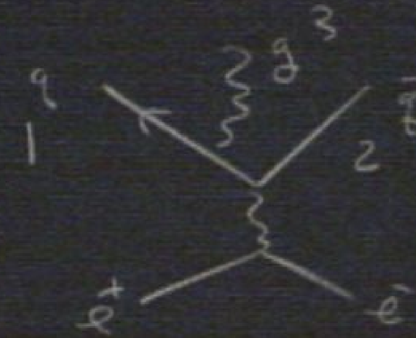
Title: Standard Model - Review (PHYS 622) - Lecture 8

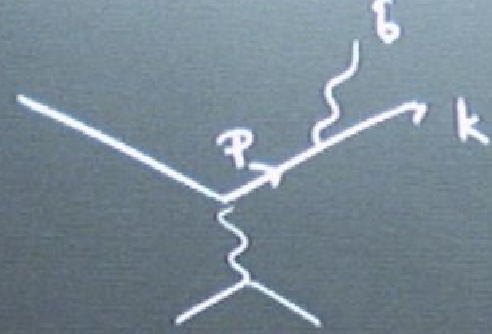
Date: Dec 09, 2009 09:00 AM

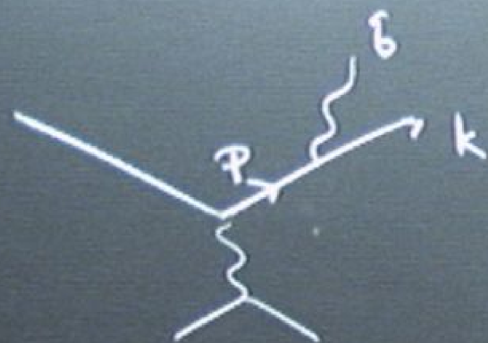
URL: http://pirsa.org/09120040

Abstract:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_1\,dx_2} = \frac{2\alpha_s}{3\pi}\,\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$



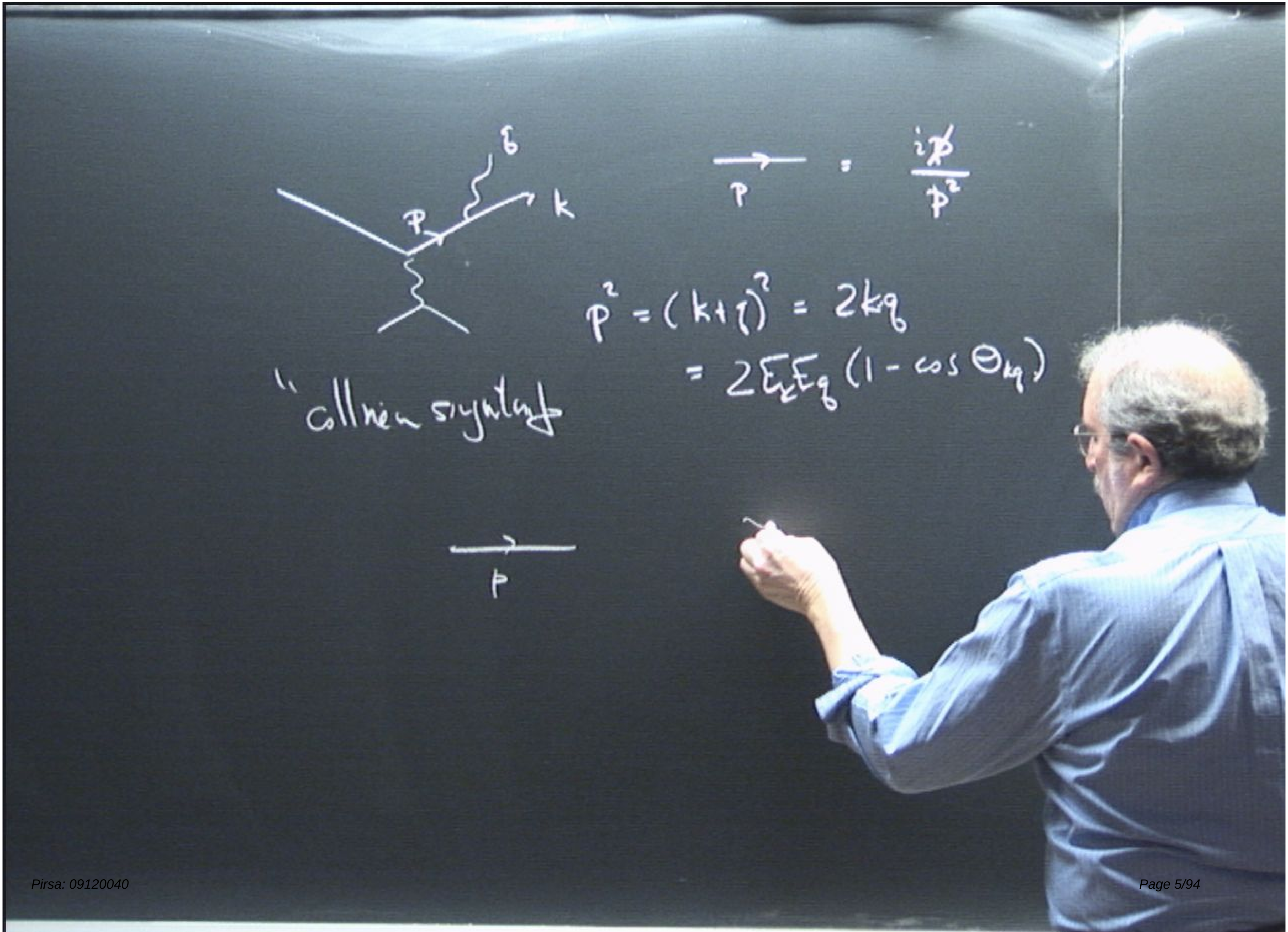

p
k

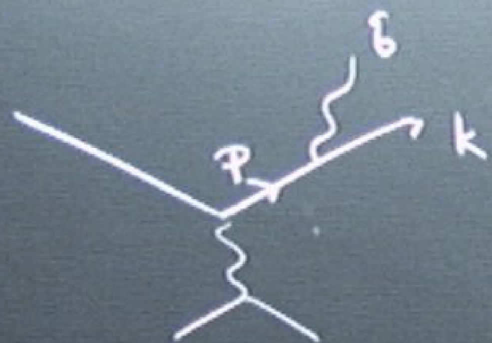


$$\xrightarrow[p]{} \; = \; \frac{i\not{p}}{p^2}$$

$$p^2 = (k+q)^2 = 2kq$$

$$= 2E_k E_q (1 - \cos\theta_{kq})$$


q
k
p
$= \frac{i\not{p}}{p^2}$
$p^2 = (k+q)^2 = 2kq$
$= 2E_k E_q (1 - \cos\theta_{kq})$
"collinear singularity
p



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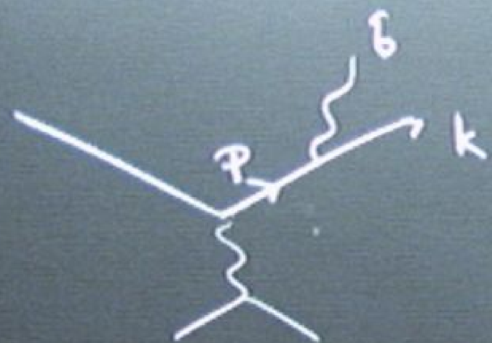
"collinear singularity"



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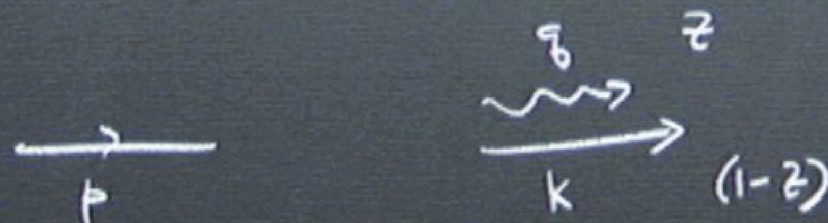

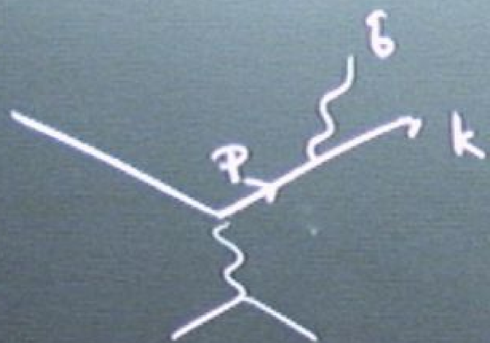

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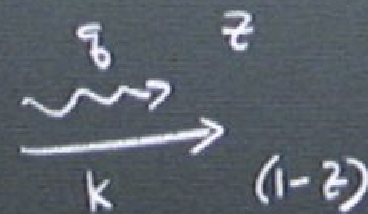
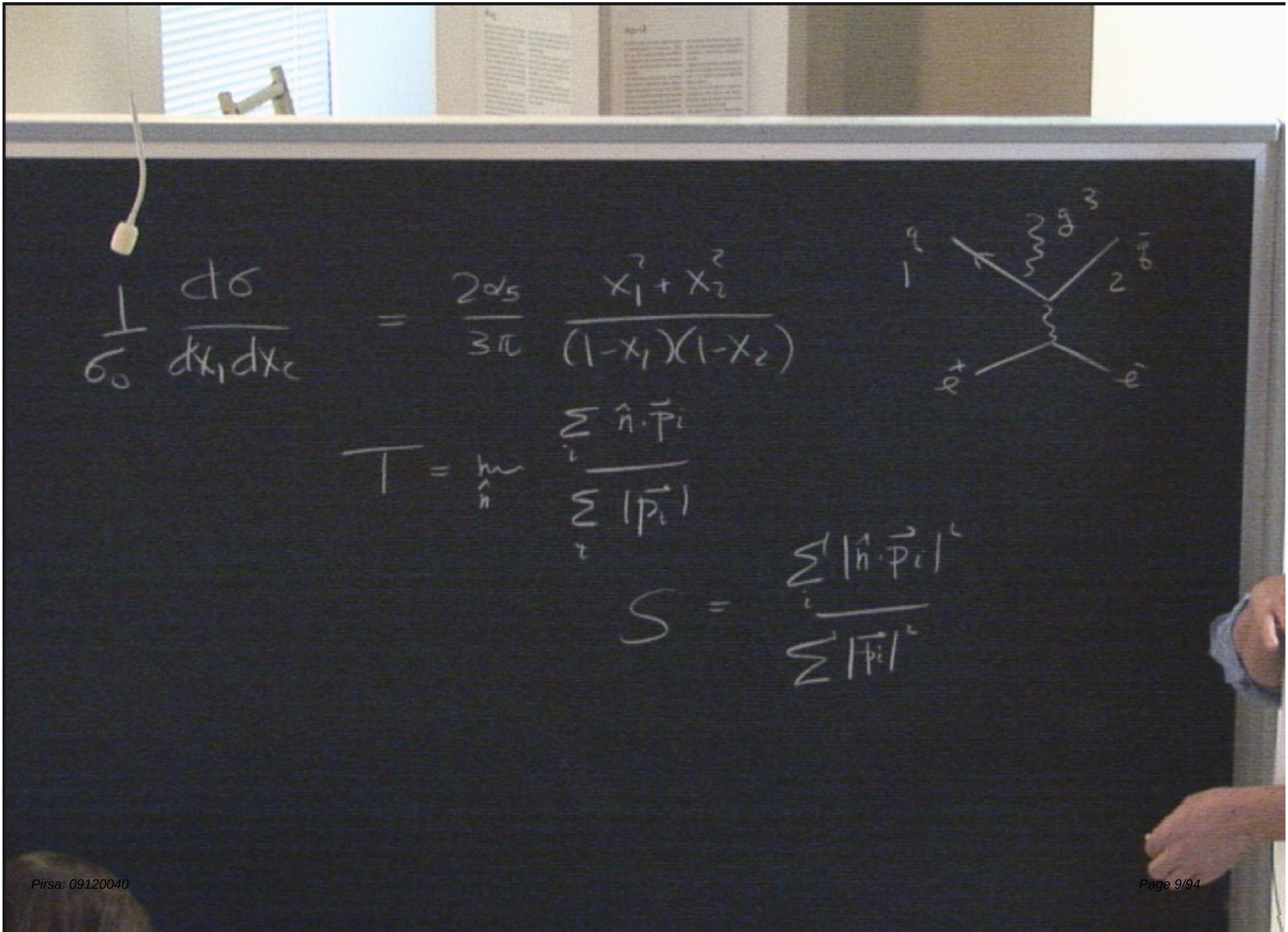
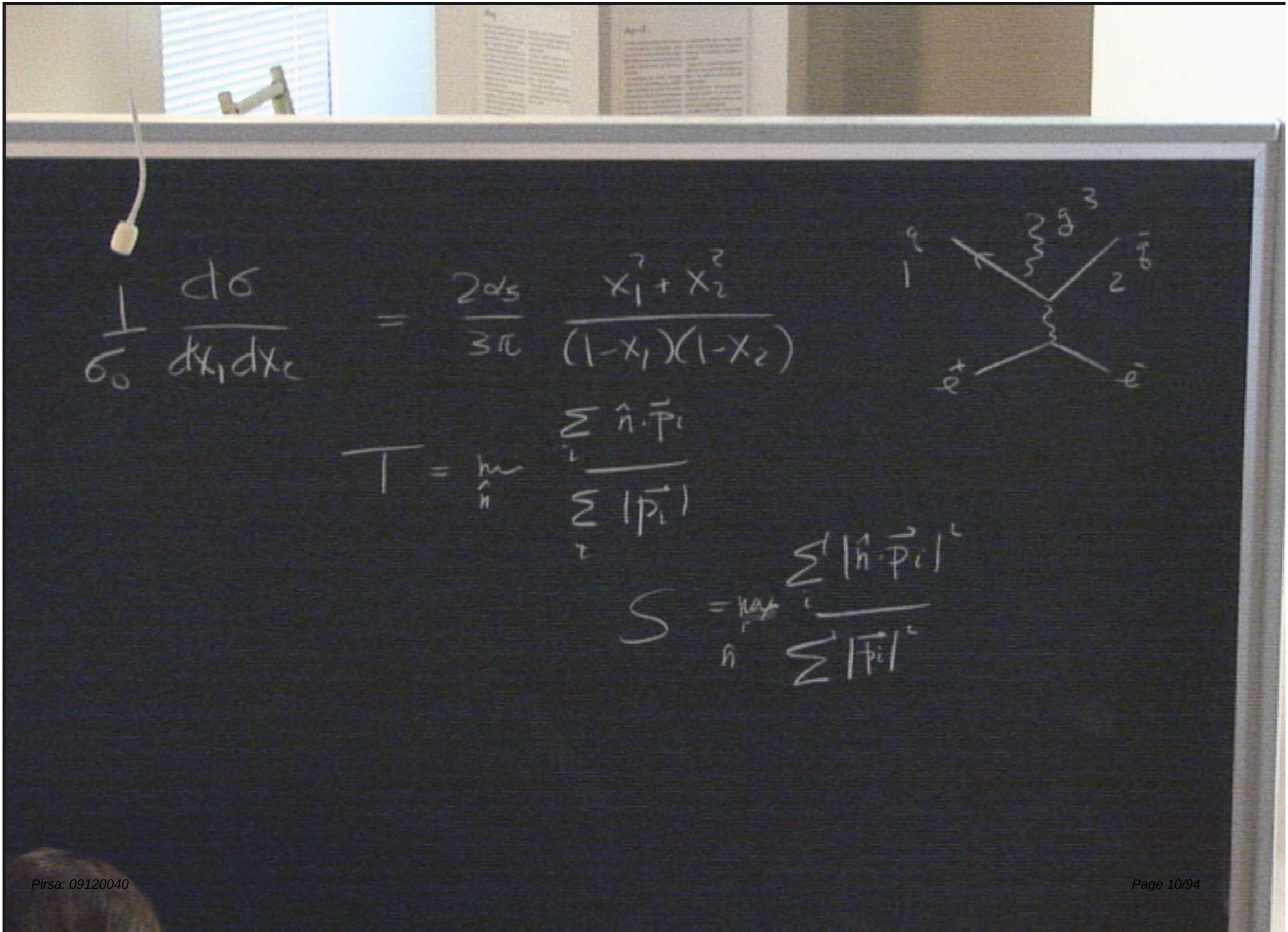




$\frac{1}{\sigma_0}\frac{d\sigma}{dx_1\,dx_2} = \frac{2\alpha_s}{3\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$
q 1
g 3
$\bar{q}$ 2
e^+
e^-
$T = \max_{\hat{n}} \frac{\sum_i \hat{n}\cdot\vec{p}_i}{\sum_i |\vec{p}_i|}$
$S = \frac{\sum_i |\hat{n}\cdot\vec{p}_i|^2}{\sum_i |\vec{p}_i|^2}$



$\frac{1}{\sigma_0}\frac{d\sigma}{dx_1 dx_2} = \frac{2\alpha_s}{3\pi}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$
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q 1
g 3
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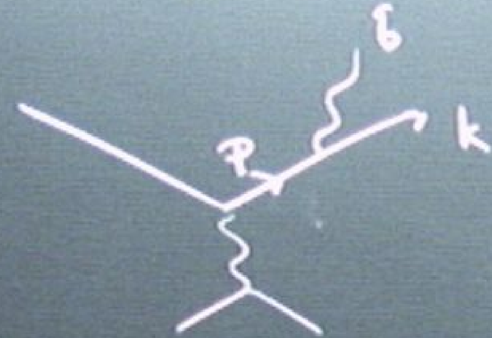

$$\xrightarrow[p]{} = \frac{i\not{p}}{p^2}$$

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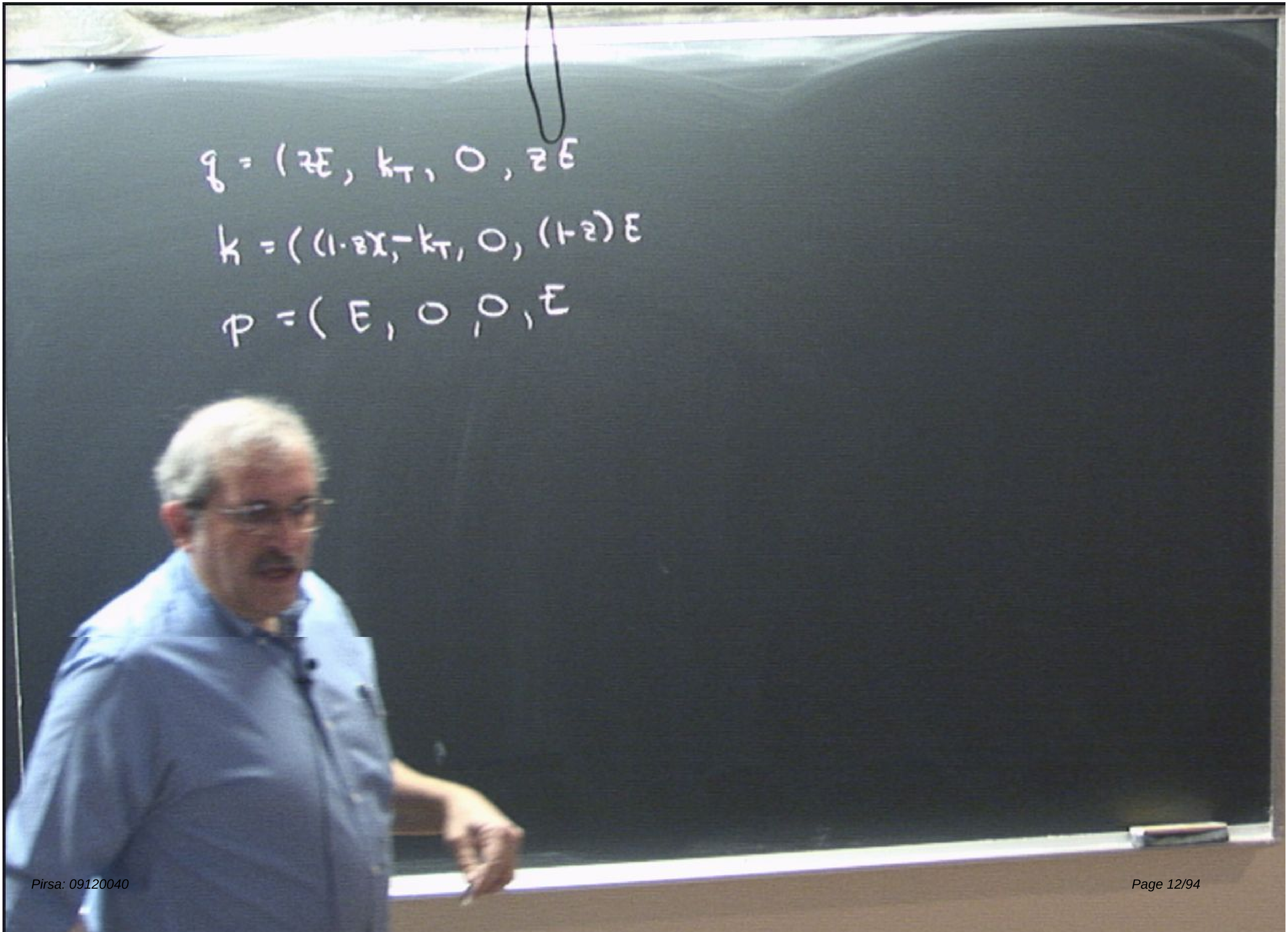
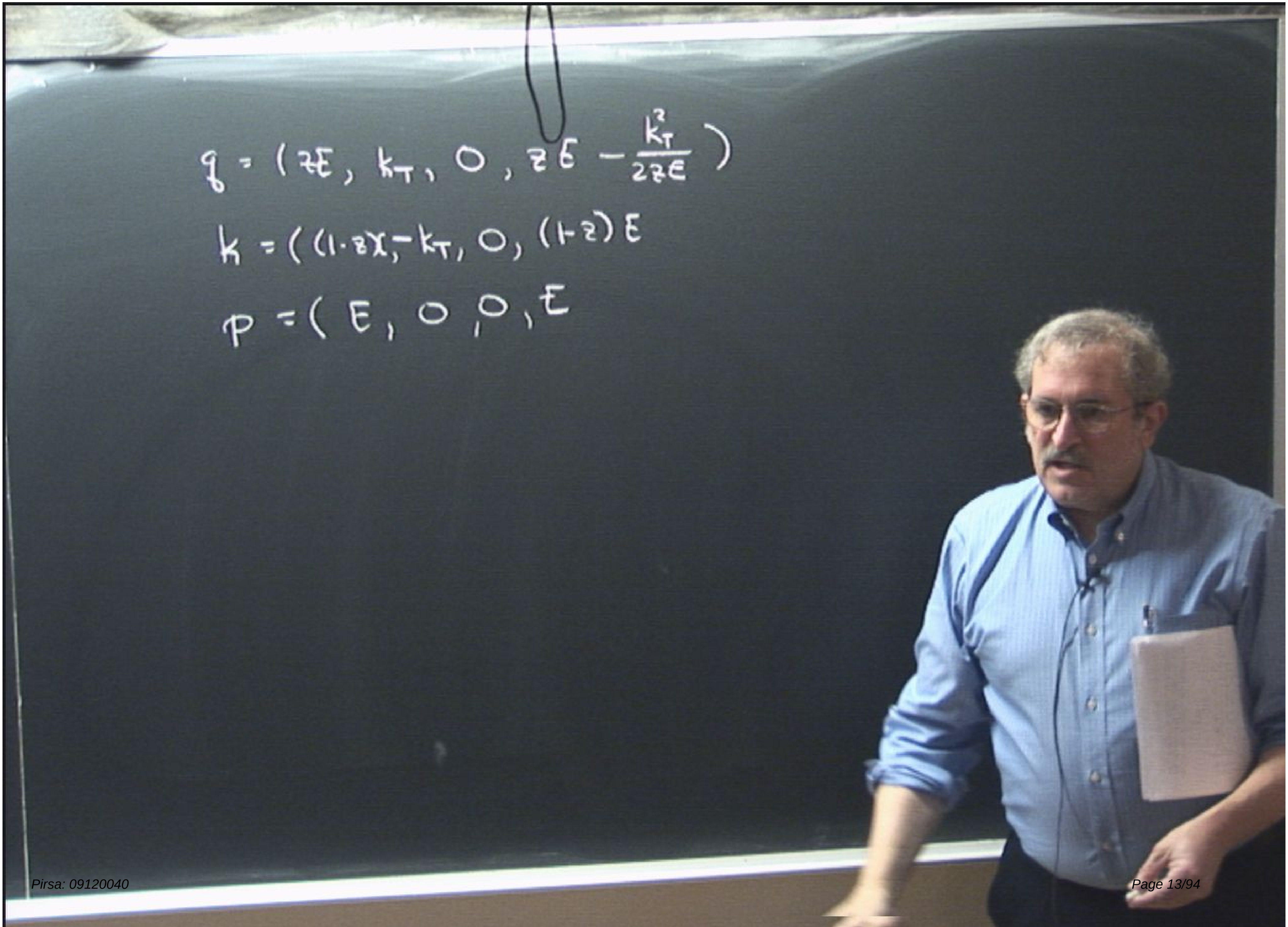
"collinear singularity"

"infrared safe"


$q = (zE, k_T, 0, zE$
$k = ((1-z)E, -k_T, 0, (1-z)E$
$p = (E, 0, 0, E$


$q = (zE, k_T, 0, zE - \frac{k_T^2}{2zE})$
$k = ((1-z)E, -k_T, 0, (1-z)E$
$p = (E, 0, 0, E$

$$q^2 = 0 \quad \text{mod. } \mathcal{O}(k_T^2)$$

$$q = \left(zE,\ k_T,\ 0,\ zE - \frac{k_T^2}{2zE} \right)$$

$$k = \left((1-z)E,\ -k_T,\ 0,\ (1-z)E - \frac{k_T^2}{2(1-z)E} \right)$$

$$p = (E,\ 0,\ 0,\ E$$

$$q = \left(zE,\ k_T,\ 0,\ zE - \frac{k_T^2}{2zE} \right)$$

$$k = \left((1-z)E,\ -k_T,\ 0,\ (1-z)E - \frac{k_T^2}{2(1-z)E} \right)$$

$$p = \left(E,\ 0,\ 0,\ E - \frac{k_T^2}{2E\, z(1-z)} \right)$$

$$p^2 = \frac{k_T^2}{z(1-z)}$$

$$q^2 = 0 \quad \text{mod.}\ \mathcal{O}(k_T^2)$$

$$\frac{1}{z} + \frac{1}{1-z} = \frac{1}{z(1-z)}$$

$$q = \left(zE,\ k_T,\ 0,\ zE - \frac{k_T^2}{2zE} \right)$$

$$k = \left((1-z)E,\ -k_T,\ 0,\ (1-z)E - \frac{k_T^2}{2(1-z)E} \right)$$

$$p = \left(E,\ 0,\ 0,\ E - \frac{k_T^2}{2E\, z(1-z)} \right)$$

$$p^2 = \frac{k_T^2}{z(1-z)}$$

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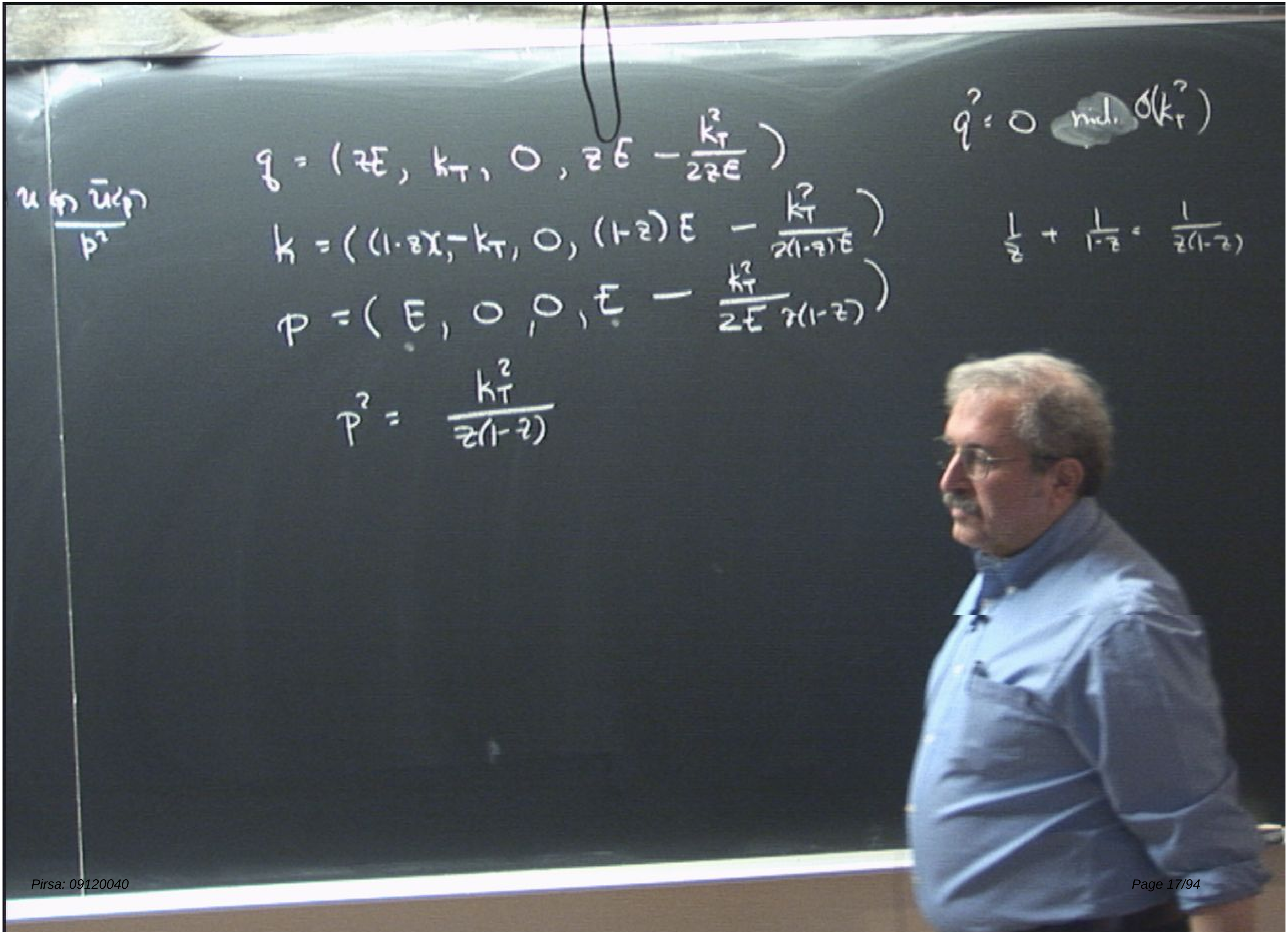
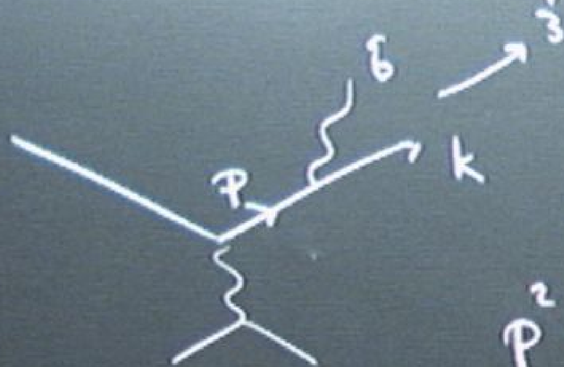
$$\frac{1}{z} + \frac{1}{1-z} = \frac{1}{z(1-z)}$$


$\frac{u(p)\,\bar{u}(p)}{p^2}$
$q = \left(zE,\ k_T,\ 0,\ zE - \frac{k_T^2}{2zE} \right)$
$k = \left((1-z)E,\ -k_T,\ 0,\ (1-z)E - \frac{k_T^2}{z(1-z)E} \right)$
$p = \left(E,\ 0,\ 0,\ E - \frac{k_T^2}{2E\,z(1-z)} \right)$
$p^2 = \frac{k_T^2}{z(1-z)}$
$q^2 = 0 \ \text{mod.}\ O(k_T^2)$
$\frac{1}{z} + \frac{1}{1-z} = \frac{1}{z(1-z)}$

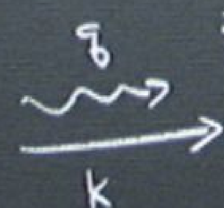


$$\underset{p}{\longrightarrow} = \frac{i\not{p}}{p^2} = \frac{i\,u(p)\,\bar{u}(p)}{p^2}$$

$$p^2 = (k+q)^2 = 2kq$$
$$= 2E_kE_q(1-\cos\Theta_{kq})$$

"collinear singularity"

$$\bar{u}(p)\,\frac{i}{p^2}\,u(p)$$



"infrared safe"

$$\frac{i\, u(q)\, \bar{u}(p)}{p^2}$$

$$q = \left(zE,\ k_T,\ 0,\ zE - \frac{k_T^2}{2zE} \right)$$

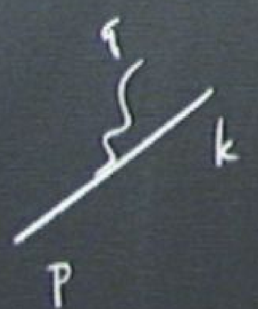
$$k = \left((1-z)E,\ -k_T,\ 0,\ (1-z)E - \frac{k_T^2}{z(1-z)E} \right)$$

$$p = \left(E,\ 0,\ 0,\ E - \frac{k_T^2}{2E\, z(1-z)} \right)$$

$$p^2 = \frac{k_T^2}{z(1-z)}$$

$q^2 = 0$

$\frac{1}{z} + \frac{1}{1-\ldots}$



$$i\mathcal{M} = i g t^a\ \bar{u}(k)\, \gamma_\mu\, \varepsilon^{*}(q)\, u(p)$$

$q^2 = 0$ incl. $\mathcal{O}(k_T^2)$

$$q = \left(zE,\ k_T,\ 0,\ zE - \frac{k_T^2}{2zE} \right)$$

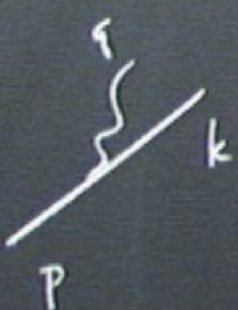
$$k = \left((1-z)E,\ -k_T,\ 0,\ (1-z)E - \frac{k_T^2}{2(1-z)E} \right)$$

$$p = \left(E,\ 0,\ 0,\ E - \frac{k_T^2}{2E\, z(1-z)} \right)$$

$$p^2 = \frac{k_T^2}{z(1-z)}$$

$$\frac{1}{z} + \frac{1}{1-z} = \frac{1}{z(1-z)}$$

$$u_L = \begin{pmatrix} \cdot \\ \cdot \\ 0 \\ 0 \end{pmatrix}$$



$$i\mathcal{M} = ig\, t^a\, \bar{u}(k)\, \gamma \cdot \varepsilon^*(q)\, u(p)$$

$$= ig\, t^a\, u_L^\dagger(k)\, \bar{\sigma} \cdot \varepsilon^*(q)\, u_L(p)$$

$$q = \left(zE,\ k_T,\ 0,\ zE - \frac{k_T^2}{2zE} \right)$$

$$k = \left((1-z)E,\ -k_T,\ 0,\ (1-z)E - \frac{k_T^2}{2(1-z)E} \right)$$

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$q^2 = 0$ incl. $O(k_T^2)$

$$\frac{1}{z} + \frac{1}{1-z} = \frac{1}{z(1-z)}$$

$$u_L = \begin{pmatrix} \vdots \\ 0 \\ 0 \end{pmatrix}$$

k

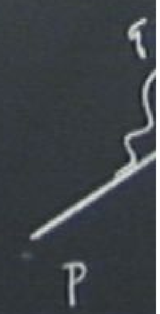
$$i\mathcal{M} = i g t^a\ \bar{u}(k)\, \gamma_i\, \varepsilon^*(q)\ u(p)$$

$$= i g t^a\ u_L^\dagger(k)\, \sigma_i\, \varepsilon^*(q)\ u_L(p)$$

$$u(p) = \sqrt{2E} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad u(k) = \sqrt{2(1-z)E} \begin{pmatrix} +\frac{k_T}{2(1-z)E} \\ 1 \end{pmatrix}$$

$$\varepsilon_L = \frac{1}{\sqrt{2}}(0,\ 1,\ -i,\ \ )\qquad \varepsilon_R = \frac{1}{\sqrt{2}}(0,\ 1,\ i,\ \ )$$

$$\varepsilon_L^* = \frac{1}{\sqrt{2}}\left(0, 1, +i, -\frac{k_T}{2E}\right) \qquad \varepsilon_R^* = \frac{1}{\sqrt{2}}\left(0, 1, -i, -\frac{k_T}{2E}\right)$$



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$$\bar{\sigma}\cdot\varepsilon_L^* = \frac{1}{\sqrt{2}}\begin{pmatrix} -\frac{k_T}{2E} & 2 \\ 0 & \frac{k_T}{2E} \end{pmatrix} \qquad \bar{\sigma}\cdot\varepsilon_R^* = \frac{1}{\sqrt{2}}\begin{pmatrix} -\frac{k_T}{2E} & 0 \\ 2 & \frac{k_T}{2E} \end{pmatrix}$$

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$$q_L \to g_R q_L$$

$$i\mathcal{M} = \Bigg\{$$

$$\varepsilon_L^* = \frac{1}{\sqrt{2}}\left(0,\ 1,\ +i,\ -\frac{k_T}{zE}\right) \qquad \varepsilon_R^* = \frac{1}{\sqrt{2}}\left(0,\ 1,\ -i,\ -\frac{k_T}{zE}\right)$$

$$\bar{\sigma}\cdot\varepsilon_L^* = \frac{1}{\sqrt{2}}\begin{pmatrix} -\frac{k_T}{zE} & 2 \\ 0 & \frac{k_T}{zE} \end{pmatrix} \qquad \bar{\sigma}\cdot\varepsilon_R^* = \frac{1}{\sqrt{2}}\begin{pmatrix} -\frac{k_T}{zE} & 0 \\ 2 & \frac{k_T}{zE} \end{pmatrix}$$

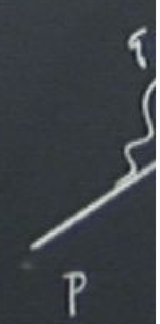
$q_L \to g_R q_L$

$$i\mathcal{M} = \left\{ \quad i g t^a \, 2E \sqrt{1-z} \, \frac{1}{\sqrt{2}} \, \frac{k_T}{zE} \right.$$

$$\varepsilon_L^* = \frac{1}{\sqrt{2}}\left(0,\ 1,\ +i,\ -\frac{k_T}{zE}\right) \qquad \varepsilon_R^* = \frac{1}{\sqrt{2}}\left(0,\ 1,\ -i,\ -\frac{k_T}{zE}\right)$$

$$\bar{\sigma}\cdot\varepsilon_L^* = \frac{1}{\sqrt{2}}\begin{pmatrix} -\frac{k_T}{zE} & 2 \\ 0 & \frac{k_T}{zE} \end{pmatrix} \qquad \bar{\sigma}\cdot\varepsilon_R^* = \frac{1}{\sqrt{2}}\begin{pmatrix} -\frac{k_T}{zE} & 0 \\ 2 & \frac{k_T}{zE} \end{pmatrix}$$

$$i\mathcal{M} = \begin{cases} q_L \to g_R q_L & i g t^a\ 2E\sqrt{1-z}\ \frac{1}{\sqrt{2}}\ \frac{k_T}{zE} \\ q_L \to g_L q_L & i g t^a\ 2E\sqrt{1-z}\ \frac{1}{\sqrt{2}} \end{cases}$$



$$\varepsilon_L^* = \frac{1}{\sqrt{2}}\left(0,\ 1,\ +i,\ -\frac{k_T}{2E}\right) \qquad \varepsilon_R^* = \frac{1}{\sqrt{2}}\left(0,\ 1,\ -i,\ -\frac{k_T}{2E}\right)$$

$$\bar{\sigma}\cdot\varepsilon_L^* = \frac{1}{\sqrt{2}}\begin{pmatrix} -\frac{k_T}{2E} & 2 \\ 0 & \frac{k_T}{2E} \end{pmatrix} \qquad \bar{\sigma}\cdot\varepsilon_R^* = \frac{1}{\sqrt{2}}\begin{pmatrix} -\frac{k_T}{2E} & 0 \\ 2 & \frac{k_T}{2E} \end{pmatrix}$$

$$i\mathcal{M} = \begin{cases} q_L \to g_R q_L & i g t^a \, 2E \sqrt{1-z} \, \frac{1}{\sqrt{2}} \, \frac{k_T}{2E} \\ q_L \to g_L q_L & i g t^a \, 2E \sqrt{1-z} \, \frac{1}{\sqrt{2}} \, \frac{k_T}{2E} \end{cases}$$

$$\varepsilon_L^* = \frac{1}{\sqrt{2}}\left(0,\ 1,\ +i,\ -\frac{k_T}{zE}\right) \qquad \varepsilon_R^* = \frac{1}{\sqrt{2}}\left(0,\ 1,\ -i,\ -\frac{k_T}{zE}\right)$$

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$$i\mathcal{M} = \begin{cases} q_L \to g_R q_L & ig t^a\, 2E\sqrt{1-z}\,\frac{1}{\sqrt{2}}\,\frac{k_T}{zE} \\ q_L \to g_L q_L & ig t^a\, 2E\sqrt{1-z}\,\frac{1}{\sqrt{2}}\,\underbrace{\left(\frac{k_T}{zE} + \frac{k_T}{(1-z)E}\right)}_{\frac{k_T}{z(1-z)E}} \end{cases}$$

$$\varepsilon_L^* = \frac{1}{\sqrt{2}}\left(0,\ 1,\ +i,\ -\frac{k_T}{zE}\right) \qquad \varepsilon_R^* = \frac{1}{\sqrt{2}}\left(0,\ 1,\ -i,\ -\frac{k_T}{zE}\right)$$

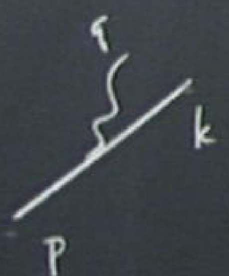
$$\bar{\sigma}\cdot\varepsilon_L^* = \frac{1}{\sqrt{2}}\begin{pmatrix} -\frac{k_T}{zE} & 2 \\ 0 & \frac{k_T}{zE} \end{pmatrix} \qquad \bar{\sigma}\cdot\varepsilon_R^* = \frac{1}{\sqrt{2}}\begin{pmatrix} -\frac{k_T}{zE} & 0 \\ 2 & \frac{k_T}{zE} \end{pmatrix}$$

$$i\mathcal{M} = \begin{cases} q_L \to g_R q_L: & i g t^a\, 2\cancel{E} \sqrt{1-z}\, \frac{1}{\sqrt{2}}\, \frac{k_T}{z\cancel{E}} \\[2ex] q_L \to g_L q_L: & i g t^a\, 2E \sqrt{1-z}\, \frac{1}{\sqrt{2}} \underbrace{\left(\frac{k_T}{zE} + \frac{k_T}{(1-z)E}\right)}_{\frac{k_T}{z(1-z)E}} \end{cases}$$

$$|\mathcal{M}|^2 = \sum \left\{ \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2 \right. \qquad g_L \quad g_R$$

$$t^a t^a = \frac{4}{3}$$

$$p^2 = \frac{k_T^2}{z(1-z)}$$



$$i\mathcal{M} = i g t^a \, \bar{u}(k) \gamma_1 \varepsilon^*(q) \, u(p)$$

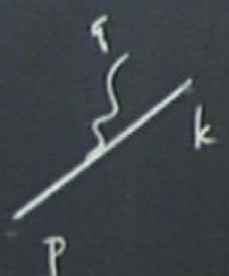
$$= i g t^a \, u_L^\dagger(k) \, \bar{\sigma}_1 \varepsilon^*(q) \, u_L(p)$$

$$u(p) = \sqrt{zE} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad u(k) = \sqrt{z(1-z)E} \begin{pmatrix} +\frac{k_T}{z(1-z)E} \\ 1 \end{pmatrix}$$

$$|\mathcal{M}|^2 = \begin{cases} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2 & g_L \\ \frac{4}{3} g^2 \frac{2(1-z)^2}{z^2(1-z)} k_T^2 & g_R \end{cases}$$

$$t^a t^a = \frac{4}{3}$$

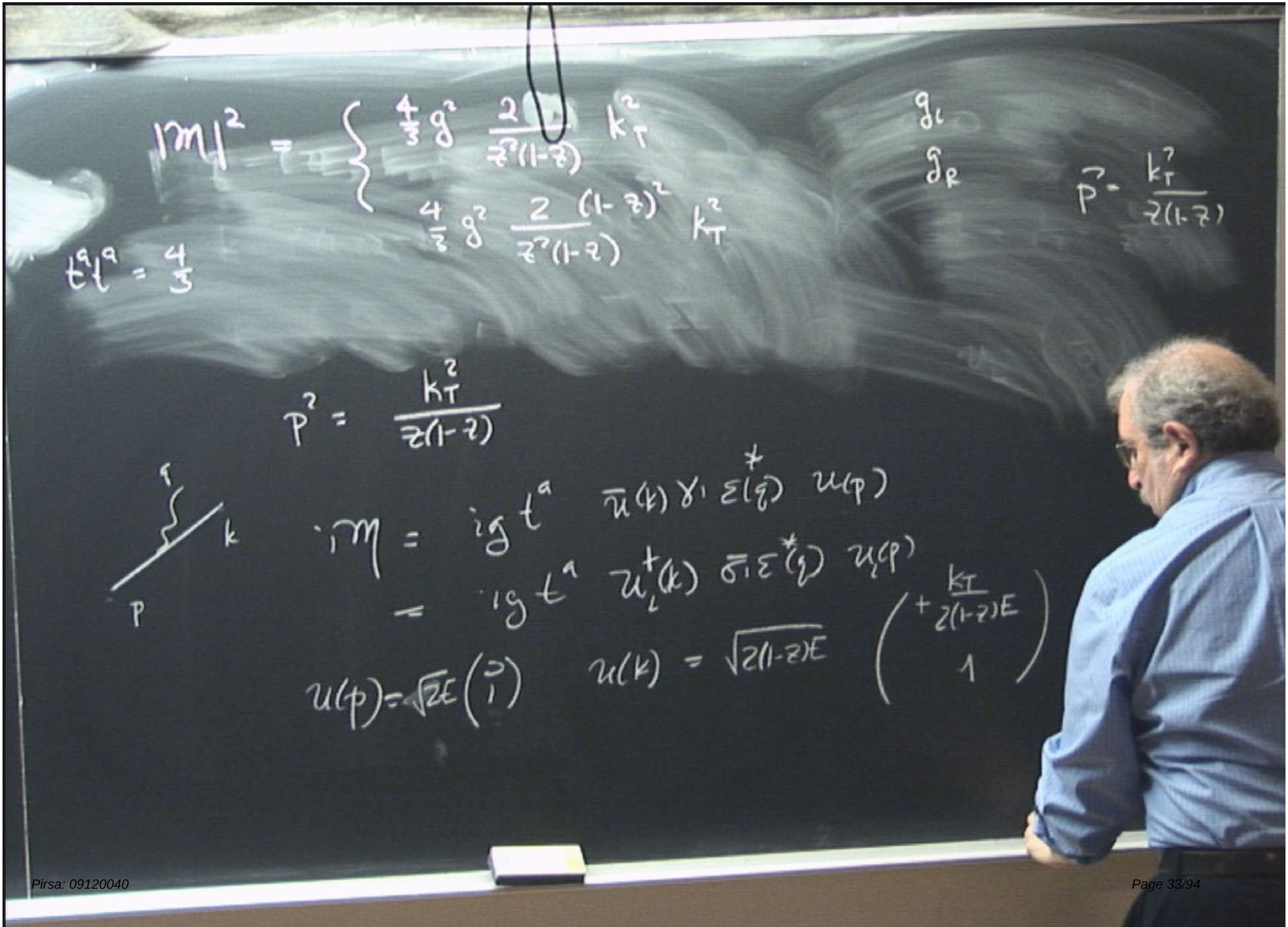
$$p^2 = \frac{k_T^2}{z(1-z)}$$

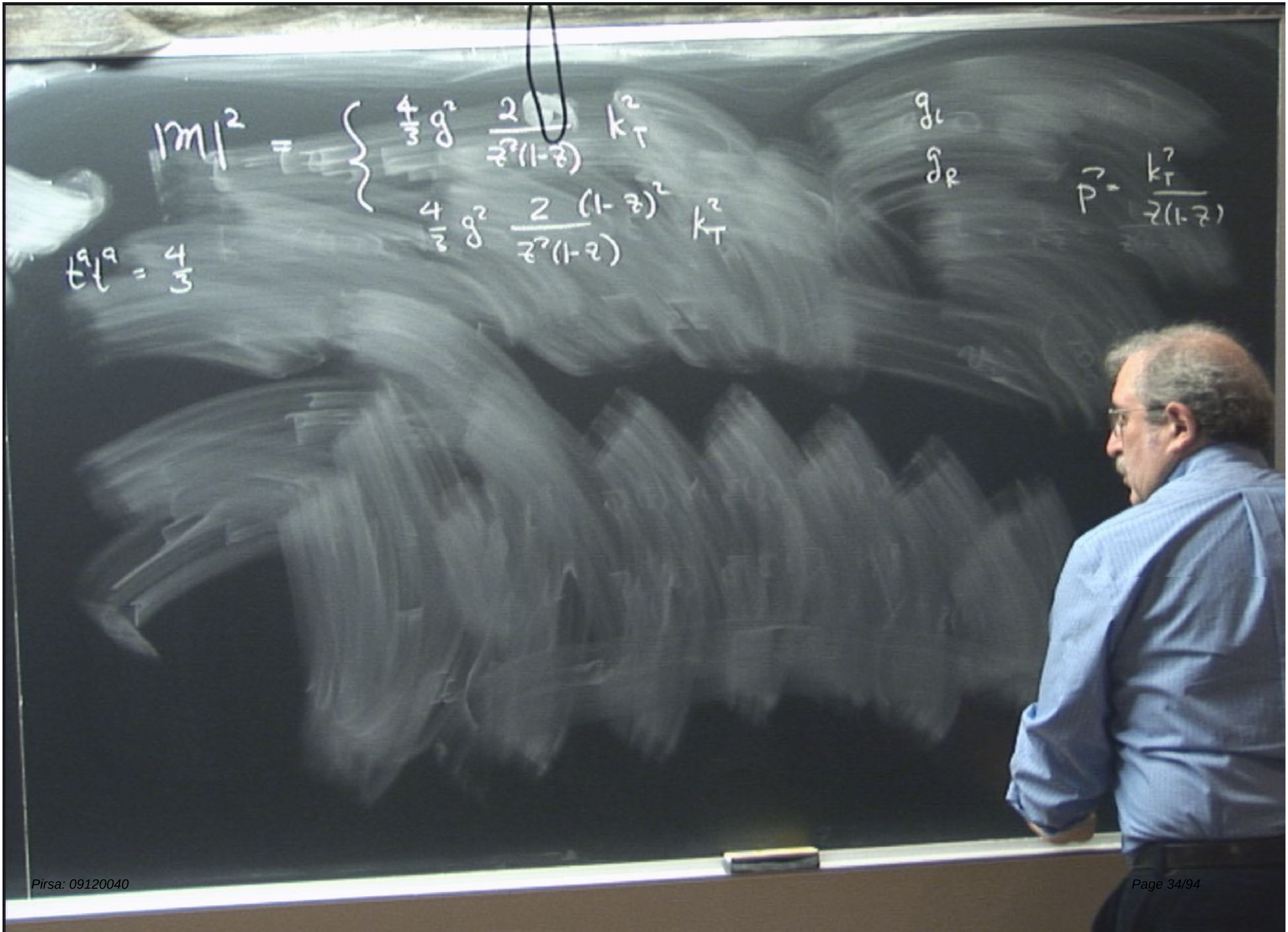


$$i\mathcal{M} = i g t^a \, \bar{u}(k) \gamma_i \varepsilon^*(q) \, u(p)$$

$$= i g t^a \, u_L^\dagger(k) \, \bar{\sigma}_i \varepsilon^*(q) \, u_L(p)$$

$$u(p) = \sqrt{2E} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad u(k) = \sqrt{z(1-z)E} \begin{pmatrix} +\frac{k_T}{z(1-z)E} \\ 1 \end{pmatrix}$$


$|\mathcal{M}|^2 = \begin{cases} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2 \\ \frac{4}{3} g^2 \frac{2(1-z)^2}{z^2(1-z)} k_T^2 \end{cases}$
g_L
g_R
$\vec{p}^2 = \frac{k_T^2}{z(1-z)}$
$t^a t^a = \frac{4}{3}$
$p^2 = \frac{k_T^2}{z(1-z)}$
q
k
p
$i\mathcal{M} = i g t^a \bar{u}(k) \gamma_i \varepsilon^*(q) u(p)$
$= i g t^a u_L^\dagger(k) \bar{\sigma}_i \varepsilon^*(q) u_L(p)$
$u(p) = \sqrt{2E}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$u(k) = \sqrt{z(1-z)E}\begin{pmatrix} +\frac{k_T}{z(1-z)E} \\ 1 \end{pmatrix}$


|M|^2 = { 4/3 g^2 2/(z^2(1-z)) k_T^2
4/3 g^2 2 (1-z)^2/(z^2(1-z)) k_T^2
t^a t^a = 4/3
g_L
g_R
p^2 = k_T^2/(z(1-z))

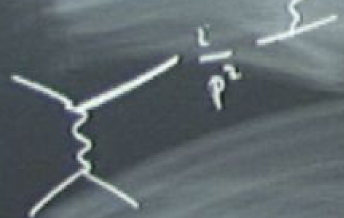
$$|\mathcal{M}|^2 = \begin{cases} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2 \\ \frac{4}{3} g^2 \frac{2(1-z)^2}{z^2(1-z)} k_T^2 \end{cases}$$

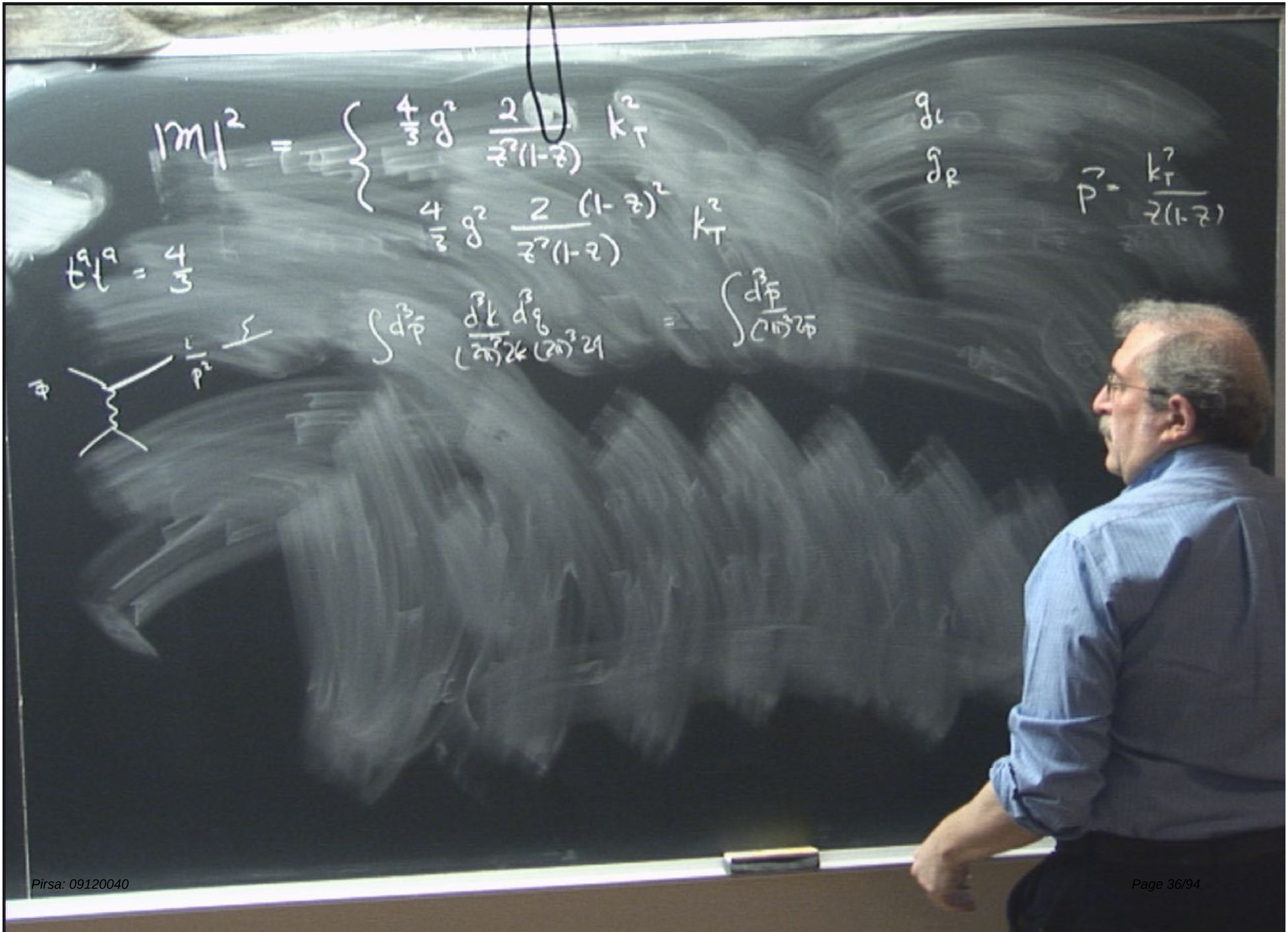
g_L

g_R

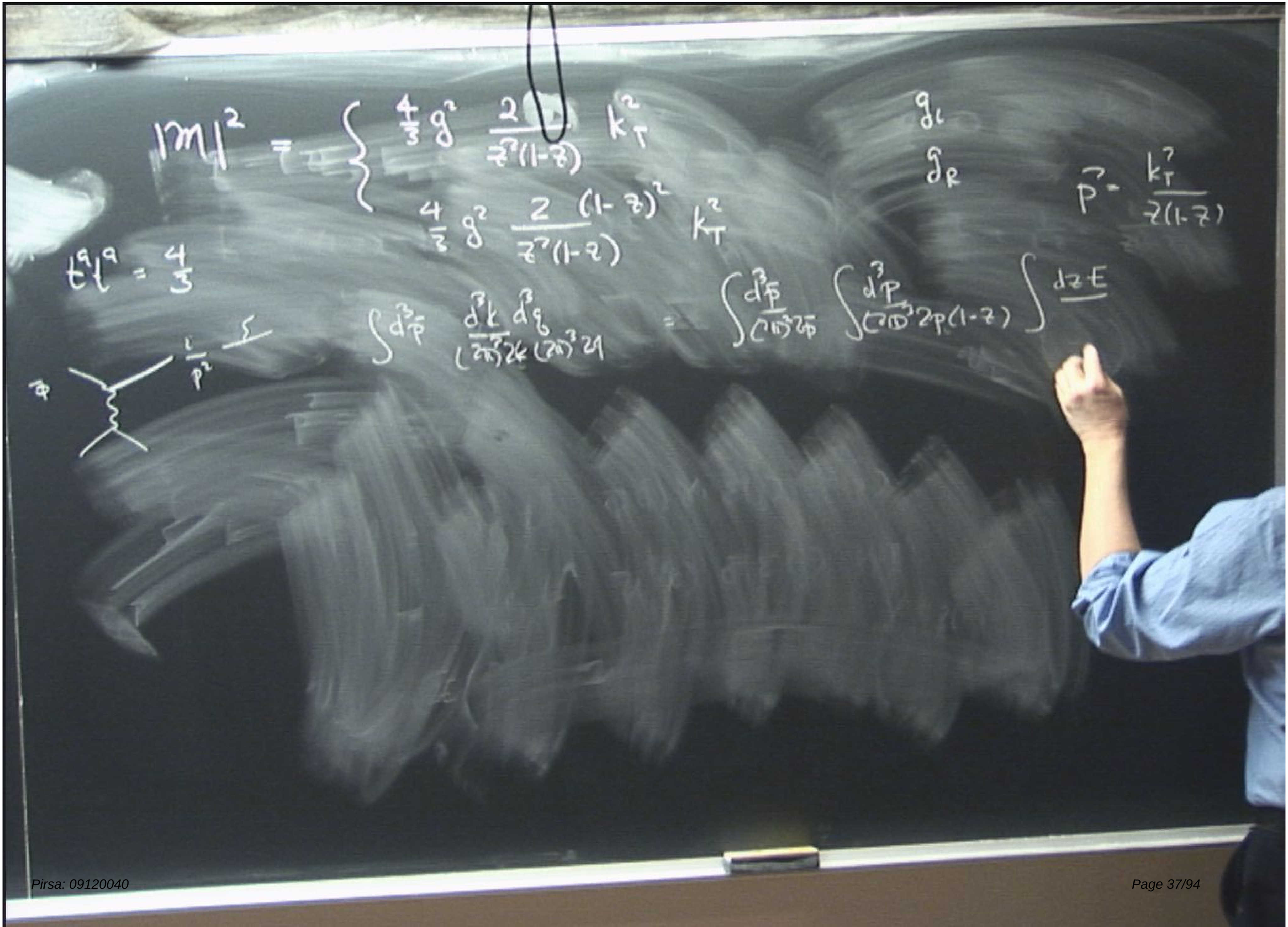
$$\vec{p}^{\,2} = \frac{k_T^2}{z(1-z)}$$

$$t^a t^a = \frac{4}{3}$$





|m|² = { 4/3 g² 2/(z²(1-z)) k_T²
4/3 g² 2(1-z)²/(z²(1-z)) k_T²
g_L
g_R
p⃗ = k_T²/(z(1-z))
t^a t^a = 4/3
∫d³p d³k/((2π)³2k) d³q/((2π)³2q) = ∫d³p/((2π)³2p)
1/p²


$|M|^2 = \begin{cases} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2 \\ \frac{4}{3} g^2 \frac{2\,(1-z)^2}{z^2(1-z)} k_T^2 \end{cases}$
g_L
g_R
$\vec{p}^2 = \frac{k_T^2}{z(1-z)}$
$t^a t^a = \frac{4}{3}$
$\frac{1}{p^2}$
Φ
$\int d^3\bar{p} \, \frac{d^3k}{(2\pi)^3 2k} \frac{d^3q}{(2\pi)^3 2q} = \int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \int \frac{d^3p}{(2\pi)^3 2p(1-z)} \int dz\, E$

$$|\mathcal{M}|^2 = \left\{ \begin{array}{l} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2 \\ \frac{4}{3} g^2 \frac{2\,(1-z)^2}{z^2(1-z)} k_T^2 \end{array} \right.$$

g_L

g_R

$$\tilde{p} = \frac{k_T^2}{z(1-z)}$$

$$t^a t^a = \frac{4}{3}$$

$$\int d^3\tilde{p} \; \frac{d^3k \; d^3q}{(2\pi)^3 2k \, (2\pi)^3 2q} = \int \frac{d^3\tilde{p}}{(2\pi)^3 2\tilde{p}} \int \frac{d^3p}{(2\pi)^3 2p(1-z)} \int \frac{dz\,E}{(2\pi)^3 2zE} \; 2\pi k_T dk_T$$

$$|\mathcal{M}|^2 = \begin{cases} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2 \\ \frac{4}{3} g^2 \frac{2\,(1-z)^2}{z^2(1-z)} k_T^2 \end{cases}$$

g_L

g_R

$$\vec{p}^2 = \frac{k_T^2}{z(1-z)}$$

$$t^a t^a = \frac{4}{3}$$

$\frac{1}{p^2}$ Σ

Φ

$$\int d^3\bar{p}\ \frac{d^3k}{(2\pi)^3 2k}\frac{d^3q}{(2\pi)^3 2q} = \int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \int \frac{d^3p}{(2\pi)^3 2p(1-z)} \int \frac{dz\,E}{(2\pi)^3 2zE}\ 2\pi k_T dk_T$$

$$= \int d\Pi_2 \int dz \int \frac{dk_T\ k_T}{z(1-z)}$$

$$|\mathcal{M}|^2 = \begin{cases} \frac{4}{3} g^2 \dfrac{2}{z^2(1-z)} k_T^2 \\ \frac{4}{3} g^2 \dfrac{2\,(1-z)^2}{z^2(1-z)} k_T^2 \end{cases}$$

g_L

g_R

$$\vec{p}^{\,2} = \frac{k_T^2}{z(1-z)}$$

$$t^a t^a = \frac{4}{3}$$

$$\int d^3\bar{p}\, \frac{d^3k}{(2\pi)^3 2k} \frac{d^3q}{(2\pi)^3 2q} = \int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \int \frac{d^3p}{(2\pi)^3 2p(1-z)} \int \frac{dz\, E \;\; 2\pi k_T dk_T}{(2\pi)^3\, 2zE}$$

$$= \int d\Pi_2 \int dz \int \frac{dk_T\, k_T}{z(1-z)\; 8\pi^2}$$

$$|\mathcal{M}|^2 = \begin{cases} \frac{4}{3} g^2 \, \frac{2}{z^2(1-z)} \, k_T^2 \\ \frac{4}{3} g^2 \, \frac{z\,(1-z)^2}{z^2(1-z)} \, k_T^2 \end{cases}$$

g_L

g_R

$$\vec{p} = \frac{k_T^2}{z(1-z)}$$

$$t^a t^a = \frac{4}{3}$$

$$\int d^3\bar{p} \, \frac{d^3k \, d^3q}{(2\pi)^3 2k \, (2\pi)^3 2q} = \int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \int \frac{d^3p}{(2\pi)^3 2p(1-z)} \int \frac{dz\,E}{(2\pi)^3 2zE} \, 2\pi k_T dk_T$$

$$= \int d\Pi_2 \int dz \int \frac{dk_T \, k_T}{z(1-z) \, 8\pi^2}$$

$$\sigma = \sigma_0$$

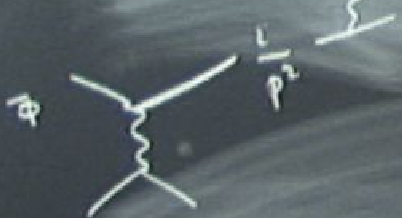
$$|m|^2 = \begin{cases} \frac{4}{3} g^2 \, \frac{2}{z^2(1-z)} \, k_T^2 \\ \frac{4}{3} g^2 \, \frac{z\,(1-z)^2}{z^2(1-z)} \, k_T^2 \end{cases}$$

g_L

g_R

$$p^2 = \frac{k_T^2}{z(1-z)}$$

$$t^a t^a = \frac{4}{3}$$



$$\int d^3\bar{p} \; \frac{d^3k \; d^3q}{(2\pi)^3 2k \, (2\pi)^3 2q} = \int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \int \frac{d^3p}{(2\pi)^3 2p(1-z)} \int \frac{dz\,E \;\; 2\pi k_T dk_T}{(2\pi)^3 \, 2zE}$$

$$= \int d\Pi_2 \int dz \int \frac{dk_T \; k_T}{z(1-z) \; 8\pi^2}$$

$$\sigma = \sigma_0 \int dz \int dk_T \; k_T \; \frac{1}{z(1-z)} \; \frac{1}{8\pi^2} \; \frac{z^2(1-z)^2}{k_T^4}$$

$$|\mathcal{M}|^2 = \begin{cases} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2 \\ \frac{4}{3} g^2 \frac{z\,(1-z)^2}{z^2(1-z)} k_T^2 \end{cases}$$

g_L

g_R

$$\vec{p}^{\,2} = \frac{k_T^2}{z(1-z)}$$

$$t^a t^a = \frac{4}{3}$$

$$\int d^3\bar{p} \, \frac{d^3k \, d^3q}{(2\pi)^3 2k \, (2\pi)^3 2q} = \int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \int \frac{d^3p}{(2\pi)^3 2p(1-z)} \int \frac{dz\, E \; 2\pi k_T dk_T}{(2\pi)^3 2zE}$$

$$= \int d\Pi_2 \int dz \int \frac{dk_T \, k_T}{z(1-z) \, 8\pi^2}$$

$$\sigma = \sigma_0 \int dz \int dk_T \, k_T \frac{1}{z(1-z)} \frac{1}{8\pi^2} \frac{z^2(1-z)^2}{k_T^4} \frac{4}{3} g^2 \frac{2}{z^2(1-z)}$$

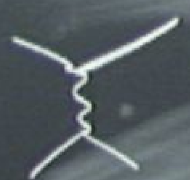
$$|\mathcal{M}|^2 = \begin{cases} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2 \\ \frac{4}{3} g^2 \frac{2\,(1-z)^2}{z^2(1-z)} k_T^2 \end{cases}$$

g_L

g_R

$$\vec{p}^{\,2} = \frac{k_T^2}{z(1-z)}$$

$$t^a t^a = \frac{4}{3}$$



Φ $\quad \frac{1}{p^2}$

$$\int d^3\bar{p}\, \frac{d^3k\, d^3q}{(2\pi)^3 2k\, (2\pi)^3 2q} = \int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \int \frac{d^3p}{(2\pi)^3 2p(1-z)} \int \frac{dz\, E\;\; 2\pi k_T dk_T}{(2\pi)^3 2zE}$$

$$= \int d\Pi_2 \int dz \int \frac{dk_T\, k_T}{z(1-z)\; 8\pi^2}$$

$$\sigma = \sigma_0 \int dz \int dk_T\, k_T \frac{1}{z(1-z)} \frac{1}{8\pi^2} \frac{z^2(1-z)^2}{k_T^4} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2$$

$$|\mathcal{M}|^2 = \begin{cases} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2 \\ \frac{4}{3} g^2 \frac{z\,(1-z)^2}{z^2(1-z)} k_T^2 \end{cases}$$

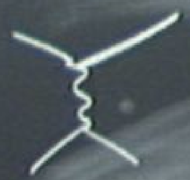
g_L

g_R

$$p^2 = \frac{k_T^2}{z(1-z)}$$

$$t^a t^a = \frac{4}{3}$$

Φ



$\frac{1}{p^2}$ Σ

$$\int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \frac{d^3k}{(2\pi)^3 2k} \frac{d^3q}{(2\pi)^3 2q} = \int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \int \frac{d^3p}{(2\pi)^3 2p(1-z)} \int \frac{dz\,E \;\; 2\pi k_T dk_T}{(2\pi)^3\, 2zE}$$

$$= \int d\Pi_2 \int dz \int \frac{dk_T\, k_T}{z(1-z)\; 8\pi^2}$$

$$\sigma = \sigma_0 \int dz \int dk_T\, k_T \frac{1}{z(1-z)} \frac{1}{8\pi^2} \frac{z^2(1-z)^2}{k_T^4} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2$$

$$|\mathcal{M}|^2 = \left\{ \begin{array}{l} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2 \\ \frac{4}{3} g^2 \frac{z\,(1-z)^2}{z^2(1-z)} k_T^2 \end{array} \right.$$

g_L

g_R

$$\vec{p}^{\,2} = \frac{k_T^2}{z(1-z)}$$

$$t^a t^a = \frac{4}{3}$$

$$\int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \frac{d^3k}{(2\pi)^3 2k} \frac{d^3q}{(2\pi)^3 2q} = \int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \int \frac{d^3p}{(2\pi)^3 2p(1-z)} \int \frac{dz\,E \quad 2\pi k_T dk_T}{(2\pi)^3\, 2zE}$$

$$= \int d\Pi_2 \int dz \int \frac{dk_T\, k_T}{z(1-z)\; 8\pi^2}$$

$1+(1-z)^2$

$$\sigma = \sigma_0 \int dz \int dk_T\, k_T \frac{1}{z\cancel{(1-z)}} \frac{1}{8\pi^2} \frac{\cancel{z^2(1-z)^2}}{k_T^4} \frac{4}{3} g^2 \frac{2}{\cancel{z^2(1-z)}} k_T^2$$

$$|\mathcal{M}|^2 = \begin{cases} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2 \\ \frac{4}{3} g^2 \frac{z\,(1-z)^2}{z^2(1-z)} k_T^2 \end{cases}$$

g_L

g_R

$$\vec{p}^{\,2} = \frac{k_T^2}{z(1-z)}$$

$$t^a t^a = \frac{4}{3}$$

$$\int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \frac{d^3k}{(2\pi)^3 2k} \frac{d^3q}{(2\pi)^3 2q} = \int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \int \frac{d^3p}{(2\pi)^3 2p(1-z)} \int \frac{dz\, E \quad 2\pi k_T dk_T}{(2\pi)^3\, 2zE}$$

$$= \int d\Pi_2 \int dz \int \frac{dk_T\, k_T}{z(1-z)\; 8\pi^2}$$

$$1+(1-z)^2$$

$$k_T^2$$

$$\sigma = \sigma_0 \int dz \int dk_T\, k_T \frac{1}{z\cancel{(1-z)}} \frac{1}{8\pi^2} \frac{\cancel{z^2(1-z)^2}}{k_T^4} \frac{4}{3} g^2 \frac{2}{\cancel{z^2(1-z)}}$$

$$\sigma(\gamma^* q \to gq) = \sigma_0 \int \frac{dk_T}{k_T} \int dz\, \frac{4}{3}\alpha_s \left[\frac{1+(1-z)^2}{z} \right]$$

$$|\mathcal{M}|^2 = \begin{cases} \frac{4}{3} g^2 \dfrac{2}{z^2(1-z)} k_T^2 & g_L \\ \frac{4}{3} g^2 \dfrac{2\,(1-z)^2}{z^2(1-z)} k_T^2 & g_R \end{cases} \qquad \vec{p}^2 = \frac{k_T^2}{z(1-z)}$$

$$t^a t^a = \frac{4}{3}$$

$$\int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \frac{d^3k}{(2\pi)^3 2k} \frac{d^3q}{(2\pi)^3 2q} = \int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \int \frac{d^3p}{(2\pi)^3 2p(1-z)} \int \frac{dz\, E \quad 2\pi k_T dk_T}{(2\pi)^3\, 2zE}$$

$$= \int d\Pi_2 \int dz \int \frac{dk_T\, k_T}{z(1-z)\; 8\pi^2} \qquad 1+(1-z)^2 \to k_T^2$$

$$\sigma = \sigma_0 \int dz \int dk_T\, k_T \frac{1}{z\cancel{(1-z)}} \frac{1}{8\pi^2} \frac{\cancel{z^2(1-z)^2}}{k_T^4} \frac{4}{3} g^2 \frac{2}{\cancel{z^2(1-z)}} k_T^2$$

$$\sigma(\gamma^* + q \to g q) = \sigma_0 \int \frac{dk_T}{k_T} \int dz\, \frac{4}{3}\alpha_s \left[\frac{1+(1-z)^2}{z}\right]$$

$$|\mathcal{M}|^2 = \begin{cases} \frac{4}{3} g^2 \frac{2}{z^2(1-z)} k_T^2 \\ \frac{4}{3} g^2 \frac{2\,(1-z)^2}{z^2(1-z)} k_T^2 \end{cases}$$

g_L

g_R

$$p^2 = \frac{k_T^2}{z(1-z)}$$

$$t^a t^a = \frac{4}{3}$$

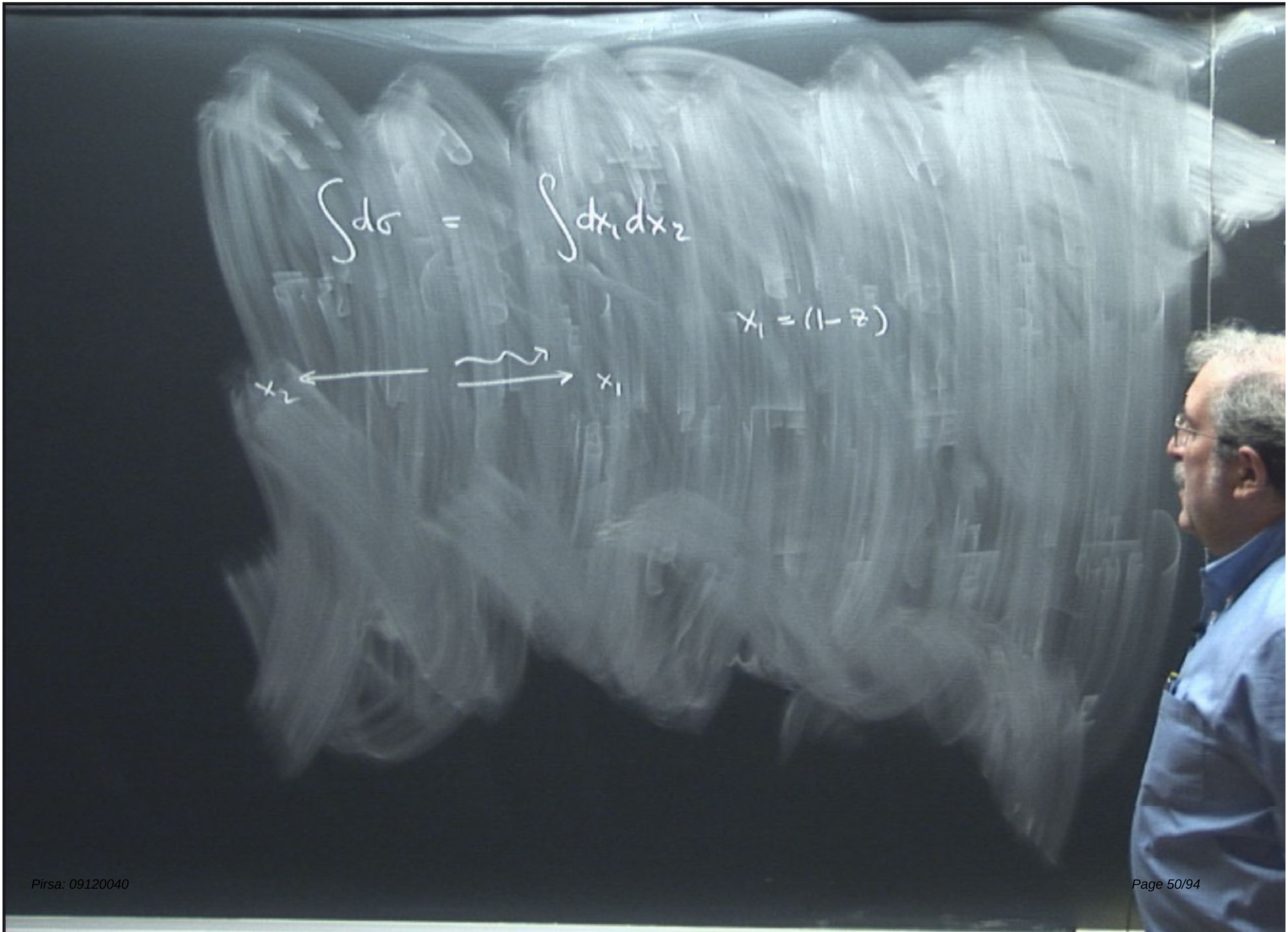
$$\int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \frac{d^3k}{(2\pi)^3 2k} \frac{d^3q}{(2\pi)^3 2q} = \int \frac{d^3\bar{p}}{(2\pi)^3 2\bar{p}} \int \frac{d^3p}{(2\pi)^3 2p(1-z)} \int \frac{dz\,E}{(2\pi)^3\, 2zE}\, 2\pi k_T dk_T$$

$$= \int d\Pi_2 \int dz \int \frac{dk_T\, k_T}{z(1-z)\; 8\pi^2}$$

$$1+(1-z)^2 \to k_T^2$$

$$\sigma = \sigma_0 \int dz \int dk_T\, k_T \frac{1}{z\cancel{(1-z)}} \frac{1}{8\pi^2} \frac{\cancel{z^2(1-z)^2}}{k_T^4} \frac{4}{3} g^2 \frac{2}{\cancel{z^2(1-z)}}$$

$$\sigma(\sigma + q \to gq) = \sigma_0 \int \frac{dk_T}{k_T} \int dz \frac{4}{3}\alpha_s \underbrace{\left[\frac{1+(1-z)^2}{z}\right]}_{P_{q\to g}(z)}$$


$\int d\sigma = \int dx_1 dx_2$
$x_1 = (1-z)$
x_2
x_1

$$\int d\sigma = \int dx_1 \, dx_2$$

$x_1 = (1-z) \qquad x_2 \approx 1$

$x_3 \approx z$

$$(Q - k_2)^2 = Q^2(1-x_2) = (k_1+k_3)^2$$

$=$

Φ

$$\int d\sigma = \int dx_1\, dx_2$$

$$x_1 = (1-z) \qquad x_2 \approx 1$$

$$x_2 \longleftarrow \qquad \longrightarrow x_1 \qquad\qquad x_3 \approx z$$

$$(Q-k_2)^2 = Q^2(1-x_2) = (k_1+k_3)^2$$

$$= \frac{k_T^2}{z(1-z)}$$

$t^a t^a =$

Φ

σ

$$\int d\sigma = \int dx_1 \, dx_2$$

$dx_1 = dz \qquad \frac{dx_2}{x_2} = \frac{dk_T^2}{k_T^2}$

$x_1 = (1-z) \qquad x_2 \approx 1$

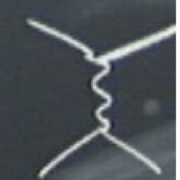
$x_3 \approx z$

$$(Q - k_2)^2 = Q^2(1-x_2) = (k_1+k_3)^2$$

$$= \frac{k_T^2}{z(1-z)}$$

$t^a t^a =$

Φ



σ

$$\int d\sigma = \int dx_1\, dx_2$$

$dx_1 = dz \qquad \frac{dx_2}{(1-x_2)} = \frac{dk_T^2}{k_T^2}$

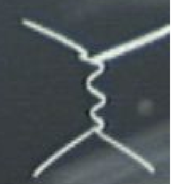
$x_1 = (1-z) \qquad x_2 \approx 1$

$x_3 \approx z$

$$(Q-k_2)^2 = Q^2(1-x_2) = (k_1+k_3)^2$$
$$= \frac{k_T^2}{z(1-z)}$$

$t^a t^a =$

Φ



σ

$$\int d\sigma = \int dx_1\, dx_2$$

$$dx_1 = dz \qquad \frac{dx_2}{(1-x_2)} = \frac{dk_T^2}{k_T^2}$$

$$x_1 = (1-z) \qquad x_2 \approx 1$$

$$x_3 \approx z$$

$$(Q - k_2)^2 = Q^2(1-x_2) = (k_1+k_3)^2$$

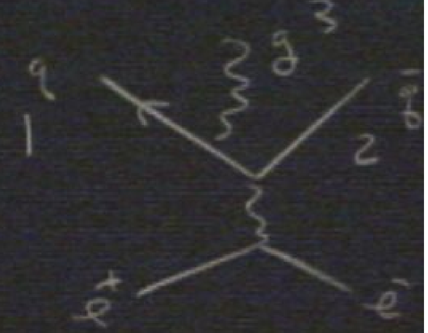
$$= \frac{k_T^2}{z(1-z)}$$

$$\frac{\sigma}{\sigma_0} = \int \frac{dk_T}{k_T} \int dz \; \frac{1+(1-z)^2}{z} \; \frac{4}{3}\frac{\alpha_s}{\pi}$$

$t^a t^a =$

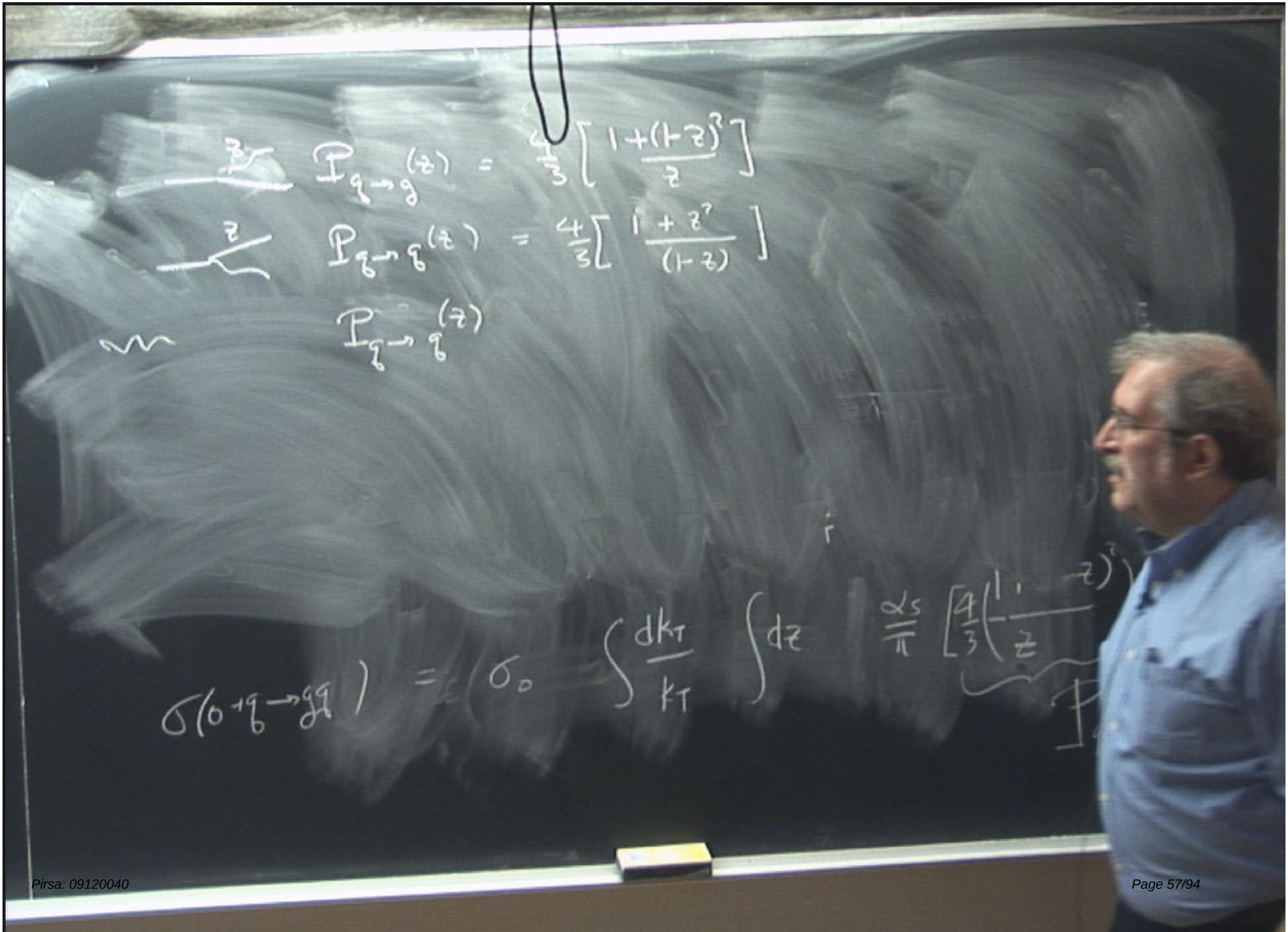
σ

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_1\,dx_2} = \frac{2\alpha_s}{3\pi}\,\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

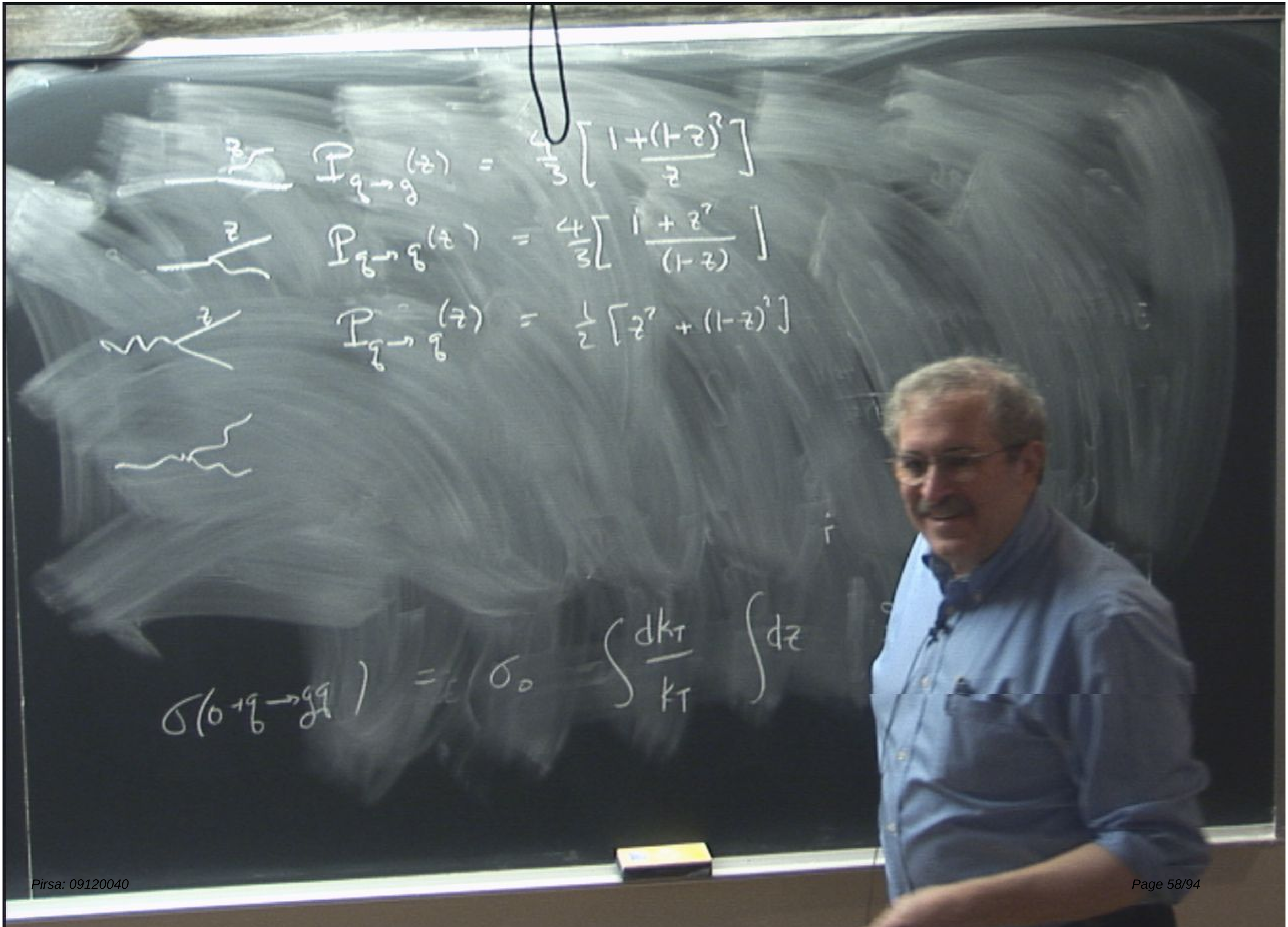


$$T = \max_{\hat{n}} \frac{\sum_i \hat{n}\cdot\vec{p}_i}{\sum_i |\vec{p}_i|}$$

$$S = \max_{\hat{n}} \frac{\sum_i |\hat{n}\cdot\vec{p}_i|^2}{\sum_i |\vec{p}_i|^2}$$



P q→g (z) = 4/3 [1+(1−z)² / z]
P q→q (z) = 4/3 [1 + z² / (1−z)]
P q→q (z)
σ(o+q→gq) = σ₀ ∫ dk_T / k_T ∫ dz α_s/π [4/3 (1 z)² / z


$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$
$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right]$
$P_{g\to q}(z) = \frac{1}{2}\left[z^2+(1-z)^2\right]$
$\sigma(\sigma + q \to g q) = \sigma_0 \int \frac{dk_T}{k_T} \int dz$

$$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$

$$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right]$$

$$P_{g\to q}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

$$P_{g\to g}(z) = 3\left[\frac{1}{z(1-z)} + \frac{z^4}{z(1-z)} + \frac{(1-z)^4}{z(1-z)}\right]$$

$$\sigma(o + q \to g q) = \sigma_0 \int \frac{dk_T}{k_T} \int dz \, \frac{\alpha_s}{\pi} \underbrace{\left[\frac{4}{3}\left(\frac{1 + (1-z)^2}{z}\right)\right]}_{P_{q\to g}(z)}$$

$$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$

$$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right]$$

$$P_{g\to q}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

$$P_{g\to g}(z) = 3\left[\frac{1}{z(1-z)} + \frac{z^4}{z(1-z)} + \frac{(1-z)^4}{z(1-z)}\right]$$

$$\sigma(\gamma + q \to g q) = \sigma_0 \int \frac{dk_T}{k_T} \int dz \, \frac{\alpha_s}{\pi} \underbrace{\left[\frac{4}{3}\left(\frac{1+(1-z)^2}{z}\right)\right]}_{P_{q\to g}(z)}$$

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$$P_{g\to g}(z) = 3\left[\frac{1}{z(1-z)} + \frac{z^4}{z(1-z)} + \frac{(1-z)^4}{z(1-z)}\right]$$

$$\sigma(0+q\to gq) = \sigma_0 \int \frac{dk_T}{k_T} \int dz \; \frac{\alpha_s}{\pi} \underbrace{\left[\frac{4}{3}\left(\frac{1+(1-z)^2}{z}\right)\right]}_{P_{q\to g}(z)}$$

$$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$

Altarelli Parisi
splitting functions

$$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right]$$

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$$\sigma(0+q\to gq) = \sigma_0 \int\frac{dk_T}{k_T}\int dz\,\frac{\alpha_s}{\pi}\underbrace{\left[\frac{4}{3}\left(\frac{1+(1-z)^2}{z}\right)\right]}_{P_{q\to g}(z)}$$

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Altarelli Parisi splitting functions

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$$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$ Altarelli Parisi

$$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right] - A\,\delta(z-1)$$ splitting functions

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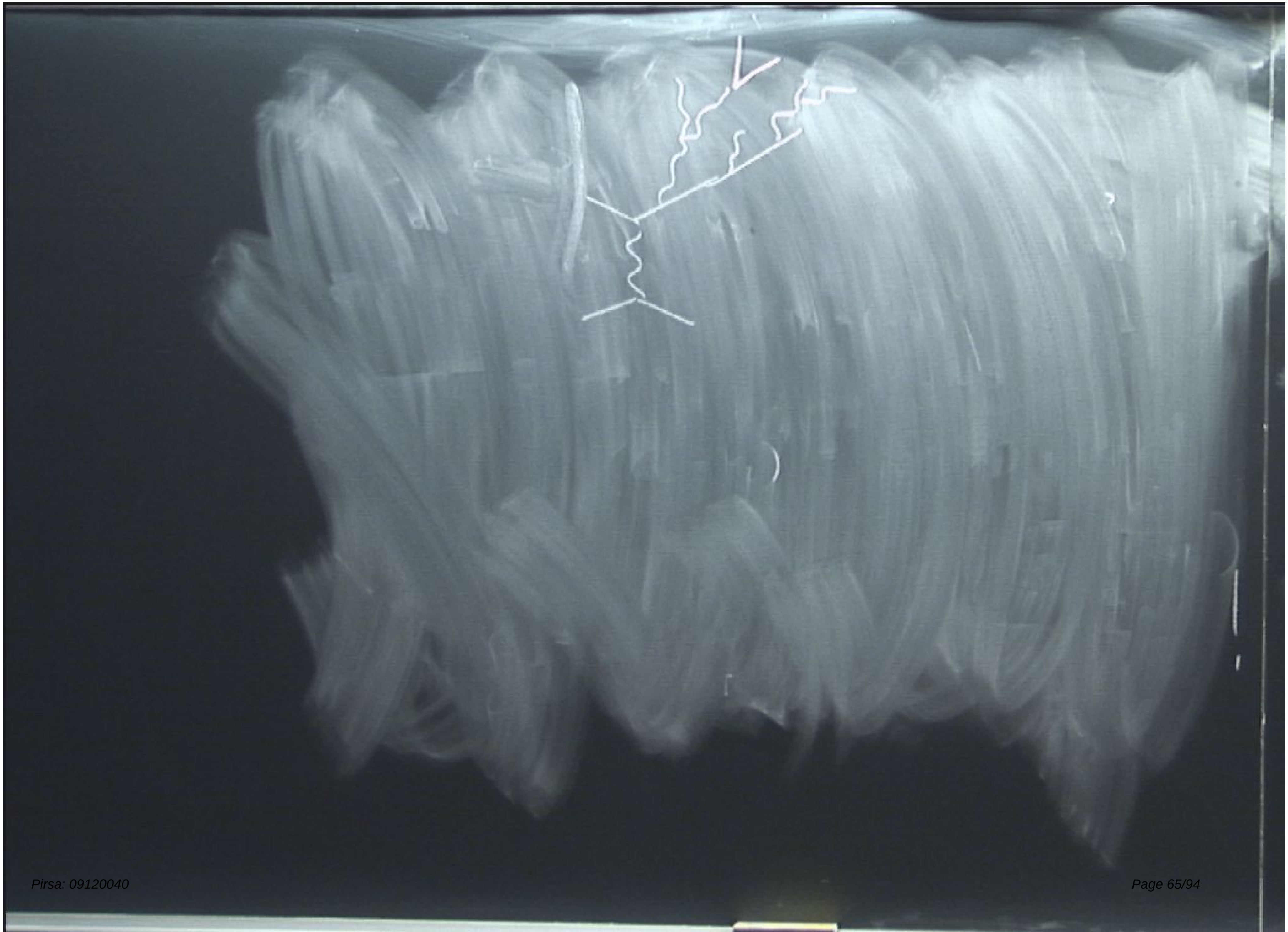
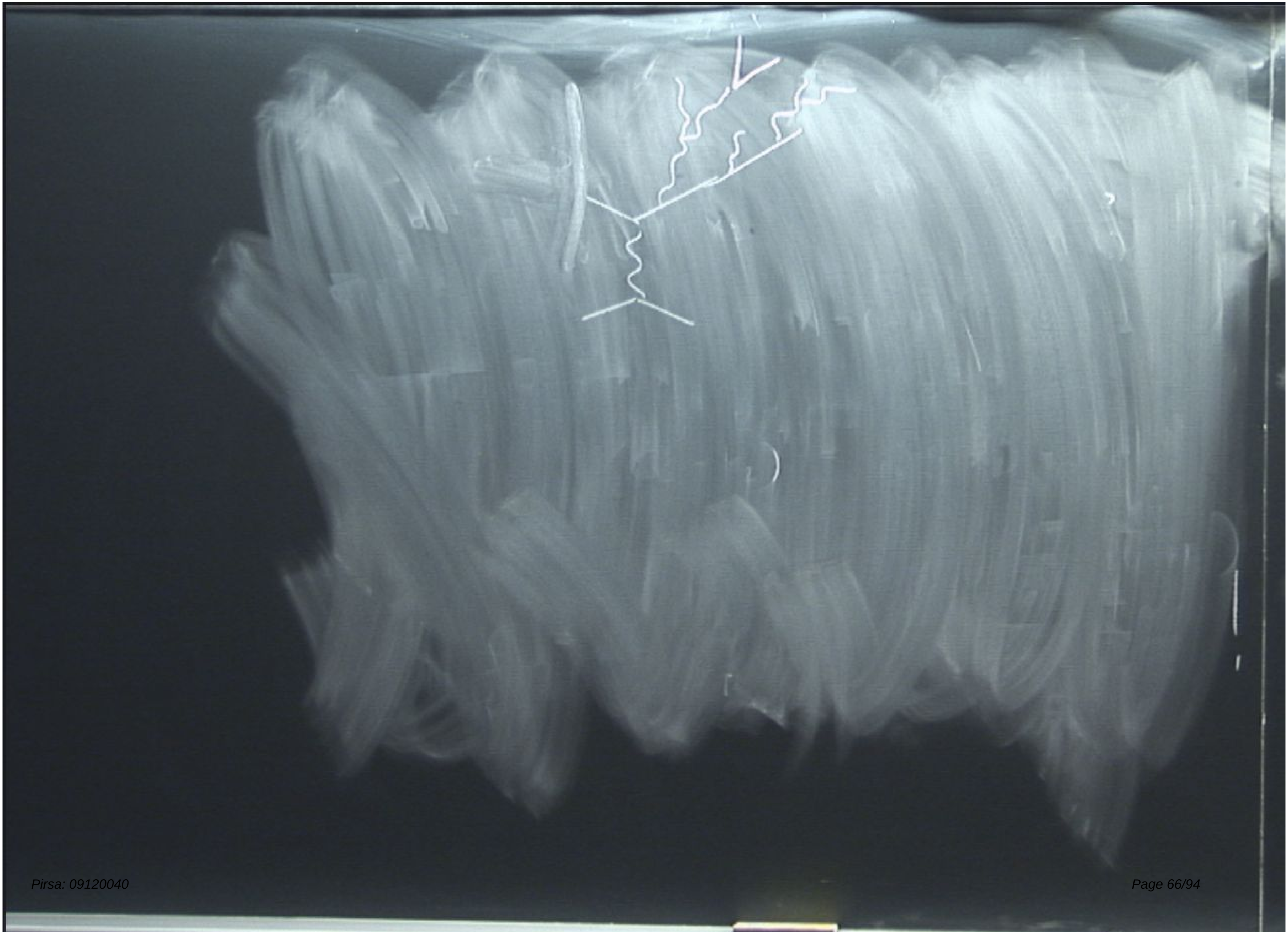
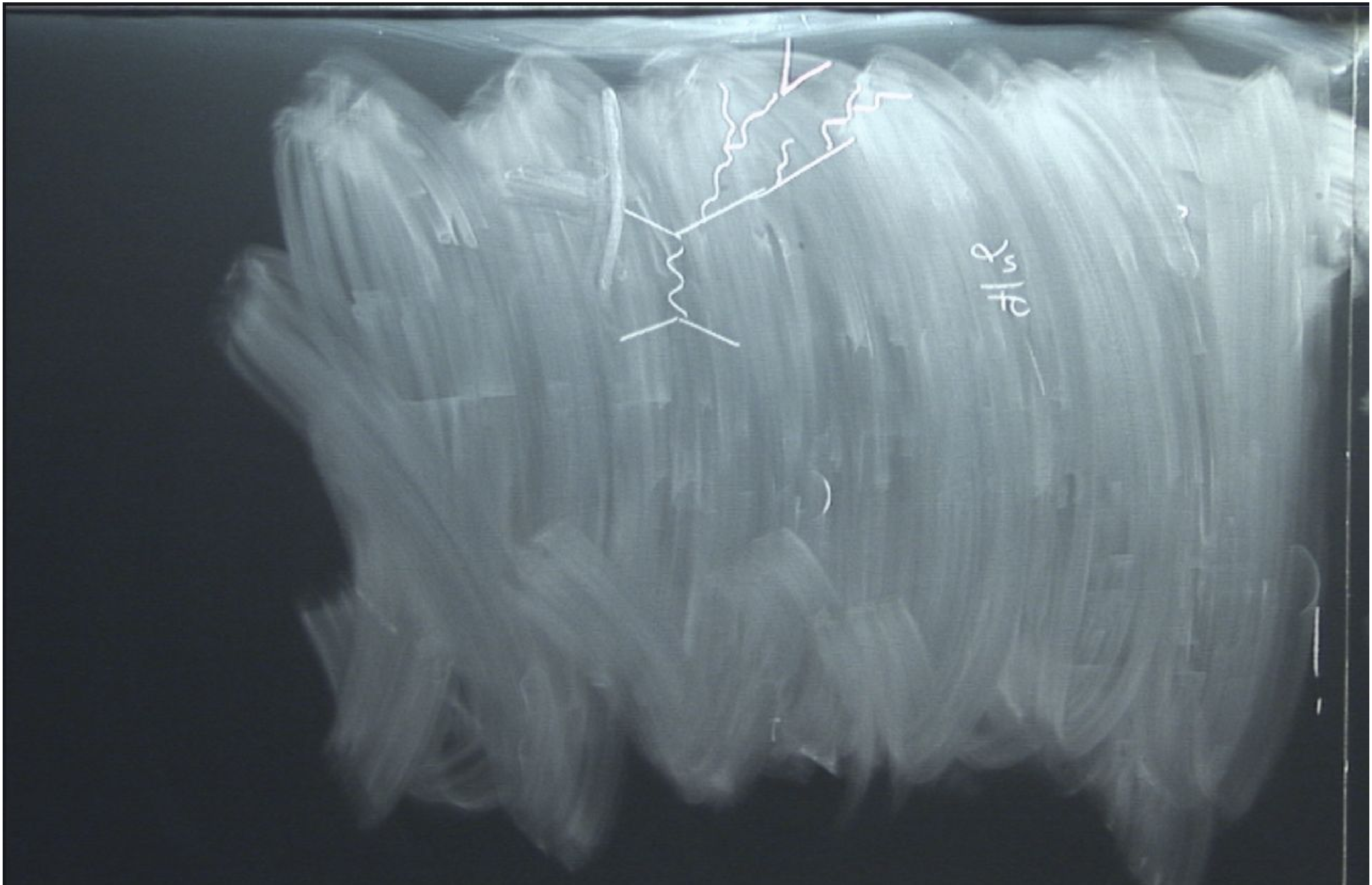
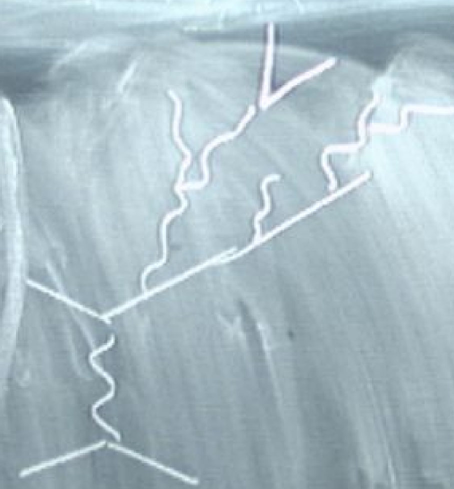
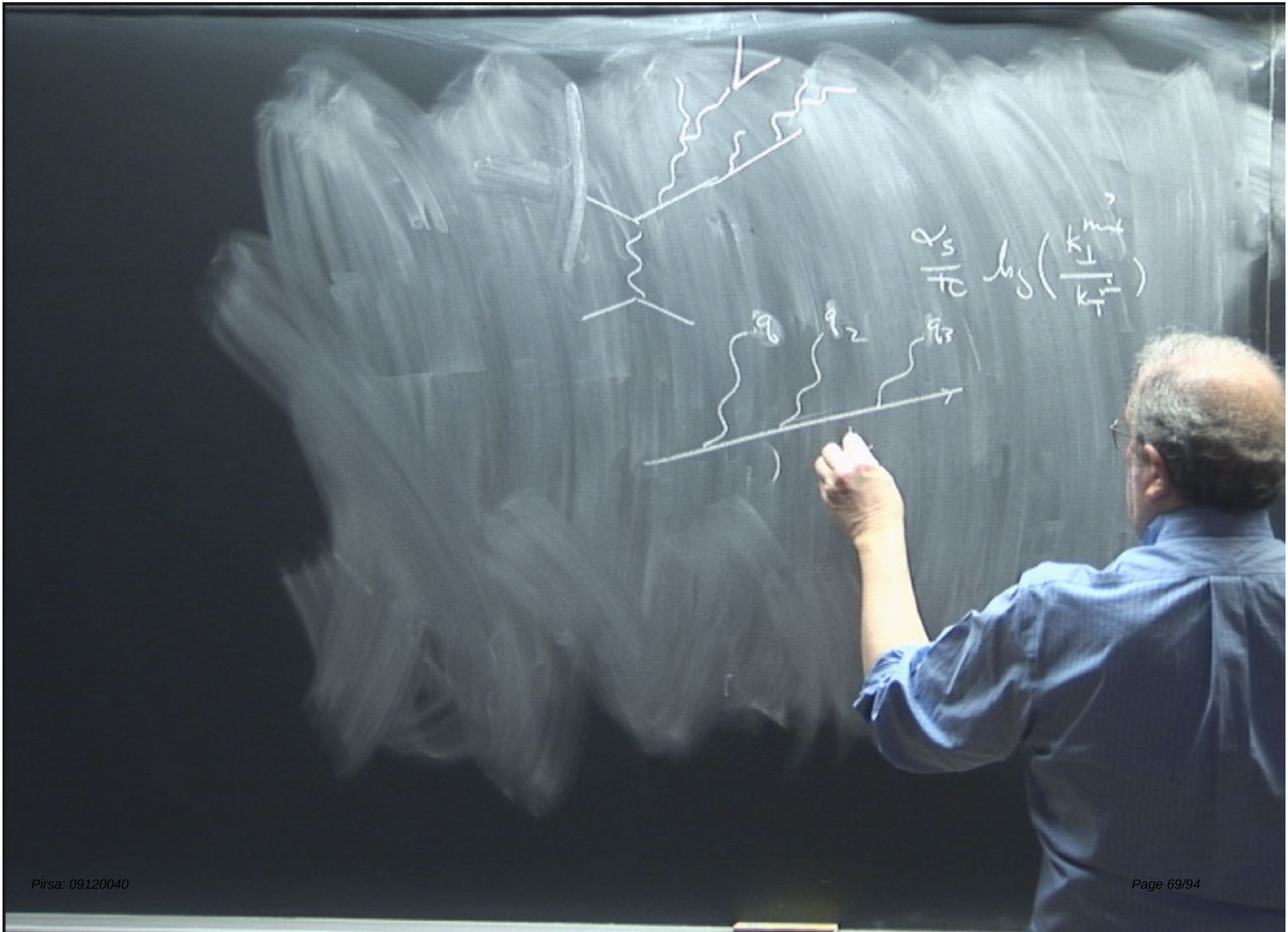


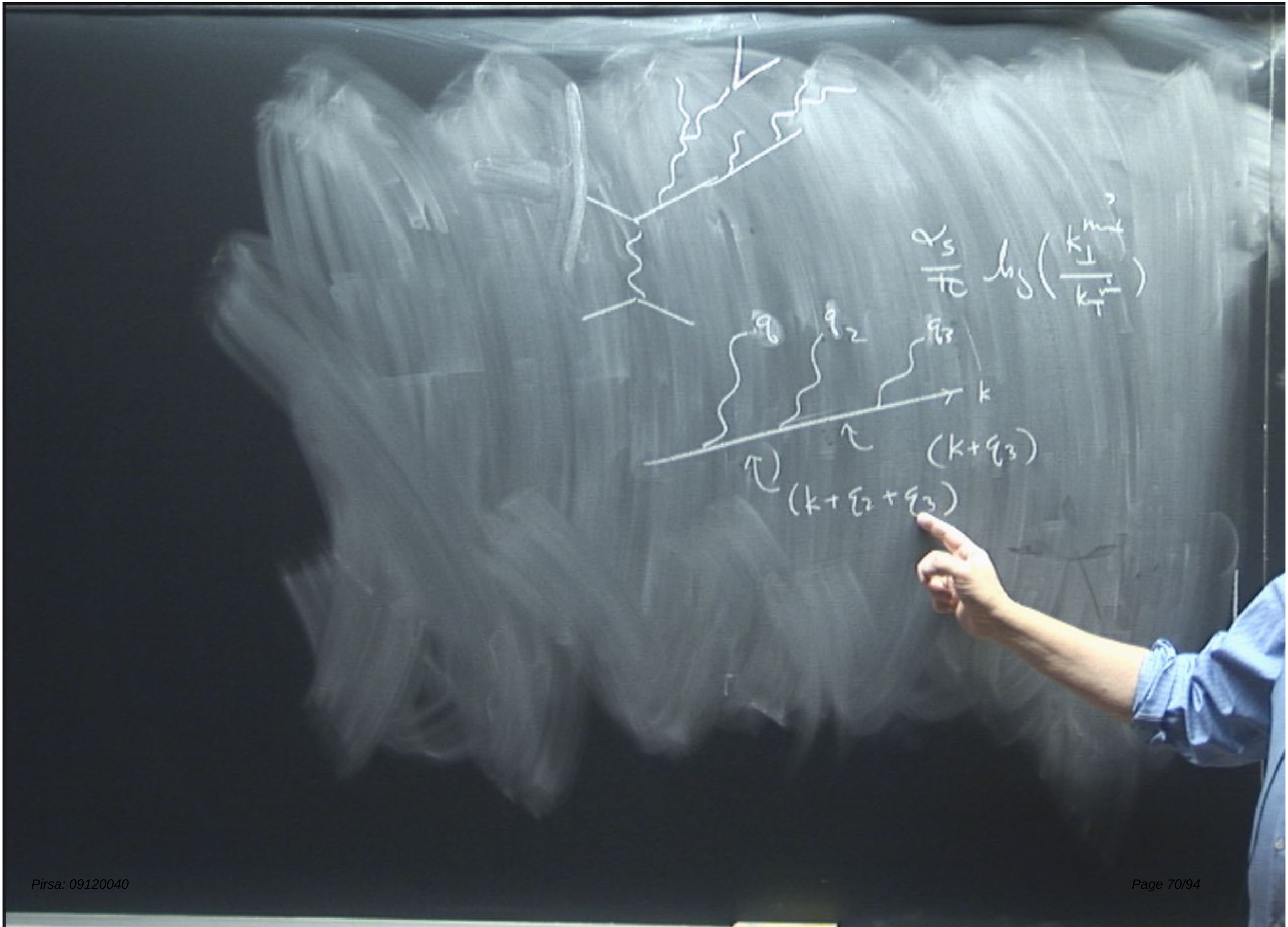


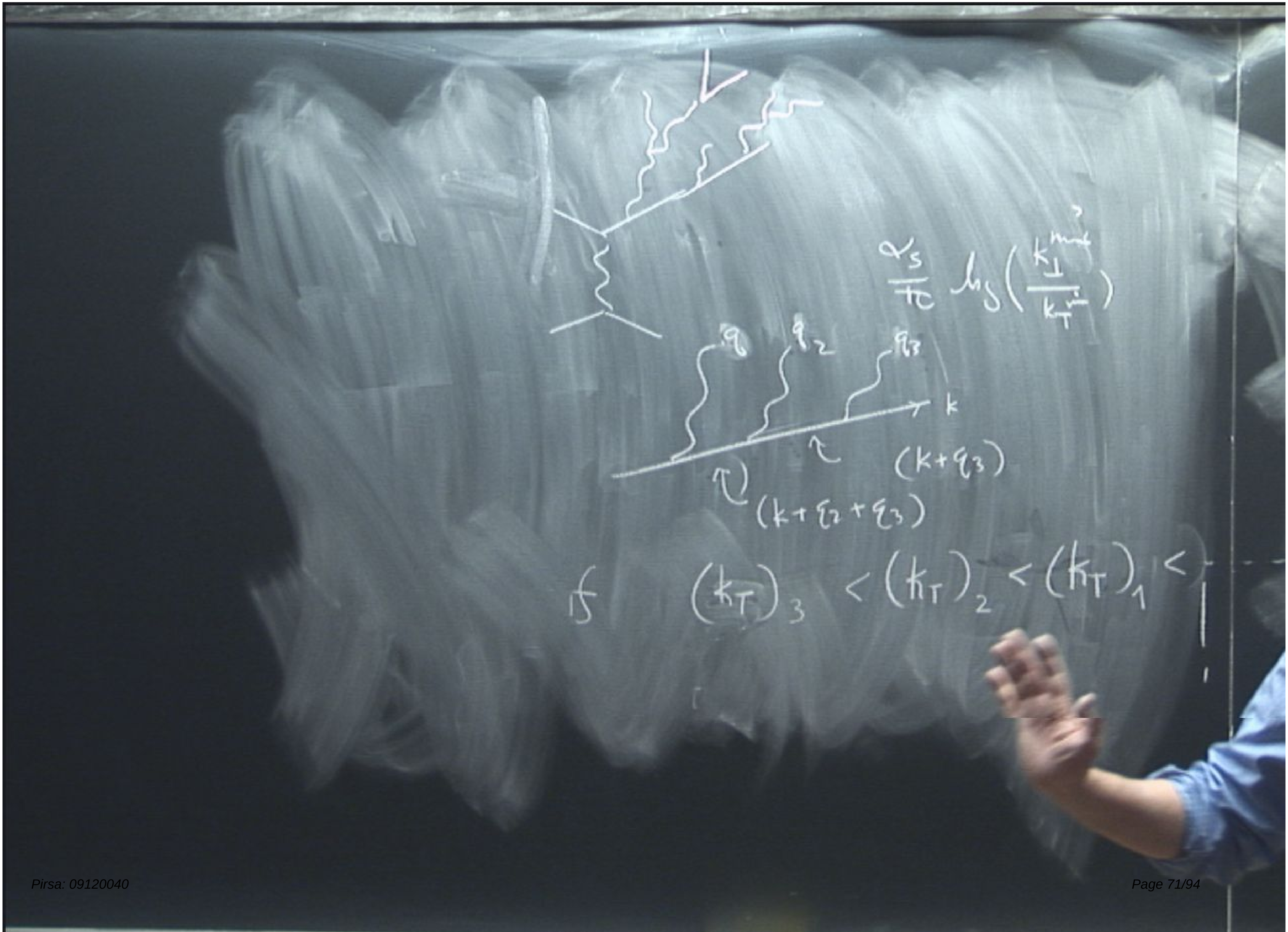

α_s
$\hbar c$



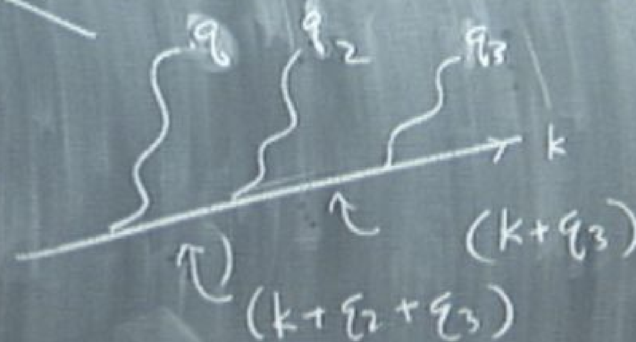
$$\frac{\alpha_s}{\pi} \ln\left(\frac{k_\perp^{max}}{k_\perp^{min}}\right)$$


$\frac{\alpha_s}{\pi} \ln\left(\frac{k_\perp^{max}}{k_\perp^{min}}\right)$
q
q_2
q_3


αs/π ln(k⊥max / k⊥min)
q
q2
q3
k
(k+q3)
(k+q2+q3)

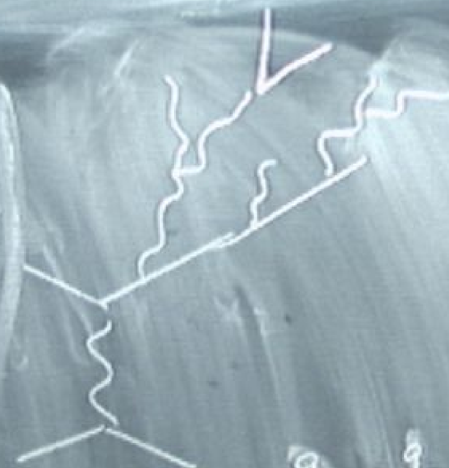

$\frac{\alpha_s}{\pi} \ln\left(\frac{k_\perp^{max}}{k_T^{min}}\right)$
q_1
q_2
q_3
k
$(k+q_3)$
$(k+q_2+q_3)$
if $(k_T)_3 < (k_T)_2 < (k_T)_1 <$

$$\frac{\alpha_s}{\pi} \ln\left(\frac{k_\perp^{max}}{k_T^{min}}\right)$$

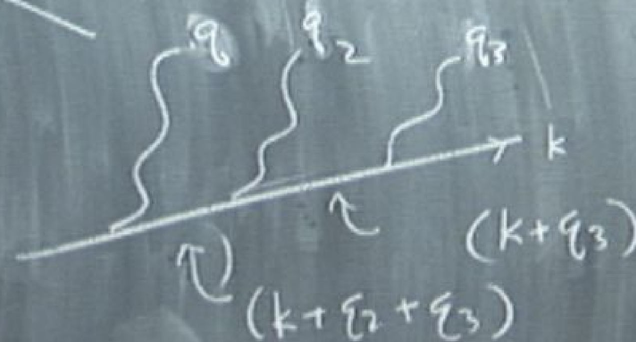


if $(k_T)_3 < (k_T)_2 < (k_T)_1 <$

"strong ordering"



$$\frac{\alpha_s}{\pi}\ln\left(\frac{k_\perp^{max}}{k_T^{min}}\right)$$



If $(k_T)_3 < (k_T)_2 < (k_T)_1 <$

"strong ordering"

$$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$

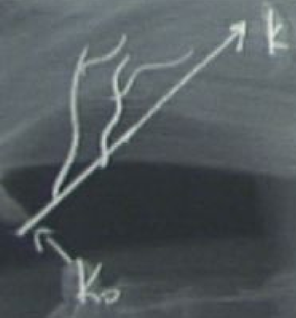
Altarelli Parisi splitting functions

$$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right] - A\delta(z-1)$$

$$P_{g\to q}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

$$P_{g\to g}(z) = 3\left[\frac{1}{z(1-z)} + \frac{z^4}{z(1-z)} + \frac{(1-z)^4}{z(1-z)}\right] - B\delta(z-1)$$

$$k = z k_0 + (\text{trans})$$



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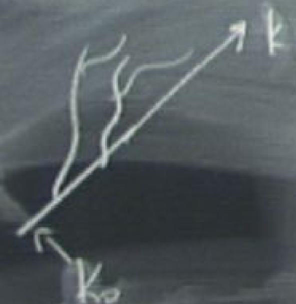
$$dz\, f_{q\to q}(z, Q)$$

including $k_T < Q$

$$k = z\,k_0 + (\text{trans})$$



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$$dz\, f_{q\to q}(z, Q)$$

including $k_T < Q$

$k = z k_0 + (\text{transverse})$

$$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$ Altarelli Parisi

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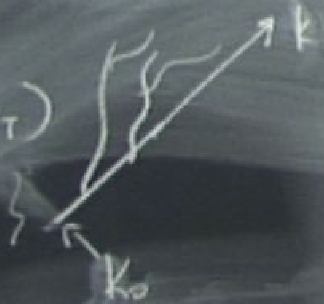
$dz\, f_{q\to q}(z, Q)$ include $k_T < Q$

$k = z k_0 + (\text{trans})$

$d(\ln k_T)$



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$$dz\, f_{q\to q}(z, Q)$$ include $k_T < Q$

$$k = z\,k_0 + (\text{transverse})$$

$d(\log k_T)$

k, k_0

$$\frac{d\, f_{q\to q}(z)}{d\log Q} = \frac{\alpha_s}{\pi}\int dz\; P_{q\to q}(z)\, f_{q\to q}(z)$$

$$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$ Altarelli Parisi

$$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right] - A\,\delta(z-1)$$ splitting functions.

$$P_{g\to q}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

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$dz\, f_{q\to q}(z, Q)$ include $k_T < Q$

$k = z\, k_0 + (\text{trans})$

$$\frac{d\, f_{q\to q}(z)}{d \log Q} = \frac{\alpha_s(Q)}{\pi}\int dz\; P_{q\to q}(z)\; f_{q\to q}(z)$$

$$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$

Altarelli Parisi splitting functions

$$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right] - A\,\delta(z-1)$$

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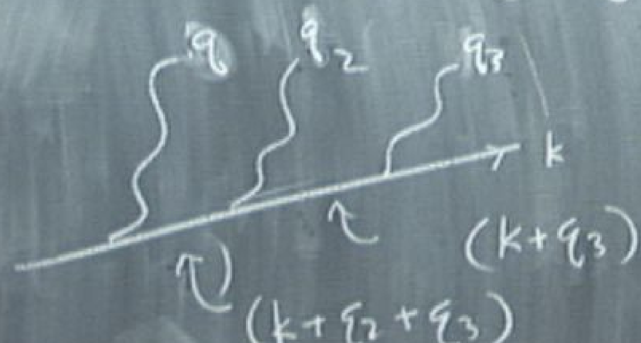
$dz\, f_{q\to q}(z, Q)$ include $k_T < Q$

$k = z k_0 + (\text{trans})$

$$\frac{d f_{q\to q}(z)}{d \log Q} = \frac{\alpha_s(Q)}{\pi}\int dw\, P_{q\to q}(w)\, f_{q\to q}(z)$$

$$\int dz = \int d\omega \int dx \; \mathcal{P}(\omega)\, f(z)$$

$$\frac{\alpha_s}{\pi} \ln\left(\frac{k_\perp^{max}}{k_T^{min}}\right)$$



if $(k_T)_3 < (k_T)_2 < (k_T)_1 <$

"strong ordering"

$d(\ln k_T)$

$z = \omega x$

$$\int dz = \int d\omega \int dx \; P(\omega)\, f(x) \int dz\, \delta(z - \omega x)$$

$$\int dz \int \frac{d\omega}{\omega} P(\omega)\, f\!\left(\frac{z}{\omega}\right)$$

q_2

$(k+q_3)$

$(k+q_2+q_3)$

if $(k_T)_3 < (k_T)_2 < (k_T)_1 <$

"strong ordering"

$d(\log k_T)$

$$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$

Altarelli Parisi splitting functions.

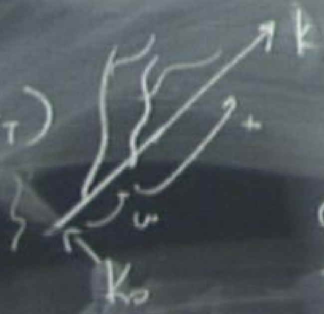
$$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right] - A\delta(z-1)$$

$$P_{g\to q}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

$$P_{g\to g}(z) = 3\left[\frac{1}{z(1-z)} + \frac{z^4}{z(1-z)} + \frac{(1-z)^4}{z(1-z)}\right] - B\delta(z-1)$$

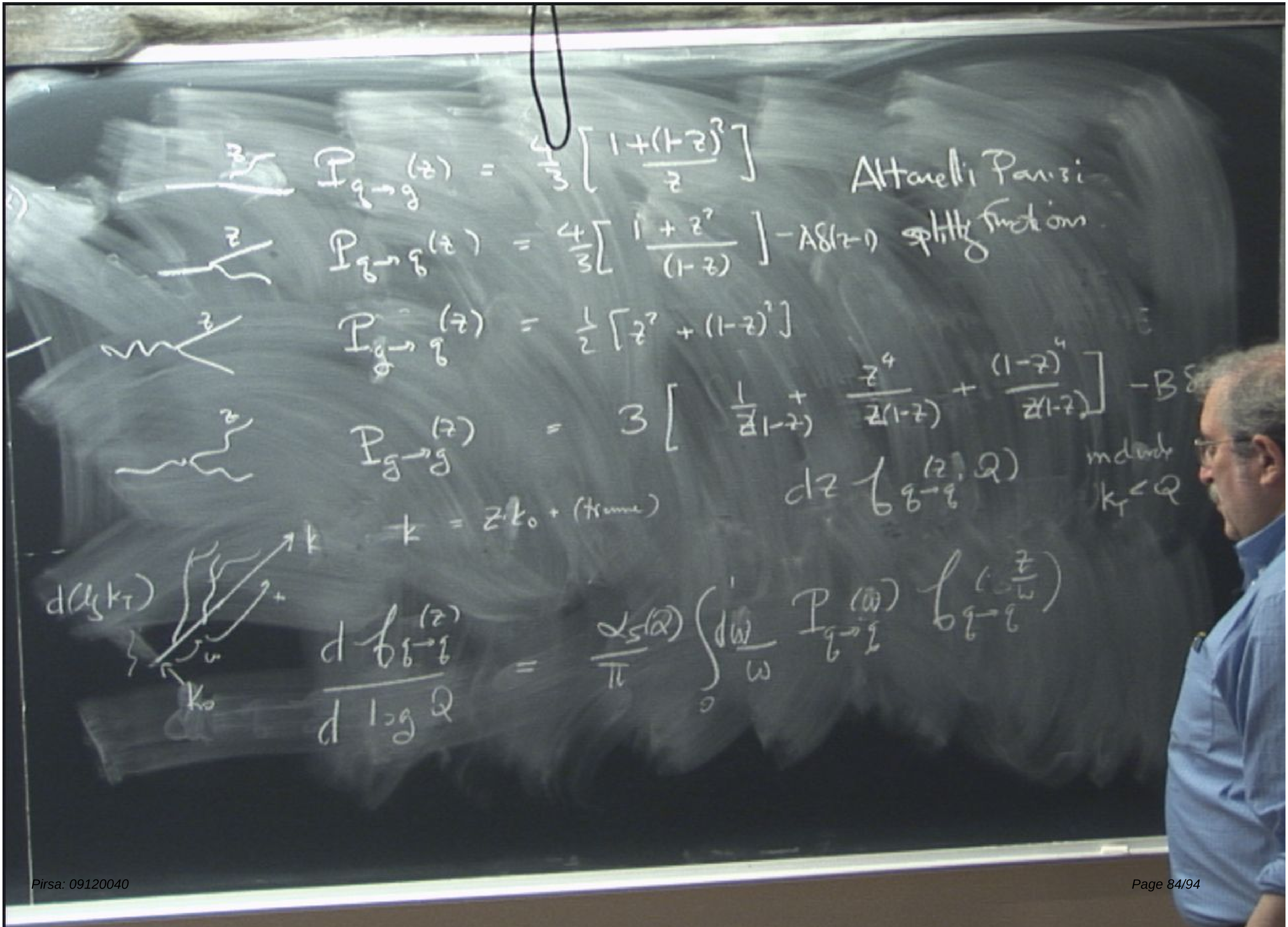
$dz\, f_{q\to q}(z, Q)$ including $k_T < Q$

$k = z\, k_0 + (\text{trans})$



$d(\alpha_s k_T)$

$$\frac{d\, f_{q\to q}(z)}{d \log Q} = \frac{\alpha_s(Q)}{\pi}\int_0^1 \frac{d\omega}{\omega}\, P_{q\to q}(\omega)\, f_{q\to q}\!\left(\frac{z}{\omega}\right)$$


$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$
Altarelli Parisi
$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right] - A\delta(z-1)$
splitting functions
$P_{g\to q}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$
$P_{g\to g}(z) = 3\left[\frac{1}{z(1-z)} + \frac{z^4}{z(1-z)} + \frac{(1-z)^4}{z(1-z)}\right] - B\delta$
$dz\, f_{q\to q}(z, Q)$
include $k_T < Q$
$k = z k_0 + (\text{trans})$
$d(\log k_T)$
k_0
$\frac{d f_{q\to q}(z)}{d \log Q} = \frac{\alpha_s(Q)}{\pi} \int_0^1 \frac{d\omega}{\omega} P_{q\to q}(\omega)\, f_{q\to q}\left(\frac{z}{\omega}\right)$

$$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$

Altarelli Parisi

$$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right] - A\delta(z-1)$$

splitting functions

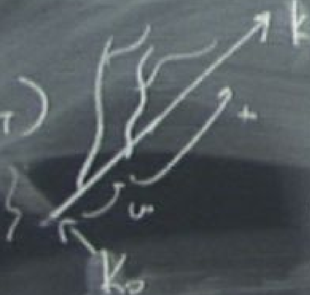
$$P_{g\to q}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

$$P_{g\to g}(z) = 3\left[\frac{1}{z(1-z)} + \frac{z^4}{z(1-z)} + \frac{(1-z)^4}{z(1-z)}\right] - B\delta(z-1)$$

$$dz\, f_{q\to q}(z, Q)$$

include $k_T < Q$

$$k = z\, k_0 + (\text{trans})$$



$d(\log k_T)$

$$\frac{d f_{q\to q}(z)}{d \log Q} = \frac{\alpha_s(Q)}{\pi}\int_0^1 \frac{d\omega}{\omega}\, P_{q\to q}(\omega)\, f_{q\to q}\!\left(\frac{z}{\omega}\right)$$

$$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$

Altarelli Parisi

$$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right] - A\delta(z-1)$$

splitting functions

$$P_{g\to q}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

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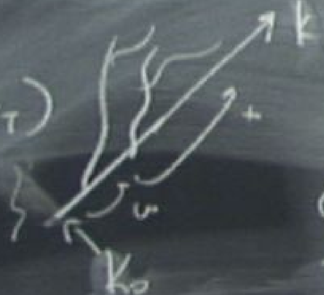
$dz\, f_{q\to q}(z, Q)$ include $k_T < Q$

$k = z\, k_0 + (\text{trans})$

$d(\log k_T)$



$$\frac{d\, f_{\beta_1\to\beta_3}(z)}{d\log Q} = \frac{\alpha_s(Q)}{\pi}\int_0^1 \frac{d\omega}{\omega}\sum_{\beta_2} P_{\beta_1\to\beta_2}(\omega)\, f_{\beta_2\to\beta_3}\!\left(\frac{z}{\omega}\right)$$

$$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$ Altarelli Parisi splitting functions

$$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right] - A\delta(z-1)$$

$$P_{g\to q}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

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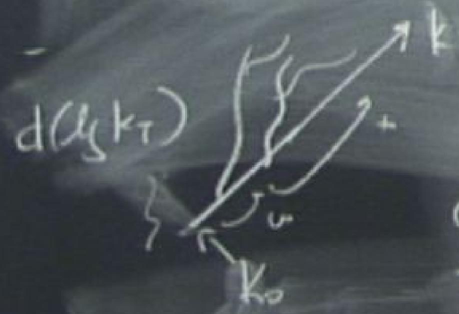
$dz\, f_{q\to q}(z, Q)$ includes $k_T < Q$

$$k = z k_0 + (\text{trans})$$



$$\frac{d f_{B_1\to B_3}(z)}{d\log Q} = \frac{\alpha_s(Q)}{\pi}\int_0^1 \frac{d\omega}{\omega}\sum_{B_2} P_{B_1\to B_2}(\omega)\, f_{B_2\to B_3}\left(\frac{z}{\omega}\right)$$ Altarelli Parisi

$$P_{q\to g}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$

Altarelli Parisi

$$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)}\right] - A\delta(z-1)$$

splitting functions

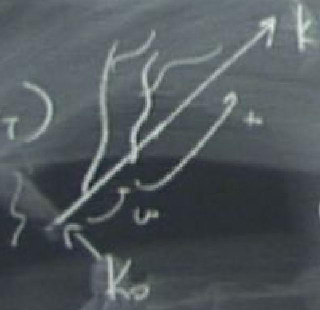
$$P_{g\to q}(z) = \frac{1}{2}\left[z^2+(1-z)^2\right]$$

$$P_{g\to g}(z) = 3\left[\frac{1}{z(1-z)} + \frac{z^4}{z(1-z)} + \frac{(1-z)^4}{z(1-z)}\right] - B\,\delta(z-1)$$

$dz\, f_{q\to q}(z, Q)$ include $k_T < Q$

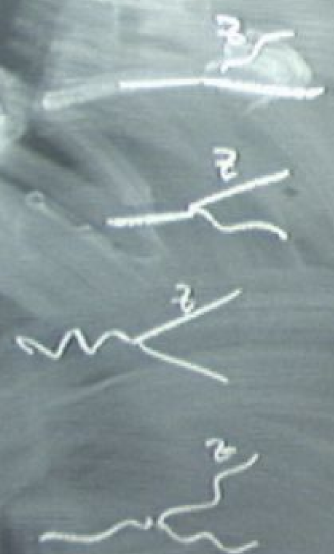
$k = z\,k_0 + (\text{trans})$

$d(\log k_T)$



$$\frac{d f_{\beta_1\to\beta_3}(z)}{d\log Q} = \frac{\alpha_s(Q)}{\pi}\int_0^1 \frac{d\omega}{\omega}\sum_{\beta_2} P_{\beta_1\to\beta_2}(\omega)\, f_{\beta_2\to\beta_3}\left(\frac{z}{\omega}\right)$$

Altarelli Parisi



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Altarelli Parisi splitting functions.

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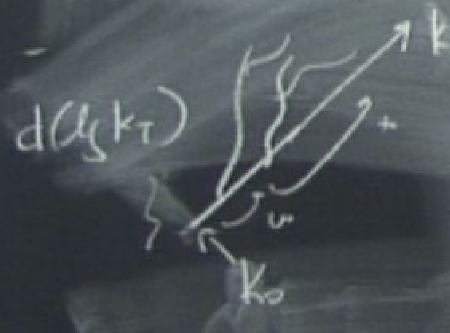
$$dz\, f_{q\to q}(z, Q)$$ include $k_T < Q$

$$k = z k_0 + (\text{trans})$$



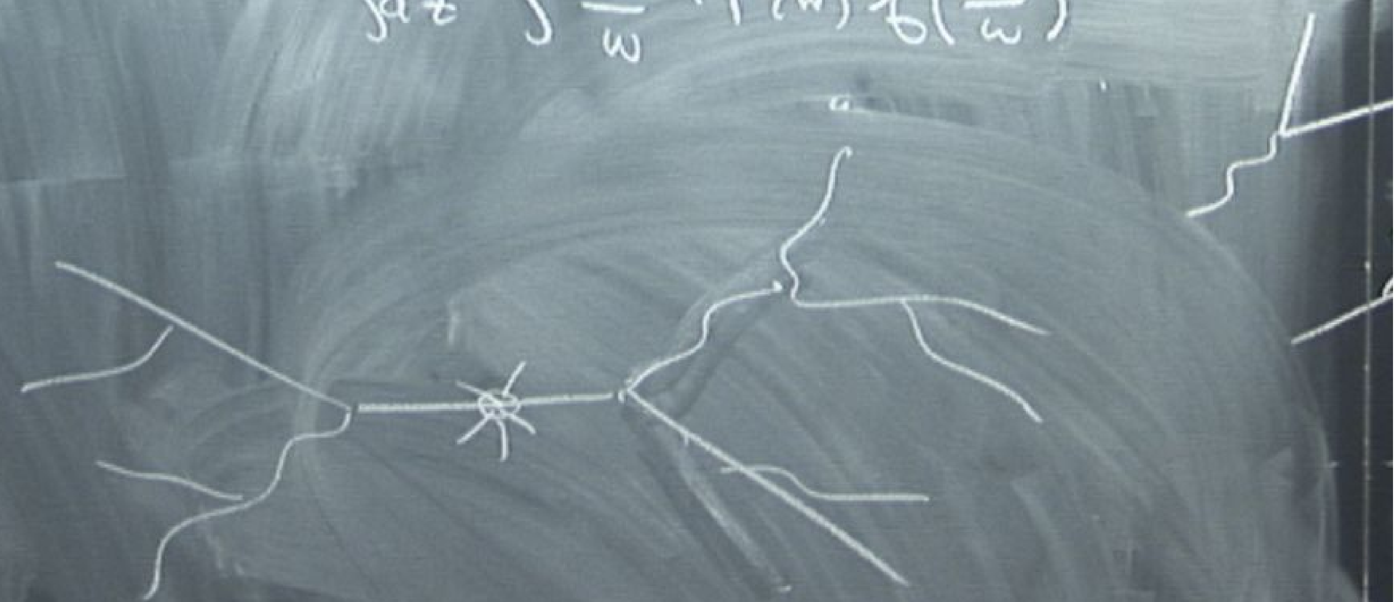
$$\frac{d f_{\theta_1\to\theta_3}(z)}{d \log Q} = \frac{\alpha_s(Q)}{\pi}\int_0^1 \frac{d\omega}{\omega}\sum_{\theta_2} P_{\theta_1\to\theta_2}(\omega)\, f_{\theta_2\to\theta_3}\left(\frac{z}{\omega}\right)$$

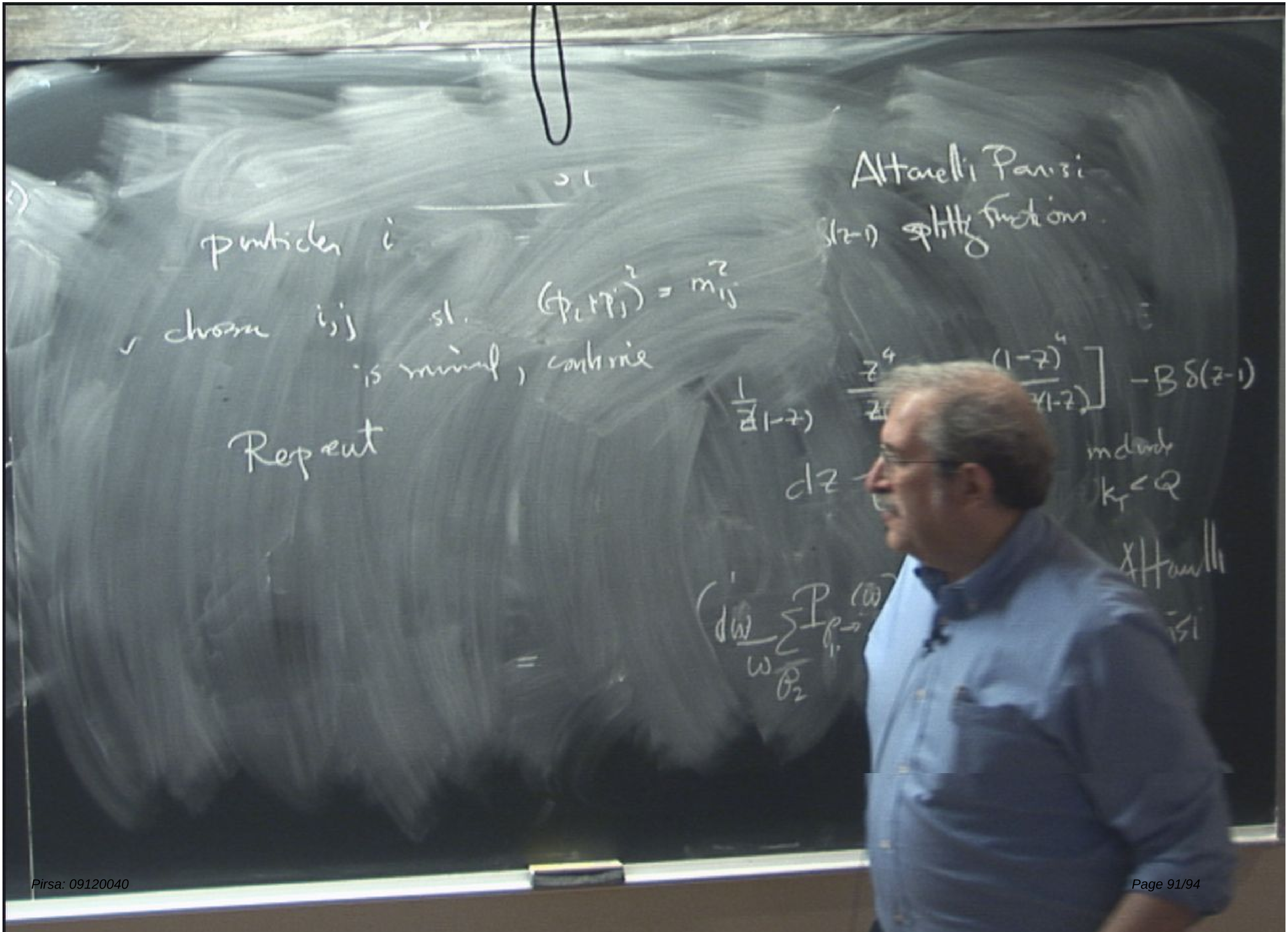
Altarelli Parisi

$$z = \omega x$$

$$\int dz \;=\; \int d\omega \int dx \; P(\omega)\, f(x) \int dz\, \delta(z-\omega x)$$

$$\int dz \int \frac{d\omega}{\omega}\, P(\omega)\, f\!\left(\frac{z}{\omega}\right)$$




Altarelli Parisi
splitting functions
particles i
choose i,j s.t. (p_i+p_j)^2 = m_ij^2
is minimal, combine
Repeat
-B δ(z-1)
k_T < Q

particles i

chosen i,j s.t. $(p_i+p_j)^2 = m_{ij}^2$

is minimal, combine

Repeat

until $m_{ij}^2 > m_{cut}^2$

then stop

Altarelli Parisi

$\delta(z-1)$ splitting functions

$$\frac{1}{z(1-z)} \quad \frac{z^4}{z(1-z)} + \frac{(1-z)^4}{z(1-z)}\Big] - B\,\delta(z-1)$$

$$dz\, f_{q\to q}(z, Q)$$

include $k_T < Q$

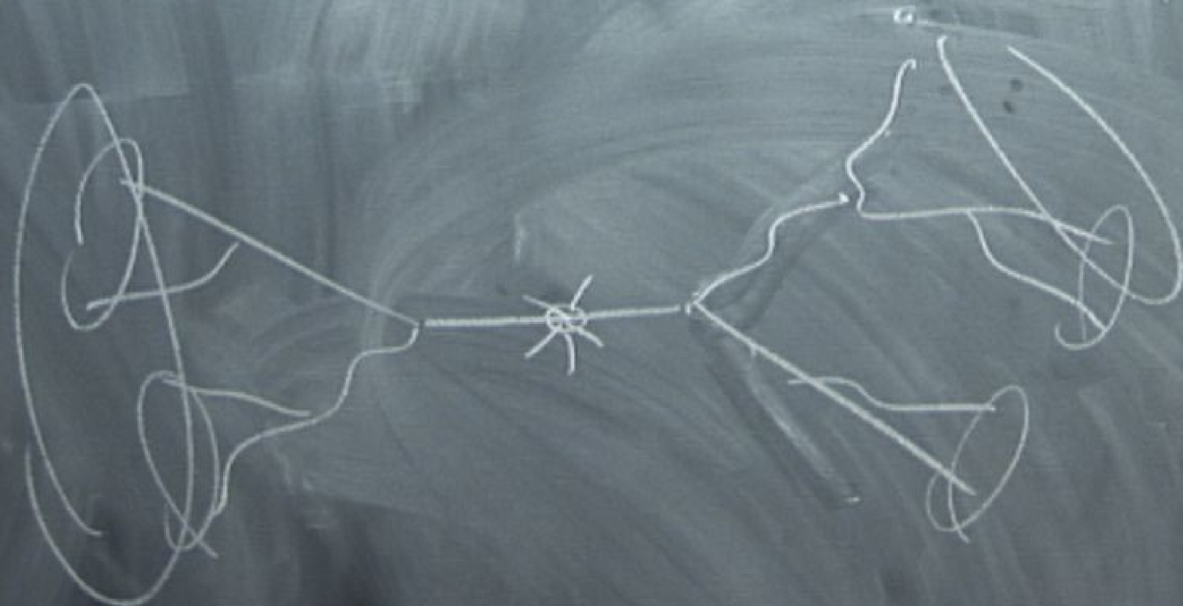
$$\int^1 \frac{d\omega}{\omega} \sum_{\beta_2} P_{\beta_1\to\beta_2}(\omega)\, f_{\beta_2\to\beta_3}\left(\frac{z}{\omega}\right)$$

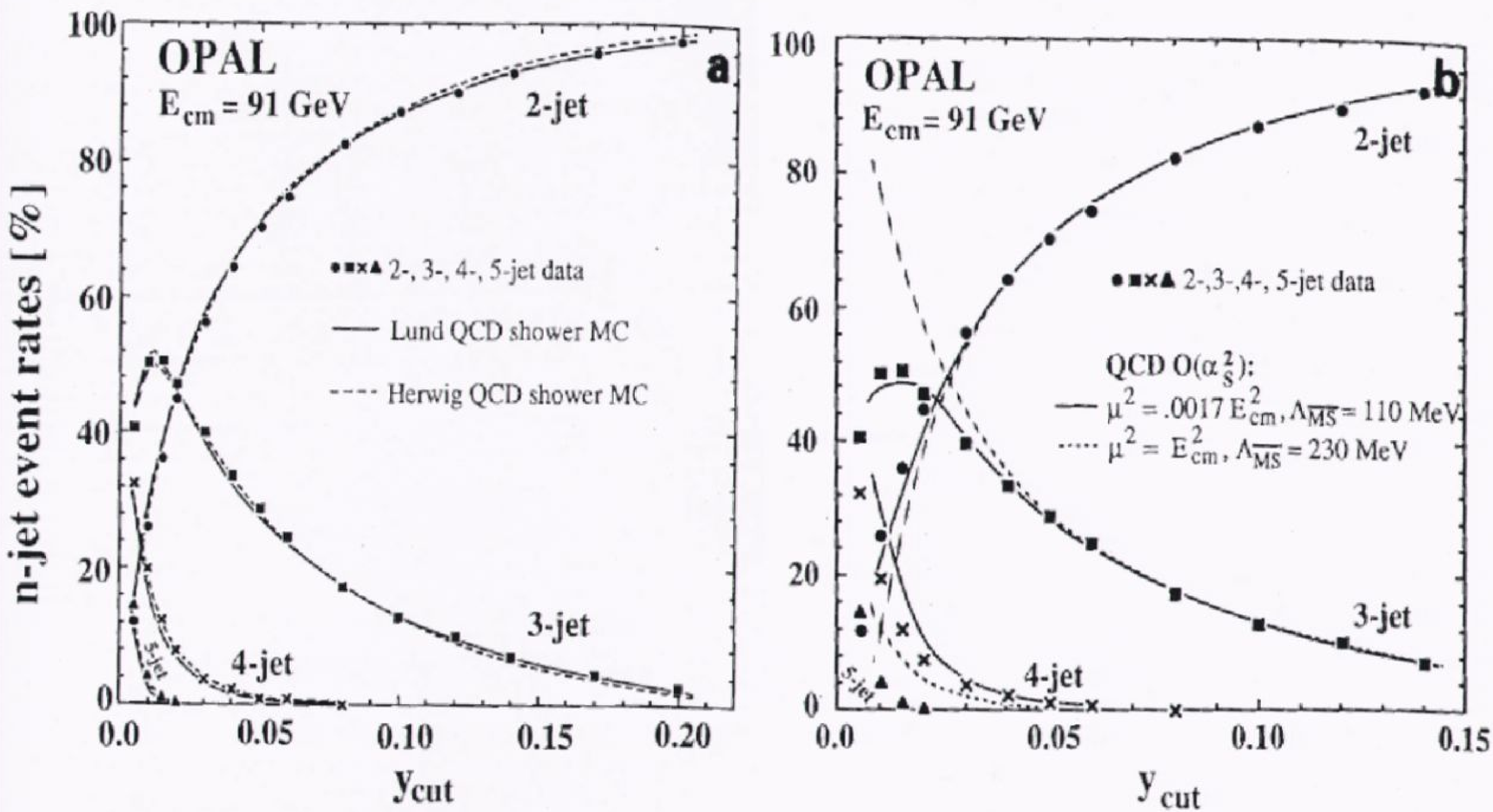
Altarelli Parisi

$$z = \omega x$$

$$\int dz = \int d\omega \int dx \; P(\omega)\, f(x) \int dz\, \delta(z - \omega x)$$

$$\int dz \int \frac{d\omega}{\omega}\, P(\omega)\, f\!\left(\frac{z}{\omega}\right)$$




OPAL
E_{cm} = 91 GeV
a
2-jet
3-jet
4-jet
5-jet
2-, 3-, 4-, 5-jet data
Lund QCD shower MC
Herwig QCD shower MC
n-jet event rates [%]
y_{cut}
OPAL
E_{cm} = 91 GeV
b
2-jet
3-jet
4-jet
5-jet
2-,3-,4-, 5-jet data
QCD $O(\alpha_s^2)$:
$\mu^2 = .0017\ E_{cm}^2,\ \Lambda_{\overline{MS}} = 110$ MeV
$\mu^2 = E_{cm}^2,\ \Lambda_{\overline{MS}} = 230$ MeV
y_{cut}