Title: The Attractiveness of Higher Dimensional Operators for Inflation

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Abstract: Scalar field models of early universe inflation are effective field theories, typically valid only up to some UV energy scale, and receive corrections through higher dimensional operators due to the UV physics. Corrections to the tree level inflationary potential by these operators can ruin an otherwise suitable model of inflation. In this talk, I will consider higher dimensional kinetic operators, and the corrections that they give to the dynamics of the inflaton field. In particular, I will show how inflationary solutions exist even when the higher dimensional operators are important and not tuned to be negligible. I will then show that these solutions, which include the usual slow roll inflationary solutions, are attractors in phase space. I will end by speculating on the role of the corrections from these higher dimensional operators in alleviating the homogeneous initial conditions problem for inflation.

The Attractiveness of Higher Dimensional Kinetic Operators for Inflation

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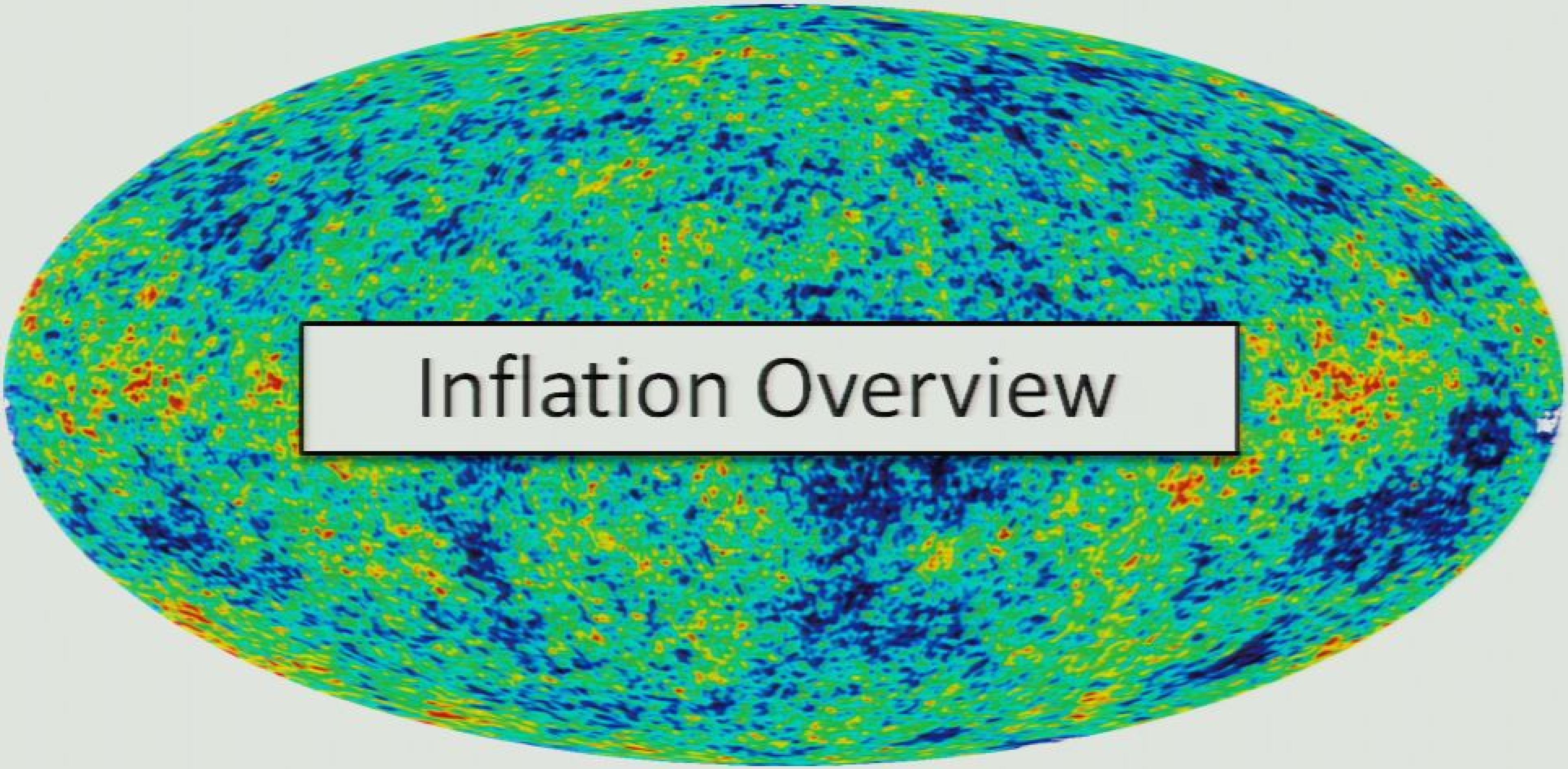
arXiv: 0912.1857
arXiv: 10xx.xxxx (in preparation)

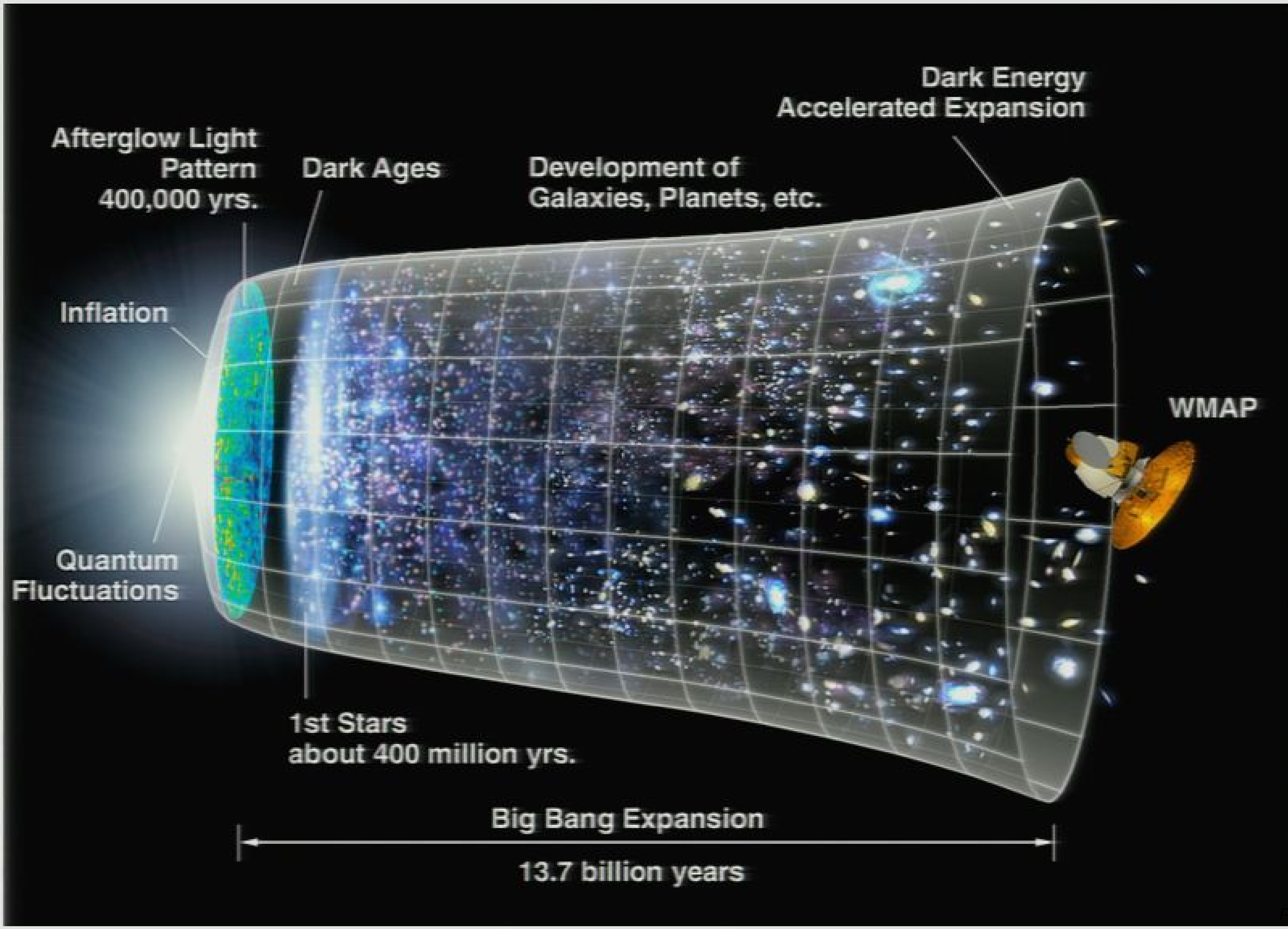
Outline

- Inflation Overview
- Motivation: EFT and Inflation
- Higher Dimensional Kinetic Operators:
  - Non-Canonical Inflationary Solutions
  - Non-Canonical Inflation is *Attractive*
  - Requirements for Non-Canonical Inflation
  - Examples
- Phase Space Dynamics & the Overshoot Problem

Inflation Overview



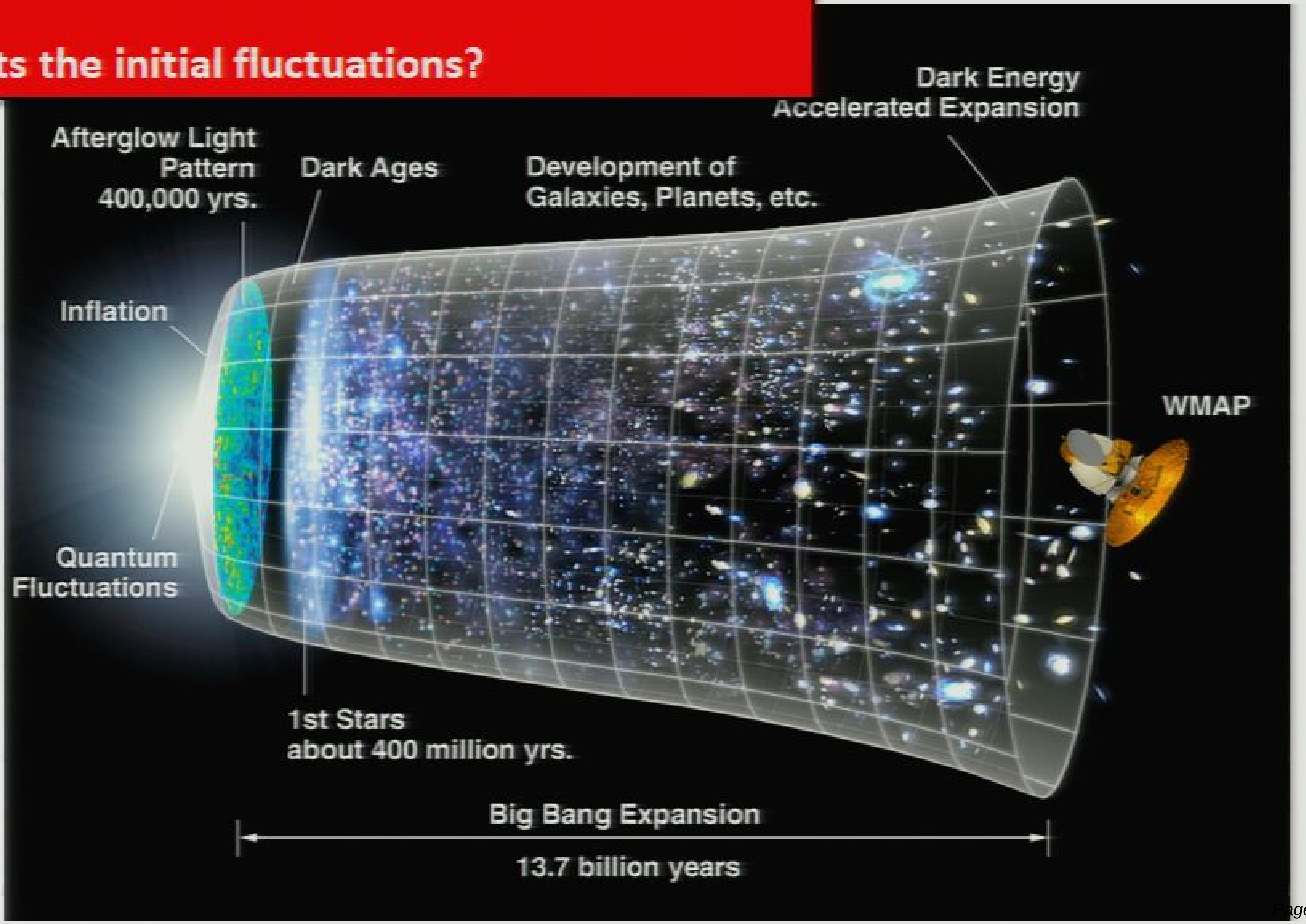
Cosmology



WMAP

Cosmology

- Why is the universe so flat and homogeneous?
- What sets the initial fluctuations?



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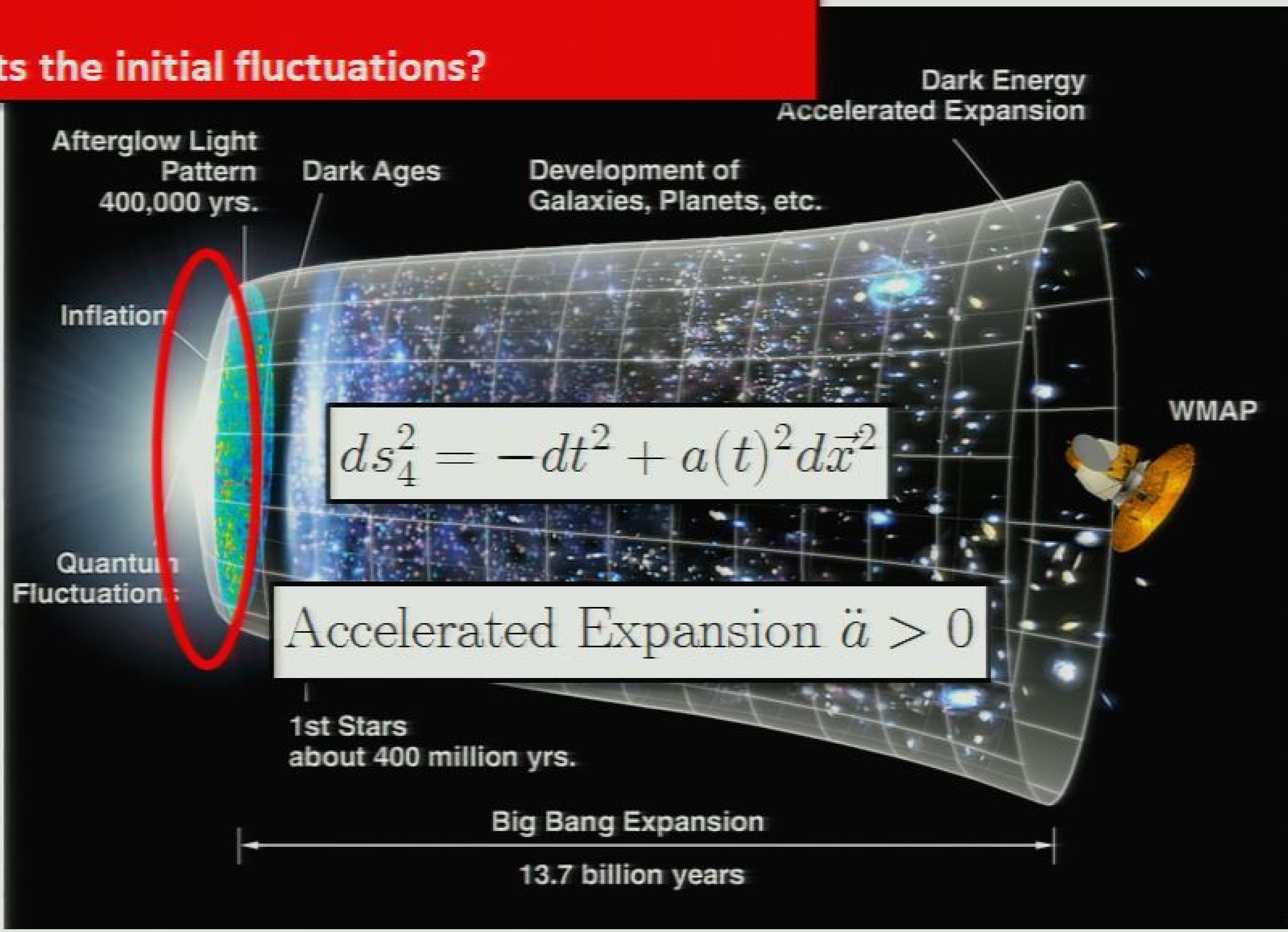

$$ds_4^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

Accelerated Expansion $\ddot{a} > 0$

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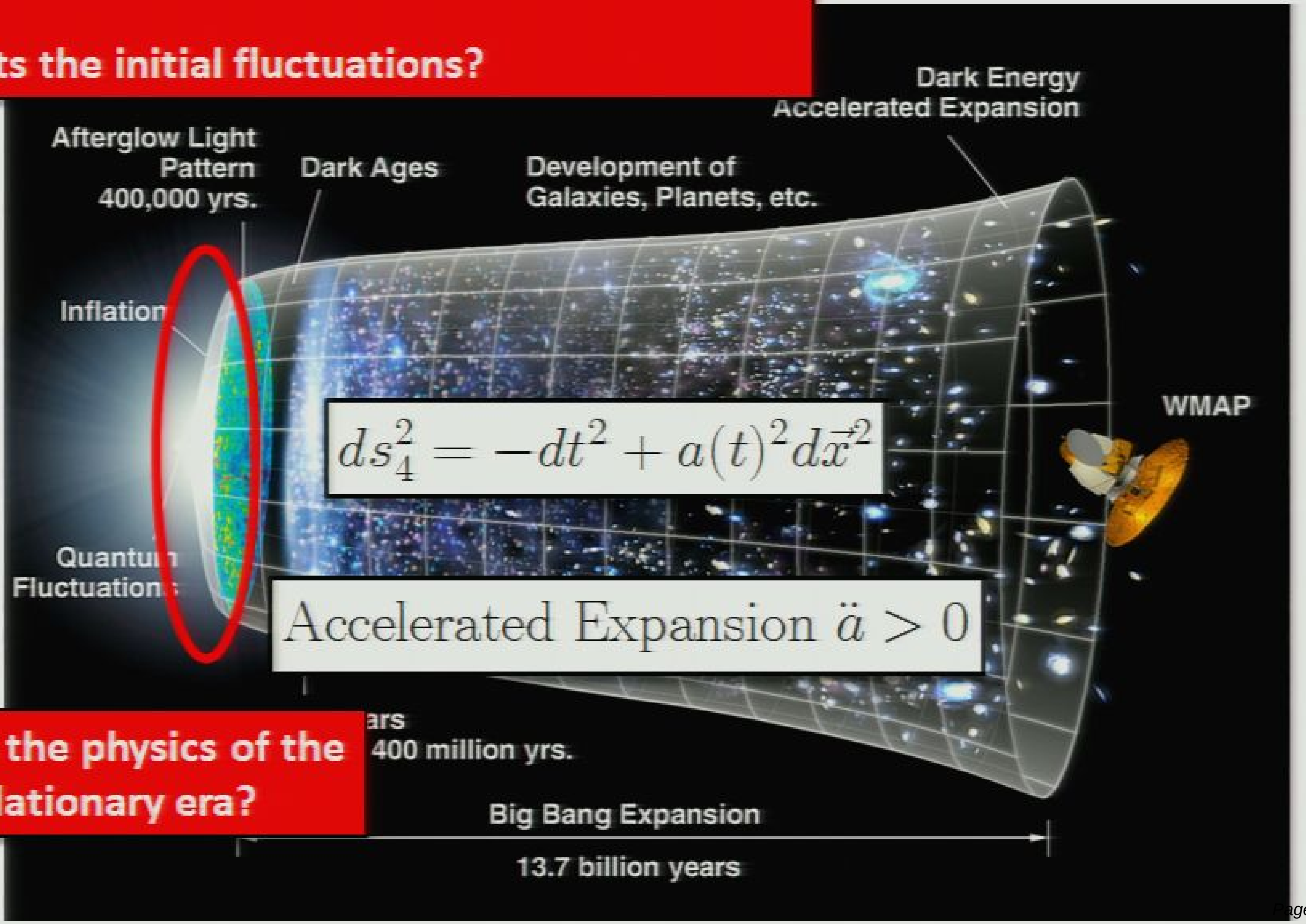


$$ds_4^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

Accelerated Expansion $\ddot{a} > 0$

What is the physics of the inflationary era?

WMAP

Classical Inflaton Physics

Why is the universe so flat and homogeneous?

$$ds_4^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

$$\text{Accelerated Expansion } \ddot{a} > 0$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_p^2}$$

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \qquad \eta \equiv \frac{\dot{\epsilon}}{H\epsilon}$$

Inflation happens… …for long enough.

When energy density/Hubble parameter is constant, we get inflation – *exponential expansion.*

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 \approx \text{ const } \Rightarrow a(t) \sim e^{Ht}$$

$$\text{Needs } \epsilon \ll 1, \quad |\eta| \ll 1$$

Classical Inflaton Physics

Parameterize Inflation by physics of a scalar field (Inflaton):

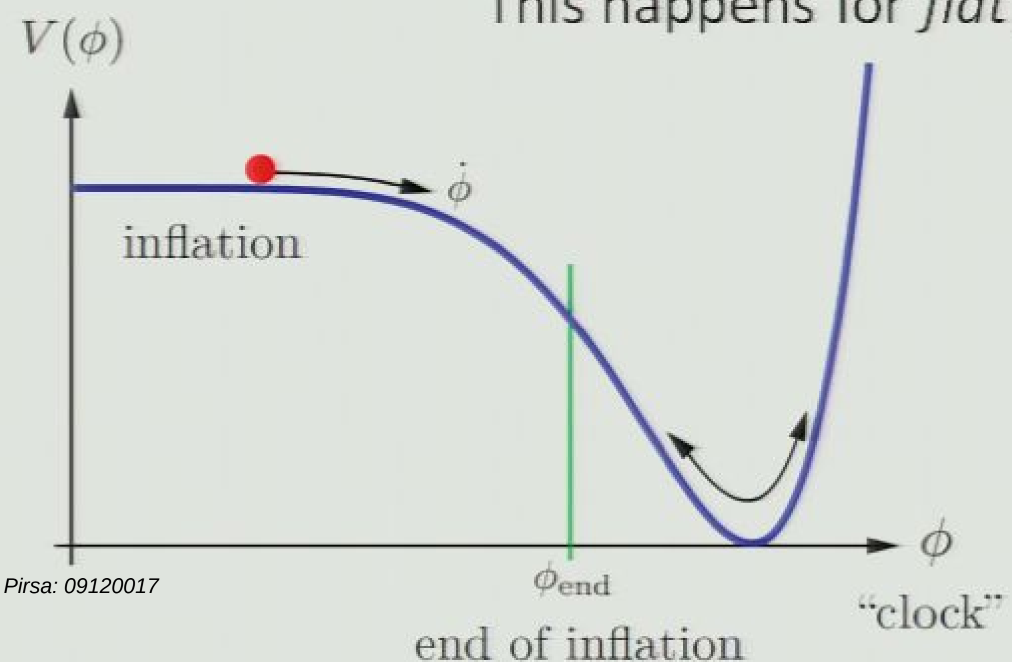
$$\mathcal{L}_{inf} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

"When energy density/Hubble parameter is constant, we get inflation."

H is constant when potential energy dominates:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \approx V(\phi)$$

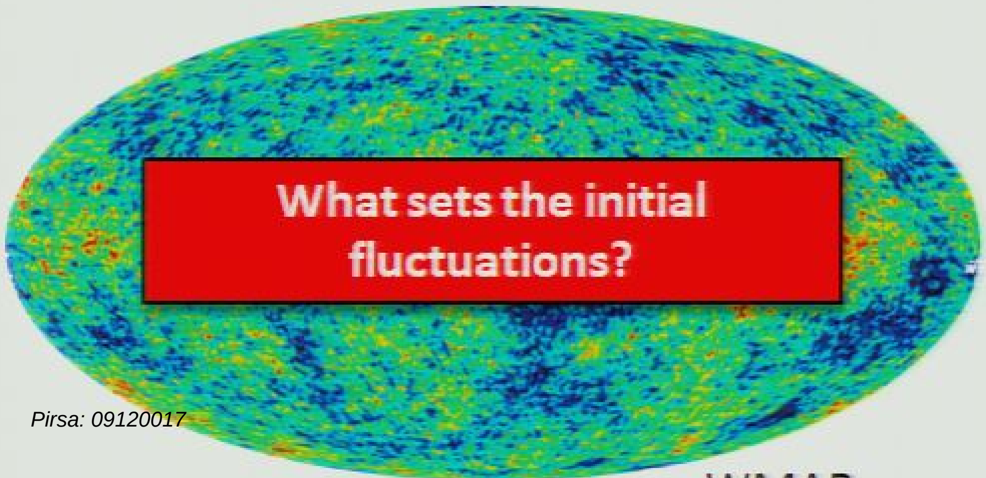
This happens for *flat potentials*:



$$\epsilon \rightarrow \epsilon_{SR} \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta \rightarrow \eta_{SR} \equiv M_p^2\left(\frac{V''}{V}\right)$$

Quantum in the Sky



WMAP

$$\delta T, \delta\zeta$$

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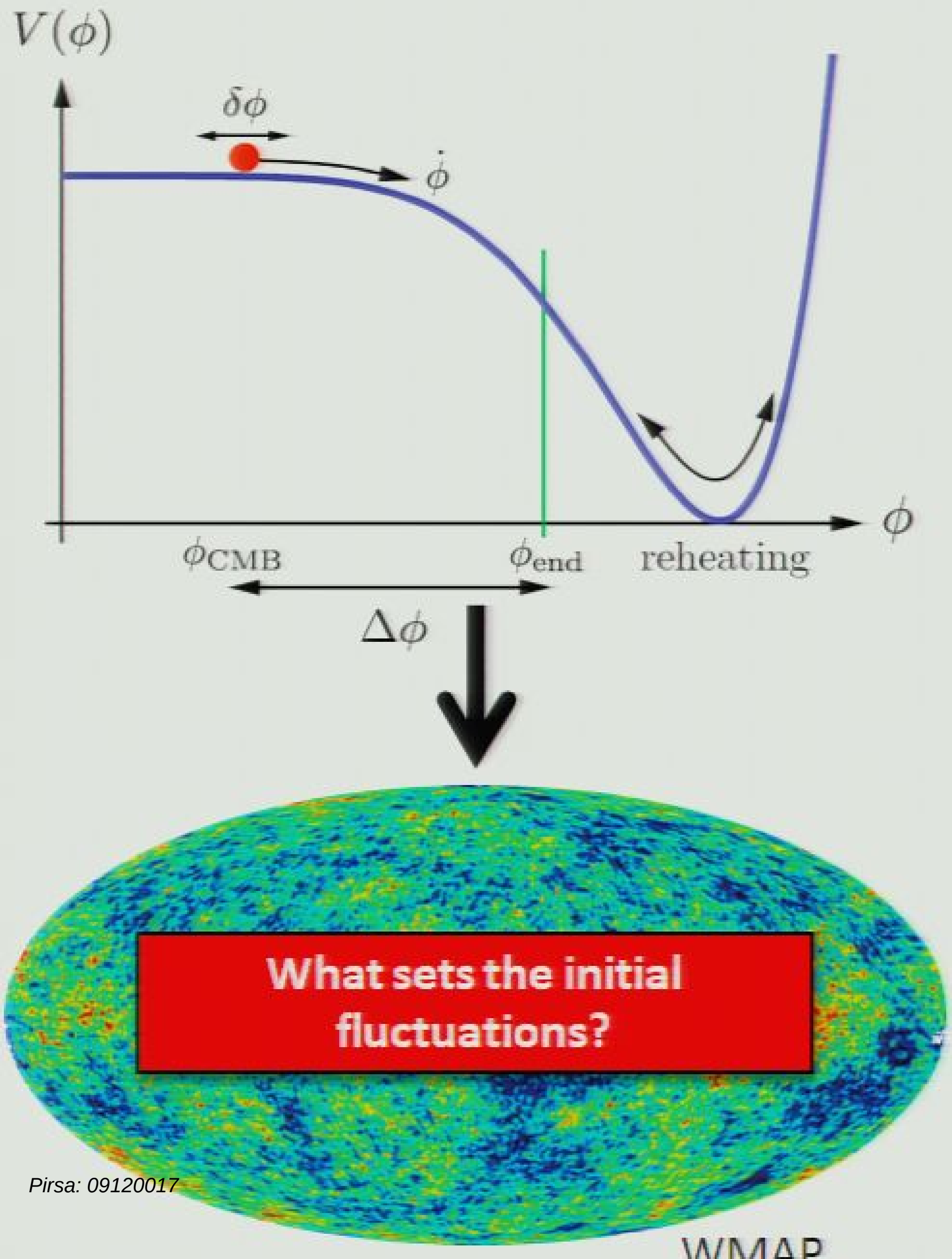


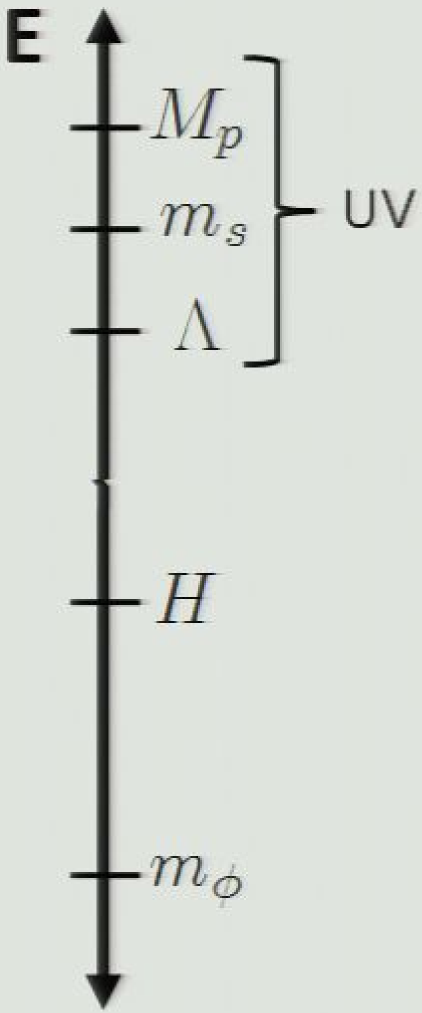
Quantum fluctuations of inflaton lead to density fluctuations in post-inflationary universe:

$$\delta\phi$$

$$\downarrow$$

$$\delta T, \delta\zeta$$

Motivation: EFT & Inflation



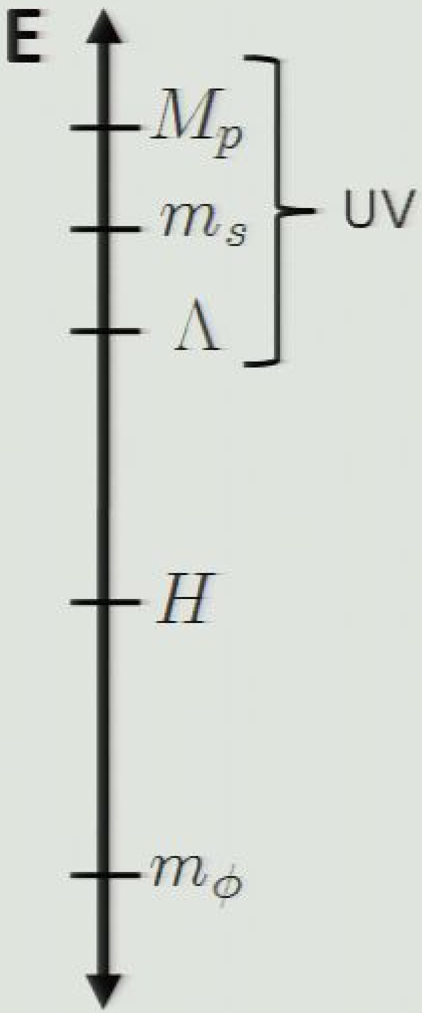
Inflation is an Effective Field Theory (EFT)

Receives corrections from UV physics through higher dim. Operators
(e.g. 4-Fermion interaction, proton decay)

$$\mathcal{L}_{eff} = \mathcal{L}_{relevant} + \sum_n c_n \frac{\mathcal{O}_n}{\Lambda^{n-4}}$$

$\mathcal{O}_n$ are all possible operators consistent with symmetries.

Motivation: EFT & Inflation



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<u>Inflation is sensitive to higher dimensional operators</u>

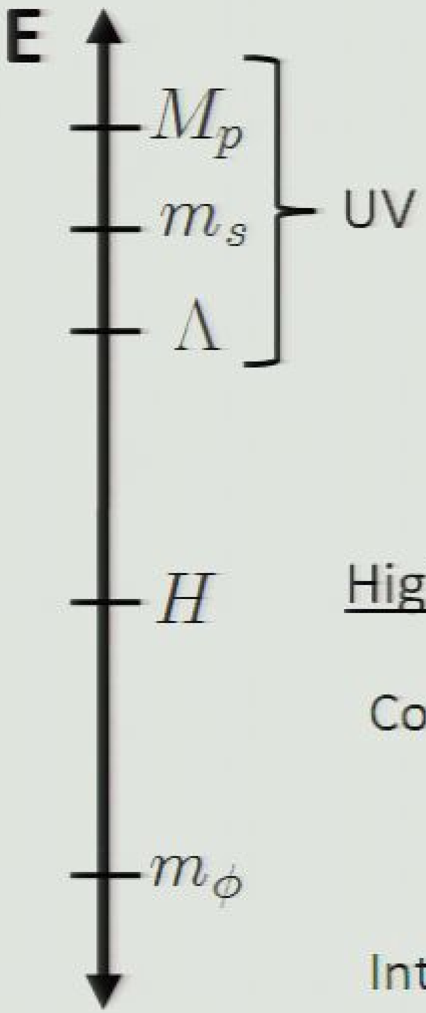
(e.g. corrections to the potential)

$$V_{eff} = V_0 + m_0^2\phi^2 + \frac{\mathcal{O}_4\phi^2}{M_p^2}$$

$$\langle \mathcal{O}_4 \rangle \sim V_0 \Rightarrow \eta \sim \mathcal{O}(1)$$

<u>Higher dimensional operators can also correct</u>
<u>*kinetic terms*</u>

Motivation: EFT & Inflation



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Higher dimensional operators can also correct *kinetic terms*

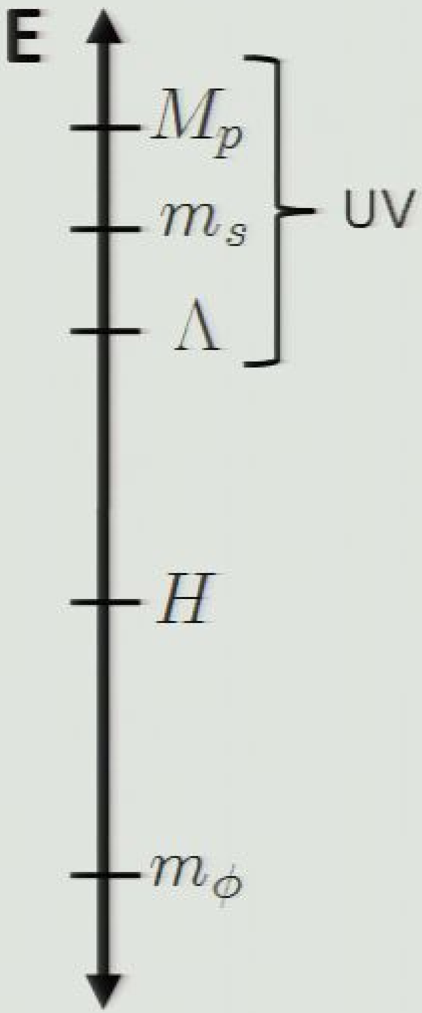
Consider a two field model

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\rho)^2 + \frac{\rho}{\Lambda}(\partial\phi)^2 - \Lambda^2\rho^2 - V_{inf}(\phi)$$

Integrating out the massive field gives (see Tolley, Wyman, 0910.1853)

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial\phi)^2 - V_{inf}(\phi) + \frac{(\partial\phi)^4}{\Lambda^4} + \ldots$$

Motivation: EFT & Inflation



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$\mathcal{O}_n$ are all possible operators consistent with symmetries.

<u>Goals:</u>

- Construct explicit inflationary solutions for non-canonical kinetic terms
- Non-canonical inflation has attractor in phase space.
- What properties must the Lagrangian have in order to support non-canonical inflation?

Non-Canonical* Inflationary Lagrangian

$$S = \int d^4x\sqrt{g_4}\left[\frac{M_p^2}{2}\mathcal{R}_4 + p(X,\phi)\right]$$

$$X \equiv -\frac{1}{2}(\partial\phi)^2 = \frac{1}{2}\dot{\phi}^2$$

*(Not really most general Lagrangian; can have higher powers of derivatives. More later...)

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Canonical: $p(X,\phi) = X - V(\phi)$

DBI: $p(X,\phi) = -\Lambda^4 \left[\sqrt{1 - 2\frac{X}{\Lambda^4}} - 1 \right] - V(\phi) \approx X - V(\phi) + \ldots$ for $2X/\Lambda^4 \ll 1$

Silverstein, Tong

Closed String Tachyon: $p(X,\phi) = -V(\phi)\sqrt{1 - 2\frac{X}{\Lambda^4}}$ A. Sen

k-inflation: $p(X,\phi) = \frac{-X + X^2}{\phi^2}$ Mukhanov, et al

Series Lagrangian: $p(X,\phi) = X - V(\phi) + c_1(\phi)\frac{X^2}{\Lambda^4} + \ldots = \sum_{n\geq 0} c_n(\phi)\frac{X^{n+1}}{\Lambda^{4n}} - V(\phi)$

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<u>My Restrictions</u>:

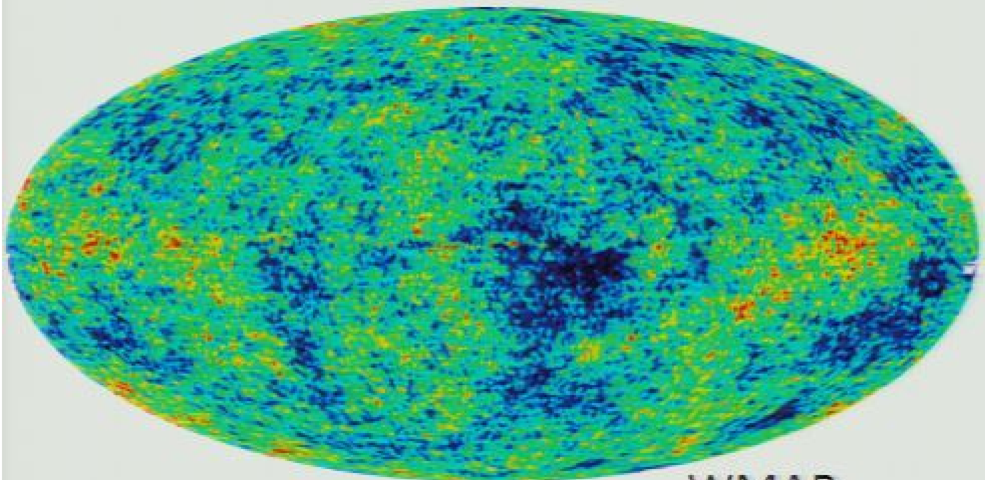
(1) Must obey the Null Energy Condition $T_{ab}k^a k^b \geq 0 \Rightarrow \partial_X p = p_X \geq 0$

(2) Must have *physical* speed of perturbations $c_s^2 = \left(1 + 2X\partial_X^2 p / \partial_X p\right)^{-1} \leq 1 \Rightarrow \partial_X^2 p \geq 0$

(3) Must reduce to canonical Lagrangian $p(X,\phi) \approx X - V(\phi) + \ldots$

Phenomenology of Non-Canonical Lagrangian

$$\mathcal{L} = p(X, \phi), \quad X \equiv -\frac{1}{2}(\partial\phi)^2$$



"Non-canonical-ness" of kinetic terms important if

$$c_s^2 = \left(1 + 2X\partial_X^2 p/\partial_X p\right)^{-1} \ll 1$$

$$(\text{e.g. } \partial_X^2 p > 0)$$

Armendáriz-Picón, Damour, Mukhanov
Garriga, Mukhanov
Chen, Huang, Kachru, Shiu

Inflationary Parameters:

$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$

$$\eta \equiv \frac{\dot{\epsilon}}{H\epsilon} = 4\epsilon - \eta_X - \eta_\Pi$$

$$\kappa \equiv \frac{\dot{c}_s}{c_s H}$$

Observables:

$P_k^\zeta = \frac{1}{8\pi^2}\frac{H^2}{M_p^2}\frac{1}{c_s\,\epsilon}\Big\vert_{c_s k = aH}$	Scalar Power Spectrum
$n_s - 1 = -2\epsilon - \eta - \kappa$	Tilt of Scalar Spectrum
$r = 16\, c_s\, \epsilon$	Tensor-to-Scalar ratio
$f_{NL}^{(equil)} \sim c_s^{-2}$	Equilateral Non-Gaussianity

But…Recall Canonical Inflation

Equations of motion

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \qquad H^2 = \frac{\frac{1}{2}\dot{\phi}^2 + V(\phi)}{3M_p^2}$$

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Slow-roll Inflationary Approximation/Solution

$$\ddot{\phi} \ll 3H\dot{\phi}, V'(\phi) \qquad \frac{1}{2}\dot{\phi}^2 \ll V(\phi)$$

$$\Rightarrow \quad \dot{\phi}_{slow-roll} = -\frac{V'(\phi)}{3H}$$

Leads to

$$\epsilon \rightarrow \epsilon_{SR} \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2$$

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What is the analog of the "Slow-roll" solution for Non-Canonical Inflation?

Phenomenology of Non-Canonical Lagrangian

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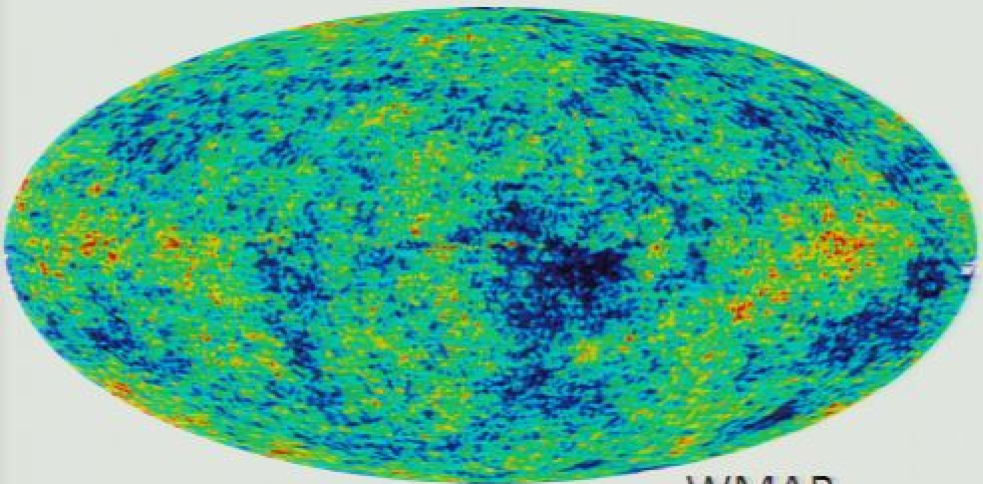

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Canonical Inflation

We will need a more systematic approach…

Equations of Motion

(1st order formulation)

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\dot{\rho} = -3H(\rho + p)$$

or

$$\Pi \equiv \dot{\phi}$$

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Inflationary Parameters:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = -\frac{1}{2}\frac{\dot{\rho}}{H\rho}$$

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Inflation happens… $\epsilon, |\eta| \ll 1$ …for long enough.

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ρ EOM:

$$p = -\rho\left(1 - \frac{2}{3}\epsilon\right)$$

$$\Rightarrow \boxed{p_{inf} \approx -\rho_{inf}}$$

Π EOM:

$$\Pi = -\frac{V'(\phi)}{3H}\left(1 + \frac{\eta}{6} - \frac{\epsilon}{3}\right)^{-1}$$

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Non-Canonical Inflation

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$$\epsilon, |\eta| \ll 1$$

In the presence of higher dimensional kinetic operators.

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Non-Canonical Inflation occurs when

$$\epsilon, |\eta| \ll 1 \quad \text{But far from slow roll} \quad \left[\begin{array}{l} \epsilon_{SR} = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \\ \eta_{SR} = M_p^2\left(\frac{V''}{V}\right) \end{array}\right] \sim \mathcal{O}(1)$$

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Non-Canonical Inflationary Solutions

$$p = -\rho\left(1 + \frac{2}{3}\epsilon\right)$$

$$\Rightarrow\ p_{inf} \approx -\rho_{inf}$$

$$\Pi = -\frac{\partial p}{\partial\phi}\frac{1}{3H}\left(1 + \frac{1}{3}(\epsilon - \eta_\Pi)\right)^{-1}$$

$$\Rightarrow \Pi_{inf} \approx \frac{\partial p}{\partial\phi}\frac{1}{3H}$$

Inflationary Parameters

General Inflationary Parameters

$$\epsilon(\phi) = 3\frac{X_{inf}(\phi)}{\rho_{inf}}\left(\frac{\partial p}{\partial X}\right)_{\Pi=\Pi_{inf}(\phi)} \qquad \eta_X(\phi) = \epsilon(\phi) + \frac{2X_{inf}(\phi)}{2H_{inf}}\frac{\partial_\phi X_{inf}(\phi)}{X_{inf}(\phi)}$$

$$\eta_\Pi(\phi) = \frac{\sqrt{2X_{inf}(\phi)}}{H_{inf}}\left(\frac{\partial^2 p/\partial\phi^2}{\partial p/\partial\phi}\right)_{\Pi=\Pi_{inf}(\phi)}$$

Given the Lagrangian, can explicitly construct inflationary solution, parameters, as functions of ϕ

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Given the Lagrangian, can explicitly construct inflationary solution, parameters, as functions of ϕ

Example: $\mathcal{L} = p(X,\phi) = \Lambda^4\left[\left(1+\frac{2}{3}\frac{X}{\Lambda^4}\right)^{3/2} - 1\right] - V(\phi)$

$$\dot{\phi}_{non-canon} = \Lambda^2\sqrt{\frac{3}{2}\left(\sqrt{1+\frac{4}{3}A(\phi)^2} - 1\right)} \qquad A(\phi) \equiv \left(\frac{2}{3}\epsilon_{SR}(\phi)\frac{V(\phi)}{\Lambda^4}\right)^{1/2}$$

$$\epsilon_{non-canon}(\phi) = \frac{3}{2^{3/2}}\frac{\epsilon_{SR}}{A^2}\left(\sqrt{1+\frac{4}{3}A^2} - 1\right)\left(1+\sqrt{1+\frac{4}{3}A^2}\right)^{1/2} \approx \begin{cases}\epsilon_{SR} & A \ll 1\\ 3^{1/4}\frac{\epsilon_{SR}}{A^{1/2}} & A \gg 1.\end{cases}$$

Comparison to Hamilton-Jacobi

When the scalar field is monotonic, can use ϕ as time variable.

Can rewrite the full EOM (no inflationary approximation) for a general scalar field as:

$$3M_p^2 H^2(\phi) = \frac{4M_p^4 (H')^2}{p_X(X(H',\phi),\phi)} - p(X(H',\phi),\phi) \qquad \left(H' \equiv \frac{dH}{d\phi}\right)$$

$$\frac{d\phi}{dt} = \sqrt{2X} = -\frac{2M_p^2}{p_X} H', \quad \frac{d\phi}{dN_e} = -\frac{2M_p^2}{p_X}\left(\frac{H'}{H}\right)$$

Bean, Chung, Geshnizjani

Slow Variation Parameters become:

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{2M_p^2}{p_X}\left(\frac{H'}{H}\right)^2$$

$$\eta = \frac{d\ln\epsilon}{dN_e} = -\frac{4M_p^2}{p_X}\left(\frac{H''}{H}\right) + 2\epsilon + 2M_p^2\left(\frac{H'}{H}\right)\left(\frac{p'_X}{p_X}\right)$$

$$\kappa = \frac{dc_s}{dN_e} = \frac{2M_p^2}{p_X}\left(\frac{H'}{H}\right)\left(\frac{(c_s^{-1})'}{(c_s^{-1})}\right)$$

Comparison to Hamilton-Jacobi

When the scalar field is monotonic, can use ϕ as time variable.

Can rewrite the full EOM (no inflationary approximation) for a general scalar field as:

$$3M_p^2 H^2(\phi) = \frac{4M_p^4 (H')^2}{p_X(X(H',\phi),\phi)} - p(X(H',\phi),\phi) \qquad \left(H' \equiv \frac{dH}{d\phi}\right)$$

$$\frac{d\phi}{dt} = \sqrt{2X} = -\frac{2M_p^2}{p_X} H', \quad \frac{d\phi}{dN_e} = -\frac{2M_p^2}{p_X}\left(\frac{H'}{H}\right)$$

Bean, Chung, Geshnizjani

Slow Variation Parameters become:

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{2M_p^2}{p_X}\left(\frac{H'}{H}\right)^2$$

$$\eta = \frac{d\ln\epsilon}{dN_e} = -\frac{4M_p^2}{p_X}\left(\frac{H''}{H}\right) + 2\epsilon + 2M_p^2\left(\frac{H'}{H}\right)\left(\frac{p_X'}{p_X}\right)$$

$$\kappa = \frac{dc_s}{dN_e} = \frac{2M_p^2}{p_X}\left(\frac{H'}{H}\right)\left(\frac{(c_s^{-1})'}{(c_s^{-1})}\right)$$

This formalism is a rewriting of the full EOM & is useful when one has

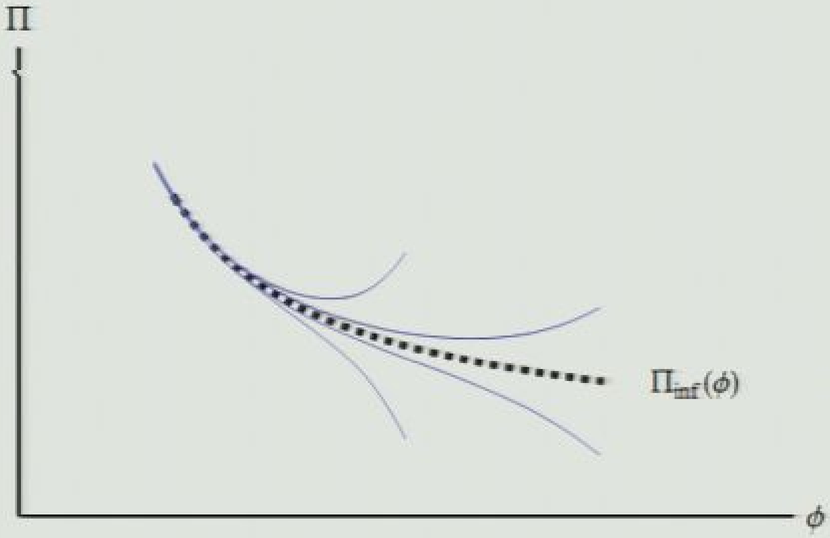
$$H = H(\phi)$$

(such as *flow equation* reconstruction of a general trajectory)

Agarwal, Bean

Does not specifically pick out inflationary trajectory solution.

Inflation is *Attractive*



Inflationary Solution

$$\Pi_{inf}(\phi) = \frac{\partial p}{\partial \phi}\frac{1}{3H}$$

Defines a trajectory in phase space.

Take perturbations about inflationary trajectory

$$\Pi = \Pi_{inf}\left(1 + \delta\Pi\right), \quad \delta\Pi \ll 1$$

$$\text{EOM} \Rightarrow \quad \frac{\delta\Pi'}{\delta\Pi} = -3 + \mathcal{O}(\epsilon, \eta_X, \eta_\Pi) \Rightarrow \delta\Pi \sim e^{-3N_e} \qquad \left(' \equiv \frac{d}{dN_e}\right)$$

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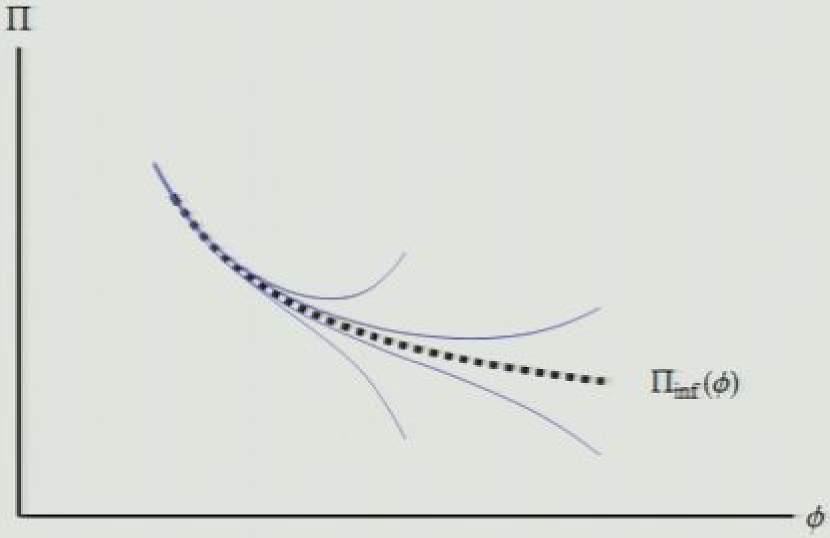

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Alternatively, in Hamilton-Jacobi formalism... $H(\phi) = H_{inf}(\phi)\left(1 + \delta H(\phi)\right)$

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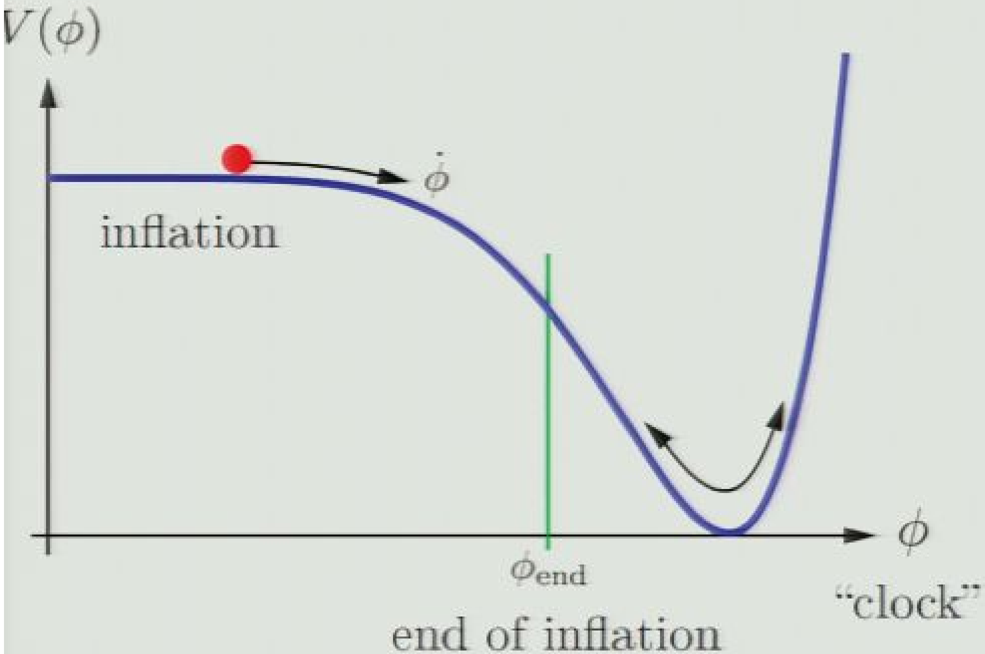
Perturbations Decay

Non-Canonical inflationary solutions are *attractors* in phase space.

Model-independent: True for all Large and Small field models.

Recall…Canonical Inflation



$$\mathcal{L}_{inf} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

Inflation occurs for *flat potentials*.

$$\epsilon_{SR} \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

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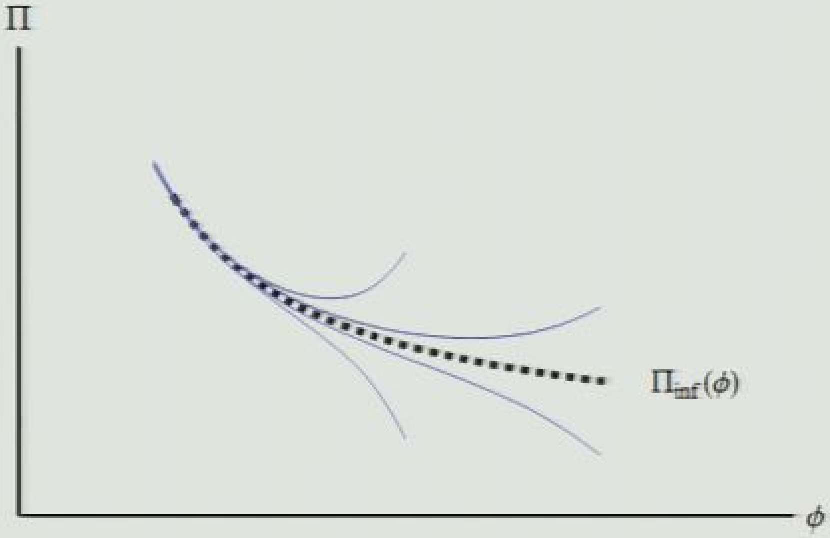
What properties must p(X,ϕ) satisfy for *Non*-Canonical Inflation?

Inflation is *Attractive*



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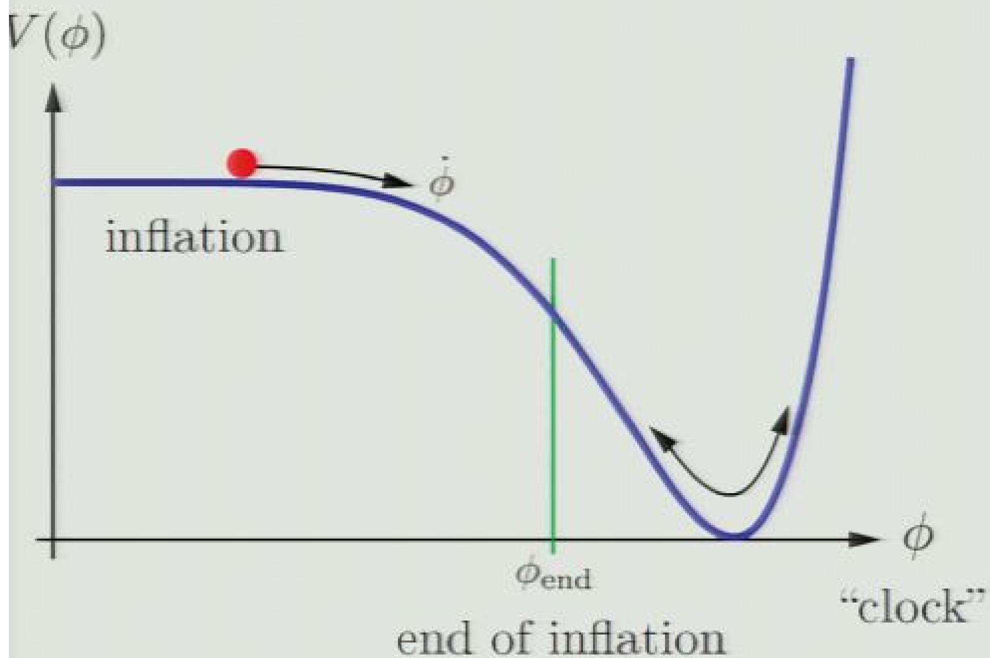

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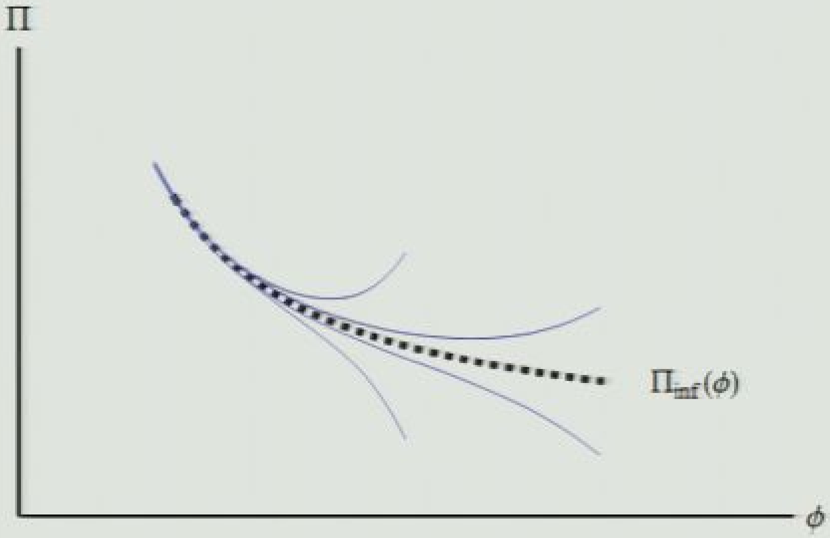

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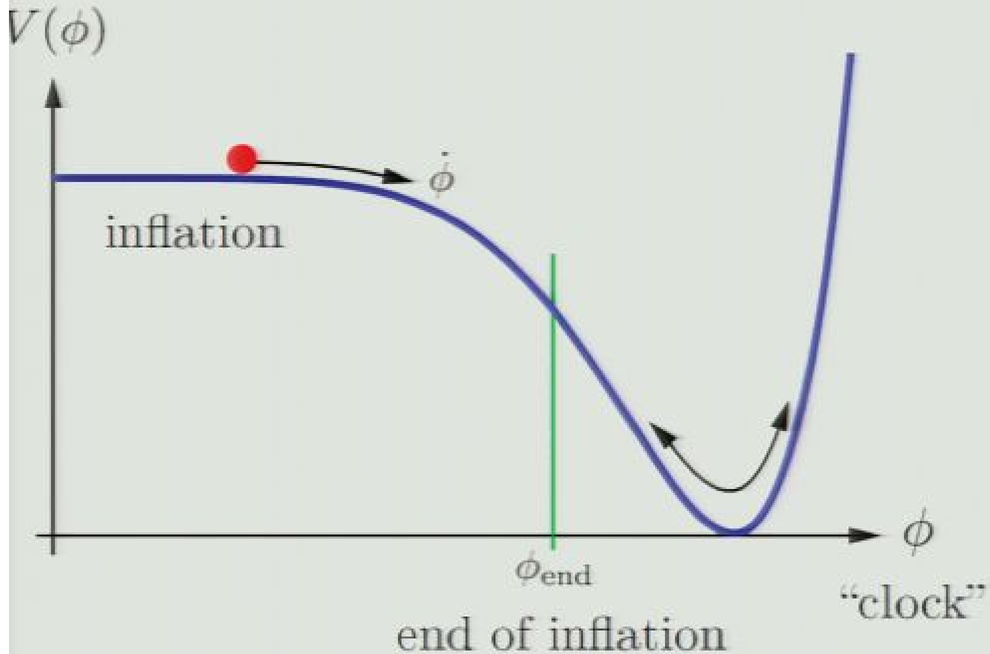

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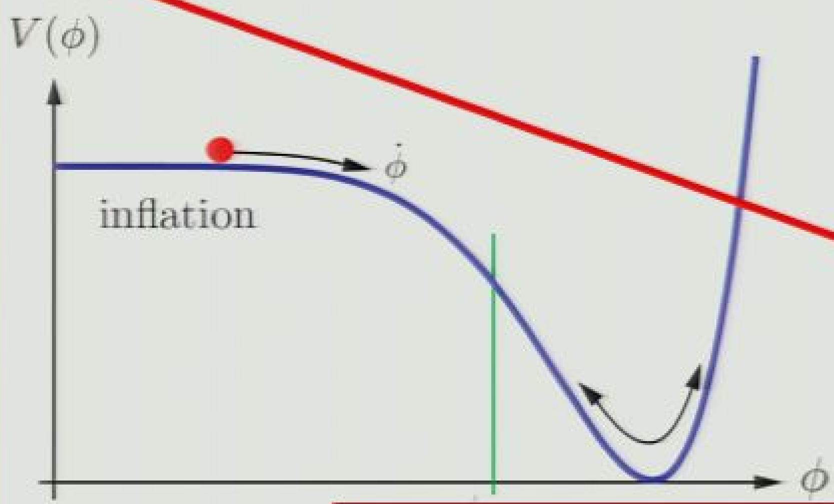

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Non-Canonical: Far from flat potential!

Parameter keeps track of *"non-canonical-ness."*

$$A = \left(\frac{2}{3}\epsilon_{SR}\frac{V}{\Lambda^4}\right)^{1/2} \begin{cases} A \ll 1 & \text{Slow Roll} \\ A \gg 1 & \text{Non-canonical} \end{cases}$$

$$\left[\Lambda \text{ typical UV scale, } p(X,\phi) \approx X - V(\phi) + \mathcal{O}\left(\frac{X^2}{\Lambda^4}\right)\right]$$

(1) Steep Potential $\epsilon_{SR} \geq 1$

(2) Large Potential $V/\Lambda^4 \gg 1$

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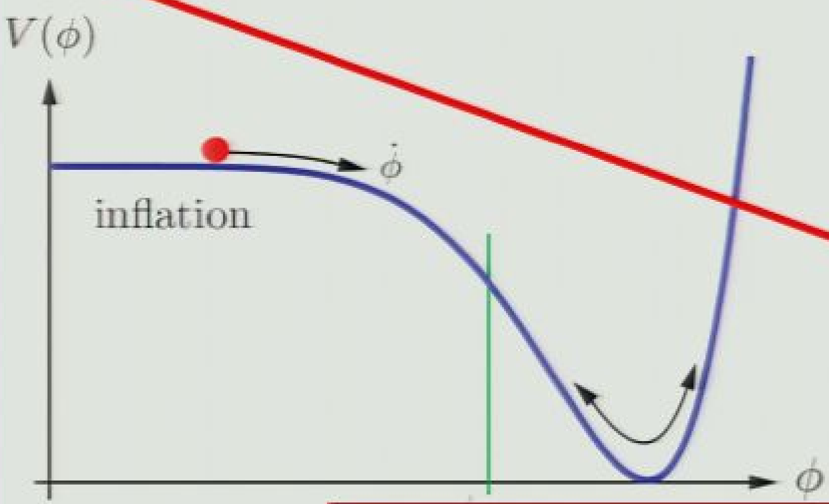

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What Properties must p(X,ϕ) satisfy for inflation?

$$\mathcal{L} = p(X,\phi), \quad X \equiv -\frac{1}{2}(\partial\phi)^2$$

Equation of Motion

$$\ddot{\phi}\left[\frac{\partial p}{\partial X} + \dot{\phi}^2\frac{\partial^2 p}{\partial X^2}\right] + \dot{\phi}^2\frac{\partial^2 p}{\partial X\partial\phi} + 3H\left(\frac{\partial p}{\partial X}\right)\dot{\phi} - \frac{\partial p}{\partial \phi} = 0$$

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Non-Canonical Inflation approximation:
Throw away "acceleration" terms

Non-Canonical Lagrangian gives "extra" Hubble Friction

(3) Positively curved kinetic term $\partial^2 p/\partial X^2 > 0$

(Already required for $0 < c_s^2 \leq 1$)

What Properties must p(X,ϕ) satisfy for inflation? *(Summary)*

$$\mathcal{L} = p(X, \phi), \quad X \equiv -\frac{1}{2}(\partial\phi)^2$$

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Then the Inflationary Parameters behave as

$$\epsilon \sim \frac{\epsilon_{SR}}{A^m}, \quad \eta \sim 3\frac{\epsilon_{SR}}{A^m} - \frac{\eta_{SR}}{A^m},$$

Suppressed by powers of A
(for some $0 \leq m \leq 1$)

What Properties must p(X,ϕ) satisfy for inflation?

$\mathcal{L} = p(X,\phi), \quad X \equiv -\frac{1}{2}(\partial\phi)^2$

$A = \left(\frac{2}{3}\epsilon_{SR}\frac{V}{\Lambda^4}\right)^{1/2}$

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There are a **large class** of examples:

Power Series

$$p(X) = \sum_{n\geq 0} c_n(\phi)\frac{X^{n+1}}{\Lambda^{4n}} - V(\phi) = X + c_1(\phi)\frac{X^2}{\Lambda^4} + \ldots - V(\phi)$$

Require:

(1) p(X) has a non-zero radius of convergence $R \equiv \lim_{n\to\infty}\left|\frac{c_n}{c_{n+1}}\right|$

$p(X)$ converges in $X/\Lambda^4 \in [0,R)$ or $[0,R]$

$\partial_X p$ converges in $X/\Lambda^4 \in [0,R)$

(2) $\partial_X p$ must diverge at the radius of convergence

Then we have, **irrespective of the details of the power series p(X)**

$$\epsilon \sim \sqrt{2R}\,\frac{\epsilon_{SR}}{A}, \quad \eta_X \sim \epsilon, \quad \eta_\Pi \sim \sqrt{2R}\,\frac{\eta_{SR}}{A} \text{ for } A \gg 1$$

What Properties must p(X,ϕ) satisfy for inflation?

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There are a **large class** of examples:

``DBI"

$$p(X) = -\Lambda^4\left[\sqrt{1-2\frac{X}{\Lambda^4}} - 1\right] - V(\phi), \quad \partial^2 p/\partial X^2 = \Lambda^{-4}\left(1-2X/\Lambda^4\right)^{-3/2} > 0$$

$$\epsilon \sim \frac{\epsilon_{SR}}{A} \text{ for } A \gg 1$$

``*Speed Limit"* not the relevant feature!

(Note we do not have any particular *stringy* realization in mind; just using this as a 4d EFT.)

``Power-like"

$$p(X) = \Lambda^4\left[\left(1+\frac{1}{n}\frac{X}{\Lambda^4}\right)^n - 1\right] - V(\phi), \quad \partial^2 p/\partial X^2 = \frac{(n-1)}{n}\Lambda^{-4}\left(1+\frac{1}{n}\frac{X}{\Lambda^4}\right)^{n-2} > 0 \text{ for } n > 1$$

$$\epsilon \sim \frac{\epsilon_{SR}}{A^{(2n-2)/(2n-1)}} \text{ for } A \gg 1$$

No speed limit, but still similar behavior!

``Exponential"

$$p(X) = \Lambda^4 e^{X/\Lambda^4} - V(\phi), \quad \partial^2 p/\partial X^2 = \Lambda^{-4}e^{X/\Lambda^4} > 0$$

$$\epsilon \sim \epsilon_{SR}\,(\log A)^{1/2}/A \text{ for } A \gg 1$$

Etc.

Consistency of Solutions…

We considered a *subset* of all possible higher dimensional operators

$$\mathcal{L}_{eff} = \mathcal{L}_{relevant} + \sum_n \frac{\mathcal{O}_n}{\Lambda^{n-4}} \supset \sum_n c_n \frac{X^{n+1}}{\Lambda^{4n}} - V(\phi)$$

What about:

- Higher powers of time derivatives? $\mathcal{L}_{eff} \supset \frac{(\partial^3\phi)^2}{\Lambda^4} + \ldots$
- Higher dimensional curvature operators? $\mathcal{L}_{eff} \supset \frac{R^2}{\Lambda^2}, \frac{(\partial R)^2}{\Lambda^4} + \ldots$

Are these important when evaluated on the *background*?

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Are these important when evaluated on the *background*?

$$\{\text{higher dimensional operators}\} \sim \mathcal{O}\left(\left(\frac{H}{\Lambda}\right)^{n-4} \varepsilon^m\right)$$

$n =$ dimension of operator $(n > 4)$

$m =$ number of derivatives on X

Higher dimensional operators suppressed

Consistency of Solutions...

Also should be concerned about perturbations...

Cheung et al, 0709.0293
Leblond, Shandera, 0802.2290
Shandera, 0812.0818

- Want perturbations to be small compared to background solution:

$$\frac{H^2}{M_p^2} < c_s^3$$

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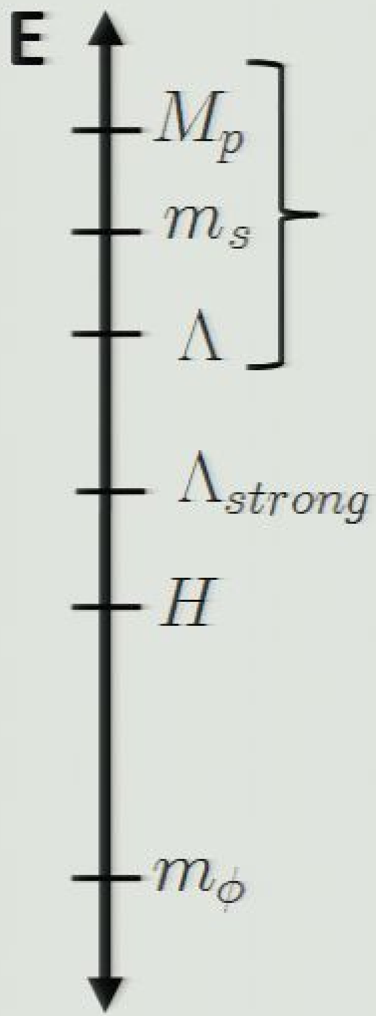

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- Due to kinetic term couplings, perturbations become strongly coupled at a scale:

$$\Lambda_{strong} \sim (M_p H)^{1/2} \epsilon^{1/4} c_s^{5/4}$$

$$H < \Lambda_{strong} \Rightarrow \frac{H^2}{M_p^2} < c_s^5 \epsilon$$

- Higher derivative terms in perturbations could become important:

$$H < c_s \Lambda \Rightarrow \frac{H^2}{M_p^2} < \frac{\Lambda^2}{M_p^2} c_s^2$$

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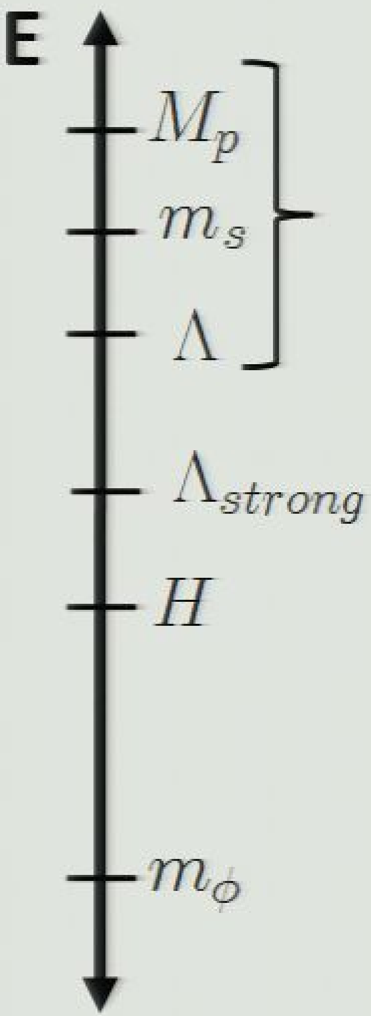

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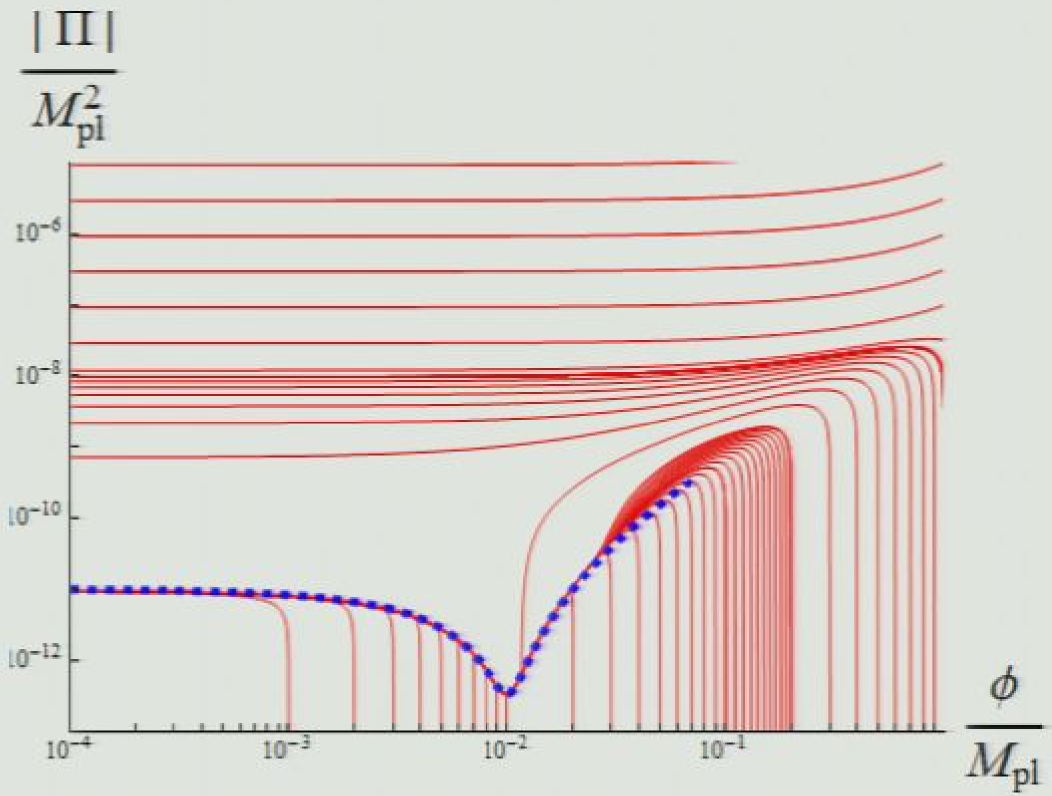
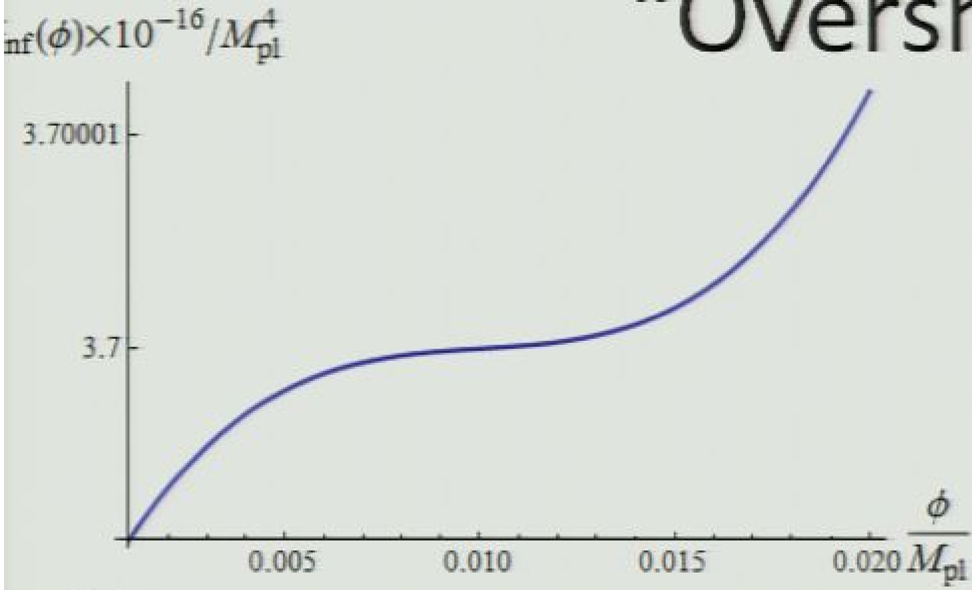
Restrict degree of *"non-canonical-ness"*.

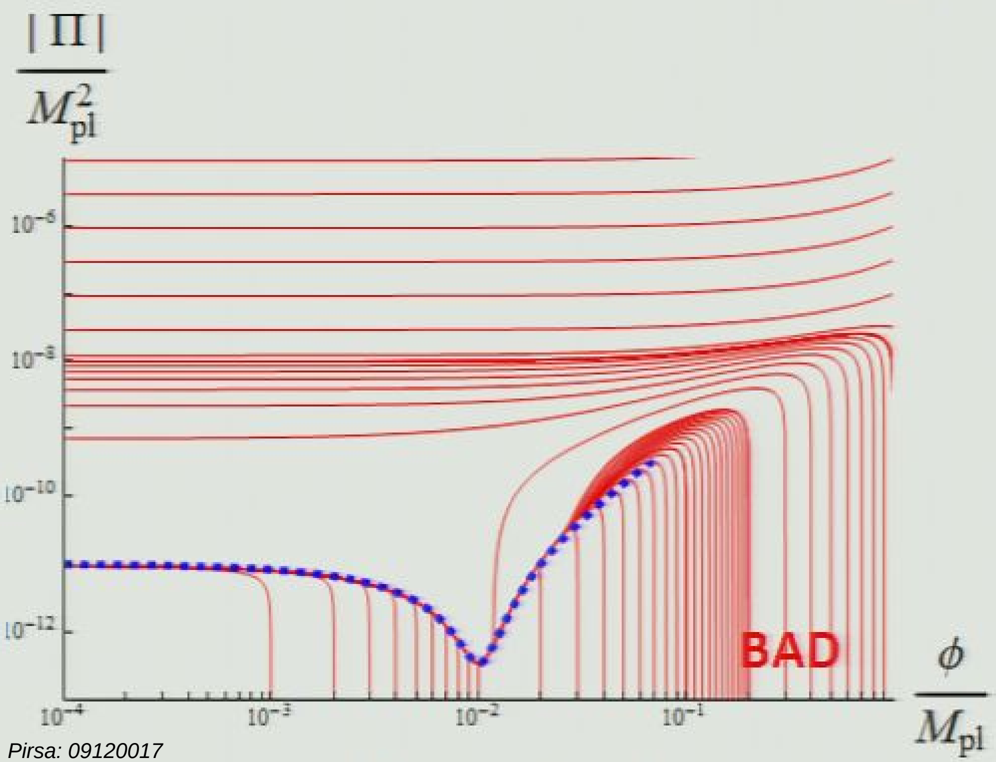
Overshoot and Motion in Phase Space



Work In Progress...

"Overshooting"





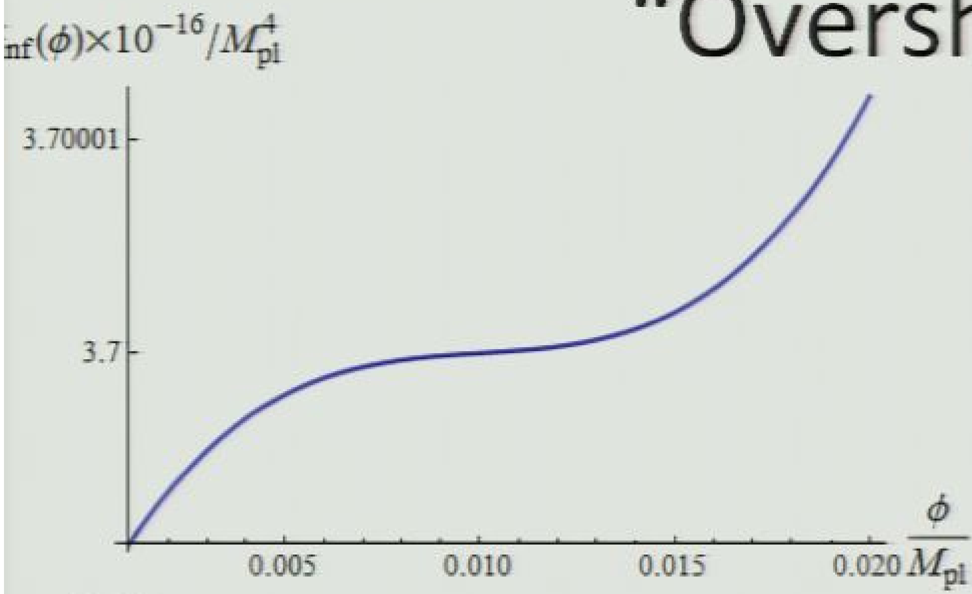
Certain initial conditions lead to inflation, while others do not:

- Start on steep part of potential at large ϕ

BAD

$$\mathcal{L} = p(X, \phi) = X - V(\phi)$$

"Overshooting"



Certain initial conditions lead to inflation, while others do not:

- Start on steep part of potential at large ϕ

BAD

- Start on flat part of potential with momentum

BAD

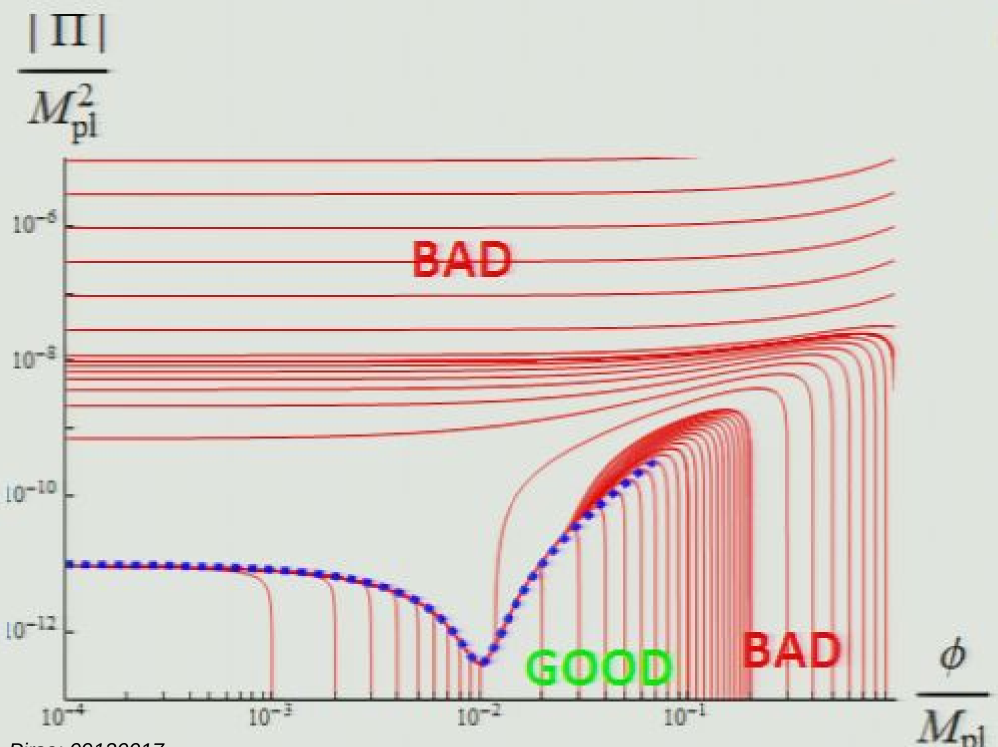
- Start on flat part of potential with very small (zero) momentum

GOOD

Generic initial conditions "overshoot" inflationary solution.

How general is this problem?

Solutions?



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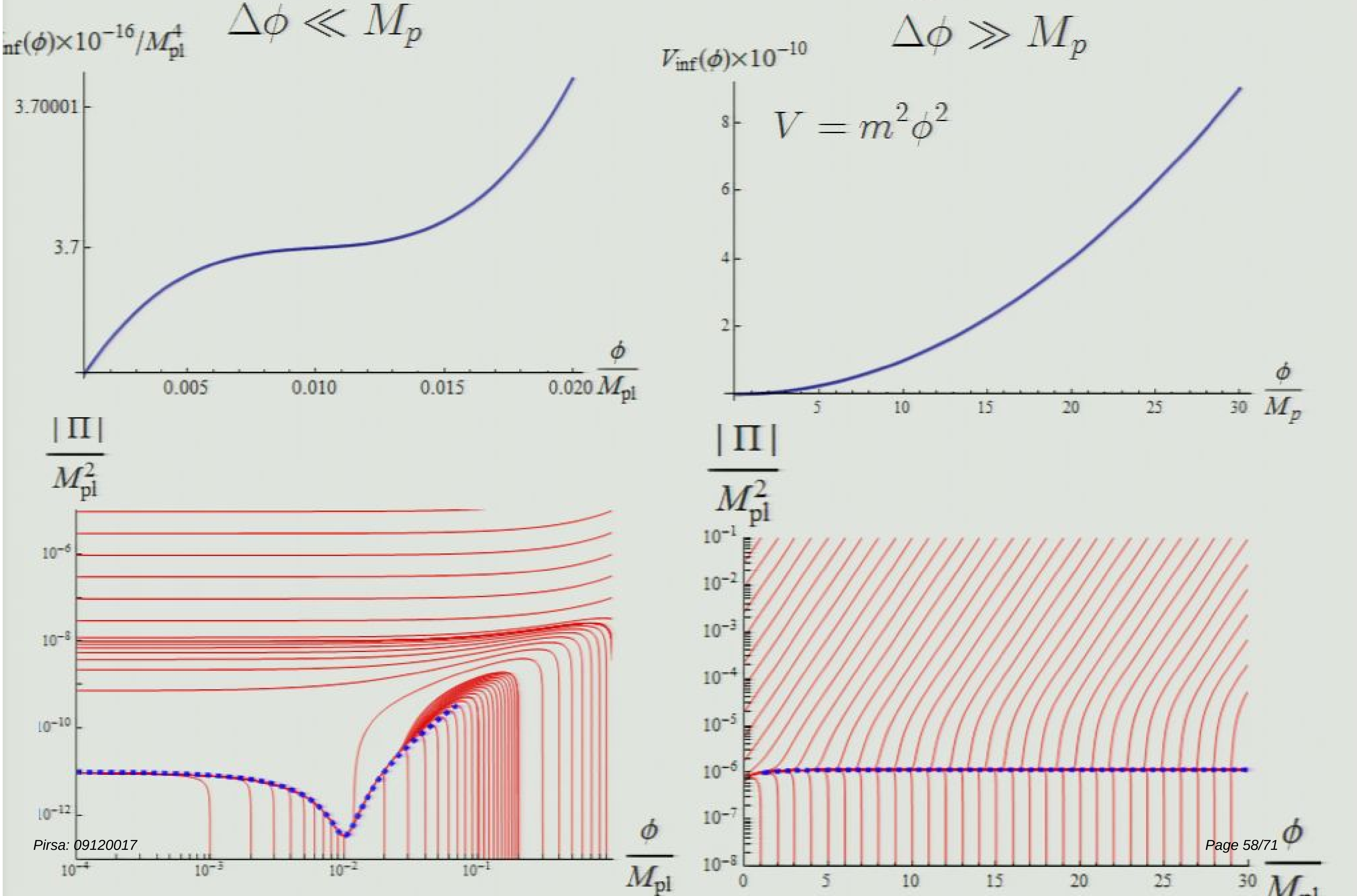
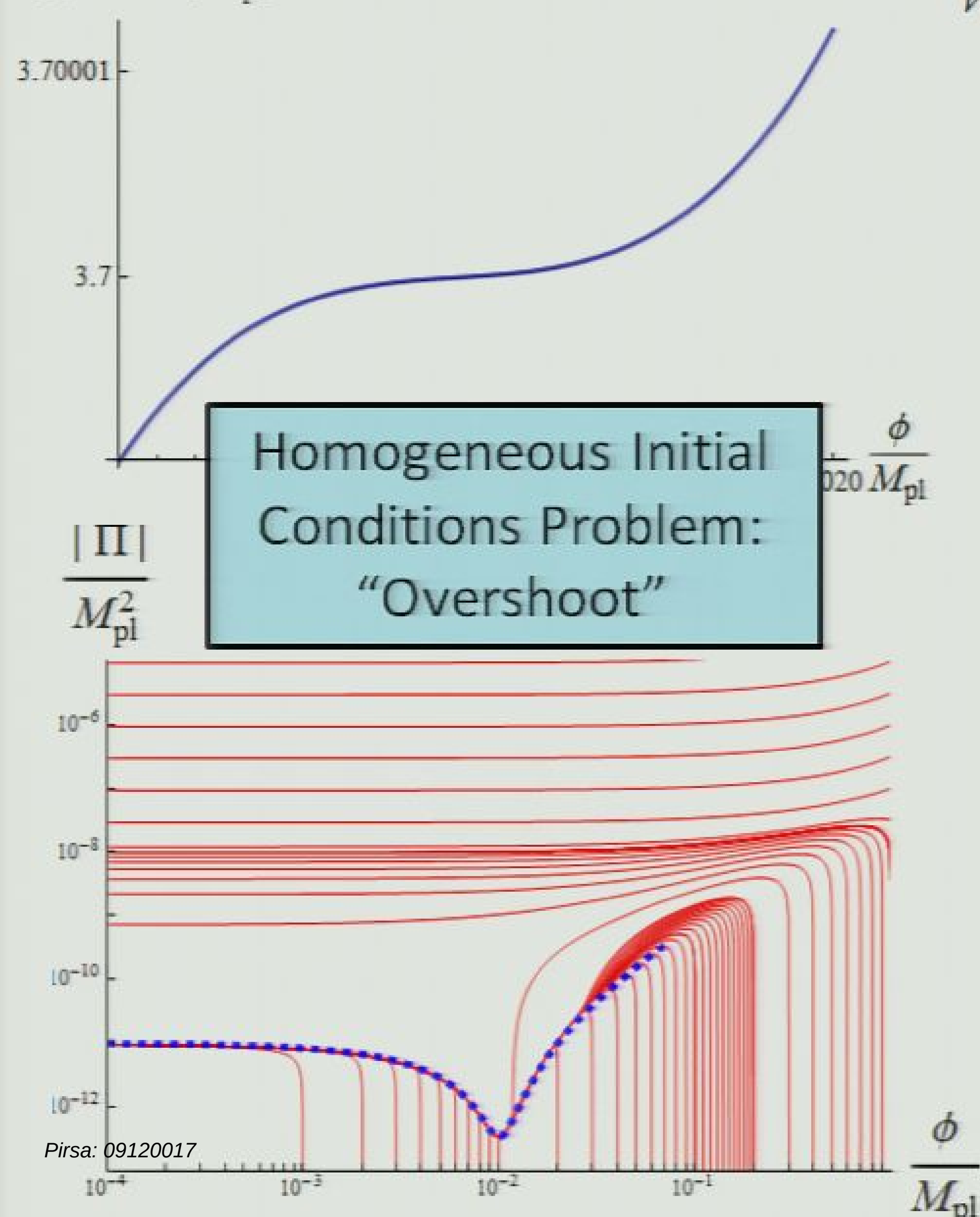
Small Field vs. Large Field



Small Field vs. Large Field

$\Delta\phi \ll M_p$



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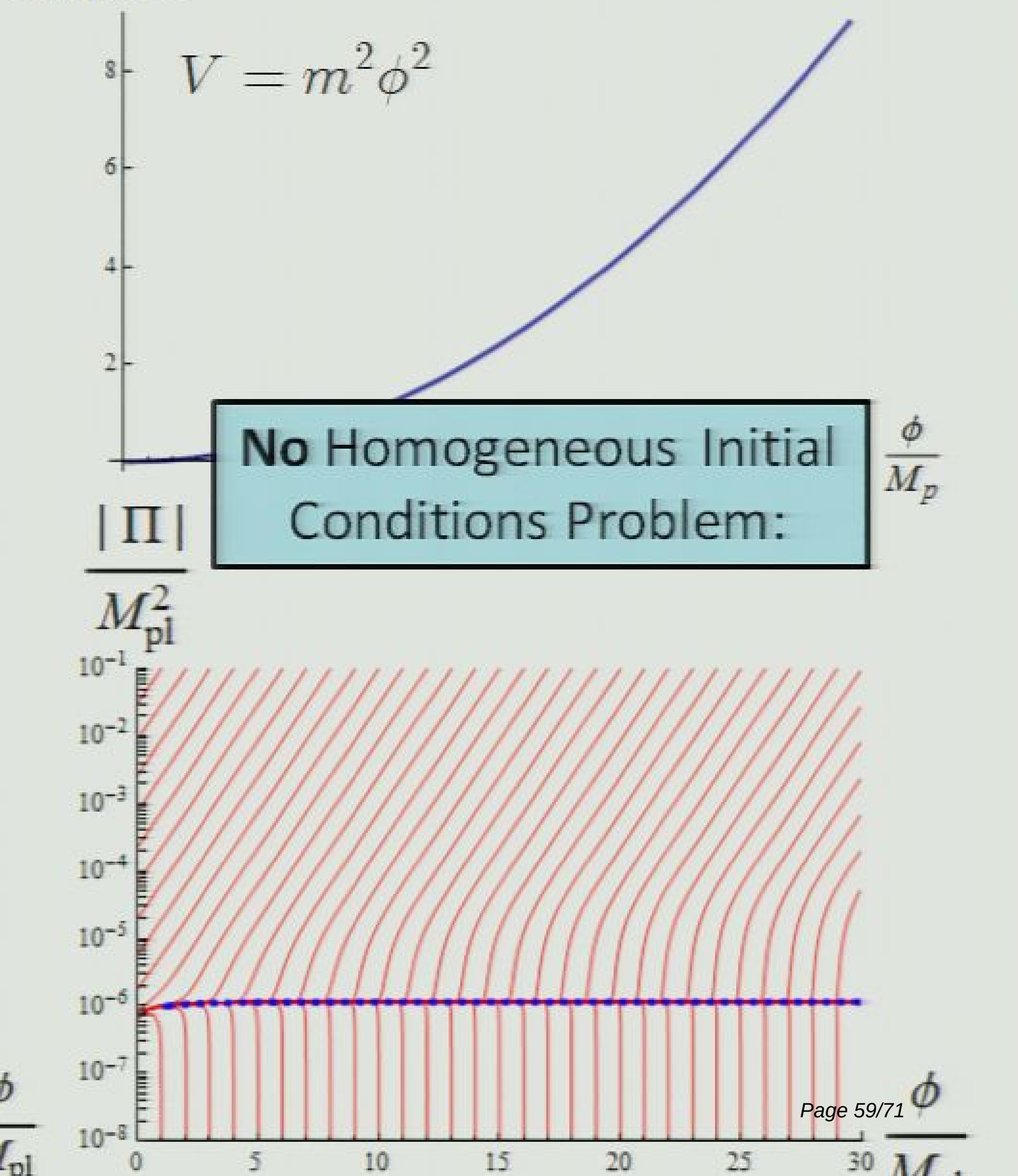

Small Field vs. Large Field

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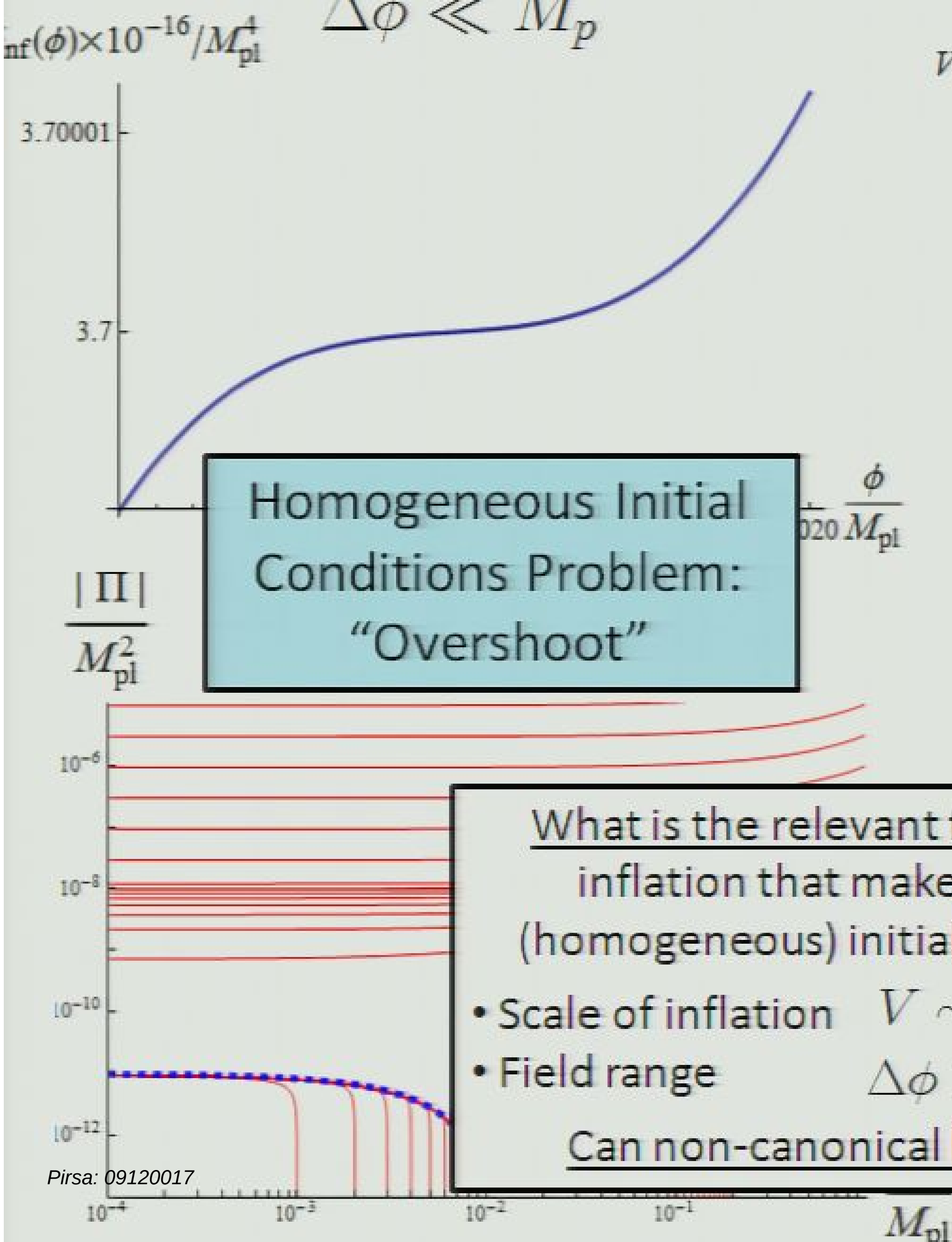
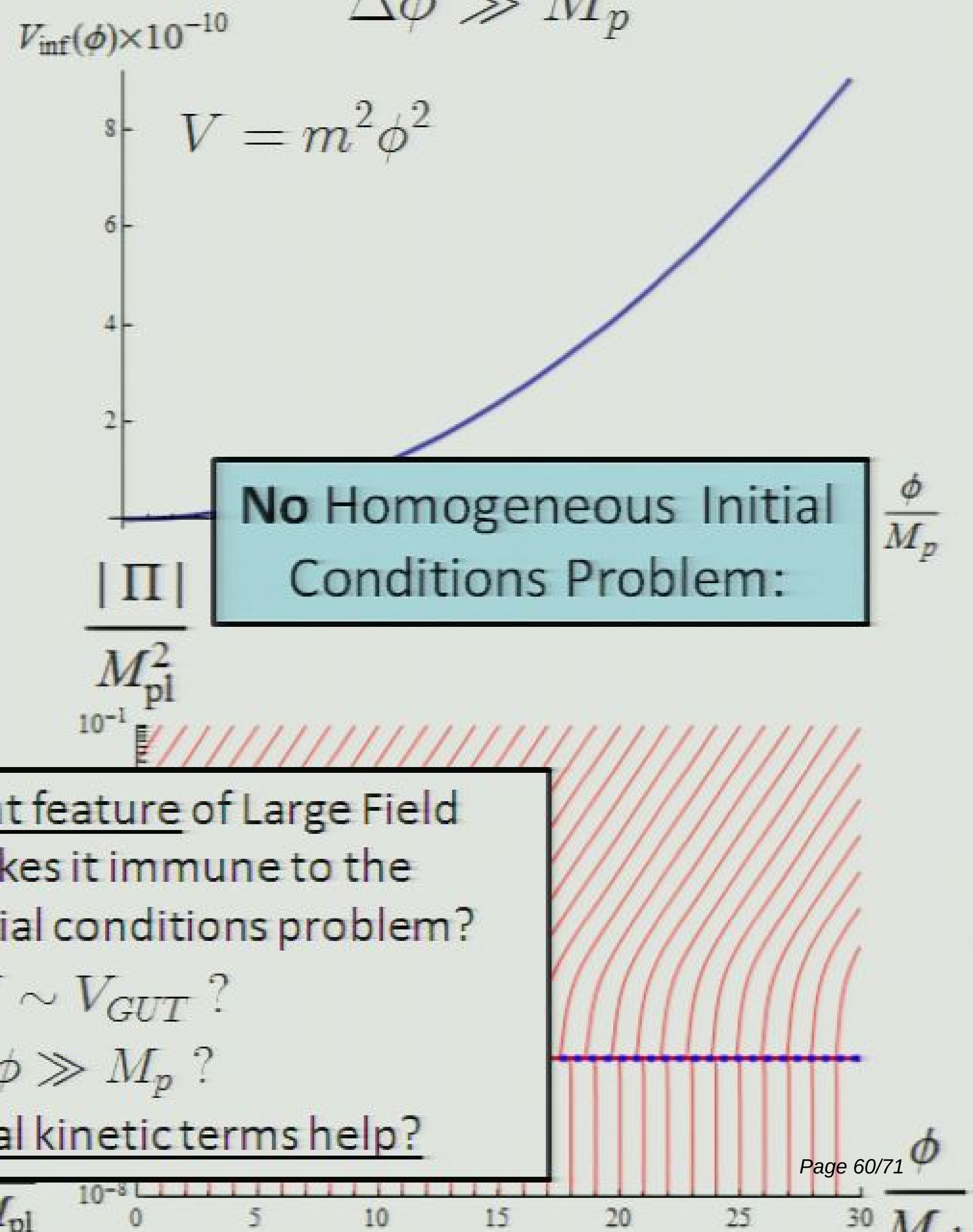


Homogeneous Initial Conditions Problem: "Overshoot"



No Homogeneous Initial Conditions Problem:

What is the relevant feature of Large Field inflation that makes it immune to the (homogeneous) initial conditions problem?

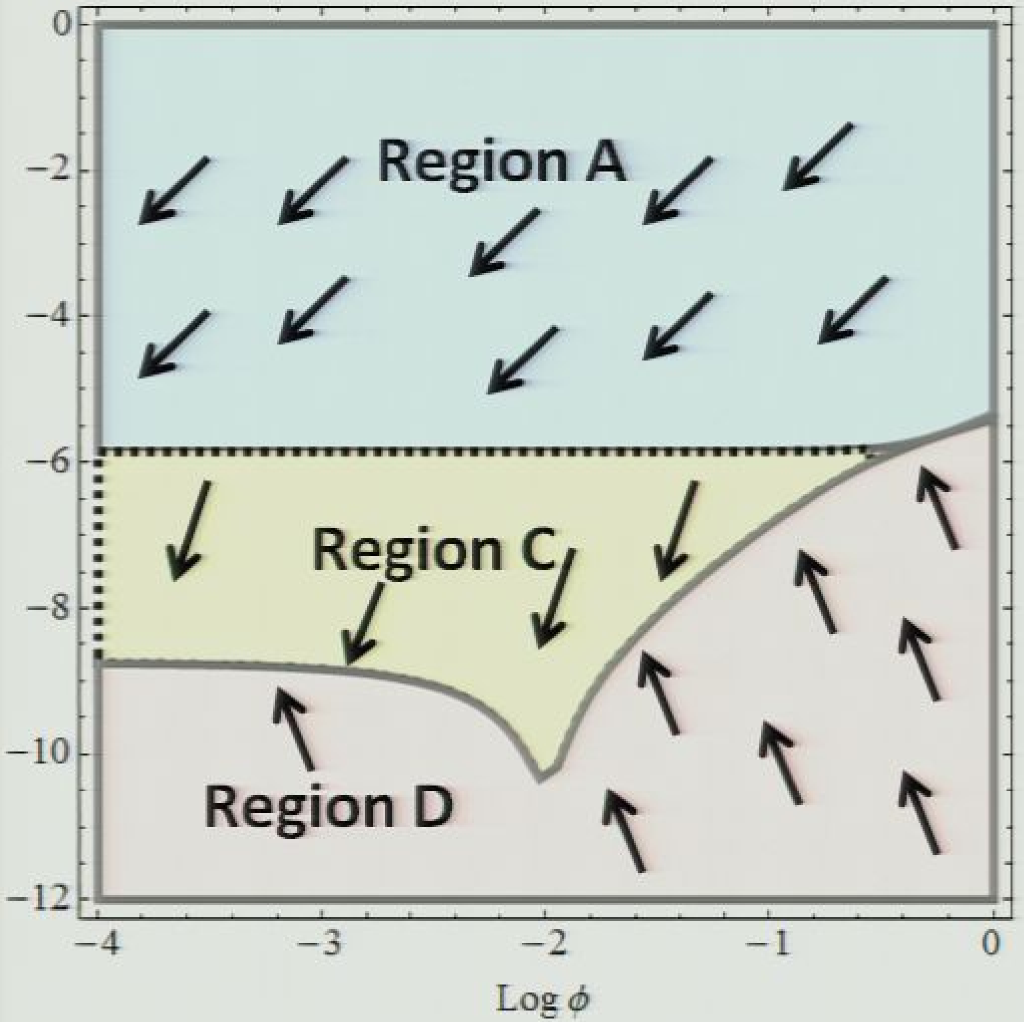
- Scale of inflation $V \sim V_{GUT}$?
- Field range $\Delta\phi \gg M_p$?

Can non-canonical kinetic terms help?

Regional Analysis of Phase Space

$\rho = (\text{kinetic energy}) + (\text{potential energy})$ $\qquad \dot{\Pi} = -3H\Pi + V'(\phi)$

Hubble Friction — Driving Force



(**Region B** not pictured)

Region A: Kinetic Energy dominates
Hubble Friction dominates

$$\dot{\Pi} = -3H\Pi < 0$$

Region B: Kinetic Energy dominates
Driving Force dominates

$$\dot{\Pi} = V'(\phi) > 0$$

Region C: Potential Energy dominates
Hubble Friction dominates

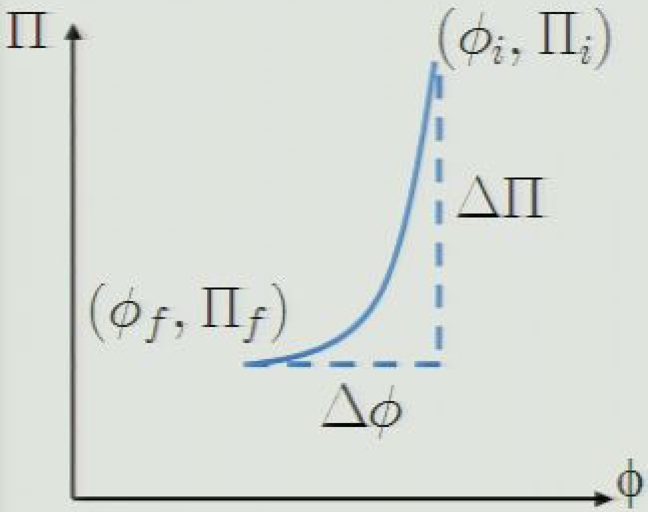
$$\dot{\Pi} = -3H\Pi < 0$$

Region D: Potential Energy dominates
Driving Force dominates

$$\dot{\Pi} = V'(\phi) > 0$$

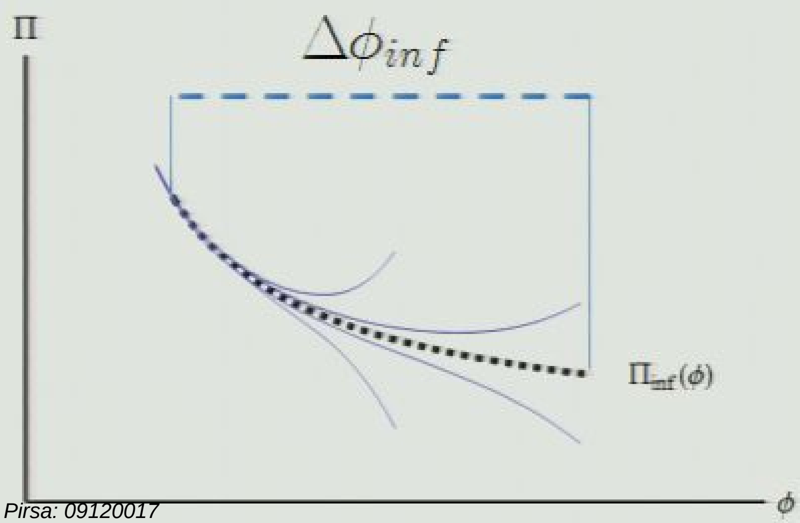
Overshooting

How far does the inflaton move while losing momentum? (Region A)



Canonical:

$$\frac{\Delta\phi}{M_p} = \sqrt{\frac{2}{3}} \log\left(\frac{\Pi_i}{\Pi_f}\right) \sim \mathcal{O}(1)$$

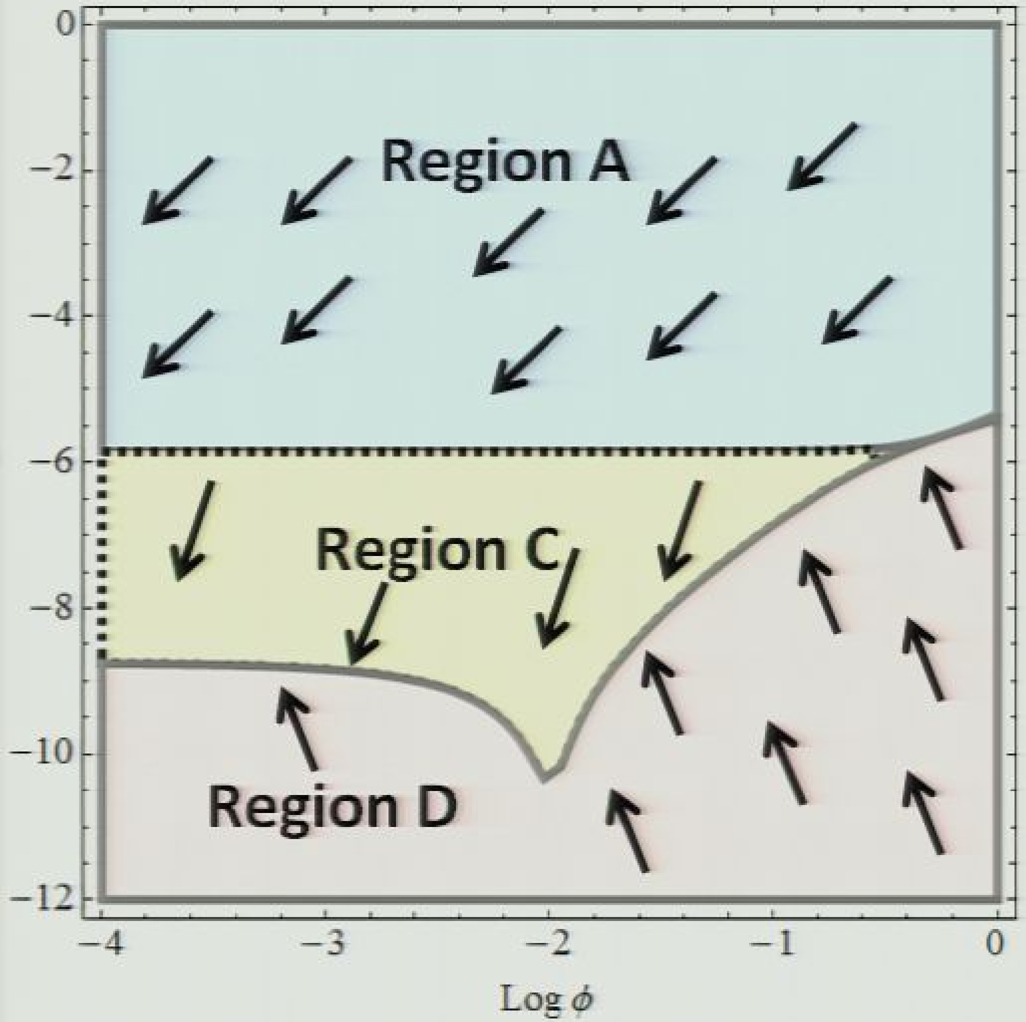


Regional Analysis of Phase Space

$$\rho = (\text{kinetic energy}) + (\text{potential energy})$$

$$\dot{\Pi} = -3H\Pi + V'(\phi)$$

Hubble Friction — Driving Force



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(**Region B** not pictured)

Overshooting

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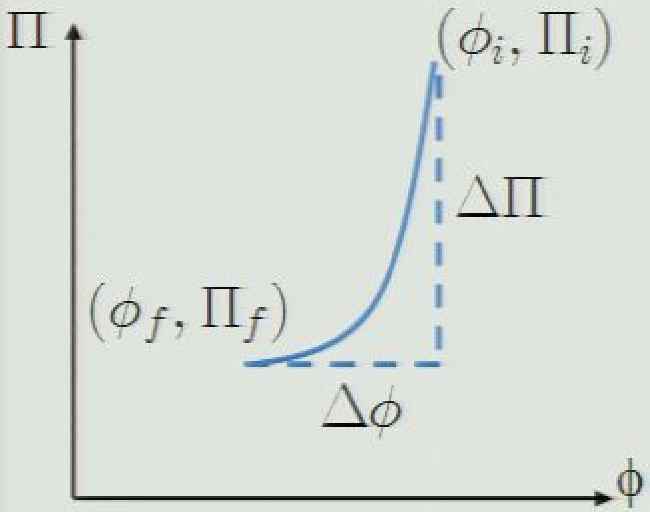
(Region A)



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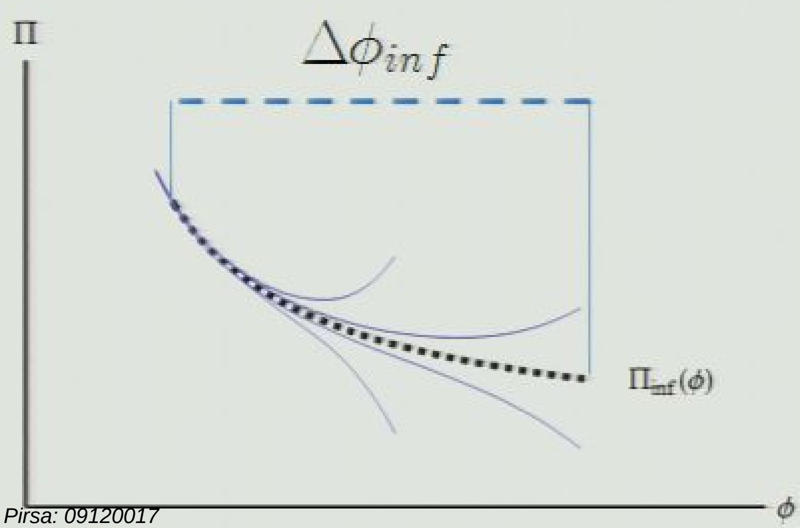
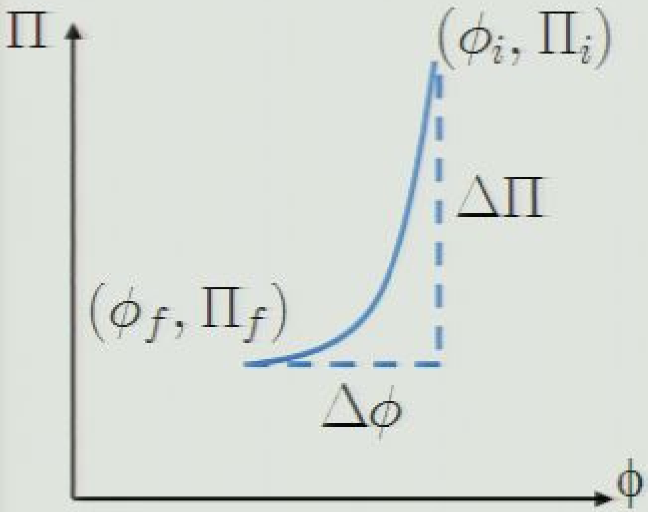


Overshooting

How far does the inflaton move while losing momentum? (Region A)



Canonical:

$$\frac{\Delta\phi}{M_p} = \sqrt{\frac{2}{3}} \log\left(\frac{\Pi_i}{\Pi_f}\right) \sim \mathcal{O}(1)$$

- Large field models have

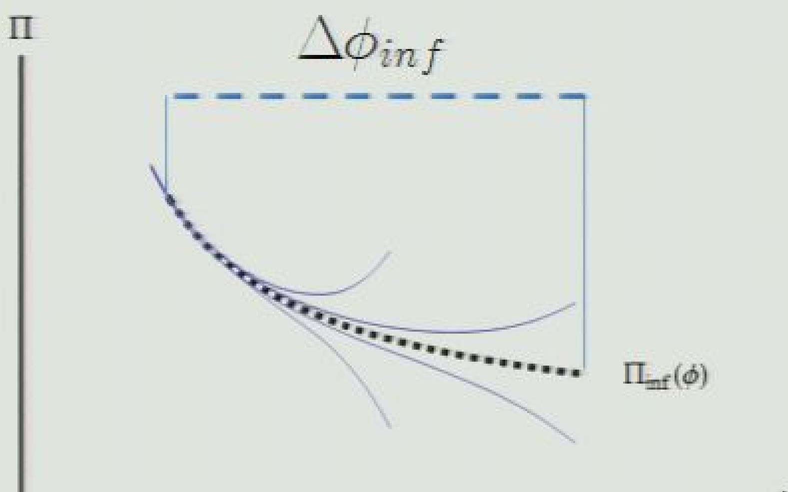
$$\Delta\phi_{inf} \sim \mathcal{O}(10) M_p \;\Rightarrow\; \frac{\Delta\phi}{\Delta\phi_{inf}} \ll 1$$

No “Overshoot” problem.

- Small field models have

$$\Delta\phi_{inf} \ll M_p \;\Rightarrow\; \frac{\Delta\phi}{\Delta\phi_{inf}} \gg 1$$

“Overshoot” problem!!



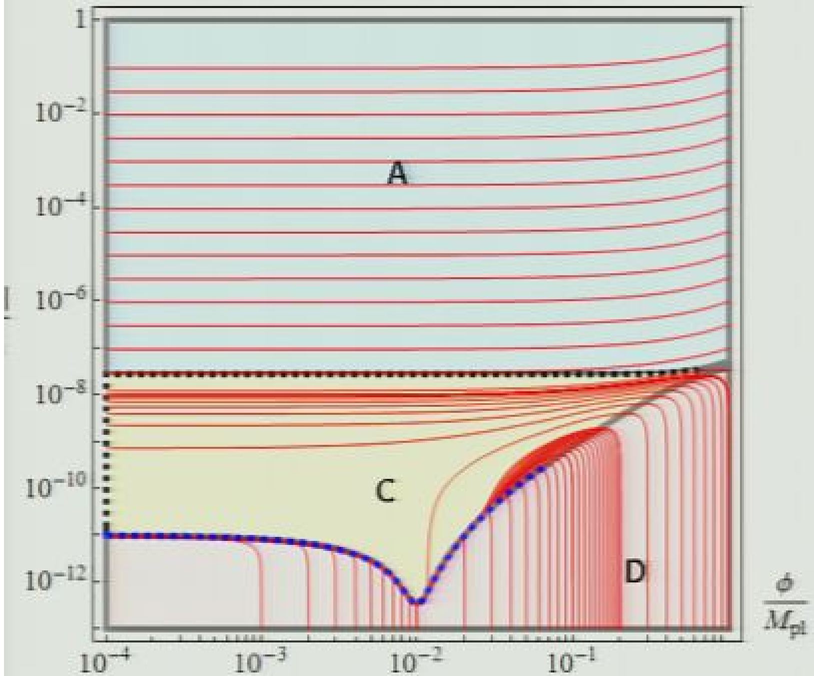
Non-Canonical:

$$\frac{\Delta\phi}{M_p} \sim \left(\frac{\Lambda^2}{\Pi_f}\right)^{1/2} \ll 1$$

No “Overshoot” problem?

Overshooting Examples

Canonical

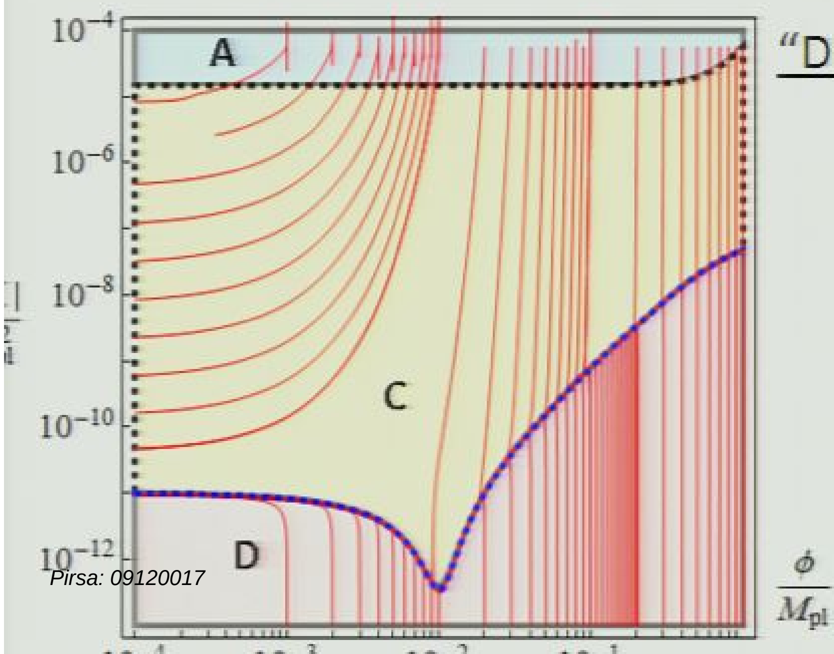


Canonical $\mathcal{L} = p(X,\phi) = X - V(\phi)$

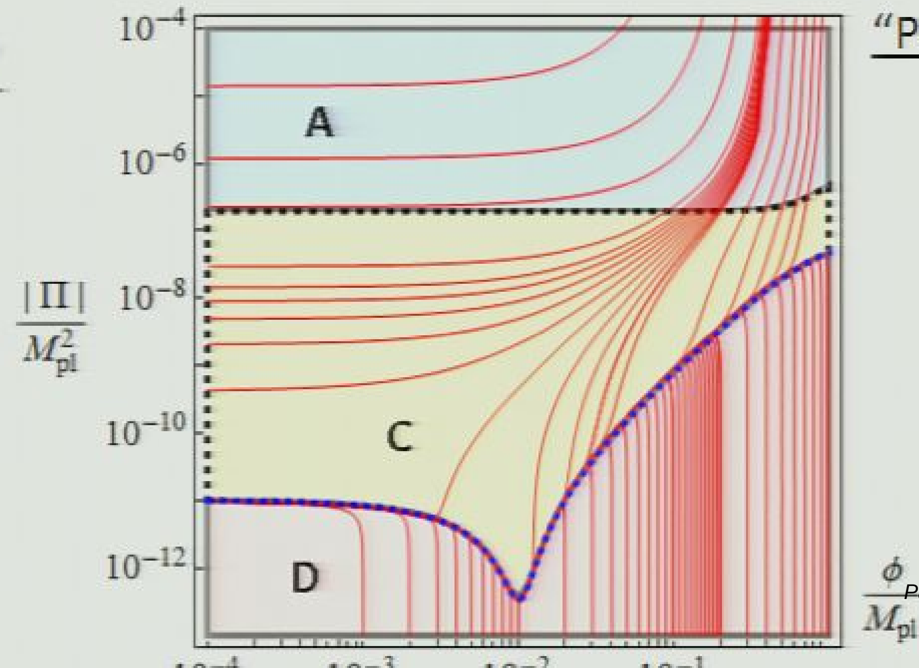
"DBI" $\mathcal{L} = p(X,\phi) = \Lambda^4\left[\sqrt{1 - 2\frac{X}{\Lambda^4}} - 1\right] - V(\phi)$

"Power-like"

$$\mathcal{L} = p(X,\phi) = \Lambda^4\left[\left(1 + \frac{2}{3}\frac{X}{\Lambda^4}\right)^{3/2} - 1\right] - V(\phi)$$



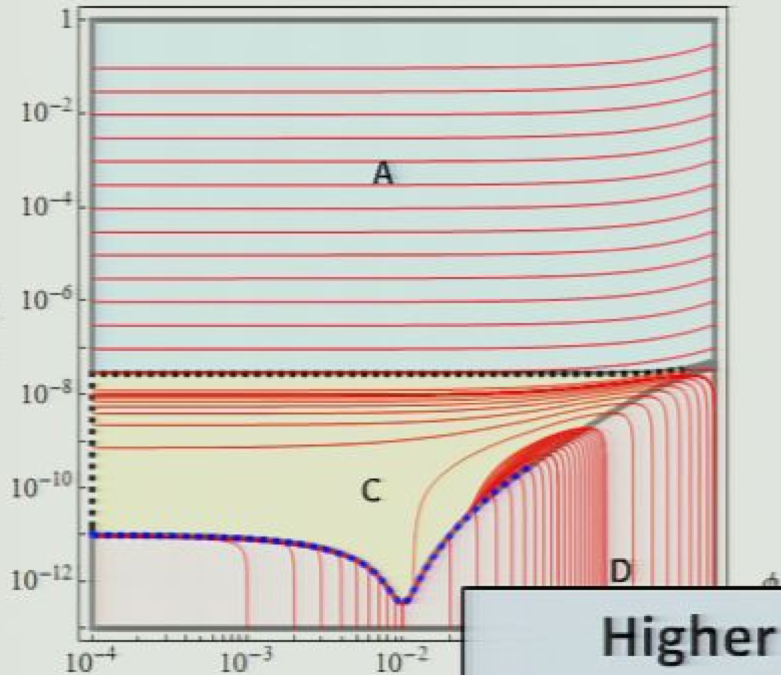
"DBI"



"Power-like"

Overshooting Examples

Canonical



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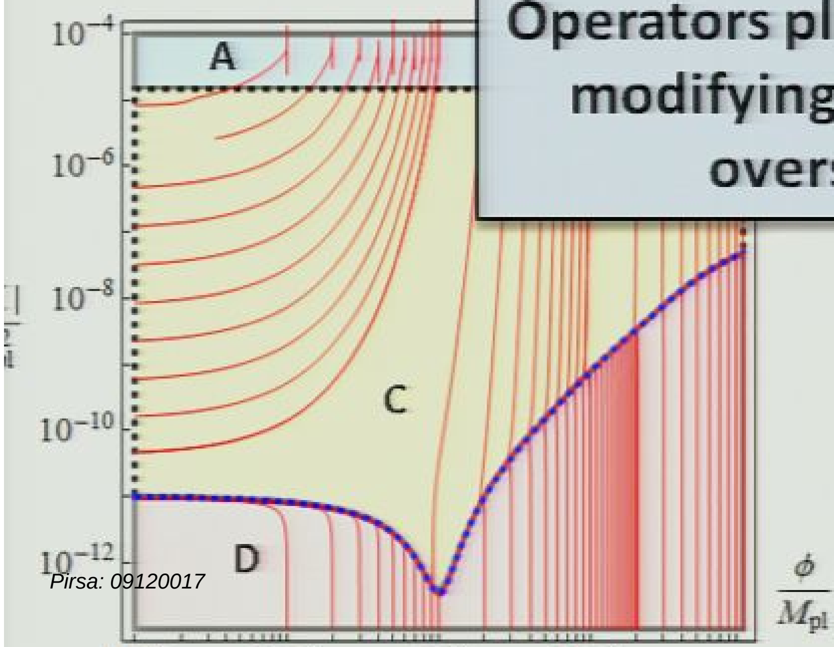
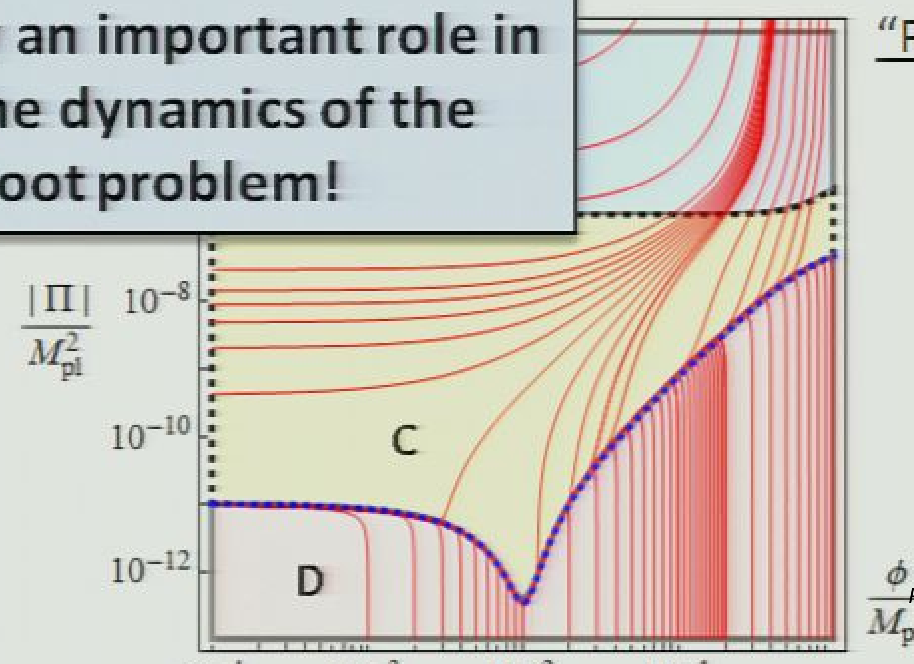
Higher Dimensional Kinetic Operators play an important role in modifying the dynamics of the overshoot problem!



"Power-like"



Small Field vs. Large Field

Small Field	Large Field
$\Delta\phi \ll M_p$	$\Delta\phi \gg M_p$

Homogeneous Initial Conditions Problem

No Homogeneous Initial Conditions Problem

Inhomogeneous Initial Conditions Problem

No Inhomogeneous Initial Conditions Problem?

Early Universe must be homogenous on scales

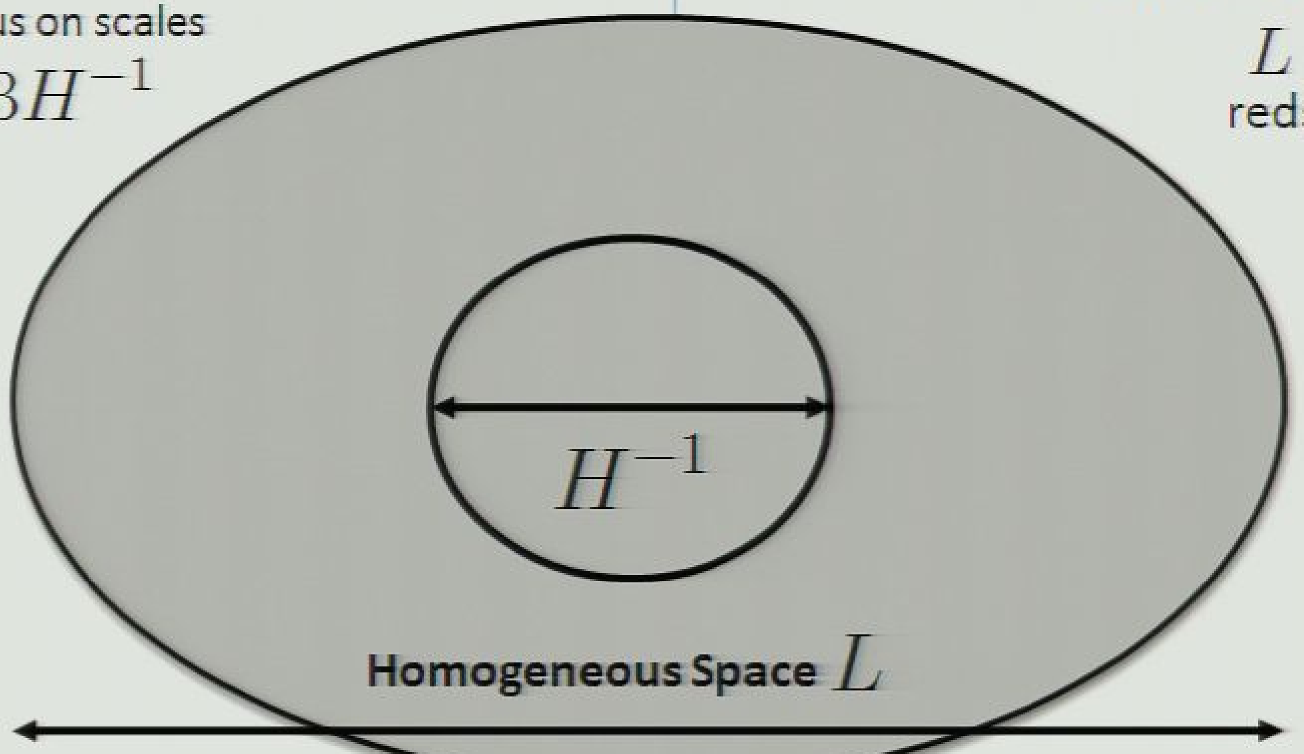
$$L \sim 3H^{-1}$$

Inhomogeneities on scales

$$L \ll H^{-1}$$

redshift away.

Brandenberger, Kung



Summary

Inflation is an EFT $\mathcal{L}_{eff} = \mathcal{L}_{relevant} + \sum_n c_n \frac{\mathcal{O}_n}{\Lambda^{n-4}}$

What is the role of Higher Dimensional Kinetic Operators?

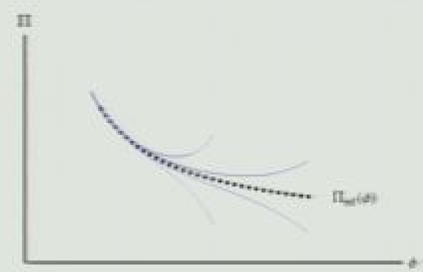
$$\mathcal{L}_{eff} = p(X,\phi) = \sum_n c_n \frac{X^{n+1}}{\Lambda^{4n}} - V(\phi), \quad X = -\frac{1}{2}(\partial\phi)^2$$

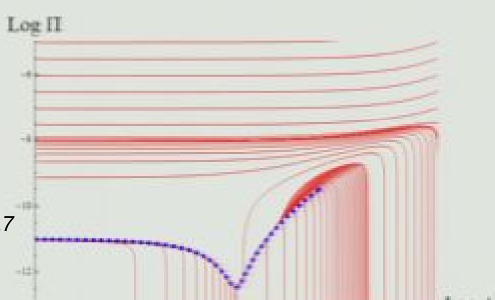
Non-Canonical Inflationary Solutions

when $\epsilon_{SR} > 1, \; V > \Lambda^4, \; \partial^2 p/\partial X^2 > 0$

(Steep potential, low EFT scale, physical propagation of perturbations)

Canonical/Non-Canonical Inflation is an *Attractor*





Higher Dimensional kinetic operators

Modify dynamics at large momentum,

new dynamical mechanisms for avoiding the initial conditions problem?

Summary

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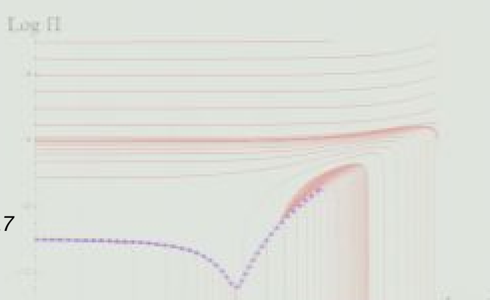
Thank You!

Canonical/Non-Canonical Inflation is an *Attractor*



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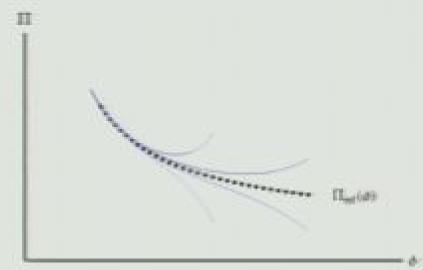

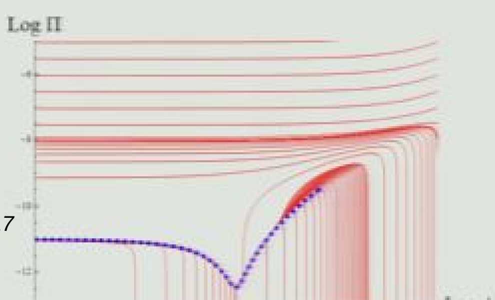

Higher Dimensional kinetic operators

Modify dynamics at large momentum, *new dynamical mechanisms for avoiding the initial conditions problem?*