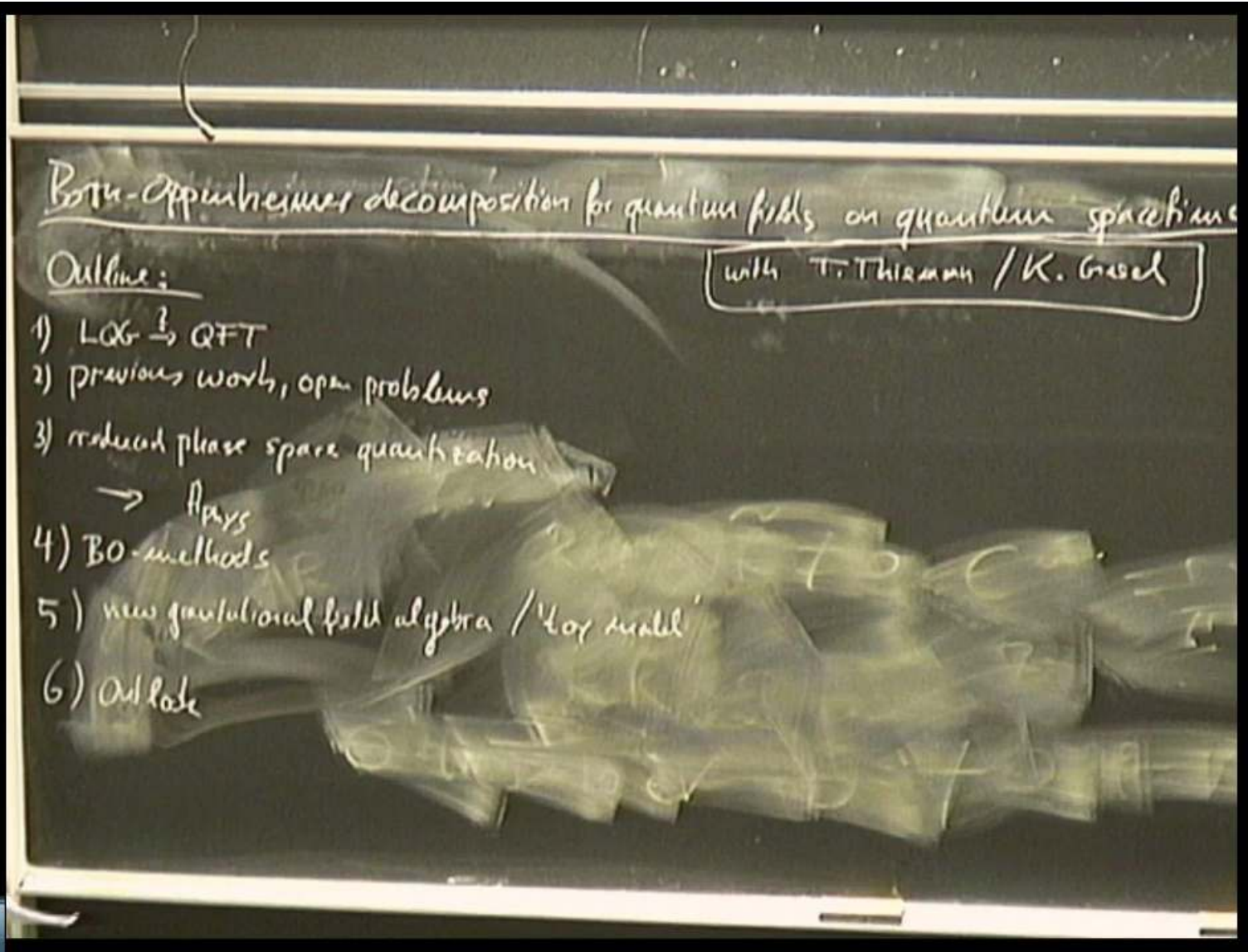
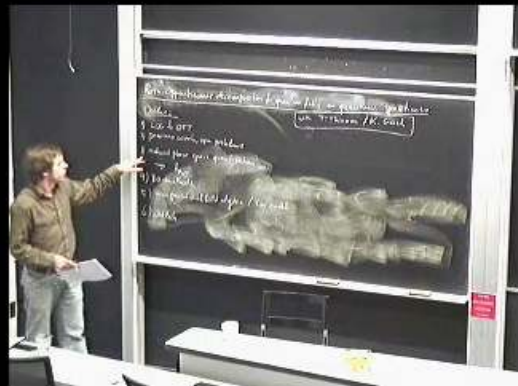


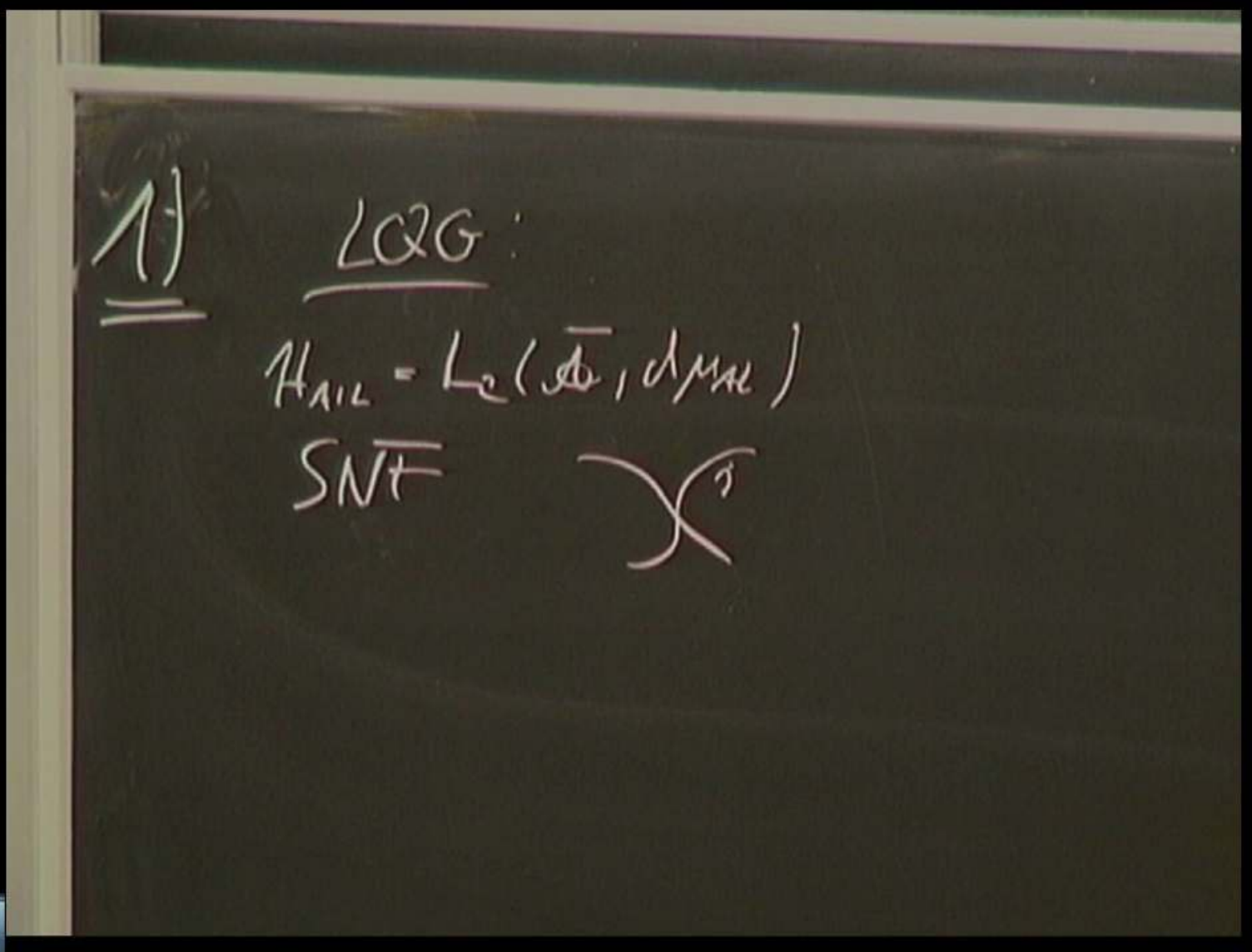
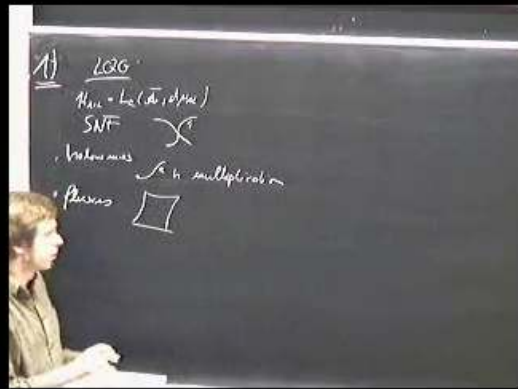
Title: Born--Oppenheimer approximation for quantum fields on quantum spacetimes

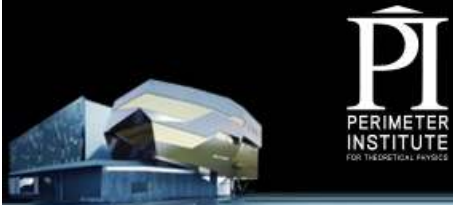
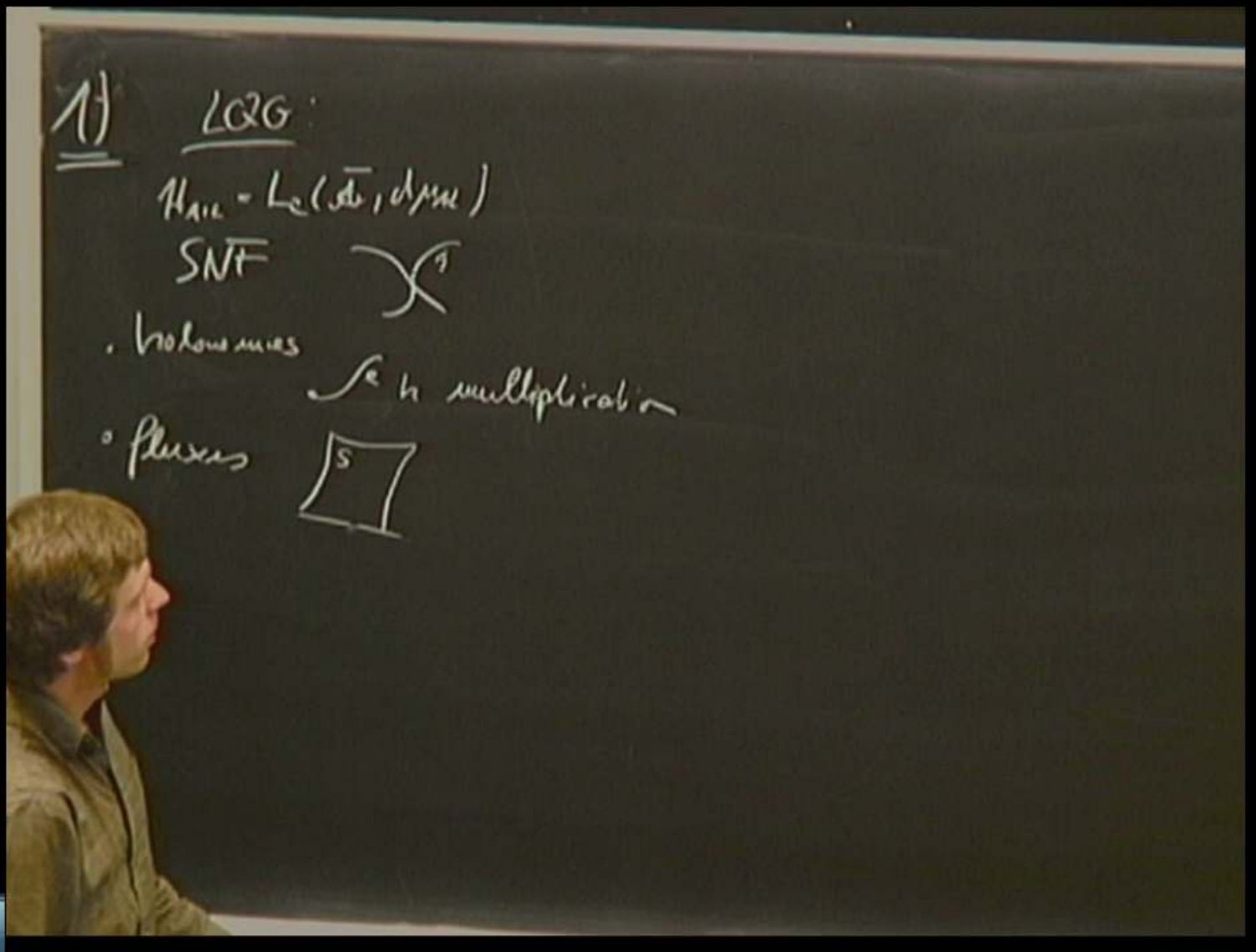
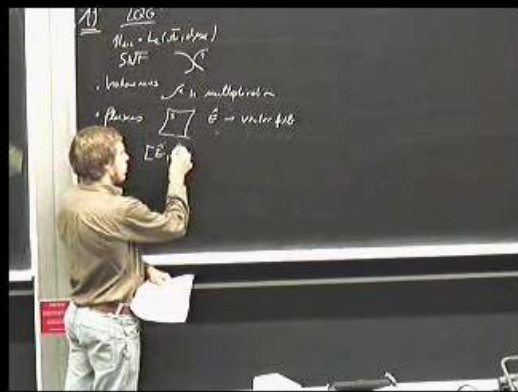
Date: Nov 25, 2009 04:00 PM

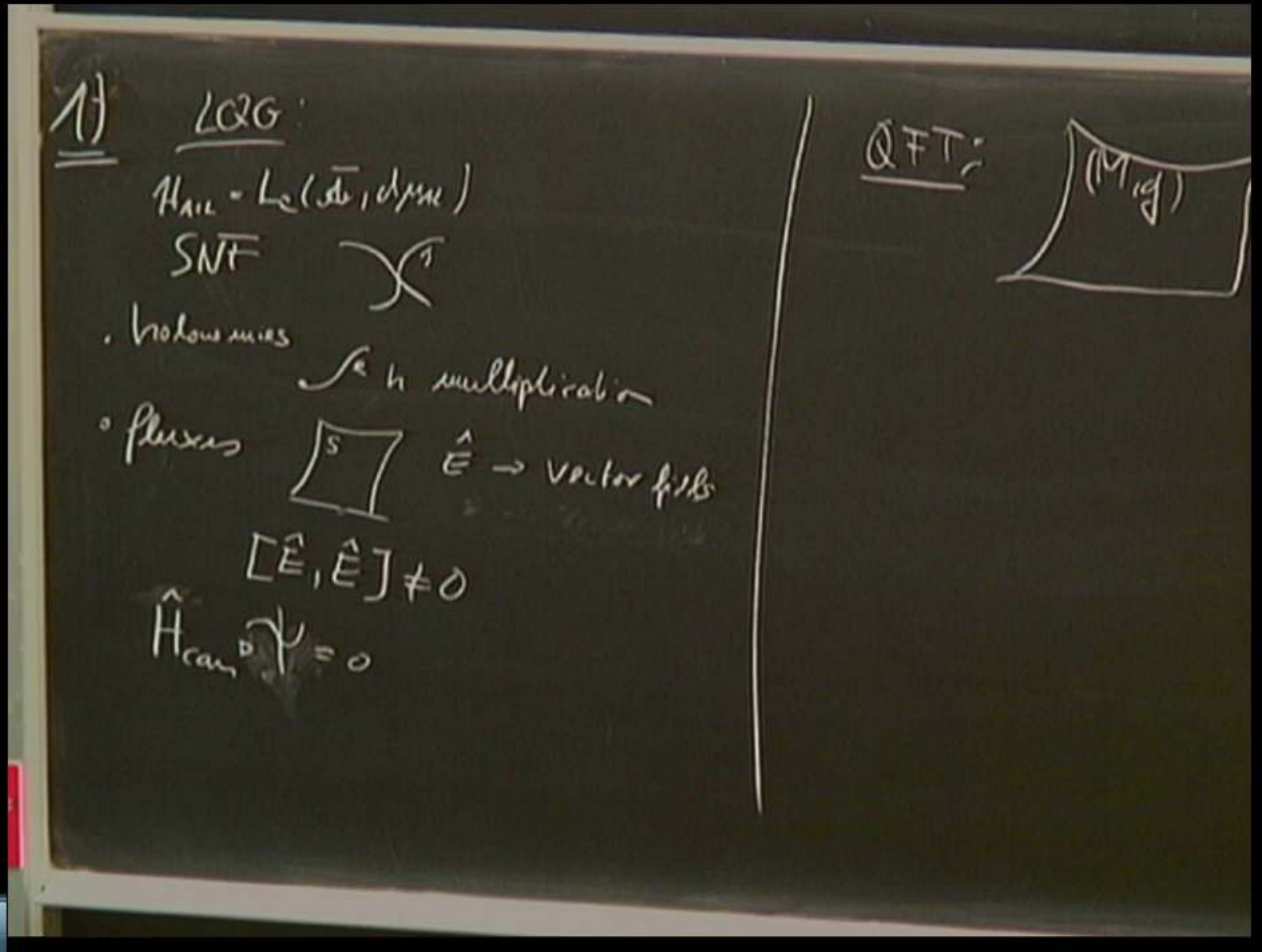
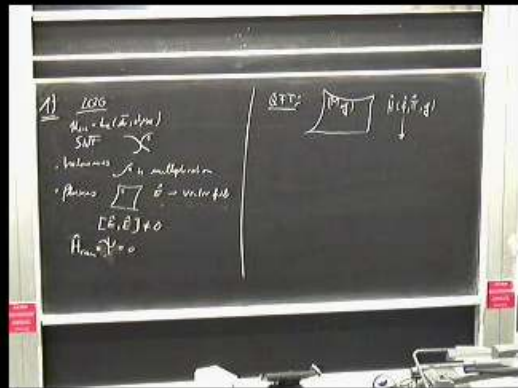
URL: <http://pirsa.org/09110142>

Abstract: The relation between loop quantum gravity (LQG) and ordinary quantum field theory (QFT) on a fixed background spacetime still bears many obstacles. When looking at LQG and ordinary QFT from a mathematical perspective it turns out that the two frameworks are rather different: Although LQG is a true continuum theory its Hilbert space is defined in terms of certain embedded graphs which are labeled by irreducible representations of $SU(2)$. The natural arena for ordinary QFT, on the other hand, is a Fock space which strongly uses the metric properties of the underlying continuum spacetime. In this talk I will review this issue and show how one can use Born--Oppenheimer methods to further progress towards an understanding of (matter) quantum field theories from first principles.











1) LQG
 $H_{\text{ADM}} = L_e(\dot{a}_i, \dot{p}_i)$
 SNF χ^i

- holonomies $\int_a^b \chi^i$ multiplication
- fluxes $\int_S \vec{E} \rightarrow$ vector fib

$[\vec{E}, \vec{E}] \neq 0$
 $\hat{H}_{\text{ADM}} \psi = 0$

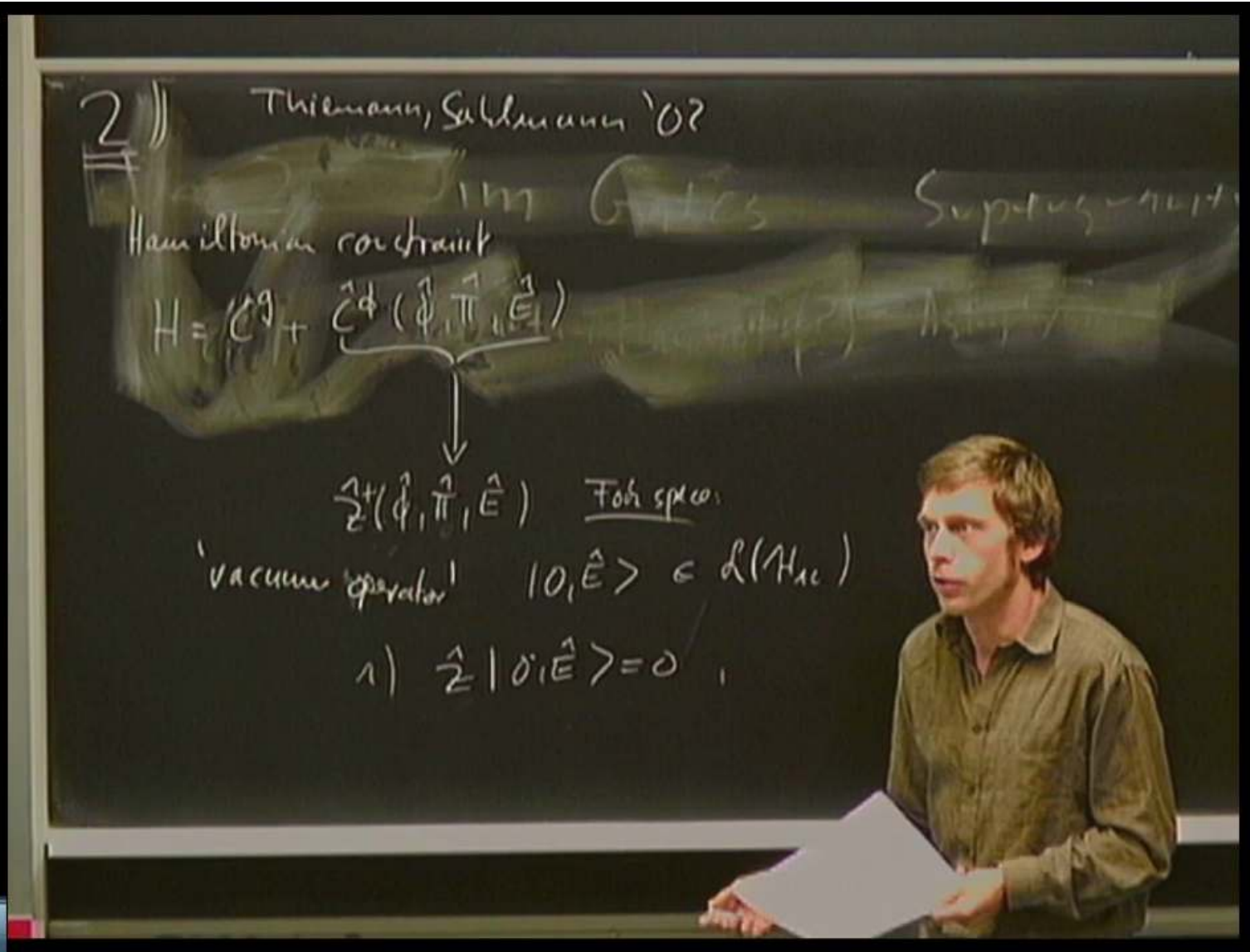
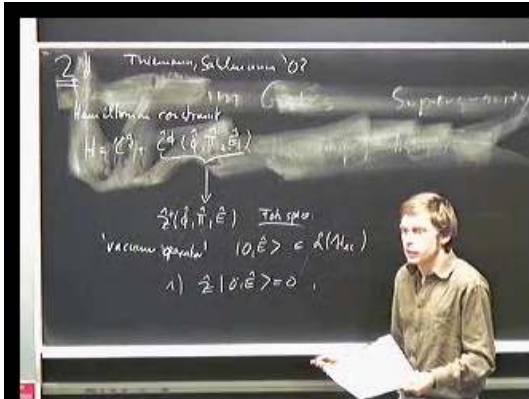
QFT: (M, g)

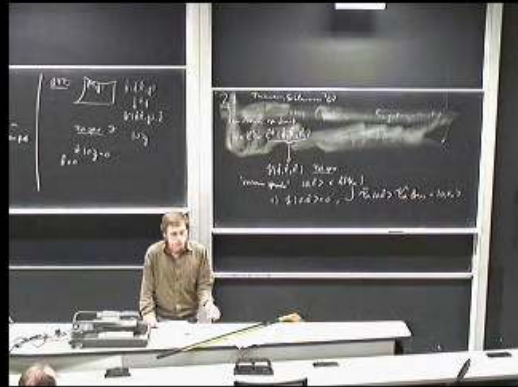
$\hat{H}(p, \pi, g)$
 $\downarrow \hat{H}_g$
 $\hat{Z}^+(p, \pi, g), \hat{Z}^-$
 $|0\rangle_g$

Fock space: \mathcal{F}

$\hat{H}|0\rangle_g = 0$
 $\hat{H}|+0\rangle = 0$







2) Thiemann, Seibener '07

im GUT's Supergravity

Hamiltonian constraint

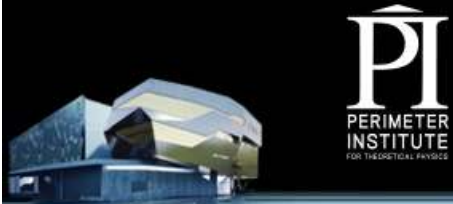
$$H = \mathcal{H} + \mathcal{H}_T(\hat{q}, \hat{\pi}, \hat{E})$$

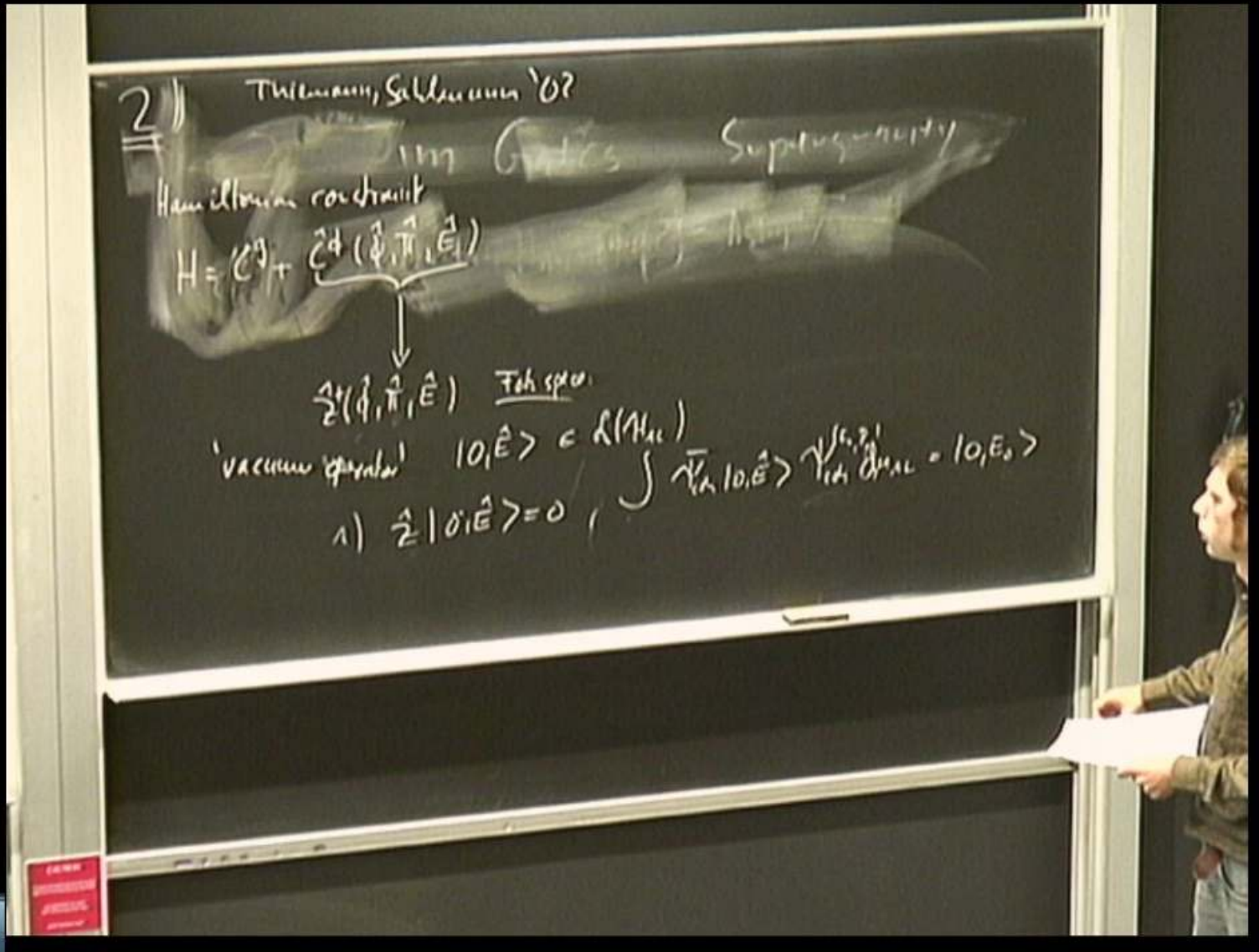
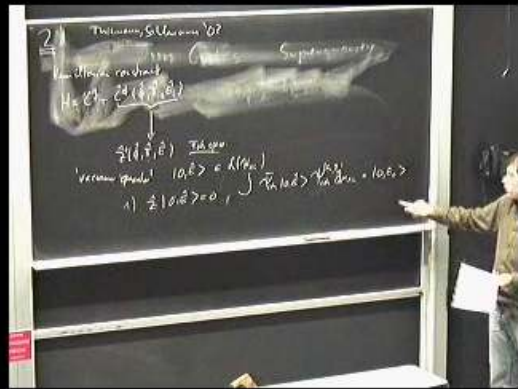
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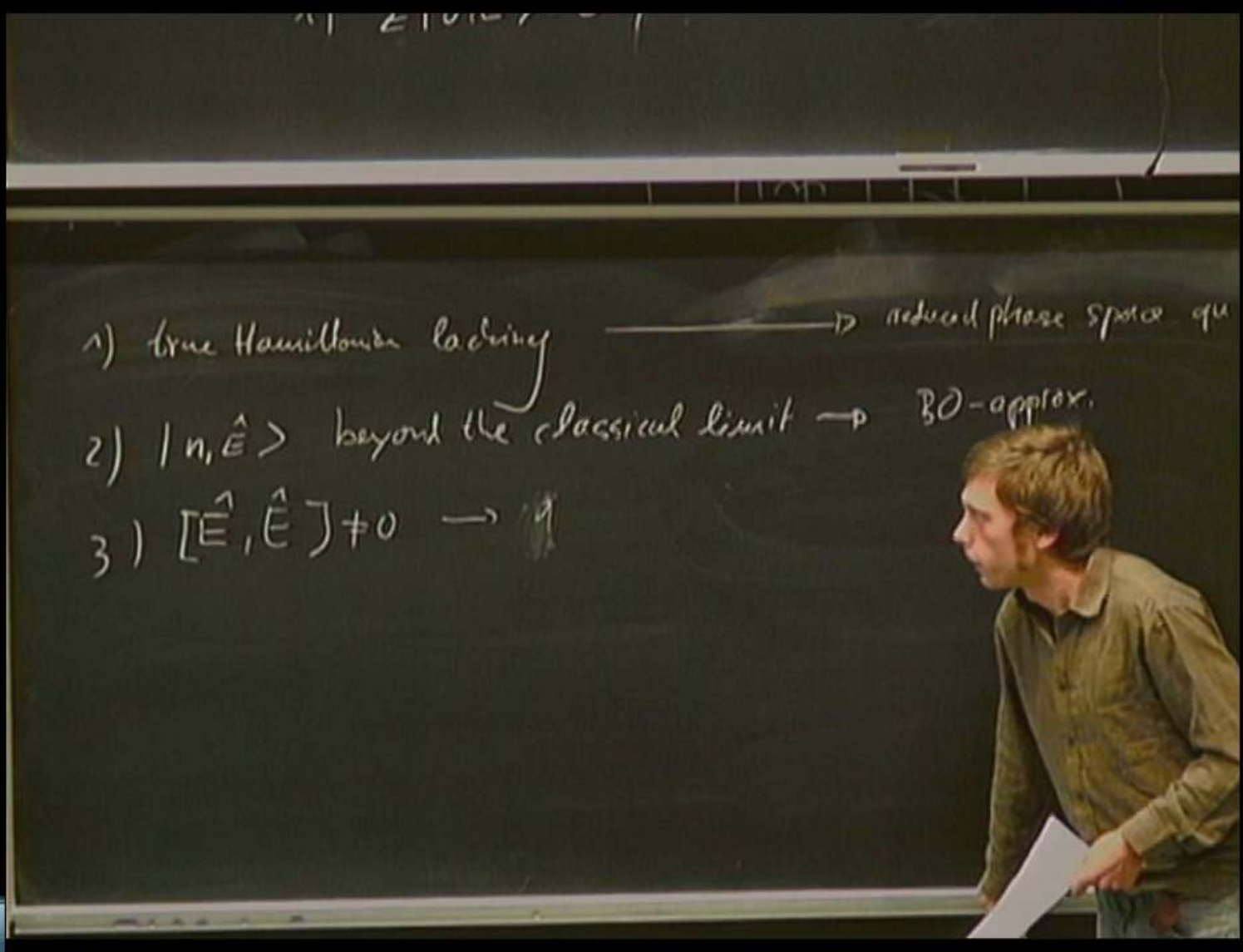
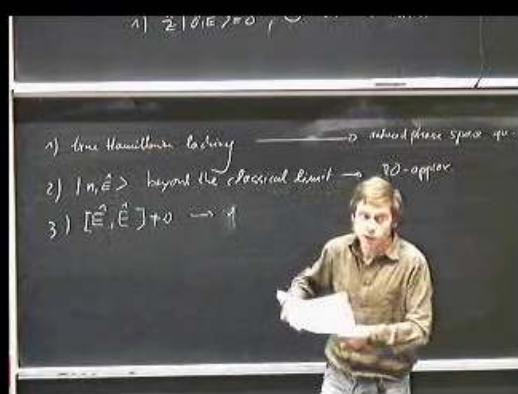
$\hat{H}(\hat{q}, \hat{\pi}, \hat{E})$ Fock space

'vacuum operator' $|0, \hat{E}\rangle \in \mathcal{L}(\mathcal{H}_{\mathcal{M}})$

1) $\hat{H}|0, \hat{E}\rangle = 0$, $\int \bar{\Psi}_{\mathcal{M}} |0, \hat{E}\rangle \Psi_{\mathcal{M}}^{\mathcal{E}} d\mathcal{M}_{\mathcal{M}} = |0, E_0\rangle$









Soil \rightarrow
 $H_2 O$

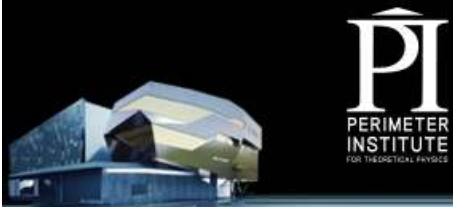
Dirac Program

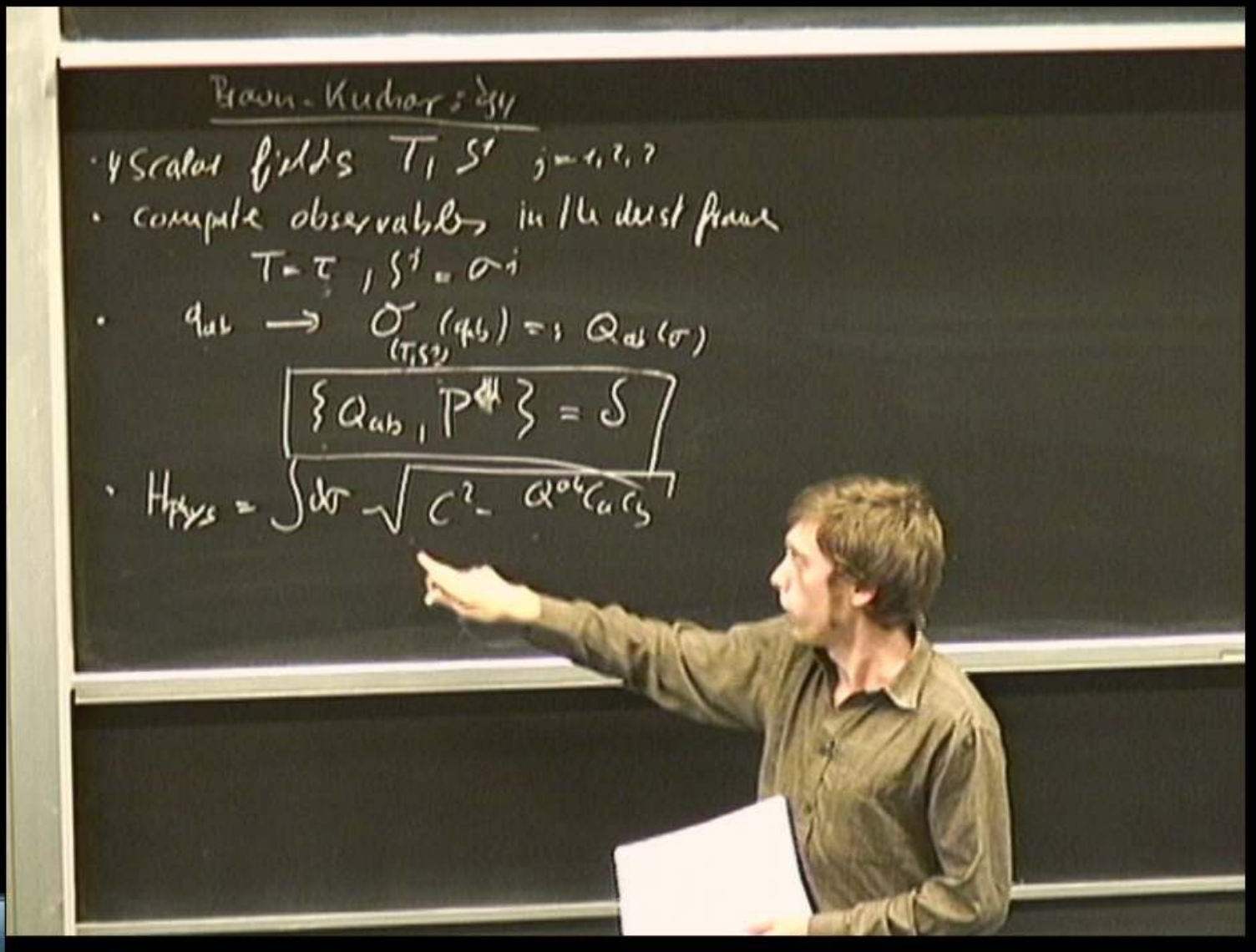
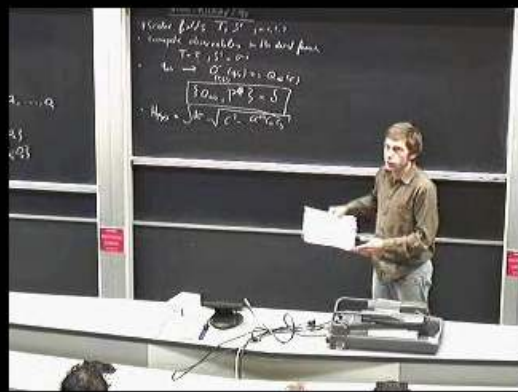
- i) quantize all dof:
- ii) $H_{\text{red}} \Psi = 0$
 Ψ
- iii) dynamics?

RPA

- i) solve constraints \rightarrow
 \hookrightarrow Dirac Observables a_1, \dots, a_n
 $\{a_1, a_2\} = ?$
- ii) representation of $\{a_1, a_2\}$
- iii) dynamics $\frac{d}{dt} a = \{H_{\text{red}}, a\}$

//





$S_{\text{EH}} + S_{\text{J}} + S_{\text{M}}$ Teves,

Brown-Kuchar: 234

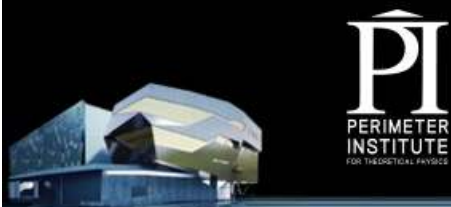
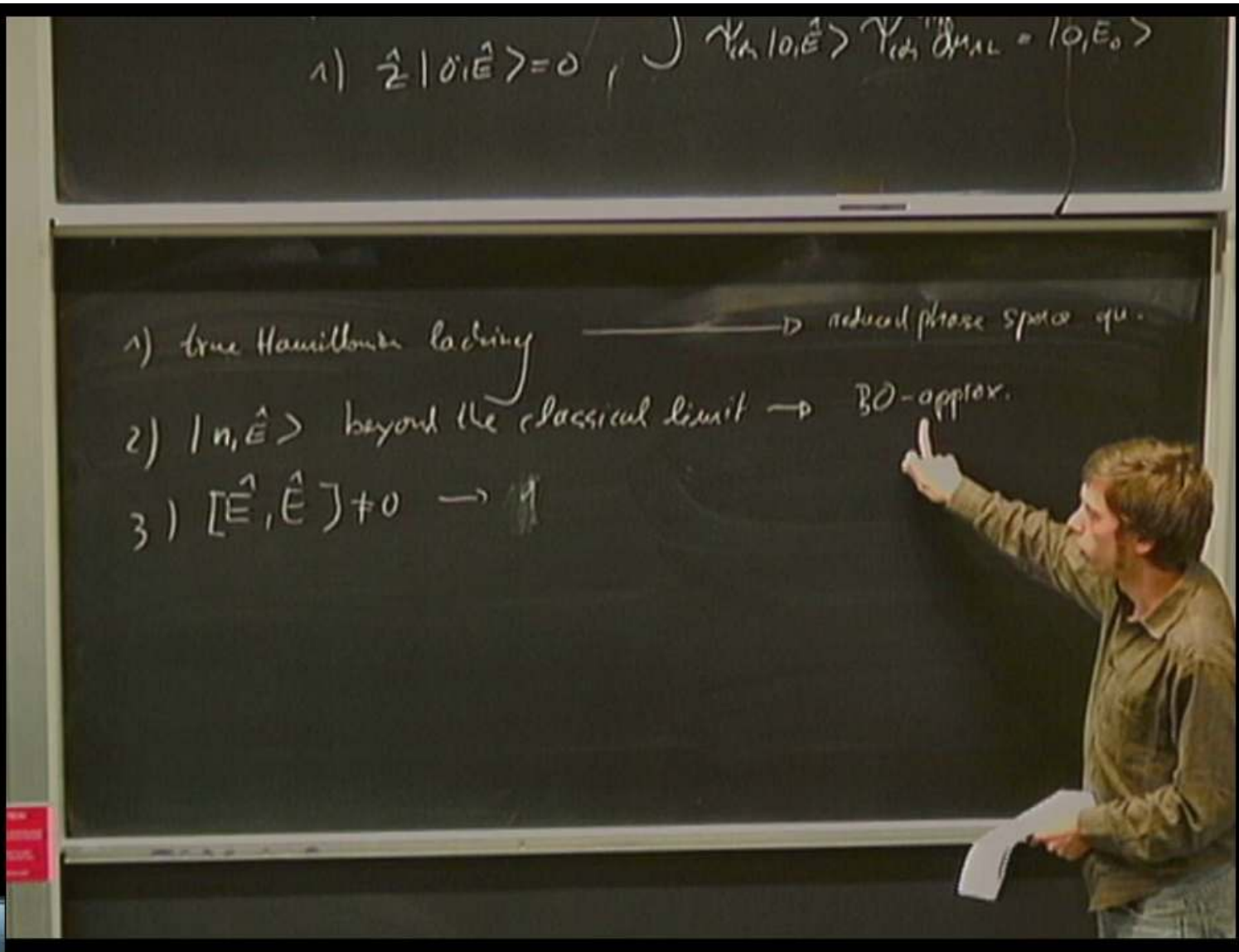
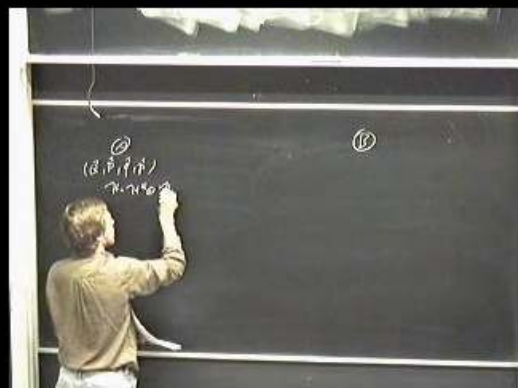
- 4 scalar fields $T_i, S^i, i=1,2,3$
- compute observables in the dust frame
 $T = \tau, S^i = \sigma^i$
- $q_{ab} \rightarrow \sigma_{(T, S^i)}^{(ab)} =: Q_{ab}(\sigma)$
- $\{Q_{ab}, P^{\mu}\} = \mathcal{D}$
- $H_{\text{phys}} = \int dV \sqrt{c^2 - Q^{ab}c_a c_b} \rightarrow c \left[1 - \frac{Q^{ab}c_a c_b}{c^2} + \dots \right]$
 $c = c_g + c_d$

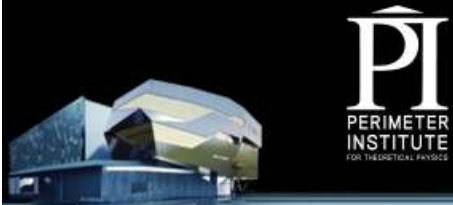
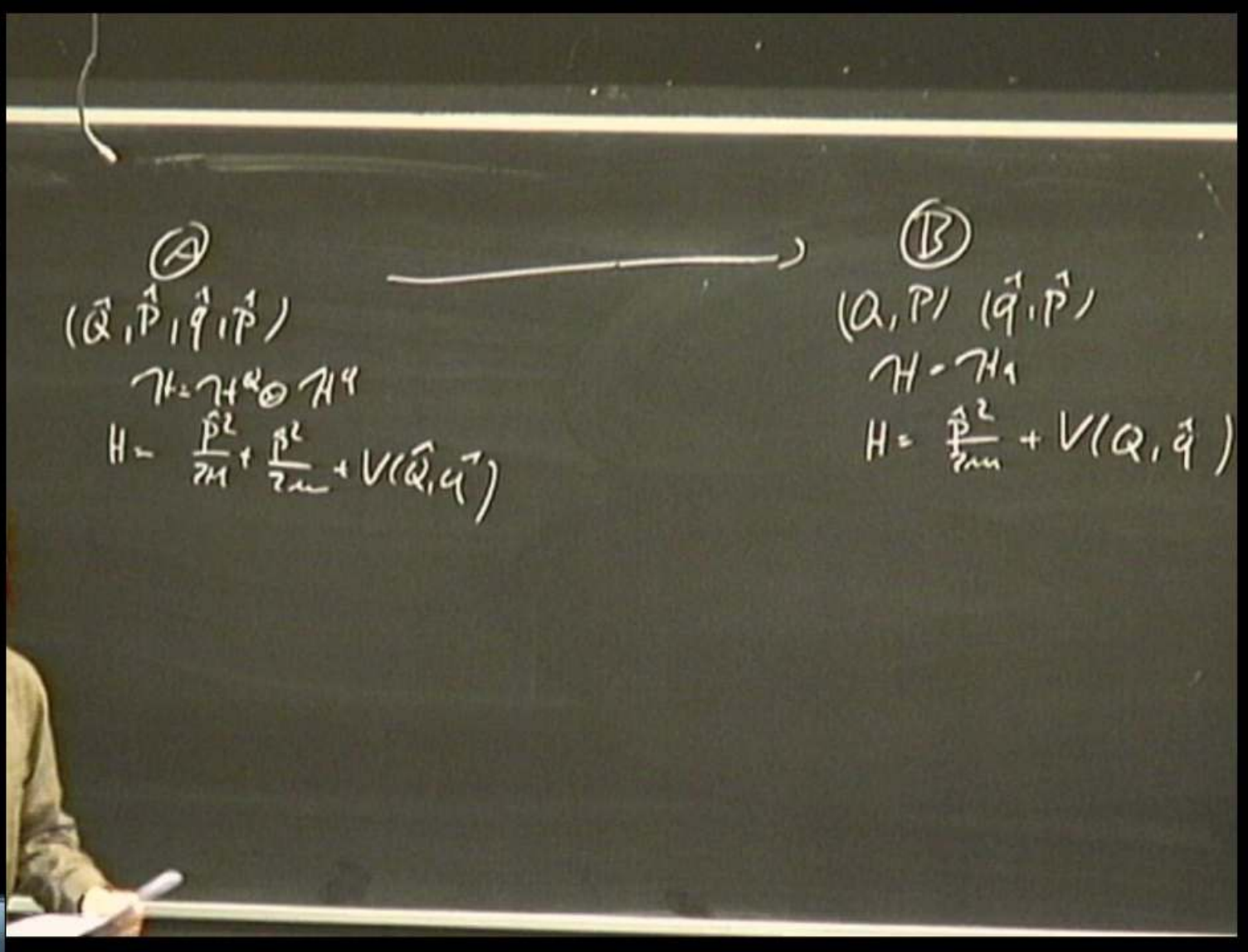
$S_{\text{EH}} + S_{\text{J}} + S_{\text{M}}$ Teves,

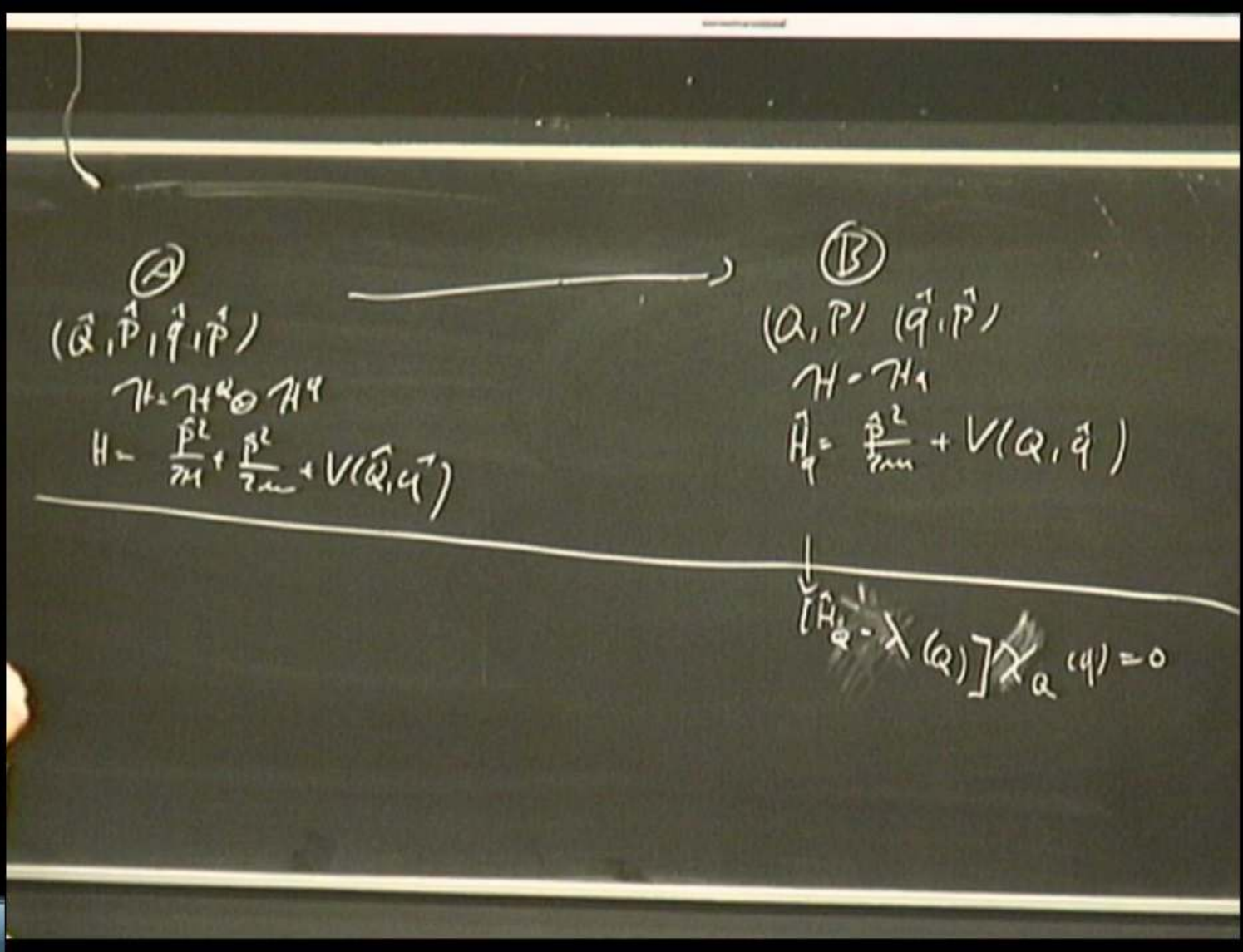
Brown-Kuchar: 234

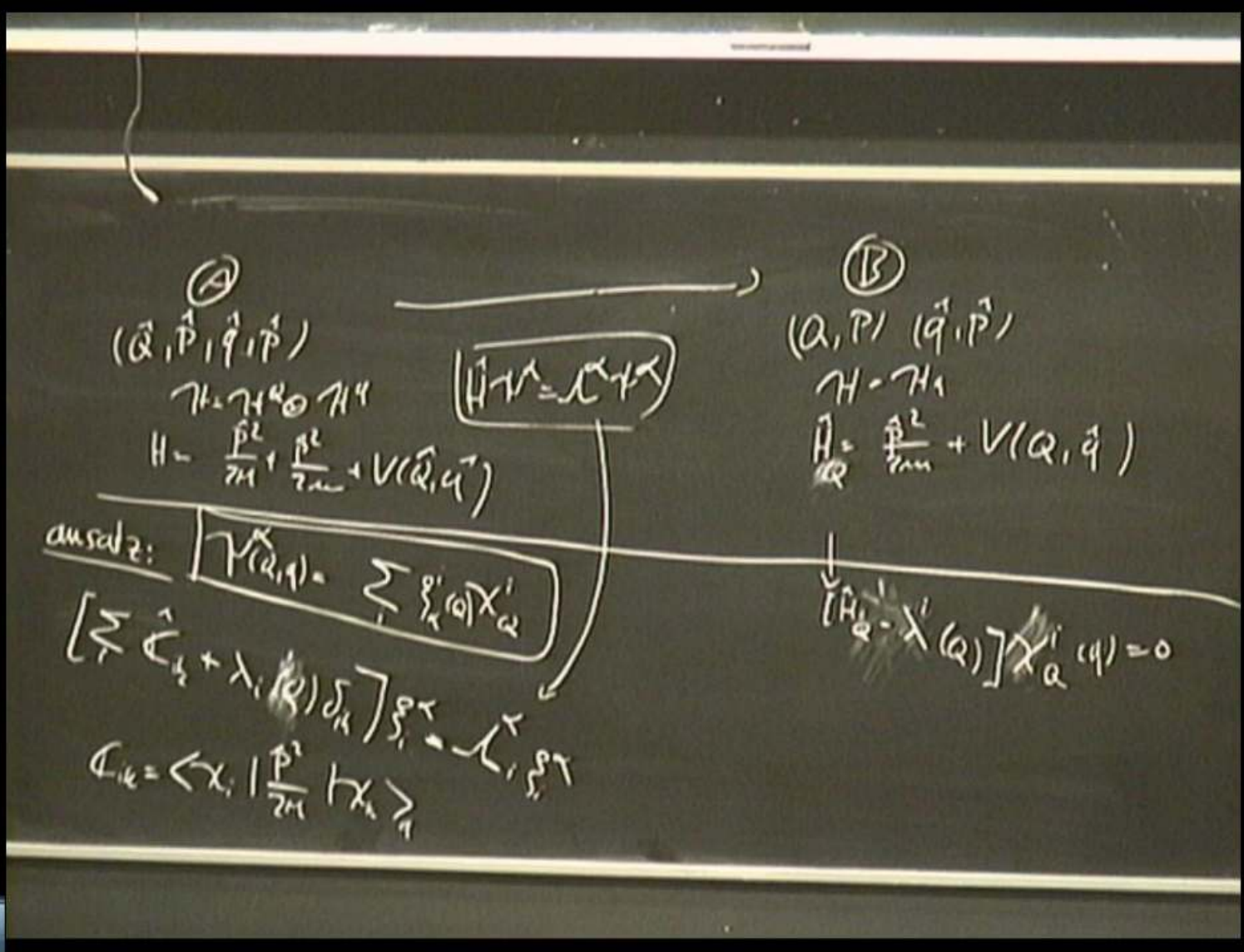
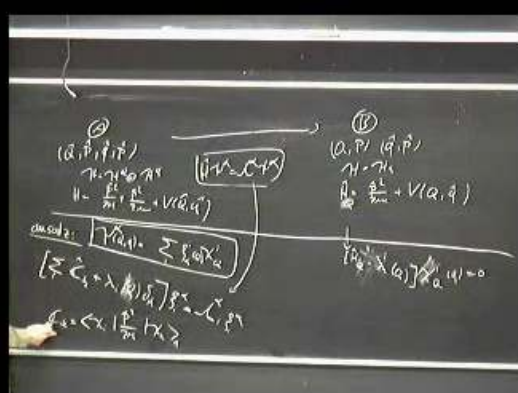
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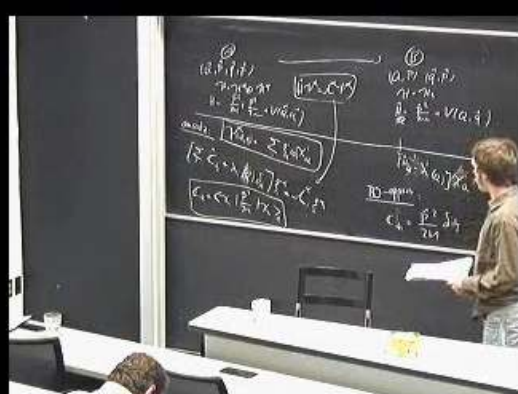












Ⓐ

(Q, P, \dot{q}, \dot{P})
 $\mathcal{H} = \mathcal{H}^0 + \mathcal{H}^1$
 $\mathcal{H} = \frac{P^2}{2M} + \frac{P^2}{2m} + V(Q, q)$

ansatz: $\Psi(Q, q) = \sum_i \psi_i(q) \chi_i(Q)$

$[\sum_i \hat{C}_i + \lambda_i \hat{P}_i] \psi_i = \mathcal{E}_i \psi_i$

$C_{ik} = \langle \chi_i | \frac{P^2}{2M} | \chi_k \rangle$

Ⓑ

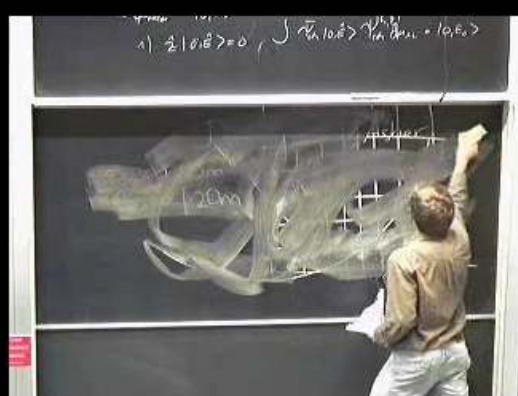
$(Q, P) | \dot{q}, \dot{P} \rangle$
 $\mathcal{H} = \mathcal{H}_1$
 $\hat{H}_Q = \frac{P^2}{2M} + V(Q, q)$

$[\hat{H}_Q - \mathcal{E}_i] \chi_i(Q) = 0$

RD-approx.

$C_{ih} = \frac{P^2}{2M}$





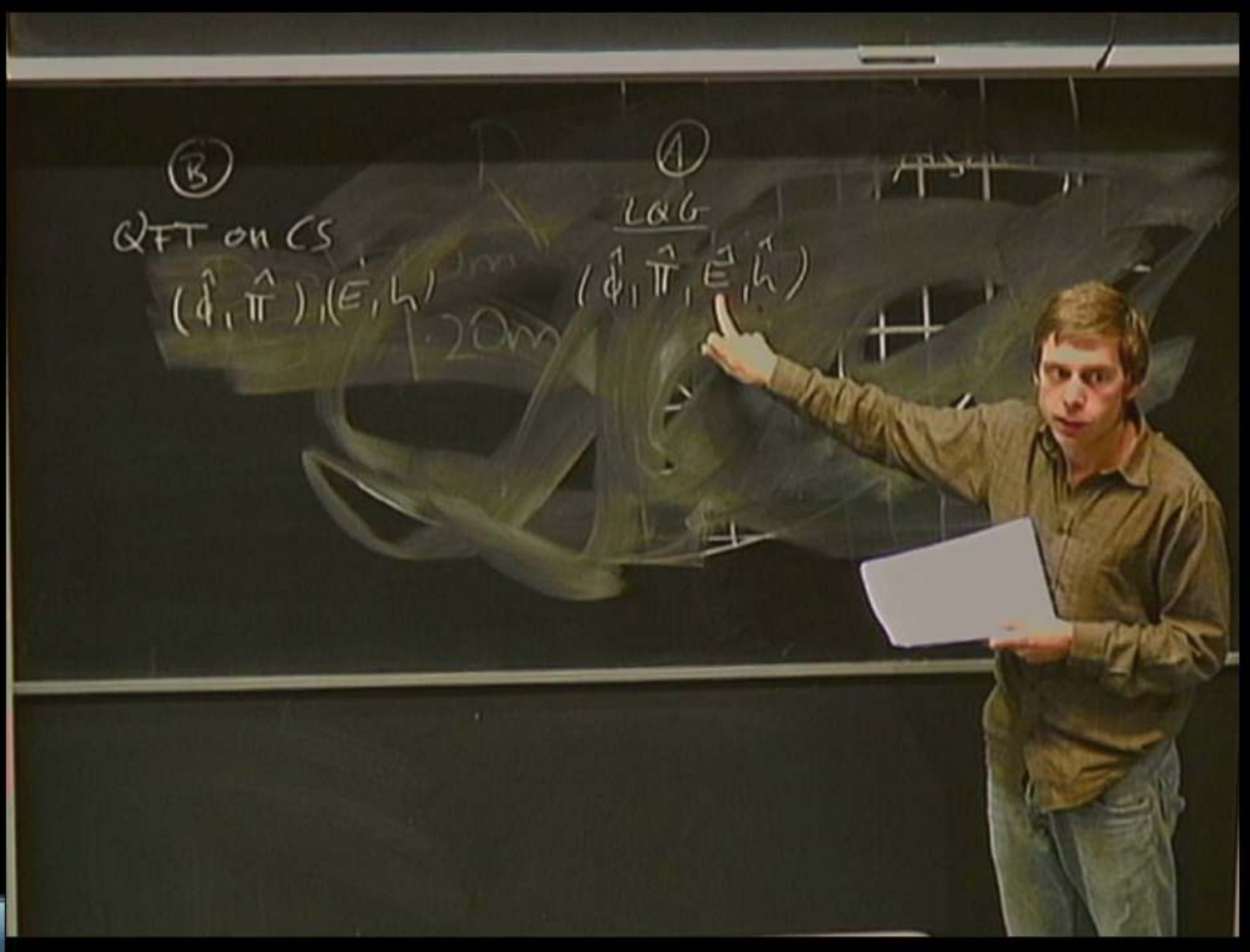
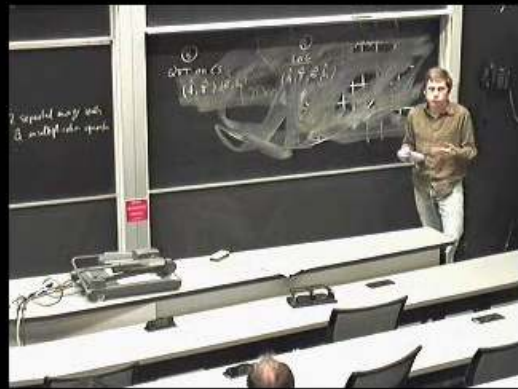
$C = C_g + C_f$

$$\left[\frac{\hat{p}^2}{2M} + \lambda_i(\hat{Q}) \right] \xi_n^i(\alpha) = \Lambda_n \xi_n^i(\alpha)$$

↑
effectively taking into account
the presence of \hat{Q}

- i) 2 separated energy scales
- ii) \hat{Q} multiplication operator







Handwritten notes on a chalkboard:

(B)
QFT on CS (lattice)
 $(\hat{\phi}, \hat{\pi}) (E, h)$

(A)
LAG
 $(\hat{\phi}, \hat{\pi}, E, h)$

Additional faint markings include "D" and "20m".

