Title: Quantum backreaction in cosmology

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Abstract: The problem of the quantum backreaction in expanding spaces is an old, as yet unresolved, question. In this talk I will consider the one-loop backreaction of a massless scalar which couples to the Ricci scalar in an expanding space with constant deceleration. I will show that the infrared divergences, which generically plague the one loop stress energy, can be removed by matching onto an earlier radiation era. An insignificant backreaction occurs, unless the coupling to the Ricci scalar is negative. Similar results hold for the graviton backreaction.

QUANTUM GRAVITATIONAL BACKREACTION

Tomislav Prokopec, ITP & Spinoza Institute, Utrecht University

Based on:

Tomas Janssen & Tomislav Prokopec, arXiv:0906.0666

Tomas Janssen, Shun-Pei Miao & Tomislav Prokopec, Richard Woodard, arXiv: 0808.2449 [gr-qc], Class. Quant. Grav. 25: 245013 (2008); 0904.1151 [gr-qc] JCAP (2009)

Tomas Janssen & Tomislav Prokopec, arXiv:0807.0477 (2008)

Jurjen F. Koksma & Tomislav Prokopec, 0901.4674 [gr-qc] Class. Quant. Grav. 26: 125003 (2009)

McGill, Nov 30 2009

WHAT IS (QUANTUM) BACKREACTION?

- Einstein's Equations

$$G_{\mu\nu}(\underbrace{g^b_{\alpha\beta}}_{BACKGD.} + \underbrace{\delta g_{\alpha\beta}}_{FLUCT.}) = 8\pi G T_{\mu\nu}(g^b_{\alpha\beta} + \delta g_{\alpha\beta}, \psi^b_i + \delta\psi_i)$$

→ $g^b_{\alpha\beta}, \delta g_{\alpha\beta}$: background gravitational fields & corresp. (quantum) fluctuations

→ $\psi^b_i, \delta\psi_i$: background matter fields & corresponding (quantum) fluctuations

▫ Classical Equations:

$$G^b_{\mu\nu} = 8\pi G T^b_{\mu\nu}, \quad G^b_{\mu\nu} = G_{\mu\nu}(g^b_{\alpha\beta})$$

are not correct in presence of strong backreaction from (quantum) fluctuations

▫ Quantum Einstein Equations:

$$G^b_{\mu\nu} + G^q_{\mu\nu} = 8\pi G(T^b_{\mu\nu} + T^q_{\mu\nu}), \qquad (\ |\Omega\rangle\text{: physical state})$$

▹ $G^q_{\mu\nu} = \langle\Omega|(\hat{G}_{\mu\nu} - G^b_{\mu\nu})|\Omega\rangle$: includes (conserved) contribution from graviton fluctuations

▹ $T^q_{\mu\nu} = \langle\Omega|(\hat{T}_{\mu\nu} - T^b_{\mu\nu})|\Omega\rangle$: includes (conserved) contribution from quantum matter fluct's

NB: Since gravitons couple to matter, it is better to write: $T^q_{\mu\nu} - G^q_{\mu\nu}/8\pi G \to T^q_{\mu\nu}$

THE PROBLEM(S) WITH BACKREACTION

- Quantum Einstein's Equations

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$\rightarrow T^q_{\mu\nu}$ has to be determined by solving dynamical equations for matter and graviton matter perturbations in the expanding Universe setting. Hopelessly hard!

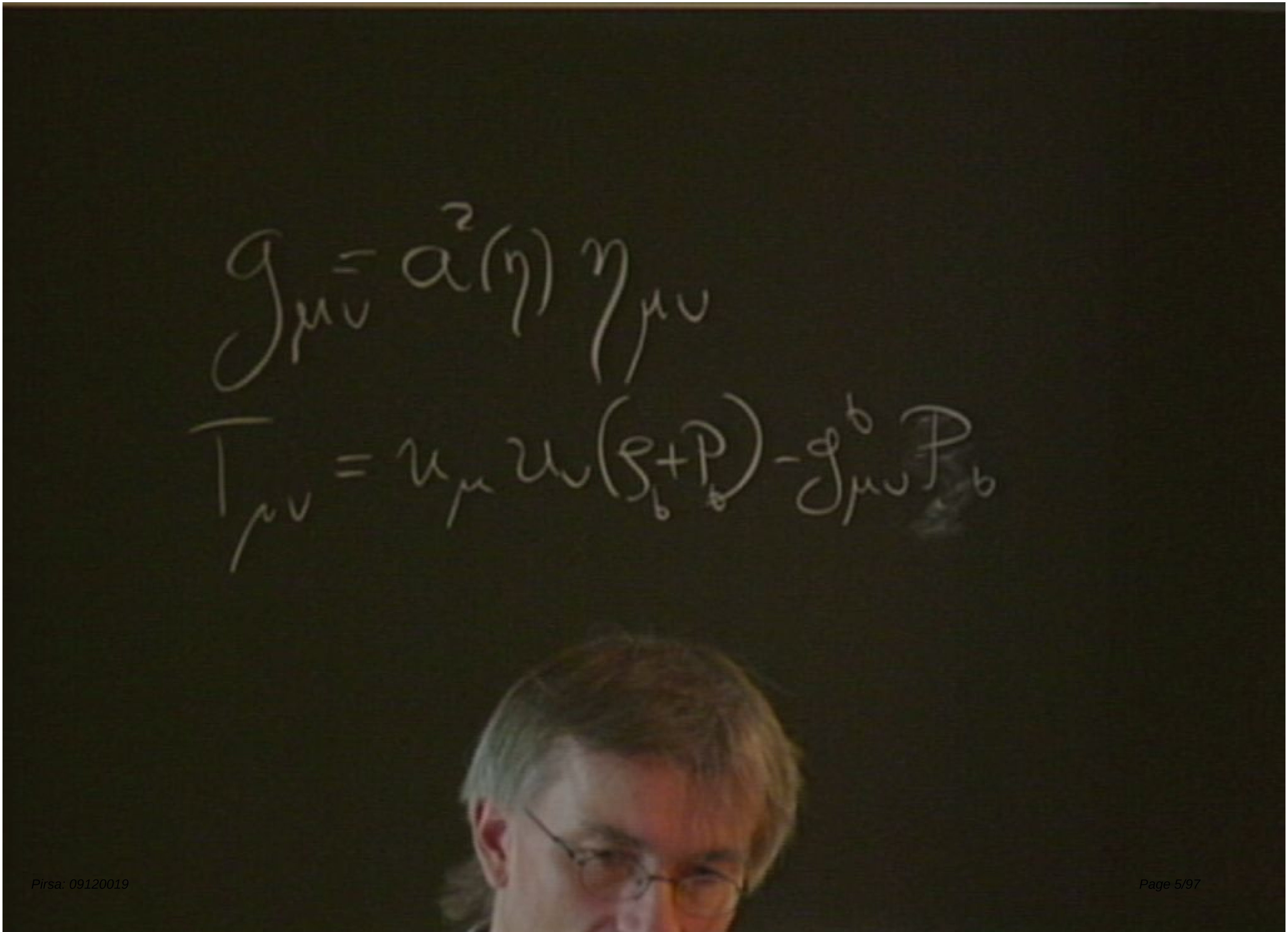
- <u>Statements</u>: **Dark energy can be (perhaps) explained by the backreaction of small scale gravitational + matter perturbations onto the background space time**

  - Hard to (dis-)prove. Naïve argument <u>against</u>: grav. potential is small: $\phi \sim 10^{-5}$
  - Maybe too naïve, because of secular (growing) terms generated by perts

<u>HERE</u>: I will discuss the simplest (!?) possible TOY MODEL:

- a homogeneous universe with constant deceleration parameter q=ε-1, ε=-(dH/dt)/H²

- a massless dynamical scalar ϕ but gravity is non-dynamical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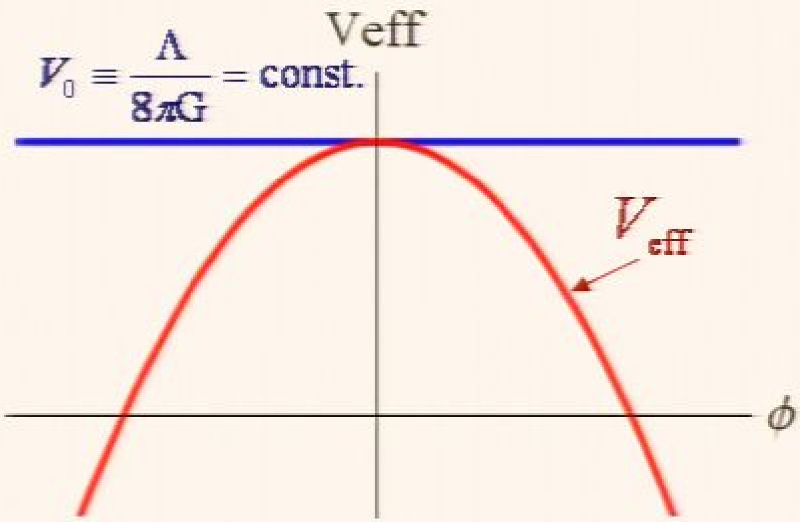
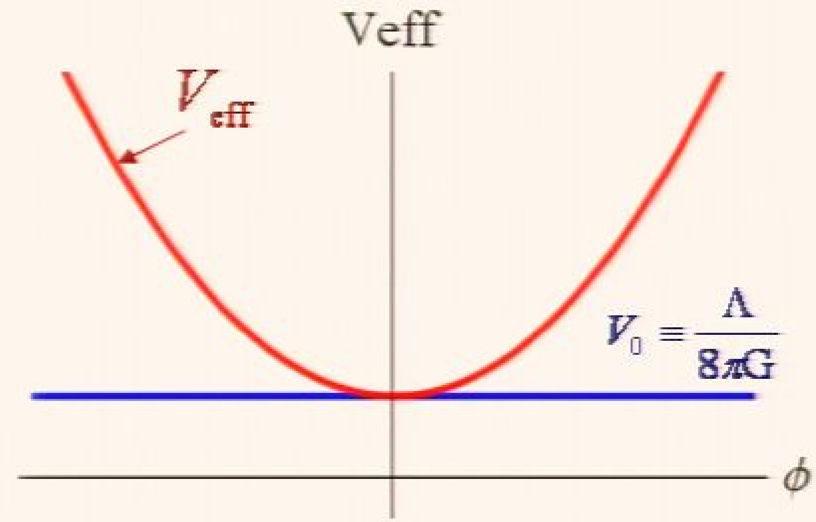
A FEW WORDS ON THE CCP PROBLEM

- Quantum corrections in scalar QED, scalar theories ($\lambda\phi$^4) are positive
- Quantum corrections from integrating fermions (QED, yukawa) are negative

FERMIONS + YUKAWA



SCALARS + VECTORS/GRAVITONS



$1000000 Q: Can solving for V_{eff} self consistently with the Friedmann equation stop the Universe from collapsing into a negative energy (`anti-de Sitter') universe?

JFK+TP, in progress

The Universe can also be stabilised by adding a sufficiently many vectors and scalars

NB: An effective potential of the form $V_{eff} \sim -\lambda(\phi^4)\ln(\phi^2/H^2)$ would solve the CCP problem (the dynamics of ϕ would drive $\Lambda \to 0$). But, how to get it?

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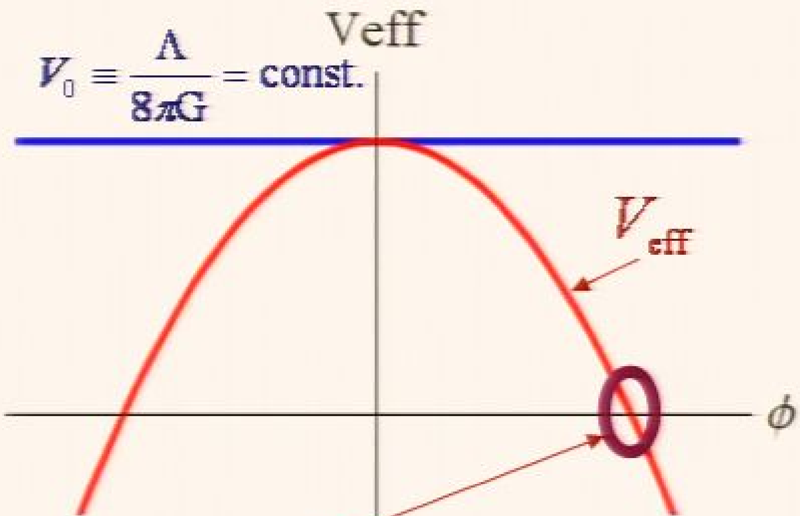

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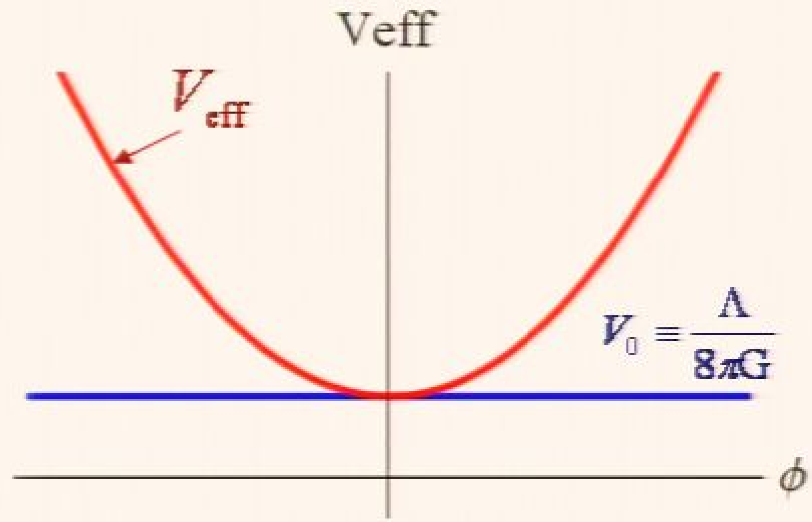

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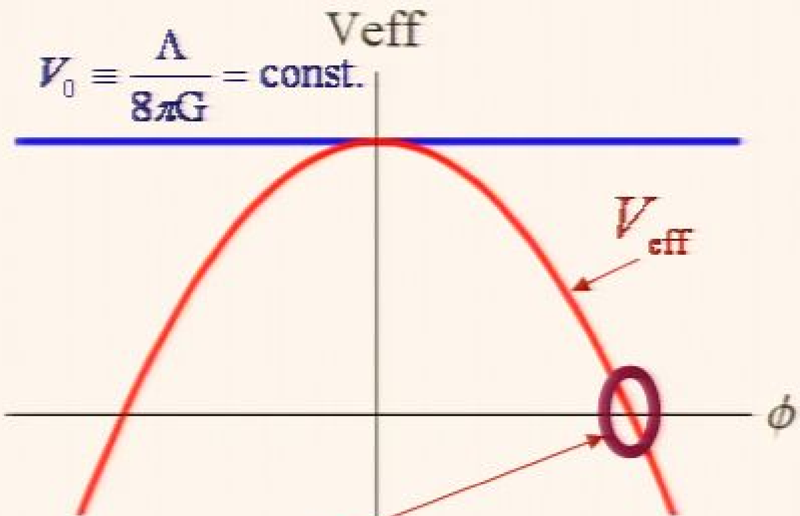

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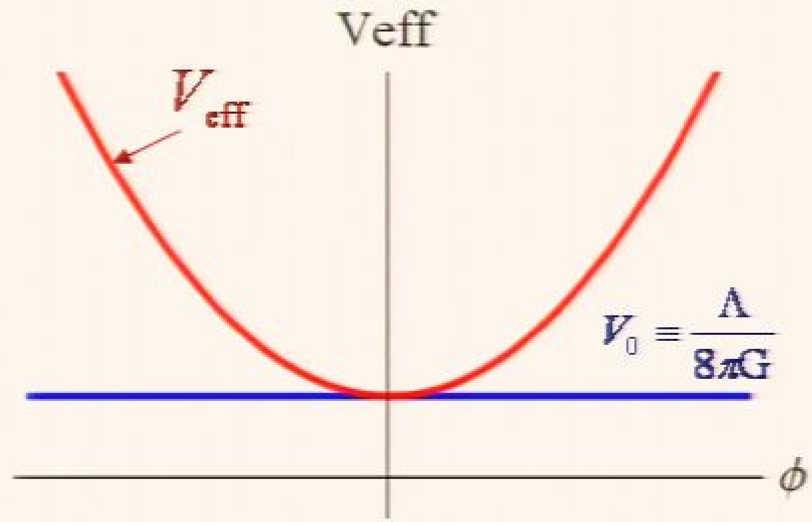

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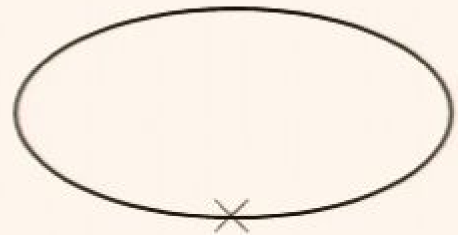
SCALAR 1 LOOP STRESS ENERGY

♦ **CENTRAL Q: Under what conditions can the backreaction from the one loop fluctuations become so large to change evolution of the (background) Universe**

▫ **QUANTUM STRESS ENERGY TRACE:**

$$T_q \equiv \langle T^\mu_\mu \rangle = -\frac{1}{2}(1-6\xi)\Box \underbrace{\langle \Omega | \phi^2(x) | \Omega \rangle}_{i\Delta(x;x)} = -\rho_q + 3p_q$$



→ $i\Delta(x;x)$: scalar propagator at coincidence

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SCALAR THEORY

- MASSLESS SCALAR FIELD ACTION ($V \to 0$, ξR plays role of a `mass')

$$S_\varphi = \int d^D x \sqrt{-g}\left(-\frac{1}{2} g^{\mu\nu}\partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2}\xi R \varphi^2 - V(\varphi)\right)$$

What is ϕ?

$\Rightarrow$ SCALAR EOM

$$\nabla_\mu \nabla^\mu \varphi - \xi R \varphi - V'(\varphi) = \frac{1}{a^2}\left(\partial^2 - (D-2)\frac{a'}{a}\partial_0 - \xi R\right)\varphi - V'(\varphi) = 0, \quad \partial^2 = \eta^{\mu\nu}\partial_\mu \partial_\nu = -\partial_0^2 + \vec{\partial}^2$$

- Field quantisation ($V \to 0$):

$$\hat{\varphi}(x) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}}\left(\varphi_k(\eta) e^{i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}} + \varphi_k^*(\eta) e^{-i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}}^+\right), \quad \hat{a}_{\vec{k}}|0\rangle = 0$$

$$\varphi_k(\eta) = \frac{1}{a}\sqrt{\frac{-\pi\eta}{4}}\left\{\alpha_k H_\nu^{(1)}(-k\eta) + \beta_k H_\nu^{(2)}(-k\eta)\right\}, \quad \nu^2|_{D=4} = \frac{1}{4} + \frac{(1-6\xi)(2-\varepsilon)}{(1-\varepsilon)^2}$$

$$(|\alpha_k|^2 - |\beta_k|^2 = 1)$$

- PROPAGATOR EQUATION

$$\left[\nabla_\mu \nabla^\mu - \xi R^b - V''(\phi^b)\right] i\Delta(x;x') = i\delta^D(x-x'),$$

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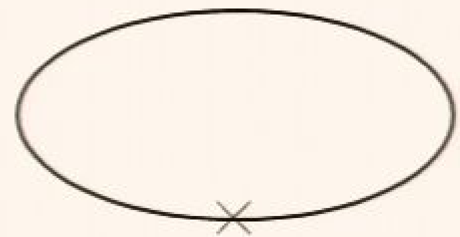

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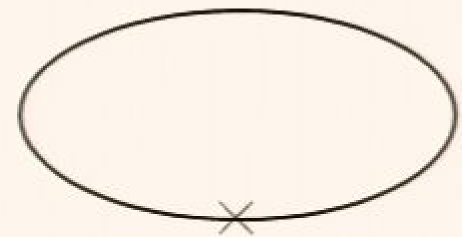

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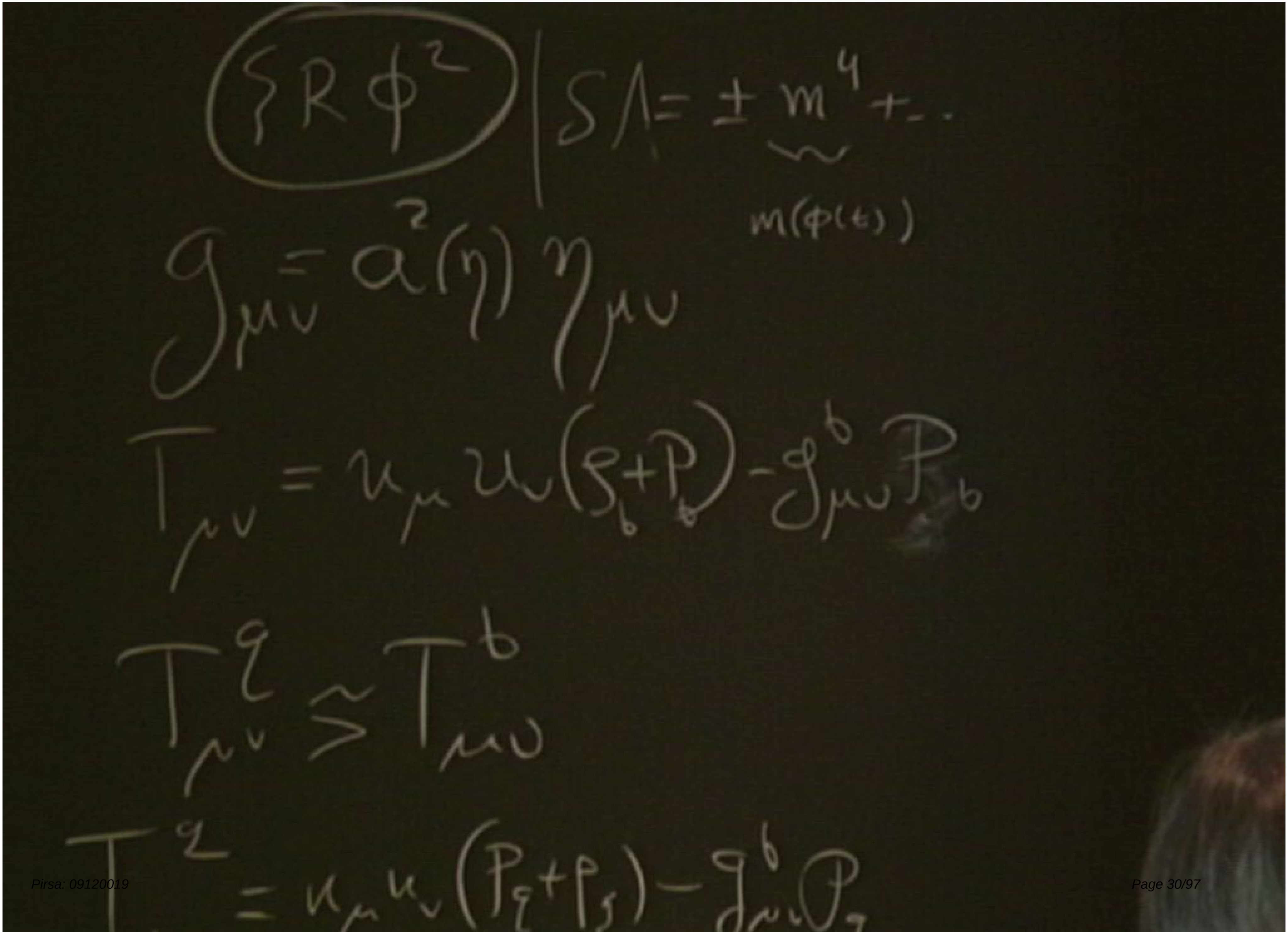
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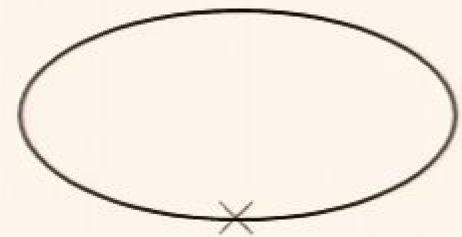

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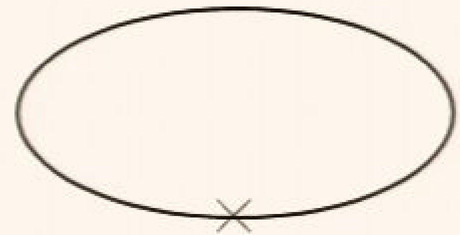

$T^{\mu\nu}(x)$

→ $i\Delta(x;x)$: scalar propagator at coincidence

→ ξ: conformal coupling

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$$\frac{1}{H}\dot{\rho}_q + 4\rho_q \equiv -T_q \to \rho_q, \quad p_q = \frac{1}{3}(\rho_q + T_q)$$

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$$\propto \frac{1}{a^4}$$

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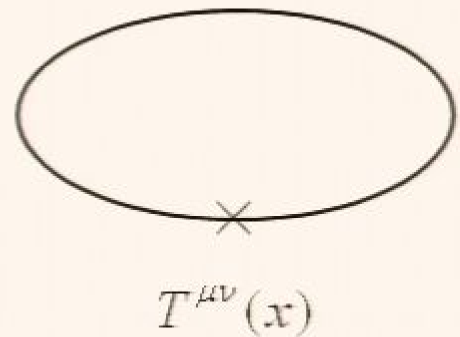

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$$S_\varphi = \int d^D x \sqrt{-g}\left(-\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}\xi R\varphi^2 - V(\varphi)\right)$$

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BACKGROUND SPACE TIME

- LINE ELEMENT (METRIC TENSOR):

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 \quad or \quad g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}, \quad \eta_{\mu\nu} = diag(-1, \underbrace{1,1,..}_{D-1})$$

- FRIEDMANN (FLRW) EQUATIONS (Λ=0):

$$H^2 = \frac{8\pi G}{3}\rho_b, \qquad \dot{H} = -4\pi G(\rho_b + p_b)$$

$$\Rightarrow \varepsilon = -\frac{\dot{H}}{H^2} = \frac{3}{2}(1 + w_b) = \text{const.}, \quad w_b = \frac{p_b}{\rho_b}$$

- for power law expansion the scale factor reads:

$$\mathrm{a} = \left(\frac{t}{t_0}\right)^{1/\varepsilon} = \left[-(1-\varepsilon)H_0\eta\right]^{-\frac{1}{1-\varepsilon}}, \quad H = H_0 a^{-\varepsilon}$$

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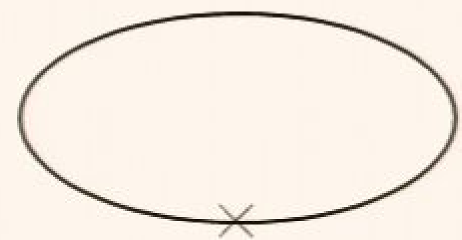

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$$\varphi_k(\eta) = \frac{1}{a}\sqrt{\frac{-\pi\eta}{4}}\left\{\alpha_k H_\nu^{(1)}(-k\eta) + \beta_k H_\nu^{(2)}(-k\eta)\right\}, \quad \nu^2|_{D=4} = \frac{1}{4} + \frac{(1-6\xi)(2-\varepsilon)}{(1-\varepsilon)^2}$$

$$(|\alpha_k|^2 - |\beta_k|^2 = 1)$$

- PROPAGATOR EQUATION

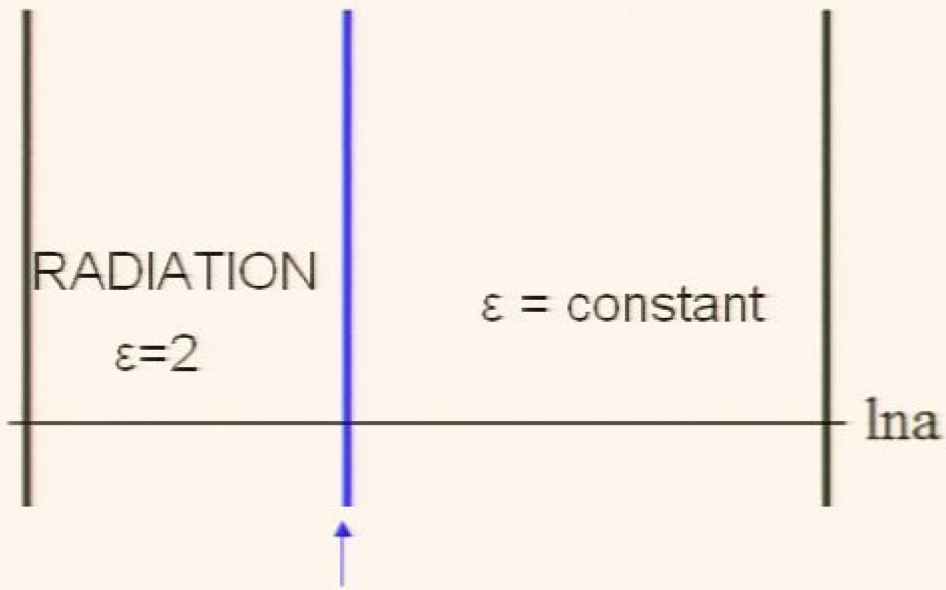
$$\left[\nabla_\mu\nabla^\mu - \xi R^b - V''(\phi^b)\right] i\Delta(x;x') = i\delta^D(x-x'),$$

MATCHING

We match radiation era (ε=2, ν=1/2) onto a constant ε homogeneous FLRW Universe

→ RECALL: observed inhomogeneities $\delta\rho/\rho \sim 1/10000$ @ cosmological scales

($0 \leq \varepsilon \leq 3$: COVERS ALL KNOWN CASES IN THE HISTORY OF THE UNIVERSE)



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$$t = \hat{t}, \ \hat{H} = H(\hat{t})$$

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- Scalar field spectrum P_φ in de Sitter ($\nu=3/2$, $\varepsilon=0$)

$$\langle 0|\hat{\varphi}(\vec{x},\eta)\hat{\varphi}(\vec{x}',\eta)|0\rangle = \int \frac{dk}{k} P_\varphi(k,\eta)\frac{\sin(k\Delta x)}{k\Delta x}, \quad \Delta x = \|\vec{x}-\vec{x}'\|$$

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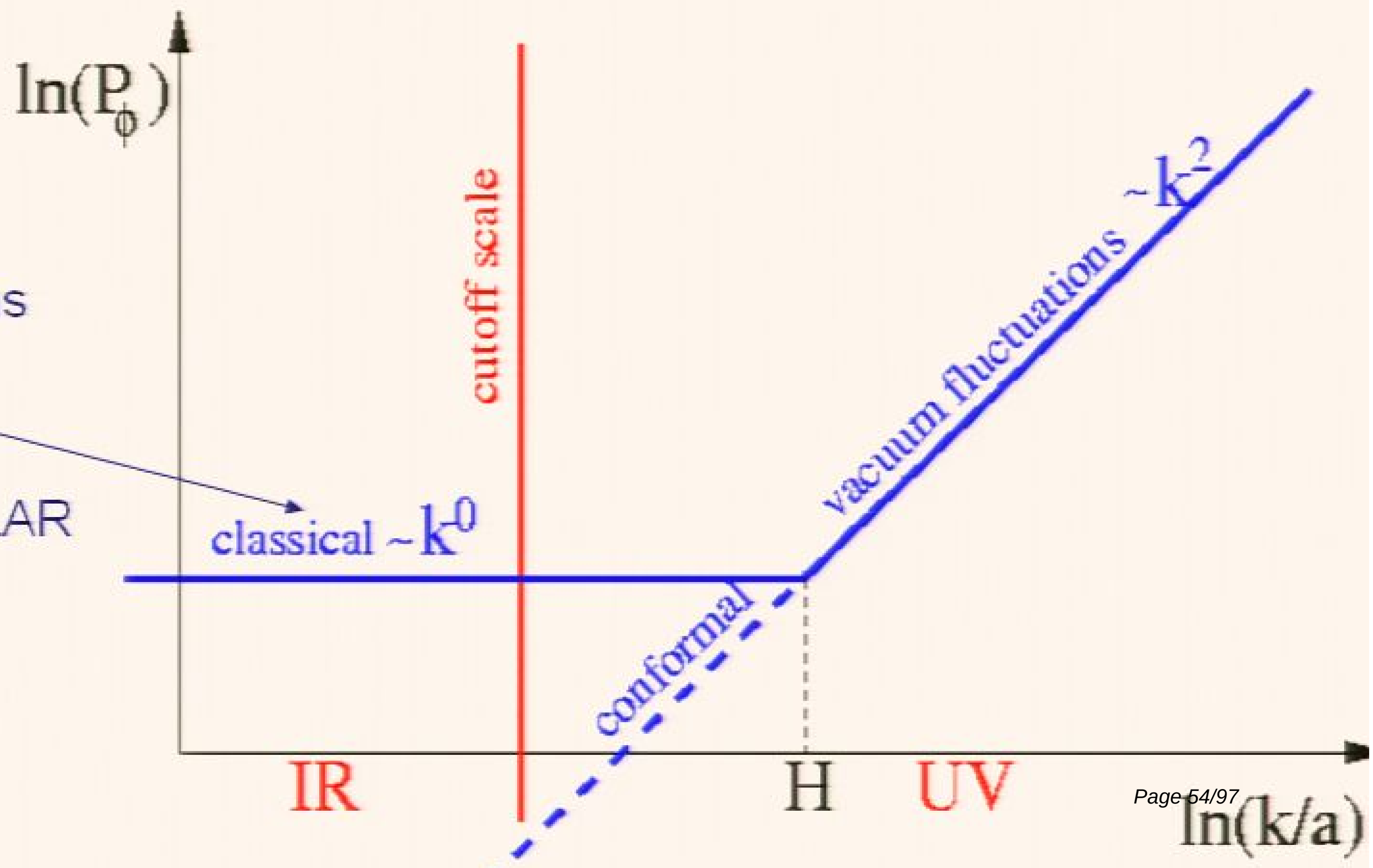

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SCALAR PROPAGATOR IN FLRW SPACES in D dimensions

- **MMC SCALAR FIELD PROPAGATOR (V''=0, ε=const)** Janssen & Prokopec 2009, 2007 Janssen, Miao & Prokopec 2008

  - **EOM** $\sqrt{-g}\left(g^{\mu\nu}\nabla_\mu\nabla_\nu - \xi R\right) i\Delta(x,x') = i\delta^D(x-x')$

  - **IR unregulated (∞ SPACE) PROPAGATOR (ε=const)**

$$i\Delta_\infty(x;x') = \frac{((1-\varepsilon)^2 HH')^{\frac{D-2}{2}}}{(4\pi)^{D/2}} \frac{\Gamma\left(\frac{D-1}{2}+\nu_D\right)\Gamma\left(\frac{D-1}{2}-\nu_D\right)}{\Gamma(D/2)} {}_2F_1\left(\frac{D-1}{2}+\nu_D, \frac{D-1}{2}-\nu_D; \frac{D}{2}; 1-\frac{y}{4}\right), \quad H = H_0 a^{-\varepsilon}$$

$$\nu_D^2 = \left(\frac{D-1-\varepsilon}{2(1-\varepsilon)}\right)^2 - \frac{(D-1)(D-2\varepsilon)\xi}{(1-\varepsilon)^2} \qquad y(x;x') = \frac{-(|\eta-\eta'|-i\delta)^2 + \|\vec{x}-\vec{x}'\|^2}{\eta\eta'}, \quad y = 4\sin^2\left(\frac{Hl}{2}\right)$$

l = geodesic distance in de Sitter space

  - **IN D=4**

$$\nu^2 = \left(\frac{3-\varepsilon}{2(1-\varepsilon)}\right)^2 - \frac{6\xi(2-\varepsilon)}{(1-\varepsilon)^2} = \frac{1}{4} - \frac{(6\xi-1)(2-\varepsilon)}{(1-\varepsilon)^2}$$

  - **NB: ν=1/2 for a conformally coupled scalar, ξ=1/6**

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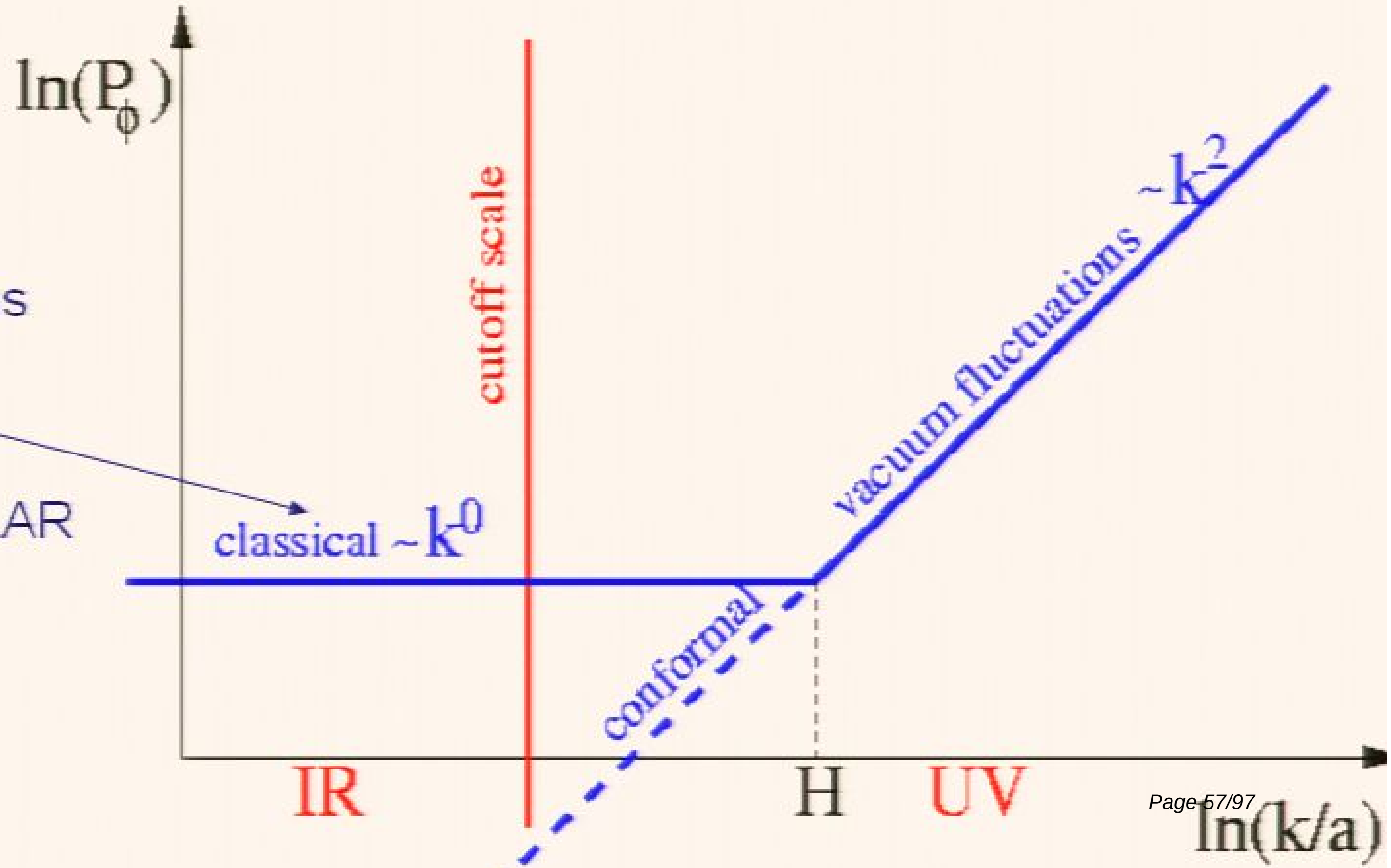

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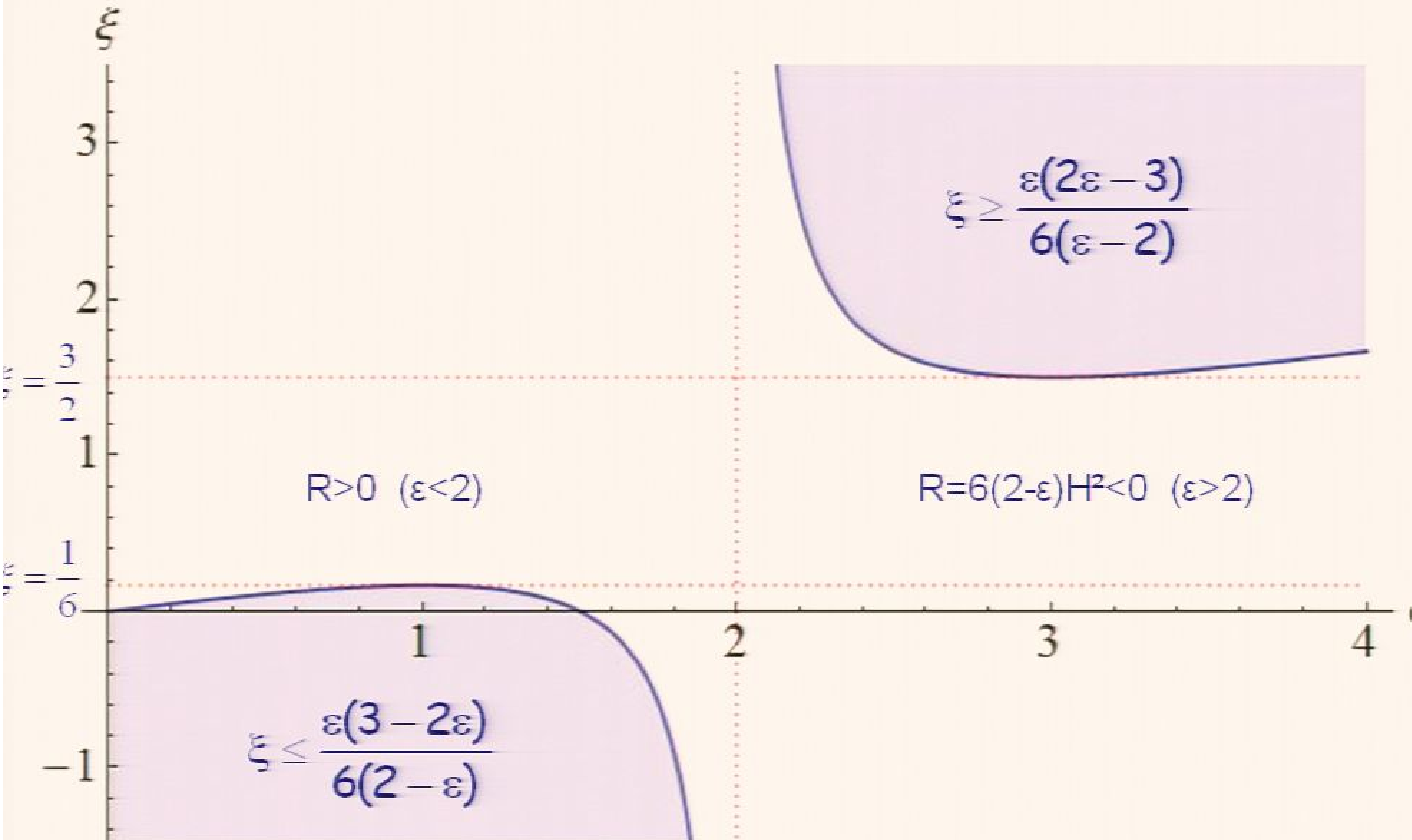

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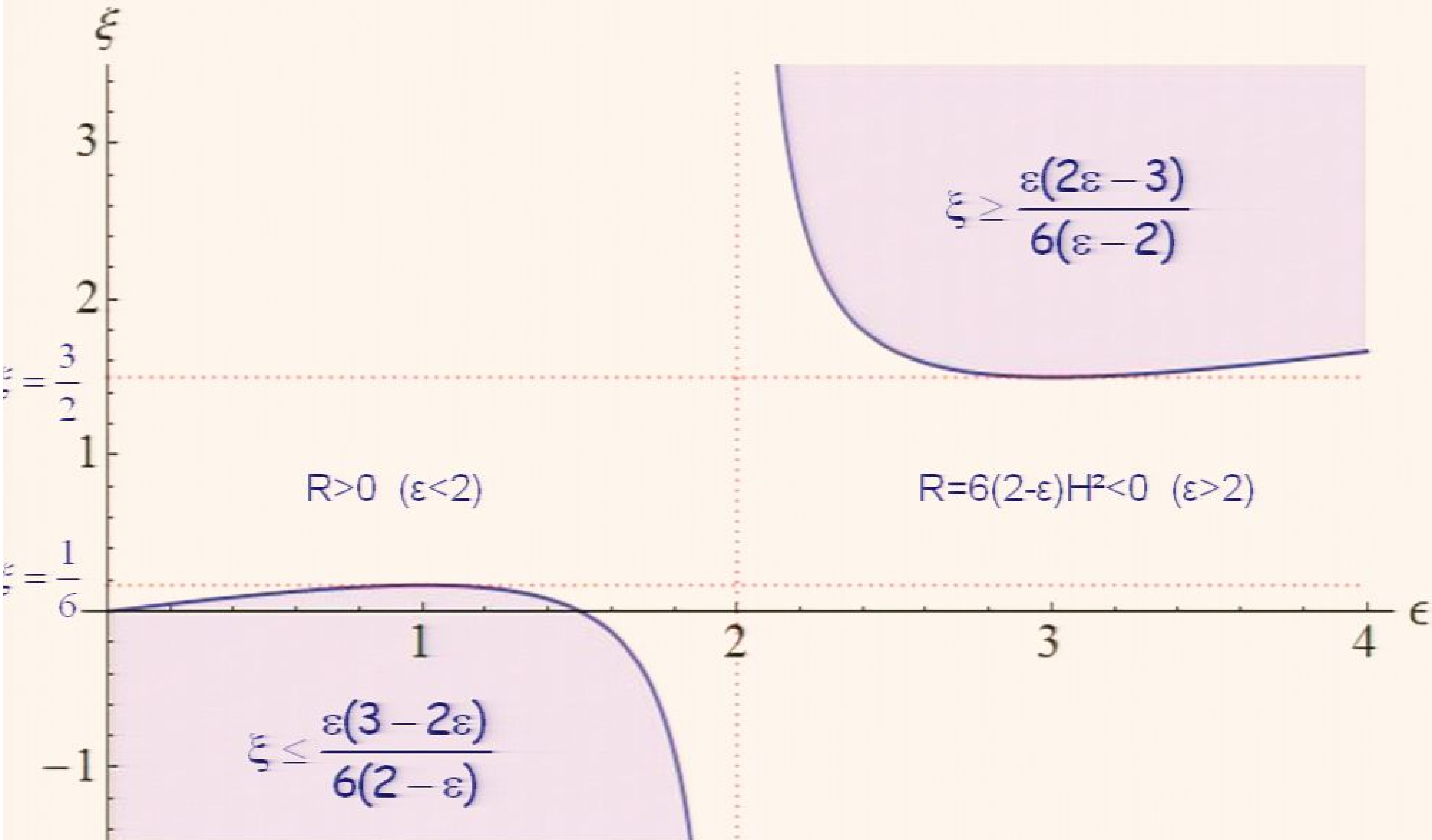

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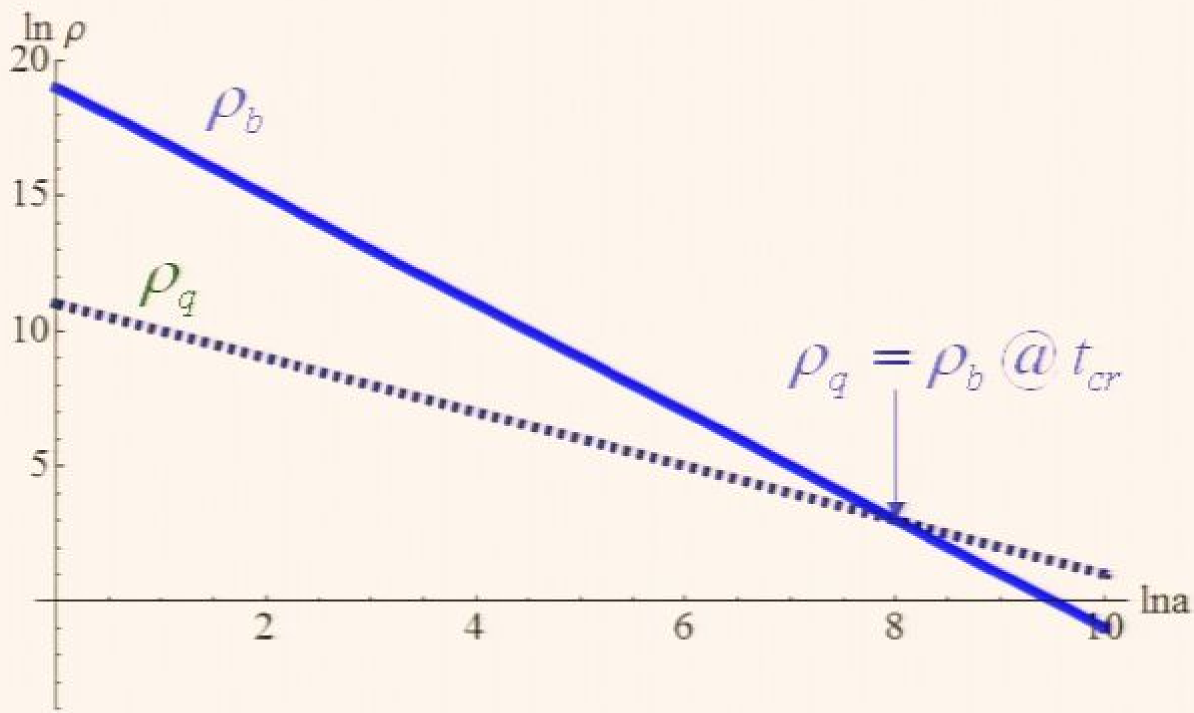

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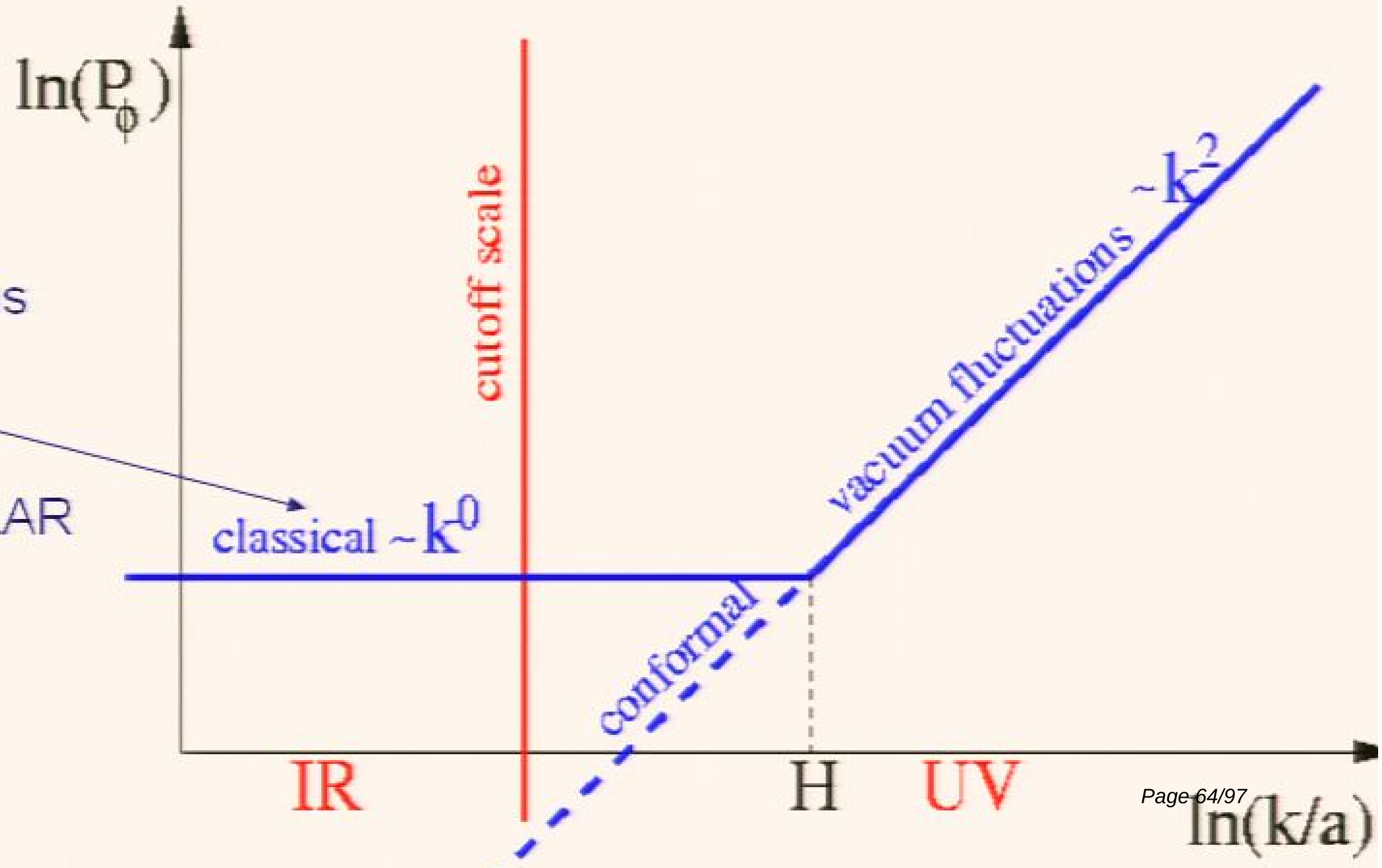

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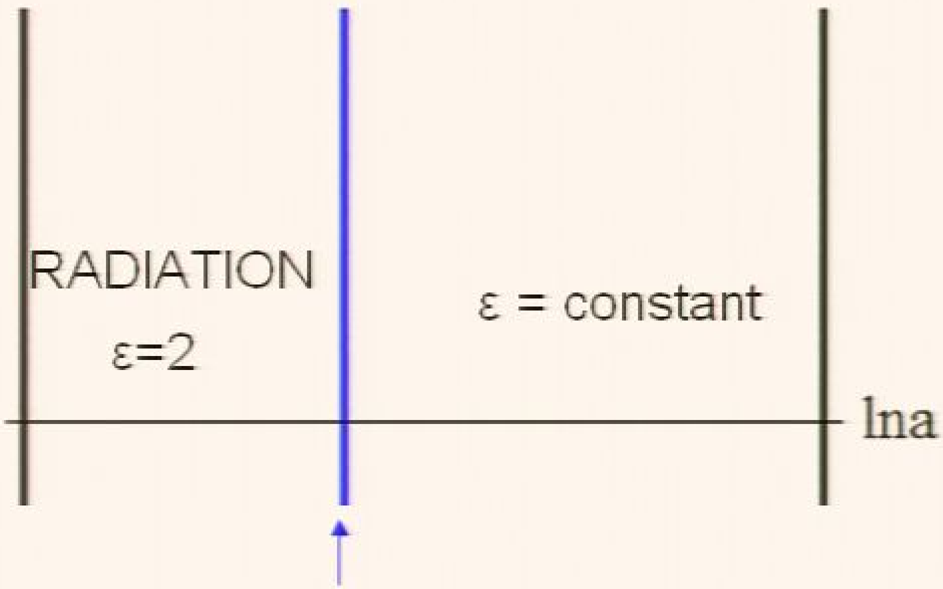

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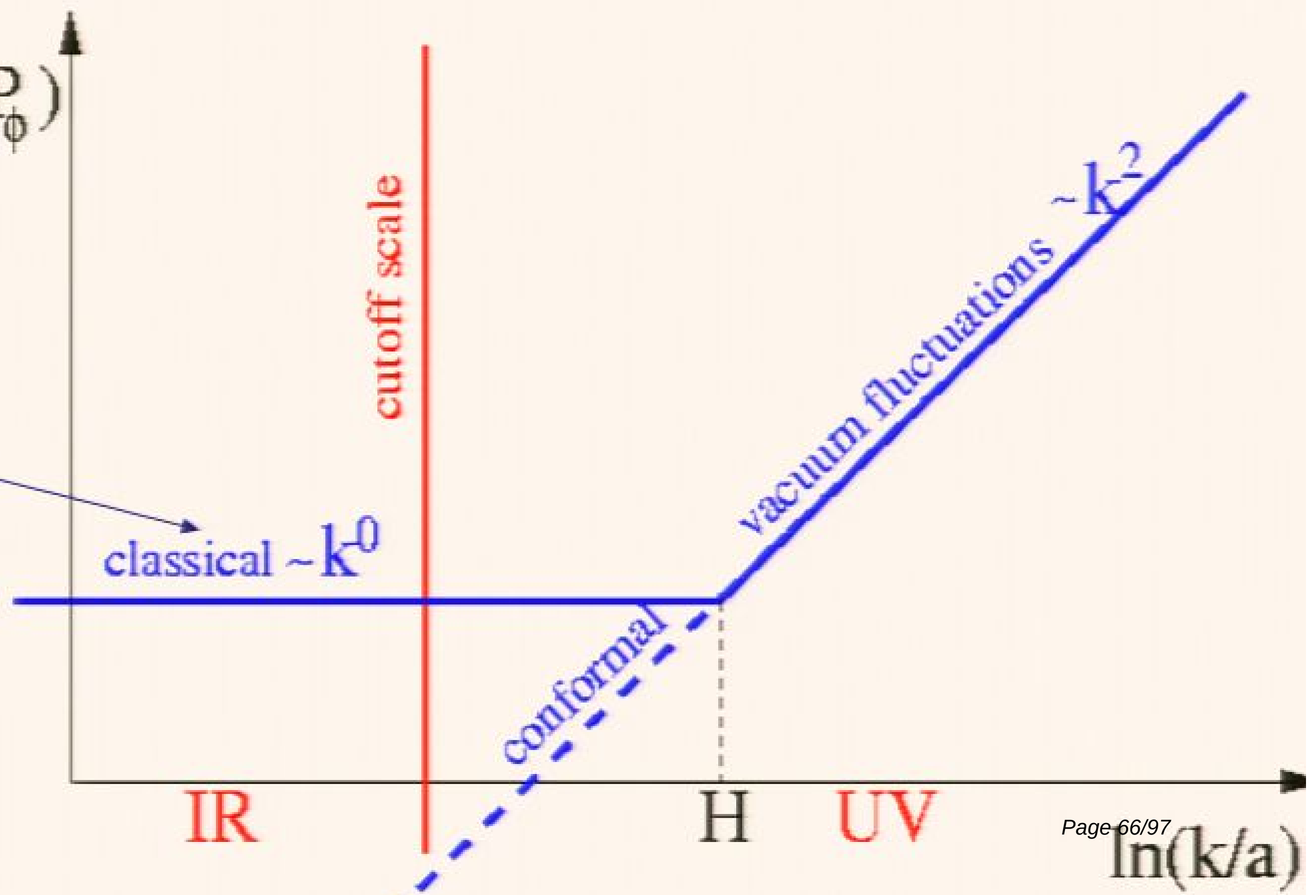

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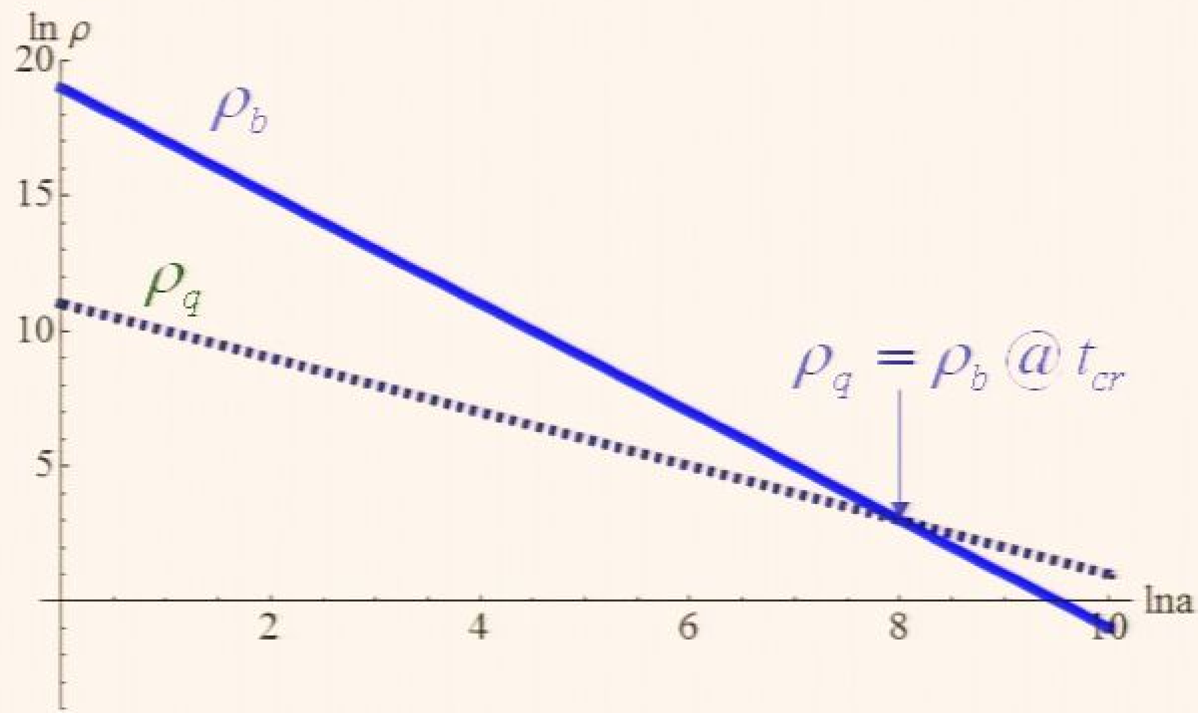

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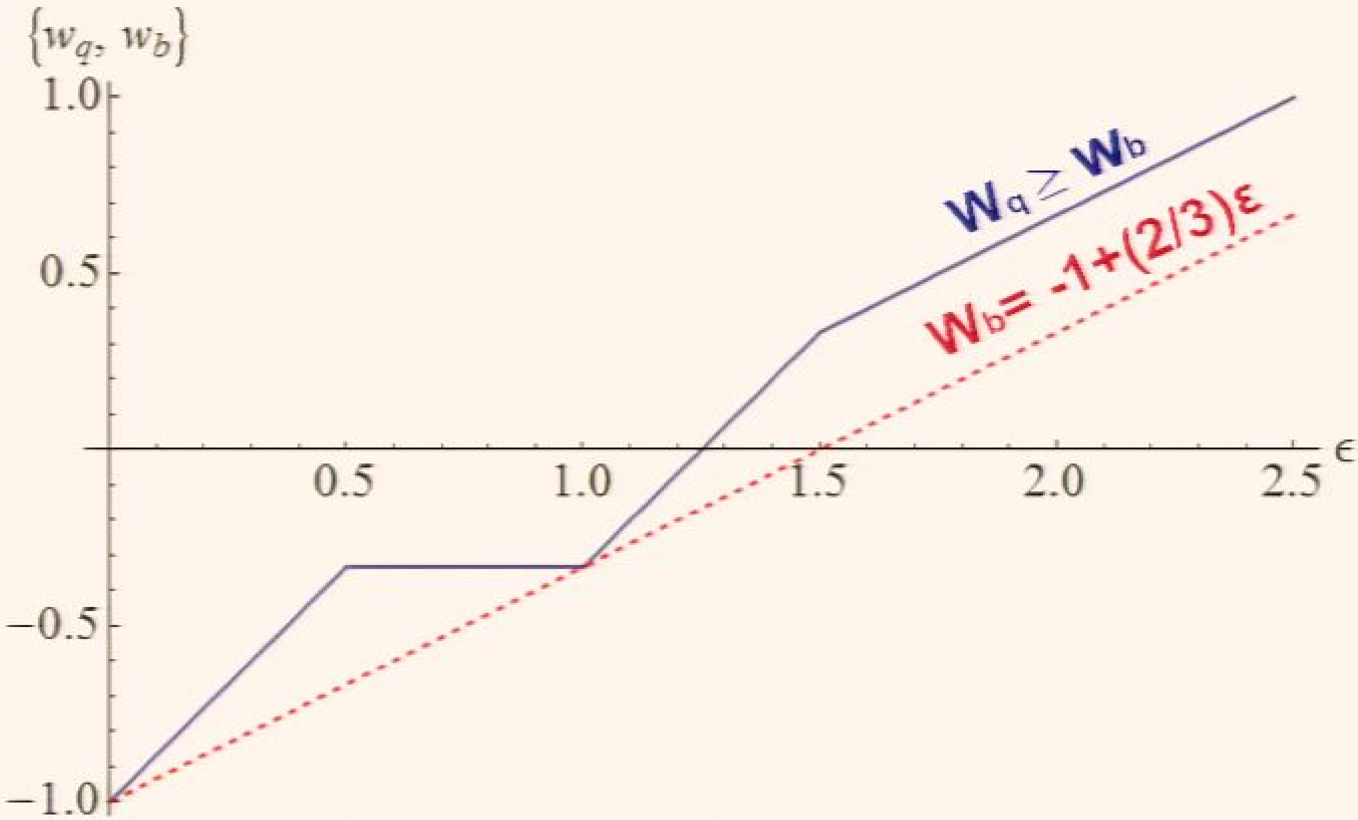

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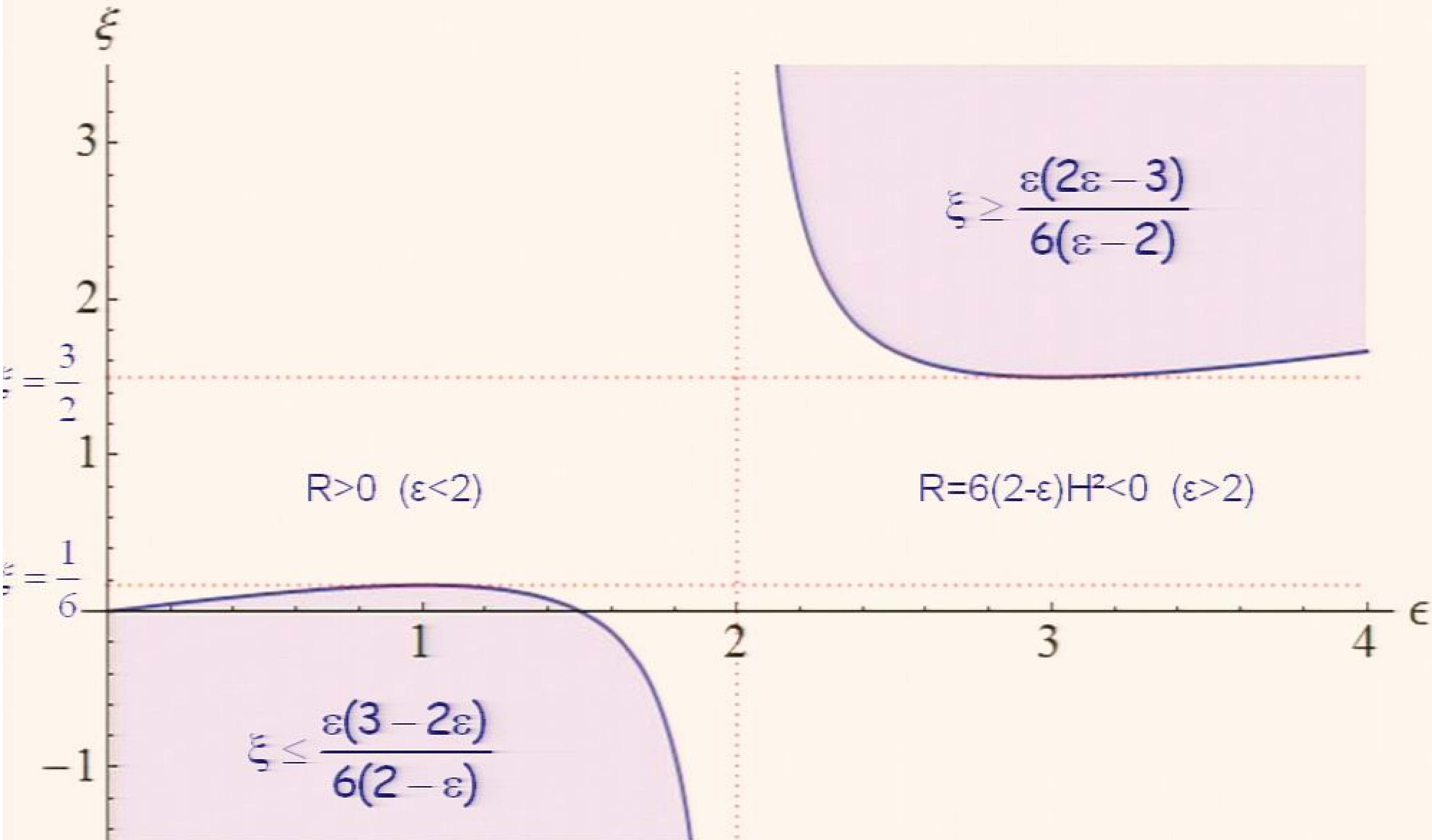

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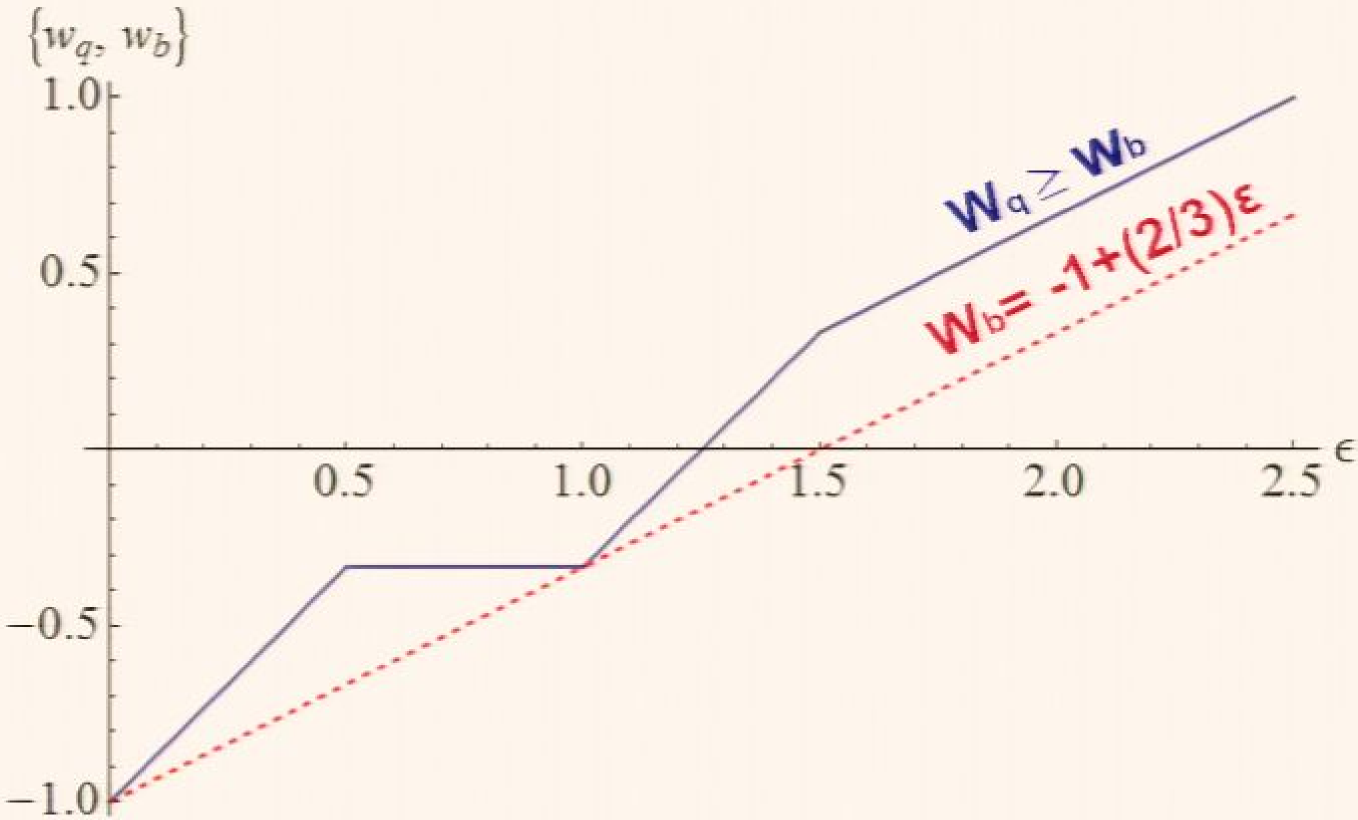

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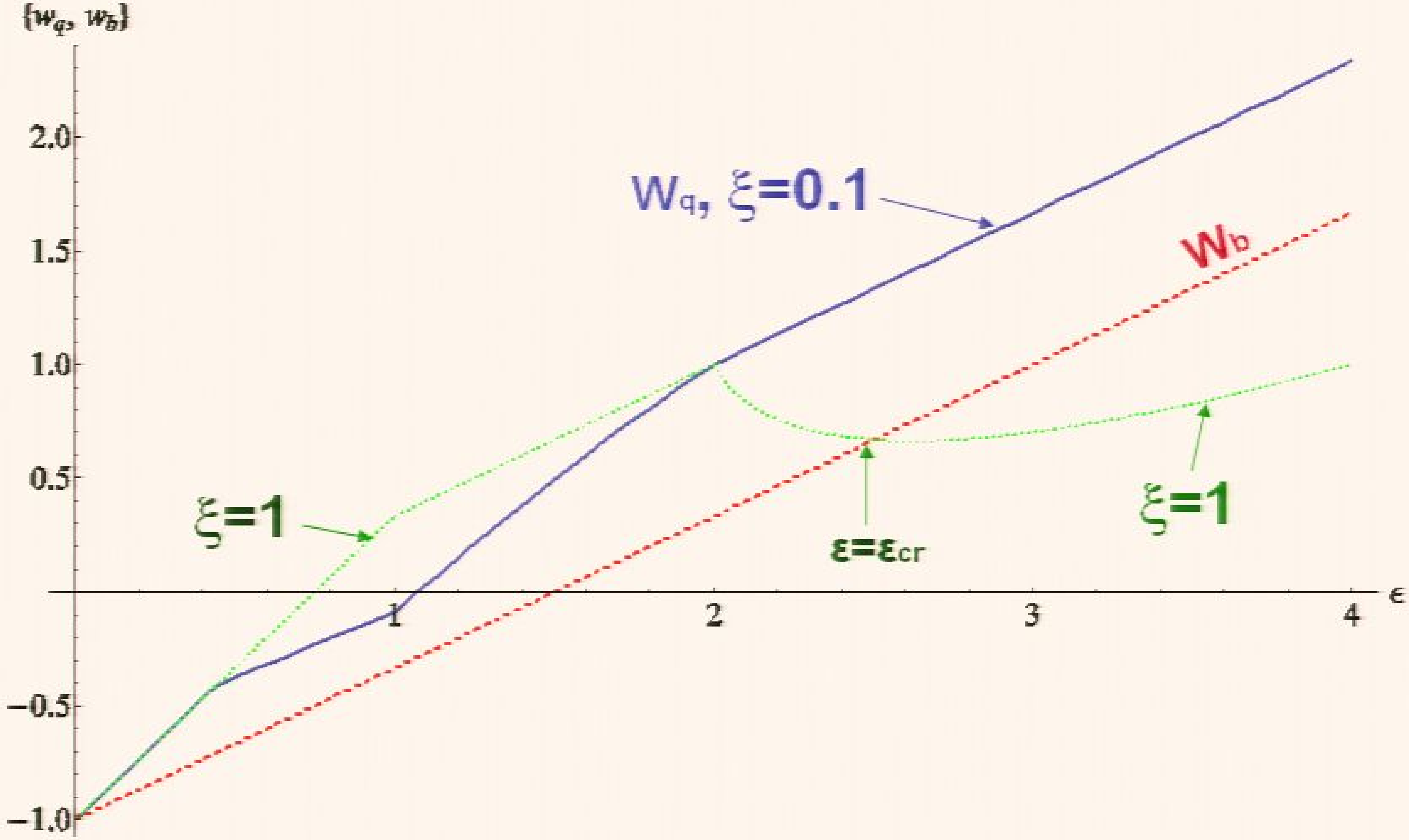

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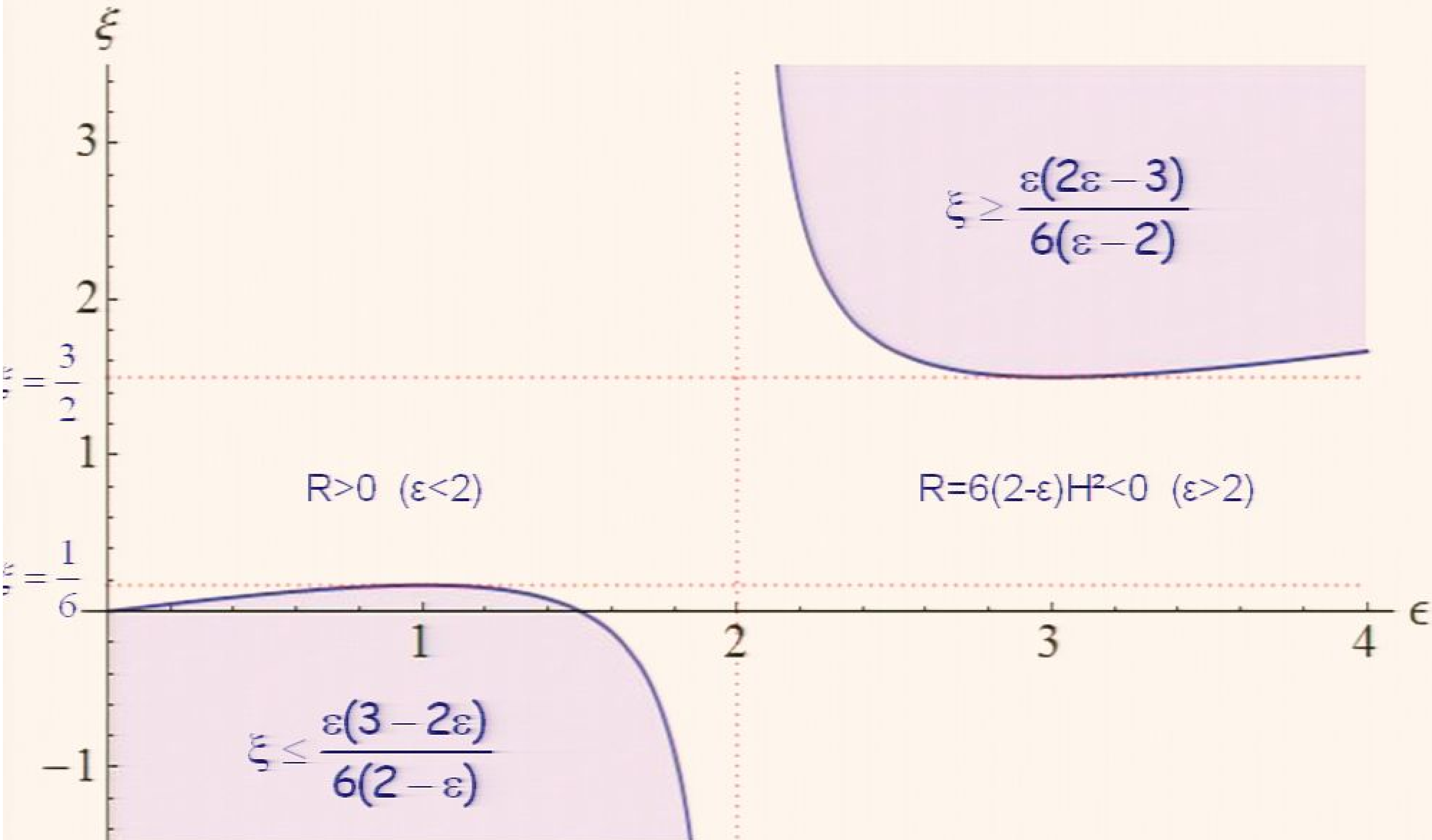

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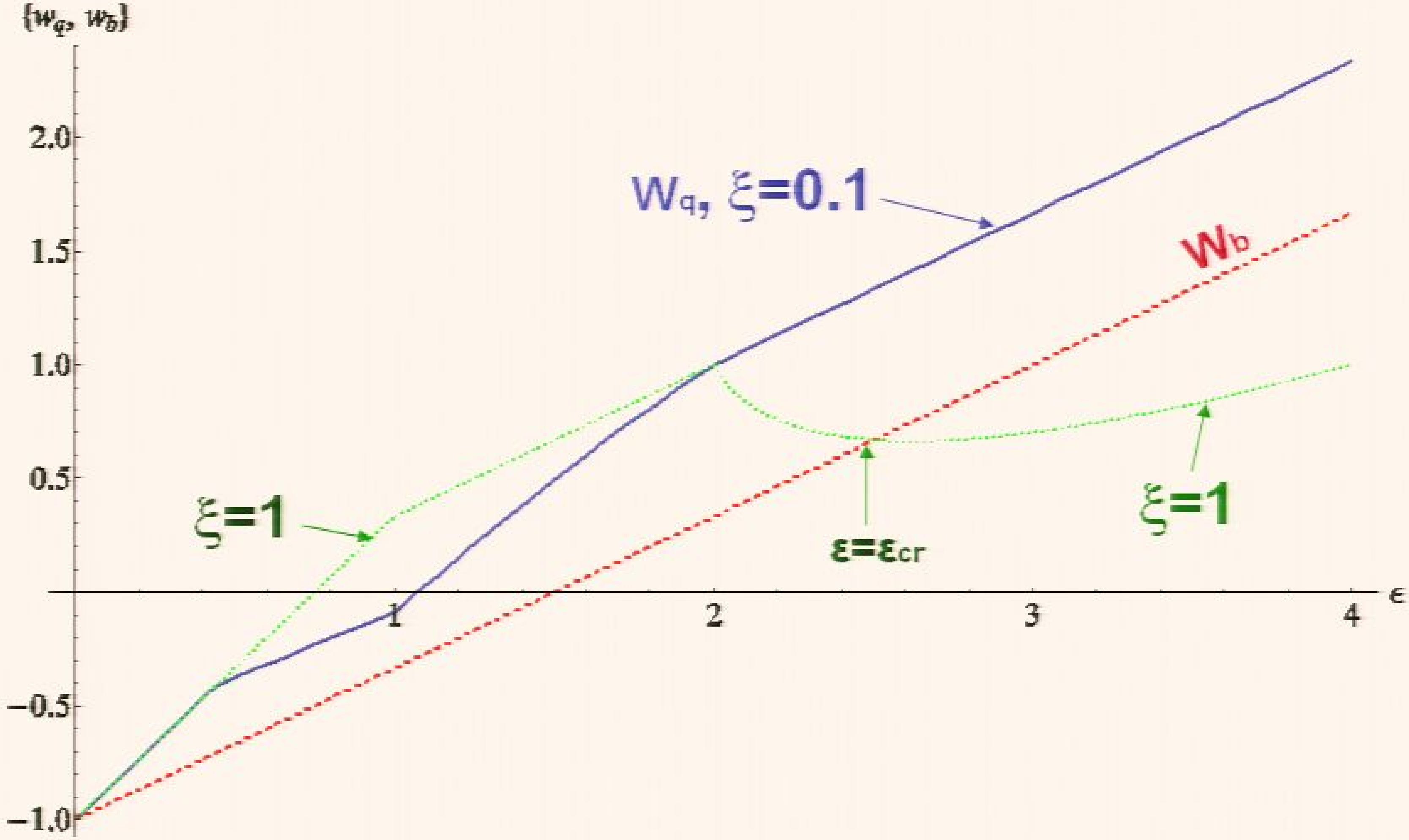

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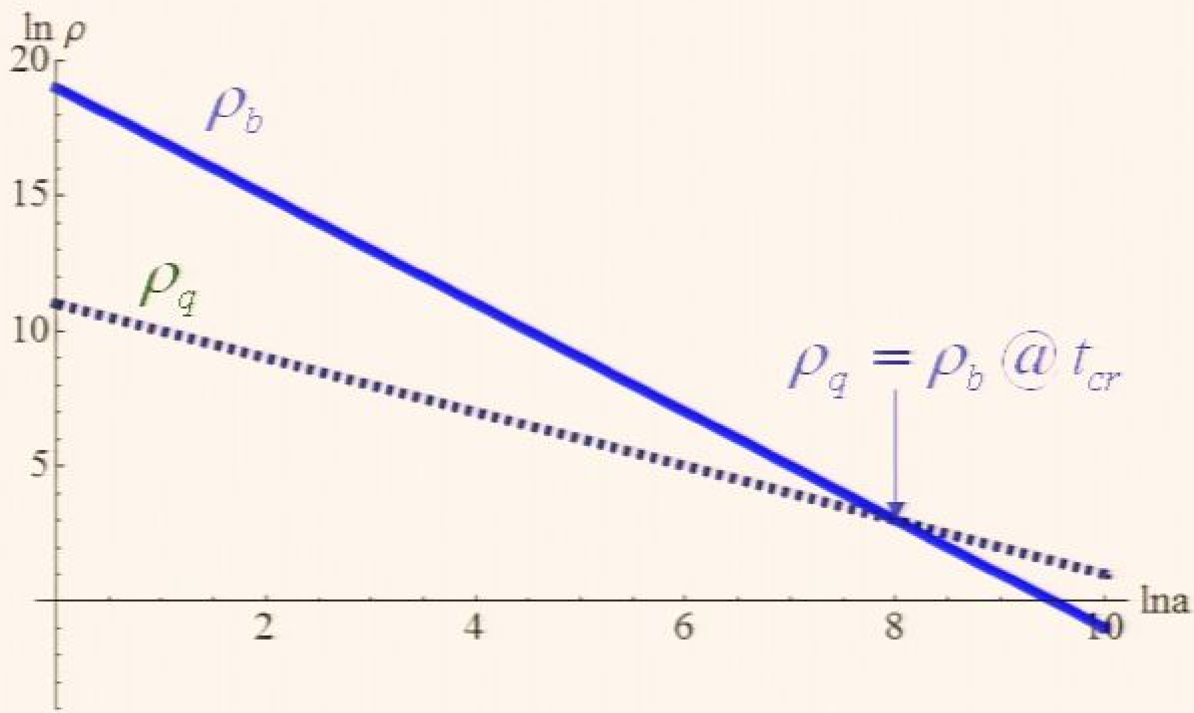

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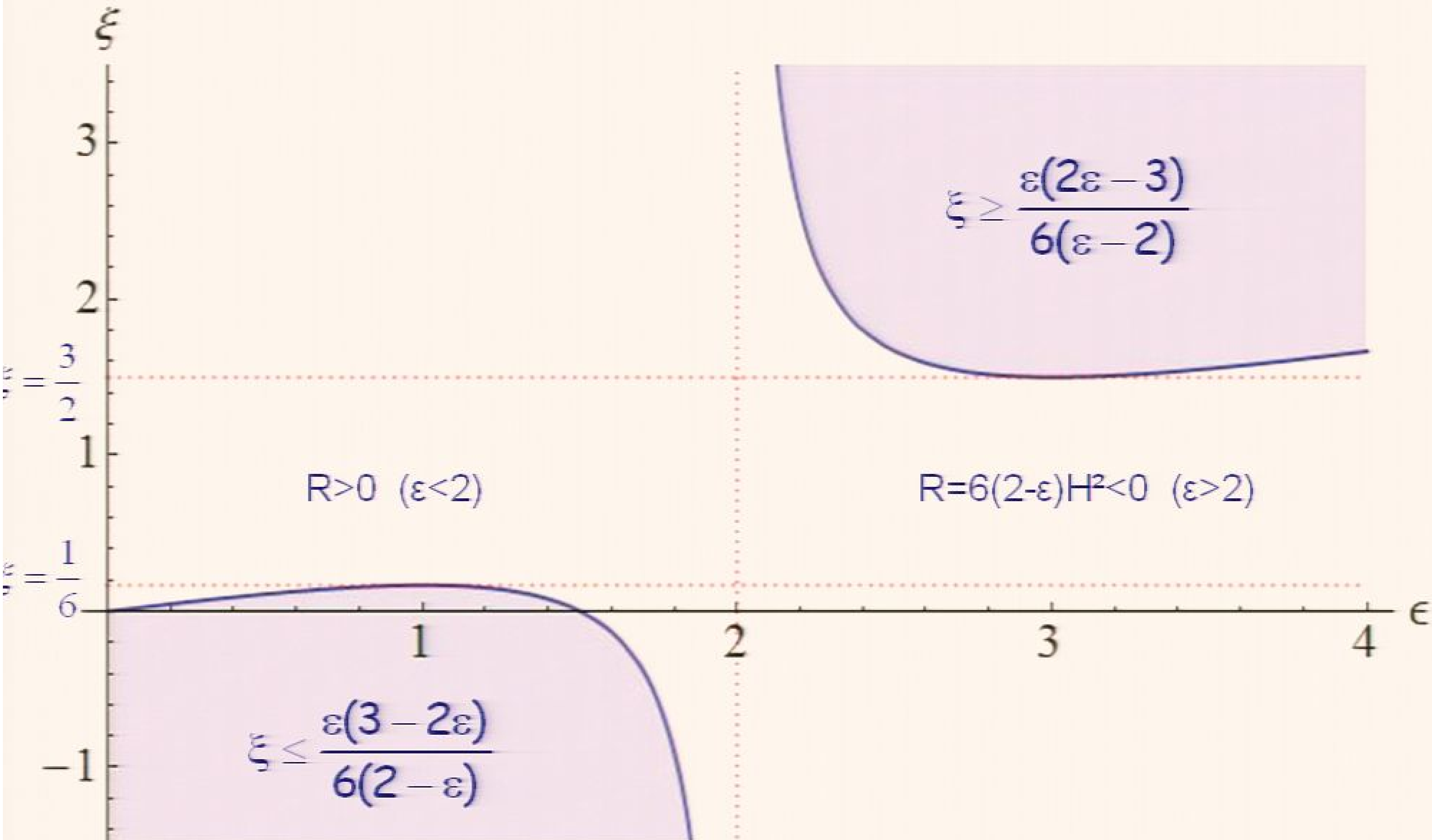

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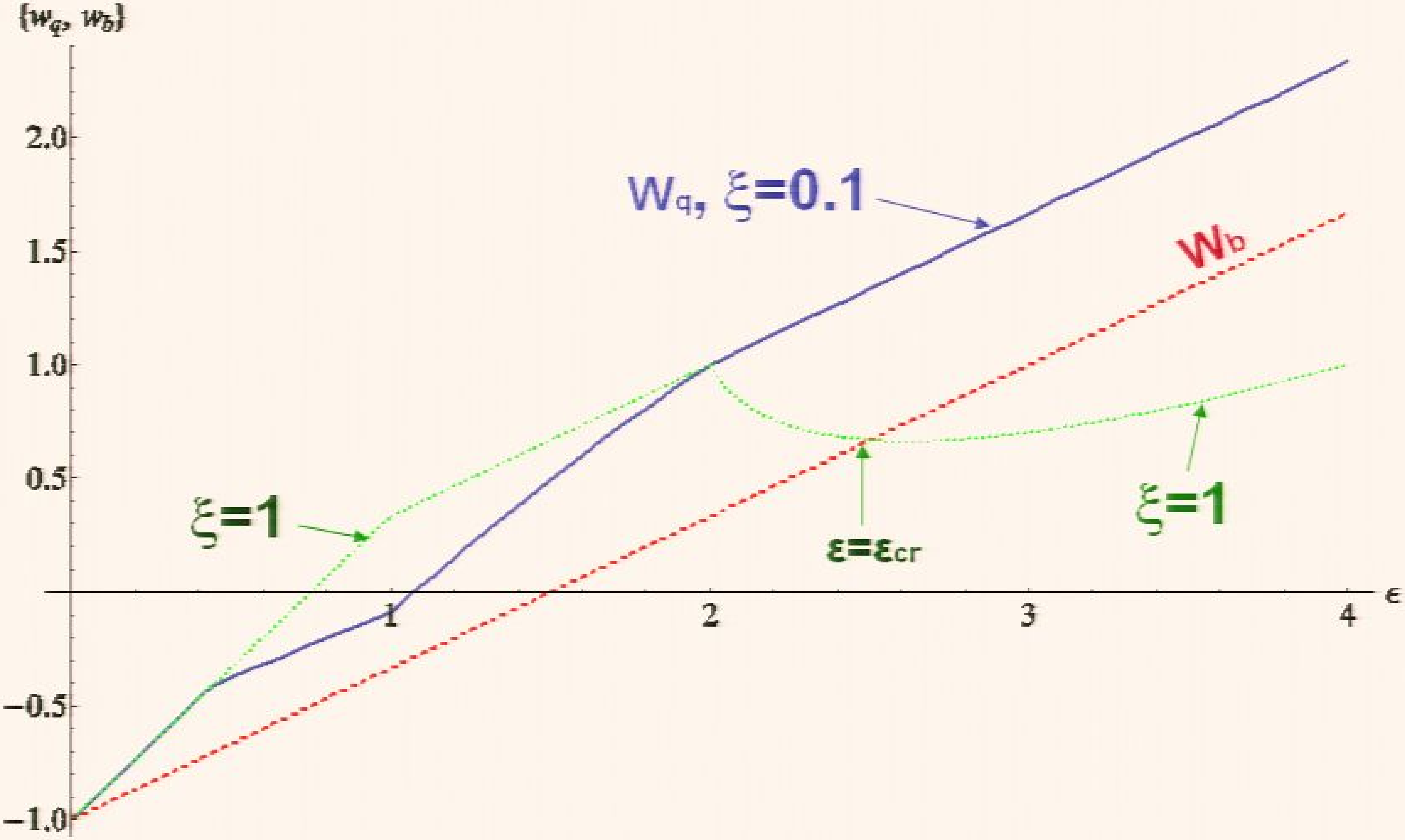

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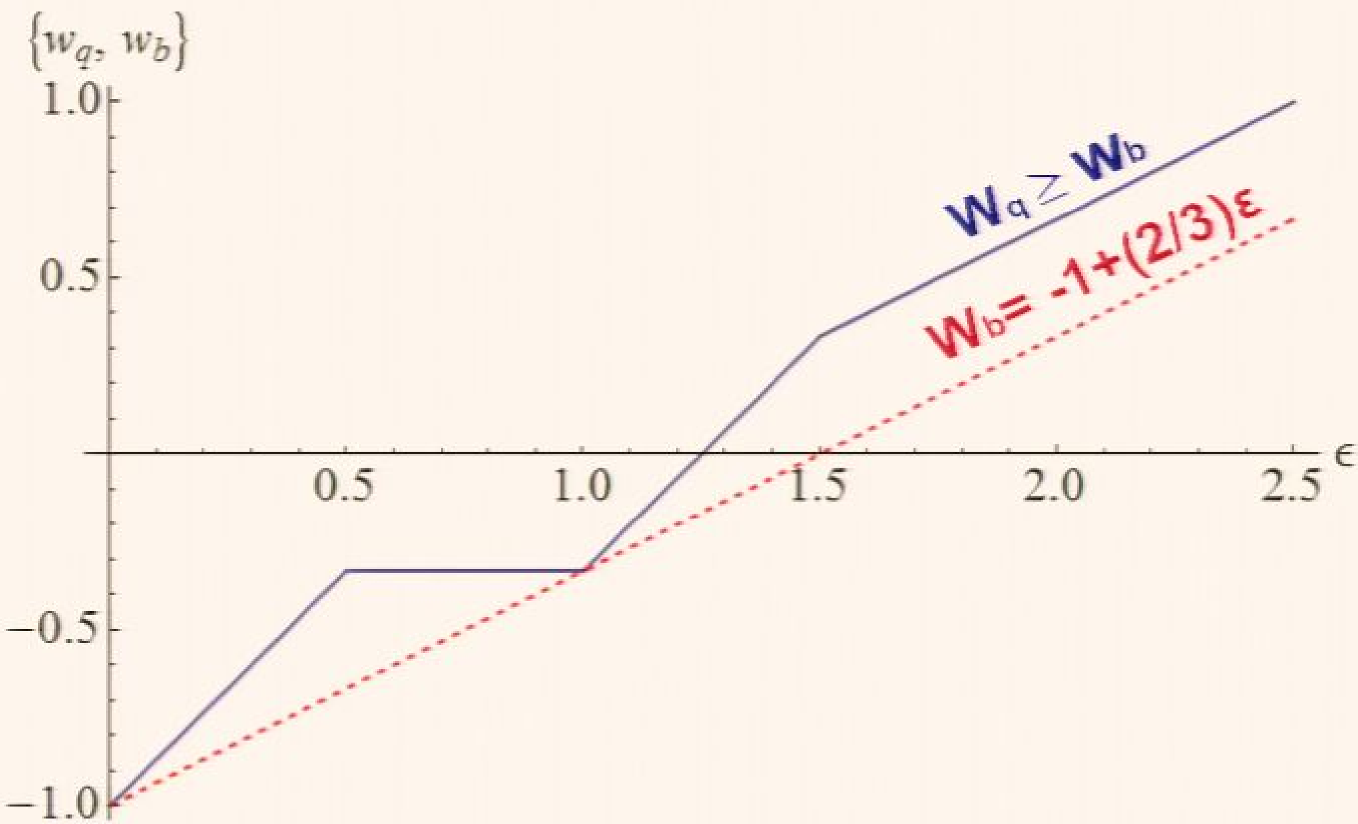

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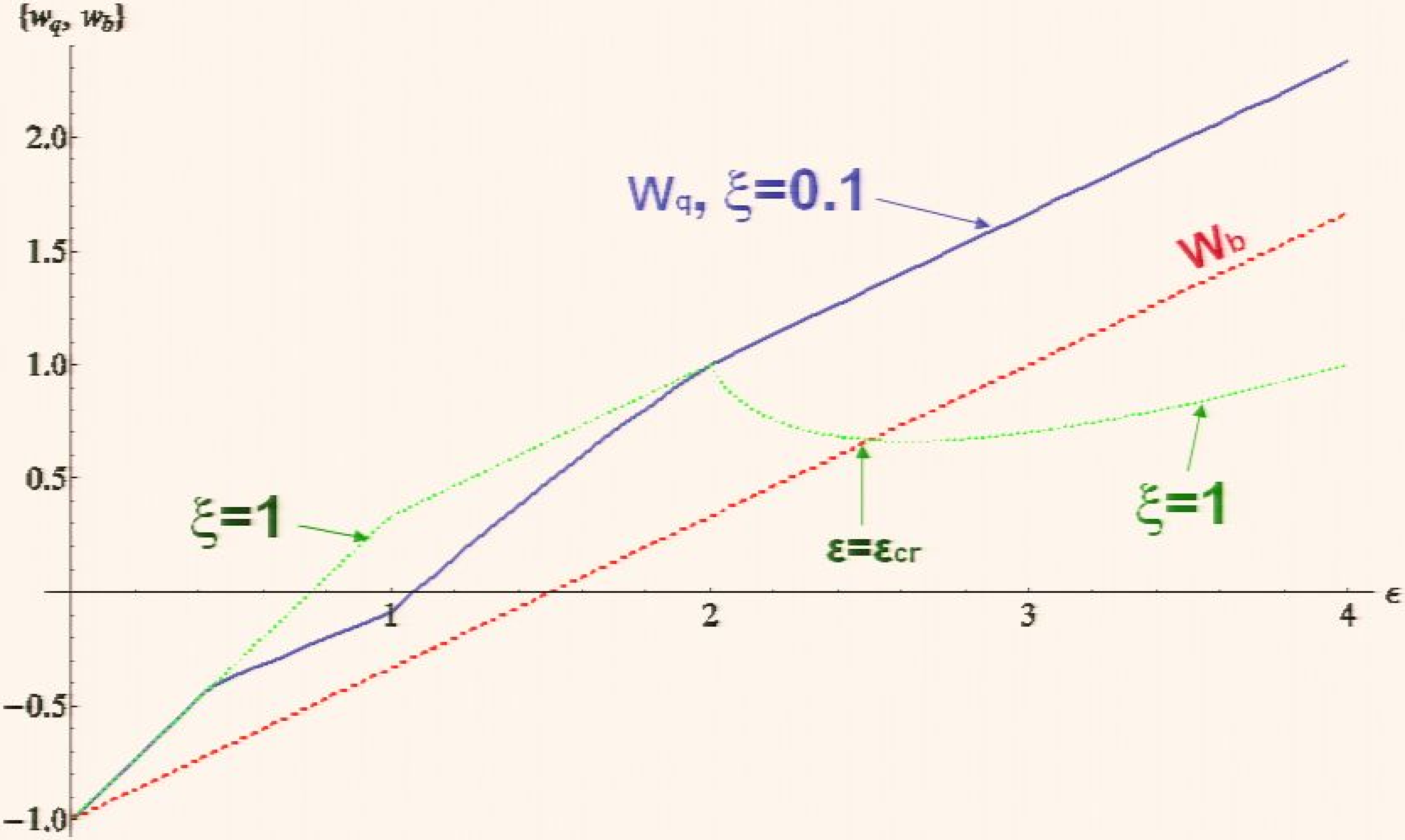

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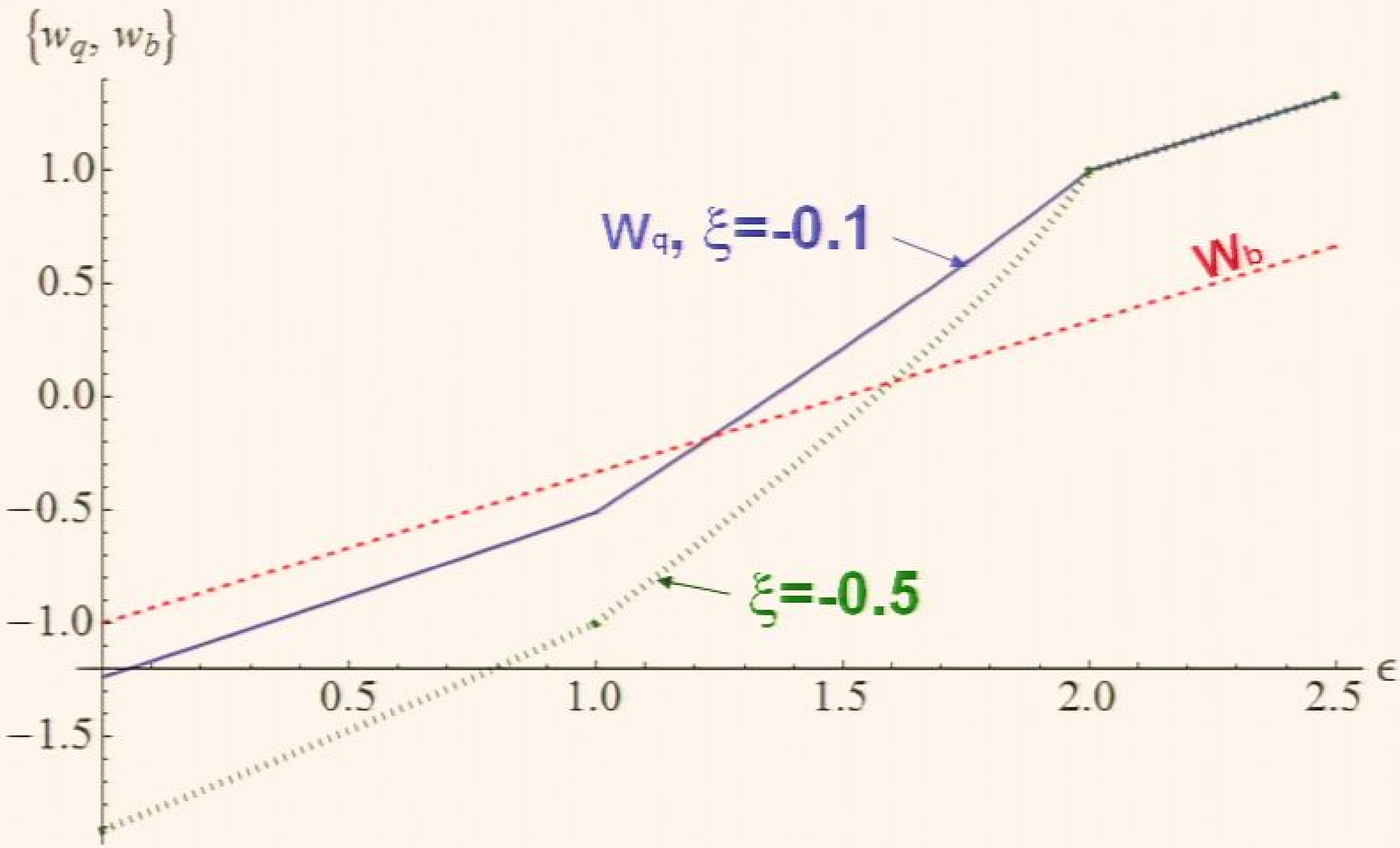

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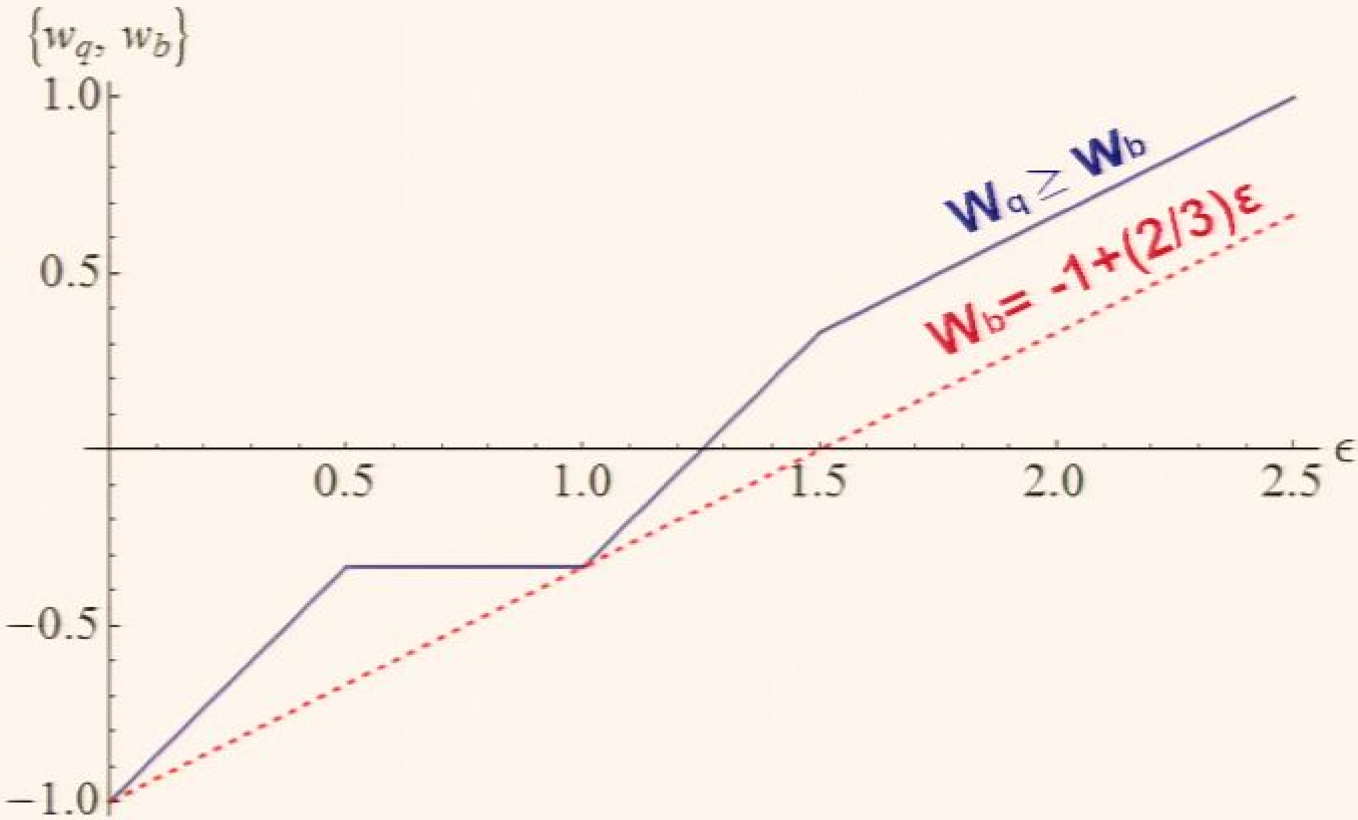

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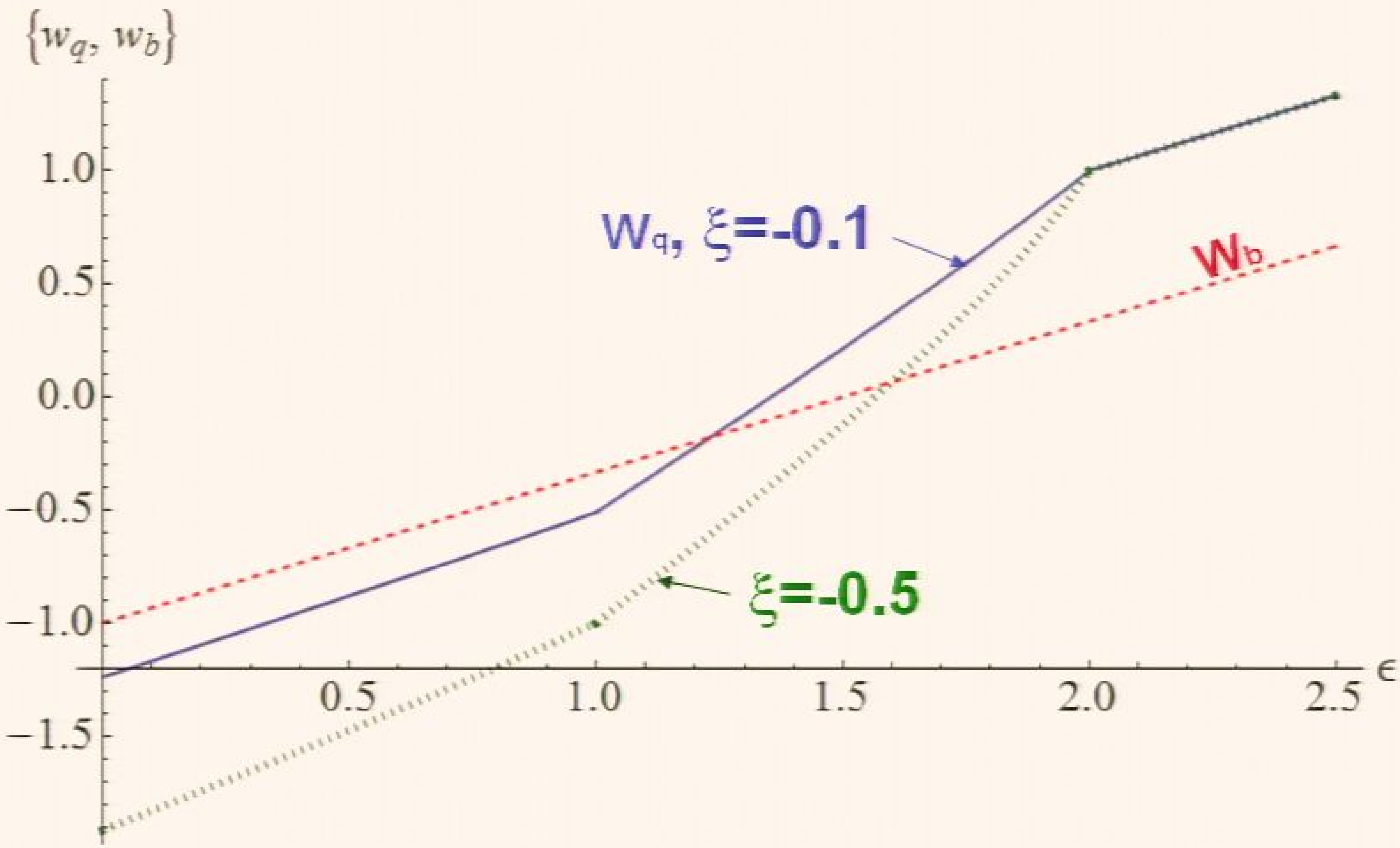

NB: For negative ξ: $w_q<w_b$: $\forall\xi<\xi_{cr}$; ξ_{cr}=0 ($0<\varepsilon<1$); **$\xi_{cr}=-(\varepsilon-1)/[3(2-\varepsilon)]$** ($1<\varepsilon<2$)

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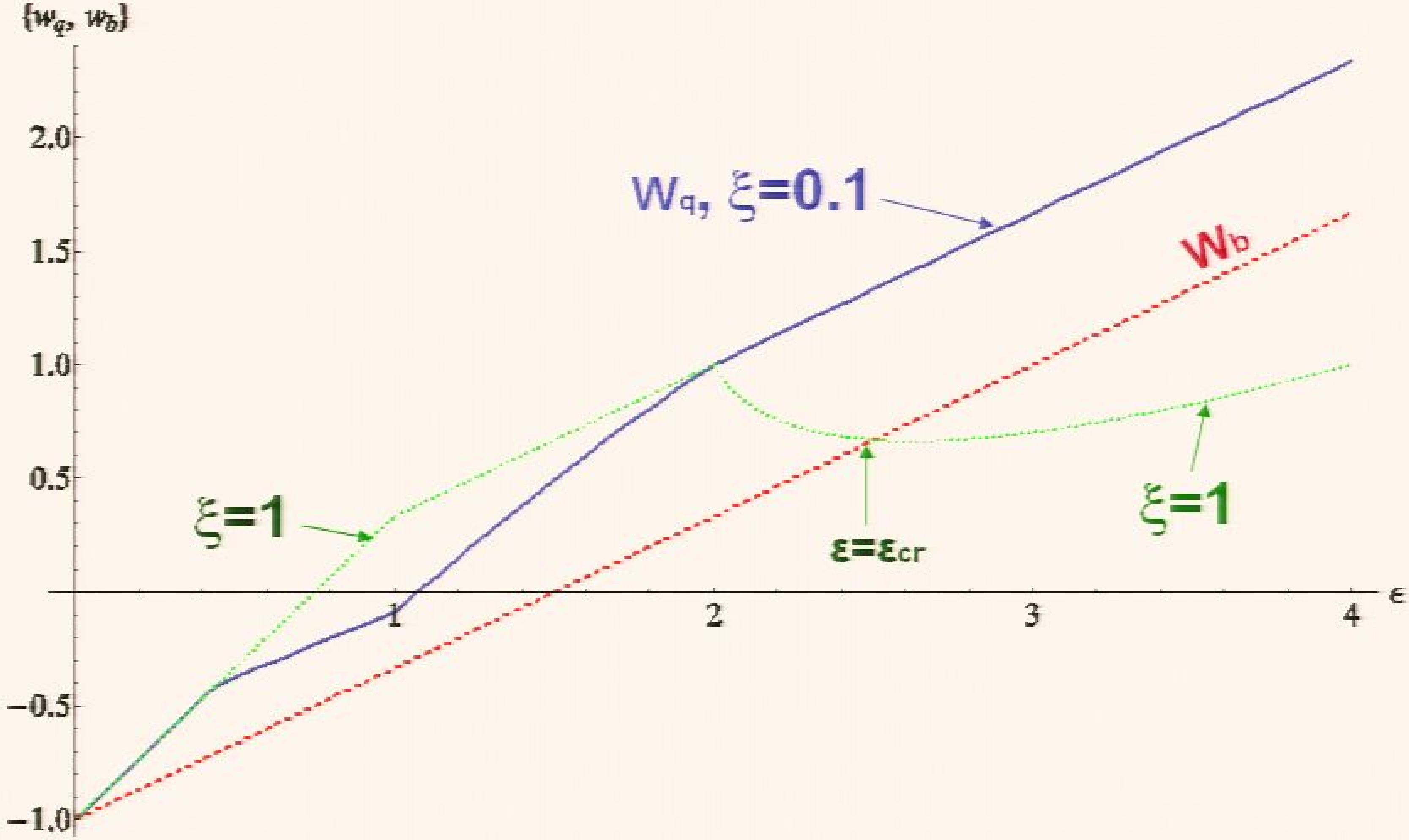
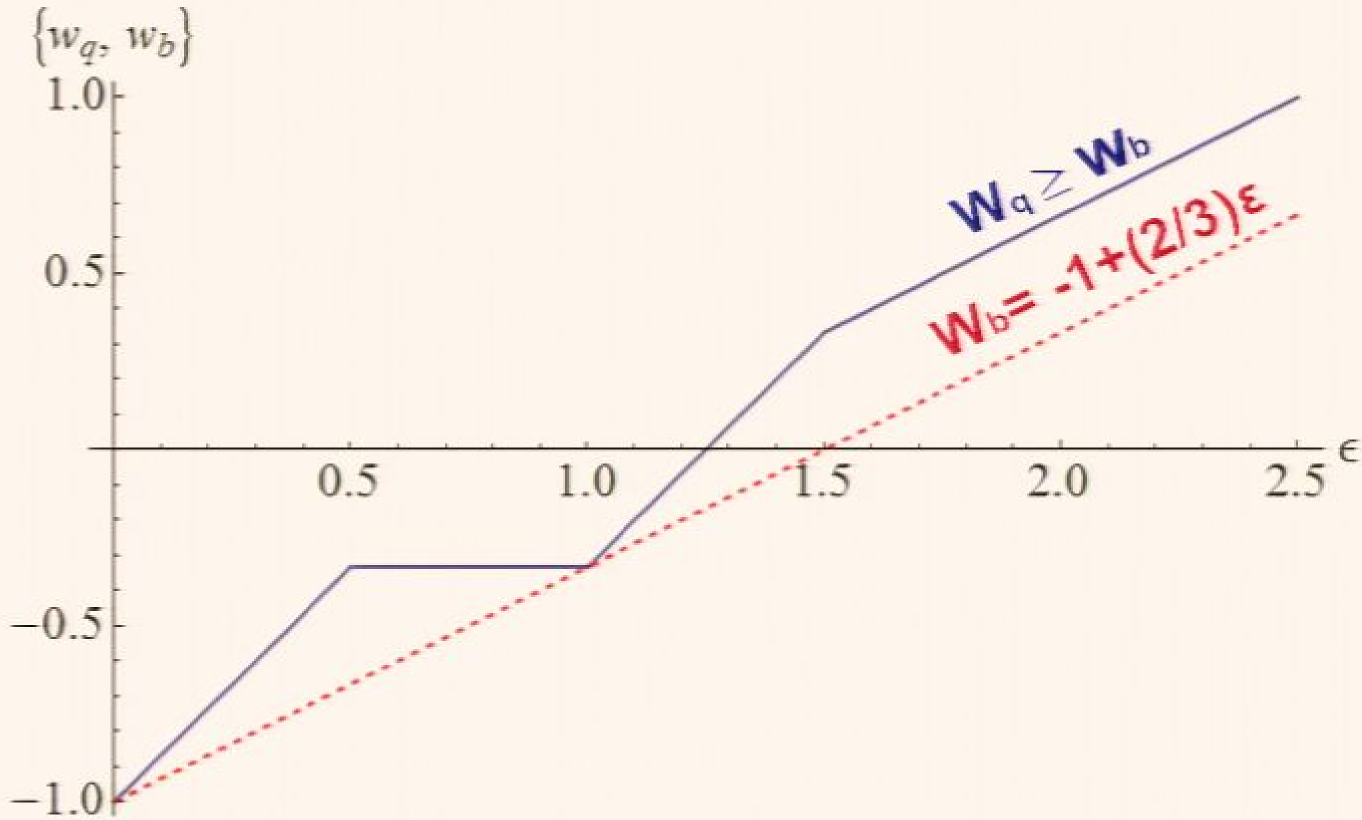


NB: For small positive ξ: $w_q>w_b$ $\forall\varepsilon$; for large $\xi>1/6$, $w_q<w_b$ if $\varepsilon>\varepsilon_{cr}>2$

CLASSICAL vs. QUANTUM DYNAMICS

- **QUANTUM & CLASSICAL ENERGY DENSITY SCALING:** w_q vs w_b



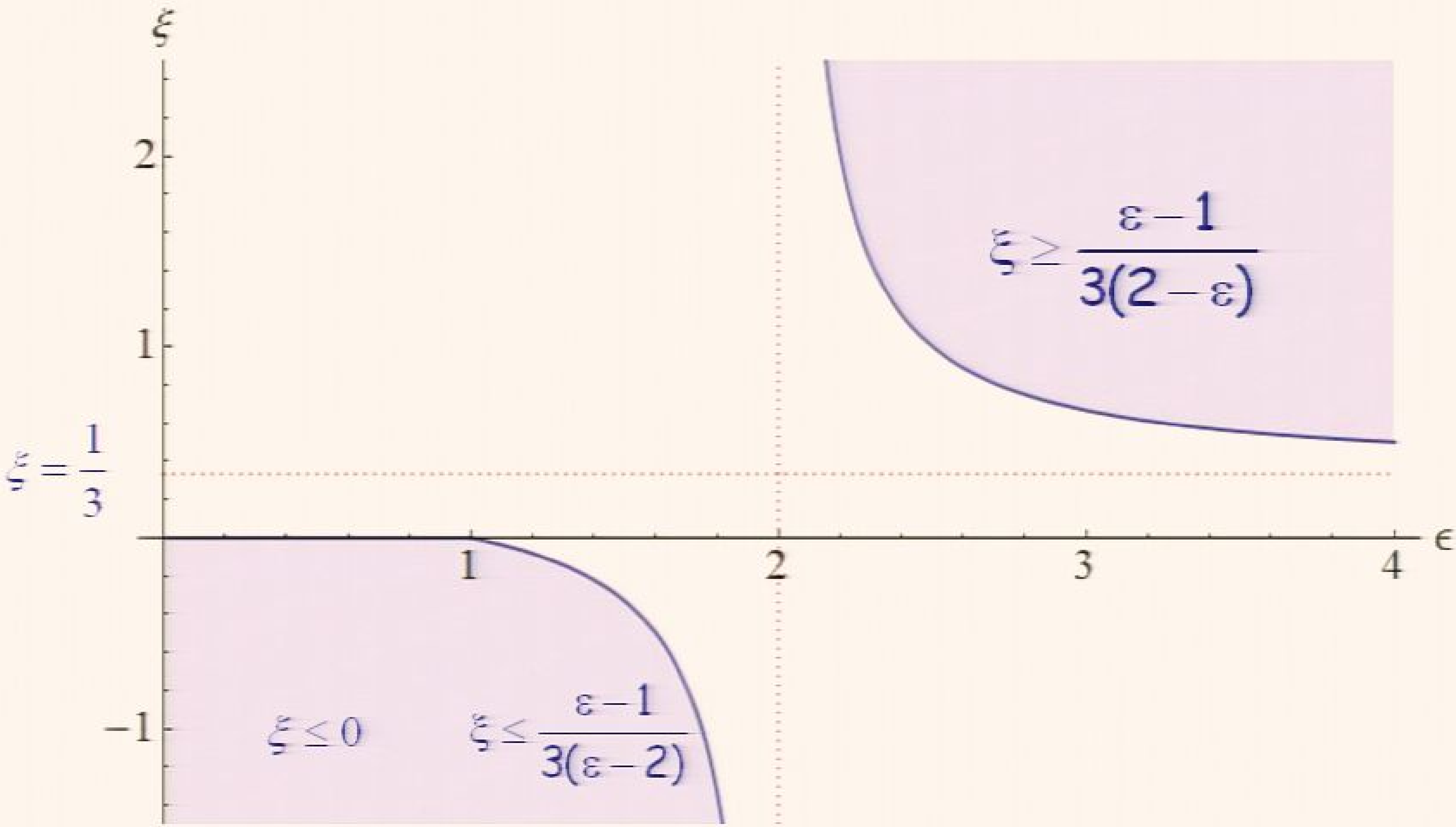
NB: We expect that for ε<1 the graviton undergoes the same scaling:

$$w_q = -1 + \frac{4}{3}\varepsilon \quad (0 < \varepsilon < 1/2), \quad w_q = -\frac{1}{3} \quad (1/2 < \varepsilon < 1)$$

- **for ε>1 we expect the ξ=0 scaling:** $w_q = -\frac{1}{3} + \frac{4}{3}(\varepsilon - 1) \quad (1 < \varepsilon < 2), \quad w_q = 1 + \frac{2}{3}(\varepsilon - 2) \quad (\varepsilon > 2)$

THE (ξ,ε) REGIONS WHERE $W_q < W_b$

▫ QUANTUM ENERGY DENSITY & PRESSURE



SHADED REGIONS: $W_q < W_b$: after some time, ρ_q will become dominant over ρ_b

CAN QUANTUM FLUCTUATIONS BE DARK ENERGY?

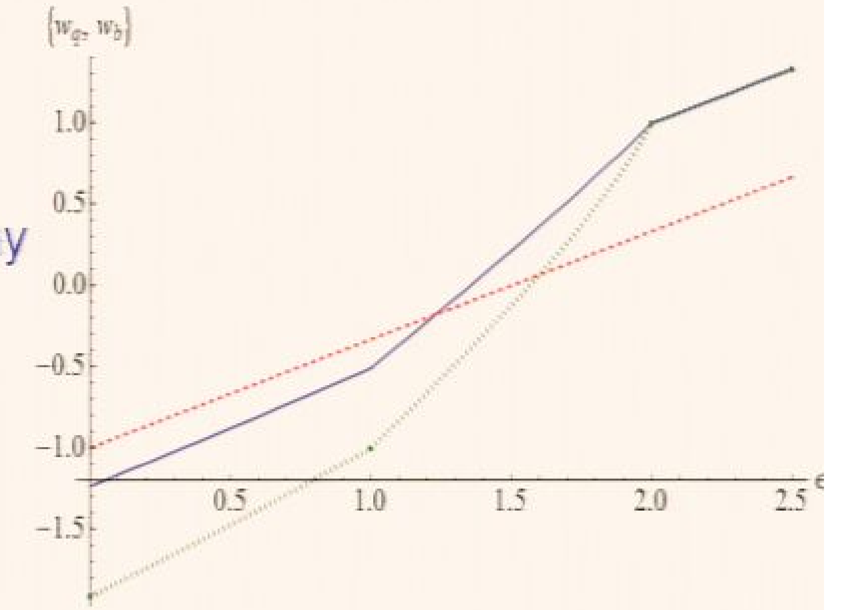
- **SIMPLE ESTIMATE: IMAGINE that quantum fluctuations generated at matter-radiation equality (z~3200) are responsible for dark energy**

$$\xi \sim -280, \quad \nu \sim 58, \quad w_q \sim -28$$

→ Ups! It does not work! A much earlier transition is needed!

→ But, from: $t_{cr} \sim \hat{t}\left(\frac{\hat{H}}{M_{Pl}}\right)^{\frac{1+w_b}{w_b-w_q}}$

we have learned that typically a large time delay occurs between the transition and $\rho_q \sim \rho_b$



- NB: It does not work for radiation $\varepsilon=2$

Q: What is the self-consistent evolution for $t > t_{cr}$, when $\rho_q \geq \rho_b$?

Q2: Can ρ_q play the role of dark energy ?

SUMMARY AND DISCUSSION

- The quantum backreaction from massless scalars in ε=const spaces can become large at 1 loop, provided conformal coupling $\xi<0$ ($\varepsilon<2$).

PEN QUESTIONS:

- What about other IR regularisations: (scalar) mass, positive curvature, finite box
- What about the backreaction from scalars/gravitons at higher loop order, non-constant ε FLRW spaces, inhomogeneous spaces, .. ?
- The backreaction from fermions is large, and distabilises the Universe, driving it to a negative energy Universe: can that be stabilised?

Koksma & Prokopec

▶ What is the effect of $d\varepsilon/dt \neq 0$ (mode mixing)?

▶ is the backreaction gauge dependent (for gravitons)? (Exact gauge?)

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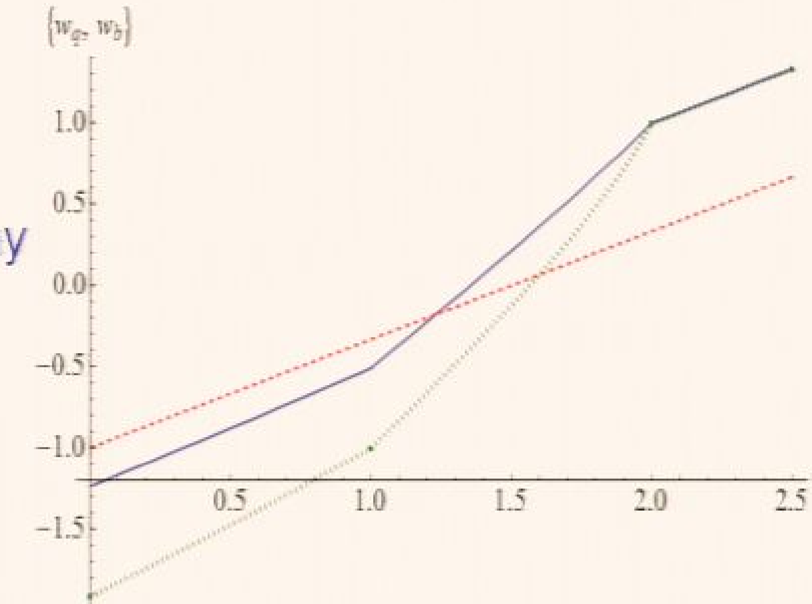

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$\propto \frac{1}{a^4}$

$\xi R \phi^2$ | $\delta\Lambda = \pm m^4 + \ldots$

$m(\phi(t))$

$|\Omega\rangle$ 3D

$$g_{\mu\nu} = a^2(\eta)\,\eta_{\mu\nu}$$

$$T_{\mu\nu} = u_\mu u_\nu(\rho_b + P_b) - g^b_{\mu\nu} P_b$$

$W_q < W_b$

$T^q_{\mu\nu} \lesssim T^b_{\mu\nu}$ | $R = 6(2-\epsilon)H^2$

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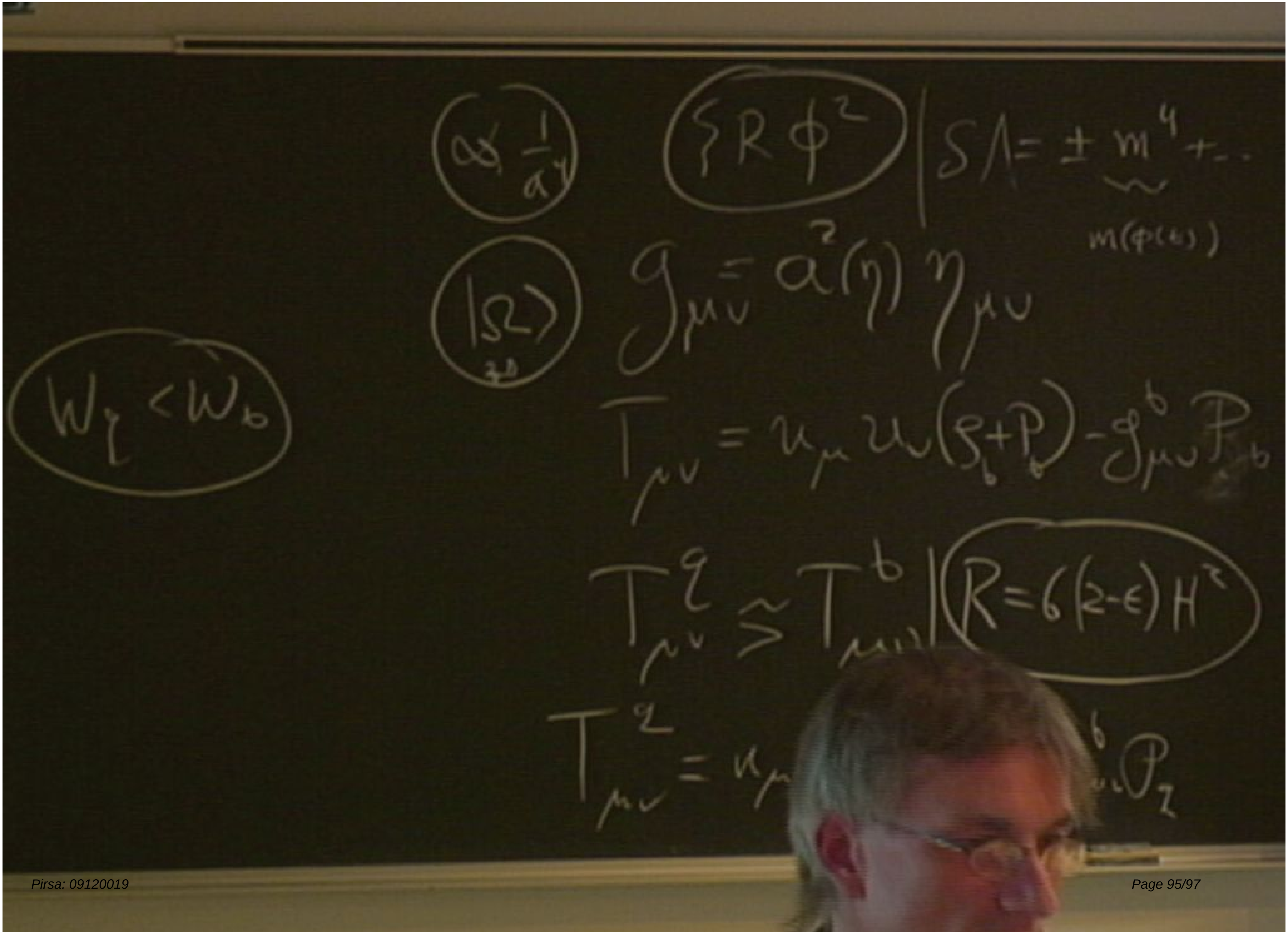
∝ 1/a⁴
ξRφ²
δΛ = ± m⁴ + ...
m(φ(x))
|Ω⟩
g_μν = a²(η) η_μν
W_ℓ < W_b
T_μν = u_μ u_ν (ρ_b + P_b) − g_μν P_b
T^ℓ_μν ≲ T^b_μν
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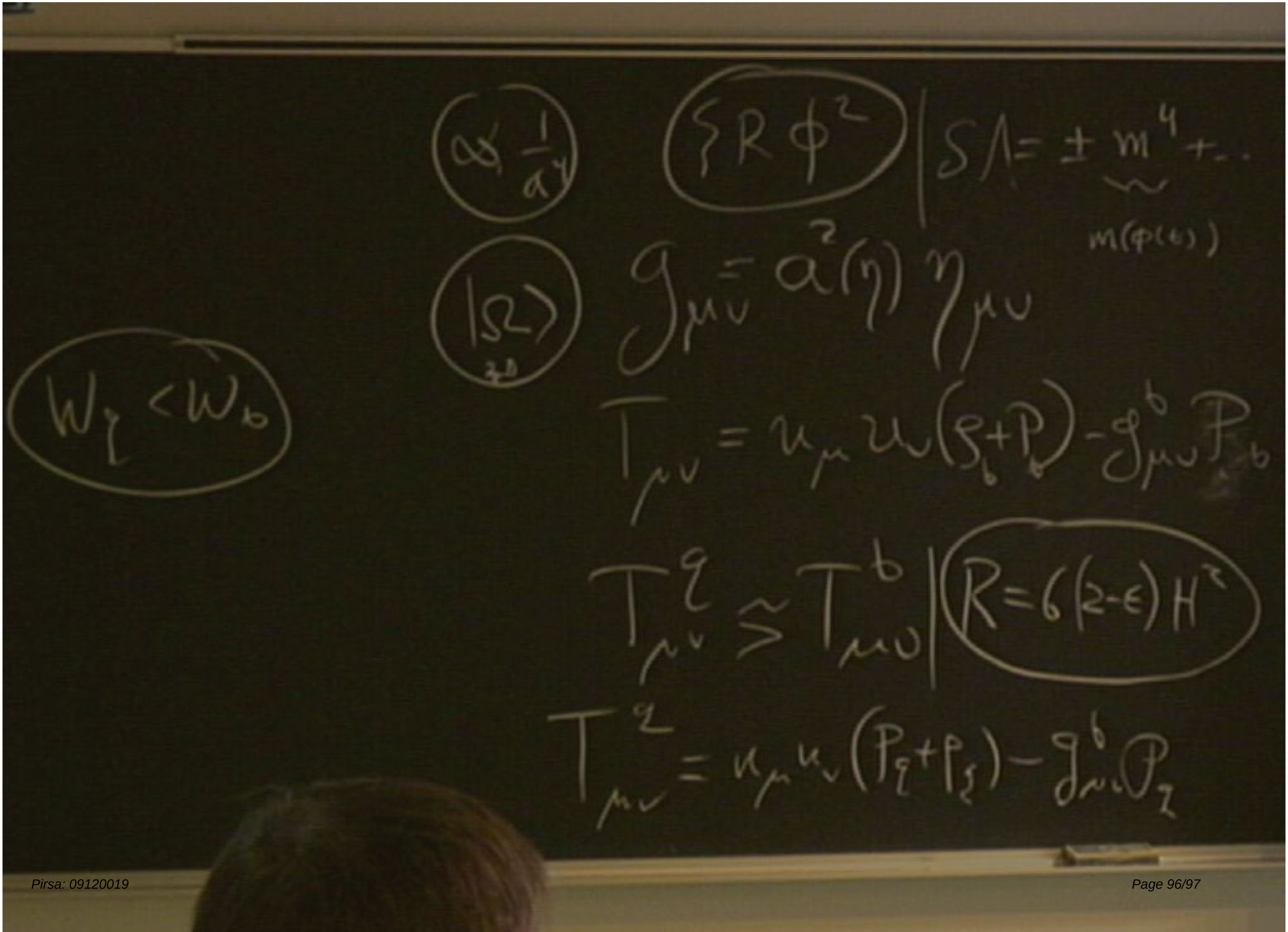
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