Title: Standard Model - Review (PHYS 622) - Lecture 10

Date: Dec 11, 2009 09:00 AM

URL: http://pirsa.org/09120042

Abstract:

PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^{a}) + \bar{q}\, i \not{D}\, q$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^{a})^2 + \bar{q}\, i \not{D}\, q$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\mathcal{L} = -\frac{1}{4}\left(F_{\mu\nu}^{a}\right)^2 + \bar{q}\, i \not{D}\, q$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$SU(2) \times SU(2) \times U(1)$

isospin

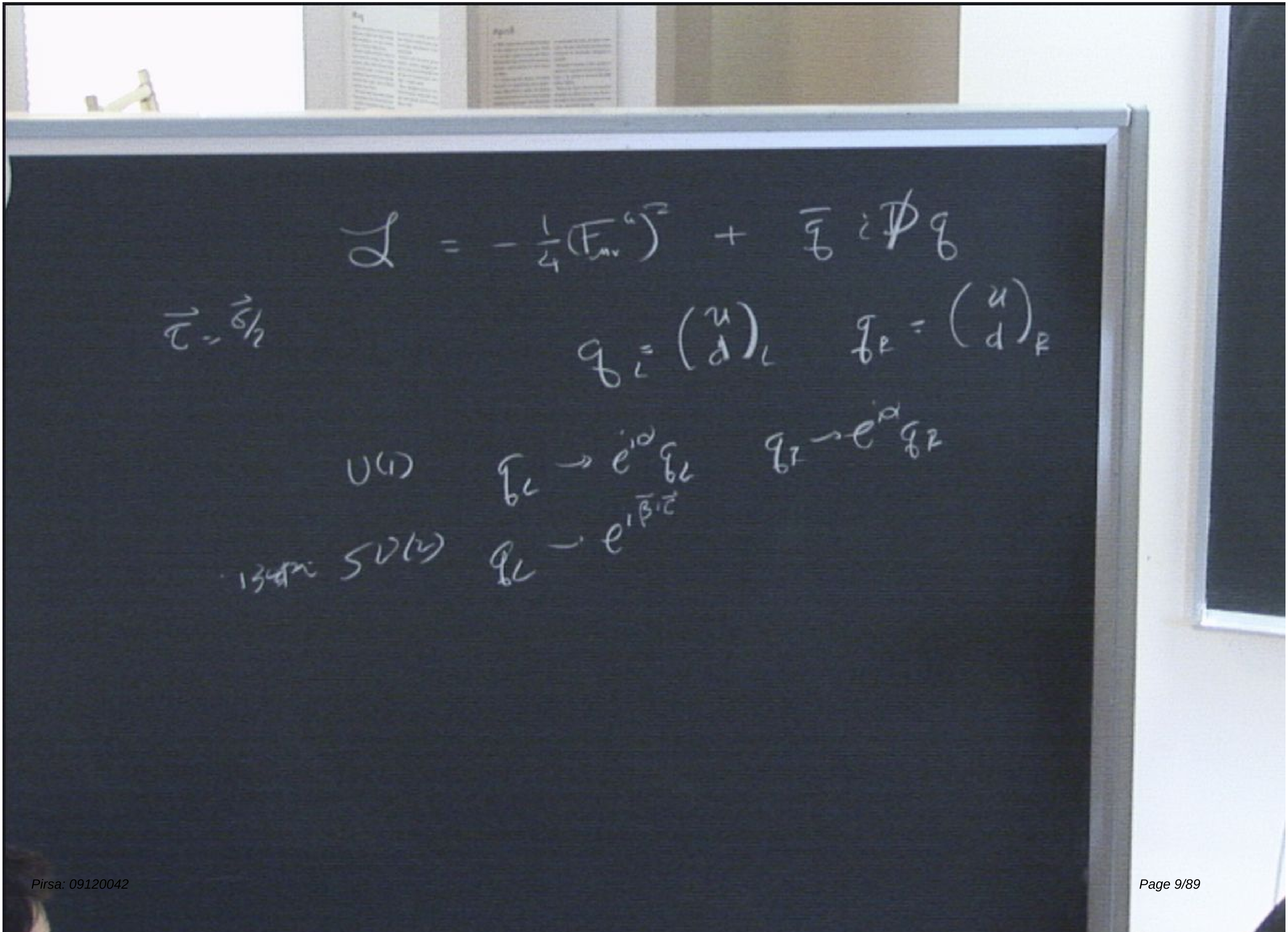
chiral isospin

baryon no.

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{q}\, i \not{D}\, q$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$U(1) \qquad q_L \to e^{i\alpha} q_L \qquad q_R \to e^{i\alpha} q_R$$


$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^a)^2 + \bar{q}\, i \not{D} q$
$\vec{\tau} = \vec{\sigma}/2$
$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$
$U(1) \quad q_L \to e^{i\alpha} q_L \quad q_R \to e^{i\alpha} q_R$
isospin $SU(2) \quad q_L \to e^{i\vec{\beta}\cdot\vec{\tau}}$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^{a})^2 + \bar{q}\, i \not{D}\, q$$

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$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$$

U(1) $\quad q_L \to e^{i\alpha} q_L \qquad q_R \to e^{i\alpha} q_R$

isospin SU(2) $\quad q_L \to e^{i\vec{\beta}\cdot\vec{\tau}} q_L \qquad q_R \to e^{i\vec{\beta}\cdot\vec{\tau}} q_R$

$$q_L \to e^{-i\vec{\gamma}\cdot\vec{\tau}} q_L \qquad q_R \to e^{i\vec{\gamma}\cdot\vec{\tau}} q_R$$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^{a})^2 + \bar{q}\, i \not{D}\, q$$

$$\vec{\tau} = \vec{\sigma}/2$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$q_L \to e^{-i\delta} q_L \qquad q_R \to e^{i\delta} q_R$$

$U(1)$ $\quad q_L \to e^{i\alpha} q_L \qquad q_R \to e^{i\alpha} q_R$

134m $SU(2)$ $\quad q_L \to e^{i\vec{\beta}\cdot\vec{\tau}} q_L \qquad q_R \to e^{i\vec{\beta}\cdot\vec{\tau}} q_R$

$$q_L \to e^{-i\vec{\gamma}\cdot\vec{\tau}} q_L \qquad q_R \to e^{i\vec{\gamma}\cdot\vec{\tau}} q_R$$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^{a})^2 + \bar{q}\, i \not{D}\, q$$

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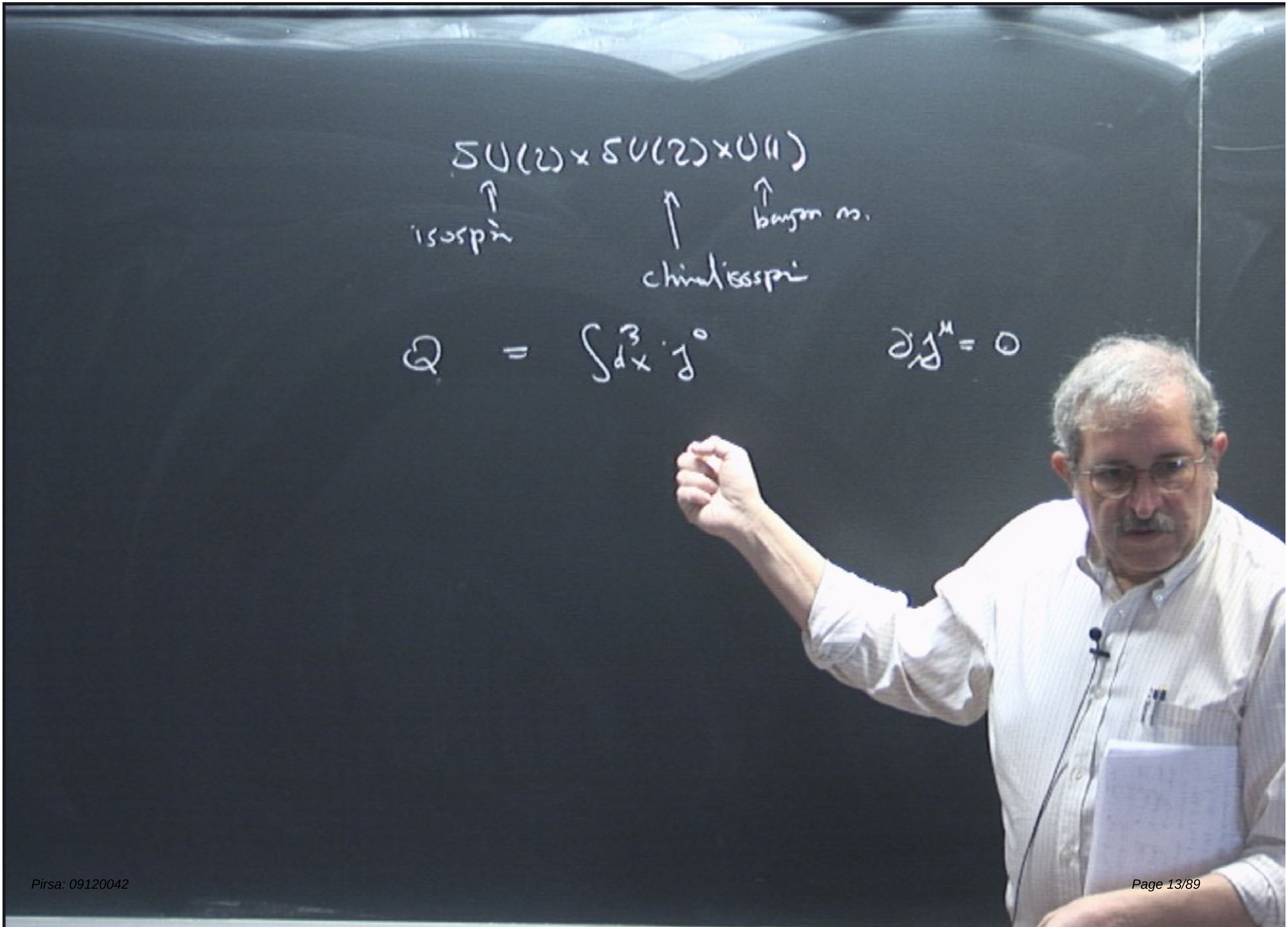
$$q_L \to e^{-i\delta} q_L \qquad q_R \to e^{i\delta} q_R$$

$$U(1) \qquad q_L \to e^{i\alpha} q_L \qquad q_R \to e^{i\alpha} q_R$$

$$\text{isospin} \quad SU(2) \qquad q_L \to e^{i\vec{\beta}\cdot\vec{\tau}} q_L \qquad q_R \to e^{i\vec{\beta}\cdot\vec{\tau}} q_R$$

$$q_L \to e^{-i\vec{\gamma}\cdot\vec{\tau}} q_L \qquad q_R \to e^{i\vec{\gamma}\cdot\vec{\tau}} q_R$$


SU(2) × SU(2) × U(1)
isospin
chiral isospin
baryon no.
Q = ∫d³x j⁰
∂μ jμ = 0

$SU(2) \times SU(2) \times U(1)$

isospin — chiral isospin — baryon no.

$$Q = \int d^3x \, j^0 \qquad \partial_\mu j^\mu = 0$$

$$j_B^\mu = \bar{q}_L \gamma^\mu q_L + \bar{q}_R \gamma^\mu q_R = \bar{q} \gamma^\mu q$$

$$j_I^{\mu a} = \bar{q}_L \gamma^\mu \tau^a q_L + \bar{q}_R \gamma^\mu \tau^a q_R = \bar{q} \gamma^\mu \tau^a q$$

$SU(2) \times SU(2) \times U(1)$

isospin ↑    chiral isospin ↑    baryon no. ↑

$$Q = \int d^3x \, j^0 \qquad \partial_\mu j^\mu = 0$$

$$j_B^\mu = \bar{q}_L \gamma^\mu q_L + \bar{q}_R \gamma^\mu q_R = \bar{q} \gamma^\mu q$$

$$j_I^{\mu a} = \bar{q}_L \gamma^\mu \tau^a q_L + \bar{q}_R \gamma^\mu \tau^a q_R = \bar{q} \gamma^\mu \tau^a q$$

$$j^{\mu 5a} = -\bar{q}_L \gamma^\mu \tau^a q_L + \bar{q}_R \gamma^\mu \tau^a q_R = \bar{q}$$

$SU(2) \times SU(2) \times U(1)$

isospin — chiral isospin — baryon no.

$$Q = \int d^3x \, j^0 \qquad \partial_\mu j^\mu = 0$$

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$$j^{\mu 5a} = -\bar{q}_L \gamma^\mu \tau^a q_L + \bar{q}_R \gamma^\mu \tau^a q_R = \bar{q} \gamma^\mu \gamma^5 \tau^a q$$

$SU(2) \times SU(2) \times U(1)$

$\uparrow$ isospin $\quad \uparrow$ chiral isospin $\quad \uparrow$ baryon no.

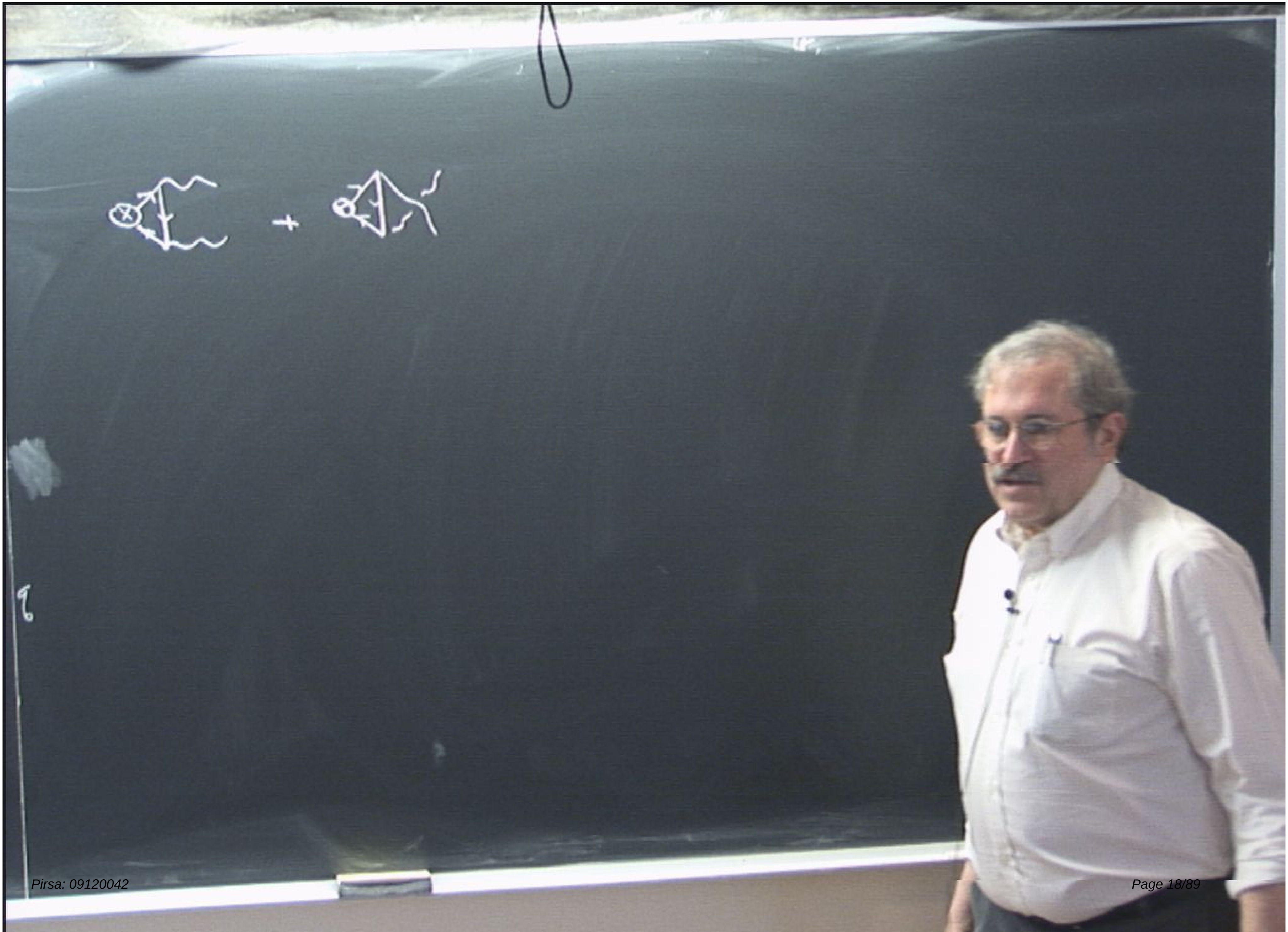
$$Q = \int d^3x \, j^0 \qquad \partial_\mu j^\mu = 0$$

$$j^\mu_B = \bar{q}_L \gamma^\mu q_L + \bar{q}_R \gamma^\mu q_R = \bar{q}\gamma^\mu q$$

$$j^{\mu\, a}_I = \bar{q}_L \gamma^\mu \tau^a q_L + \bar{q}_R \gamma^\mu \tau^a q_R = \bar{q}\gamma^\mu \tau^a q$$

$$j^{\mu 5 a} = -\bar{q}_L \gamma^\mu \tau^a q_L + \bar{q}_R \gamma^\mu \tau^a q_R = \bar{q}\gamma^\mu \gamma^5 \tau^a q$$

$$j^{5\mu} = \bar{q}\gamma^\mu \gamma^5 q$$



$\Rightarrow \quad \partial_\mu j^{5\mu} \neq 0$

axial vector anomaly

$$\Rightarrow \quad \partial_\mu j^{5\mu} \neq 0$$

axial vector
anomaly

$$m\bar{q}q = m(\bar{q}_L q_R + \bar{q}_R q_L)$$

q

 $\Rightarrow$ $\partial_\mu j^{5\mu} \neq 0$

axial vector anomaly

$$m\bar{q}q = m(\bar{q}_L q_R + \bar{q}_R q_L)$$

Nambu + Jona Lasinio

 +  $\Rightarrow \quad \partial_\mu j^{5\mu} \neq 0$

axial vector anomaly

$$m\bar{q}q = m(\bar{q}_L q_R + \bar{q}_R q_L)$$

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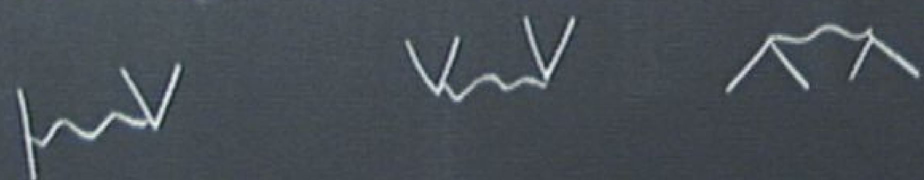
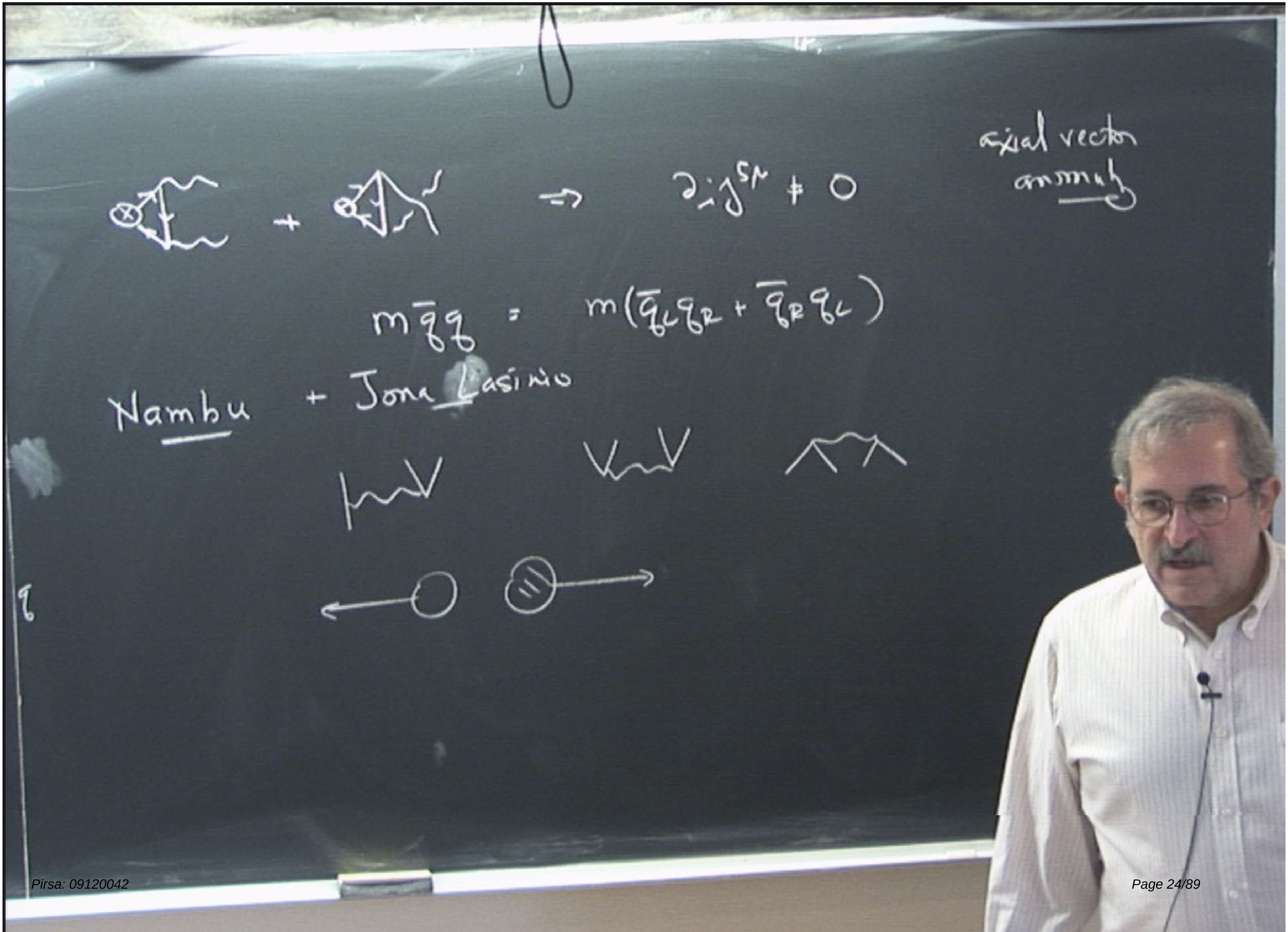
$\Rightarrow \quad \partial_\mu j^{5\mu} \neq 0$

axial vector anomaly

$$m\bar{q}q = m(\bar{q}_L q_R + \bar{q}_R q_L)$$

Nambu + Jona Lasinio



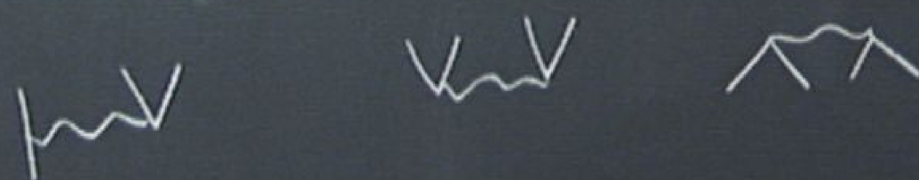

axial vector anomaly
$\Rightarrow \quad \partial_\mu j^{5\mu} \neq 0$
$m\bar{q}q = m(\bar{q}_L q_R + \bar{q}_R q_L)$
Nambu + Jona Lasinio

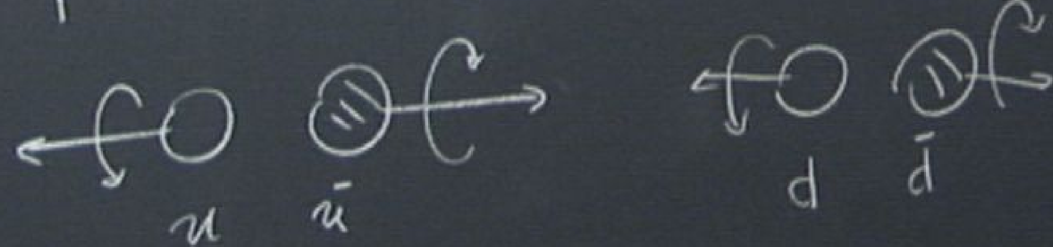
$$\Rightarrow \quad \partial_\mu j^{5\mu} \neq 0$$

axial vector anomaly

$$m\bar{q}q = m(\bar{q}_L q_R + \bar{q}_R q_L)$$

Nambu + Jona Lasinio



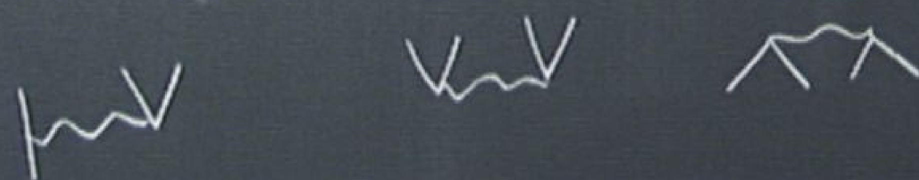



$+ (L \to R)$

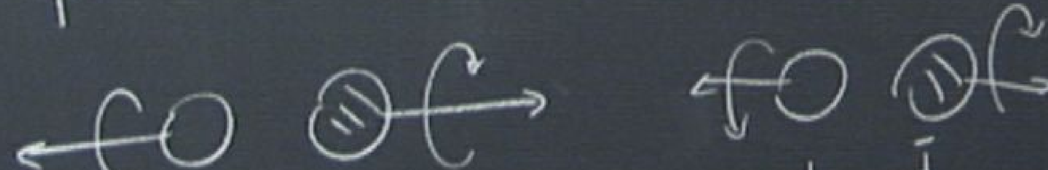
 $\Rightarrow \quad \partial_\mu j^{5\mu} \neq 0$ axial vector anomaly

$$m\bar{q}q = m(\bar{q}_L q_R + \bar{q}_R q_L)$$

Nambu + Jona Lasinio



$q$

 + (L → R)

$u_L$ $\bar{u}_L$ ← antiparticle of $u_R$

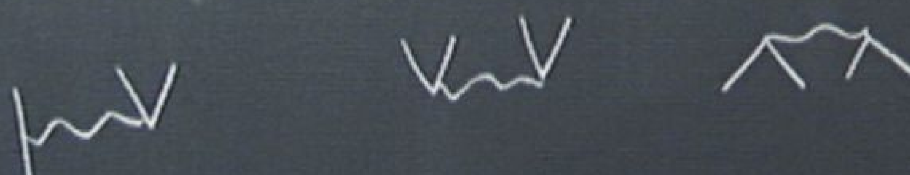
d $\bar{d}$

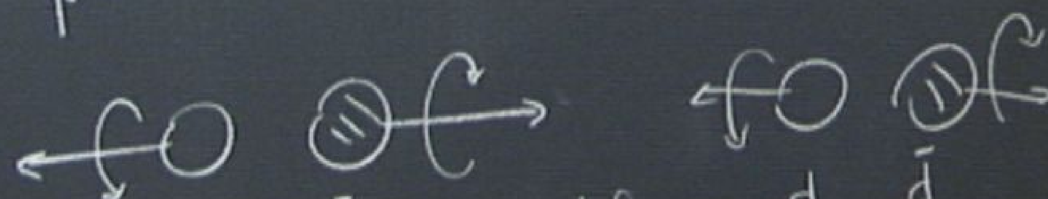
 $\Rightarrow$ $\partial_\mu j^{5\mu} \neq 0$

axial vector anomaly

$$m\bar{q}q = m(\bar{q}_L q_R + \bar{q}_R q_L)$$

Nambu + Jona Lasinio





+ $(L \to R)$

$u_L$ $\bar{u}_L$ d $\bar{d}$

antiparticle of $u_R$

$$\bar{q} q = \bar{q}_L q_R$$

$$\langle \Omega | \bar{u} u | \Omega \rangle = \langle \Omega | \bar{d} d | \Omega \rangle = -\Delta$$

$$\bar{q} q = \bar{q}_L q_R$$

$$\langle \Omega | \bar{u} u | \Omega \rangle = \langle \Omega | \bar{d} d | \Omega \rangle = -\Delta$$

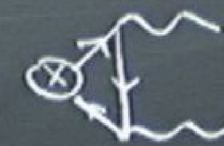
$$\bar{q} q = \bar{q}_L q_R$$

$$\langle \Omega | \bar{u} u | \Omega \rangle = \langle \Omega | \bar{d} d | \Omega \rangle = -\Delta \qquad (3$$

$$\bar{q}q = \bar{q}_L q_R$$

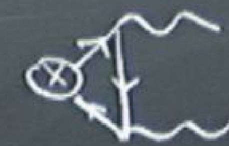
$$\langle \Omega | \bar{u} u | \Omega \rangle = \langle \Omega | \bar{d} d | \Omega \rangle = -\Delta \neq 0$$

$$(300\,\mathrm{MeV})^3$$

 +

Nambu +

$$\bar q q = \bar q_L q_R$$

 +

$$\langle\Omega|\,\bar u\, u\,|\Omega\rangle = \langle\Omega|\,\bar d\, d\,|\Omega\rangle = -2\Delta \neq 0$$

$$(300\,\mathrm{MeV})^3$$

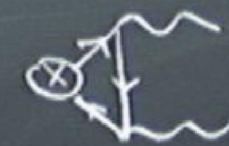
$i\ j = (u,d)$ $\quad \langle\Omega|\,\bar q_{L i}\, q_{R j}\,|\Omega\rangle = -\ \delta_{ij}\,\Delta$

Nambu +

$$\bar{q}q = \bar{q}_L q_R$$

$$\langle\Omega|\,\bar{u}\,u\,|\Omega\rangle = \langle\Omega|\,\bar{d}\,d\,|\Omega\rangle = -\Delta \neq 0$$

$$(300\,\mathrm{MeV})^3$$



$i\,j = (u,d)$ $\quad$ $\langle\Omega|\,\bar{q}_{Li}\,q_{Rj}\,|\Omega\rangle = -\frac{1}{2}\delta_{ij}\,\Delta$

Nambu

$$\bar{q} q = \bar{q}_L q_R$$

$$\langle \Omega | \bar{u} u | \Omega \rangle = \langle \Omega | \bar{d} d | \Omega \rangle = -\Delta \neq 0$$

$(300\,\mathrm{MeV})^3$

 +

$i\, j = (u, d)$

$$\langle \Omega | \bar{q}_{L i} q_{R j} | \Omega \rangle = -\tfrac{1}{2} \delta_{ji} \Delta$$

Nambu +

$$q_{R j} \to U_{j \bar{j}} q_{R \bar{j}} \qquad q_{L j \bar{j}} \to (U^{-1})_{\bar{j} j} q_{j}$$

$$\bar{q} \to$$

$$\bar q q = \bar q_L q_R$$

 +

$$\langle \Omega | \bar u u | \Omega \rangle = \langle \Omega | \bar d d | \Omega \rangle = -\Delta \neq 0$$

$$(300\,\mathrm{MeV})^3$$

$i\, j = (u, d)$

$$\langle \Omega | \bar q_{Li} q_{Rj} | \Omega \rangle = -\tfrac{1}{2} \delta_{ji} \Delta$$

Nambu +

$$q_{Rj} \to U_{j\bar j}\, q_{R\bar j} \qquad q_{L j\bar j} \to (U^{-1})_{\bar j j}\, q_{L j}$$

$$\bar q \to \bar q\, U$$

$$\langle \Omega | \bar q_{Li} q_{jR} | \Omega \rangle = -\tfrac{1}{2} (U^2)_{ji} \Delta$$

$$\bar{q} q = \bar{q}_L q_R$$

 +

$$\langle \Omega | \bar{u} u | \Omega \rangle = \langle \Omega | \bar{d} d | \Omega \rangle = -\Delta \neq 0$$

$(300\,\mathrm{MeV})^3$

$i\ j = (u, d)$

$$\langle \Omega | \bar{q}_{Li} q_{Rj} | \Omega \rangle = -\tfrac{1}{2} \delta_{ji} \Delta$$

Nambu

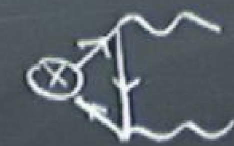
$$q_{Rj} \to U_{jj} q_{Rj} \qquad q_{Ljj} \to (U^{-1})_{jj} q_j$$

$$\bar{q} \to \bar{q} U$$

$$\langle \Omega | \bar{q}_{Li} q_{jR} | \Omega \rangle = -\frac{1}{2} (U^2)_{ji} \Delta$$

$S^3$ 

$$\bar q q = \bar q_L q_R$$

 $+$

$$\langle \Omega | \bar u u | \Omega \rangle = \langle \Omega | \bar d d | \Omega \rangle = -\Delta \neq 0$$

$(300\,\mathrm{MeV})^3$

$i\ j = (u, d)$

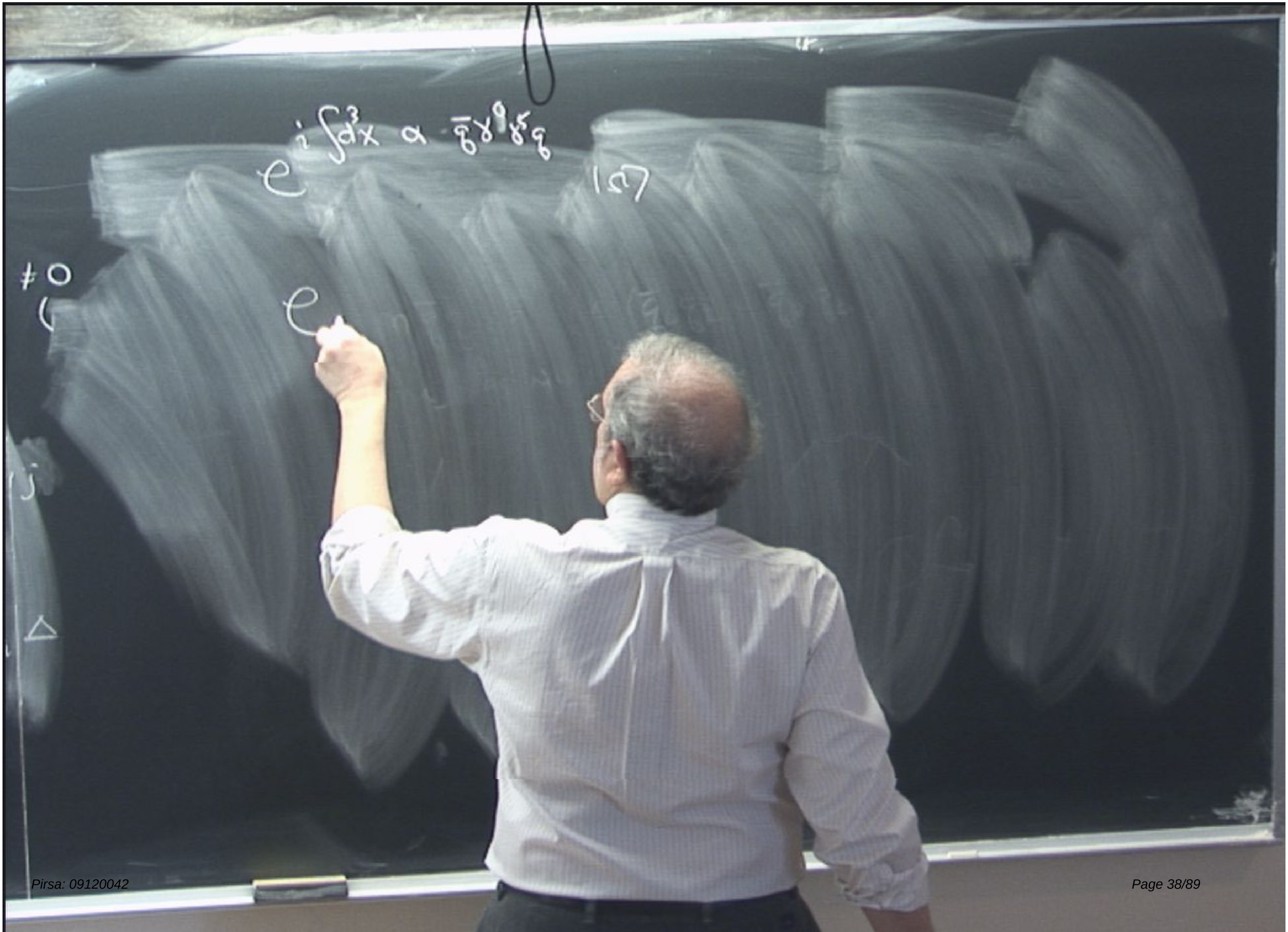
$$\langle \Omega | \bar q_{Li} q_{Rj} | \Omega \rangle = -\tfrac{1}{2}\delta_{ji}\,\Delta$$

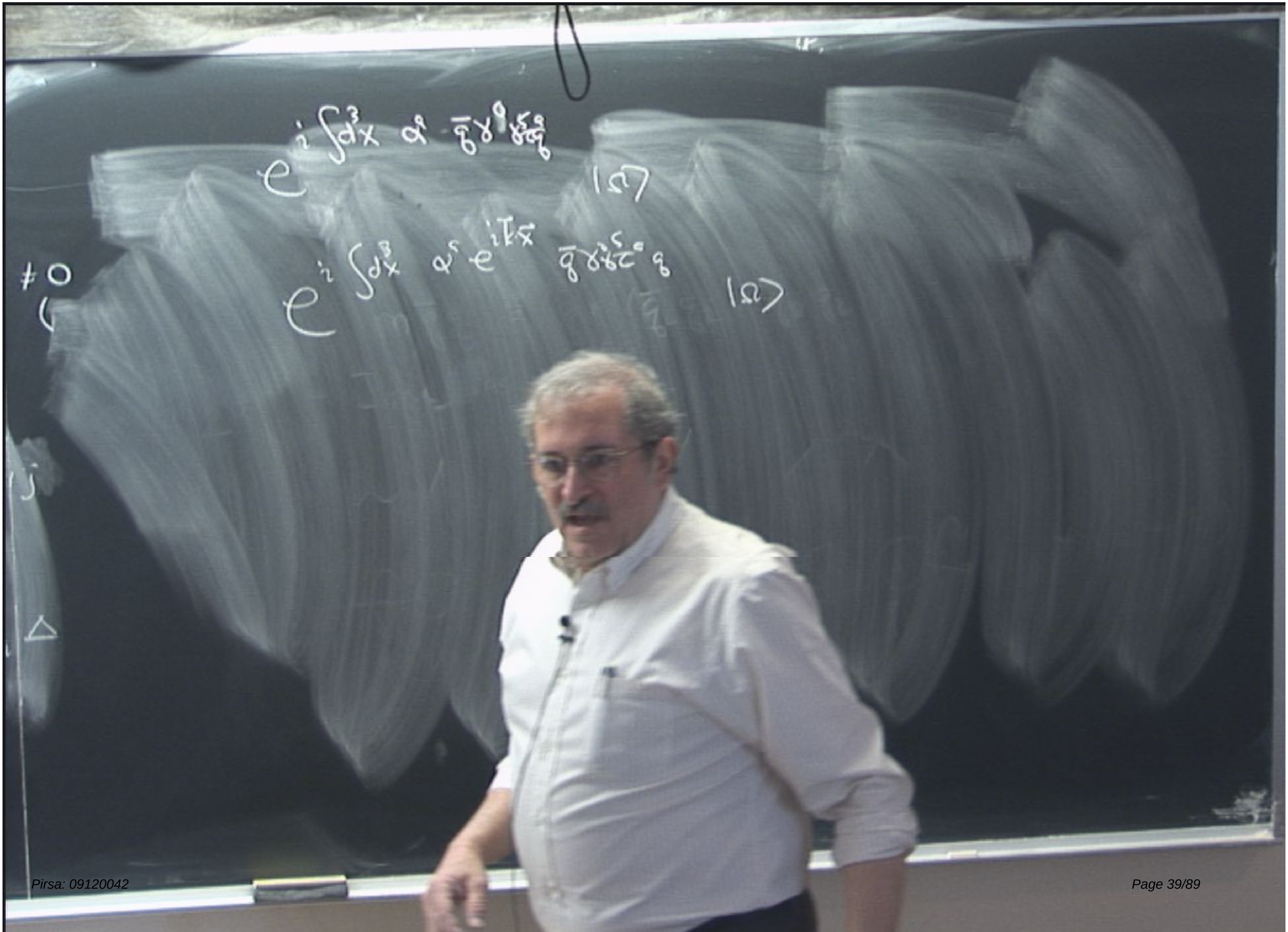
Nambu

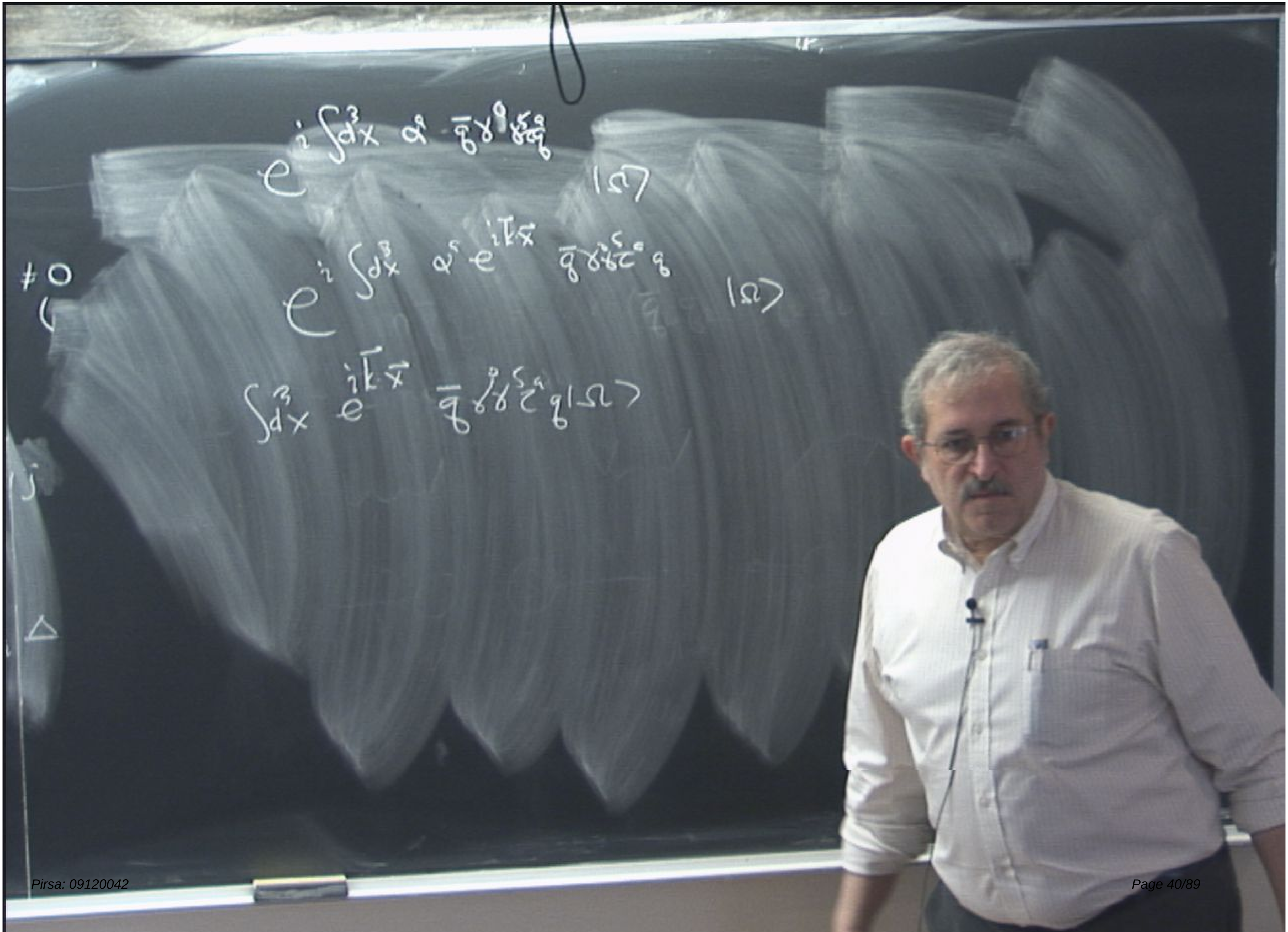
$$q_{Rj} \to U_{jj'}\, q_{Rj'} \qquad q_{Lj} \to (U^{-1})_{jj'}\, q_{j'}$$

$$\bar q \to \bar q\, U$$

$$\langle \Omega | \bar q_{Li} q_{jR} | \Omega \rangle = -\tfrac{1}{2}\,(U^2)_{ji}\,\Delta$$


$e^{i\int d^3x\, \alpha\, \bar{q}\gamma^0\gamma^5 q}$




$\neq 0$
$e^{i\int d^3x\, \alpha^a \bar{q}\gamma^0\gamma^5\tau^a q}|\Omega\rangle$
$e^{i\int d^3x\, \alpha^a e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q}|\Omega\rangle$
$\int d^3x\, e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q|\Omega\rangle$

$\neq 0$

$$e^{i\int d^3x\, \alpha^a \bar{q}\gamma^0\gamma^5\tau^a q}\,|\Omega\rangle$$

$$e^{i\int d^3x\, \alpha^a e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q}\,|\Omega\rangle$$

$$\int d^3x\, e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q|\Omega\rangle \qquad |\pi^a(k)\rangle$$

$\neq 0$

$$e^{i\int d^3x\, \alpha^a \bar{q}\gamma^0\gamma^5\tau^a q}\,|\Omega\rangle$$

$$e^{i\int d^3x\, \alpha^a e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q}\,|\Omega\rangle$$

$$\int d^3x\, e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q\,|\Omega\rangle$$

$|\pi^a(k)\rangle$ $\quad m = 0$

$P = -1 \quad I = 1$

$\neq 0$

$$e^{i\int d^3x\, \alpha^a \bar{q}\gamma^0\gamma^5\tau^a q}|\Omega\rangle$$

Goldstone boson

$$e^{i\int d^3x\, \alpha^a e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q}|\Omega\rangle$$

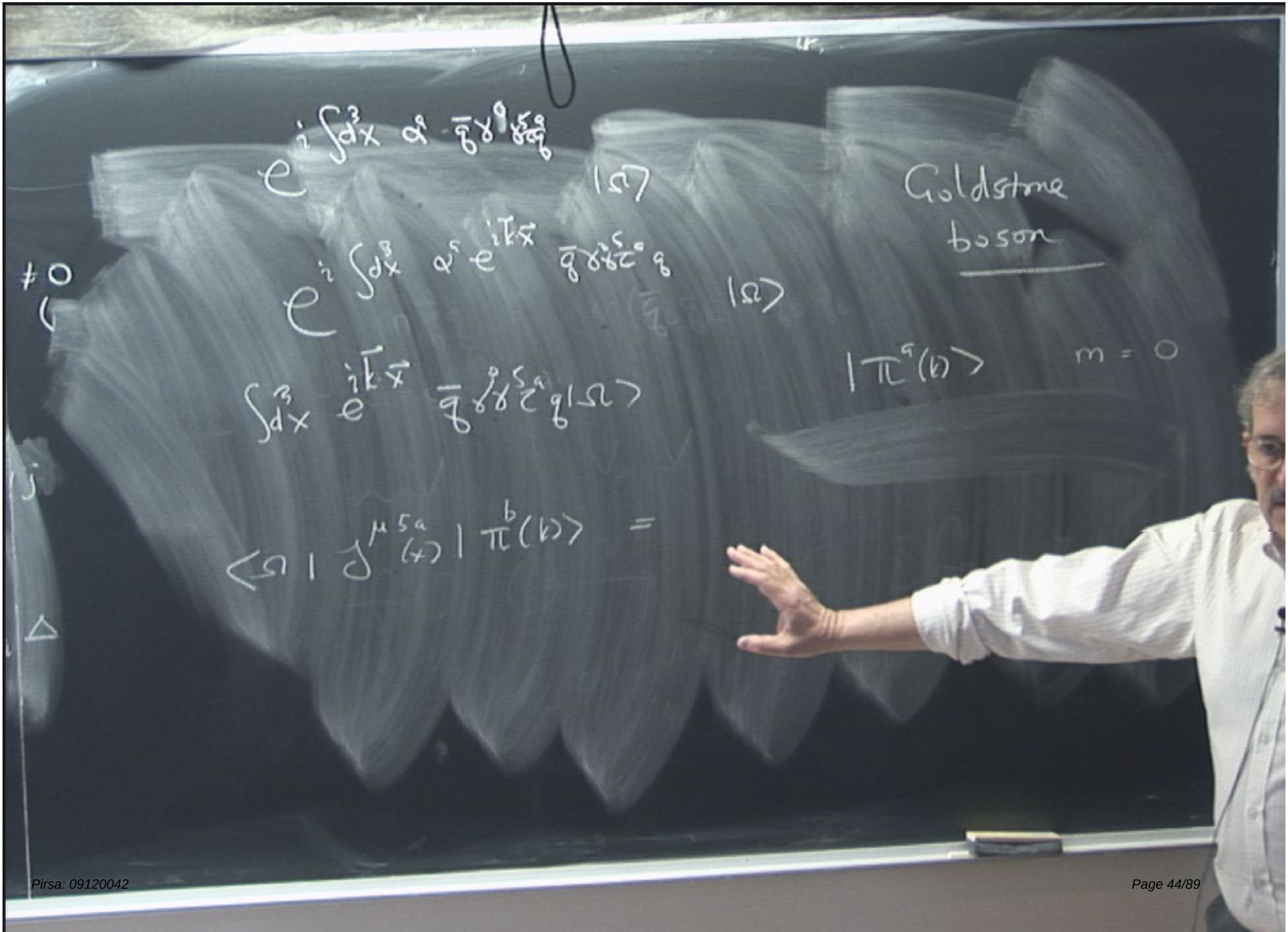
$$\int d^3x\, e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q|\Omega\rangle$$

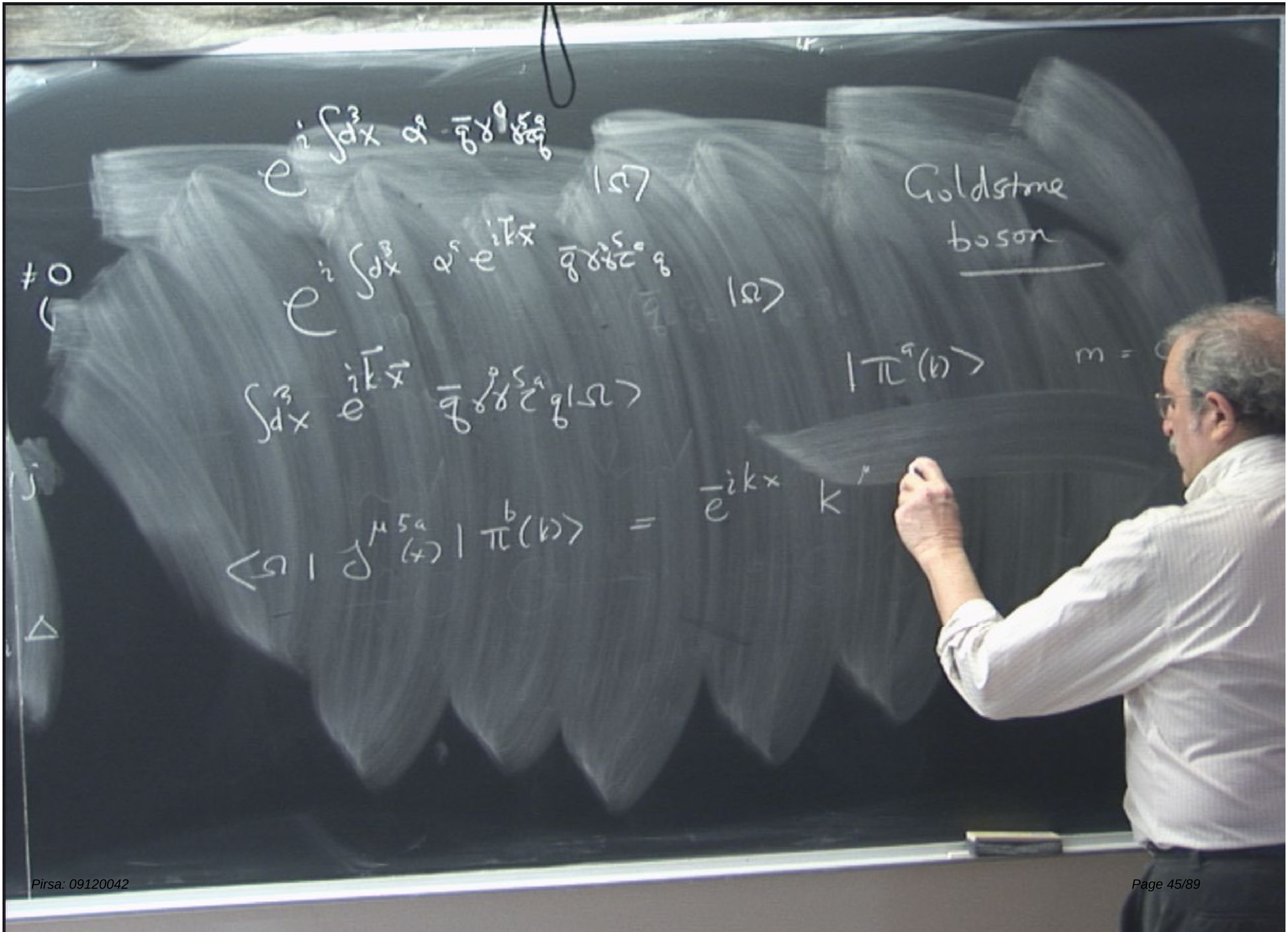
$|\pi^a(k)\rangle$ $\quad m = 0$

$P = -1 \quad I = 1$

$$\langle\Omega| J^{\mu 5a}(x) |\pi^b(k)\rangle$$

$\Delta$


$e^{i\int d^3x\, \alpha^a \bar{q}\gamma^0\gamma^5\tau^a q}|\Omega\rangle$
$e^{i\int d^3x\, \alpha^a e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q}|\Omega\rangle$
$\int d^3x\, e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q|\Omega\rangle$
Goldstone boson
$|\pi^a(k)\rangle$
$m = 0$
$\langle\Omega|J^{\mu 5a}(x)|\pi^b(k)\rangle =$


$e^{i\int d^3x\, \alpha^a \bar{q}\gamma^0\gamma^5\tau^a q}|\Omega\rangle$
Goldstone boson
$e^{i\int d^3x\, \alpha^a e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q}|\Omega\rangle$
$\neq 0$
$\int d^3x\, e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q|\Omega\rangle$
$|\pi^a(k)\rangle$
$m =$
$\langle\Omega|J^{\mu 5a}(x)|\pi^b(k)\rangle = e^{-ikx}\, k^\mu$

$\neq 0$

$$e^{i\int d^3x\, \alpha^a \bar{q}\gamma^0\gamma^5\tau^a q}\,|\Omega\rangle$$

Goldstone boson

$$e^{i\int d^3x\, \alpha^a e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q}\,|\Omega\rangle$$

$$\int d^3x\, e^{i\vec{k}\cdot\vec{x}}\, \bar{q}\gamma^0\gamma^5\tau^a q\,|\Omega\rangle \qquad |\pi^a(k)\rangle \qquad m=0$$

$$\langle\Omega|\, J^{\mu 5a}(x)\,|\pi^b(k)\rangle = e^{-ikx}\, k^\mu f_\pi\, \delta^{ab}$$

$\neq 0$

$$e^{i\int d^3x\, \alpha^a\, \bar q \gamma^0 \gamma^5 \tau^a q}\,|\Omega\rangle$$

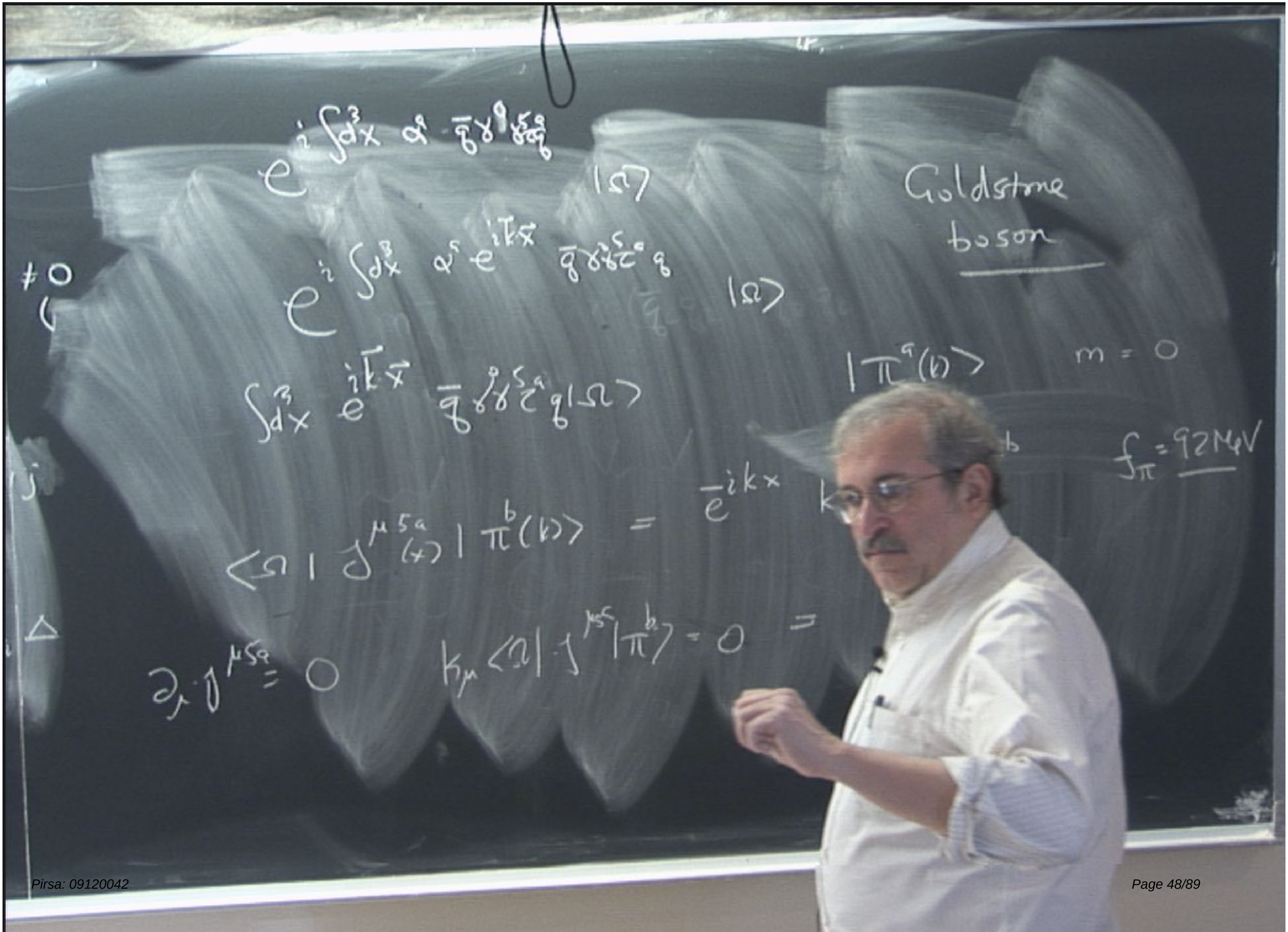
Goldstone boson

$$e^{i\int d^3x\, \alpha^a e^{i\vec k\cdot\vec x}\, \bar q \gamma^0 \gamma^5 \tau^a q}\,|\Omega\rangle$$

$$\int d^3x\, e^{i\vec k\cdot\vec x}\, \bar q \gamma^0 \gamma^5 \tau^a q\,|\Omega\rangle \qquad |\pi^a(k)\rangle \qquad m = 0$$

$$f_\pi = 92\,\mathrm{MeV}$$

$$\langle\Omega|\, J^{\mu 5a}(x)\, |\pi^b(k)\rangle = e^{-ikx}\, k^\mu f_\pi\, \delta^{ab}$$


Goldstone boson
$|\Omega\rangle$
$|\pi^a(k)\rangle$
$m = 0$
$f_\pi = 92\,\text{MeV}$
$\langle\Omega|\, j^{\mu 5a}(x)\,|\pi^b(k)\rangle = e^{-ikx}$
$\partial_\mu j^{\mu 5a} = 0$
$k_\mu \langle\Omega|\, j^{\mu 5}\,|\pi^b\rangle = 0 =$

$$e^{i\int d^3x\, \alpha^a \bar{q}\gamma^0\gamma^5\tau^a q}|\Omega\rangle$$

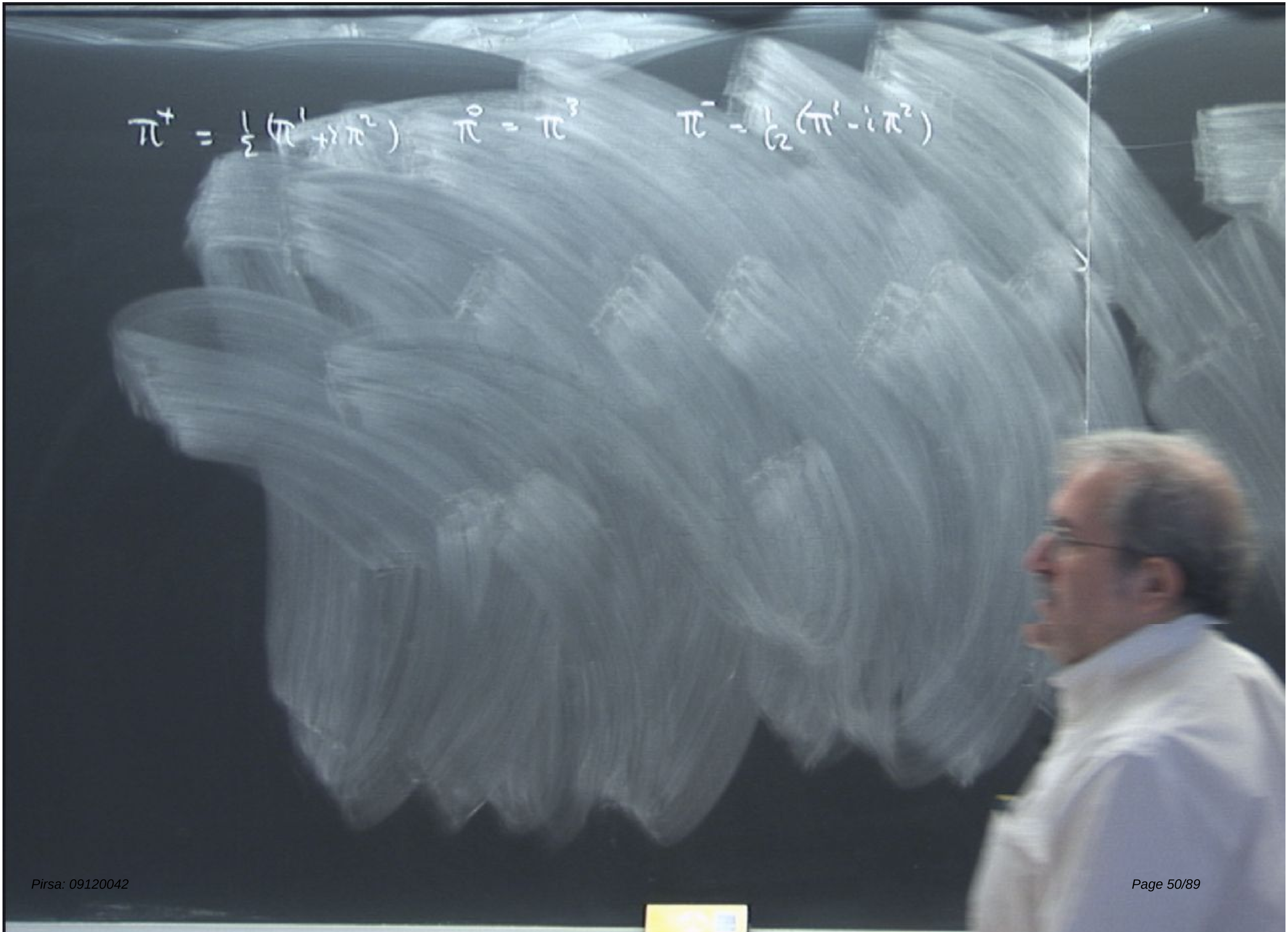
Goldstone boson

$$e^{i\int d^3x\, \alpha^a e^{i\vec{k}\cdot\vec{x}}\bar{q}\gamma^0\gamma^5\tau^a q}|\Omega\rangle$$

$$\int d^3x\, e^{i\vec{k}\cdot\vec{x}}\bar{q}\gamma^0\gamma^5\tau^a q|\Omega\rangle \qquad |\pi^a(k)\rangle \qquad m=0$$

$$\langle\Omega|J^{\mu 5a}(x)|\pi^b(k)\rangle = e^{-ikx}k^\mu f_\pi \delta^{ab} \qquad f_\pi = 92\,\text{MeV}$$

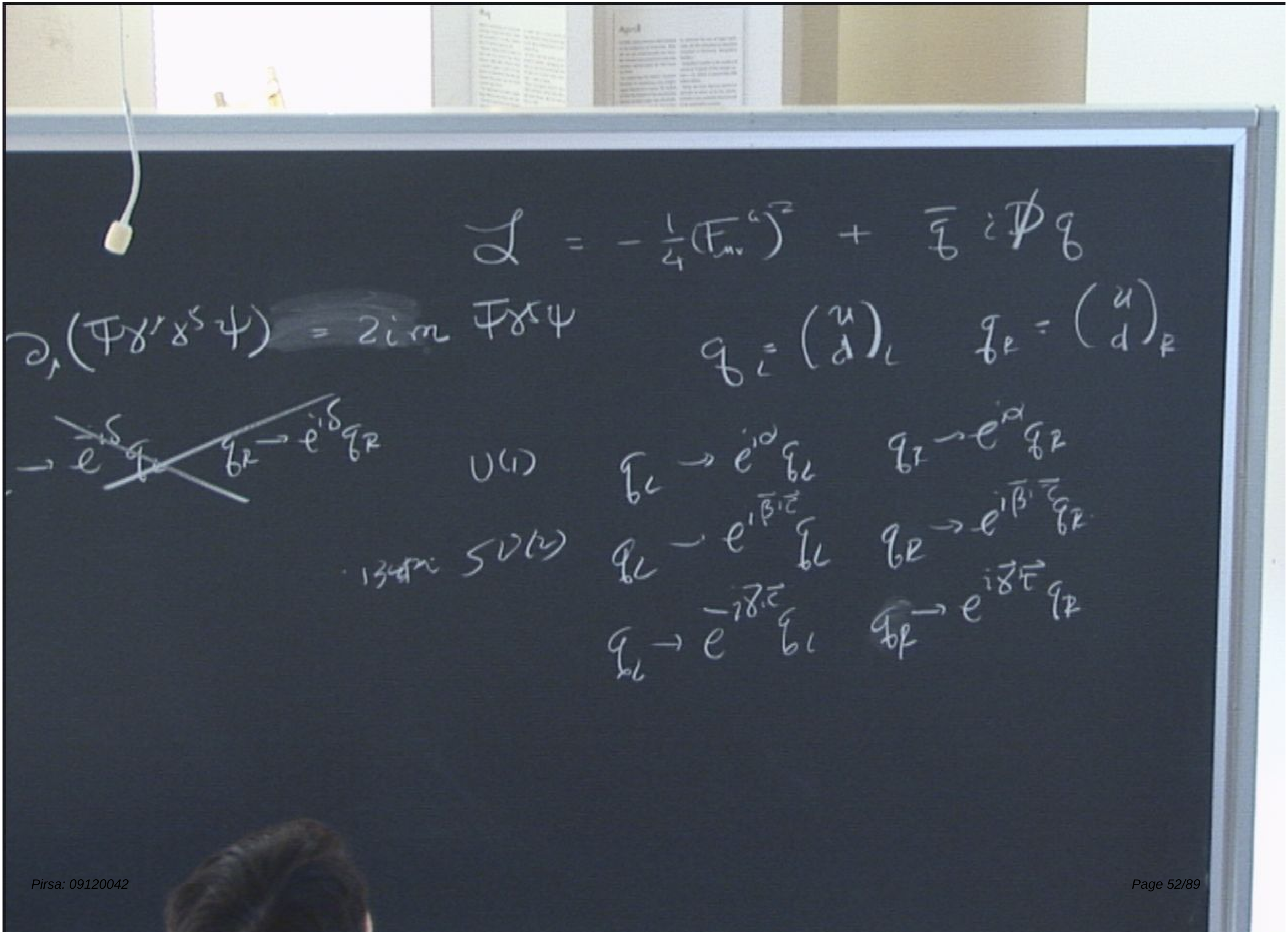
$$\partial_\mu J^{\mu 5a} = 0 \qquad k_\mu\langle\Omega|J^{\mu 5}|\pi^b\rangle = 0 = k^2 f_\pi$$


$\pi^+ = \frac{1}{\sqrt{2}}(\pi^1 + i\pi^2)$
$\pi^0 = \pi^3$
$\pi^- = \frac{1}{\sqrt{2}}(\pi^1 - i\pi^2)$

$$\pi^+ = \frac{1}{\sqrt{2}}(\pi^1 + i\pi^2) \qquad \pi^0 = \pi^3 \qquad \pi^- = \frac{1}{\sqrt{2}}(\pi^1 - i\pi^2)$$

add small mass for u, d

$$\partial_\mu J^{\mu 5 3} =$$


$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^a)^2 + \bar{q}\, i\not{D}\, q$
$\partial_\mu(\bar\psi\gamma^\mu\gamma^5\psi) = 2im\,\bar\psi\gamma^5\psi$
$q_L = \begin{pmatrix}u\\d\end{pmatrix}_L$ $q_R = \begin{pmatrix}u\\d\end{pmatrix}_R$
$\cancel{\to e^{i\delta} q_L \quad q_R \to e^{i\delta} q_R}$
U(1) $q_L \to e^{i\alpha} q_L$ $q_R \to e^{i\alpha} q_R$
SU(2) $q_L \to e^{i\vec\beta\cdot\vec\tau} q_L$ $q_R \to e^{i\vec\beta\cdot\vec\tau} q_R$
$q_L \to e^{-i\vec\gamma\cdot\vec\tau} q_L$ $q_R \to e^{i\vec\gamma\cdot\vec\tau} q_R$

$\pi^+ = \frac{1}{\sqrt{2}}(\pi^1 + i\pi^2) \quad \pi^0 = \pi^3 \quad \pi^- = \frac{1}{\sqrt{2}}(\pi^1 - i\pi^2)$

$\frac{\sigma^3}{2}$

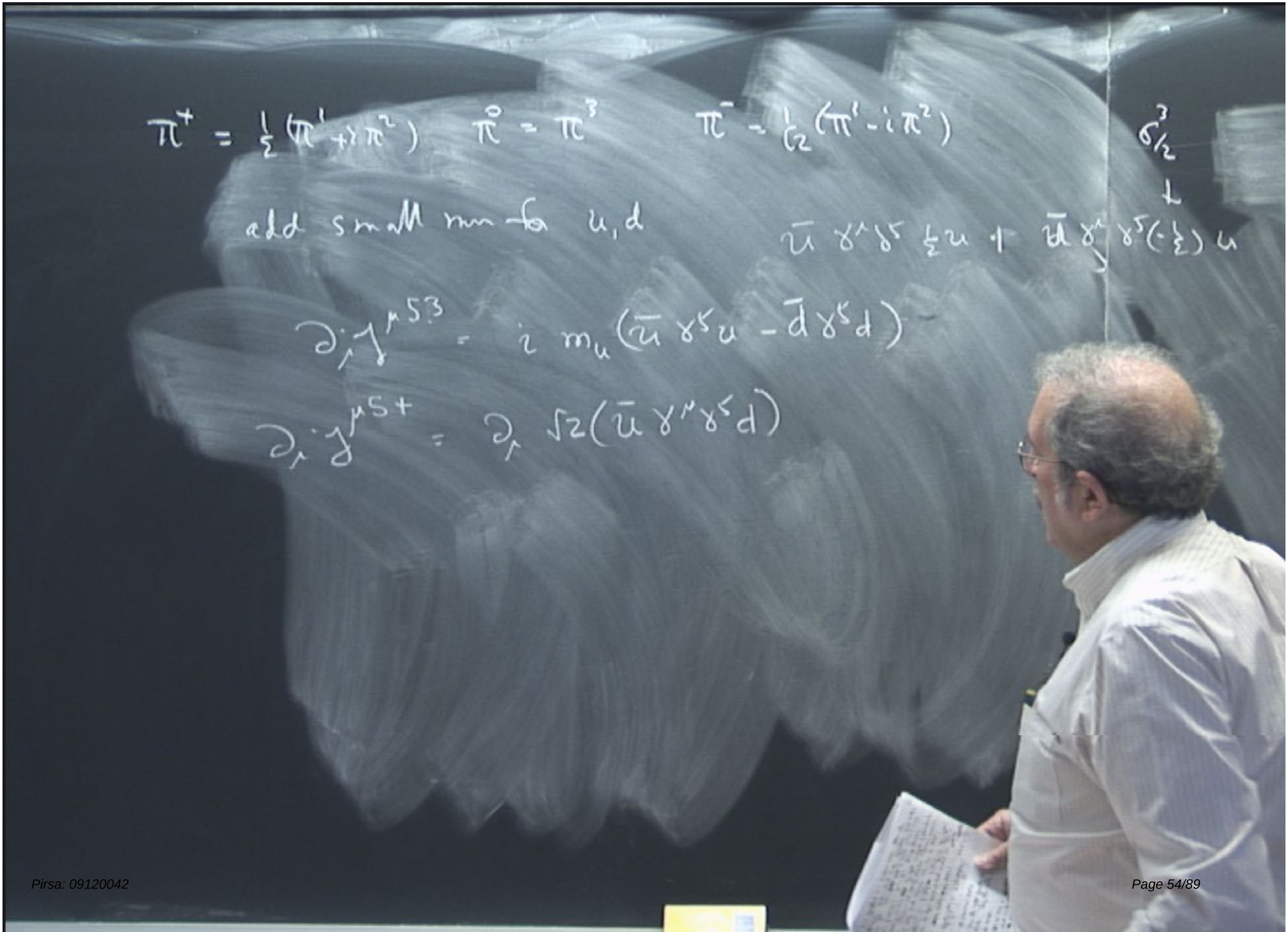
$\downarrow$

add small mass for u, d

$\bar{u}\gamma^\mu\gamma^5 \frac{1}{2} u + \bar{d}\gamma^\mu\gamma^5 (-\frac{1}{2}) d$

$\partial_\mu J^{\mu 5 3} =$

$\partial_\mu$


$\pi^+ = \frac{1}{\sqrt{2}}(\pi^1 + i\pi^2)$ $\pi^0 = \pi^3$ $\pi^- = \frac{1}{\sqrt{2}}(\pi^1 - i\pi^2)$
$\sigma^3/2$
add small m for u, d
$\bar{u}\gamma^\mu\gamma^5 \frac{1}{2} u + \bar{d}\gamma^\mu\gamma^5(-\frac{1}{2}) d$
$\partial_\mu J^{\mu 5 3} = i\, m_u(\bar{u}\gamma^5 u - \bar{d}\gamma^5 d)$
$\partial_\mu J^{\mu 5 +} = \partial_\mu \sqrt{2}(\bar{u}\gamma^\mu\gamma^5 d)$

$\pi^+ = \frac{1}{\sqrt{2}}(\pi^1 + i\pi^2)$ $\quad \pi^0 = \pi^3$ $\quad \pi^- = \frac{1}{\sqrt{2}}(\pi^1 - i\pi^2)$ $\quad \sigma^3/2$

add small m for u, d

$\bar{u}\gamma^\mu\gamma^5 \frac{1}{2} u + \bar{d}\gamma^\mu\gamma^5(-\frac{1}{2}) d$

$$\partial_\mu J^{\mu 5 3} = i\,(m_u \bar{u}\gamma^5 u - m_d \bar{d}\gamma^5 d)$$

$$\partial_\mu J^{\mu 5 +} = \partial_\mu \sqrt{2}(\bar{u}\gamma^\mu\gamma^5 d) = i$$

$$\pi^+ = \frac{1}{\sqrt{2}}(\pi^1 + i\pi^2) \qquad \pi^0 = \pi^3 \qquad \pi^- = \frac{1}{\sqrt{2}}(\pi^1 - i\pi^2)$$

$\sigma^3/2$

add small mass for u, d

$$\bar{u}\gamma^\mu\gamma^5 \tfrac{1}{2} u + \bar{u}\gamma^\mu\gamma^5(-\tfrac{1}{2})u$$

$$\partial_\mu J^{\mu 5 3} = i(m_u \bar{u}\gamma^5 u - m_d \bar{d}\gamma^5 d)$$

$$\partial_\mu J^{\mu 5 +} = \partial_\mu \sqrt{2}(\bar{u}\gamma^\mu\gamma^5 d) = i(m_u + m_d) \quad \bar{u}\gamma^5 d$$

$\partial_\mu$

$$\pi^+ = \frac{1}{2}(\pi^1 + i\pi^2) \qquad \pi^0 = \pi^3 \qquad \pi^- = \frac{1}{\sqrt{2}}(\pi^1 - i\pi^2)$$

$\frac{\sigma^3}{2}$

add small m for u, d

$$\bar{u}\gamma^\mu\gamma^5 \tfrac{1}{2} u + \bar{d}\gamma^\mu\gamma^5(-\tfrac{1}{2}) u$$

$$\partial_\mu j^{\mu 5 3} = i(m_u \bar{u}\gamma^5 u - m_d \bar{d}\gamma^5 d)$$

$$\partial_\mu j^{\mu 5 +} = \partial_\mu \sqrt{2}(\bar{u}\gamma^\mu\gamma^5 d) = i(m_u + m_d) \quad \bar{u}\gamma^5 d$$

$\partial_\mu$

$\pi^+ = \frac{1}{\sqrt{2}}(\pi^1 + i\pi^2) \quad \pi^0 = \pi^3 \quad \pi^- = \frac{1}{\sqrt{2}}(\pi^1 - i\pi^2)$ $\quad \sigma^3/2$

add small mm for u, d $\quad \bar{u}\gamma^\mu\gamma^5 \frac{1}{2} u + \bar{d}\gamma^\mu\gamma^5(-\frac{1}{2}) d$

$$\partial_\mu j^{\mu 5 3} = i(m_u \bar{u}\gamma^5 u - m_d \bar{d}\gamma^5 d)$$

$$\partial_\mu j^{\mu 5 +} = \partial_\mu \sqrt{2}(\bar{u}\gamma^\mu\gamma^5 d) = i(m_u + m_d) \quad \bar{u}\gamma^5 d$$

$$\langle 0 | j^{\mu 5 3} | \pi^0 \rangle = i k^\mu f_\pi$$

$k^\mu$

$$\langle 0 | (m_u \bar{u}\gamma^5 u + m_d \bar{d}\gamma^5 d) | 0 \rangle = k^2 f_\pi = m_\pi^2 f_\pi$$

$\partial_\mu$

$$\pi^+ = \tfrac{1}{\sqrt{2}}(\pi^1 + i\pi^2) \qquad \pi^0 = \pi^3 \qquad \pi^- = \tfrac{1}{\sqrt{2}}(\pi^1 - i\pi^2)$$

$$\sigma^3/2$$

add small mm for u, d

$$\bar{u}\gamma^\mu\gamma^5 \tfrac{1}{2} u + \bar{d}\gamma^\mu\gamma^5(-\tfrac{1}{2}) d$$

$$\partial_\mu J^{\mu 5 3} = i\,(m_u \bar{u}\gamma^5 u - m_d \bar{d}\gamma^5 d)$$

$$\partial_\mu J^{\mu 5 +} = \partial_\mu \sqrt{2}(\bar{u}\gamma^\mu\gamma^5 d) = i\,(m_u + m_d) \quad \bar{u}\gamma^5 d$$

$$k^\mu \qquad \langle 0 | J^{\mu 5 3} | \pi^0 \rangle = i k^\mu f_\pi$$

$$\langle 0 | (m_u \bar{u}\gamma^5 u - m_d \bar{d}\gamma^5 d) | \pi \rangle = k^2 f_\pi = m_\pi^2 f_\pi$$

$\partial_\mu$

$\delta^3_{1/2}$

$\downarrow$

$\gamma^5 (\cdot \tfrac{1}{2})\, u$

$\langle 0 | \bar{u} \gamma^5 u | \pi^0 \rangle = \langle 0 | \frac{\bar{u} u}{\sqrt{\pi}} | 0 \rangle$

Goldstone boson

$\bar{u} \gamma^5 d$

$\int d^3x \; e^{i\vec{k}\cdot\vec{x}} \; \bar{q} \gamma^\mu \gamma^5 \tau^a q | \Omega \rangle$

$| \pi^a(k) \rangle \qquad m = 0$

$f_\pi = 92\,\text{MeV}$

$\langle \Omega | J^{\mu 5 a}(x) | \pi^b(k) \rangle = e^{-ikx} \, k^\mu \, f_\pi \, \delta^{ab}$

$k_\mu \langle \Omega | J^{\mu 5} | \pi^b \rangle = 0 = k^2 f_\pi$

$\partial_\mu J^{\mu 5 a} = 0$

$$\pi^+ = \frac{1}{\sqrt{2}}(\pi^1 + i\pi^2) \qquad \pi^0 = \pi^3 \qquad \pi^- = \frac{1}{\sqrt{2}}(\pi^1 - i\pi^2)$$

$\frac{\sigma^3}{2}$

add small mm for u, d

$$\bar{u}\gamma^\mu\gamma^5 \tfrac{1}{2} u + \bar{d}\gamma^\mu\gamma^5(-\tfrac{1}{2}$$

$$\partial_\mu j^{\mu 5 3} = i(m_u \bar{u}\gamma^5 u - m_d \bar{d}\gamma^5 d)$$

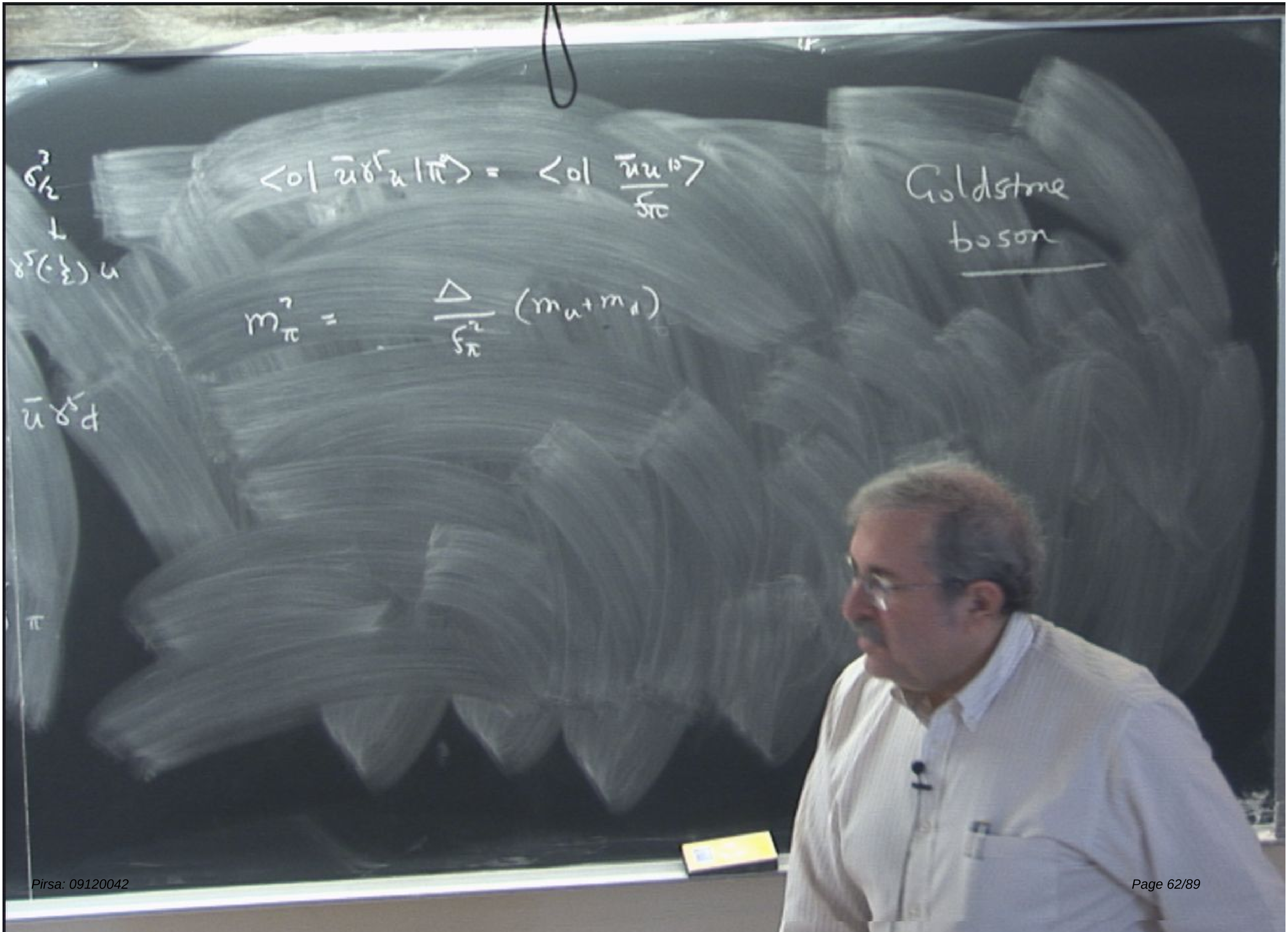
$$\partial_\mu j^{\mu 5 +} = \partial_\mu \sqrt{2}(\bar{u}\gamma^\mu\gamma^5 d) = i(m_u + m_d) \quad \bar{u}\gamma^5$$

$k^\mu$

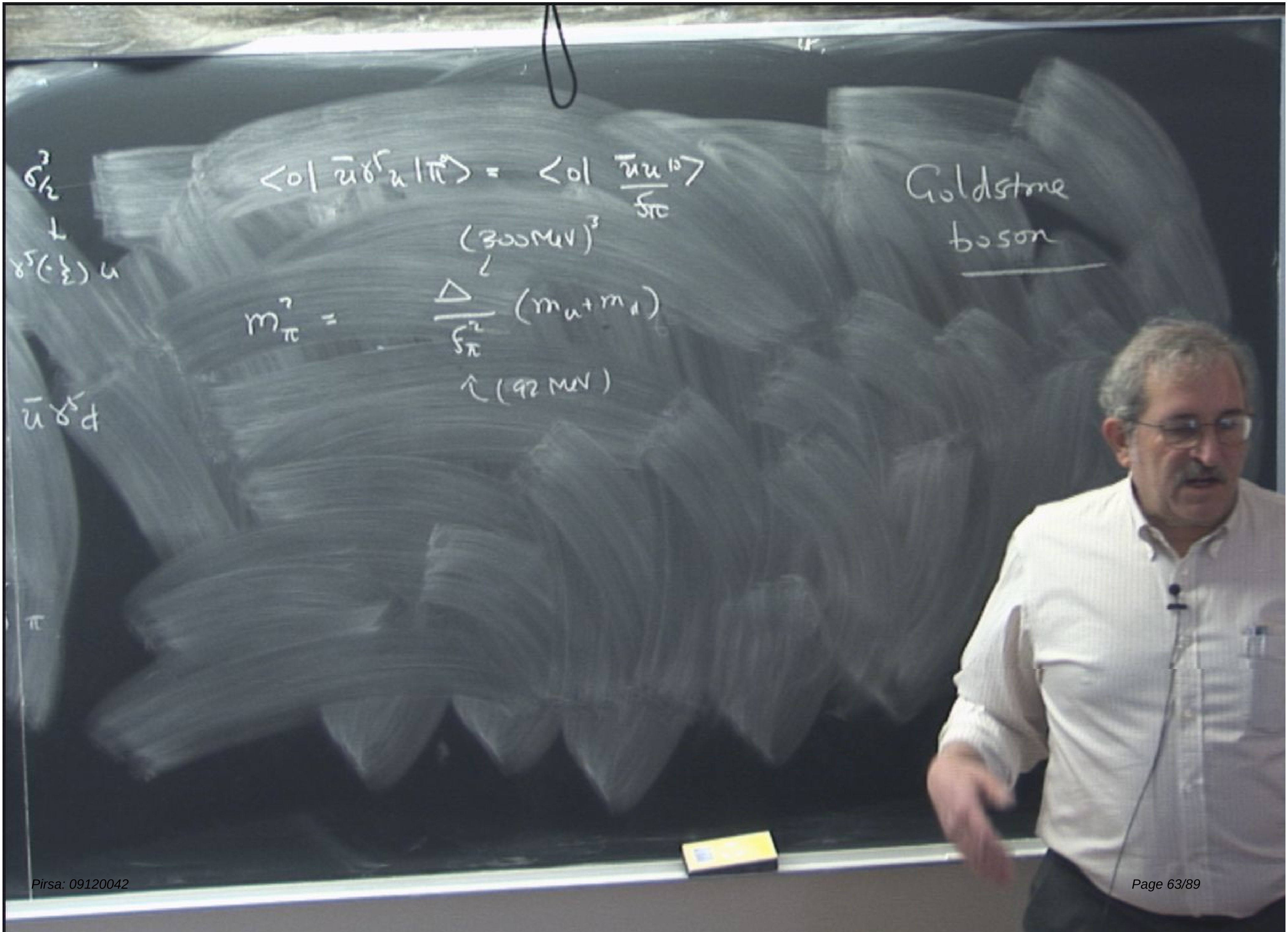
$$\langle 0 | j^{\mu 5 3} | \pi^0 \rangle = i k^\mu f_\pi$$

$$\langle 0 | (m_u \bar{u}\gamma^5 u - m_d \bar{d}\gamma^5 d) | \pi \rangle = k^2 f_\pi = m_\pi^2 f_\pi$$

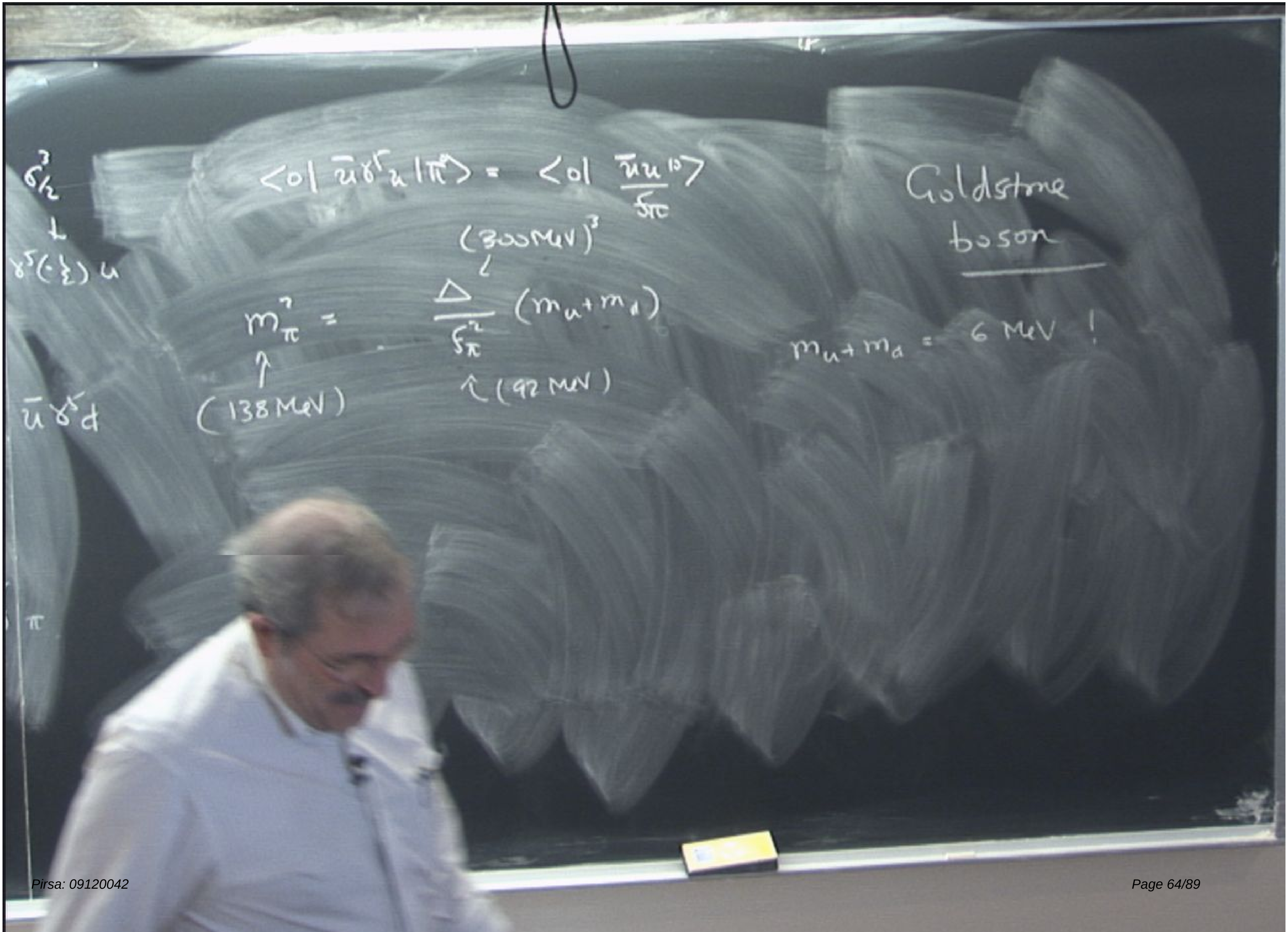
$$\frac{(m_u + m_d)}{f_\pi} \Delta$$



$\langle 0 | \bar{u}\gamma^5 u | \pi^0 \rangle = \langle 0 | \frac{\bar{u}u}{f_\pi} | 0 \rangle$
Goldstone boson
$m_\pi^2 = \frac{\Delta}{f_\pi^2}(m_u + m_d)$
$\bar{u}\gamma^5 d$
$\pi$



$\langle 0|\bar{u}\gamma^5 u|\pi^0\rangle = \langle 0|\frac{\bar{u}u}{f_\pi}|0\rangle$
$(300\,\text{MeV})^3$
$m_\pi^2 = \frac{\Delta}{f_\pi^2}(m_u+m_d)$
$(92\,\text{MeV})$
Goldstone boson
$\bar{u}\gamma^5 d$



<0| ū γ5 u |π0> = <0| ūu |0> / fπ
(300 MeV)^3
mπ^2 = Δ / fπ^2 (mu + md)
(138 MeV)
(92 MeV)
Goldstone boson
mu + md = 6 MeV !
ū γ5 d
π

$\sigma^3_{1/2}$

$\downarrow$

$\gamma^5 (-\frac{1}{2}) u$

$\bar{u} \gamma^5 d$

$\pi$

$\langle 0 | \bar{u} \gamma^5 u | \pi^0 \rangle = \langle 0 | \frac{\bar{u} u}{f_\pi} | 0 \rangle$

$(300 \text{ MeV})^3$

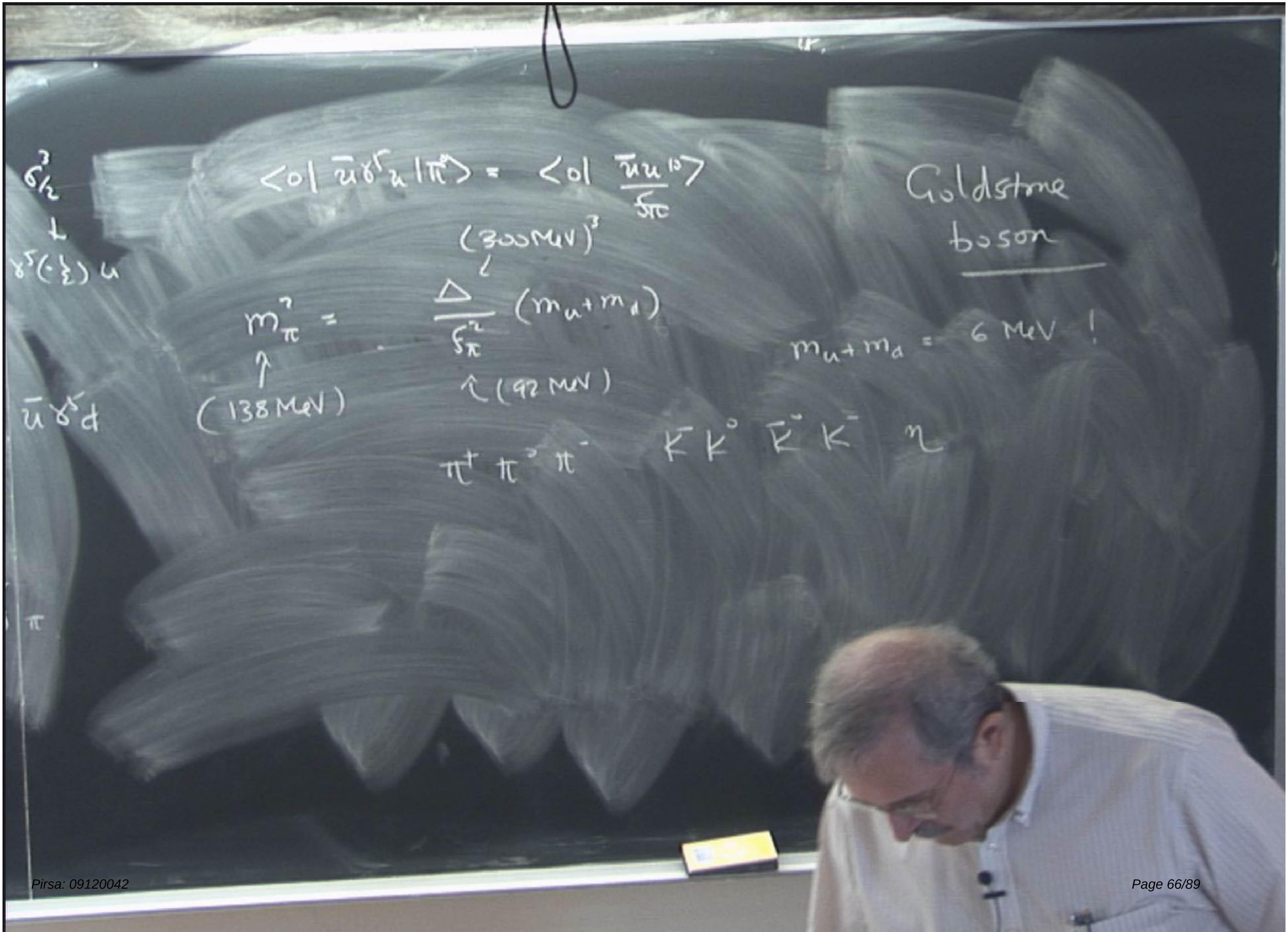
$$m_\pi^2 = \frac{\Delta}{f_\pi^2} (m_u + m_d)$$

$\uparrow$ (138 MeV)

$\uparrow$ (92 MeV)

Goldstone boson

$m_u + m_d = 6 \text{ MeV}$ !


$\frac{3}{2}$
$\gamma^5(-\frac{1}{2})u$
$\bar{u}\gamma^5 d$
$\pi$
$\langle 0|\bar{u}\gamma^5 u|\pi^0\rangle = \langle 0|\frac{\bar{u}u}{f_\pi}|0\rangle$
$(300\,\text{MeV})^3$
$m_\pi^2 = \frac{\Delta}{f_\pi^2}(m_u+m_d)$
$(138\,\text{MeV})$
$(92\,\text{MeV})$
$m_u+m_d = 6\,\text{MeV}$ !
Goldstone boson
$\pi^+\ \pi^0\ \pi^-\quad \bar{K}\ K^0\ \bar{K}^0\ K^-\quad \eta$

$\sigma^3/2$

$\gamma^5(-\frac{1}{2})u$

$\bar u \gamma^5 d$

$\pi$

$$\langle 0|\bar u \gamma^5 u|\pi^0\rangle = \langle 0|\frac{\bar u u}{f_\pi}|0\rangle$$

$(300\,\text{MeV})^3$

$$m_\pi^2 = \frac{\Delta}{f_\pi^2}(m_u+m_d)$$

$m_\pi$: $(138\,\text{MeV})$ $f_\pi$: $(92\,\text{MeV})$

Goldstone boson

$m_u+m_d = 6$ MeV !

$\pi^+\ \pi^0\ \pi^-\quad K^+\ K^0\ \bar K^0\ K^-\quad \eta$

$$m_{K^+}^2 = (m_u+m_s)\frac{\Delta}{f_\pi^2}$$

$$m_{K^0}^2 = \frac{\Delta}{f_\pi^2}(m_d+m_s)$$

$$m_\eta^2 =$$

$\sigma^3_{1/2}$

$\downarrow$

$\gamma^5(-\frac{1}{2})\,u$

$\bar{u}\gamma^5 d$

$\pi$

$$\langle 0|\,\bar{u}\gamma^5 u\,|\pi^0\rangle = \langle 0|\,\frac{\bar{u}u}{f_\pi}|0\rangle$$

$(300\,\text{MeV})^3$

Goldstone boson

$$m_\pi^2 = \frac{\Delta}{f_\pi^2}(m_u + m_d)$$

$\uparrow$ $(138\,\text{MeV})$

$\uparrow$ $(92\,\text{MeV})$

$m_u + m_d = 6\,\text{MeV}$ !

$\pi^+\ \pi^0\ \pi^-\quad \bar{K}\ K^0\ \bar{K}\ K^-\quad \eta$

$$m_{K^0}^2 = \frac{\Delta}{f_\pi^2}(m_d + m_s)$$

$$m_{K^+}^2 = (m_u + m_s)\frac{\Delta}{f_\pi^2}$$

$$m_\eta^2 = \left(\frac{4m_s + m_u + m_d}{3}\right)\frac{\Delta}{f_\pi^2}$$

$\frac{\sigma^3}{2}$

$\gamma^5(-\frac{1}{2})\,u$

$\langle 0|\,\bar u\gamma^5 u\,|\pi^0\rangle = \langle 0|\,\frac{\bar u u}{f_\pi}\,|0\rangle$

$(300\,\mathrm{MeV})^3$

Goldstone boson

$$m_\pi^2 = \frac{\Delta}{f_\pi^2}(m_u+m_d)$$

(138 MeV) (92 MeV)

$m_u + m_d = 6$ MeV !

$\bar u\gamma^5 d$

$\pi^+\ \pi^0\ \pi^-$ $\bar K\ K^0\ \bar K^0\ K^-$ $\eta$

$$m_{K^0}^2 = \frac{\Delta}{f_\pi^2}(m_d+m_s)$$

$$m_{K^+}^2 = (m_u+m_s)\frac{\Delta}{f_\pi^2}$$

$\pi$

$$m_\eta^2 = \left(\frac{4m_s + m_u + m_d}{3}\right)\frac{\Delta}{f_\pi^2}$$

$\sigma^3_{1/2}$

$\downarrow$

$\gamma^5(-\frac{1}{2})u$

$\langle 0|\bar{u}\gamma^5 u|\pi^0\rangle = \langle 0|\frac{\bar{u}u}{f_\pi}|0\rangle$

$(300\,\text{MeV})^3$

Goldstone boson

$$m_\pi^2 = \frac{\Delta}{f_\pi^2}(m_u+m_d)$$

$(138\,\text{MeV})$ $\quad$ $(92\,\text{MeV})$

$m_u+m_d \simeq 6\,\text{MeV}$ !

$\bar{u}\gamma^5 d$

$\frac{m_d}{m_u} \sim 2$

$\pi^+\ \pi^0\ \pi^-\quad K^+\ K^0\ \bar{K}^0\ K^-\quad \eta$

$$m_{K^0}^2 = \frac{\Delta}{f_\pi^2}(m_d+m_s)$$

$\frac{m_s}{m_d} \sim 20$

$$m_{K^+}^2 = (m_u+m_s)\frac{\Delta}{f_\pi^2}$$

$\pi$

$$m_\eta^2 = \left(\frac{4m_s+m_u+m_d}{3}\right)\frac{\Delta}{f_\pi^2}$$

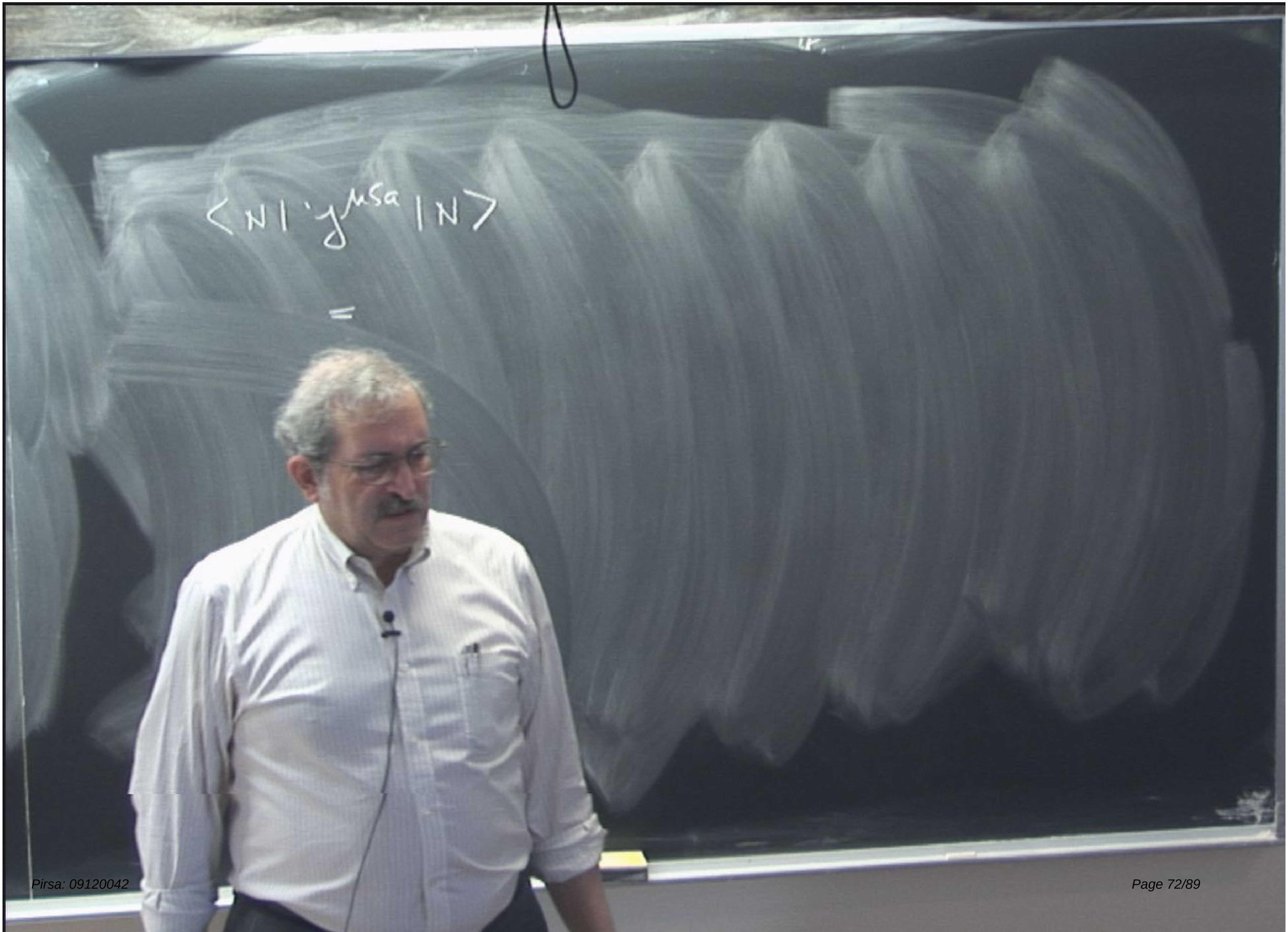
Gell Mann Okubo

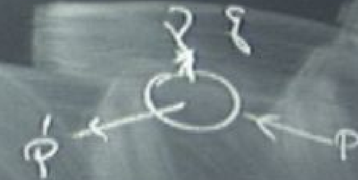
$$2\left(m_{K^+}^2 + m_{K^0}^2\right) = 3\, m_\eta^2 + m_\pi^2$$

567. MeV

vs.

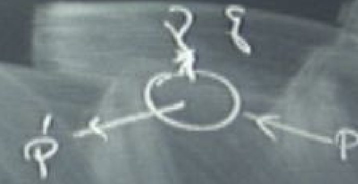
548 MeV


$\langle N | \gamma^{\mu 5 a} | N \rangle$
$=$




$$\langle N | j^{\mu 5 a} | N \rangle$$

$$= \bar{u}(p') \left[ \gamma^\mu \gamma^5 F_1(q^2) + \frac{i \sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) + q^\mu \gamma^5 F_3 \right] u(p),$$




$$\langle N | j^{\mu 5 a} | N \rangle$$

$$= \bar{u}(p') \left[ \gamma^\mu \gamma^5 F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) + q^\mu \gamma^5 F_3 \right] u(p)$$

$$F_1(0) = g_A$$

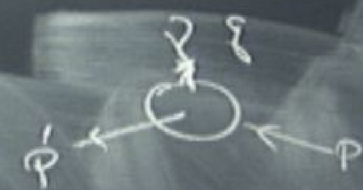

$$\langle N | j^{\mu 5 a} | N \rangle$$

$$= \bar{u}(p') \left[ \gamma^\mu \gamma^5 F_1(q^2) + \frac{i \sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) + q^\mu \gamma^5 F_3 \right] u(p)$$

$$F_1(0) = g_A$$

$$\mathcal{O} = q_\mu j^\mu$$

$$= \bar{u}(p') \left[ \not{q} \gamma^5 F_1(q^2) + q^2 \gamma^5 F_3(q^2) \right] u(p)$$




$$\langle N | j^{\mu 5 a} | N \rangle$$

$$= \bar{u}(p')\left[\gamma^\mu \gamma^5 \underbrace{F_1(q^2)} + i\frac{\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) + q^\mu \gamma^5 F_3\right] u(p)$$

$$F_1(0) = g_A$$

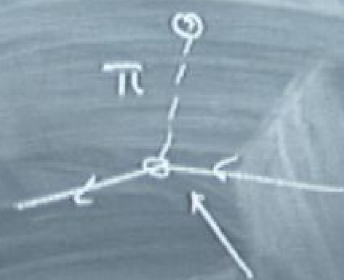
$$\mathcal{O} = q_\mu j^\mu$$

$$= \bar{u}(p')\left[\not{q}\, \gamma^5 F_1(q^2) + q^2 \gamma^5 F_3(q^2)\right] u(p)$$

$$= \bar{u}\gamma^5 u \, 2m_N F_1(q^2) + q^2 \bar{u}\gamma^5 u \, F_3(q^2)$$

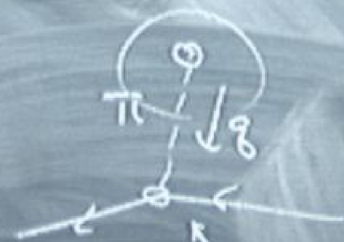
$$\mathcal{O} =$$

$$2m_N \bar{u}\gamma^5 u$$




$$2\, g_{\pi NN} \; \bar{u}\, \gamma^5 \tau^a u \; \frac{i}{q^2}$$

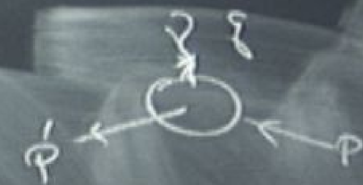

$$2\, g_{\pi NN}\ \bar{u}\, \gamma^5 \tau^a u\ \frac{i}{q^2}\ i f_\pi\, q^\mu$$




$$2\, g_{\pi NN}\ \bar{u}\, \gamma^5 \tau^a u\ \frac{i}{q^2}\ i f_\pi\, q^\mu$$

$$g_{\pi NN}\, \bar{u}\, \gamma^5\, \tau^a u\ \frac{q^\mu}{q^2}\, f_\pi$$

$$F_3 = \frac{f_\pi}{q^2}$$




$$\langle N | j^{\mu 5 a} | N \rangle$$

Goldberg Treiman

$$= \bar{u}(p')\left[ \gamma^\mu \gamma^5 \underbrace{F_1(q^2)} + i \frac{\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) + q^\mu \gamma^5 F_3 \right] u(p)$$

$$g_A = \underbrace{\frac{f_\pi}{m_N}}\, g_{\pi NN} \leftarrow 13.3 \qquad F_1(0) = g_A$$

$\downarrow$

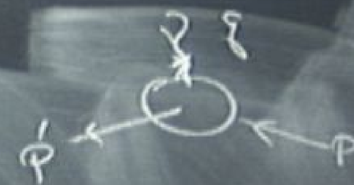
$1.25 \cong 1.31$

$$= \bar{u}(p')\left[ \not{q} \gamma^5 F_1(q^2) + q^2 \gamma^5 F_3(q^2) \right] u(p)$$

$$= \bar{u} \gamma^5 u \; 2 m_N F_1(q^2) + q^2 \, \bar{u} \gamma^5 u \; F_3(q^2)$$

$$0 =$$

$$= 2 m_N g_A - f_\pi g_{\pi NN} = 0$$




Goldberger Treiman

$$\langle N | j^{\mu 5 a} | N \rangle = \bar{u}(p')\left[ \gamma^\mu \gamma^5 \underbrace{F_1(q^2)} + i \frac{\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) + q^\mu \gamma^5 F_3 \right] u(p)$$

$$F_1(0) = g_A$$

$$g_A = \underbrace{\frac{f_\pi}{m_N}\, g_{\pi NN}} \leftarrow 13.3$$

$$\downarrow$$

$$1.25 \cong 1.31$$

$$= \bar{u}(p')\left[ \not{q} \gamma^5 F_1(q^2) + q^2 \gamma^5 F_3(q^2) \right] u(p)$$

$$= \bar{u} \gamma^5 u \, 2 m_N F_1(q^2) + q^2 \bar{u} \gamma^5 u \, F_3(q^2)$$

$$0 = 2 m_N g_A - f_\pi g_{\pi NN} = 0$$

$t\zeta(x)$

$G_0$

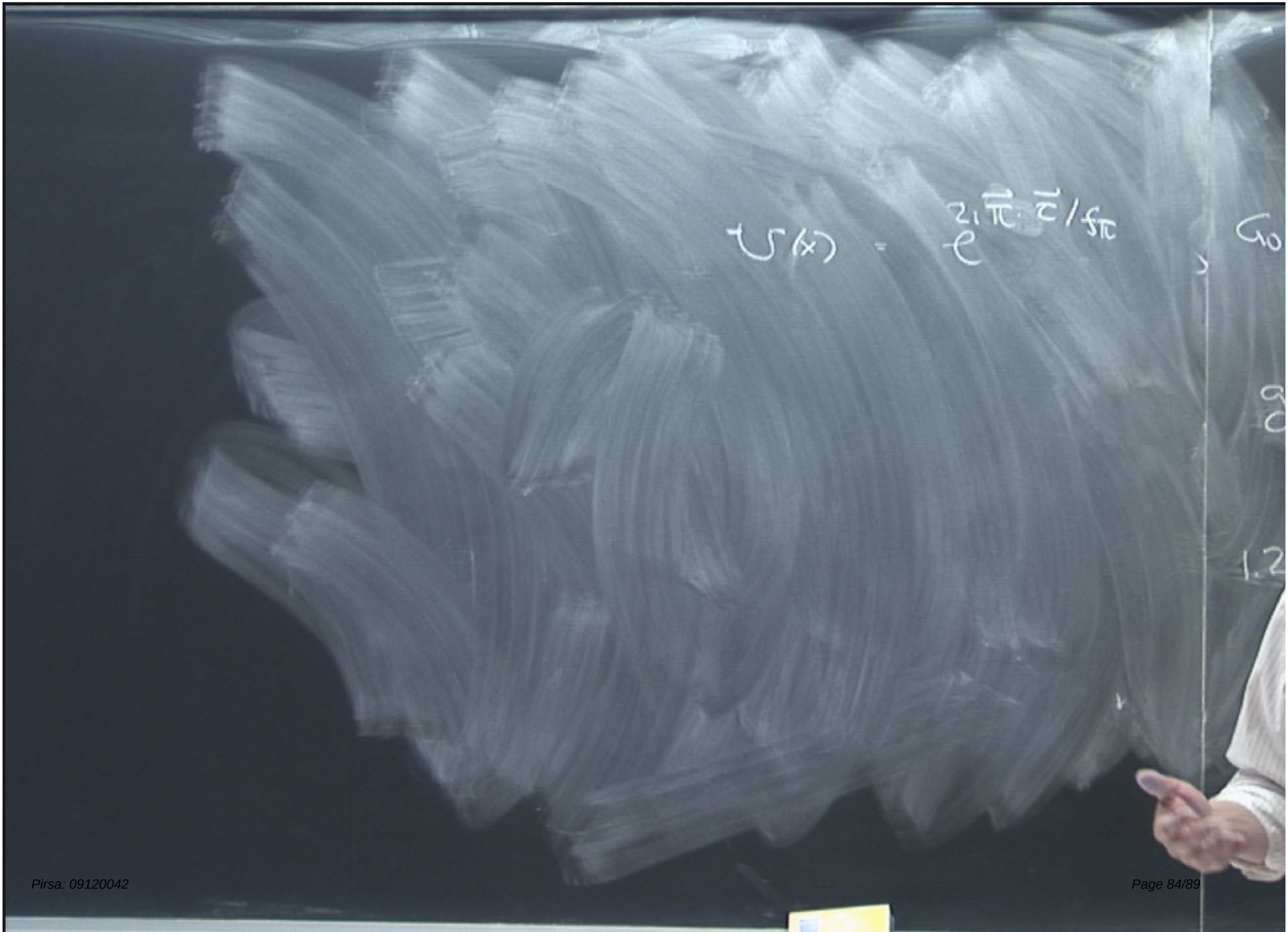
$g$
$c$

$1.2$

$$U(x) = e^{2i\vec{\pi}\cdot\vec{\tau}/f_\pi}$$

$G_0$

$1.2$


U(x) = e^{2i π⃗·τ⃗/fπ}

$$\Sigma(x) = e^{2i\vec{\pi}\cdot\vec{\tau}/f_\pi}$$

$$\Sigma \to V \Sigma V^\dagger$$

$$U \to W U W$$

$G_0$

$g$

$1.2$

$$U(x) = e^{2i\vec{\pi}\cdot\vec{\tau}/f_\pi}$$

Go

$$U \to V U V^{+}$$

$$U \to W U W$$

$$\mathcal{L} = \frac{1}{2} f_\pi^2 \, \mathrm{tr} \, \partial^\mu U^{+} \partial_\mu U$$

1.2

$$U^\dagger U = 1$$

$$U(x) = e^{2i\vec{\pi}\cdot\vec{\tau}/f_\pi}$$

$$U \to V U V^\dagger$$

$$U \to W U W$$

$$\mathcal{L} = \frac{1}{2} f_\pi^2 \, \mathrm{tr}\, \partial^\mu U^\dagger \partial_\mu U$$

Go

1.2

$$U^{\dagger} U = 1$$

$$U(x) = e^{2i\vec{\pi}\cdot\vec{\tau}/f_\pi}$$

$$U \to V U V^{\dagger}$$

$$U \to W U W$$

$$\mathcal{L} = \frac{1}{2} f_\pi^2 \, \mathrm{tr}\, \partial^\mu U^{\dagger} \partial_\mu U$$

$U^\dagger U = 1$

$U(x) = e^{2i\vec{\pi}\cdot\vec{\tau}/f_\pi}$

$U \to V U V^\dagger$

$U \to W U W$

$\mathcal{L} = \frac{1}{2} f_\pi^2 \operatorname{tr} \partial^\mu U^\dagger \partial_\mu U$

$I = 0$ $I = 1$ $I = 2$

Same Phase Sma

$+$ mainly

Go

1.2