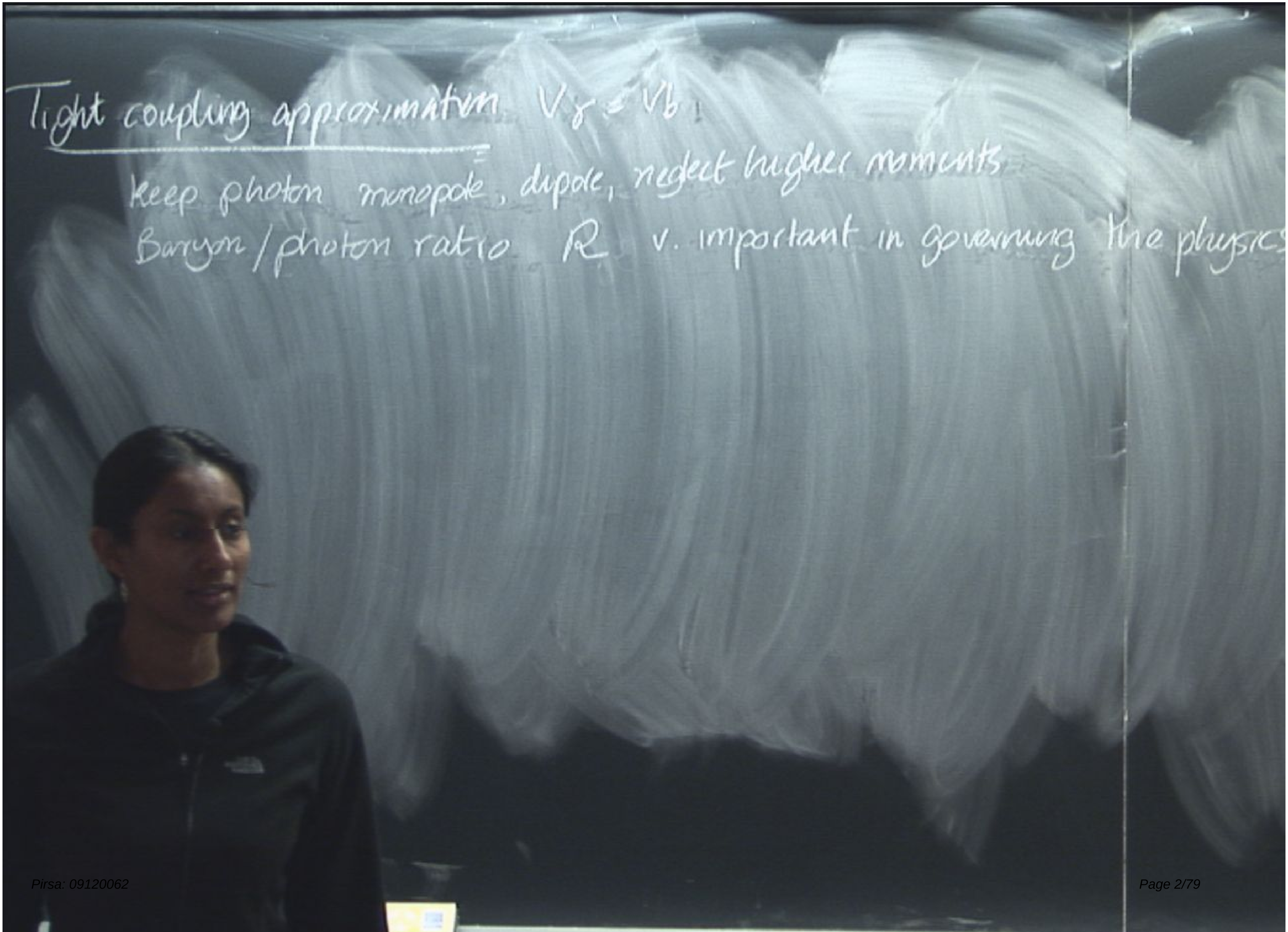
Title: Cosmology - Review (PHYS 621) - Lecture 8

Date: Dec 09, 2009 10:00 AM

URL: http://www.pirsa.org/09120062

Abstract:


Tight coupling approximation $v_\gamma = v_b$
keep photon monopole, dipole, neglect higher moments
Baryon/photon ratio R v. important in governing the physics

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(1) Number continuity

γ's are not created or destroyed!

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no expansion

$$\dot{n}_\gamma + \vec{\nabla} \cdot (n_\gamma \vec{V}_\gamma) = 0$$

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$$n_\gamma \propto a$$

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$$\dot{n}_\gamma + \vec{\nabla}\cdot(n_\gamma \vec{V}_\gamma) = 0$$

$n_\gamma \propto a^{-3}$

$$\dot{n}_\gamma + 3 n_\gamma \frac{\dot{a}}{a} + \vec{\nabla}\cdot(n_\gamma$$

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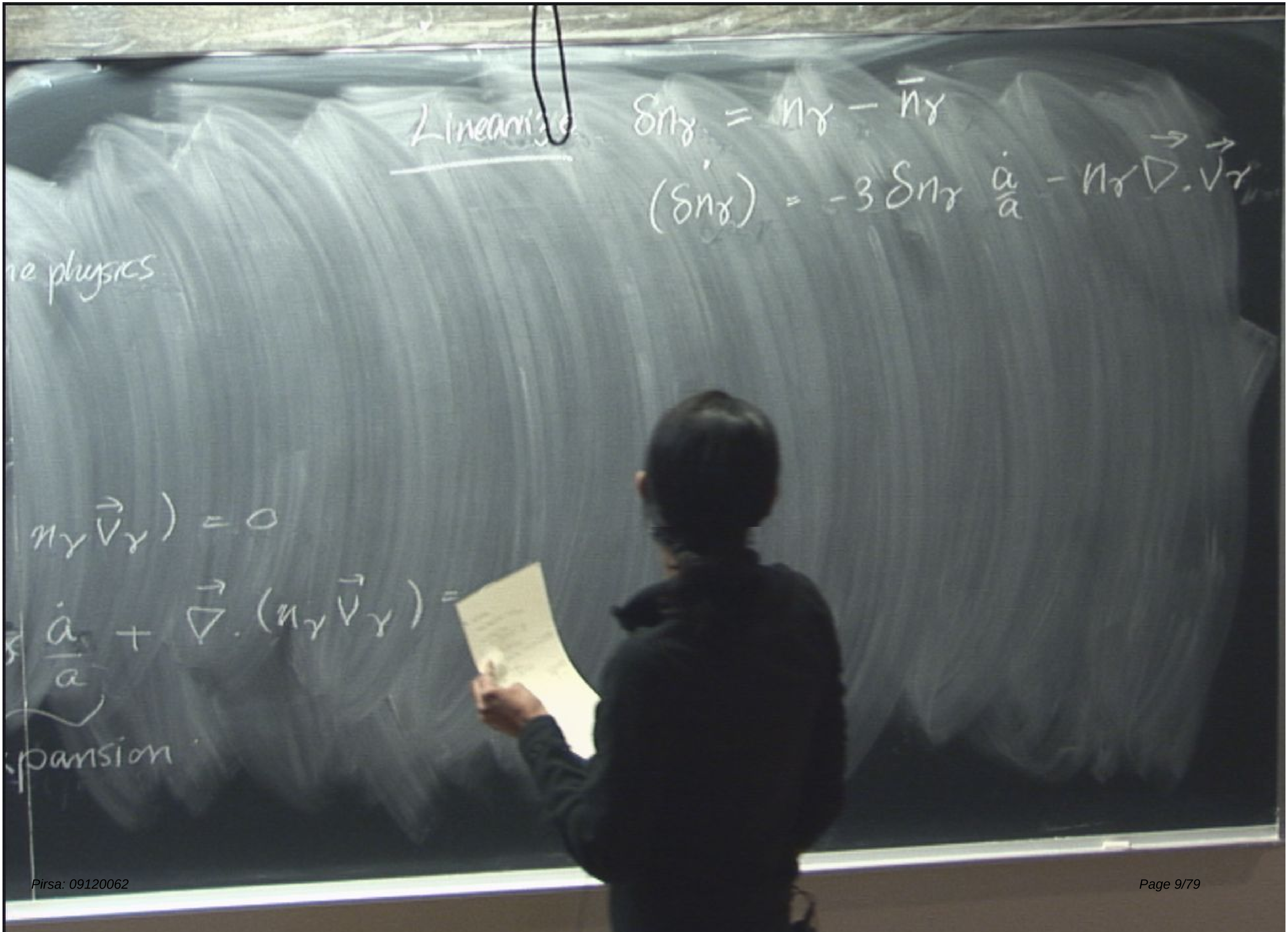
$$n_\gamma \propto a^{-3} \qquad \dot{n}_\gamma + \underbrace{3 n_\gamma \frac{\dot{a}}{a}}_{\text{expansion}} + \vec{\nabla}\cdot(n_\gamma \vec{v}_\gamma)$$

Linear


Linearize $\delta n_\gamma = n_\gamma - \bar{n}_\gamma$
$(\delta n_\gamma)' = -3\,\delta n_\gamma \frac{\dot{a}}{a} - n_\gamma \vec{\nabla}\cdot\vec{v}_\gamma$
physics
$n_\gamma \vec{v}_\gamma) = 0$
$\frac{\dot{a}}{a} + \vec{\nabla}\cdot(n_\gamma \vec{v}_\gamma) =$
pansion

Linearize $\delta n_\gamma = n_\gamma - \bar{n}_\gamma$

$$(\dot{\delta n_\gamma}) = -3\,\delta n_\gamma \frac{\dot{a}}{a} - n_\gamma \vec{\nabla}\cdot\vec{v}_\gamma$$

$$\boxed{\left(\dot{\frac{\delta n_\gamma}{n_\gamma}}\right) = -\vec{\nabla}\cdot\vec{v}_\gamma}$$

...e physics

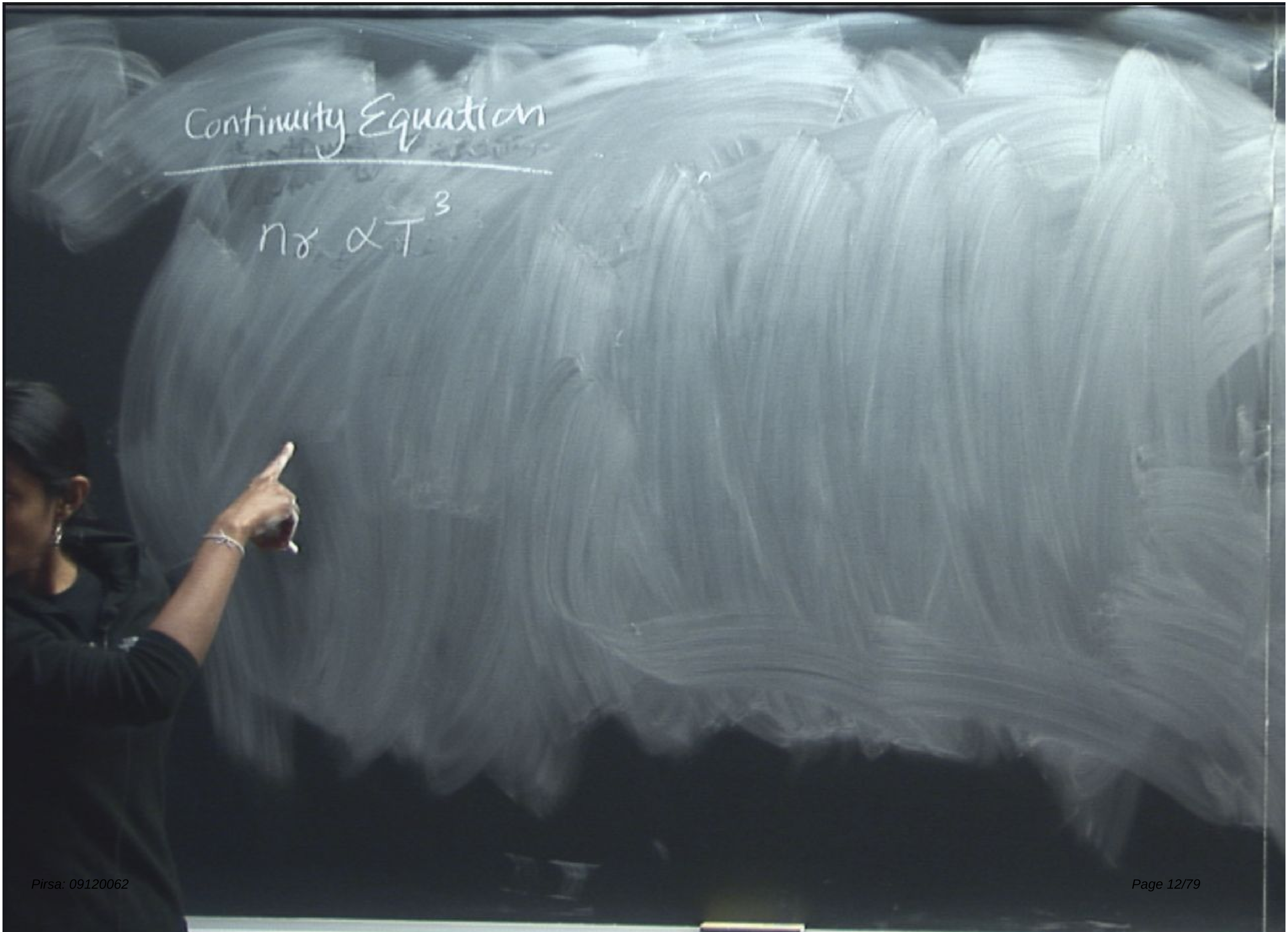
$n_\gamma \vec{v}_\gamma) = 0$

$\frac{\dot{a}}{a} + \vec{\nabla}\cdot(n_\gamma \vec{v}_\gamma) = 0$

...pansion

Continuity Equation

n_γ


Continuity Equation
$n_\gamma \propto T^3$

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define temperature fluctuation Θ

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$$\frac{\delta n_\gamma}{n_\gamma} = 3\,\frac{\delta T}{T} \equiv 3\,\Theta$$

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$$\boxed{\frac{\delta n_\gamma}{n_\gamma} = 3\,\frac{\delta T}{T} \equiv 3\,\Theta}$$

Real space

$$\dot{\Theta} = -\frac{1}{3}\vec{\nabla}\cdot\vec{v}_\gamma$$

Continuity Equation

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define temperature fluctuation Θ

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Real space

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Fourier spa

$$\dot{\Theta} = -\frac{1}{3}$$

y Equation

$\propto T^3$

temperature fluctuation Θ

$$\boxed{\frac{\delta n_\gamma}{n_\gamma} = 3\,\frac{\delta T}{T} \equiv 3\,\Theta}$$

Real space

$$\dot{\Theta} = -\frac{1}{3}\vec{\nabla}\cdot\vec{v}_\gamma$$

Fourier space

$$\dot{\Theta} = -\frac{1}{3}\,i\,\vec{k}\cdot\vec{v}_\gamma$$

$$\dot{\Theta}_1 = -\frac{1}{3}\, i\, \vec{k}\cdot\vec{v}_\gamma$$

Momentum conservation

- no expansion $\dot{\vec{q}} = \vec{F}$

- wavelength stretches w/ expansion

$$\dot{\vec{q}} + \frac{\dot{a}}{a}\vec{q} = \vec{F}$$

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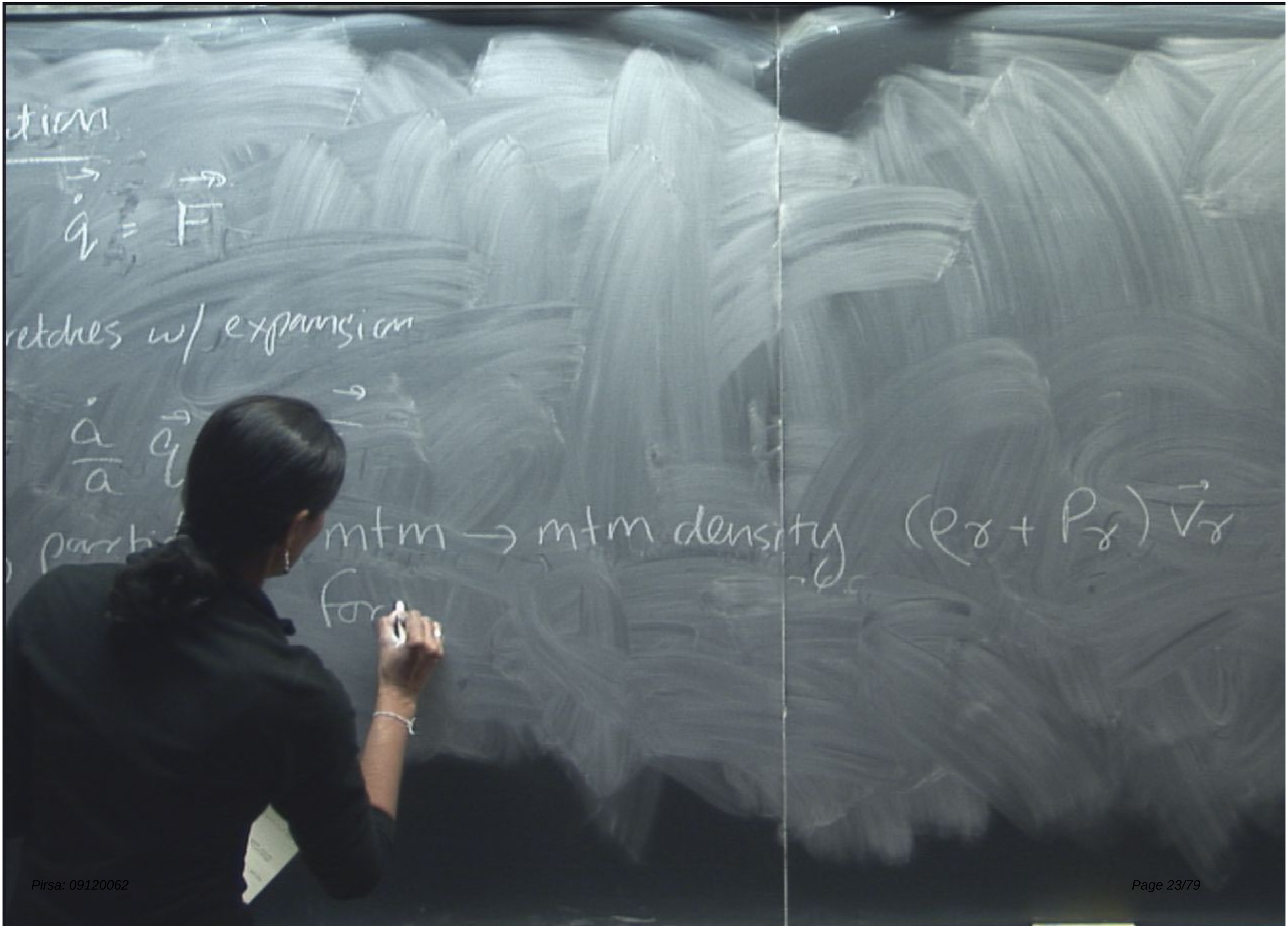
collection of particles : mtm $\rightarrow$ mtm density $(\rho_\gamma +$


tion
$\dot{\vec{q}} = \vec{F}$
retches w/ expansion
$\frac{\dot{a}}{a} \vec{q}$
mtm → mtm density $(\rho_\gamma + P_\gamma)\vec{v}_\gamma$
for

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particles : mtm → mtm density $(\rho_\gamma + P_\gamma)\vec{v}_\gamma$

force → pressure gradient ∇P

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force $\rightarrow$ pressure gradient ∇

$$[(\rho_\gamma + P_\gamma)\vec{v}_\gamma]^{\cdot}$$

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ollection of particles : mtm $\rightarrow$ mtm density $(\rho_\gamma +$ ρ_c

force $\rightarrow$ pressure gradient ∇

$$[(\rho_\gamma + P_\gamma)\vec{v}_\gamma]^{\cdot} = -4\frac{\dot{a}}{a}(\rho_\gamma + P_\gamma)\vec{v}_\gamma + \vec{\nabla} P_\gamma$$

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$$\frac{4}{3}\rho_\gamma \dot{\vec{v}}_\gamma = \frac{1}{3}\vec{\nabla}\rho_\gamma$$

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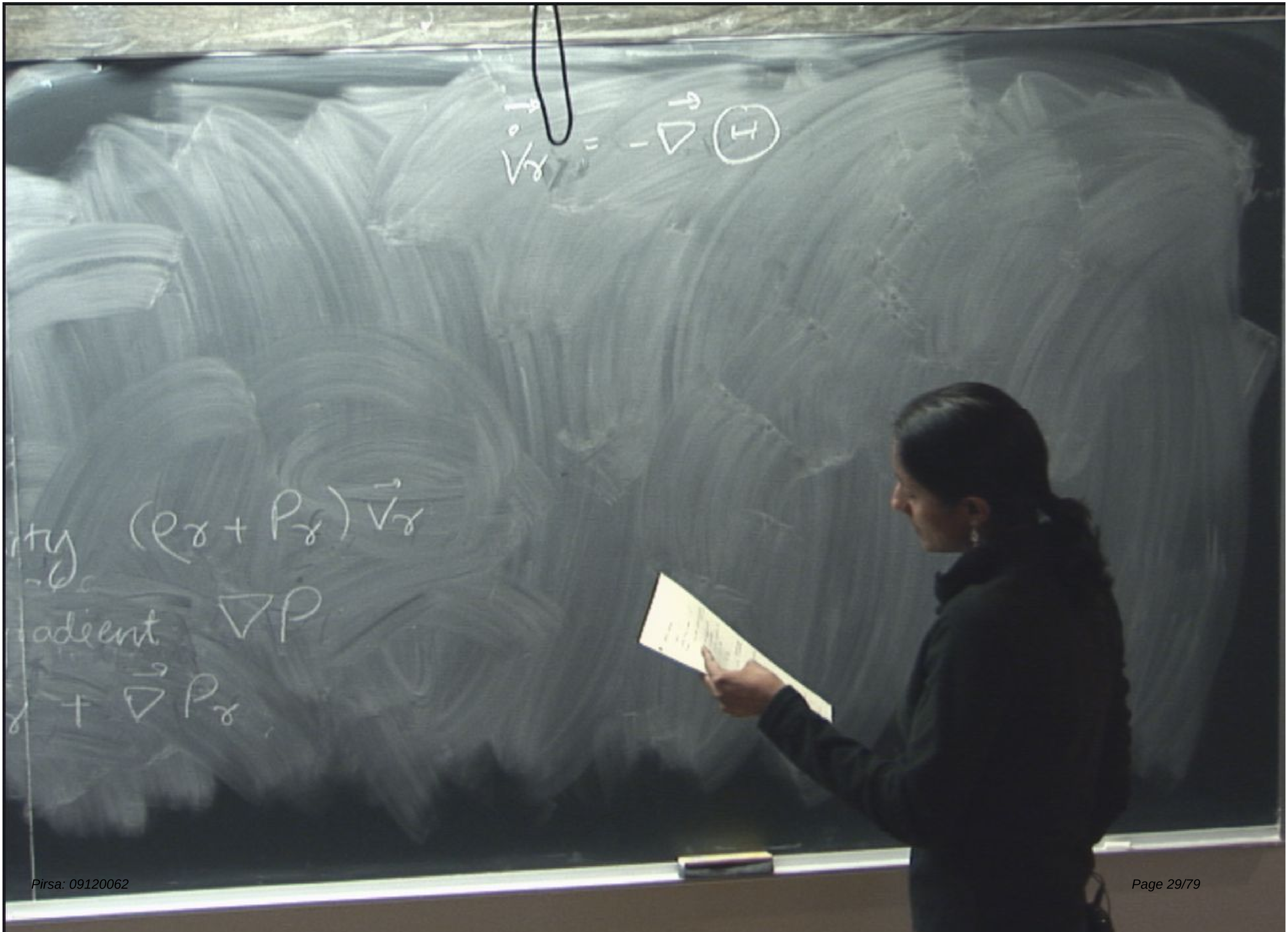
$\dot{\vec{V}}_\gamma = -\vec{\nabla}\Theta$
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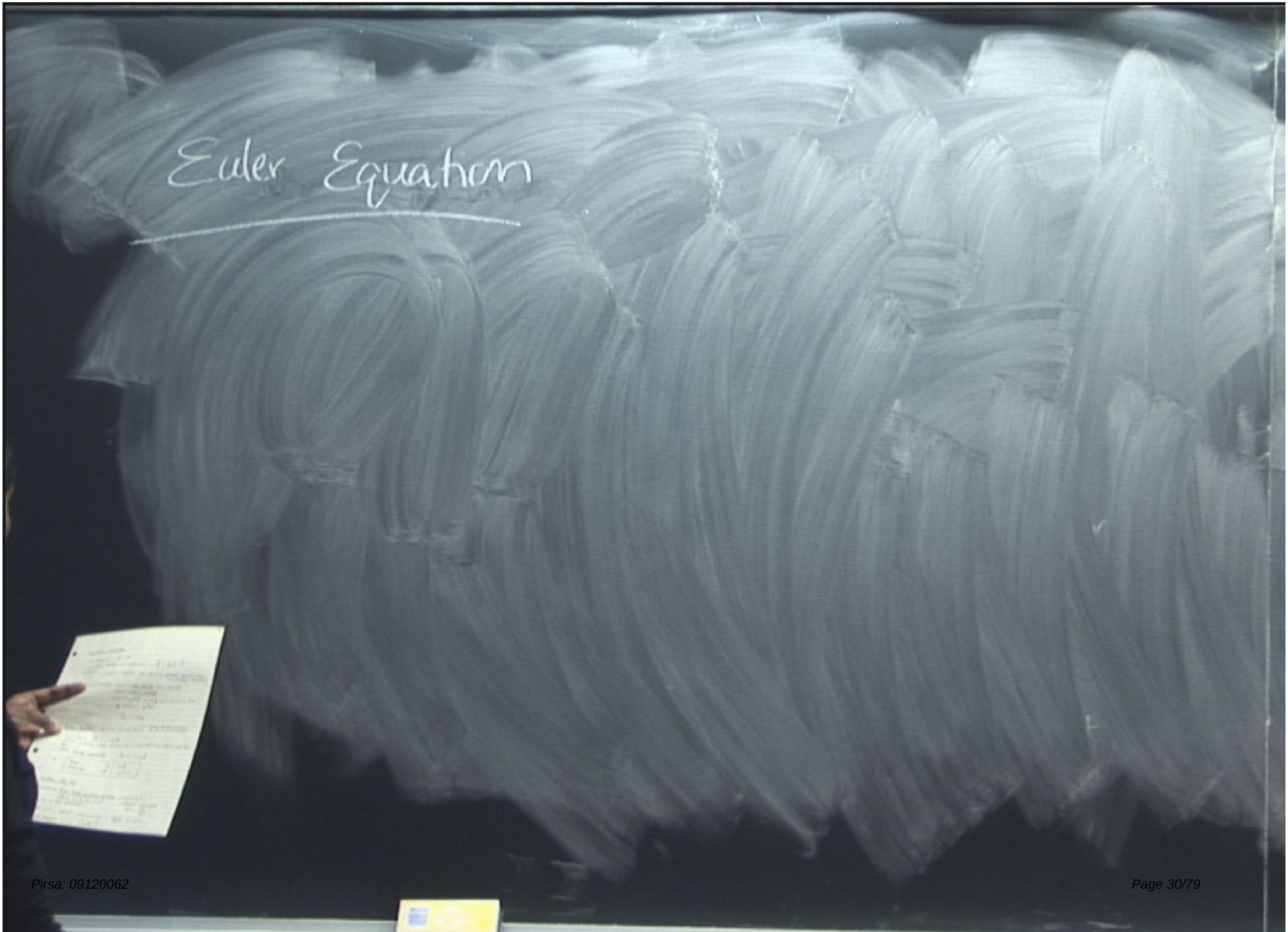
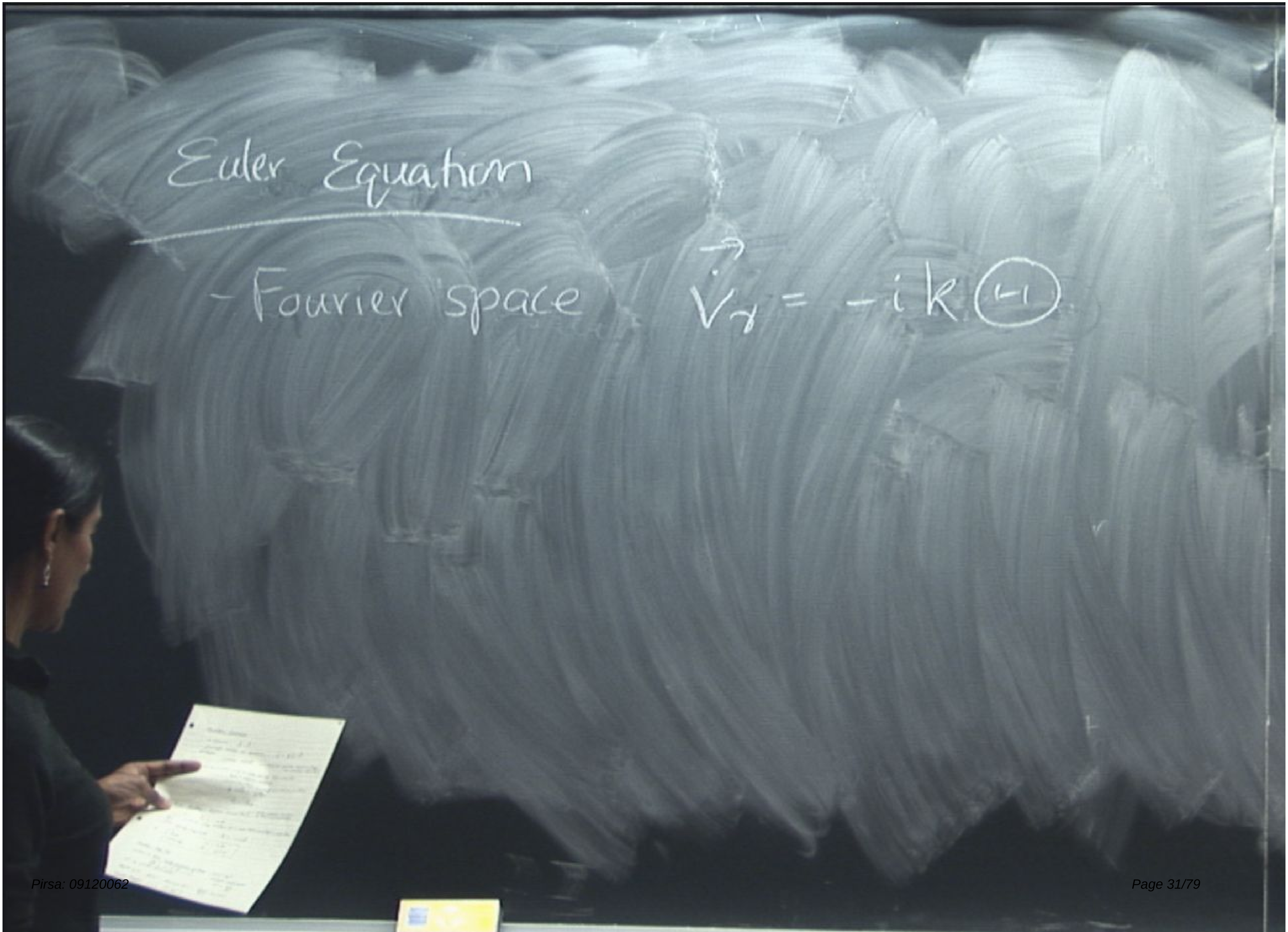
$\gamma + \vec{\nabla} P_\gamma$

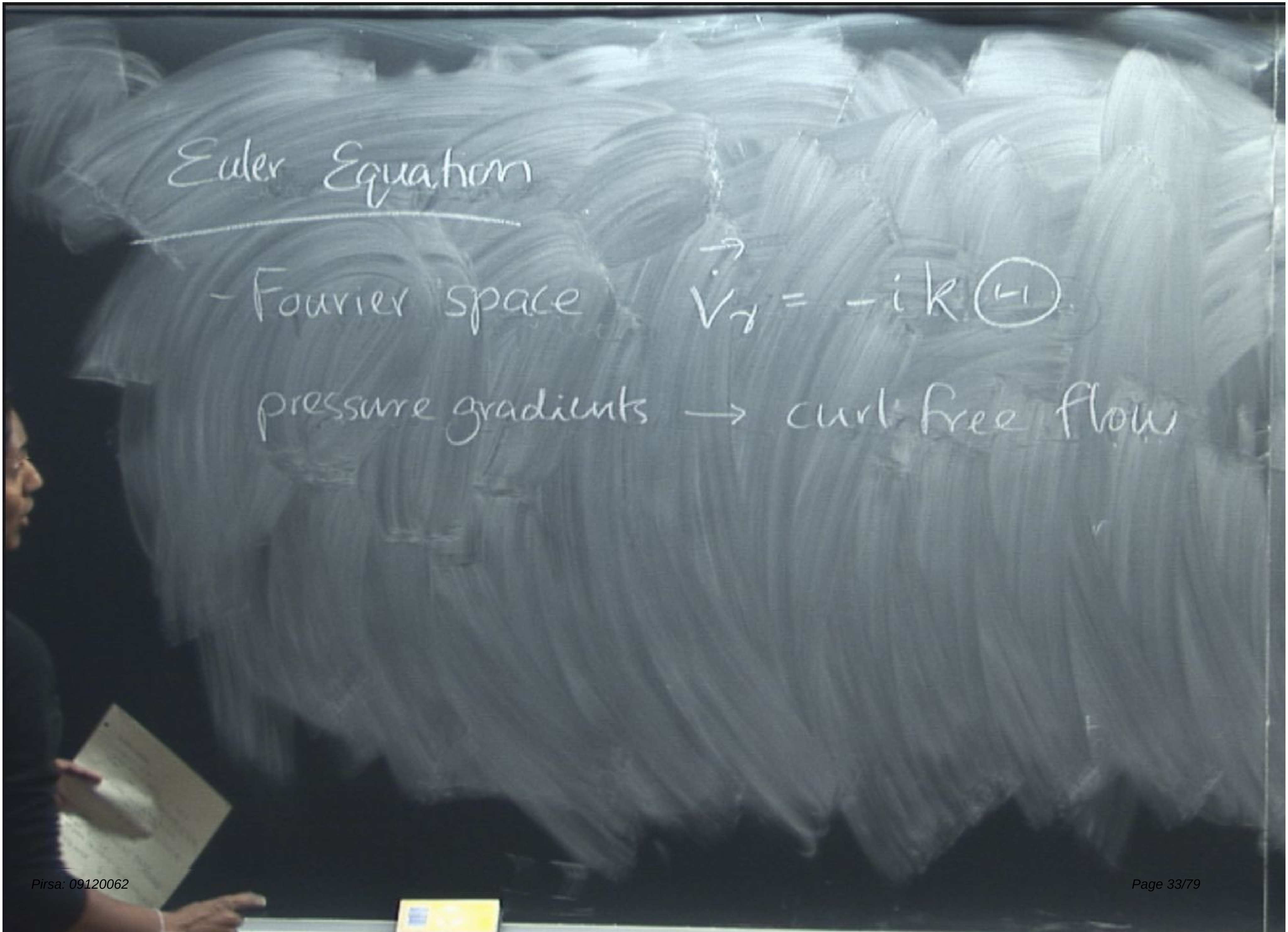

Euler Equation


Euler Equation
- Fourier space
$\vec{V}_\gamma = -ik$

Euler Equation

- Fourier space $\quad \dot{\vec{V}}_\gamma = -ik \, (\ldots)$

pressure gradients

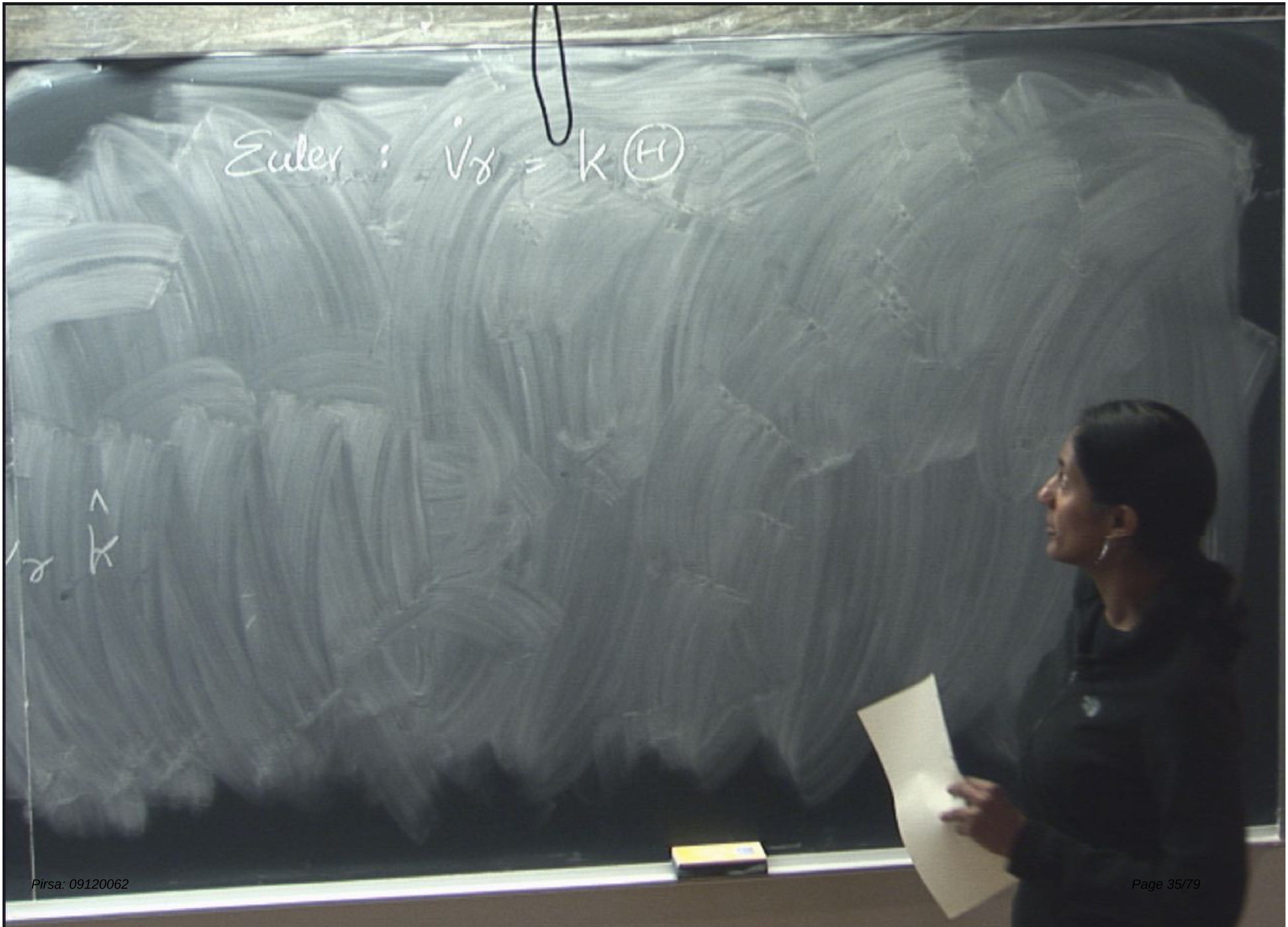

Euler Equation
- Fourier space $\dot{\vec{v}}_\gamma = -ik\,(-1)$
pressure gradients → curl free flow

Euler Equation

- Fourier space $\quad \dot{\vec{V}}_\gamma = -i\vec{k}\ \text{4-1}$

pressure gradients $\longrightarrow$ curl free flow

define velocity amplitude $\quad \vec{V}_\gamma \equiv -iV_\gamma \hat{k}$


Euler : $\dot{V}_\gamma = k$ (H)

Euler : $\dot{V}_\gamma = k\, \Theta$

Continuity : $\dot{\Theta} = -\frac{1}{3} k . V_\gamma$

$V_\gamma\, \hat{k}$

Euler: $\dot{V}_\gamma = k\,\Theta$

Continuity: $\dot{\Theta} = -\frac{1}{3} k\, V_\gamma$

Oscillator: Take One

Combine these equations

simple harmonic oscillator

$$\ddot{\Theta} + c_s^2 k^2 \Theta = 0$$

$c_s^2 = \frac{1}{3}$ (photon dominated)

Oscillator: Take One

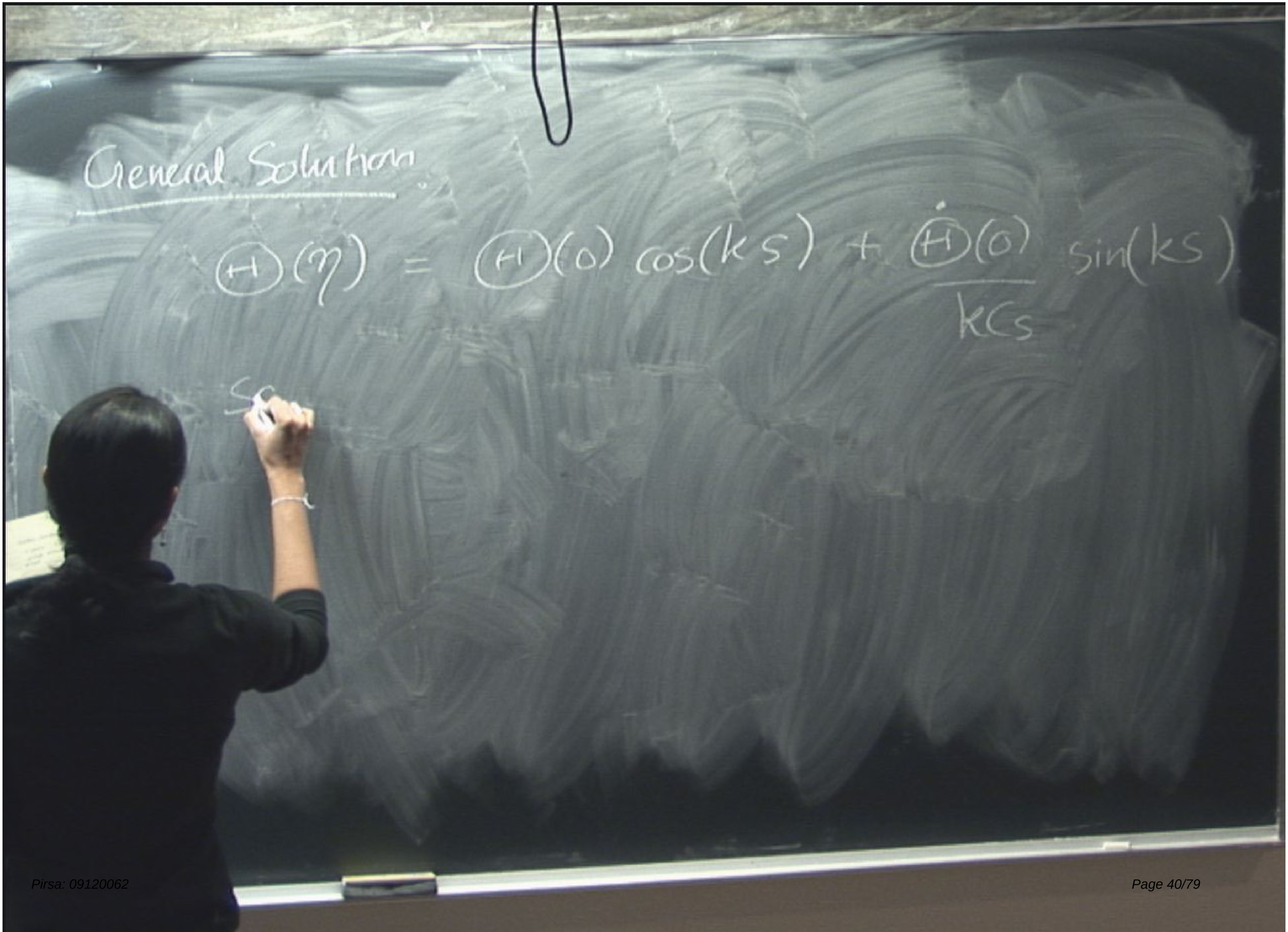
Combine these equations

simple harmonic oscillator

$$\ddot{\Theta} + c_s^2 k^2 \Theta = 0$$

$c_s^2 = \frac{1}{3}$ (photon dominated)

adiabatic sound speed $\quad c_s^2 \equiv \dfrac{\dot{P}_\gamma}{\dot{\rho}_\gamma}$


General Solution
$\Theta(\eta) = \Theta(0)\cos(ks) + \frac{\dot{\Theta}(0)}{kc_s}\sin(ks)$

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sound horizon $\quad s \equiv \int c_s \, d\eta$

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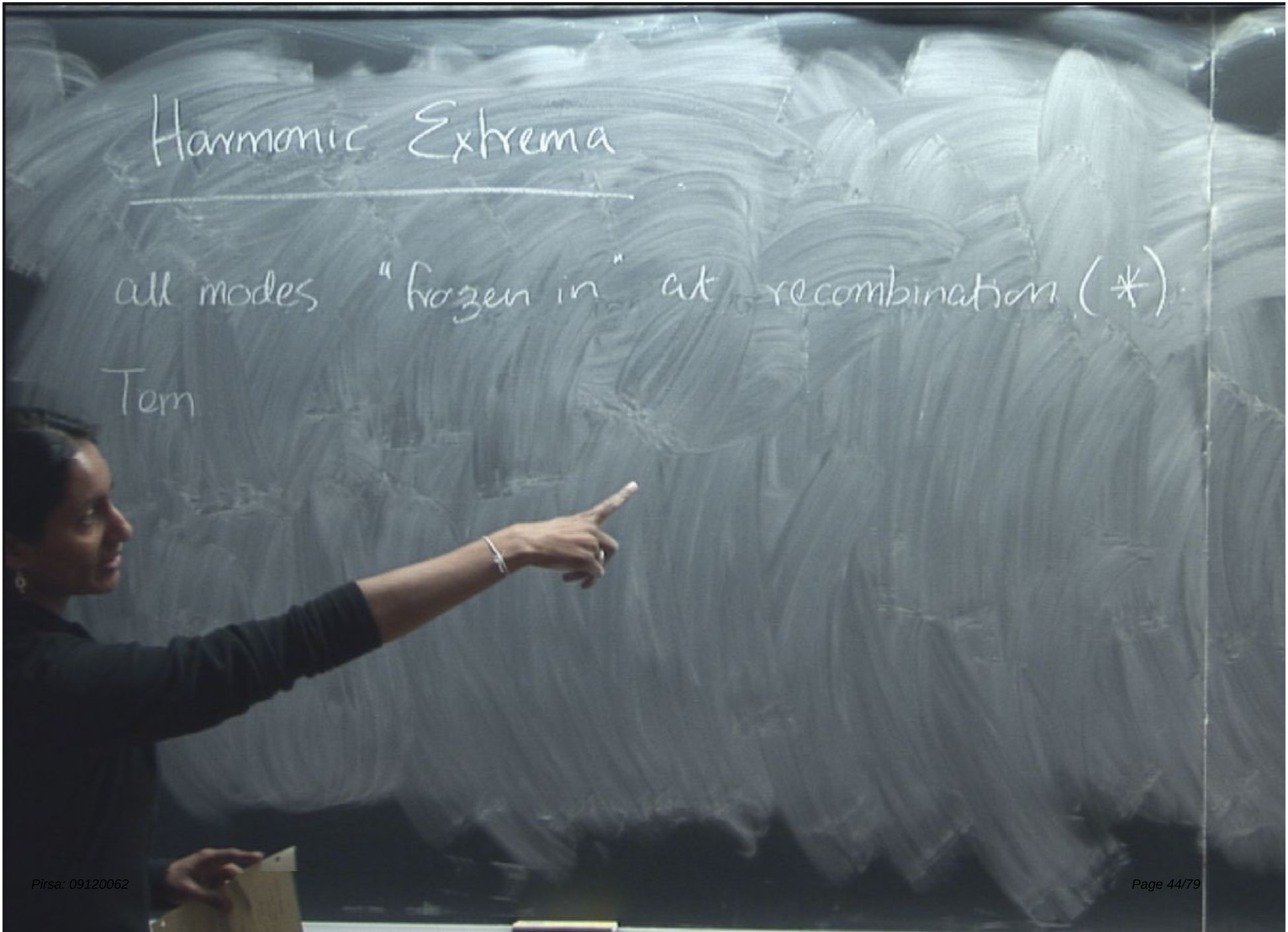
Harmonic Extrema


Harmonic Extrema
all modes "frozen in" at recombination (*).
Tem

Harmonic Extrema

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Temperature perturbations have different amplitud

for differen modes

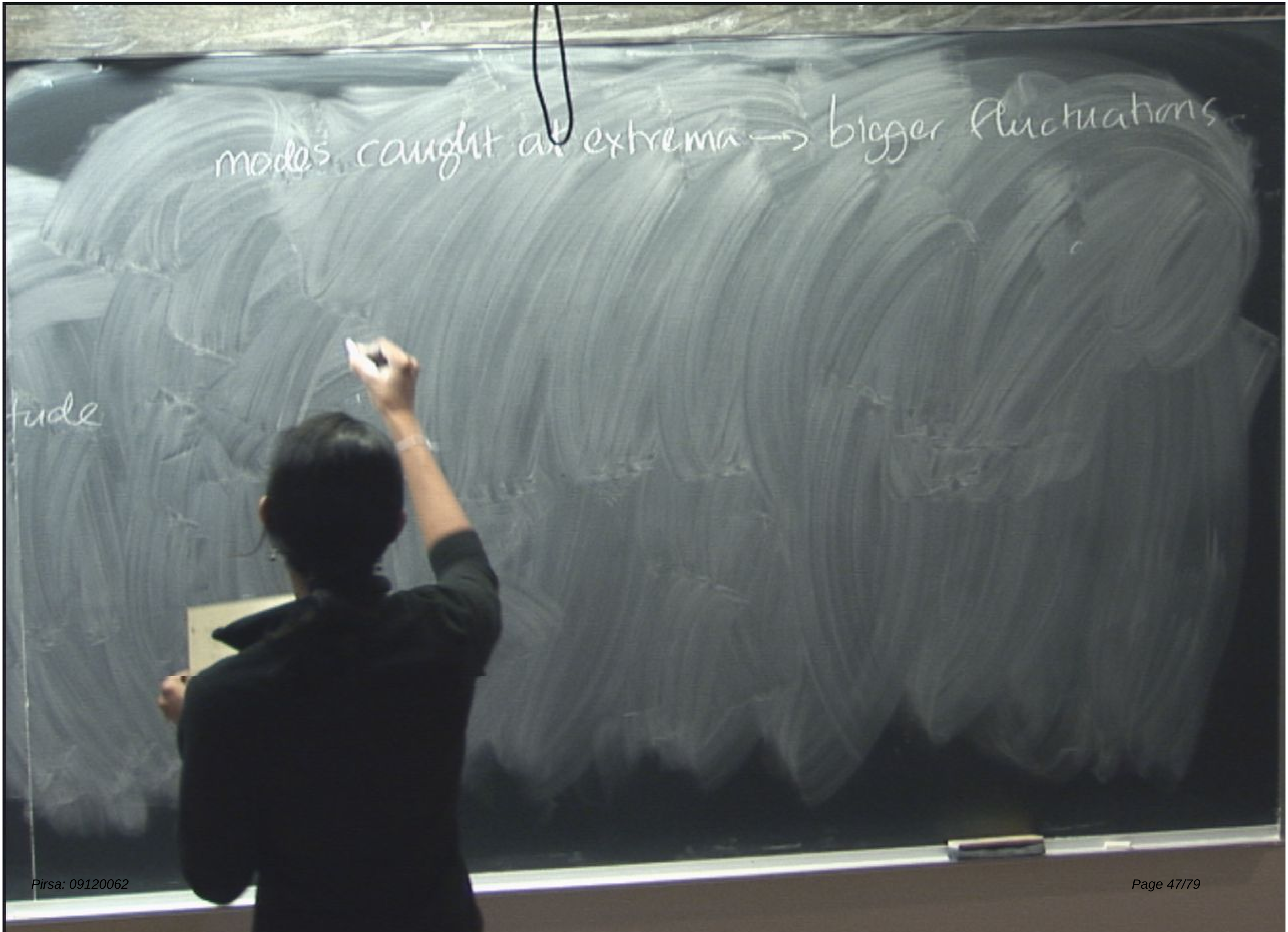
Harmonic Extrema

all modes "frozen in" at recombination ($*$).

Temperature perturbations have different amplitude
for different modes

adiabatic (curvature) mode $\dot{\Theta}(0) = 0$

$$\Theta(\eta_*) = \Theta(0) \cos(k s_*)$$


modes caught at extrema → bigger fluctuations
tude

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$$k_n s_* = n\pi$$

tude

modes caught at extrema → bigger fluctuations

$$k_n S_* = n\pi$$

fundamental frequency $= k_A = \frac{\pi}{S_*}$

tude

modes caught at extrema → bigger fluctuations

$k_n S_* = n\pi$

fundamental frequency = $\boxed{k_A = \frac{\pi}{S_*}}$

tude

harmonic relationship

to other extrema as 1 : 2 : 3

Peak location

fundamental physical scale

→ angular scale by simple projection at angular diameter distance D_A

Peak location

fundamental physical scale

→ angular scale by simple projection at angular diameter distance D_A

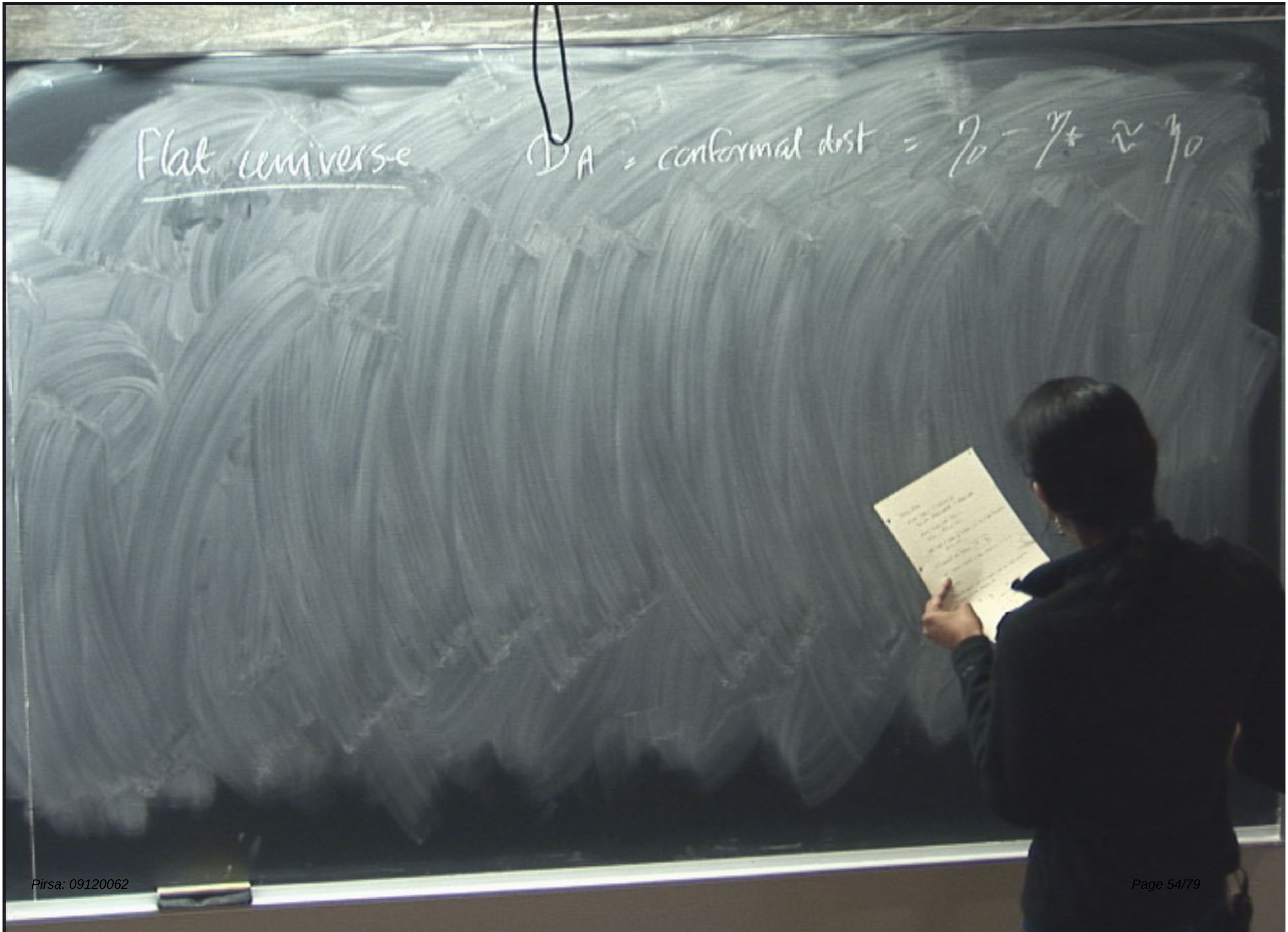
$$\Theta_A = \frac{x_A}{D_A}$$

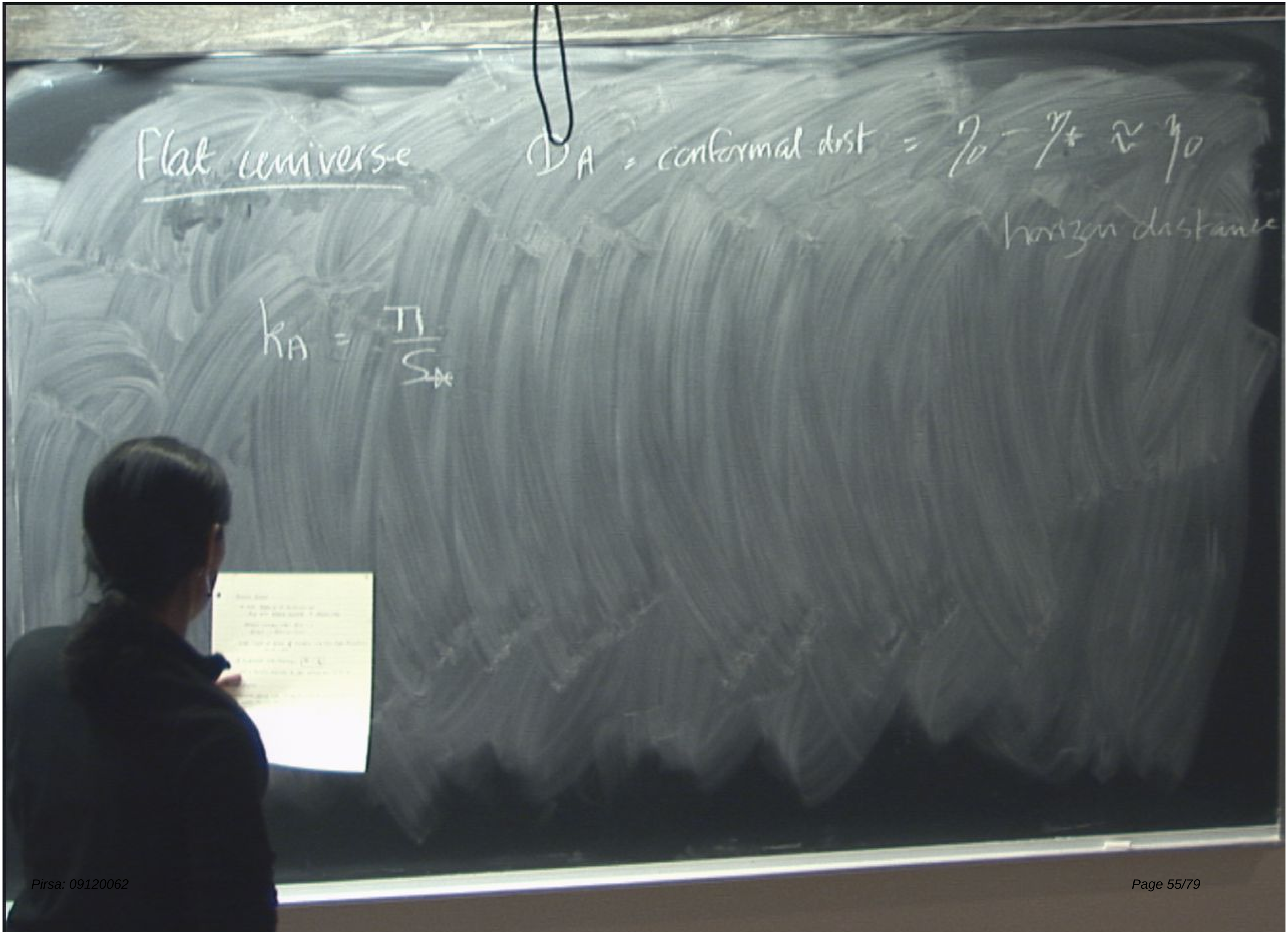
Peak location

fundamental physical scale

→ angular scale by simple projection at angular diameter distance D_A

$$\Theta_A = \frac{x_A}{D_A} \qquad l_A = k_A D_A$$


Flat universe
D_A = conformal dist = $\eta_0 - \eta_* \approx \eta_0$


Flat universe
D_A = conformal dist = $\eta_0 - \eta_* \simeq \eta_0$
horizon distance
$k_A = \frac{\pi}{s_{*}}$

Flat universe

D_A = conformal dist $= \eta_0 - \eta_* \simeq \eta_0$

horizon distance

$$k_A = \frac{\pi}{s_*} = \sqrt{3}\,\frac{\pi}{\eta_*}$$

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$$k_A = \frac{\pi}{s_*} = \sqrt{3}\,\frac{\pi}{\eta_*}$$

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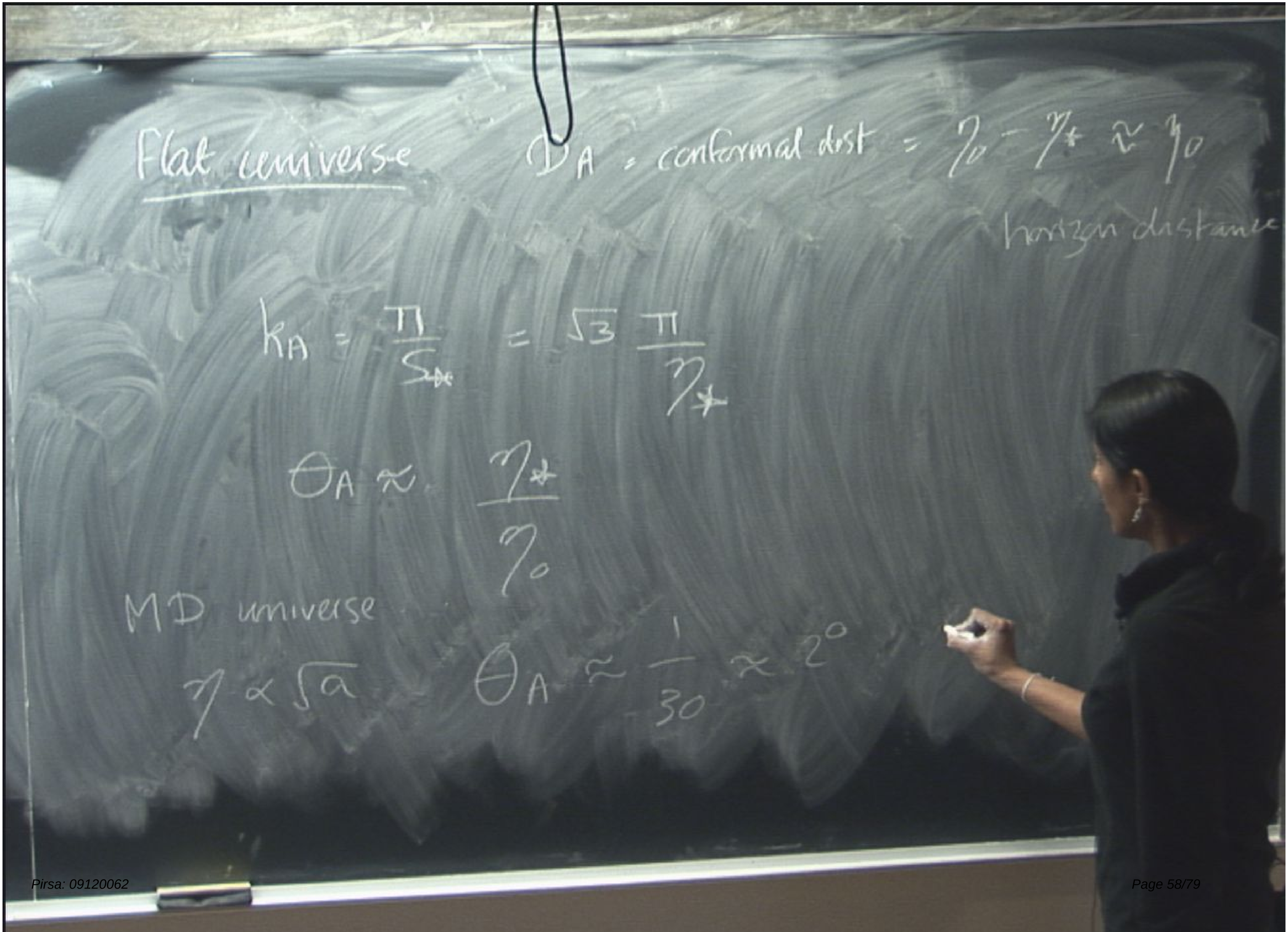
Flat universe
D_A = conformal dist = $\eta_0 - \eta_* \simeq \eta_0$
horizon distance
$k_A = \frac{\pi}{s_*} = \sqrt{3}\,\frac{\pi}{\eta_*}$
$\theta_A \approx \frac{\eta_*}{\eta_0}$
MD universe
$\eta \propto \sqrt{a}$
$\theta_A \approx \frac{1}{30} \approx 2^\circ$

Flat universe $\quad D_A = \text{conformal dist} = \eta_0 - \eta_* \simeq \eta_0$

horizon distance

$$k_A = \frac{\pi}{s_*} = \sqrt{3}\,\frac{\pi}{\eta_*}$$

$$\theta_A \approx \frac{\eta_*}{\eta_0}$$

MD universe

$\eta \propto \sqrt{a} \qquad \theta_A \simeq \frac{1}{30} \approx 2^\circ \qquad \ell_A \sim 200$

Doppler effect

Doppler effect

Bulk motion of fluid changes observed temperature via doppler shift

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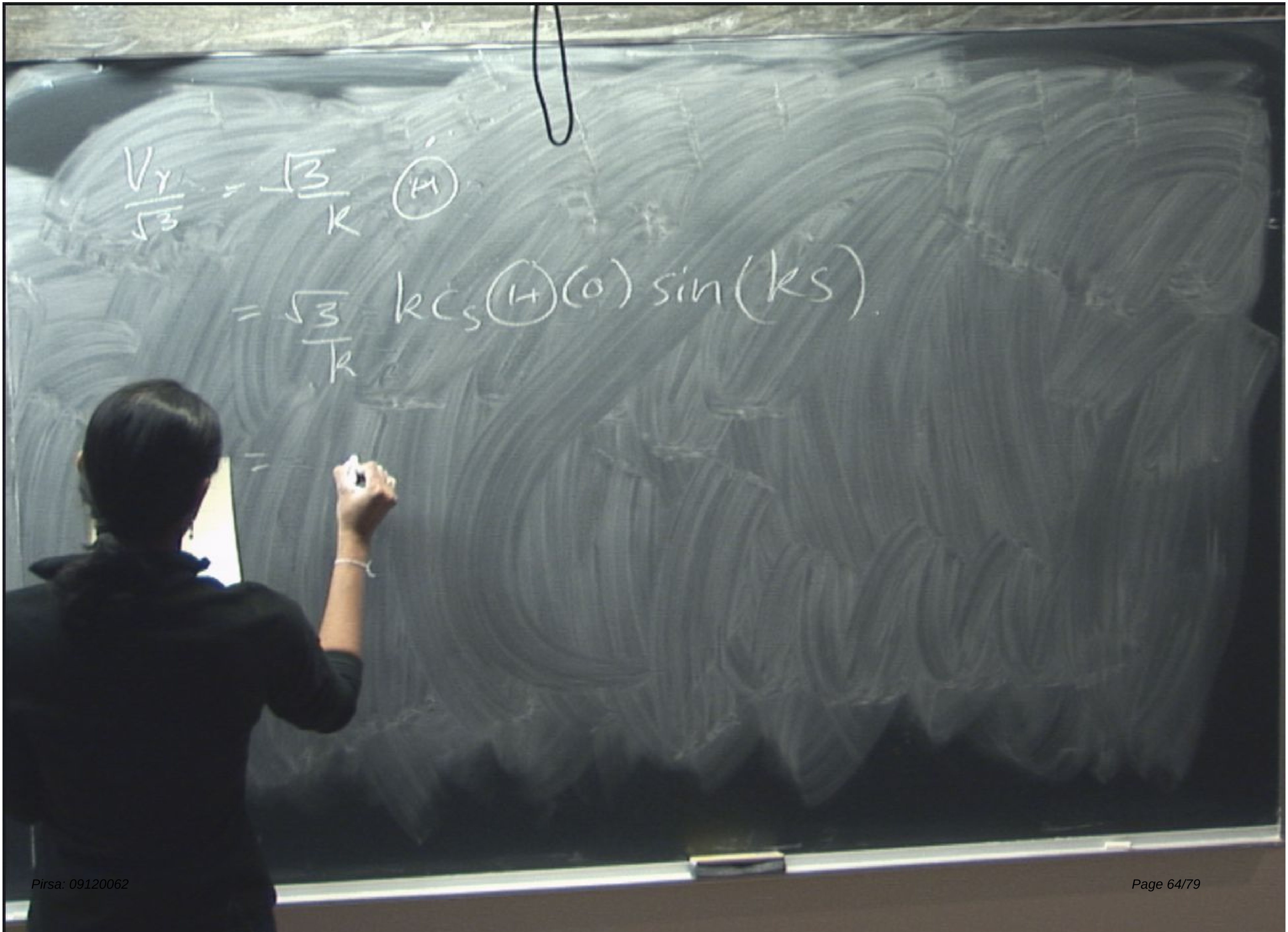
$$\left(\frac{\Delta T}{T}\right)_{\text{Doppler}} = \hat{n} \cdot \vec{V}_\gamma$$

Doppler effect

Bulk motion of fluid changes observed temperature via doppler shift

$$\left(\frac{\Delta T}{T}\right)_{\text{Doppler}} = \hat{n} \cdot \vec{V}_\gamma$$

avg over dir$\hat{n}$'s $\left(\frac{\Delta T}{T}\right)_{RMS} = \frac{V_\gamma}{\sqrt{3}}$



$\frac{V_\gamma}{\sqrt{3}} = \frac{\sqrt{3}}{k}\,\Theta_1$
$= \frac{\sqrt{3}}{k}\,k c_s\,\Theta_1(0)\sin(ks)$
$=$

$$\frac{V_Y}{\sqrt{3}} = \frac{\sqrt{3}}{k} \Theta$$

$$= \frac{\sqrt{3}}{k} k c_s \Theta(0) \sin(ks)$$

$$= \Theta(0) \sin(ks)$$

$$\frac{V_\gamma}{\sqrt{3}} = \frac{\sqrt{3}}{k}\dot{\Theta}$$

$$= \frac{\sqrt{3}}{k}\, k c_s \Theta(0)\sin(ks)$$

$$= \Theta(0)\sin(ks)$$

Doppler Peaks?

equal in amplitude $\frac{\pi}{2}$ out of phase r Peaks?

in amplitude $\frac{\pi}{2}$ out of phase

add in quadrature

$$\left(\frac{\Delta T}{T}\right)^2 = \Theta^2(0)\left[\cos^2(ks) + \sin^2(ks)\right] = \Theta^2(0)$$

$$\frac{V_\gamma}{\sqrt{3}} = \frac{\sqrt{3}}{k}\dot{\Theta}$$

$$= \frac{\sqrt{3}}{k} k c_s \Theta$$

$$= \Theta(0) \sin$$

Doppler Peaks?

dipole ⇒ doppler shifts in temp due to motion of the photon baryon plasma along l.o.s.

curl free ⇒ no contribution to Doppler

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oppler shifts in temp due to motion of

e photon baryon plasma along L.O.S.

ree $\Rightarrow$ no contribution to Doppler $\hat{n} \perp \hat{k}$

pler Peaks?

e $\Rightarrow$ doppler shifts in temp. due to motion of
the photon baryon plasma along l.o.s.

curl free $\Rightarrow$ no contribution to Doppler $\hat{n} \perp$

mathematically Doppler $\to$ dipole $\hat{n} \cdot \hat{k} \; \alpha \; Y_{1m}$

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e $\Rightarrow$ doppler shifts in temp due to motion of the photon baryon plasma along l.o.s.

curl free $\Rightarrow$ no contribution to Doppler $\hat{n} \perp$

mathematically Doppler $\rightarrow$ dipole $\hat{n}\cdot\hat{k} \propto Y_{1m}$

couple $Y_{\ell m}$ (projection) : $Y_{1m}\, Y_{\ell' m'}$

...es?

...pler shifts in temp due to motion of photon baryon plasma along l.o.s.

...e ⇒ no contribution to Doppler $\hat{n} \perp \hat{k}$

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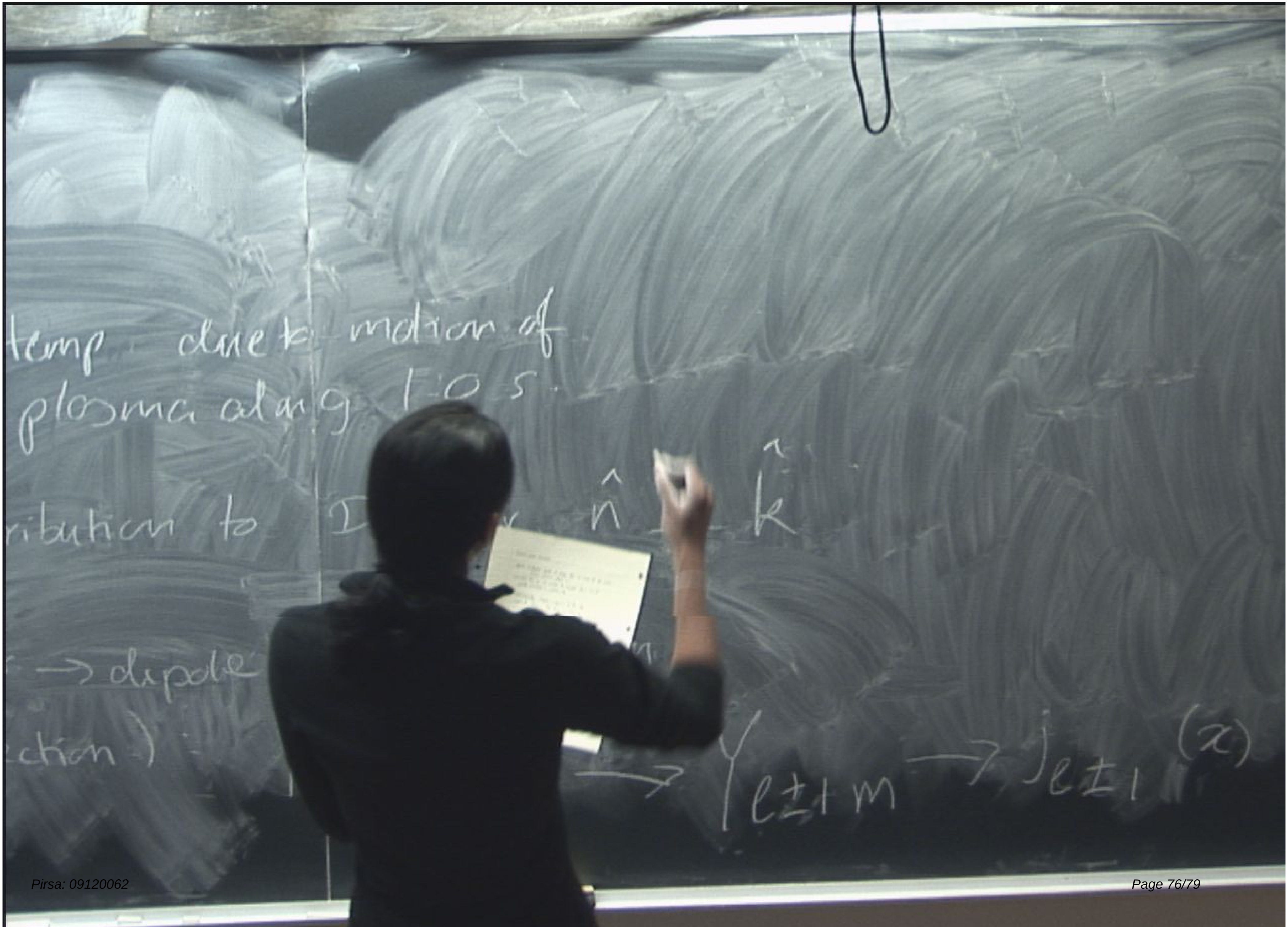
...ple $Y_{\ell m}$ (projection) : $Y_{1m} Y_{\ell m} \longrightarrow Y_{\ell \pm 1\, m}$

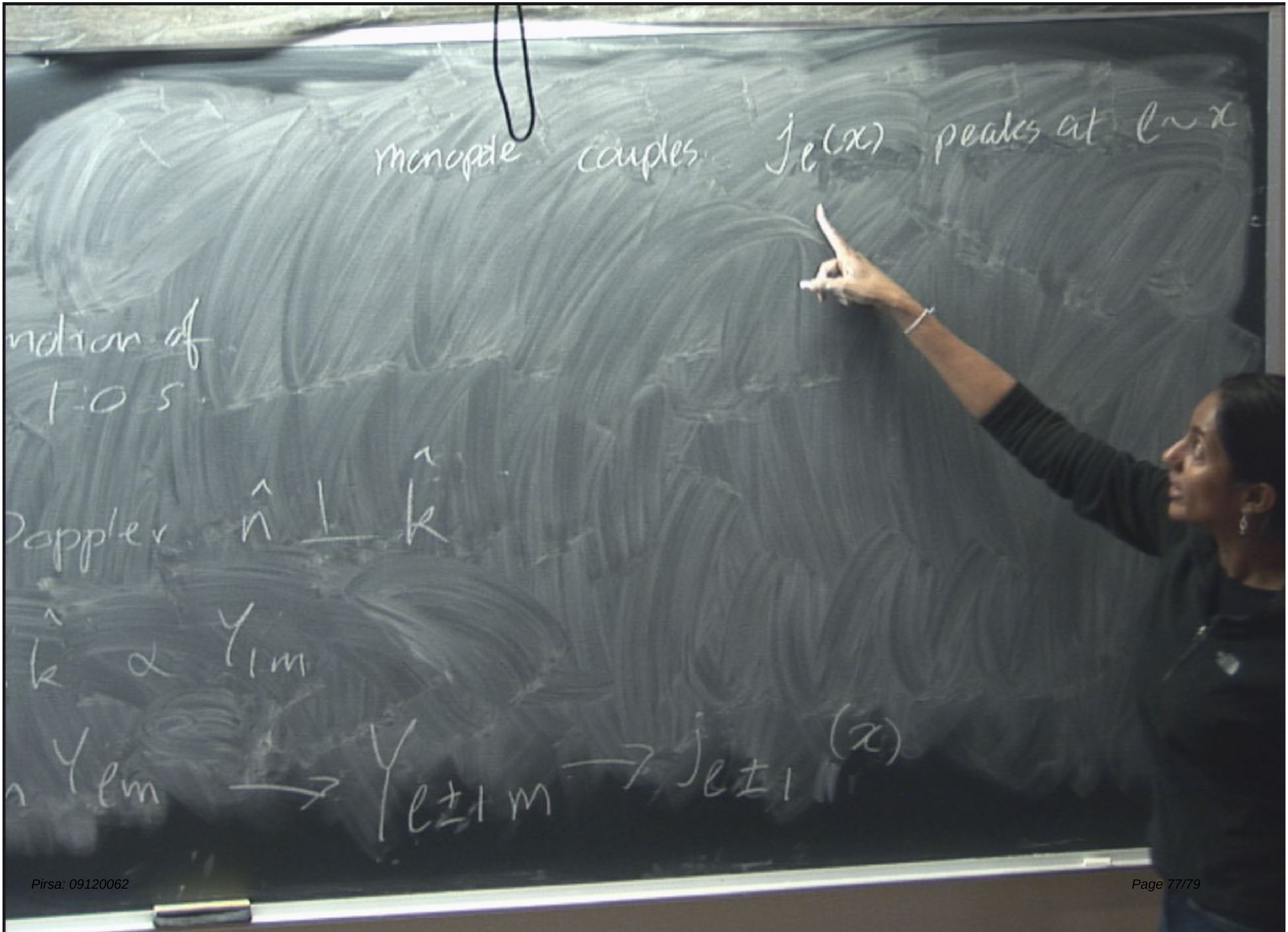
temp due to motion of
plasma along l.o.s.

ribution to Doppler $\hat{n} \perp \hat{k}$

→ dipole $\hat{n} \cdot \hat{k} \propto Y_{1m}$

ction) : $Y_{1m} Y_{\ell m} \longrightarrow Y_{\ell \pm 1\, m} \longrightarrow j_{\ell \pm 1}$


temp due to motion of
plasma along LOS
ribution to
→ dipole
ction)
→ Y l±1 m → j l±1 (x)


monopole couples $j_\ell(x)$ peaks at $\ell \sim x$
otion of
L.O.S.
Doppler $\hat{n} \perp \hat{k}$
$\hat{k} \propto Y_{1m}$
$Y_{\ell m} \longrightarrow Y_{\ell \pm 1\, m} \longrightarrow j_{\ell \pm 1}(x)$

monopole couples $j_\ell(x)$ peaks at $\ell \sim x$

dipole $\rightarrow$ $j_{\ell\pm1}(x)$

notion of
L.O.S.

Doppler $\hat{n} \perp \hat{k}$

$\hat{k} \propto Y_{1m}$

n $Y_{\ell m} \longrightarrow Y_{\ell\pm1\, m} \longrightarrow j_{\ell\pm1}(x)$

monopole couples $j_\ell(x)$ peaks at $\ell \sim x$

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...ppler $\hat{n} \perp \hat{k}$

$\propto Y_{1m}$

$Y_{\ell m} \longrightarrow Y_{\ell\pm1\, m} \longrightarrow j_{\ell\pm1}(x)$