Title: Deep Inelastic Scattering in Conformal QCD

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Abstract:

Deep Inelastic Scattering in Conformal QCD

Miguel S. Costa

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0911.0043 [hep-th], 0804.1562 [hep-ph], 0801.3002 [hep-th]

Work with L. Cornalba and J. Penedones

Perimeter Institute, December 2009

Motivation

- QCD is approximately conformal at high energies, for very small probes with

$$r_{probe}\Lambda_{QCD} \ll 1$$

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- QCD is approximately conformal at high energies, for very small probes with

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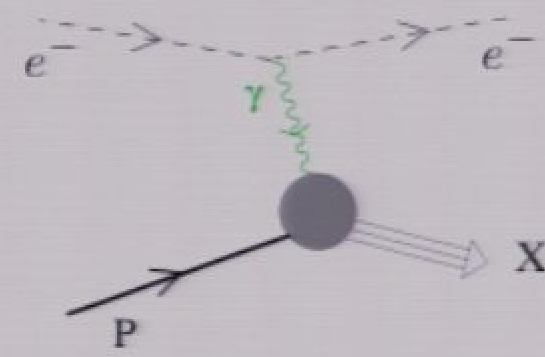
- Regge limit of high center of mass energy with other kinematic invariants fixed ($s \gg -t, \Lambda^2_{QCD}$) corresponds to low Bjorken - x in DIS.

- Use conformal symmetry and AdS/CFT to make general predictions for conformal limit of DIS in QCD.

Deep Inelastic Scattering

Deep Inelastic Scattering

- Electron interacts with proton via exchange of off-shell photon γ



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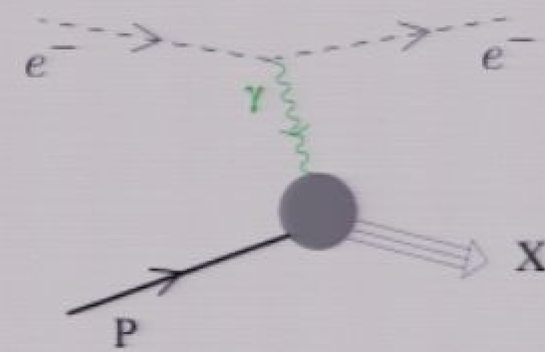
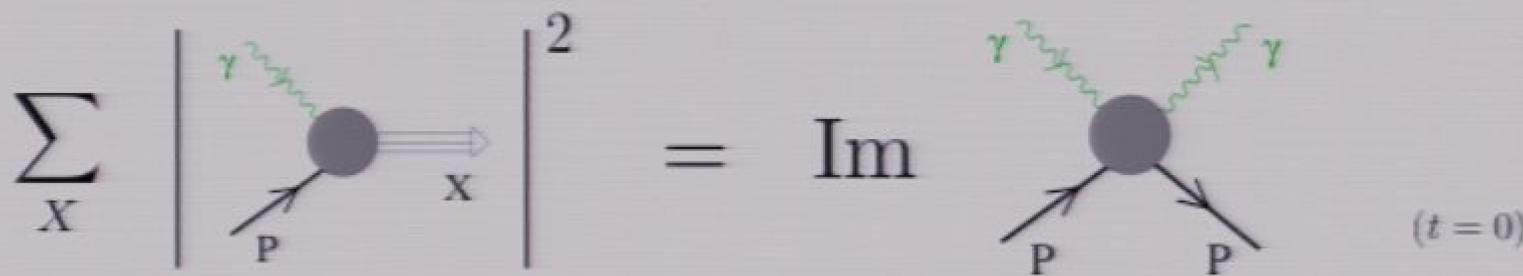
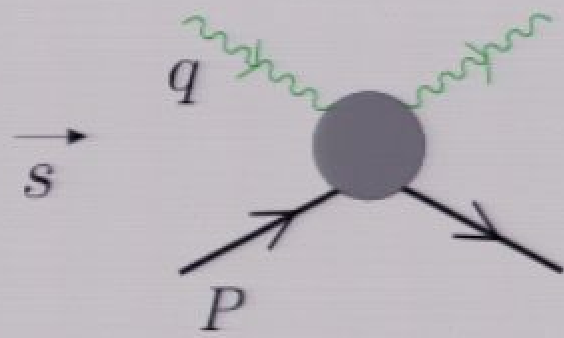


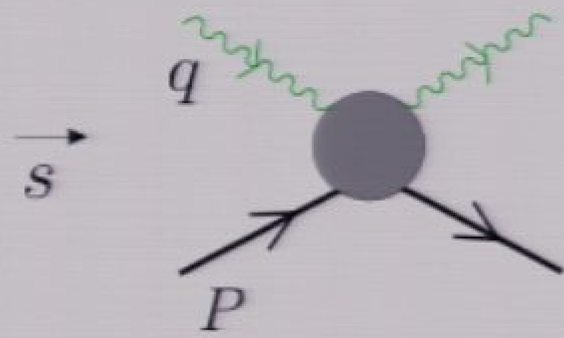
- Optical theorem





$$s = -(q+P)^2$$

$$Q^2 = -q^2$$



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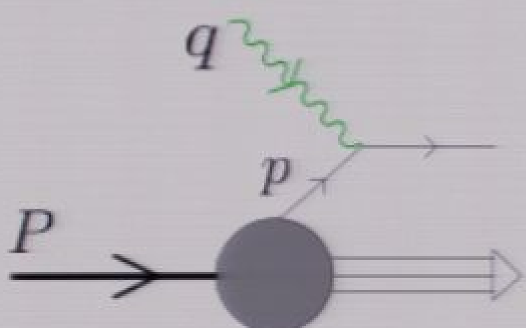
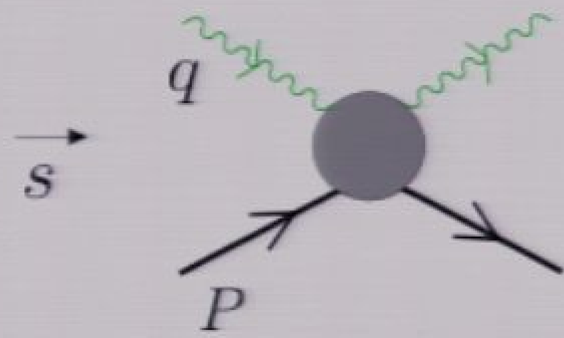
- Bjorken x



$p = xP$ $\qquad s \simeq \frac{Q^2}{x}$



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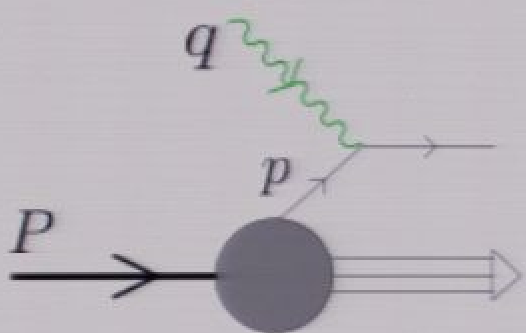
- Bjorken x
- Transverse resolution $1/Q$



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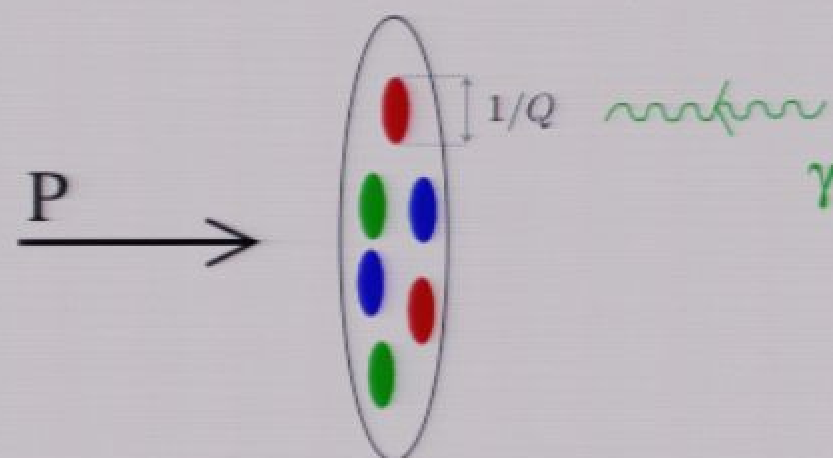

- Hadronic tensor

$$T^{ab} = \int d^4y\, e^{iq\cdot y} \langle P|\, j^a(y_1)\, j^b(y_3)\, |P\rangle$$

$$T^{ab} = \left(\eta^{ab} - \frac{q^a q^b}{q^2}\right) \Pi_1(\mathrm{x}, Q^2) - \frac{2\mathrm{x}}{Q^2} \left(p^a + \frac{q^a}{2\mathrm{x}}\right) \left(p^b + \frac{q^b}{2\mathrm{x}}\right) \Pi_2(\mathrm{x}, Q^2)$$

structure functions

$$F_i\left(\mathrm{x}, Q^2\right) = \frac{1}{2\pi}\, \mathrm{Im}\, \Pi_i$$

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- Simulate proton with scalar operator $\mathcal{O}$ of dimension Δ and confinement with off-shellness $-p^2=\bar{Q}^2\sim\Lambda^2_{QCD}$

$$(2\pi)^d\,\delta\left(\sum k_j\right)T^{ab}(k_j)=\langle j^a(k_1)\,\mathcal{O}(k_2)\,j^b(k_3)\,\mathcal{O}(k_4)\rangle$$

j^a with dimension $\xi=3$

Regge Kinematics in CFTs

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- Consider correlator with EMG current and scalar operators in position space

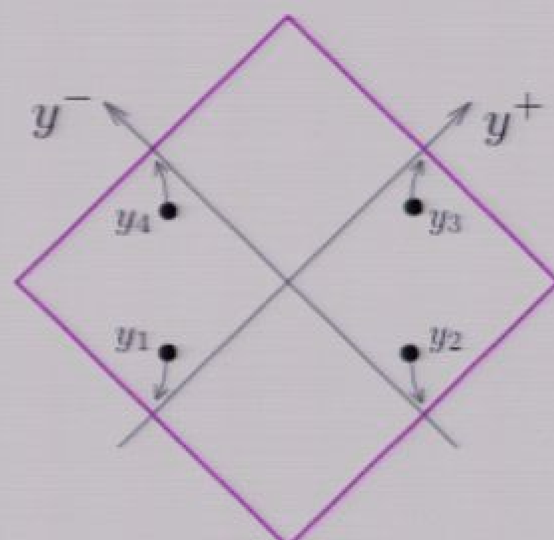
$$A^{ab}(y_i) = \langle j^a(y_1)\,\mathcal{O}(y_2)\,j^b(y_3)\,\mathcal{O}(y_4)\rangle$$

- Regge limit $\quad y = (y^+, y^-, y_\perp)$

$y_1^+ \to -\infty \qquad y_2^- \to -\infty$

$y_3^+ \to +\infty \qquad y_4^- \to +\infty$

$y_i^2,\ y_{i\perp}^2$ fixed



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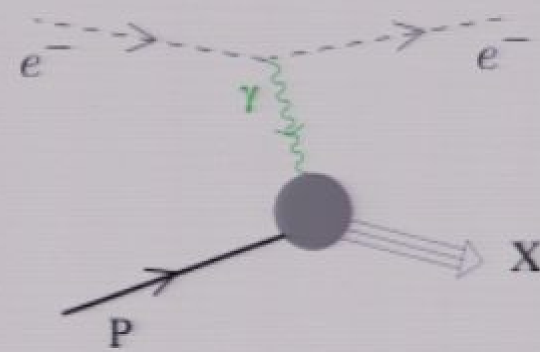

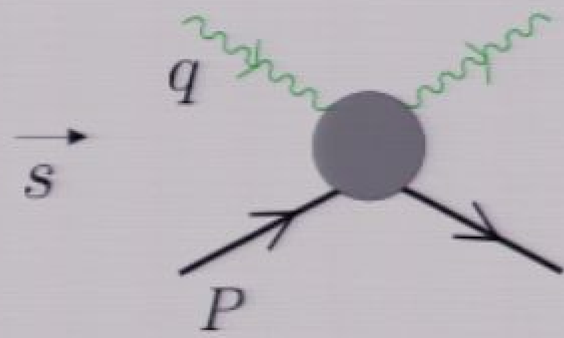

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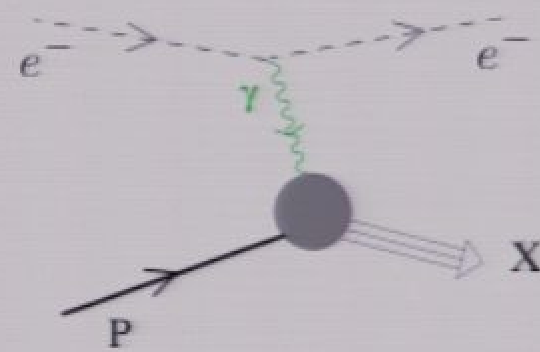

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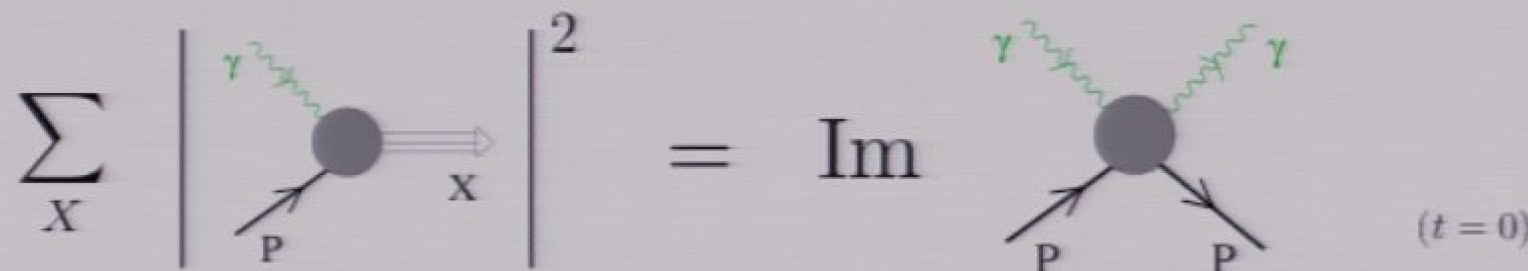

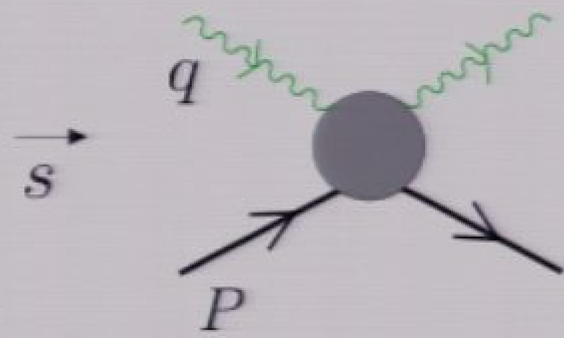

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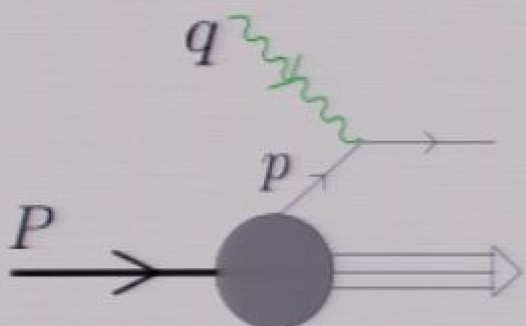
- Bjorken x



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Regge Kinematics in CFTs

- Consider correlator with EMG current and scalar operators in position space

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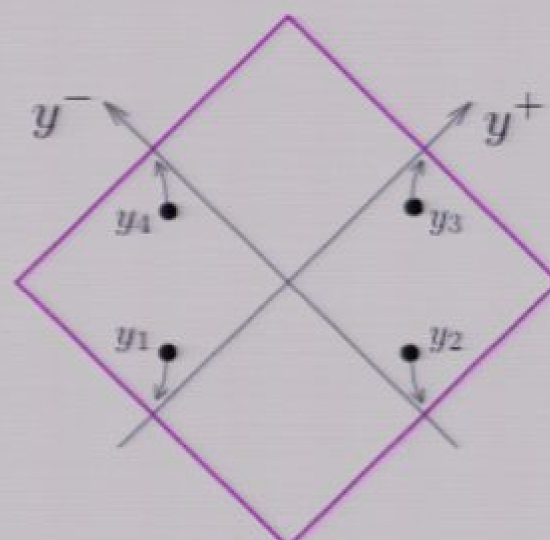
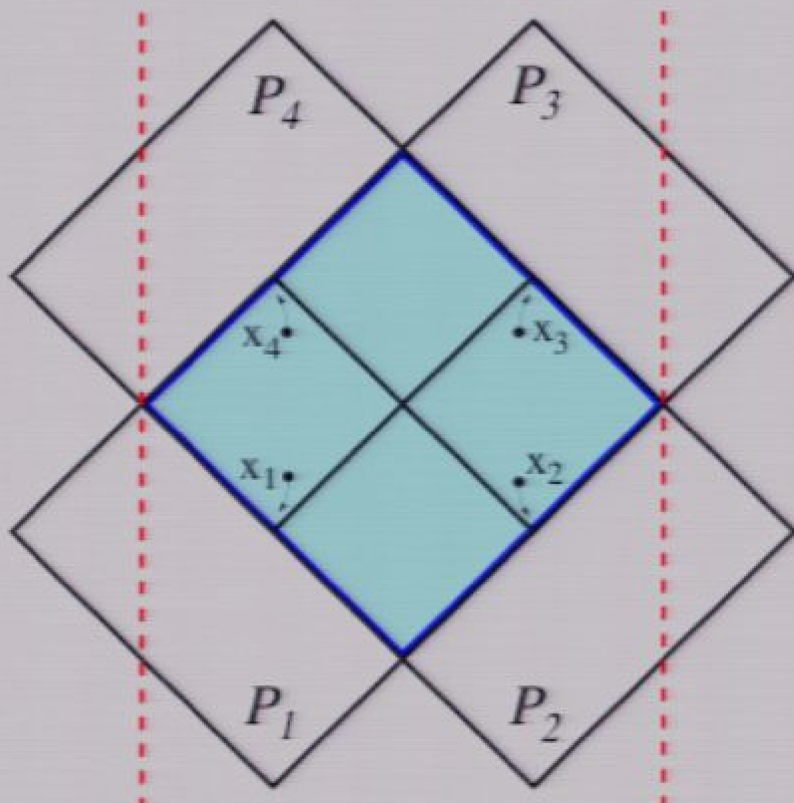
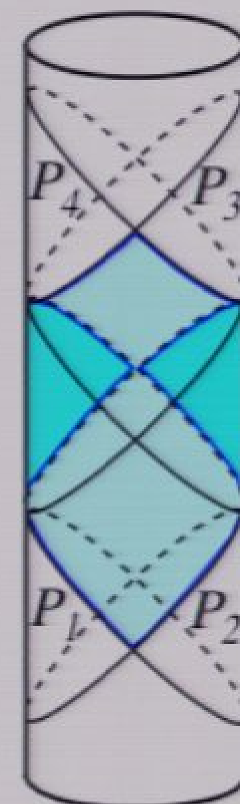
$$y_i^2,\ y_{i\perp}^2 \quad \text{fixed}$$



• Use different Poincaré patches to cover each operator



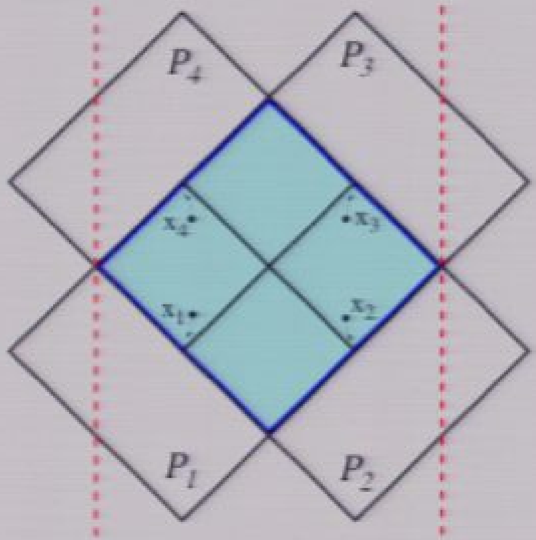


- Conformal transformation for each operator

$$x_i = (x_i^+, x_i^-, x_{i\perp}) = -\frac{1}{y_i^+}(1, y_i^2, y_{i\perp}) \,, \qquad i = 1,3$$

$$x_i = (x_i^+, x_i^-, x_{i\perp}) = -\frac{1}{y_i^-}(1, y_i^2, y_{i\perp}) \,, \qquad i = 2,4$$

$$-dy^+ dy^- + dy_\perp^2 = \frac{1}{(x^+)^2}(-dx^+ dx^- + dx_\perp^2)$$



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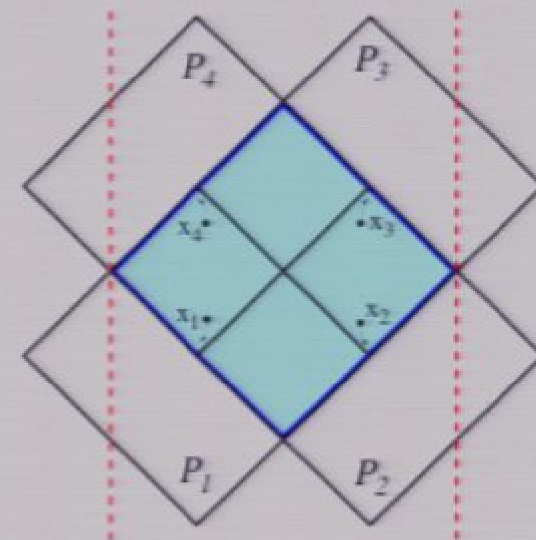


- In CFT Regge limit useful to consider correlator

$$A^{mn}(x_i) = \langle j^m(x_1)\, \mathcal{O}(x_2)\, j^n(x_3)\, \mathcal{O}(x_4)\rangle$$

Regge limit $x_i \to 0$

$$j^a(y) = \left|\frac{\partial x}{\partial y}\right|^{\frac{\xi+1}{d}} \frac{\partial y^a}{\partial x^m}\, j^m(x)$$

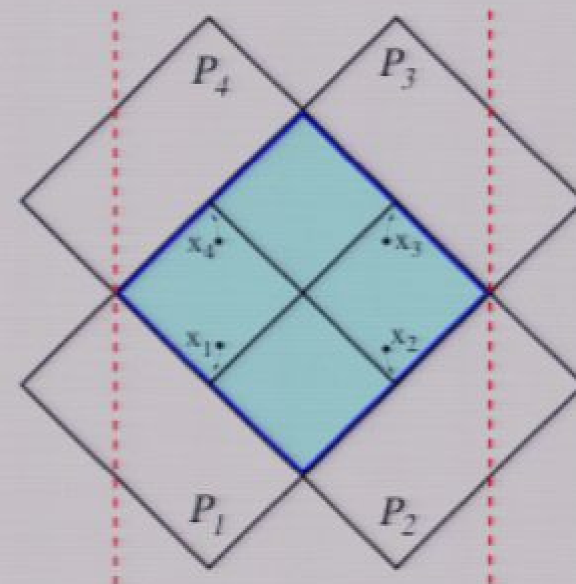
- Cross ratios

Translations in P_1, P_3 or Special Conformal in P_2, P_4

$$x_{1,3} \rightarrow x_{1,3} + a\,, \qquad x_{2,4} \rightarrow x_{2,4} + O(a^2)$$

Translations in P_2, P_4 or Special Conformal in P_1, P_3

$$x_{1,3} \rightarrow x_{1,3} + O(b^2)\,, \qquad x_{2,4} \rightarrow x_{2,4} + b$$



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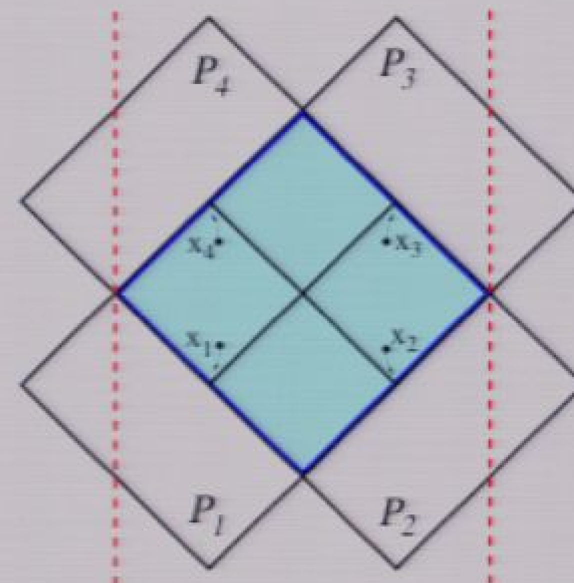
$$x_{1,3} \to x_{1,3} + O(b^2)\,, \qquad x_{2,4} \to x_{2,4} + b$$

$$x \approx x_1 - x_3\,, \qquad \bar{x} \approx x_2 - x_4$$

Lorentz Transformations $x \to \Lambda x\,, \quad \bar{x} \to \Lambda \bar{x}$

Dilatations $x \to \lambda x\,, \quad \bar{x} \to \frac{1}{\lambda}\bar{x}$



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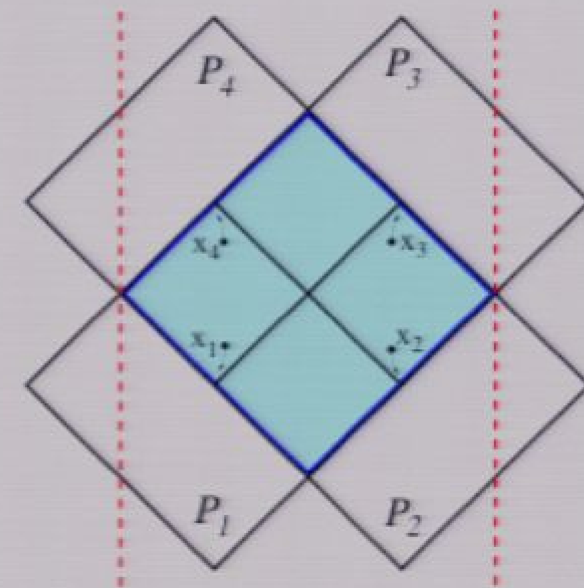
Residual transverse conformal group $SO(1,1) \times SO(3,1)$



- Cross ratios

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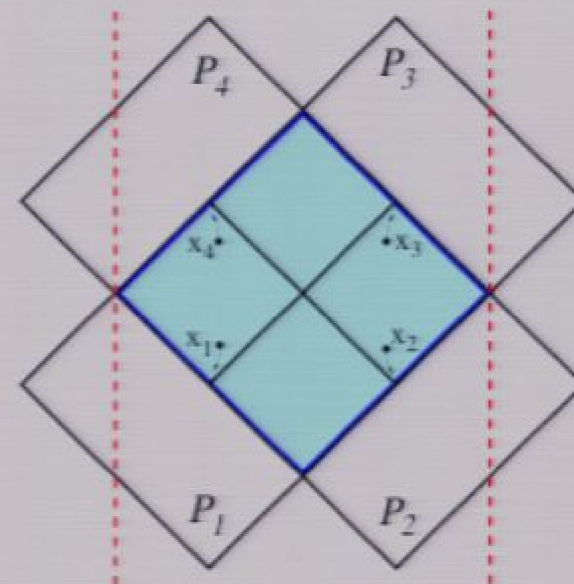


Invariants

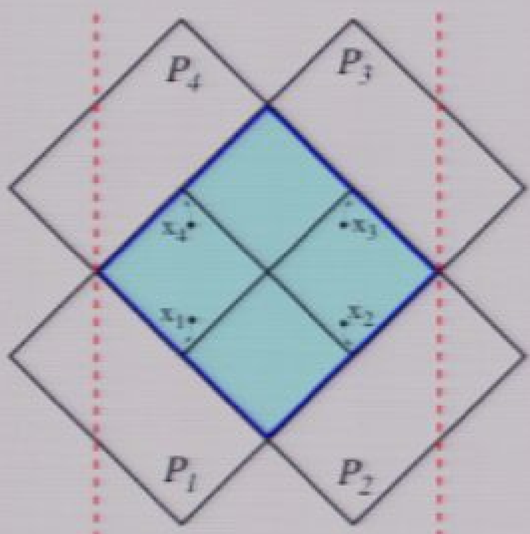
$$\sigma^2 = x^2 \bar{x}^2\,, \qquad \cosh\rho = -\frac{x \cdot \bar{x}}{|x||\bar{x}|}$$

Regge limit $\sigma \to 0$ fixed ρ

Regge Theory in CFTs

- Conformal symmetry restricts $A^{mn}(x_i)$ to have the form

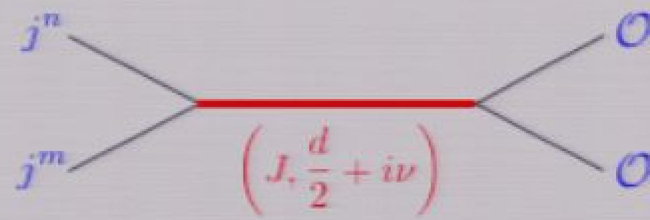
$$A^{mn} = \frac{\mathcal{A}^{mn}(x,\bar{x})}{(x^2 - i\epsilon_x)^{\xi}(\bar{x}^2 - i\epsilon_{\bar{x}})^{\Delta}}\,, \qquad \mathcal{A}^{mn} = \sum_{k=1}^{4} f_k(\sigma,\rho)\, t_k^{mn}(x,\bar{x})$$



$t_k^{mn}(x,\bar{x})$ invariant under scale invariance and $SO(3,1)$ tensors, e.g.

$$t_1^{mn} = \eta^{mn} - 2\,\frac{x^m x^n}{x^2}$$

- Single t-channel conformal partial wave gives in Regge limit



$$\mathcal{A}^{mn} \approx \sigma^{1-J}\left[E(\rho)\,\eta^{mn} + F(\rho)\,\frac{x^m x^n}{x^2} + G(\rho)\,\frac{\bar{x}^m\bar{x}^n}{\bar{x}^2} + H(\rho)\,\frac{x^m\bar{x}^n+\bar{x}^m x^n}{|x||\bar{x}|}\right]$$

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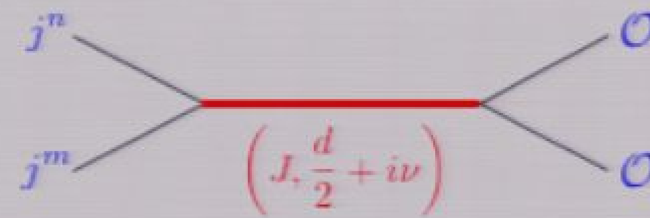

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- Sum over spins and dimensions gives for a Regge pole $j(\nu)$ [Cornalba 07]

$$\mathcal{A}^{mn} \approx 2\pi i \sum_{k=1}^{4} \int d\nu\, \sigma^{1-j(\nu)} \alpha_k(\nu)\, \mathcal{D}_k^{mn}\, \Omega_{i\nu}(\rho)$$

$\Omega_{i\nu}(\rho)$ harmonic functions on H_3

$$\left(\Box_{H_3} + \nu^2 + 1\right)\,\Omega_{i\nu}(\rho) = 0$$

$$\Omega_{i\nu}(\rho) = \frac{\nu}{4\pi^2}\,\frac{\sin \nu\rho}{\sinh \rho}$$

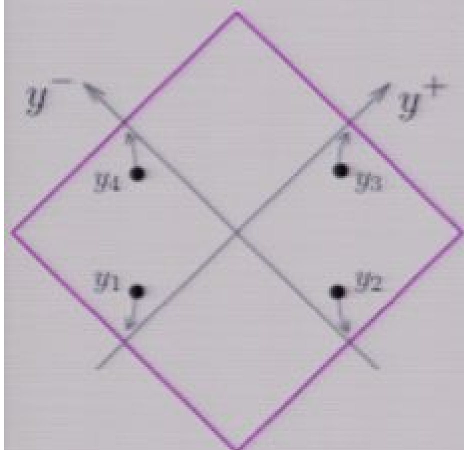
$$\mathcal{D}_1^{mn} = \eta^{mn} - \frac{x^m x^n}{x^2}$$

$$\mathcal{D}_2^{mn} = \frac{x^m x^n}{x^2}$$

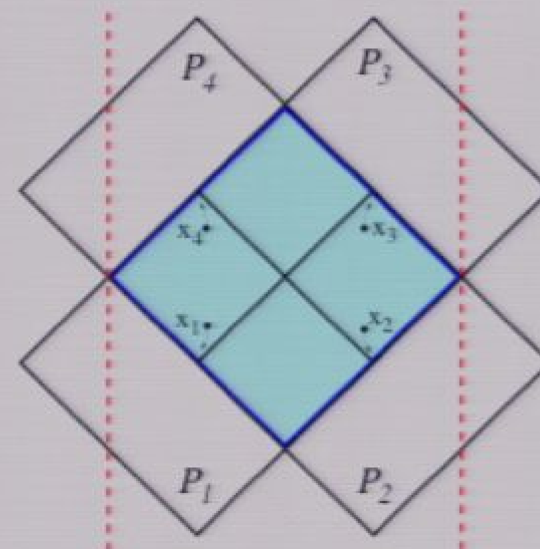
$$\mathcal{D}_3^{mn} = x^m \partial^n + x^n \partial^m$$

$$\mathcal{D}_4^{mn} = x^2 \partial^m \partial^n + (x^m \partial^n + x^n \partial^m) - \frac{1}{3}\left(\eta^{mn} - \frac{x^m x^n}{x^2}\right) x^2 \Box_x$$

Overview

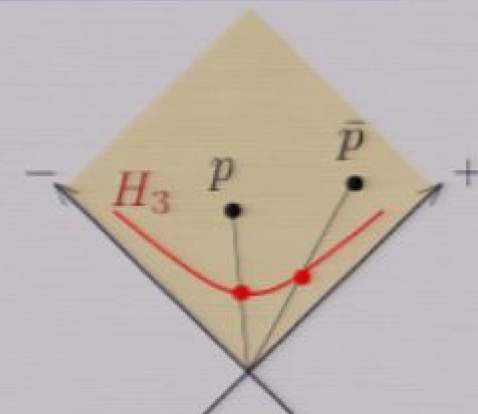


$$\begin{array}{ccc} A^{ab}(y_i) & \xleftrightarrow{\text{C.T.}} & A^{mn}(x,\bar{x}) \\ \updownarrow \text{F.T.} & & \text{F.T.} \updownarrow \\ T^{ab}(k_i) & \longleftrightarrow & B^{mn}(p,\bar{p}) \end{array}$$



AdS Phase-shift

$$A^{mn}(x,\bar{x}) = \int dp\, d\bar{p}\, e^{-2ip\cdot x - 2i\bar{p}\cdot\bar{x}} \frac{\mathcal{B}^{mn}(p,\bar{p})}{(-p^2)^{\frac{d}{2}-\xi}(-\bar{p}^2)^{\frac{d}{2}-\Delta}}$$



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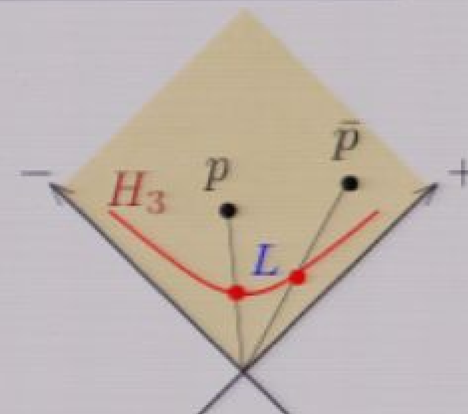


- Current conservation

$p_m \mathcal{B}^{mn} = 0$ ➡ $\mathcal{B}^i_{\ j}$ matrix on H_3 polarization space

$$\mathcal{B}^i_{\ j} \approx 2\pi i \int d\nu\, S^{j(\nu)-1} \left[\beta_1(\nu)\,\delta^i_{\ j} + \beta_4(\nu)\left(\nabla^i\nabla_j - \frac{1}{3}\delta^i_{\ j}\right)\right]\Omega_{i\nu}(L)$$

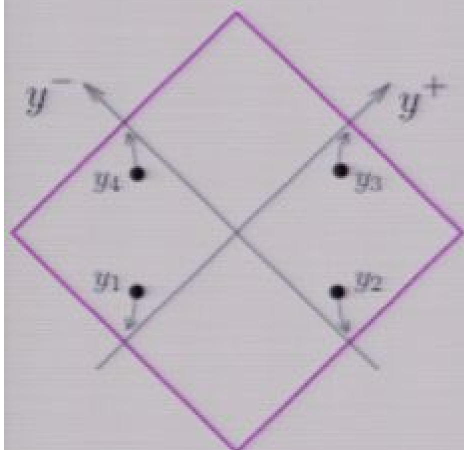
$$S = 4|p||\bar{p}|$$

$$\cosh L = -\frac{p\cdot\bar{p}}{|p||\bar{p}|}$$

- Relation to amplitude in momentum space

$$T^{ab}(k_j) \approx 2s \int dl_\perp e^{iq_\perp \cdot l_\perp} \int \frac{dr}{r^3} \frac{d\bar{r}}{\bar{r}^3} F^{a}_{1\,i}(r)\, F_3^{bj}(r)\, F_2(\bar{r})\, F_4(\bar{r})\, \mathcal{B}^i{}_j(S, L)$$

Overview



$$
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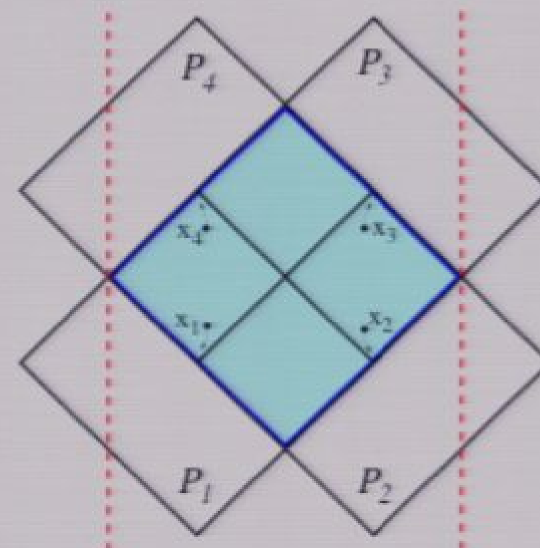
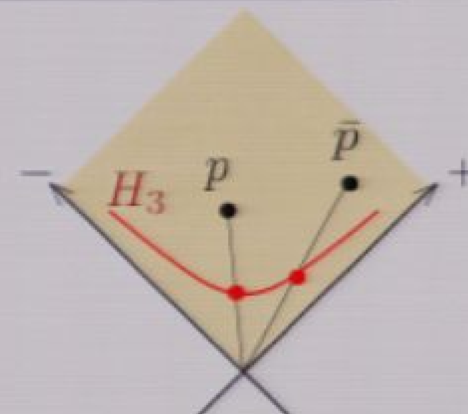


- Where is AdS?

AdS Phase-shift



$$A^{mn}(x,\bar{x}) = \int dp\, d\bar{p}\, e^{-2ip\cdot x - 2i\bar{p}\cdot\bar{x}} \frac{\mathcal{B}^{mn}(p,\bar{p})}{(-p^2)^{\frac{d}{2}-\xi}(-\bar{p}^2)^{\frac{d}{2}-\Delta}}$$

- Current conservation

$p_m\, \mathcal{B}^{mn} = 0$ ➡ $\mathcal{B}^i_{\ j}$ matrix on H_3 polarization space

- Relation to amplitude in momentum space

$$T^{ab}(k_j) \approx 2s \int dl_\perp e^{iq_\perp \cdot l_\perp} \int \frac{dr}{r^3} \frac{d\bar{r}}{\bar{r}^3} F^{a}_{1\,i}(r)\, F^{bj}_{3}(r)\, F_2(\bar{r})\, F_4(\bar{r})\, \mathcal{B}^i{}_j(S, L)$$

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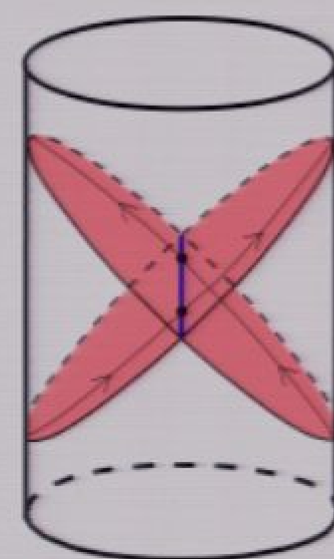

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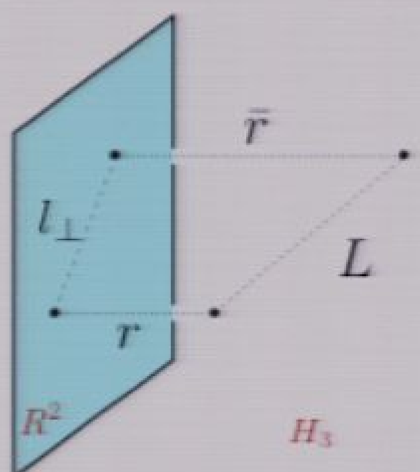
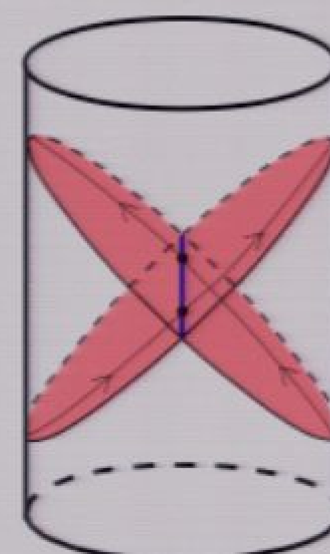


$S = r\bar{r}s$, AdS energy squared

$\cosh L = \dfrac{r^2+\bar{r}^2+l_\perp^2}{2r\bar{r}}$, impact parameter



- Relation to amplitude in momentum space

$$\left[e^{i\delta(s,l_\perp)}\right]^{ab}$$

AdS scalar wave function

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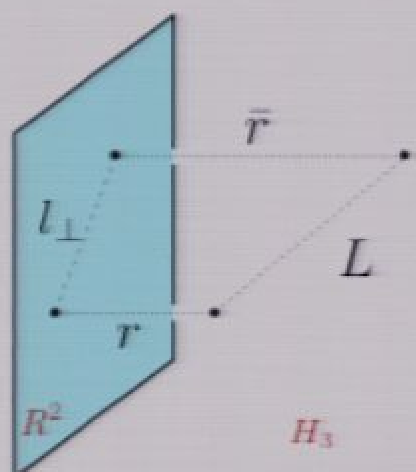
AdS gauge field wave function



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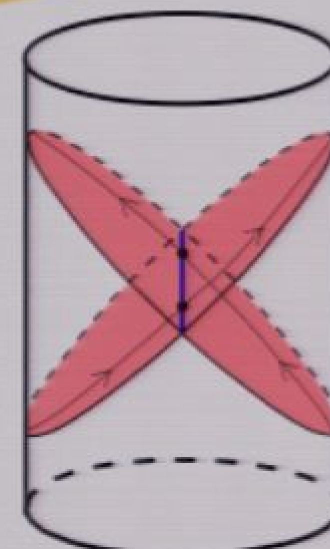
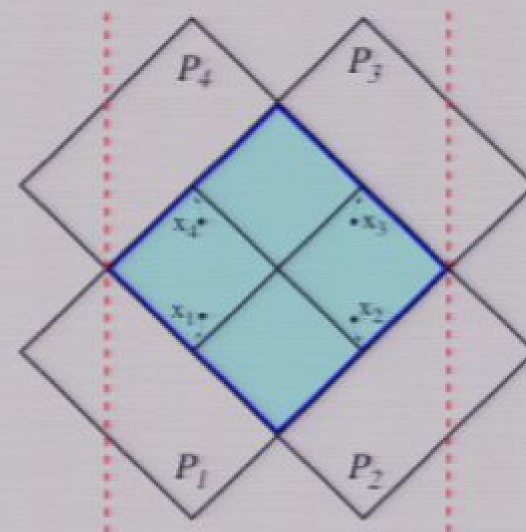


Summary so far



Up to now, results are valid at
any value of the 't Hooft coupling

CFT Regge pole

$$\mathcal{A}^{mn} \approx 2\pi i \sum_{k=1}^{4} \int d\nu \, \sigma^{1-j(\nu)} \alpha_k(\nu) \, \mathcal{D}_k^{mn} \, \Omega_{i\nu}(\rho)$$

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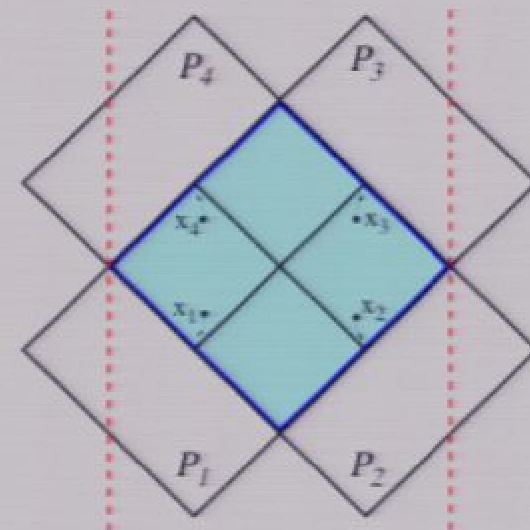

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- CFT impact parameter representation (AdS phase shift)
- Can also derive Regge representation of structure functions

Graviton/Pomeron Regge trajectory [Brower, Polchinski, Strassler, Tan 06]

- Trajectory depends on t'Hooft coupling $j = j(\nu, \bar{\alpha_s})$ $\alpha_k = \alpha_k(\nu, \bar{\alpha_s})$

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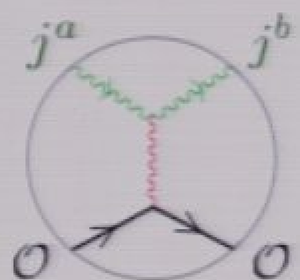
- Strong coupling
(N=4 graviton trajectory)

$$j \approx 2 - \frac{4+\nu^2}{4\pi\sqrt{\bar{\alpha}_s}}$$

α_k real



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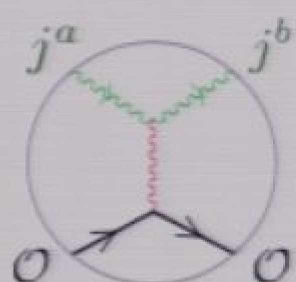

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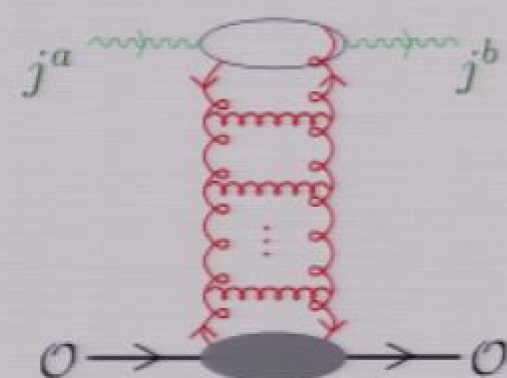

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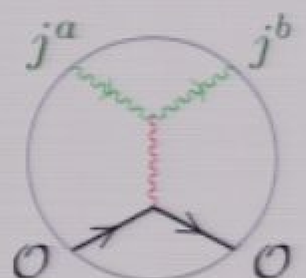

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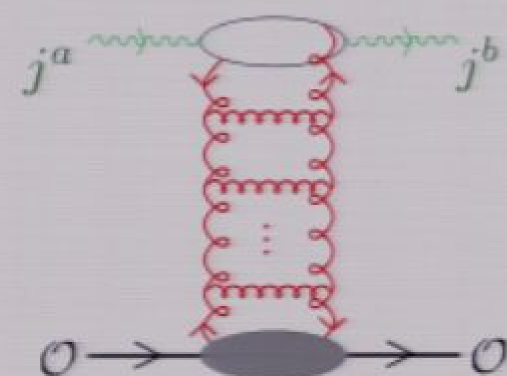

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Hard Pomeron & Transverse conformal symmetry [Balitsky, Fadin, Kuraev, Lipatov]

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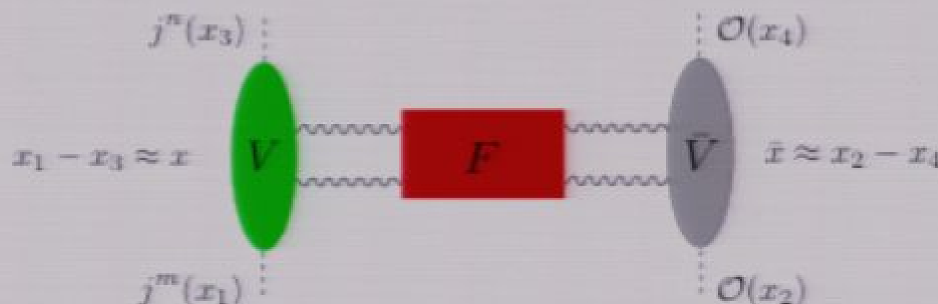

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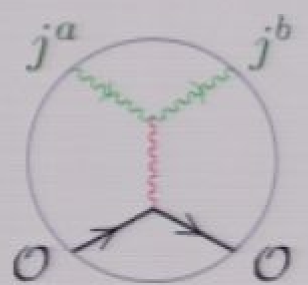

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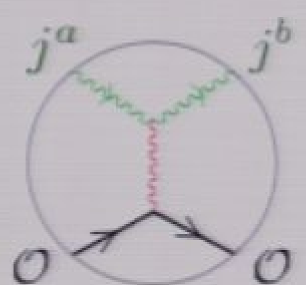

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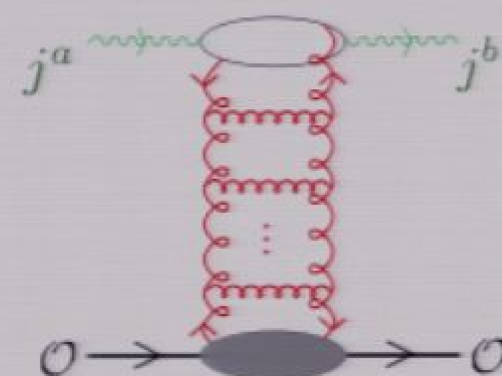

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Hard Pomeron & Transverse conformal symmetry [Balitsky, Fadin, Kuraev, Lipatov]

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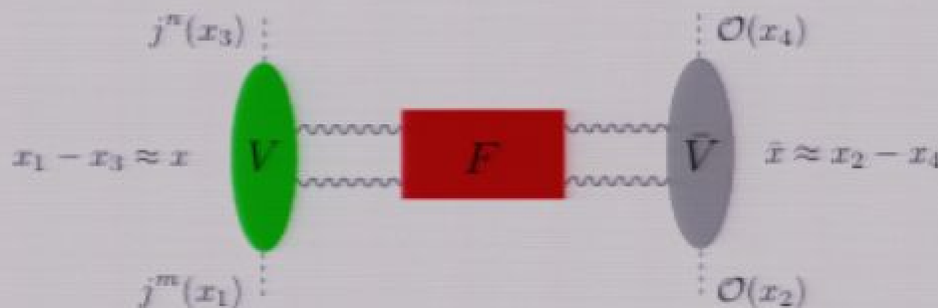

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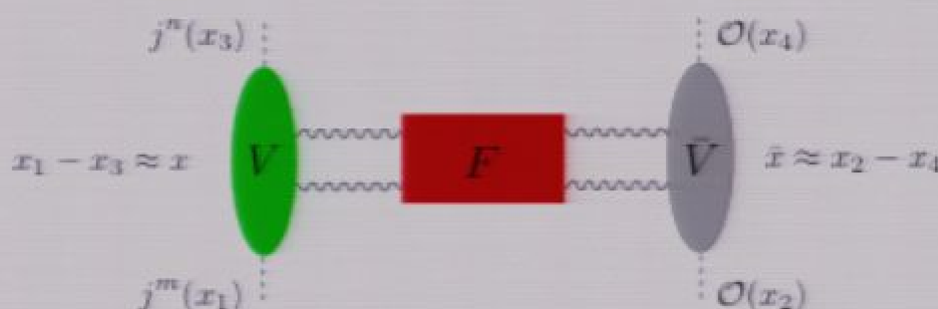

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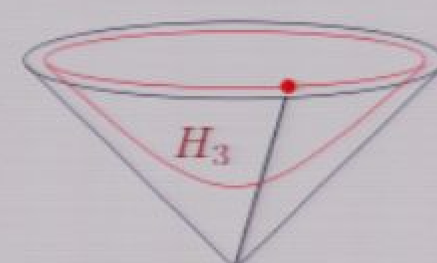
- Transverse conformal symmetry SO(3,1) manifest



$$z^m = (z^+, z^-, z_\perp) = (1, z_\perp^2, z_\perp) \qquad z \sim \lambda z$$

$$z_{ij} = -2 z_i \cdot z_j = (z_{i\perp} - z_{j\perp})^2 \qquad z^2 = 0$$

BFKL propagator

- $F(z_1, z_3, z_2, z_4)$ transforms as 4pt-function of scalar primaries of zero dimension.

For two gluons

$$F(z_1, z_3, z_2, z_4) = \ln \frac{z_{13} z_{24}}{z_{12} z_{34}} \ln \frac{z_{14} z_{23}}{z_{12} z_{34}}$$

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[Lipatov 86]

- Decomposition in transverse conformal partial waves of spin n and dimension $E = 1 + (n \pm 1)$

$$F(z_1, z_2, z_3, z_4) = \frac{4}{\pi^2} \sum_{n=0}^{\infty} 2^n \int d\nu \, \frac{\nu^2 + n^2}{(\nu^2 + (n-1)^2)(\nu^2 + (n+1)^2)} \times$$

$$\int dz_7$$

(spin,dimension)

Impact factors

- Conformal symmetry $\Rightarrow V^{mn}(x, z_1, z_3)$ homogeneous of weight 0

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- A single conformal invariant cross ratio

$$u = \frac{(-x^2) z_{13}}{(-2x \cdot z_1)(-2x \cdot z_3)}$$

- Five possible tensor structures

$$\begin{aligned}
\mathcal{I}_1^{mn} &= \eta^{mn} \\
\mathcal{I}_2^{mn} &= \frac{x^m x^n}{x^2} \\
\mathcal{I}_3^{mn} &= \frac{x^m z_1^n + x^n z_1^m}{-2x \cdot z_1} + \frac{x^m z_3^n + x^n z_3^m}{-2x \cdot z_3} \\
\mathcal{I}_4^{mn} &= \frac{z_1^m z_1^n (-x^2)}{(-2x \cdot z_1)^2} + \frac{z_3^m z_3^n (-x^2)}{(-2x \cdot z_3)^2} \\
\mathcal{I}_5^{mn} &= \frac{z_1^m z_3^n + z_3^m z_1^n}{z_{13}}
\end{aligned}$$

- Basis with definite transverse spin (n=0 and n=2)

$$V^{mn}(x, z_1, z_3) = \int d\mu \left[T(\mu)\, \psi_\mu^{mn}(u) + \sum_{k=1}^{4} S_k(\mu)\, \mathcal{D}_k^{mn} \chi_\mu(u) \right]$$

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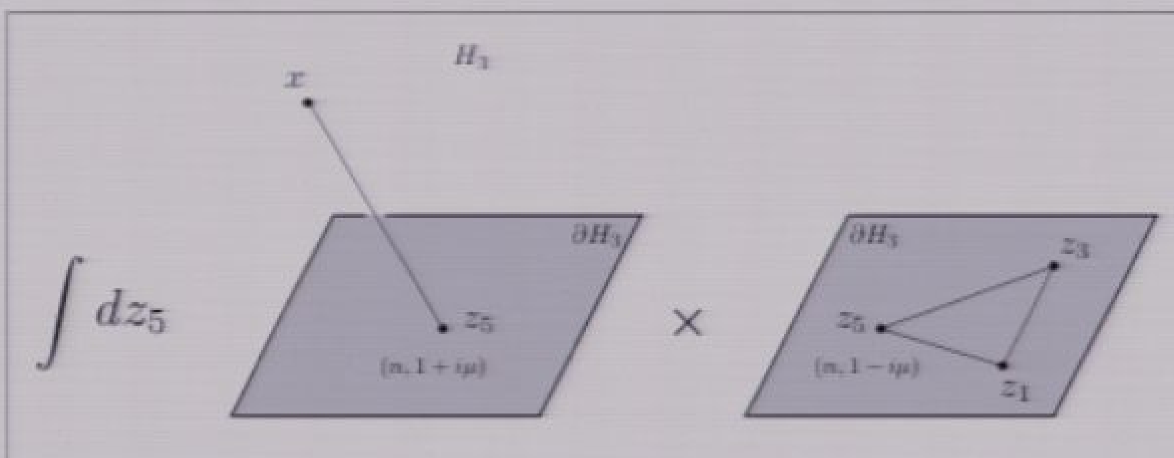
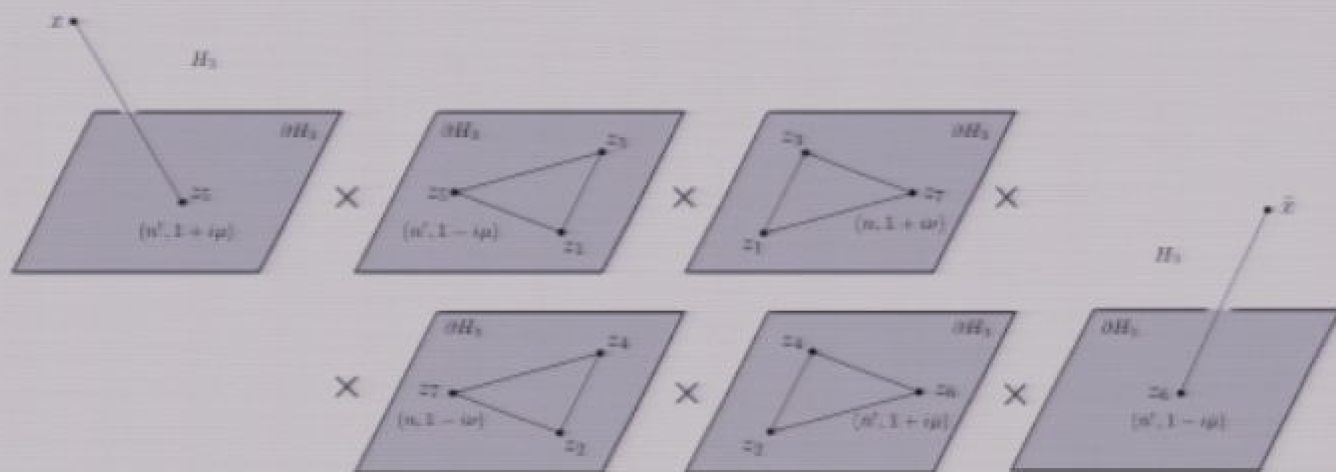


$$= \begin{cases} \chi_\mu(u)\,, & n=0 \\ \psi^{mn}_\mu(u)\,, & n=2 \end{cases}$$

Back to Regge theory

$$\int \prod_{i=1}^{7} dz_i$$



$$\bar{n}' = 0 \quad \Rightarrow \quad n = 0 \quad \Rightarrow \quad n' = 0$$

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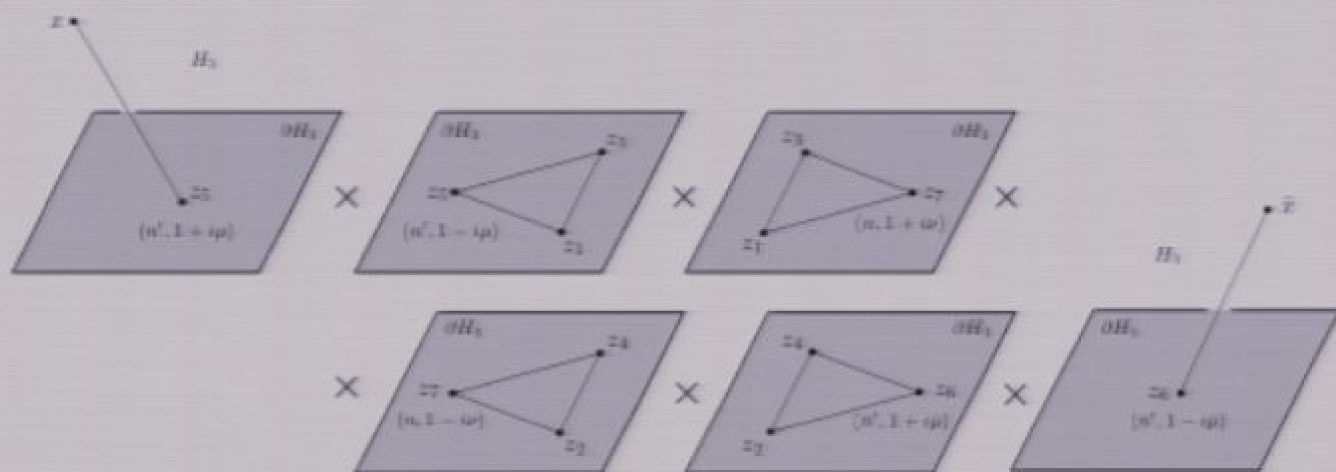


Obtain general Regge form

$\bar{n'} = 0 \Rightarrow n = 0 \Rightarrow n' = 0$

$$\mathcal{A}^{mn} \approx 2\pi i \sum_{k=1}^{4} \int d\nu \, \sigma^{1-j(\nu)} \alpha_k(\nu) \, \mathcal{D}_k^{mn} \, \Omega_{i\nu}(\rho)$$

$$j(\nu) = 1 + \bar{\alpha}_s \, \chi(\nu) + \cdots$$

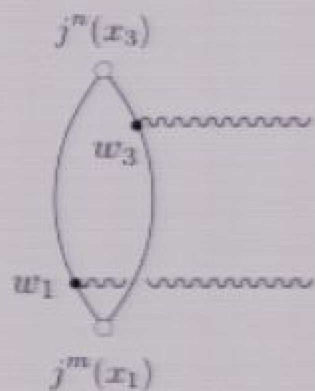
$$\alpha_k(\nu) = S_k(\nu) \, \frac{\tanh \frac{\pi\nu}{2}}{\nu} \, \bar{V}(\nu)$$

transverse spin n=2 drops out

Impact factor in QCD (massless quark)

$j^m = \bar{\psi}\gamma^m\psi$

$x_1 - x_3 \approx x$



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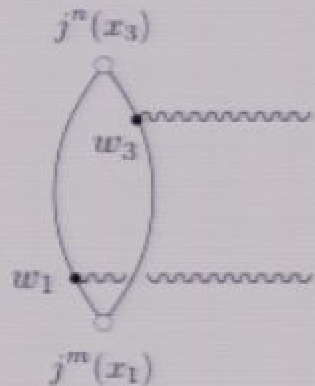

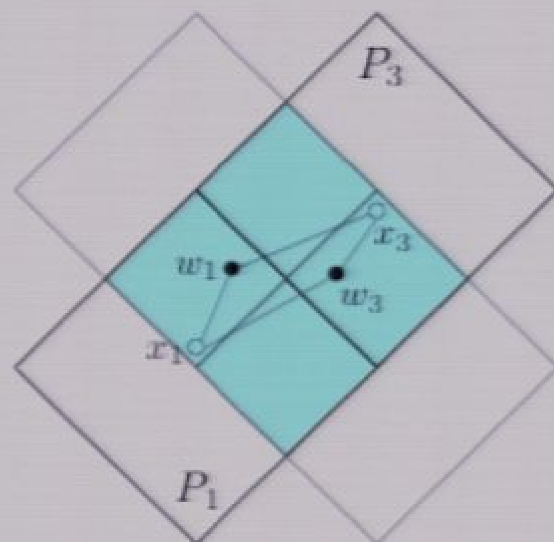

parameterize w_i with

$$w_i = \sigma_i z_i + \lambda_i n$$

$$z_i = (1, z_{i\perp}^2, z_{i\perp})$$

$$n = (0, 1, 0)$$

Simple result

$$V^{mn} = \frac{2}{\pi}\bar{\alpha_s}\, u^3 \left(\eta^{mn} + 2\,\frac{z_1^m z_3^n + z_1^n z_3^m}{z_{13}}\right)$$

$$u = \frac{(-x^2)z_{13}}{(-2x\cdot z_1)(-2x\cdot z_3)}$$

Transverse spin 0 and spin 2 - a conjecture for N=4 SYM

$$V^{mn}(x, z_1, z_3) = a \int \frac{d\mu}{\cosh\frac{\pi\mu}{2}} \left[t(\mu)\, \psi_\mu^{mn}(u) + \sum_{k=1}^{4} s_k(\mu)\, \mathcal{D}_k^{mn} \chi_\mu(u) \right]$$

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Weyl fermion

$$\begin{aligned} s_1 &= \frac{\mu^2+25}{48} \\ s_2 &= \frac{\mu^2-7}{16} \\ s_3 &= \frac{1}{4} \\ s_4 &= -\frac{1}{32}\frac{\mu^2+1}{\mu^2+4} \\ t &= -\frac{(1+\mu^2)^2(9+\mu^2)}{256(4+\mu^2)} \end{aligned}$$

Complex scalar

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R-current spin 2 component vanishes

Saturation

$$\sigma = 2 \int d^2 l_\perp \, \mathrm{Re}\left(1 - e^{2i\delta(s, l_\perp)}\right)$$

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Showed in conformal limit

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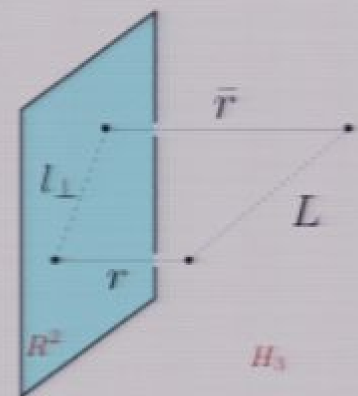
$r \sim 1/Q \qquad r \sim 1/\bar{Q} \qquad S \sim \frac{Q}{x}$



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$$e^{2i\delta(s,l_\perp)} = \int \frac{dr}{r^3}\frac{d\bar{r}}{\bar{r}^3}\, F_1(r)\,F_3(r)\; F_2(\bar{r})\,F_4(\bar{r})\; e^{2i\Delta(S,L)}$$

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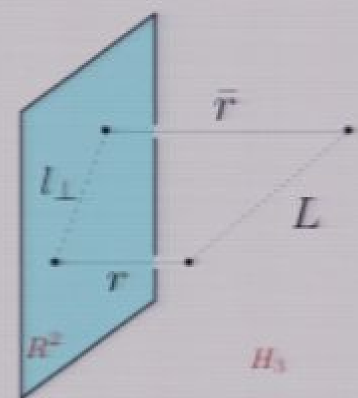


$S = r\bar{r}s$

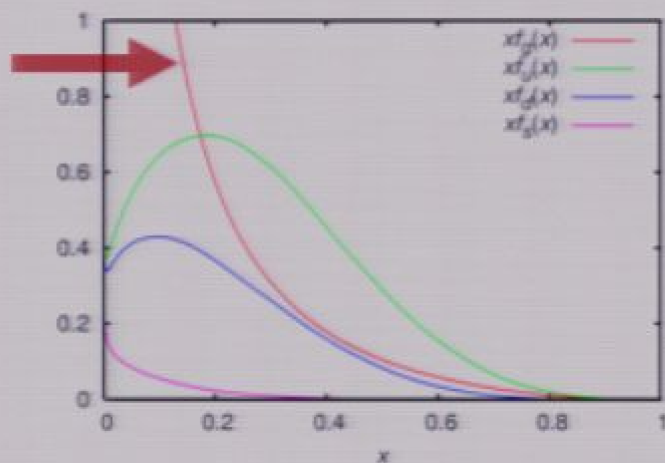
$\cosh L = \frac{r^2 + \bar{r}^2 + l_\perp^2}{2r\bar{r}}$

Write as integral in L

$$\sigma(x,Q^2) \sim \frac{1}{Q\bar{Q}}\int_{|\ln\frac{Q}{\bar{Q}}|}^{\infty} dL\,\sinh L\,\mathrm{Re}\left(1 - e^{2i\Delta(S,L)}\right)$$

- Cross section grows for small x

Gluons dominate at small x



$$F_2(x, Q^2) \sim Q^2 \sigma(x, Q^2)$$

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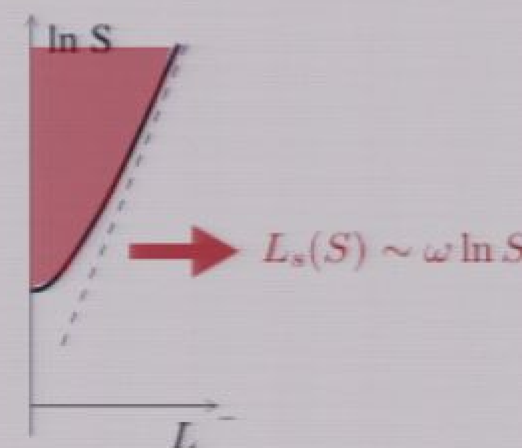
$$S \sim \frac{Q}{x}$$

- Saturation starts for when $\operatorname{Im}\Delta(S, L) \sim 1$. Can understand growth from single pomeron exchange. Saturation line

$$L_s(S) \sim \omega \ln S$$

From DIS geometric scaling $\omega \simeq 0.14$



Deep Saturation

$\mathrm{Im}\,\Delta(S,L) \gg 1$ → Black disk in AdS (or in conformal QCD)

L

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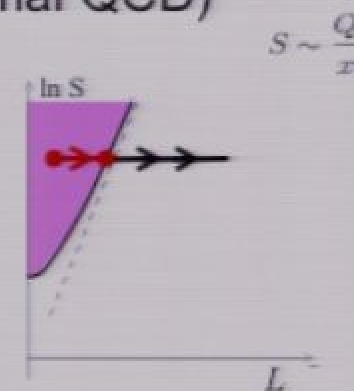
$\mathrm{Im}\,\Delta(S,L) \gg 1$ → Black disk in AdS (or in conformal QCD)

$$\sigma(x,Q^2) \sim \frac{1}{Q\bar{Q}} \int_{|\ln \frac{Q}{\bar{Q}}|}^{L_s} dL\, \sinh L \cdot 1$$

Black disk



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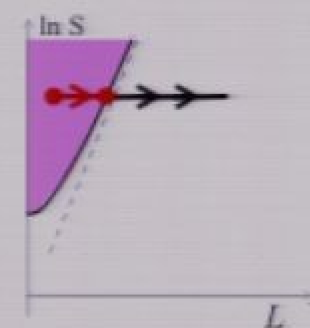
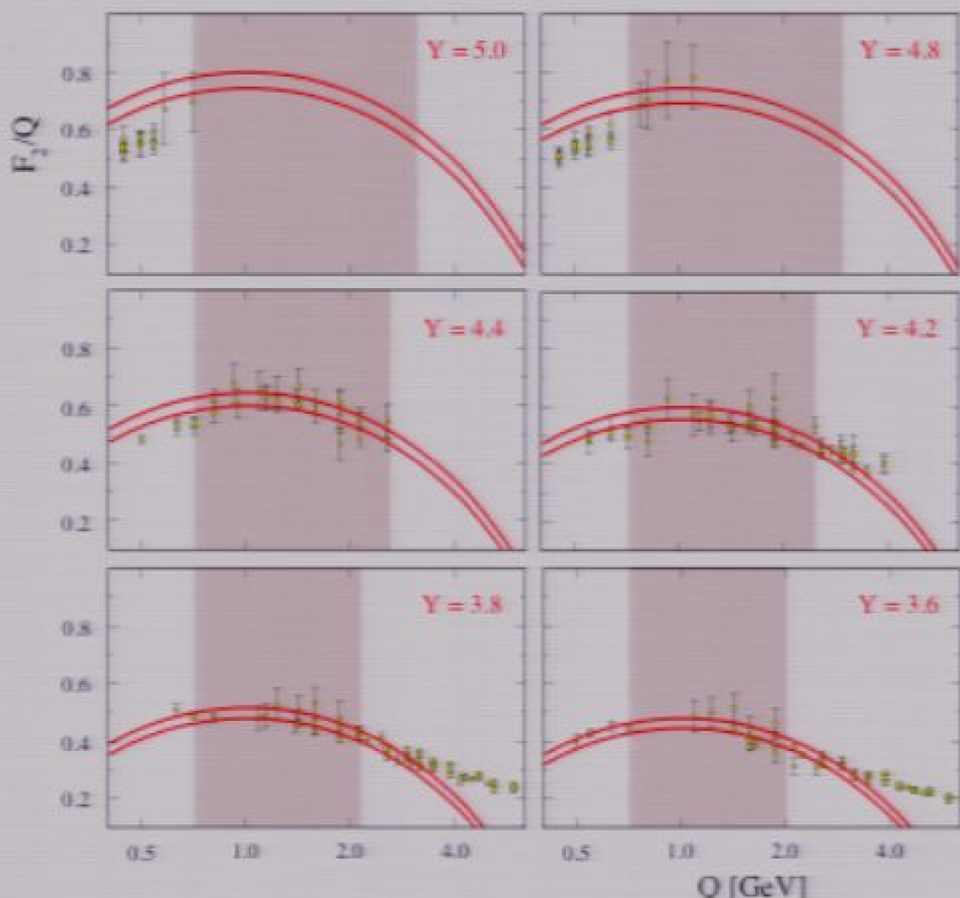
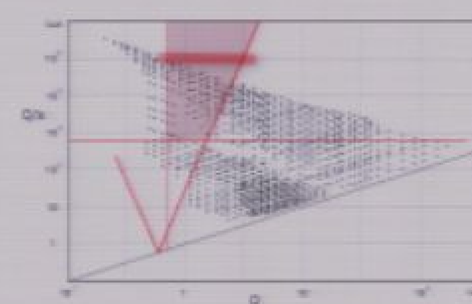


- It is all AdS (or CFT) kinematics. Only dynamical information is the on-set of black disk region.

Compare with Data



$$Y = \ln\left(\frac{Q\ [\mathrm{GeV}]}{x}\right)$$



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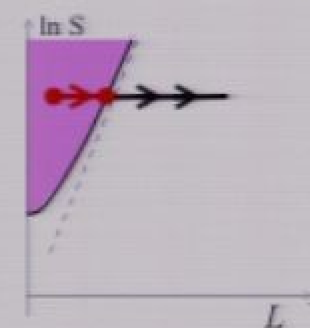

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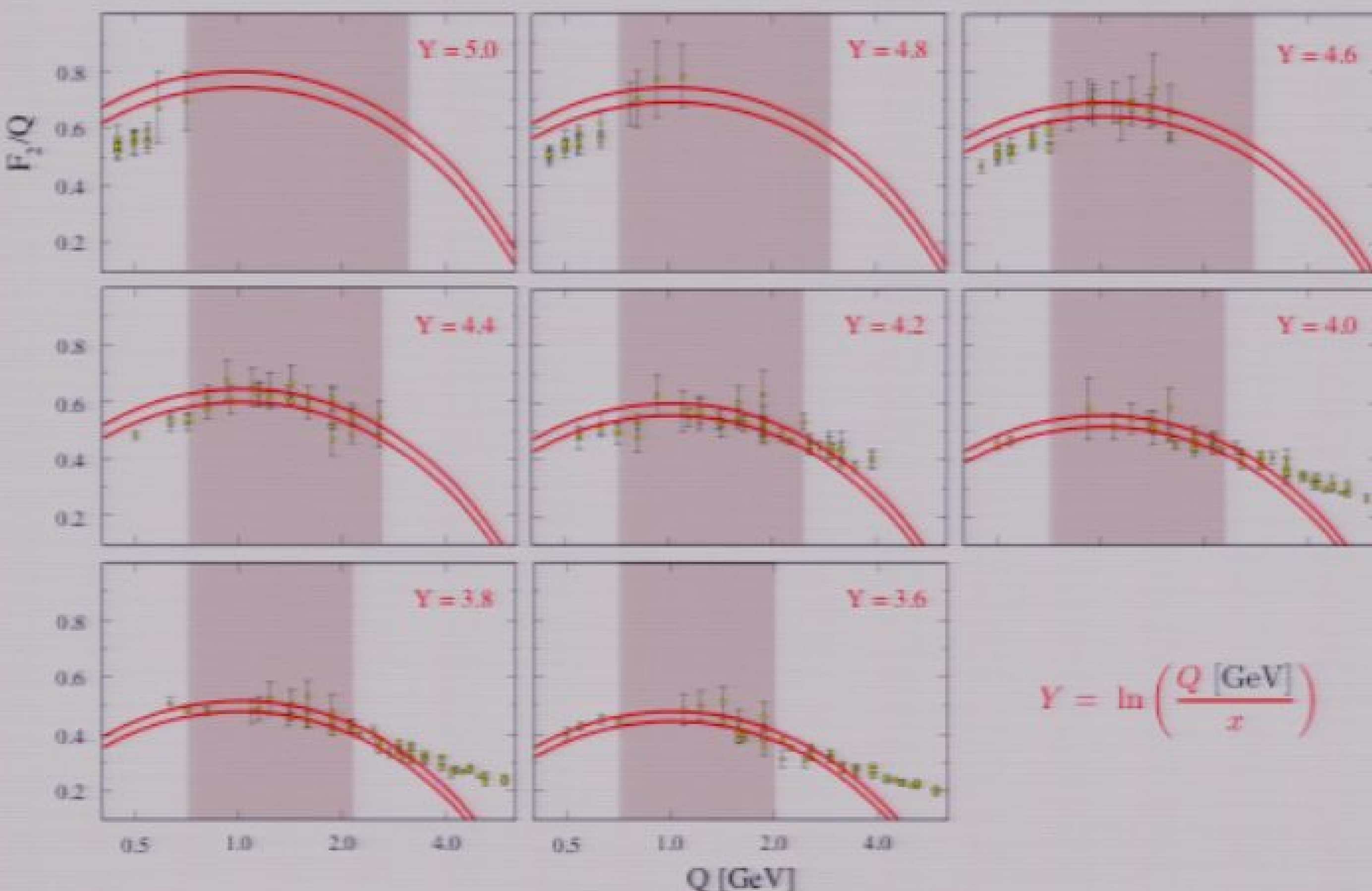

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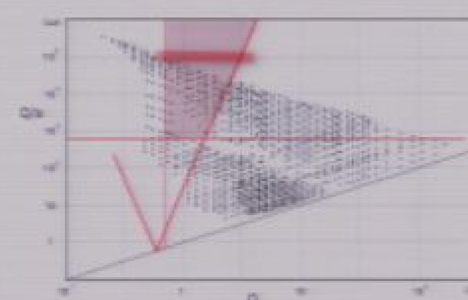

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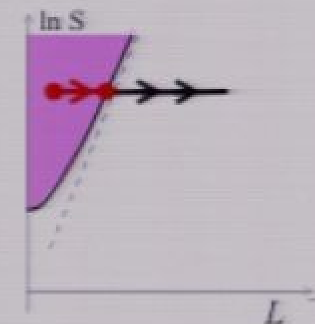

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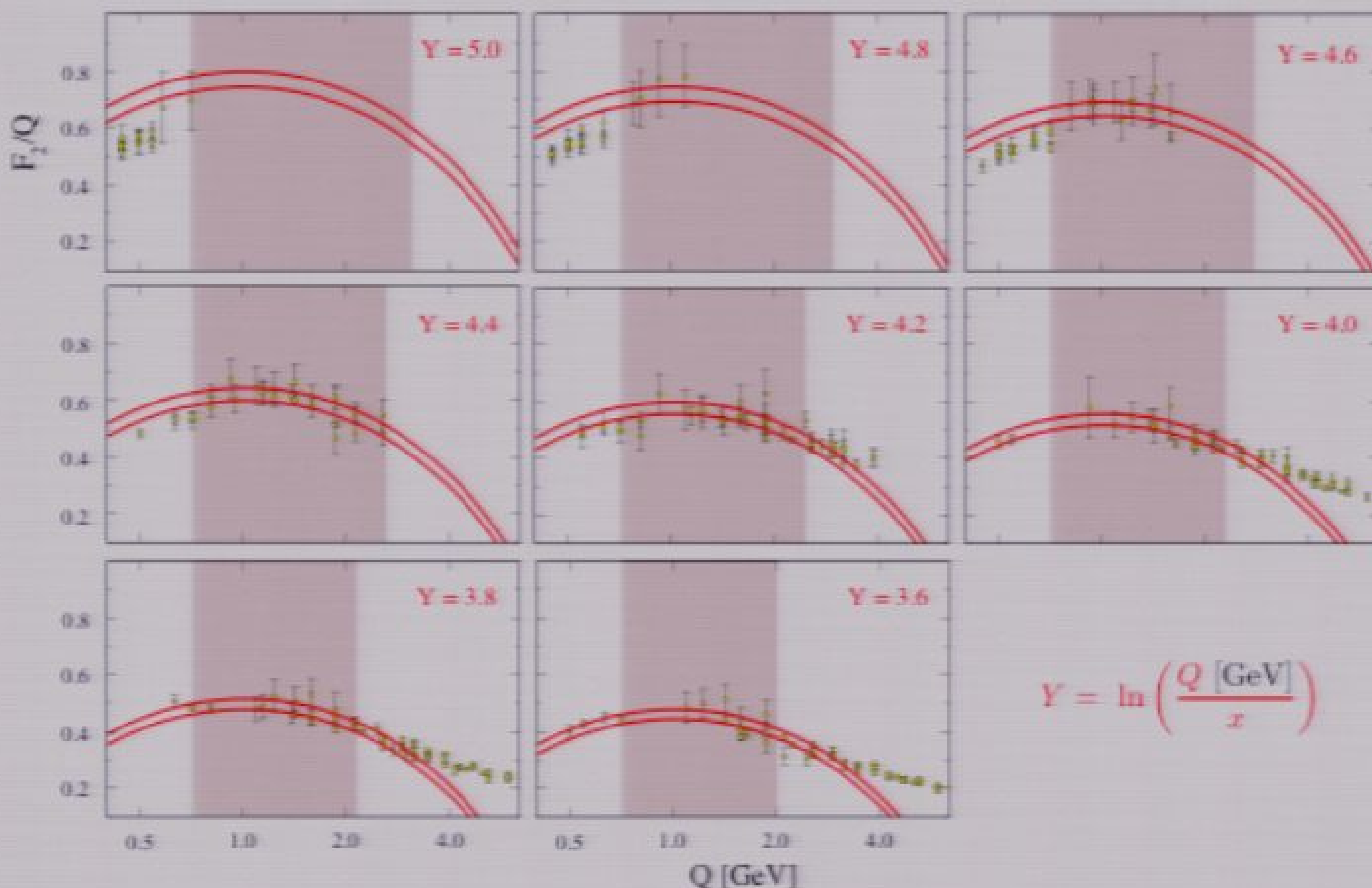

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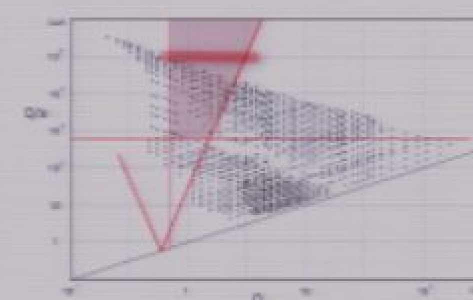

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(Extend saturation analysis to include polarization. Non-conformal extension to holographic QCD models)

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- Impact factors at next-to-leading order and at strong coupling. Integrability of impact factors.