Title: Born--Oppenheimer approximation for quantum fields on quantum spacetimes

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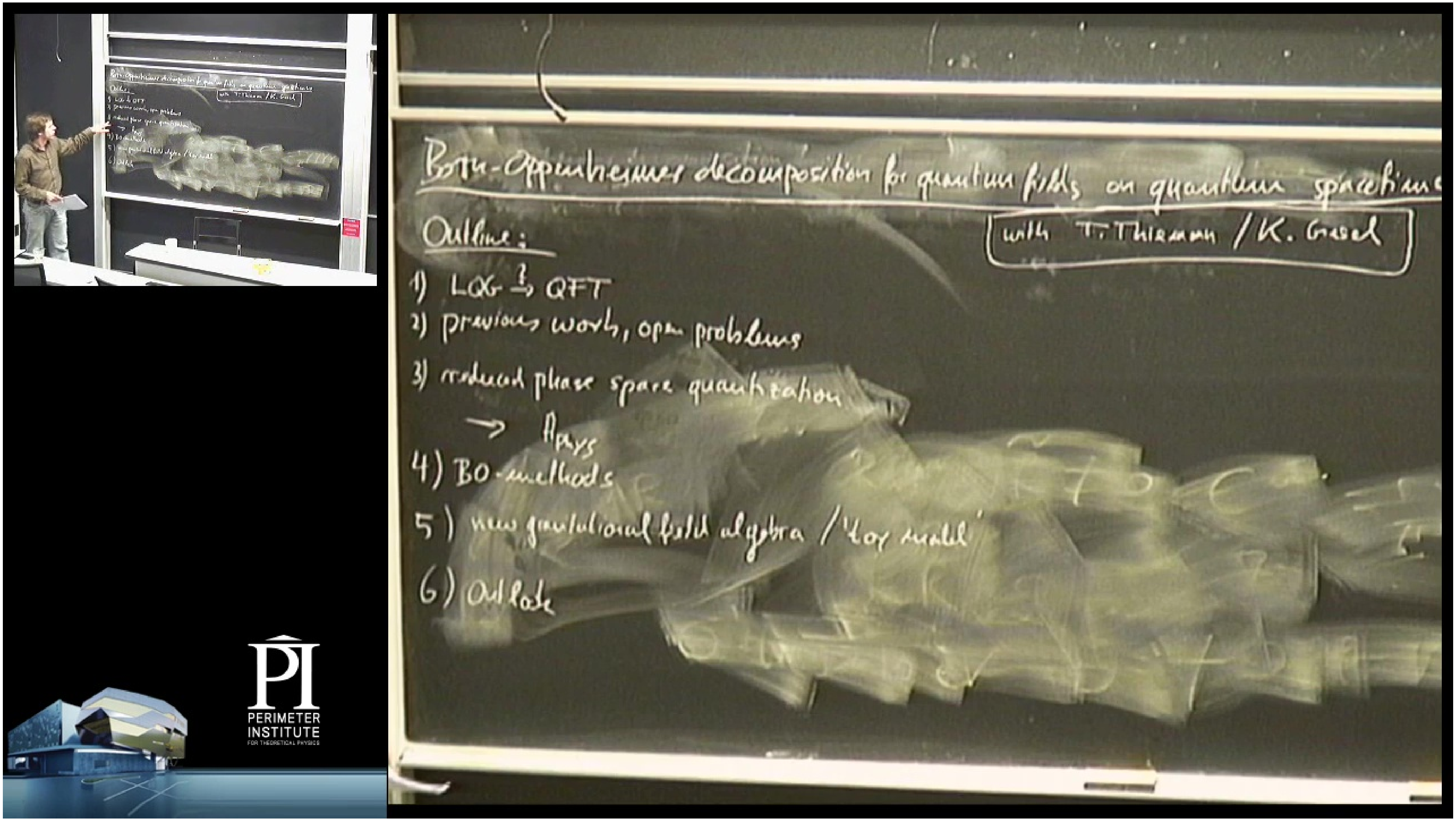
Abstract: The relation between loop quantum gravity (LQG) and ordinary quantum field theory (QFT) on a fixed background spacetime still bears many obstacles. When looking at LQG and ordinary QFT from a mathematical perspective it turns out that the two frameworks are rather different: Although LQG is a true continuum theory its Hilbert space is defined in terms of certain embedded graphs which are labeled by irreducible representations of SU(2). The natural arena for ordinary QFT, on the other hand, is a Fock space which strongly uses the metric properties of the underlying continuum spacetime. In this talk I will review this issue and show how one can use Born--Oppenheimer methods to further progress towards an understanding of (matter) quantum field theories from first principles.


Born-Oppenheimer decomposition for quantum fields on quantum spacetime
with T. Thiemann / K. Giesel
Outline:
1) LQG →? QFT
2) previous work, open problems
3) reduced phase space quantization
→ A_phys
4) BO-methods
5) new gravitational field algebra / 'toy model'
6) Outlook
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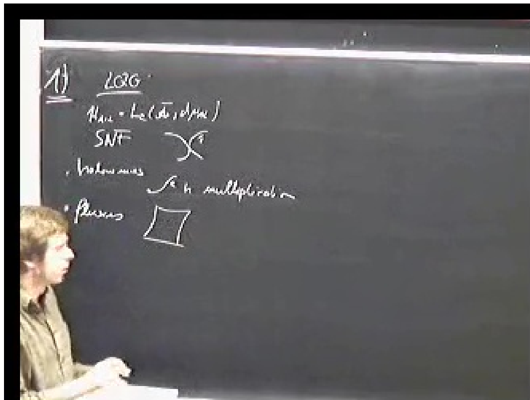
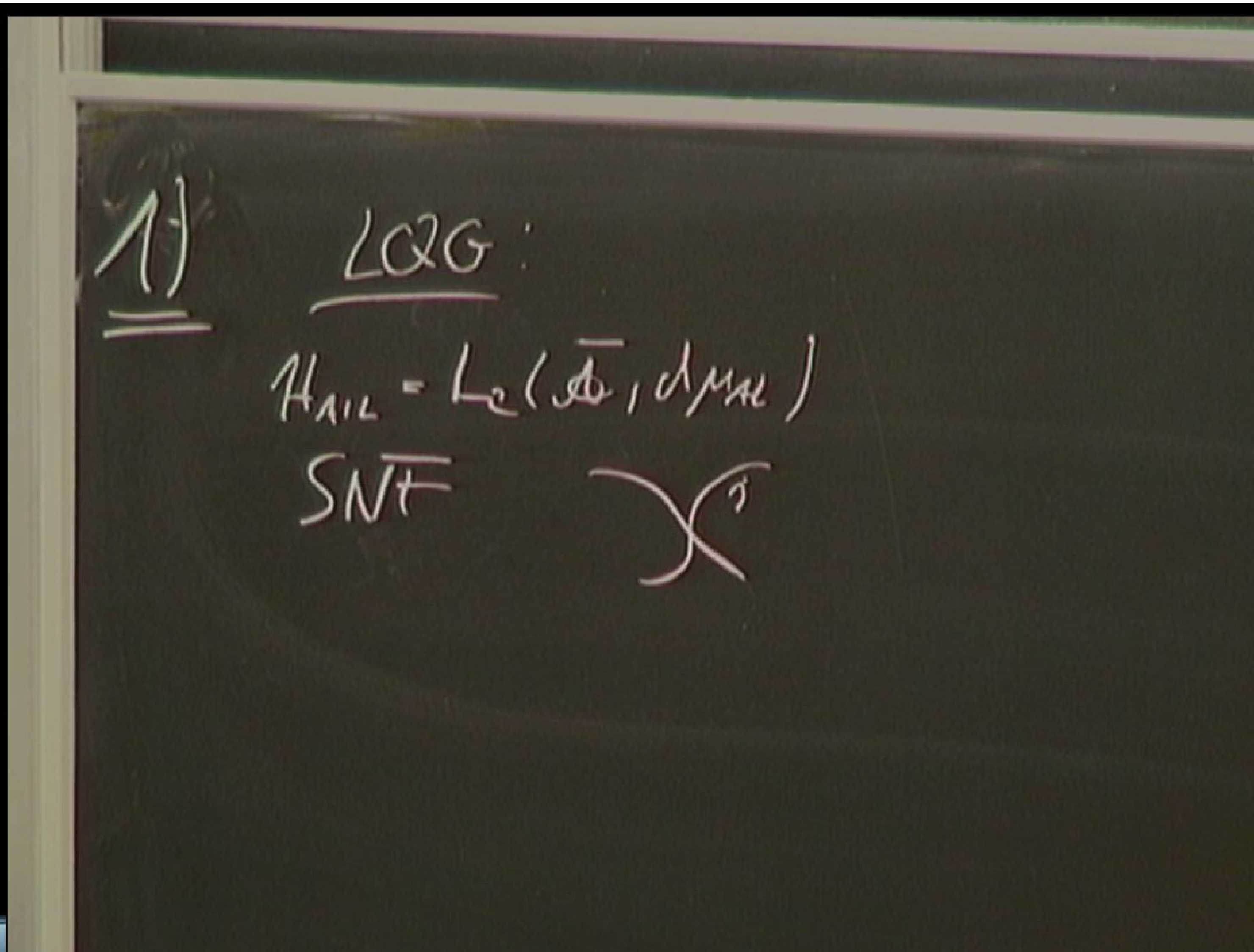
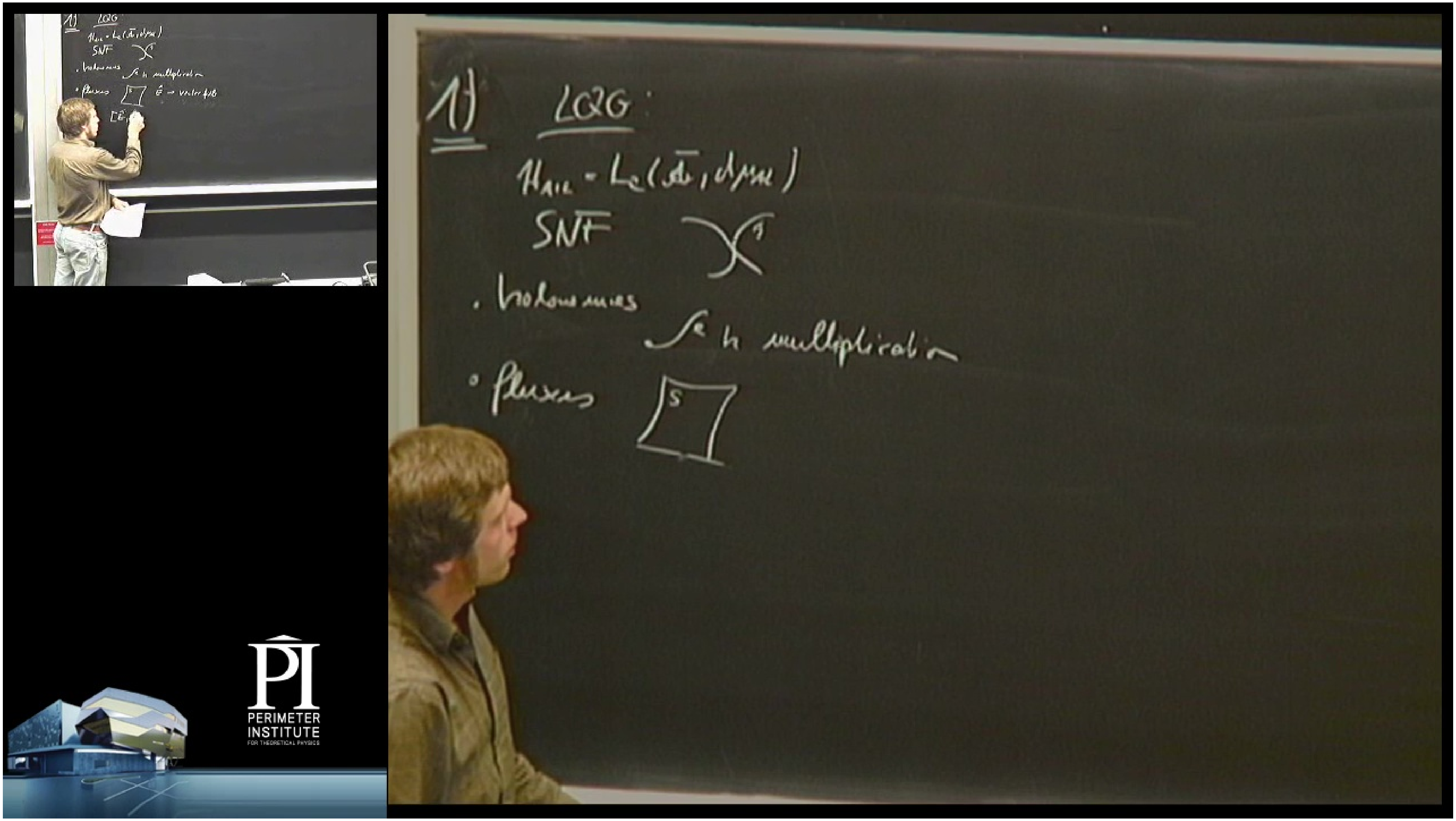



1)
LQG:
$\mathcal{H}_{AIL} = L_2(\bar{\mathcal{A}}, d\mu_{AL})$
SNF


1) LQG:
SNF
. holonomies
la multiplication
° fluxes
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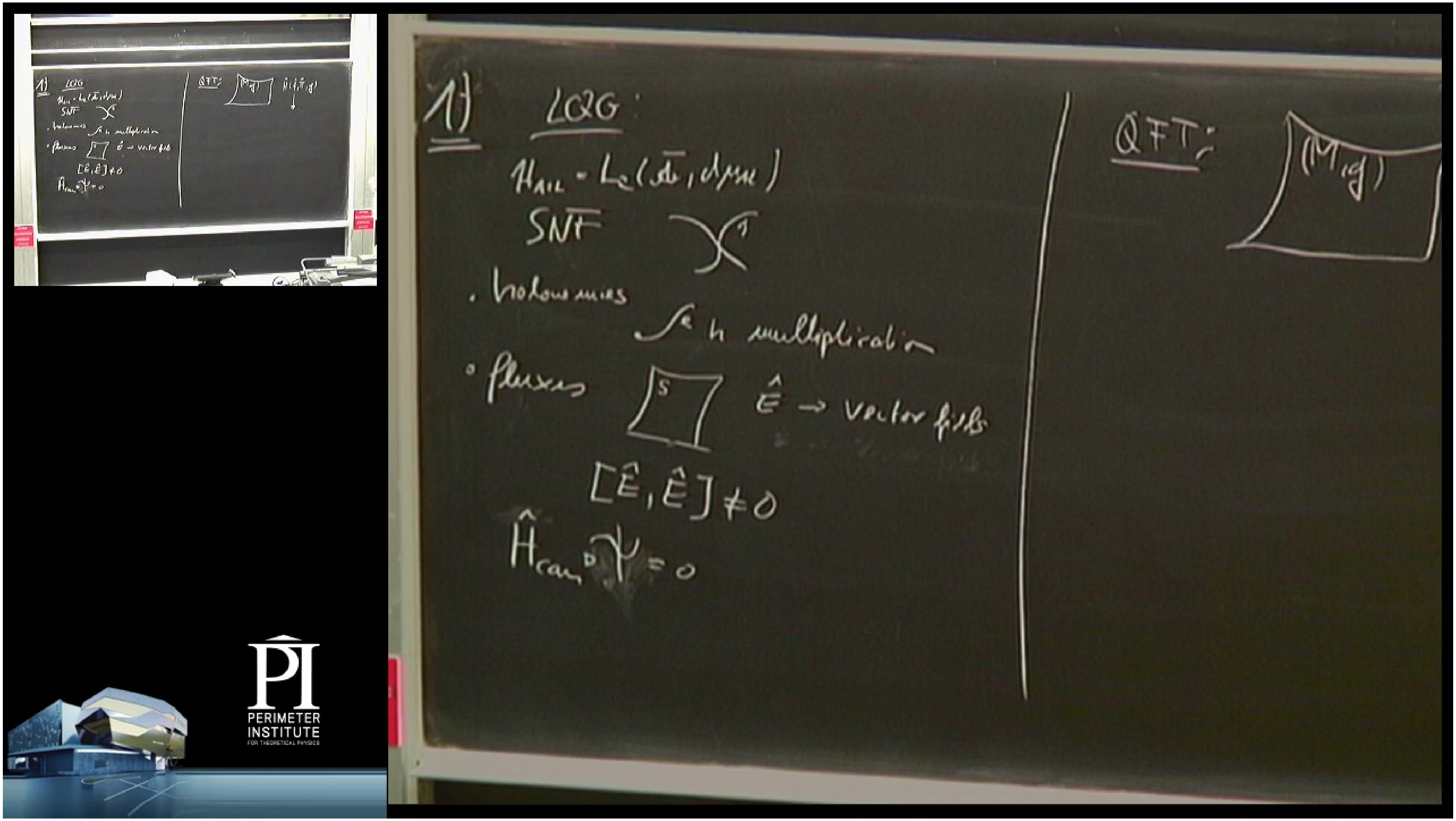
1) LQG:
$H_{AIL} = L_2(\bar{A}, d\mu_{AL})$
SNF
. holonomies
act by multiplication
∘ fluxes S $\hat{E}$ → vector fields
$[\hat{E}, \hat{E}] \neq 0$
$\hat{H}_{can} \Psi = 0$
QFT: (M, g)
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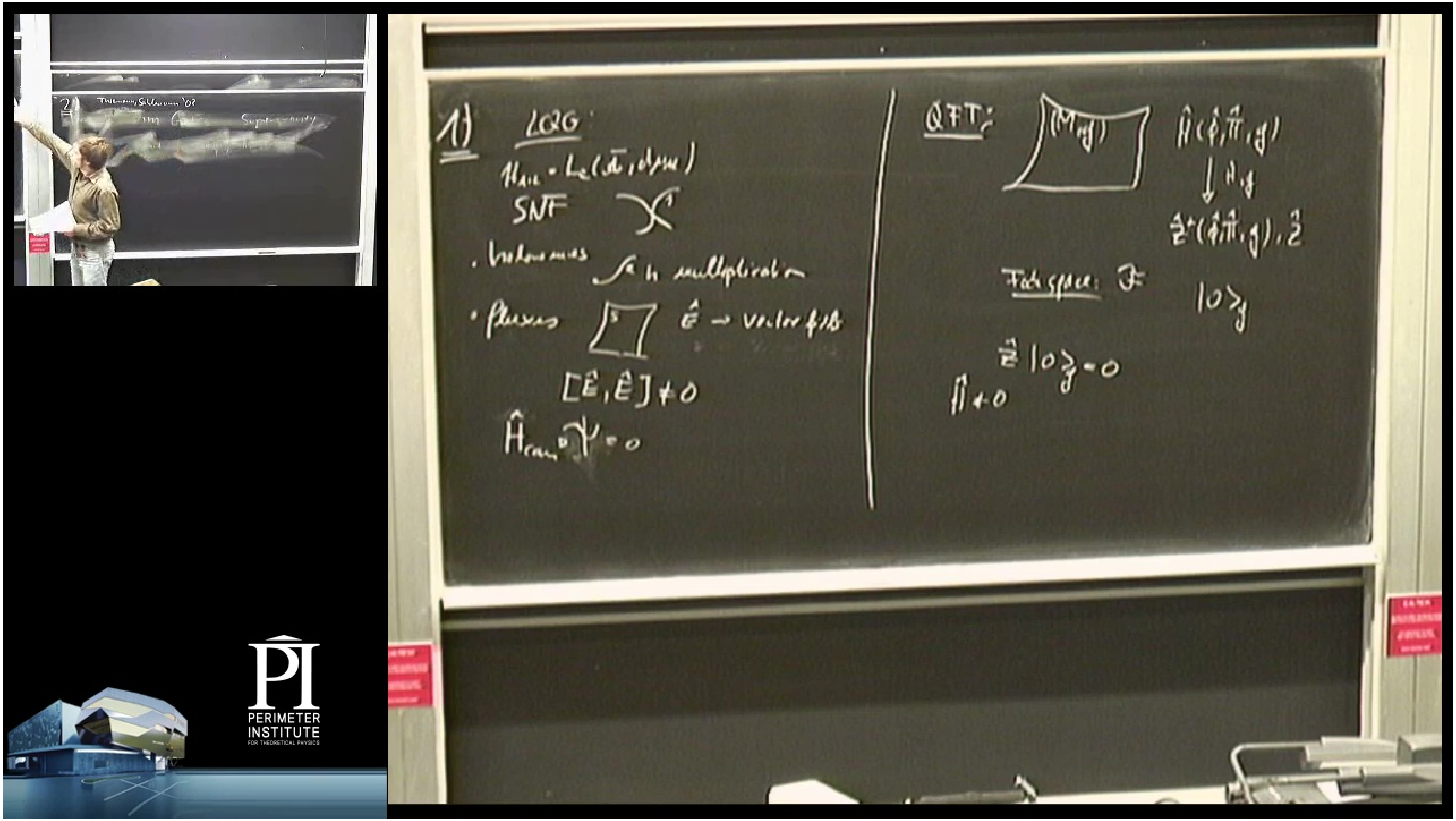
1)
LQG
SNF
· holonomies
· fluxes
[Ê, Ê] ≠ 0
QFT:
Fock space: F
|0>
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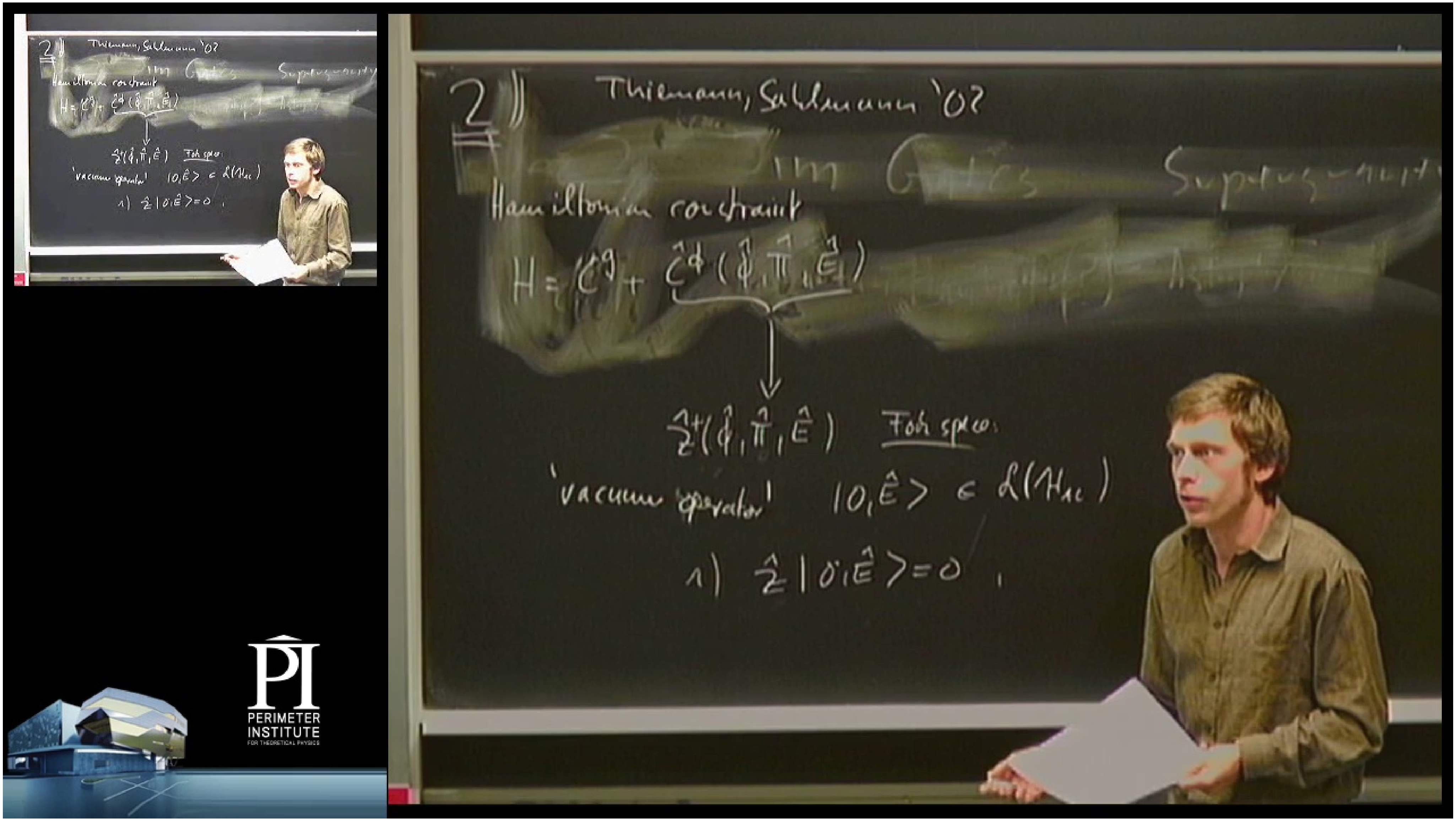

Thiemann, Sahlmann '02
Hamiltonian constraint
H = C^g + C^φ(φ̂, π̂, Ê)
ẑ(φ̂, π̂, Ê) Fock space
'vacuum operator' |0, Ê> ∈ L(H_kin)
1) ẑ|0, Ê>=0
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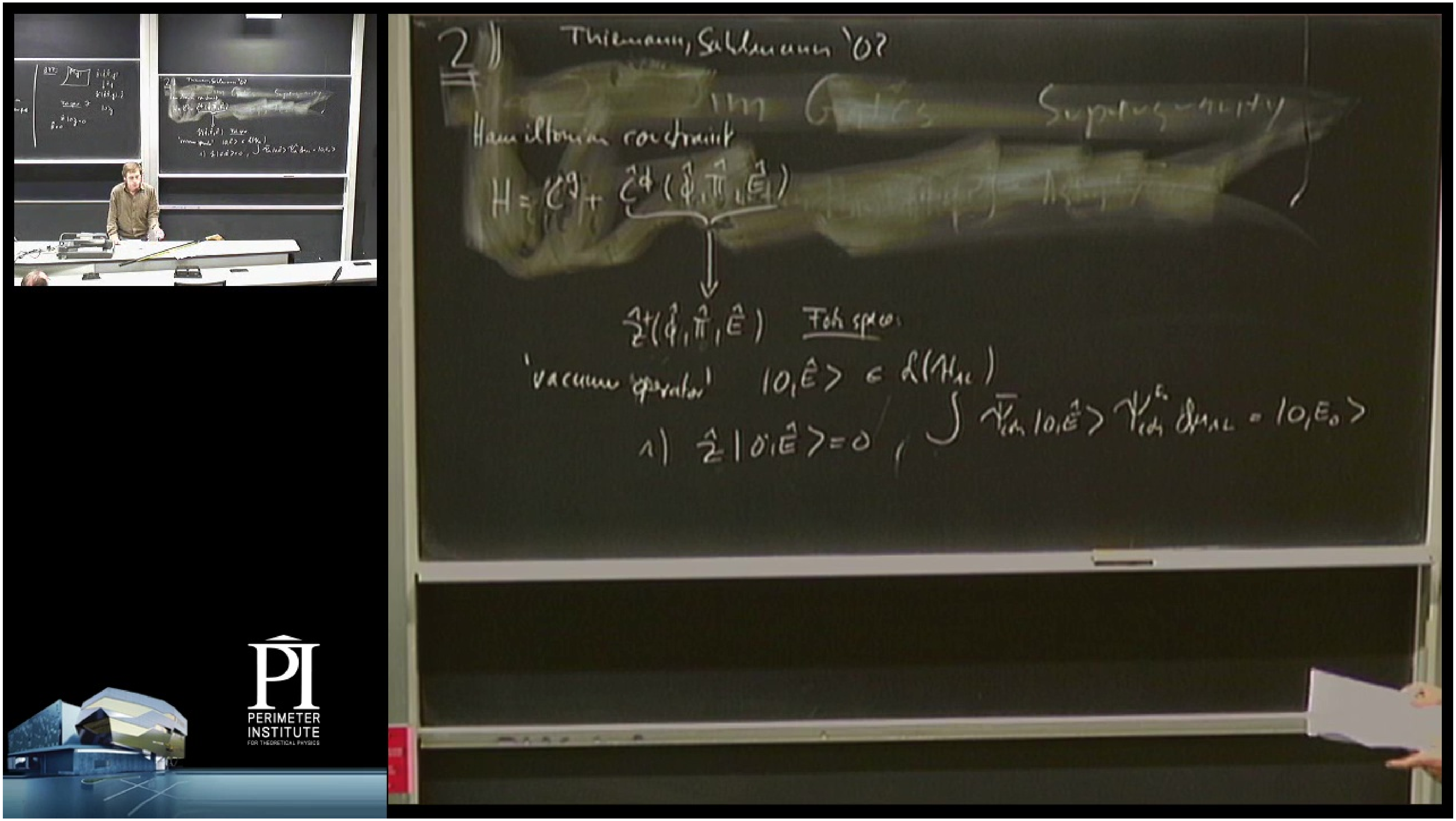

Thiemann, Sahlmann '02
Hamiltonian constraint
Fock space
'vacuum operator'
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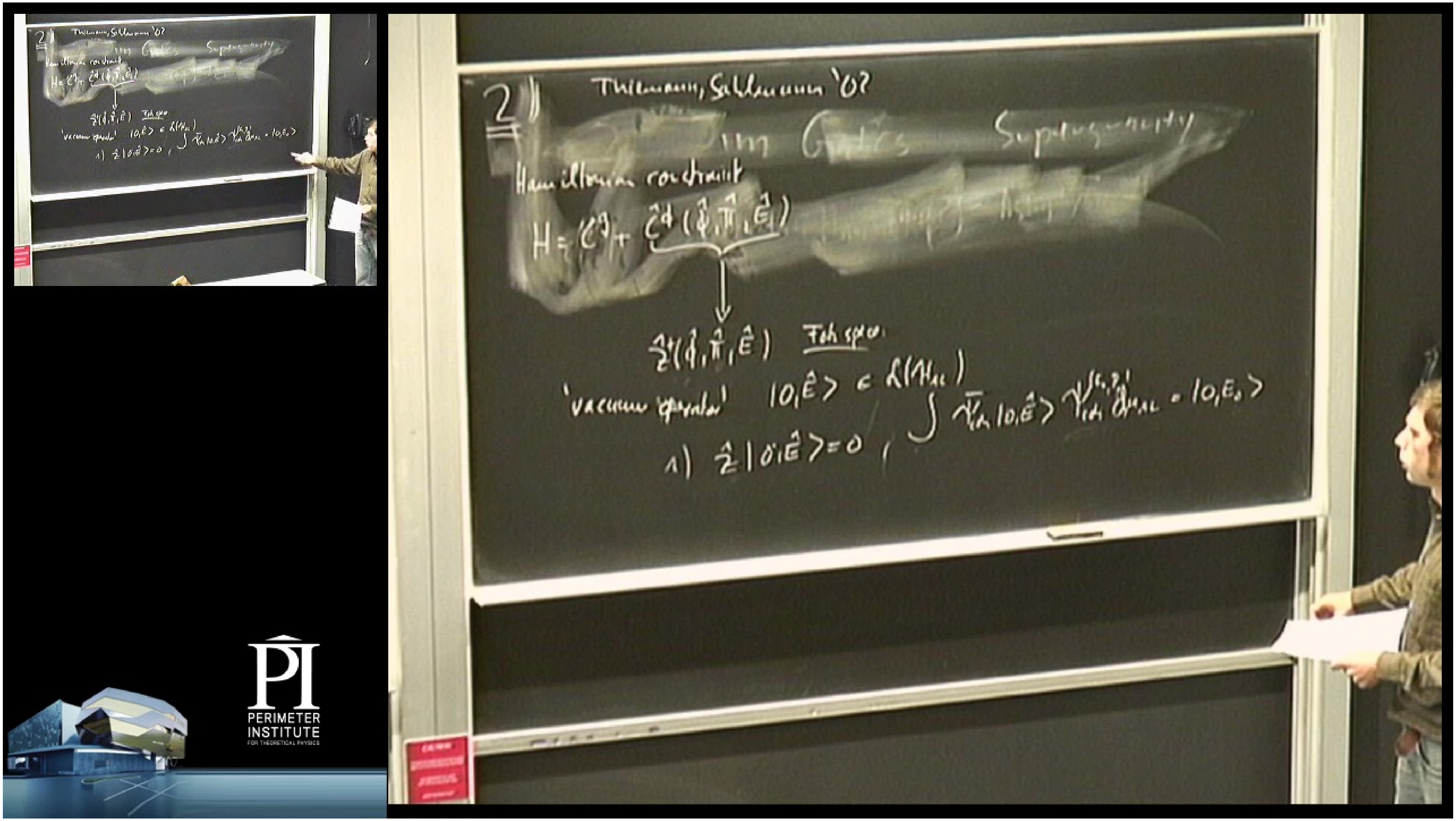
2)
Thiemann, Sahlmann '07?
Hamiltonian constraint
Fock space
'vacuum operator'
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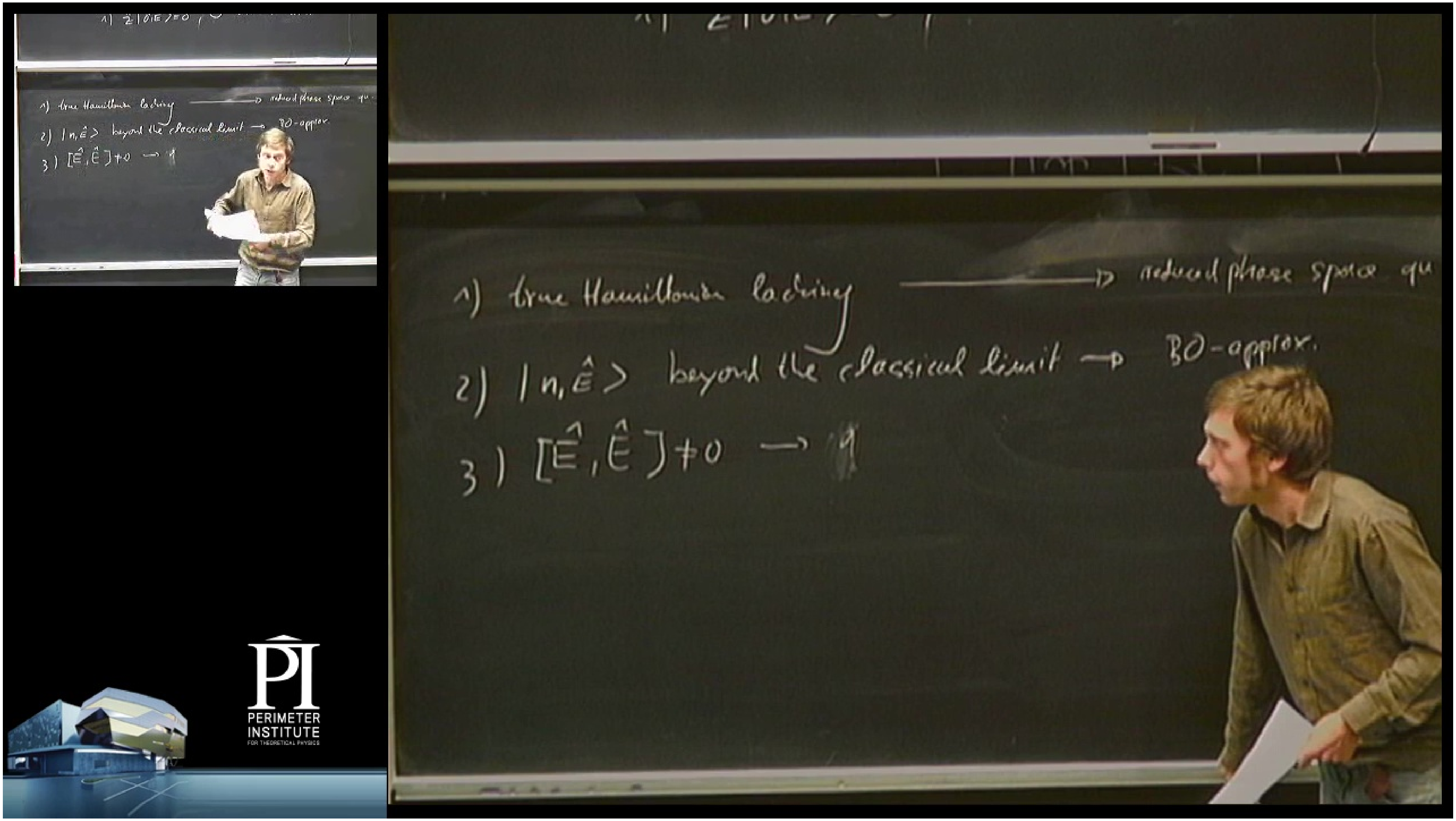
1) true Hamiltonian ladder —▷ reduced phase space qu
2) |n,Ê> beyond the classical limit → BO-approx.
3) [Ê,Ê] ≠ 0 → 1
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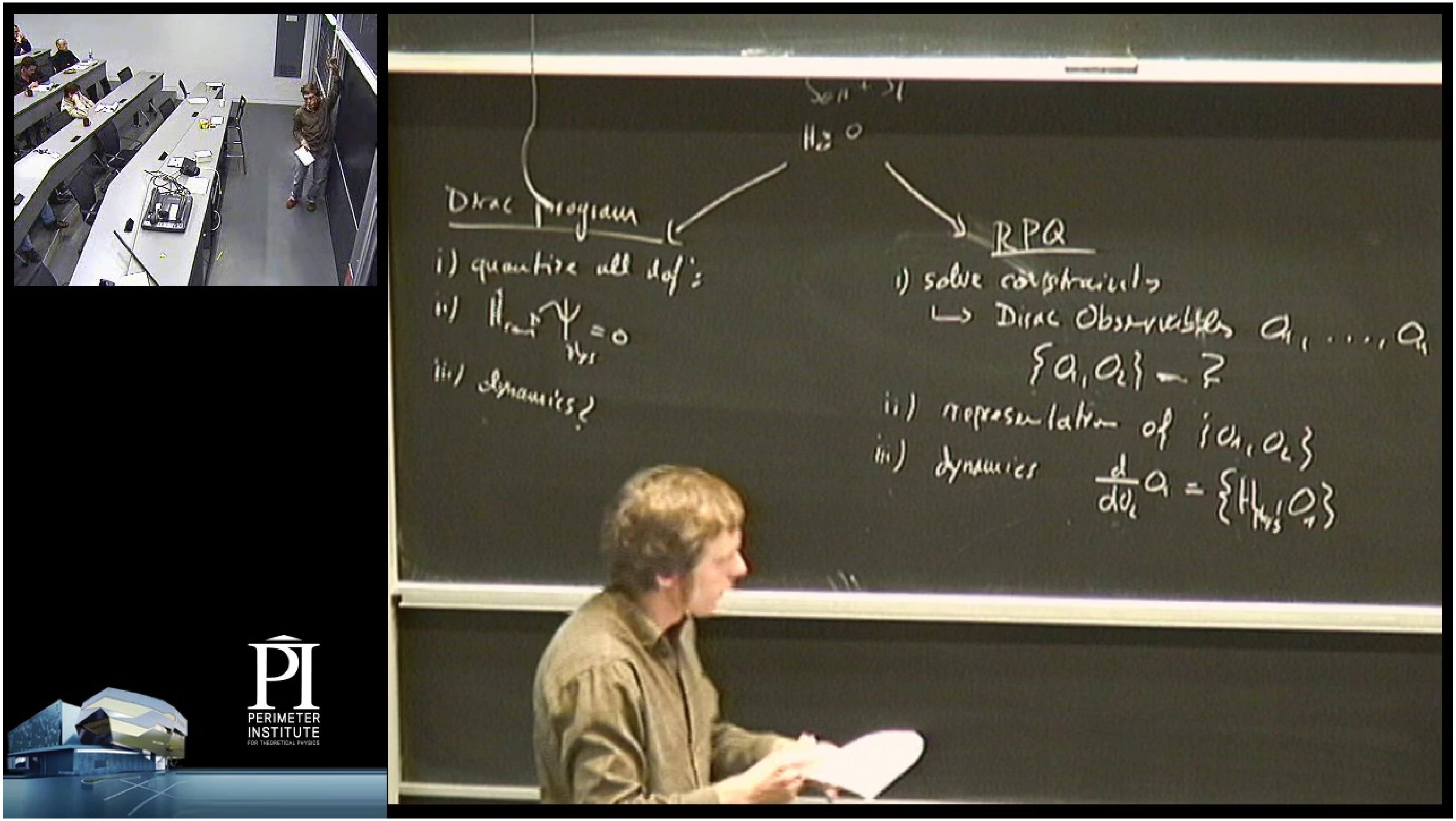
Dirac program
i) quantize all dof's
iii) dynamics?
RPQ
i) solve constraints
→ Dirac Observables
ii) representation of
iii) dynamics
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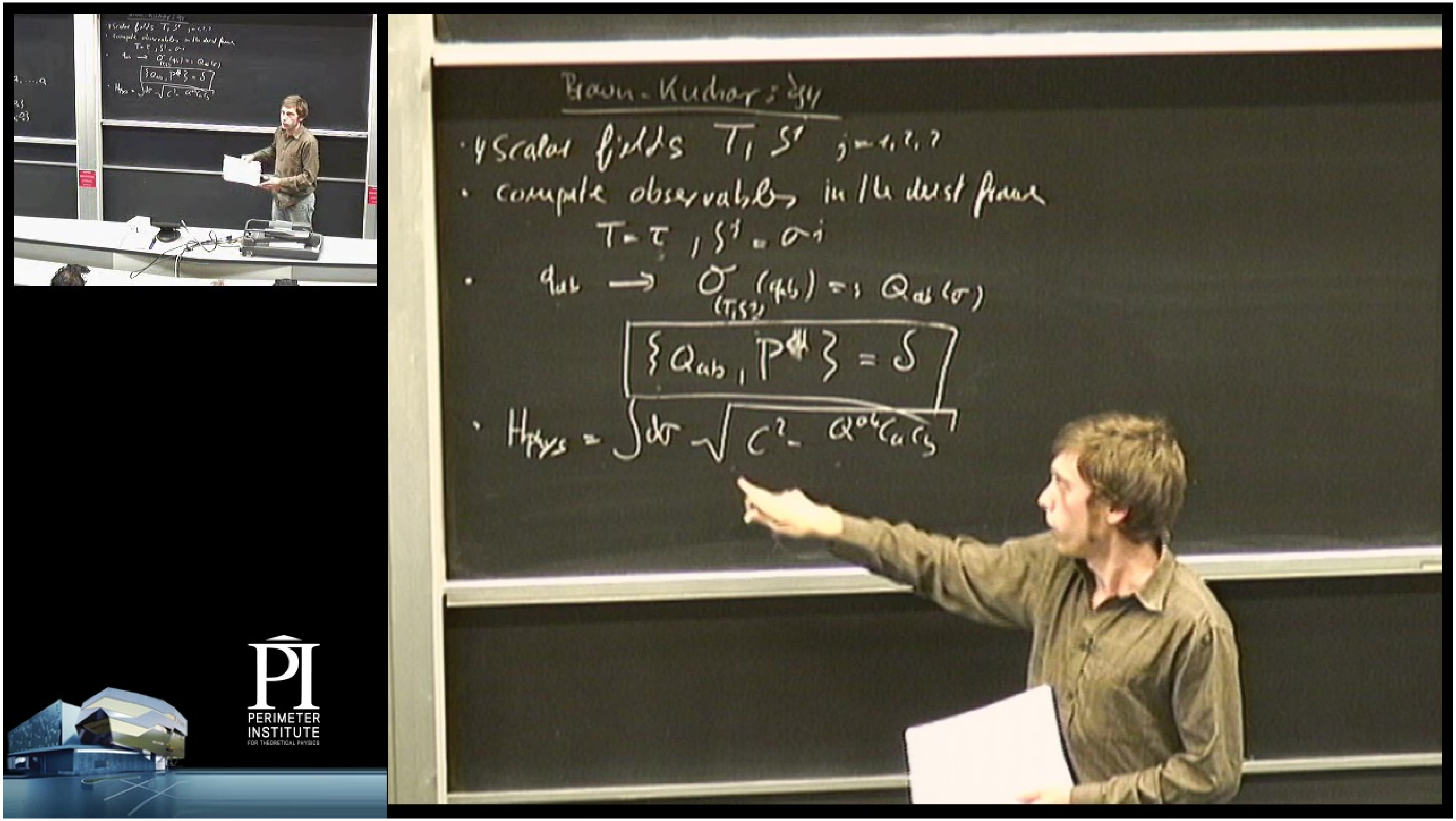
· 4 scalar fields T, S^j j=1,2,3
· complete observables in the dust frame
T = τ , S^j = σ^j
· q_ab → O_(T,S^j)(q_ab) =: Q_ab(σ)
{Q_ab , P^cd} = δ
· H_phys = ∫dσ √(C² − Q^ab C_a C_b)
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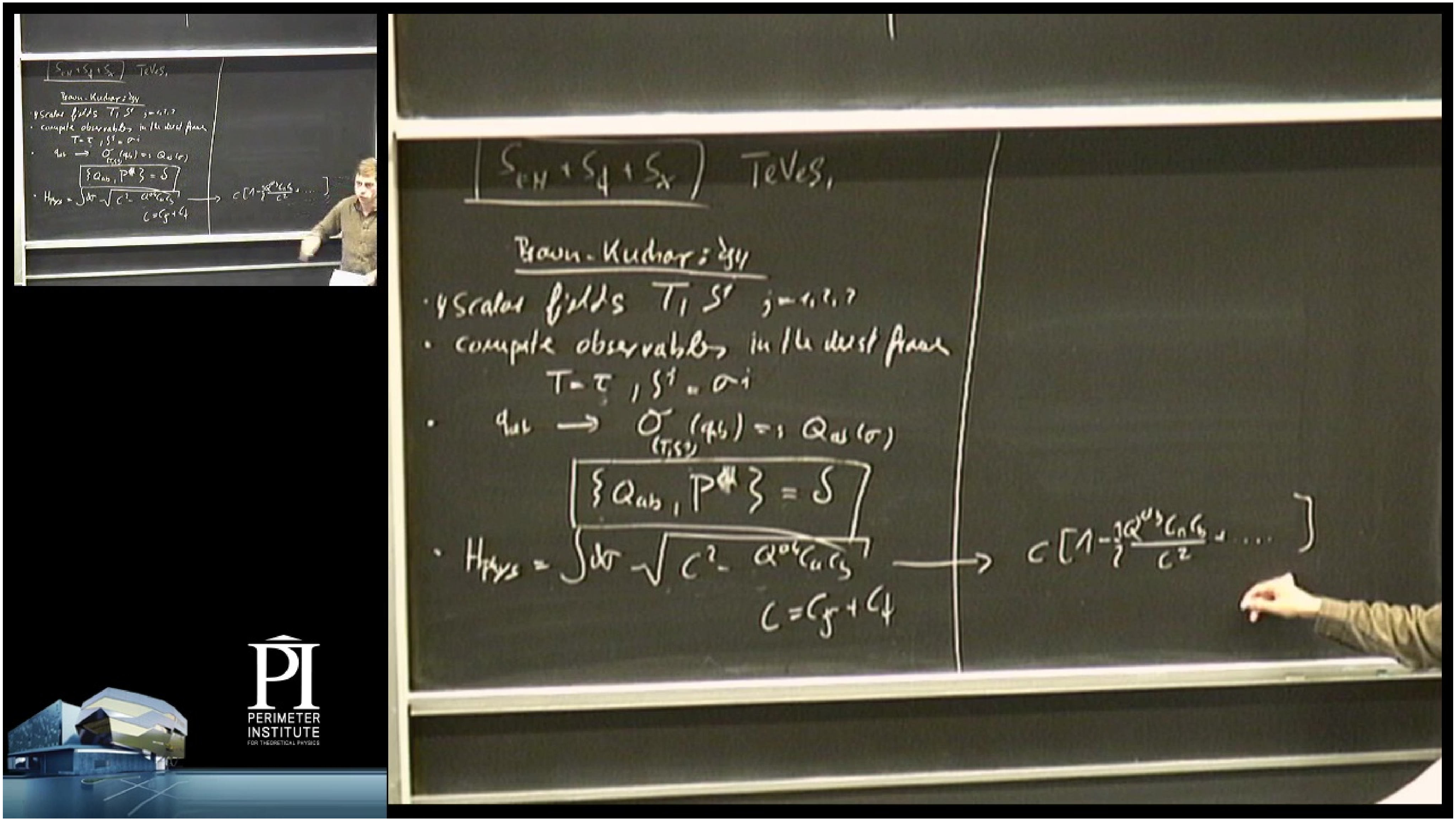
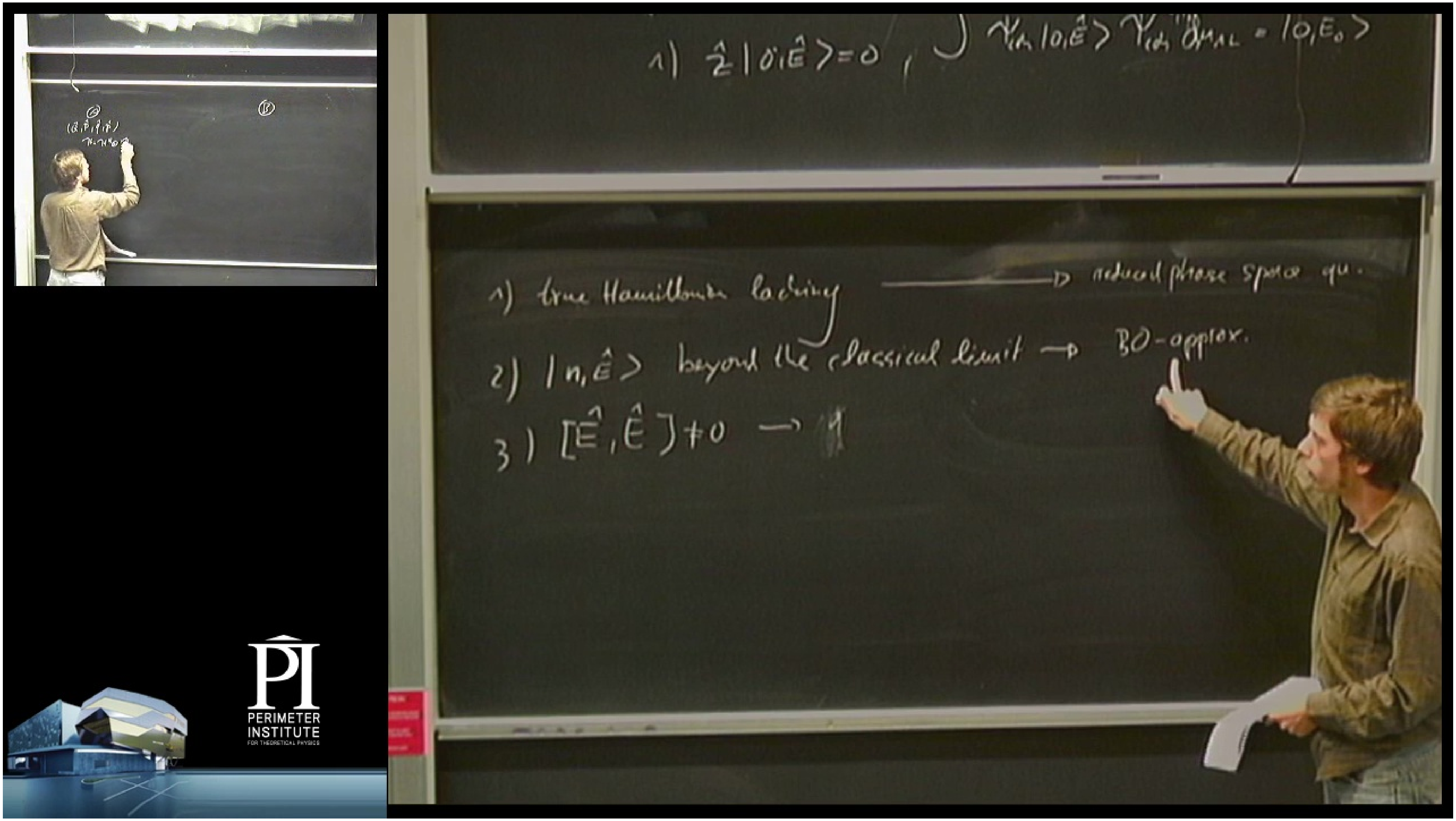

$S_{EH} + S_\phi + S_\chi$ TeVeS,
Brown-Kuchar: dust
· 4 scalar fields T, S^j $j=1,2,3$
· compute observables in the dust frame
$T=\tau$, $S^j=\sigma^j$
· $q_{ab} \longrightarrow O_{(T,S^j)}(q_{ab}) =: Q_{ab}(\sigma)$
$\{Q_{ab}, P^{cd}\} = \delta$
· $H_{phys} = \int d\sigma \sqrt{C^2 - Q^{ab}C_aC_b} \longrightarrow C[1-\frac{1}{2}\frac{Q^{ab}C_aC_b}{C^2}+\dots]$
$C = C_g + C_\phi$


1) true Hamiltonian lacking → reduced phase space qu.
2) |n, Ê> beyond the classical limit → BO-approx.
3) [Ê, Ê] ≠ 0 →
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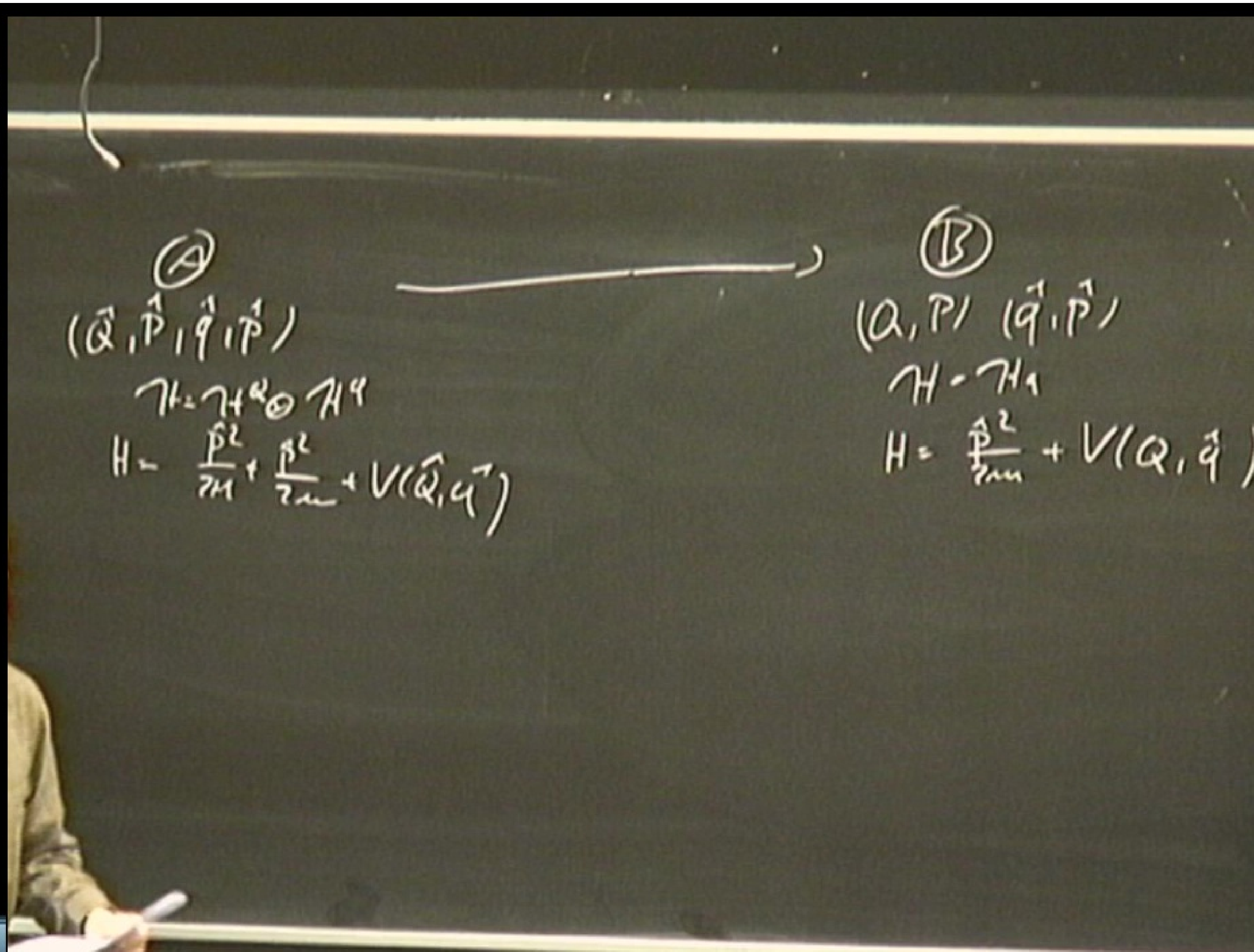
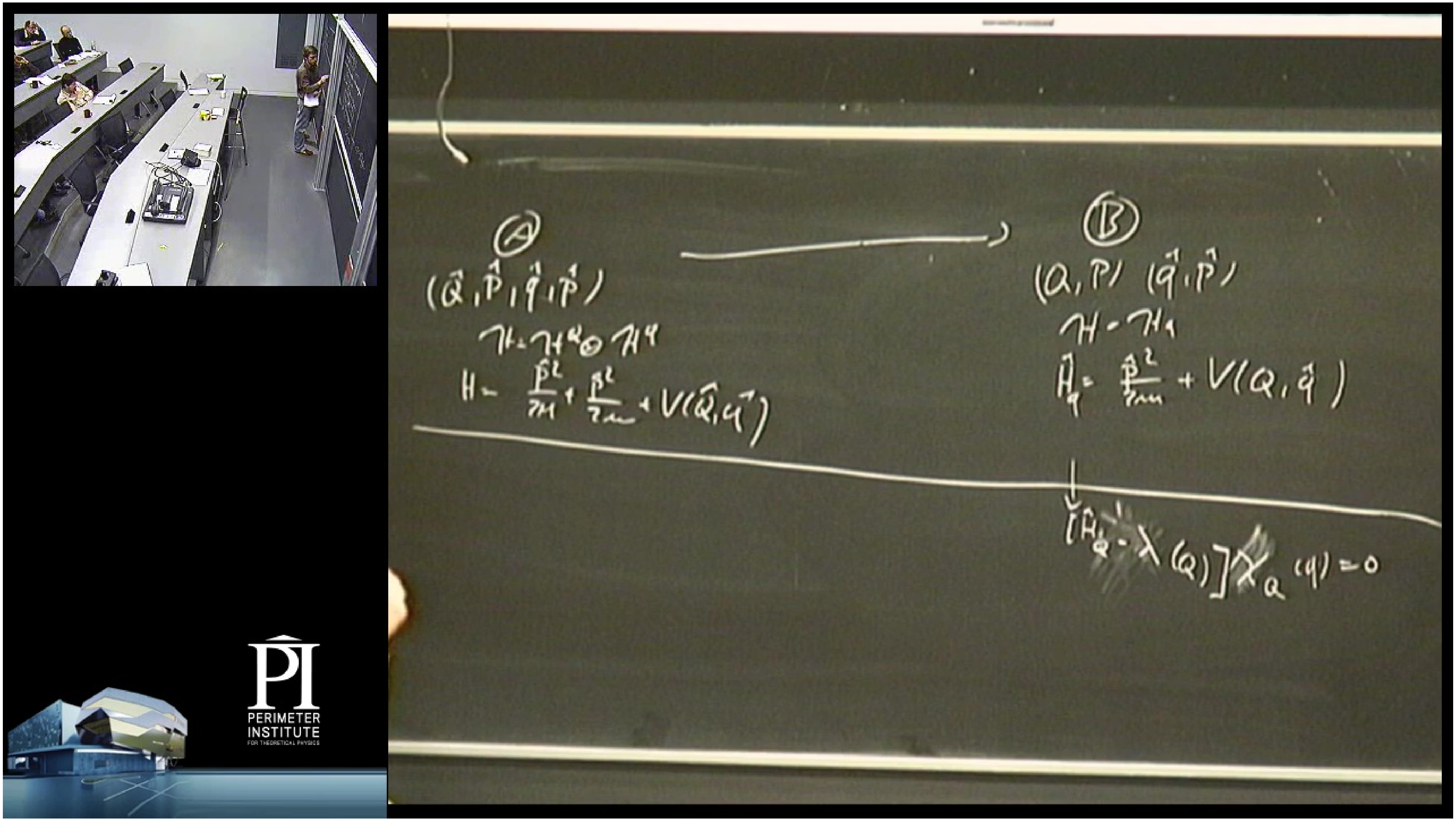

A
$(\vec{Q}, \vec{P}, \vec{q}, \vec{p})$
$\mathcal{H} = \mathcal{H}^Q \otimes \mathcal{H}^q$
$H = \frac{P^2}{2M} + \frac{p^2}{2m} + V(\hat{Q}, \vec{q})$
B
(Q, P) $(\vec{q}, \vec{p})$
$\mathcal{H} = \mathcal{H}_q$
$H = \frac{\vec{p}^2}{2m} + V(Q, \vec{q})$



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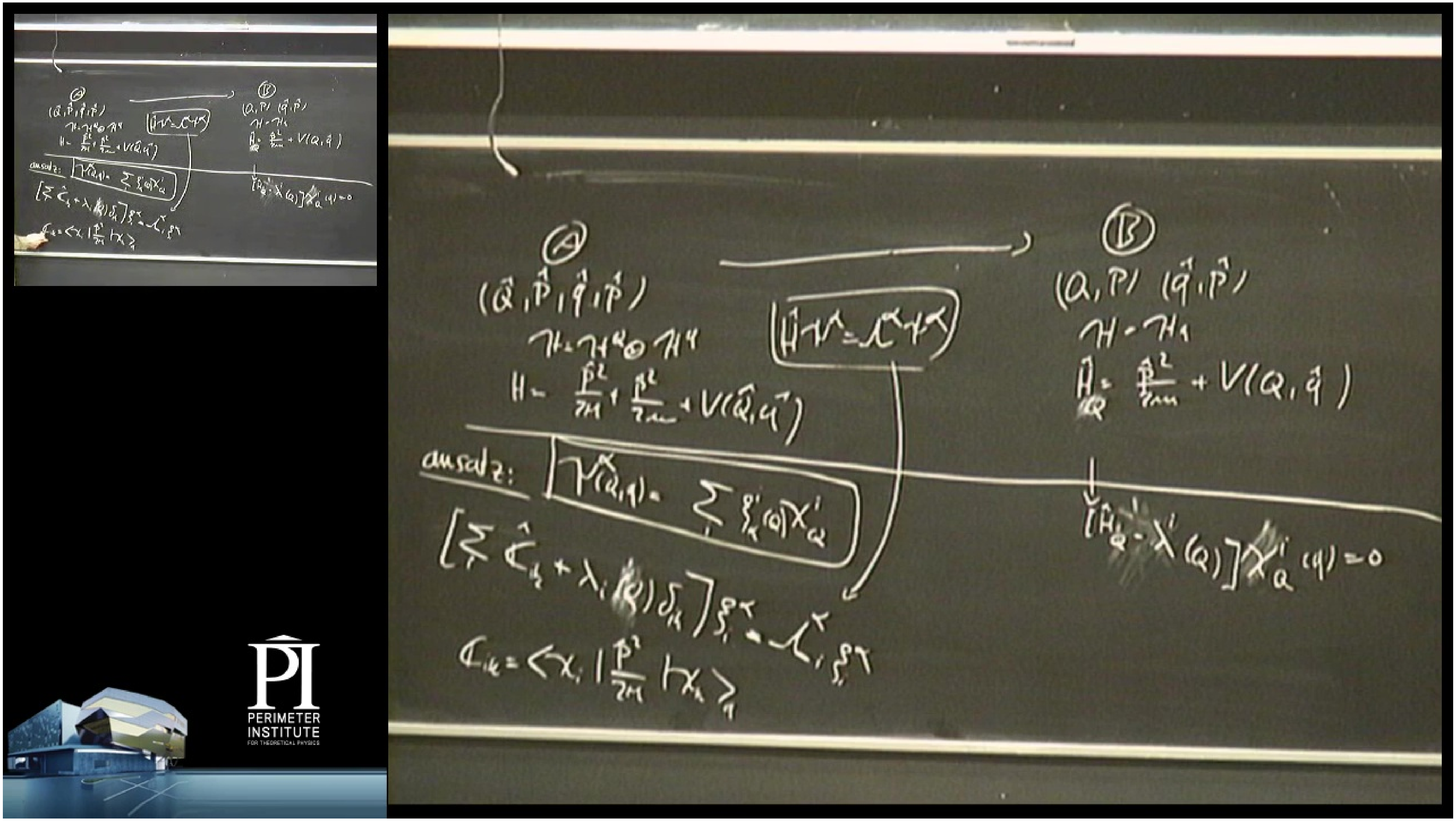
(A)
$(\vec{Q}, \vec{P}, \vec{q}, \vec{p})$
$\mathcal{H} = \mathcal{H}^Q \otimes \mathcal{H}^q$
$H = \frac{\vec{P}^2}{2M} + \frac{\vec{p}^2}{2m} + V(\vec{Q}, \vec{q})$
(B)
$(Q, P)\ (\vec{q}, \vec{p})$
$\mathcal{H} = \mathcal{H}_q$
$\hat{H}_Q = \frac{\vec{p}^2}{2m} + V(Q, \vec{q})$
$[\hat{H}_Q - \lambda(Q)]\chi_Q(q) = 0$


A
B
$(\vec{Q}, \vec{P}, \vec{q}, \vec{p})$
$(Q, P)\ (\vec{q}, \vec{p})$
$\hat{H} = \frac{\vec{p}^2}{2m} + V(Q, \vec{q})$
ansatz:
$C_{ik} = \langle \chi_i | \frac{P^2}{2M} | \chi_k \rangle_q$
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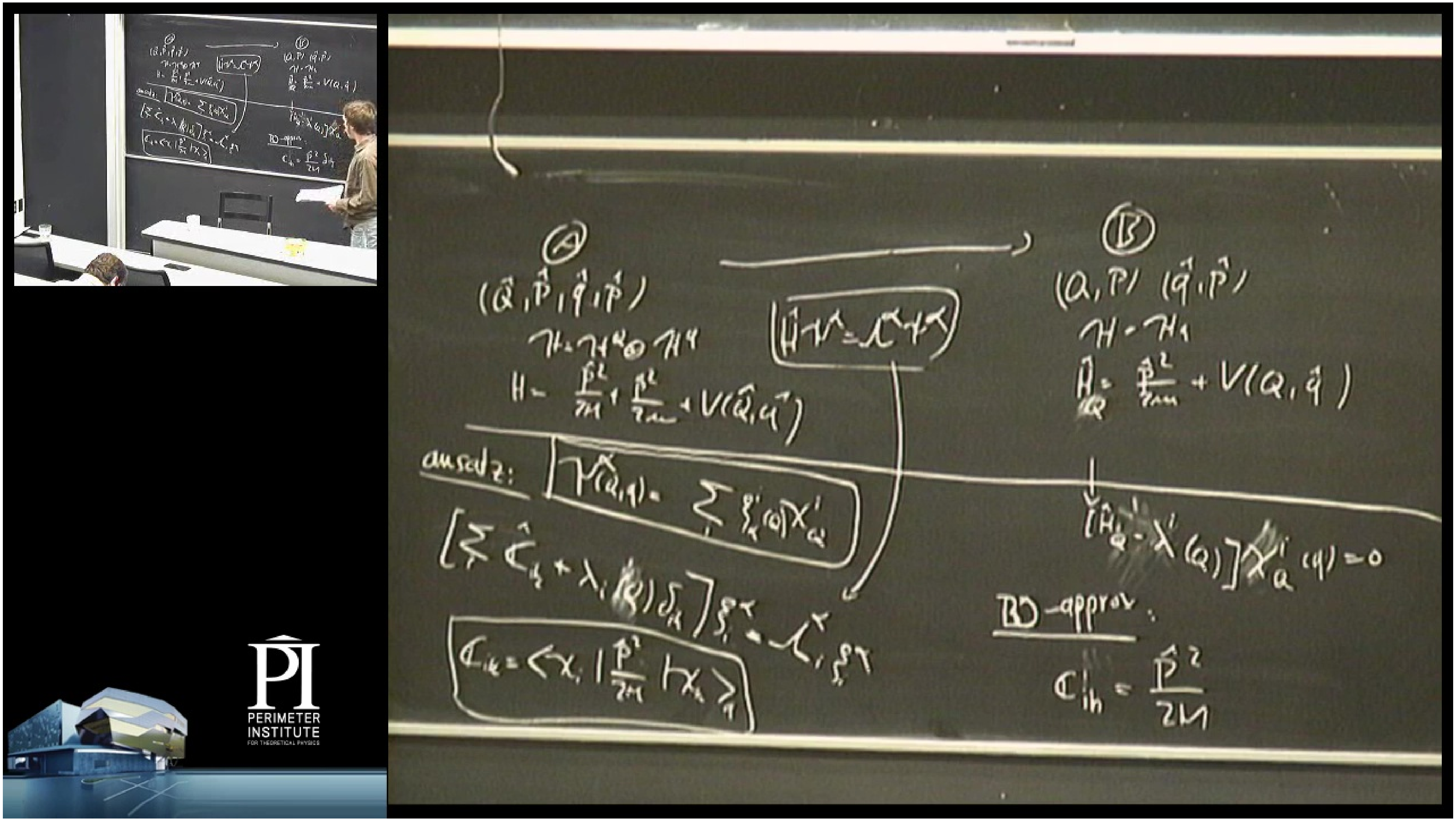
A
B
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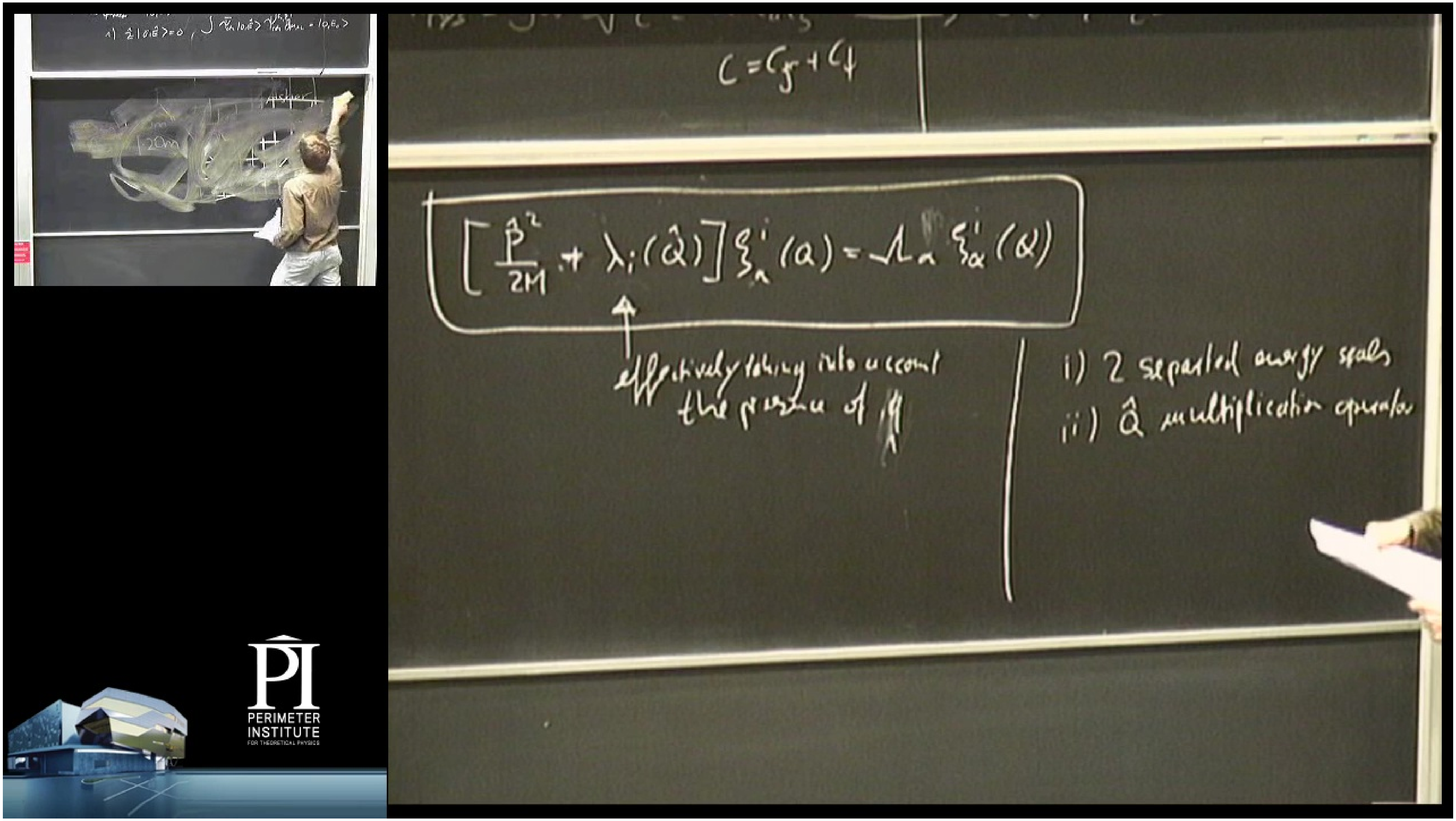
ansatz:
BO-approx.


C = C_g + C_f
i) 2 separated energy scales
ii) Q̂ multiplication operator
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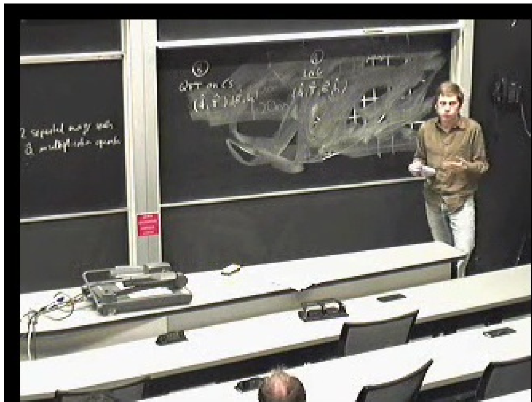

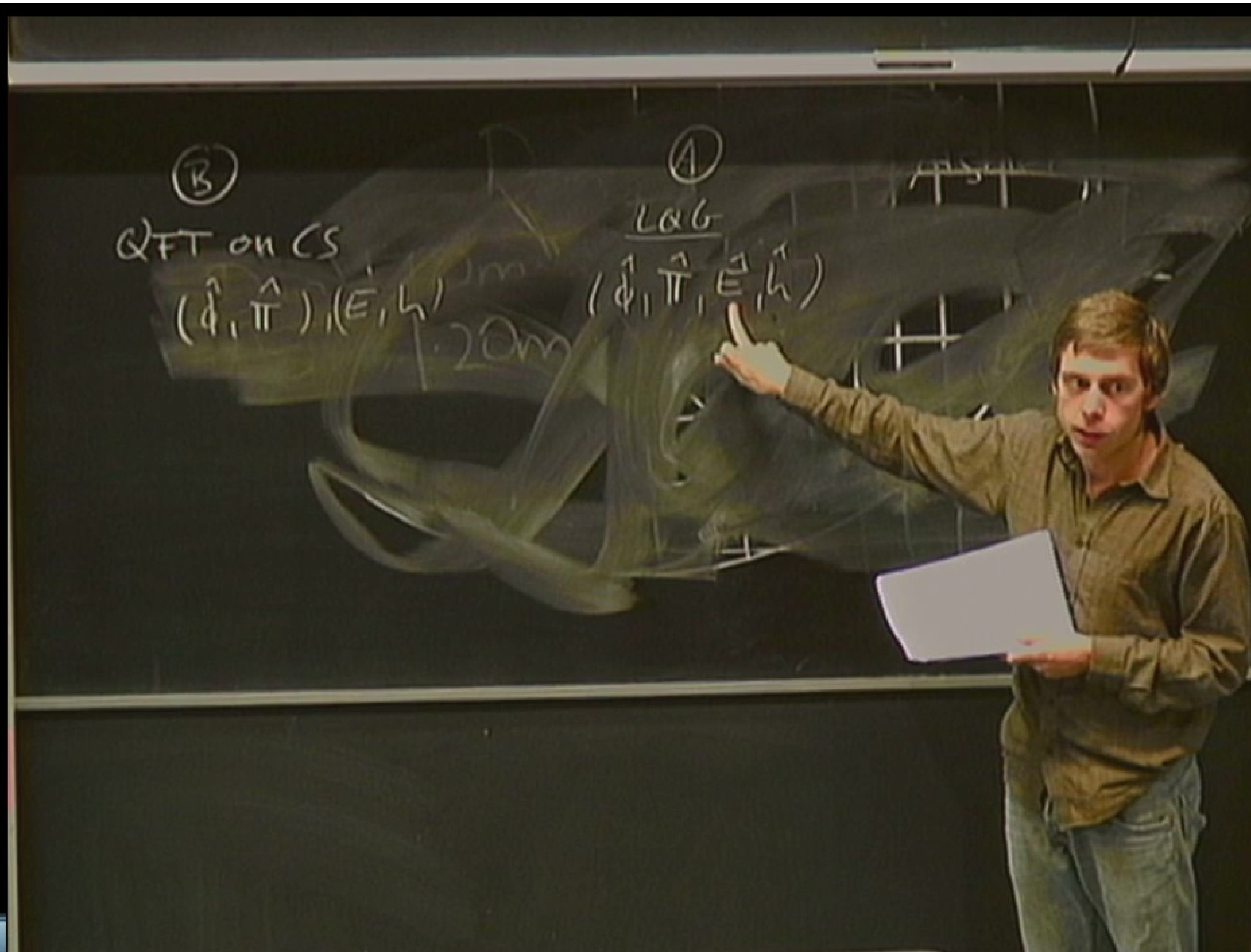

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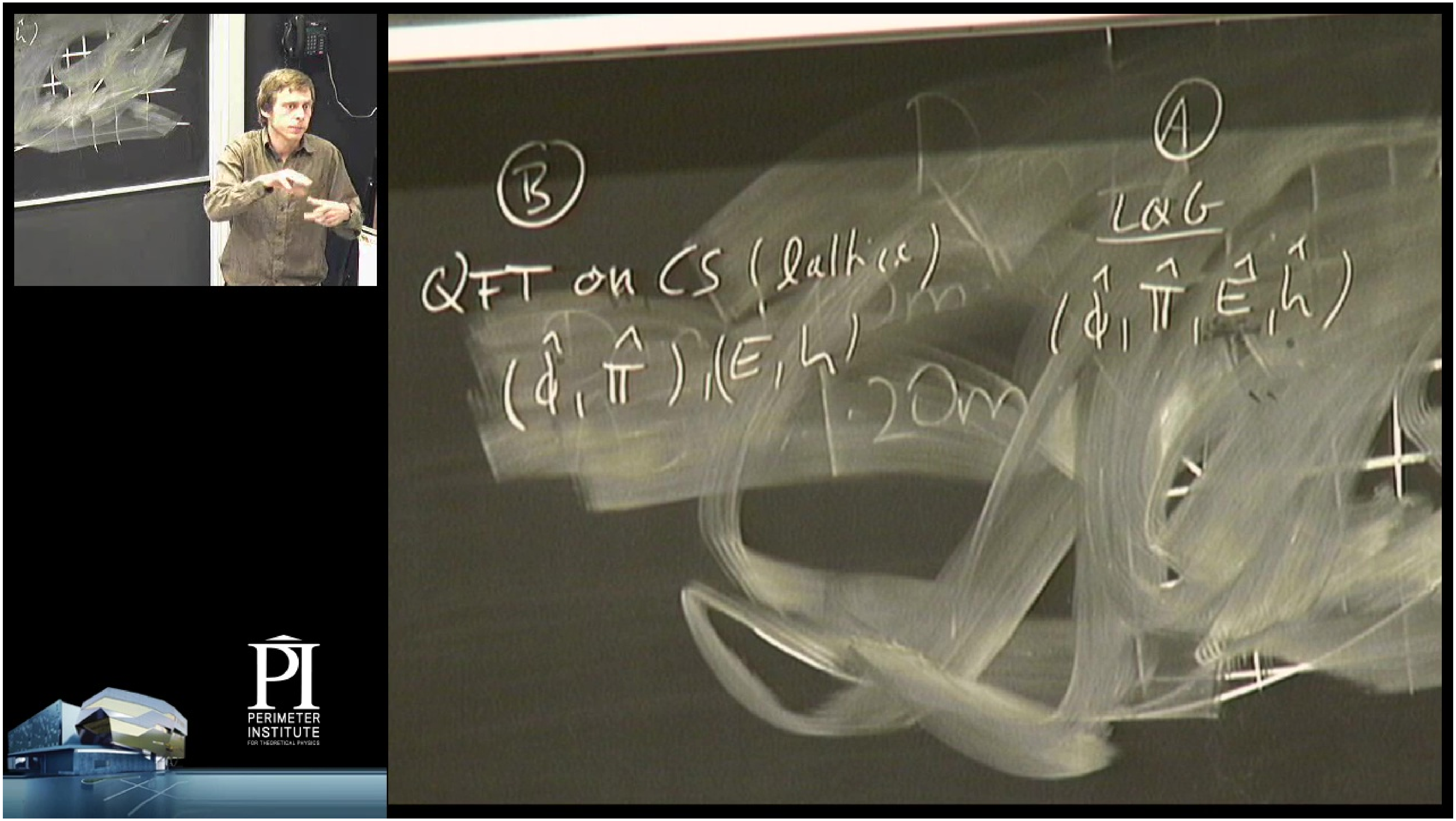
B
QFT on CS (lattice)
$(\hat{\phi}, \hat{\pi}), (E, h)$
20m
A
LQG
$(\hat{\phi}, \hat{\pi}, \hat{E}, \hat{h})$
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