

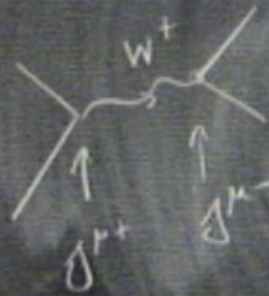
Title: Standard Model - Review (PHYS 622) - Lecture 12

Date: Dec 15, 2009 09:00 AM

URL: <http://pirsa.org/09120046>

Abstract:

$$S_{\mathcal{L}} = -\frac{4G_F}{\sqrt{2}} \left[ \underbrace{\delta_{\mu\nu}^+ J_{\mu}^{\nu}} + \left( J_{\mu}^3 - s_W^2 J_{\mu}^{EM} \right)^2 \right]$$



$$\partial_{\lambda} F^{\mu\nu} = -e j^{\mu} \delta^{\nu}_{\lambda}$$

$$\partial^{\lambda} A^{\nu} - \partial^{\nu} A^{\lambda} = -e j^{\nu} \delta^{\lambda}_{\nu}$$

$$\partial_\lambda F^{\lambda\nu} = -e_j^{\nu}$$

$$\partial^\lambda A^\nu - \partial^\nu A^\lambda = -e_j^{\nu}$$

$$\partial_\lambda A^\lambda = 0$$

$$\partial^\lambda A^\nu = -e_j^{\nu}$$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + D_\mu \phi^\dagger D^\mu \phi$$

$$D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

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$$D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

$$j^\mu = \frac{1}{e} \frac{\delta \mathcal{L}}{\delta A_\mu} = i(\partial_\mu \phi^\dagger \phi - \phi^\dagger \partial_\mu \phi)$$

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$$D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

$$j^\mu = \frac{1}{e} \frac{\delta \mathcal{L}}{\delta A_\mu} = i(\partial_\mu \phi^\dagger \phi - \phi^\dagger \partial_\mu \phi) + 2e^2 \phi^\dagger \phi A_\mu$$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + D_\mu \phi^\dagger D^\mu \phi - m^2 \phi^\dagger \phi \quad D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

$$j^\mu = \frac{1}{e} \frac{\delta \mathcal{L}}{\delta A_\mu} = i(\partial_\mu \phi^\dagger \phi - \phi^\dagger \partial_\mu \phi) + 2e^2 \phi^\dagger \phi A_\mu$$



$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + D_\mu \phi^\dagger D^\mu \phi - V(\phi) \quad D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

$$j^\mu = \frac{1}{e} \frac{\delta \mathcal{L}}{\delta A_\mu} = i(\partial_\mu \phi^\dagger \phi - \phi^\dagger \partial_\mu \phi) + 2e^2 \phi^\dagger \phi A_\mu$$



$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + D_\mu \phi^\dagger D^\mu \phi - V(\phi) \quad D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

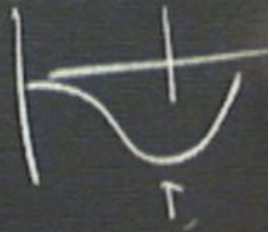
$$j^\mu = \frac{1}{e} \frac{\delta \mathcal{L}}{\delta A_\mu} = i(\partial_\mu \phi^\dagger \phi - \phi^\dagger \partial_\mu \phi) + 2e^2 \phi^\dagger \phi A_\mu$$

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + D_\mu \phi^\dagger D^\mu \phi - \mu^2 |\phi|^2 \quad D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

$$j^\mu = \frac{1}{e} \frac{\delta \mathcal{L}}{\delta A_\mu} = i(\partial_\mu \phi^\dagger \phi - \phi^\dagger \partial_\mu \phi) + 2e^2 \phi^\dagger \phi A_\mu$$

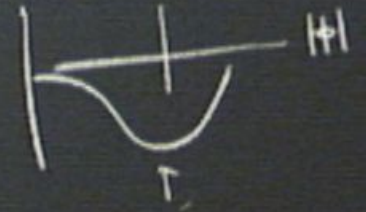
$$V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$



$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + D_\mu \phi^\dagger D^\mu \phi - \mu^2 |\phi|^2 \quad D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

$$j^\mu = \frac{1}{e} \frac{\delta \mathcal{L}}{\delta A_\mu} = i(\partial_\mu \phi^\dagger \phi - \phi^\dagger \partial_\mu \phi) + 2e^2 \phi^\dagger \phi A_\mu$$

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$



$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu \phi)^\dagger D^\mu \phi - \mu^2 |\phi|^2 \quad D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

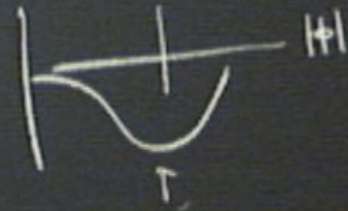
$$eA_\mu \gamma^\mu = \frac{\delta \mathcal{L}}{\delta A_\mu} = ie(\partial_\mu \phi^\dagger \phi - \phi^\dagger \partial_\mu \phi) + 2e^2 \phi^\dagger \phi A_\mu$$

$$eA_\mu \gamma^\mu$$

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$

$$\text{at min} \quad \phi = \frac{\mu}{\sqrt{2}\lambda}$$

$$\frac{\mu}{\sqrt{2}} e^{i\alpha}$$



$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu \phi)^\dagger D^\mu \phi - V(\phi) \quad D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

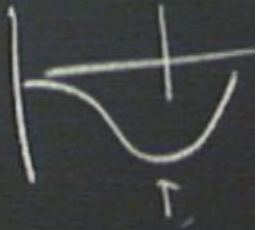
$$e_j^\mu = \frac{\delta \mathcal{L}}{\delta A_\mu} \Big|_{\text{min}} = ie(\partial_\mu \phi^\dagger \phi - \phi^\dagger \partial_\mu \phi) + 2e^2 \phi^\dagger \phi A_\mu$$

$$eA_\mu j^\mu$$

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$

$$\text{at min} \quad \phi = \frac{\mu}{\sqrt{2}\lambda} e^{i\alpha}$$

$$\frac{\mu}{\sqrt{2}\lambda} e^{i\alpha}$$



$$\partial_\mu F^{\mu\nu} = -e j^\nu$$

$$\partial^\lambda A^\nu - \partial^\nu A^\lambda = -e j^\nu$$

$$j^\nu =$$


$$\partial_\lambda F^{\lambda\nu} = -e j^\nu$$

$$\partial^\lambda A^\nu - \partial^\nu A^\lambda = -e j^\nu$$

$$\partial_\lambda A^\lambda = 0$$

$$e j^\nu = e^2 v^2 A_\nu$$

$$\partial^2 A^\nu = -e^2 v^2 A_\nu$$

$$(\partial^2 + e^2 v^2) A_\nu = 0$$



$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu \phi)^\dagger D^\mu \phi - V(\phi) \quad D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

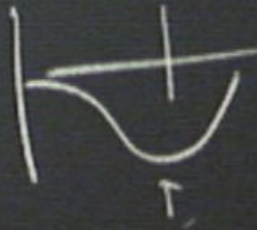
$$e_j^\mu = \frac{\delta \mathcal{L}}{\delta A_\mu} \Big|_{\text{r.h.s}} = ie(\partial_\mu \phi^\dagger \phi - \phi^\dagger \partial_\mu \phi) + 2e^2 \phi^\dagger \phi A_\mu$$

$$eA_\mu j^\mu$$

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$

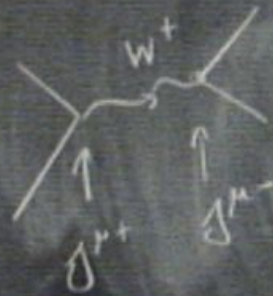
at min  $\phi = \frac{\mu}{\sqrt{2}\lambda}$

$$\frac{\mu}{\sqrt{2}\lambda} e^{i\alpha}$$



$$S_L = -\frac{4G_F}{\sqrt{2}} \left[ \underbrace{\delta_M^+ J^{\mu-}}_{\text{Diagram}} + (\underbrace{J_M^3 - g_W^2 J_M^{\text{EM}}}_{\text{Diagram}})^2 \right]$$

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 Guralnik Hagen  
 Brant Englert



$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu \phi)^\dagger D^\mu \phi - \mu^2 |\phi|^2 - \lambda |\phi|^4 \quad D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

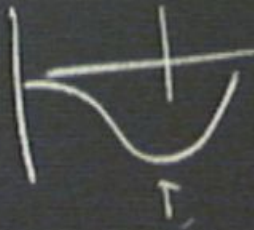
$$eA_\mu \mathcal{L} = \frac{\delta \mathcal{L}}{\delta A_\mu} = ie(\partial_\mu \phi^\dagger \phi - \phi^\dagger \partial_\mu \phi) + 2e^2 \phi^\dagger \phi A_\mu$$

$$eA_\mu \mathcal{L}$$

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$

at min  $\phi = \frac{\mu}{\sqrt{2}\lambda}$

$$\frac{\mu}{\sqrt{2}\lambda} e^{i\alpha}$$



$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu\phi)^\dagger D^\mu\phi - V(\phi) \quad D_\mu\phi = (\partial_\mu + ieA_\mu)\phi$$

$$\phi(x) \rightarrow e^{i\psi(x)} \phi(x)$$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu \phi)^\dagger D^\mu \phi - V(\phi) \quad D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

$$\phi(x) \rightarrow e^{i\sigma(x)} \phi(x)$$

choose a gauge where  $\phi$  is real everywhere

$$\phi(x) = \frac{1}{\sqrt{2}} (\nu + \sigma(x))$$

$$\mathcal{L} = -\frac{1}{2} F_{\mu\nu}^2 + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} e^2 (\nu + \sigma(x))^2 A_\mu^2 -$$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu \phi)^\dagger D^\mu \phi - V(\phi) \quad D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

$$\phi(x) \rightarrow e^{i\alpha(x)} \phi(x)$$

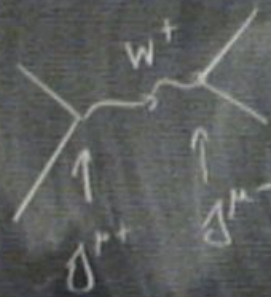
choose a gauge where  $\phi$  is real everywhere

$$\phi(x) = \frac{1}{\sqrt{2}} (v + \sigma(x))$$

$$\mathcal{L} = -\frac{1}{4} A_\mu (\partial^\mu \sigma - \partial^\nu \sigma) A_\nu + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} e^2 (v + \sigma(x))^2 A_\mu^2 - V(\sigma)$$

$$S_L = -\frac{4G_F}{\sqrt{2}} \left[ \delta_{\mu}^+ J^{\mu-} + \left( J_{\mu}^3 - g_W^{-1} J_{\mu}^{EM} \right)^2 \right]$$

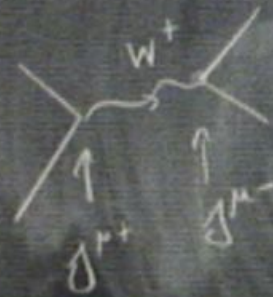
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$$[a_k^{\nu} a_k^{\nu+}] = g^{\mu\nu}$$

$$S_L = -\frac{4G_F}{\sqrt{2}} \left[ \underbrace{\delta_{\mu\nu}^+}_{\text{Higgs}} + (\delta_{\mu\nu}^3 - g_W^2 J_{\mu\nu}^{EM})^2 \right]$$

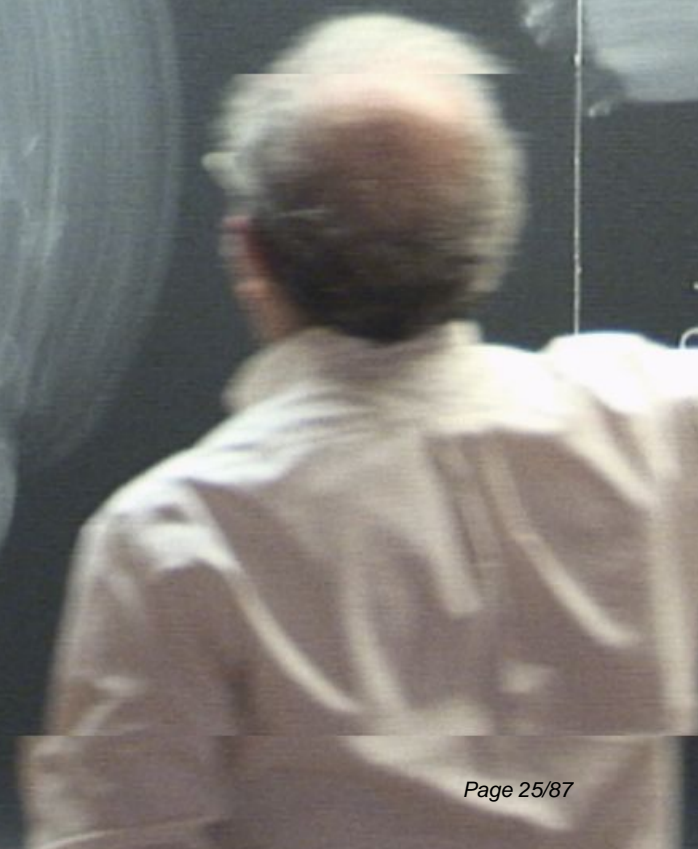
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$$[a_k^{\nu+} a_{k'}^{\nu+}] = g^{\mu\nu} \delta(\mathbf{k} - \mathbf{k}')$$



$SU(2)$  n. Higgs  $\phi^a$  red.



SU(2) n. Higgsi  $\phi^a$  real.

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^c)^2 + \frac{1}{2}(D_\mu \phi^a)^2 - V(\phi) \quad D_\mu \phi^a = \partial_\mu \phi^a + g \epsilon^{abc} A_\mu^b \phi^c$$

SU(2) v. Higgs  $\phi^a$  red.

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^c)^2 + \frac{1}{2}(D_\mu \phi^a)^2 - V(\phi) \quad D_\mu \phi^a = \partial_\mu \phi^a + g \epsilon^{abc} A_\mu^b \phi^c$$

SU(2) v. Higgs  $\phi^a$  red.

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^a)^2 + \frac{1}{2}(D_\mu \phi^a)^2 - V(\phi) \quad D_\mu \phi^a = \partial_\mu \phi^a + g \epsilon^{abc} A_\mu^b \phi^c$$

$$m \phi^a \langle \phi^a \rangle = v \sigma^{a3}$$

SU(2) n. Higgs  $\phi^a$  real.

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^a)^2 + \frac{1}{2}(D_\mu \phi^a)^2 - V(\phi) \quad D_\mu \phi^a = \partial_\mu \phi^a + g \epsilon^{abc} A_\mu^b \phi^c$$

$$m = 0 \quad \langle \phi^a \rangle = v \delta^{a3}$$

$$\frac{1}{2}(D_\mu \phi)^2 = \frac{1}{2} (g \epsilon^{abc} A_\mu^b \delta^{c3} v)^2$$

SU(2) n. Higgs  $\phi^a$  red.

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^a)^2 + \frac{1}{2}(D_\mu \phi^a)^2 - V(\phi) \quad D_\mu \phi^a = \partial_\mu \phi^a + g \epsilon^{abc} A_\mu^b \phi^c$$

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$$\frac{1}{2}(D_\mu \phi)^2 = \frac{1}{2} (g \epsilon^{abc} A_\mu^b \delta^{c3} v)^2$$

$$\epsilon^{abc} \epsilon^{ade} = \delta^{bd} \delta^{ce} - \delta^{be} \delta^{cd}$$

SU(2) n. Higgs  $\phi^a$  red.

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^a)^2 + \frac{1}{2}(D_\mu \phi^a)^2 - V(\phi) \quad D_\mu \phi^a = \partial_\mu \phi^a + g \epsilon^{abc} A_\mu^b \phi^c$$

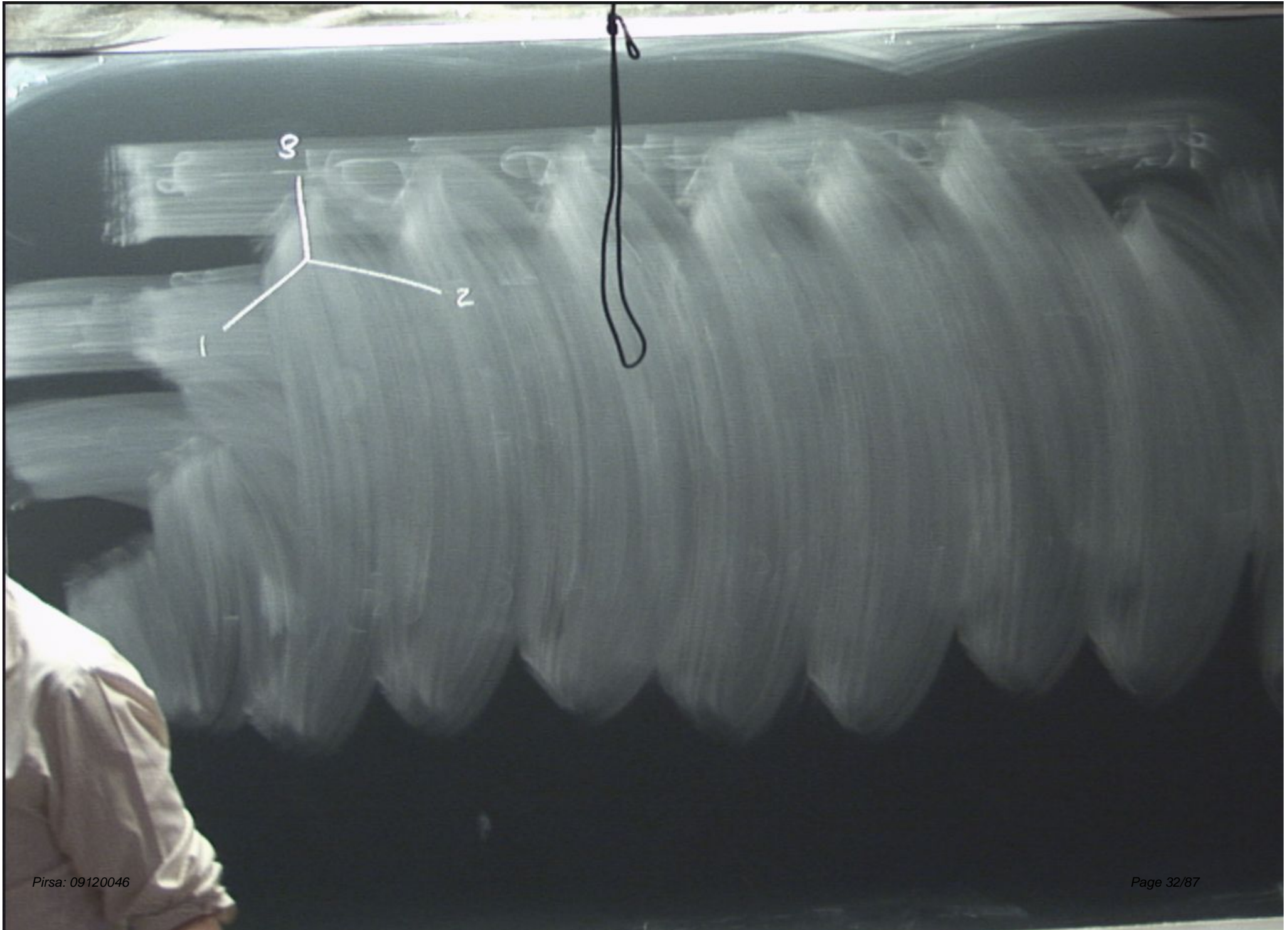
$$m = 0 \quad \langle \phi^a \rangle = v \delta^{a3}$$

$$\frac{1}{2}(D_\mu \phi)^2 = \frac{1}{2} (g \epsilon^{abc} A_\mu^b \delta^{c3} v)^2$$

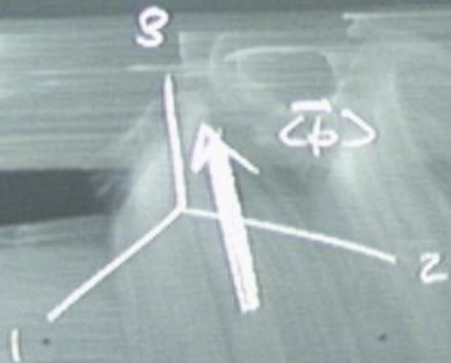
$$\epsilon^{abc} \epsilon^{ade} = \delta^{bd} \delta^{ce} - \delta^{be} \delta^{cd}$$

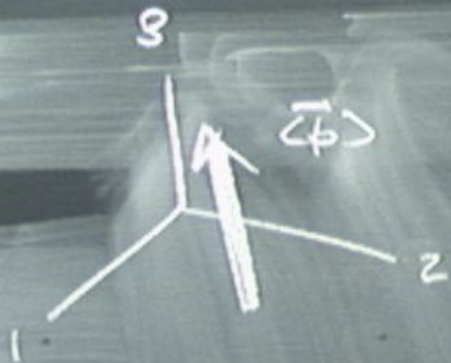
$$= \frac{1}{2} (gv)^2 (A_\mu^a A^{\mu a} - A_\mu^3 A^{\mu 3})$$

$$= \frac{1}{2} (gv)^2 [(A^1)^2 + (A^2)^2]$$

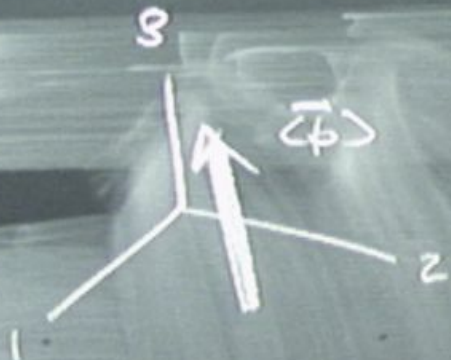




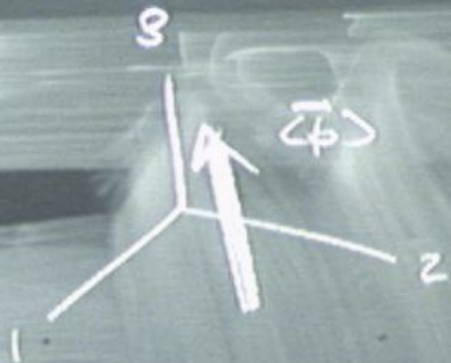




$$W^\ddagger = \frac{1}{\sqrt{2}} (A^1 \mp iA^2)$$

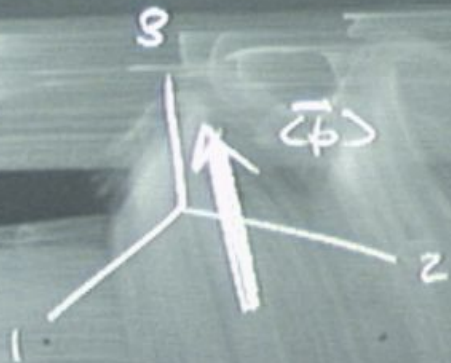


$$W^\pm = \frac{1}{\sqrt{2}} (A^1 \mp iA^2)$$



$$W_{\mu}^{\#} = \frac{1}{\sqrt{2}} (A^1 \mp iA^2)$$

$$A = A^3$$

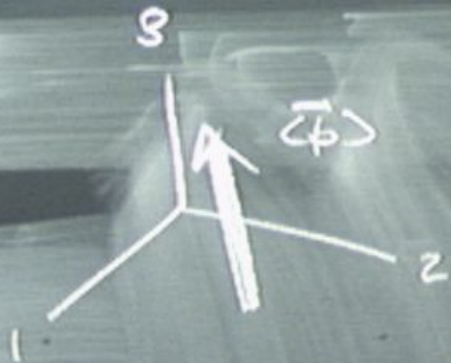


Glashow Weinberg Salam

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (A^1 \mp iA^2)$$

$$A = A^3$$

$$SU(2) \times U(1)$$



Glashow

Wernig

Slam

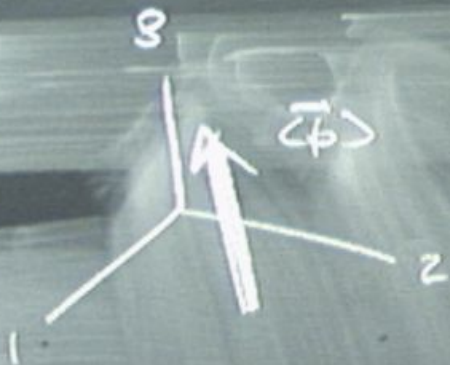
$SU(2) \times U(1)$

$\phi_\alpha$

2 of  $SU(2)$

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (A^1 \mp iA^2)$$

$$A = A^3$$



$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (A^1 \mp iA^2)$$

$$A = A^3$$

$$SU(2) \times U(1)$$

Glashow Weinberg Salam

$$\phi_{\alpha}$$

2 of SU(2)

+ 1/2 under U(1)

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$\tau^a = \frac{\sigma^a}{2}$$

SU(2) v. Higgs  $\phi^a$  red.

$$D_\mu \phi = (\partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi)$$



$$\tau^a = \frac{\sigma^a}{2}$$

SU(2) v. Higgsi  $\phi^a$  red.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\tau^a = \frac{\sigma^a}{2}$$

SU(2) v. Higgsi  $\phi^a$  red.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$|D_\mu \phi|^2$$

$T^a = \frac{\sigma^a}{2}$  SU(2) n. Higgs  $\phi^a$  red.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$|D_\mu \phi|^2 = \frac{1}{2} (0 \ 0) \left[ ig A_\mu^a \frac{\sigma^a}{2} + ig' B_\mu \frac{1}{2} \right] \left[ -ig A_\mu^a \frac{\sigma^a}{2} - ig' B_\mu \frac{1}{2} \right] \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$\tau^a = \frac{\sigma^a}{2}$  SU(2) v. Higgs  $\phi^a$  red.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$|D_\mu \phi|^2 = \frac{1}{2} (0 \ 0) \left[ ig A_\mu^a \frac{\sigma^a}{2} + ig' B_\mu \frac{1}{2} \right] \left[ -ig A_\mu^a \frac{\sigma^a}{2} - ig' B_\mu \frac{1}{2} \right] \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{1}{2} \frac{g^2}{4} v^2 \left[ (A_\mu^1)^2 + (A_\mu^2)^2 \right]$$

$\tau^a = \frac{\sigma^a}{2}$  SU(2) v. Higgs  $\phi^a$  red.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$|D_\mu \phi|^2 = \frac{1}{2} (0 \ 0) \left[ ig A_\mu^a \frac{\sigma^a}{2} + ig' B_\mu \frac{1}{2} \right] \left[ -ig A_\mu^a \frac{\sigma^a}{2} - ig' B_\mu \frac{1}{2} \right] \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{1}{2} \frac{1}{4} v^2 \left[ (g A_\mu^a)^2 + (g' B_\mu)^2 \right]$$

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (A^{\pm} \mp iA^2)$$

$$Z_{\mu}^0 =$$

$$\frac{gA_{\mu} - g'B_{\mu}}{\sqrt{g^2 + g'^2}}$$

$$B_{\mu} \left[ \begin{matrix} 0 \\ 1 \end{matrix} \right]$$

$$W_{\mu}^{\#} = \frac{1}{\sqrt{2}} (A^1 \mp iA^2)$$

$$Z_{\mu}^0 =$$

$$\frac{gA_{\mu}^3 - g'B_{\mu}}{\sqrt{g^2 + g'^2}}$$

$$A_{\mu} =$$

$$\frac{g'A_{\mu}^3 + gB_{\mu}}{\sqrt{g^2 + g'^2}}$$

$$B_{\mu} = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$Z_1^0 =$$

$$A_1 =$$

$$\frac{g A_1^3 - g' B_1}{\sqrt{g^2 + g'^2}}$$

$$\frac{g' A_1^3 + g B_1}{\sqrt{g^2 + g'^2}}$$

$$W_1^\pm = \frac{1}{\sqrt{2}} (A^1 \mp i A^2)$$

$$m_W^2 = \left(\frac{g v}{2}\right)^2$$

$$m_Z^2 = \frac{v^2}{2} (g^2 + g'^2)$$



$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (A^{\pm} \mp iA^3) \quad m_W^2 = \left(\frac{g v}{2}\right)^2$$

$$Z_{\mu}^0 = \frac{g A_{\mu}^3 - g' B_{\mu}}{\sqrt{g^2 + g'^2}} \quad m_Z^2 = \frac{v^2}{2} (g^2 + g'^2)$$

$$A_{\mu} = \frac{g' A_{\mu}^3 + g B_{\mu}}{\sqrt{g^2 + g'^2}} \quad m_A = 0$$

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (A^{\pm} \mp iA^3) \quad m_W^2 = \left(\frac{g v}{2}\right)^2$$

$$Z_{\mu}^0 = \frac{g A_{\mu}^3 - g' B_{\mu}}{\sqrt{g^2 + g'^2}} \quad m_Z^2 = \frac{v^2}{2} (g^2 + g'^2)$$

$$A_{\mu} = \frac{g' A_{\mu}^3 + g B_{\mu}}{\sqrt{g^2 + g'^2}} \quad m_A = 0$$

$$\phi \rightarrow e^{-i\alpha\sigma^3/2} e^{+i\alpha/2} \phi = e^{i\alpha} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \phi$$

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (A^{\pm} \mp iA^3) \quad m_W^2 = \left(\frac{g v}{2}\right)^2$$

$$Z_{\mu}^0 = \frac{g A_{\mu}^3 - g' B_{\mu}}{\sqrt{g^2 + g'^2}} \quad m_Z^2 = \frac{v^2}{2} (g^2 + g'^2)$$

$$A_{\mu} = \frac{g' A_{\mu}^3 + g B_{\mu}}{\sqrt{g^2 + g'^2}} \quad m_A = 0$$

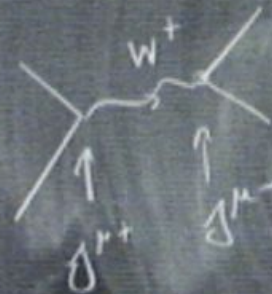
$$\phi \rightarrow e^{-i\alpha\sigma^3/2} e^{+i\alpha/2} \phi = e^{i\alpha} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \phi$$

$$\cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}}$$

$$\sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}} \left[ \underbrace{\delta_m^+ \delta_m^-}_{\text{Higgs}} + (\delta_m^3 - s_w^2 j_m^{EM})^2 \right]$$

Higgs, Kibble  
 Guralnik, Hagen  
 Brant, Englert



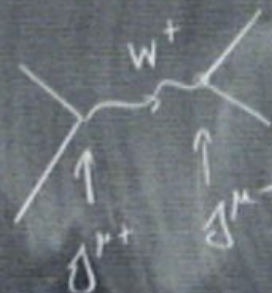
$$[a_k^{\nu+}, a_{k'}^{\nu+}] = -g^{\mu\nu} \delta(A-k')$$

$$\cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}}$$

$$\sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}} \left[ \underbrace{\delta_{ij}^+ J_i^\mu}_{\text{hadronic}} + (\delta_{ij}^3 - g_W^2 J_i^{\text{EM}})^\mu \right]$$

Higgs Kibble  
 Guralnik Hagen  
 Brant Englert



$$[a_k^\mu, a_{k'}^{\nu\dagger}] = g^{\mu\nu} \delta(\mathbf{k} - \mathbf{k}')$$

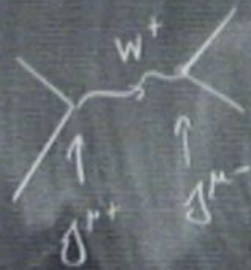
$$\cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}}$$

$$\sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$m_W = m_Z \cos \theta_w$$

$$S\mathcal{L} = -\frac{4G_F}{\sqrt{2}} \left[ \delta_{ij}^+ J_i^{\mu-} + (\delta_{ij}^3 - g_W^2 J_i^{\mu EM})^2 \right]$$

Higgs Kibble  
 Guralnik Hagen  
 Brant Englert



$$[a_k^\mu, a_{k'}^{\nu\dagger}] = -g^{\mu\nu} \delta(k-k')$$

$\tau^a = \frac{\sigma^a}{2}$  SU(2) v. Higgs  $\phi^a$  red.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$D_\mu \psi = \partial_\mu \psi - ig A_\mu^a t^a$$

$\tau^a = \frac{\sigma^a}{2}$  SU(2) v. Higgs  $\phi^a$  real.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$D_\mu \psi = \left( \partial_\mu \psi - ig A_\mu^a t^a - ig' B_\mu Y \right) \psi$$

↑  
"weak isospin"



$\tau^a = \frac{\sigma^a}{2}$  SU(2) v. Higgs  $\phi^a$  real.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$D_\mu \psi = \left( \partial_\mu \psi - ig A_\mu^a \overset{\uparrow}{t^a} - ig' B_\mu \overset{\uparrow}{Y} \right) \psi$$

"weak isospin"                      "hypercharge"

$$\begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$\begin{pmatrix} \nu \\ e^- \end{pmatrix}_L$$

$\tau^a = \frac{\sigma^a}{2}$  SU(2) v. Higgs  $\phi^a$  real.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$D_\mu \psi = \left( \partial_\mu \psi - ig A_\mu^a t^a - ig' B_\mu Y \right) \psi$$

↑
↑  
 "weak isospin"      "hypercharge"

$\begin{pmatrix} u \\ d \end{pmatrix}_L$        $\begin{pmatrix} \nu \\ e \end{pmatrix}_L$        $\nu_R$     $d_R$     $e_R^-$

$$\tau^a = \frac{\sigma^a}{2}$$

SU(2) v. Higgs  $\phi^a$  real.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$D_\mu \psi = \left( \partial_\mu \psi - ig A_\mu^3 t^3 - ig' B_\mu Y \right) \psi$$

↑  
"weak isospin"

↑  
"hypercharge"

$$\begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$\begin{pmatrix} \nu \\ e^- \end{pmatrix}_L$$

$$u_R \quad d_R \quad e^-_R$$

$\tau^a = \frac{\sigma^a}{2}$  SU(2) v. Higgs  $\phi^a$  real.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$D_\mu \psi = \left( \partial_\mu \psi - ig A_\mu^3 \frac{\sigma^3}{2} - ig' B_\mu Y \right) \psi$$

↑
↑  
 "weak isospin"      "hypercharge"

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L \quad \nu_R \quad d_R \quad e^-_R$$

$$u \quad I^3 = +\frac{1}{2} \quad d \quad I^3 = -\frac{1}{2}$$

$\tau^a = \frac{\sigma^a}{2}$  SU(2) v. Higgs  $\phi^a$  real.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$D_\mu \psi = \left( \partial_\mu \psi - ig A_\mu^3 \frac{\sigma^3}{2} - ig' B_\mu Y \right) \psi$$

↑
↑  
 "weak isospin"                      "hypercharge"

$$\begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$\begin{pmatrix} \nu \\ e^- \end{pmatrix}_L$$

$$u_R \quad d_R \quad e^-_R$$

$$u_L \quad I^3 = +\frac{1}{2}$$

$$u_R \quad I^3 = 0$$

$$d_L \quad I^3 = -\frac{1}{2}$$

$$d_R \quad I^3 = 0$$

$$\partial_\mu - ig(\cos\theta_w Z - \sin\theta_w A)I^3 - ig'(\sin\theta_w Z + \cos\theta_w A)Y$$

$$\sqrt{g^2 + g'^2} \cos\theta_w$$

$$Z^0 =$$

$$\frac{g A^3 - g' B^1}{\sqrt{g^2 + g'^2}}$$

$$m_Z^2 = \frac{g^2}{4} (g^2 + g'^2)$$

$$A_\mu =$$

$$\frac{g' A^3 + g B^1}{\sqrt{g^2 + g'^2}}$$

$$m_A = 0$$

$$\partial_\mu = \frac{1}{\sqrt{g_1 g_2} \cos \theta_\omega} \left( \cos \theta_\omega \partial_\omega Z - \sin \theta_\omega A \right) I^3 - i g' \left( \sin \theta_\omega Z + \cos \theta_\omega A \right) Y$$

$$\partial_\mu = -i \sqrt{g_1^2 + g_2^2} Z, I^3 -$$



$$\partial_1 = \frac{1}{\sqrt{g_1 g_2} \cos \theta_w} \left[ \cos \theta_w Z - \sin \theta_w A \right] I^3 - i g' \left( \sin \theta_w Z + \cos \theta_w A \right) Y$$

$$\partial_1 = -i \sqrt{g_1^2 + g_2^2} Z_1 \left[ I^3 - \sin^2 \theta_w (I^3 + Y) \right]$$



$$\partial_x = \frac{1}{\sqrt{g_1 g_2} \cos \theta_w} \left[ \cos \theta_w Z - \sin \theta_w A \right] I^3 - i g' (\sin \theta_w Z + \cos \theta_w A) Y$$

$$\partial_x = -i \sqrt{g_1^2 + g_2^2} Z \left[ I^3 - \sin^2 \theta_w (I^3 + Y) \right]$$

$$-i \sqrt{g_1^2 + g_2^2} \cos \theta_w \sin \theta_w A (I^3 + Y)$$

$$\partial_1 = \frac{1}{\sqrt{g_1 g_2} \cos \theta_w} \left[ \cos \theta_w Z - \sin \theta_w A \right] I^3 - ig' (\sin \theta_w Z + \cos \theta_w A) Y$$

$$\partial_1 = -i \sqrt{g_1^2 + g_2^2} Z_1 \left[ I^3 - \sin^2 \theta_w (I^3 + Y) \right]$$

$$-i \sqrt{g_1^2 + g_2^2} \cos \theta_w \sin \theta_w A_1 (I^3 + Y)$$

$$\partial_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left[ \cos\theta_\omega Z - \sin\theta_\omega A \right] I^3 - ig' \left( \sin\theta_\omega Z + \cos\theta_\omega A \right) Y$$

$$\partial_\mu = -i \sqrt{g^2 + g'^2} Z_\mu \left[ I^3 - \sin^2\theta_\omega (I^3 + Y) \right] - i \sqrt{g^2 + g'^2} \cos\theta_\omega \sin\theta_\omega A_\mu (I^3 + Y)$$

$$e = \sqrt{g^2 + g'^2} \cos\theta \sin\theta = \frac{gg'}{\sqrt{g^2 + g'^2}}$$

$$Q = I^3 + Y$$

$$\partial_x = \frac{1}{\sqrt{g^2 + g'^2}} \left[ ig (\cos \theta_w Z - \sin \theta_w A) I^3 - ig' (\sin \theta_w Z + \cos \theta_w A) Y \right]$$

$$\partial_x = -i \sqrt{g^2 + g'^2} Z \left[ I^3 - \sin^2 \theta_w (I^3 + Y) \right] - i \sqrt{g^2 + g'^2} \cos \theta_w \sin \theta_w A (I^3 + Y)$$

$$e = \sqrt{g^2 + g'^2} \cos \theta_w \sin \theta_w = \frac{gg'}{\sqrt{g^2 + g'^2}}$$

$$Q = I^3 + Y$$

$$\tau^a = \frac{\sigma^a}{2} \quad \text{SU(2) v. Higgs } \phi^a \text{ red.}$$

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$D_\mu \psi = \left( \partial_\mu \psi - ig A_\mu^3 \frac{\sigma^3}{2} - ig' B_\mu Y \right) \psi$$

$$= \left[ \partial_\mu - ie A_\mu Q - i \left( \frac{e}{\cos \theta_w \sin \theta_w} \right) (I^3 - \sin^2 \theta_w Q) \right] \psi$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad Y = -\frac{1}{2}$$

$$e_R \quad Y = -1$$

$$u_L \quad I^3 = +\frac{1}{2}$$

$$u_R \quad I^3 = 0$$

$$Y = \frac{2}{3}$$

$$d_L \quad I^3 = -\frac{1}{2} \quad Y = \frac{1}{6}$$

$$d_R \quad I^3 = 0$$

$$Y = -\frac{1}{3}$$

$\tau^a = \frac{\sigma^a}{2}$  SU(2) n. Higgs  $\phi^a$  red.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$D_\mu \psi = \left( \partial_\mu \psi - ig A_\mu^3 \frac{\sigma^3}{2} - ig' B_\mu Y \right) \psi$$

$$= \left( \partial_\mu - ie A_\mu Q - (ig' B_\mu) \left( I^3 - \sin^2 \theta_w Q \right) \right) \left[ \begin{matrix} \psi \\ \chi \end{matrix} \right] \psi$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad Y = -\frac{1}{2}$$

$$e_R \quad Y = -1$$

$$u_L \quad I^3 = +\frac{1}{2}$$

$$u_R \quad I^3 = 0$$

$$Y = \frac{2}{3}$$

$$Y = \frac{1}{2}$$

$\tau^a = \frac{\sigma^a}{2}$  SU(2) n. Higgs  $\phi^a$  red.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$D_\mu \psi = \left( \partial_\mu \psi - ig A_\mu^3 \frac{\sigma^3}{2} - ig' B_\mu Y \right) \psi$$

$$= \left[ \partial_\mu - ie A_\mu Q - i \left( \frac{e}{\sin\theta_w \cos\theta_w} \right) (I^3 - \sin^2\theta_w Q) Z_\mu \right] \psi$$

$\nu_e$	$Y = -\frac{1}{2}$	$u_L$	$I^3 = +\frac{1}{2}$	$d_L$	$I^3 = -\frac{1}{2}$	$Y = \frac{1}{6}$
$e_L$	$Y = -\frac{1}{2}$	$u_R$	$I^3 = 0$	$d_R$	$I^3 = 0$	
	$Y = -1$		$Y = \frac{2}{3}$		$Y = -\frac{1}{3}$	

$\tau^a = \frac{\sigma^a}{2}$  SU(2) n. Higgs  $\phi^a$  red.

$$D_\mu \phi = \left( \partial_\mu \phi - ig A_\mu^a \tau^a \phi - ig' B_\mu \frac{1}{2} \phi \right)$$

$$D_\mu \psi = \left( \partial_\mu \psi - ig A_\mu^3 \frac{\sigma^3}{2} - ig' B_\mu Y \right) \psi$$

$$= \left[ \partial_\mu - ie A_\mu Q - i \left( \frac{e}{\cos \theta_w \sin \theta_w} \right) \left( \underbrace{I^3 - \sin^2 \theta_w Q}_{Q_2} \right) \right] \psi$$

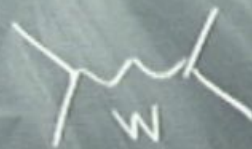
$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$	$Y = -\frac{1}{2}$	$u_L$	$I^3 = +\frac{1}{2}$	$d_L$	$I^3 = -\frac{1}{2}$	$Y = \frac{1}{6}$
		$u_R$	$I^3 = 0$	$d_R$	$I^3 = 0$	
$e_R$	$Y = -1$		$Y = \frac{2}{3}$		$Y = -\frac{1}{3}$	



$$\partial_r = -i \frac{\partial}{\partial \phi} \left[ W_{\uparrow}^+ j_{\uparrow}^+ + W_{\uparrow}^- j_{\uparrow}^- \right]$$

$(z_{\uparrow})_4$

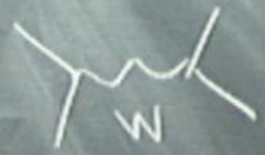
$$\left( \partial_\mu - i \frac{g}{2} \left[ W_\mu^+ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + W_\mu^- \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right] \right) \begin{pmatrix} \psi \\ \psi' \end{pmatrix}$$



$$\left( \frac{ig}{2} \right)^2 \frac{-i}{(q^2 - m_W^2)} \not{p}^+ \not{p}^-$$

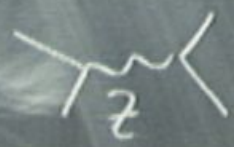
$\psi$  ] 4

$$\left( \partial_\mu - i \frac{g}{\sqrt{2}} \left[ W_\mu^+ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + W_\mu^- \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right] \right) \begin{pmatrix} \nu \\ d \end{pmatrix}$$



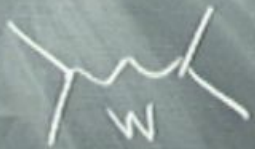
$$\left( \frac{ig}{\sqrt{2}} \right)^2 \frac{-i}{q^2 - m_W^2} j_\mu^+ j_\mu^-$$

] 4

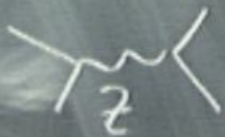
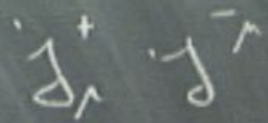


$$\frac{1}{2} \left( \frac{ig}{\cos\theta_W} \right)^2 \frac{-i}{q^2 - m_Z^2} \left( j_\mu^3 - \sin^2\theta_W j_\mu^{\text{EM}} \right)^2$$

$$\left( \rho_{\uparrow} - i \frac{g^2}{\sqrt{2}} \left[ W_{\uparrow}^+ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + W_{\uparrow}^- \sigma^- \right] \right) \begin{pmatrix} 2 \\ a \end{pmatrix}$$



$$\left( \frac{ig}{\sqrt{2}} \right)^2 \left( \frac{-i}{q^2 - m_W^2} \right)$$



$$\frac{1}{2} \left( \frac{ig}{\cos\theta_W} \right)^2$$

$$\left( \frac{-i}{q^2 - m_Z^2} \right)$$

$$q^2 \rightarrow 0$$

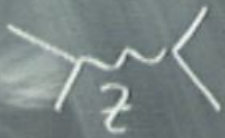
$$\frac{g^2}{2m_W^2}$$

$$= \frac{4G_F}{\sqrt{2}}$$

$$\left( \frac{1}{2} - i \frac{g^2}{\sqrt{2}} \left[ W_{\mu\nu}^+ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + W_{\mu\nu}^- \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right] \right) \begin{pmatrix} 2 \\ a \end{pmatrix}$$



$$\left( \frac{ig}{\sqrt{2}} \right)^2 \frac{-i}{g^2 - m_W^2} \begin{matrix} \nearrow \\ \searrow \end{matrix}$$



$$\frac{1}{2} \left( \frac{ig}{\cos\theta_W} \right)^2 \frac{-i}{g^2 - m_Z^2} \begin{matrix} \nearrow \\ \searrow \end{matrix}$$

$$g^2 \rightarrow 0$$

$$\frac{e^2}{2m_W^2 \sin^2\theta_W} = \frac{g^2}{2m_W^2} = \frac{4G_F}{\sqrt{2}}$$

$$Z^{\mu} = g_{1/2}^{\mu}$$

SU(2) v. Higgs  $\phi^a$  real.

$$e^2 = 4\pi\alpha$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad Y = -\frac{1}{2}$$

$$e_R \quad Y = -1$$

$u_L$   
 $u_R$

$$I^3 = +\frac{1}{2}$$

$$I^3 = 0$$

$$Y = \frac{2}{3}$$

$d_L$

$$I^3 = -\frac{1}{2}$$

$d_R$

$$I^3 = 0$$

$$Y = -\frac{1}{3}$$

$$Z^{\mu} = \frac{g_1}{\sqrt{2}}$$

SU(2) v. Higgs  $\phi^a$  real.

$$e^2 = 4\pi\alpha = 1/129$$

$$\sin^2 \theta_w = 0.23$$

$$m_W = \left[ \frac{4\pi\alpha}{4\sqrt{2} G_F \sin^2 \theta_w} \right]^{\frac{1}{2}} =$$

$$m_Z = \left[ \frac{4\pi\alpha}{\sqrt{2} G_F \sin 2\theta_w} \right]^{\frac{1}{2}} =$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad Y = -\frac{1}{2}$$

$$e_R \quad Y = -1$$

$$u_L$$

$$I^3 = +\frac{1}{2}$$

$$I^3 = 0$$

$$Y = \frac{2}{3}$$

$$u_R$$

$$d_L$$

$$I^3 = -\frac{1}{2}$$

$$d_R$$

$$I^3 = 0$$

$$Y = -\frac{1}{3}$$

$$Z^{\mu} = g_{\mu\nu}^{\rho} \nu^{\rho}$$

SU(2) v. Higgs  $\phi^a$  real.

$$e^2 = 4\pi\alpha = 1/129$$

$$\sin^2 \theta_w = 0.23$$

$$m_W = \left[ \frac{4\pi\alpha}{4\sqrt{2} G_F \sin^2 \theta_w} \right]^{\frac{1}{2}} = 80.1 \text{ GeV}$$

$$m_Z = \left[ \frac{4\pi\alpha}{\sqrt{2} G_F \sin 2\theta_w} \right]^{\frac{1}{2}} = 91.3 \text{ GeV}$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad Y = -\frac{1}{2}$$

$$e_R \quad Y = -1$$

$$\begin{matrix} u_L \\ u_R \end{matrix}$$

$$I^3 = +\frac{1}{2}$$

$$I^3 = 0$$

$$Y = \frac{2}{3}$$

$$d_L$$

$$I^3 = -\frac{1}{2}$$

$$d_R$$

$$I^3 = 0$$

$$Y = -\frac{1}{3}$$



$$m_n \bar{u}_L u_R + h.c.$$

$$m_n \bar{u}_L u_R + h.c.$$

...

$$m_n \bar{u}_L u_R + h.c.$$

$$L = (\bar{\psi})_L$$

$$\delta \mathcal{L} = - \lambda e \bar{L}_L \cdot b$$

$$m_n \bar{u}_L u_R + h.c.$$

$$L = (\bar{\psi} \gamma^\mu \psi)$$

$\delta \mathcal{L}$

$$= - \partial_\mu \bar{L} \cdot \gamma^\mu e_R + h.c.$$

$$+ \frac{1}{2} \frac{1}{2} - 1 = 0$$

$$m_n \bar{u}_L u_R + h.c.$$

$$L = (\bar{\nu} e)_L$$

$\delta \mathcal{L} =$

$$- \lambda_e \bar{L} \cdot \sigma e_R + h.c.$$

$$+ \frac{1}{2} \frac{1}{2} - 1 = 0$$

$$m_e = \frac{\lambda_e v}{\sqrt{2}}$$

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= - \frac{\lambda_e v}{\sqrt{2}} \bar{e}_L e_R + h.c.$$

$$Q = \begin{pmatrix} 2 \\ 9 \end{pmatrix}$$

$$\mathcal{L} = -\lambda_d \overline{Q} \cdot \phi d_R - \lambda_u \overline{Q}_a \epsilon_{ab} \phi_b^c u_R$$

$$m_W = \left[ \frac{4\pi\alpha}{4\sqrt{2} G_F \sin^2 \theta_W} \right]^{\frac{1}{2}} = 80 \text{ GeV}$$

$$m_Z = \left[ \frac{4\pi\alpha}{\sqrt{2} G_F \sin 2\theta_W} \right]^{\frac{1}{2}} = 91.3 \text{ GeV}$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$Y = -\frac{1}{2}$$

$$e_R \quad Y = -1$$

$$u_L$$

$$I^3 = +\frac{1}{2}$$

$$I^3 = 0$$

$$Y = \frac{2}{3}$$

$$u_R$$

$$d_L$$

$$I^3 = -\frac{1}{2}$$

$$d_R$$

$$I^3 = 0$$

$$Y = -\frac{1}{3}$$

$$Q = \begin{pmatrix} 2 \\ 9 \end{pmatrix}$$

$$S_{\mathcal{L}} = -\lambda_d \overline{Q} \cdot \phi d_R - \lambda_u \overline{Q}_a \epsilon_{ab} \phi_b^* u_R$$

$\underbrace{\quad\quad\quad}_{\text{...}}$

$-\frac{1}{6} \quad -\frac{1}{2} \quad +\frac{2}{3}$

$$m_W = \left[ \frac{4\pi\alpha}{4\sqrt{2} G_F \sin^2 \theta_W} \right]^{\frac{1}{2}} = 80.1 \text{ GeV}$$

$$m_Z = \left[ \frac{4\pi\alpha}{\sqrt{2} G_F \sin 2\theta_W} \right]^{\frac{1}{2}} = 91.3 \text{ GeV}$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad Y = -\frac{1}{2}$$

$$e_R \quad Y = -1$$

$$u_L \quad I^3 = +\frac{1}{2}$$

$$u_R \quad I^3 = 0$$

$$u_R \quad Y = \frac{2}{3}$$

$$d_L \quad I^3 = -\frac{1}{2}$$

$$d_R \quad I^3 = 0$$

$$d_R \quad Y = -\frac{1}{3}$$