Title: Colored group field theory: Scaling properties and positivity

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Abstract: The scaling analysis in the large spin limit of Feynman amplitudes for the Bosonic colored group field theory are considered in any dimension starting with dimension 4. By an explicit integration of two colors, we show that the model is positive. This formulation could be useful for the constructive analysis of this type of models.

Colored group field theory: Scaling properties and positivity

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Outline

1. Introduction
2. The 4D colored Ooguri model
3. Colored GFT in dimension D
4. Integration of two colors: a new representation
5. Conclusion

Generalities

Group Field Theories (GFTs)

- Beginning in the 90's: Boulatov [MPL **A7** (1992)] and Ooguri [MPL **A7** (1992)] group models.
- (Tensor) quantum field theories over group manifolds.
- A fundamental framework for background free quantum gravity. [D. Oriti in *Quantum Gravity*, Ed. Fauser *et al* (2007); gr-qc/0607032; L Freidel, IJTP **44** (2005); C. Rovelli, *Quantum Gravity* (2004) chap 9].
- Loosely speaking, *background independence* means that one should resum over all geometry and topology.

Salient features

- Gauge invariance;
- Non locality: the arguments of the fields are paired in a specific way $\xleftrightarrow{\text{dual}}$ Gluing of simplices.

Some known $G = SU(2)$-FTs

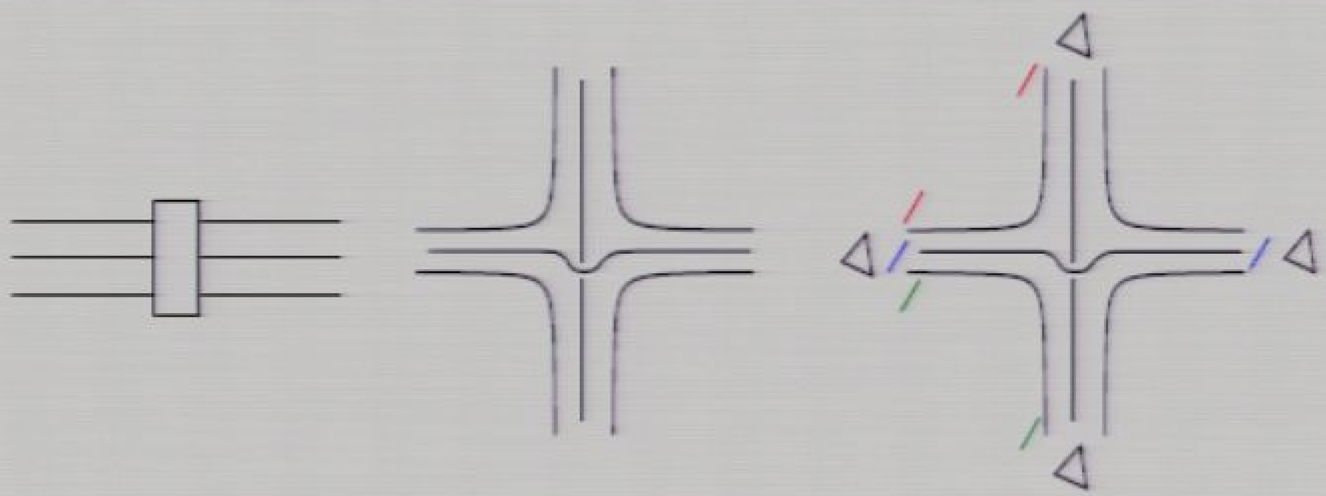

Figure: 3D GFT: Boulatov propagator and vertex.

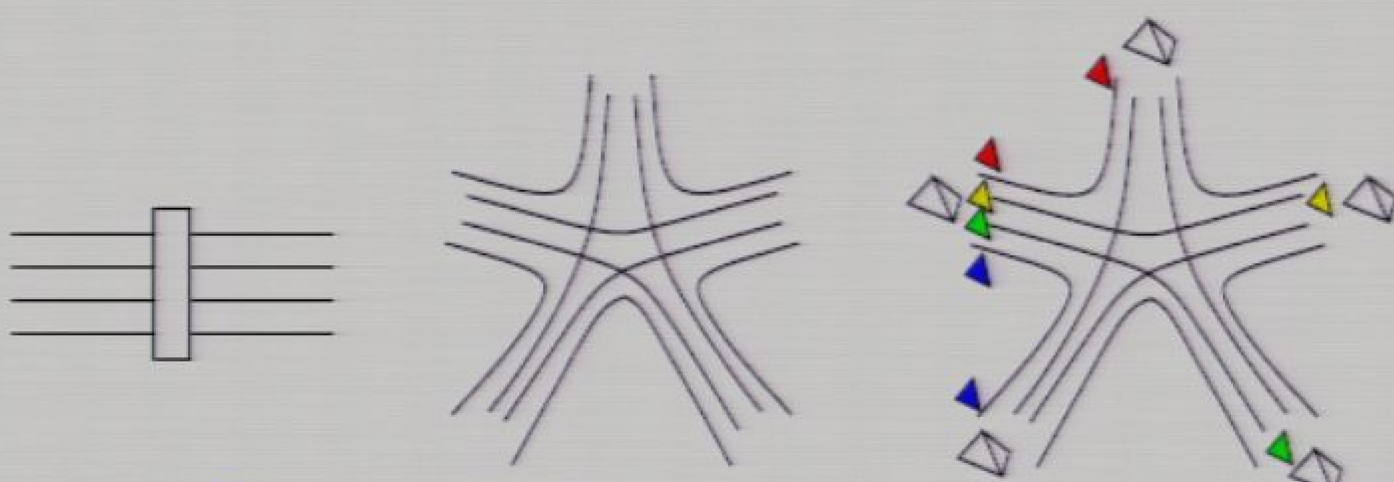

Figure: 4D GFT: Ooguri propagator and vertex.

Heuristic principles of a renormalization group analysis

1. A scale analysis: To give an orientation of the RG; assignment to "low" and "high"; "small" and "long", "left" and "right".
2. A power counting theorem: To classify according to their topology which of the Feynman graphs are the most divergent $\Rightarrow$ a specific treatment.
3. A "generalized locality" principle: A selection principle based on the definition of scales above, stating which of the diagrams can be recast in the form of a term already present in the theory plus a smaller "irrelevant" remainder.

First steps of a renormalization program

- Systematic analysis of Boulatov's model [Freidel, Gurau, Oriti, PR **D80** (2009)]; "Type I" graphs: full contraction procedure is possible; exact power counting$= \Lambda^{B-1}$, B the number of bubbles (graph with dual consisting in a simplex with a closed collection of surfaces); A conjecture: *Type I graphs will dominate in the large spin regime.*
- Scaling behaviour of Boulatov's model and its FL - constructive regularization [Magnen, Noui, Rivasseau, Smerlak, CQG **26** (2009)]; Feynman amplitudes studied in the large cutoff limit $\Lambda_B^{3n/2}$; optimal bounds for graphs without generalized tadpoles; the FL-model is perturbatively more divergent Λ_{FL}^{3n} than the ordinary one; Borel summability of the connected functions in the coupling constant established.

First steps of a renormalization program

However, some difficulties with (generalized) tadpoles: the power counting of the most general topological models is governed by "generalized tadpoles".

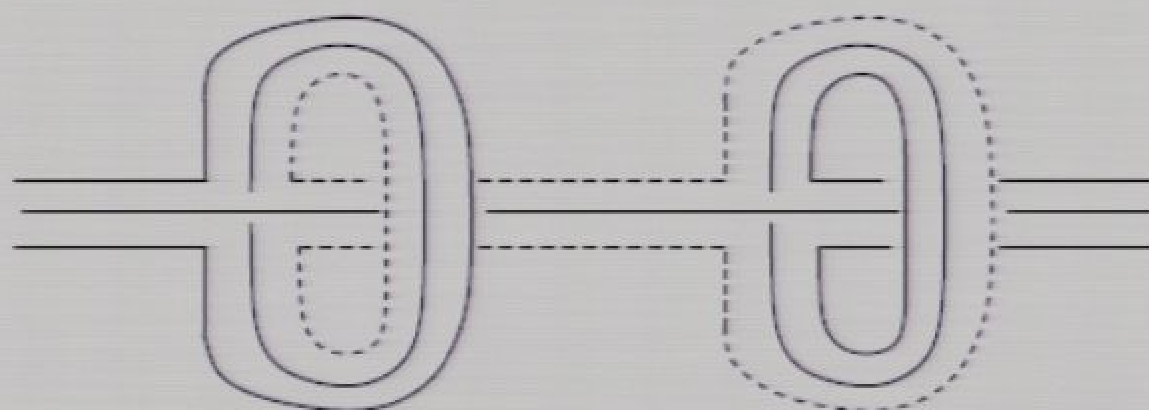

Figure: Additional divergence of nonplanar tadpoles when inserted in more involved graphs.

• Rough bound $\Lambda^{3/2}$ violated; this type of graphs $\supset$ these tadpoles $\rightsquigarrow$ singular manifolds (?).

First steps of a renormalization program

In the meantime,

• Fermionic colored GFT model in dimension D [Gurau, 0907.2582]: $SU(D+1)$ symmetric, colored graphs have computable homology; $\neq$ in general GFT theory: the "bubbles" can be easily identified; and for the Colored model...

First steps of a renormalization program

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- Fermionic colored GFT model in dimension D [Gurau, 0907.2582]: $SU(D+1)$ symmetric, colored graphs have computable homology; $\neq$ in general GFT theory: the "bubbles" can be easily identified; and for the Colored model...

No tadpoles !

Goals

- Bosonic version of the Gurau colored model,
- Study perturbative bounds large spin limit in any dimension, starting by 4.
- Explicit integration of two colors and the Matthews-Salam field representation (revealing an interesting hidden positivity of the model encouraging for a constructive analysis)

[BGMR, arXiv 0911.1719[hep-th]]

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The 4D colored Ooguri model

- The fields: dynamical variables of 4D $G = SU(2)$-FT:

$$\phi^\ell : G^4 \to \mathbb{C} \tag{1}$$

$$\phi^\ell(g_1 h, g_2 h, g_3 h, g_4 h) = \phi^\ell(g_1, g_2, g_3, g_4), \quad h \in G,$$
$$\bar\phi^\ell(g_1 h, g_2 h, g_3 h, g_4 h) = \bar\phi^\ell(g_1, g_2, g_3, g_4), \quad h \in G, \tag{2}$$

$$\phi^\ell_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} := \phi^\ell(g_{\alpha_1}, g_{\alpha_2}, g_{\alpha_3}, g_{\alpha_4}) \tag{3}$$

Action + GI const.

$$\begin{aligned} S[\phi] \ :=\ & \int \prod_{i=1}^{4} dg_i \ \sum_{\ell=1}^{5} \bar\phi^\ell_{1,2,3,4}\ \phi^\ell_{4,3,2,1} \\ & + \lambda_1 \int \prod_{i=1}^{10} dg_i\ \phi^1_{1,2,3,4}\ \phi^5_{4,5,6,7}\ \phi^4_{7,3,8,9}\ \phi^3_{9,6,2,10}\ \phi^2_{10,8,5,1} \\ & + \lambda_2 \int \prod_{i=1}^{10} dg_i\ \bar\phi^1_{1,2,3,4}\ \bar\phi^5_{4,5,6,7}\ \bar\phi^4_{7,3,8,9}\ \bar\phi^3_{9,6,2,10}\ \bar\phi^2_{10,8,5,1} \end{aligned} \tag{4}$$

The 4D colored Ooguri model

Partition function

$$Z(\lambda_1, \lambda_2) = \int d\mu_C[\bar{\phi}, \phi]\, e^{-\lambda_1 T_1[\phi] - \lambda_2 T_2[\bar{\phi}]}, \qquad (5)$$

• $d\mu_C[\bar{\phi}, \phi]$ degenerate Gaussian measure $\supset D[\bar{\phi}, \phi] = \prod_\ell d\bar{\phi}^\ell d\phi^\ell$ + GI constraint + Mass terms $\sum_\ell \bar{\phi}^\ell_{1,2,3,4}\ \phi^\ell$.
• The covariance (or propagator) C

$$\begin{aligned} C_{\ell\ell'}(g_1, g_2, g_3, g_4; g_4', g_3', g_2', g_1') &= \int \bar{\phi}^\ell_{1,2,3,4}\, \phi^{\ell'}_{4',3',2',1'}\, d\mu_C[\bar{\phi}, \phi] \\ &= \delta_{\ell\ell'} \int dh \prod_{i=1}^{4} \delta(g_i h (g_i')^{-1}). \end{aligned} \qquad (6)$$

The 4D colored Ooguri model

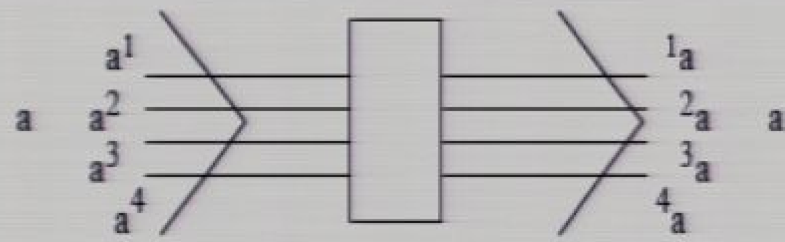


Figure: The propagator or covariance of the colored model.

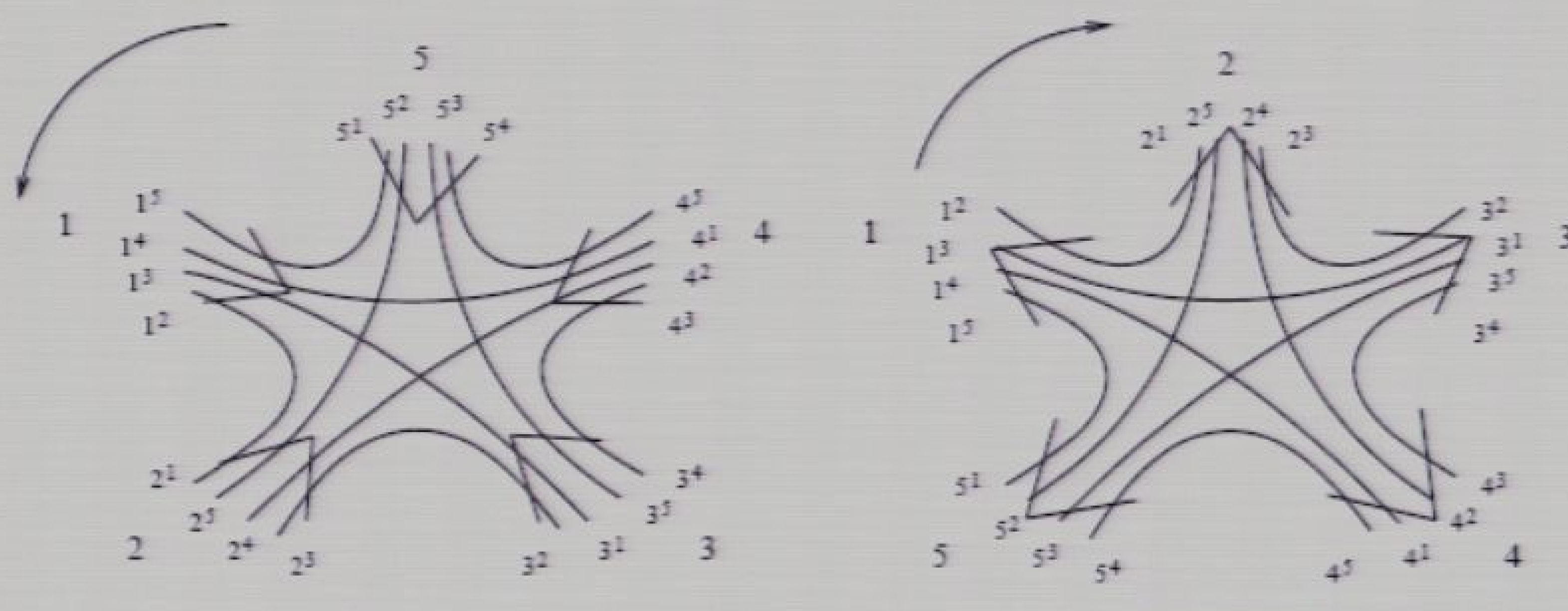


Figure: Vertices: Pentachores I (ϕ^5) and II ($\bar{\phi}^5$).

Graph properties

(i) A N-point graph with n internal vertices: if one color is missing on the external legs, then (a) n is a even number; (b) N is also even and external legs have colors which appear in pair;

(ii) There is no odd $N < 5$-point function in the colored Ooguri model.

(ii) a face (or closed cycle) is bi-colored with an even number of lines;

(iii) a chain (open cycle) of length > 1 is bi-colored.

(iv) $\nexists$ generalized tadpole in the colored (Ooguri) group field model.

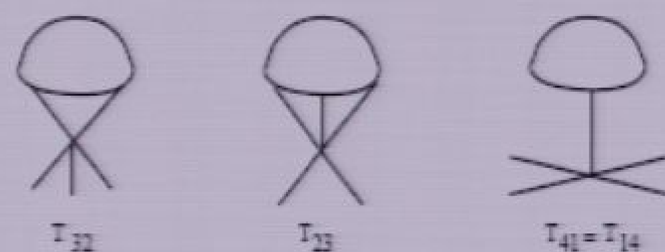


Figure: Generalized tadpoles.

(v) For $\mathcal{G}$ a colored connected two-point graph, $\exists$ an exhausting sequence of cuts for $\mathcal{G}$ (an ordering of vertices such that each vertex can be 'pulled' successively through a 'frontier' from a part $B_{\mathcal{G}}$ to another part $A_{\mathcal{G}}$, without disconnecting these parts.)

Cutoff and main theorem in 4D

- Truncate the Peter-Weyl field expansion as

$$\phi^{\ell}_{1,2,3,4} = \sum_{j_1,j_2,j_3,j_4}^{\Lambda} \mathrm{tr}\left(\Phi^{\ell}_{j_1,j_2,j_3,j_4} D^{j_1}(g_1) D^{j_2}(g_2) D^{j_3}(g_3) D^{j_4}(g_4)\right), \tag{7}$$

- $\delta_\Lambda(h) = \sum_j^\Lambda (2j+1)\,\mathrm{tr}\, D^j(h)$ and diverges as $\sum_j^\Lambda j^2 \sim \Lambda^3$.

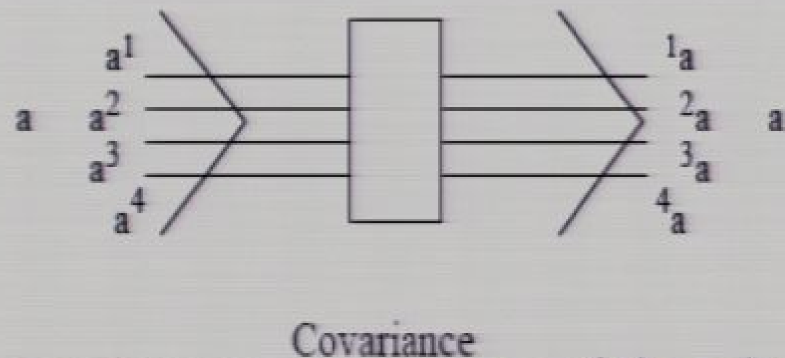
Theorem

There exists a constant K such that for any connected colored vacuum graph $\mathcal{G}$ of the Ooguri model with n internal vertices, we have

$$|\mathcal{A}_{\mathcal{G}}| \leq K^n \Lambda^{9n/2+9}. \tag{8}$$

This bound is optimal in the sense that there exists a graph $\mathcal{G}_n$ with n internal vertices such that $|\mathcal{A}_{\mathcal{G}}| \simeq K^n \Lambda^{9n/2+9}$.

The 4D colored Ooguri model



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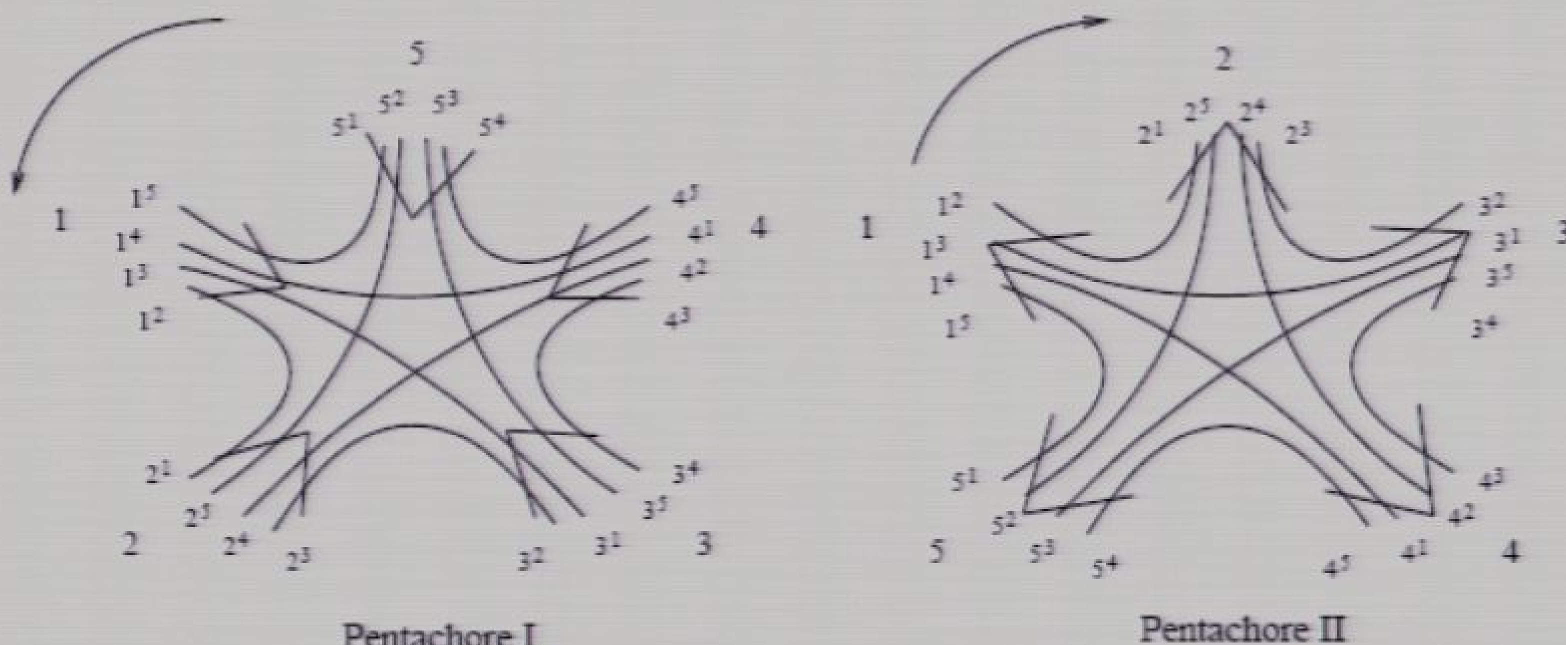

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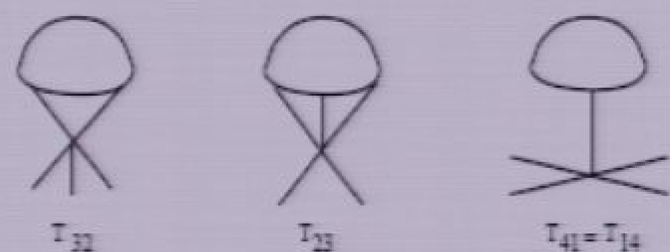

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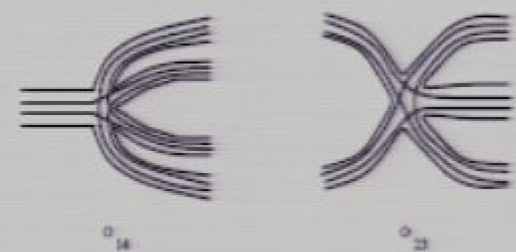
Streamlined proof

- Vertex operators from an exhausting sequence of cuts:



- $O_{14} : \mathcal{H}_0^\Lambda \longrightarrow \mathcal{H}_0^\Lambda \otimes \mathcal{H}_0^\Lambda \otimes \mathcal{H}_0^\Lambda \otimes \mathcal{H}_0^\Lambda$
- $||H|| = \lim_{n\to\infty} \left(\mathrm{tr}[H^\dagger H]^n\right)^{1/2n}$

$$\mathrm{tr}(O_{14}O_{41})^n = \int \prod_{l\in\mathcal{L}_{G_n^{14}}} dh_l \prod_{f\in\mathcal{F}_{G_n^{14}}} \delta_\Lambda\left(\vec{\prod}_{l\in\partial f} h_l\right) \tag{9}$$

G_n^{14} =

$$\begin{aligned} \mathrm{tr}(O_{14}O_{41})^n &\leq \Lambda^{9n+9} \\ ||O_{14}|| &\leq \Lambda^{9/2}. \end{aligned} \tag{10}$$

- Similarly $\mathrm{tr}(O_{23}O_{32})^{2n} \leq \Lambda^{6n+15}$ and $||O_{23}|| \leq \Lambda^{3/2}$. □

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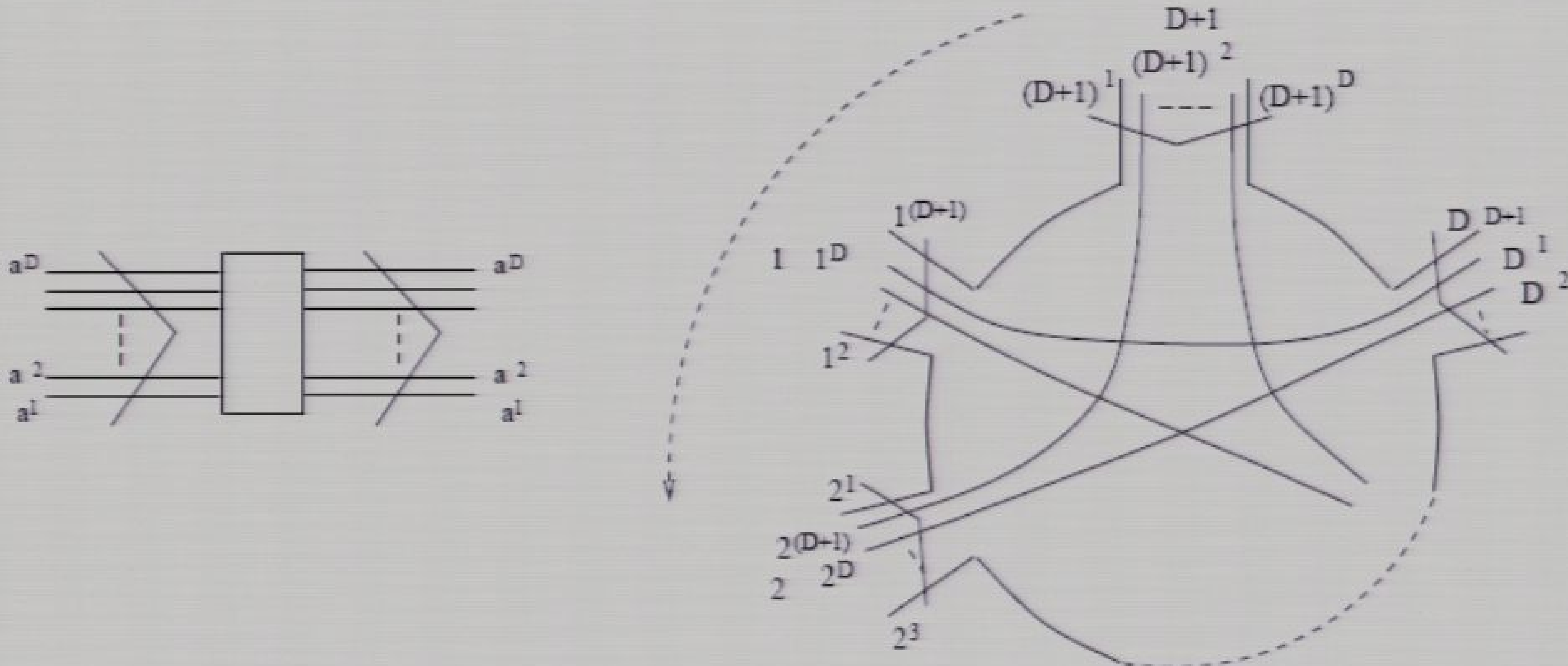
The action

- The fields $\phi^\ell : G^D \to \mathbb{C}$, $\ell = 1, 2, \dots, D+1$.

The Action

$$S^D[\phi] := \int \prod_{i=1}^{D} dg_i \sum_{\ell=1}^{D+1} \bar{\phi}^\ell_{1,2,\dots,D} \; \phi^\ell_{D,\dots,2,1}$$
$$+\lambda_1 \int \prod dg_{ij} \; \phi^1_{1^2,1^3,\dots,1^{(D+1)}} \; \phi^{(D+1)}_{(D+1)^1,(D+1)^2,(D+1)^3,\dots,(D+1)^D} \; \phi^D_{D^{D+1},D^1,D^2,\dots,D^{D-1}}$$
$$\dots\dots \phi^3_{3^4,3^5,\dots,3^{D+1},3^1,3^2} \; \phi^2_{2^3,2^4,\dots,2^{D+1},2^1} \prod_{j\neq i}^{D+1} \delta(g_{ij}(g_{ji})^{-1})$$
$$+\lambda_2 \int \prod dg_{ij} \; \bar{\phi}^1_{1^2,1^3,\dots,1^{(D+1)}} \; \bar{\phi}^{D+1}_{(D+1)^1,(D+1)^2,(D+1)^3,\dots,(D+1)^D} \; \bar{\phi}^D_{D^{D+1},D^1,D^2,\dots,D^{D-1}}$$
$$\dots\dots \bar{\phi}^3_{3^4,3^5,\dots,3^{D+1},3^1,3^2} \; \bar{\phi}^2_{2^3,2^4,\dots,2^{D+1},2^1} \prod_{j\neq i}^{D+1} \delta(g_{ij}(g_{ji})^{-1}) \qquad (11)$$

Propagator and vertex



Propagator Vertex

Figure: Propagator and vertex ϕ^{D+1} in D dimensional GFT.

Results

Theorem

There exists a constant K such that for any connected colored vacuum graph $\mathcal{G}$ of the D dimensional GFT model with n internal vertices, we have

$$|\mathcal{A}_{\mathcal{G}}| \leq K^n \Lambda^{3(D-1)(D-2)n/4+3(D-1)}. \tag{12}$$

This bound is optimal in the sense that there exists a graph $\mathcal{G}_n$ with n internal vertices such that $|\mathcal{A}_{\mathcal{G}}| \simeq K^n \Lambda^{3(D-1)(D-2)n/4+3(D-1)}$.

For $0 < p \leq D$, vertex operator has $D+1-p$ legs in a part A and remaining p legs in a part B, for a (A, B)-cut for $\mathcal{G}$.

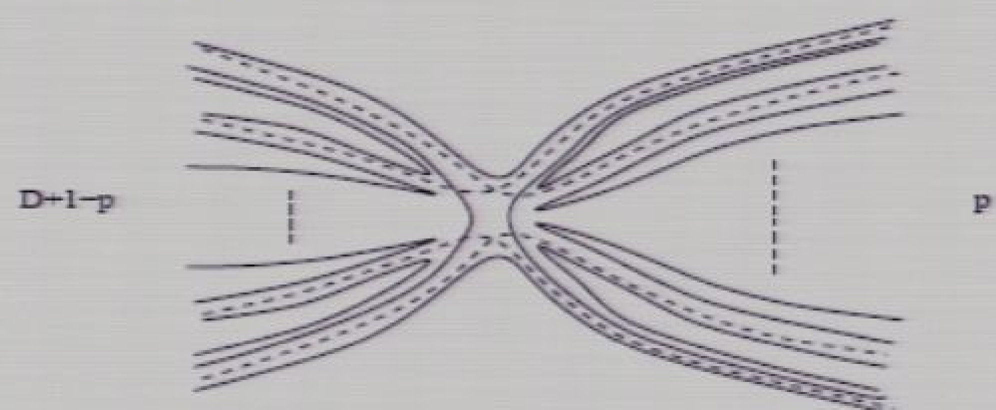


Figure: The operator $O(D+1-p, p)$.

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Re-expressing Z

- Restriction to $4D$ but the result can be shown in general.
- For $\lambda_1 = \lambda_2 = \lambda$

$$H(g_1, g_2, g_3, g_4; g_4', g_3', g_2', g_1') = \delta(g_1(g_1')^{-1}) \int dg_5 dg_6 dg_7 \; \phi^5_{4,2',5,6} \, \phi^4_{6,3,3',7} \, \phi^3_{7,5,2,4'} \;, \quad (13)$$

$$H^*(g_1, g_2, g_3, g_4; g_4', g_3', g_2', g_1') = \delta(g_4(g_4')^{-1}) \int dg_5 dg_6 dg_7 \; \bar{\phi}^5_{1',3,5,6} \, \bar{\phi}^4_{6,2,2',7} \, \bar{\phi}^3_{7,5,3',1} \quad (14)$$

Partition function

$$Z(\lambda) = \int d\mu_C[\bar{\phi}, \phi] \times \exp[-\lambda[\int \prod_{i=1}^{4} dg_i dg_i' \;\; \phi^1_{1,2,3,4} \, H(g_1, g_2, g_3, g_4; g_4', g_3', g_2', g_1') \, \phi^2_{4',3',2',1'} + \bar{\phi}^2_{1,2,3,4} \, H^*(g_1, g_2, g_3, g_4; g_4', g_3', g_2', g_1') \, \bar{\phi}^1_{4',3',2',1'}]]. \quad (15)$$

Integration of two colors and Matthews-Salam determinant

- Introduce $v = (\mathrm{Re}\phi^1, \mathrm{Im}\phi^1, \mathrm{Re}\phi^2, \mathrm{Im}\phi^2)$

$$Z(\lambda) = \int d\mu_C[\bar\phi, \phi]\, e^{-\lambda v^t A v}, \tag{16}$$

$$A = \begin{pmatrix} 0 & 0 & H & iH \\ 0 & 0 & iH & -H \\ H^* & -iH^* & 0 & 0 \\ -iH^* & -H^* & 0 & 0 \end{pmatrix}. \tag{17}$$

- Partition function integrated

Partition function as a MS-determinant

$$\begin{aligned} Z(\lambda) &= \int d\mu'_{C'}[\bar\phi^{3,4,5}, \phi^{3,4,5}]\, K [\det(1+\lambda CA)]^{-1} \\ &= \int d\mu'_{C'}[\bar\phi^{3,4,5}, \phi^{3,4,5}]\, K e^{-\mathrm{tr}\log(1+\lambda CA)}, \end{aligned} \tag{18}$$

- CA composed by CH and CH^*.

Integration of two colors and Matthews-Salam determinant

- CA a sum of two matrices $\mathbb{H} + \mathbb{H}^*$

$$
\begin{aligned}
& e^{-\mathrm{tr}\log(1+\lambda(\mathbb{H}+\mathbb{H}^*))} = e^{+\mathrm{tr}\sum_{n=1}^{\infty}\frac{(-\lambda)^n}{n}(\mathbb{H}+\mathbb{H}^*)^n} \\
& = e^{+\mathrm{tr}\sum_{p=1}^{\infty}\frac{\lambda^{2p}}{2p}([\mathbb{H}+\mathbb{H}^*)^2])^p} = e^{\frac{1}{2}\mathrm{tr}\sum_{p=1}^{\infty}\frac{\lambda^{2p}}{p}(Q)^p} = e^{-\frac{1}{2}\mathrm{tr}\log(1-\lambda^2 Q)}
\end{aligned} \quad (19)
$$

use the fact that $\mathrm{tr}((\mathbb{H} + \mathbb{H}^*)^{2p+1}) = 0$ for all p;
$Q := (\mathbb{H} + \mathbb{H}^*)^2 = \mathbb{H}\mathbb{H}^* + \mathbb{H}^*\mathbb{H}$, since $\mathbb{H}^2 = 0 = (\mathbb{H}^*)^2$.

- Q is Hermitian

$$
Q \quad = \quad \begin{pmatrix} 2\mathcal{H}\mathcal{H}^* & -2i\mathcal{H}\mathcal{H}^* & 0 & 0 \\ 2i\mathcal{H}\mathcal{H}^* & 2\mathcal{H}\mathcal{H}^* & 0 & 0 \\ 0 & 0 & 2\mathcal{H}^*\mathcal{H} & 2i\mathcal{H}^*\mathcal{H} \\ 0 & 0 & -2i\mathcal{H}^*\mathcal{H} & 2\mathcal{H}^*\mathcal{H} \end{pmatrix} . \quad (20)
$$

- Q has positive real eigenvalues and $-\lambda^2 Q$ is positive for $\lambda = ic$, $c \in \mathbb{R}$.

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$Q := (\mathbb{H}+\mathbb{H}^*)^2 = \mathbb{H}\mathbb{H}^* + \mathbb{H}^*\mathbb{H}$, since $\mathbb{H}^2 = 0 = (\mathbb{H}^*)^2$.

- Q is Hermitian

$$Q = \begin{pmatrix} 2\mathcal{H}\mathcal{H}^* & -2i\mathcal{H}\mathcal{H}^* & 0 & 0 \\ 2i\mathcal{H}\mathcal{H}^* & 2\mathcal{H}\mathcal{H}^* & 0 & 0 \\ 0 & 0 & 2\mathcal{H}^*\mathcal{H} & 2i\mathcal{H}^*\mathcal{H} \\ 0 & 0 & -2i\mathcal{H}^*\mathcal{H} & 2\mathcal{H}^*\mathcal{H} \end{pmatrix}. \qquad (20)$$

- Q has positive real eigenvalues and $-\lambda^2 Q$ is positive for $\lambda = ic$, $c \in \mathbb{R}$.

Outline

1. Introduction
2. The 4D colored Ooguri model
3. Colored GFT in dimension D
4. Integration of two colors: a new representation
5. Conclusion

Results...

(i) Perturbative bounds of a general graph of colored GFT have been obtained in any dimension $K^n \Lambda^{3(D-1)(D-2)n/4+3(D-1)}$.

(ii) It may $\exists$ bounds of Feynman amplitudes; allows to know which types of graphs (generalized tadpoles) require a specific attention; basic topological features of colored graphs; good introduction in the renormalization domain.

(iii) The model is positive if the coupling constant is purely imaginary; No need of a regularization procedure, such as an inclusion of a Freidel-Louapre term.

(iv) Loop vertex expansion $\leadsto$ (at least the constant modes of the fields) a convergent series.

.... and Perspectives

(i) Bounds obtained for colored GFT models should now be completed into a more precise power counting and scaling analysis.

(ii) To start this program: • Establish the power counting of a simplified colored Ooguri model with a commutative group. • the "linearized" colored Ooguri model: the power counting is given by a homology formula [work in progress] (this should help for the more complicated study of the "non-linear" models).

(iii) Extension to the physically more interesting models: the so called EPRL-FK model [Engle *et al.* Nucl. Phys. **B799** (2008); Freidel and Krasnov Class. Quant. Grav. **25** (2008)]:
a) to find the group field formulation of the EPRL-FK model $\rightsquigarrow$ linearization; to perform an analysis on this linear model...
b) ... then study the ordinary "nonlinear" models.

Thank you

Colored group field theory: Scaling properties and positivity

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