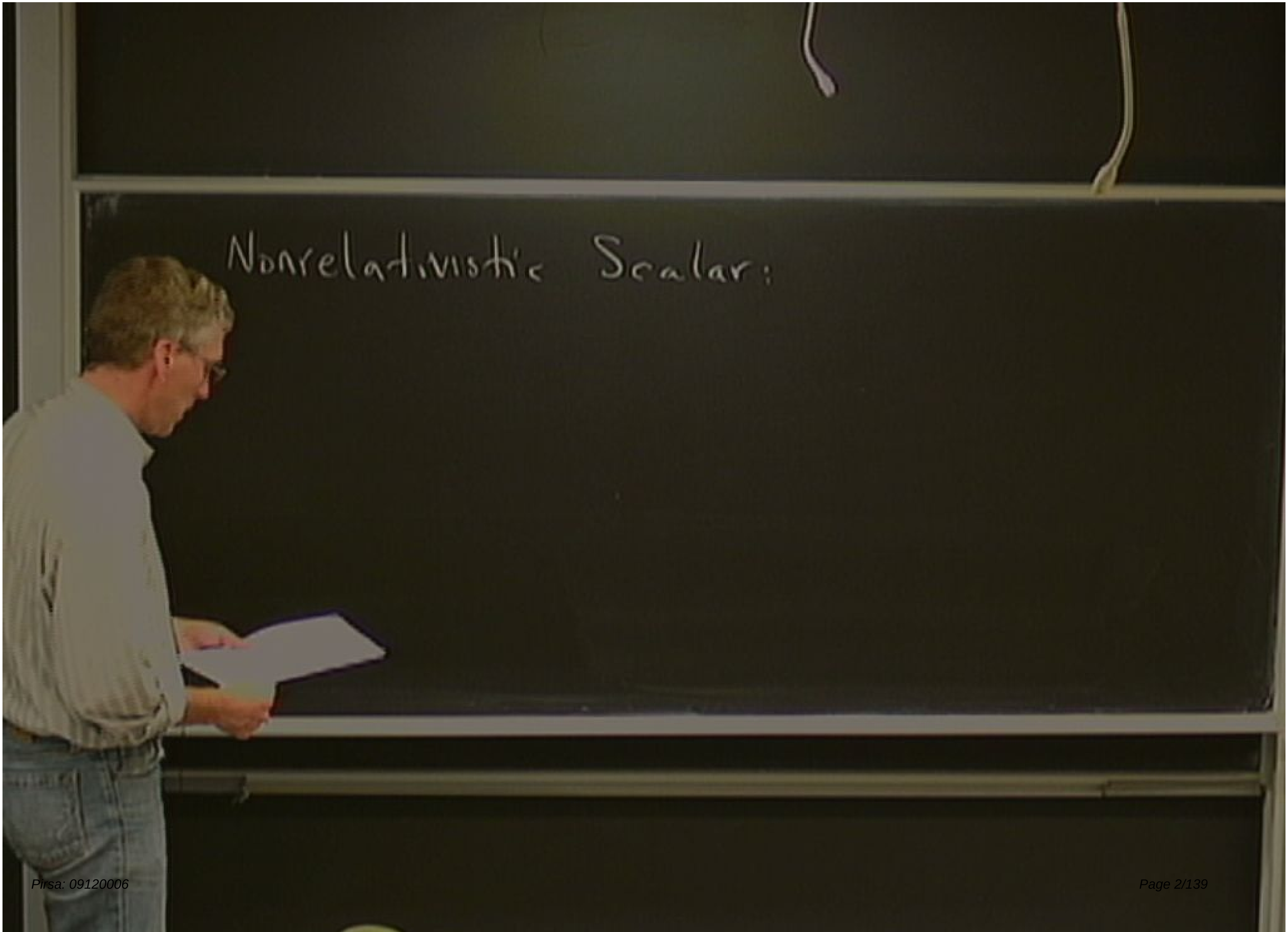
Title: Introduction to Effective Field Theory - Lecture 12

Date: Dec 09, 2009 10:00 AM

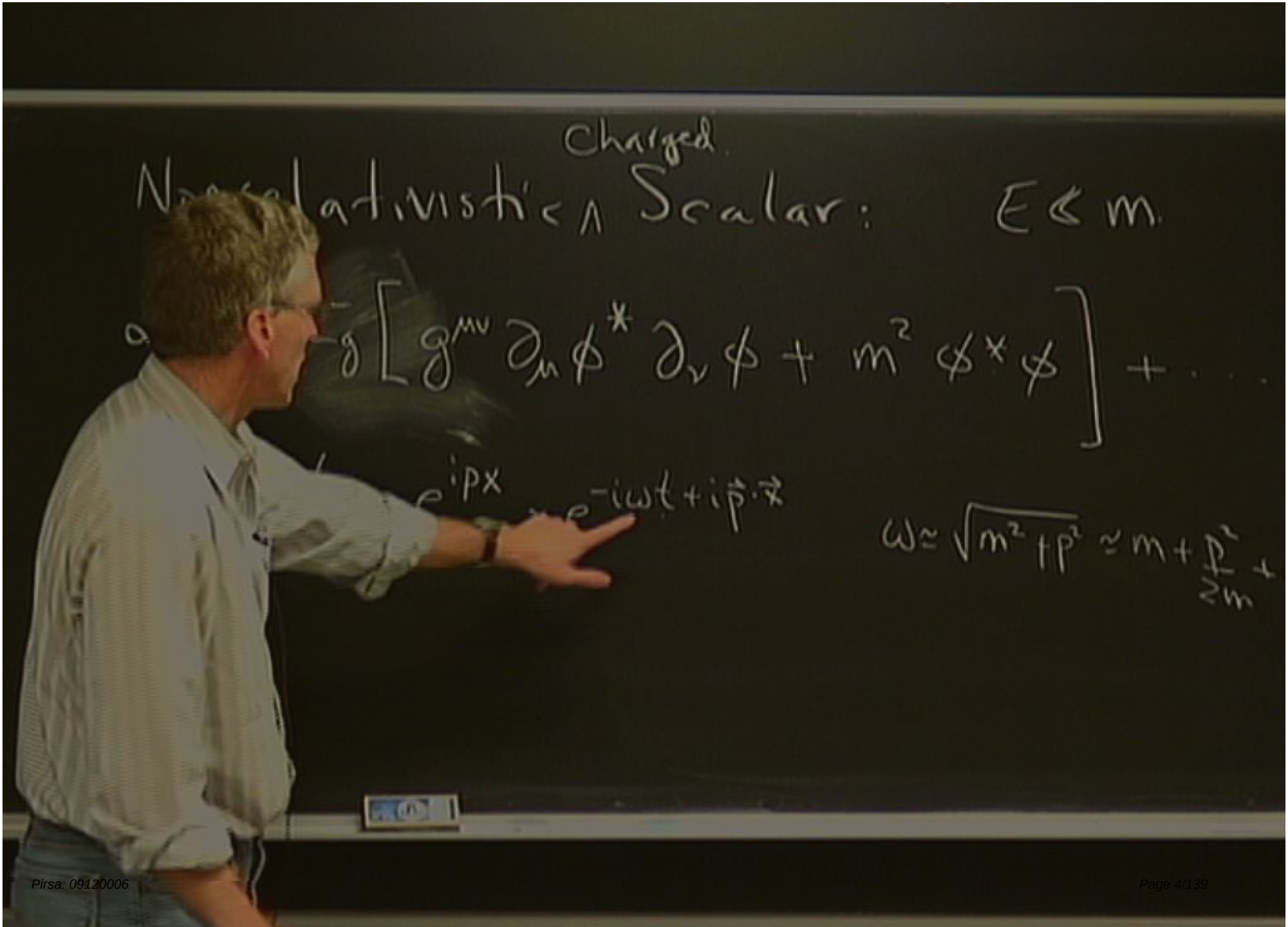
URL: http://pirsa.org/09120006

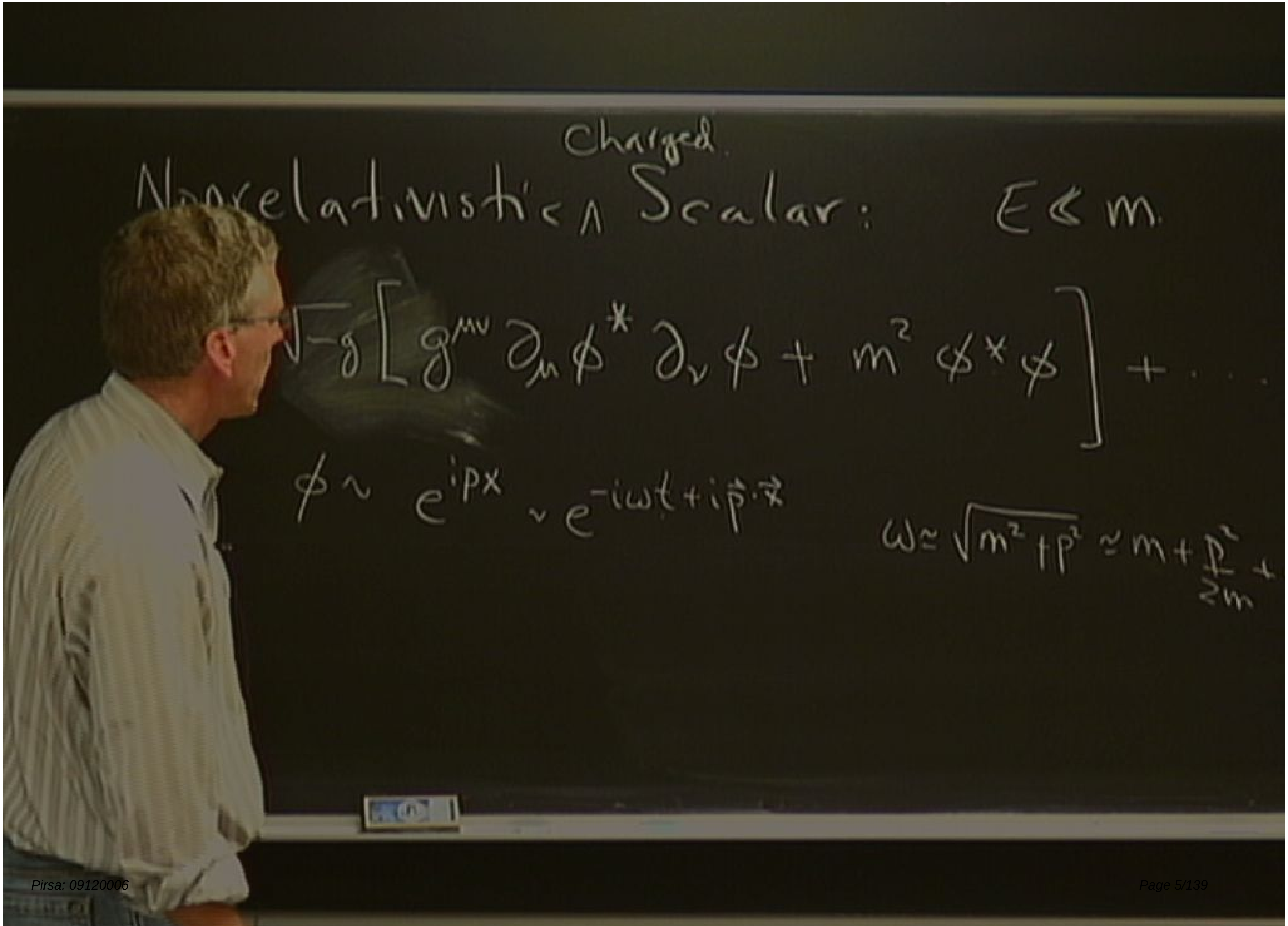
Abstract:

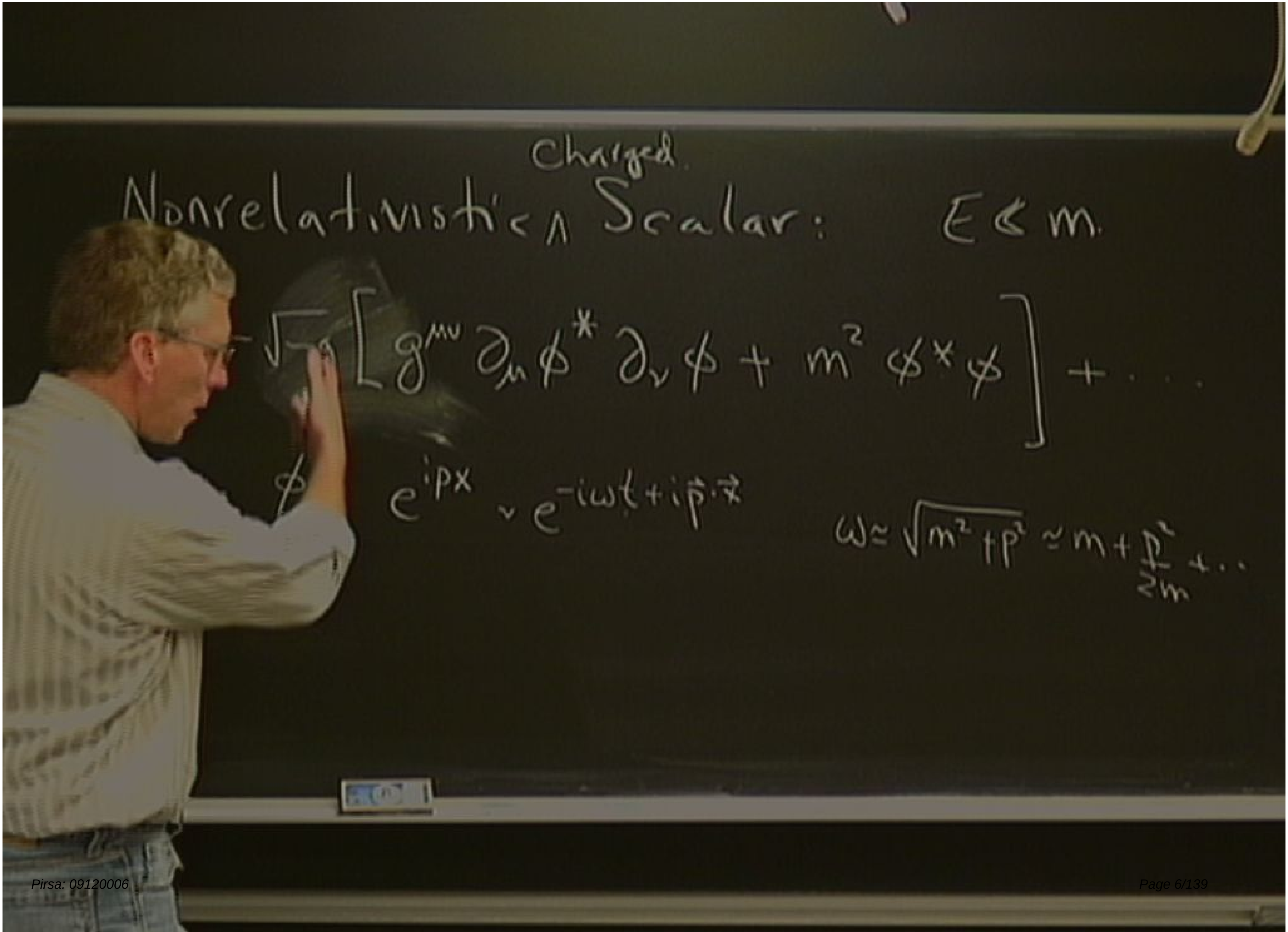

Nonrelativistic Scalar:

Nonrelativistic Charged Scalar: $E \ll m$

$$\mathcal{L} = -\sqrt{-g}\,[$$


Charged
N lativistic Scalar: E ≪ m
g[g^μν ∂_μ φ* ∂_ν φ + m² φ* φ] + . . .
e^ipx
e^-iωt + i p⃗·x⃗
ω ≃ √(m² + p²) ≃ m + p²/2m +


Charged
Nonrelativistic Scalar: $E \ll m$
$\sqrt{-g}\left[g^{\mu\nu}\partial_\mu \phi^* \partial_\nu \phi + m^2 \phi^* \phi\right] + \ldots$
$\phi \sim e^{ipx} \sim e^{-i\omega t + i\vec{p}\cdot\vec{x}}$
$\omega \simeq \sqrt{m^2+p^2} \simeq m + \frac{p^2}{2m} + \ldots$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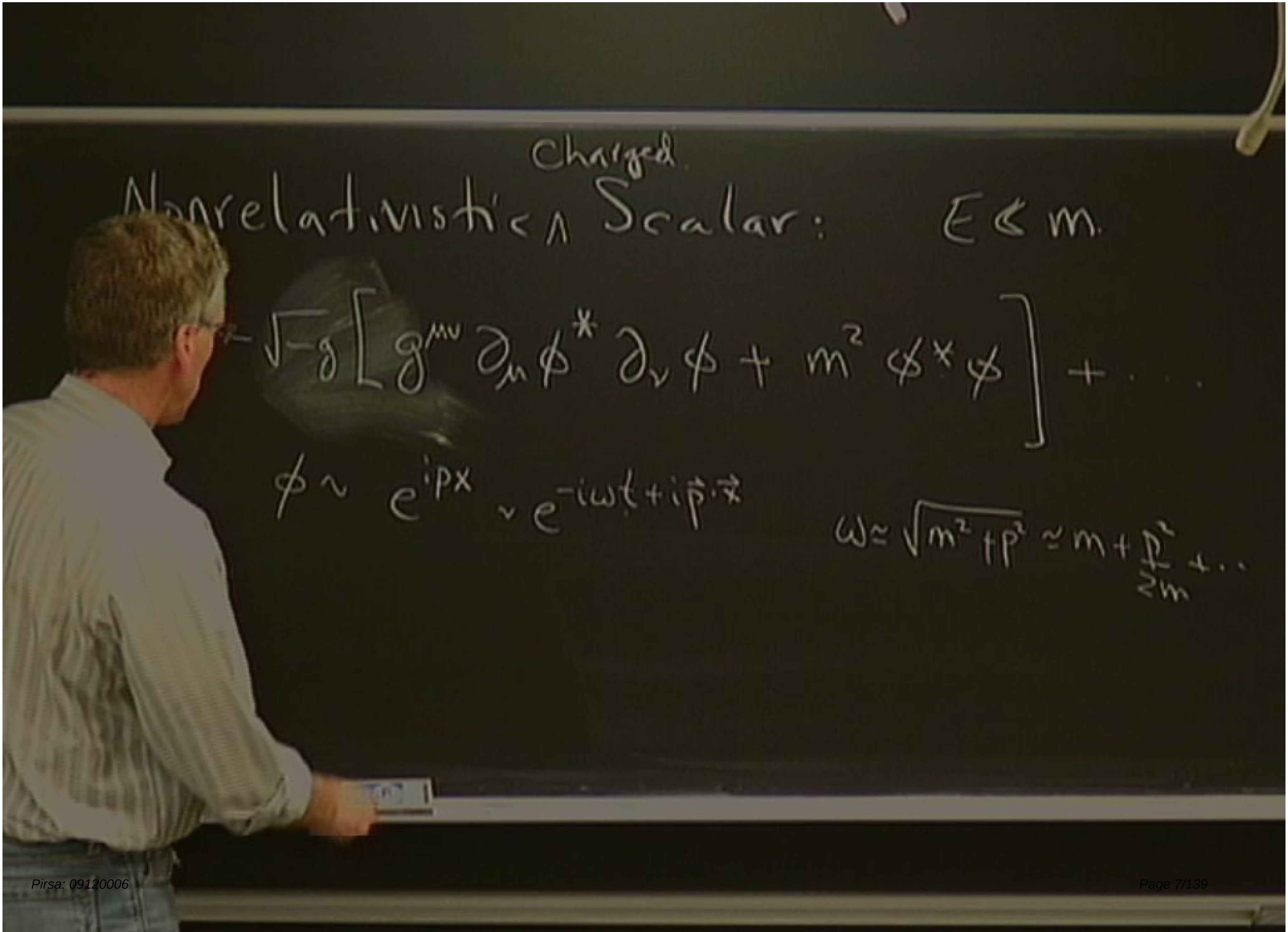
Nonrelativistic Scalar: $E \ll m$
$\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + m^2\phi^*\phi\right] + \ldots$
$\phi \; e^{ipx} \sim e^{-i\omega t + i\vec{p}\cdot\vec{x}}$
$\omega \approx \sqrt{m^2 + p^2} \approx m + \frac{p^2}{2m} + \ldots$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Nonrelativistic Scalar: $E \ll m$
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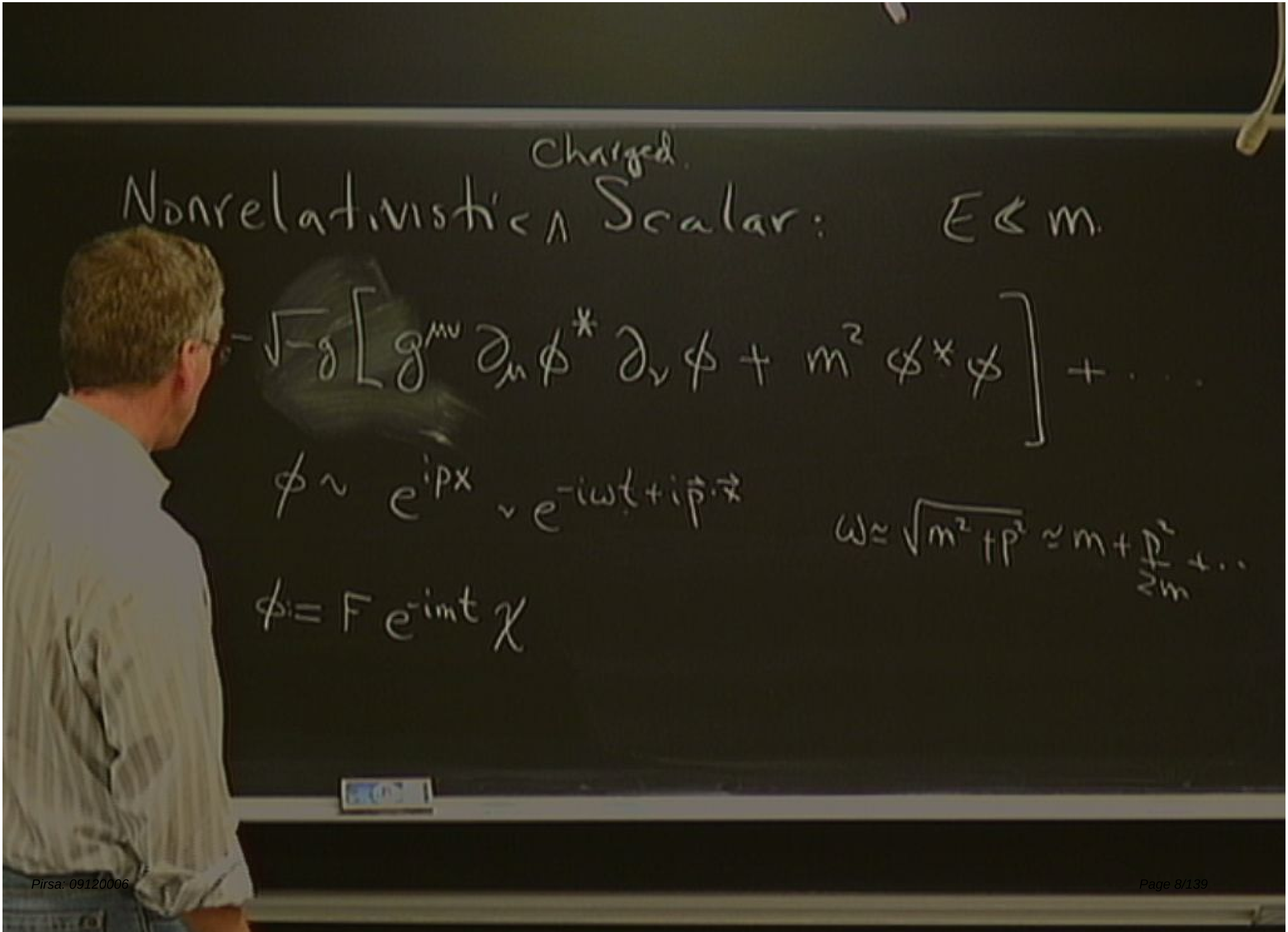
Charged
Nonrelativistic Scalar: $E \ll m$
$-\sqrt{-g}\,[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + m^2\phi^*\phi] + \ldots$
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$\omega \simeq \sqrt{m^2+p^2} \simeq m + \frac{p^2}{2m} + \ldots$
$\phi := F e^{-imt}\chi$

Nonrelativistic Charged Scalar: $E \ll m$

$$\mathcal{L} = -\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + m^2\phi^*\phi\right] + \ldots$$

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$\phi := F e^{-imt}\chi$

$\partial_t\phi = F e^{-imt}(\partial_t\chi - im\chi)$

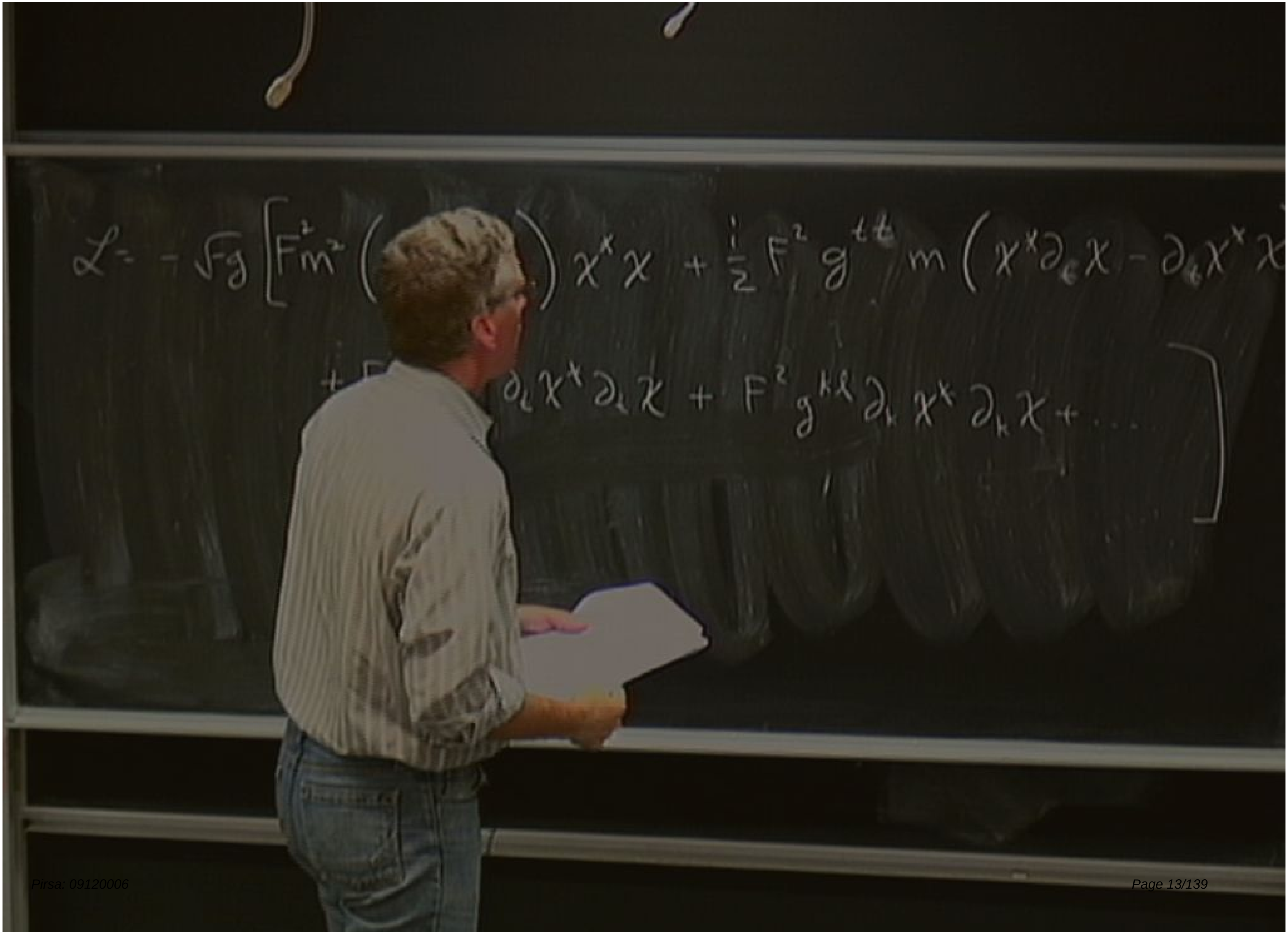
$$\mathcal{L} = -\sqrt{-g}\left[F^2 m^2 \left(1 + g^{tt}\right)\chi^*\chi + \frac{1}{2}F^2\right.$$

$$\mathcal{L} = -\sqrt{-g}\left[F^2 m^2 \left(1+g^{tt}\right)\chi^* \chi + \frac{i}{2} F^2 g^{tk} m \left(\chi^* \partial_k \chi - \partial_k \chi^* \chi\right)\right.$$

$$\left. + F^2 g^{tt} \partial_t \chi^* \partial_t \chi + F^2 g^{kl} \partial_k \chi^* \partial_l \chi + \ldots \right]$$

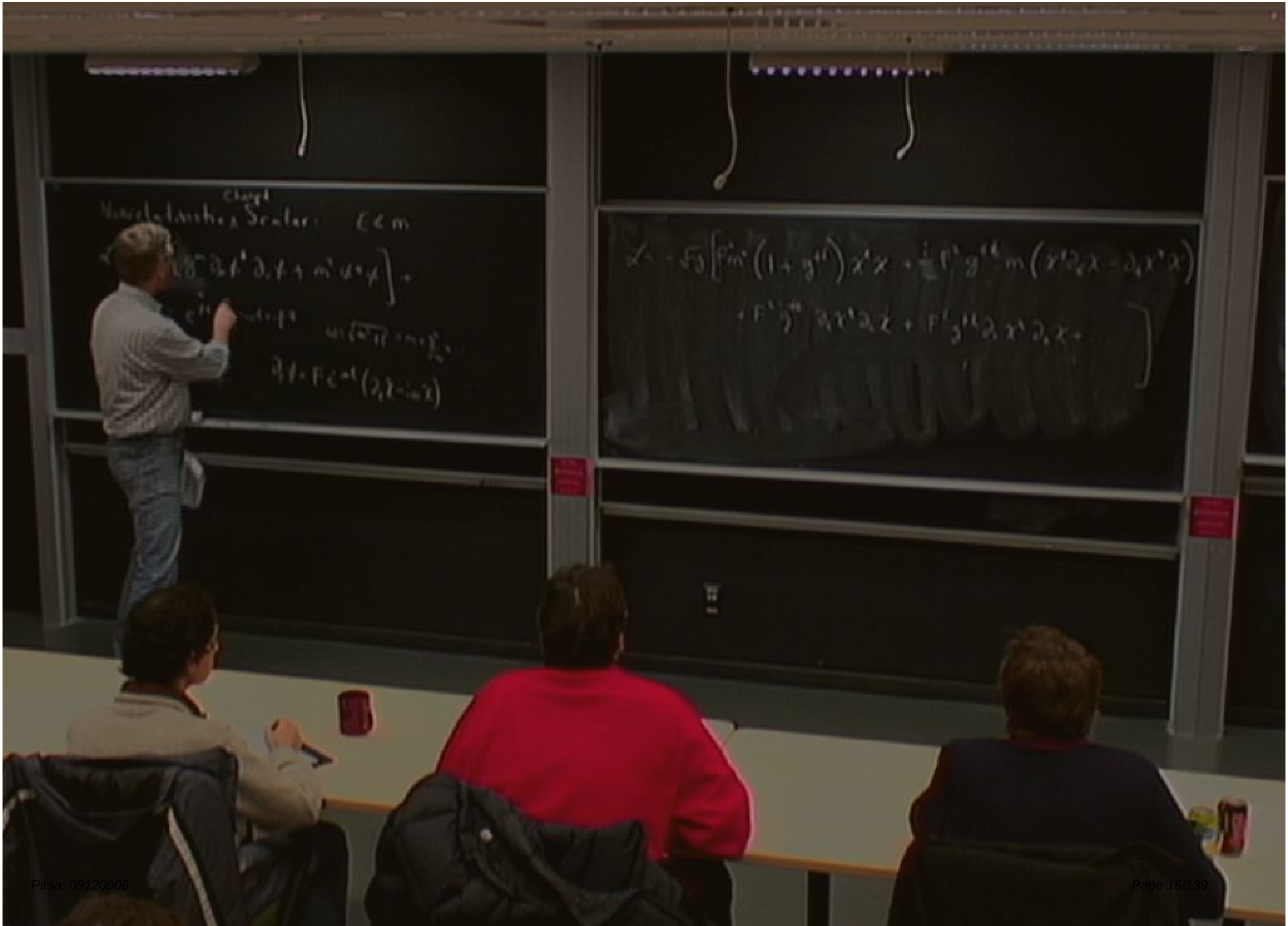
$$\mathcal{L} = -\sqrt{g}\Big[F^2 m^2 (1 + g^{tt})\chi^* \chi + \frac{i}{2} F^2 g^{tk} m (\chi^* \partial_k \chi - \partial_k \chi^* \chi$$

$$+ F^2 g^{tt} \partial_t \chi^* \partial_t \chi + F^2 g^{kl} \partial_k \chi^* \partial_l \chi + \dots \Big]$$



$$\mathcal{L} = -\sqrt{-g}\Big[F^2 m^2 \left(1 + g^{tt}\right)\chi^*\chi + \frac{i}{2}F^2 g^{tt}\, m \left(\chi^*\partial_t\chi - \partial_t\chi^*\chi\right.$$

$$+ F^2 g^{tt}\,\partial_t\chi^*\partial_t\chi + F^2 g^{kl}\,\partial_k\chi^*\partial_l\chi + \ldots\Big]$$



$$\mathcal{L} = -\sqrt{g}\Big[F^2 m^2 \left(1 + g^{tt}\right)\chi^* \chi + i F^2 g^{tt} m \left(\chi^* \partial_t \chi - \partial_t \chi^* \chi\right)$$

$$+ F^2 g^{tt} \partial_t \chi^* \partial_t \chi + F^2 g^{kl} \partial_k \chi^* \partial_l \chi + \dots \Big]$$

Nonrelativistic Charged Scalar: $E \ll m$

$$\mathcal{L} = -\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + m^2\phi^*\phi\right] + \ldots \qquad x^\mu = t, x^i$$

$$\phi \sim e^{ipx} \sim e^{-i\omega t + i\vec{p}\cdot\vec{x}}$$

$$\omega \simeq \sqrt{m^2 + p^2} \simeq m + \frac{p^2}{2m} + \ldots$$

$$\phi = F e^{-imt}\chi$$

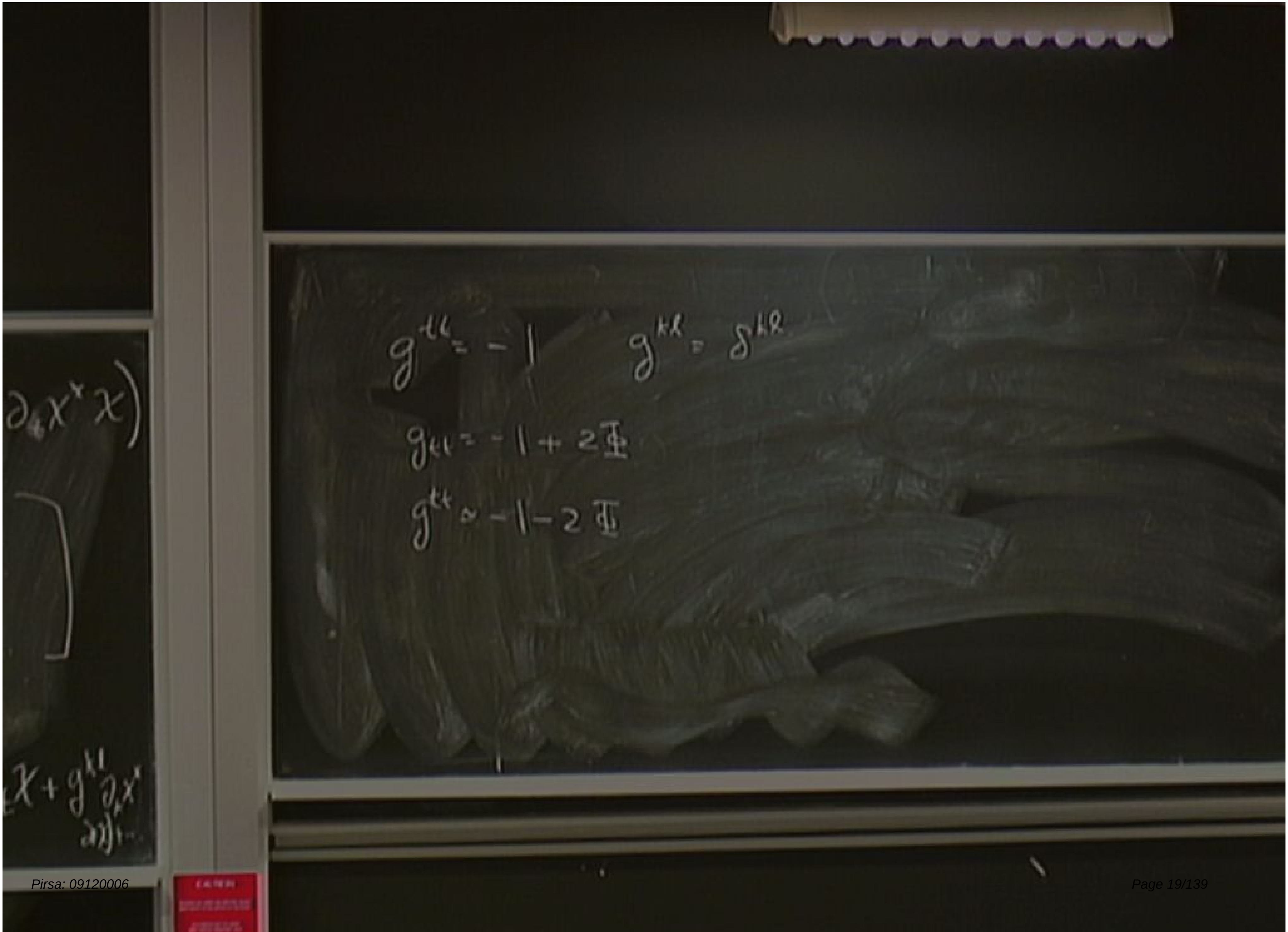
$$\partial_t\phi = F e^{-imt}\left(\partial_t\chi - im\chi\right)$$

$$\mathcal{L} = -\sqrt{-g}\Big[F^2 m^2 (1+g^{tt})\,\chi^* \chi + i F^2 g^{tt} m \left(\chi^* \partial_t \chi - \partial_t \chi^* \chi\right)$$

$$+ F^2 g^{tt} \partial_t \chi^* \partial_t \chi + F^2 g^{kl} \partial_k \chi^* \partial_l \chi + \dots \Big]$$

Choose: $F^2 = \frac{1}{2m}$ so

$$\mathcal{L} = -\sqrt{-g}\Big[\frac{i}{2} g^{tt}\left(\chi^* \partial_t \chi - \partial_t \chi^* \chi\right) + \frac{m}{2}(1+g^{tt})\,\chi^* \chi + \frac{1}{2m}\Big[g^{tt}\partial_t \chi^* \partial_t \chi + g^{kl} \partial_k \chi^* \partial_l \chi\Big] \dots$$


$g^{tt} = -1 \quad g^{kl} = \delta^{kl}$
$g_{tt} = -1 + 2\Phi$
$g^{tt} \simeq -1 - 2\Phi$
$\partial_t x^x \chi)$
$\chi + g^{kl}\partial_k x^x$

$$g^{tt} = -1 \qquad g^{kl} = \delta^{kl}$$

$$g_{tt} = -1 + 2\Phi$$

$$g^{tt} \simeq -1 - 2\Phi$$

$$\mathcal{L} = -\sqrt{-g}\Big[F^2 m^2 \left(1 + g^{tt}\right) \chi^* \chi + i F^2 g^{tt} m \left(\chi^* \partial_t \chi - \partial_t \chi^* \chi\right)$$

$$+ F^2 g^{tt} \partial_t \chi^* \partial_t \chi + F^2 g^{kl} \partial_k \chi^* \partial_l \chi + \dots \Big]$$

choose: $F^2 = \frac{1}{2m}$

-2Φ

$$\mathcal{L} = -\sqrt{-g}\Big[\frac{i}{2} g^{tt}\left(\chi^* \partial_t \chi - \partial_t \chi^* \chi\right) + \frac{m}{2}\left(1 + g^{tt}\right) \chi^* \chi + \frac{1}{2m}\Big[g^{tt} \partial_t \chi^* \partial_t \chi + g^{kl} \partial_k \chi^* \partial_l \chi + \dots$$

$$g^{tt} = -1 \qquad g^{kl} = \delta^{kl}$$

$$g_{tt} = -1 + 2\Phi$$

$$g^{tt} \simeq -1 - 2\Phi$$

$$\mathcal{L} = -\sqrt{-g}\left[\frac{i}{2}\left(\chi^{*}\partial_t \chi - \partial_t \chi^{*} \chi\right) + \frac{1}{2m}\nabla\chi^{*}\cdot\nabla\chi + \ldots\right]$$

$$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{p^2}{2m} \qquad \chi \sim e^{-iEt + i\vec{p}\cdot\vec{x}}$$

Nonrelativistic Charged Scalar: $E \ll m$

$$\mathcal{L} = -\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + m^2\phi^*\phi \right] + \dots \qquad x^\mu = t, x^i$$

$$\phi \sim e^{ipx} \sim e^{-i\omega t + i\vec{p}\cdot\vec{x}} \qquad E \sim p$$

$$\omega \simeq \sqrt{m^2 + p^2} \simeq m + \frac{p^2}{2m} + \dots$$

$$\underline{\phi = F e^{-imt}\chi}$$

$$\partial_t\phi = F e^{-imt}\left(\partial_t\chi - im\chi\right)$$

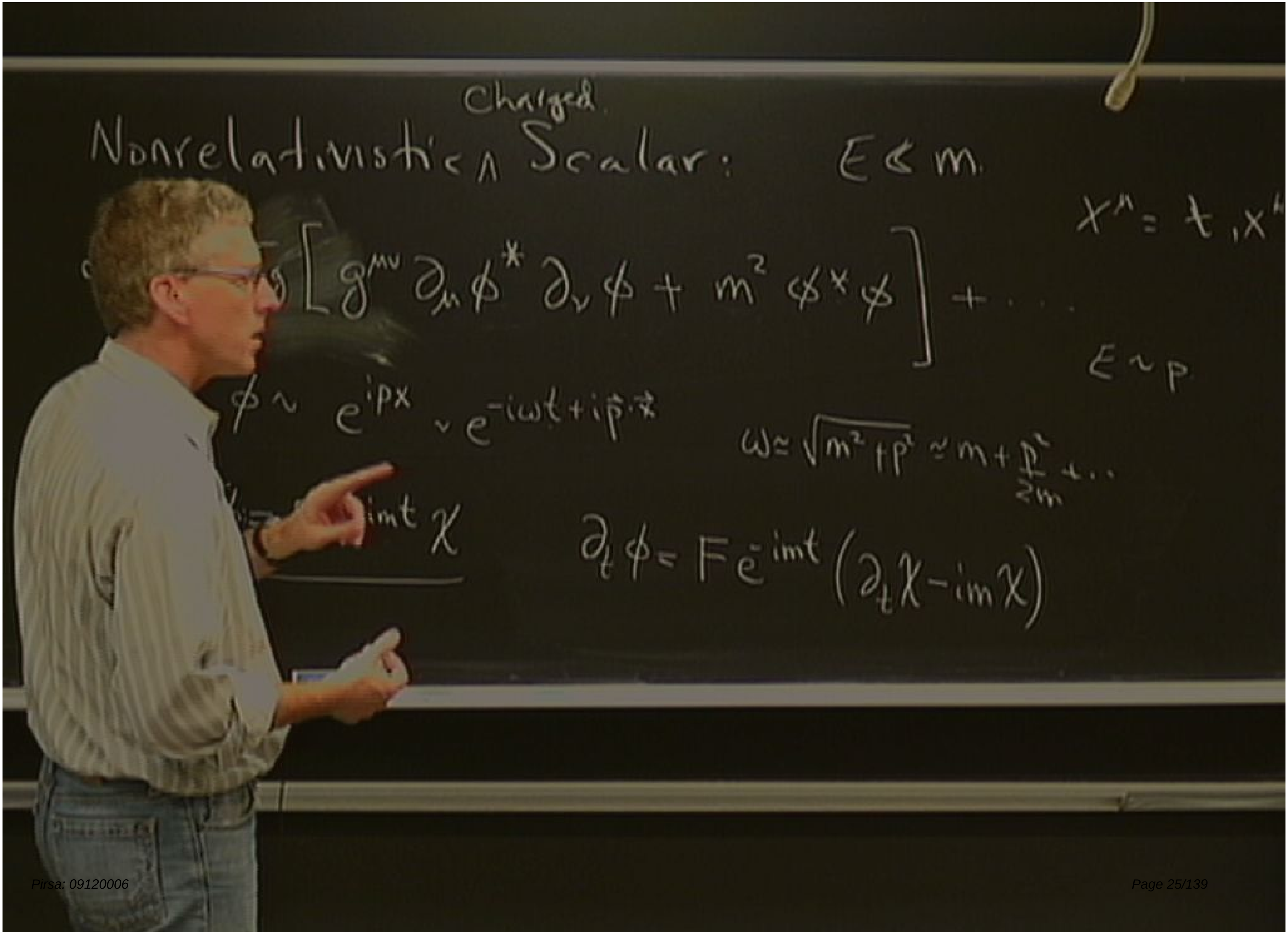
$$\mathcal{L} = -\sqrt{g}\Big[F^2 m^2 \left(1 + g^{tt}\right)\chi^\dagger \chi + i F^2 g^{tt} m \left(\chi^\dagger \partial_t \chi - \partial_t \chi^\dagger \chi\right)$$

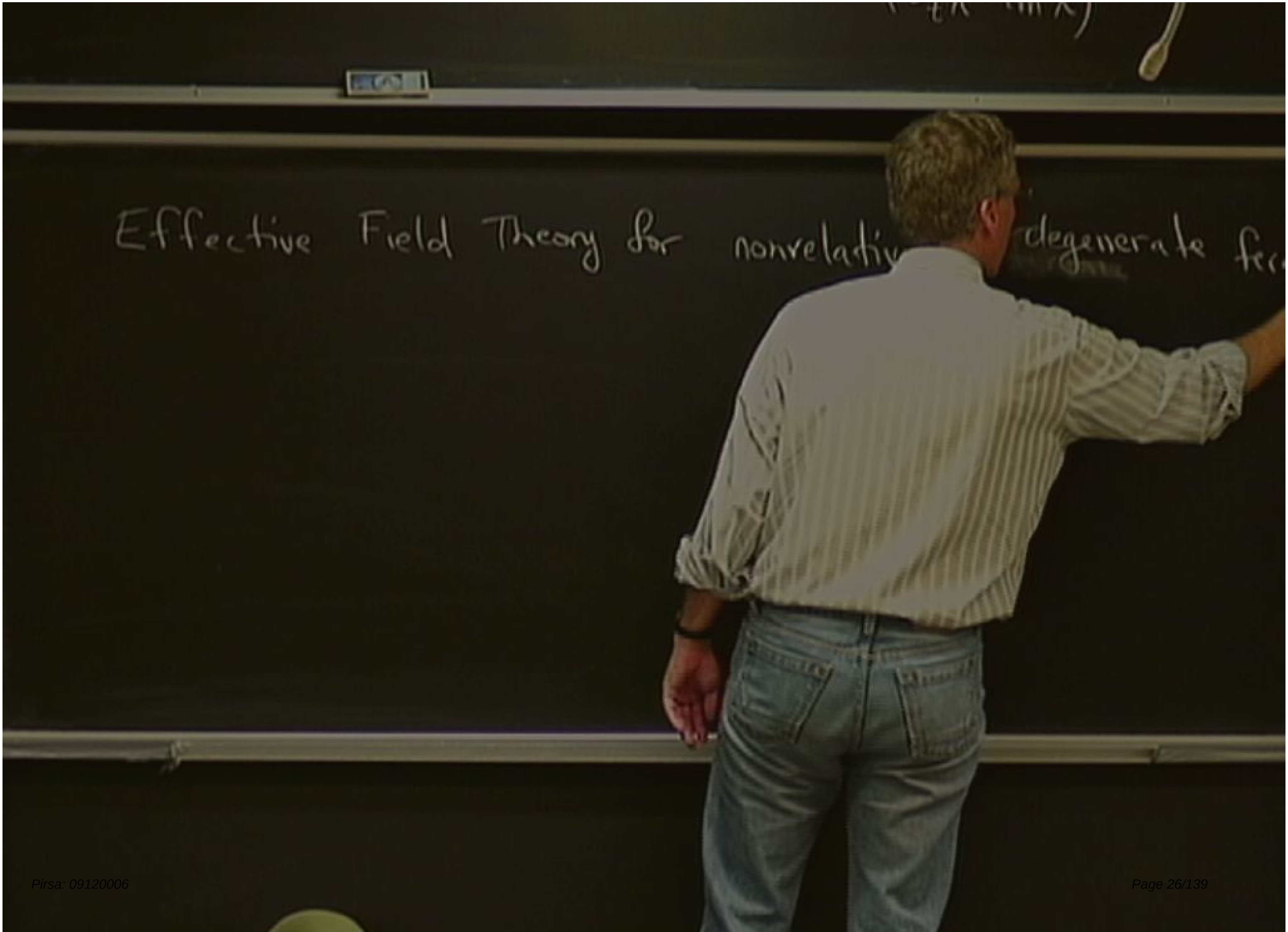
$$+ F^2 g^{tt} \partial_t \chi^\dagger \partial_t \chi + F^2 g^{k\ell} \partial_k \chi^\dagger \partial_\ell \chi + \dots \Big]$$

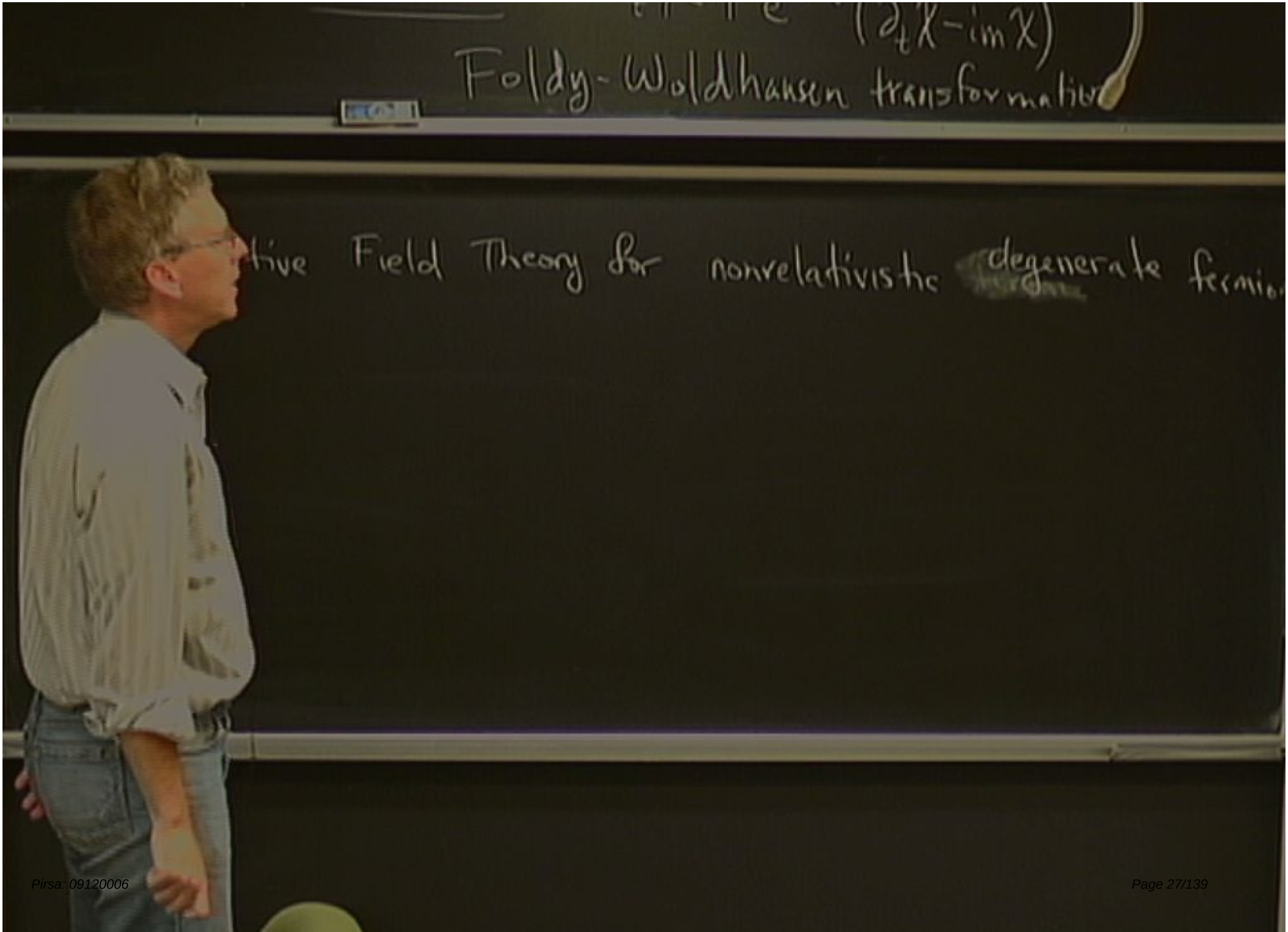
choose: $F^2 = \frac{1}{2m}$

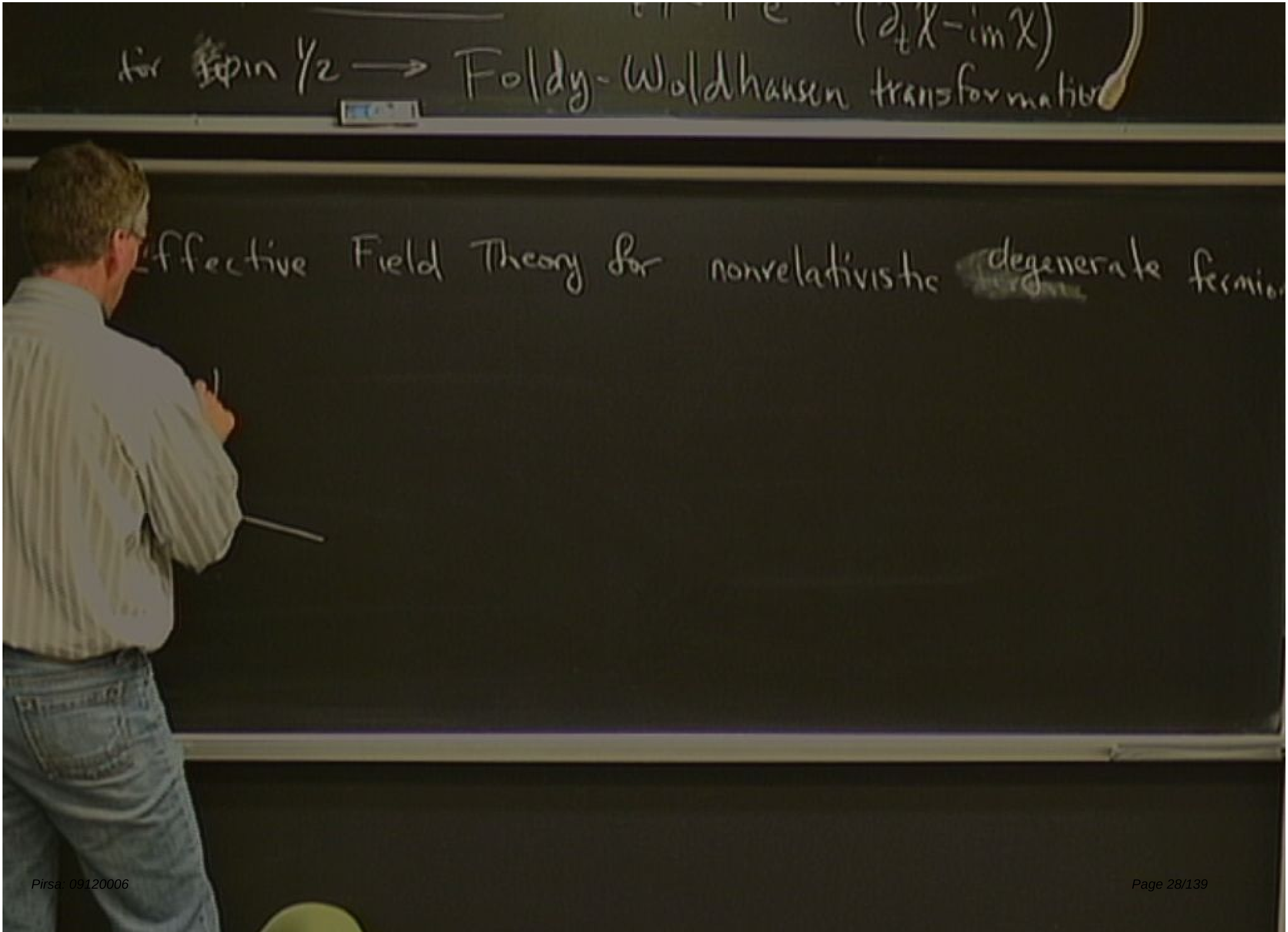
$-2i$

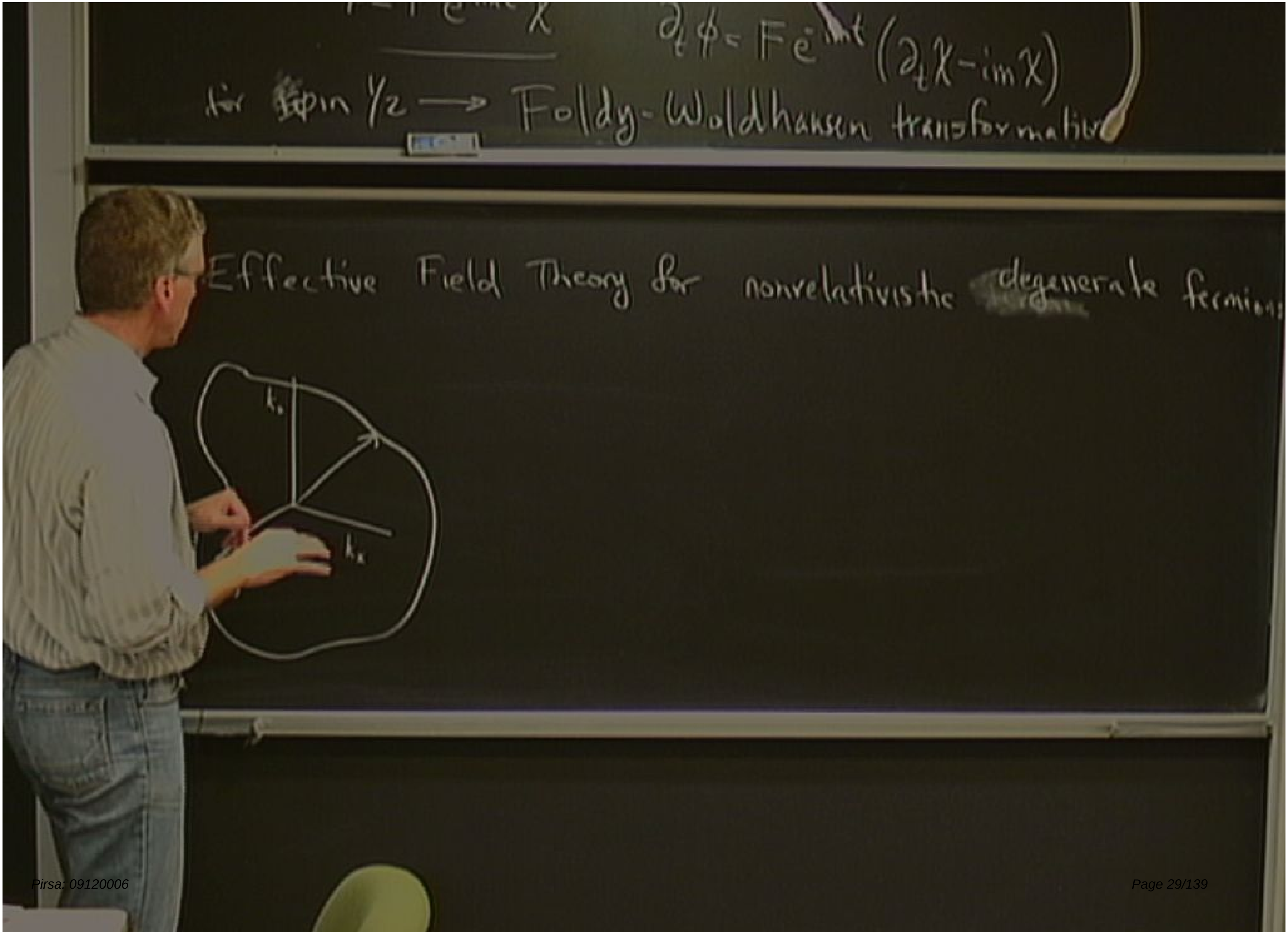
$$\mathcal{L} = -\sqrt{g}\Big[\frac{i}{2} g^{tt}\left(\chi^\dagger \partial_t \chi - \partial_t \chi^\dagger \chi\right) + \frac{m}{2}\left(1 + g^{tt}\right)\chi^\dagger \chi + \frac{1}{2m}\Big[g^{tt} \underline{\partial_t \chi^\dagger \partial_t \chi} + g^{k\ell} \partial_k \chi^\dagger \partial_\ell \chi\Big] \dots$$


Charged
Nonrelativistic Scalar: $E \ll m$
$x^\mu = t, x^i$
$[g^{\mu\nu}\partial_\mu \phi^* \partial_\nu \phi + m^2 \phi^* \phi] + \ldots$
$E \sim p$
$\phi \sim e^{ipx} \sim e^{-i\omega t + i\vec{p}\cdot\vec{x}}$
$\omega \simeq \sqrt{m^2 + p^2} \simeq m + \frac{p^2}{2m} + \ldots$
$e^{-imt}\chi$
$\partial_t \phi = F e^{-imt}(\partial_t \chi - im\chi)$


Effective Field Theory for nonrelativ
degenerate fer


$(\partial_t \chi - im\chi)$
Foldy-Woldhausen transformation
tive Field Theory for nonrelativisthc degenerate fermio


$(\partial_t \chi - im\chi)$
for spin 1/2 ⟶ Foldy-Woldhausen transformation
Effective Field Theory for nonrelativistic degenerate fermions


∂_t φ = F e^{-imt} (∂_t χ − im χ)
for spin 1/2 ⟶ Foldy-Woldhausen transformation
Effective Field Theory for nonrelativistic degenerate fermions
k_z
k_x

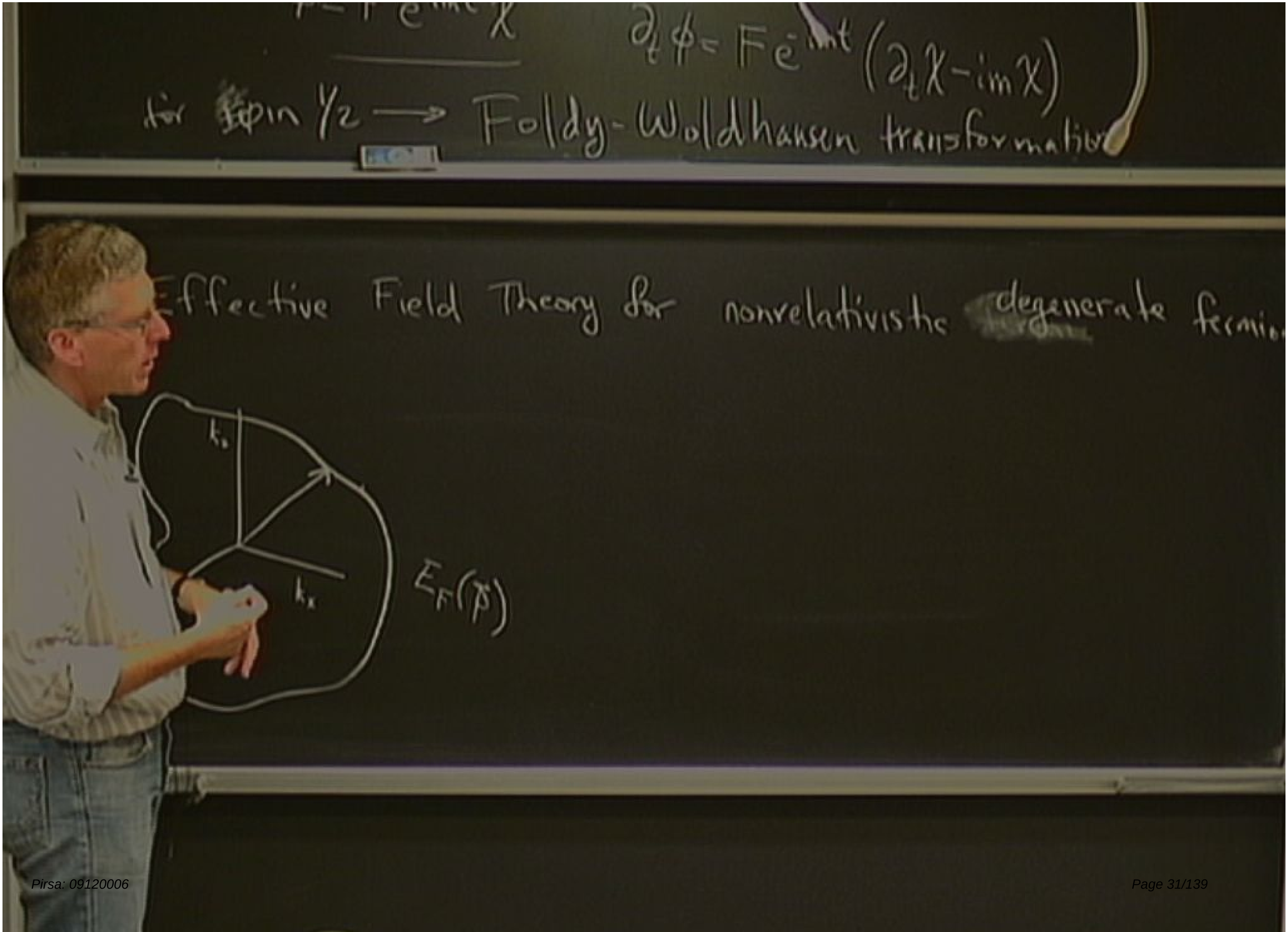
$$g^{tt} = -1 \qquad g^{k\ell} = \delta^{k\ell}$$

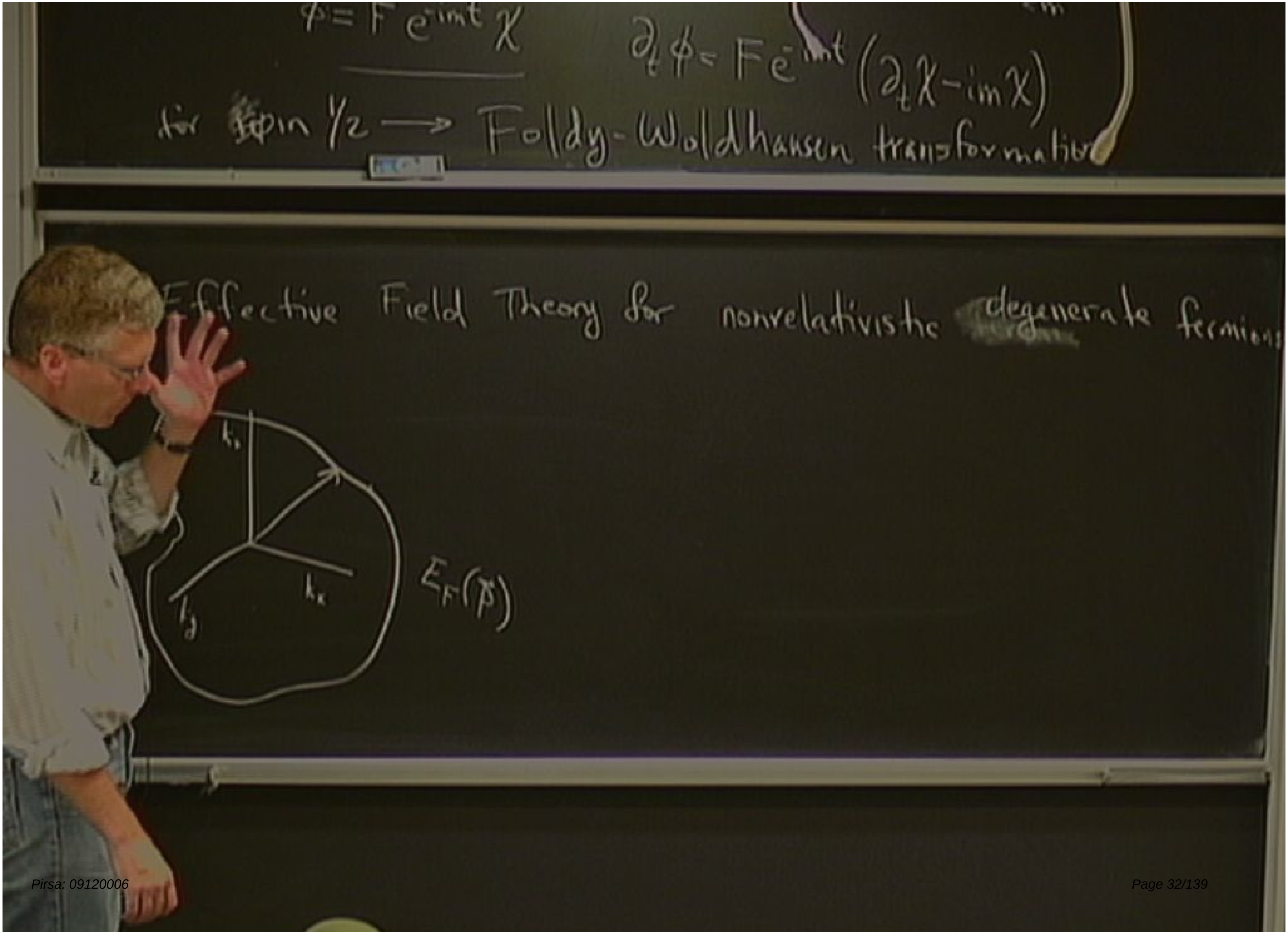
$$g_{tt} = -1 + 2\Phi$$

$$g^{tt} \simeq -1 - 2\Phi$$

$$\mathcal{L} = -\sqrt{-g}\left[\frac{i}{2}\left(\chi^* \partial_t \chi - \partial_t \chi^* \chi\right) + \frac{1}{2m} \nabla \chi^* \cdot \nabla \chi + \ldots\right]$$

$$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \longrightarrow E = \frac{p^2}{2m} \qquad \chi \sim e^{-iEt + i\vec{p}\cdot\vec{x}}$$


∂t φ = Fe^-imt (∂t χ − im χ)
for spin 1/2 → Foldy-Woldhausen transformatio
Effective Field Theory for nonrelativistic degenerate fermi
kz
kx
EF(p)

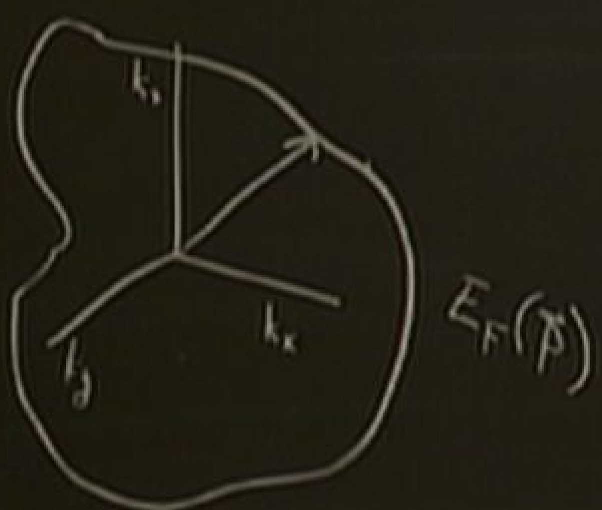


$\phi = F e^{-imt} \chi$
$\partial_t \phi = F e^{-imt}(\partial_t \chi - im\chi)$
for spin 1/2 ⟶ Foldy-Woldhausen transformation
Effective Field Theory for nonrelativistic degenerate fermions
k_z
k_x
k_y
$E_F(\vec{p})$

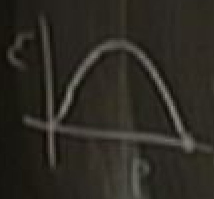
$$\phi = F e^{-imt}\chi \qquad \partial_t \phi = F e^{-imt}(\partial_t \chi - im\chi)$$

for spin 1/2 $\longrightarrow$ Foldy-Woldhausen transformation

Effective Field Theory for nonrelativistic degenerate fermions



f small E does not also mean small $|\vec{p}|$, because the particles involved have a dispersion relation $E(p)$ which $\to 0$ for large p as well as small p, If small E does not also mean small $|\vec{p}|$, because the particles involved have a dispersion relation $E(p)$ which $\to 0$ for large p as well as small p,



eg $E \sim \sin(pa)$

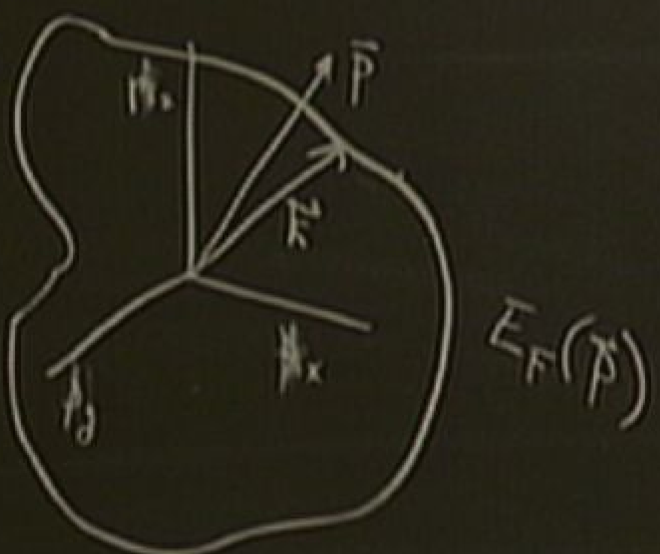
If small E does not also mean small $|\vec{p}|$, because the particles involved have a dispersion relation $E(p)$ which $\to 0$ for large p as well as small p,

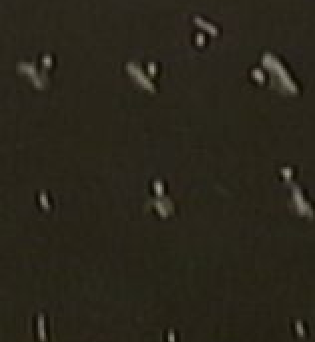
eg $E \propto \sin(pa)$

$\phi = F e^{-imt} \chi$ $\qquad \partial_t \phi = F e^{-imt}(\partial_t \chi - im\chi)$

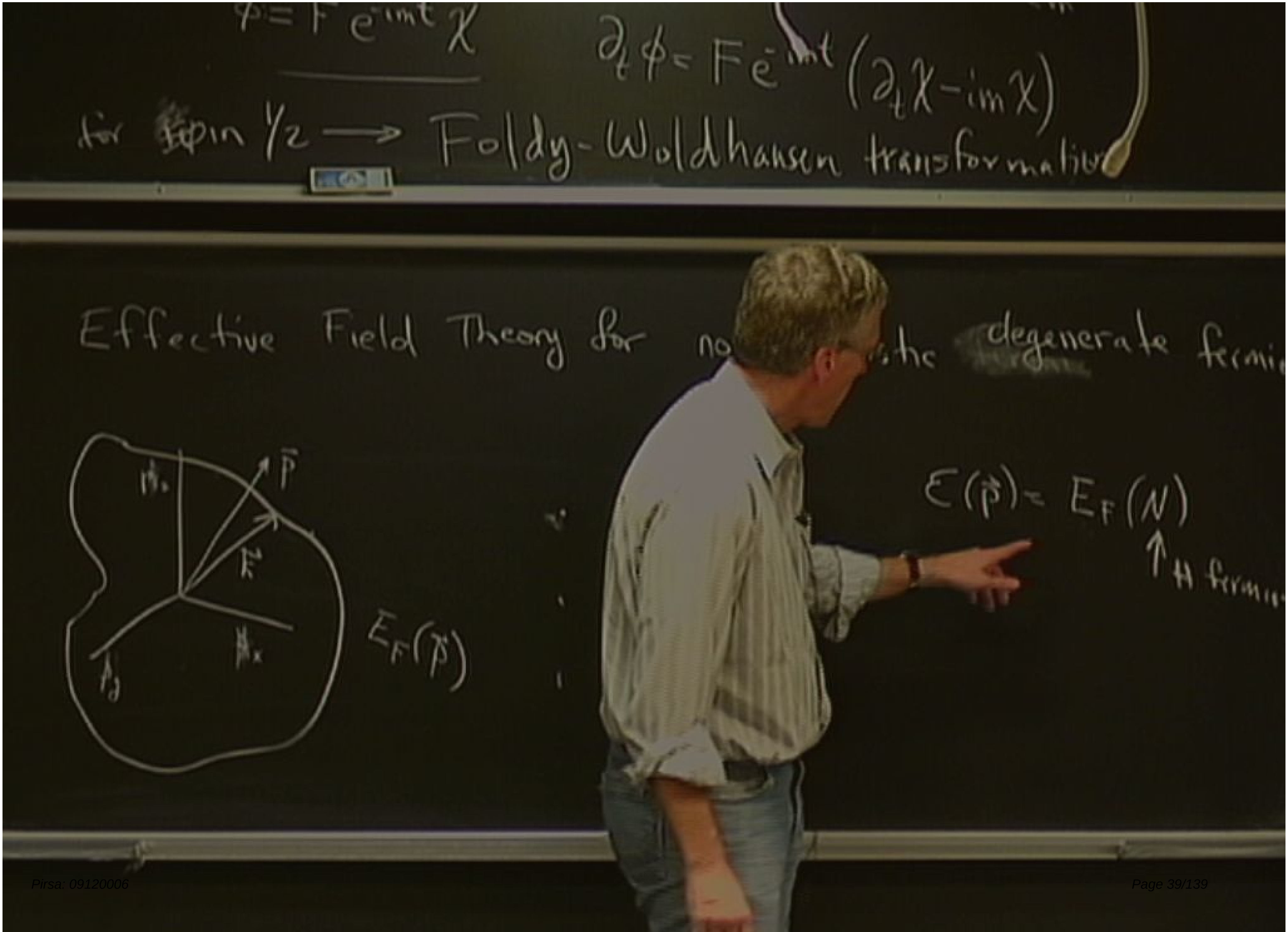
for spin 1/2 $\longrightarrow$ Foldy-Woldhausen transformatio

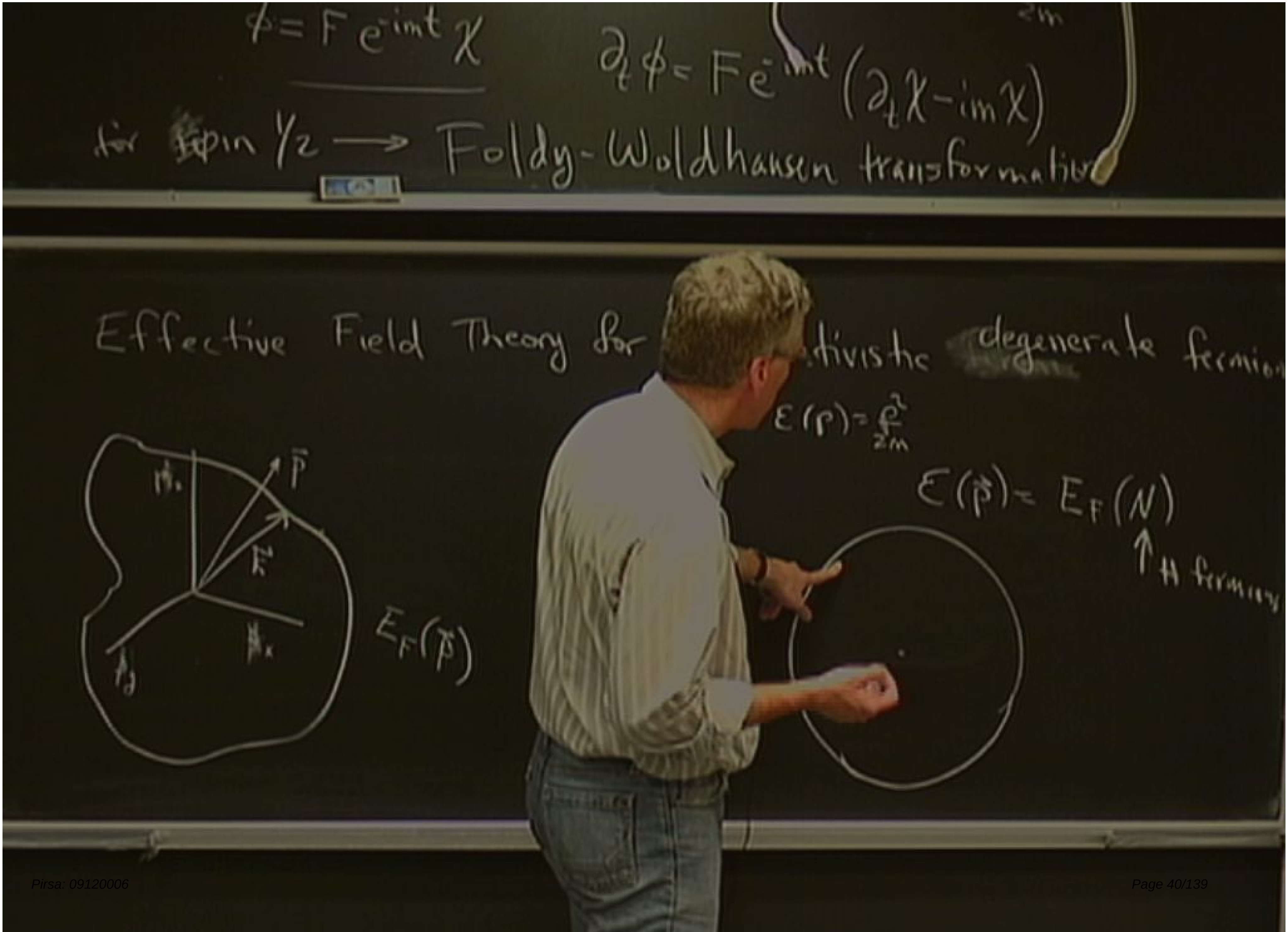
Effective Field Theory for nonrelativistic degenerate fermi

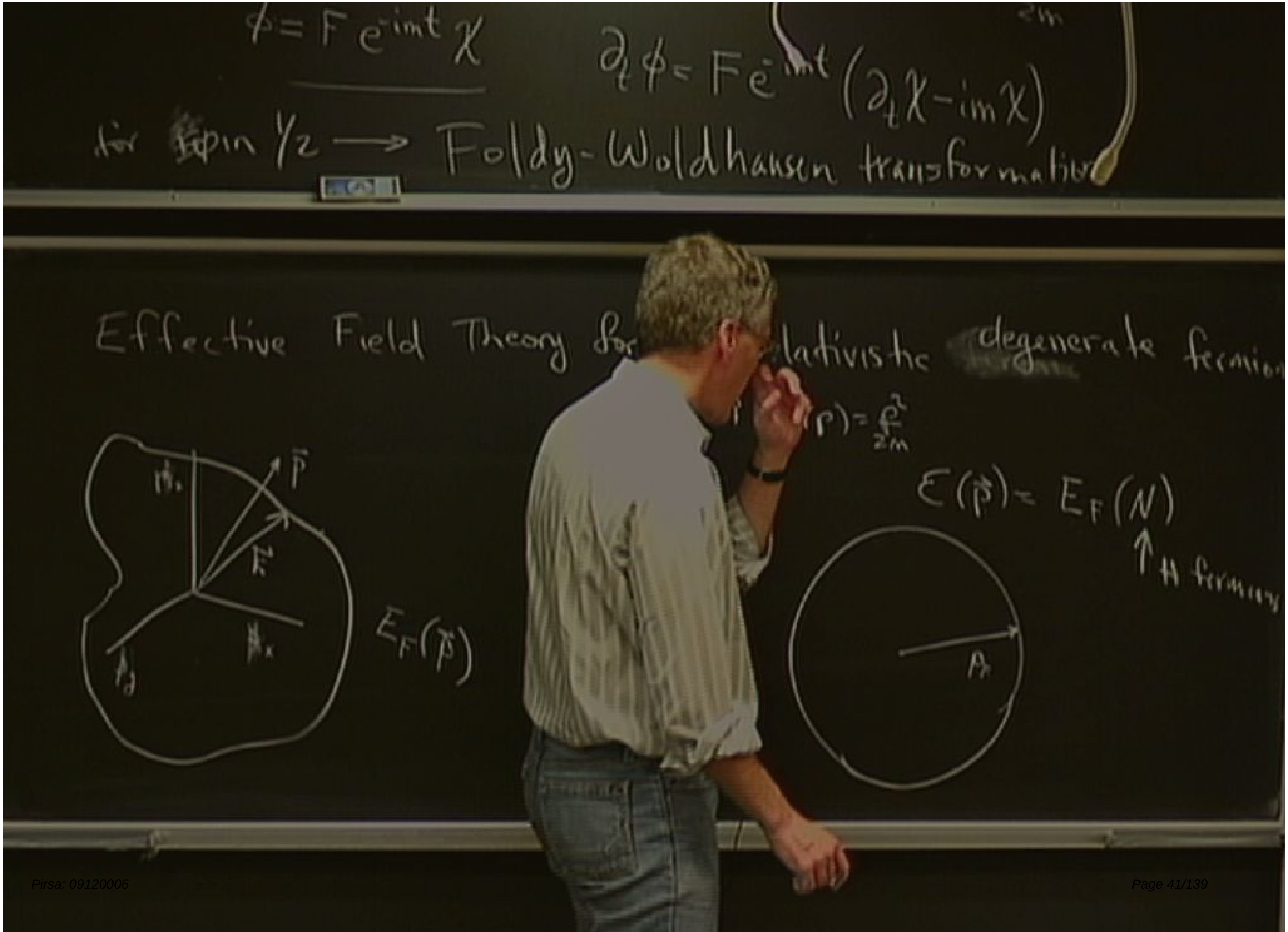




Take zero of energy to be the Fermi surface defined by $\vec{p} = \vec{k}$ where $\varepsilon_1(\vec{k}) = 0$.


$\partial_t \phi = F e^{-imt} (\partial_t \chi - im \chi)$
for spin 1/2 → Foldy-Woldhausen transformation
Effective Field Theory for
degenerate
$E(\vec{p}) = E_F(N)$
$\vec{p}$
$\vec{k}$
$E_F(\vec{p})$

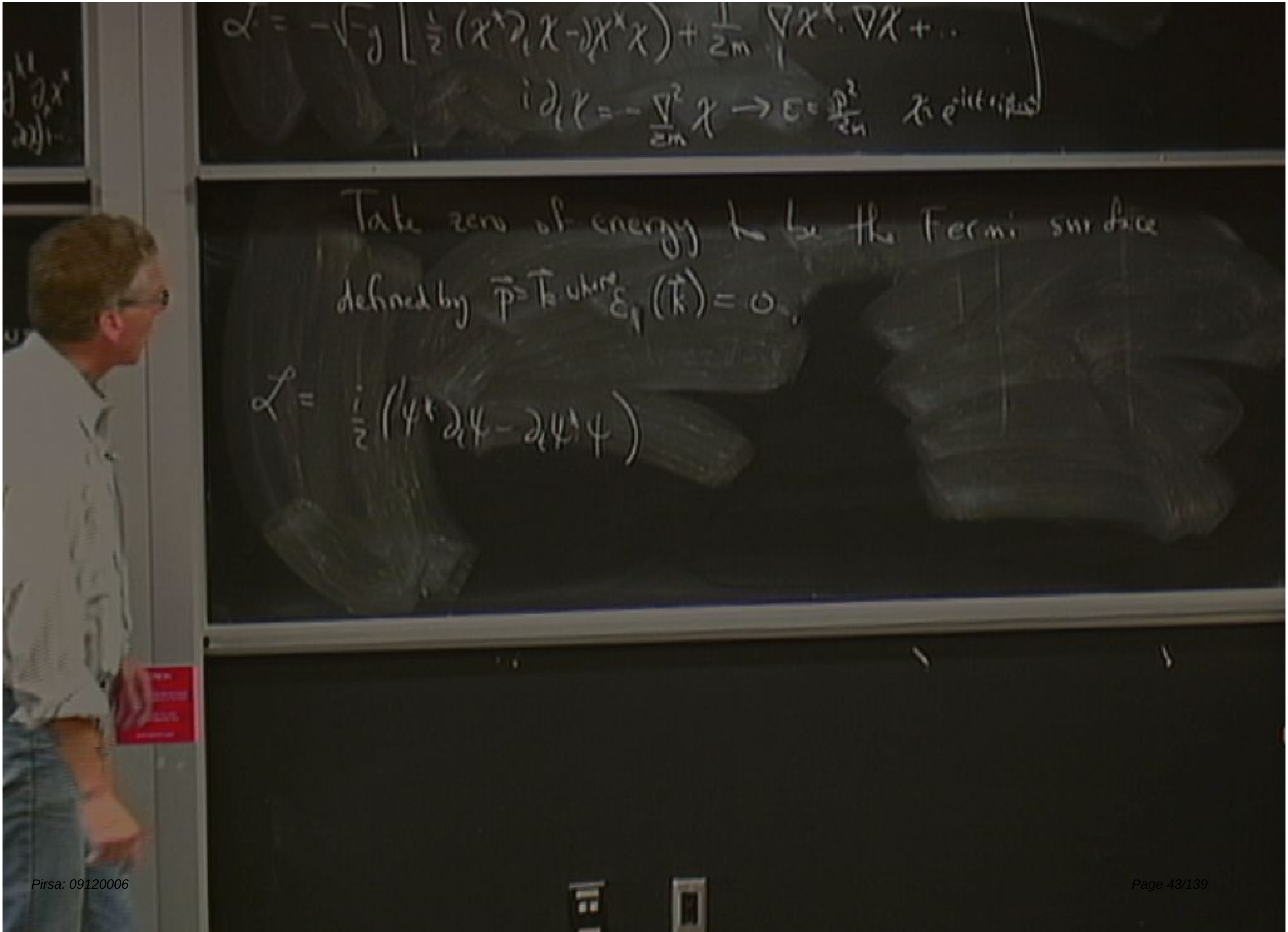

φ = F e^{-imt} χ
∂_t φ = F e^{-imt} (∂_t χ − im χ)
for spin 1/2 → Foldy-Woldhausen transformation
Effective Field Theory for ... tivistic degenerate fermions
E(p) = p²/2m
E(p) = E_F(N)
# fermions
E_F(p)



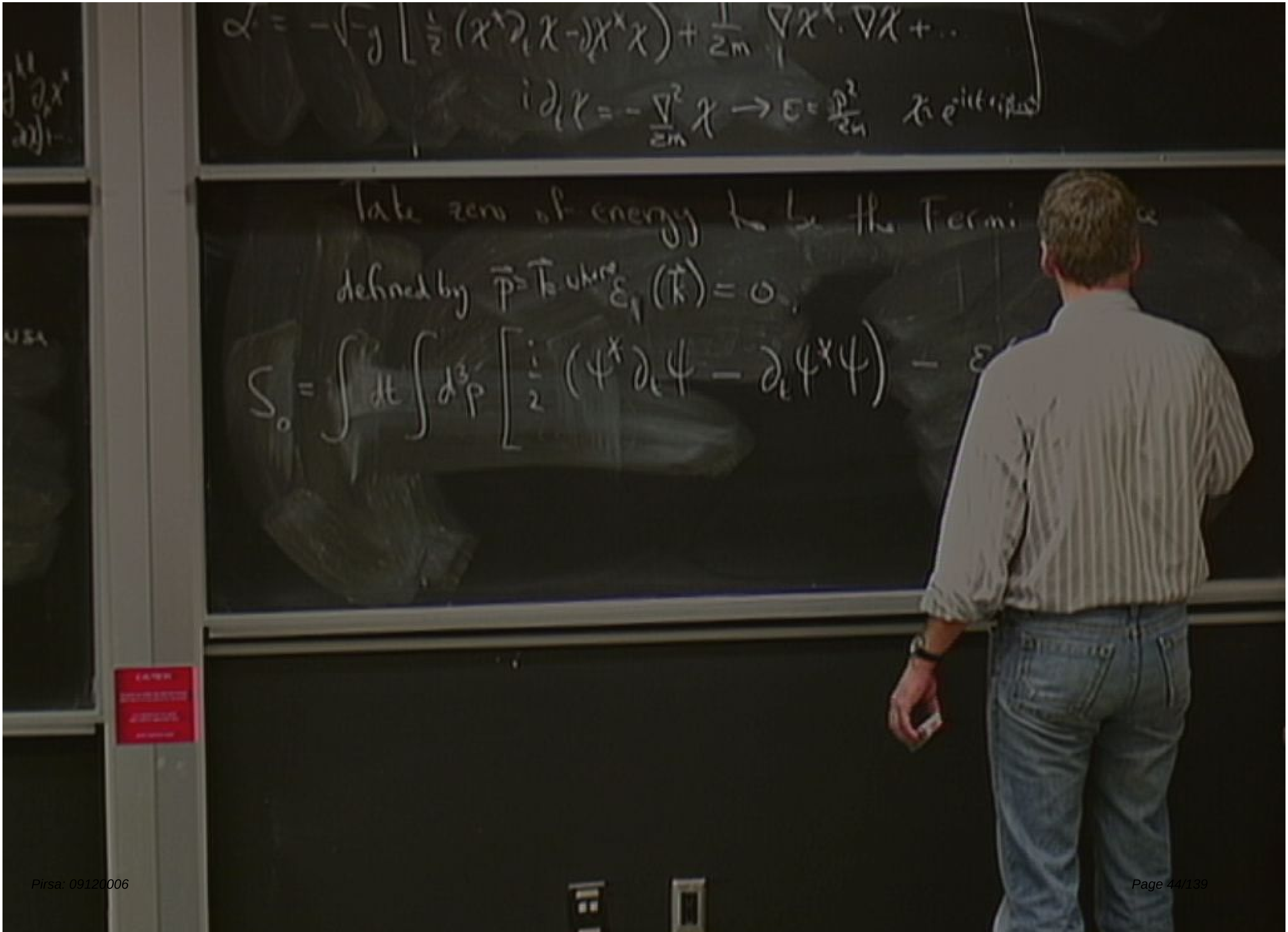
φ = F e^{-imt} χ
∂_t φ = F e^{-imt} (∂_t χ − im χ)
for spin 1/2 → Foldy-Woldhausen transformation
Effective Field Theory for [non-re]lativistic degenerate fermions
E(p) = p²/2m
ε(p) = E_F(N)
# fermions
E_F(p)
p
p_F

$$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow \epsilon = \frac{p^2}{2m} \quad \chi \sim e^{-i\epsilon t + i\vec{p}\cdot\vec{x}}$$

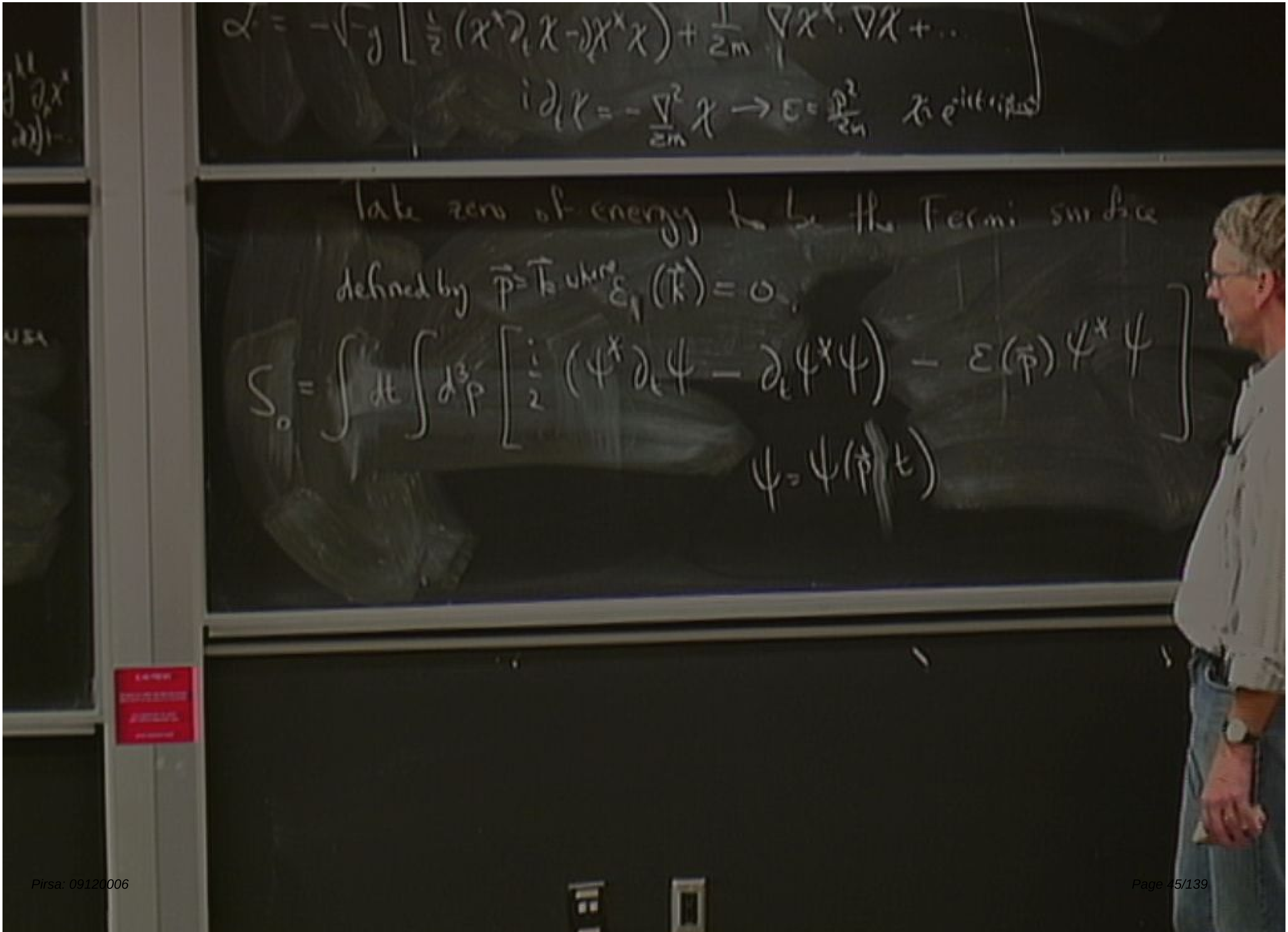
Take zero of energy to be the Fermi surface defined by $\vec{p} = \vec{k}$ where $\epsilon_F(\vec{k}) = 0$.



$\mathcal{L} = -\sqrt{-g}\left[\frac{i}{2}(\chi^\dagger\partial_t\chi - \partial_t\chi^\dagger\chi) + \frac{1}{2m}\nabla\chi^\dagger\cdot\nabla\chi + \dots\right.$
$i\partial_t\chi = -\frac{\nabla^2}{2m}\chi \to E = \frac{p^2}{2m}$ $\chi \sim e^{iEt + i\vec{p}\cdot\vec{x}}$
Take zero of energy to be the Fermi surface
defined by $\vec{p} = \vec{k}$ where $\epsilon_1(\vec{k}) = 0$.
$\mathcal{L} = \frac{i}{2}(\psi^\dagger\partial_t\psi - \partial_t\psi^\dagger\psi)$



$\mathcal{L} = -\sqrt{-g}\left[\frac{i}{2}(\chi^\dagger\partial_t\chi - \partial_t\chi^\dagger\chi) + \frac{1}{2m}\nabla\chi^\dagger\cdot\nabla\chi + \dots\right.$
$i\partial_t\chi = -\frac{\nabla^2}{2m}\chi \to E = \frac{p^2}{2m}$
Take zero of energy to be the Fermi
defined by $\vec{p} = \vec{k}$ where $\varepsilon_1(\vec{k}) = 0$.
$S_0 = \int dt \int d^3p \left[\frac{i}{2}(\psi^\dagger\partial_t\psi - \partial_t\psi^\dagger\psi) - \varepsilon\right.$


$\mathcal{L} = -\sqrt{-g}\left[\frac{i}{2}(\chi^*\partial_t\chi - \partial_t\chi^*\chi) + \frac{1}{2m}\nabla\chi^*\cdot\nabla\chi + \dots\right.$
$i\partial_t\chi = -\frac{\nabla^2}{2m}\chi \rightarrow \varepsilon = \frac{p^2}{2m}$
take zero of energy to be the Fermi surface
defined by $\vec{p} = \vec{k}_F$ where $\varepsilon_1(\vec{k}) = 0$.
$S_0 = \int dt \int d^3p \left[\frac{i}{2}(\psi^*\partial_t\psi - \partial_t\psi^*\psi) - \varepsilon(\vec{p})\psi^*\psi\right]$
$\psi = \psi(\vec{p}, t)$

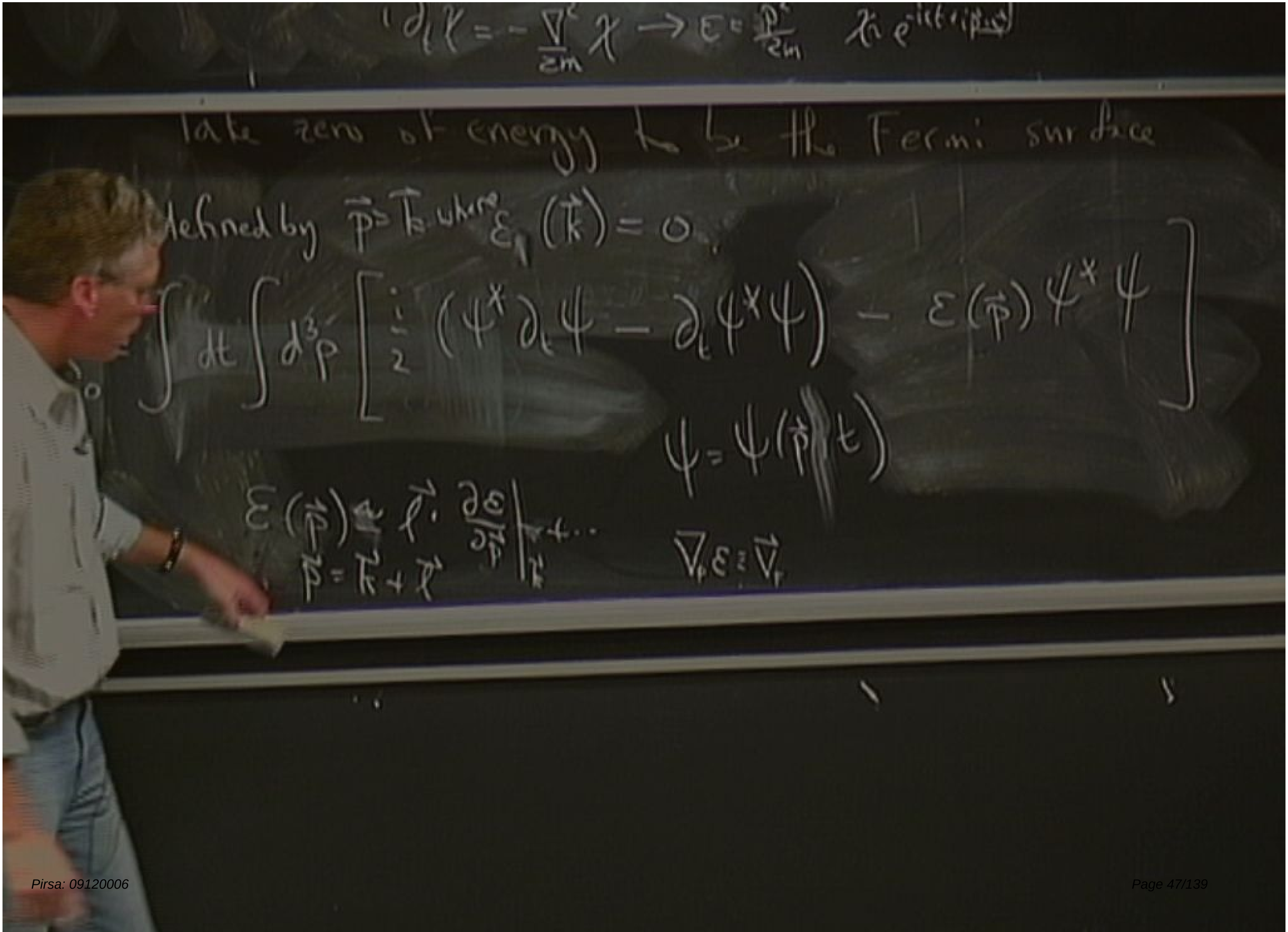
$$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{\vec{p}^2}{2m} \qquad \chi \sim e^{-i(Et - \vec{p}\cdot\vec{x})}$$

Take zero of energy to be the Fermi surface

...d by $\vec{p} = \vec{k}$ where $\varepsilon_F(\vec{k}) = 0$.

$$\int dt \int d^3p \left[\frac{i}{2}\left(\psi^* \partial_t \psi - \partial_t \psi^* \psi\right) - \varepsilon(\vec{p})\, \psi^* \psi \right]$$

$$\psi = \psi(\vec{p}, t)$$


$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{p^2}{2m}$ $\chi \sim e^{i(Et + i\vec{p}\cdot\vec{x})}$
Take zero of energy to be the Fermi surface
defined by $\vec{p} = \vec{k}$ where $\varepsilon(\vec{k}) = 0$
$\int dt \int d^3p \left[\frac{i}{2}\left(\psi^* \partial_t \psi - \partial_t \psi^* \psi\right) - \varepsilon(\vec{p})\psi^*\psi \right]$
$\psi = \psi(\vec{p}, t)$
$\varepsilon(\vec{p}) = \vec{\ell}\cdot \frac{\partial \varepsilon}{\partial \vec{p}}\Big|_{\vec{k}} + \dots$
$\vec{p} = \vec{k} + \vec{\ell}$
$\nabla_p \varepsilon = \vec{v}_F$

$$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{p^2}{2m} \qquad \chi \sim e^{-i(Et - \vec{p}\cdot\vec{x})}$$

Take zero of energy to be the Fermi surface

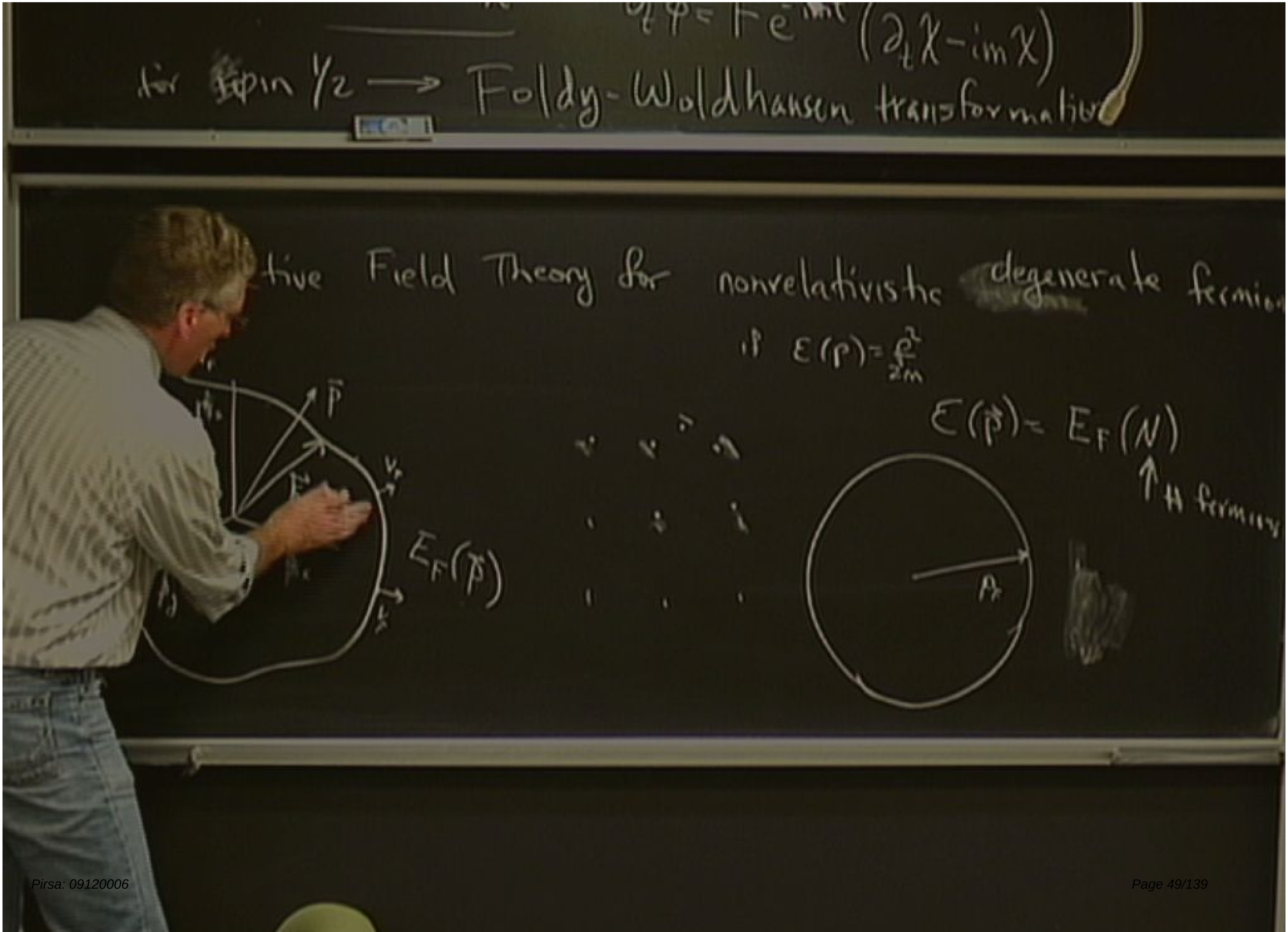
defined by $\vec{p} = \vec{k}$ where $\varepsilon_1(\vec{k}) = 0$

$$S_0 = \int dt \int d^3p \left[\frac{i}{2}\left(\psi^* \partial_t \psi - \partial_t \psi^* \psi\right) - \varepsilon(\vec{p})\, \psi^* \psi \right]$$

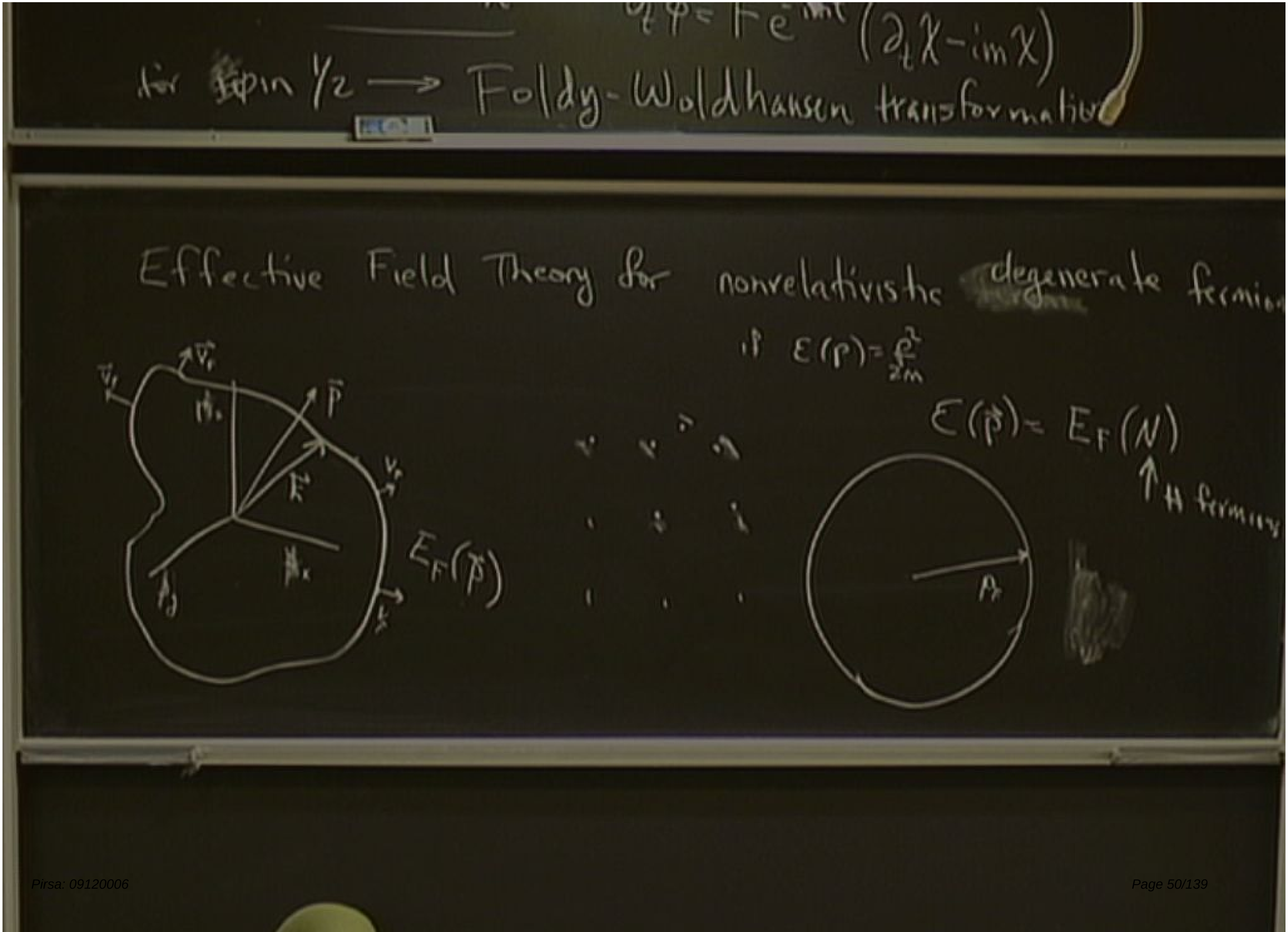
$$\psi = \psi(\vec{p}, t)$$

$$\varepsilon(\vec{p}) \approx \vec{\ell} \cdot \frac{\partial \varepsilon}{\partial \vec{p}}\bigg|_{\vec{k}} + \dots \qquad \vec{p} = \vec{k} + \vec{\ell}$$

$$\nabla_p \varepsilon = \vec{v}_F \qquad \varepsilon \approx \vec{\ell} \cdot \vec{v}_F(\vec{k})$$



∂t φ = F e^-imt (∂t χ - im χ)
for spin 1/2 → Foldy-Woldhausen transformation
tive Field Theory for nonrelativistic degenerate fermions
ε(p) = p²/2m
ε(p) = E_F(N)
# fermions
E_F(p)


$\partial_t \varphi = F e^{-imt} (\partial_t \chi - im\chi)$
for spin 1/2 → Foldy-Woldhausen transformation
Effective Field Theory for nonrelativistic degenerate fermions
$\varepsilon(p) = \frac{p^2}{2m}$
$\varepsilon(\vec{p}) = E_F(N)$
fermions
$E_F(\vec{p})$

$$\mathcal{L} = -\sqrt{-g}\left[\frac{i}{2}\left(\chi^* \partial_t \chi - \partial_t \chi^* \chi\right) + \frac{1}{2m}\nabla\chi^* \cdot \nabla\chi + \dots\right]$$

$$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \to \varepsilon = \frac{p^2}{2m} \qquad \chi \propto e^{-i\varepsilon t + i\vec p\cdot\vec x}$$

... zero of energy to be the Fermi surface

defined by $\vec p = \vec k$ where $\varepsilon(\vec k) = 0$.

$$S_0 = \int dt \int d^3p \left[\frac{i}{2}\left(\psi^*\partial_t\psi - \partial_t\psi^*\psi\right) - \varepsilon(\vec p)\,\psi^*\psi\right]$$

$$\psi = \psi(\vec p, t)$$

$$\varepsilon(\vec p) \simeq \vec\ell \cdot \left.\frac{\partial\varepsilon}{\partial\vec p}\right|_{\vec k} + \dots \qquad \vec p = \vec k + \vec\ell$$

$$\nabla_p \varepsilon = \vec v_F \qquad \varepsilon \simeq \vec\ell\cdot\vec v_F(\vec k) \qquad \vec\ell = \{\vec\ell_\parallel, \ell\}$$

$$\mathcal{L} = -\sqrt{-g}\left[\frac{i}{2}\left(\chi^* \partial_t \chi - \partial_t \chi^* \chi\right) + \frac{1}{2m}\nabla\chi^* \cdot \nabla\chi + \dots\right]$$

$$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \to \varepsilon = \frac{p^2}{2m} \qquad \chi \sim e^{-i\varepsilon t + i\vec p\cdot\vec x}$$

... zero of energy to be the Fermi surface

defined by $\vec p = \vec k$ where $\varepsilon_F(\vec k) = 0$.

$$S_0 = \int dt \int d^3p \left[\frac{i}{2}\left(\psi^* \partial_t \psi - \partial_t \psi^* \psi\right) - \varepsilon(\vec p)\,\psi^* \psi\right]$$

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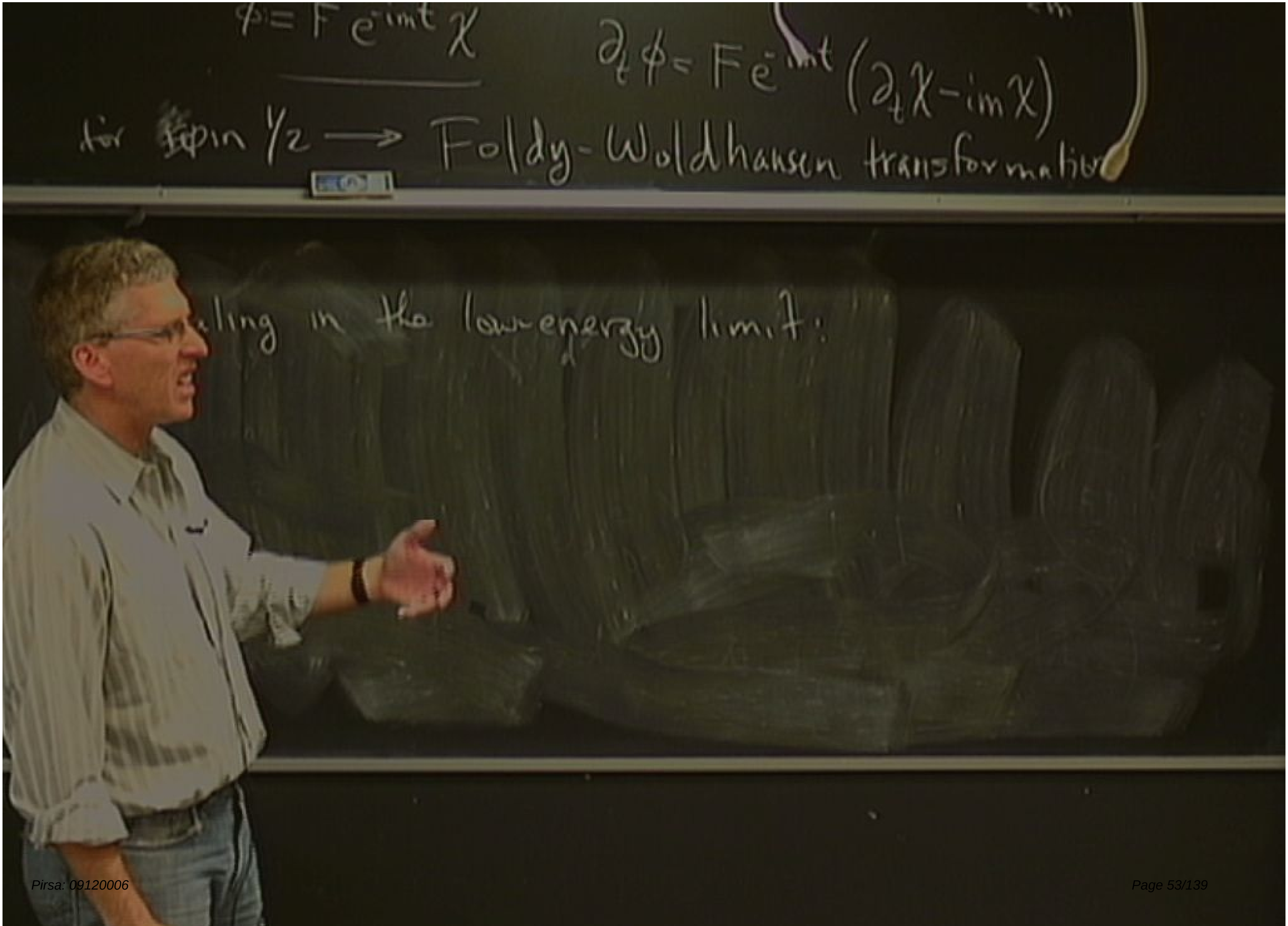
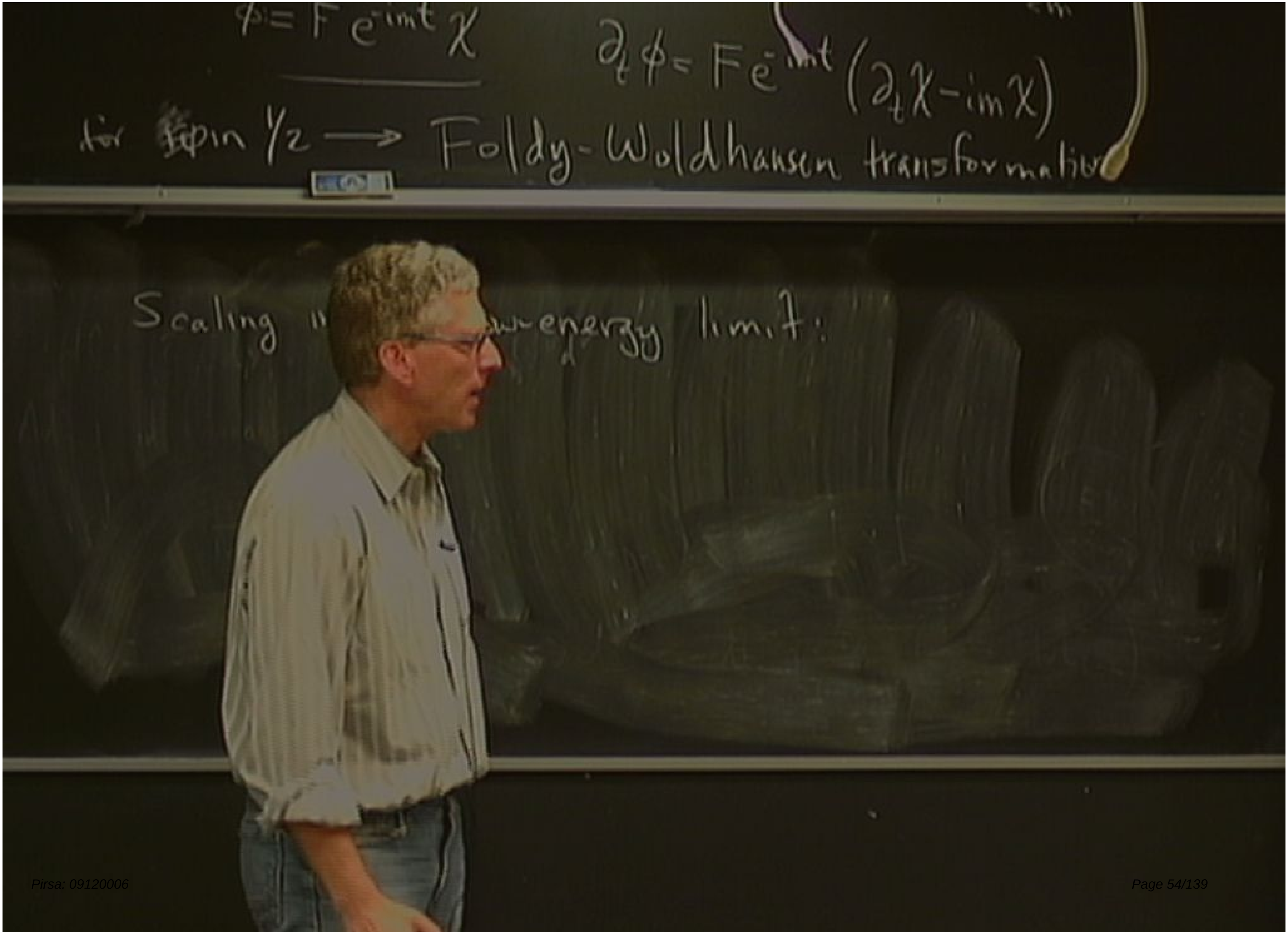
$$\nabla_p \varepsilon = \vec v_F \qquad \varepsilon \approx \vec\ell \cdot \vec v_F(\vec k) \qquad \vec\ell = \{\vec\ell_\parallel, \ell\}$$


$\phi = F e^{-imt} \chi$
$\partial_t \phi = F e^{-imt} (\partial_t \chi - im\chi)$
for spin 1/2 → Foldy-Woldhausen transformation
ling in the low energy limit:



$\phi = F e^{-imt} \chi$
$\partial_t \phi = F e^{-imt} (\partial_t \chi - im \chi)$
for spin 1/2 → Foldy-Woldhausen transformatio
Scaling
energy limit:

$$\phi = F e^{-imt} \chi \qquad \partial_t \phi = F e^{-imt} (\partial_t \chi - im \chi)$$

for spin $1/2 \longrightarrow$ Foldy-Woldhausen transformation

Scaling in the low-energy limit:

$$\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow \varepsilon = \frac{p^2}{2m} \qquad \chi \sim e^{i(\ldots)}$$

Take zero of energy to be the Fermi surface

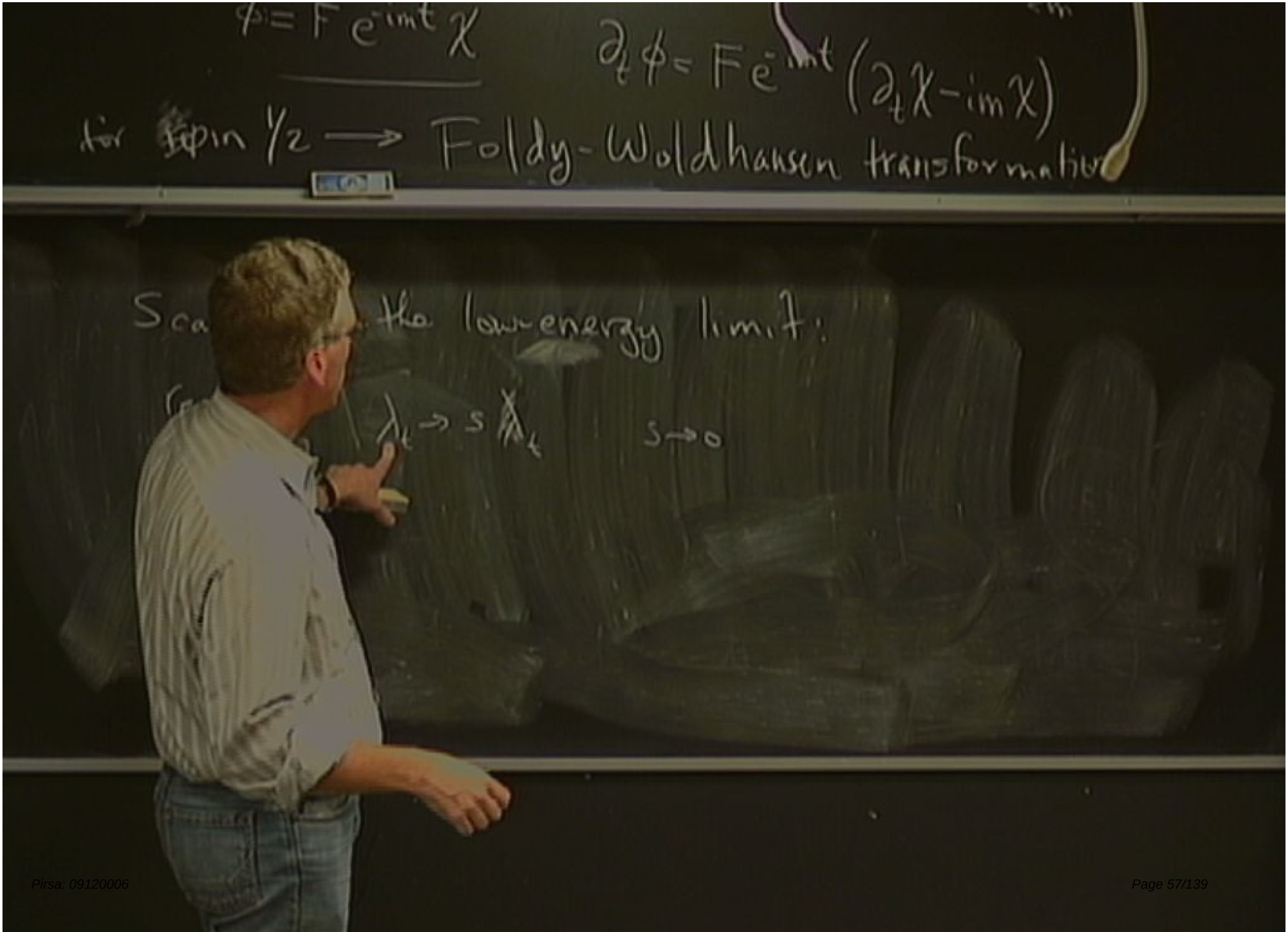
defined by $\vec{p} = \vec{k}$ where $\varepsilon_1(\vec{k}) = 0$.

$$S_0 = \int dt \int_{\Lambda/\lambda_c}^{\lambda_c} d^3p \left[\frac{i}{2}\left(\psi^* \partial_t \psi - \partial_t \psi^* \psi\right) - \varepsilon(\vec{p})\,\psi^* \psi \right]$$

$$\psi = \psi(\vec{p}, t)$$

$$\varepsilon(\vec{p}) = \vec{\ell} \cdot \left.\frac{\partial \varepsilon}{\partial \vec{p}}\right|_{\vec{k}} + \ldots \qquad \vec{p} = \vec{k} + \vec{\ell}$$

$$\nabla_p \varepsilon = \vec{v}_F \qquad \varepsilon \approx \vec{\ell} \cdot \vec{v}_F(\vec{k}) = \ell v_F \qquad \vec{\ell} = \xi \ldots$$



$\phi = F e^{-imt} \chi$
$\partial_t \phi = F e^{-imt} (\partial_t \chi - im\chi)$
for spin 1/2 → Foldy-Woldhausen transformation
the low-energy limit:
$\partial_t \to s\, \partial_t$
$s \to 0$

$\phi = F e^{-imt} \chi$

$\partial_t \phi = F e^{-imt} (\partial_t \chi - i m \chi)$

for spin 1/2 $\longrightarrow$ Foldy-Woldhausen transformation

Scaling in the low-energy limit:

rescale $\lambda_t \to s\, \lambda_t$ $\quad s \to 0$

$\phi = F e^{-imt} \chi$

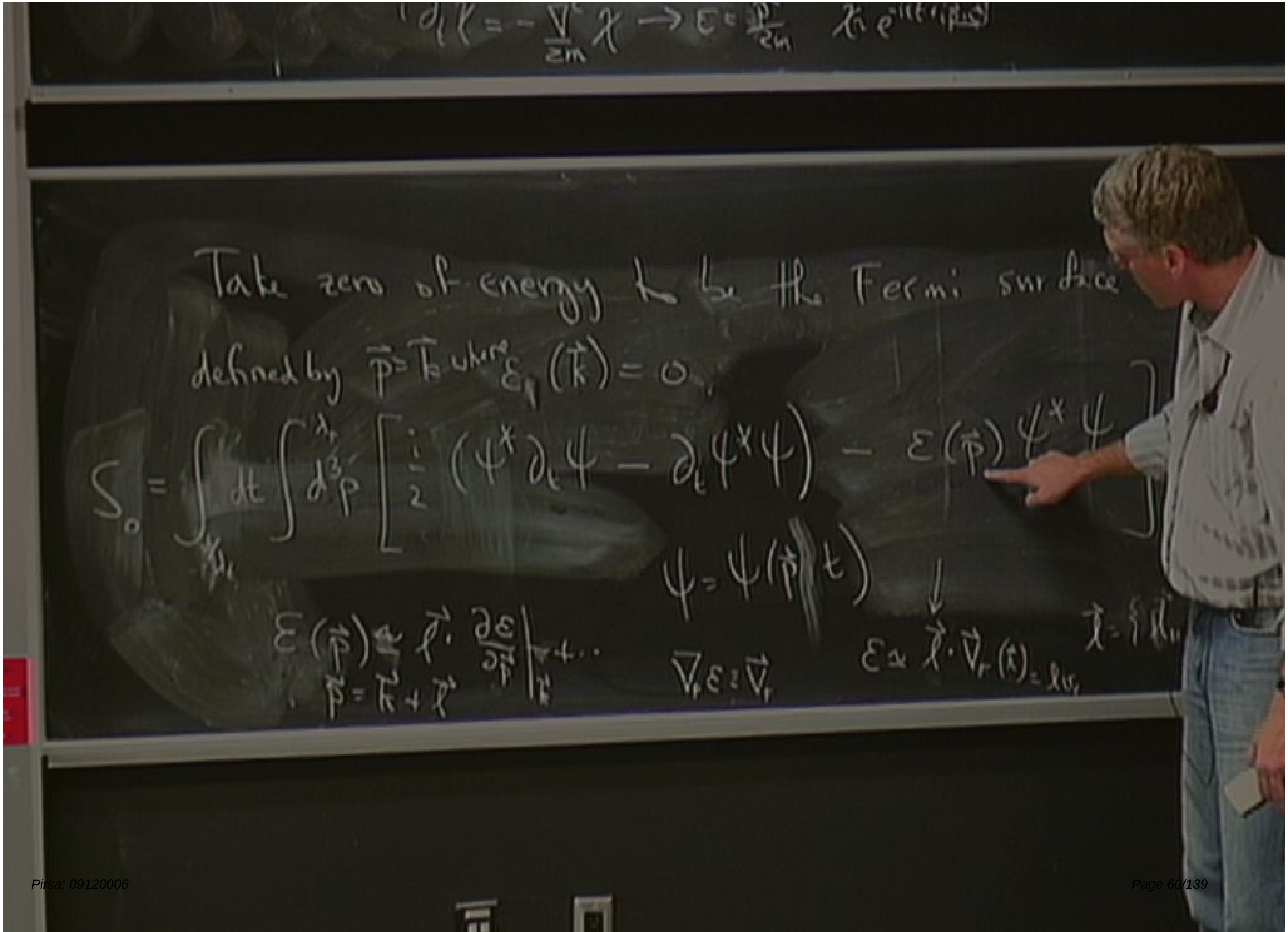
$\partial_t \phi = F e^{-imt} (\partial_t \chi - im\chi)$

for spin 1/2 $\longrightarrow$ Foldy-Wouthuysen transformation

Scaling in the low-energy limit:

rescale $\lambda_t \to s\lambda_t$ $\quad s \to 0$ $\quad \mathcal{E} = \frac{p^2}{2m}$

$\lambda_p \to s^{1/2}\lambda_p$



$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{p^2}{2m}$ $\chi \sim e^{i(Et + i\vec p\cdot\vec x)}$
Take zero of energy to be the Fermi surface
defined by $\vec p = \vec k$ where $\varepsilon_1(\vec k) = 0$.
$S_0 = \int dt \int_{k_F}^{\lambda_1} d^3p \left[\frac{i}{2}\left(\psi^* \partial_t \psi - \partial_t \psi^* \psi\right) - \varepsilon(\vec p)\, \psi^* \psi \right]$
$\psi = \psi(\vec p, t)$
$\varepsilon(\vec p) = \vec\ell \cdot \frac{\partial \varepsilon}{\partial \vec p}\Big|_{\vec k} + \dots$
$\vec p = \vec k + \vec \ell$
$\nabla_p \varepsilon = \vec v_F$
$\varepsilon \simeq \vec\ell \cdot \vec v_F(\vec k) = \ell v_F$

$$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \to \mathcal{E} = \frac{p^2}{2m} \qquad \chi \sim e^{i(Et + i\vec p\cdot\vec x)}$$

Take zero of energy to be the Fermi surface

defined by $\vec p = \vec k$ where $\mathcal{E}_1(\vec k) = 0$.

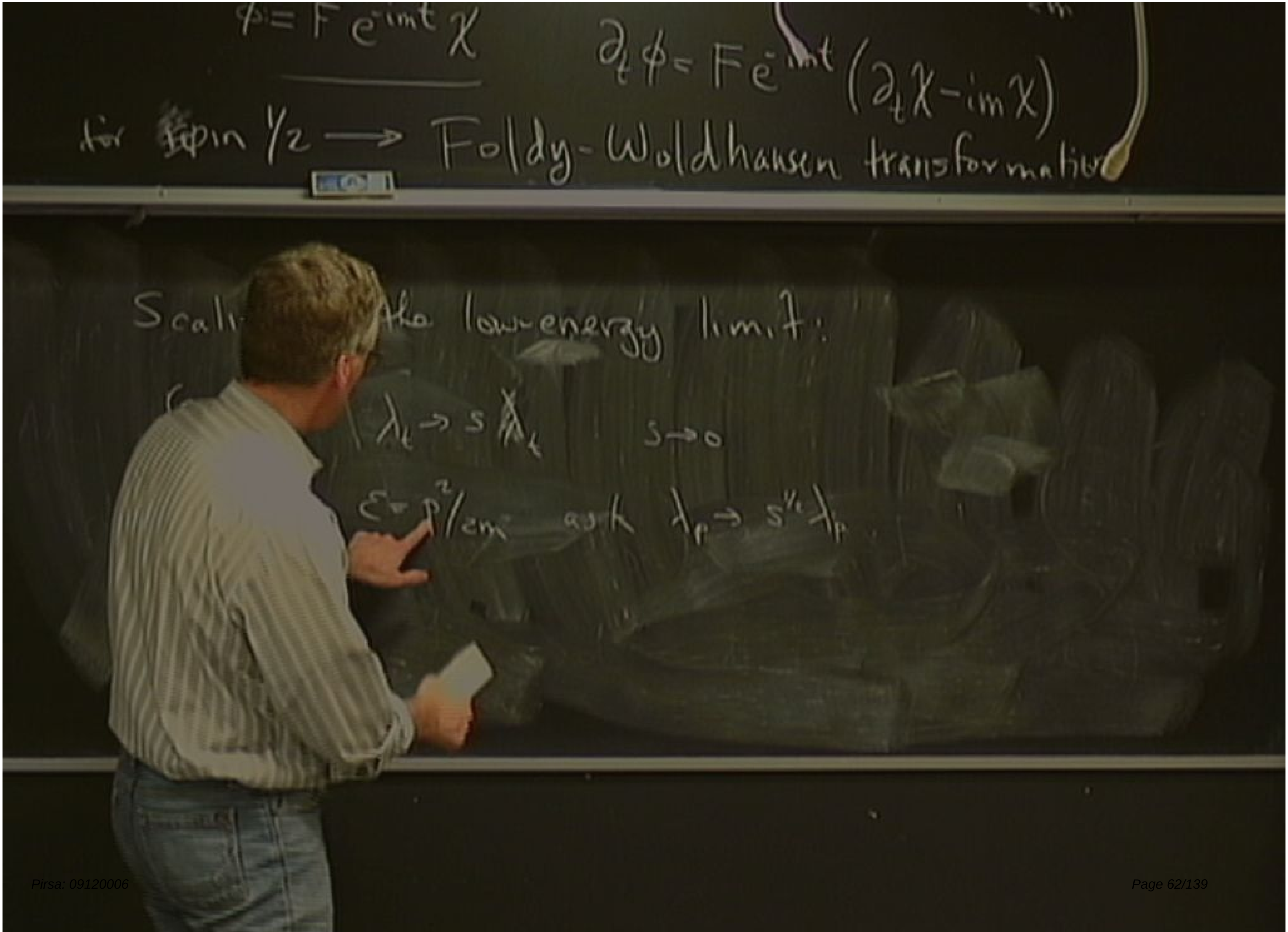
$$S_0 = \int dt \int_{\lambda_F}^{\lambda_1} d^3p \left[\frac{i}{2}\left(\psi^* \partial_t \psi - \partial_t \psi^* \psi\right) - \mathcal{E}(\vec p)\, \psi^* \psi \right]$$

$$\psi = \psi(\vec p, t)$$

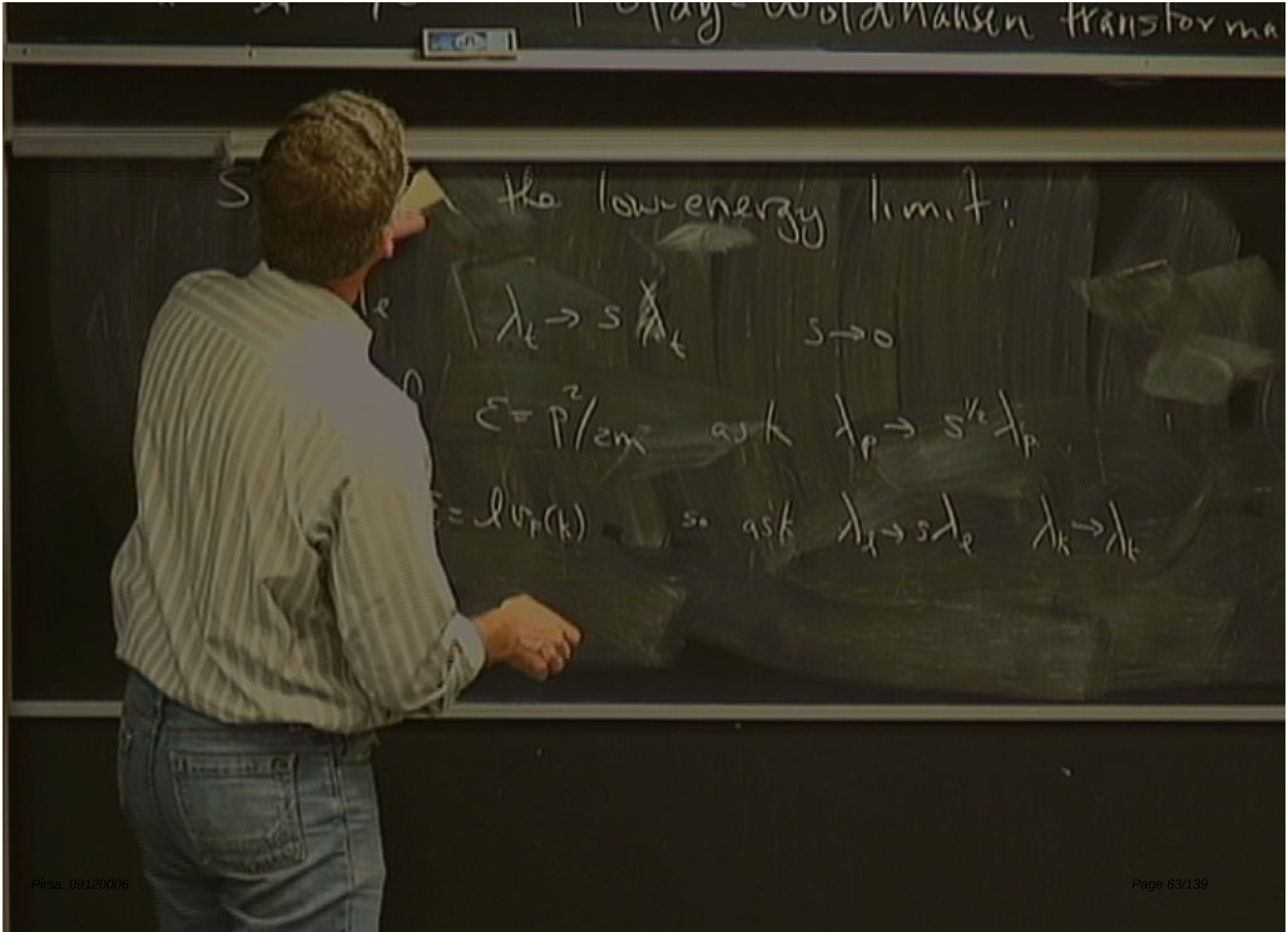
$$\mathcal{E}(\vec p) \approx \vec \ell \cdot \frac{\partial \mathcal{E}}{\partial \vec p}\Big|_{\vec k} + \dots \qquad \vec p = \vec k + \vec \ell$$

$$\nabla_p \mathcal{E} = \vec v_F$$

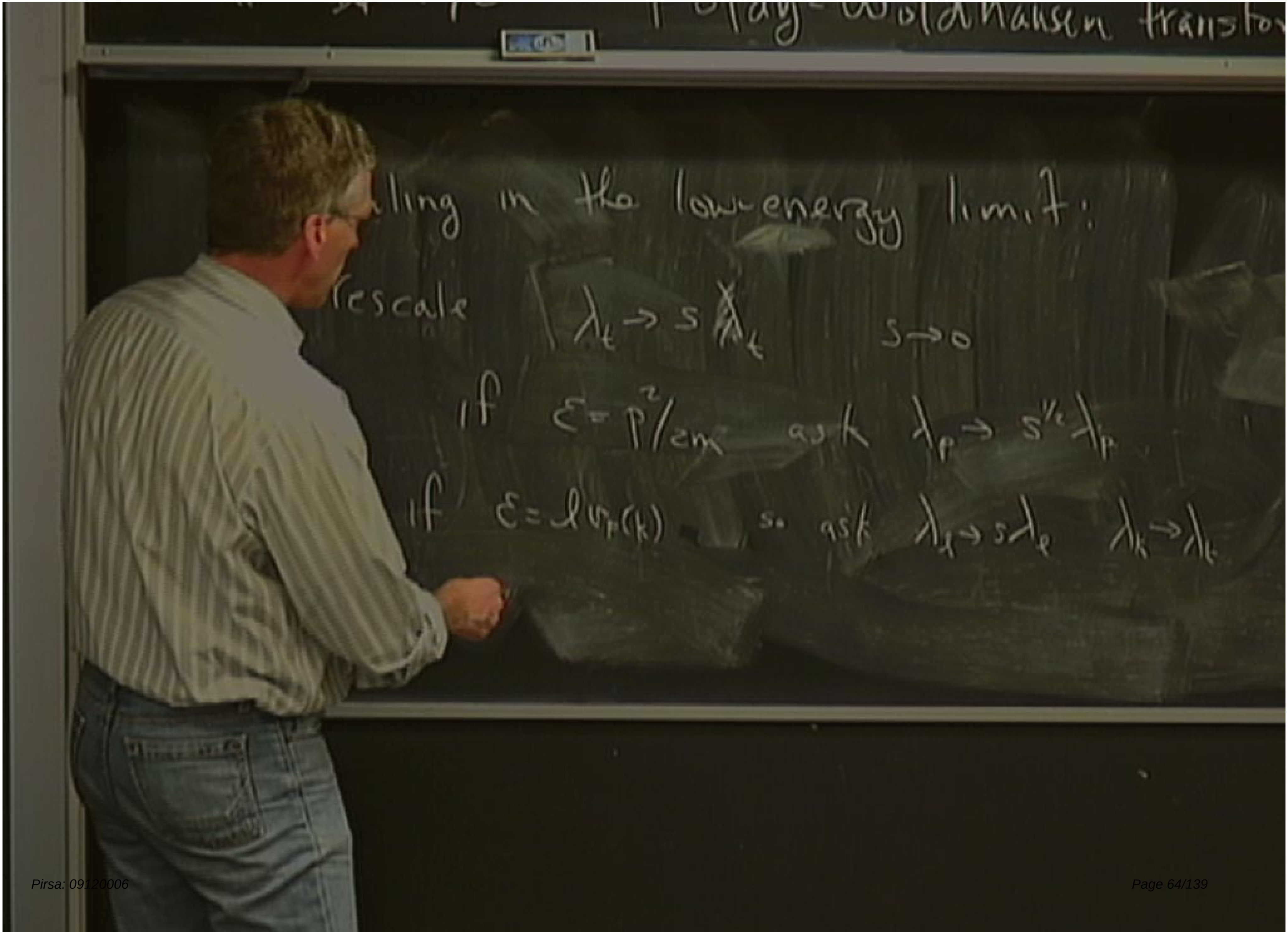
$$\mathcal{E} \approx \vec\ell \cdot \vec v_F(\vec k) = \ell v_F \qquad \vec \ell = \{\vec \ell_\parallel, \ell\}$$

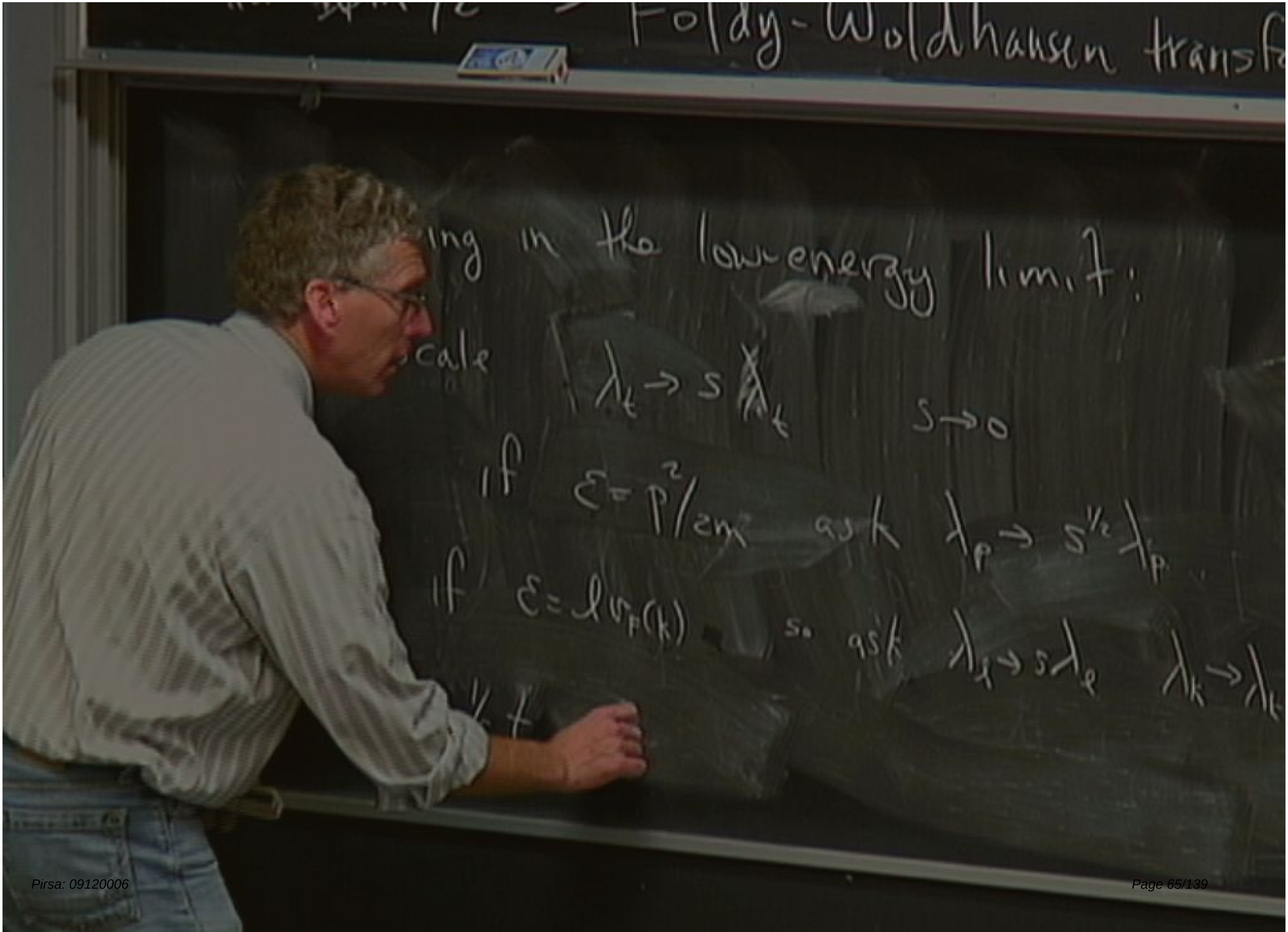


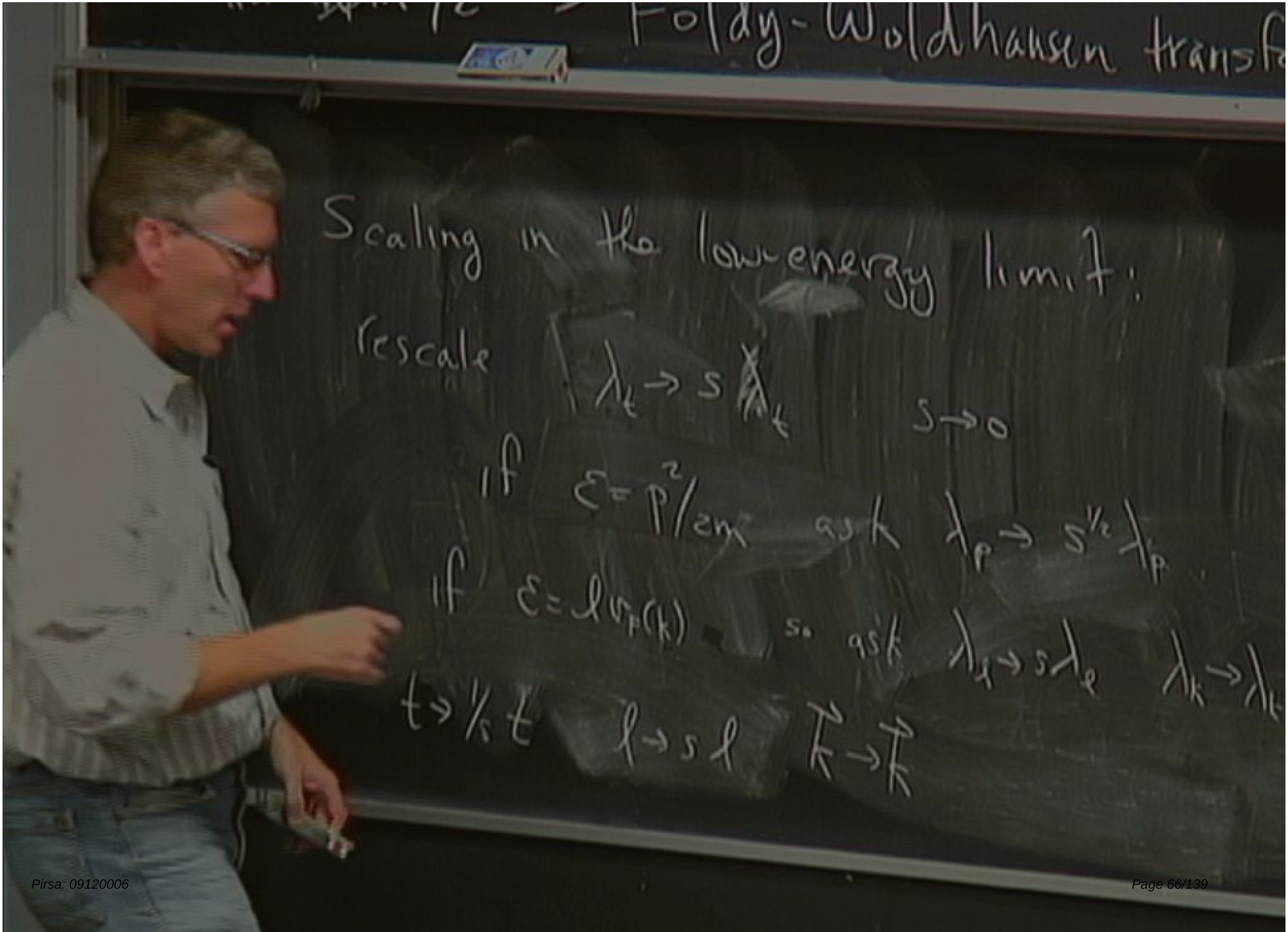
φ = F e^-imt χ
∂t φ = F e^-imt (∂t χ - im χ)
for spin 1/2 → Foldy-Woldhausen transformation
the low-energy limit:
λt → s λt   s → 0
E = p²/2m   λp → s^1/2 λp



the low-energy limit:
λt → s λt   s → 0
ε = p²/2m as k λp → s½ λp
so as k λℓ → s λℓ   λk → λk


Woldhausen transfor
lling in the low-energy limit:
rescale λt → s λt   s → 0
if ε = p²/2m as k λp → s^1/2 λp
if ε = ℏ vF(k) as k λℓ → s λℓ   λk → λk


Foldy-Woldhausen transf
ing in the low-energy limit:
scale
λt → s λt
s → 0
if ε = p²/2m as k λp → s½ λp
if ε = ℓ vF(k) so as k λℓ → s λℓ λk → λk


Foldy-Woldhausen
Scaling in the low-energy limit:
rescale $\lambda_t \to s\,\lambda_t$
$s \to 0$
if $\varepsilon = p^2/2m$ as k $\lambda_p \to s^{1/2}\lambda_p$
if $\varepsilon = \ell\, v_F(k)$ so as k $\lambda_\ell \to s\lambda_\ell$
$t \to 1/s\; t$ $\ell \to s\,\ell$ $\vec{k} \to \vec{k}$

for spin $1/2 \longrightarrow$ Foldy-Woldhausen transformation

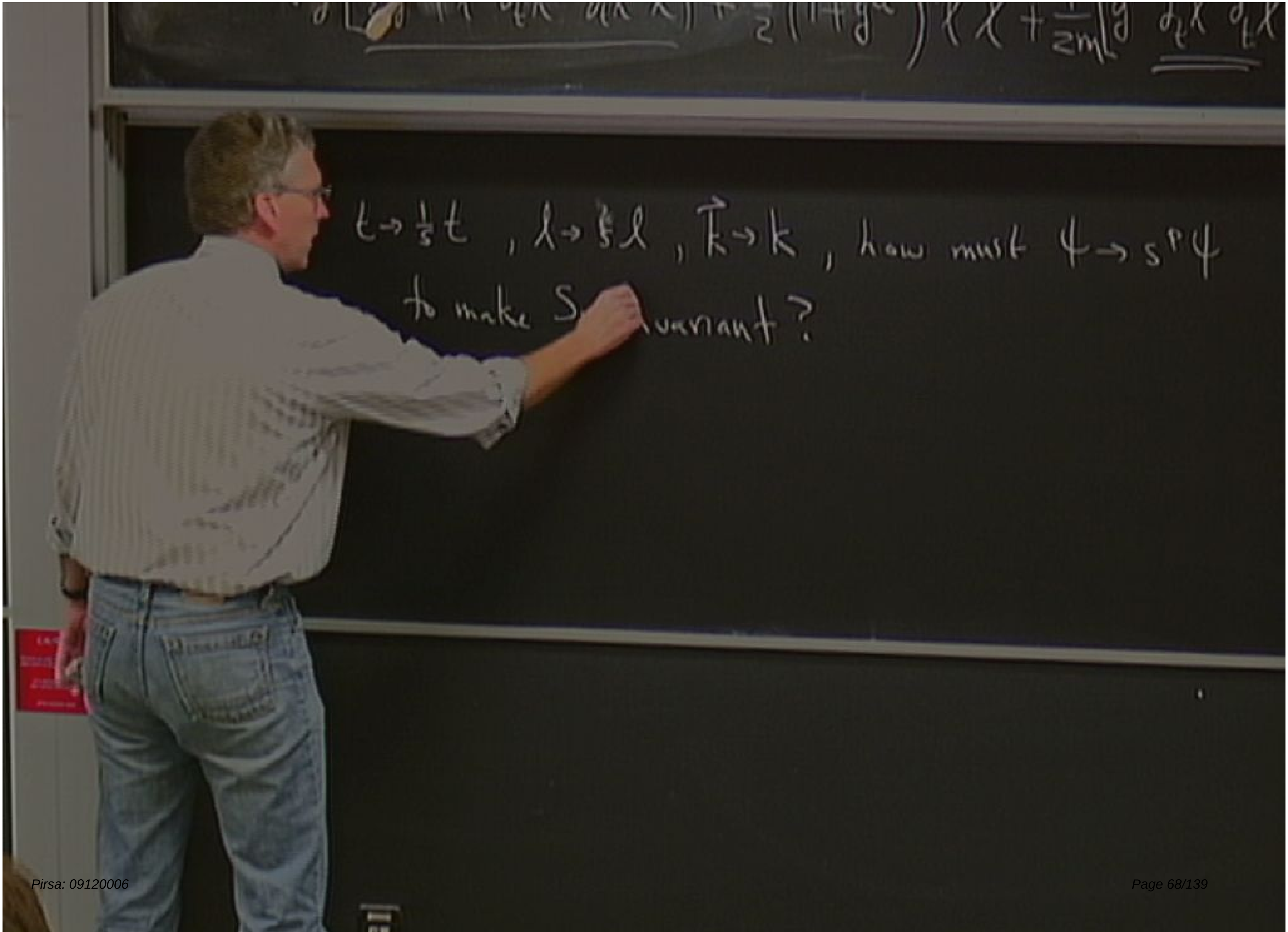
Scaling in the low energy limit:

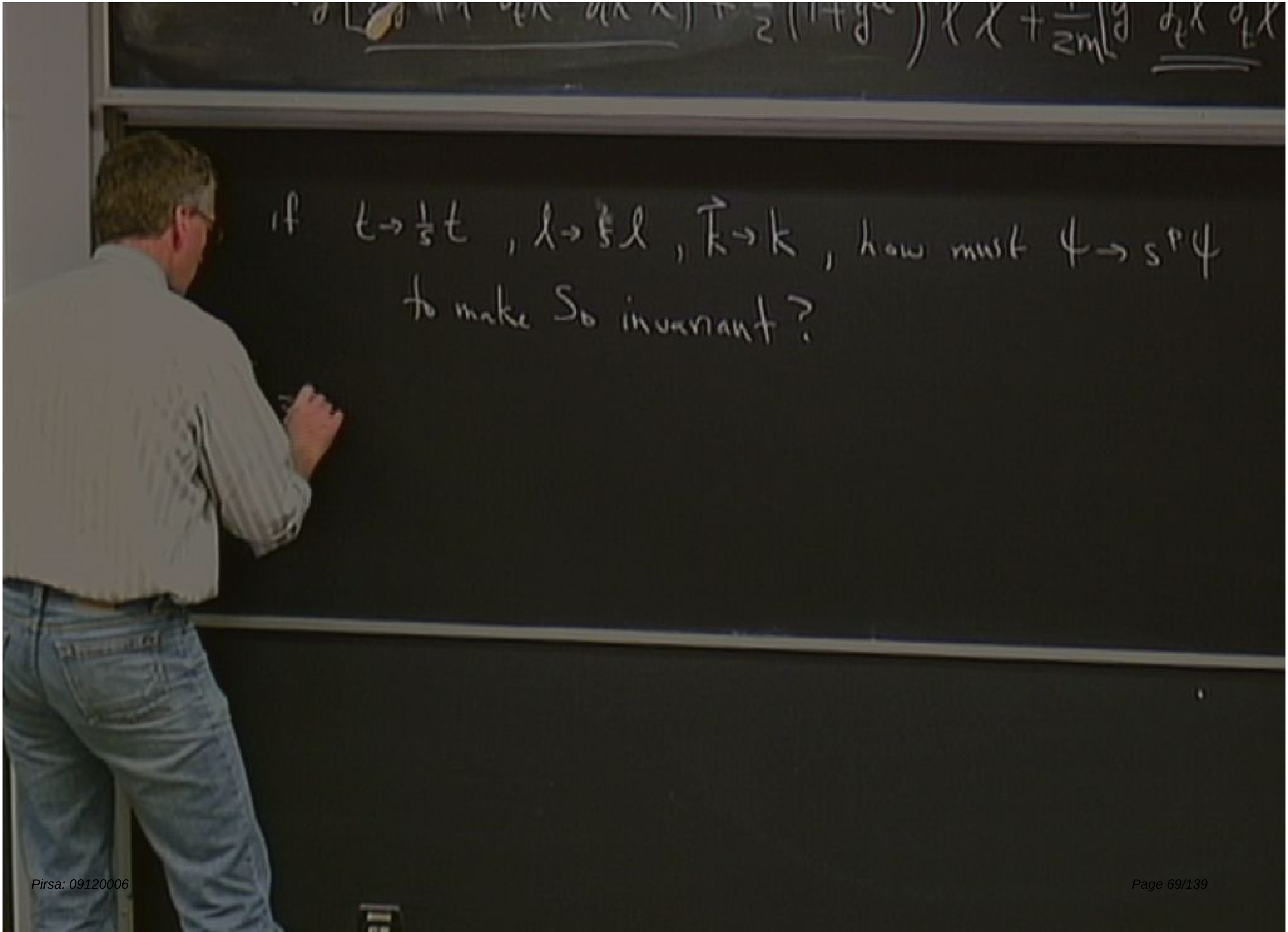
rescale $\lambda_t \to s\lambda_t$ $s \to 0$ if $\varepsilon \sim p$ $\lambda_p \to s\lambda_p$

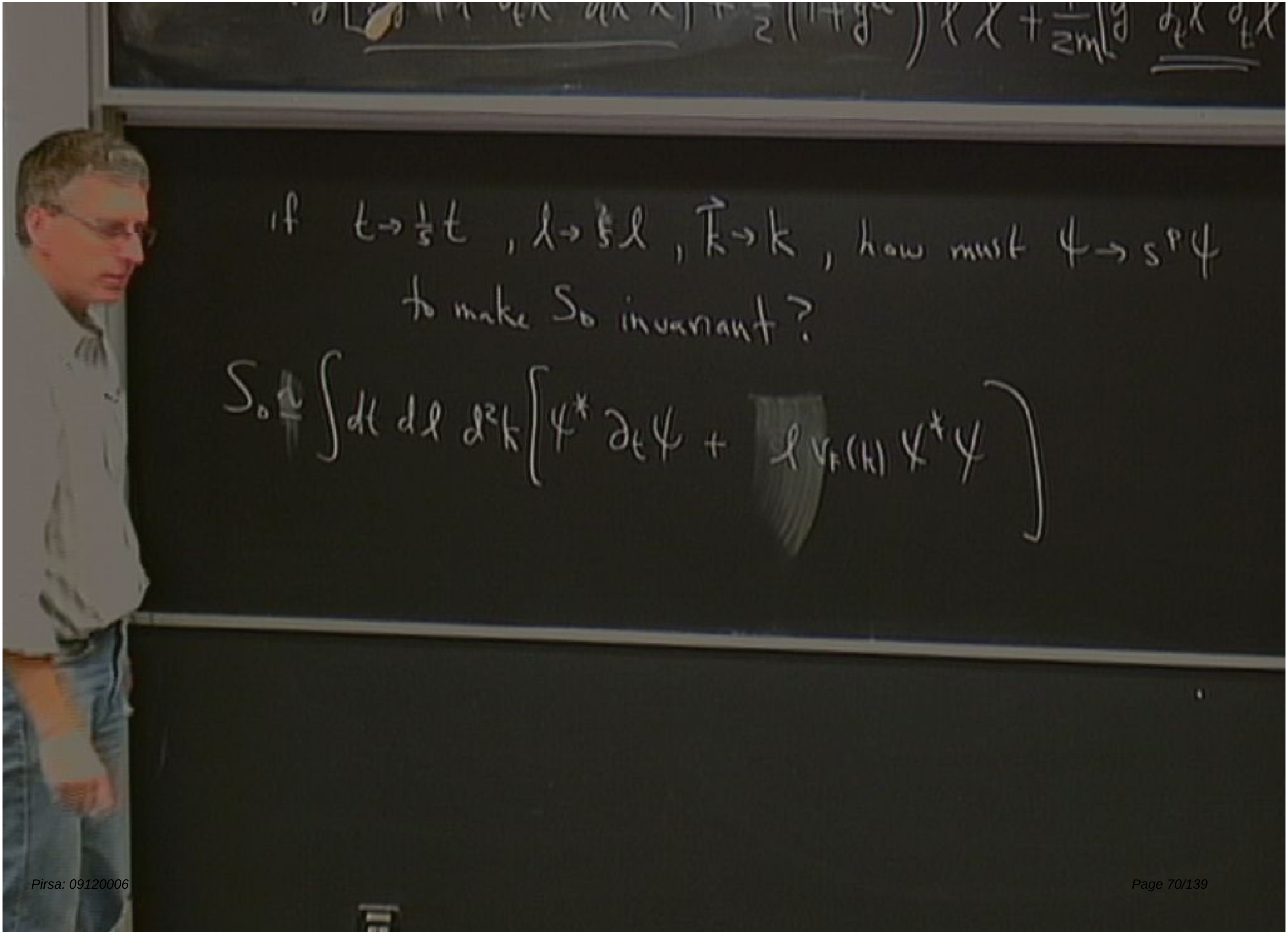
if $\varepsilon = p^2/2m$ as k $\lambda_p \to s^{1/2}\lambda_p$

if $\varepsilon = \ell\, v_p(k)$ so as k $\lambda_\ell \to s\lambda_\ell$ $\lambda_k \to \lambda_k$

instead change to $t \to 1/s\, t$ $\ell \to s\,\ell$ $\vec{k} \to \vec{k}$ so λ_t, λ_ℓ don't change


$t \to \frac{1}{s} t$, $\lambda \to \frac{1}{s} \ell$, $\vec{k} \to k$, how must $\psi \to s^p \psi$
to make S ... variant ?


if $t \to \frac{1}{s} t$, $\ell \to \frac{1}{s} \ell$, $\vec{k} \to k$, how must $\psi \to s^{p} \psi$
to make S_0 invariant?


if t → (1/s) t , ℓ → (1/s) ℓ , k⃗ → k , how must ψ → s^p ψ
to make S₀ invariant?
S₀ = ∫ dt dℓ d²k [ψ* ∂_t ψ + ℓ v_F(k) ψ* ψ]

if $t \to \frac{1}{s} t$, $\ell \to \frac{1}{s} \ell$, $\vec{k} \to k$, how must $\psi \to s^p \psi$

to make S_0 invariant?

$$S_0 = \int dt \; d\ell \; d^2k \left[\psi^* \partial_t \psi + \ell \, v_F(k) \, \psi^* \psi \right]$$

$$\begin{array}{cccccccccc} 1/s & s & s^0 & s^p & s & s^p & s & s^0 & s^p & s^p \end{array}$$

for spin $1/2 \longrightarrow$ Foldy-Woldhausen transformation

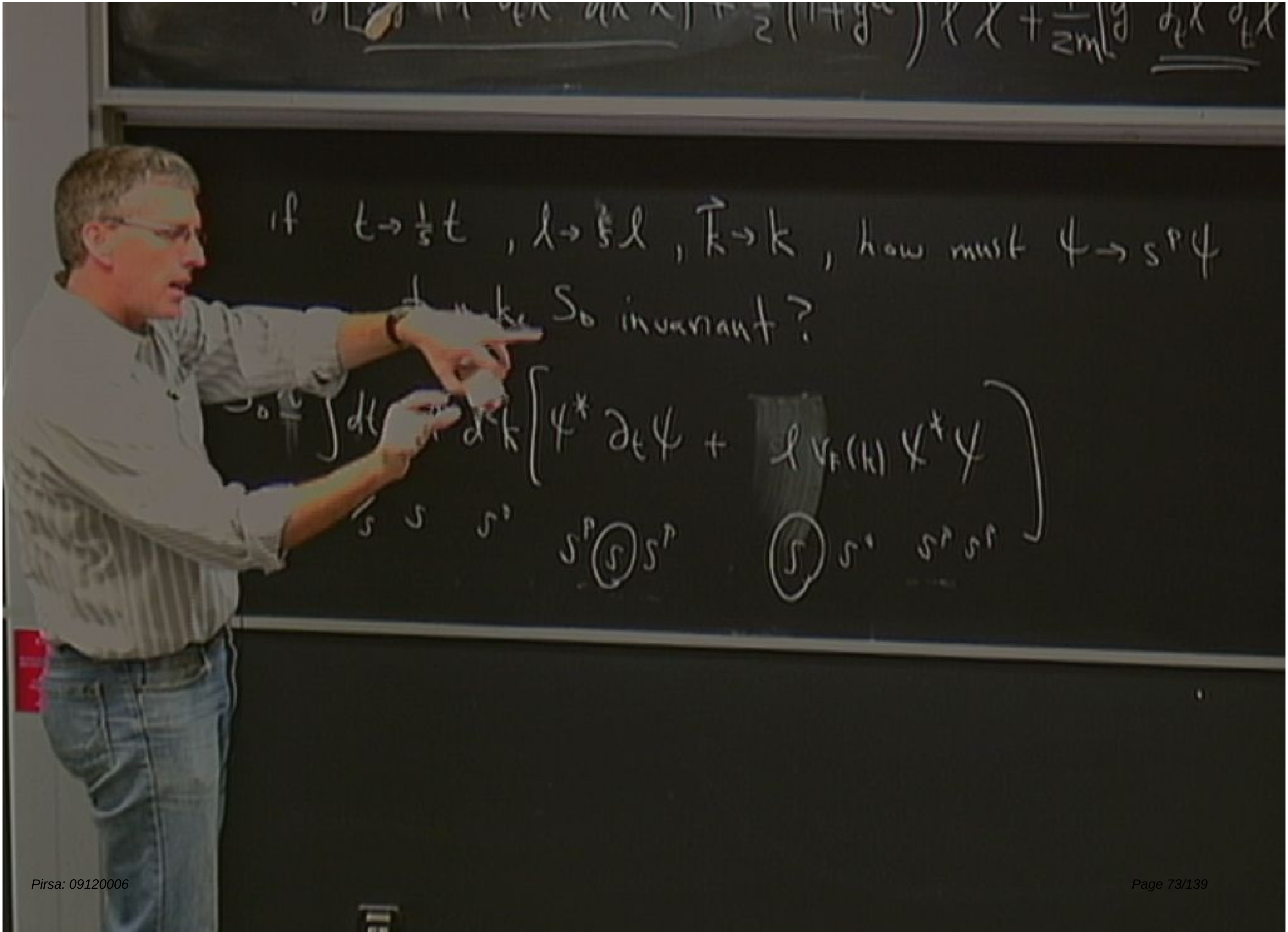
Scaling in the low energy limit:

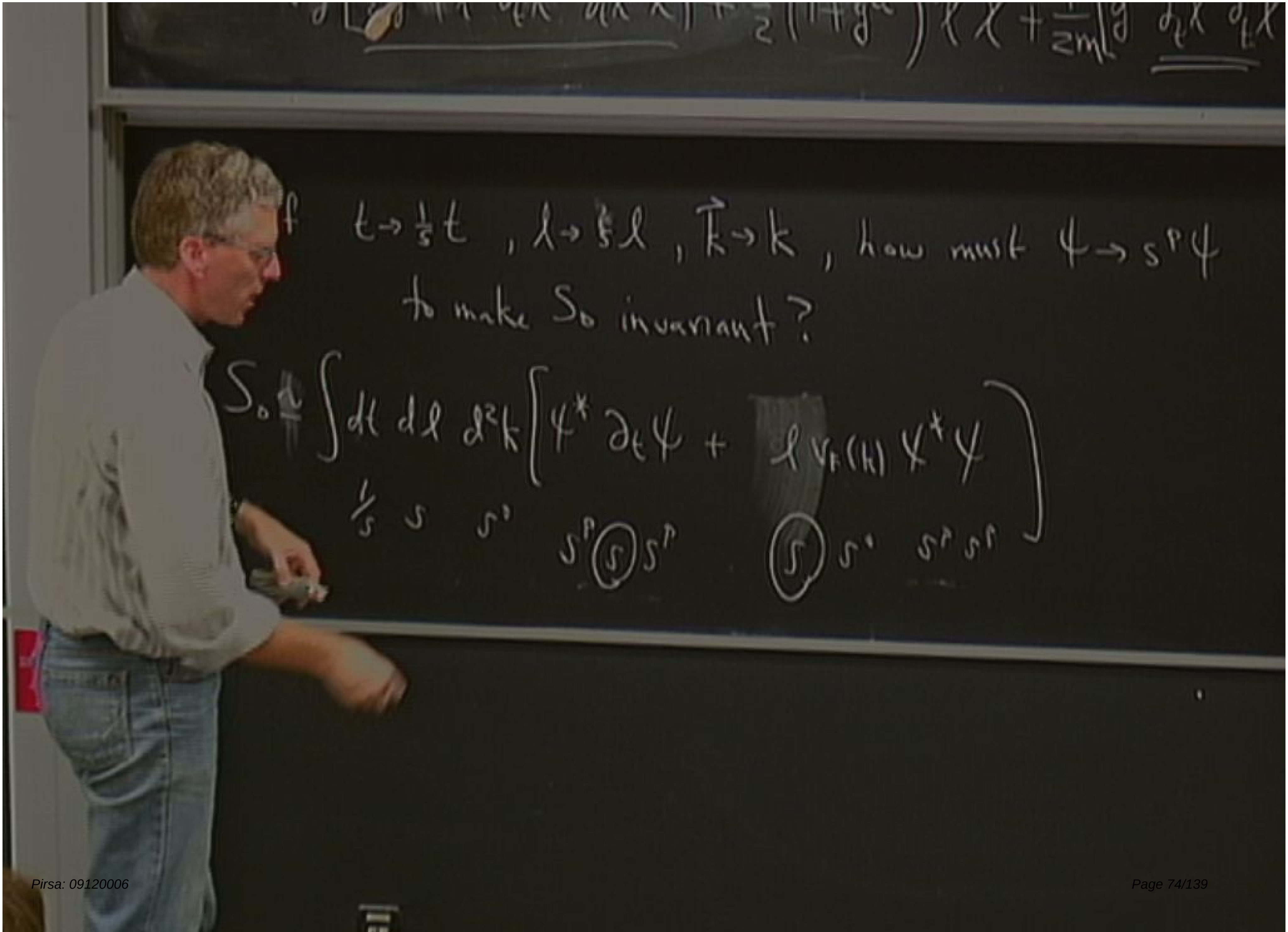
rescale $\boxed{\lambda_t \to s\lambda_t}$ $s \to 0$ if $\varepsilon \sim p$ $\lambda_p \to s\lambda_p$

if $\varepsilon = p^2/2m$ ask $\lambda_p \to s^{1/2}\lambda_p$

if $\varepsilon = \ell v_p(k)$ so ask $\boxed{\lambda_\ell \to s\lambda_\ell}$ $\lambda_k \to \lambda_k$

instead change to $t \to 1/s\, t$ $\ell \to s\,\ell$ $\vec{k} \to \vec{k}$ so λ_t, λ_ℓ don't change


if $t \to \frac{1}{s} t$, $\ell \to s \ell$, $\vec{k} \to k$, how must $\psi \to s^p \psi$
S_0 invariant?
$\left[\psi^* \partial_t \psi + \ell\, v_F(k)\, \psi^* \psi \right]$


If t → 1/s t , l → 1/s l , k → k , how must ψ → s^p ψ
to make S₀ invariant?
S₀ = ∫ dt dl d²k [ψ* ∂t ψ + l vF(k) ψ* ψ]
1/s s s⁰ s^p s s^p s s⁰ s^p s^p

if $t \to \frac{1}{s} t$, $\ell \to s \ell$, $\vec{k} \to k$, how must $\psi \to s^{p} \psi$

to make S_0 invariant?

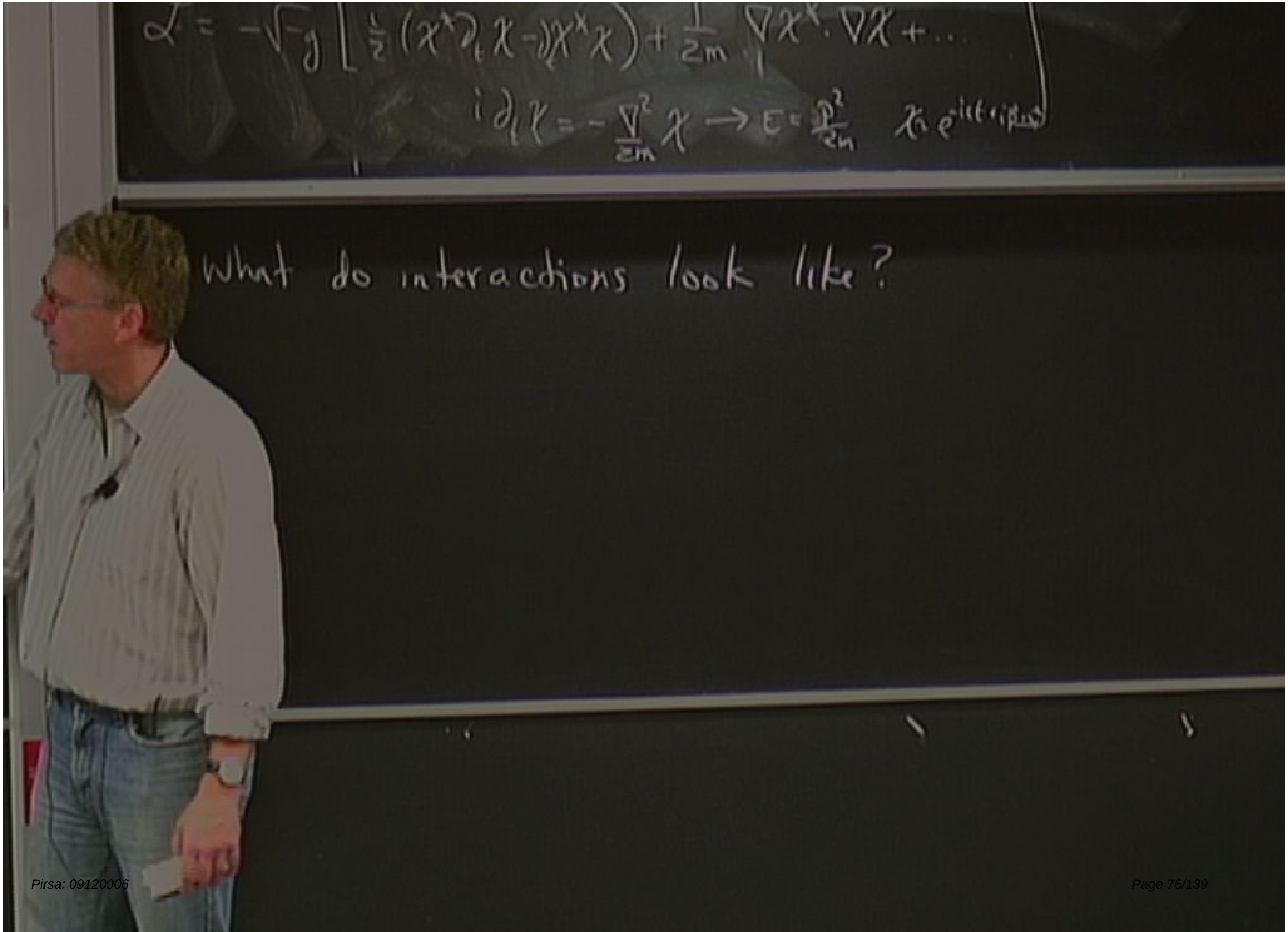
$\psi \to s^{-1/2} \psi$

$$S_0 = \int dt \, d\ell \, d^2k \left[\psi^* \partial_t \psi + \ell \, v_F(k) \, \psi^* \psi \right] \sim s^{-1+1+2p+1}$$

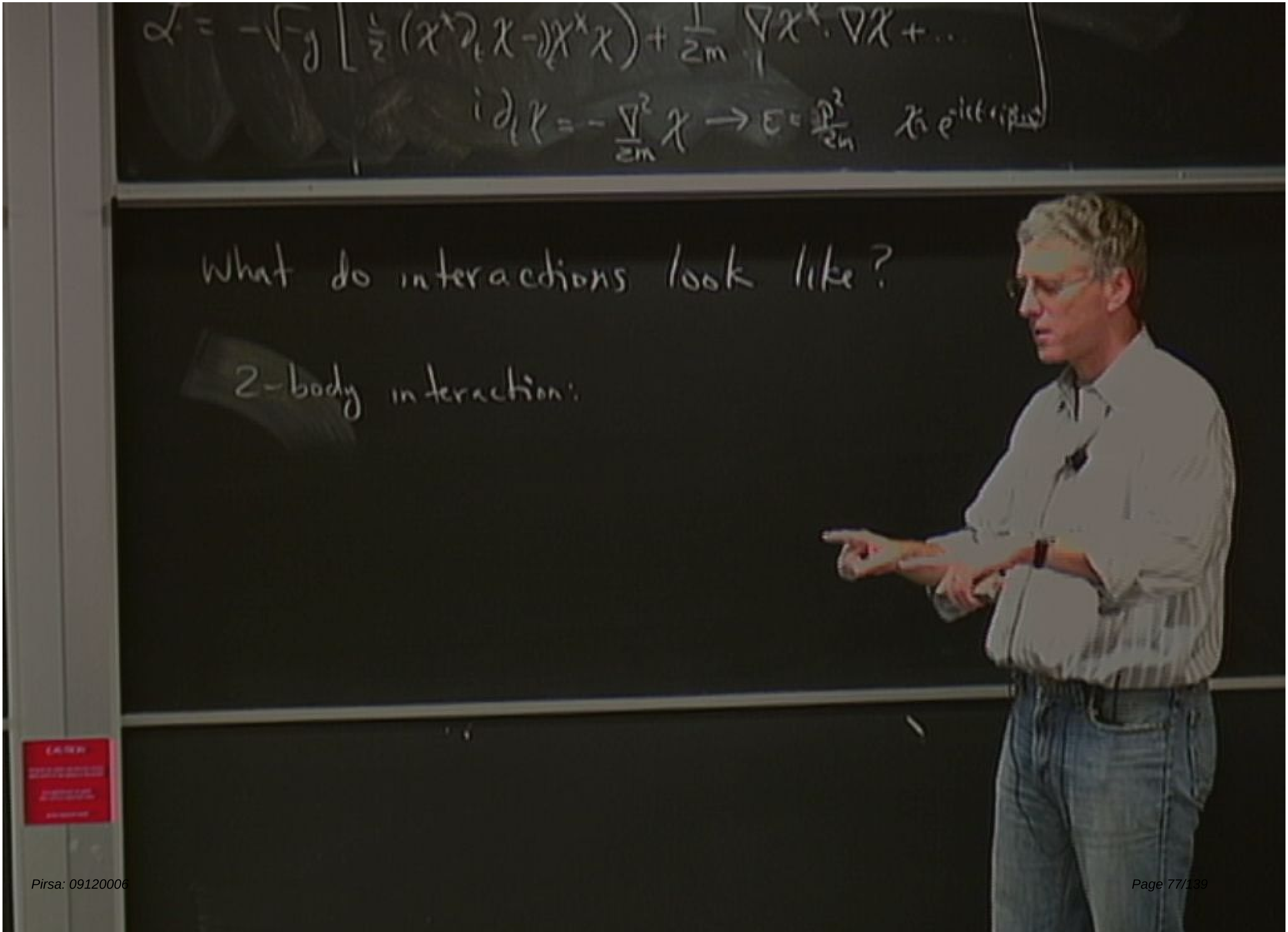
$$\frac{1}{s} \quad s \quad s^0 \qquad s^p \; s \; s^p \qquad s \; s^0 \; s^p \, s^p$$

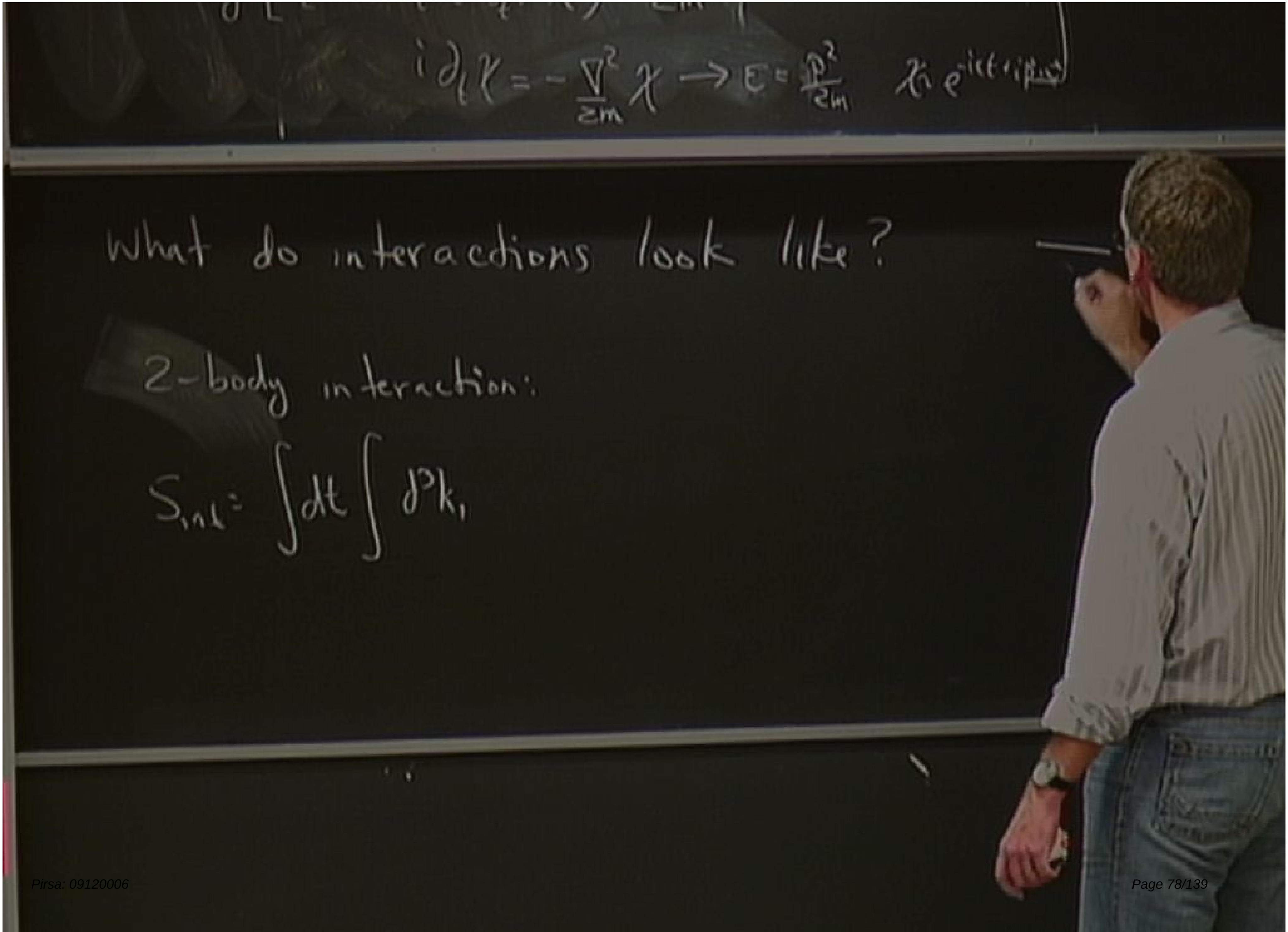
s^{2p+1}

$p = -1/2$



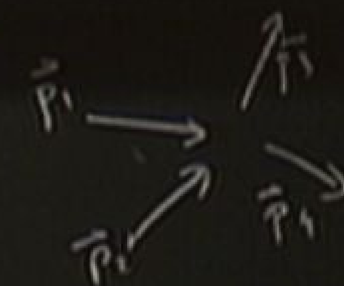
$\mathcal{L} = -\sqrt{-g}\left[\frac{i}{2}(\chi^* \partial_t \chi - \partial_t \chi^* \chi) + \frac{1}{2m} \nabla \chi^* \cdot \nabla \chi + \dots\right]$
$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{p^2}{2m} \quad \chi \sim e^{i(Et + ip \cdot x)}$
What do interactions look like?


What do interactions look like?
2-body interaction:


$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{p^2}{2m}$ $\chi \sim e^{-iEt+ip\cdot x}$
What do interactions look like?
2-body interaction:
$S_{int} = \int dt \int d^3k_1$

$$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{\vec{p}^2}{2m} \qquad \chi \sim e^{-iEt + i\vec{p}\cdot\vec{x}}$$

What do interactions look like?



2-body interaction:

$$S_{int} = \int dt \int d^3p_1\, d^3p_2\, d^3p_3\, d^3p_4$$

$$\mathcal{L} = -\sqrt{-g}\left[\frac{i}{2}\left(\chi^*\partial_t\chi - \partial_t\chi^*\chi\right) + \frac{1}{2m}\nabla\chi^*\cdot\nabla\chi + \dots\right]$$

$$i\partial_t\chi = -\frac{\nabla^2}{2m}\chi \to E = \frac{p^2}{2m} \qquad \chi \sim e^{-iEt + i\vec{p}\cdot\vec{x}}$$

What do interactions look like?

2-body interaction:

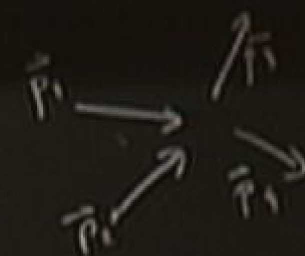
$$S_{int} = \int dt \int d^3p_1\, d^3p_2\, d^3p_3\, d^3p_4\, \delta^3(p_1+p_2-p_3-p_4)\, V(p_1,p_2,p_3,p_4)\, \psi(p_1)\psi(p_2)\psi^*(p_3)\psi^*(p_4)$$

$+$ more powers of $\psi \dots$

$$\mathcal{L} = -\sqrt{-g}\left[\frac{i}{2}\left(\chi^*\partial_t\chi - \partial_t\chi^*\chi\right) + \frac{1}{2m}\nabla\chi^*\cdot\nabla\chi + \dots\right]$$

$$i\partial_t\chi = -\frac{\nabla^2}{2m}\chi \to E = \frac{p^2}{2m} \qquad \chi \sim e^{-iEt + i\vec{p}\cdot\vec{x}}$$

What do interactions look like?



2-body interaction:

$$S_{int} = \int dt \int d^3p_1\, d^3p_2\, d^3p_3\, d^3p_4\; \delta^3(p_1 + p_2 - p_3 - p_4)\, V(p_1, p_2, p_3, p_4)$$
$$\psi(p_1)\psi(p_2)\psi^*(p_3)\psi^*(p_4)$$

+ more powers of ψ...

Cha

How does S_{int} scale?

How does S_{int} scale? $V(p_1, p_2, p_3, p_4) \simeq V(k_1, k_2, k_3, k_4)$

$\cancel{+ \text{ powers of } l_1, l_2, l_3, l_4.}$

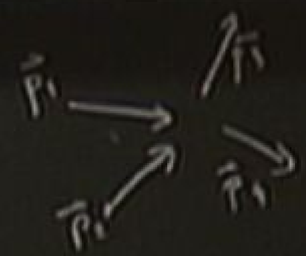
subdominant as $\delta \to 0$.

$d^3 p_1 = dl_1\, d^2 k_1$ etc.

$$\mathcal{L} = -\sqrt{-g}\left[\frac{i}{2}\left(\chi^*\partial_t\chi - \partial_t\chi^*\chi\right) + \frac{1}{2m}\nabla\chi^*\cdot\nabla\chi + \dots\right.$$

$$i\partial_t\chi = -\frac{\nabla^2}{2m}\chi \rightarrow \epsilon = \frac{p^2}{2m} \qquad \chi \sim e^{i(\epsilon t + i p\cdot x)}$$

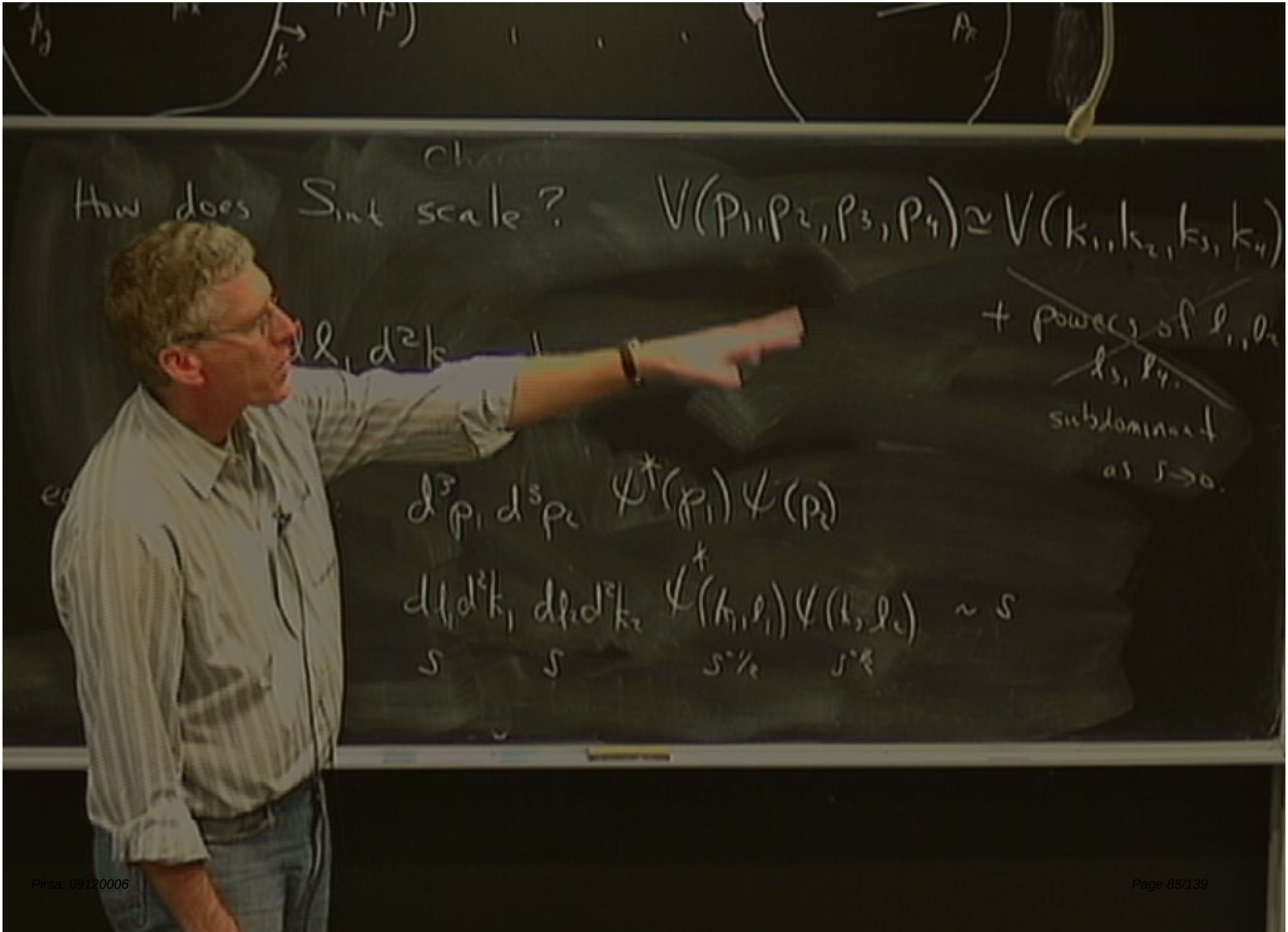
What do interactions look like?

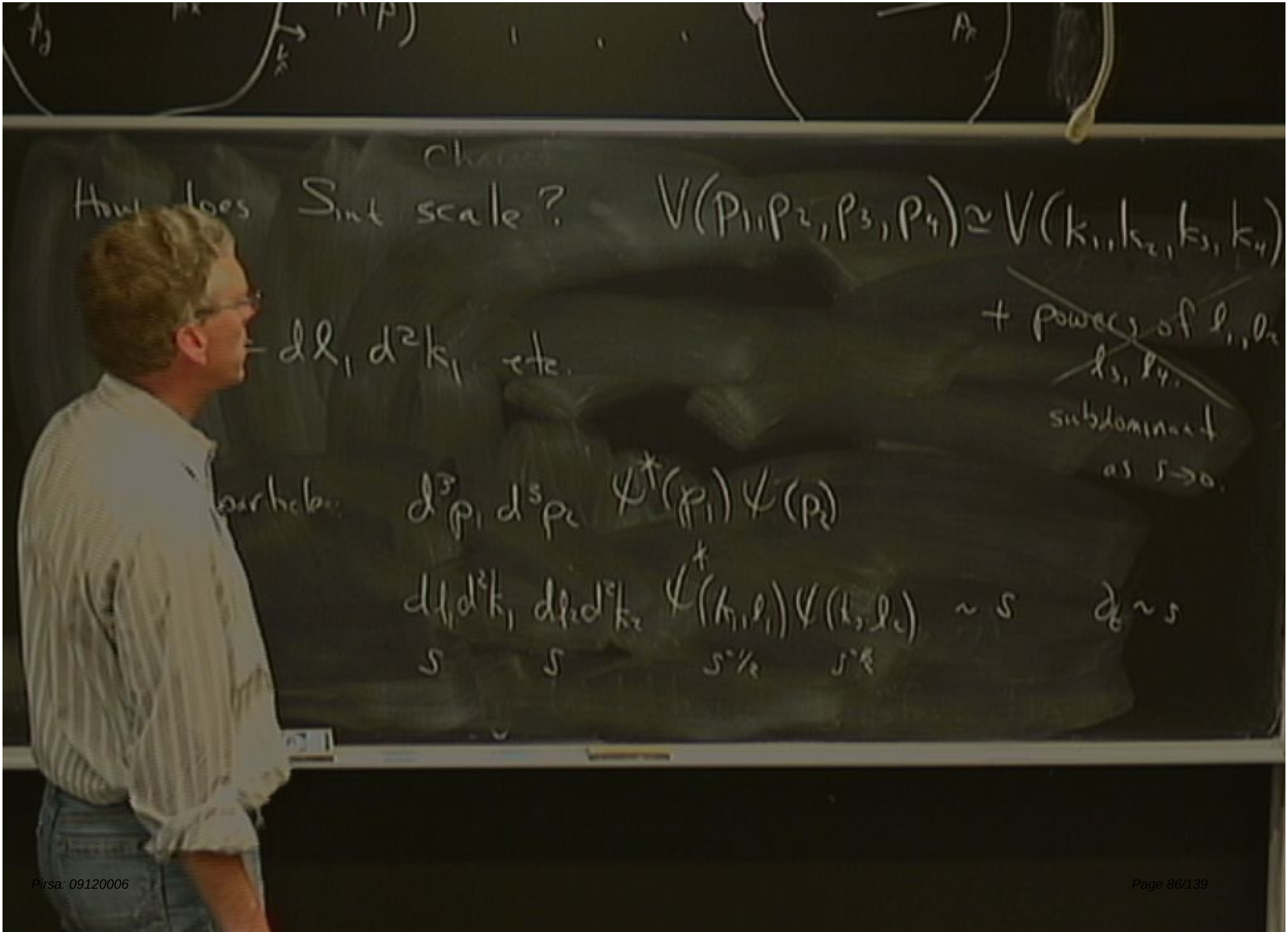


2-body interaction:

$$S_{int} = \int dt \int d^3p_1\, d^3p_2\, d^3p_3\, d^3p_4\, \delta^3(p_1+p_2-p_3-p_4)\, V(p_1,p_2,p_3,p_4)$$
$$\psi(p_1)\psi(p_2)\psi^*(p_3)\psi^*(p_4)$$

$+$ more powers of ψ...


How does S_int scale?
V(p1, p2, p3, p4) ≃ V(k1, k2, k3, k4)
+ powers of l1, l2, l3, l4.
subdominant as S→0.
d³p1 d³p2 ψ†(p1) ψ(p2)
dl1 d²k1 dl2 d²k2 ψ†(k1, l1) ψ(k2, l2) ~ S
S S S^-1/2 S^-1/2


How does S_{int} scale?
$V(p_1, p_2, p_3, p_4) \simeq V(k_1, k_2, k_3, k_4)$
~~+ powers of l_1, l_2, l_3, l_4~~
subdominant as $s \to 0$.
$dl_1\, d^2k_1$ etc.
particle: $d^3p_1\, d^3p_2\, \psi^*(p_1)\psi(p_2)$
$dl_1 d^2k_1\, dl_2 d^2k_2\, \psi^*(k_1,l_1)\psi(k_2,l_2) \sim s$
$\partial_t \sim s$
$s \quad s \quad s^{-1/2} \quad s^{-1/2}$

How does S_{int} scale? $V(p_1, p_2, p_3, p_4) \simeq V(k_1, k_2, k_3, k_4)$

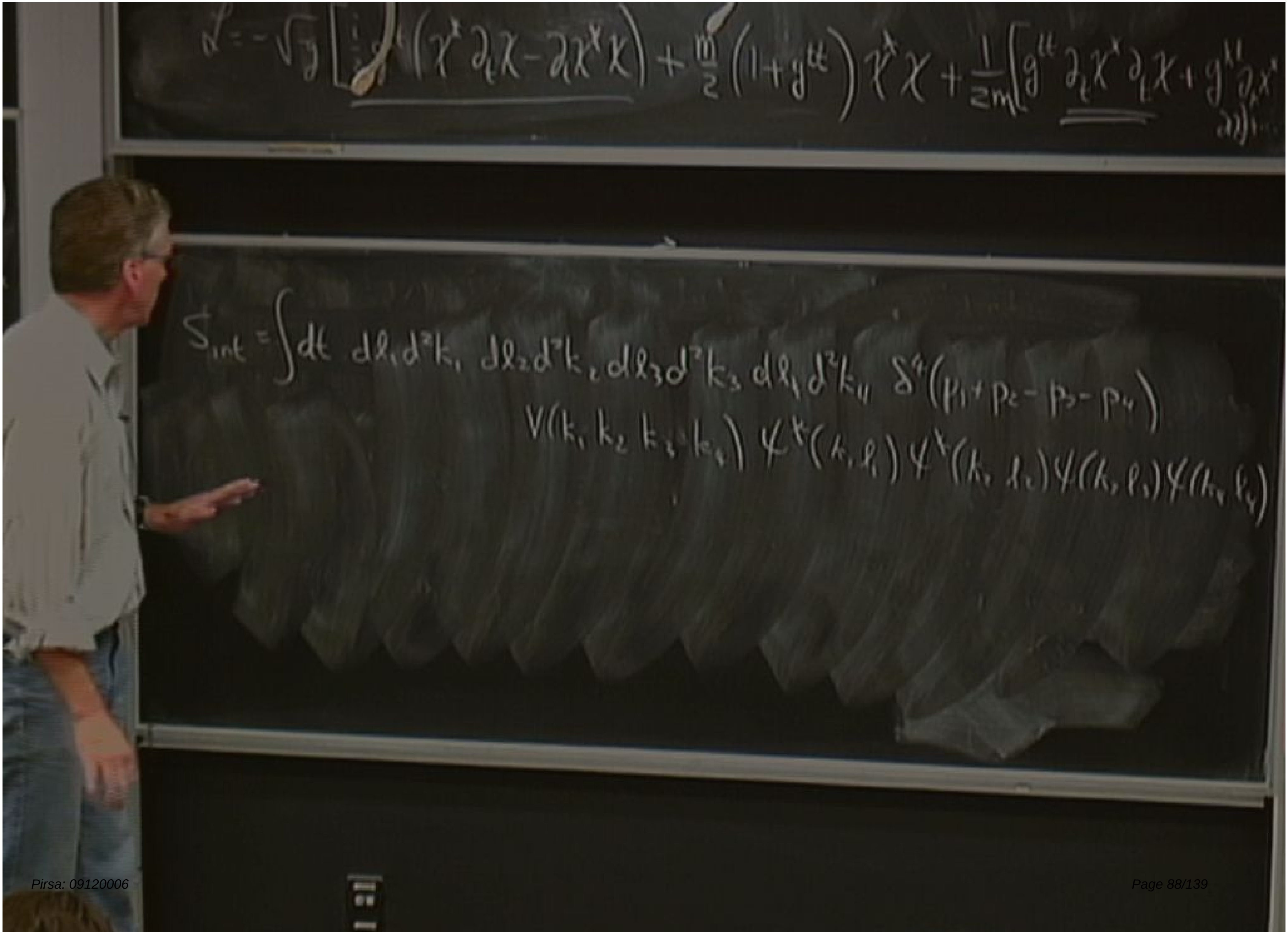
$\cancel{+ \text{ powers of } \ell_1, \ell_2, \ell_3, \ell_4.}$

subdominant as $s \to 0$.

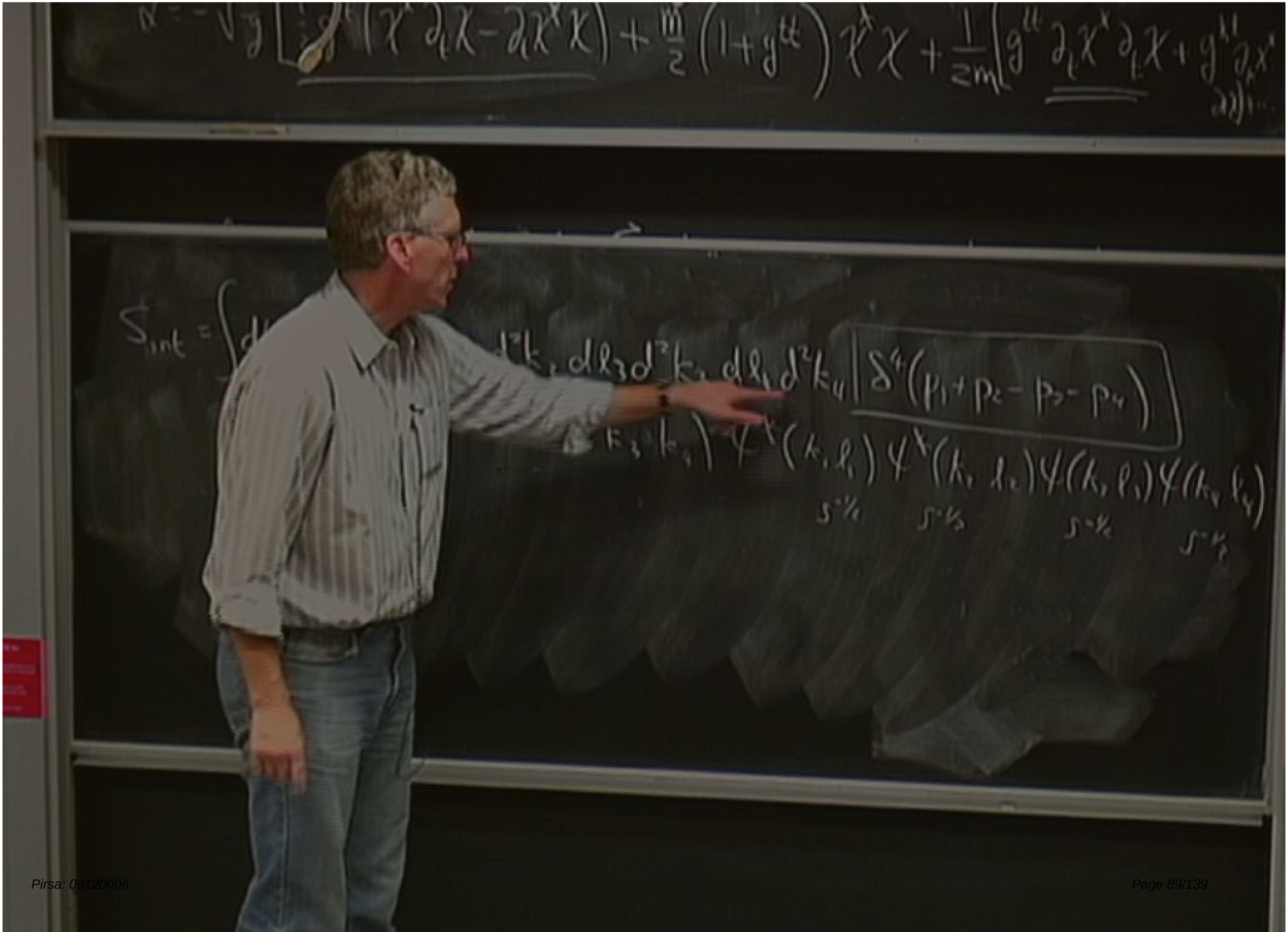
$d^3p_1 = d\ell_1\, d^2k_1$ etc.

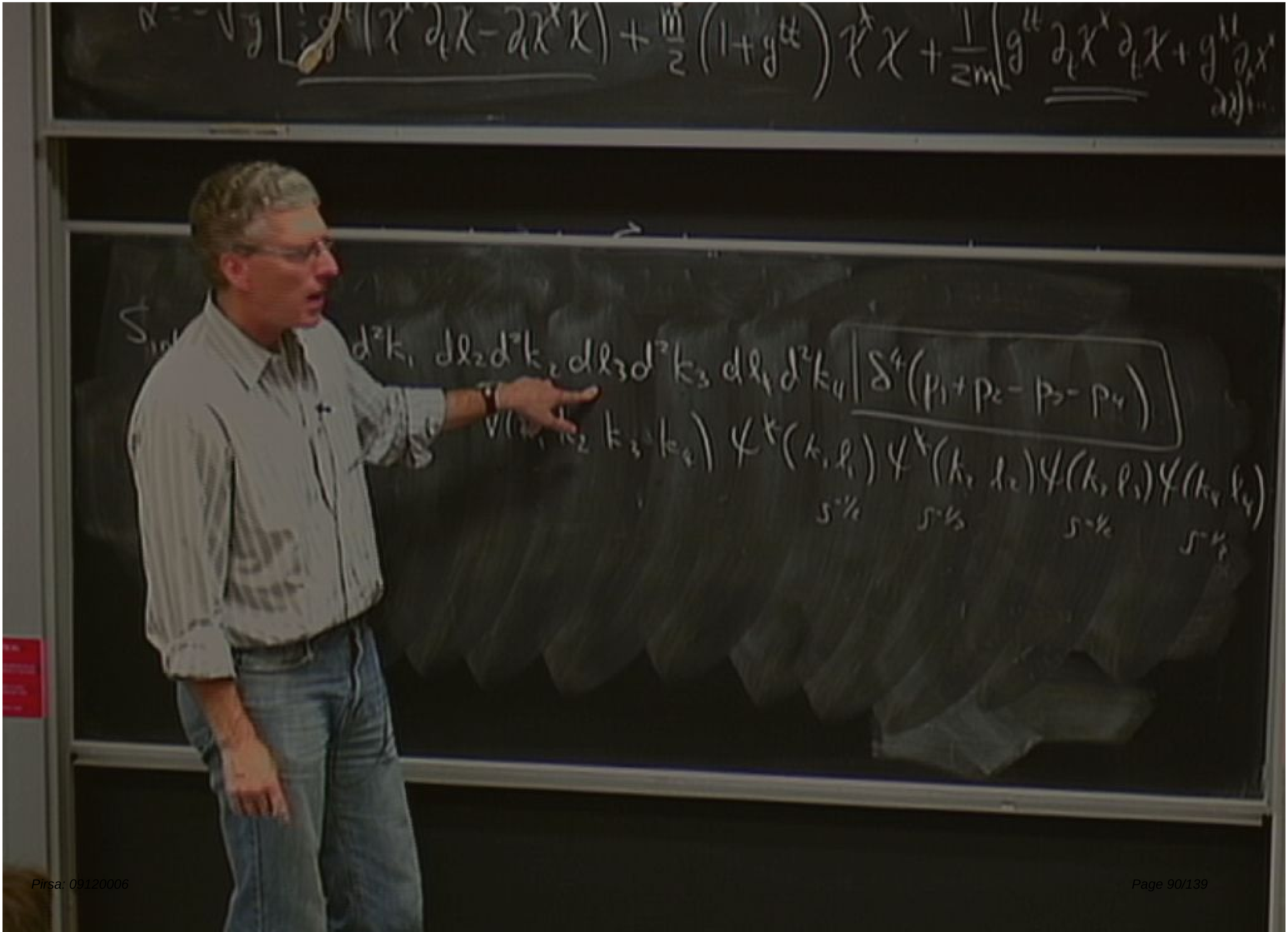
each new particle: $d^3p_1\, d^3p_2\, \psi^{\dagger}(p_1)\psi(p_2)$

$$\underset{s}{d\ell_1 d^2k_1}\ \underset{s}{d\ell_2 d^2k_2}\ \underset{s^{-1/2}}{\psi^{*}(k_1,\ell_1)}\ \underset{s^{-1/2}}{\psi(k_2,\ell_2)} \sim s \qquad \partial_t \sim s$$



$\mathcal{L} = -\sqrt{g}\left[\frac{i}{2}g^{tt}(\chi^\dagger\partial_t\chi - \partial_t\chi^\dagger\chi) + \frac{m}{2}(1+g^{tt})\chi^\dagger\chi + \frac{1}{2m}(g^{tt}\partial_t\chi^\dagger\partial_t\chi + g^{ij}\partial_i\chi^\dagger\right.$
$S_{int} = \int dt\, d\ell_1 d^2k_1\, d\ell_2 d^2k_2\, d\ell_3 d^2k_3\, d\ell_4 d^2k_4\, \delta^4(p_1+p_2-p_3-p_4)$
$V(k_1, k_2, k_3, k_4)\, \psi^\dagger(k_1,\ell_1)\, \psi^\dagger(k_2,\ell_2)\, \psi(k_3,\ell_3)\, \psi(k_4,\ell_4)$




d²k₁ dl₂ d²k₂ dl₃ d²k₃ dl₄ d²k₄ δ⁴(p₁ + p₂ − p₃ − p₄)
ψ*(k₁, l₁) ψ*(k₂, l₂) ψ(k₃, l₃) ψ(k₄, l₄)

$$\mathcal{L} = -\sqrt{g}\left[\tfrac{i}{2}\, g^{t}\left(\chi^\dagger \partial_t \chi - \partial_t \chi^\dagger \chi\right) + \frac{m}{2}\left(1 + g^{tt}\right)\chi^\dagger \chi + \frac{1}{2m}\left[g^{tt}\, \partial_t \chi^\dagger \partial_t \chi + g^{x1}\partial_x \chi^\dagger \ldots\right.\right.$$

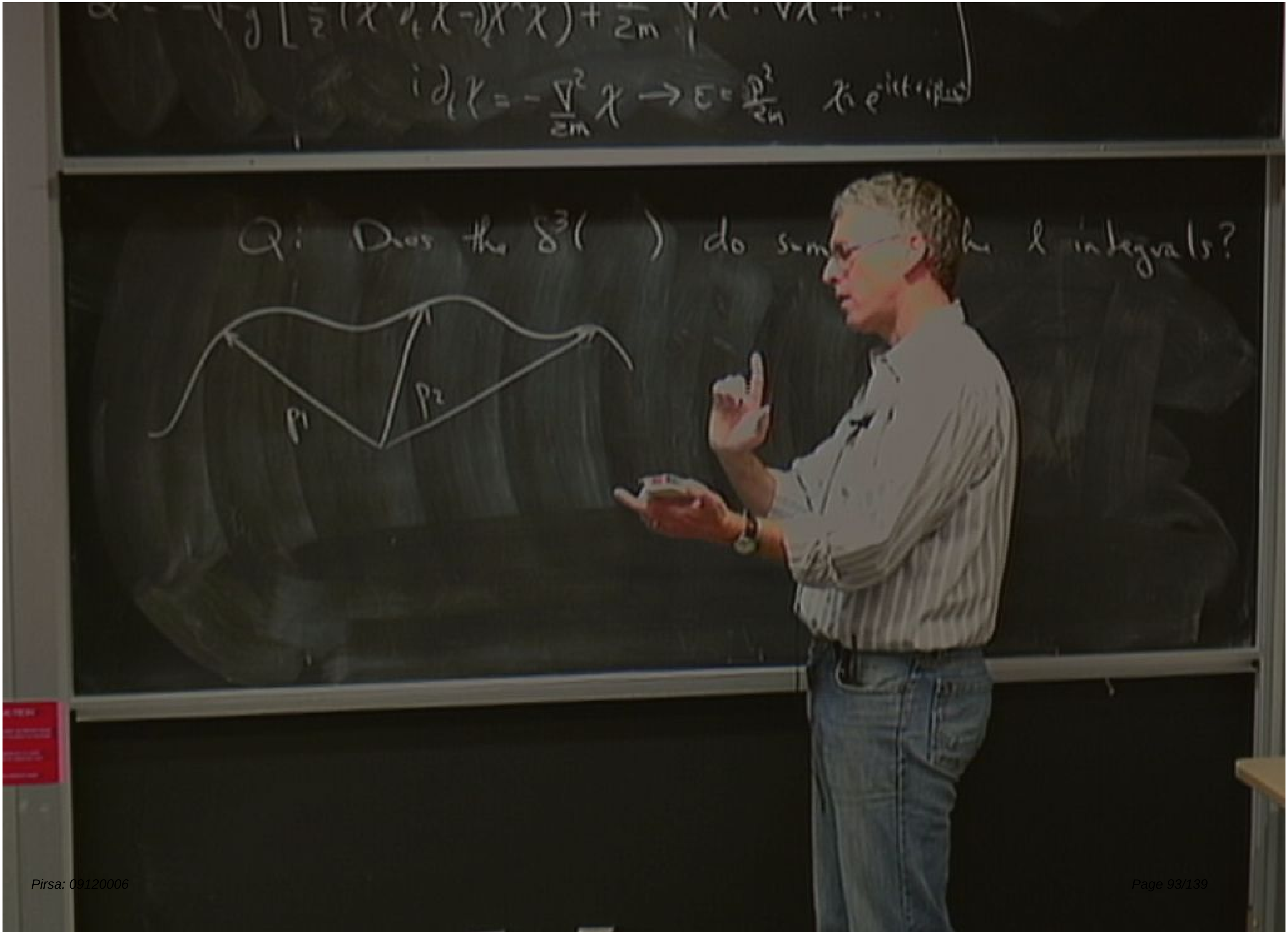
$$S_{int} = \int dt \; \overset{s}{d\ell_1}\, d^2k_1 \; \overset{s}{d\ell_2}\, d^2k_2 \; \overset{s}{d\ell_3}\, d^2k_3 \; \overset{s}{d\ell_4}\, d^2k_4 \; \boxed{\delta^{3}\left(p_1 + p_2 - p_3 - p_4\right)}$$

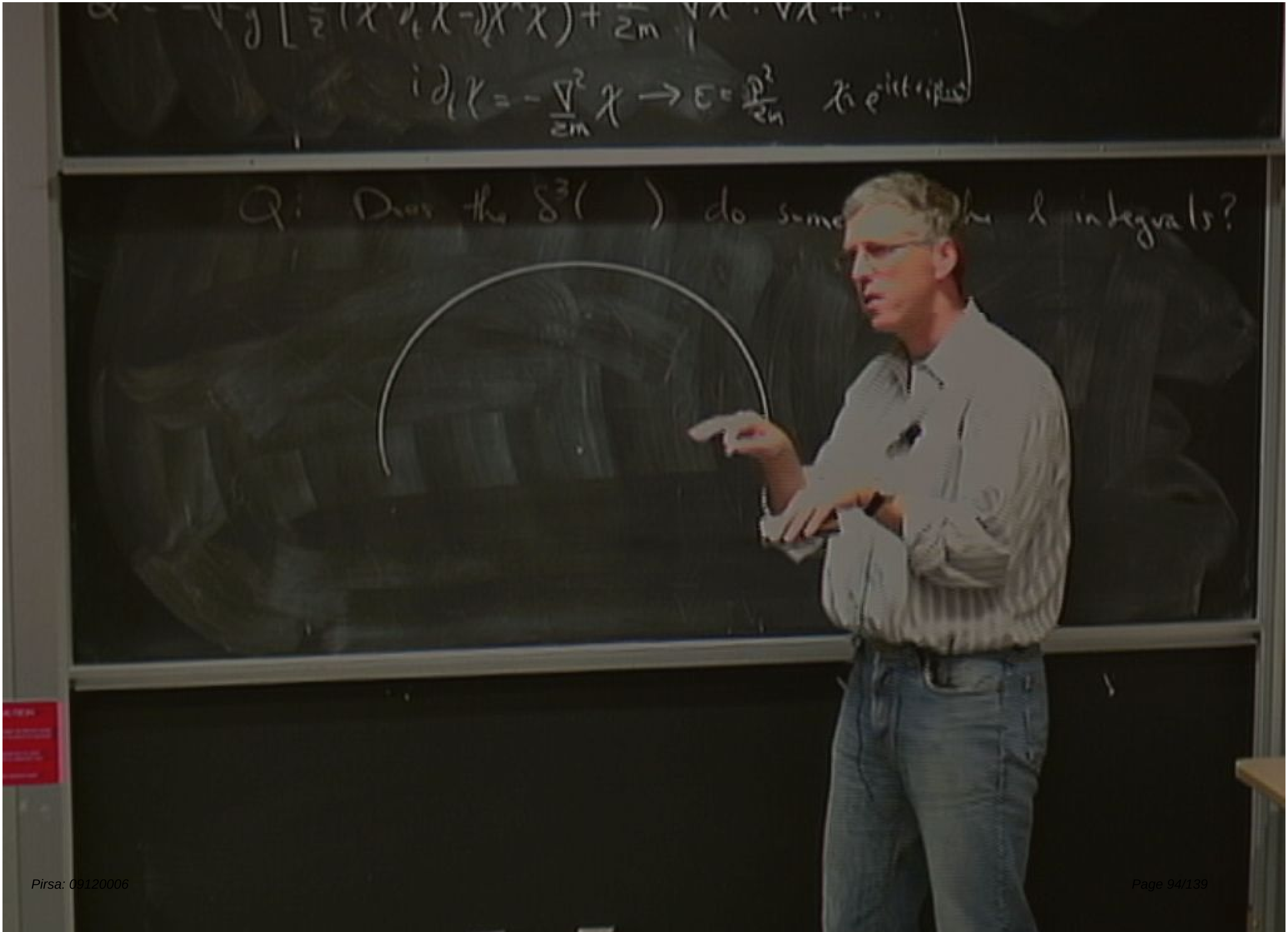
$$\underset{1/s}{} \quad V(k_1, k_2, k_3, k_4)\; \underset{s^{-1/2}}{\psi^\dagger(k_1,\ell_1)}\; \underset{s^{-1/2}}{\psi^\dagger(k_2,\ell_2)}\; \underset{s^{-1/2}}{\psi(k_3,\ell_3)}\; \underset{s^{-1/2}}{\psi(k_4,\ell_4)}$$

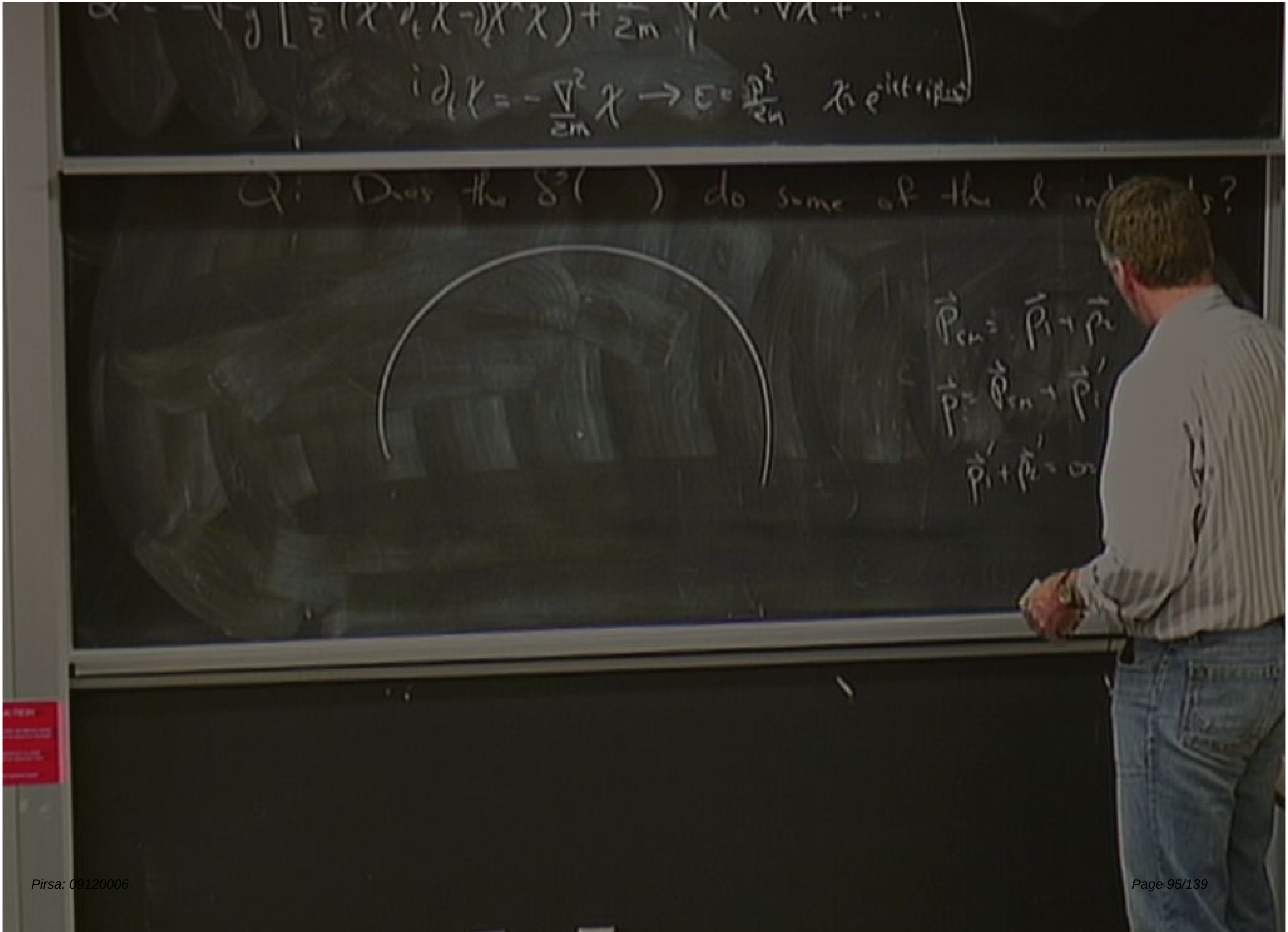
$$\mathcal{L} = -\sqrt{g}\Big[\tfrac{i}{2} g^{t}\left(\chi^{\dagger}\partial_t\chi - \partial_t\chi^{\dagger}\chi\right) + \frac{m}{2}\left(1+g^{tt}\right)\chi^{\dagger}\chi + \frac{1}{2m}\Big[g^{tt}\,\partial_t\chi^{\dagger}\partial_t\chi + g^{ij}\partial_i\chi^{\dagger}\partial_j\chi\Big] + \ldots$$

$$S_{int} = \int dt\ \overset{s}{d\ell_1} d^2k_1\ \overset{s}{d\ell_2} d^2k_2\ \overset{s}{d\ell_3} d^2k_3\ \overset{s}{d\ell_4} d^2k_4\ \boxed{\delta^3\left(p_1+p_2-p_3-p_4\right)}$$

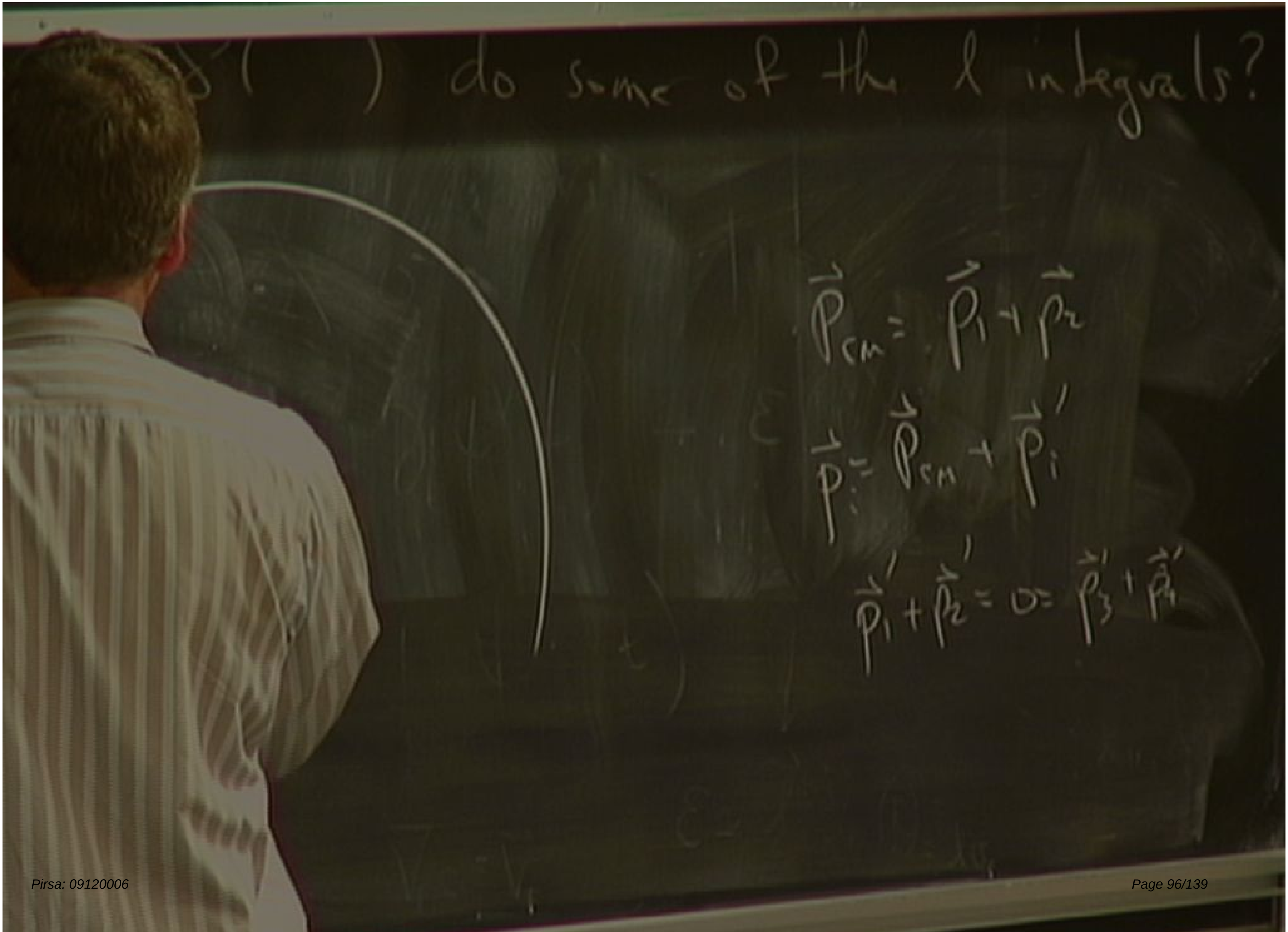
$$\underset{1/s}{} \quad V(k_1, k_2, k_3, k_4)\ \underset{s^{-1/2}}{\psi^{*}(k_1,\ell_1)}\ \underset{s^{-1/2}}{\psi^{*}(k_2,\ell_2)}\ \underset{s^{-1/2}}{\psi(k_3,\ell_3)}\ \underset{s^{-1/2}}{\psi(k_4,\ell_4)}$$

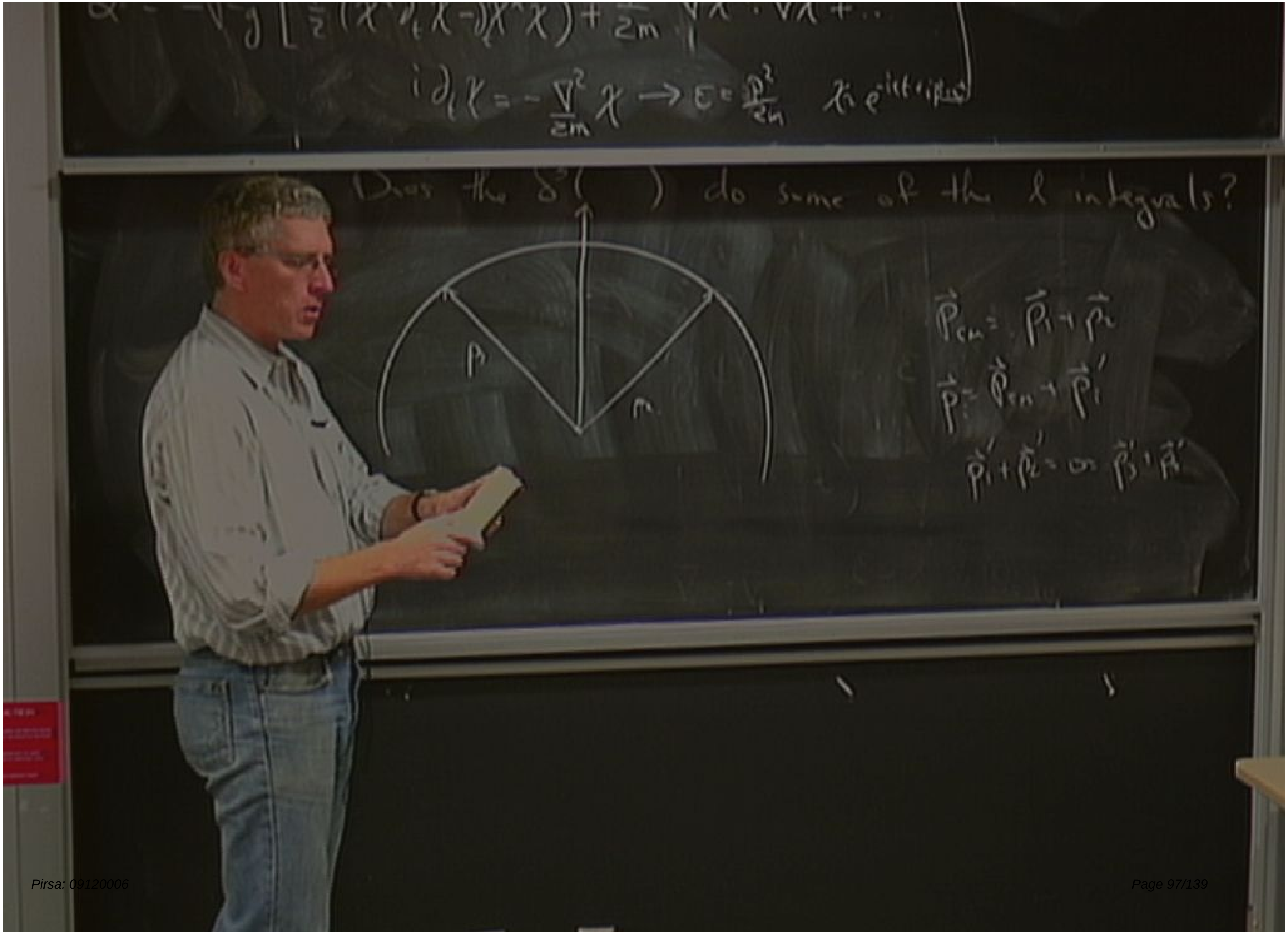

$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{p^2}{2m}$
Q: Does the $\delta^3($ $)$ do som
ℓ integrals?
p_1
p_2

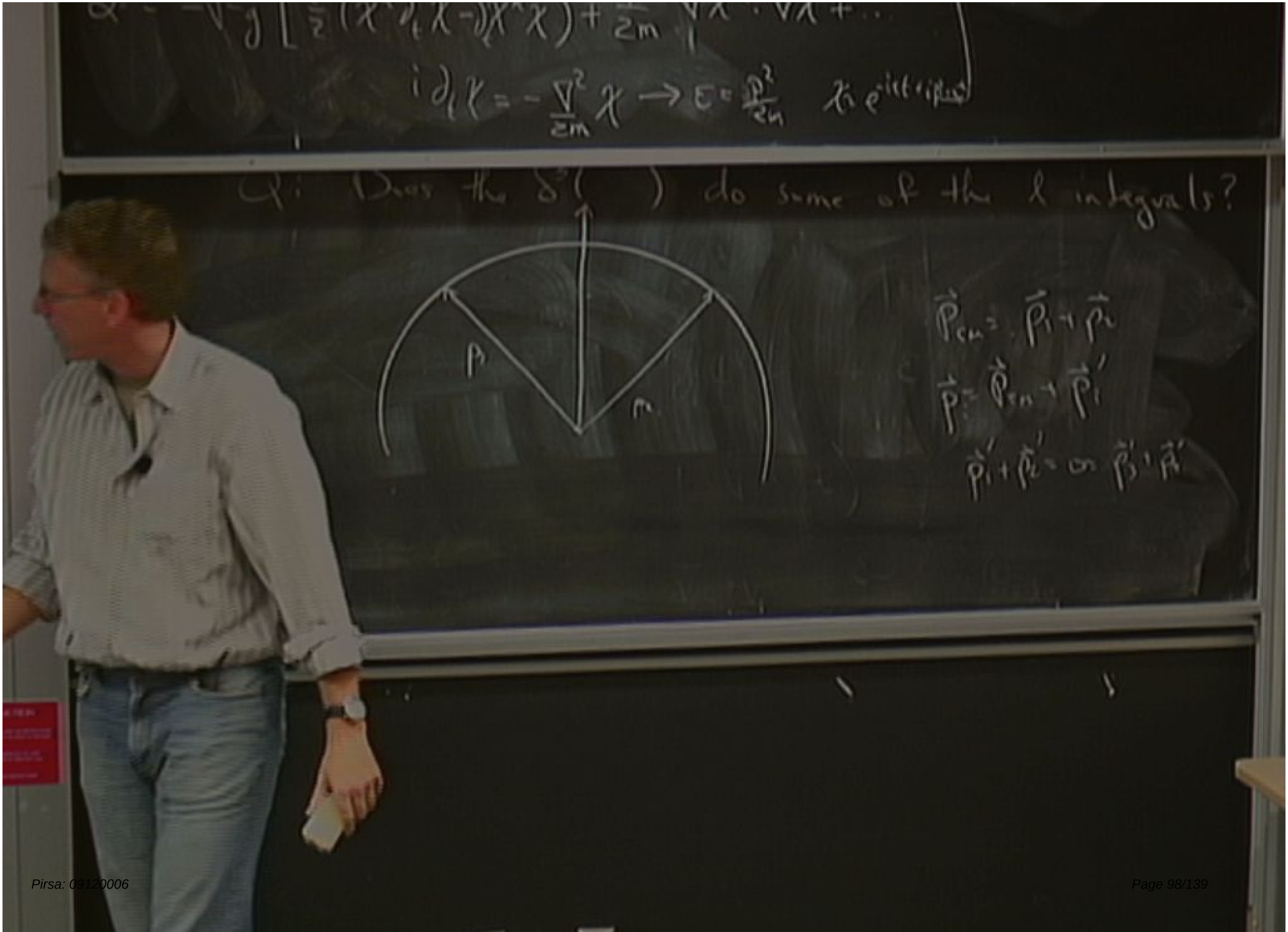

$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{p^2}{2m}$
Q: Does the $\delta^3(\quad)$ do some
l integrals?



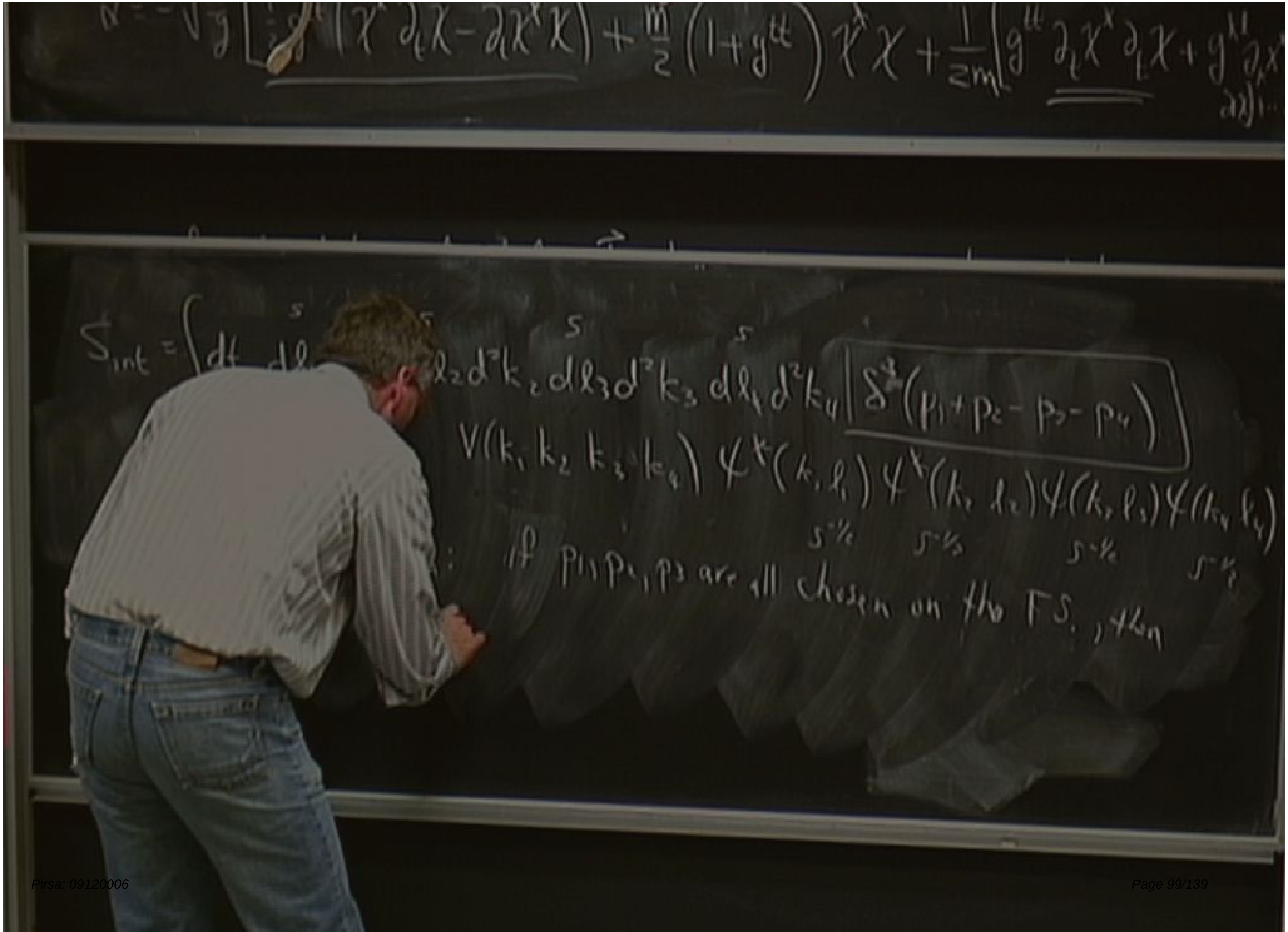
i∂ₜχ = −∇²/2m χ → E = p²/2m
Q: Does the δ³( ) do some of the l in…s?
P_cm = p₁ + p₂


δ( ) do some of the l integrals?
$\vec{P}_{cm} = \vec{p}_1 + \vec{p}_2$
$\vec{p}_i = \vec{P}_{cm} + \vec{p}_i\,'$
$\vec{p}_1\,' + \vec{p}_2\,' = 0 = \vec{p}_3\,' + \vec{p}_4\,'$


$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{p^2}{2m}$
Does the $\delta^3($ $)$ do some of the l integrals?
$\vec{P}_{cm} = \vec{p}_1 + \vec{p}_2$
$\vec{p}_1' + \vec{p}_2' = \vec{p}_3' + \vec{p}_4'$



$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \to E = \frac{p^2}{2m}$
Q: Does the $\delta^3(\ \ )$ do some of the l integrals?
$\vec{P}_{cm} = \vec{p}_1 + \vec{p}_2$
$\vec{p} = \vec{P}_{cm} + \vec{p}_1'$
$\vec{p}_1' + \vec{p}_2' = $ or $\vec{p}_3' + \vec{p}_4'$


S_int = ∫dt dl … l2 d²k2 dl3 d²k3 dl4 d²k4 δ(p1 + p2 − p3 − p4)
V(k1 k2 k3 k4) ψ*(k1 l1) ψ*(k2 l2) ψ(k3 l3) ψ(k4 l4)
if p1, p2, p3 are all chosen on the F.S., then

$$\mathcal{L} = -\sqrt{g}\left[\frac{i}{2} g^{t}\left(\chi^\dagger \partial_t \chi - \partial_t \chi^\dagger \chi\right) + \frac{m}{2}\left(1+g^{tt}\right)\chi^\dagger \chi + \frac{1}{2m}\left[g^{tt}\, \partial_t \chi^\dagger\, \partial_t \chi + g^{ij}\partial_i \chi^\dagger \ldots\right.\right.$$

$$\int dt\, \overset{s}{d\ell_1}\, d^2k_1\, \overset{s}{d\ell_2}\, d^2k_2\, \overset{s}{d\ell_3}\, d^2k_3\, \overset{s}{d\ell_4}\, d^2k_4\, \boxed{\delta^3(p_1+p_2-p_3-p_4)}$$

$$\frac{1}{s} \qquad V(k_1,k_2,k_3,k_4)\, \overset{s^{-1/2}}{\psi^*(k_1,\ell_1)}\, \overset{s^{-1/2}}{\psi^*(k_2,\ell_2)}\, \overset{s^{-1/2}}{\psi(k_3,\ell_3)}\, \overset{s^{-1/2}}{\psi(k_4,\ell_4)}$$

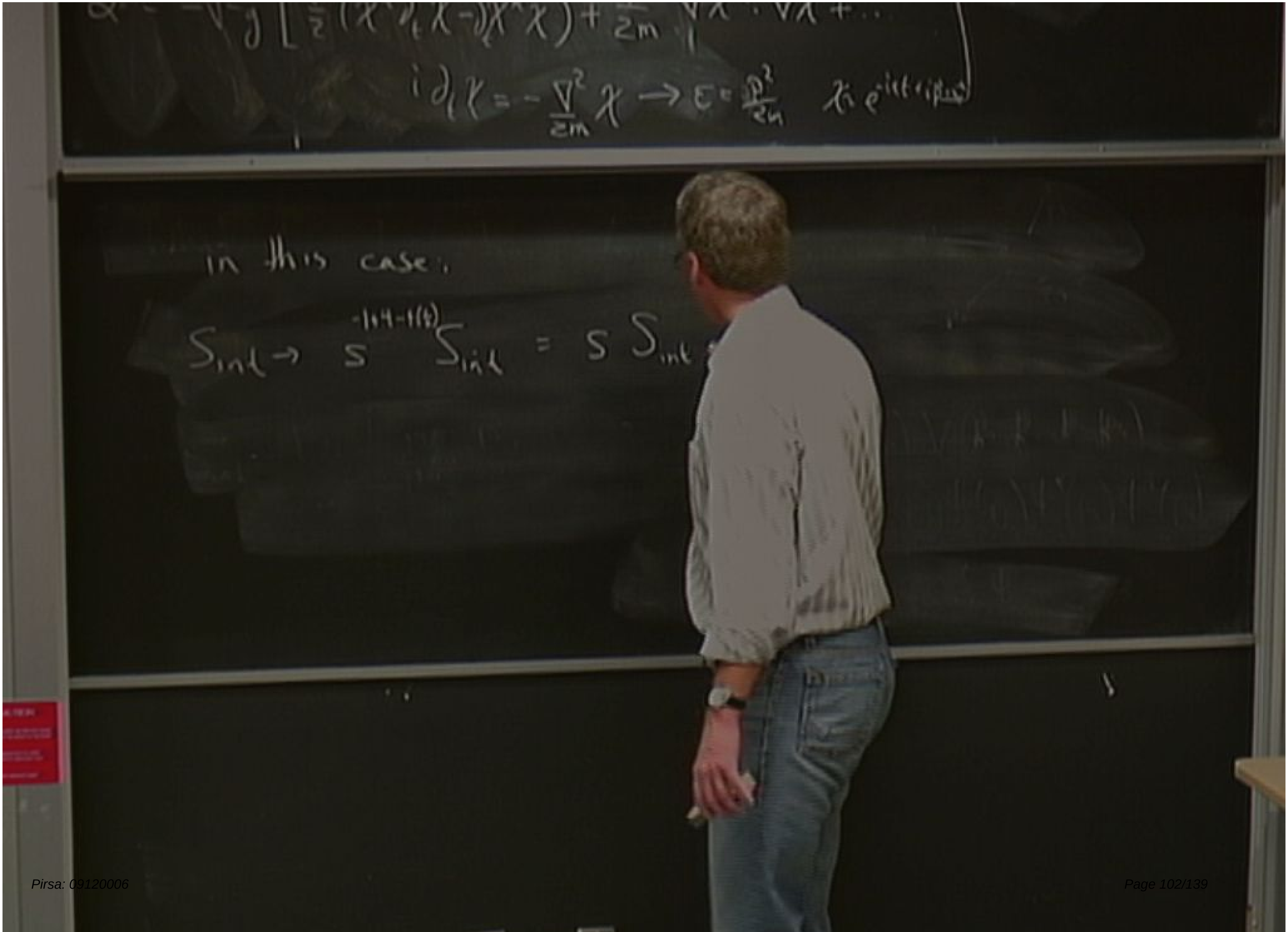
Generic situation: if p_1, p_2, p_3 are all chosen on the F.S., then $p_4 = p_1+p_2-p_3$ is too. So all 3 parts of $\delta^3(p_1+p_2-p_3-p_4)$ are doing d^2k integrals & not $d\ell$ integrals.

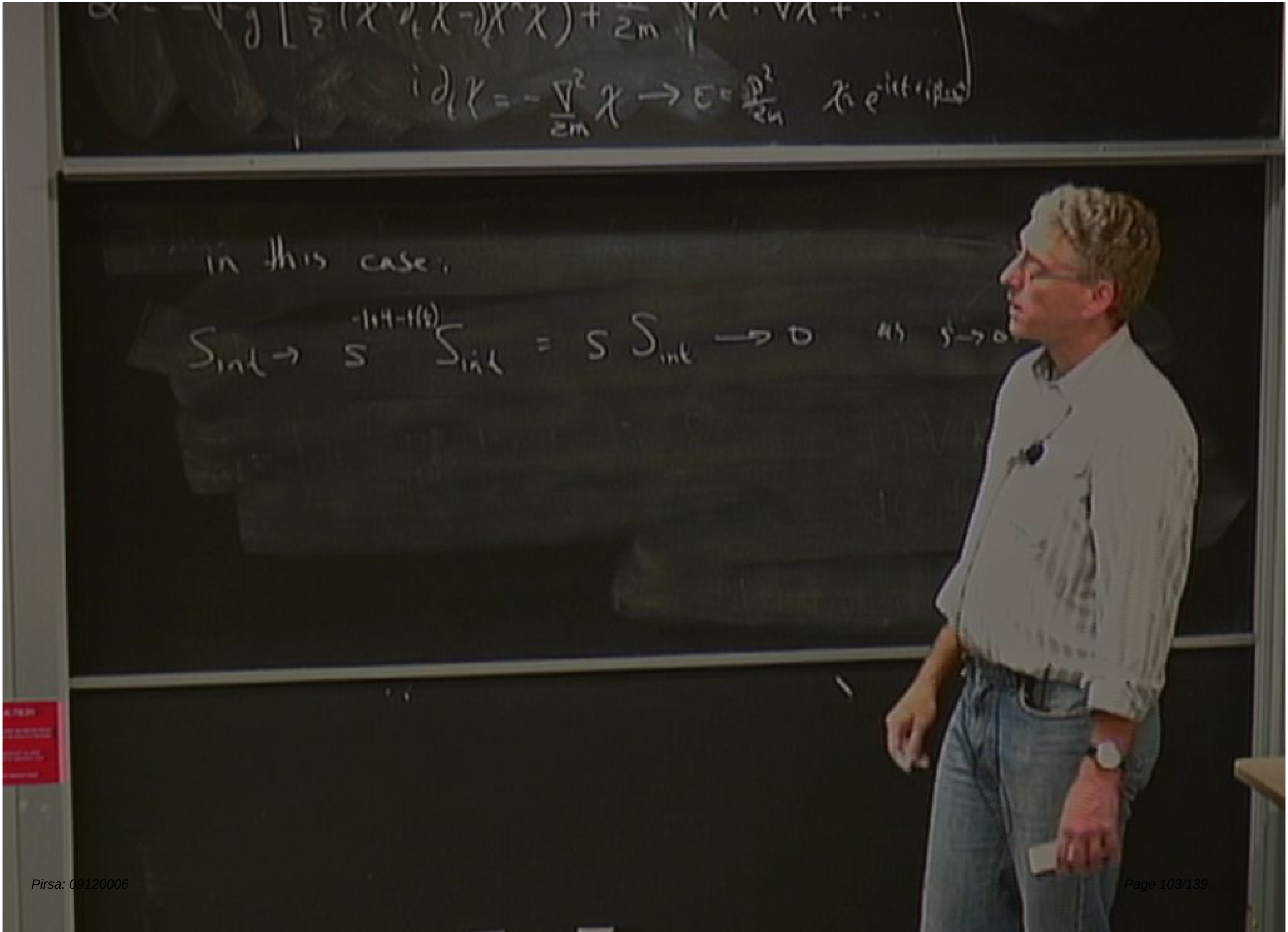
$$\mathcal{L} = -\sqrt{g}\left[\frac{i}{2} g^{t}\left(\chi^\dagger \partial_t \chi - \partial_t \chi^\dagger \chi\right) + \frac{m}{2}\left(1+g^{tt}\right)\chi^\dagger \chi + \frac{1}{2m}\left[g^{tt}\, \partial_t \chi^\dagger \partial_t \chi + g^{ti} \partial_t \chi^\dagger \partial_i \chi \ldots\right.\right.$$

$$S_{int} = \int dt\, \overset{s}{d\ell_1} d^2k_1\, \overset{s}{d\ell_2} d^2k_2\, \overset{s}{d\ell_3} d^2k_3\, \overset{s}{d\ell_4} d^2k_4\, \boxed{\delta^3(p_1+p_2-p_3-p_4)}$$

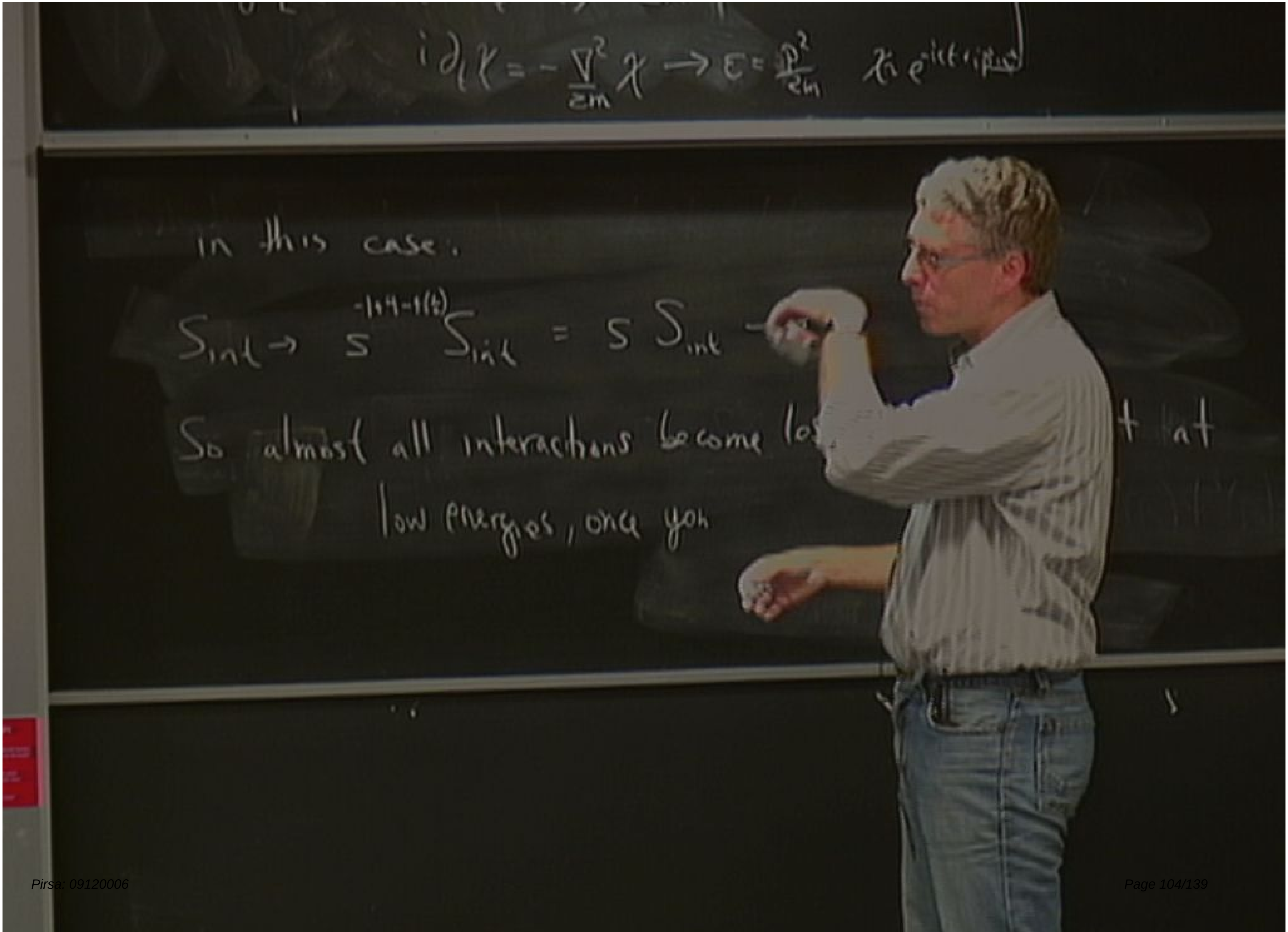
$$1/s \qquad V(k_1, k_2, k_3, k_4)\, \underset{s^{-1/2}}{\psi^\dagger(k_1, \ell_1)}\, \underset{s^{-1/2}}{\psi^\dagger(k_2, \ell_2)}\, \underset{s^{-1/2}}{\psi(k_3, \ell_3)}\, \underset{s^{-1/2}}{\psi(k_4, \ell_4)}$$

Generic situation: if p_1, p_2, p_3 are all chosen on the F.S., then $p_4 = p_1 + p_2 - p_3$ is too. So all 3 parts of $\delta^3(p_1+p_2-p_3-p_4)$ are doing d^2k integrals & not $d\ell$ integrals.


in this case:



in this case:
S_int → s^(-1+4-1(t)) S_int = s S_int → 0 as s → 0



$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{p^2}{2m}$
in this case:
$S_{int} \rightarrow s^{-1+4-4(\frac{1}{2})} S_{int} = s\, S_{int}$
So almost all interactions become lo
t at
low energies, once you

$$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{p^2}{2m} \quad \chi \sim e^{-iEt + ip\cdot x}$$

in this case:

$$S_{int} \rightarrow s^{-1+4-4(t)} S_{int} = s\, S_{int} \longrightarrow 0 \quad \text{as } s \rightarrow 0.$$

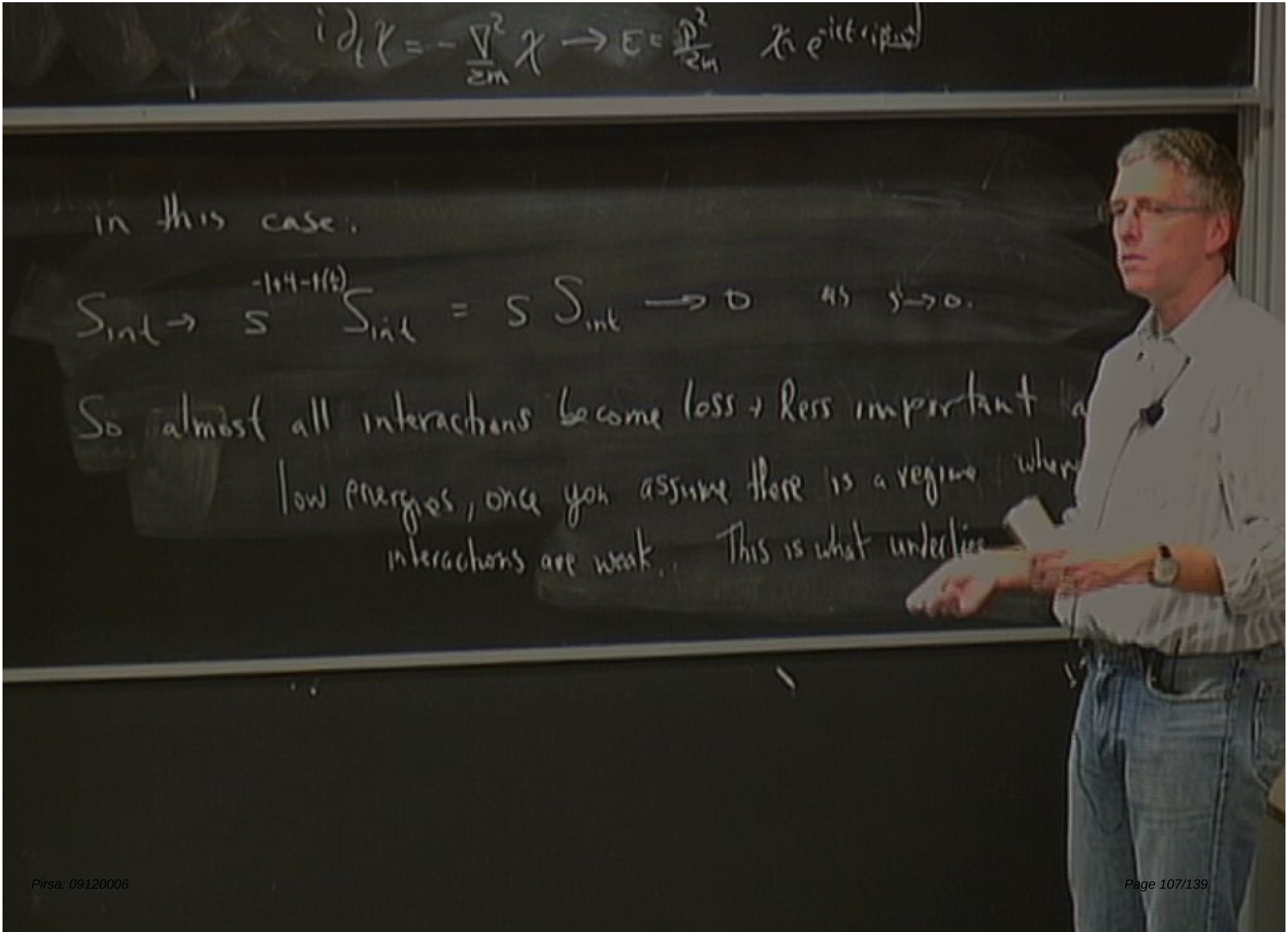
So almost all interactions become less + less important at low energies, once you

$$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \to E = \frac{p^2}{2m} \quad \chi \sim e^{-iEt+ip\cdot x}$$

in this case:

$$S_{int} \to s^{-1+4-4(\frac{1}{2})} S_{int} = s\, S_{int} \to 0 \quad \text{as } s \to 0.$$

So almost all interactions become less + less important at low energies, once you assume there is a regime wh
interactions are weak.



$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \to E = \frac{p^2}{2m}$
in this case:
$S_{int} \to s^{-1+4-1(t)} S_{int} = s\, S_{int} \to 0$ as $s \to 0$.
So almost all interactions become less + less important
low energies, once you assume there is a regime
interactions are weak. This is what

$$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \to E = \frac{p^2}{2m} \quad \chi \sim e^{i(Et + ip\cdot x)}$$

n this case:

$$\to s^{-1+4-4(\frac{1}{2})} S_{int} = s\, S_{int} \to 0 \quad \text{as } s \to 0.$$

nost all interactions become less & less important at low energies, once you assume there is a regime where interactions are weak. This is what underlies the free electrons description of metals.

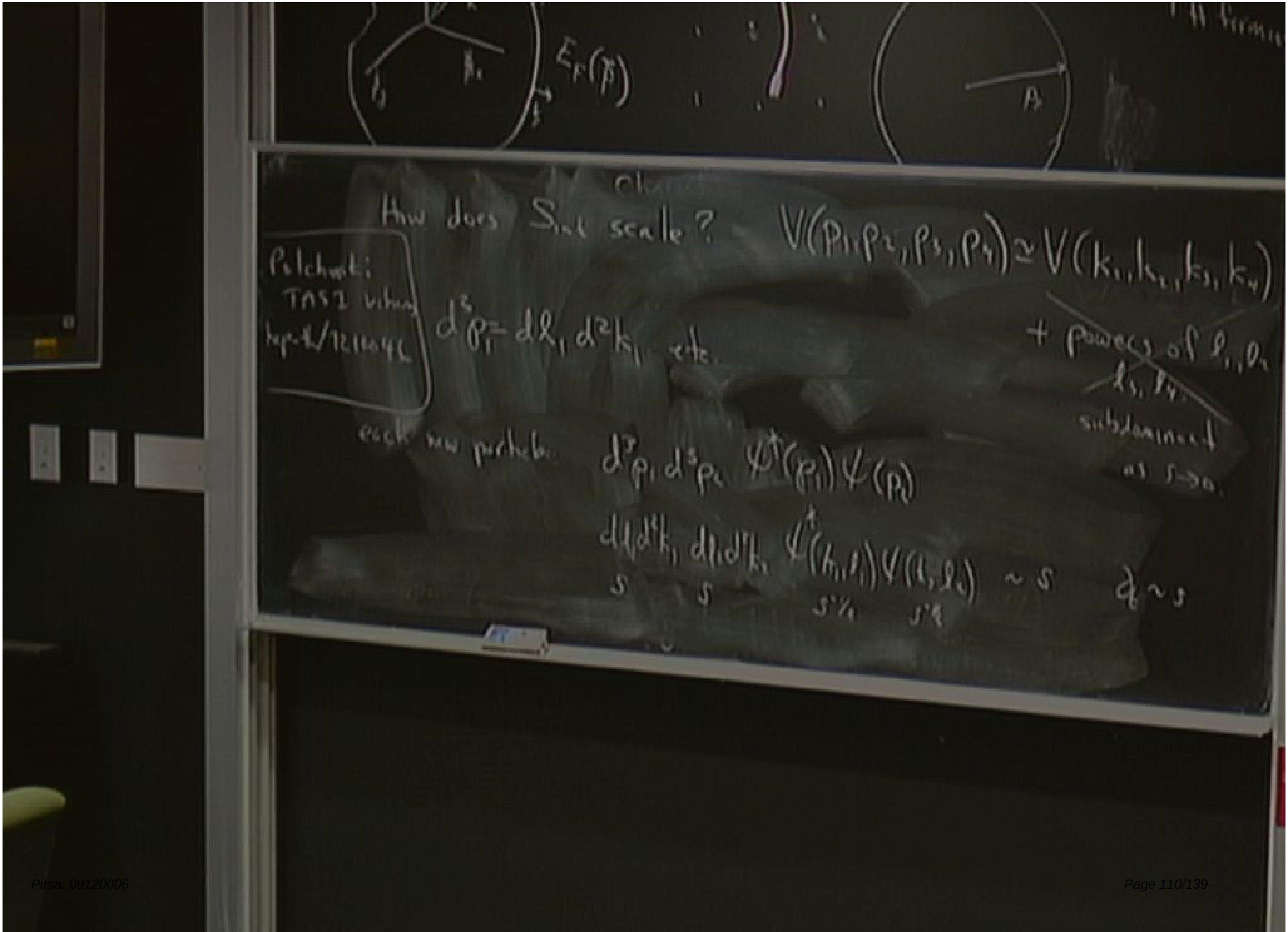
$$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{p^2}{2m} \quad \chi \sim e^{i(Et + \vec{p}\cdot\vec{x})}$$

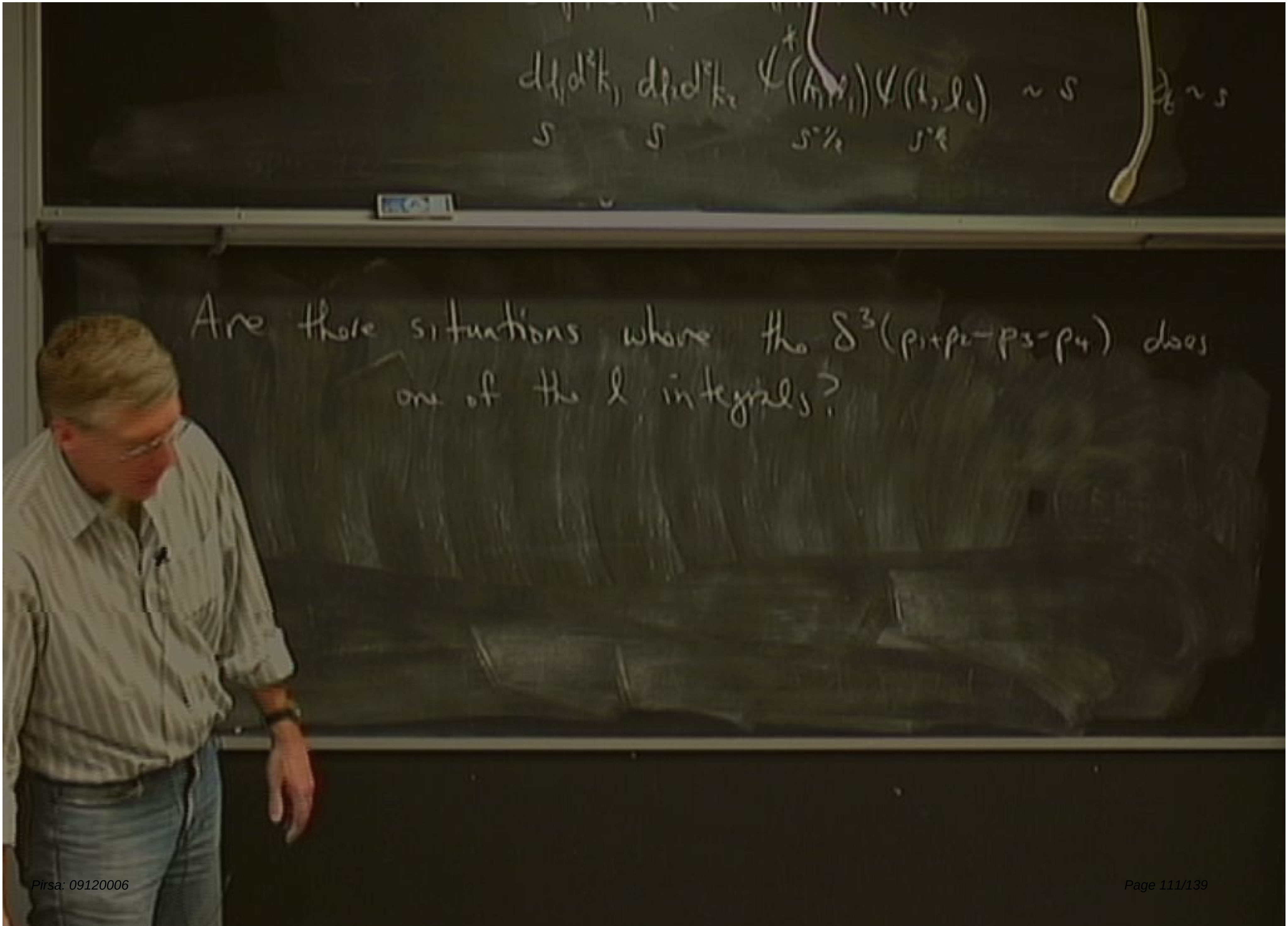
in this case:

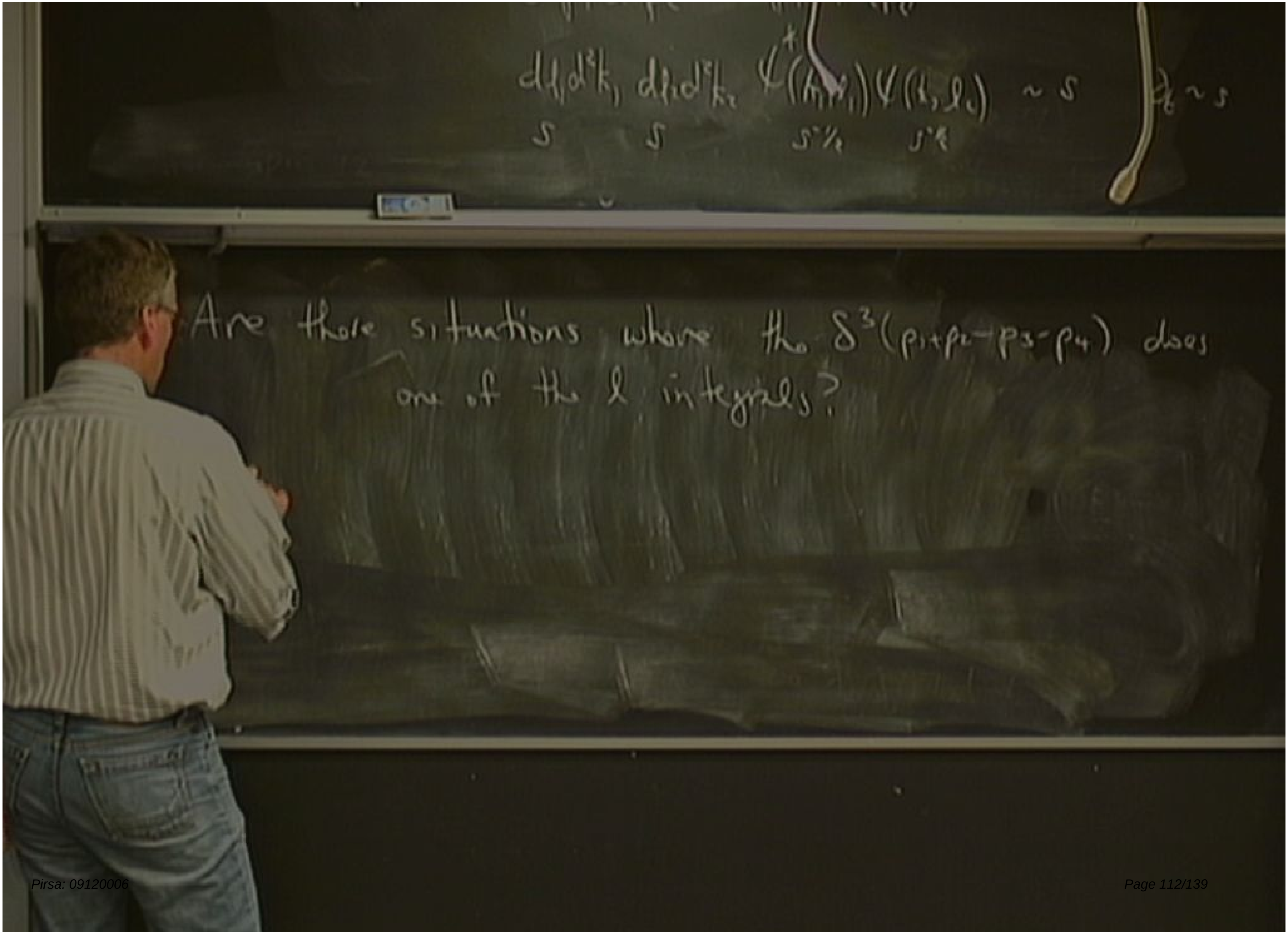
$$S_{int} \rightarrow s^{-1+4-f(t)} S_{int} = s\, S_{int} \longrightarrow 0 \quad \text{as } s \rightarrow 0.$$

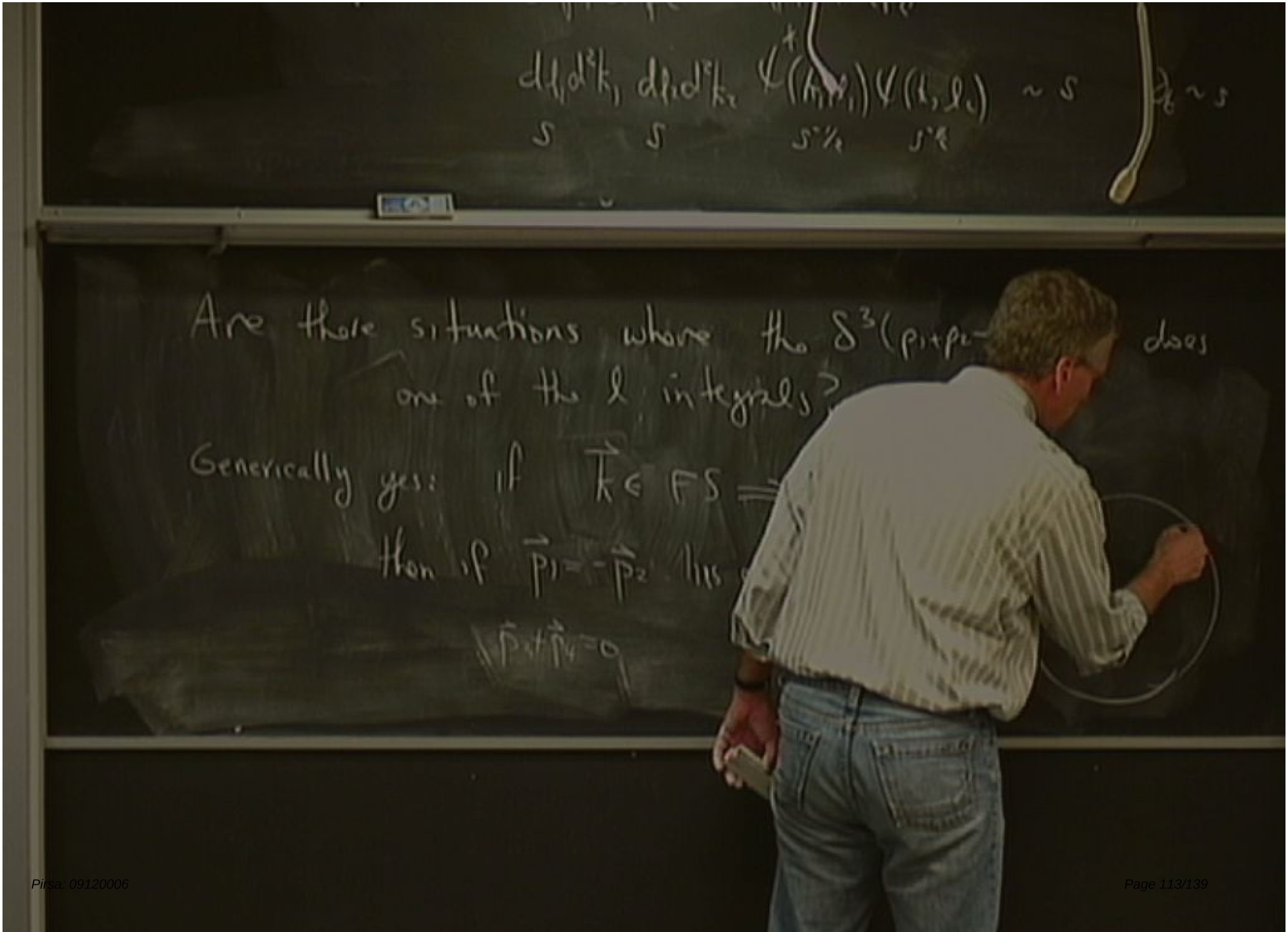
So almost all interactions become less + less important at low energies, once you assume there is a regime where interactions are weak. This is what underlies the free electrons description of metals.

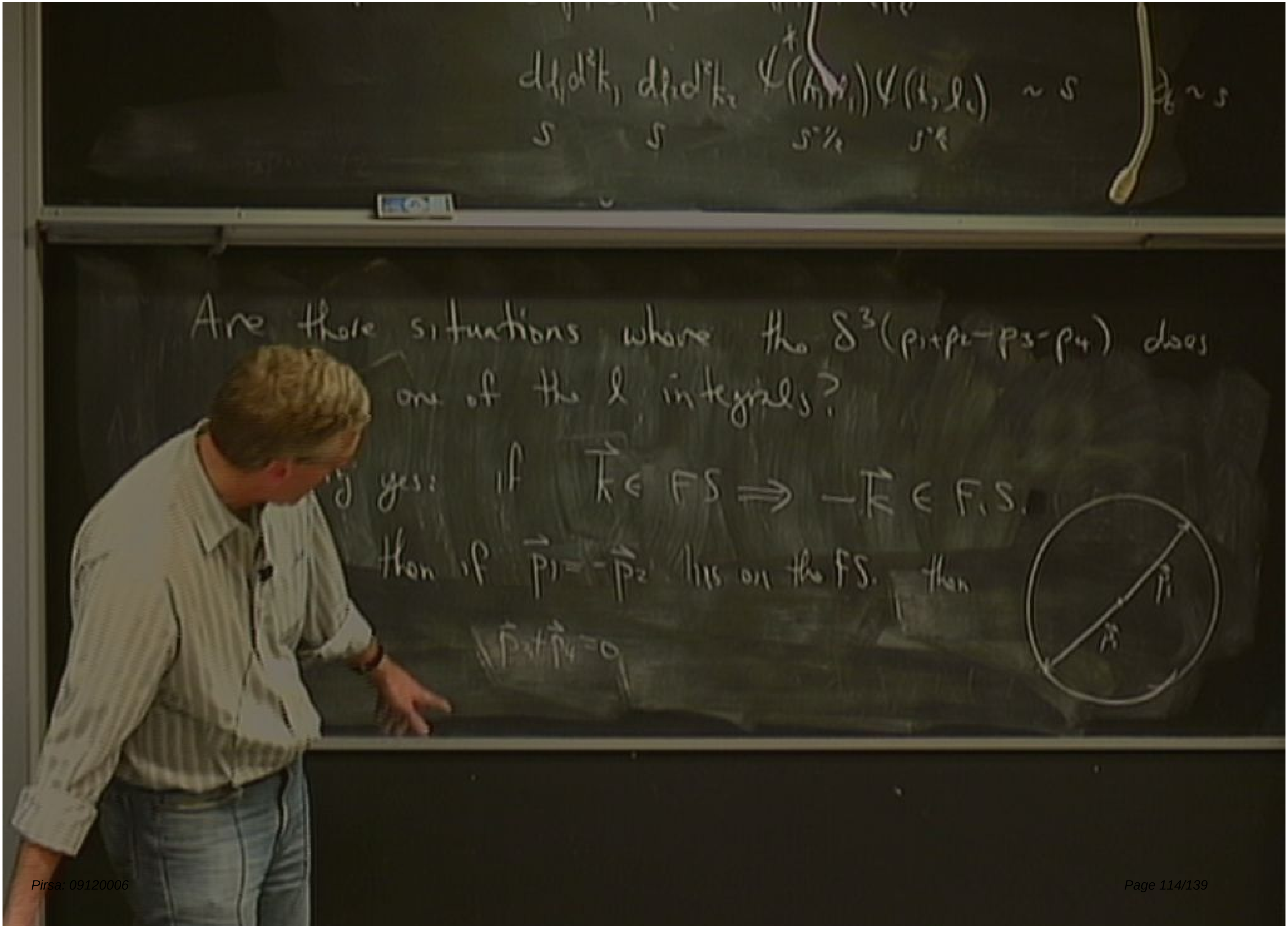
Landau fermi liquid

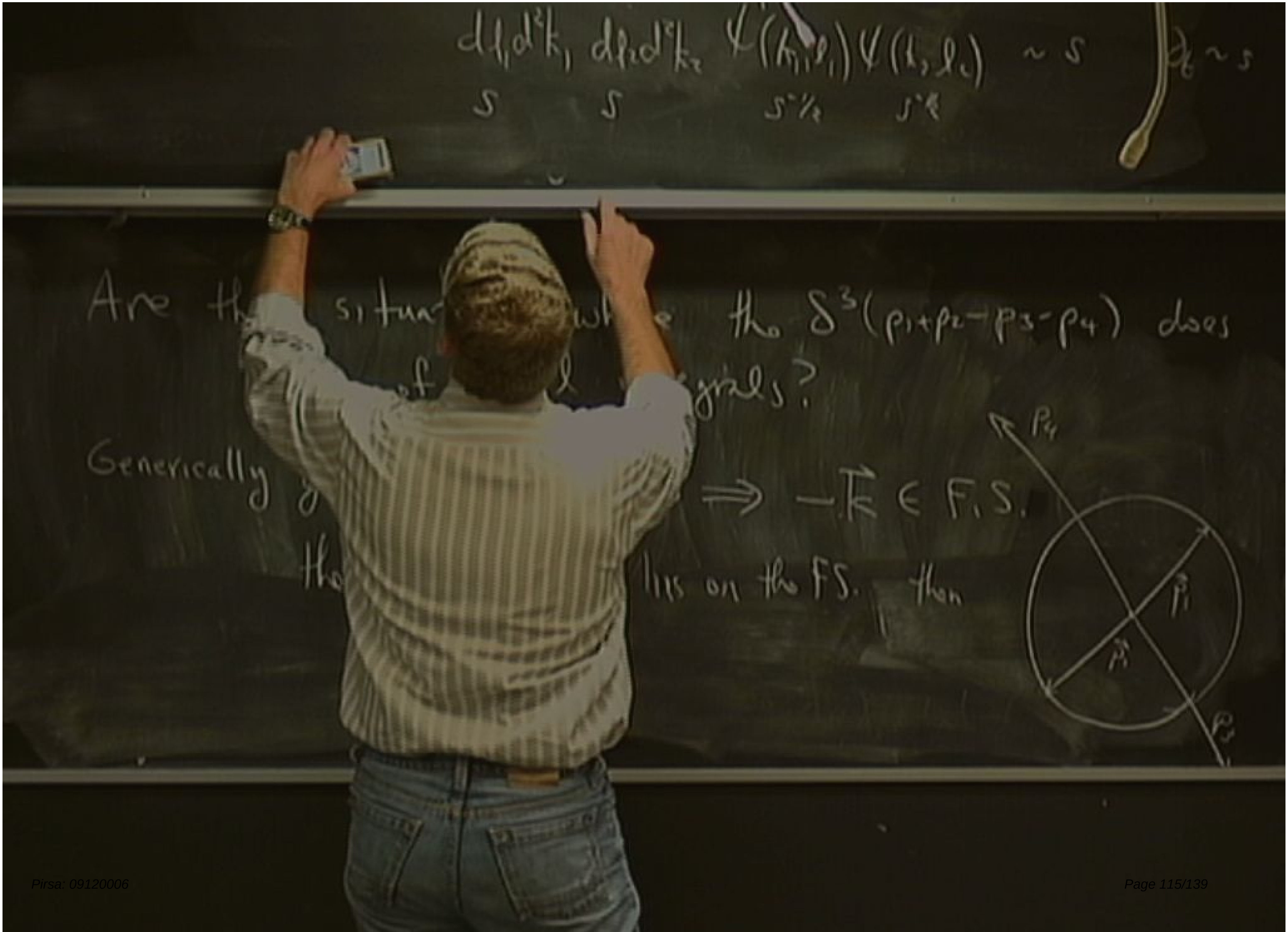

E_F(p)
How does S_int scale?
V(p1, p2, p3, p4) ≃ V(k1, k2, k3, k4)
Polchinski:
TASI lectures
hep-th/9210046
d³p1 = dl1 d²k1 etc.
+ powers of l1, l2, l3, l4
subdominant
as s → 0.
each new particle: d³p1 d³p2 ψ†(p1) ψ(p2)
dl1 d²k1 dl2 d²k2 ψ†(k1, l1) ψ(k2, l2) ~ s
s s s^-1/2 s^-1/2
∂t ~ s


dl1 d²k1 dl2 d²k2 ψ*((k1,l1)) ψ((k2,l2)) ~ s
s s s^-1/2 s^-1/2
∂t ~ s
Are there situations where the δ³(p1+p2−p3−p4) does
one of the l integrals?


$dl_1 d^2k_1\, dl_2 d^2k_2\, \psi^*((k_1,l_1))\, \psi((k_2,l_2)) \sim s$
$s \quad s \quad s^{-1/2} \quad s^{-1/2}$
Are there situations where the $\delta^3(p_1+p_2-p_3-p_4)$ does
one of the l integrals?


dl₁d²k₁ dl₂d²k₂ ψ*((k₁,l₁))ψ((k₂,l₂)) ~ s
Are there situations where the δ³(p₁+p₂−
does
one of the l integrals?
Generically yes: if k ∈ FS ⇒
then if p₁ = −p₂ lies
p₃+p₄=0


$dl_1 d^2k_1 \; dl_2 d^2k_2 \; \psi^*((k_1,l_1))\,\psi((k_2,l_2)) \sim s$
$s \quad s \quad s^{-1/2} \quad s^{-1/2}$
$\partial_t \sim s$
Are there situations where the $\delta^3(p_1+p_2-p_3-p_4)$ does
one of the l integrals?
yes: if $\vec{k} \in FS \Rightarrow -\vec{k} \in F.S.$
then if $\vec{p}_1 = -\vec{p}_2$ lies on the F.S. then
$\vec{p}_3+\vec{p}_4=0$
$\vec{p}_1$
$\vec{p}_2$


$dl_1 d^2k_1 \; dl_2 d^2k_2 \; \psi(k_1,l_1)\,\psi(k_2,l_2) \sim s$
$s \quad s \quad s^{-1/2} \quad s^{-1/2}$
Are th situa wh the $\delta^3(p_1+p_2-p_3-p_4)$ does
Generically $\Rightarrow -\vec{k} \in$ F.S.
lies on the F.S. then
p_4
p_1
p_3

$d\ell_1 d^2k_1 \, d\ell_2 d^2k_2 \; \Psi(k_1,\ell_1)\,\Psi(k_2,\ell_2) \sim s \qquad \partial_t \sim s$

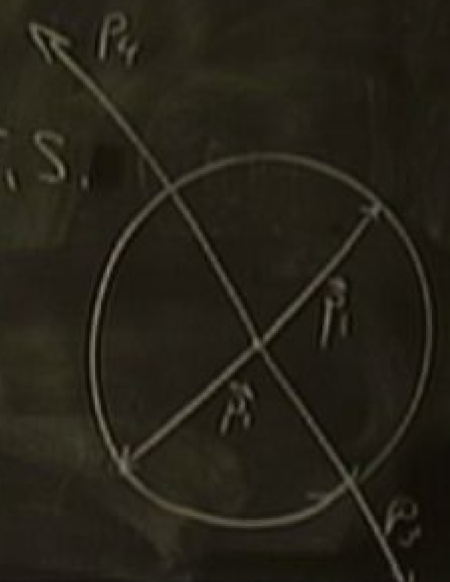
$s \qquad s \qquad s^{-1/2} \qquad s^{-1/2}$

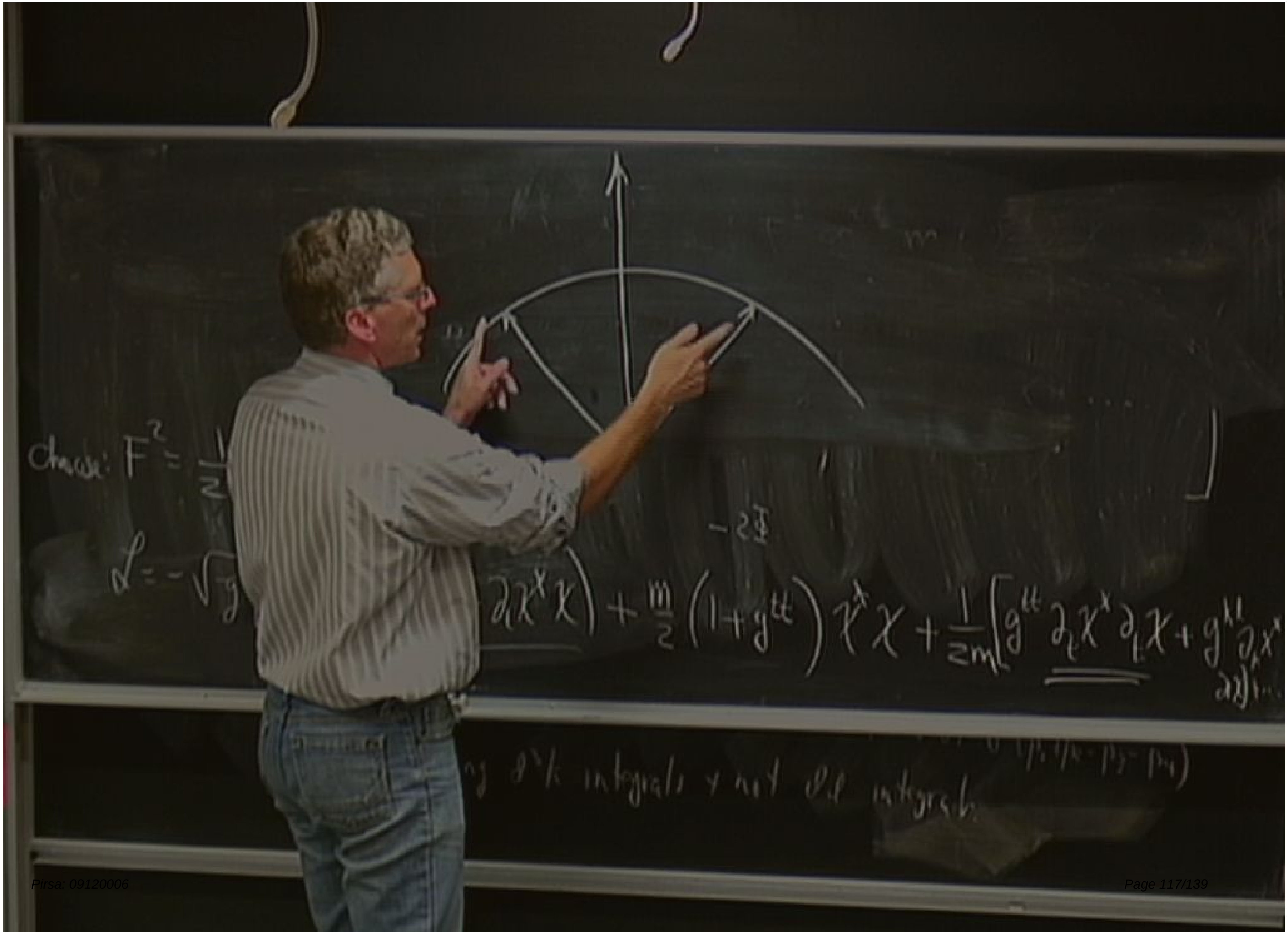
Are there situations where the $\delta^3(p_1+p_2-p_3-p_4)$ does one of the ℓ integrals?

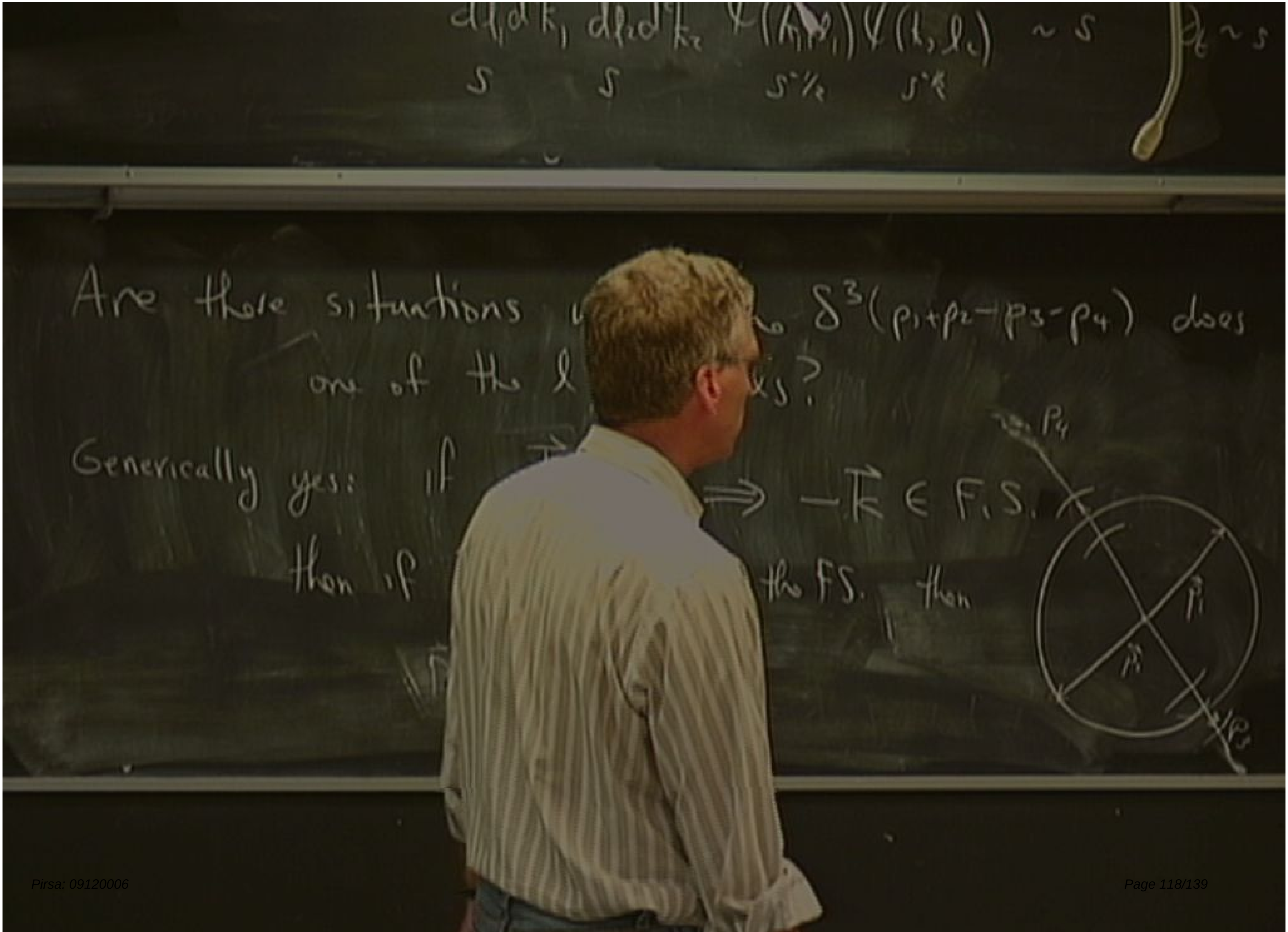
Generically yes: if $\vec{k} \in FS \Rightarrow -\vec{k} \in F.S.$

then if $\vec{p}_1 = -\vec{p}_2$ lies on the FS. then

$\vec{p}_3 + \vec{p}_4 = 0$






$d\ell_1 d^d k_1 \; d\ell_2 d^d k_2 \; \psi(k_1,\ell_1)\,\psi(k_2,\ell_2) \sim s$
$s \quad s \quad s^{-1/2} \quad s^{-1/2}$
Are there situations
$\delta^3(p_1+p_2-p_3-p_4)$ does
one of the ℓ
Generically yes: if
$\Rightarrow -\vec{k} \in$ F.S.
then if
the F.S. then
p_4
$\vec{p}_1$
$\vec{p}_2$
p_3

$d l_1 d^d k_1 \; d l_2 d^d k_2 \; \psi(k_1, l_1)\, \psi(k_2, l_2) \sim s$ $\quad \partial_t \sim s$

$s \quad s \quad s^{-1/2} \quad s^{-1/2}$

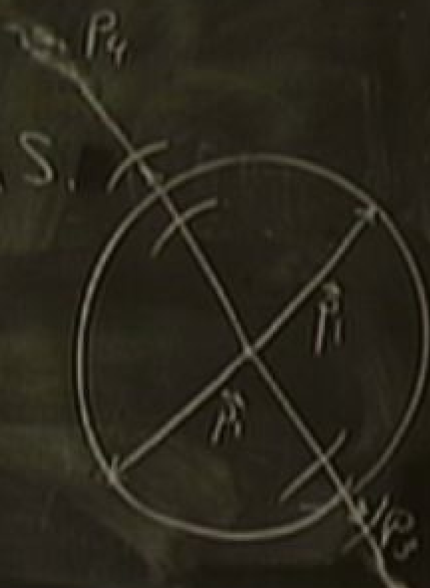
Are there situations where the $\delta^3(p_1+p_2-p_3-p_4)$ does one of the l integrals?

Generically yes: if $\vec{k} \in FS \Rightarrow -\vec{k} \in F.S.$

then if $\vec{p}_1 = -\vec{p}_2$ lies on the FS. then

$\vec{p}_3 + \vec{p}_4 = 0$

so $\delta^3(\Sigma p)$ does exactly one l integral.



$$S_{int} \to s^{-1+3-4(\frac{1}{2})} S_{int} = S_{int}.$$

"marginal" interaction

check: $F^2 = \frac{1}{2m}$ so

$$\mathcal{L} = -\sqrt{g}\left[\frac{i}{2} g^{tt}\left(\chi^\dagger \partial_t \chi - \partial_t \chi^\dagger \chi\right) + \frac{m}{2}\left(1+g^{tt}\right)\chi^\dagger \chi + \frac{1}{2m}\left[g^{tt}\partial_t \chi^\dagger \partial_t \chi + g^{ij}\partial_i \chi^\dagger \partial_j \ldots\right.\right.$$

-2Φ

we doing d^3k integrals & not d^4k integrals

$\delta(p_1 + p_2 - p_3 - p_4)$

$$S_{int} \to s^{-1+3-4\left(\frac{3}{4}\right)} S_{int} = S_{int}.$$

"marginal" interaction

if, $S_{int} \to s^{-k} S_{int}$ $k>0$ ...

"relevant" interaction

choose: $F^2 = \frac{1}{2m}$

$$\mathcal{L} = -\sqrt{g}\left[\frac{i}{2} g^{tt}\left(\chi^* \partial_t \chi - \partial_t \chi^* \chi\right) + \frac{m}{2}\left(1 + g^{tt}\right)\chi^*\chi + \frac{1}{2m}\left[g^{tt}\,\partial_t\chi^*\partial_t\chi + g^{xt}\partial_x\chi^* \ldots\right.\right.$$

we doing $d^d k$ integrals & not $d^d x$ integrals.

$S_{int} \to s^{-1+3-4(\frac{1}{4})} S_{int} = S_{int}$.

"marginal" interaction

Another situation where $\delta^3(\Sigma p)$ does an ℓ integral is when the FS is nested $\vec{n}$ (eg cubic)

$s^{-1/2}$ $s^{-1/2}$ $s^{-1/2}$ $s^{-1/2}$

Generic situation: if p_1, p_2, p_3 are all chosen on the FS, then $p_4 = p_1 + p_2 - p_3$ is too. So all 3 parts of $\delta^3(p_1 + p_2 - p_3 - p_4)$ are doing d^2k integrals & not $d\ell$ integrals.

$S_{int} \to s^{-1+3-4(\frac{1}{2})} S_{int} = S_{int}$.

"marginal" interaction

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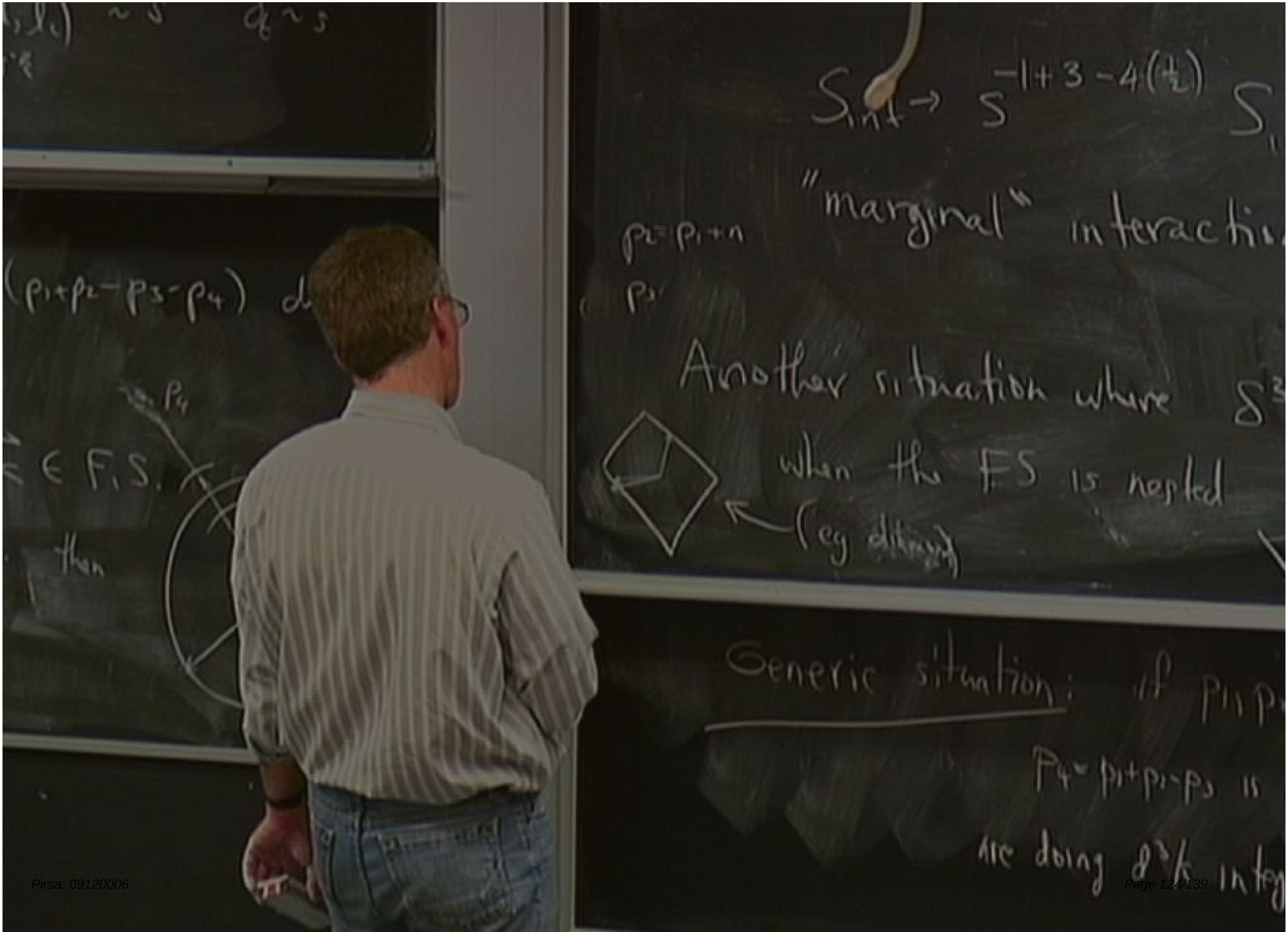
$s^{-1/2}$ $s^{-1/2}$ $s^{-1/2}$ $s^{-1/2}$


"marginal"
Another situation where
when the FS is nested
Generic situation:
then

"marginal" interac

$p_2+p_1 = n$

$p_3+p_4 = \vec{n}$

Another situation where

When the FS is nested

(eg diamond)

$S_{int} \to s^{-1+3-4(\frac{1}{2})} \, S_{int} = S_{int}$.

"marginal" interaction

$p_1 + p_2 = n$

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Another situation where $\delta^3(\Sigma p)$ ... l integral is

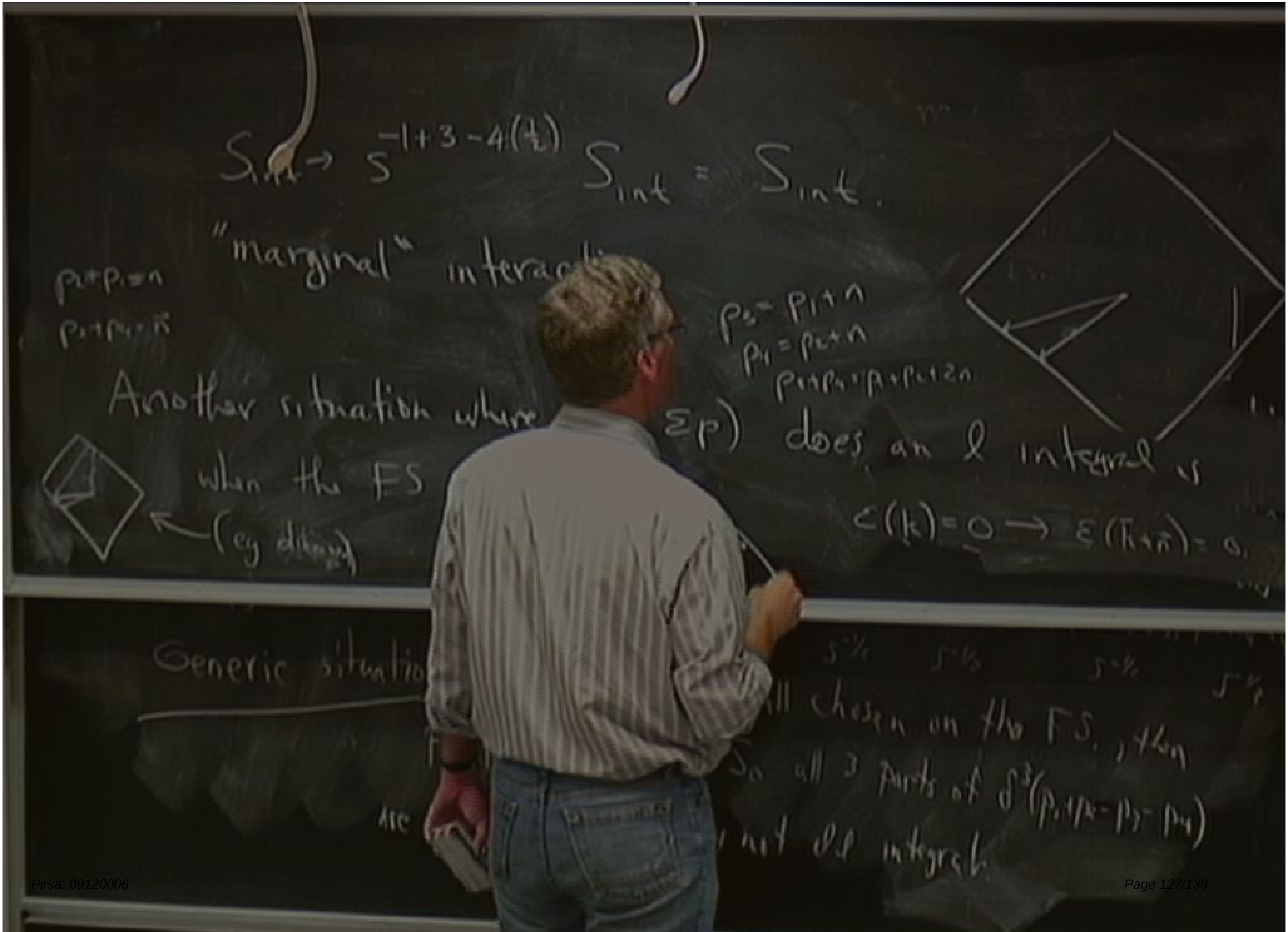
when the FS is nested ... $\to \varepsilon(\vec{k} + \vec{n}) = \varepsilon$.

(eg diamond)

Generic situation: if p_1, p_2 ... the FS, then

$p_4 = p_1 + p_2 - p_3$ is ... of $\delta^3(p_1 + p_2 - p_3 - p_4)$

we doing d^3k int... ch.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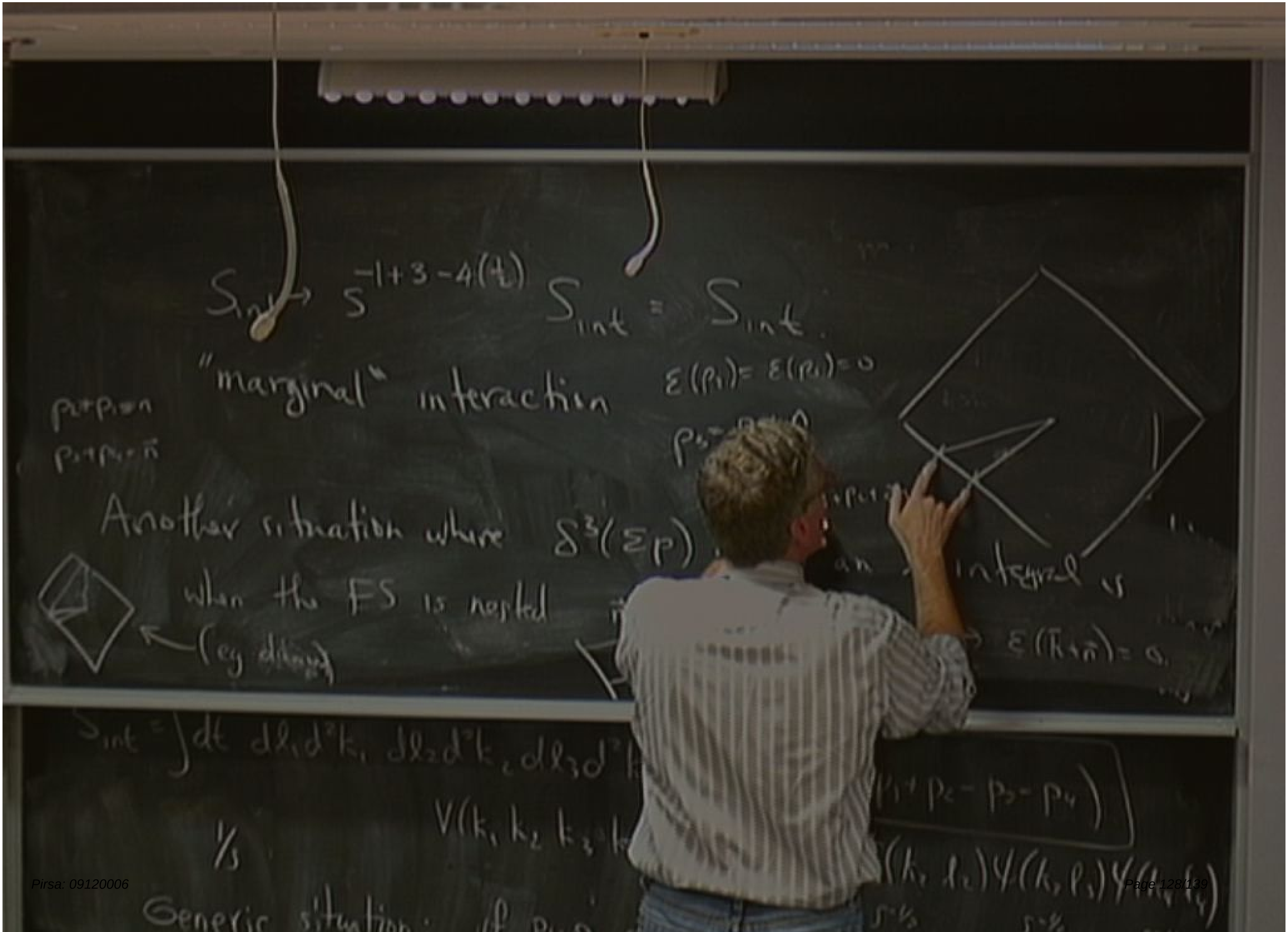
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$$S_{int} \to s^{-1+3-4(\frac{1}{2})} S_{int} = S_{int}.$$

"marginal" interaction $\quad \varepsilon(p_1) = \varepsilon(p_2) = 0$

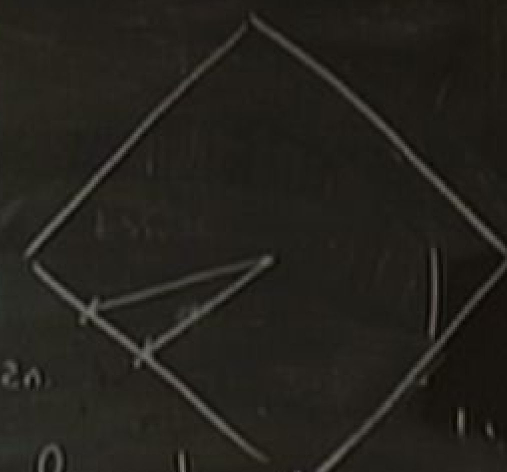
$p_2 + p_1 = n$

$p_3 + p_4 = \vec{n}$

$p_3 = p_1 + n$

$p_4 = p_2 + n$

$p_3 + p_4 = p_1 + p_2 + 2n$



Another situation where $\delta^3(\Sigma p)$ does an ℓ integral is when the FS is nested $\vec{n}$

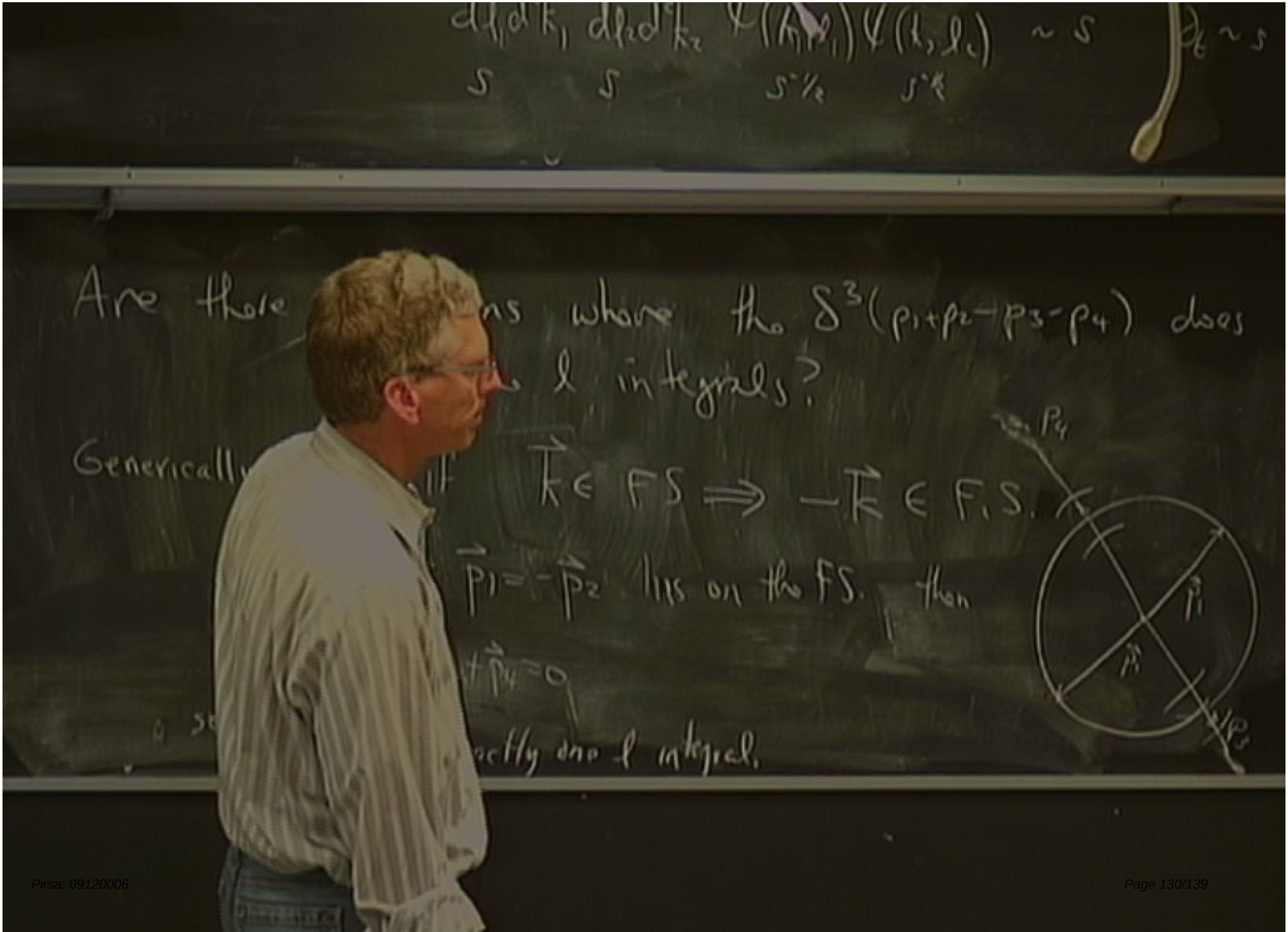
(e.g. diamond)

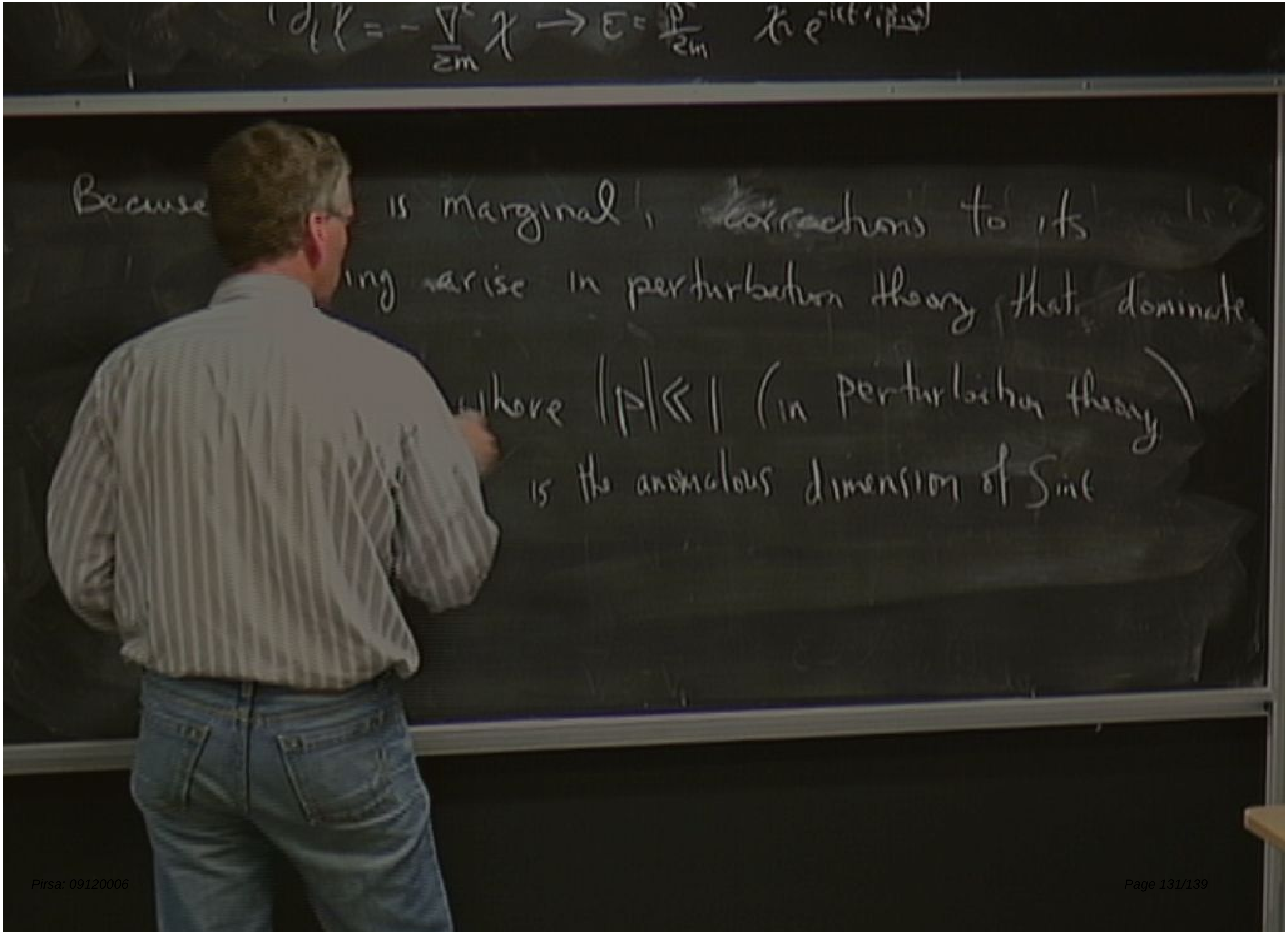
$\varepsilon(k) = 0 \to \varepsilon(\vec{k} + \vec{n}) = 0.$

$$S_{int} = \int dt\, d\ell_1 d^2k_1\, d\ell_2 d^2k_2\, d\ell_3 d^2k_3\, d\ell_4 d^2k_4 \boxed{\delta^3(p_1 + p_2 - p_3 - p_4)}$$

$$\tfrac{1}{s} \quad V(k_1, k_2, k_3, k_4)\, \psi^\dagger(k_1, \ell_1)\, \psi^\dagger(k_2, \ell_2)\, \psi(k_3, \ell_3)\, \psi(k_4, \ell_4)$$

Generic situation: if $p_1 \ldots$ $\quad s^{-1/2} \quad s^{-1/2} \quad s^{-1/2}$


Are there
ns where the $\delta^3(p_1+p_2-p_3-p_4)$ does
l integrals?
Generically
$\vec{k} \in FS \Rightarrow -\vec{k} \in F.S.$
$\vec{p}_1 = -\vec{p}_2$ lies on the FS. then
$+\vec{p}_4 = 0$
ctly one l integral.
p_4
$\vec{p}_1$
$\vec{p}_3$


is marginal, corrections to its
arise in perturbation theory that dominate
where ||p|| ≪ 1 (in perturbation theory)
is the anomalous dimension of Sint

$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \to E = \frac{p^2}{2m}$ $\chi \sim e^{i(Et + ip\cdot x)}$

Because S_{int} is marginal, corrections to its scaling arise in perturbation theory that dominate

$S_{int} \to s^{\Delta} S_{int}$ where $|\Delta| \ll 1$ (in perturbation theory)

is the anomalous dimension of S_{int}

then if $\Delta > 0$

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then if $\Delta > 0$

$i\partial_t \chi = -\frac{\nabla^2}{2m}\chi \rightarrow E = \frac{p^2}{2m} \quad \chi \propto e^{i(Et + i\vec{p}\cdot\vec{x})}$

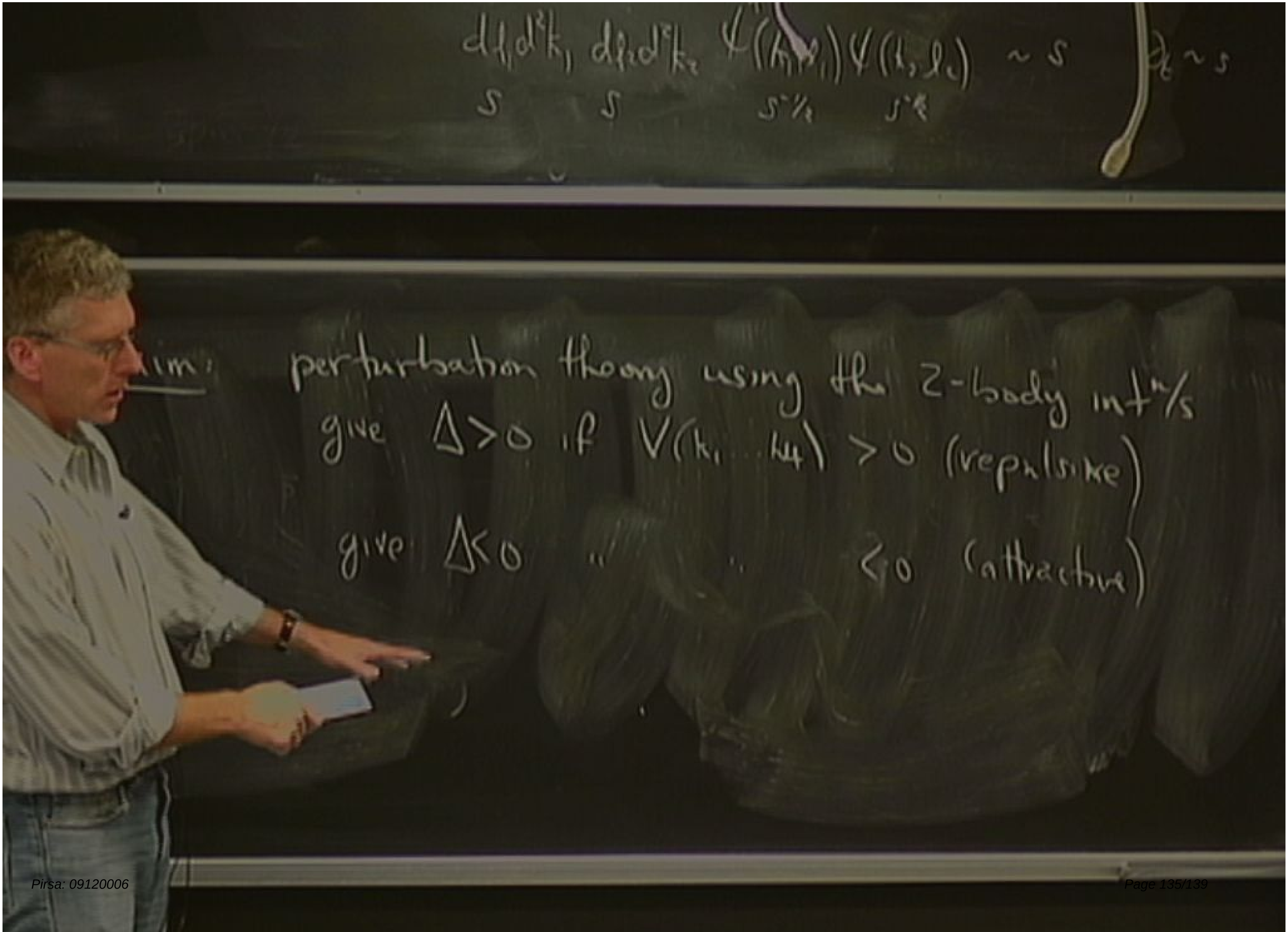
Because S_{int} is marginal, corrections to its scaling arise in perturbation theory that dominate

$S_{int} \rightarrow s^{\Delta} S_{int}$ where $|\Delta| \ll 1$ (in perturbation theory)

Δ is the anomalous dimension of S_{int}

then if $\Delta > 0$ $S_{int} \rightarrow 0$ is called "marginally irrelevant"

" " $\Delta < 0$ $S_{int} \rightarrow \infty$ " " "relevant"


$dt_1 d^2k_1 \; dt_2 d^2k_2 \; \psi(k_1, l_1)\, \psi(k_2, l_2) \sim s$ $\partial_t \sim s$
s s $s^{-1/2}$ $s^{-1/2}$
im: perturbation theory using the 2-body int^n/s
give $\Delta > 0$ if $V(k_1 \ldots k_4) > 0$ (repulsive)
give $\Delta < 0$ " " < 0 (attractive)

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$$s \qquad s \qquad s^{-1/2} \qquad s^{-1/2}$$

<u>Claim:</u> perturbation theory using the 2-body int^n/s
give $\Delta > 0$ if $V(k_1 \ldots k_4) > 0$ (repulsive)
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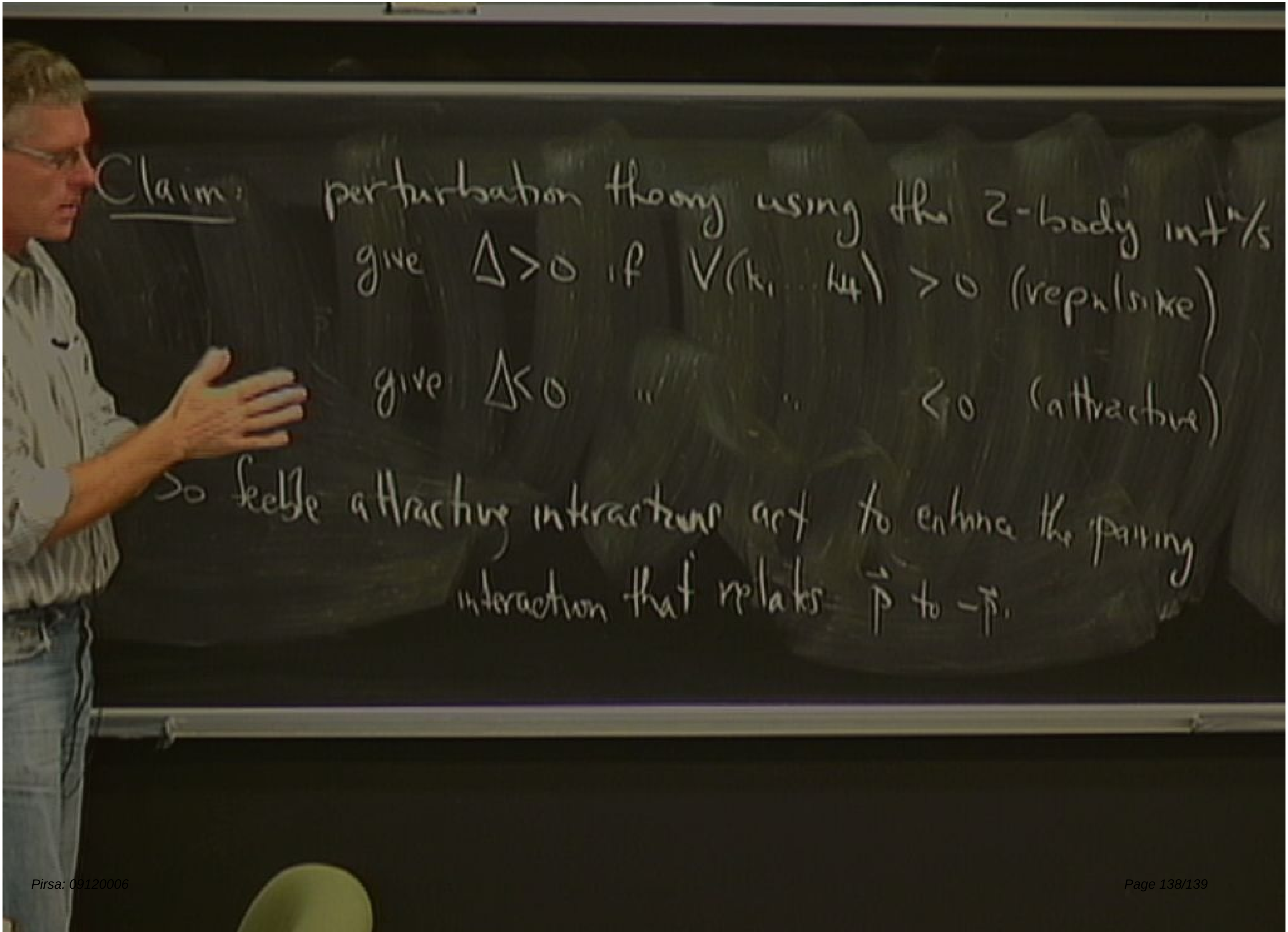
So feeble attractive interactions act to enhance the pairing interaction that relates $\vec{p}$ to $-\vec{p}$.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