

Title: Foundations of Quantum Mech. (PHYS 639) - Lecture 1

Date: Nov 30, 2009 11:00 AM

URL: <http://pirsa.org/09110168>

Abstract:

Foundations of Quantum Theory



Provide an adequate interpretation

Explore nonclassical phenomena

Determine principles from which
quantum theory may be derived

What's the problem?

“Orthodox” postulates of quantum theory

Representational completeness of ψ . The rays of Hilbert space correspond one-to-one with the **physical states** of the system.

Measurement. If the Hermitian operator A with spectral projectors $\{P_k\}$ is measured, the probability of outcome k is $\langle \psi | P_k | \psi \rangle$. These **probabilities are objective -- indeterminism.**

Evolution of isolated systems. It is unitary, $|\psi\rangle \rightarrow U|\psi\rangle = e^{-\frac{i}{\hbar}Ht}|\psi\rangle$ therefore **deterministic and continuous.**

Evolution of systems undergoing measurement. If Hermitian operator A with spectral projectors $\{P_k\}$ is measured and outcome k is obtained, the physical state of the system **changes discontinuously,**

$$|\psi\rangle \rightarrow |\psi_k\rangle = \frac{P_k|\psi\rangle}{\sqrt{\langle \psi | P_k | \psi \rangle}}$$

First problem: the term “measurement” is not defined in terms of the more primitive “physical states of systems”. Isn’t a measurement just another kind of physical interaction?

Two strategies:

- (1) **Realist strategy:** Eliminate measurement as a primitive concept and describe everything in terms of physical states
- (2) **Operational strategy:** Eliminate “the physical state of a system” as a primitive concept and describe everything in terms of operational concepts

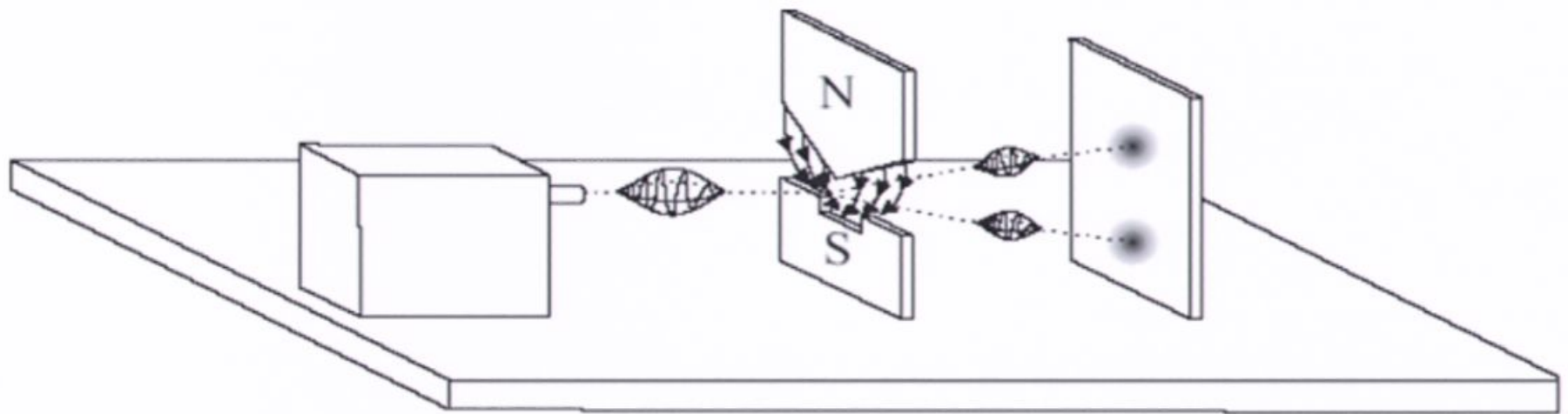
“It would seem that the theory is exclusively concerned about "results of measurement", and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of "measurer"? ”

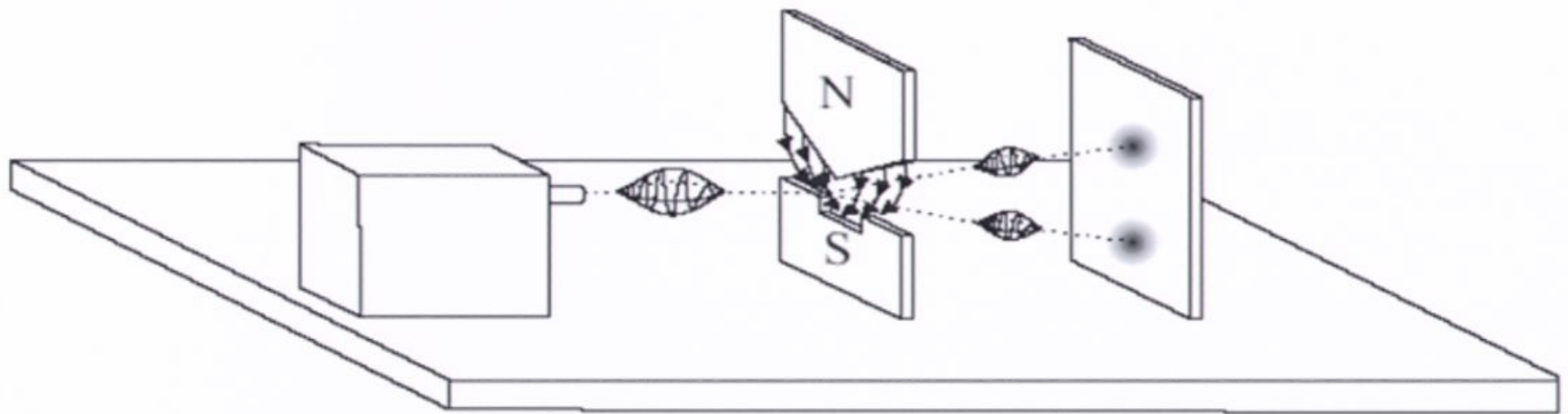
- John Bell

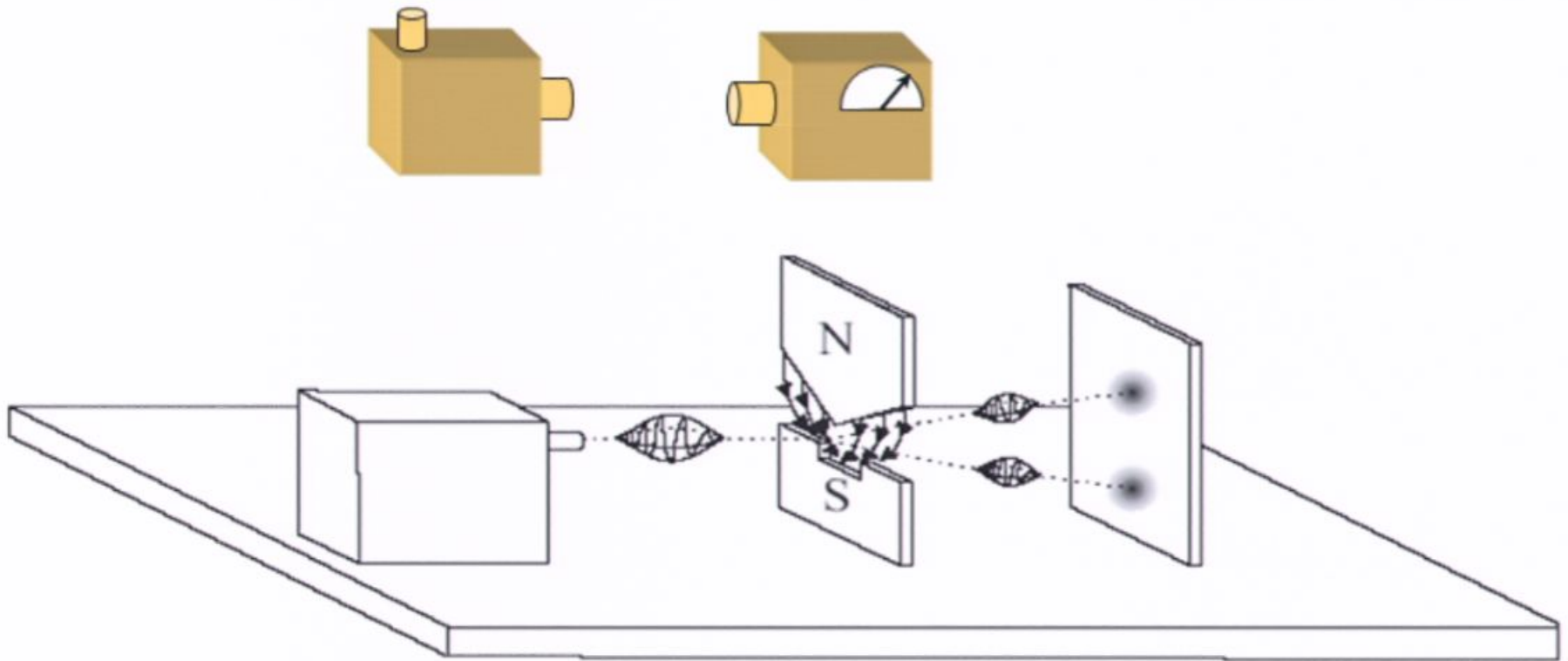
“In a strict sense, quantum theory is a set of rules allowing the computation of probabilities for the outcomes of tests which follow specified preparations.”

- Asher Peres

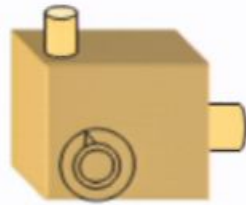
The operational strategy





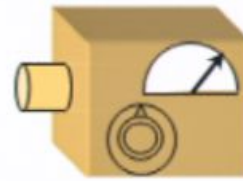


Operational Quantum Mechanics



Preparation
 P

Vector
 $|\psi\rangle$

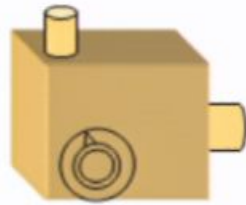


Measurement
 M

Hermitian operator
 A
 $A = \sum_k a_k P_k$

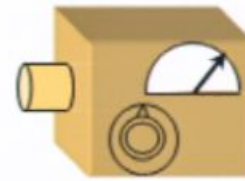
$$Pr(k|P, M) = \langle \psi | P_k | \psi \rangle$$

Operational Quantum Mechanics



Preparation
 P

Vector
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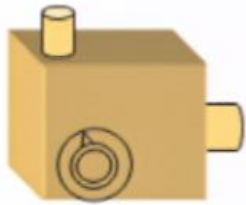


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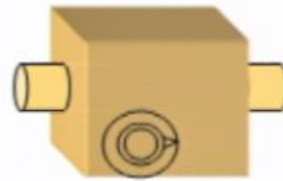
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Operational Quantum Mechanics



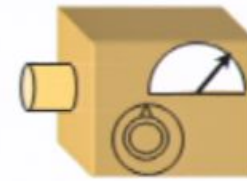
Preparation
 P

Vector
 $|\psi\rangle$



Transformation
 T

Unitary map
 U

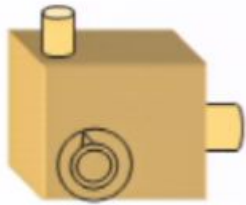


Measurement
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Hermitian operator
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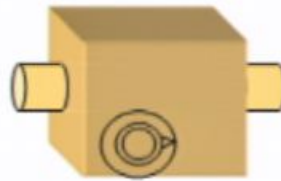
$$Pr(k|P, T, M) = \langle \psi | U^\dagger P_k U | \psi \rangle$$

Operational Quantum Mechanics



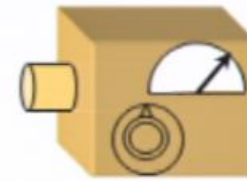
Preparation

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Transformation

T



Measurement

M



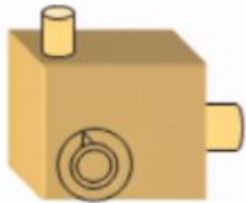
Effective preparation

P'

$$|\psi\rangle \rightarrow |\psi'\rangle = U|\psi\rangle$$

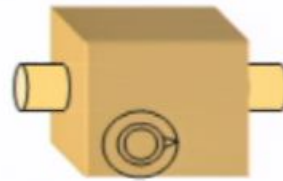
$$P_{\text{ex}}(k|P', M) = \langle e|P'|P|e\rangle = \langle e|U^\dagger P U|e\rangle$$

Operational Quantum Mechanics



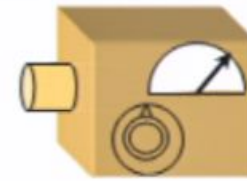
Preparation

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Transformation

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Measurement

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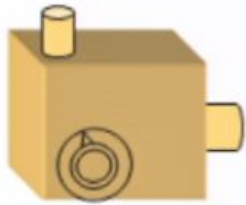
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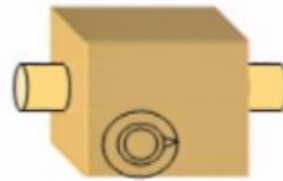
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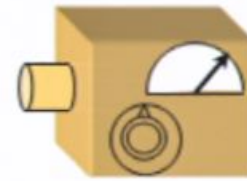
Operational Quantum Mechanics



Preparation
P



Transformation
T



Measurement
M



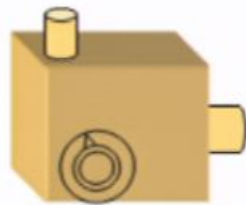
Effective Measurement
M'

$$A \rightarrow A' = U^\dagger A U$$

$$A' = \sum_k a_k P'_k$$

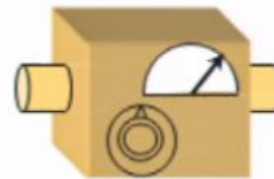
$$P_m(k|P, M) = \langle a|_k P'_k |a\rangle = \langle a|_k U^\dagger P U |a\rangle$$

Operational Quantum Mechanics



Preparation

P



Measurement

M

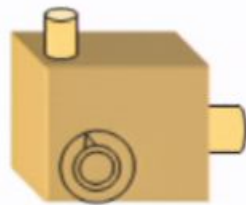
Effective preparation

P_k

Update map

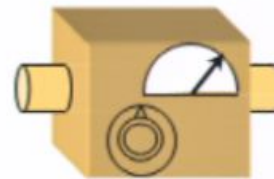
$$|\psi\rangle \rightarrow |\psi_k\rangle = \frac{P_k|\psi\rangle}{\sqrt{\langle\psi|P_k|\psi\rangle}}$$

Operational Quantum Mechanics



Preparation

P



Measurement

M

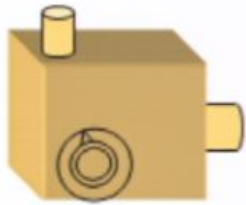
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Operational Quantum Mechanics

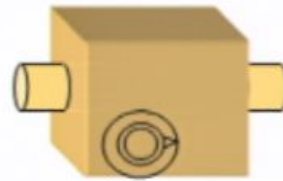


Preparation

\mathcal{P}

Density operator

ρ

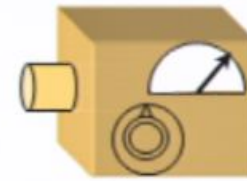


Transformation

\mathcal{T}

Trace-preserving
completely positive
linear map (CP map)

\mathcal{T}



Measurement

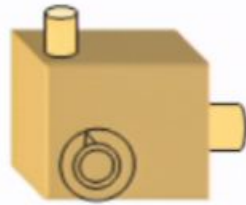
\mathcal{M}

Positive operator-valued
measure (POVM)

$\{E_k\}$

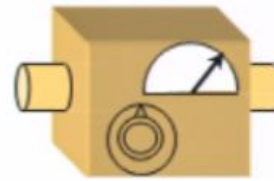
$$Pr(k|\mathcal{P}, \mathcal{T}, \mathcal{M}) = \text{Tr}[E_k \mathcal{T}(\rho)]$$

Operational Quantum Mechanics



Preparation

P



Measurement

M



Effective preparation

P_k

Update map

$$\rho \rightarrow \rho_k = \frac{\mathcal{I}_k(\rho)}{\text{Tr}[\mathcal{I}_k(\rho)]}$$

Trace-decreasing
completely positive
linear map

Operational postulates of quantum theory

Every preparation P is associated with a density operator ρ

Every measurement M is associated with a positive operator-valued measure $\{E_k\}$. The probability of M yielding outcome k given a preparation P is $p_k = \text{Tr}(E_k \rho)$.

Every transformation is associated with a trace-preserving completely-positive linear map $\rho \rightarrow \rho' = T(\rho)$,

Every measurement outcome k is associated with a trace-nonincreasing completely-positive linear map T_k such that $\rho \rightarrow \rho_k = T_k(\rho) / \text{Tr}[T_k(\rho)]$.

Is the operational interpretation satisfactory?

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Inconsistencies of the orthodox interpretation

By the collapse postulate
(applied to the system)

By unitary evolution postulate
(applied to isolated system that
includes the apparatus)

Indeterministic and
discontinuous evolution

Deterministic and
continuous evolution

Determinate properties

Indeterminate properties

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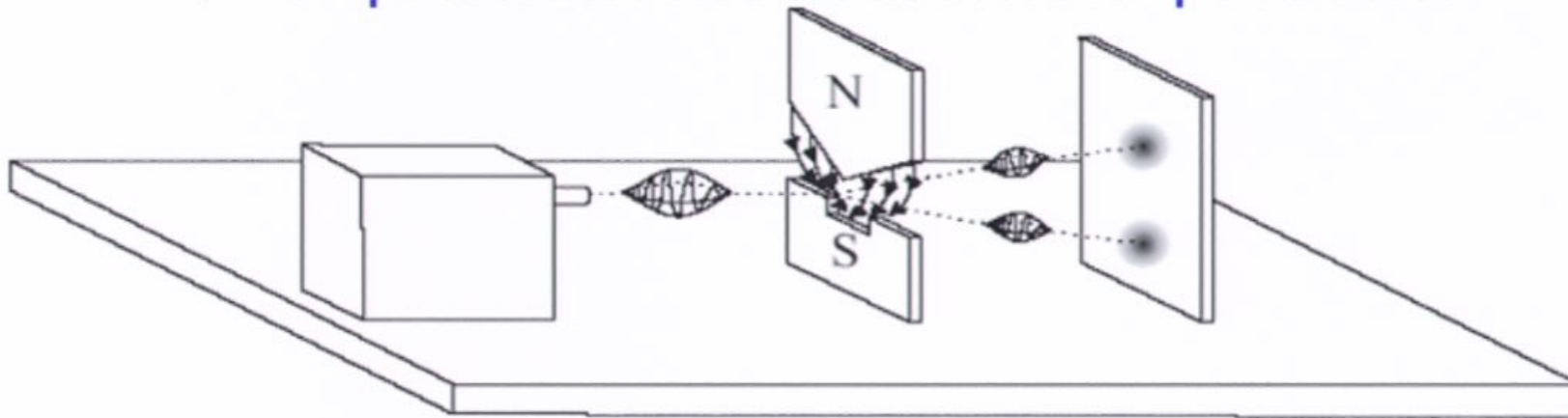
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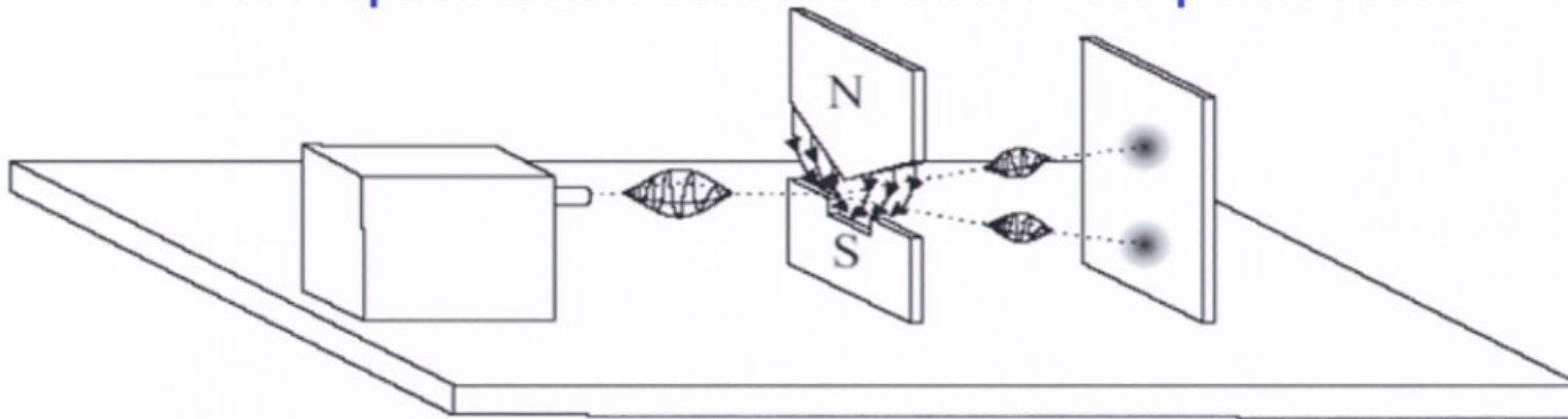
Determinate properties

Indeterminate properties

The quantum measurement problem



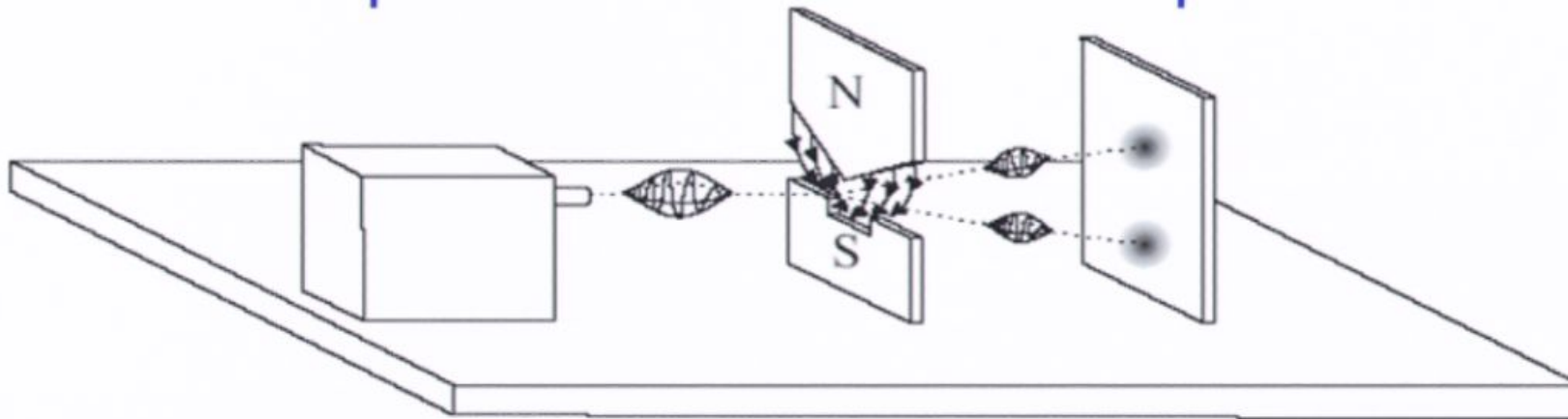
The quantum measurement problem



If the measurement apparatus is treated **externally**

$$a|\uparrow\rangle + b|\downarrow\rangle \rightarrow |\uparrow\rangle \text{ with probability } |a|^2$$
$$\rightarrow |\downarrow\rangle \text{ with probability } |b|^2$$

The quantum measurement problem



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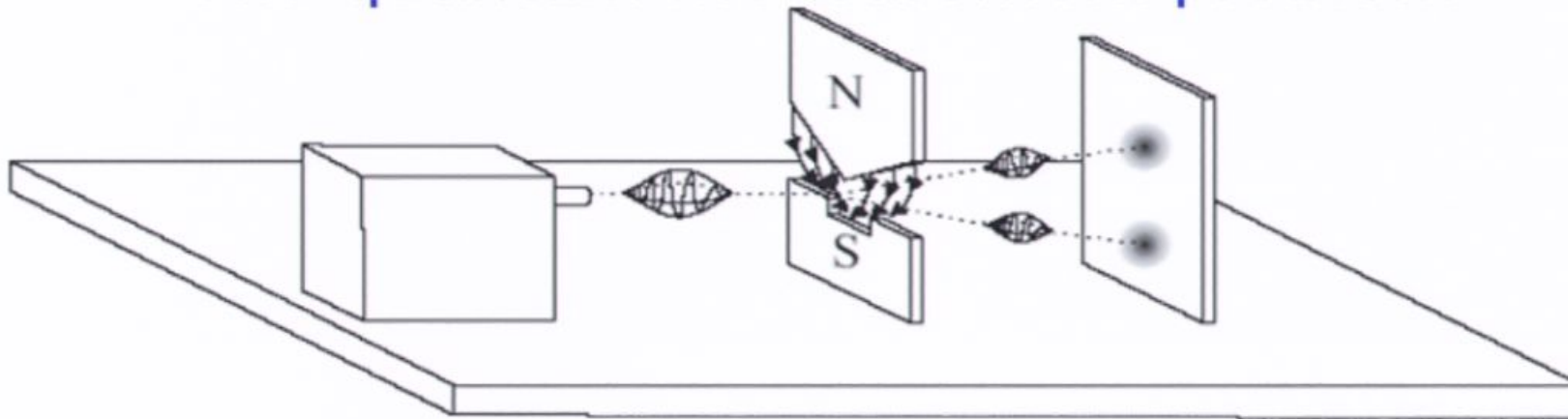
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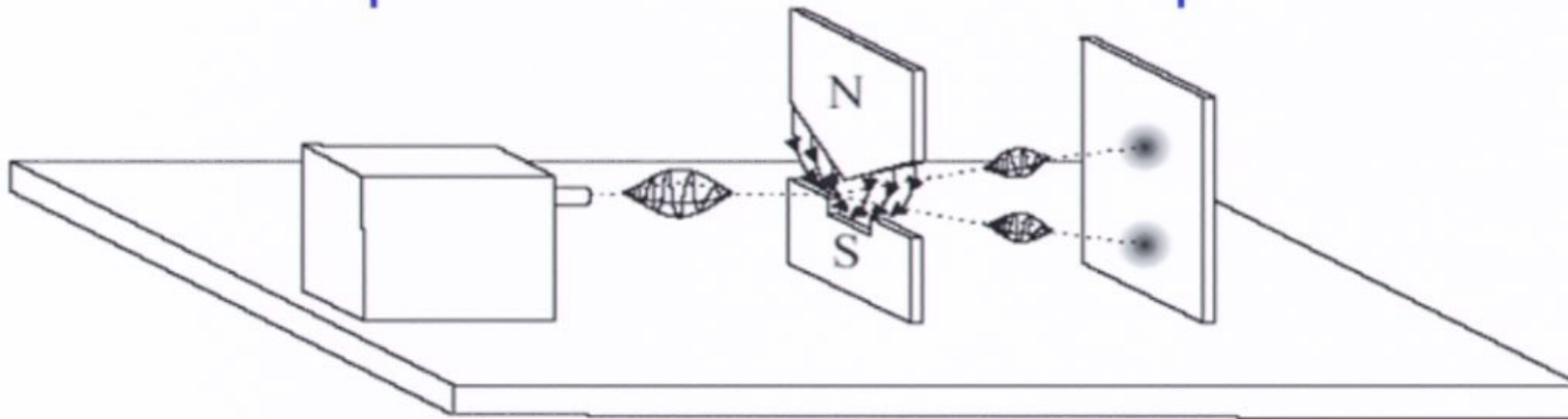
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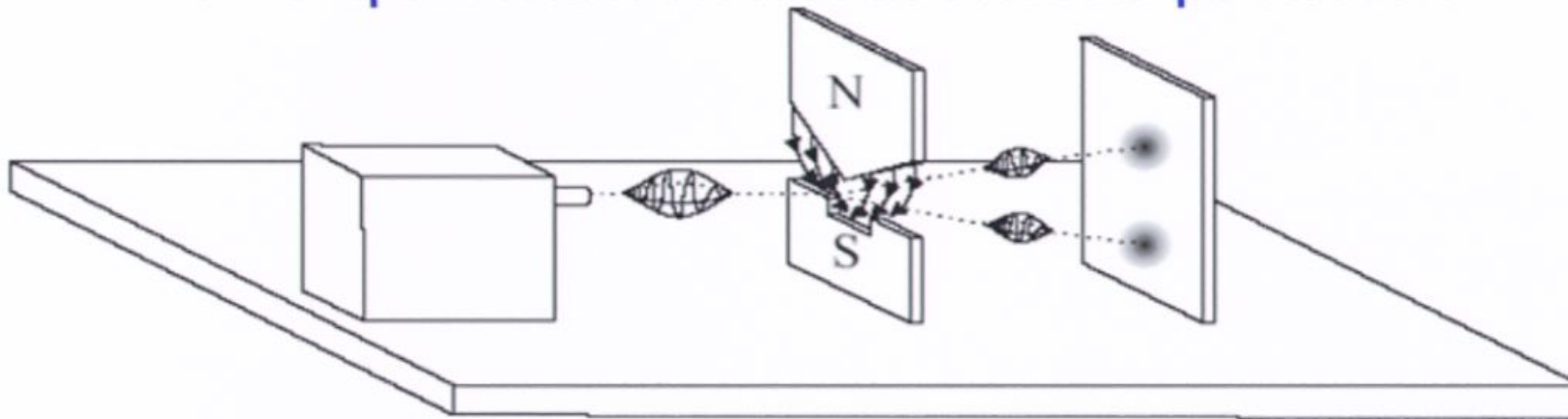
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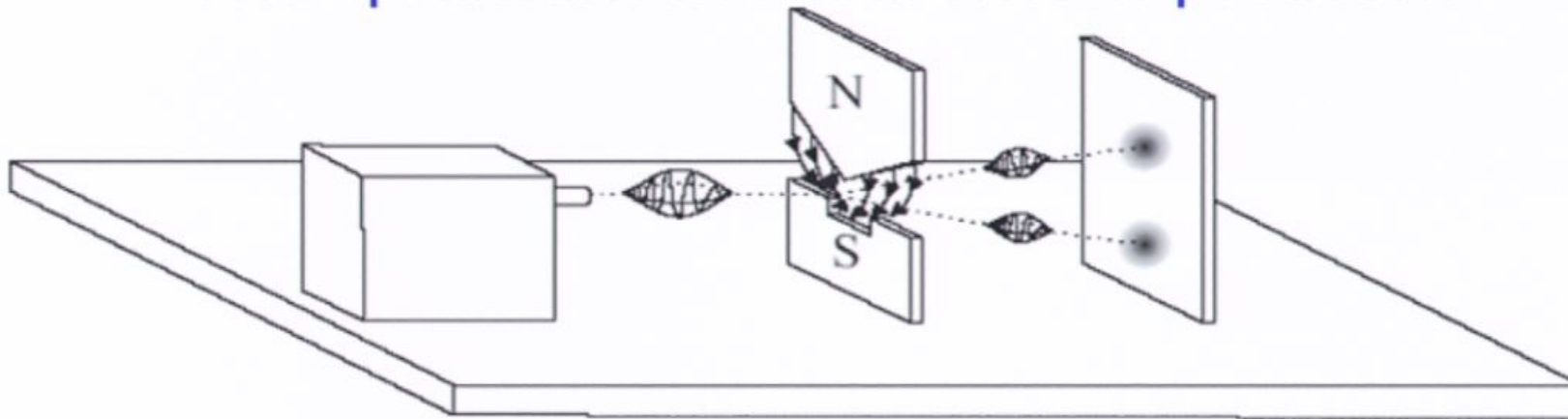
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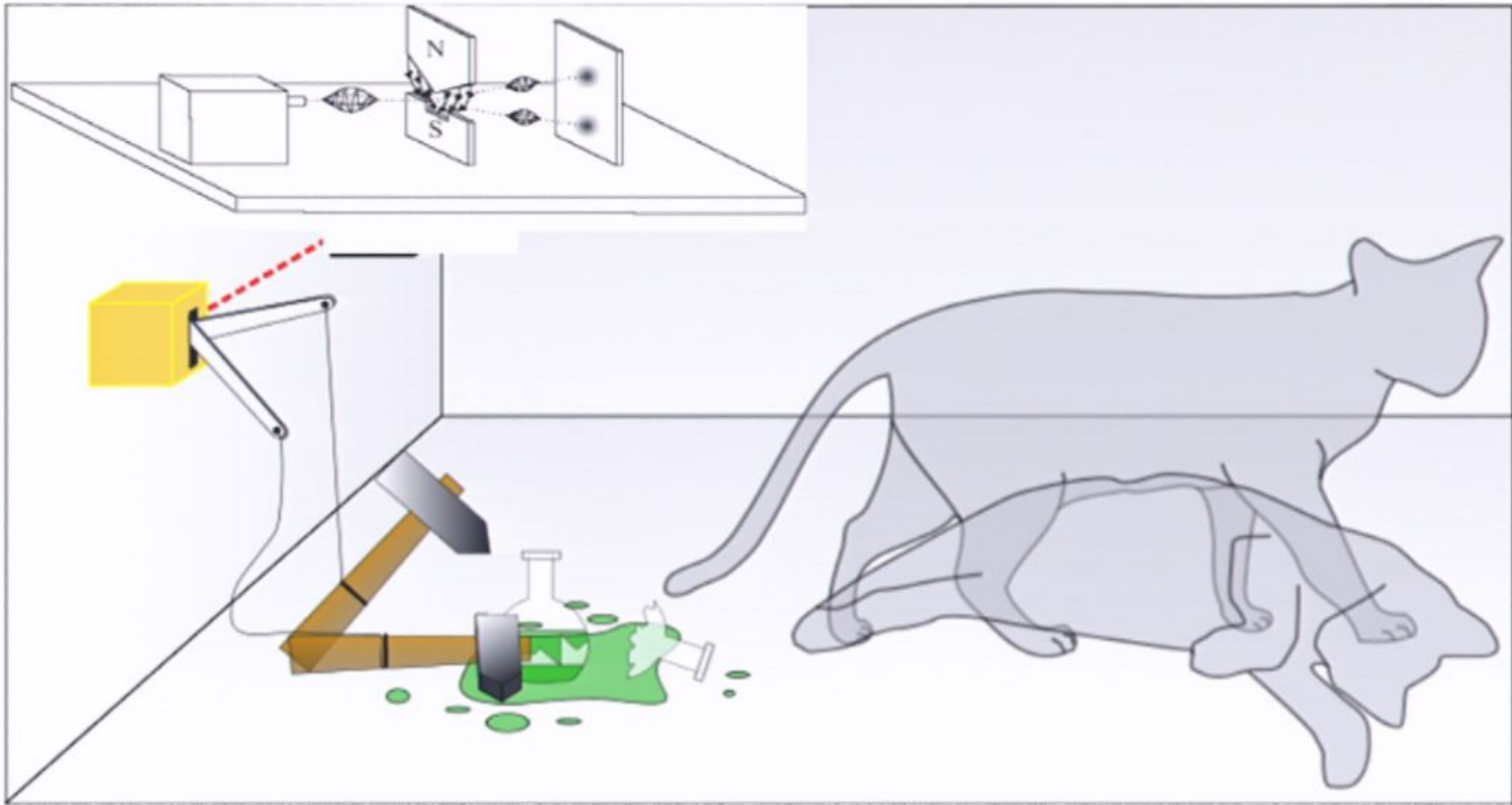
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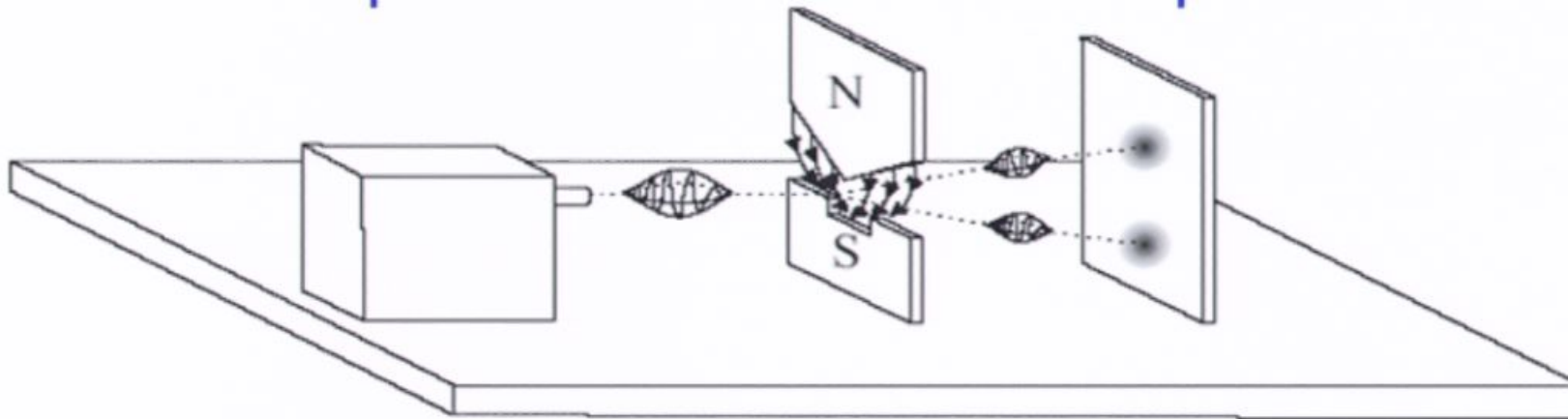
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False starts on the measurement problem

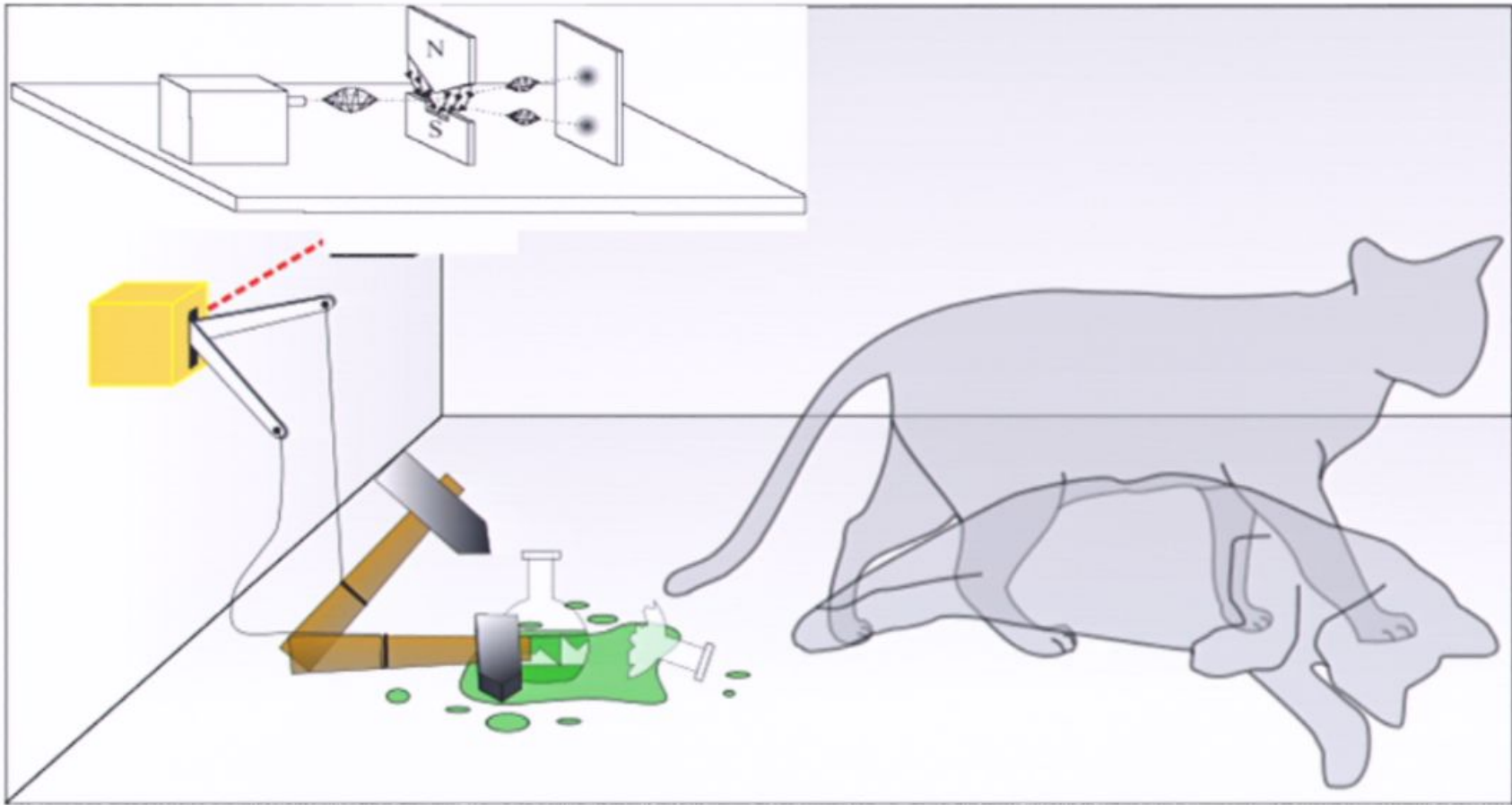
- Interpret coherent superposition as disjunction

$$a|\uparrow\rangle \otimes |\text{"up"}\rangle + b|\downarrow\rangle \otimes |\text{"down"}\rangle$$

Means either $|\uparrow\rangle \otimes |\text{"up"}\rangle$

or $|\downarrow\rangle \otimes |\text{"down"}\rangle$

with probabilities $|a|^2$ and $|b|^2$
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This is a denial of the representational completeness of ψ

False starts on the measurement problem

- Interpret the reduced density operator as a proper mixture

$$a|\uparrow\rangle \otimes |\text{"up"}\rangle + b|\downarrow\rangle \otimes |\text{"down"}\rangle$$

$$\rho = |a|^2 |\text{"up"}\rangle \langle \text{"up"}| + |b|^2 |\text{"down"}\rangle \langle \text{"down"}|$$

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False starts on the measurement problem

- Appeal to environment-induced decoherence

$$(a|\uparrow\rangle + b|\downarrow\rangle) \otimes |\text{"ready"}\rangle \otimes |E_0\rangle$$

$$\rightarrow (a|\uparrow\rangle \otimes |\text{"up"}\rangle + b|\downarrow\rangle \otimes |\text{"down"}\rangle) \otimes |E_0\rangle$$

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False starts on the measurement problem

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And by linearity

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5. Deny some other feature of the realist framework?