

Title: Standard Model - Review (PHYS 622) - Lecture 7

Date: Dec 08, 2009 09:00 AM

URL: <http://pirsa.org/09120039>

Abstract:

$$\mathcal{L} = -\frac{1}{4g^2} (\vec{F}_m^a)^2$$

$$\mathcal{L} = -\frac{1}{4g^2} (\vec{F}_M^a)^2 - \frac{1}{2} \bar{a}_\mu^a \Delta_{ab}^{\mu\nu} a_\nu^b - \bar{c} \Delta_{ab} c^b + \bar{\psi} \Delta_{ab} \psi^b$$

$$\mathcal{L} = -\frac{1}{4g^2} (\mathbb{F}_{\mu\nu}^a)^2 - \frac{1}{2} \bar{\psi}_\mu \Delta_{\nu\sigma} \psi_\nu + \bar{c} \Delta_{\nu\sigma} c + \bar{\psi} \Delta_{\nu\sigma} \psi + \phi^* \Delta_{\nu\sigma} \phi$$

$$\Delta = -\mathbb{D}^2 + 2 \frac{1}{2} \mathbb{F}_{\mu\nu}^a \mathbb{Q}^{\mu\nu} t_r$$

$$\mathcal{L} = -\frac{1}{4g^2} (\mathbb{F}_{\mu\nu}^a)^2 - \frac{1}{2} \bar{\psi}_\lambda \Delta_{\mu\nu}^{\lambda\sigma} \psi^\sigma + \bar{c} \Delta_{c_0} c + \bar{\psi} \Delta_{\psi_0} \psi + \phi^\dagger \Delta_{\phi_0} \phi$$

$$\Delta = -\mathbb{D}^2 + 2 \frac{1}{2} \mathbb{F}_{\mu\nu}^a \mathbb{Q}^{\mu\nu} t_r^a$$

$$\mathbb{Q}^{\mu\nu} = \frac{i}{4} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) \quad \text{Dirac}$$

$$= i (\delta_\alpha^\mu \delta_\beta^\nu - \delta_\alpha^\nu \delta_\beta^\mu) \quad \text{4-vectors}$$

$$\mathcal{L} = -\frac{1}{4g^2} (\mathbb{F}_{\mu\nu}^a)^2 - \frac{1}{2} \bar{\psi} \gamma^\mu \Delta_\mu \psi + \bar{c} \Delta_{c_0} c + \bar{\psi} \Delta_{c_0} \psi + \phi^\dagger \Delta_{c_0} \phi$$

$$\Delta = -\mathbb{D}^2 + 2 \frac{1}{2} \mathbb{F}_{\mu\nu}^a \mathbb{Q}^{\mu\nu} t_r$$

$$\mathbb{Q}^{\mu\nu} = \frac{i}{4} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) \quad \text{Dirac}$$

$$= i (\delta_\alpha^\mu \delta_\beta^\nu - \delta_\alpha^\nu \delta_\beta^\mu) \quad \text{4-vector}$$

$$\psi + \psi^\dagger \Delta_{co} \psi$$

δ^j Dirac

$\delta_p^\nu - \delta_a^\nu \delta_p^u$ 4-vector

$$\vec{\mu} = g \frac{Q e \hbar}{m} \vec{\sigma}$$

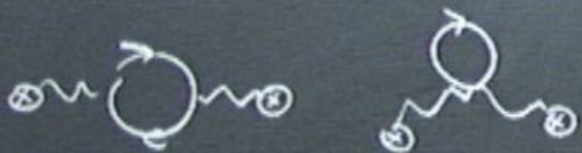


$$\mathcal{L} = -\frac{1}{4g^2} (\mathbb{F}_{\mu\nu})^2 - \frac{1}{2} \bar{\psi} \gamma^\mu \Delta_\mu \psi + \bar{c} \Delta_{\alpha\beta} c + \bar{\psi} \Delta_{\alpha\beta} \psi$$

$$\Delta = -\mathbb{D}^2 + 2 \frac{1}{2} \mathbb{F}_{\mu\nu} \sigma^{\mu\nu} t_r$$

$$\mathcal{J} = \frac{i}{4} (\psi^\dagger \psi)$$

$$= i (\psi^\dagger \psi)$$



$$\psi + \phi^\dagger \Delta_{co} \phi$$

$\delta^{\nu\mu}$ Dirac

$\delta_p^\nu - \delta_a^\nu \delta_p^a$ 4-vector

$$\mathcal{D}_\mu = \partial_\mu - ig A_\mu^a T_r^a$$

$$\vec{\mu} = g \frac{Q \hbar}{m} \vec{S}$$



$$\mathcal{L} = -\frac{1}{4g^2} (\mathbb{F}_M^{\mu\nu})^2 - \frac{1}{2} A_\mu^a \Delta_{ab}^{\mu\nu} \partial_\nu^c + \bar{c} \Delta_{a_0} c + \mathbb{F} \Delta_{a_0} c$$

$$\Delta = -\mathbb{D}^2 + 2 \frac{1}{2} \mathbb{F}_{\mu\nu}^a \otimes^{\mu\nu} t_r^a$$

$$\mathcal{J}^{\mu\nu} = \frac{i}{4} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) = i(\sigma_a^{\mu\nu} S^a)$$



$$\frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} A_{\mu}^a(-q) A_{\nu}^b(q) i(\not{q} \gamma^{\mu\nu} - \not{q}^{\mu} \gamma^{\nu}) \frac{1}{48\pi^2} \text{tr}(-i) t_r^a t_r^b$$

$$i \bar{\psi} \gamma^\mu \partial_\mu \psi + \phi^\dagger \Delta_{co} \phi$$

8) Dirac

$(\delta_\mu^\nu - \delta_\alpha^\nu \delta_\mu^\alpha)$ 4-vector

$$D_\mu = \partial_\mu - ig A_\mu^a T_r^a$$

$$\text{tr } T_r^a T_r^b = C(r) \delta^{ab}$$

$$\text{tr } g^{\mu\nu} g^{\alpha\beta} = C(S) (\delta^{\mu\nu} \delta^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta})$$

$$\vec{\mu} = g \frac{Q e \hbar}{m} \vec{S}$$

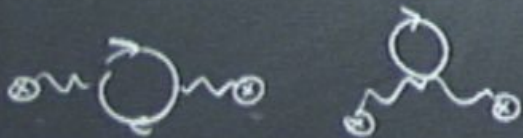


$$\mathcal{L} = -\frac{1}{4g^2} (\mathbb{F}_{\mu\nu}^a)^2 - \frac{1}{2} \bar{\psi} \gamma^\mu \Delta_\mu \psi + \bar{c} \Delta_{a_0} c + \bar{\psi} \Delta_{r_1} \psi + \psi^\dagger \Delta_{r_2} \psi$$

$$\Delta = -\mathbb{D}^2 + 2 \frac{1}{2} \mathbb{F}_{\mu\nu}^a \mathbb{Q}^{\mu\nu} t^a$$

$$\mathcal{J}^{\mu\nu} = \frac{i}{4} (\psi^\dagger \delta^\mu \psi - \psi \delta^\mu \psi^\dagger) \text{Dirac}$$

$$= i (\delta_\alpha^\mu \delta_\beta^\nu - \delta_\alpha^\nu \delta_\beta^\mu) \psi^\alpha \psi^\beta$$



$$\frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} A_{\mu(r)}^a A_{\nu(r)}^b i (q^\mu \delta^{\nu\lambda} - q^\nu \delta^{\mu\lambda}) \frac{1}{48\pi^2} \mathcal{L}_j(-i) \text{tr} \left(\frac{t_r^a t_r^b}{(r) \text{sol}} \right)$$

$$\frac{1}{4} \int \frac{d^4 q}{(2\pi)^4} F_{\mu\nu}^a F_{\mu\nu}^a \frac{C(r)}{48\pi^2} \mathcal{L}_j(-i^2)$$

$$\mathcal{L} = -\frac{1}{4g^2} (\mathbb{F}_{\mu\nu}^c)^2 - \frac{1}{2} A_\mu^c \Delta_{\mu\nu}^c \psi^\nu + \bar{c} \Delta_{\mu\nu}^c c + \bar{\psi} \Delta_{\mu\nu}^c \psi + \psi^\dagger \Delta_{\mu\nu}^c \psi$$

$$\Delta = -\mathbb{D}^2 + 2 \frac{1}{2} \mathbb{F}_{\mu\nu}^c \mathbb{Q}^{\mu\nu} t^c$$

$$\mathbb{D}^{\mu\nu} = \frac{i}{4} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) \mathbb{D} \text{Dirac}$$

$$= i (\delta_\alpha^\mu \delta_\beta^\nu - \delta_\alpha^\nu \delta_\beta^\mu)$$



$$1 + \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} A_{\mu\nu}^c(q) A^{\mu\nu c}(q) i (q^\mu \gamma^\nu - q^\nu \gamma^\mu) \frac{1}{48\pi^2} \gamma(-q) \text{tr} \left(\frac{t_r^c t_r^c}{(tr) \text{sol}} \right)$$

$$1 + \frac{i}{4} \int \frac{d^4 q}{(2\pi)^4} F_{\mu\nu}^a F^{\mu\nu a} \frac{C(r)}{48\pi^2} \gamma(-q)$$

$$\bar{\psi} \gamma^\nu + \bar{c} \Delta_{c,p} c + \bar{\psi} \Delta_{r,t} \psi + \phi^\dagger \Delta_{c,0} \phi$$

$$D_\mu = \partial_\mu - ig A_\mu^a t_r^a$$

$$F^{\mu\nu} = \frac{i}{4} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) \text{Dirac}$$

$$= i (\delta_\alpha^\mu \delta_\beta^\nu - \delta_\alpha^\nu \delta_\beta^\mu) \text{ 4-vector}$$

$$t_r^a$$

$$(g^{\mu\nu} - g^{\nu\mu}) \frac{1}{48\pi^2} \int (-i) \text{tr} \left(\frac{t_r^a t_r^b}{C(r) \text{sol}} \right)$$

$$- \frac{1}{4g^2} (F_{\mu\nu})^2 \left[1 - \frac{g^2 C(r)}{48\pi^2} \int (-i) \right]$$

$$\frac{C(r)}{48\pi^2} \int (-i)$$

$$\bar{\psi} \Delta_{\mu\nu} \psi + \bar{\psi} \Delta_{r,t} \psi + \psi^\dagger \Delta_{r,t} \psi$$

$$\mathcal{F} = \frac{i}{4} (\gamma^\mu \gamma^\nu) \text{Dirac}$$

$$= i (\sigma_\alpha^\mu \sigma_\rho^\nu - \sigma_\alpha^\nu \sigma_\rho^\mu) \quad \text{4-vector}$$

$$m q^{\nu} \frac{1}{48\pi^2} \mathcal{L}_3(-t) + \frac{t^a t^b}{C(r) \text{sol}}$$

$$\mathcal{L}_3(-t^2)$$

$$\mathbb{D}_\mu = \partial_\mu - ig A_\mu^a T_r^a$$

$$- \frac{1}{4g^2} (F_{\mu\nu})^2 \left[1 - \frac{g^2 C(r)}{48\pi^2} \mathcal{L}_3(-t) \right]$$

$$\bar{c} \Delta_{\mu\nu} c + \bar{\psi} \Delta_{\mu\nu} \psi + \phi^* \Delta_{\mu\nu} \phi$$

$$\mathcal{F} = \frac{i}{4} (\gamma^\mu \gamma^\nu) \text{Dirac}$$

$$= i (\delta_\mu^\alpha \delta_\nu^\beta - \delta_\mu^\beta \delta_\nu^\alpha) \quad \text{4-vector}$$

$$\mu\nu) \frac{1}{48\pi^2} \mathcal{L}_g(-\hat{t}) + \frac{t_r^a t_r^b}{C(r) \text{sol}}$$

$$\mathcal{L}_g(-\hat{t})$$

$$\mathcal{D}_\mu = \partial_\mu - ig A_\mu^a \tau_r^a$$

$$- \frac{1}{4g^2} (F_{\mu\nu})^2 \left[1 - \frac{g^2 C(r)}{48\pi^2} \mathcal{L}_g(-\hat{t}) \right]$$

$$g^2(-\hat{t}) = \frac{g^2}{1 - \frac{g^2 C(r)}{48\pi^2} \mathcal{L}_g(-\hat{t})}$$

$$\bar{\psi} \Delta_{\mu\nu} \psi + \bar{\psi} \Delta_{r_1} \psi + \psi^\dagger \Delta_{r_0} \psi$$

$$\mathcal{F} = \frac{i}{4} (\gamma^\mu \gamma^\nu) \text{Dirac}$$

$$= i (\delta_\mu^\alpha \delta_\nu^\beta - \delta_\mu^\beta \delta_\nu^\alpha) \quad \text{4-vector}$$

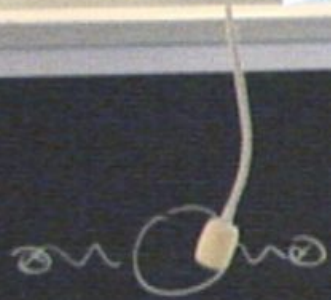
$$m_{\mu\nu} = \frac{1}{48\pi^2} \mathcal{L}_g(-\dot{t}^2) + \frac{t_r^a t_r^b}{c(r) \text{sol}}$$

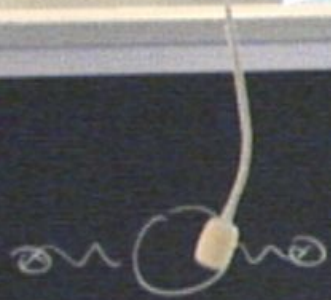
$$= \mathcal{L}_g(-\dot{t}^2)$$

$$\mathbb{D}_\mu = \partial_\mu - ig A_\mu^a \tau_r^a$$

$$- \frac{1}{4g^2} (\mathbb{F}_{\mu\nu})^2 \left[1 - \frac{g^2 c(r)}{48\pi^2} \mathcal{L}_g(-\dot{t}^2) \right]$$

$$g^2(-\dot{t}^2) = \frac{g^2}{1 - \frac{g^2 c(r)}{48\pi^2} \mathcal{L}_g(-\dot{t}^2)}$$









$$i \int \frac{d^4 p}{(2\pi)^4} \frac{g^{\mu\nu}}{p^2}$$



$$+ \partial^{\mu\nu} \partial^{\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$$

$$i \Delta_{\mu\nu}^{-1} g^{\mu\nu}$$



$$+ \partial^{\mu\nu} \partial^{\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$$

$$i \overline{D}_{\mu\nu} \partial^{\mu\nu}$$

$$\frac{1}{4} \int \frac{d^4 z}{(2\pi)^4} F_{\mu\nu} F^{\mu\nu} \} (1r)$$



$$i \bar{\psi} \gamma_{\mu\nu} \psi$$

$$+ \partial^{\mu\nu} \partial^{\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$$

$$\frac{1}{4} \int \frac{d^4 p}{(2\pi)^4}$$

$$F_{\mu\nu} F^{\mu\nu}$$

$$\} C(p)$$

$$\left[\frac{4}{48\pi^2} \right]$$

$$- \frac{4C(J)}{16\pi^2} \} g(-p')$$



$$i \bar{\psi} \gamma_{\mu\nu} \psi$$

$$+ \partial^{\mu\nu} \partial^{\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$$

$$\frac{1}{4} \int \frac{d^4 p}{(2\pi)^4} F_{\mu\nu} F^{\mu\nu} \left\{ c(r) \frac{(-1)}{2} \left[\frac{4}{48\pi^2} - \frac{4C(J)}{16\pi^2} \right] g(-\epsilon^4) \right\}$$

$$\psi + \psi^\dagger \Delta_{r0} \psi$$

$\delta^{\nu\lambda}$ Dirac

$\delta^\nu_\mu - \delta^\nu_\alpha \delta^\mu_\rho$ 4. vector

$$D_\mu = \partial_\mu - ig A_\mu^a T^a$$

$C(S) = \begin{cases} 0 & \text{scal} \\ 1 & \text{Dirac} \\ 2 & \text{vector} \end{cases}$

$$T^a T^b = C(r) S^{ab}$$

$$T^a T^b T^c = C(S) (S^a S^b - S^b S^a)$$

$$\vec{\mu} = g \frac{Q e \hbar}{m} \vec{S}$$

$$-\frac{1}{4g^2} (F_{\mu\nu})^2 \left[1 - \frac{g^2 C(r)}{48\pi^2} L_S(-\tilde{t}) \right]$$



$$g^2(-\tilde{t}) = \frac{g^2}{1 - \frac{g^2 C(r)}{48\pi^2} L_S(-\tilde{t})}$$



$$i \overline{\sigma}_{\mu\nu} \partial^{\mu\nu}$$

$$+ \partial^{\mu\nu} \partial^{\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$$

$$\frac{1}{4} \int \frac{d^4 p}{(2\pi)^4} F_{\mu\nu} F^{\mu\nu} \left\{ C(p) \frac{(-1)}{2} \left[\frac{4}{48\pi^2} - \frac{4C(J)}{16\pi^2} \right] g(-p^2) \right\}$$

$\underbrace{\hspace{10em}}_{\frac{1}{12\pi^2} !}$



$$i \bar{\psi} \gamma^{\mu\nu} \psi$$

$$+ \partial^{\mu\nu} \partial^{\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$$

$$\frac{1}{4} \int \frac{d^4 p}{(2\pi)^4} F_{\mu\nu} F^{\mu\nu} \left\{ C(p) \left(\frac{-1}{2} \left[\frac{4}{48\pi^2} - \frac{4C(J)}{16\pi^2} \right] \log(-\epsilon^2) \right) \right\}$$

$$\int d^4 x F_{\mu\nu}(x) F^{\mu\nu}(x)$$

$$\frac{1}{12\pi^2}$$

$$\int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2)^2 k^2}$$

$$= \frac{i}{16\pi^2} \left[+ \log\left(\frac{\Lambda^2}{\mu^2}\right) \right]$$



$$i \bar{\psi} \gamma^{\mu\nu} \psi$$

$$+ \partial^{\mu\nu} \partial^{\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$$

$$\frac{1}{4} \int \frac{d^4 q}{(2\pi)^4} F_{\mu\nu} F^{\mu\nu} \left\{ C(q) \left(\frac{-1}{2} \left[\frac{4}{48\pi^2} - \frac{4C(J)}{16\pi^2} \right] \log(-q^2) \right) \right\}$$

$$\int d^4 x F_{\mu\nu}(x) F^{\mu\nu}(x)$$

$$\frac{1}{12\pi^2}$$

$$\int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + i0)^2 k^2}$$

$$= \frac{i}{16\pi^2} \left[+ 15 \left(\frac{\Lambda^2}{q^2} \right) \right]$$

$$\psi + \psi^\dagger \Delta_{r0} \psi$$

$\delta^{\mu\nu}$ Dira
 $(\delta^{\nu\rho} - \delta^{\nu\alpha} \delta^{\alpha\rho})$ 4-vector

$$D_\mu = \partial_\mu - ig A_\mu^a T_r^a$$

$C(S) = \begin{cases} 0 & \text{scal} \\ 1 & \text{Dirac} \\ 2 & \text{vector} \end{cases}$

$$\begin{aligned}
 \text{tr } T_r^a T_r^b &= C(r) \delta^{ab} \\
 \text{tr } Q^\mu Q^\nu &= C(S) (g^{\mu\nu} - g^{\mu c} g^{\nu c})
 \end{aligned}$$

$$\vec{\mu} = g \frac{Q \text{et} \vec{S}}{m}$$

$$- \frac{i}{4g^2} (F_{\mu\nu})^2 \left[1 - \frac{g^2 C(r)}{48\pi^2} \mathcal{L}_g(-\vec{t}) - \frac{g^2 C(S)}{12\pi^2} \mathcal{L}_g(-\vec{t}) \right]$$

$$g^2(-\vec{t}) = \frac{g^2}{1 - \frac{g^2 C(r)}{48\pi^2} \mathcal{L}_g(-\vec{t})}$$

$$\mathcal{L} = -\frac{1}{4g^2} (\mathbb{F}_{\mu\nu}^a)^2 - \frac{1}{2} \bar{\psi}_\mu \Delta_{\mu\nu}^a \psi_\nu + \bar{c} \Delta_{a,0} c + \bar{\psi} \Delta_{\mu\nu}^a \psi$$

$$\Delta = -\mathbb{D}^2 + 2 \frac{1}{2} \mathbb{F}_{\mu\nu}^a \mathbb{Q}^{\mu\nu} t_r^a$$

$$\mathcal{J}^{\mu\nu} = \frac{i}{4} (\psi^\mu \psi^\nu - \psi^\nu \psi^\mu)$$



$$1 + \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} (\mathbb{F}_{\mu\nu}^a \mathbb{F}^{\mu\nu a})$$

$$\mathcal{L} = -\frac{1}{4g^2} (\mathbb{F}_m^a)^2 - \frac{1}{2} \bar{\psi}_\mu \Delta_{\mu\nu}^m \psi_\nu^s + \bar{c} \Delta_{a,0} c + \mathbb{F} \Delta_{r,1}$$

$$\Delta = -\mathbb{D}^2 + 2 \frac{1}{2} \mathbb{F}_{\mu\nu}^s \mathbb{Q}^{\mu\nu} t_r^s$$

$$\mathcal{J}^m = \frac{i}{4} (\gamma^\mu \psi^\nu - \gamma^\nu \psi^\mu)$$

$$= i (\delta_{\alpha\beta}^{\mu\nu} \psi^\alpha \psi^\beta)$$



$$1 + \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} (\mathbb{F}_{\mu\nu}^a \mathbb{F}^{\mu\nu a}) \frac{1}{48\pi^2}$$

$$\mathcal{L} = -\frac{1}{4g^2} (\mathbb{F}_{\mu\nu}^a)^2 - \frac{1}{2} \bar{\psi}_\mu \Delta_{\mu\nu}^a \psi_\nu + \bar{c} \Delta_{a,0} c + \bar{\psi} \Delta_{\mu\nu}^a \psi$$

$$\Delta = -\mathbb{D}^2 + 2 \frac{1}{2} \mathbb{F}_{\mu\nu}^a \mathbb{Q}^{\mu\nu} t_r^a$$

$$\mathcal{L}^{\text{gh}} = \frac{i}{4} (\bar{\psi} \gamma^\mu \psi) \partial_\mu c$$

$$= i (\delta_{\alpha\beta}^{\gamma\delta} \psi^\alpha \psi^\beta \partial_\mu c)$$



$$1 + \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} (\mathbb{F}_{\mu\nu}^a \mathbb{F}^{\mu\nu a}) \frac{1}{2} \left(\frac{4}{48\pi^2} - \frac{4 \cdot 2}{16\pi^2} \right)$$

$$\mathcal{L} = -\frac{1}{4g^2} (\mathbb{F}_{\mu\nu}^a)^2 - \frac{1}{2} \bar{\psi}_\mu \Delta_{\mu\nu}^a \psi_\nu + \bar{c} \Delta_{a,0} c + \bar{\psi} \Delta_{\mu\nu}^a \psi$$

$$\Delta = -\mathbb{D}^2 + 2 \frac{1}{2} \mathbb{F}_{\mu\nu}^a \sigma^{\mu\nu} t_r^a$$

$$\mathcal{J} = \frac{i}{4} (\psi^\dagger \psi) = i (\delta_{\alpha\beta} \psi^\alpha \psi^\beta)$$

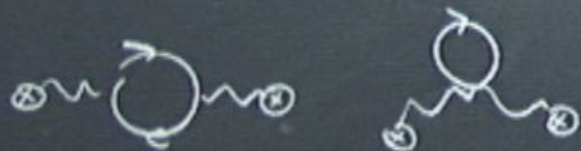


$$1 + \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} (\mathbb{F}_{\mu\nu}^a \mathbb{F}^{\mu\nu a}) \left\{ \frac{L}{2} \left(\frac{4}{48\pi^2} - \frac{4 \cdot 2}{16\pi^2} \right) \mathcal{J}_2(-q^2) \right\}$$

$$\mathcal{L} = -\frac{1}{4g^2} (\mathbb{F}_M^{\mu\nu})^2 - \frac{1}{2} A_\mu^a \Delta_{ab}^{\mu\nu} A_\nu^b + \bar{c} \Delta_{ab}^{\mu\nu} c + \Phi \Delta_{ab}^{\mu\nu} \psi$$

$$\Delta = -\mathbb{D}^2 + 2 \frac{1}{2} \mathbb{F}_{\mu\nu}^a \mathbb{Q}^{\mu\nu} t_r^a$$

$$\mathcal{J}^{\mu\nu} = \frac{i}{4} (\psi^\dagger \gamma^{\mu\nu} \psi) = i (\delta_\alpha^\mu \delta_\beta^\nu - \delta_\alpha^\nu \delta_\beta^\mu)$$



$$1 + \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} (\mathbb{F}_{\mu\nu}^a \mathbb{F}^{\mu\nu a}) \left\{ \frac{1}{2} \left(\frac{4}{48\pi^2} - \frac{4 \cdot 2}{16\pi^2} \right) \mathcal{J}_3(-q^2) \right.$$

$$\left. - \frac{1}{48\pi^2} \mathcal{G}(q) \mathcal{J}_3(-q^2) \right\}$$

$$\psi + \psi^\dagger \Delta_{r,0} \psi$$

8) Dirac

($\delta^{\mu\nu} - \delta^{\nu\mu} \delta^{\rho\sigma}$) 4-vector

$$D_\mu = \partial_\mu - ig A_\mu^a t_r^a$$

$C(S) = \begin{cases} 0 & \text{scal} \\ 1 & \text{Dirac} \\ 2 & \text{vector} \end{cases}$

$$\text{tr } t_r^a t_r^b = C(r) \delta^{ab}$$

$$\text{tr } g^{\mu\nu} g^{\rho\sigma} = C(S) (\delta^{\mu\nu} \delta^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})$$

$$\vec{\mu} = g \frac{Q \hbar}{m} \vec{S}$$

$$- \frac{i}{4g^2} (F_{\mu\nu})^2 \left[1 - \frac{g^2 C(r)}{48\pi^2} \ln(-i) - \frac{g^2 C(r)}{12\pi^2} \ln(-i) \right]$$

$$- C(r) \ln(-i) \Big\}$$

$$\frac{1}{4g^2} (\mathbb{F}_M^{\mu\nu})^2 - \frac{1}{2} Q_\mu^a \Delta_{ab}^{\mu\nu} Q_\nu^b + \bar{c} \Delta_{a0} c + \underbrace{\Phi \Delta_{r1} \Psi}_{\frac{1}{2} n_3} + \phi^k \Delta_{r0}$$

$$\Delta = -\mathbb{D}^2 + 2 \frac{1}{2} \mathbb{F}_{\mu\nu}^c Q^{\mu\nu} t_r^c$$

$$\mathcal{J}^{\mu\nu} = \frac{i}{4} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) \text{Dirac}$$

$$= i (\delta_\alpha^\mu \delta_\beta^\nu - \delta_\alpha^\nu \delta_\beta^\mu) \quad 4 \cdot 4$$



$$\frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} (\mathbb{F}_{\mu\nu}^c \mathbb{F}^{\mu\nu a}) \left\{ \frac{1}{2} \left(\frac{4}{48\pi^2} - \frac{4 \cdot 2}{16\pi^2} \right) \mathcal{J}(-i^2) C(G) \right\}$$

$$= \frac{1}{48\pi^2} C(G) \mathcal{J}(-i^2)$$

$$\psi + \psi^\dagger \Delta_{\text{co}} \psi$$

8) Dirac

$(\delta_\mu^\nu - \delta_\mu^\nu \delta_\rho^\mu)$ 4. vector

$$D_\mu = \partial_\mu - ig A_\mu^a t_r^a$$

$C(S) = \begin{cases} 0 & \text{scal} \\ 1 & \text{Dirac} \\ 2 & \text{vector} \end{cases}$

$$\text{tr } t_r^a t_r^b = C(r) \delta^{ab}$$

$$\text{tr } g^{\mu\nu} g^{\rho\sigma} = C(S) (\delta^{\mu\nu} \delta^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})$$

$$\vec{m} = g \frac{q \text{etk}}{m} \vec{S}$$

$C(G)$

$$- \frac{i}{4g^2} (F_{\mu\nu})^2 \left[1 - \frac{g^2 C(r)}{48\pi^2} \text{tr}(-\not{t}) - \frac{g^2 C(r)}{12\pi^2} \text{tr}(-\not{t}) \right]$$

$$- C(G) \text{tr}(-\not{t}) \left\{ \frac{10}{24} + \frac{1}{24} \right\}$$

$$\beta(\omega) = + \frac{\int_0^3 C(r) n_f}{12\pi^2} + \frac{\int_0^3 C(r) n_b}{48\pi^2} + \dots$$

$$1 + \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3} \left(\overline{TF}_{Av} \overline{TF}_{Av} \right) \left\{ \frac{L}{2} \left(\frac{4}{48\pi^2} - \frac{4 \cdot 2}{16\pi^2} \right) \mathcal{L}_3(-\omega^2) - \frac{1}{48\pi^2} \mathcal{G}(\omega) \mathcal{L}_3(-\omega^2) \right\}$$

$$P(s) = + \frac{9^3}{12\pi^2} C(s) \eta_f + \frac{9^3}{48\pi^2} C(s) \eta_b \rightarrow \frac{11}{24\pi^2}$$

$$1 + \frac{1}{2} \int \frac{d\eta}{(2\eta)^n} \left(\mathbb{F}_{\Lambda^c}^{\leftarrow} \mathbb{F}_{\Lambda^c}^{\rightarrow} \right) \left\{ \frac{1}{2} \left(\frac{4}{48\pi^2} - \frac{4 \cdot 2}{16\pi^2} \right) \mathcal{J}_s(-i^2) C(s) \right. \\ \left. - \frac{1}{48\pi^2} C(s) \mathcal{J}_s(-i^2) \right\}$$

$$\text{tr } t_r^c t_r^b = C(r) \delta^{cb}$$

$$\text{tr } g^{mv} g^{ar} = C(S) (\delta^{mv} \delta^{ar} - g^{mv} g^{ar})$$

$$D_\mu = \partial_\mu - ig A_\mu^a t_r^a$$

$C(S) = \begin{cases} 0 & \text{scalar} \\ 1 & \text{Dirac} \\ 2 & \text{vector} \end{cases}$

$$\vec{\mu} = g \frac{Q \text{det}}{m} \vec{S}$$

$$(F_{\mu\nu})^2 \left[1 - \frac{g^2 C(r)}{48\pi^2} \ln(-t) - \frac{g^2 C(r)}{12\pi^2} \ln(-t) \right]$$

$$- \frac{g^2 C(r)}{48} \ln(-t) \left\{ \frac{10}{48} + \frac{1}{48} \right\}$$

$$P(s) = + \frac{g^3}{12\pi^2} C(s) n_f + \frac{g^3}{48\pi^2} C(s) n_b \rightarrow \frac{11}{48\pi^2} C(s)$$

$$1 + \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} (TF_{Av}^{\leftarrow} TF_{Av}^{\rightarrow}) \left\{ \frac{1}{2} \left(\frac{4}{48\pi^2} - \frac{4 \cdot 2}{16\pi^2} \right) \mathcal{L}_s(-i^2) C(s) \right. \\ \left. - \frac{1}{48\pi^2} C(s) \mathcal{L}_s(-i^2) \right\}$$

$$P(g) = + \frac{g^3}{12\pi^2} C(r) n_f + \frac{g^3}{48\pi^2} C(r) n_b - \frac{11}{48\pi^2} g^3 C(a)$$

$$= - \frac{g^3}{(4\pi)^2} \left[\frac{11}{3} C(a) - \frac{4}{3} C(r) n_f \right]$$

$$1 + \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} \left(\overline{F}_{Av}^c F^{Av a} \right) \left\{ \frac{1}{2} \left(\frac{4}{48\pi^2} - \frac{4 \cdot 2}{16\pi^2} \right) \mathcal{I}_3(-i^2) C(a) \right.$$

$$\left. - \frac{1}{48\pi^2} C(a) \mathcal{I}_3(-i^2) \right\}$$

$$P(s) = + \frac{g^3}{12\pi^2} C(r) n_f + \frac{g^3}{48\pi^2} C(r) n_b \rightarrow \frac{11}{48\pi^2} g^3 C(a)$$

$$= - \frac{g^3}{(4\pi)^2} \left[\frac{11}{3} C(a) - \frac{4}{3} C(r) n_f \right]$$

$$1 + \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} \left(\overline{TF}^{Av} \overline{TF}^{Av} \right) \left\{ \frac{1}{2} \left(\frac{4}{48\pi^2} - \frac{4 \cdot 2}{16\pi^2} \right) \mathcal{I}_2(-l^2) C(a) \right. \\ \left. - \frac{1}{48\pi^2} C(a) \mathcal{I}_2(-l^2) \right\}$$

$$P(g) = + \frac{g^3}{12\pi^2} C(r) n_f + \frac{g^3}{48\pi^2} C(r) n_b - \frac{11}{48\pi^2} g^3 C(G)$$

$$= - \frac{g^3}{(4\pi)^2} \left[\frac{11}{3} C(G) - \frac{4}{3} C(r) n_f \right]$$

$SU(N)$

$C(G) = N$

$C(r)$

$$\beta(g) = + \frac{g^3}{12\pi^2} C(r) n_f + \frac{g^3}{48\pi^2} C(r) n_b - \frac{11}{48\pi^2} g^3 C(G)$$

$$= - \frac{g^3}{(4\pi)^2} \left[\frac{11}{3} C(G) - \frac{4}{3} C(r) n_f \right]$$

$$SU(N) \quad C(G) = N \quad C(r) = \frac{1}{2}$$

$$\beta(g) = - \frac{g^3}{(4\pi)^2} \left[\frac{11}{3} N - \frac{2}{3} n_f \right]$$

$$P(g) = + \frac{g^3}{12\pi^2} C(r) n_f + \frac{g^3}{48\pi^2} C(r) n_b - \frac{11}{48\pi^2} g^3 C(G)$$

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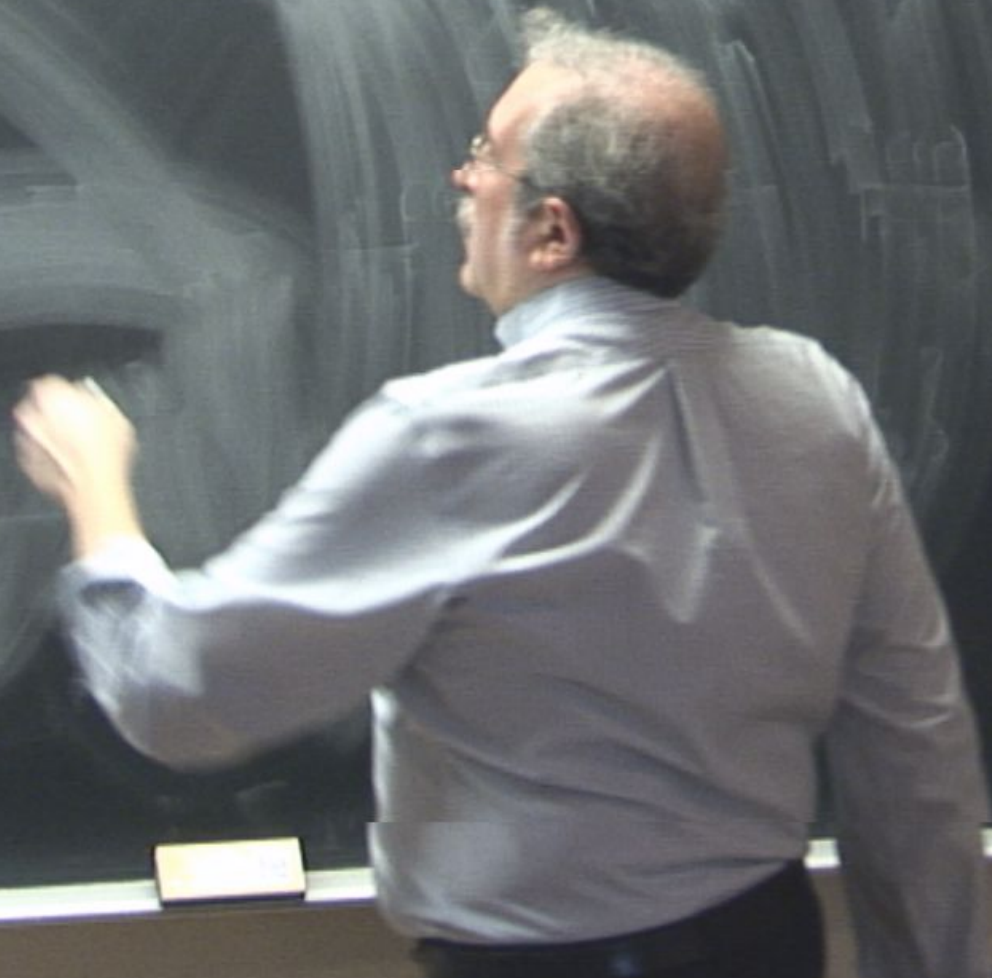
$$= - \frac{g^3}{(4\pi)^2} \left[\frac{11}{3} C(G) - \frac{4}{3} C(r) n_f \right]$$

$$SU(N) \quad C(G) = N \quad C(r) = \frac{1}{2}$$

$$\beta(g) = - \frac{g^3}{(4\pi)^2} \left[\frac{11}{3} N - \frac{2}{3} n_f \right]$$

"asymptotic freedom"

Yang Mills groups in 3 of $SU(3)$



Yang Mills gauge in 3 of $SU(3)$

Quantum Chromodynamics

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^a)^2 + \sum_f \bar{q}_f (i\not{D} - m_f) q_f$$

QCD

Yang Mills gauge in 3 of $SU(3)$

Quantum Chromodynamics

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^a)^2 + \sum_f \bar{q}_f (i\not{D} - m_f) q_f$$

QCD

~~$SU(3)$~~ S_{ij} $q_i q_j$

Yang Mills gauge in 3 of $SU(3)$

Quantum Chromodynamics

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^a)^2 + \sum_f \bar{q}_f (i\not{D} - m_f) q_f$$

QCD

~~$SU(3)$~~ S_{ij} $q_i q_j$

$(t_a^b)_{ac} = i f^{abc}$

$\text{tr}(t_a t_b) = \frac{1}{2} \delta^{ab}$



Yang Mills gauge in 3 of $SU(3)$

Quantum Chromodynamics

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^a)^2 + \sum_f \bar{q}_f (i\not{D} - m_f) q_f$$

QCD

~~S_{ij}~~ $g_i g_j$

$(t_a^b)_{ac} = i f^{abc}$

$(\text{tr } t_a^d t_a^b) = f^{cad} f^{bcd}$

Yang Mills gauge in 3 of $SU(3)$

Quantum Chromodynamics

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^a)^2 + \sum_f \bar{\psi}_f (i\not{D} - m_f) \psi_f$$

QCD

~~S_{ij}~~ $g_i g_j$

$$(t_a^b)_{ac} = i f^{abc}$$

$$\begin{aligned} \text{tr}(t_a t_b) &= \text{tr} t_a^d t_d^b \\ &= f^{cad} f^{dbc} \\ &= f^{acd} f^{bcd} \end{aligned}$$

Yang Mills gauge in 3 of $SU(3)$

Quantum Chromodynamics

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^a)^2 + \sum_f \bar{q}_f (i\not{D} - m_f) q_f$$

QCD

~~$SU(3)$ S_{ij} $q_i q_j$~~

$g \rightarrow \infty$

$(t_a^b)_{ac} = i f^{abc}$

$$\begin{aligned} \text{tr}(t_a^d t_b^c) &= f^{cad} f^{dbc} \\ &= f^{acd} f^{bcd} \end{aligned}$$

Yang Mills gauge in 3 of SU(3)

Quantum Chromodynamics

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^a)^2 + \sum_f \bar{\psi}_f (i\not{D} - m_f) \psi_f$$

QCD

$$10 \text{ GeV} \leftarrow Q < 200 \text{ GeV}$$

Yang Mills gauge in 3 of SU(3)

Quantum Chromodynamics

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QCD

$$10 \text{ GeV} \leftarrow Q < 200 \text{ GeV}$$

$$\alpha_s = \frac{g^2}{4\pi}$$

Yang Mills gauge in 3 of SU(3)

Quantum Chromodynamics

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^a)^2 + \sum_f \bar{q}_f (i\not{D} - m_f) q_f$$

QCD

$$10 \text{ GeV} < Q < 200 \text{ GeV}$$

$$\alpha_s = \frac{g^2}{4\pi}$$

$$\alpha_s(Q) = \frac{\alpha_s(Q_0)}{1 + b_0 \frac{\alpha_s(Q_0)}{2\pi} \ln \frac{Q}{Q_0}}$$

$$b_0 = \frac{11}{3} \cdot 3 - \frac{2}{3} \cdot 5$$

Yang Mills gauge in 3 of SU(3)

Quantum Chromodynamics

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^a)^2 + \sum_f \bar{q}_f (i\not{D} - m_f) q_f$$

QCD

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Yang Mills gauge in 3 of SU(3)

Quantum Chromodynamics

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$$b_0 = \frac{11}{3} \cdot 3 - \frac{2}{3} \cdot 5 = \frac{23}{3}$$

$$= 0.18$$

$$0.12$$

$$10 \text{ GeV}$$

$$100 \text{ GeV}$$

Yang Mills gauge in 3 of SU(3)

Quantum Chromodynamics

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QCD

$$10 \text{ GeV} < Q < 200 \text{ GeV}$$

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$$\alpha_s(Q) = \frac{\alpha_s(Q_0)}{1 + b_0 \frac{\alpha_s(Q_0)}{2\pi} \ln \frac{Q}{Q_0}}$$

$$b_0 = \frac{11}{3} \cdot 3 - \frac{2}{3} \cdot 5 = \frac{23}{3}$$

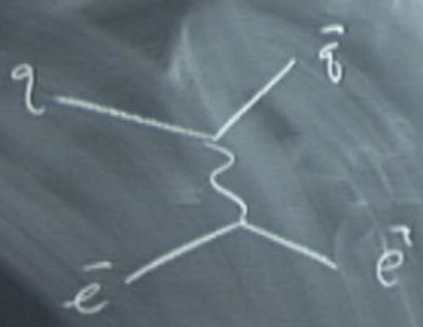
$$= 0.18$$

10 GeV

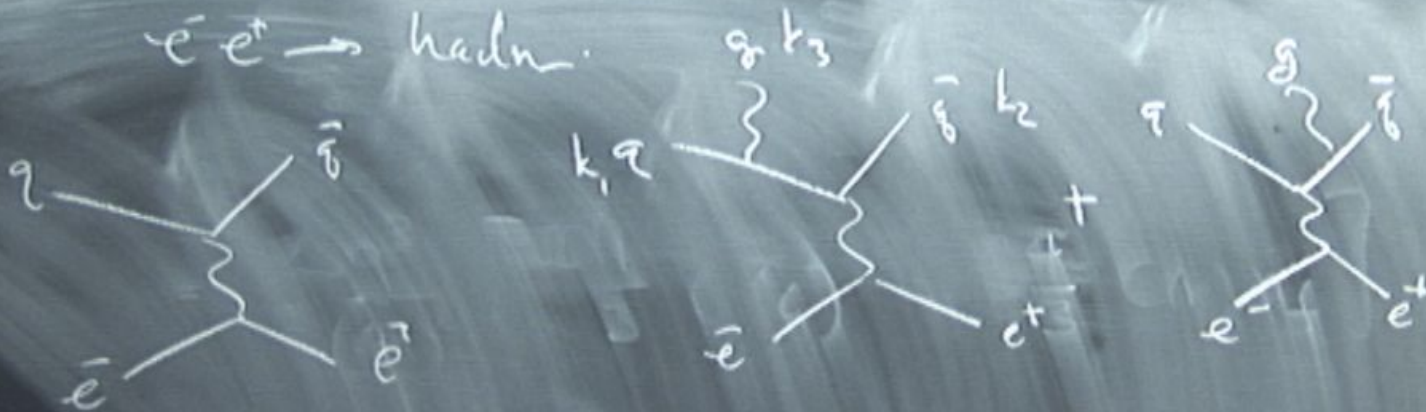
$$0.12$$

100 GeV

$e^- e^+ \rightarrow \text{hadrons}$



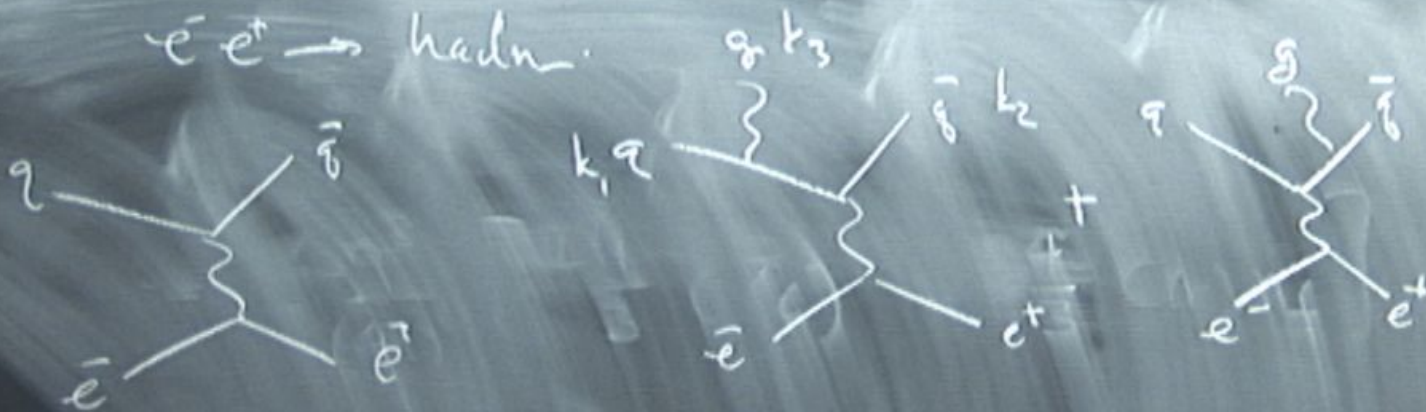
$e^- e^+ \rightarrow \text{hadrons}$



$$Q = k_1 + k_2 = k_3$$

$$X_i = \frac{2\alpha k_i}{Q^2}$$

$e^- e^+ \rightarrow \text{hadron}$

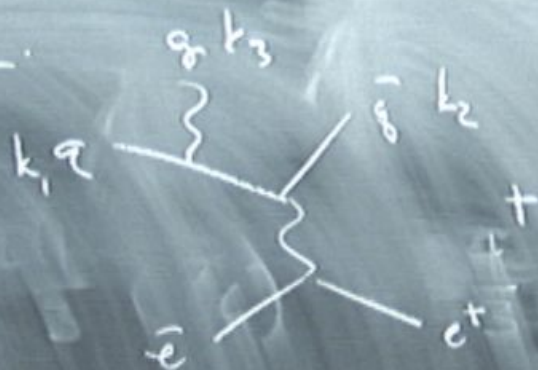
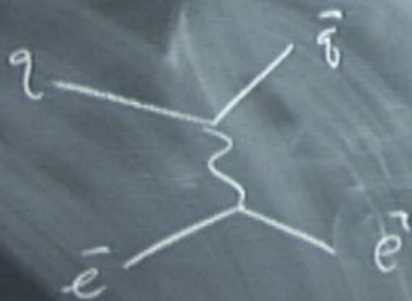


$$\sigma_0 = \frac{4\pi}{3} \frac{\alpha^2}{s} \sum_f 3Q_f^2$$

$$Q = k_1 + k_2 + k_3 \quad x_i = \frac{2Q \cdot k_i}{Q^2}$$

$$x_1 + x_2 + x_3 = 1$$

$e^- e^+ \rightarrow \text{hadrons}$



$$Q = k_1 + k_2 + k_3$$

$$x_i = \frac{2Q k_i}{Q^2}$$

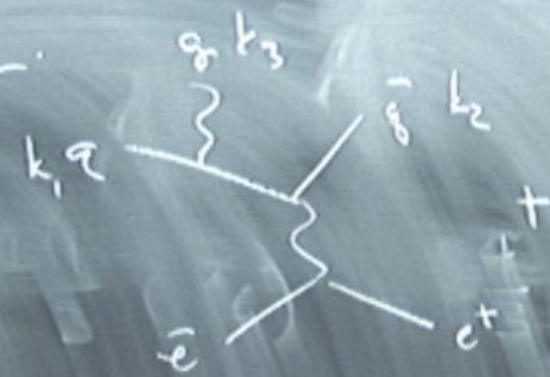
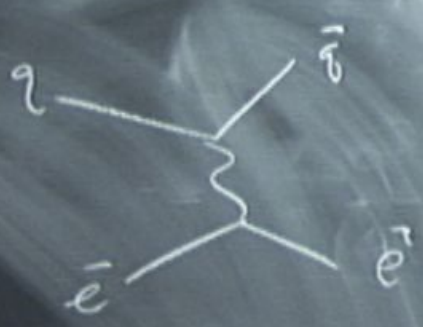
$$x_1 + x_2 + x_3 = 1$$

$$\sigma_0 = \frac{4\pi}{3} \sum_f \frac{q_f^2}{N_c} 3Q_f^2$$

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \cdot \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$\frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$e^- e^+ \rightarrow \text{hadron}$



$$Q = k_1 + k_2 = k_3 \quad x_i = \frac{2Q \cdot k_i}{Q^2}$$

$$x_1 + x_2 + x_3 = 1$$

$$\sigma_0 = \frac{4\pi}{3} \sum_f \frac{q_f^2}{3} 3Q_f^2$$

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \cdot \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Farhi

T =



Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \cdot \vec{\phi}_i|}{\sum_i |\vec{\phi}_i|}$$



Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \phi_i|}{\sum_i |\phi_i|}$$



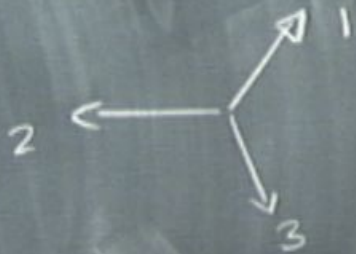
Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \cdot \vec{\phi}_i|}{\sum_i |\vec{\phi}_i|}$$



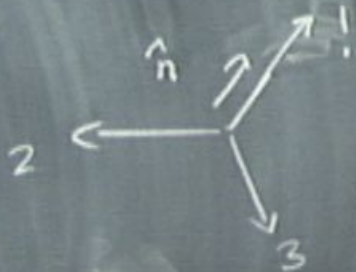
Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \vec{\phi}_i|}{\sum_i |\vec{\phi}_i|}$$



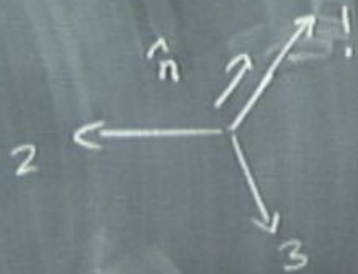
Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \cdot \vec{\phi}_i|}{\sum_i |\vec{\phi}_i|}$$



Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \cdot \vec{\phi}_i|}{\sum_i |\vec{\phi}_i|}$$



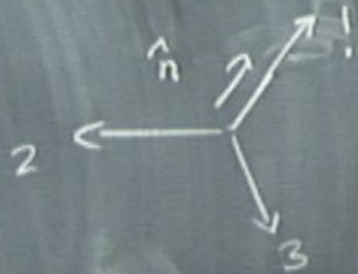
reg

$$T = \max \{x_1, x_2, x_3\}$$



Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \cdot \vec{\phi}_i|}{\sum_i |\vec{\phi}_i|}$$



qeg

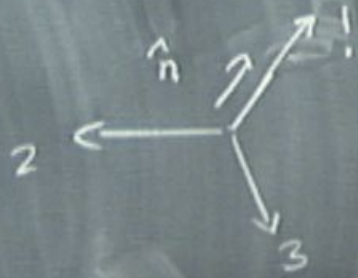
$$T = \max \{x_1, x_2, x_3\}$$

$$\frac{2}{3} < T < 1$$



Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \cdot \vec{\phi}_i|}{\sum_i |\vec{\phi}_i|}$$



reg

$$T = \max \{x_1, x_2, x_3\}$$

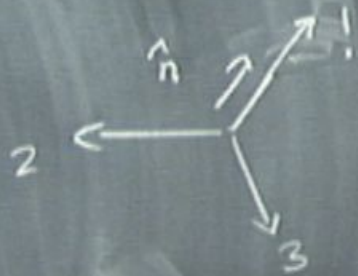
random

$$T = \frac{1}{3}$$



Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \phi_i|}{\sum_i |\phi_i|}$$



reg

$$T = \max \{x_1, x_2, x_3\}$$

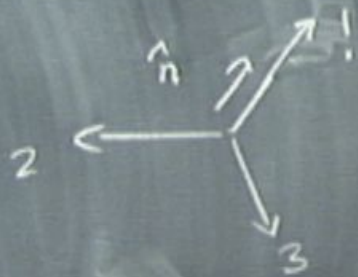
$$T = \frac{1}{3}$$

random

$$\frac{2}{3} < T < 1$$

Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \cdot \vec{\phi}_i|}{\sum_i |\vec{\phi}_i|}$$



reg

$$T = \max \{x_1, x_2, x_3\}$$

$$T = \frac{1}{3}$$

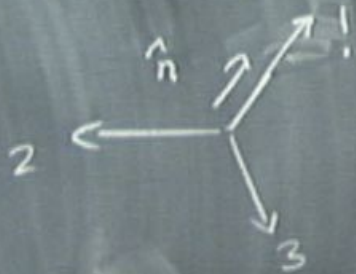
random

$$\frac{2}{3} < T < 1$$

$$\frac{1}{c_0} \frac{dc}{dT} = \frac{2dc}{3\pi} \left\{ \frac{2(3T^2 - 3T + 2)}{T(1-T)} \right\} \left\{ \frac{2T-1}{1-T} - 3 \frac{(3T-2)(2-T)}{1-T} \right\}$$

Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \cdot \vec{\phi}_i|}{\sum_i |\vec{\phi}_i|}$$



2EG

$$\frac{2}{3} < T < 1$$



$$T = \max \{x_1, x_2, x_3\}$$

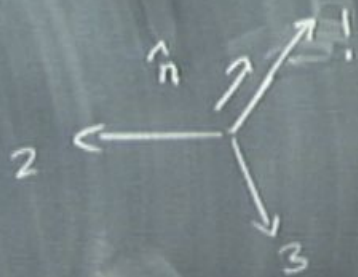
random

$$T = \frac{1}{3}$$

$$\frac{1}{\zeta_0} \frac{d\zeta}{dT} = \frac{2d\zeta}{3\pi} \left\{ \frac{2(3T^2 - 3T + 2)}{T(1-T)} \right\} \left\{ \frac{2T-1}{1-T} - 3 \frac{(3T-2)(2-T)}{1-T} \right\}$$

Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \cdot \vec{\phi}_i|}{\sum_i |\vec{\phi}_i|}$$



reg

$$\frac{2}{3} < T < 1$$

$\frac{d\epsilon}{dT}$



$$T = \max \{x_1, x_2, x_3\}$$

random

$$T = \frac{1}{3}$$

$$\frac{1}{\epsilon_0} \frac{d\epsilon}{dT} =$$

$$\frac{2d\epsilon}{3\pi}$$

$$\frac{2(3T^2 - 3T + 2)}{T(1-T)}$$

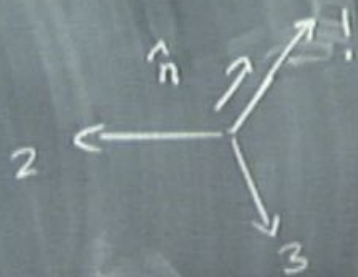
$$\frac{2T-1}{1-T}$$

$$-3$$

$$\frac{(3T-2)(2-T)}{1-T}$$

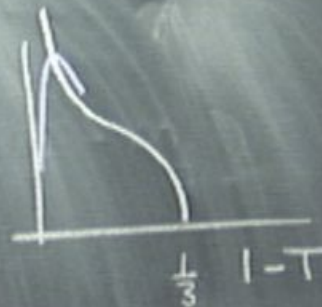
Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \cdot \vec{\phi}_i|}{\sum_i |\vec{\phi}_i|}$$



avg

$\frac{ds}{dT}$



$$T = \max \{x_1, x_2, x_3\}$$

random

$$T = \frac{1}{3}$$

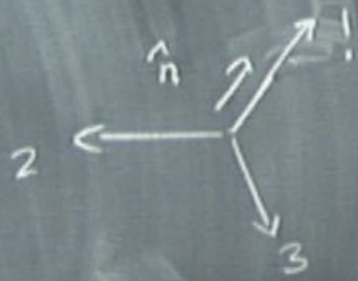
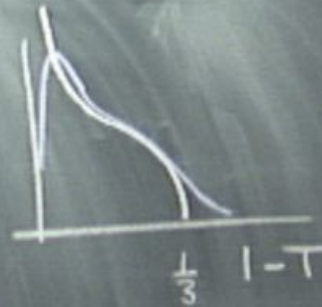
$$\frac{2}{3} < T < 1$$

$$\frac{1}{c_0} \frac{ds}{dT} = \frac{2ds}{3\pi} \left\{ \frac{2(3T^2 - 3T + 2)}{T(1-T)} \right\} \left\{ \frac{2T-1}{1-T} - 3 \frac{(3T-2)(2-T)}{1-T} \right\}$$

Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \cdot \vec{\phi}_i|}{\sum_i |\vec{\phi}_i|}$$

$\frac{d\epsilon}{dT}$



reg

$$T = \max \{x_1, x_2, x_3\}$$

random

$$T = \frac{1}{3}$$

$$\frac{2}{3} < T < 1$$

$$\frac{1}{3} \frac{d\epsilon}{dT} =$$

$$\frac{2d\epsilon}{3\pi}$$

$$\frac{2(3T^2 - 3T + 2)}{T(1-T)}$$

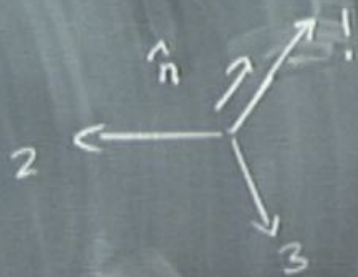
$$\frac{2T-1}{1-T}$$

-3

$$\frac{(3T-2)(2-T)}{1-T}$$

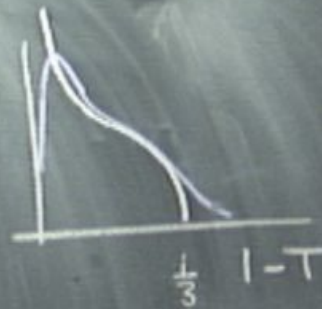
Farhi

$$T = \max_{\hat{n}} \frac{\sum_i |\hat{n} \cdot \vec{\phi}_i|}{\sum_i |\vec{\phi}_i|}$$



reg

$\frac{ds}{dT}$



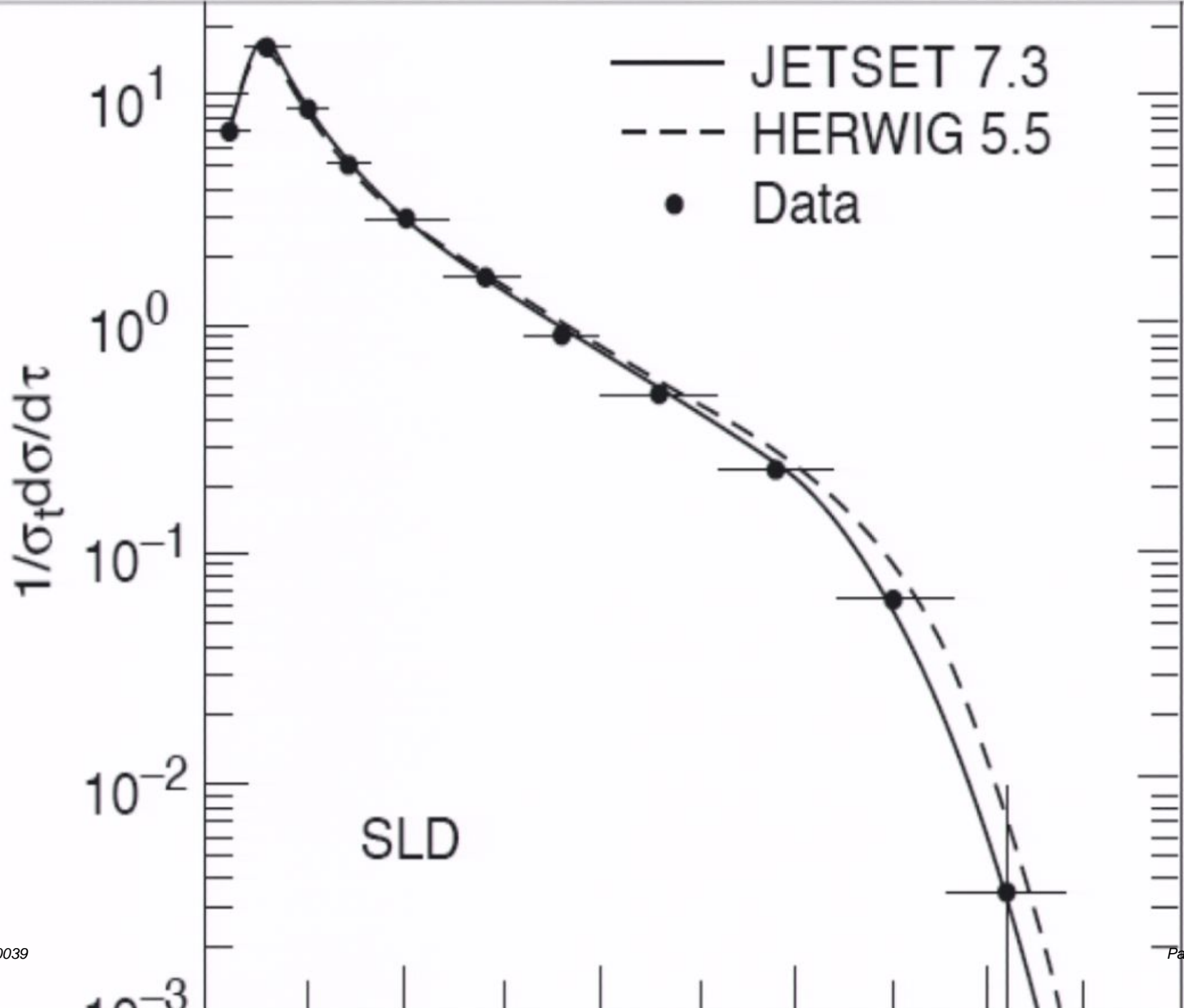
$$T = \max \{x_1, x_2, x_3\}$$

random

$$T = \frac{1}{3}$$

$$\frac{2}{3} < T < 1$$

$$\frac{1}{6} \frac{ds}{dT} = \frac{2ds}{3\pi} \left\{ \frac{2(3T^2 - 3T + 2)}{T(1-T)} \right\} \left\{ \frac{2T-1}{1-T} - 3 \frac{(3T-2)(2-T)}{1-T} \right\}$$



$\alpha_s(Q)$

0.4

0.3

0.2

0.1

\triangle Deep Inelastic Scattering

\circ e^+e^- Annihilation

\diamond Hadron Collisions

\boxtimes Heavy Quarkonia

QCD $\alpha_s(M_Z) = 0.1189 \pm 0.0010$

