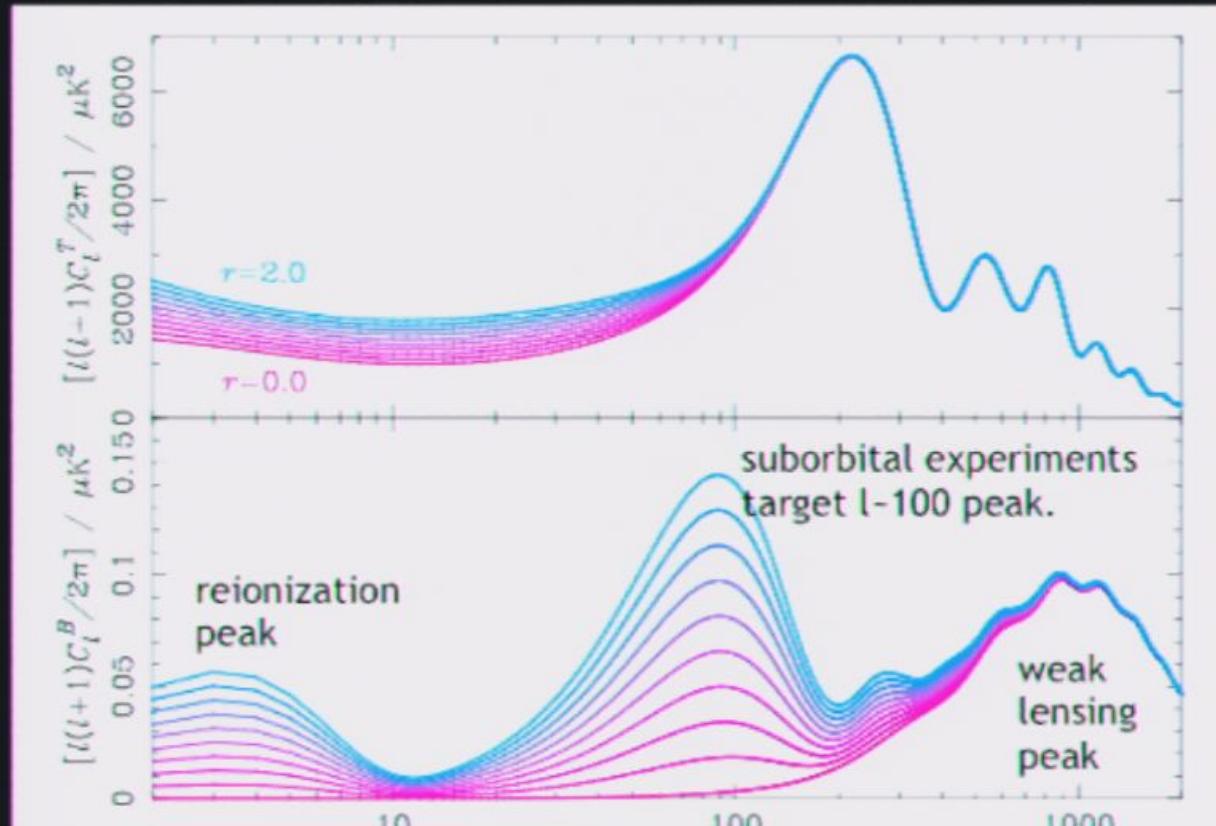


Relative Amplitudes of CMB power spectra



Challenge II: Weak Lensing

- ▶ Generated by weak lensing of the E-mode by large scale structure; subdominant on large scales, dominates on small scales.
(Seljak & Zaldarriaga 1998)
- ▶ Use cross-correlation/ map non-Gaussianity to “de-lens”?
(Okamoto & Hu/Lewis/Knox & Song/Smith)



Challenge III: Detectors

- ▶ Polarization-sensitive bolometers
 - Good above ~ 100 GHz. (e.g. BOOMERanG, DASI)

- ▶ HEMT polarimeters
 - Good below ~100 GHz (e.g. WMAP)

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$$\Delta T_{\text{rms}} = \frac{T_{\text{RMS}} + T_{\text{receiver}}}{\sqrt{\Delta\nu\Delta t}}$$

Need **detector arrays** to beat down noise limit/detector

Wide frequency coverage to keep foregrounds in check

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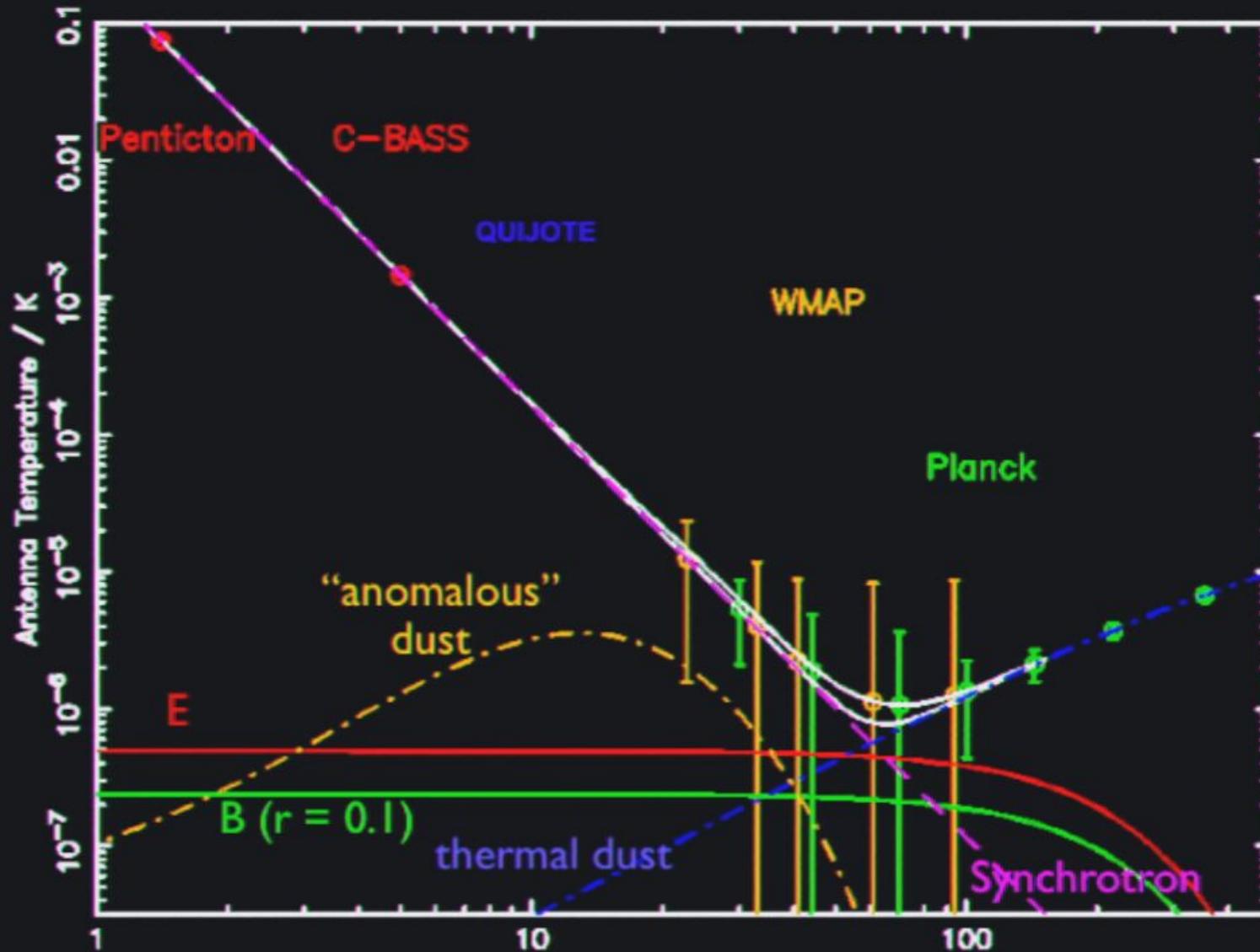
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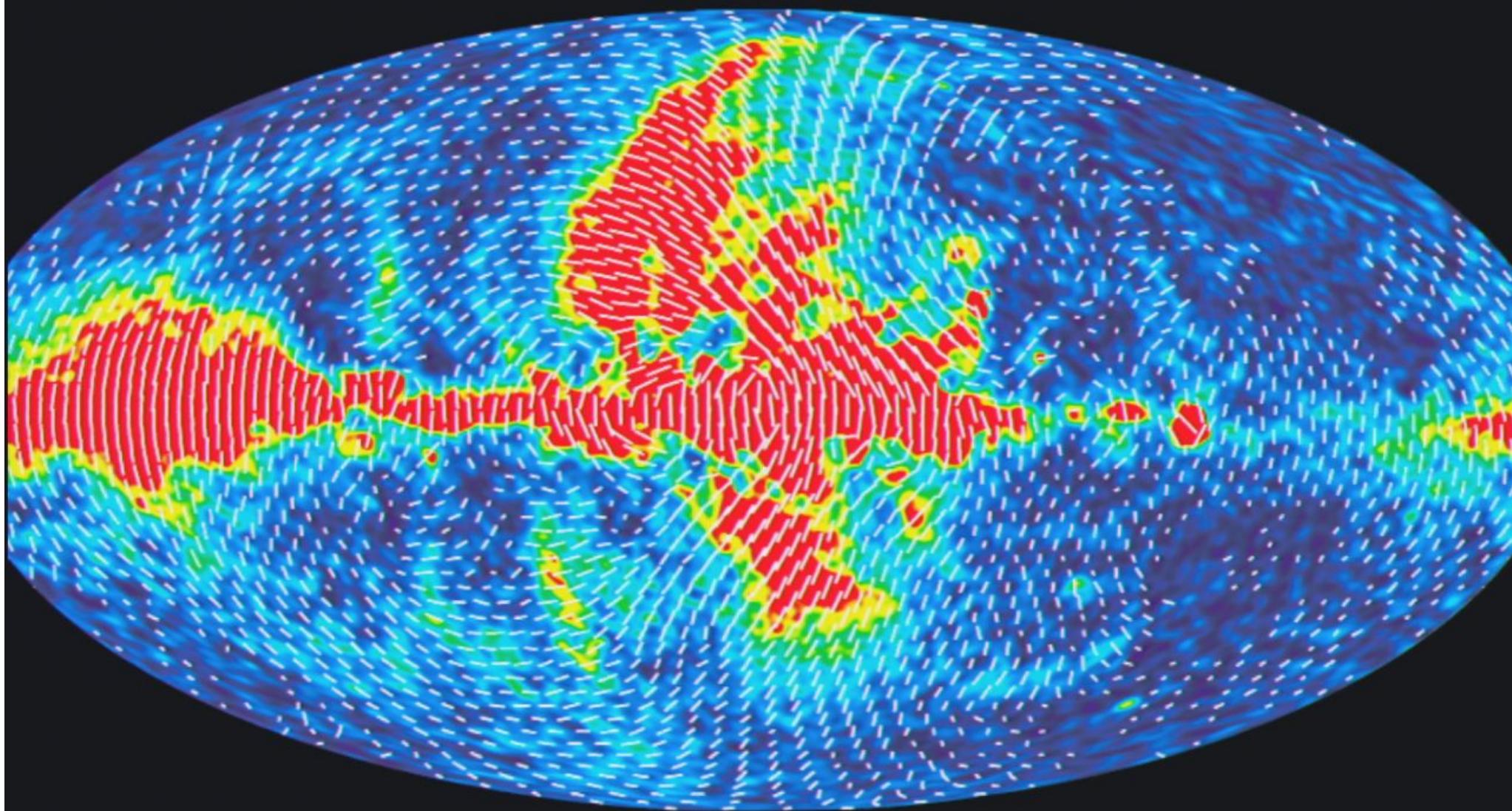
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Challenge IV: we live inside a galaxy!

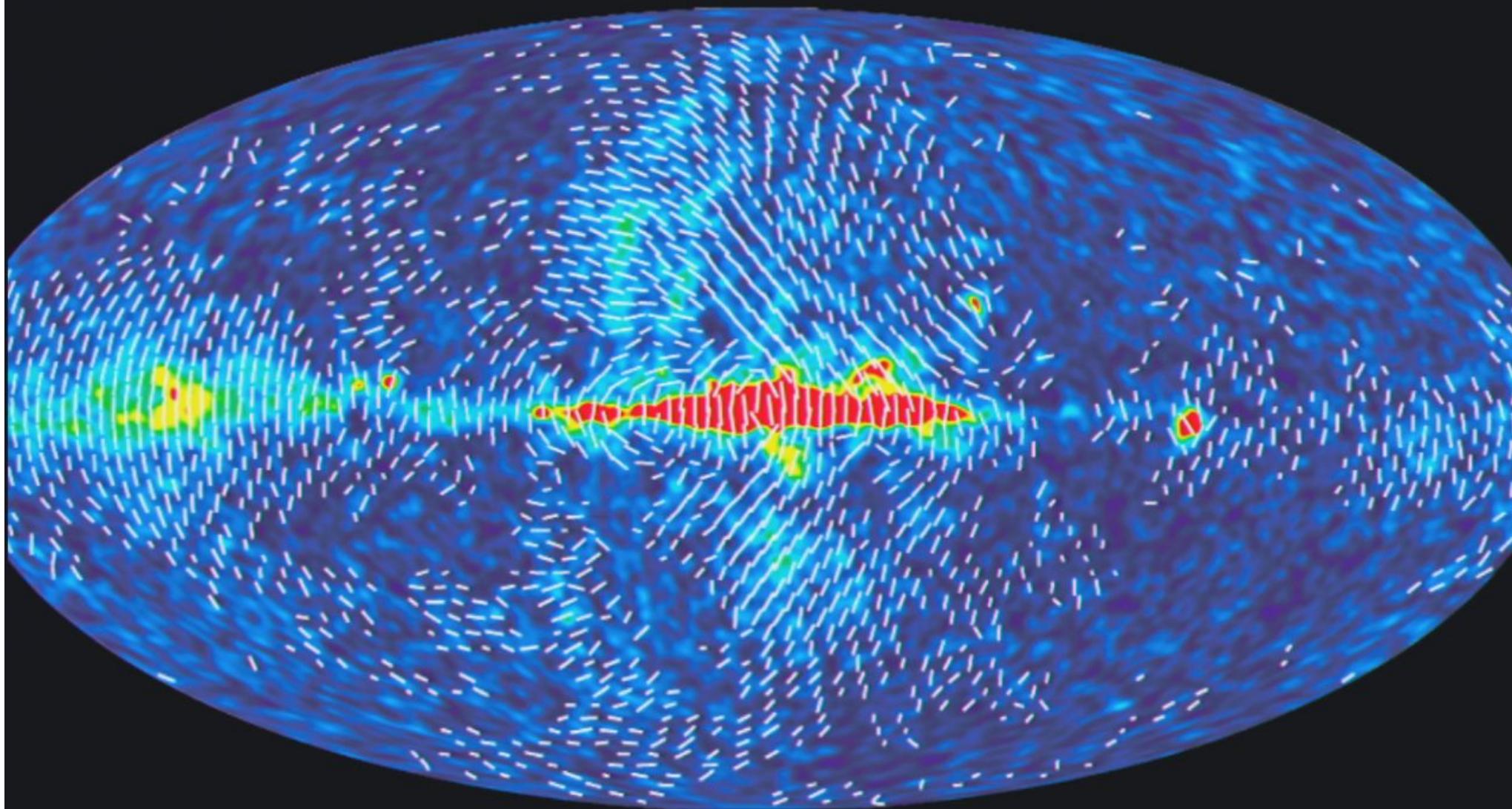
RMS polarized Galactic emission at 1 deg.



K Band (23 GHz)

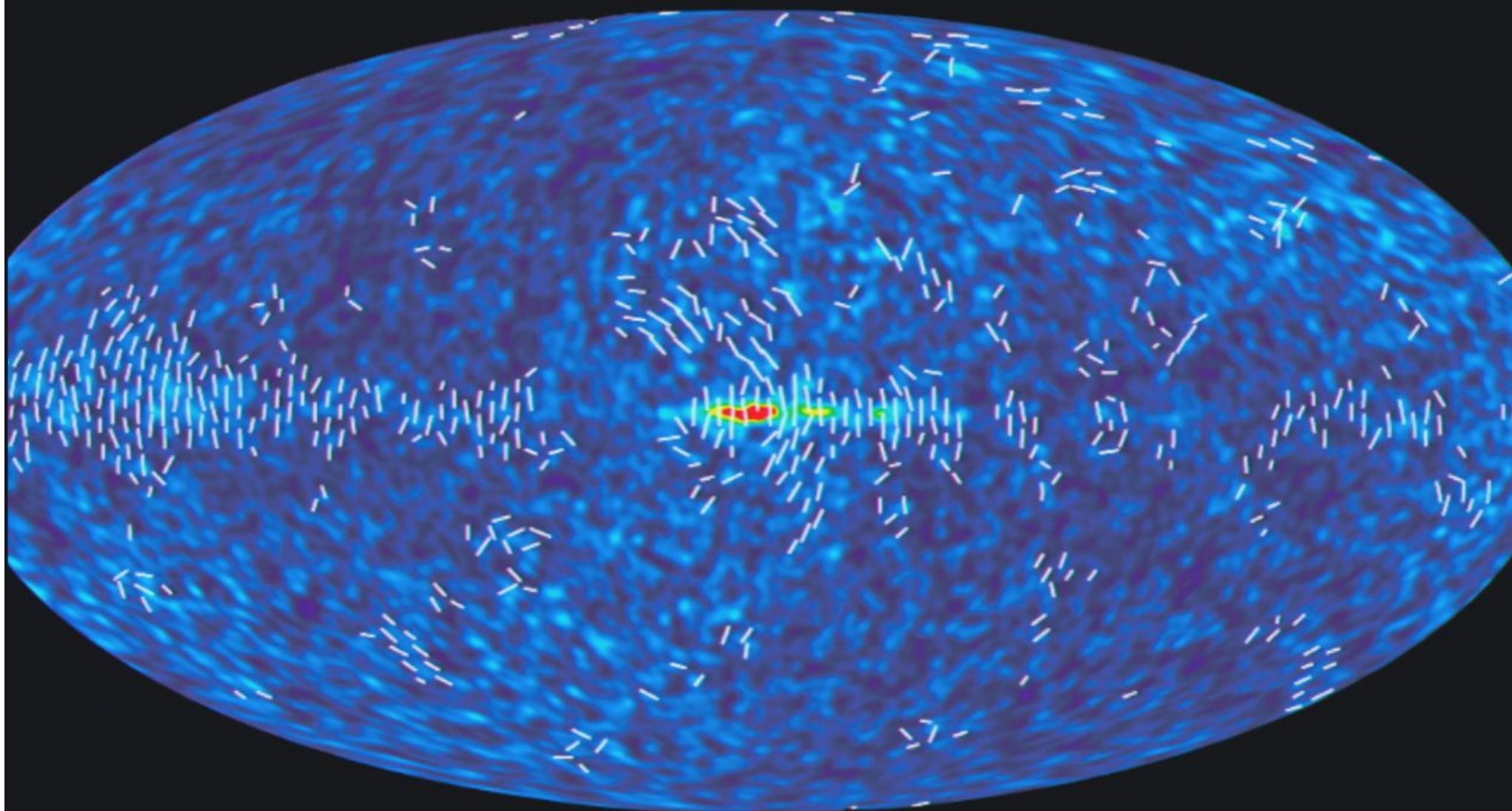


Ka Band (33 GHz)

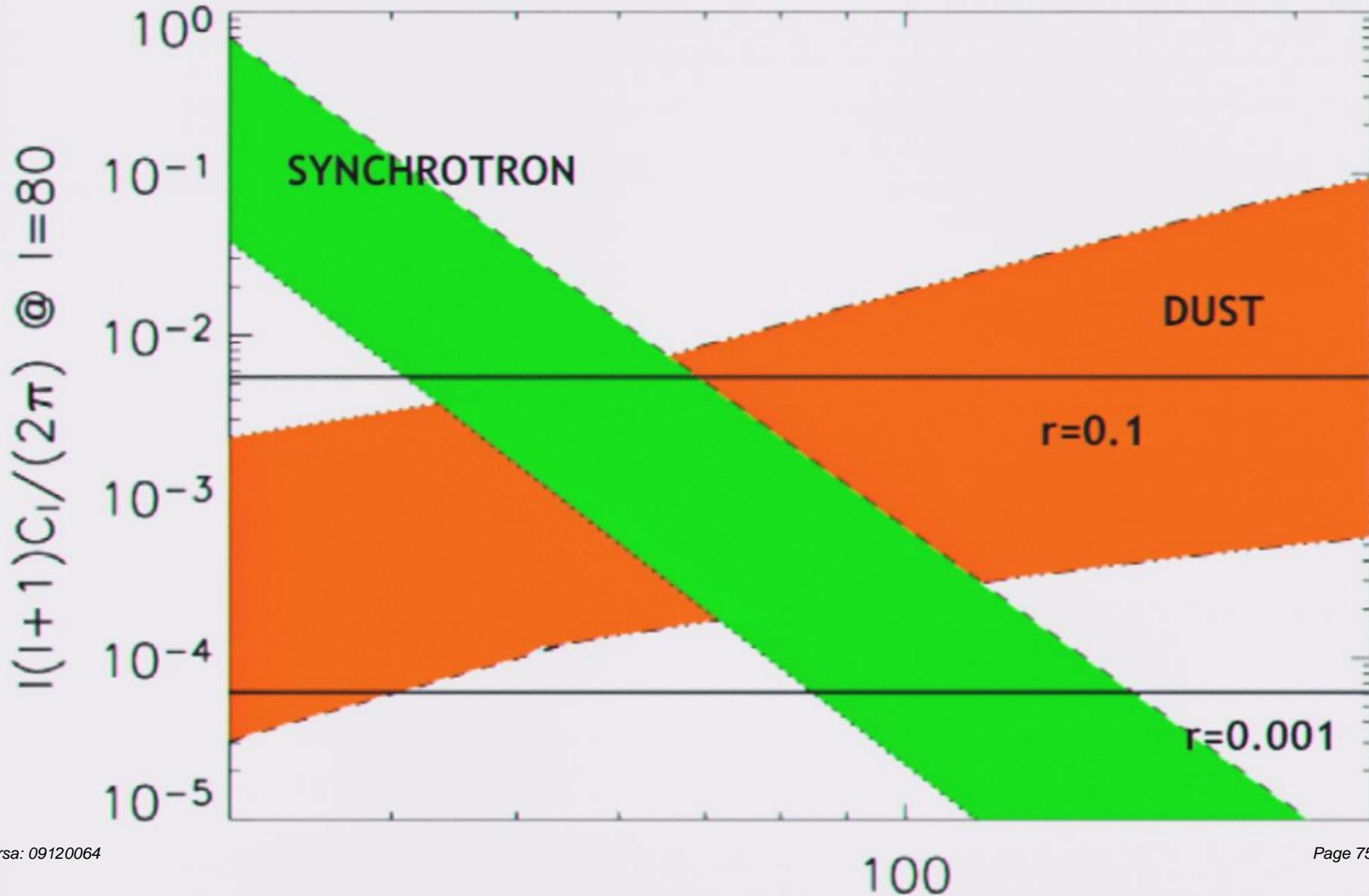


Synchrotron drops as $v^{-3.2}$ from K to Ka.

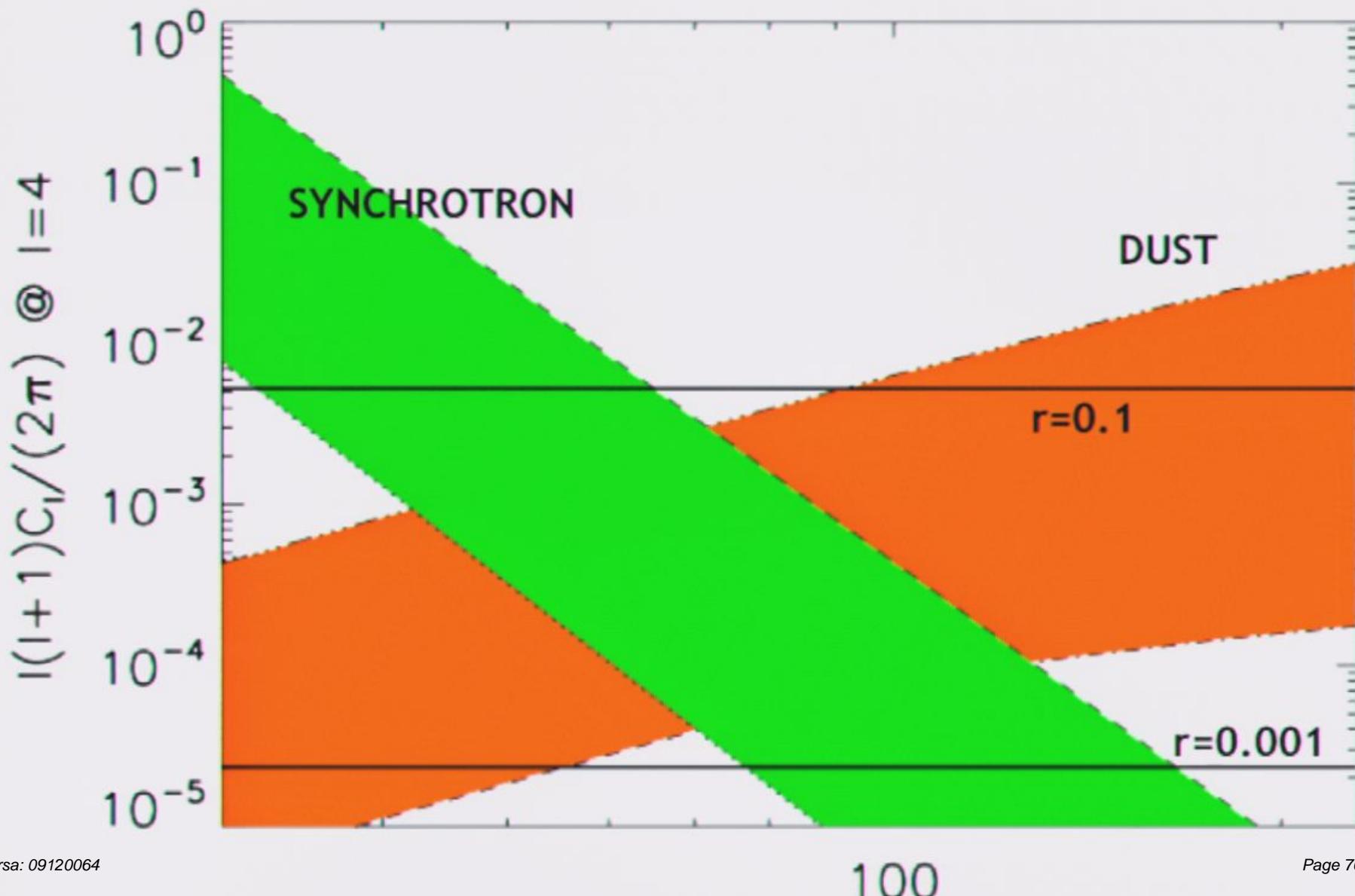
V Band (61 GHz)



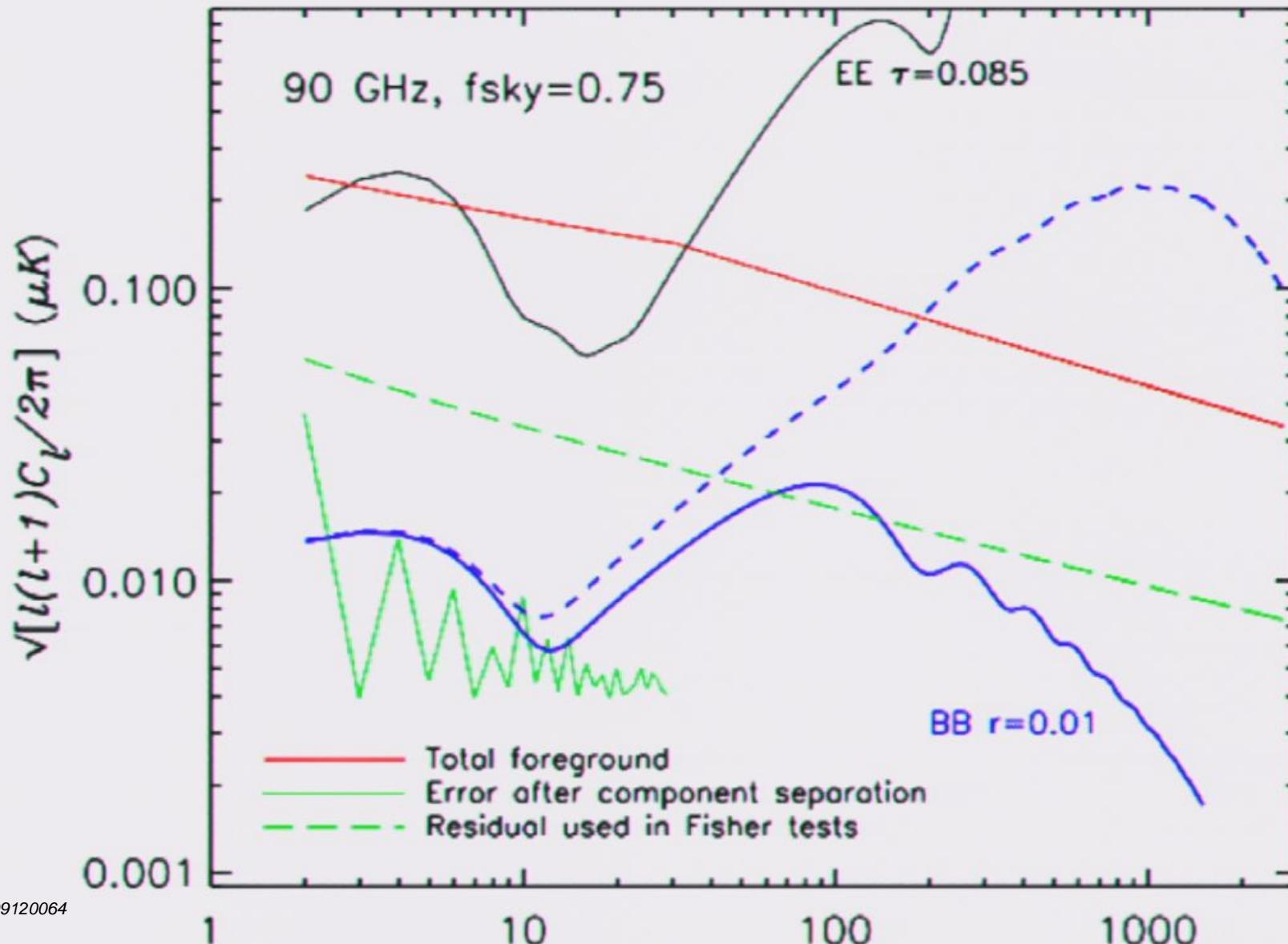
Foreground uncertainties vs CMB at $l=80$



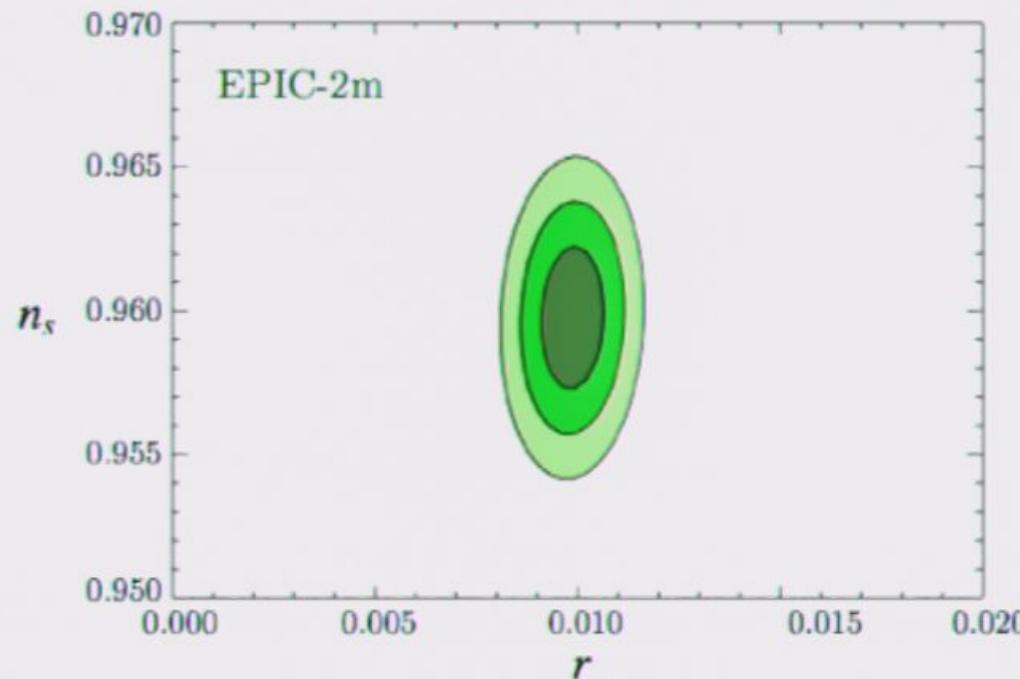
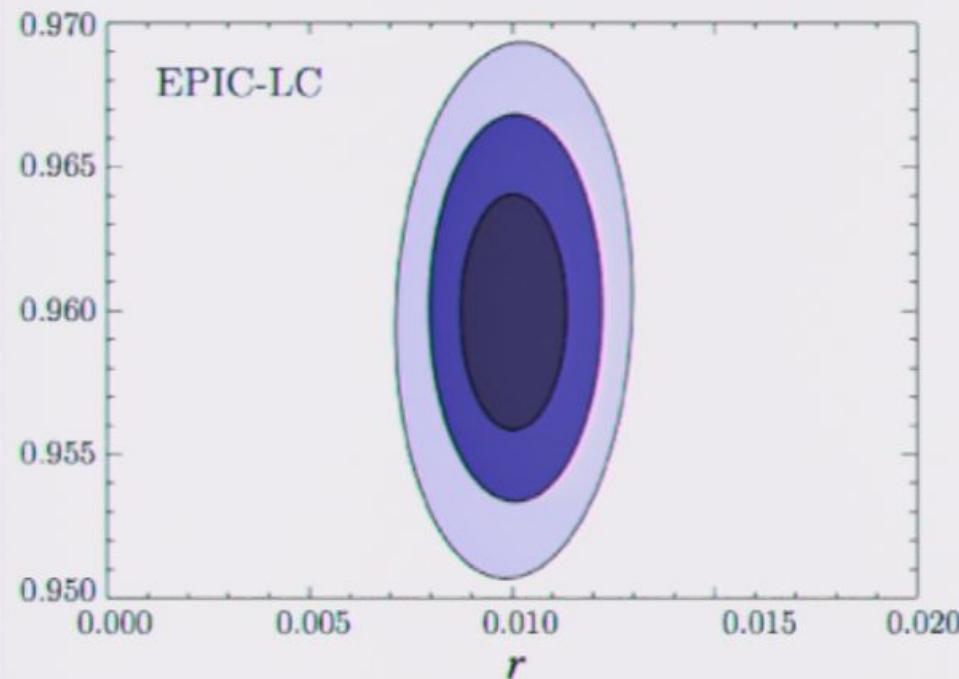
Foreground uncertainties vs CMB at $l=4$



FG cleaning residuals from simulated maps

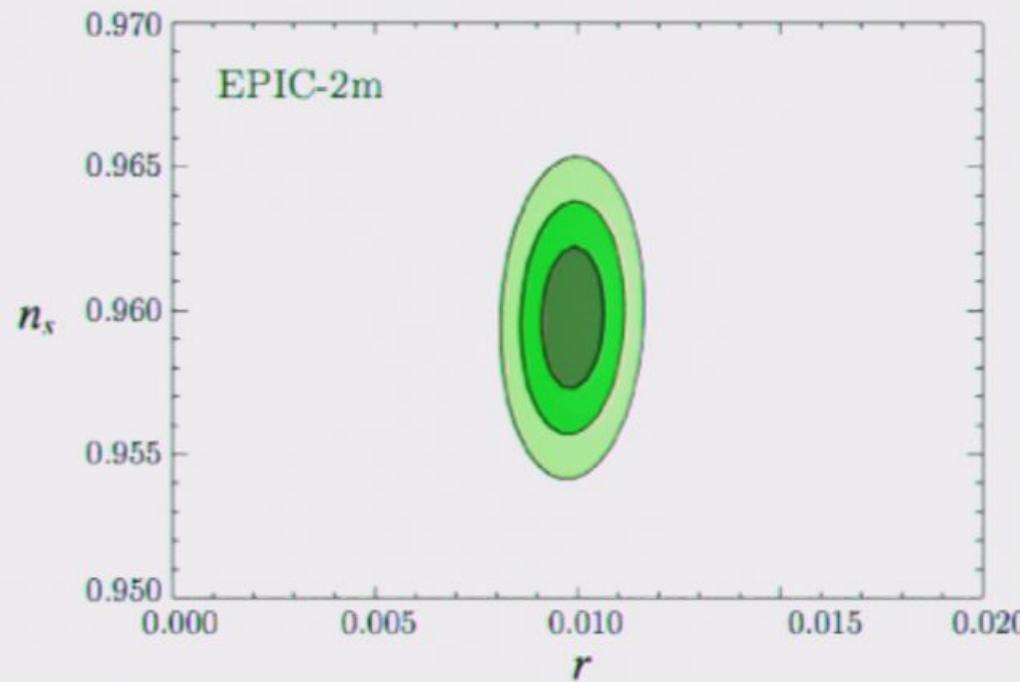
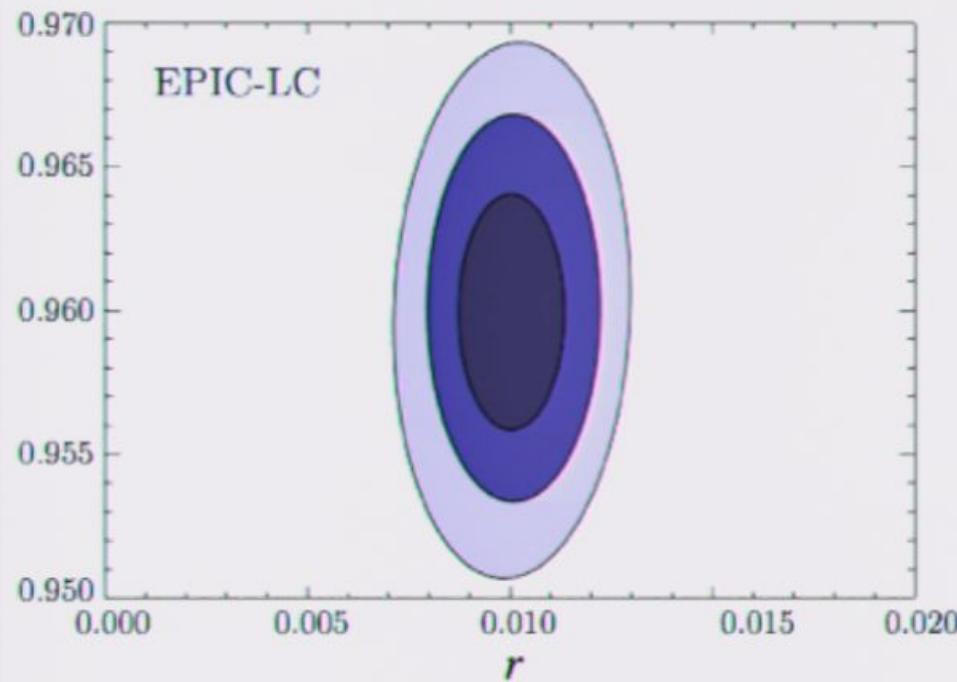


Forecasted CMBpol constraints for $r=0.01$



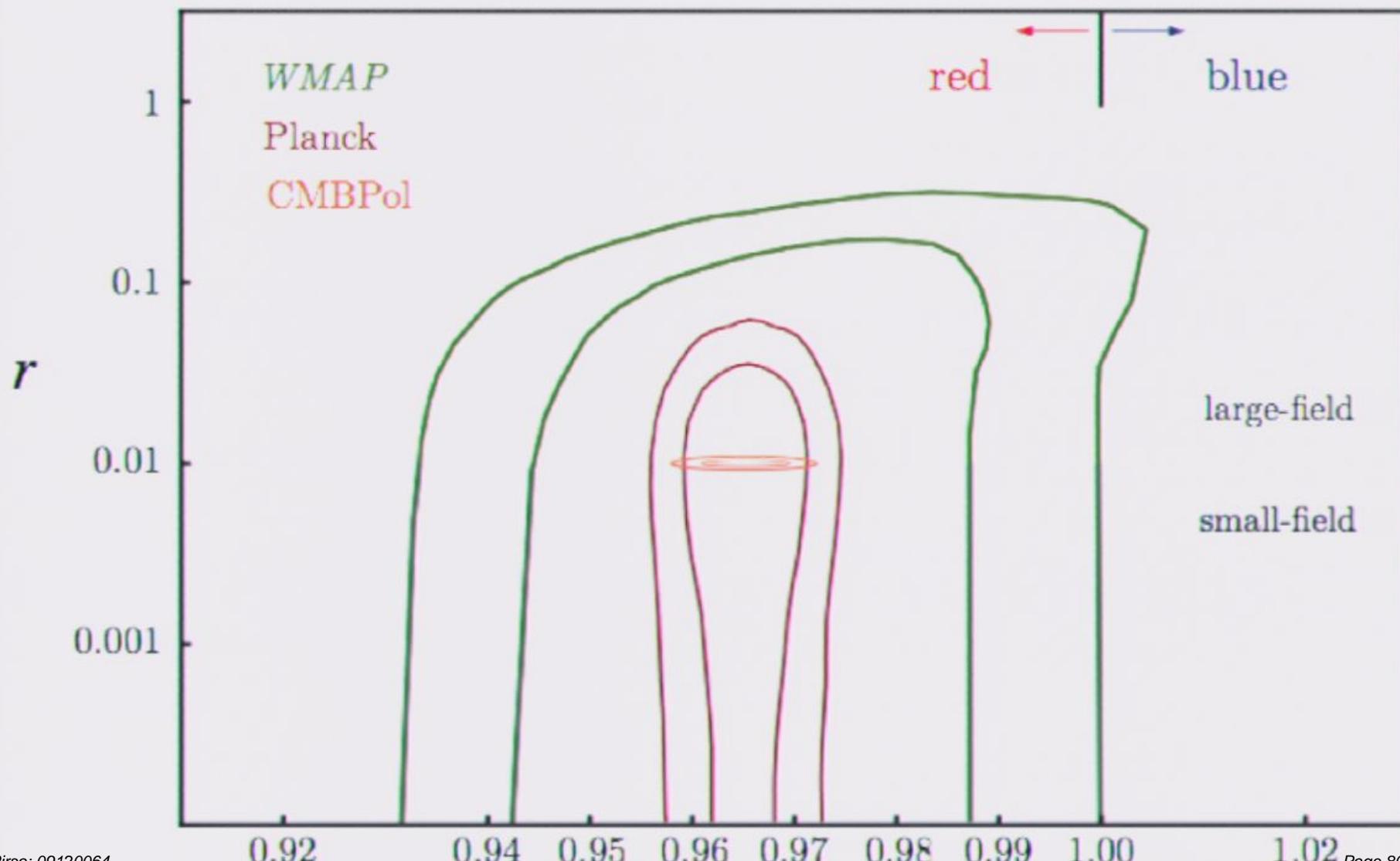
- ▶ EPIC-LC: “low cost” CMBpol proposal
- ▶ EPIC-2m: “mid cost” CMBpol proposal

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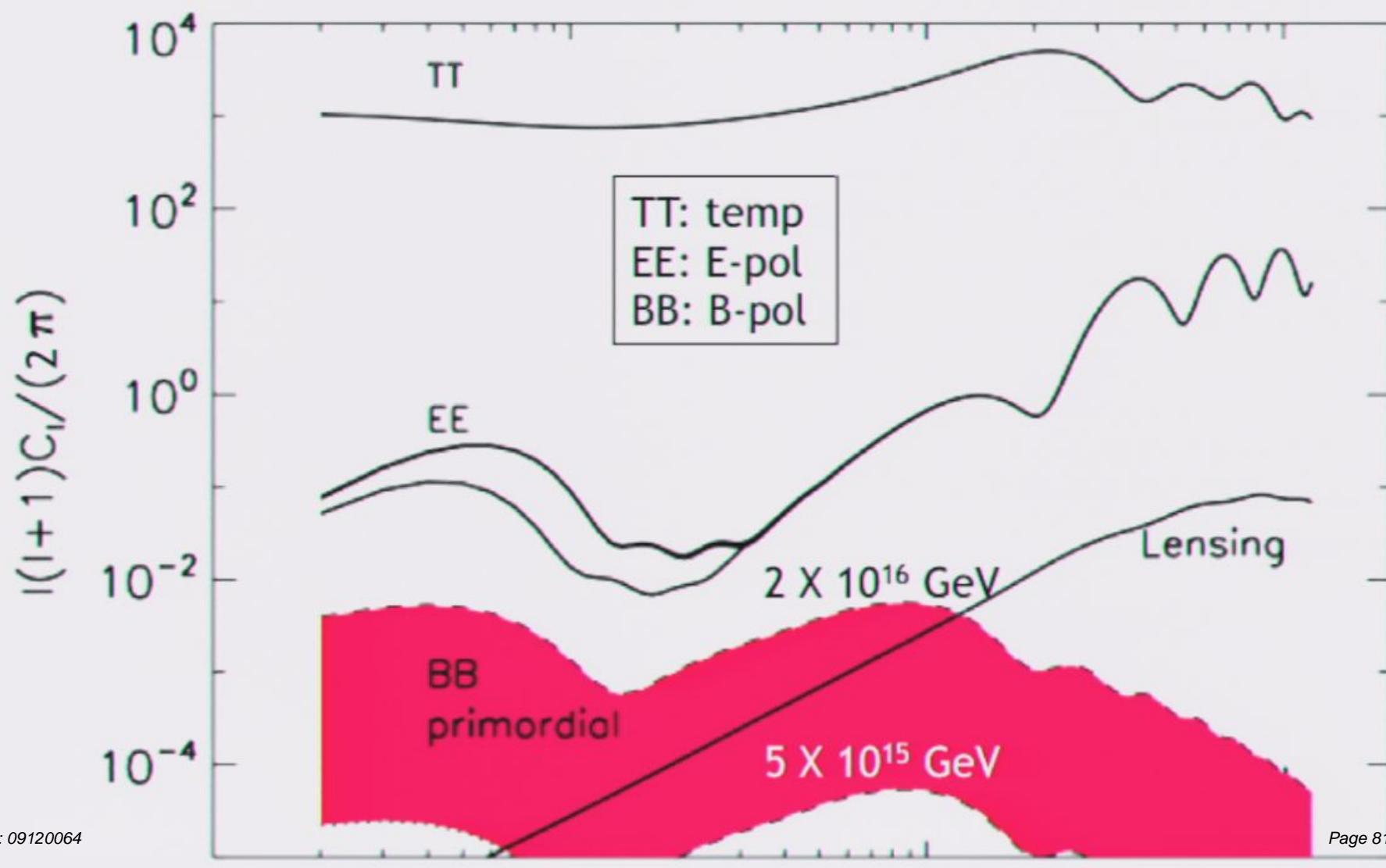
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Approximate range of primordial tensors accessible to upcoming experiments

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