Title: Is reality a "matrix"?

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Abstract: I will describe a new connection between supersymmetry, geometry and computer science. An exploration of the equations of supersymmetry has revealed a geometrical sub-structure whose classification depends on self-dual error correcting codes.

Is Reality A 'Matrix?'

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Codes, Cubes, & Space-time Supersymmetry Representations

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A Statement of the Problem

Fields

Space time supersymmetry begins from the space of fields.

$$\mathcal{F} = \{\mathcal{F}\}_b \oplus \{\mathcal{F}\}_f$$

scalar vector graviton

$$\mathcal{F}_b = \{ \ \phi(x) \, , \ A_{\underline{a}}(x) \, , \ h_{\underline{a}\,\underline{b}}(x), \dots \ \}$$

spin − 0, spin − 1, spin − 2

spinor gravitino

$$\mathcal{F}_f = \{ \ \ \lambda^{\alpha}(x) \, , \ \psi_{\underline{a}}{}^{\beta}(x), \dots \ \}$$

spin − 1/2, spin − 3/2

SUSY Variations

$$\delta_Q(\epsilon^\alpha)\,\mathcal{F} = \{\mathcal{F}\}_f \,\oplus\, \{\mathcal{F}\}_b$$

Elements of $\{\mathcal{F}\}_f$ are:

linear in ϵ^α,

linear in the elements of $\{\mathcal{F}\}_f$,

involve invariant tensors (γ, η, etc.) and

may involve first derivatives.

Elements of $\{\mathcal{F}\}_b$ are:

linear in ϵ^α,

linear in the elements of $\{\mathcal{F}\}_b$,

involve invariant tensors (γ, η, etc.) and

and may involve first derivatives.

Translations

$$\delta_P(\xi^a)\,\mathcal{F} = (\xi^a \partial_a \{\mathcal{F}\}_b) \,\oplus\, (\xi^a \partial_a \{\mathcal{F}\}_f)$$

Off-Shell SUSY Representation

$$\delta_Q(\epsilon_1^{\alpha})\,\delta_Q(\epsilon_2^{\beta}) - \delta_Q(\epsilon_2^{\alpha})\,\delta_Q(\epsilon_1^{\beta}) = \delta_P(\xi^a) \quad ,$$

where $\xi^a = i2\,\epsilon_1^{\alpha}\gamma^a{}_{\alpha\beta}\epsilon_2^{\beta}$.

Even now thirty years after its first statement, the general solution to this problem is still *not* known.

On-Shell SUSY Representation

$$\delta_Q(\epsilon_1^{\alpha})\,\delta_Q(\epsilon_2^{\beta}) - \delta_Q(\epsilon_2^{\alpha})\,\delta_Q(\epsilon_1^{\beta}) = \delta_P(\xi^a) + \partial S$$

But in many theories, such as low-energy string effective actions, the action is not even known completely!

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But in many theories, such as low-energy string effective actions, the action is not even known completely!

$$\mathcal{S}_{off-shell} = \int d^4x \, [\; -\tfrac{1}{2}(\partial^{\underline{a}}\overline{A})(\partial_{\underline{a}}A) \; - \; i\overline{\psi}^{\dot{\alpha}}\partial_{\underline{a}}\psi^{\alpha} \; + \; \overline{F}F$$

$$+ \; \tfrac{1}{2}m(\,\psi^{\alpha}\psi_{\alpha} + \overline{\psi}^{\dot{\alpha}}\overline{\psi}_{\dot{\alpha}}) \; + \; m(\,AF + \overline{A}\overline{F}\,)\;] \quad .$$

$$\Box A \; + \; m\overline{F} \; = \; 0 \quad ,$$

$$-i\partial_{\underline{a}}\psi^{\alpha} \; + \; m\overline{\psi}_{\dot{\alpha}} \; = \; 0 \quad ,$$

$$F \; + \; m\overline{A} \; = \; 0 \quad .$$

$$F \; = \; -m\overline{A}$$

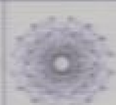
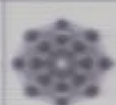
$$(\,\Box \; - \; m^2\,)\,A \; = \; 0 \quad ,$$

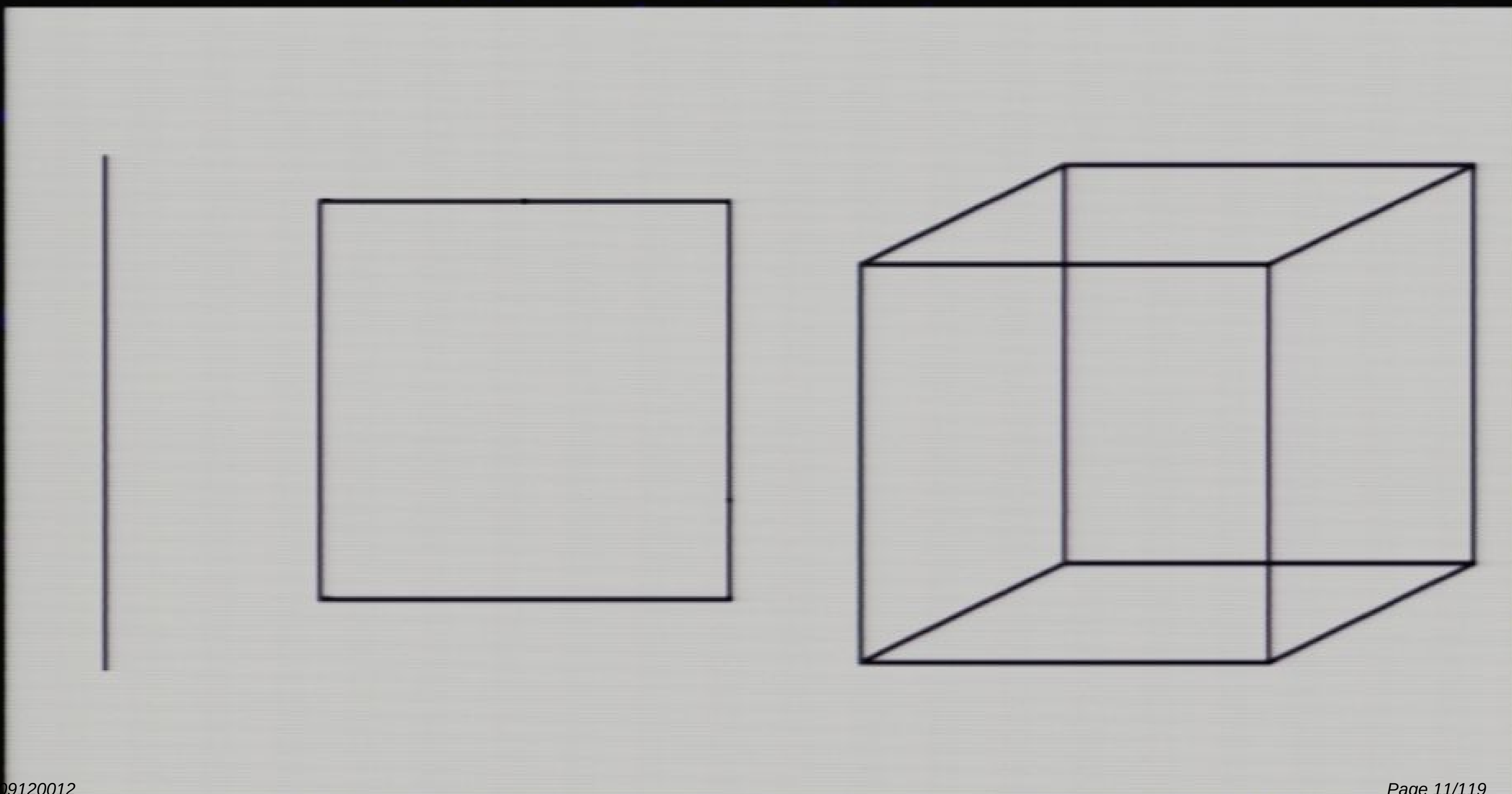
$$-i\partial_{\underline{a}}\psi^{\alpha} \; + \; m\overline{\psi}_{\dot{\alpha}} \; = \; 0 \quad .$$

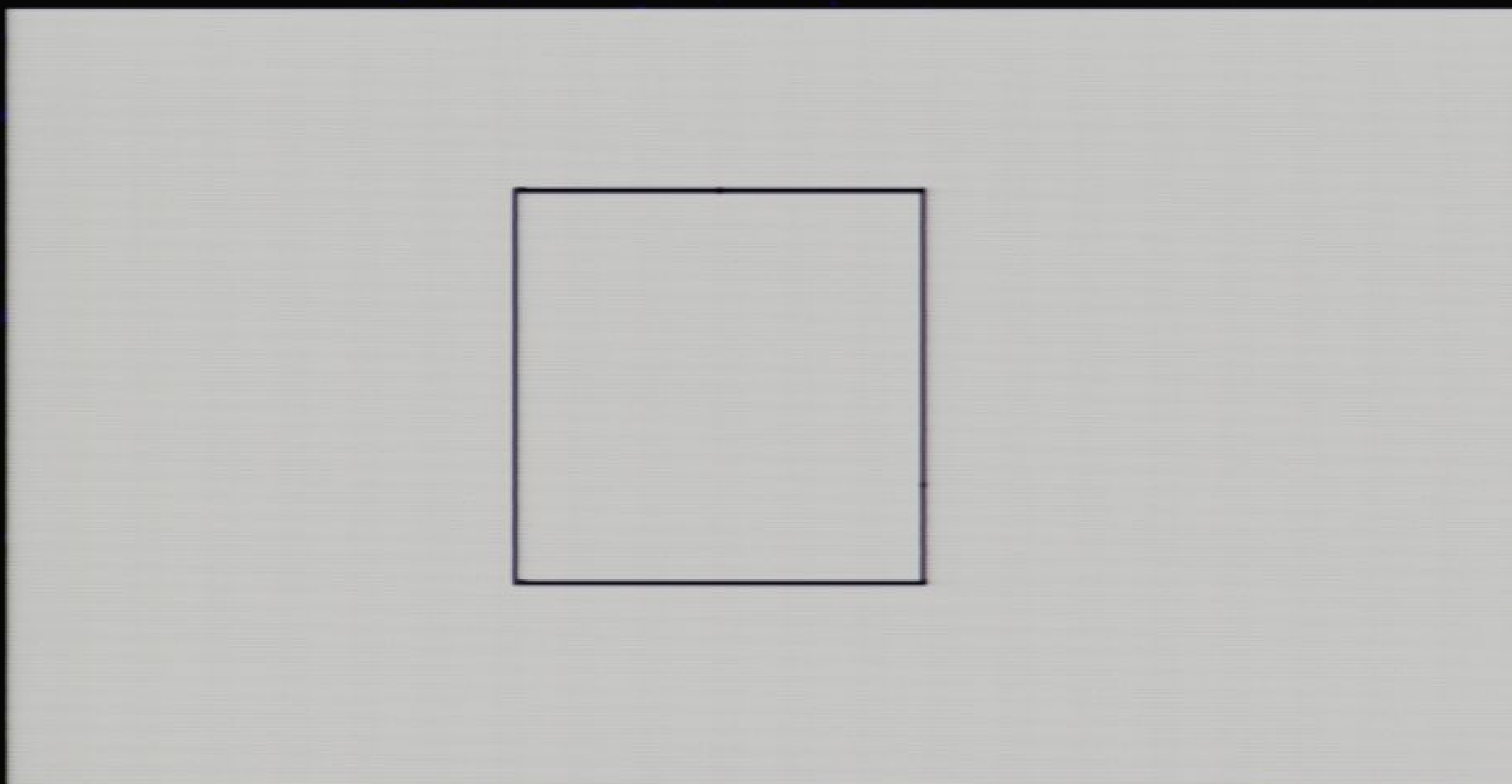
$$\mathcal{S}_{on-shell} = \int d^4x \, [\; -\tfrac{1}{2}(\partial^{\underline{a}}\overline{A})(\partial_{\underline{a}}A) \; + \; m^2\overline{A}A$$

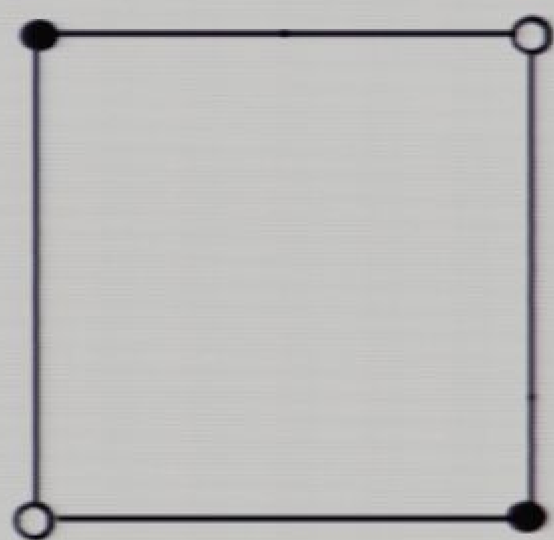
$$- \; i\overline{\psi}^{\dot{\alpha}}\partial_{\underline{a}}\psi^{\alpha} \; + \; \tfrac{1}{2}m(\,\psi^{\alpha}\psi_{\alpha} + \overline{\psi}^{\dot{\alpha}}\overline{\psi}_{\dot{\alpha}})\;] \quad .$$

Hypercube elements

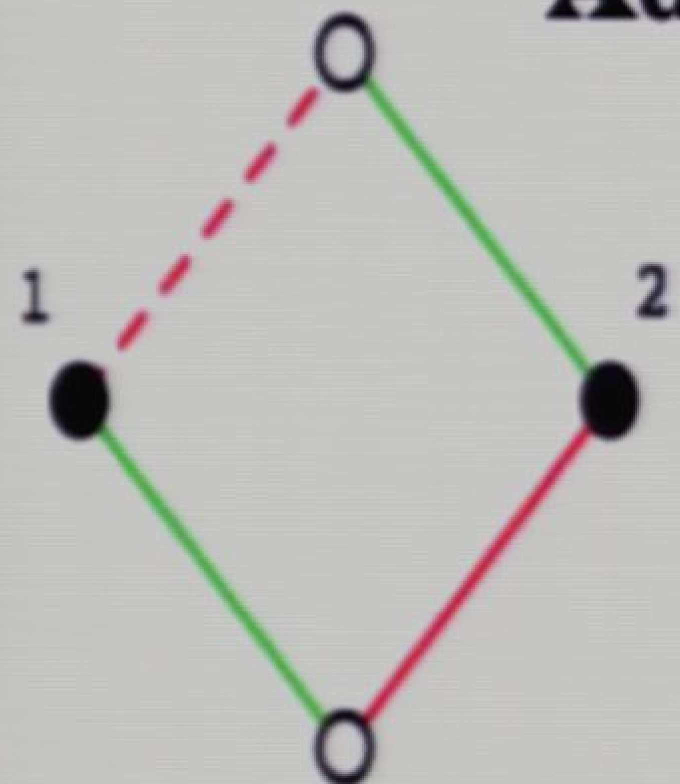
n-cube	Graph	Names Schläfli symbol Coxeter-Dynkin	Vertices (*0*-faces)	Edges (*1*-faces)	Faces (*2*-faces)	Cells (*3*-faces)	(*4*-faces)	(*5*-faces)	(*6*-faces)
0-cube		Point -	1						
1-cube		Digon {} or {2}	2	1					
2-cube		Square **Tetragon** {4}	4	4	1				
3-cube		Cube **Hexahedron** {4,3}	8	12	6	1			
4-cube		Tesseract **octachoron** {4,3,3}	16	32	24	8	1		
5-cube		Penteract **decateron** {4,3,3,3}	32	80	80	40	10	1	
6-cube		Hexeract **dodecapeton** {4,3,3,3,3}	64	192	240	160	60	12	1
7-cube		**Hepteract** **tetradeca-7-tope** {4,3,3,3,3,3}	128	448	672	560	280	84	14
8-cube		**Octeract** **hexadeca-8-tope** {4,3,3,3,3,3,3}	256	1024	1792	1792	1120	448	112
9-cube		**Enneract** **octadeca-9-tope** {4,3,3,3,3,3,3,3}	512	2304	4608	5376	4032	2016	672







Adinkra



$$D_1 A = i\Psi_1 \quad , \quad D_2 A = i\Psi_2 \ ,$$

$$D_1 F = i\partial_\tau \Psi_2 \quad , \quad D_2 F = -i\partial_\tau \Psi_1 \ ,$$

$$D_1 \Psi_1 = \partial_\tau A \quad , \quad D_2 \Psi_1 = -F \ ,$$

$$D_1 \Psi_2 = F \quad , \quad D_2 \Psi_2 = \partial_\tau A \ .$$

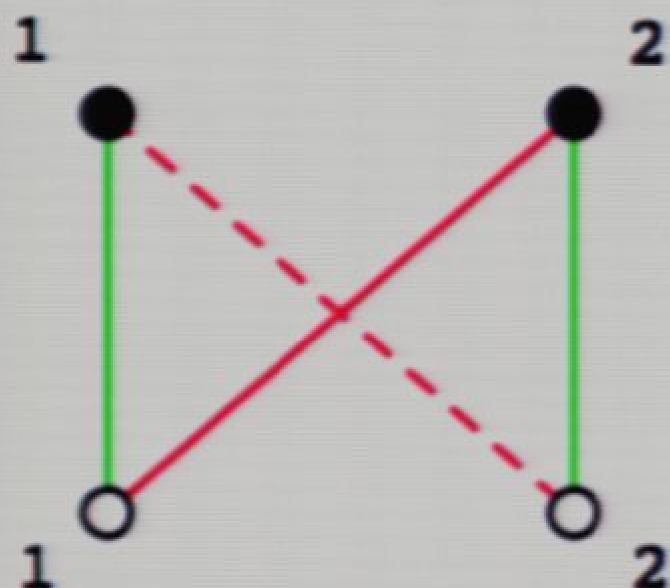
Color indicates direction,

Height indicates engineering dimension,

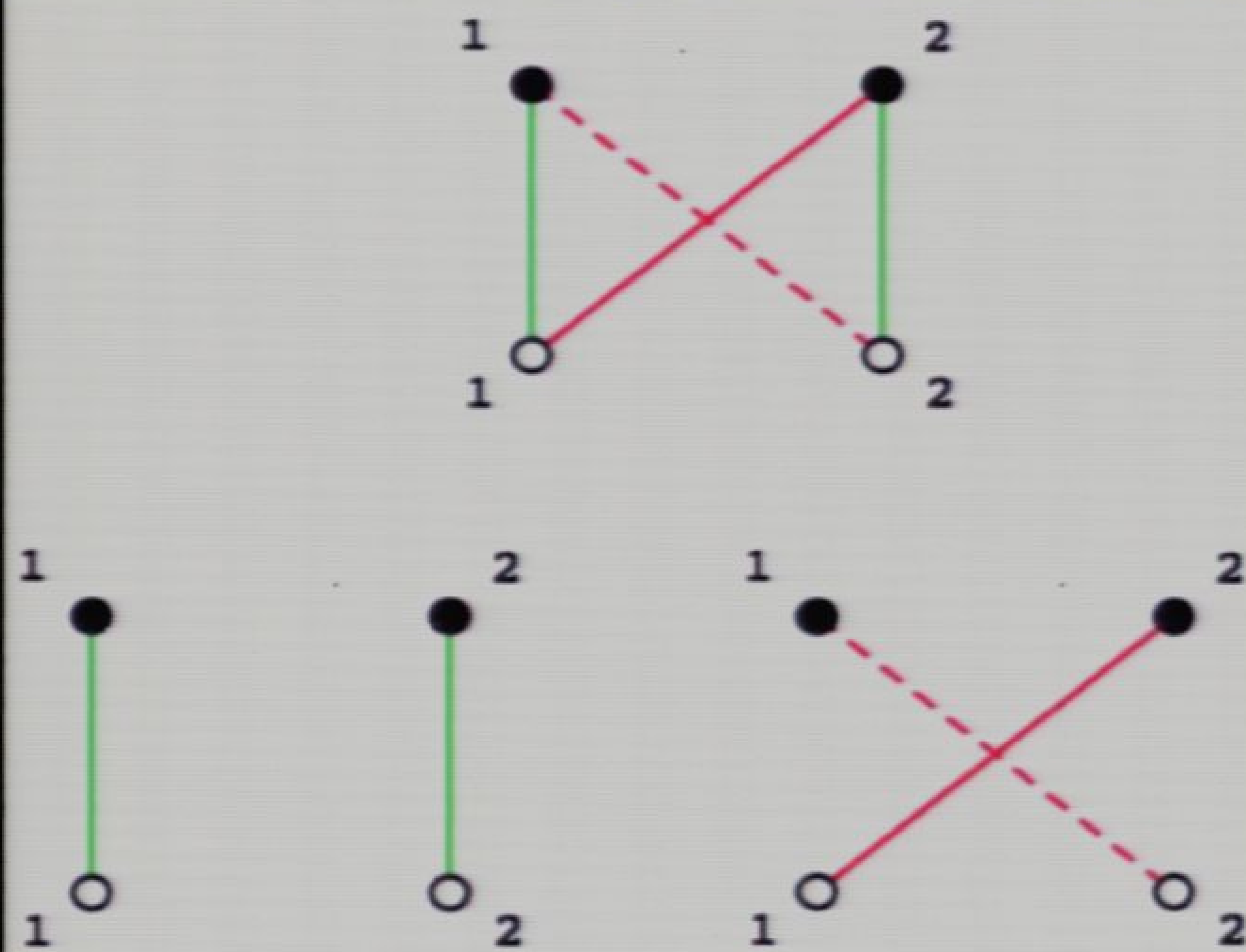
Solid lines indicate positive coefficients,

Dashed lines indicate negative coefficients.

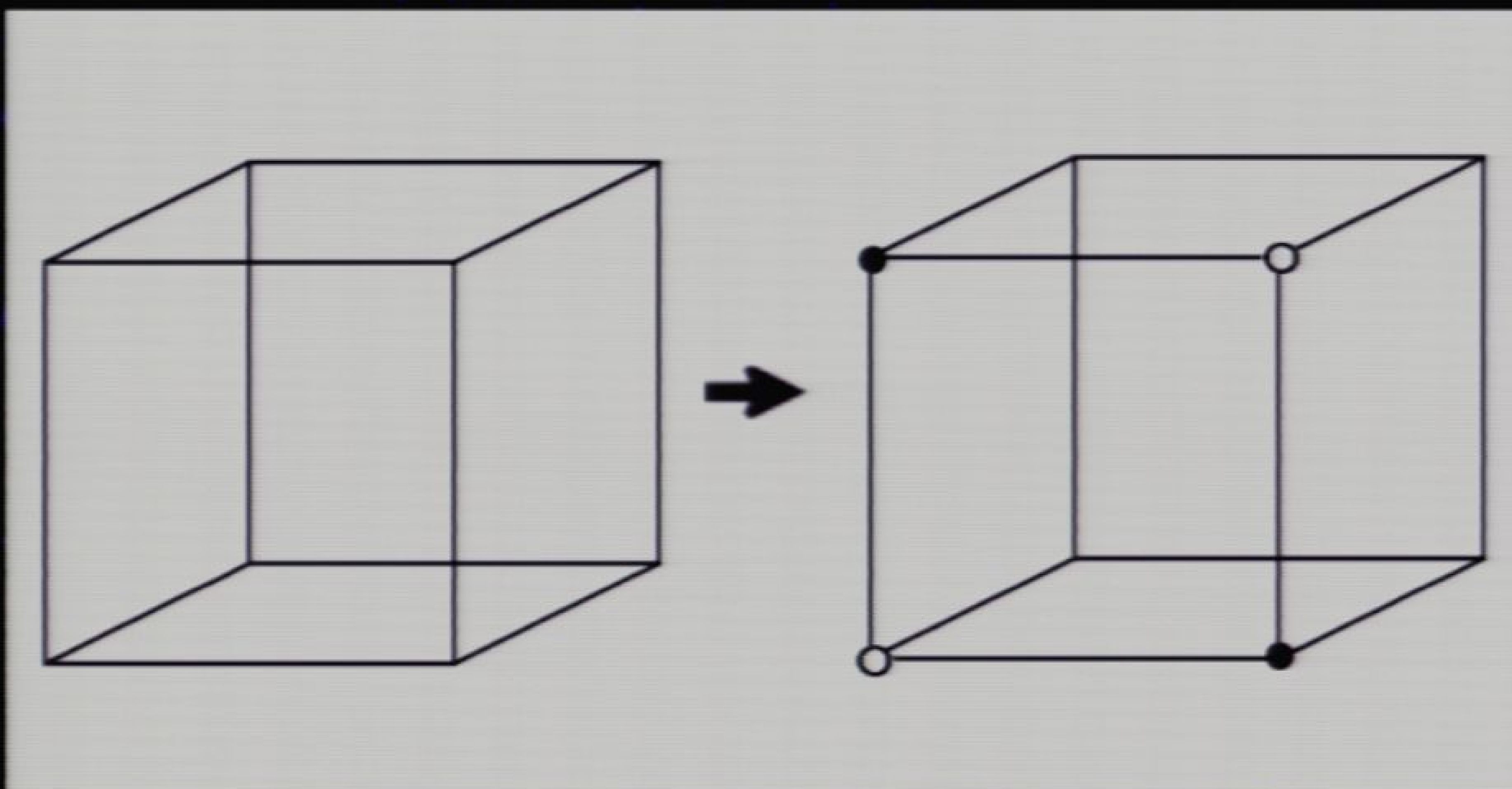
Adinkra

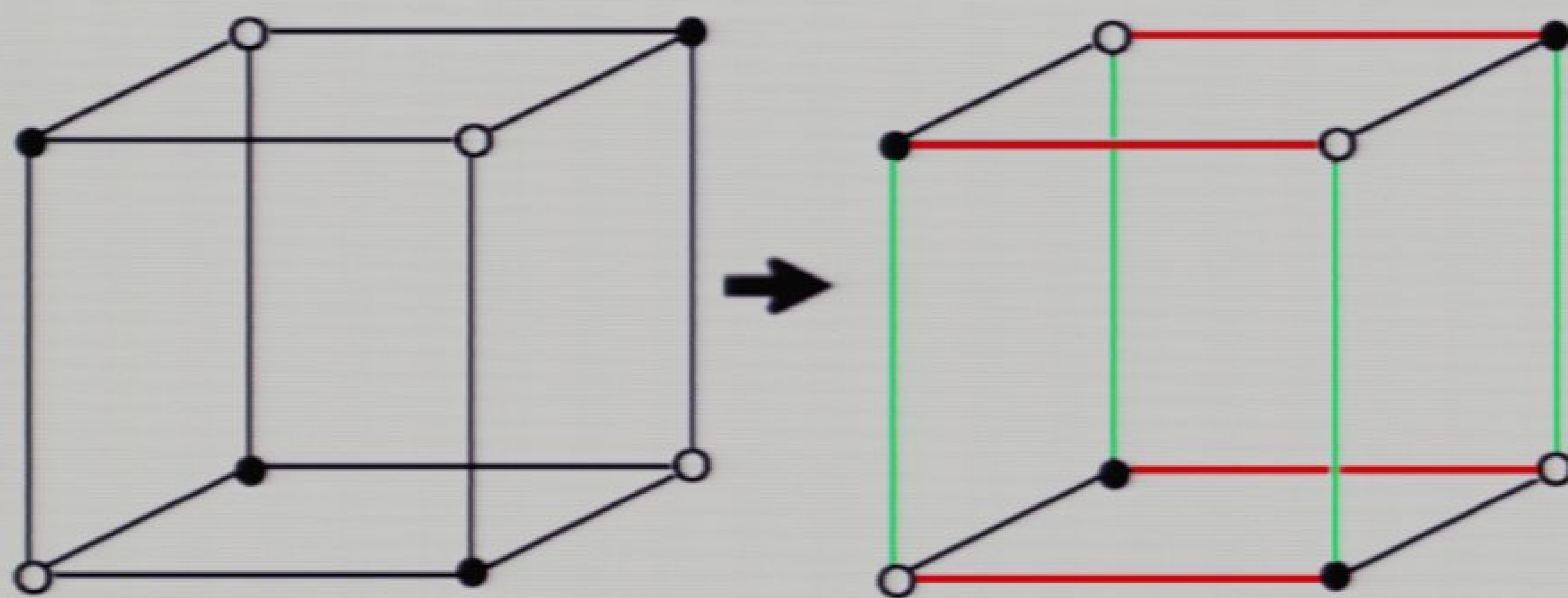


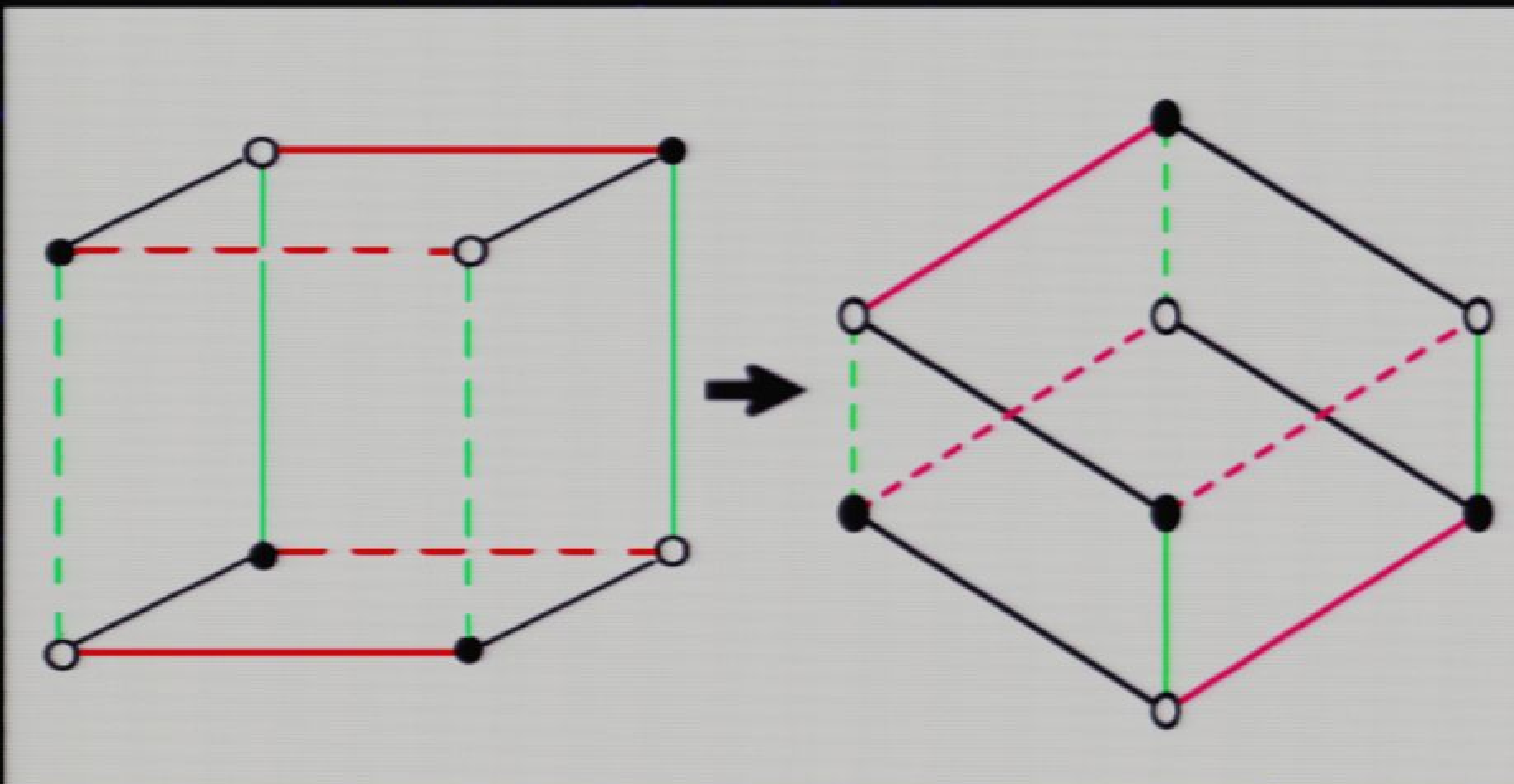
$$D_1\Phi_1 = i\Psi_1 \ , \ D_2\Phi_1 = i\Psi_2 \ ,$$
$$D_1\Phi_2 = i\Psi_2 \ , \ D_2\Phi_2 = -i\Psi_1 \ ,$$
$$D_1\Psi_1 = \partial_\tau\Phi_1 \ , \ D_2\Psi_1 = -\partial_\tau\Phi_2 \ ,$$
$$D_1\Psi_2 = \partial_\tau\Phi_2 \ , \ D_2\Psi_2 = \partial_\tau\Phi_1 \ .$$

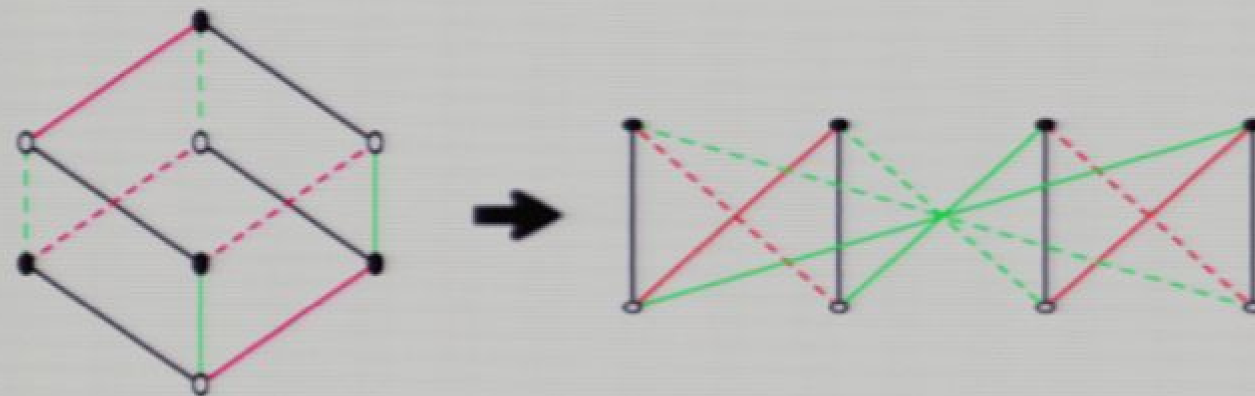


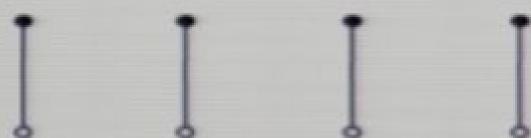
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$







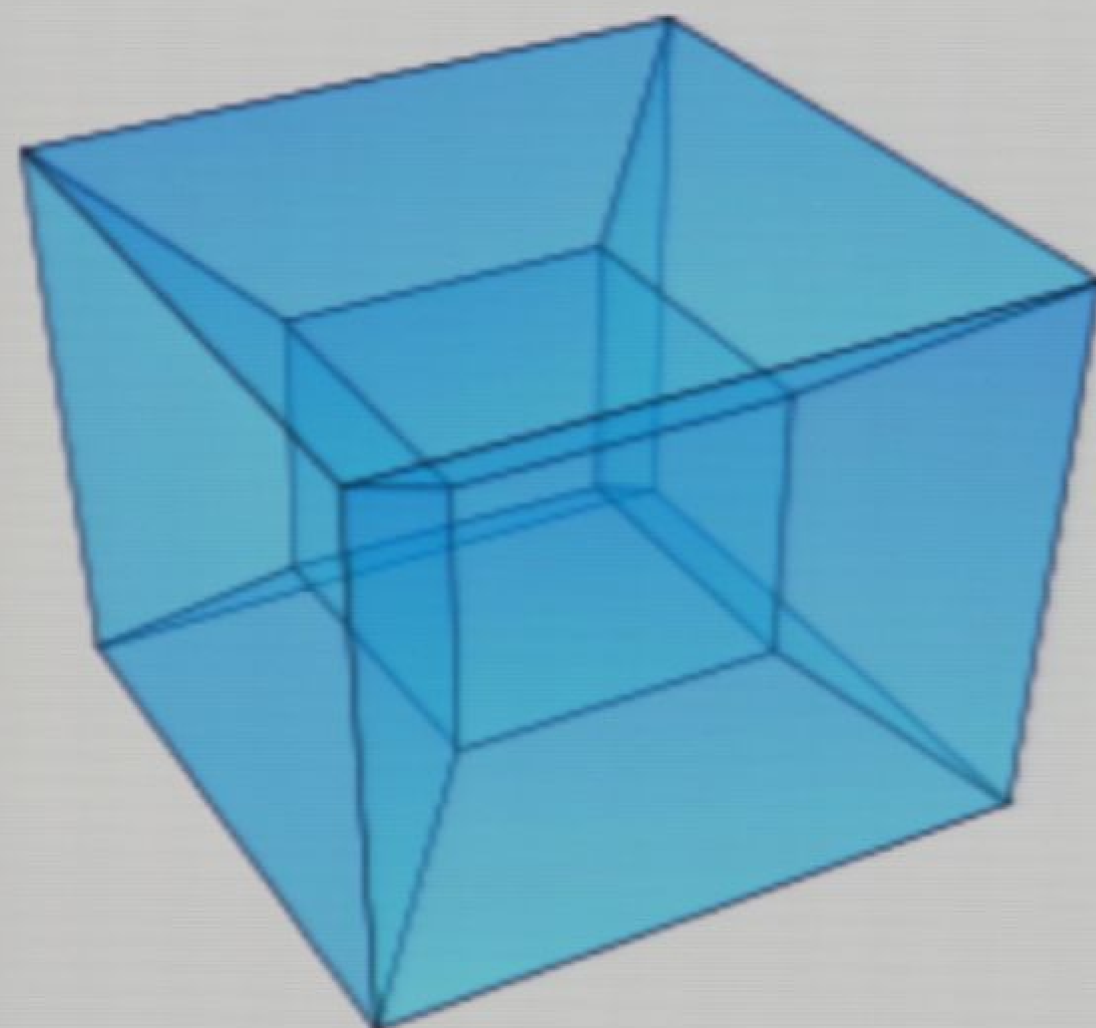




$$= \mathbf{I}_2 \otimes \mathbf{I}_2$$

$$= i\,\mathbf{I}_2 \otimes \sigma^2$$

$$= i\,\sigma^2 \otimes \sigma^1$$



Vertex 'Polarization' of a Tesseract

"$\mathcal{GR}(\mathrm{d}, \mathcal{N})$ Algebras" or "Garden Algebras."

$$(\mathrm{L}_{\mathrm{I}})_i{}^j (\mathrm{R}_{\mathrm{J}})_j{}^k + (\mathrm{L}_{\mathrm{J}})_i{}^j (\mathrm{R}_{\mathrm{I}})_j{}^k = 2\,\delta_{\mathrm{IJ}}\,\delta_i{}^k \quad ,$$

$$(\mathrm{R}_{\mathrm{J}})_{\hat{i}}{}^j (\mathrm{L}_{\mathrm{I}})_j{}^{\hat{k}} + (\mathrm{R}_{\mathrm{I}})_{\hat{i}}{}^j (\mathrm{L}_{\mathrm{J}})_j{}^{\hat{k}} = 2\,\delta_{\mathrm{IJ}}\,\delta_{\hat{i}}{}^{\hat{k}} \quad .$$

$$(\mathrm{R}_{\mathrm{I}})_{\hat{j}}{}^k \,\delta_{ik} = (\mathrm{L}_{\mathrm{I}})_i{}^{\hat{k}}\,\delta_{\hat{j}\hat{k}} \quad .$$

Using A Matrix Representation to Define Basis Elements

Consider real matrices (with $I = 1, \ldots, \mathcal{N}+1$):

$$\gamma^I \gamma^J + \gamma^J \gamma^I = 2\delta^{IJ}\,\mathbf{I}$$

Projection Operator

$$P_{\pm} = \tfrac{1}{2}(\mathbf{I} \pm \gamma^{\mathcal{N}+1})$$

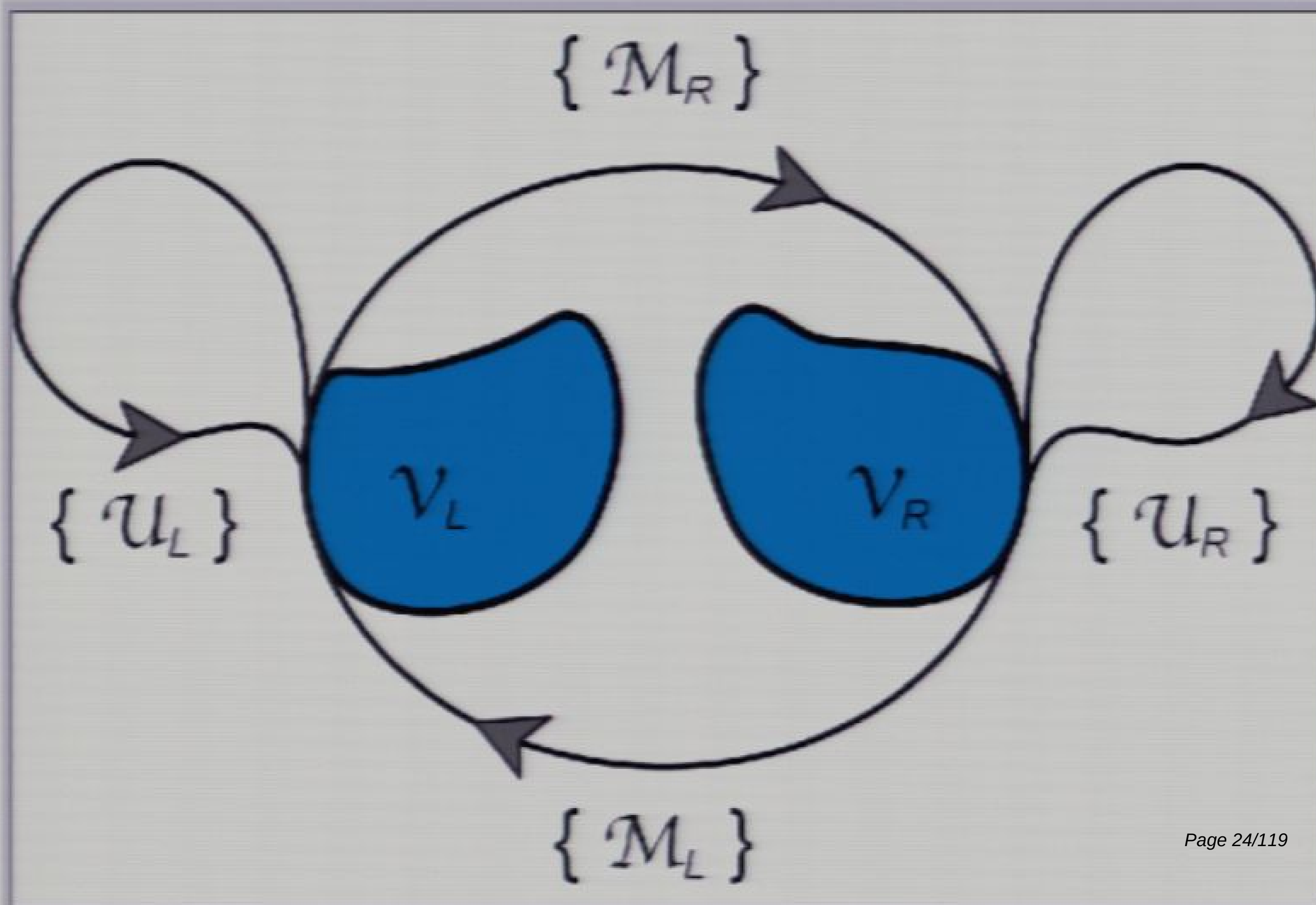
L/R Definitions

$$L_{\rm I} \equiv P_+ \gamma_{\rm I} P_- \quad , \quad R_{\rm I} \equiv P_- \gamma_{\rm I} P_+ \quad ,$$

with $\mathrm{I} = 1, \ldots \mathcal{N}$

$$L_{\rm I} R_{\rm K} + L_{\rm K} R_{\rm I} = 2\delta_{\rm IK} P_+ \quad ,$$
$$R_{\rm I} L_{\rm K} + R_{\rm K} L_{\rm I} = 2\delta_{\rm IK} P_- \quad ,$$
$$\forall\, \mathrm{I}, \mathrm{K} = 1, \cdots, \mathcal{N} \quad .$$

Venn Diagram



For a fixed value of $\mathcal{N}$ there is a minimum value $d_{\mathcal{N}}$ such that $d_{\mathcal{N}} \times d_{\mathcal{N}}$ matrices faithfully represent this algebra. With $\mathcal{N} = 8m + n$, $1 \leq n \leq 8$ and using the definition if $\mathcal{N} = 8k \rightarrow m = k - 1$ for $k = 1, 2, 3, \ldots \infty$, this minimum value is shown in the following table

$$\mathrm{d}_{\mathcal{N}} = 16^m F_{\mathcal{R}\mathcal{H}}(n)$$

n	$F_{\mathcal{R}\mathcal{H}}(n)$
1	1
2	2
3	4
4	4
5	8
6	8
7	8
8	8

Conjectures On Minimal Off-shell Multiplets

$\mathcal{N}$	N	#(bosons)	#(fermions)	Number of Adinkra Topologies
1	4	4	4	1 (D_4)
2	8	8	8	1 (E_8)
3	12	64	64	2 (D_{12}, $E_8 \times D_4$)
4	16	128	128	2 (E_{16}, $E_8 \times E_8$)
5	20	1,024	1,024	10
6	24	2,048	2,048	9
7	28	16,384	16,384	151
8	32	32,768	32,768	85

Minimal Off-Shell 4D, $\mathcal{N} \leq 8$ Supermultiplets

Adinkras:

Definition of Spacetime SUSY ?

...lorized Transformation Law...

Example

Example (3)

$$\{\mathcal{F}\} : \mathbb{R}^1 \to \mathcal{M}_L \oplus \mathcal{M}_R$$

$$\{\mathcal{B}\} : \mathbb{R}^1 \to \mathcal{U}_L \oplus \mathcal{U}_R .$$

$$\delta_Q \Phi_{k\,l} = i \left[\epsilon^{\rm I} ({\rm L}^{\rm I})_k{}^{\hat\ell}\, \Psi_{\hat\ell\, l} + \bar\epsilon^{\rm I} ({\rm L}^{\rm I})_l{}^{\hat\ell}\, \widehat\Psi_{k\,\hat\ell} \right] \quad ,$$

$$\delta_Q \Psi_{\hat k\, l} = \left[- \epsilon^{\rm I} ({\rm R}^{\rm I})_{\hat k}{}^{\ell}\, \partial_\tau \Phi_{\ell\, l} + \bar\epsilon^{I} ({\rm L}^{\rm I})_l{}^{\hat\ell}\, \partial_\tau \widehat\Phi_{\hat k\,\hat\ell} \right]$$

$$\delta_Q \widehat\Psi_{k\,\hat l} = \left[- \epsilon^{\rm I} ({\rm L}^{\rm I})_k{}^{\hat\ell}\, \partial_\tau \widehat\Phi_{\hat\ell\,\hat l} - \bar\epsilon^{\rm I} ({\rm R}^{\rm I})_{\hat l}{}^{\ell}\, \partial_\tau \Phi_{k\,\ell} \right]$$

$$\delta_Q \widehat\Phi_{\hat k\,\hat l} = i \left[\epsilon^{\rm I} ({\rm R}^{\rm I})_{\hat k}{}^{\ell}\, \widehat\Psi_{\ell\,\hat l} - \bar\epsilon^{I} ({\rm R}^{\rm I})_{\hat l}{}^{\ell}\, \Psi_{\hat k\,\ell} \right] \quad ,$$

Colorized Transformation Laws

$$\delta_Q A = -i\epsilon\hat{\psi}^1 - i\epsilon\hat{\psi}^2 + i\epsilon\psi^1 + i\epsilon\psi^2 \ ,$$

$$\delta_Q B = i\epsilon\psi^1 + i\epsilon\psi^2 + i\epsilon\hat{\psi}^1 + i\epsilon\hat{\psi}^2 \ ,$$

$$\delta_Q \psi^1 = \epsilon\partial_\tau B + \epsilon G + \epsilon\partial_\tau A - \epsilon F \ ,$$

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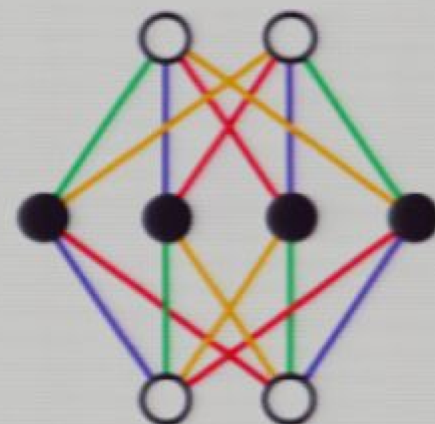
$$\delta_Q \hat{\psi}^1 = -\epsilon\partial_\tau A - \epsilon F + \epsilon\partial_\tau B - \epsilon G \ ,$$

$$\delta_Q \hat{\psi}^2 = -\epsilon\partial_\tau A + \epsilon F + \epsilon\partial_\tau B + \epsilon G \ ,$$

$$\delta_Q F = i\epsilon\partial_\tau\hat{\psi}^2 - i\epsilon\partial_\tau\hat{\psi}^1 + i\epsilon\partial_\tau\psi^2 - i\epsilon\partial_\tau\psi^1 \ ,$$

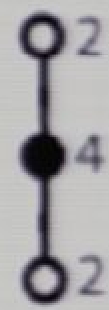
$$\delta_Q G = -i\epsilon\partial_\tau\psi^2 + i\epsilon\partial_\tau\psi^1 + i\epsilon\partial_\tau\hat{\psi}^2 - i\epsilon\partial_\tau\hat{\psi}^1 \ .$$

laws

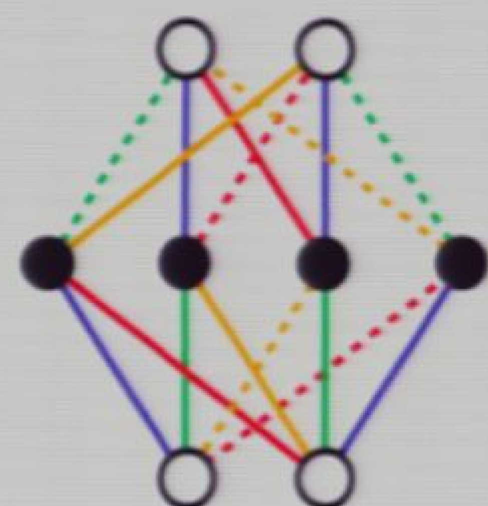


'Peacock mode' of the Adinkra

For some purposes, it is not necessary to display this level of detail. In this case, nodes may be 'collapsed' upon one another. For example, the fully collapsed version of this Adinkra is given by



where the numbers next to the nodes signify their respective multiplicities.



'Rampant peacock mode' of the Adinkra

Colorized Transformation Laws

$$\delta_Q A = -i\epsilon\hat{\psi}^1 - i\epsilon\hat{\psi}^2 + i\epsilon\psi^1 + i\epsilon\psi^2 \ ,$$

$$\delta_Q B = i\epsilon\psi^1 + i\epsilon\psi^2 + i\epsilon\hat{\psi}^1 + i\epsilon\hat{\psi}^2 \ ,$$

$$\delta_Q \psi^1 = \epsilon\partial_\tau B + \epsilon G + \epsilon\partial_\tau A - \epsilon F \ ,$$

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$$\delta_Q \hat{\psi}^2 = -\epsilon\partial_\tau A + \epsilon F + \epsilon\partial_\tau B + \epsilon G \ ,$$

$$\delta_Q F = i\epsilon\partial_\tau\hat{\psi}^2 - i\epsilon\partial_\tau\hat{\psi}^1 + i\epsilon\partial_\tau\psi^2 - i\epsilon\partial_\tau\psi^1 \ ,$$

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this Adinkra is given by

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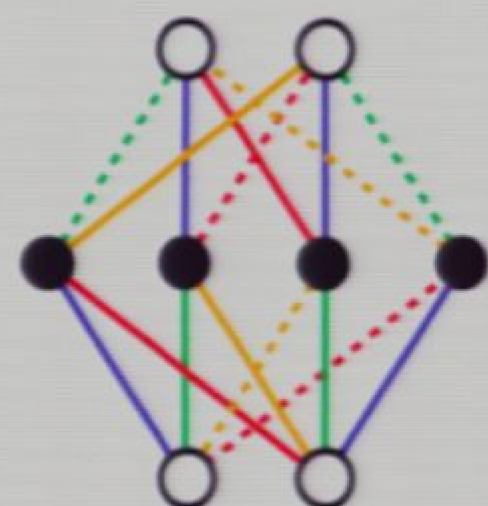

'Rampant peacock mode' of the Adinkra

$$\mathrm{D}_a A = \psi_a \quad ,$$

$$\mathrm{D}_a B = i(\gamma^5)_a{}^b \psi_b \quad ,$$

$$\mathrm{D}_a \psi_b = i(\gamma^\mu)_{ab} \partial_\mu A - (\gamma^5 \gamma^\mu)_{ab} \partial_\mu B - i C_{ab} F + (\gamma^5)_{ab} G \quad ,$$

$$\mathrm{D}_a F = (\gamma^\mu)_a{}^b \partial_\mu \psi_b \quad ,$$

$$\mathrm{D}_a G = i(\gamma^5 \gamma^\mu)_a{}^b \partial_\mu \psi_b \quad .$$

Root Superfields & SUSY Holography

Colorized Transformation Laws

$$\delta_Q A = -i\epsilon\hat{\psi}^1 - i\epsilon\hat{\psi}^2 + i\epsilon\psi^1 + i\epsilon\psi^2 ,$$

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Colorized Transformation Laws

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$$\delta_Q G = -i\epsilon\partial_\tau\psi^2 + i\epsilon\partial_\tau\psi^1 + i\epsilon\partial_\tau\hat{\psi}^2 - i\epsilon\partial_\tau\hat{\psi}^1 \ .$$

Root Superfields & SUSY Holography

$$\mathrm{L}^{\mathrm{I}}\mathrm{R}^{\mathrm{J}} = -\delta^{\mathrm{IJ}}\mathrm{I} + \epsilon^{\mathrm{IJ}} f^* \;,\quad \mathrm{R}^{\mathrm{I}}\mathrm{L}^{\mathrm{J}} = -\delta^{\mathrm{IJ}}\mathrm{I} + \epsilon^{\mathrm{IJ}} \hat{f}^* \;,$$

$$Tr[f^*] = Tr[\hat{f}^*] = 0 \;,\quad (f^*)^2 = (\hat{f}^*)^2 = -\mathrm{I} \;.$$

$$\Phi_{k\,l} = \tfrac{1}{2}\delta_{k\,l}B + \tfrac{1}{2}(f^*)_{k\,l}(\partial_\tau)^{-1}G \;,\quad \Psi_{\hat{k}\,l} = -\tfrac{1}{2}\mathrm{R}^{\mathrm{I}}_{\hat{k}l}\psi^{\mathrm{I}} \;,$$

$$\widehat{\Phi}_{\hat{k}\,\hat{l}} = \tfrac{1}{2}\hat{\delta}_{\hat{k}\hat{l}}A + \tfrac{1}{2}(\hat{f}^*)_{\hat{k}\hat{l}}(\partial_\tau)^{-1}F \;,\quad \widehat{\Psi}_{k\,\hat{l}} = \tfrac{1}{2}\mathrm{L}^{\mathrm{I}}_{k\,\hat{l}}\hat{\psi}^{\mathrm{I}} \;,$$

$$[\Phi_{k\,l}]_{\mathcal{R}}(a_1, a_2) = \tfrac{1}{2}\delta_{k\,l}(\partial_\tau)^{-a_1}B_0 + \tfrac{1}{2}(f^*)_{k\,l}(\partial_\tau)^{-a_2}B_2 \;,$$

$$[\Psi_{\hat{k}\,l}]_{\mathcal{R}}(a_3) = -\tfrac{1}{2}\mathrm{R}^{\mathrm{I}}_{\hat{k}l}(\partial_\tau)^{-a_3}\varphi^{\mathrm{I}} \;,$$

$$[\widehat{\Phi}_{\hat{k}\,\hat{l}}]_{\mathcal{R}}(a_4, a_5) = \tfrac{1}{2}\hat{\delta}_{\hat{k}\hat{l}}(\partial_\tau)^{-a_4}\widehat{B}_0 + \tfrac{1}{2}(\hat{f}^*)_{\hat{k}\hat{l}}(\partial_\tau)^{-a_5}\widehat{B}_2 \;,$$

$$[\widehat{\Psi}_{k\,\hat{l}}]_{\mathcal{R}}(a_6) = \tfrac{1}{2}\mathrm{L}^{\mathrm{I}}_{k\,\hat{l}}(\partial_\tau)^{-a_6}\hat{\varphi}^{\mathrm{I}} \;.$$

Garden Algebra & Chromocharacters

$$\varphi^{(p)}{}_{\mathrm{I}_1 \mathrm{J}_1 \cdots \mathrm{I}_p \mathrm{J}_p} \;=\; \mathrm{Tr}\left[\, \mathrm{L}_{\mathrm{I}_1} (\mathrm{L}_{\mathrm{J}_1})^t \cdots \mathrm{L}_{\mathrm{I}_p} (\mathrm{L}_{\mathrm{J}_p})^t \,\right]$$

$$\tilde{\varphi}^{(p)}{}_{\mathrm{I}_1 \mathrm{J}_1 \cdots \mathrm{I}_p \mathrm{J}_p} \;=\; \mathrm{Tr}\left[\, (\mathrm{L}_{\mathrm{I}_1})^t \mathrm{L}_{\mathrm{J}_1} \cdots (\mathrm{L}_{\mathrm{I}_p})^t \mathrm{L}_{\mathrm{J}_p} \,\right]$$

For all off-shell 4D, $\mathcal{N} = 1$ *multiplets, the* $p = 1$ *chromocharacters take the form*

$$\varphi^{(1)}{}_{\mathrm{IJ}} \;=\; \mathrm{d}\,\delta_{\mathrm{IJ}} \quad .$$

For all off-shell 4D, $\mathcal{N} = 1$ *multiplets, the* $p = 2$ *chromocharacters take the form*

$$\varphi^{(2)}{}_{\mathrm{IJKL}} \;=\; \mathrm{d}\left[\, \delta_{\mathrm{IJ}}\delta_{\mathrm{KL}} \;-\; \delta_{\mathrm{IK}}\delta_{\mathrm{JL}} \;+\; \delta_{\mathrm{IL}}\delta_{\mathrm{JK}} \,\right] \;+\; \chi_0\, \epsilon_{\mathrm{IJKL}} \quad .$$

$$\phi^{(p)}{}_{I_1 J_1 \cdots I_p J_p} = \mathrm{Tr}\left[L_{I_1} (L_{J_1})^t \cdots L_{I_p} (L_{J_p})^t \right]$$

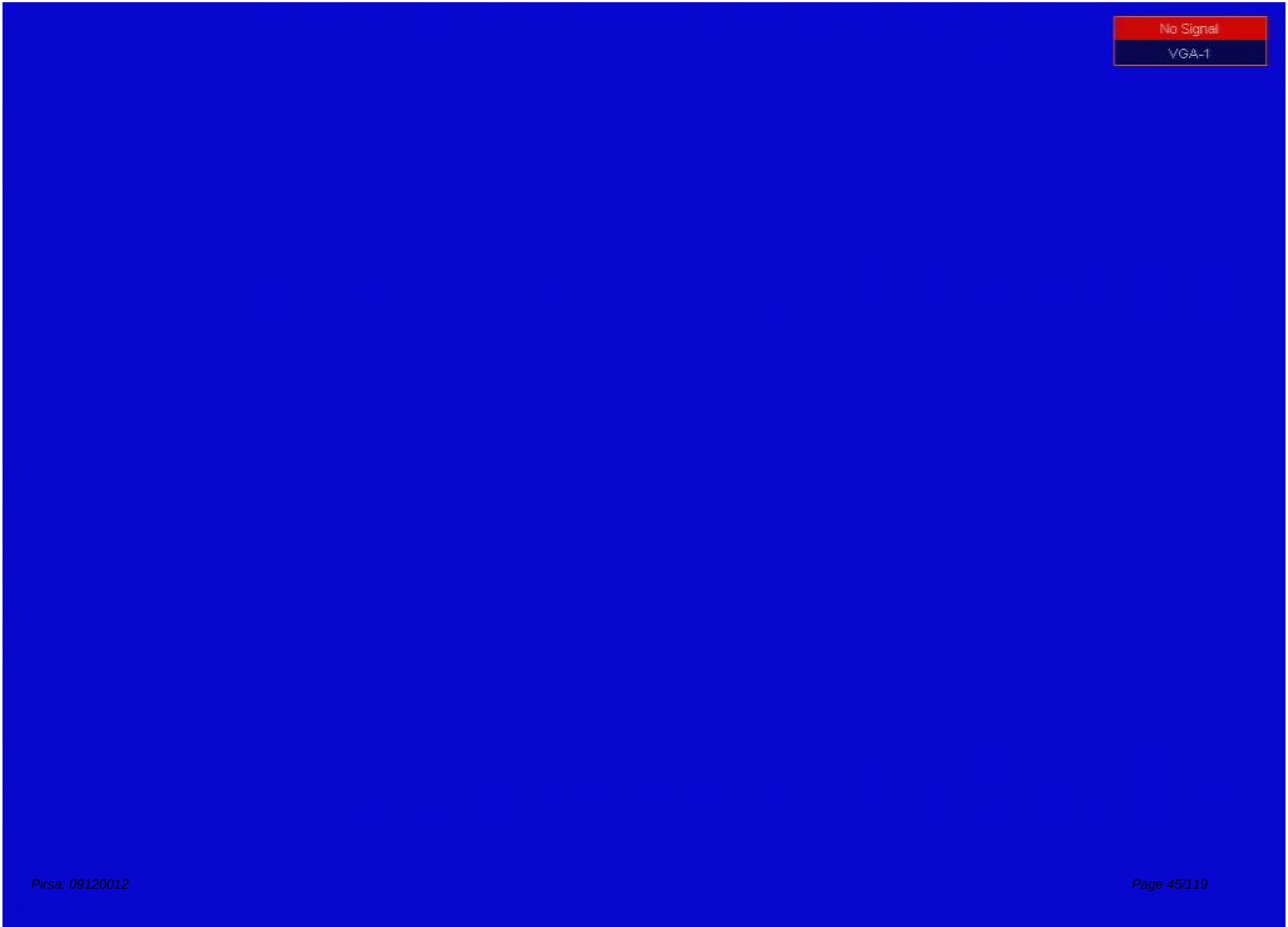
$$\tilde{\phi}^{(p)}{}_{I_1 J_1 \cdots I_p J_p} = \mathrm{Tr}\left[(L_{I_1})^t L_{J_1} \cdots (L_{I_p})^t L_{J_p} \right]$$

For all off-shell 4D, $\mathcal{N} = 1$ *multiplets, the* $p = 1$ *chromocharacters take the form*

$$\phi^{(1)}{}_{IJ} = d\,\delta_{IJ} \quad .$$

For all off-shell 4D, $\mathcal{N} = 1$ *multiplets, the* $p = 2$ *chromocharacters take the form*

$$\phi^{(2)}{}_{IJKL} = d \left[\delta_{IJ}\delta_{KL} - \delta_{IK}\delta_{JL} - \delta_{IL}\delta_{JK} \right] + \chi_0\, \epsilon_{IJKL} \quad .$$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$$\varphi^{(p)}{}_{I_1 J_1 \cdots I_p J_p} = \mathrm{Tr}\left[L_{I_1} (L_{J_1})^t \cdots L_{I_p} (L_{J_p})^t \right]$$

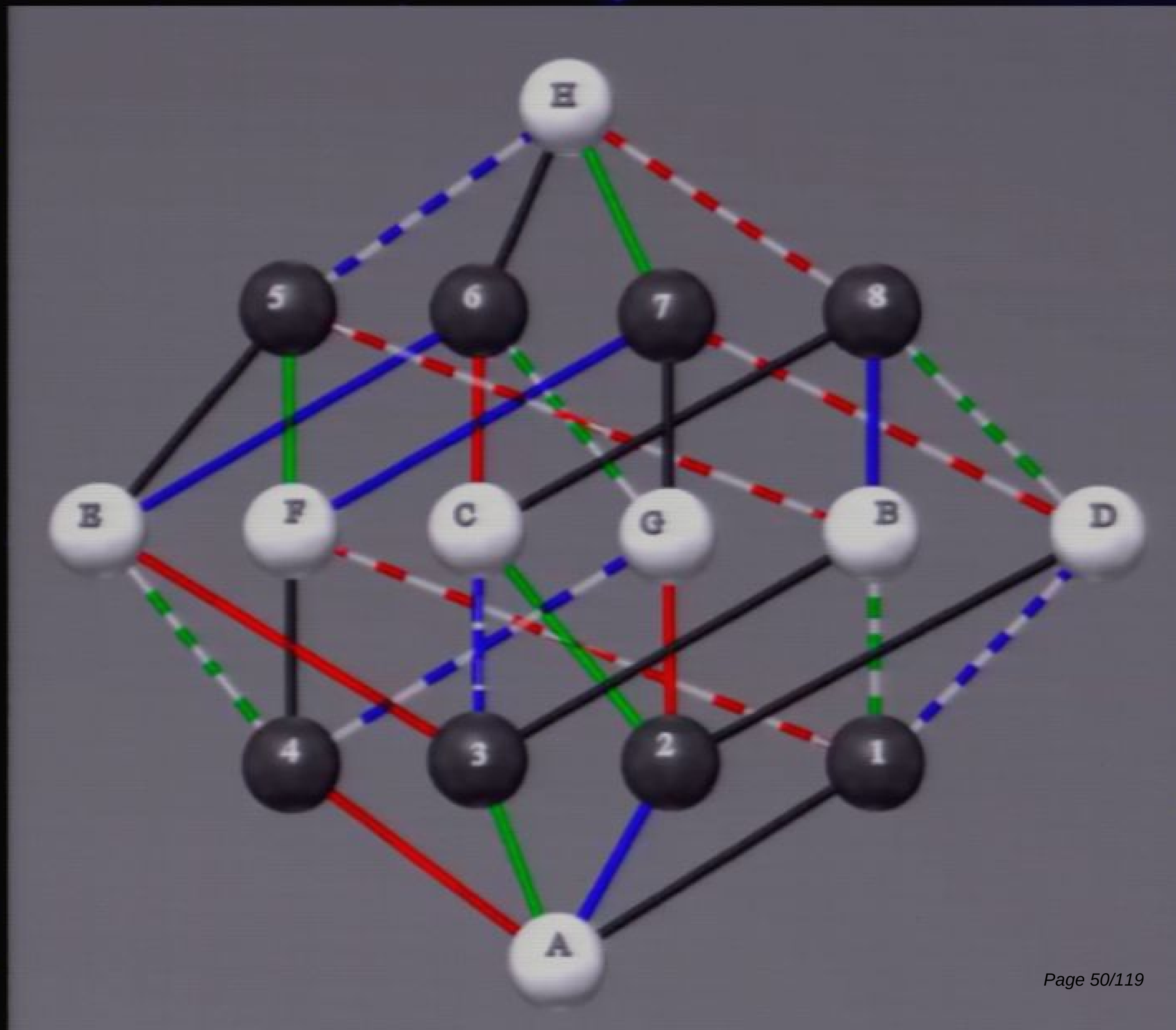
$$\widetilde{\varphi}^{(p)}{}_{I_1 J_1 \cdots I_p J_p} = \mathrm{Tr}\left[(L_{I_1})^t L_{J_1} \cdots (L_{I_p})^t L_{J_p} \right]$$

For all off-shell 4D, $\mathcal{N} = 1$ *multiplets, the* $p = 1$ *chromocharacters take the form*

$$\varphi^{(1)}{}_{IJ} = d\,\delta_{IJ} \quad .$$

For all off-shell 4D, $\mathcal{N} = 1$ *multiplets, the* $p = 2$ *chromocharacters take the form*

$$\varphi^{(2)}{}_{IJKL} = d\left[\delta_{IJ}\delta_{KL} - \delta_{IK}\delta_{JL} + \delta_{IL}\delta_{JK} \right] + \chi_0\,\epsilon_{IJKL} \quad .$$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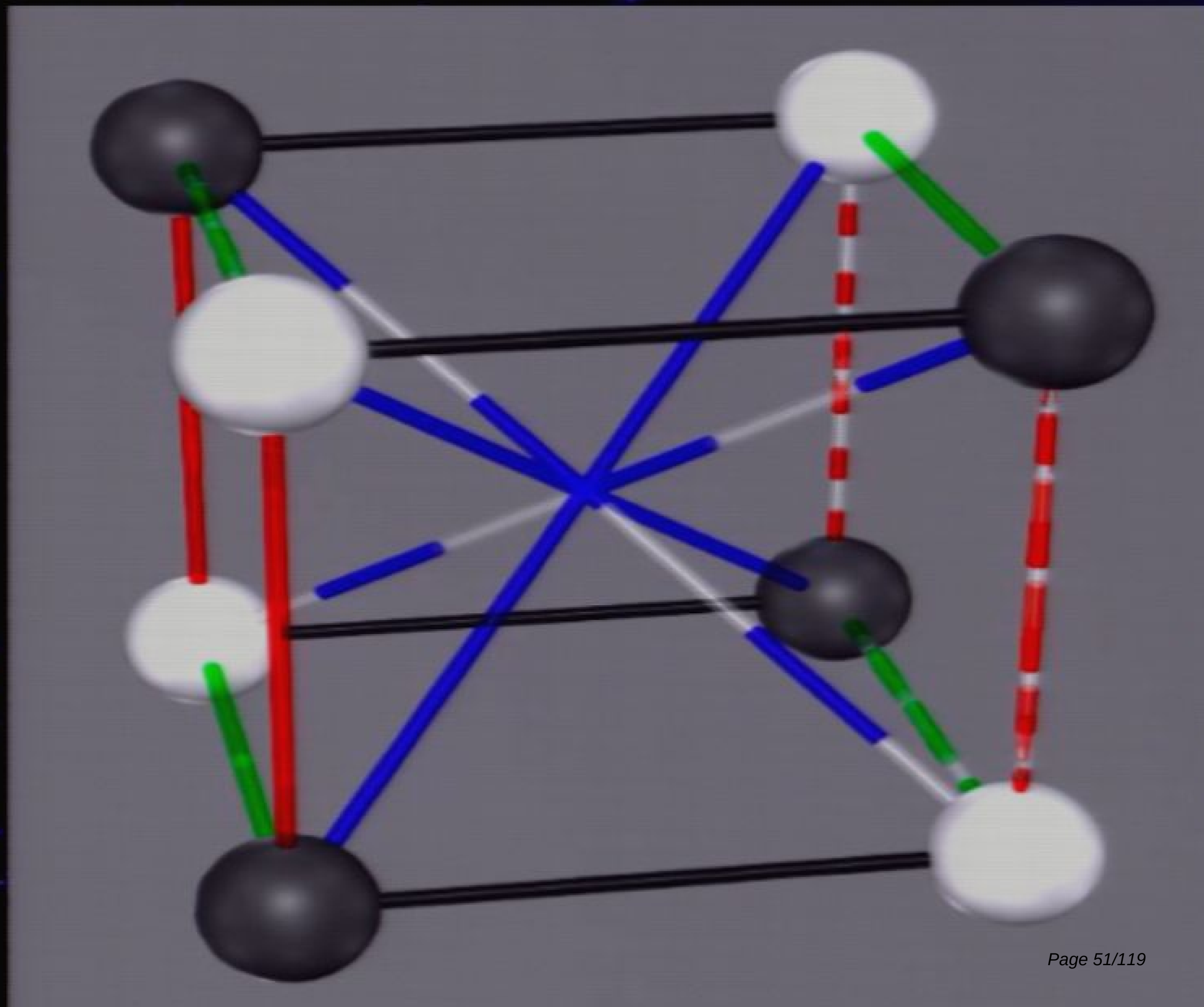


$$D_a A_\mu = (\gamma^\mu)_a{}^b \lambda_b \ ,$$

$$D_a \lambda_b = -i\frac{1}{4}([\gamma^\mu, \gamma^\nu])_{ab}(\partial_\mu A_\nu - \partial_\nu A_\mu) + (\gamma^5)_{ab}\, d \ ,$$

$$D_a d = i(\gamma^5 \gamma^\mu)_a{}^b \partial_\mu \lambda_b \ .$$

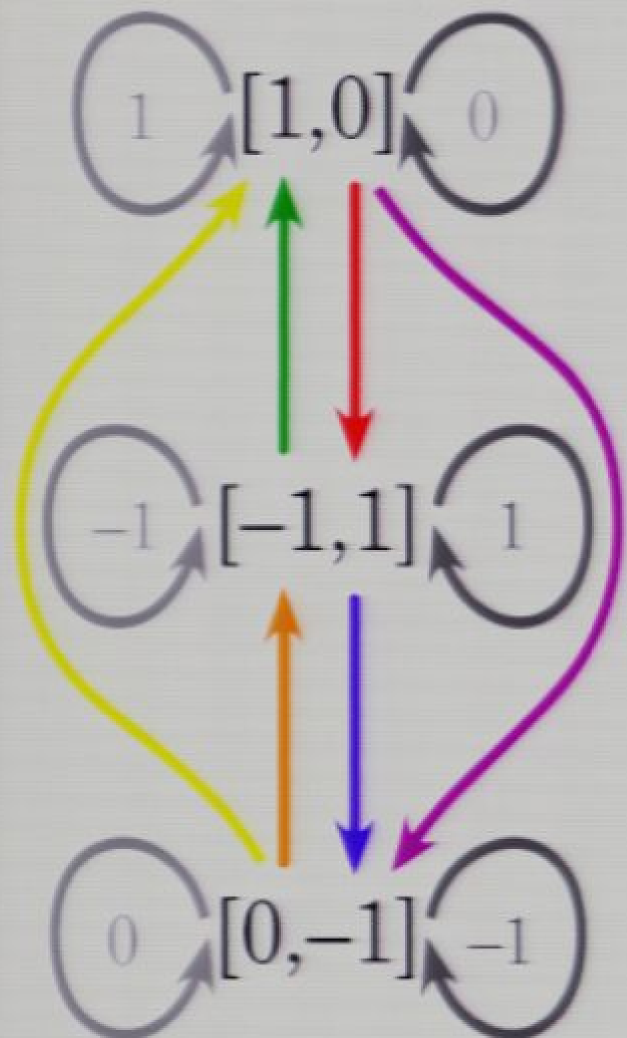
4D, $\mathcal{N} = 1$ Chromocharacter Conjecture

$$\varphi^{(1)}{}_{IJ} = 4\left(N_c + N_t\right)\delta_{IJ} \quad ,$$

$$\varphi^{(2)}{}_{IJKL} = 4\left(N_c + N_t\right)\left[\,\delta_{IJ}\delta_{KL} - \delta_{IK}\delta_{JL} + \delta_{IL}\delta_{JK}\,\right] + 4\left(N_c - N_t\right)\epsilon_{IJKL} \quad ,$$

where N_c and N_t are the number of cis-Adinkras and trans-Adinkras contained in the given supersymmetry representation.

Adinkras in Compact Lie Algebras



The fundamental, 3-dimensional representation of $su(3)$

$E_3 = [1,1]$

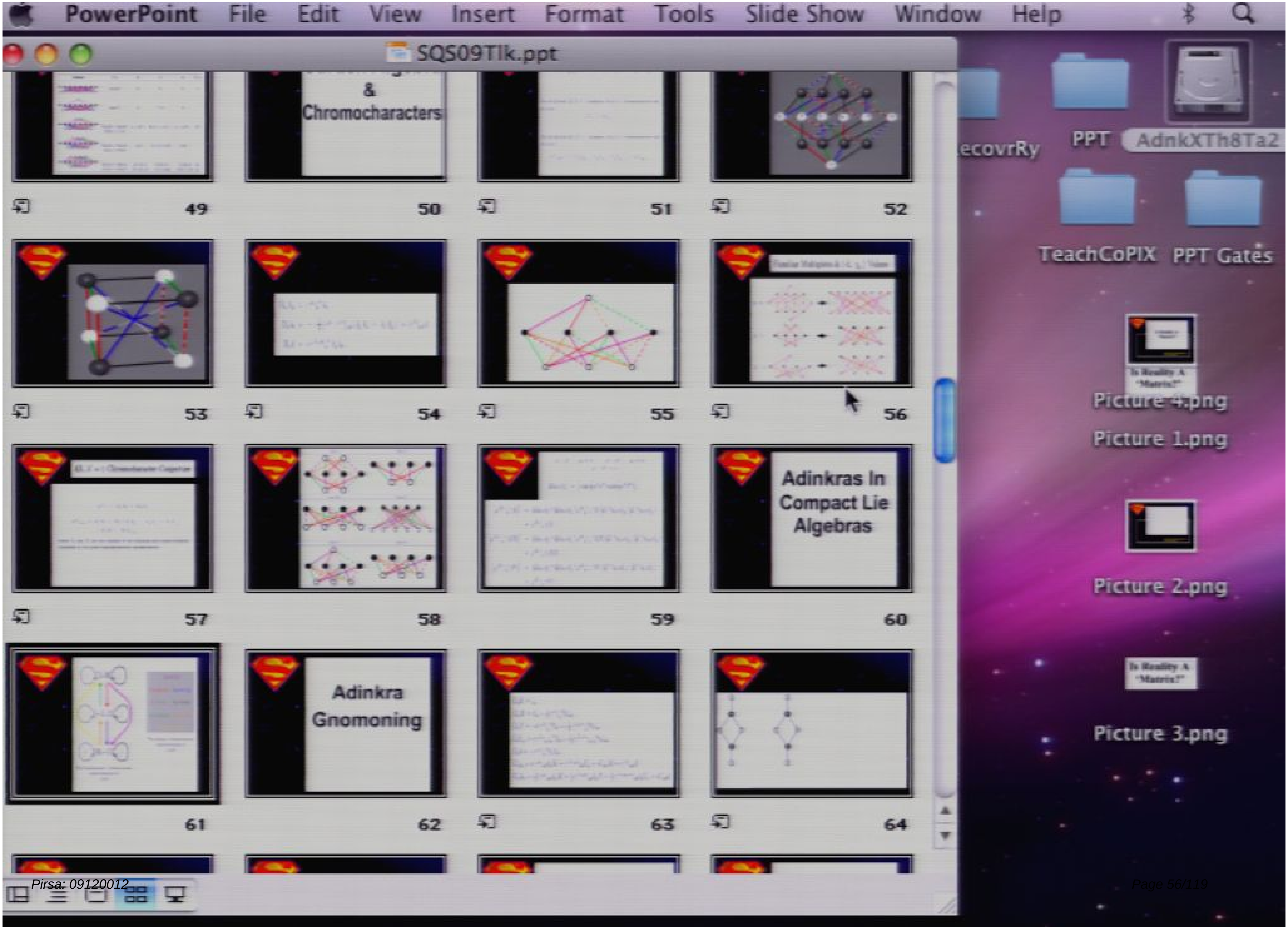
$E_1 = [2,-1]$ $E_2 = [-1,2]$

$H_1 = [0,0]$ $H_2 = [0,0]$

$E_{-1} = [-2,1]$ $E_{-2} = [1,-2]$

$E_{-3} = [-1,-1]$

The adjoint, 8-dimensional representation of $su(3)$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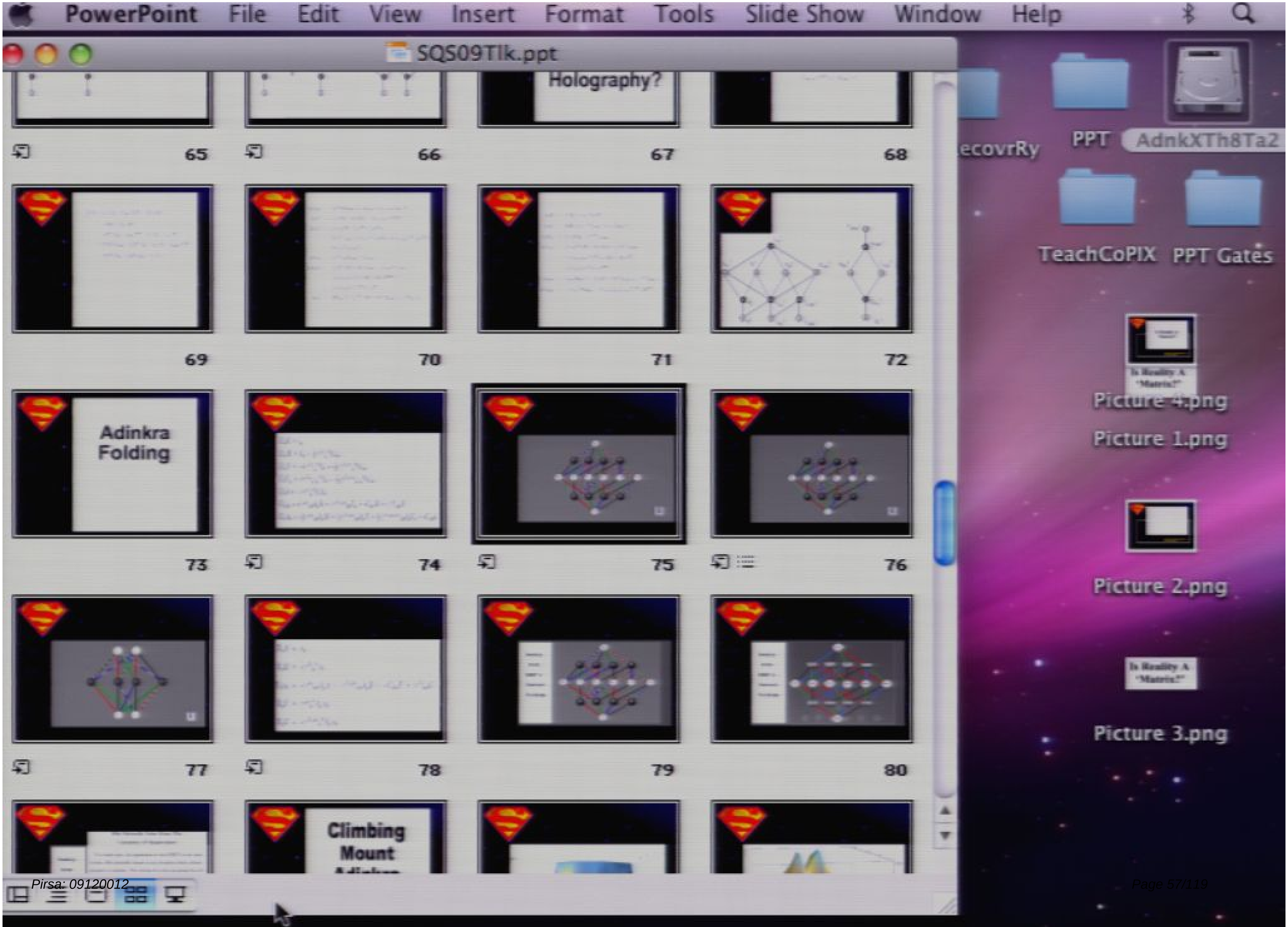
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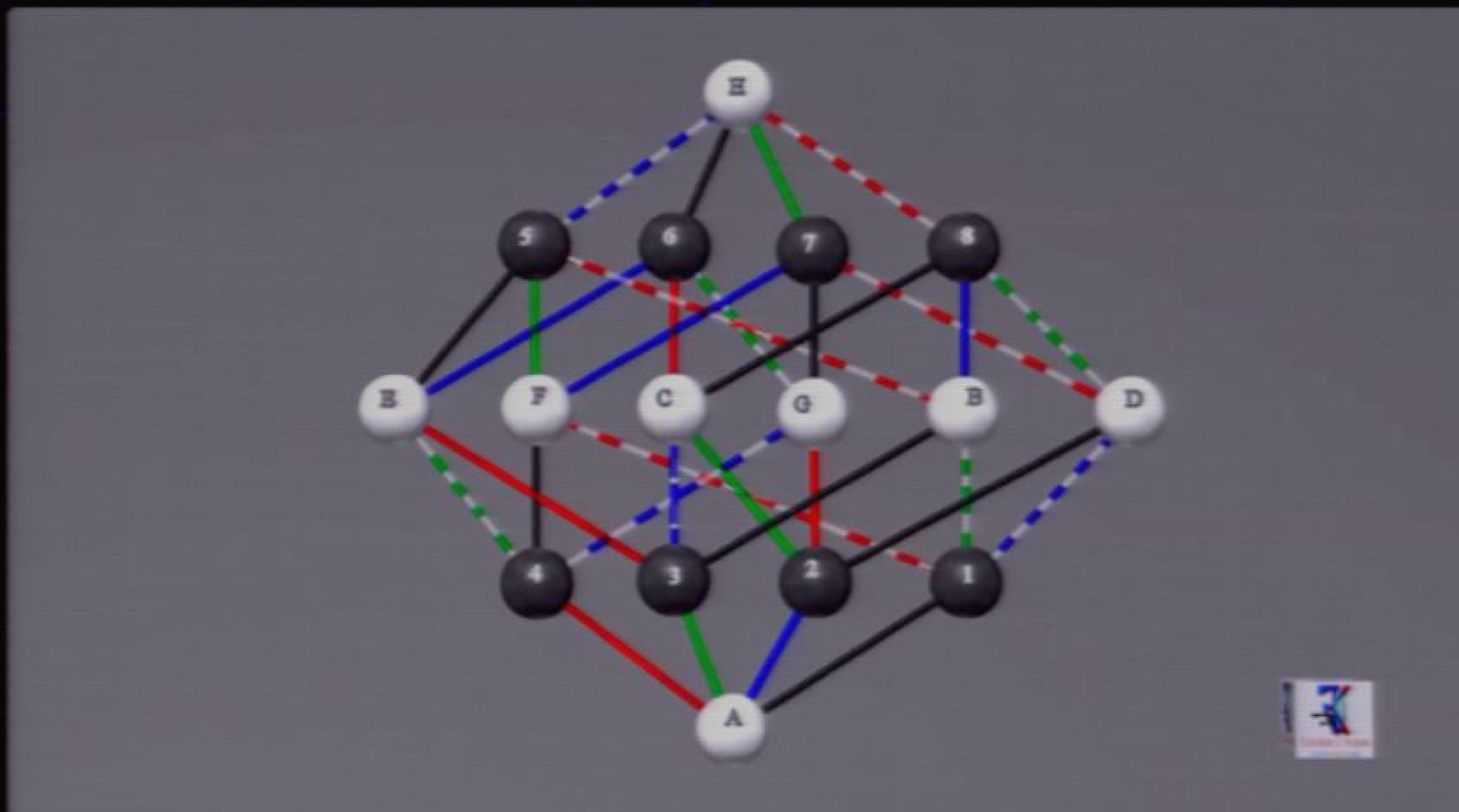
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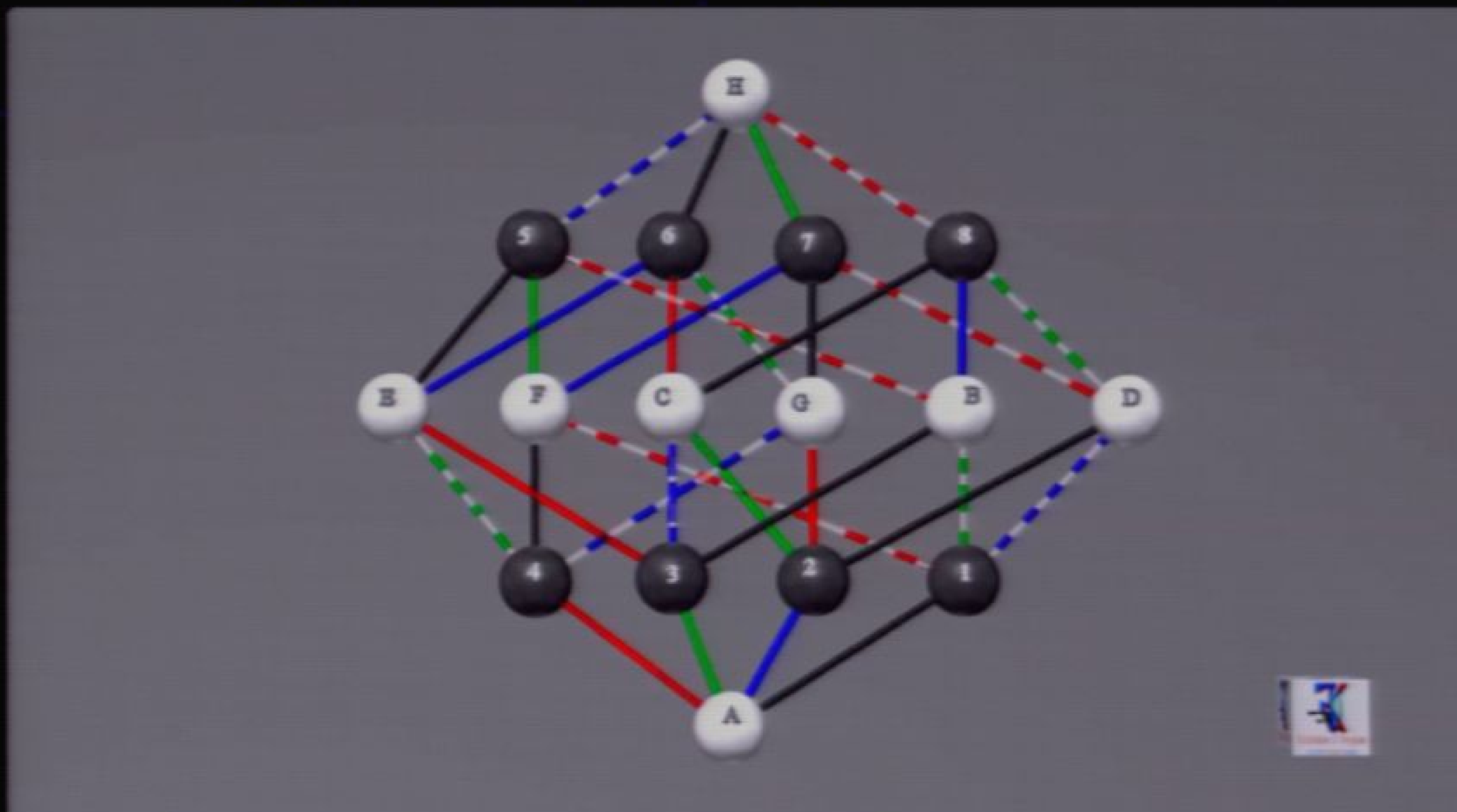
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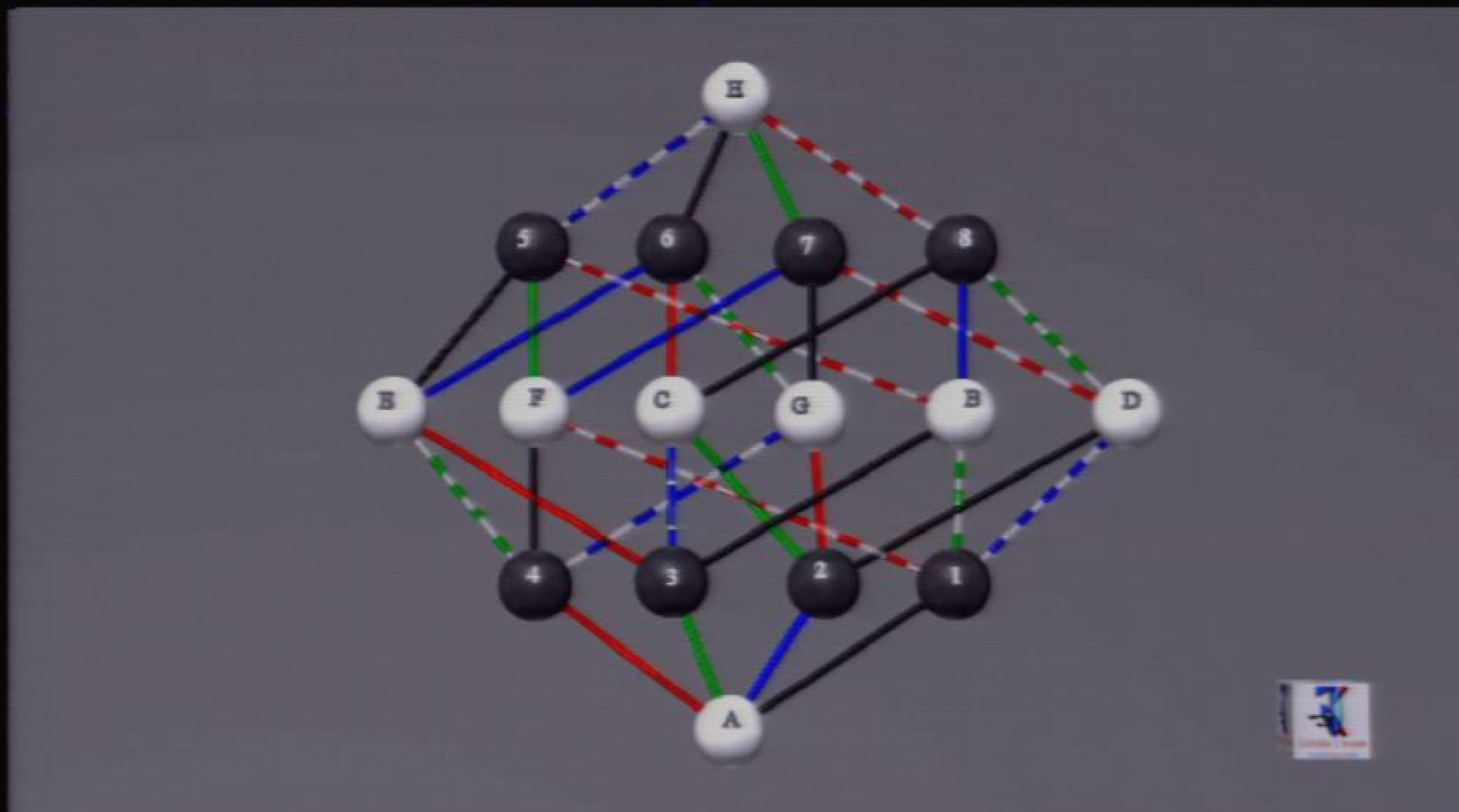
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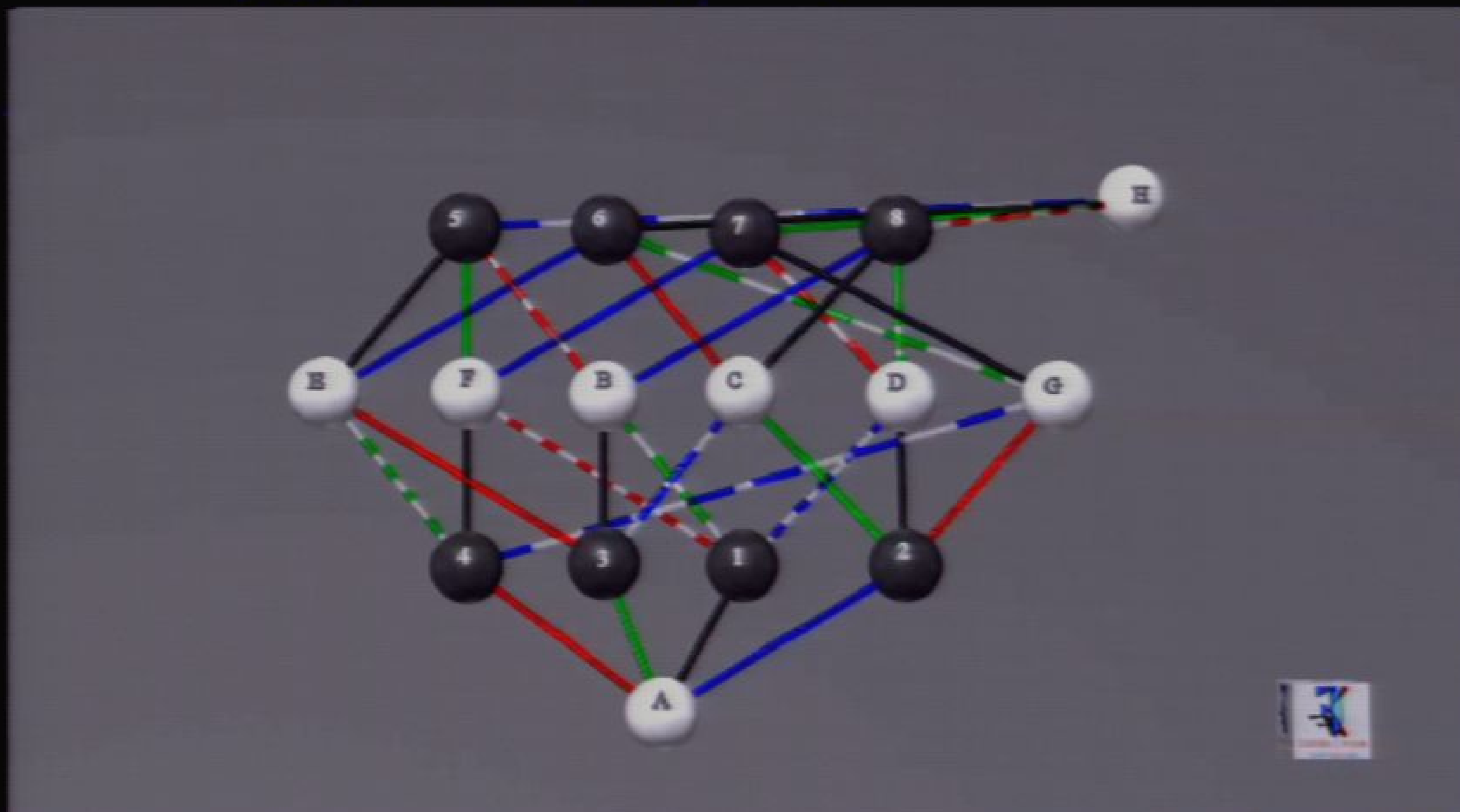
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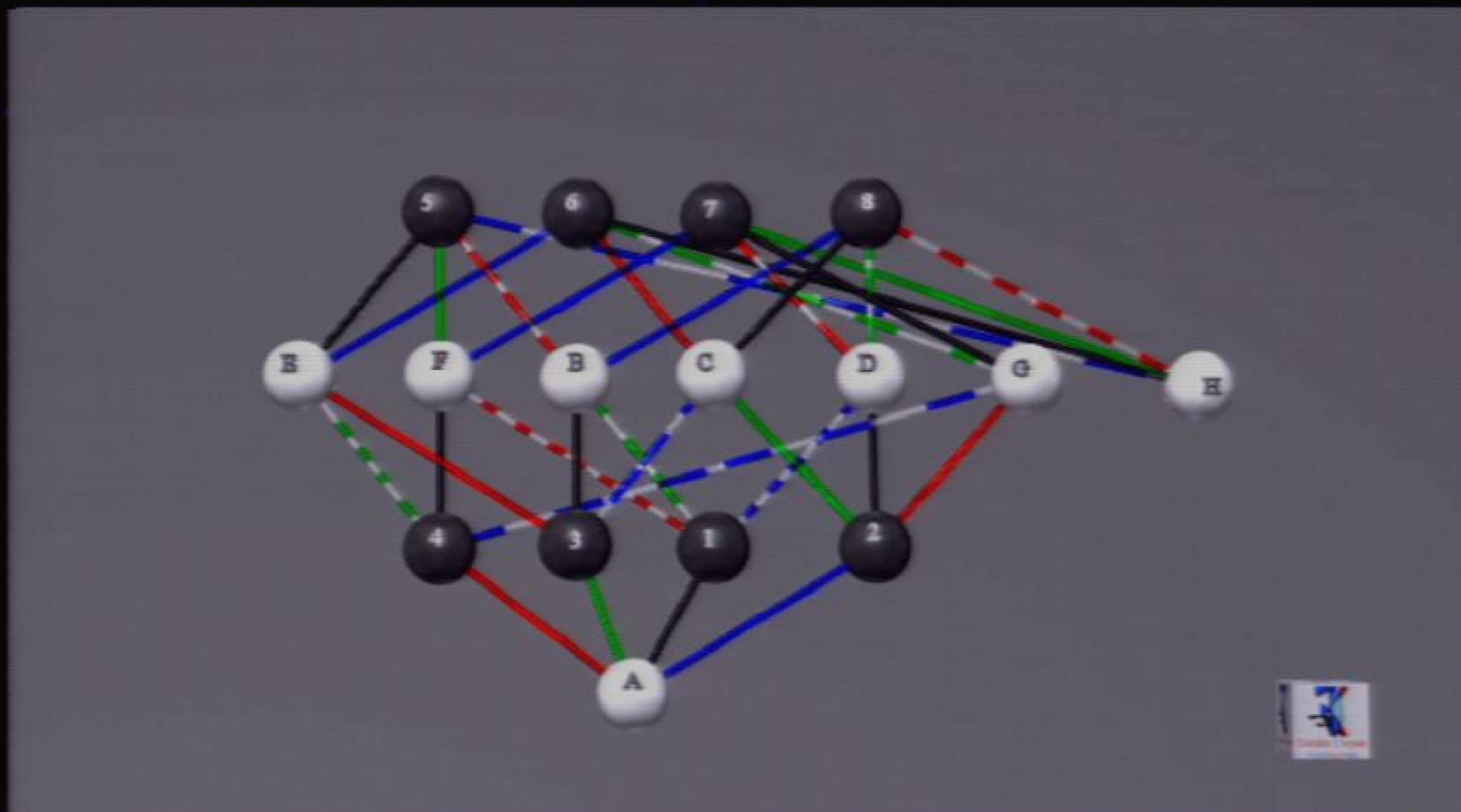
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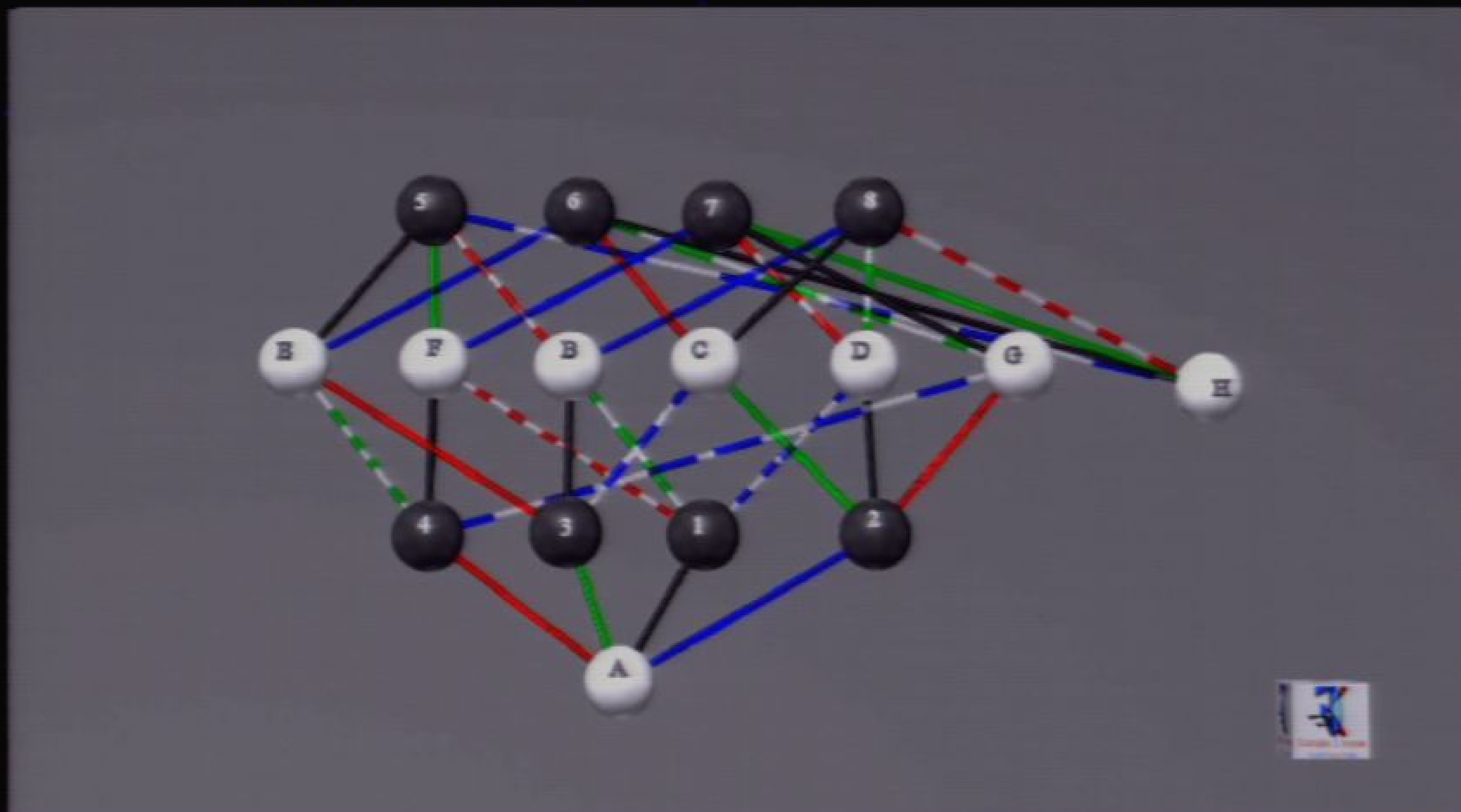
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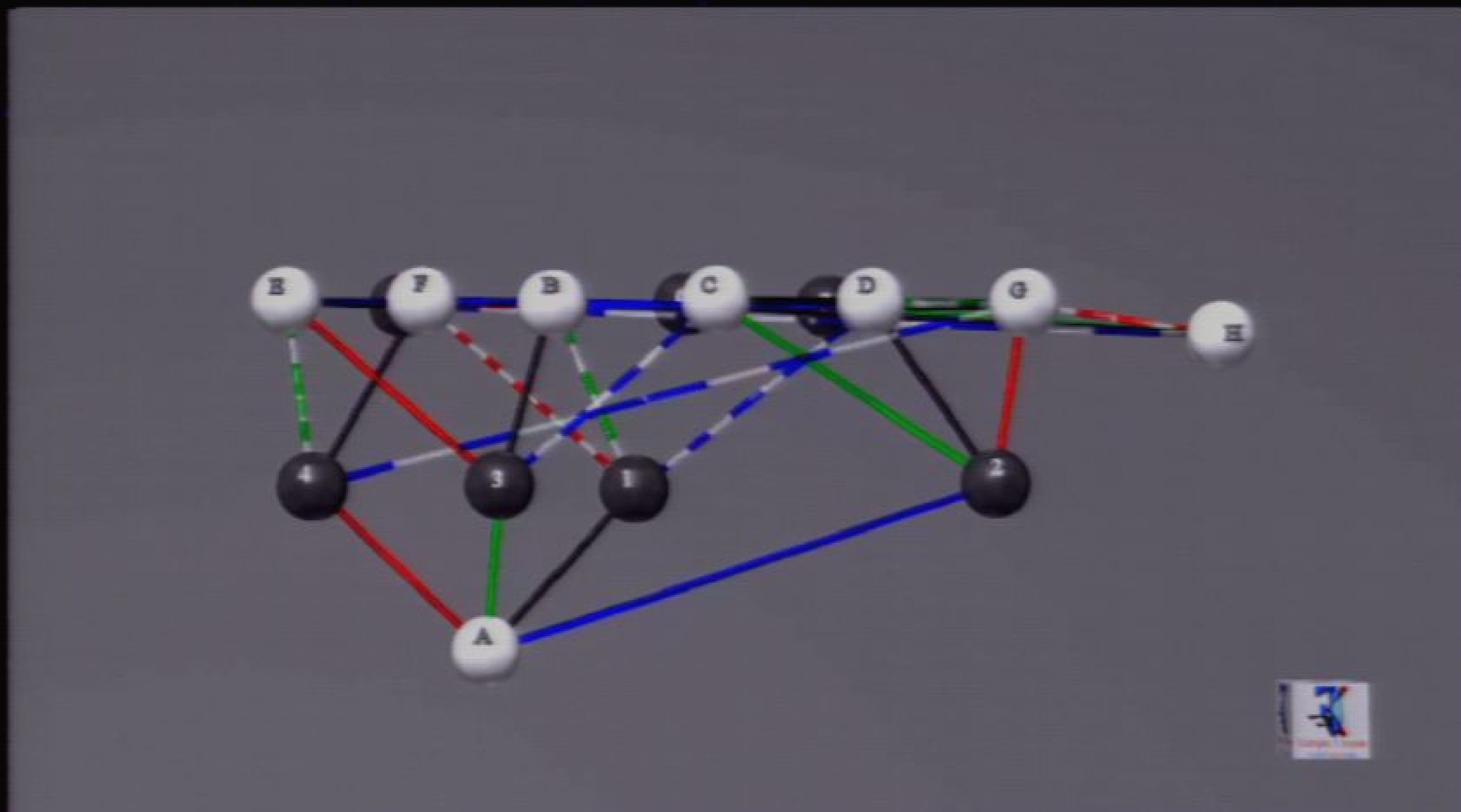
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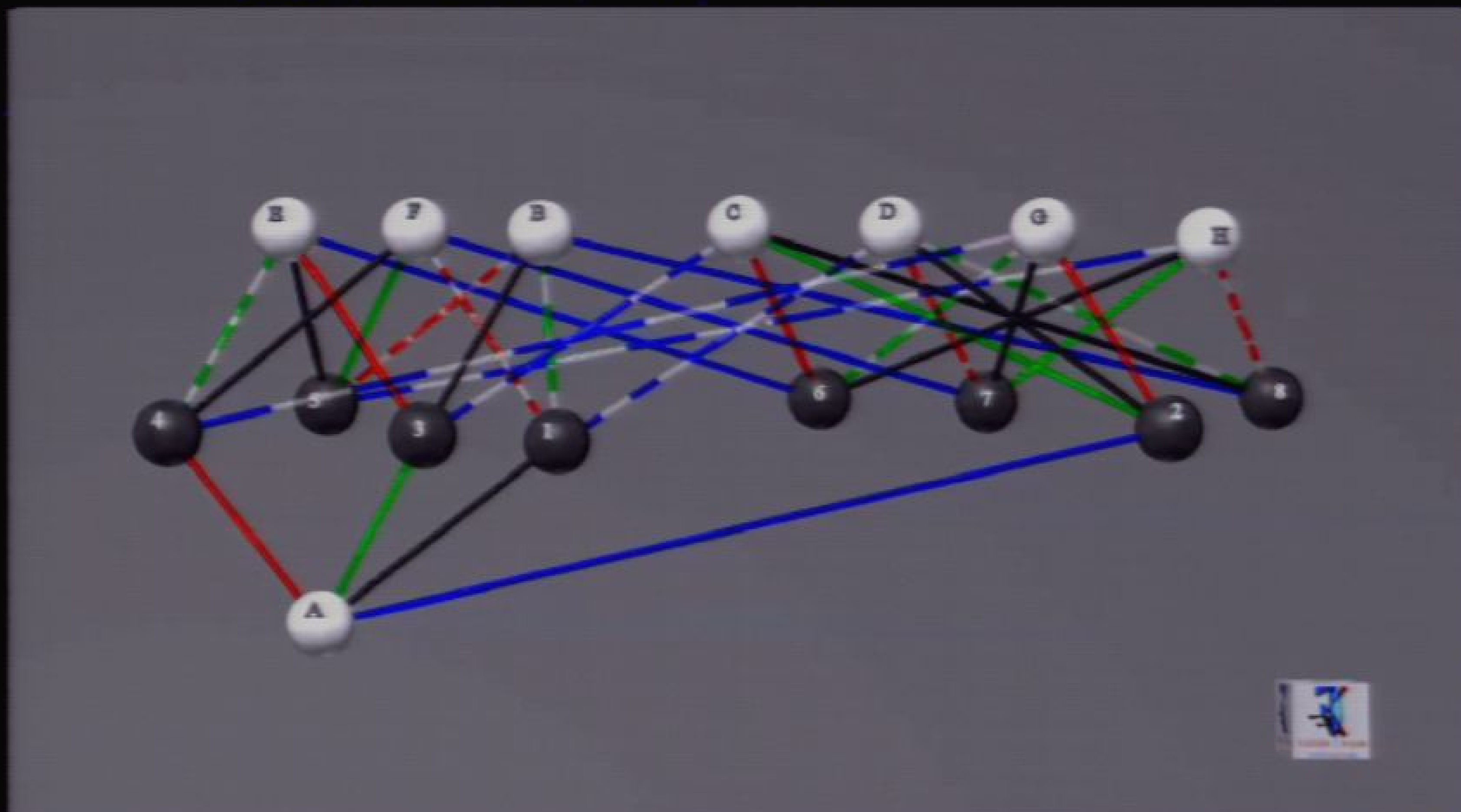
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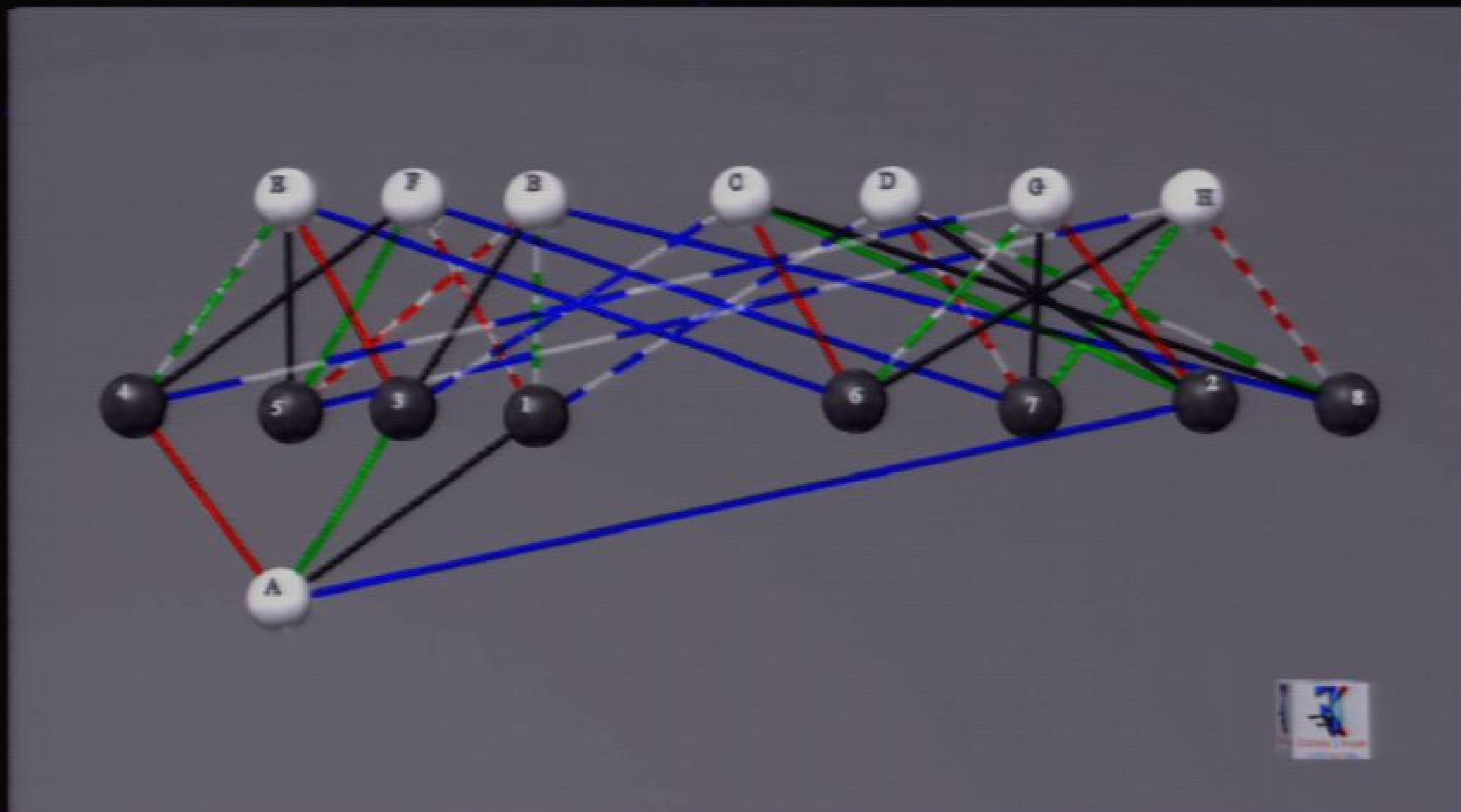
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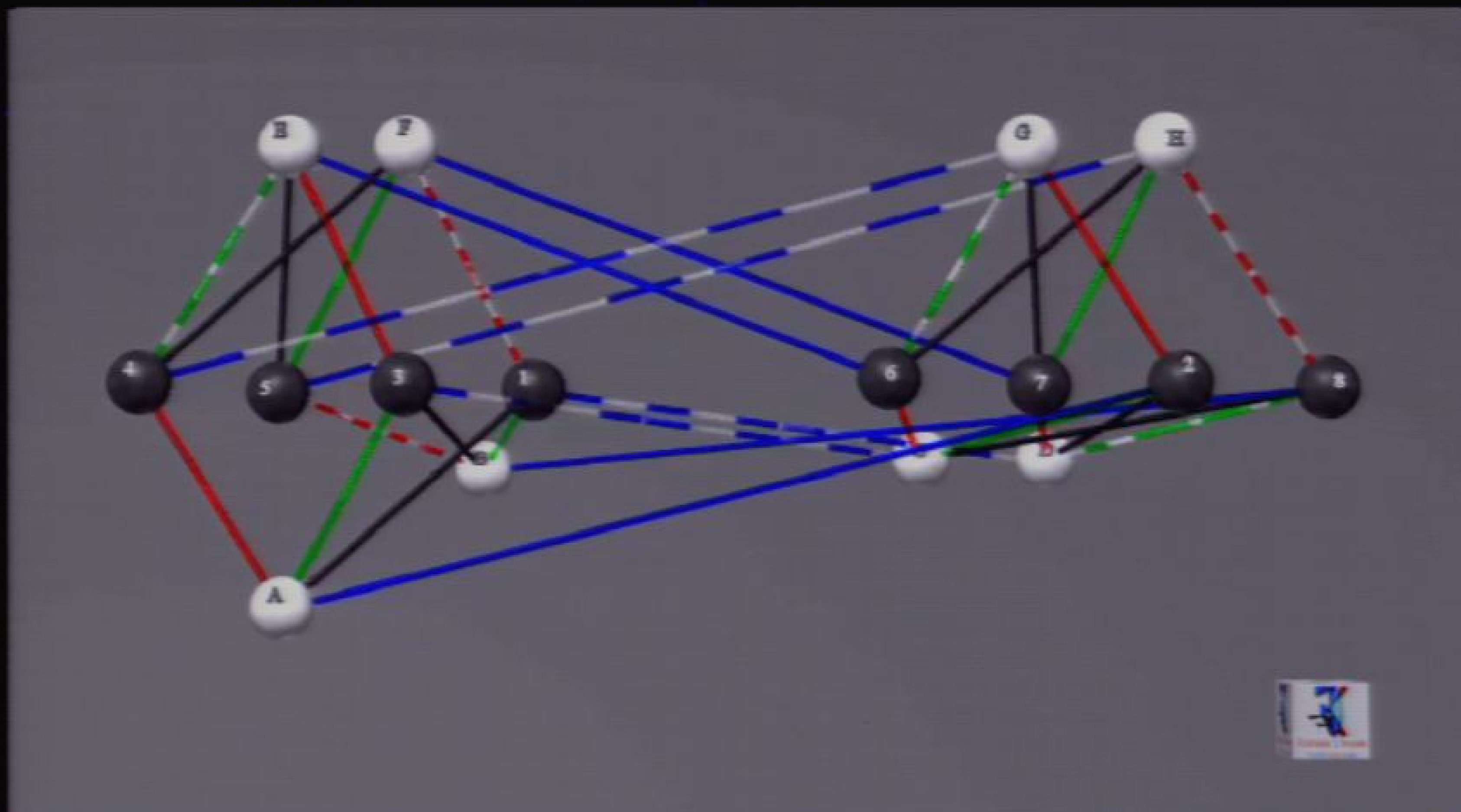
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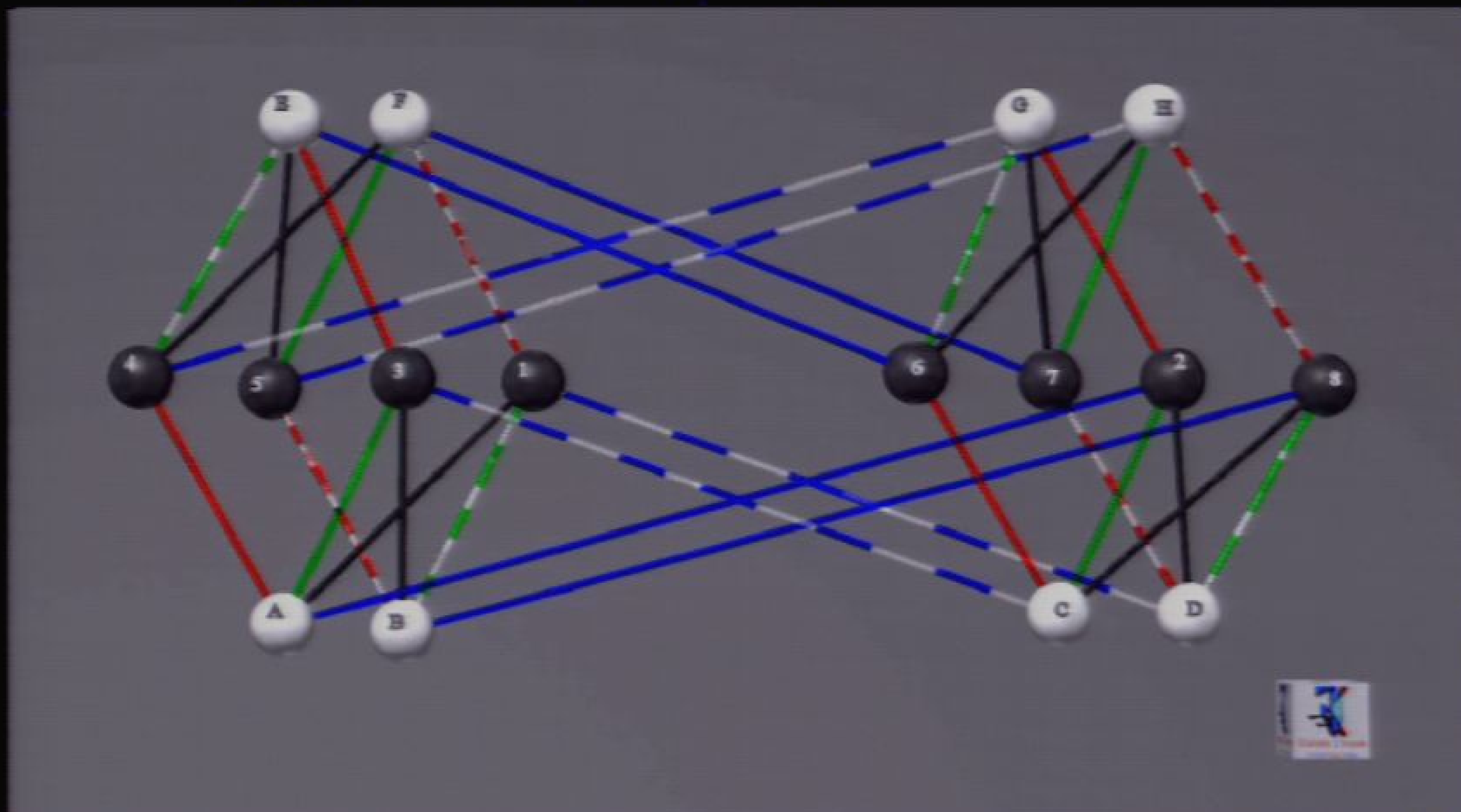
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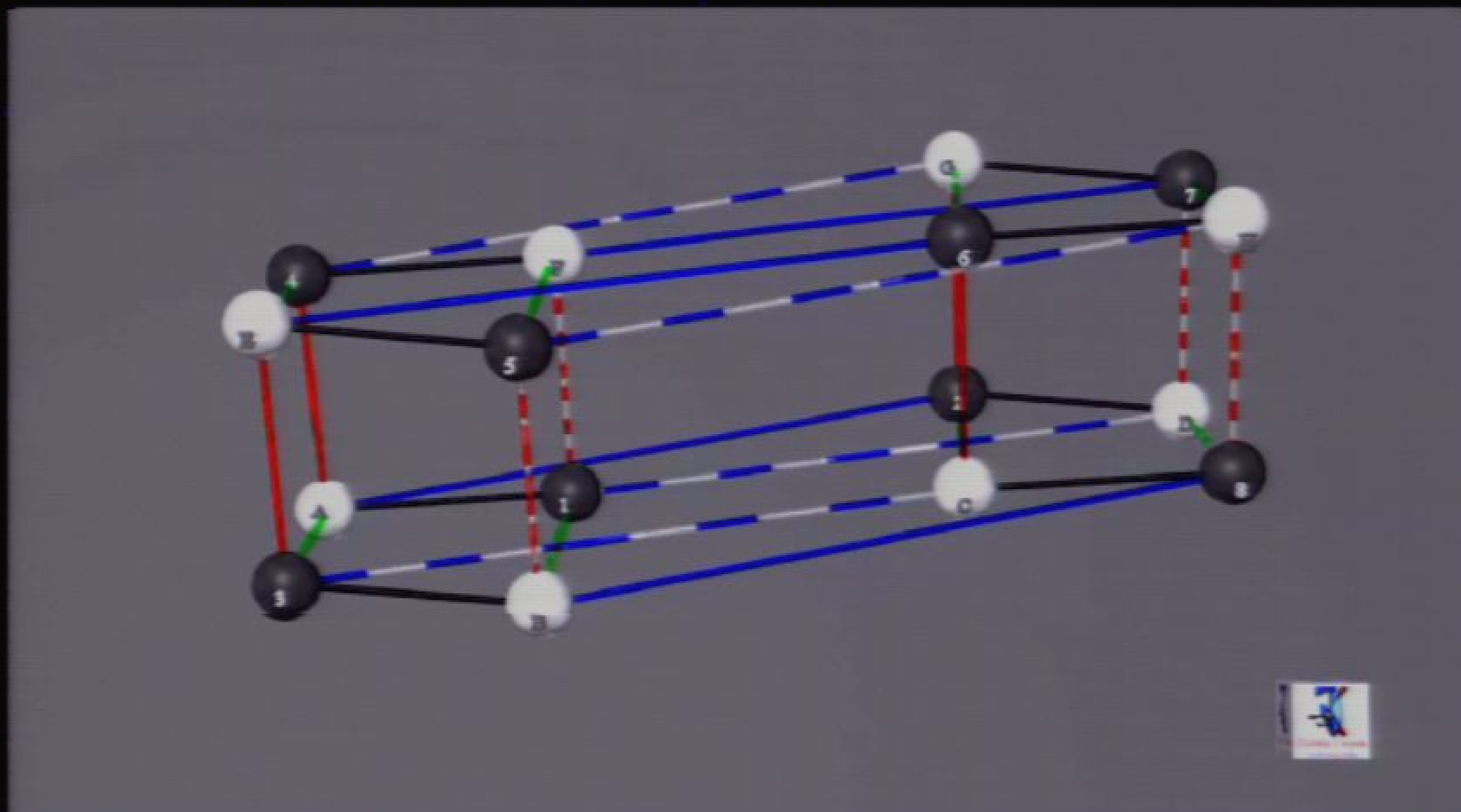
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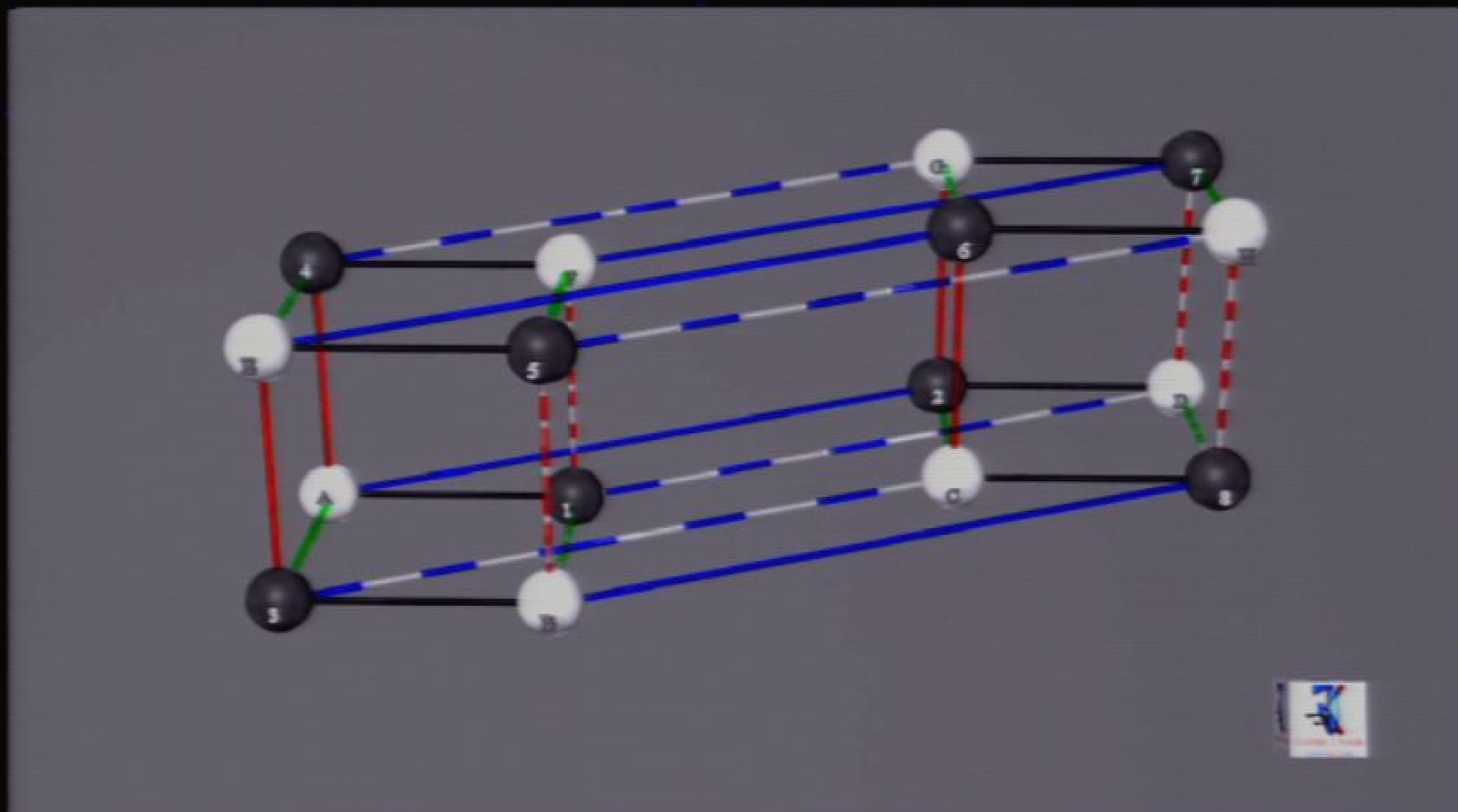
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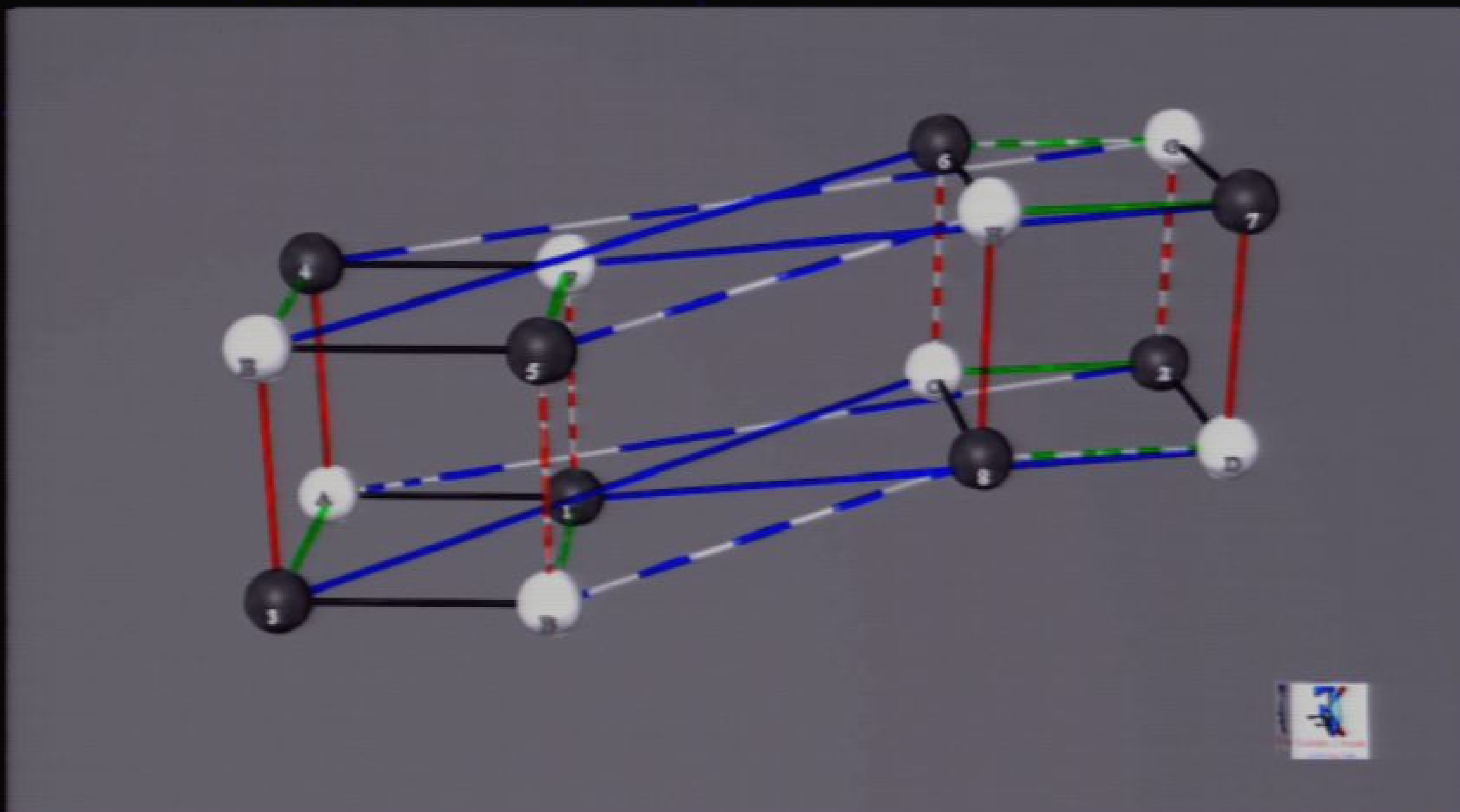
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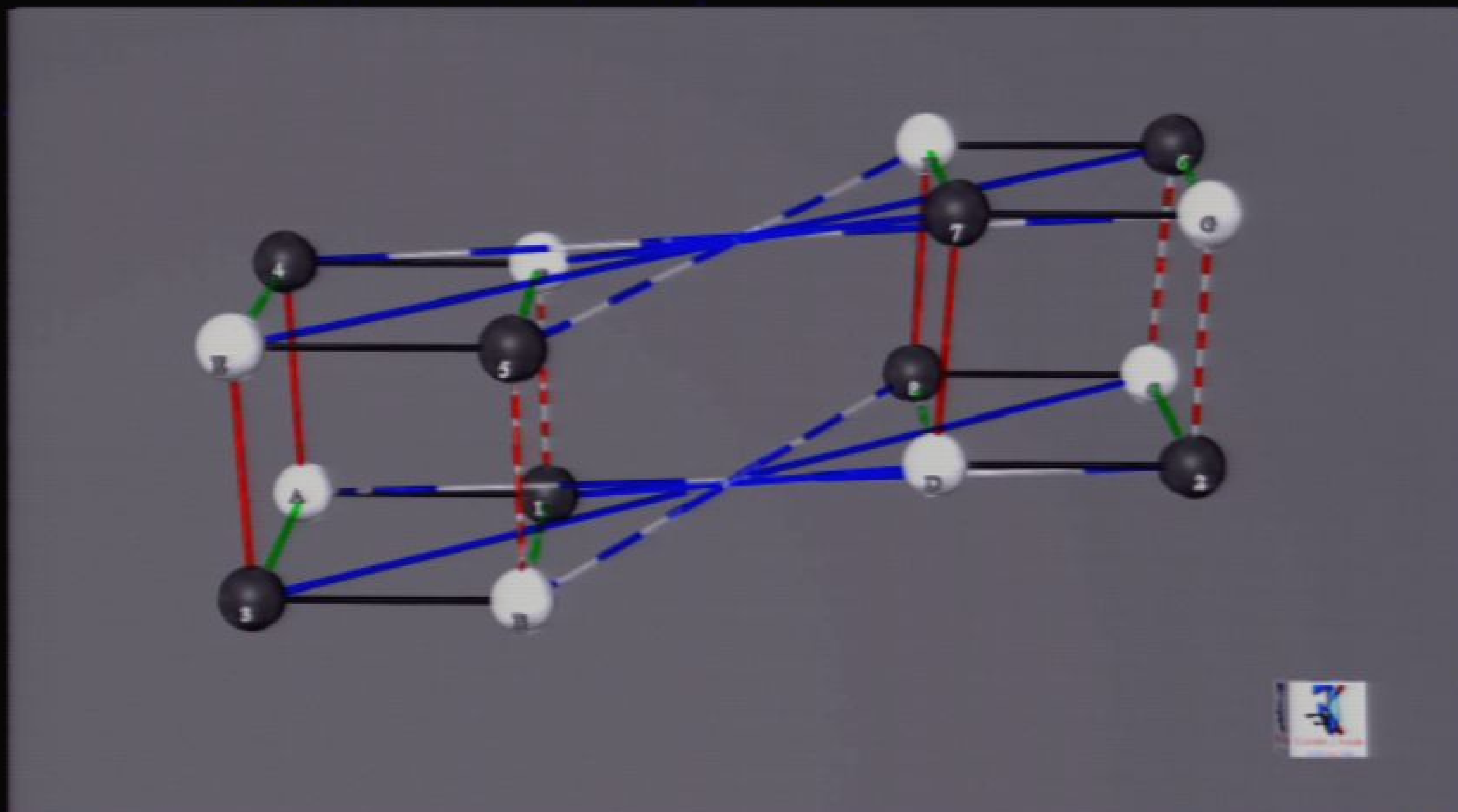

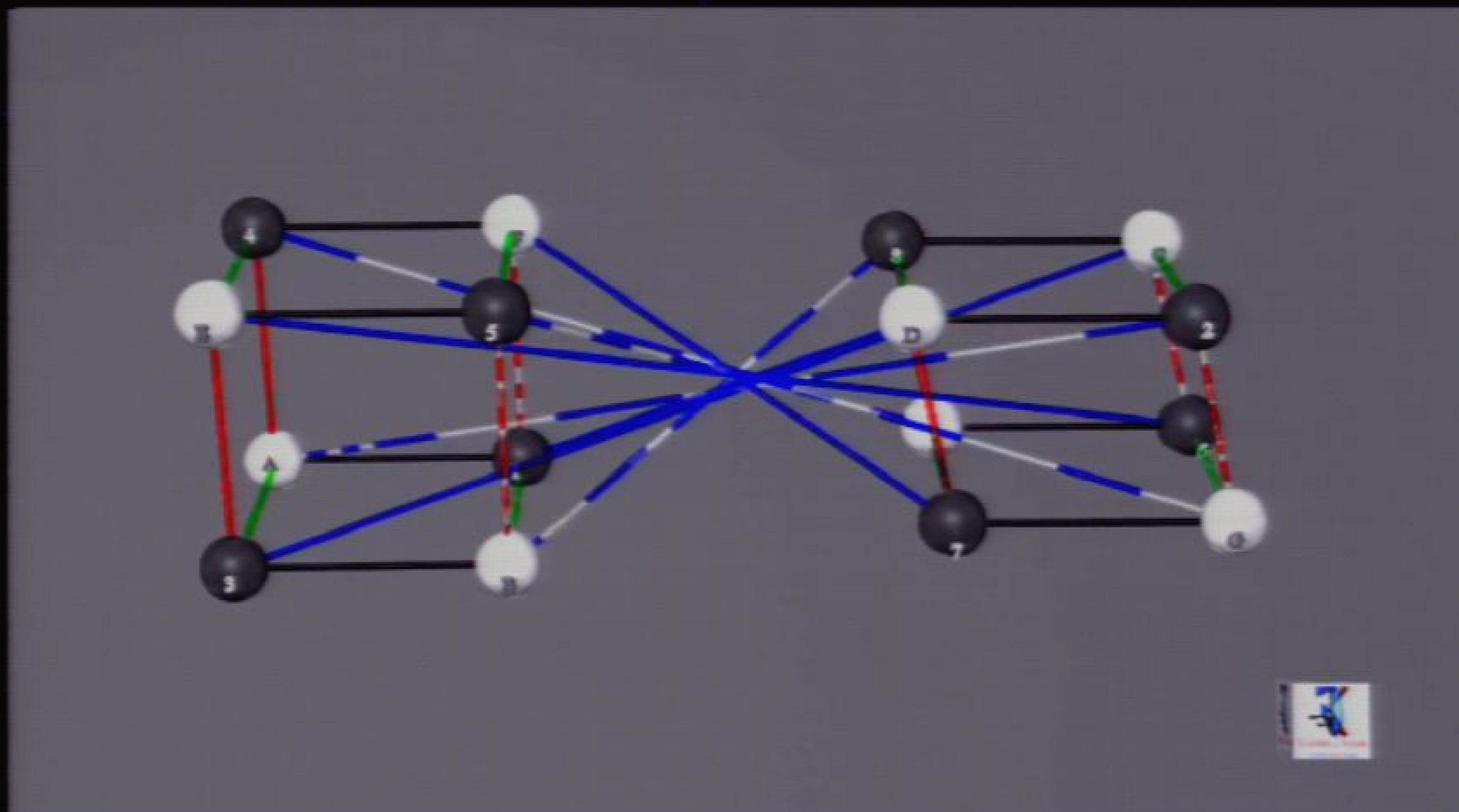

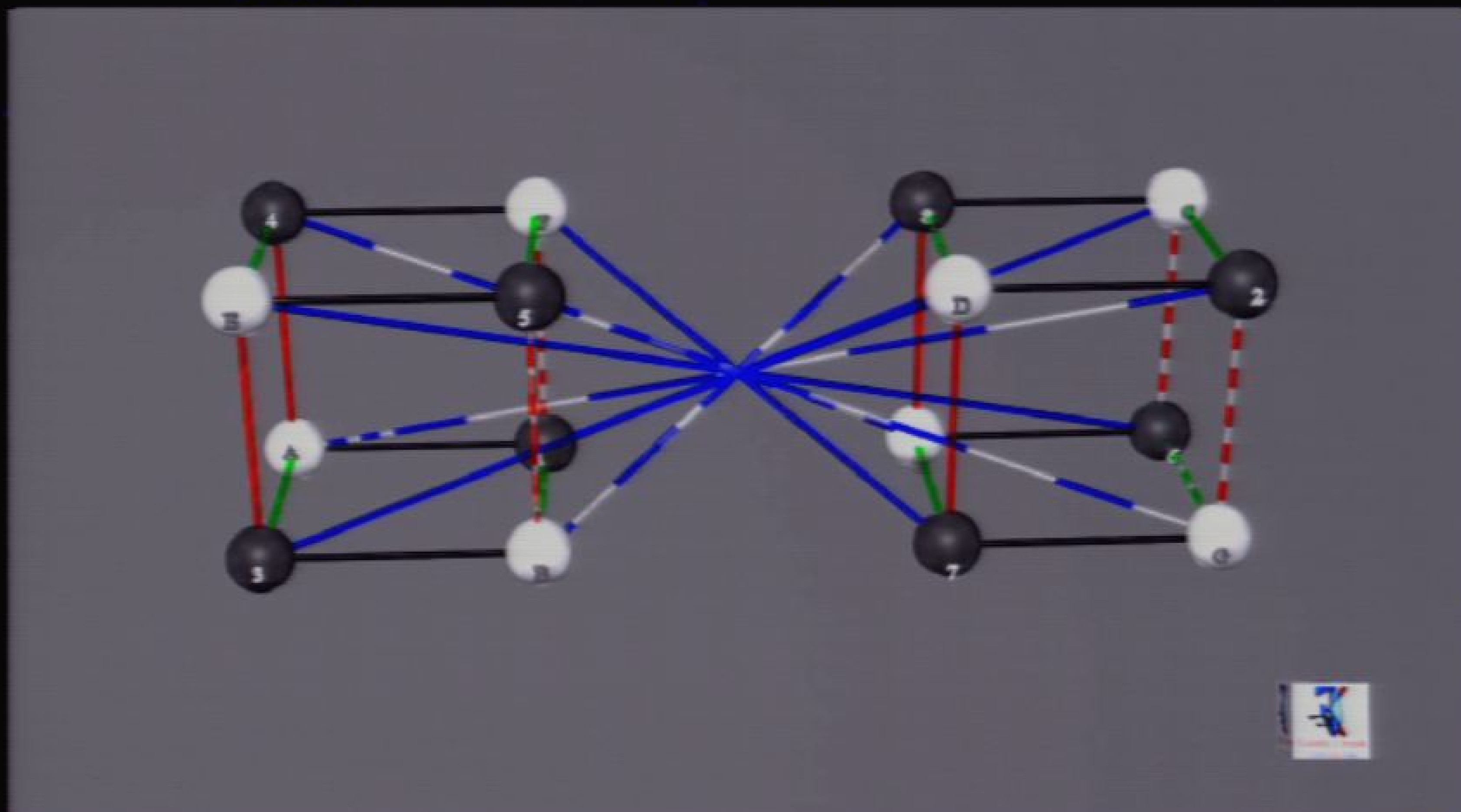
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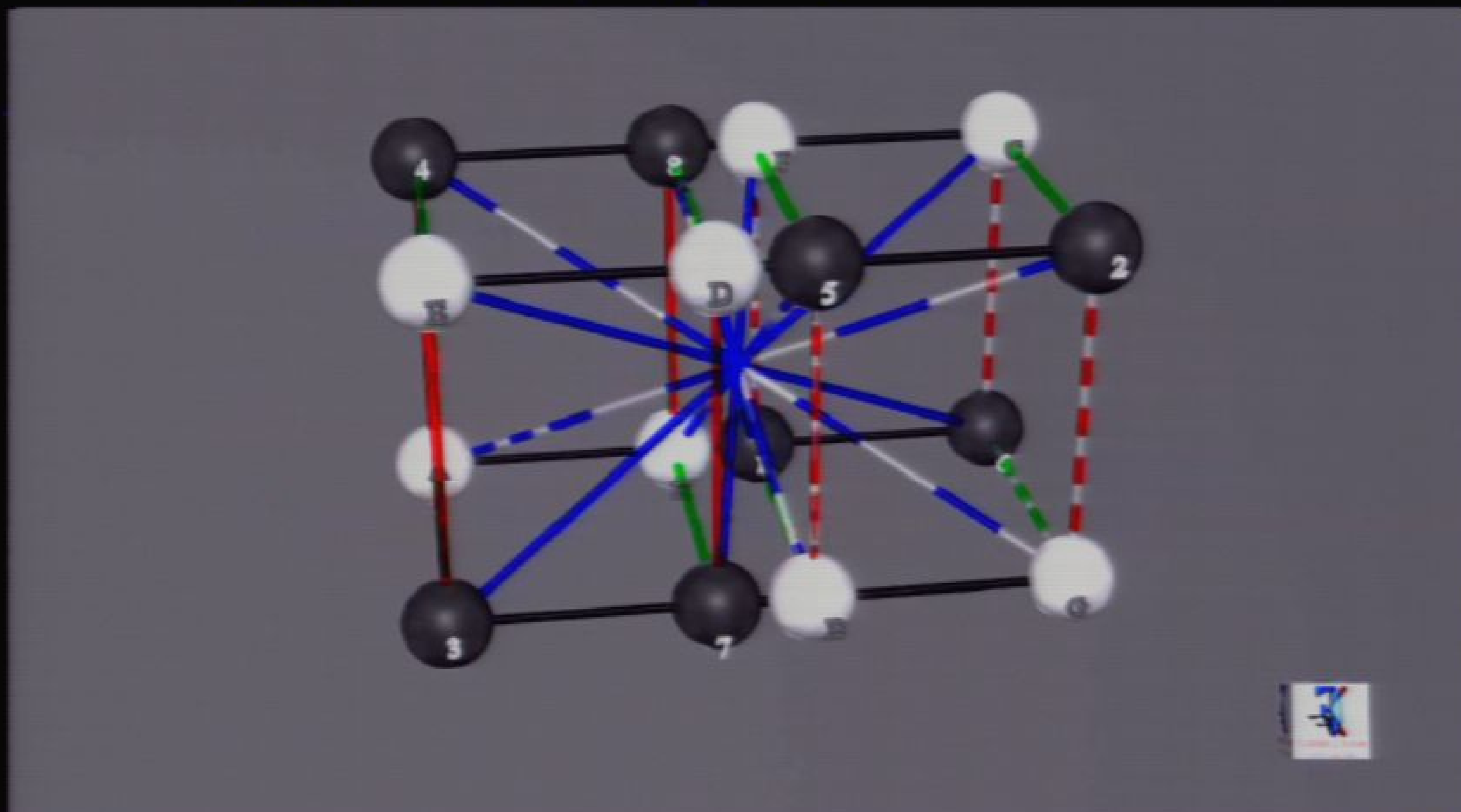
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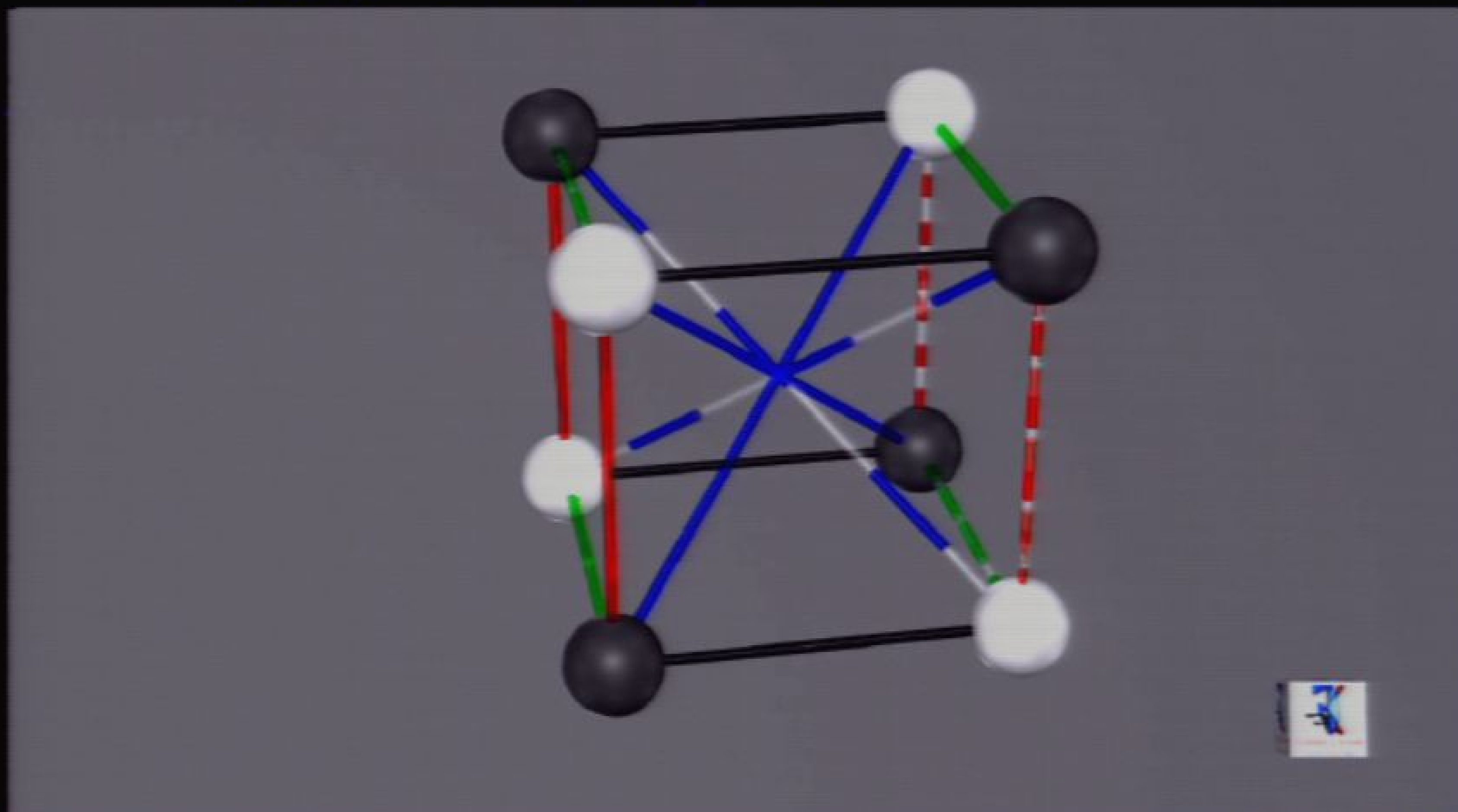

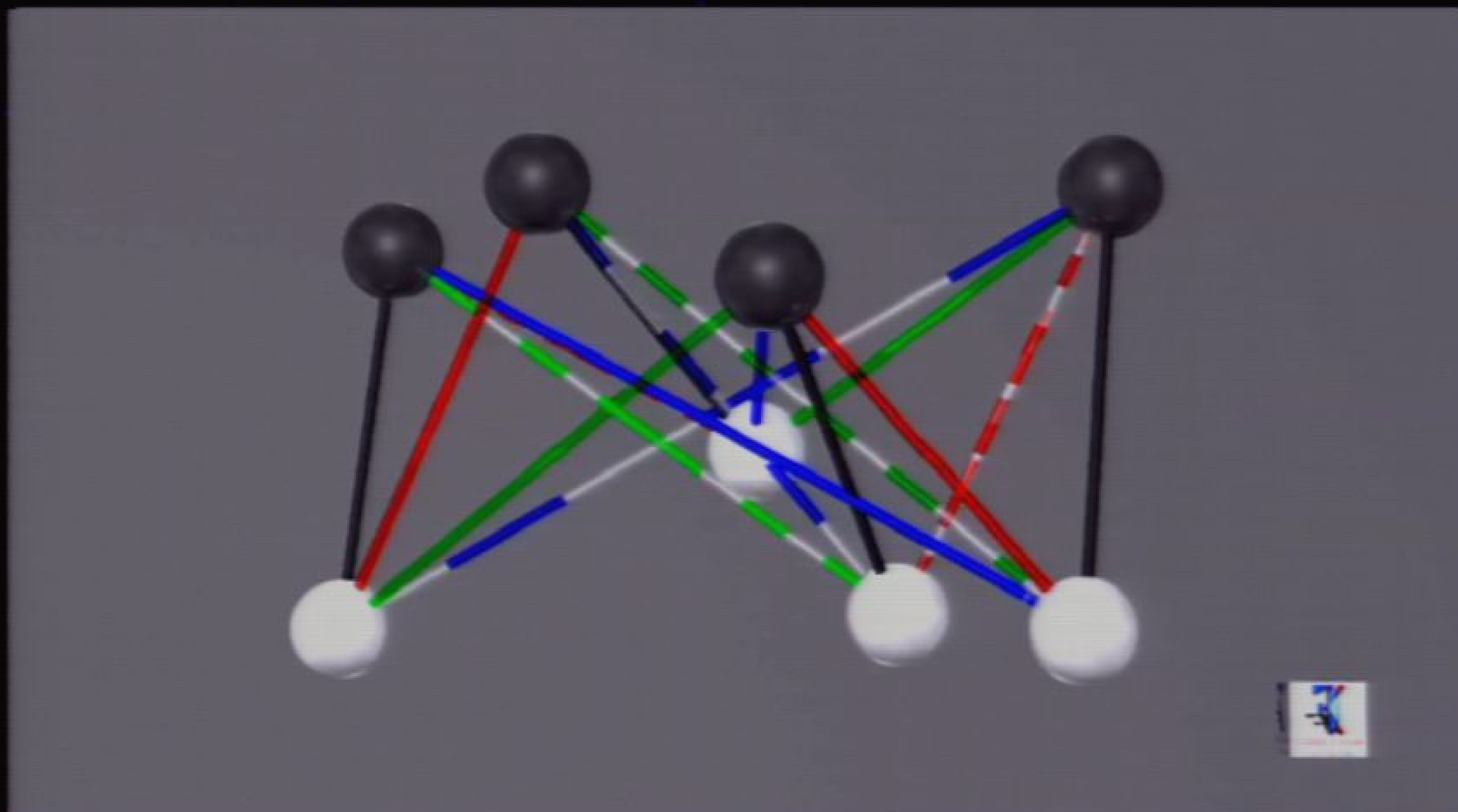

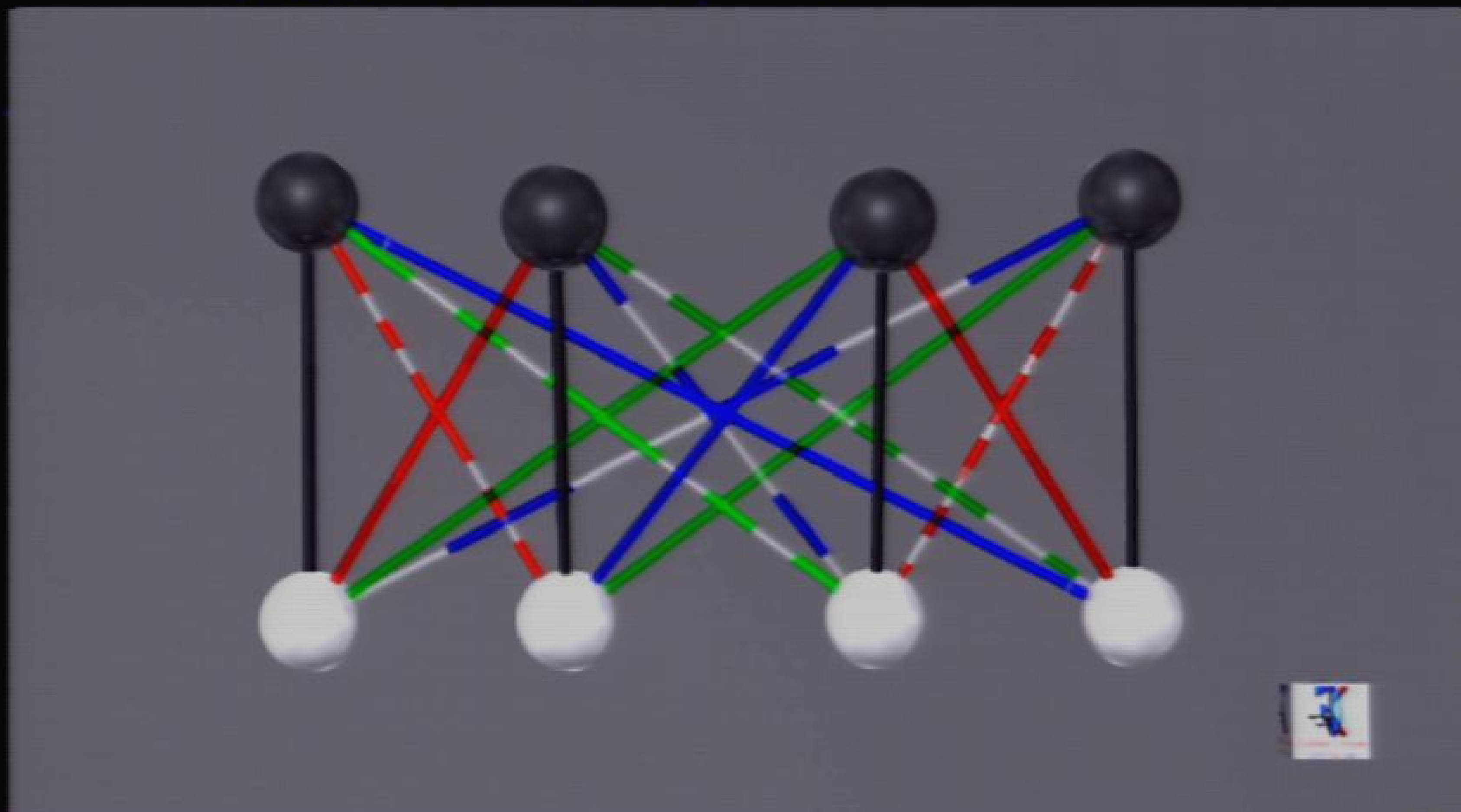

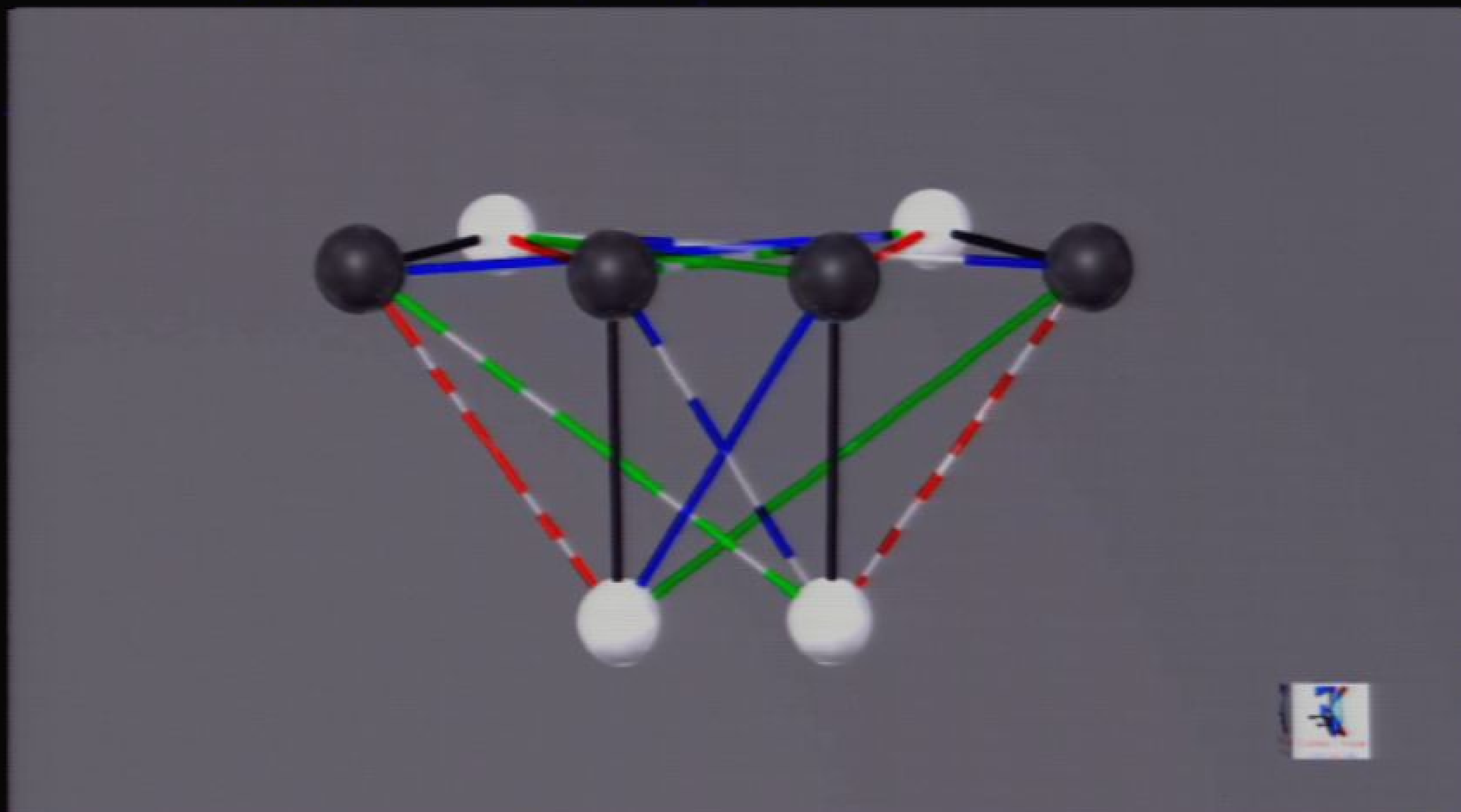

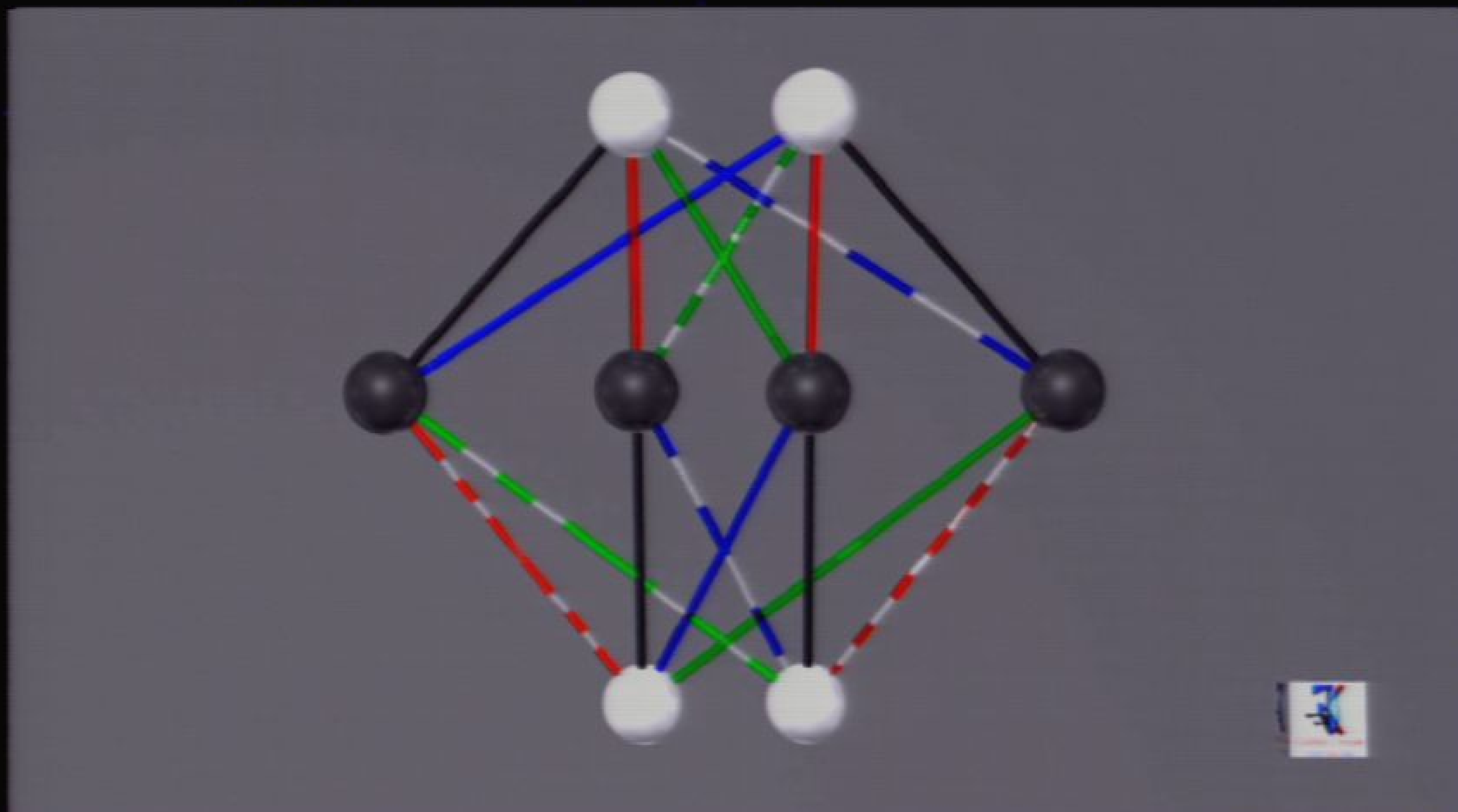

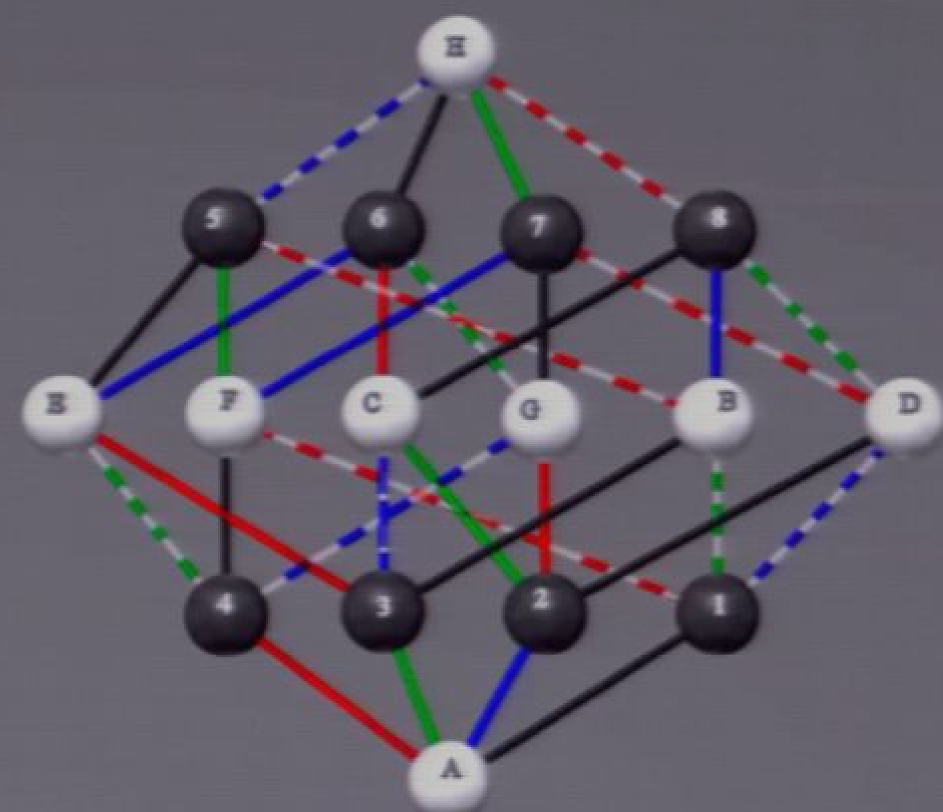
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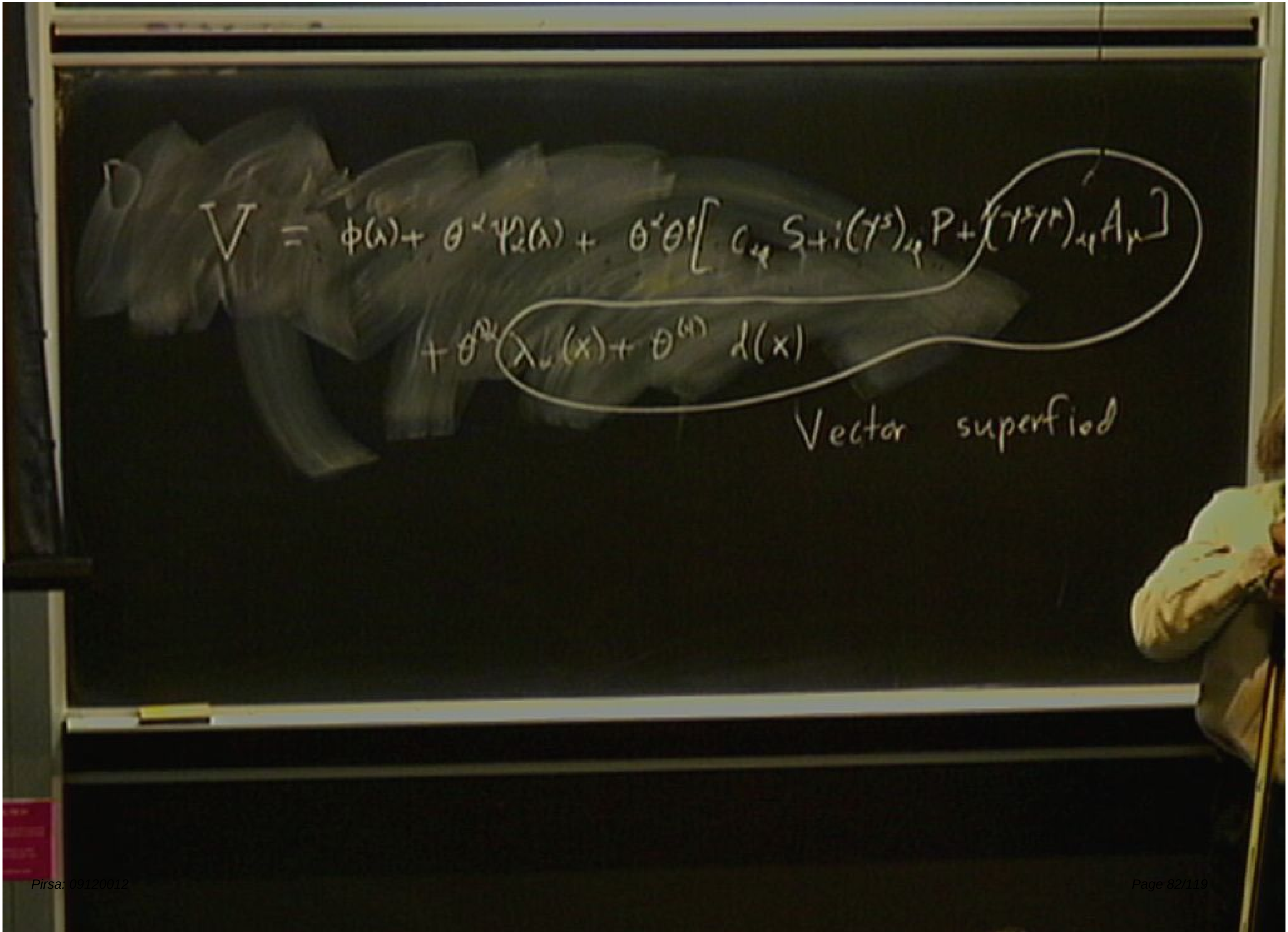
$V = \phi(x) + \theta^{\alpha}\psi_{\alpha}(x) + \theta^{\alpha}\theta^{\beta}\left[C_{\alpha\beta} S + i(\gamma^5)_{\alpha\beta} P + (\gamma^5\gamma^{\mu})_{\alpha\beta} A_{\mu} \right]$
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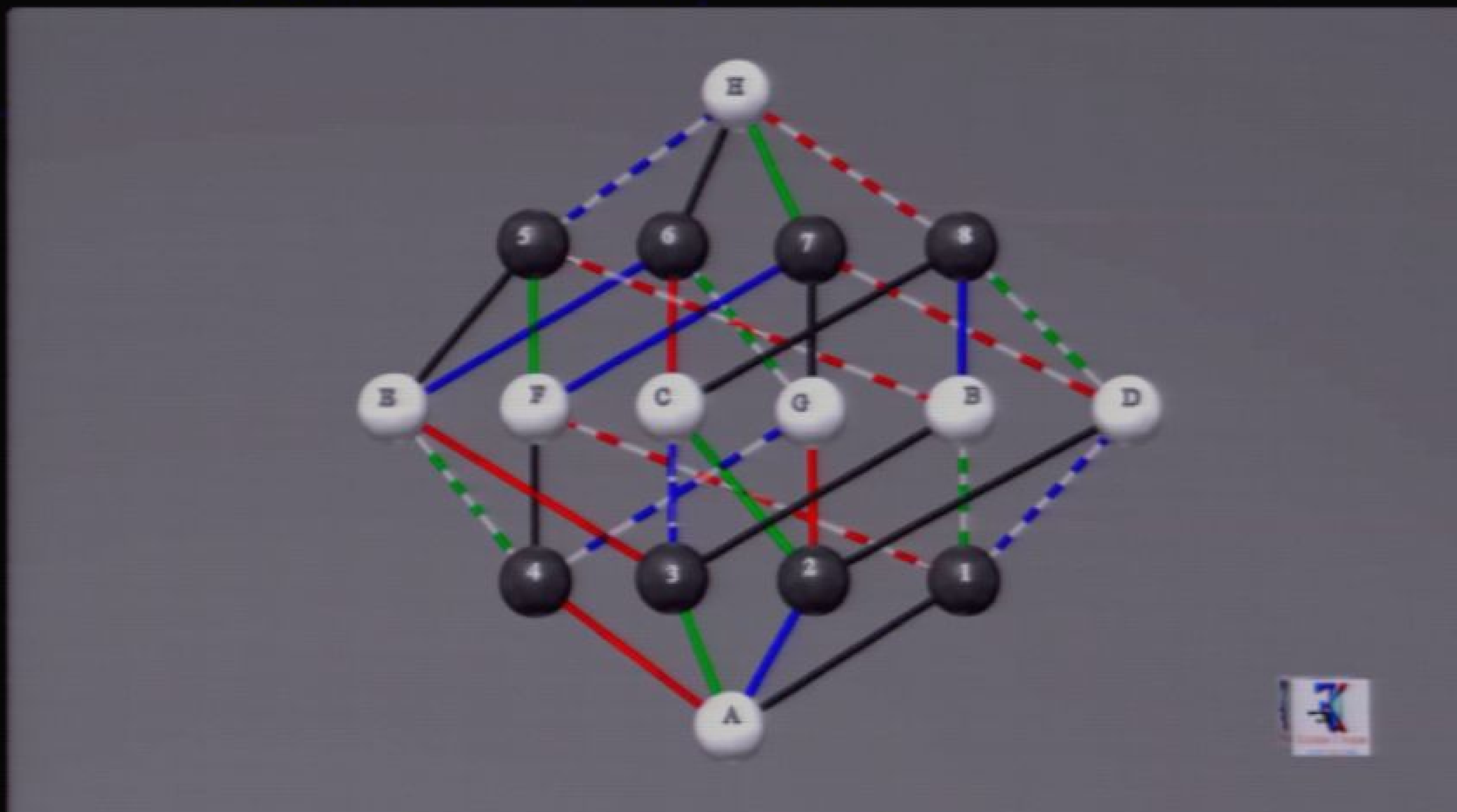
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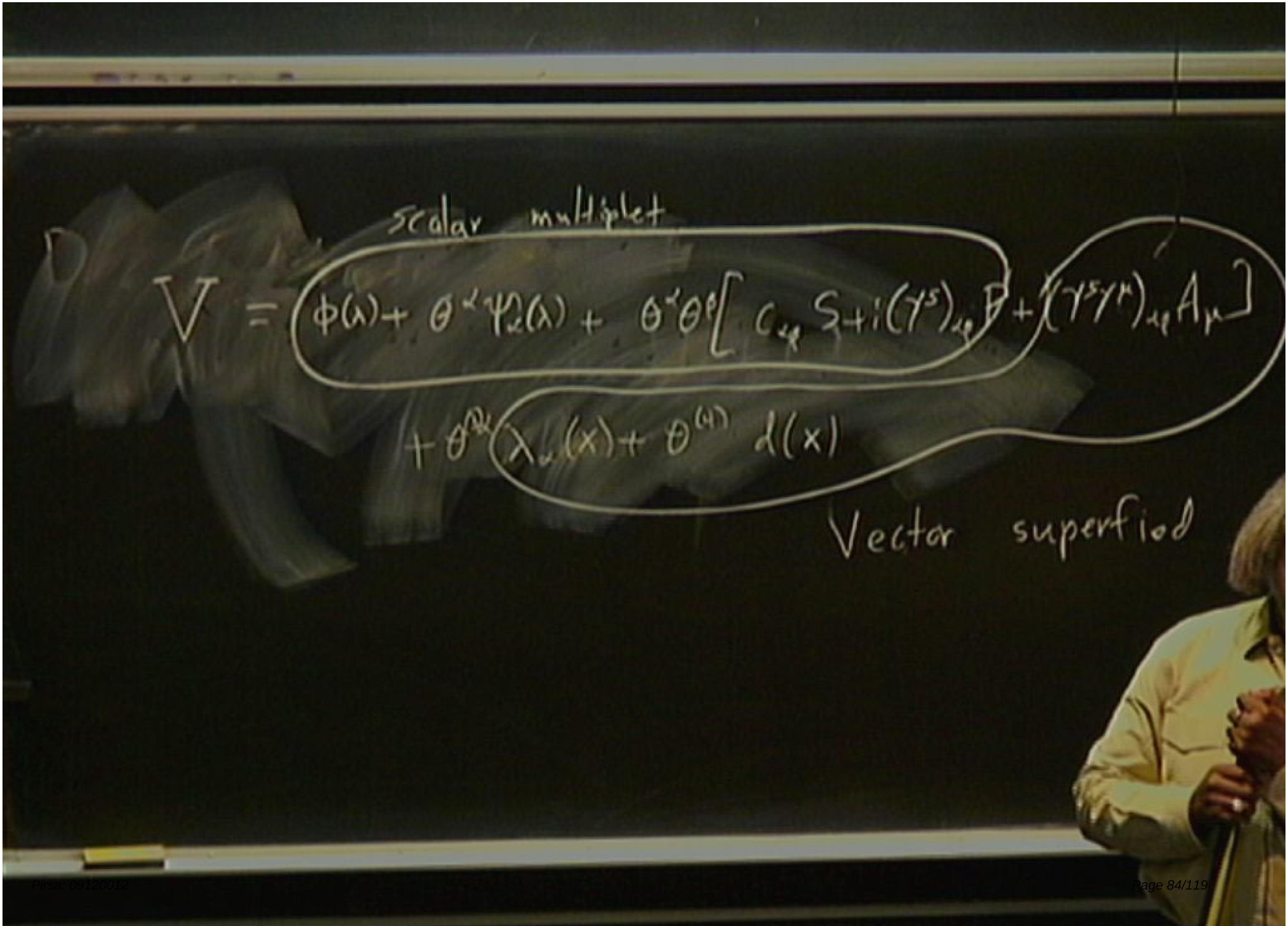
scalar multiplet
$V = \phi(x) + \theta^\alpha \psi_\alpha(x) + \theta^\alpha\theta^\beta [C_{\alpha\beta} S + i(\gamma^5)_{\alpha\beta} P + (\gamma^5\gamma^\mu)_{\alpha\beta} A_\mu]$
$+ \theta^{(3)\alpha} \lambda_\alpha(x) + \theta^{(4)} d(x)$
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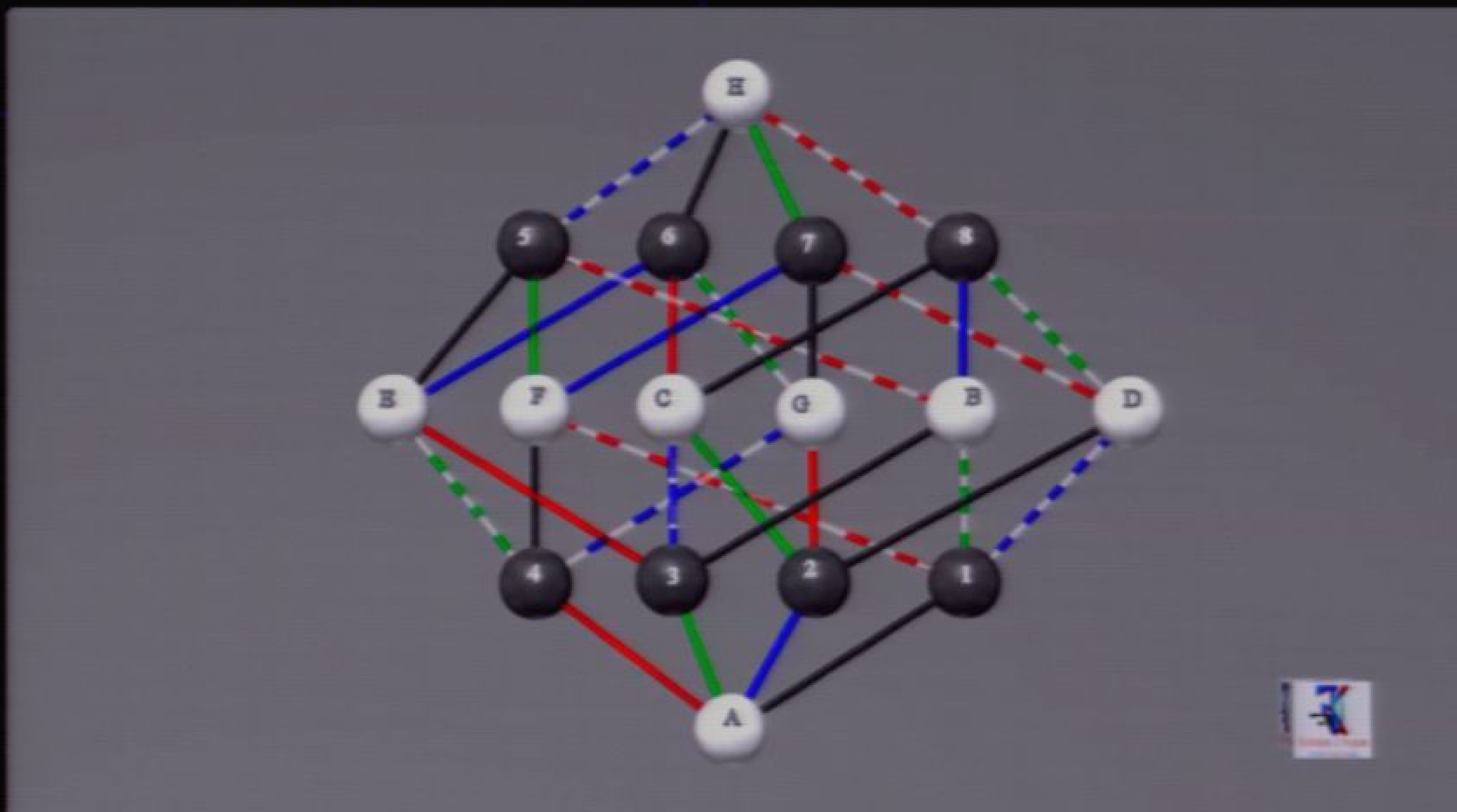
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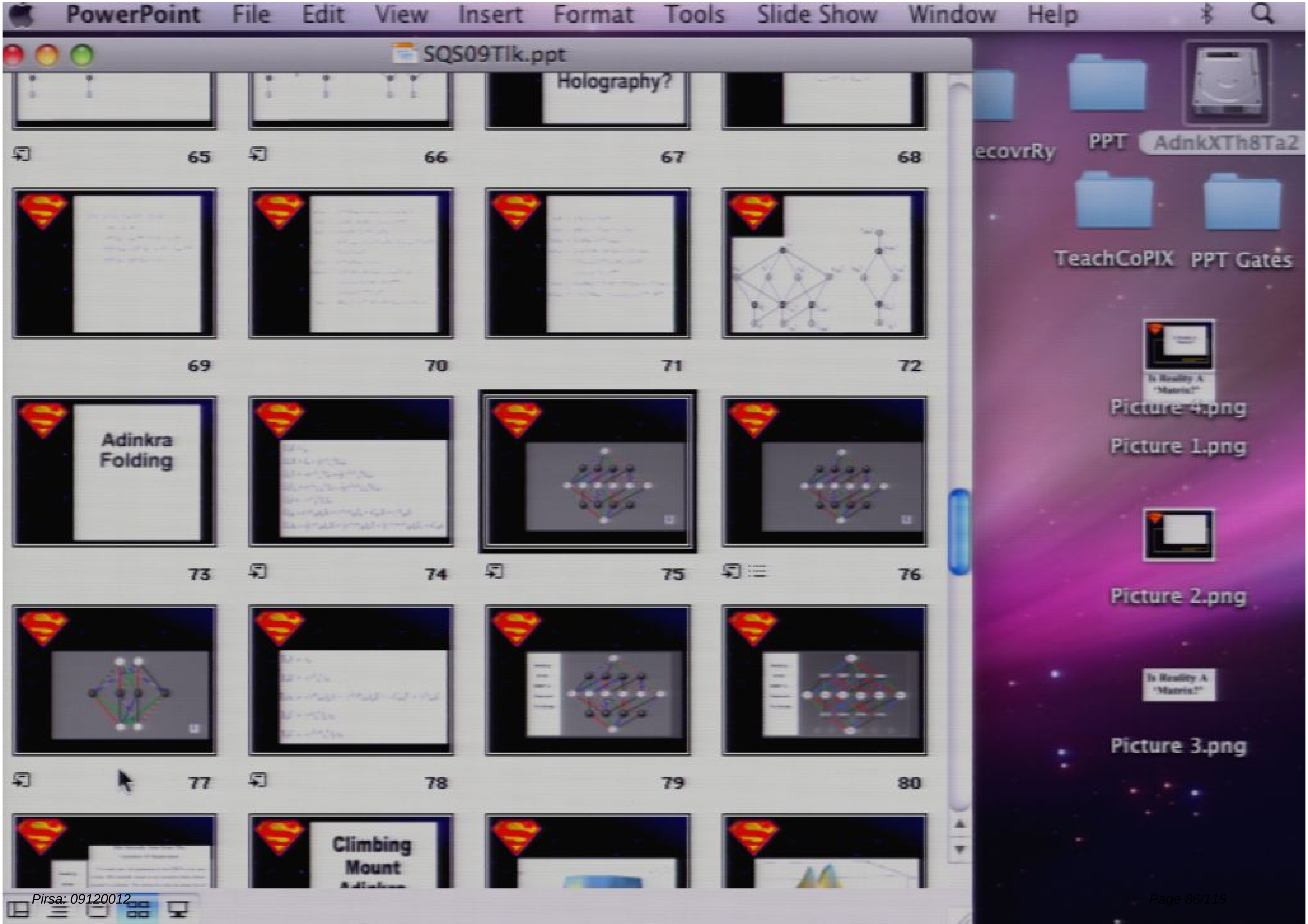
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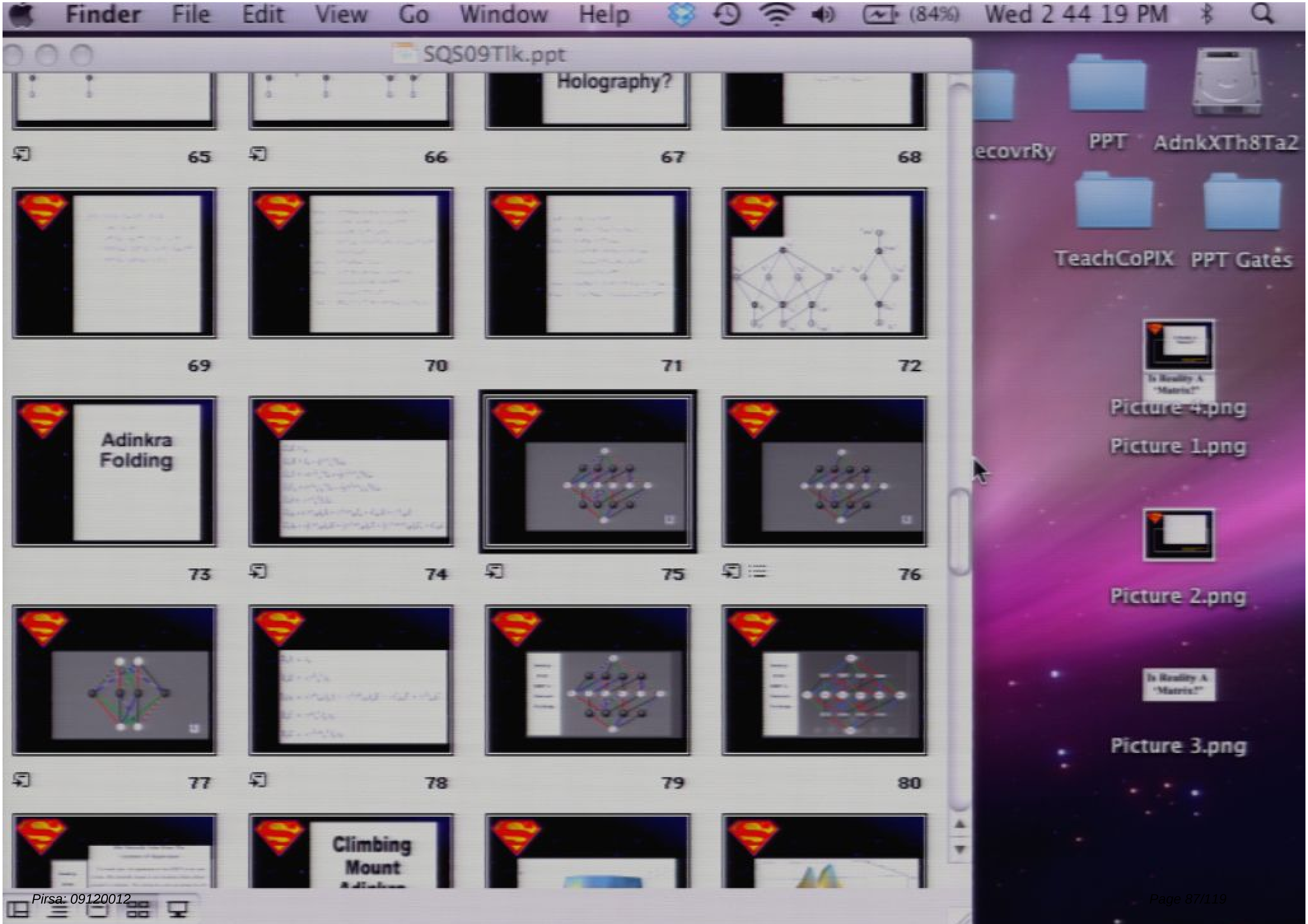
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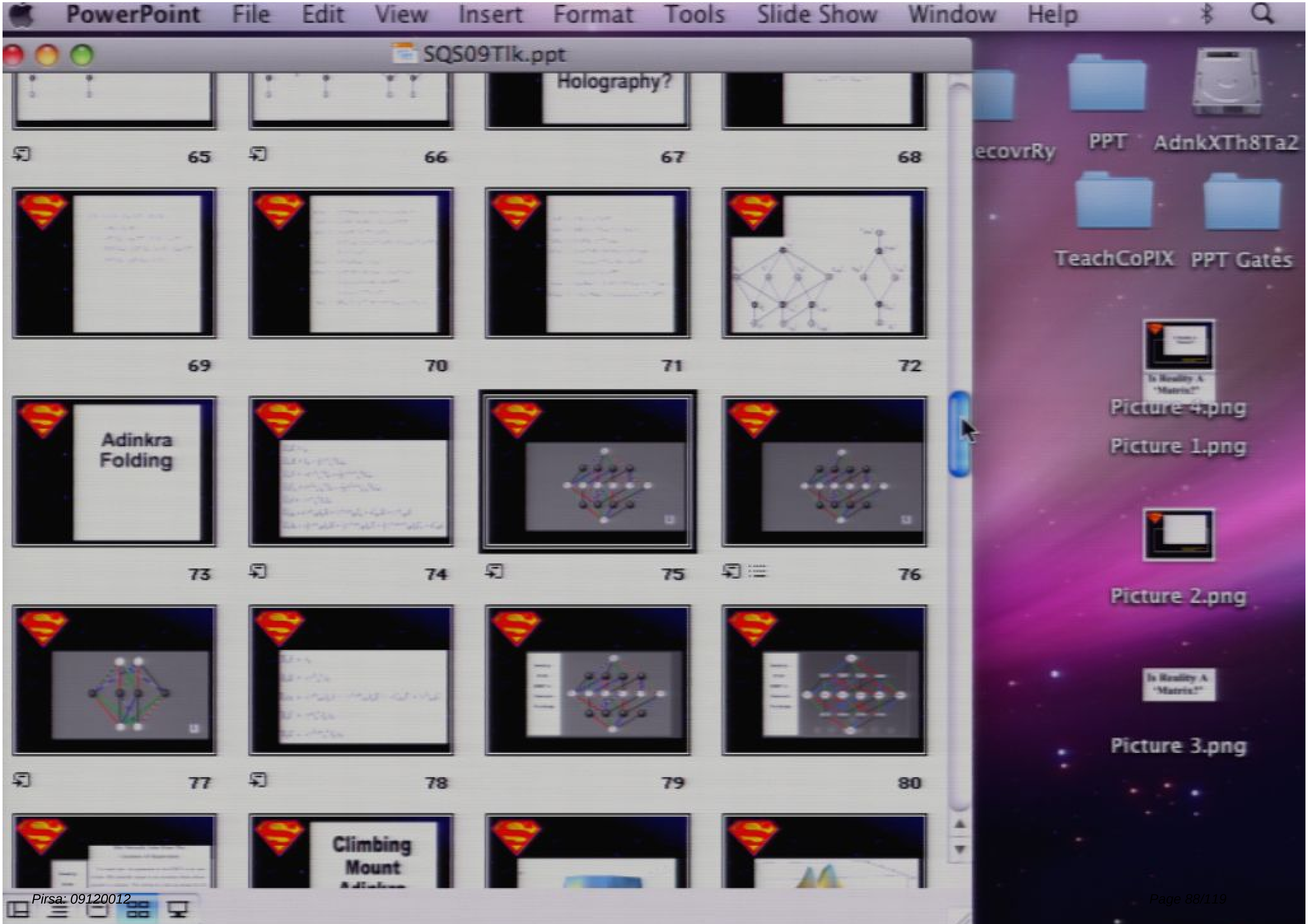
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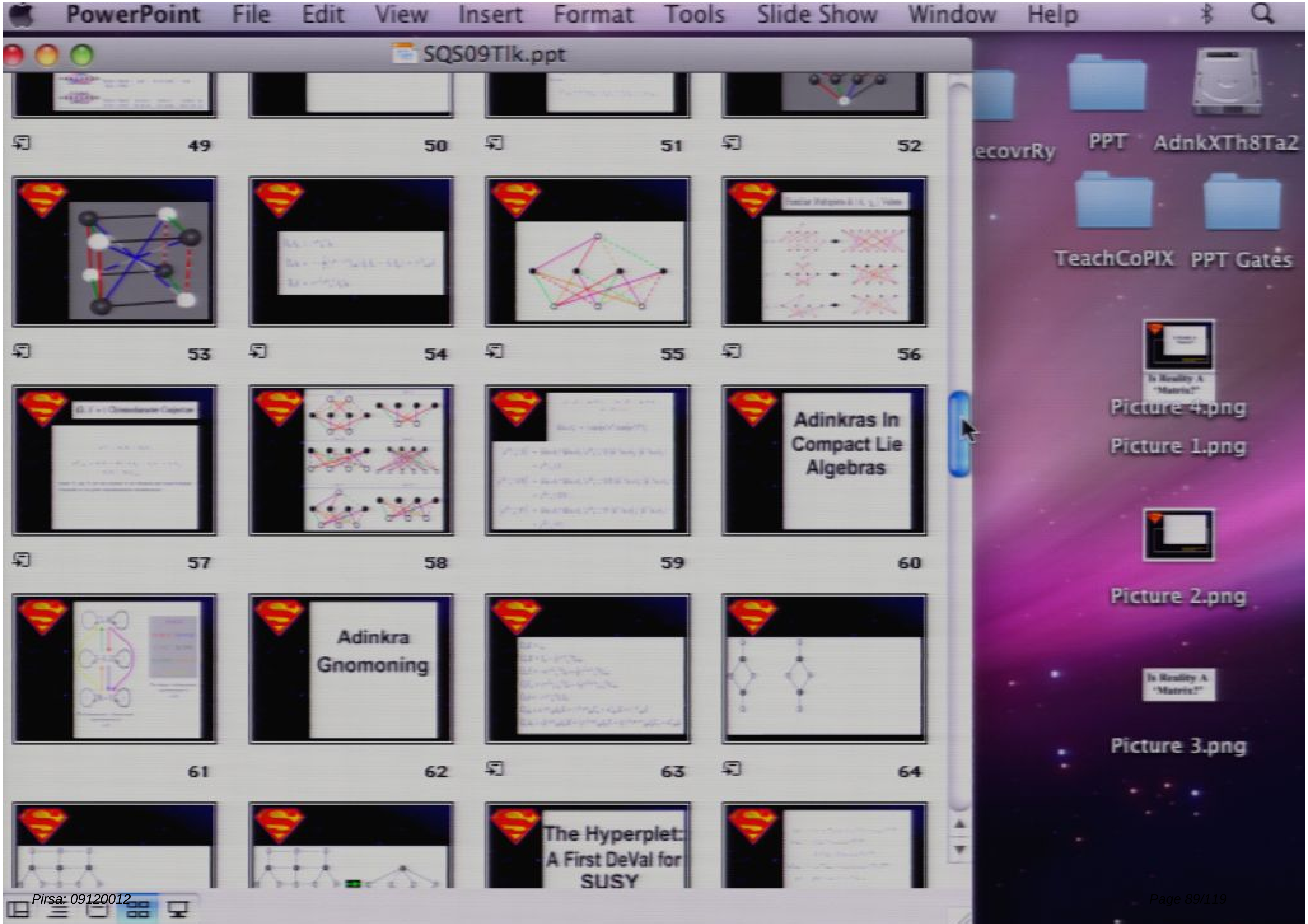

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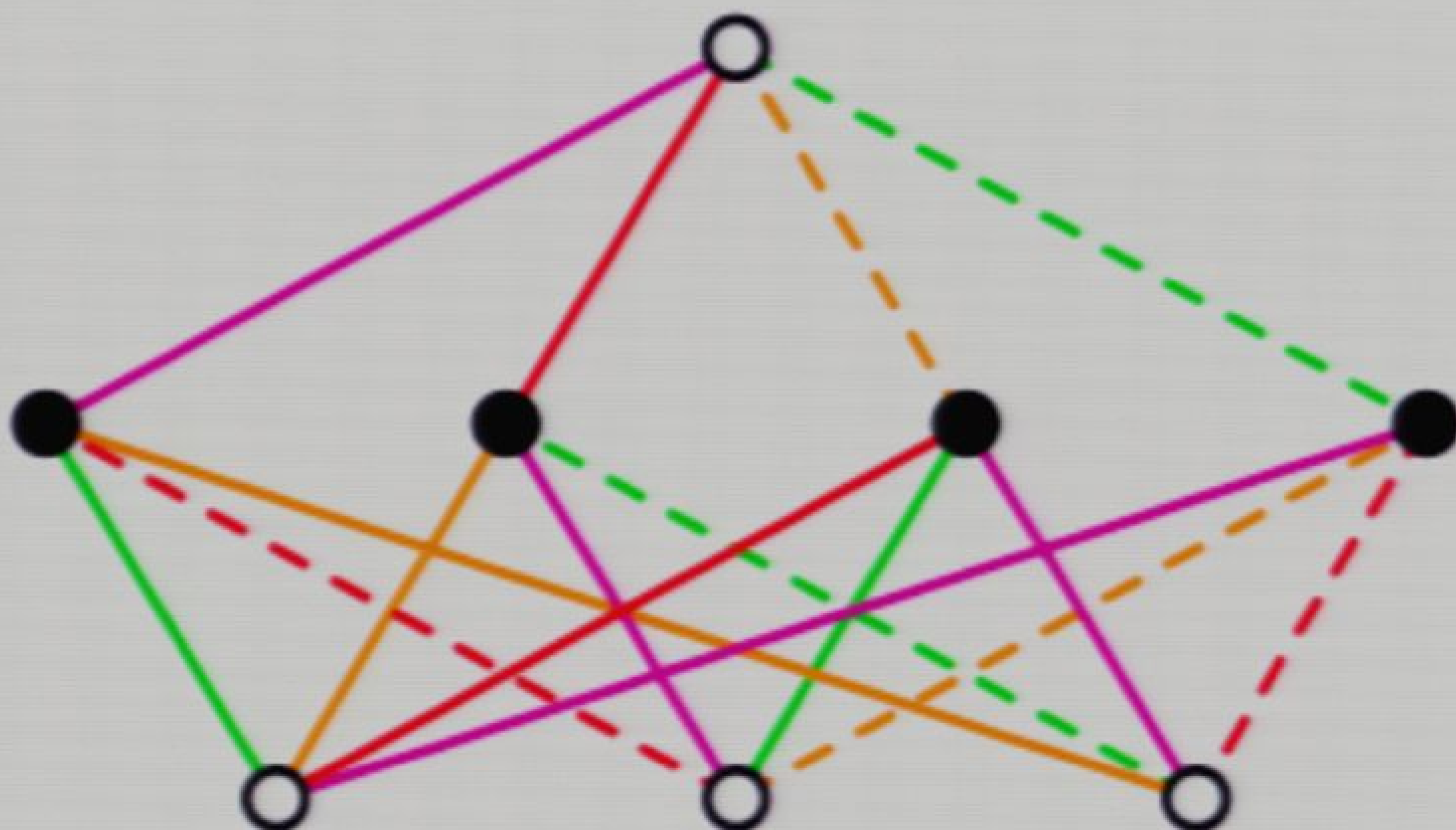
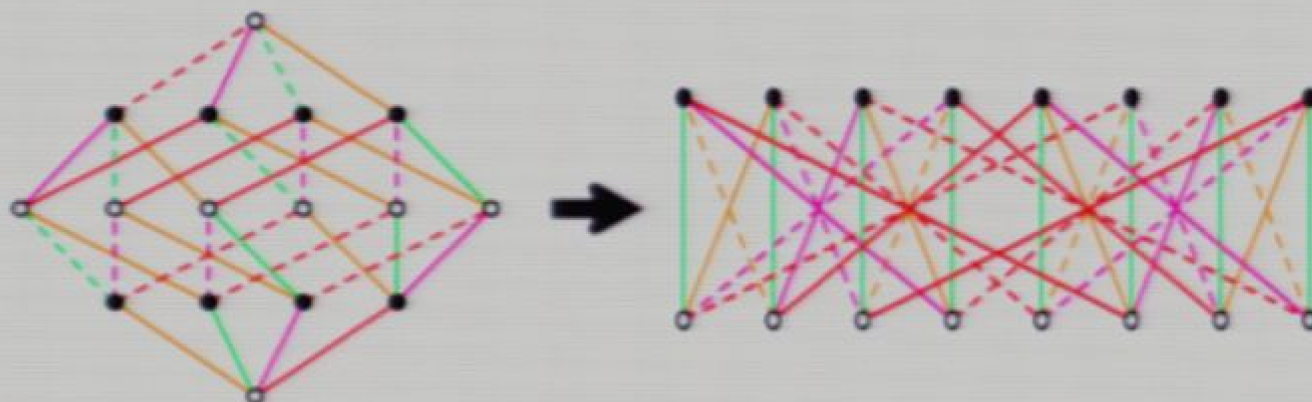
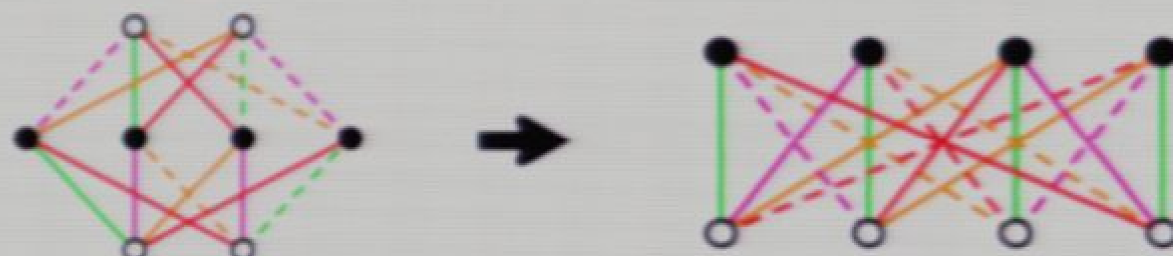


Familiar Multiplets & (d, χ_0) Values

(8, 0)



(4, 1)



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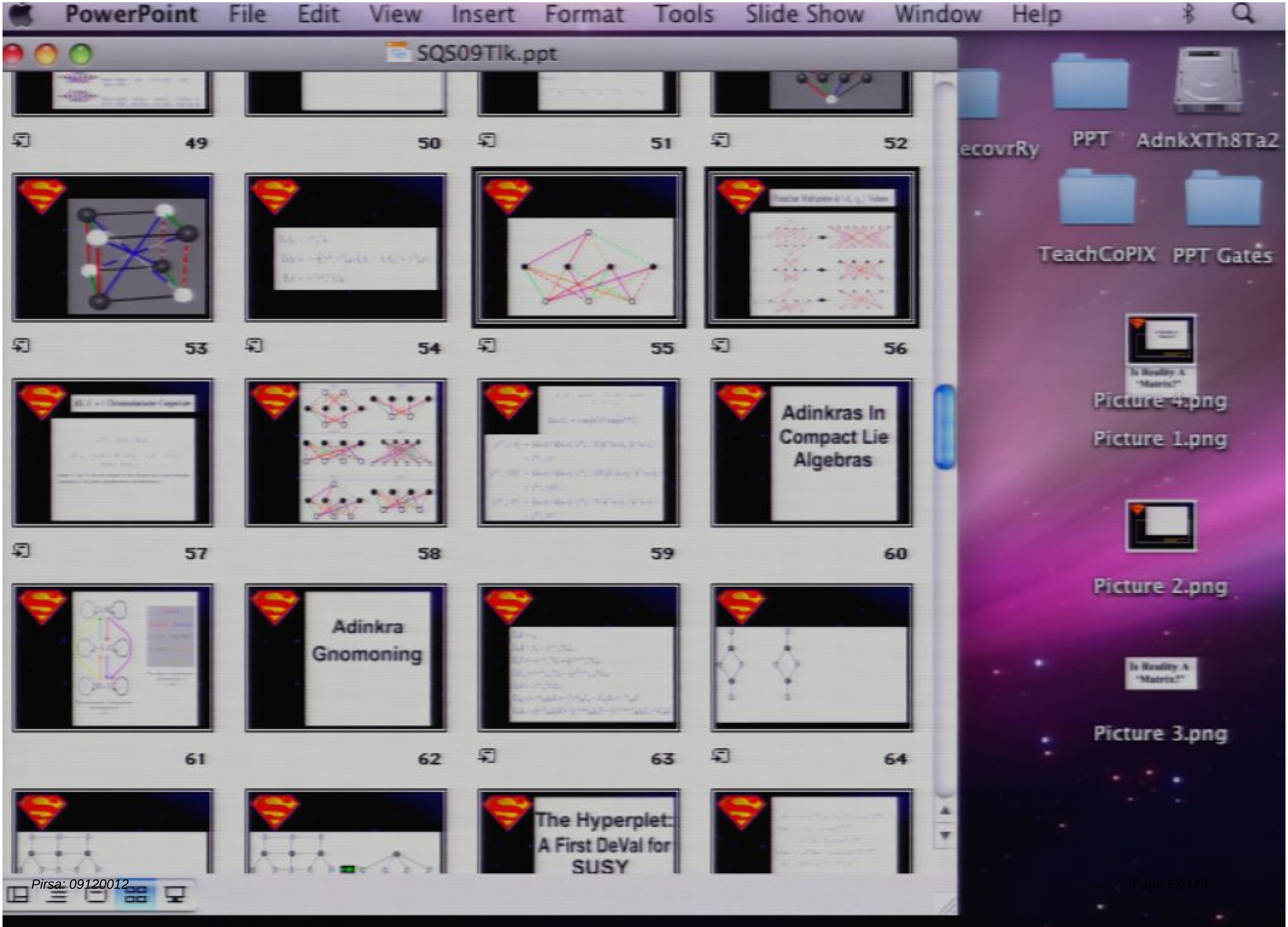
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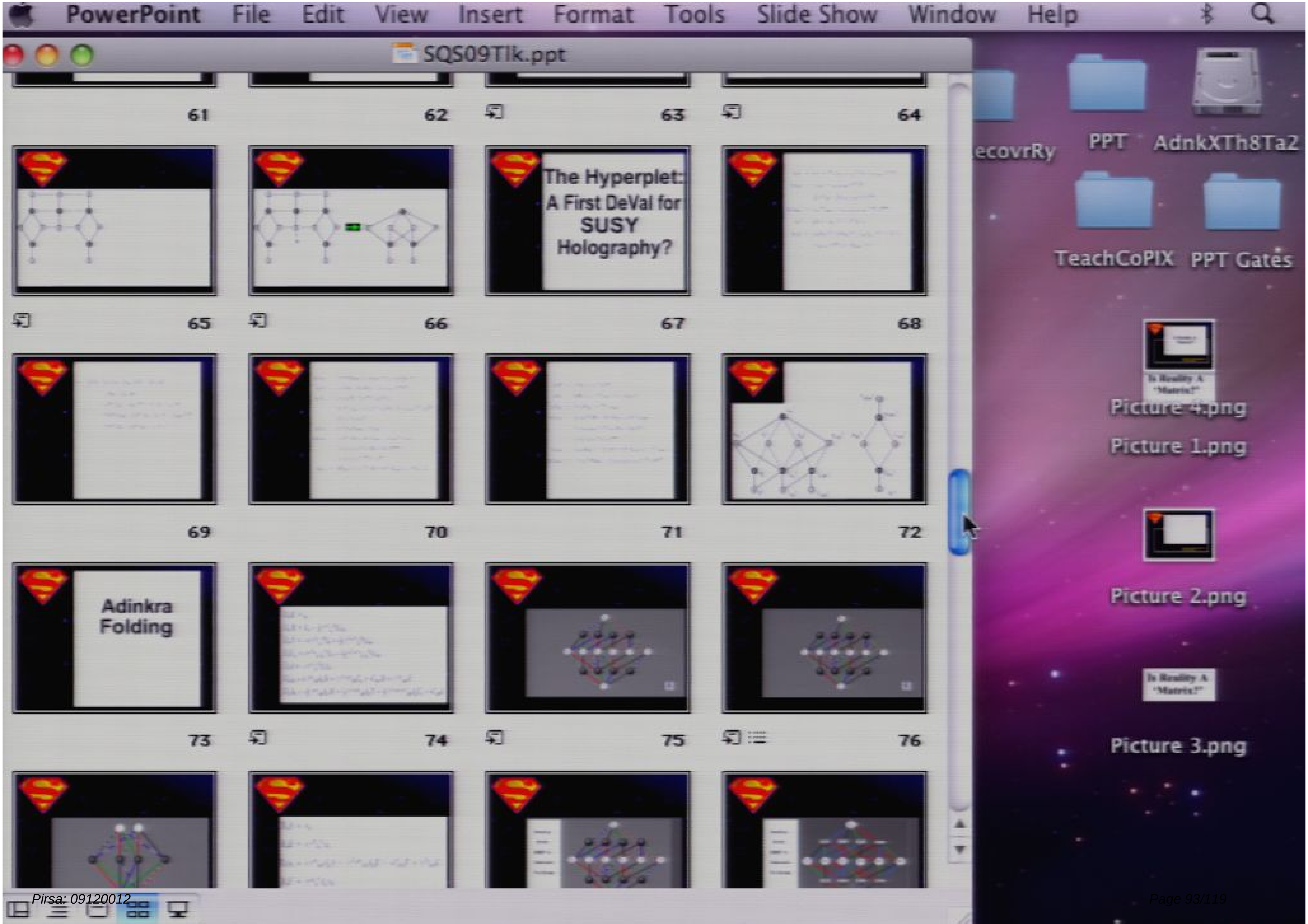

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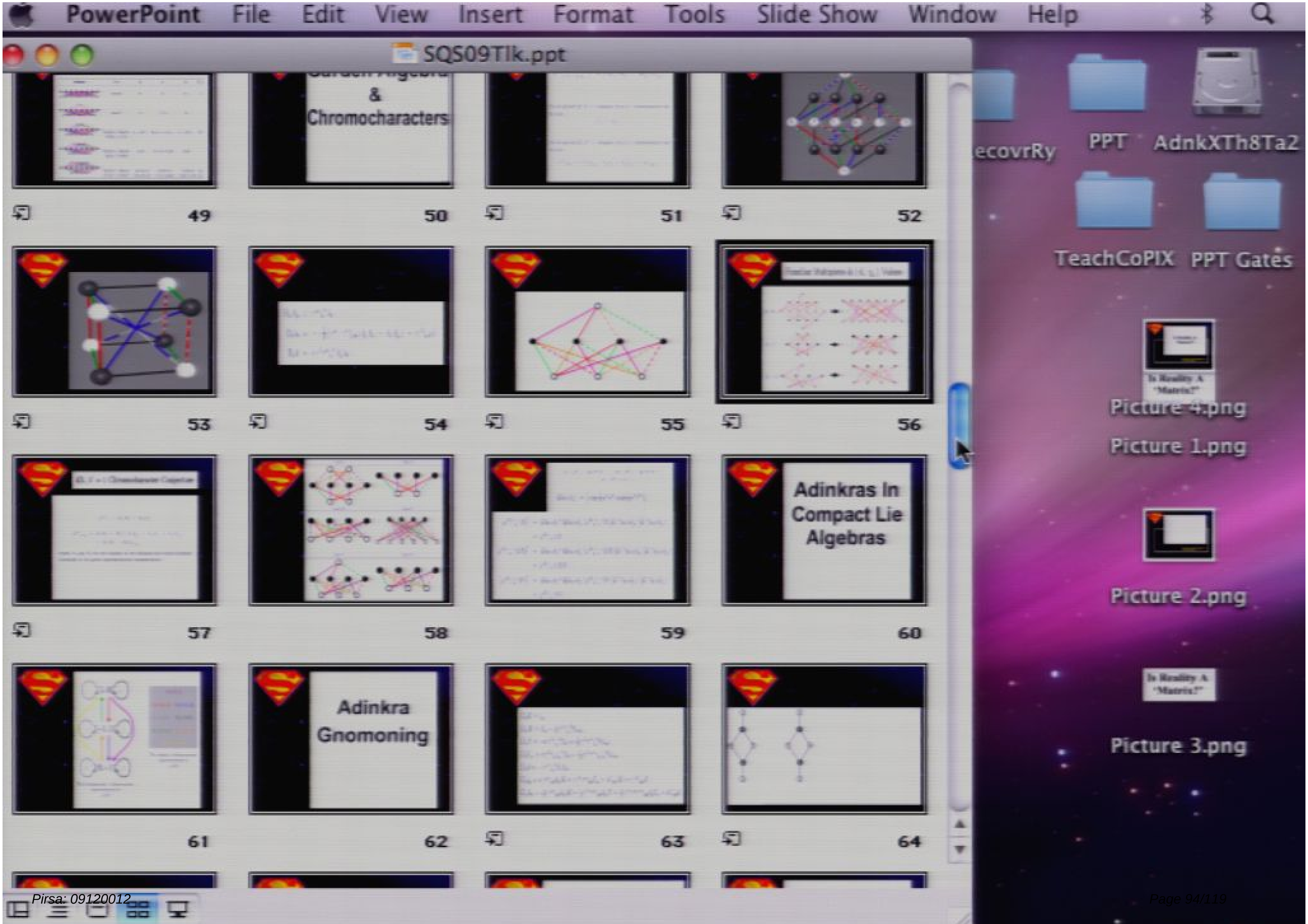
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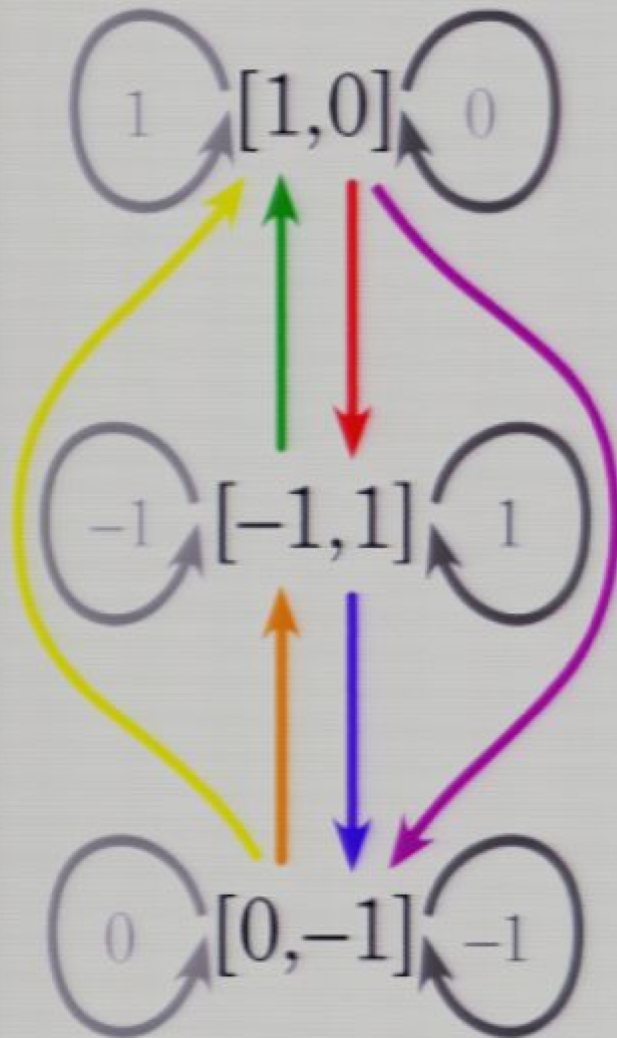


The fundamental, 3-dimensional representation of $su(3)$

$E_3 = [1,1]$

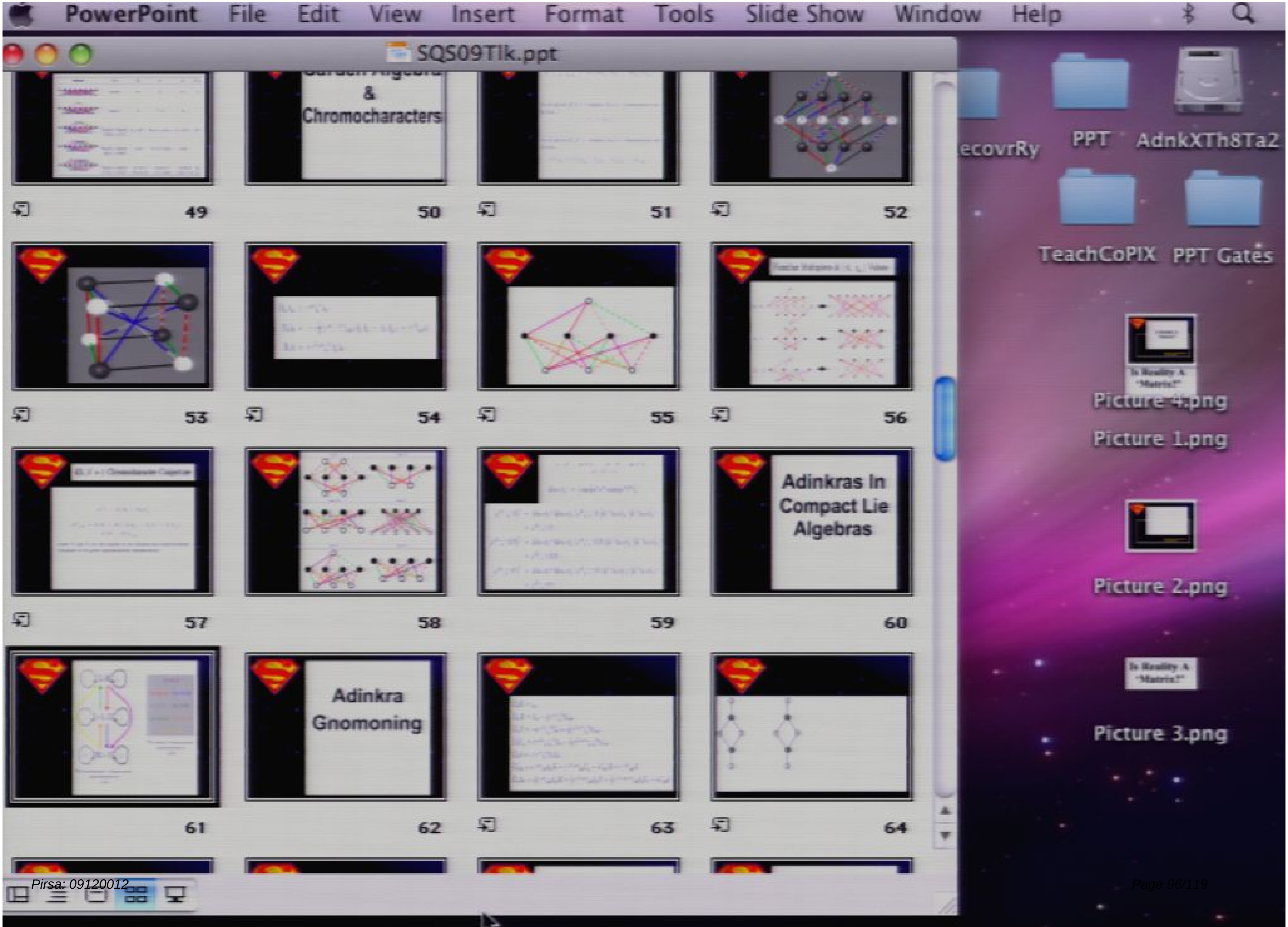
$E_1 = [2,-1]$ $E_2 = [-1,2]$

$H_1 = [0,0]$ $H_2 = [0,0]$

$E_{-1} = [-2,1]$ $E_{-2} = [1,-2]$

$E_{-3} = [-1,-1]$

The adjoint, 8-dimensional representation of $su(3)$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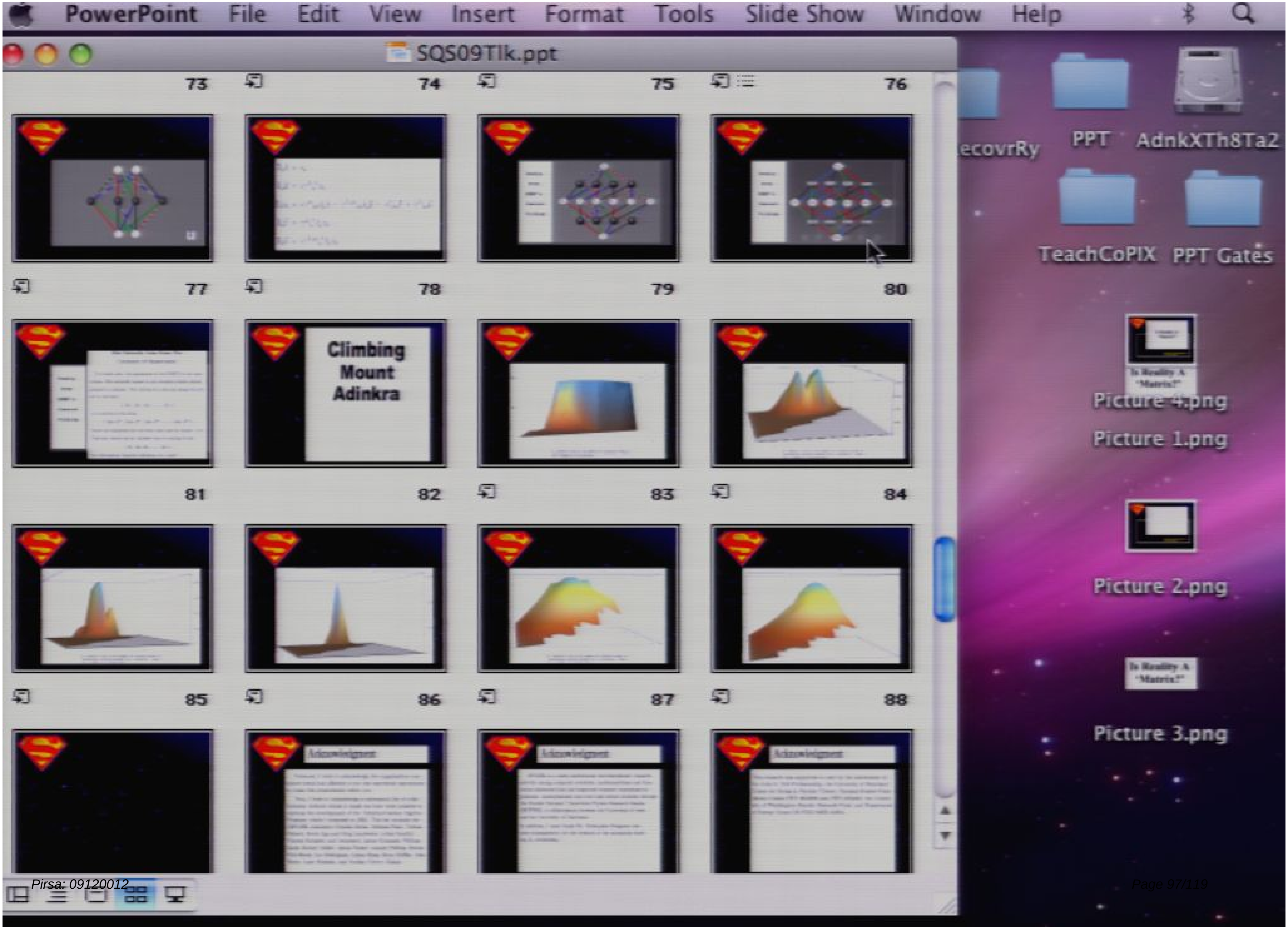
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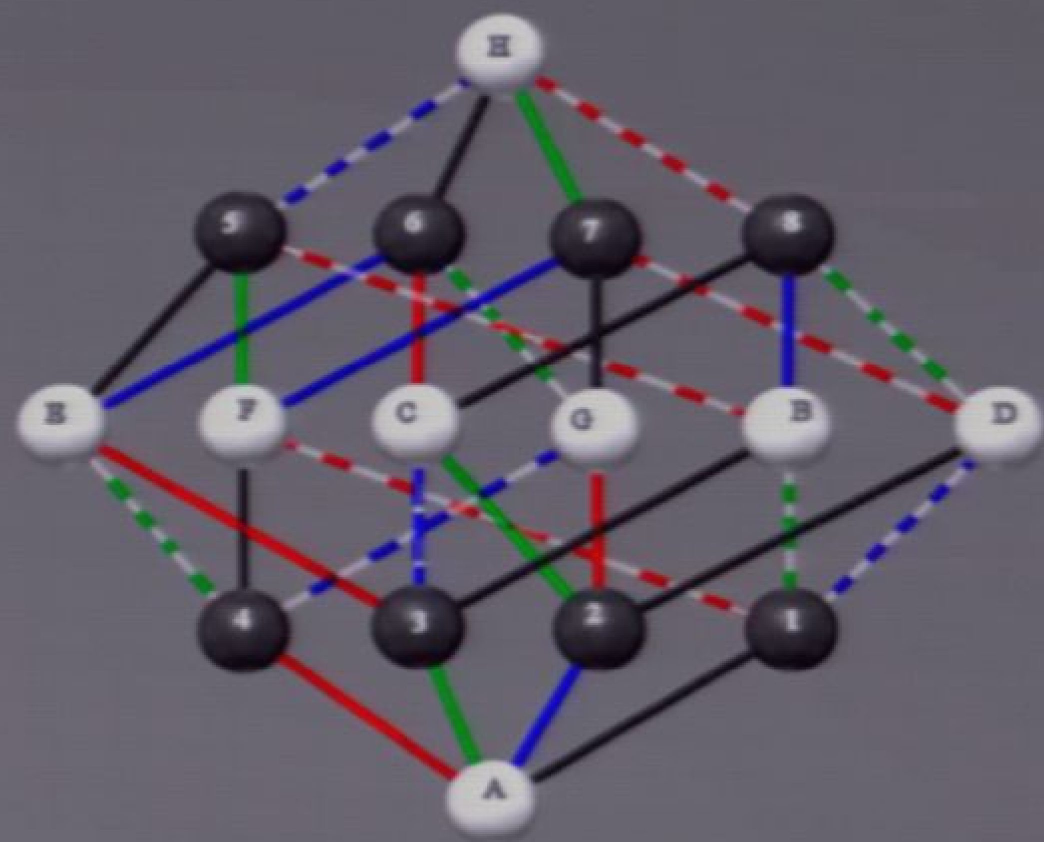
Doubly

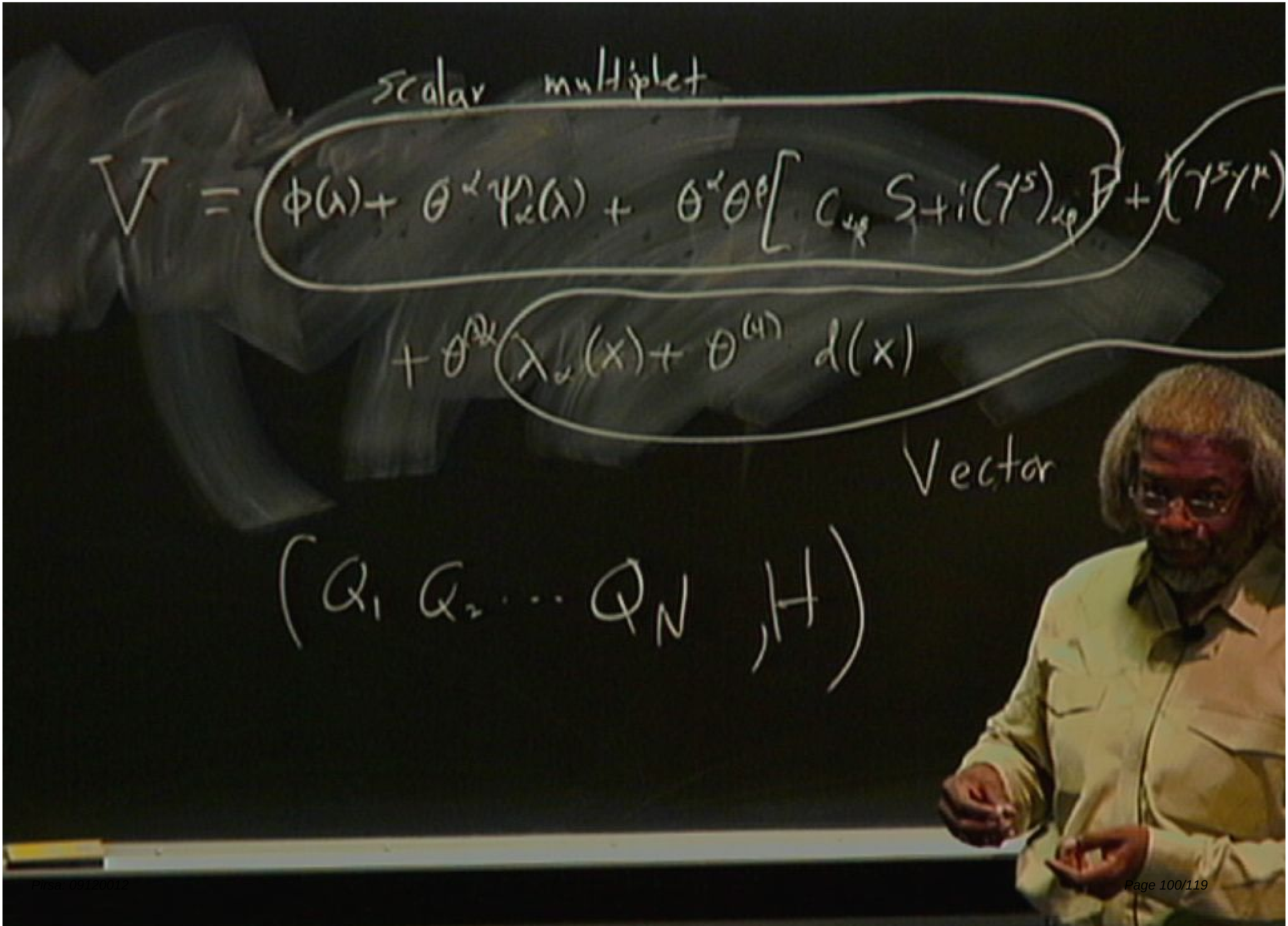
Even

SDEC's

Control

Folding




scalar multiplet
V = φ(x) + θ^α ψ_α(x) + θ^α θ^β [C_αβ S + i(γ^5)_αβ P + (γ^5 γ^μ)
+ θ^(3)α λ_α(x) + θ^(4) d(x)
Vector
(Q_1 Q_2 ... Q_N , H)

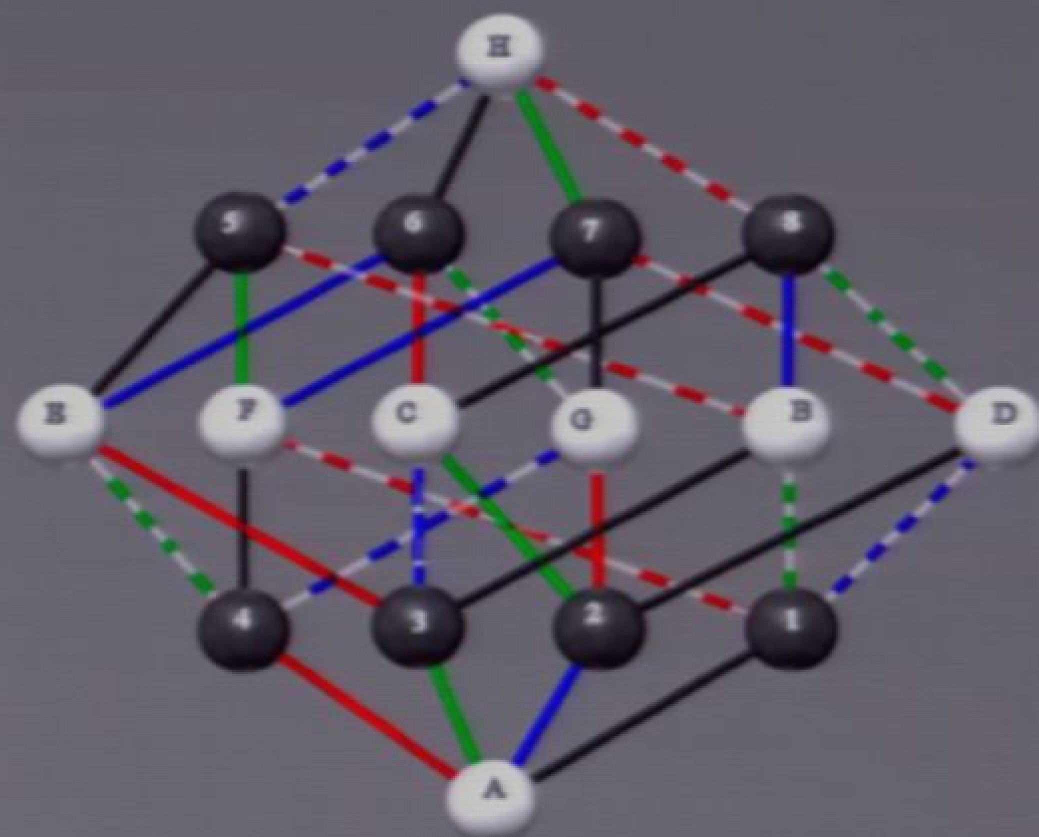
Doubly

Even

SDEC's

Control

Folding



Doubly

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SDEC's

Control

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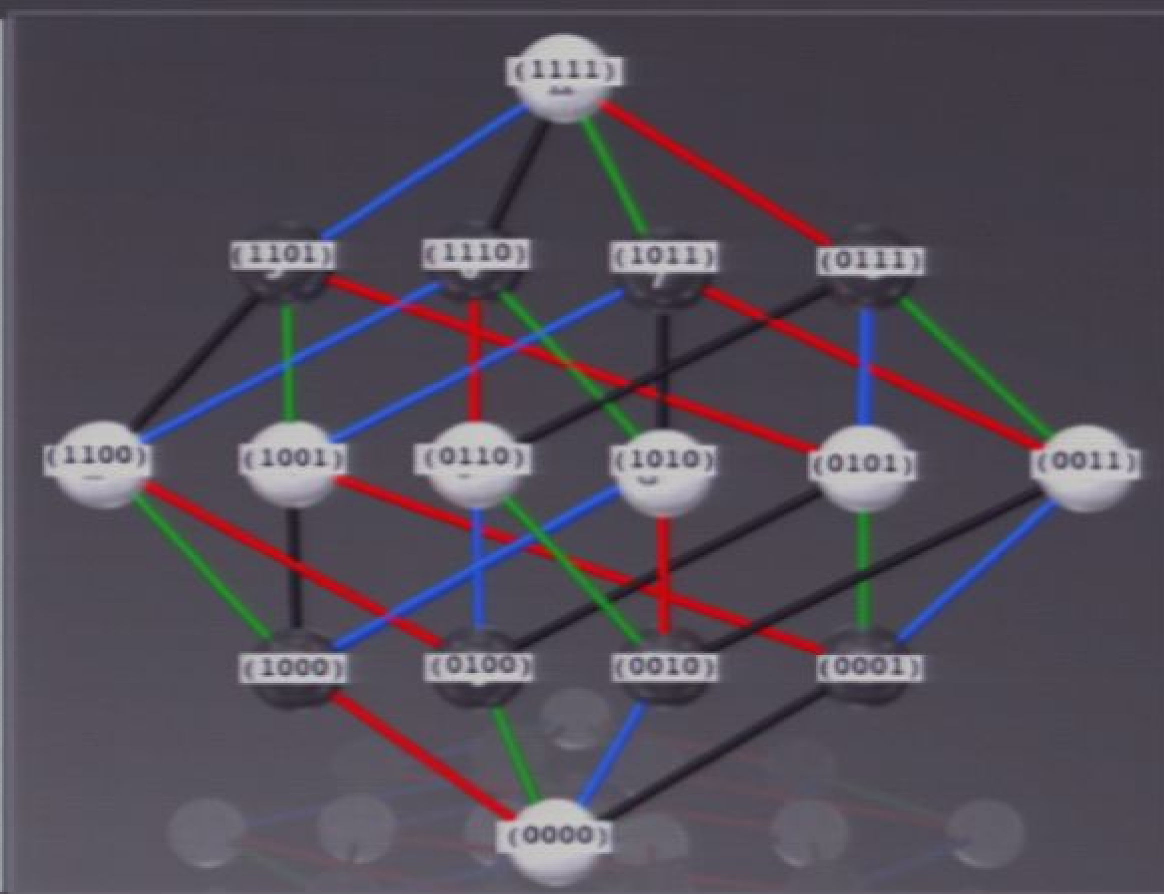


Doubly

Even

SDEC's

Control

Folding

Bits Naturally Arise From The Geometry Of Hypercubes

To a small part, the appearance of the SDEC's is not mysterious. Bits naturally appear in any situation where cubical geometry is relevant. The vertices of a cube can always be written in the form

$$(\pm 1,\ \pm 1,\ \pm 1,\ \dots,\ \pm 1)$$

or re-written in the form

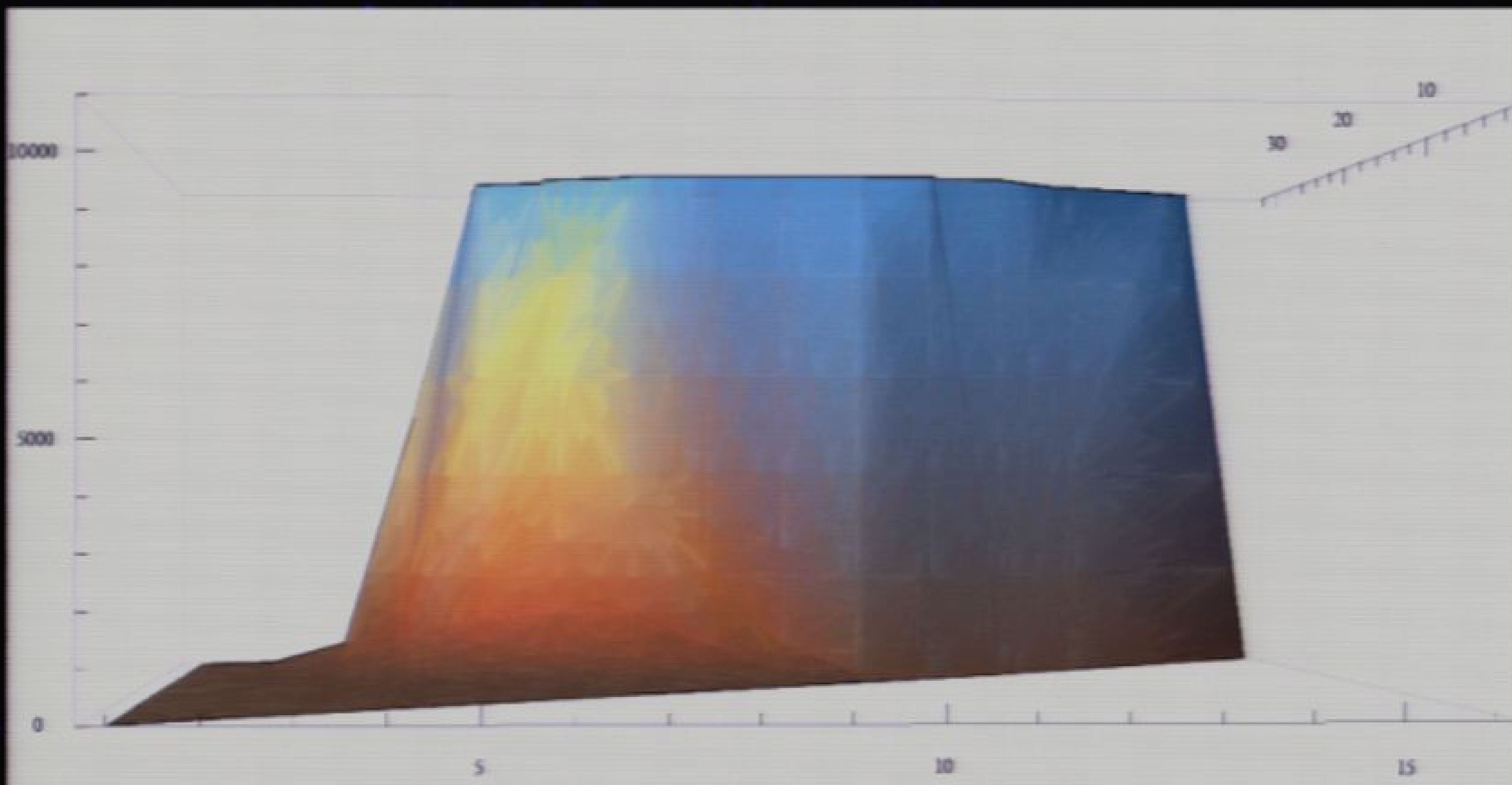
$$((\pm 1,)^{p_1},\ (\pm 1,)^{p_2},\ (\pm 1,)^{p_3},\ \dots,\ (\pm 1,)^{p_d})$$

where the exponents are bits since they take on values 1 or 0. Thus any vertex has an 'address' that is a string of bits

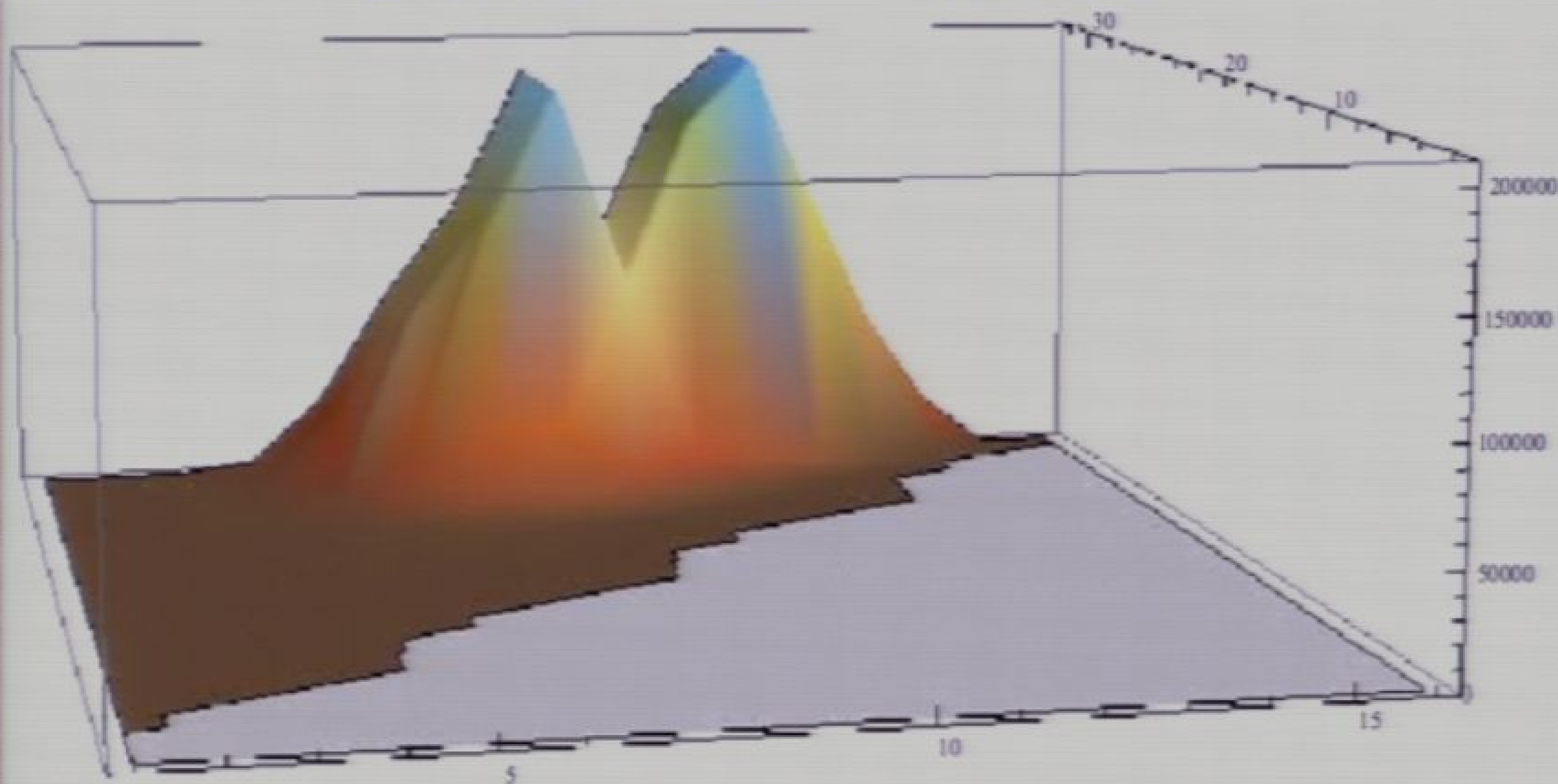
$$(\boldsymbol{p}_1,\ \boldsymbol{p}_2,\ \boldsymbol{p}_3,\ \dots,\ \boldsymbol{p}_d)$$

the information theoretic definition of a 'word.'

Climbing Mount Adinkra



Mt. Adinkra: a plot of the numbers (as currently known) of topologically distinct Adinkras for N (plotted as ``depth'') and k (number of Z2 projections)



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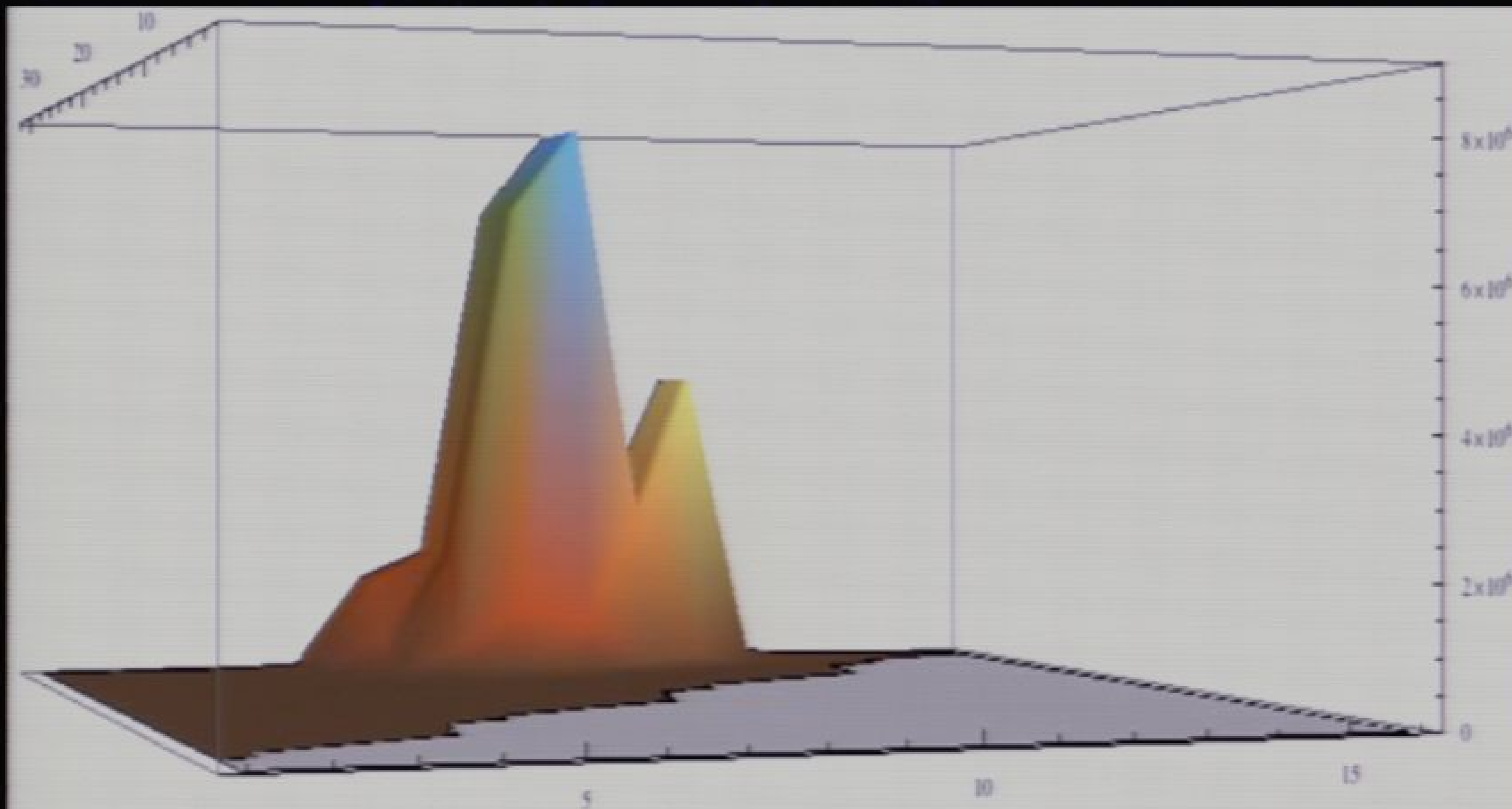

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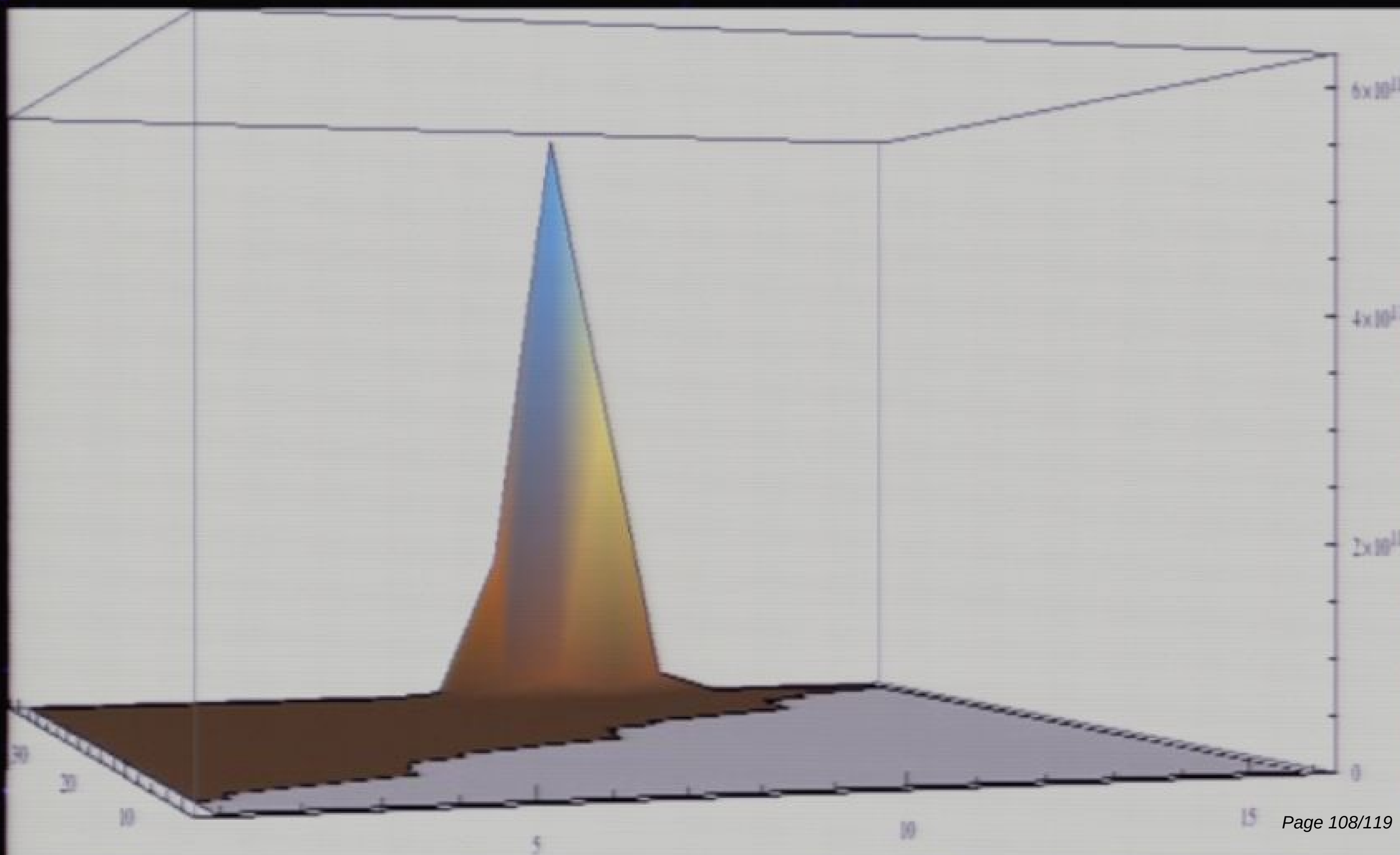
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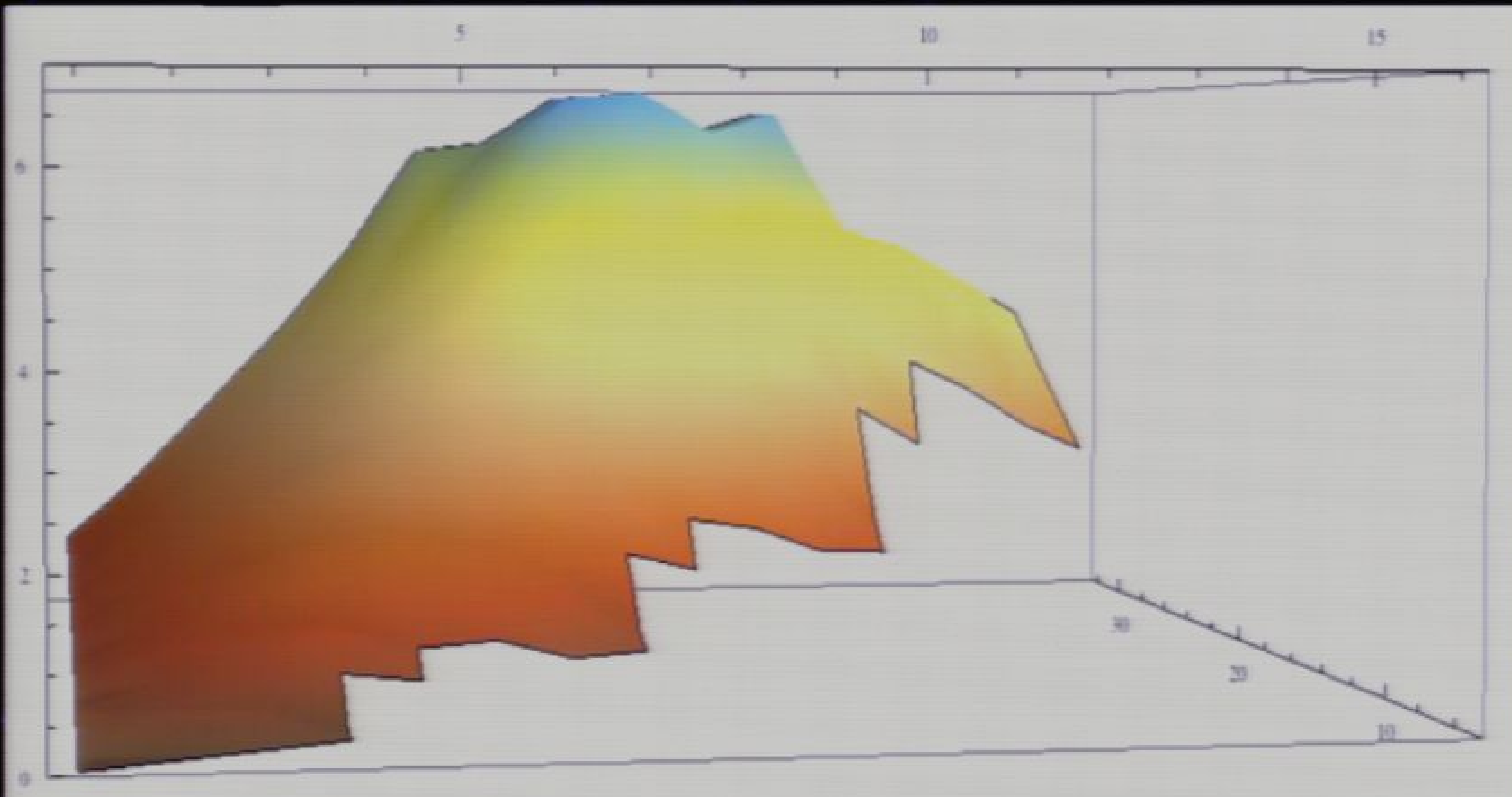

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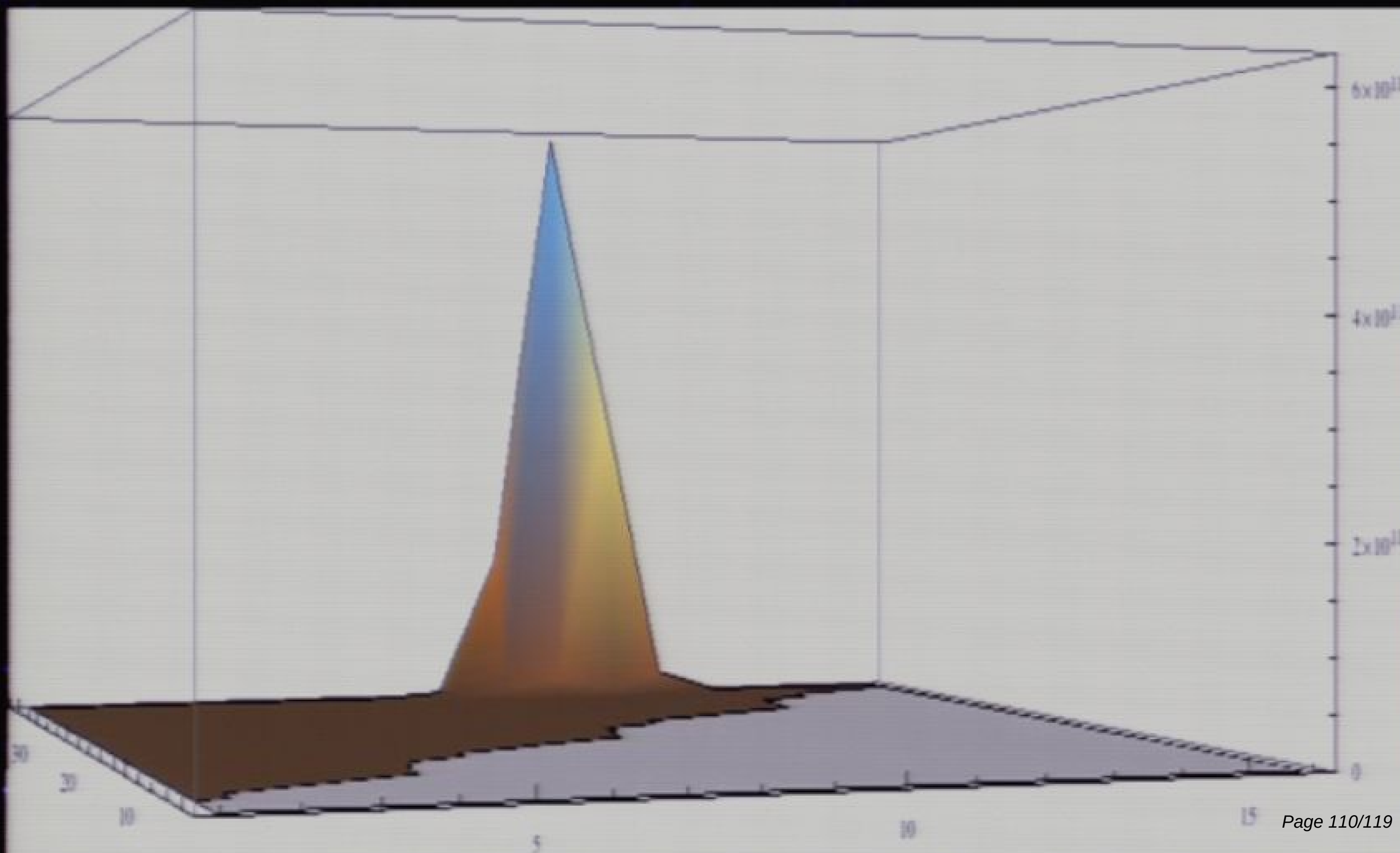
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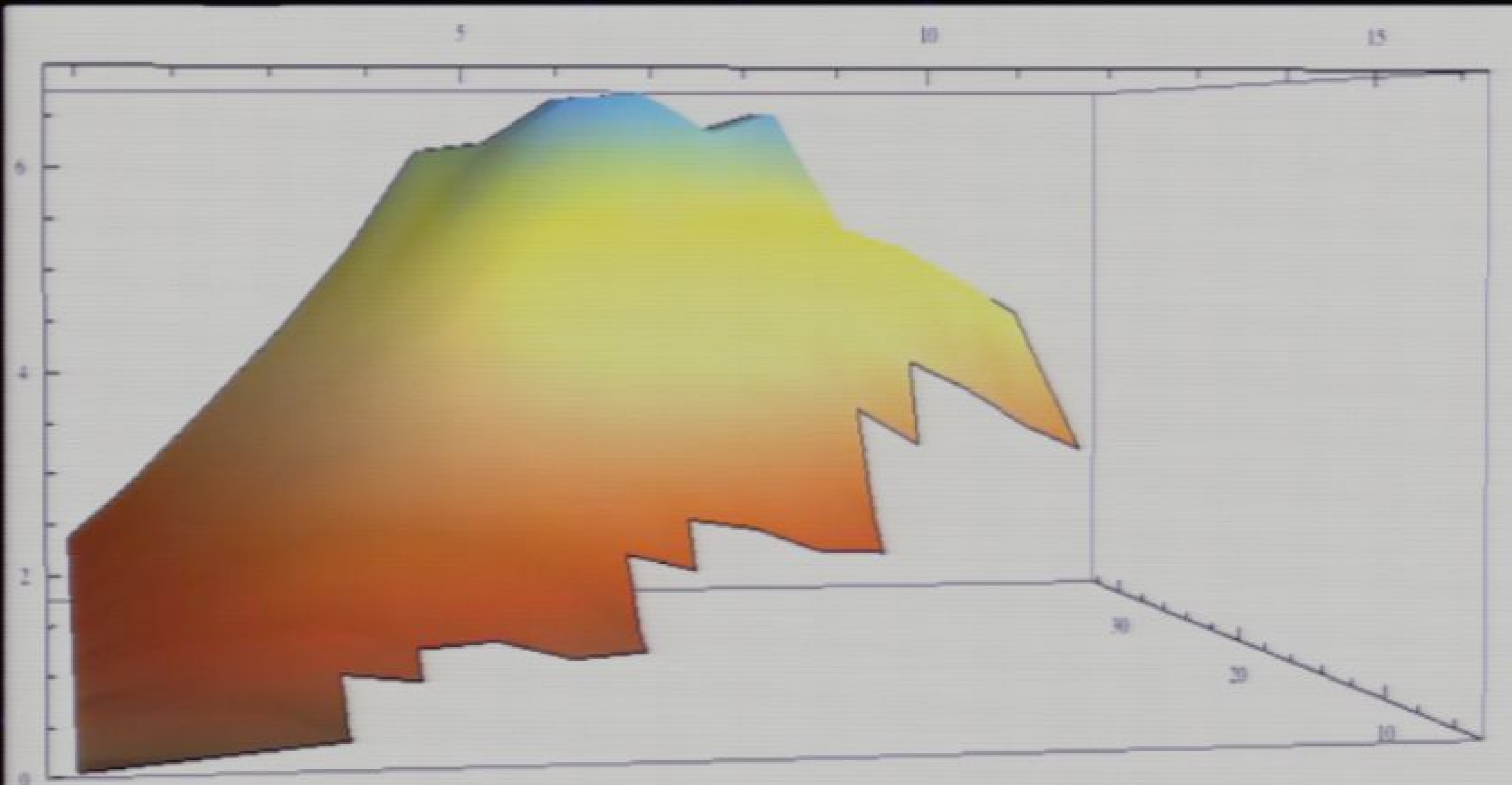

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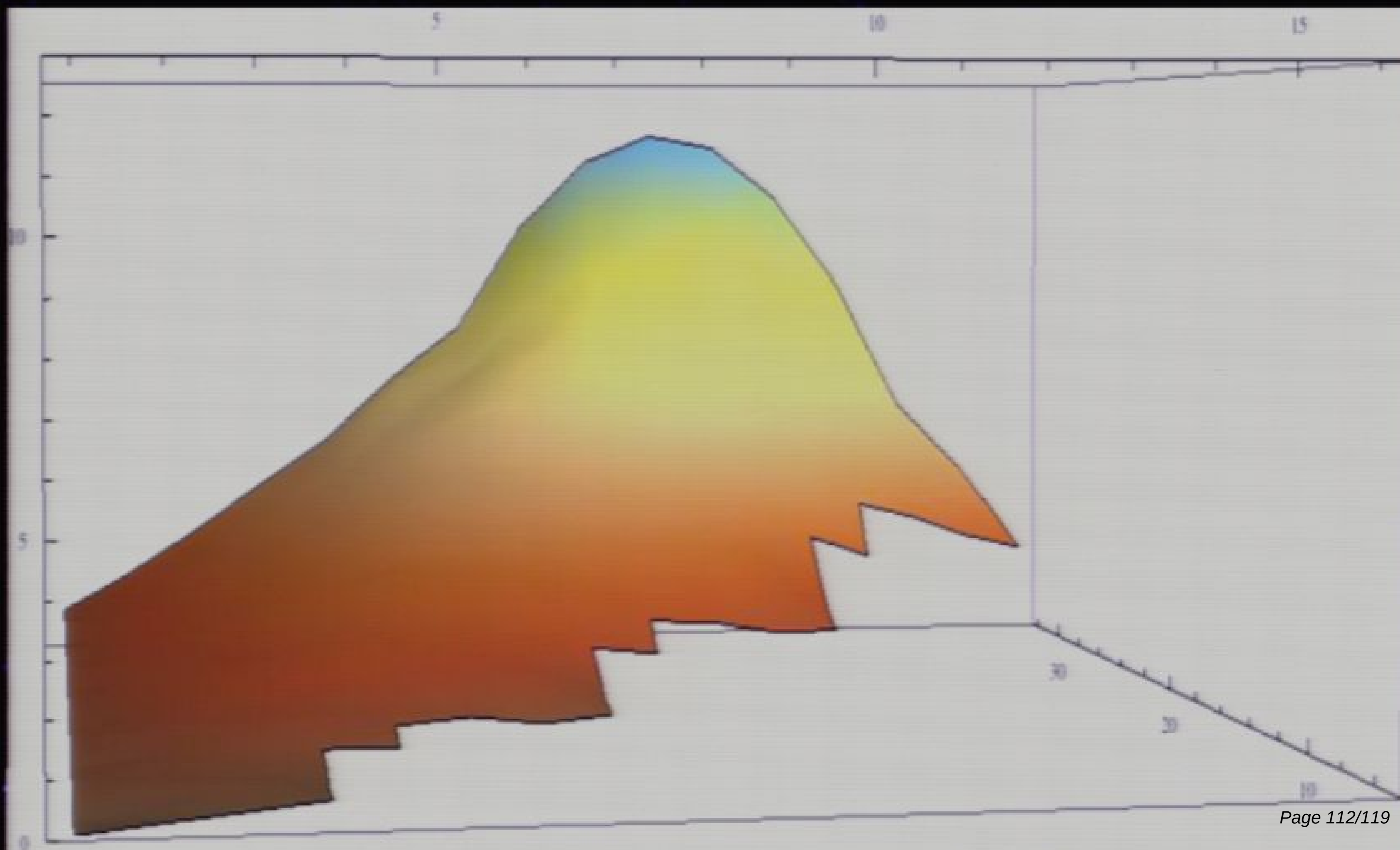
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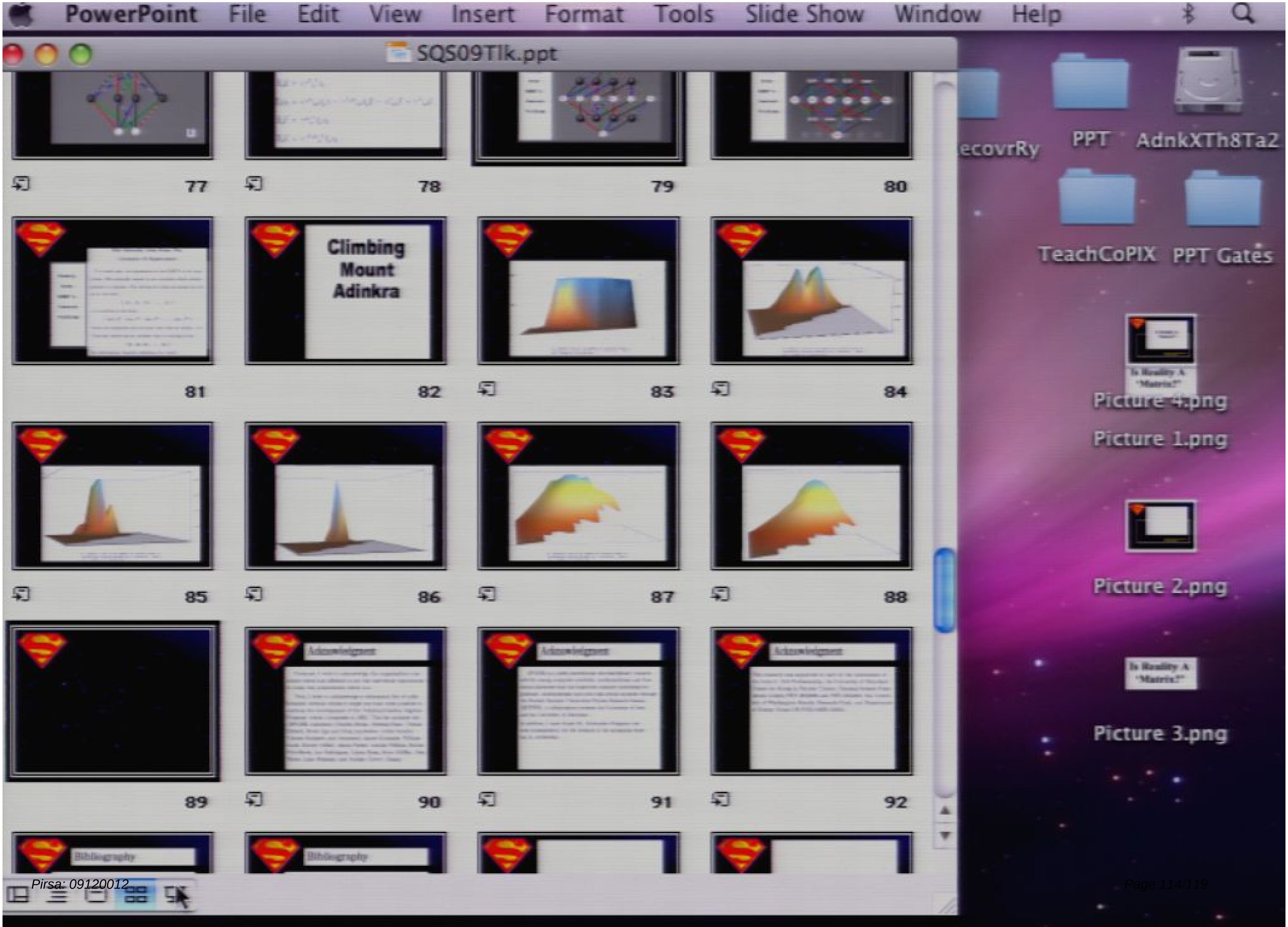
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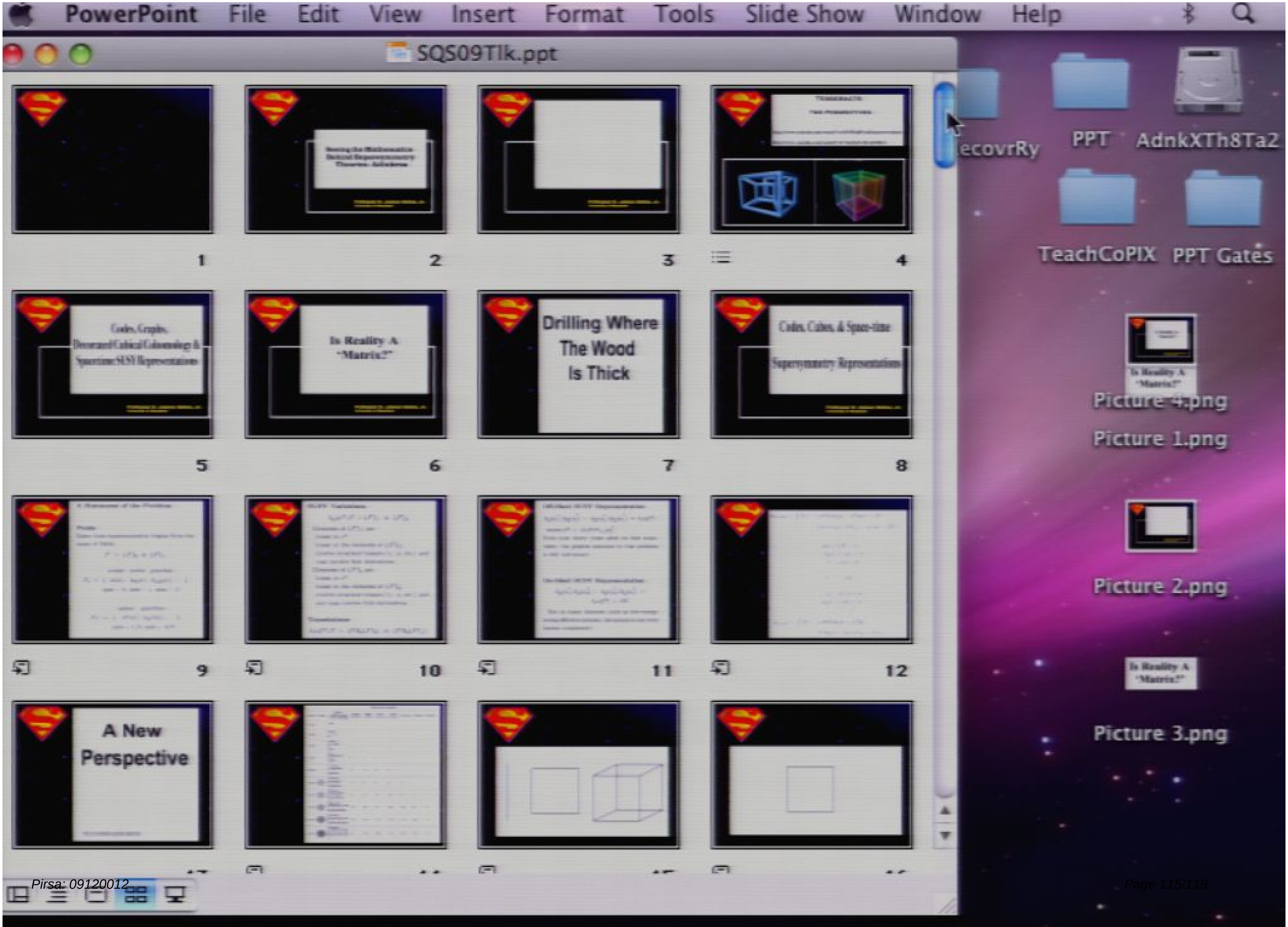

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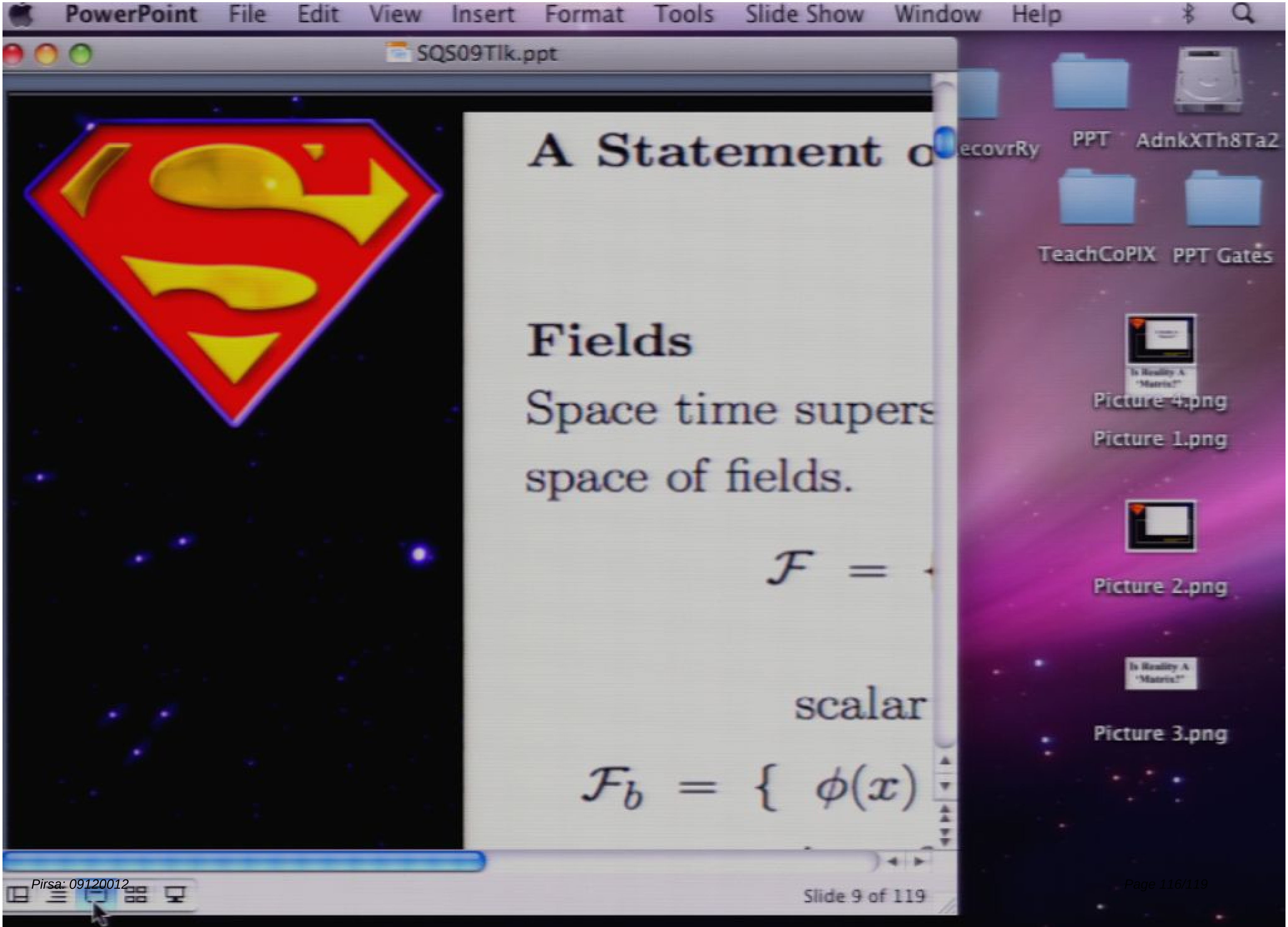
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A Statement o
Fields
Space time supers
space of fields.
$\mathcal{F} =$
scalar
$\mathcal{F}_b = \{\ \phi(x)$
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A Statement of the Problem

Fields

Space time supersymmetry begins from the space of fields.

$$\mathcal{F} = \{\mathcal{F}\}_b \oplus \{\mathcal{F}\}_f$$

scalar vector graviton

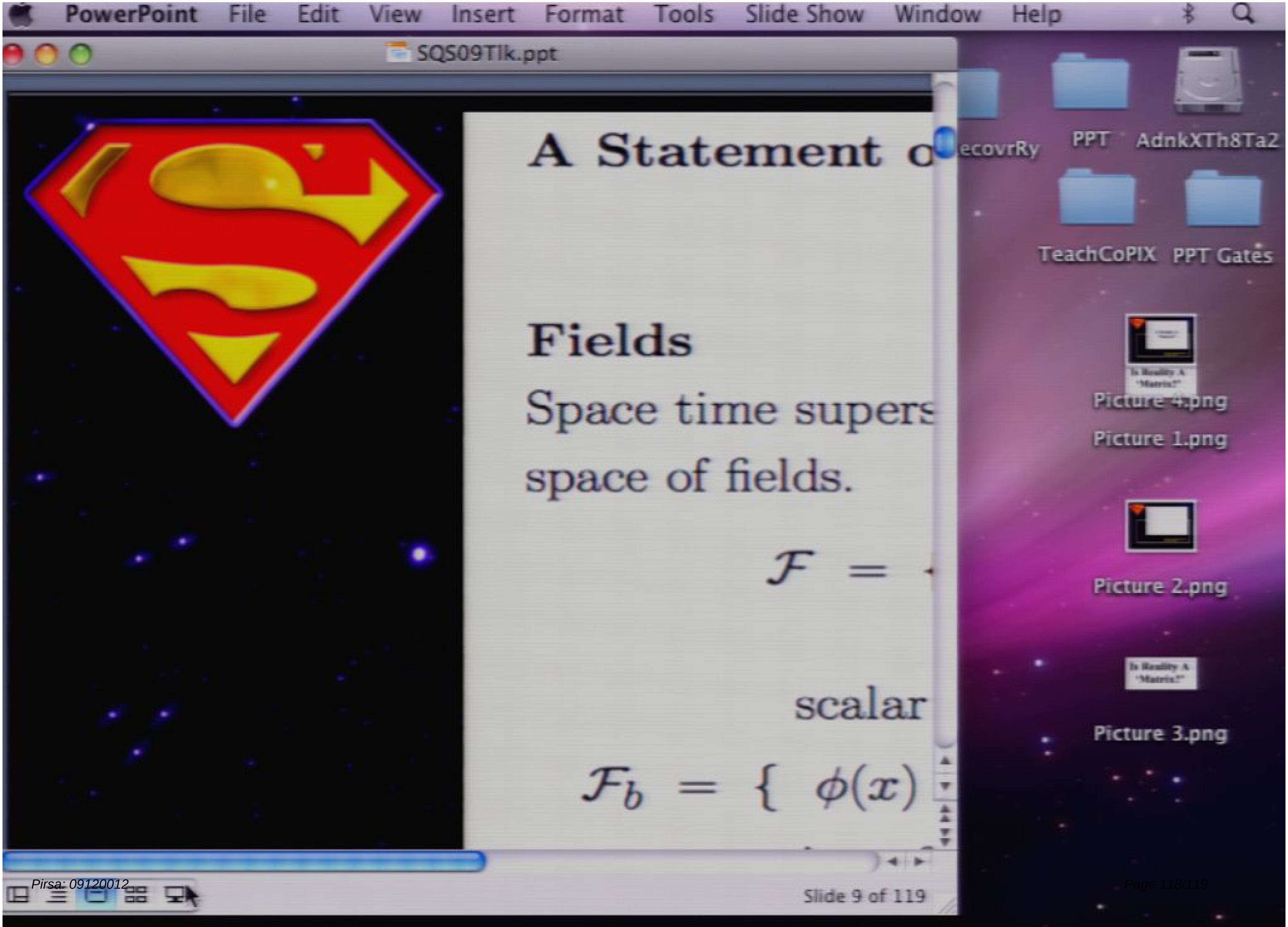
$$\mathcal{F}_b = \{ \ \phi(x) \, , \ A_{\underline{a}}(x) \, , \ h_{\underline{a}\,\underline{b}}(x), \dots \ \}$$

spin − 0, spin − 1, spin − 2

spinor gravitino

$$\mathcal{F}_f = \{ \ \ \lambda^{\alpha}(x) \, , \ \psi_{\underline{a}}{}^{\beta}(x), \dots \ \}$$

spin − 1/2, spin − 3/2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A Statement o
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