Title: Evaporating black holes in the presence of a minimal length

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Abstract: After implementing an effective minimal length, we will present a new class of spacetimes, describing both neutral and charged black holes. As a result, we will improve the conventional Schwarzschild and Reisner-Nordstroem spacetimes, smearing out their singularities at the origin. On the thermodynamic side, we will show how the new black holes admit a maximum temperature, followed by the ``SCRAM phase'', a thermodynamic stable shut down, characterized by a positive black hole heat capacity. As a consequence, also for the neutral solution, in place of the runaway behavior of the temperature, one finds that the evaporation ends up with a zero temperature extremal black hole, i.e. a final configuration entirely governed by the minimal length. For the charged case, both the Hawking and Schwinger pair creation will be discussed in this new scenario. We will also analyze the above solutions in the presence of extra dimensions and the connections with the production of mini black holes, which is foreseen in the extreme energy hadron collisions at the LHC in the next few months. Finally we will discuss further developments and possible connections with other approaches in this field.

Evaporating black holes in the presence of a minimal length

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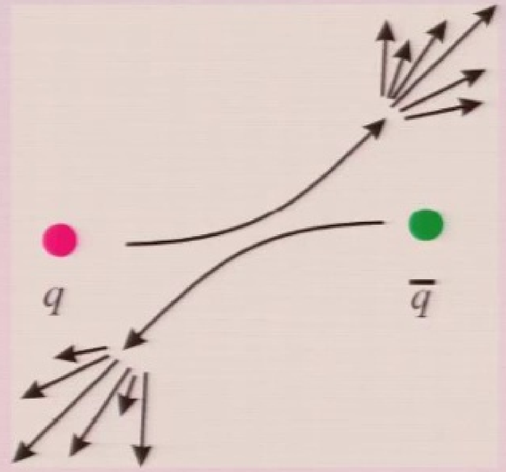
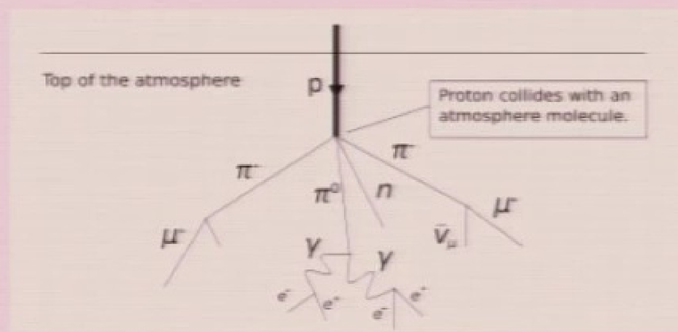
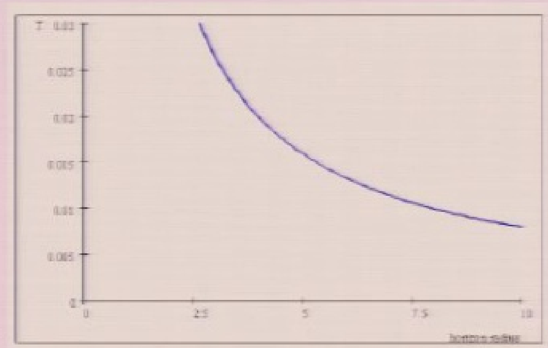
(Mini) Black holes @ LHC & Cosmic ray showers





(Mini) Black holes @ LHC & Cosmic ray showers

Black hole life



T_H vs r_H

- Balding phase
- Spin down phase
- Schwarzschild phase $T_H \sim 1/r_H$
- Planck phase (?)

Noncommutative geometry

The emergence of a minimal length ℓ

- $$[\mathbf{x}^\mu \,, \mathbf{x}^\nu] = i\,\theta^{\mu\nu} \Longrightarrow \Delta x^\mu \Delta x^\nu \sim \ell^2 \equiv ||\theta^{\mu\nu}|| \qquad (1)$$

Noncommutative Gravity. Question

What is the noncommutative equivalent of General Relativity?

Answer

No complete and fully compelling theory of this type yet exits.

The noncommutative quasi-coordinates

Coordinate coherent states

- NCG$\rightarrow$

$$f(x) \longrightarrow f(\langle \mathbf{x} \rangle) \tag{2}$$

- $\langle \mathbf{x} \rangle$ are expectation values
-

$$f(\langle \mathbf{x} \rangle) = \int d^d p e^{-\ell^2 p^2} e^{ip\langle \mathbf{x} \rangle} f(p) \tag{3}$$

-

$$\left[\triangle_{\langle \mathbf{x} \rangle} + X(\langle \mathbf{x} \rangle)\right] \; f(\langle \mathbf{x} \rangle) = \int d^d p e^{-\ell^2 p^2} e^{ip\langle \mathbf{x} \rangle} s(p) \tag{4}$$

- $s(p) = const.$

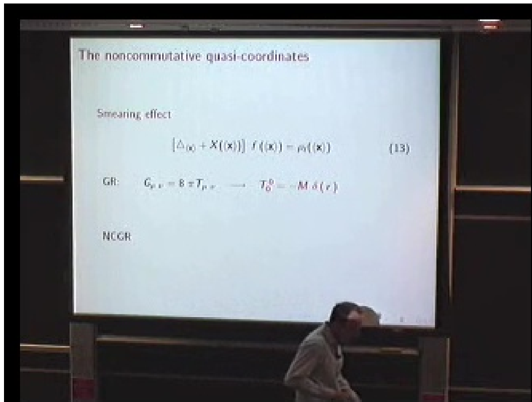
$$\rho_\ell(\langle \mathbf{x} \rangle) = \frac{1}{(4\pi\ell^2)^{d/2}} \exp\left(-\frac{\langle \mathbf{x} \rangle^2}{4\ell^2}\right) = \int d^d p e^{-\ell^2 p^2} e^{ip\langle \mathbf{x} \rangle} s(p) \tag{5}$$

The noncommutative quasi-coordinates

Smearing effect

$$\left[\triangle_{\langle \mathbf{x} \rangle} + X(\langle \mathbf{x} \rangle)\right] \; f\left(\langle \mathbf{x} \rangle\right) = \rho_\ell(\langle \mathbf{x} \rangle) \qquad (6)$$

GR



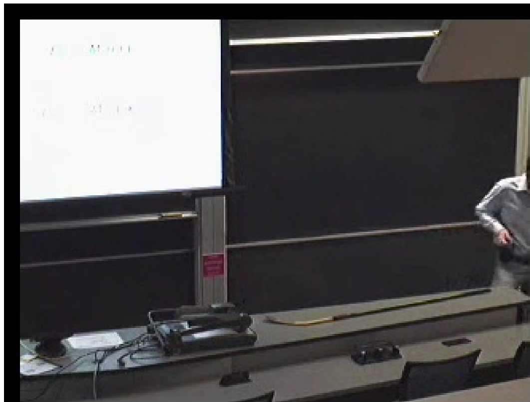
The noncommutative quasi-coordinates

Smearing effect

$$\left[\triangle_{\langle \mathbf{x} \rangle} + X(\langle \mathbf{x} \rangle)\right] \; f\left(\langle \mathbf{x} \rangle\right) = \rho_{\ell}(\langle \mathbf{x} \rangle) \qquad (12)$$

GR: $\quad G_{\mu\ \nu} = 8\ \pi\, T_{\mu\ \nu} \quad \longrightarrow$

NCGR



The noncommutative quasi-coordinates

Smearing effect

$$\left[\triangle_{\langle \mathbf{x} \rangle} + X(\langle \mathbf{x} \rangle)\right] \; f\left(\langle \mathbf{x} \rangle\right) = \rho_\ell(\langle \mathbf{x} \rangle) \qquad (16)$$

GR: $\quad G_{\mu\,\nu} = 8\,\pi\, T_{\mu\,\nu} \quad \longrightarrow \quad T_0^0 = -M\,\delta\,(\,r\,)$

$$\updownarrow$$

NCGR $\quad T_0^0\,\big|_\ell = -M\,\rho_\ell\,(\,\langle \mathbf{x} \rangle\,)$

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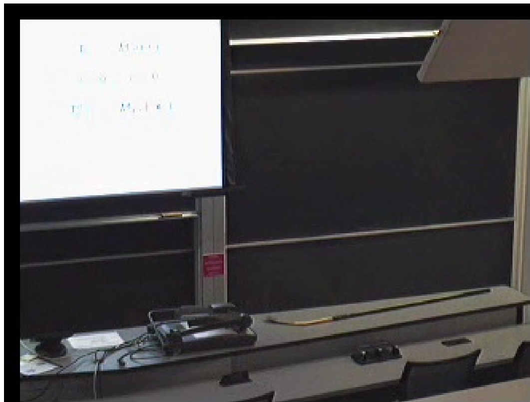

The noncommutative quasi-coordinates

Smearing effect

$$\left[\triangle_{\langle \mathbf{x}\rangle} + X(\langle \mathbf{x}\rangle)\right]\ f\left(\langle \mathbf{x}\rangle\right) = \rho_\ell(\langle \mathbf{x}\rangle) \qquad (19)$$

GR: $\quad G_{\mu\ \nu} = 8\ \pi\, T_{\mu\ \nu} \quad \longrightarrow \quad T_0^0 = -M\ \delta\left(\, r\,\right)$

$$\ell \neq 0 \downarrow \uparrow \ell \to 0$$

NCGR: $\quad G_{\mu\ \nu}\big|_\ell = 8\ \pi\ T_{\mu\ \nu}\big|_\ell \quad \longleftarrow \quad T_0^0\,\big|_\ell = -M\,\rho_\ell\left(\,\langle \mathbf{x}\rangle\,\right)$

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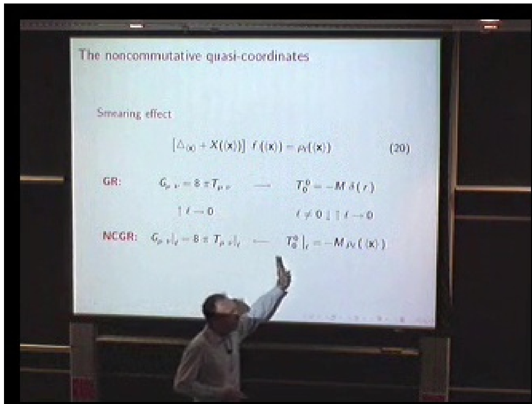


The noncommutative quasi-coordinates

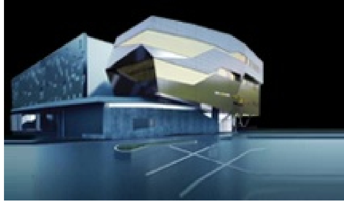
Smearing effect

$$\left[\triangle_{\langle \mathbf{x} \rangle} + X(\langle \mathbf{x} \rangle)\right] \ f(\langle \mathbf{x} \rangle) = \rho_\ell(\langle \mathbf{x} \rangle) \tag{20}$$

GR: $G_{\mu\,\nu} = 8\,\pi T_{\mu\,\nu} \quad \longrightarrow \quad T_0^0 = -M\,\delta(\,r\,)$

$\uparrow \ell \to 0 \qquad\qquad \ell \neq 0 \downarrow \uparrow \ell \to 0$

NCGR: $G_{\mu\,\nu}|_\ell = 8\,\pi\,T_{\mu\,\nu}|_\ell \quad \longleftarrow \quad T_0^0\,\big|_\ell = -M\,\rho_\ell(\,\langle \mathbf{x} \rangle\,)$



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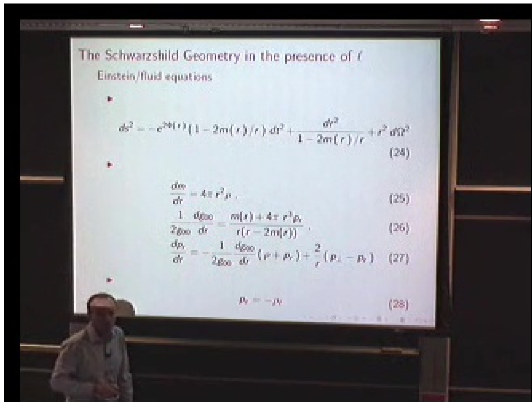
The Schwarzshild Geometry in the presence of ℓ

The energy-momentum tensor

- $$T^{\mu}{}_{\nu} = \mathrm{Diag}\left(-\rho_\ell\left(\langle \mathbf{r} \rangle \right) \,, p_r\left(\langle \mathbf{r} \rangle \right) \,, p_\perp\left(\langle \mathbf{r} \rangle \right) \,, p_\perp\left(\langle \mathbf{r} \rangle \right) \right) \qquad (21)$$

- $$T_0^0 = -\rho_\ell(\langle \mathbf{r} \rangle) = -\frac{M}{(4\pi\ell^2)^{3/2}} \exp\left(-\frac{\langle \mathbf{r} \rangle^2}{4\ell^2} \right) \qquad (22)$$

- $$T^{\mu\nu}\,;\,\nu = 0 \qquad (23)$$



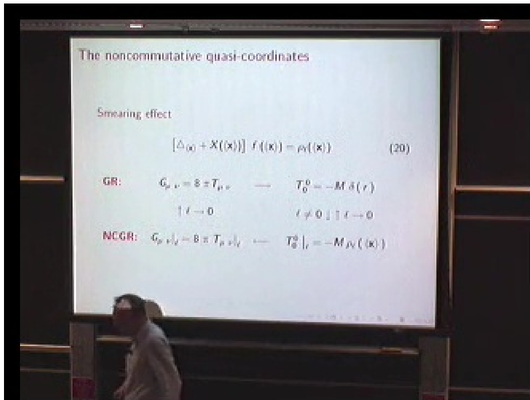
The Schwarzshild Geometry in the presence of ℓ

Einstein/fluid equations

- $$ds^2 = -e^{2\Phi(r)}\,(1 - 2m(r)/r)\,dt^2 + \frac{dr^2}{1 - 2m(r)/r} + r^2\,d\Omega^2 \quad (24)$$

- $$\frac{dm}{dr} = 4\pi\,r^2\rho\,, \quad (25)$$
$$\frac{1}{2g_{00}}\frac{dg_{00}}{dr} = \frac{m(r) + 4\pi\;r^3 p_r}{r(r - 2m(r))}\,, \quad (26)$$
$$\frac{dp_r}{dr} = -\frac{1}{2g_{00}}\frac{dg_{00}}{dr}\,(\rho + p_r) + \frac{2}{r}\,(p_\perp - p_r) \quad (27)$$

- $$p_r = -\rho_\ell \quad (28)$$



The noncommutative quasi-coordinates

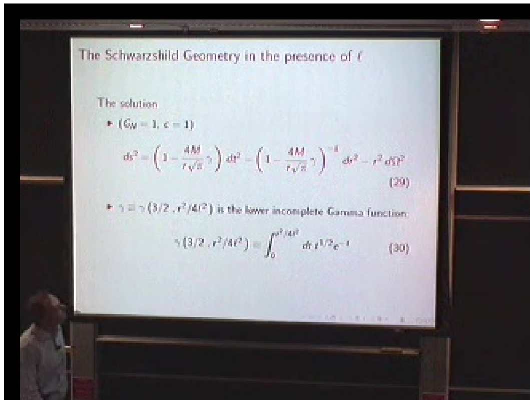
Smearing effect

$$\left[\triangle_{\langle \mathbf{x}\rangle} + X(\langle \mathbf{x}\rangle)\right]\ f\left(\langle \mathbf{x}\rangle\right) = \rho_\ell(\langle \mathbf{x}\rangle) \qquad (20)$$

GR: $G_{\mu\ \nu} = 8\ \pi T_{\mu\ \nu} \quad \longrightarrow \quad T^0_0 = -M\ \delta\left(\, r\,\right)$

$\uparrow \ell \rightarrow 0 \qquad\qquad \ell \neq 0 \downarrow \uparrow \ell \rightarrow 0$

NCGR: $G_{\mu\ \nu}|_\ell = 8\ \pi\ T_{\mu\ \nu}|_\ell \quad \longleftarrow \quad T^0_0\ \big|_\ell = -M\,\rho_\ell\left(\,\langle \mathbf{x}\rangle\,\right)$



The Schwarzshild Geometry in the presence of ℓ

The solution

- $(G_N = 1,\, c = 1)$

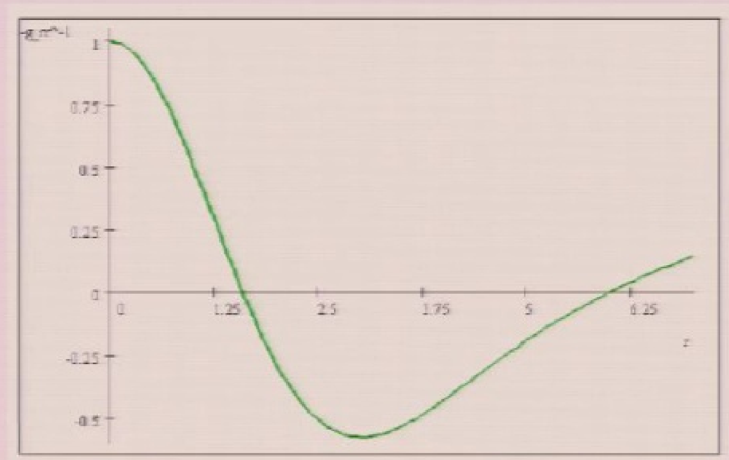
$$ds^2 = \left(1 - \frac{4M}{r\sqrt{\pi}}\gamma\right) dt^2 - \left(1 - \frac{4M}{r\sqrt{\pi}}\gamma\right)^{-1} dr^2 - r^2\, d\Omega^2 \tag{29}$$

- $\gamma \equiv \gamma\left(3/2\,,\, r^2/4\ell^2\right)$ is the lower incomplete Gamma function:

$$\gamma\left(3/2\,,\, r^2/4\ell^2\right) \equiv \int_0^{r^2/4\ell^2} dt\, t^{1/2} e^{-t} \tag{30}$$

The Noncommutative Schwarzshild Geometry

The horizon equation $g_{00}(r_H) = -g_{rr}^{-1}(r_H) = 0$

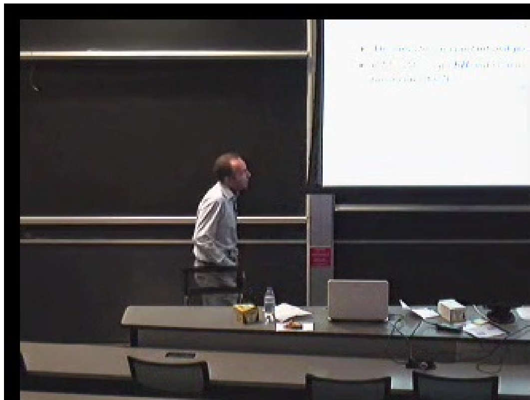


$-g_{rr}^{-1}$ vs r, for various values of M/ℓ.

$M = 3\ell \Rightarrow$ two horizons;

$M = \ell \Rightarrow$ ...;

$M = 1.9\ell \Rightarrow$ ...;



The Schwarzshild Geometry in the presence of ℓ

At the black hole centre

- *The Ricci scalar near the origin is*

$$R(0) = \frac{4M}{\sqrt{\pi}\,\ell^3} \qquad (31)$$

- *The curvature is constant and positive (deSitter geometry)*
- *If $M < M_0 \Rightarrow$ no BH and no naked singularity (mini-gravastar?)*

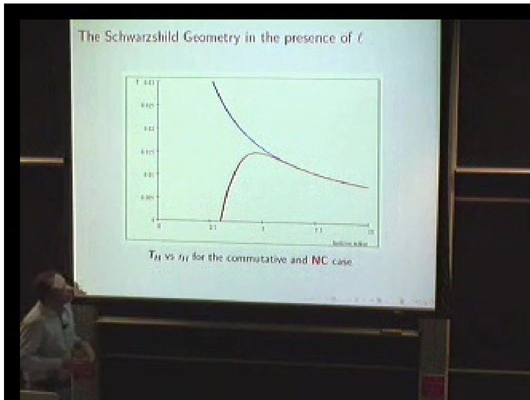
Large mass regime, $M \gg M_0$

The Schwarzshild Geometry in the presence of ℓ

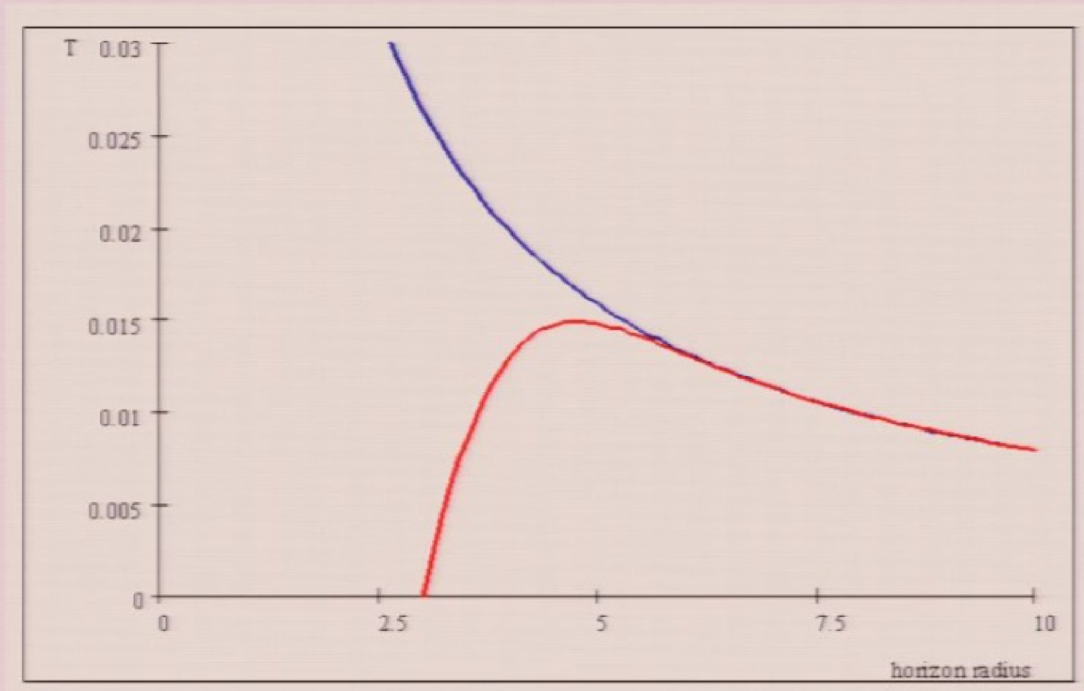
The Hawking temperature

$$T_H = \frac{1}{4\pi\, r_H}\left[1 - \frac{r_H^3}{4\,\ell^3}\,\frac{e^{-r_H^2/4\ell^2}}{\gamma\left(3/2\,;\,r_H^2/4\ell^2\right)}\right] \tag{32}$$

- *If* $r_H^2/4\ell^2 >> 1 \Rightarrow T_H = \frac{1}{4\pi\, r_H}$ *coincides with the Hawking result*
- *If* $r_H \simeq \ell \Rightarrow T_H$ *reaches a maximum* $\simeq 0.015 \times 1/\ell$ *corresponds to a mass* $M \simeq 2.4 \times \ell$ *and* $r_H \simeq 4.7\ell$
- **SCRAM phase**: *cooling down to absolute zero at* $r_H = r_0 = 3.0\ell$ *and* $M = M_0 = 1.9\,\ell$, *the extremal BH*
- *If* $r < r_0$ *there is no black hole.*



The Schwarzshild Geometry in the presence of ℓ



T_H vs r_H for the commutative and **NC** case.

The Schwarzshild Geometry in the presence of ℓ

Back reaction

- *relevant back-reaction in Planck phase.*
- *SCRAM phase* $\Rightarrow$ *a suppression of quantum back-reaction*
- *At maximum temperature, the thermal energy is* $E = T_H^{Max} \simeq 0.015\,/\ell$, *while the mass is* $M \simeq 2.4\,\ell\,M_P^2$
- $E \sim M \Rightarrow \ell \approx 0.2\,L_P \sim 10^{-34}\,cm.$
- *For this reason we can safely use unmodified form of the metric during all the evaporation process.*

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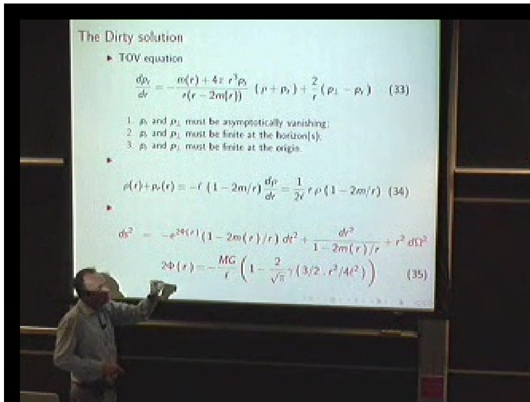


The Dirty solution

- TOV equation

$$\frac{dp_r}{dr} = -\frac{m(r) + 4\pi\, r^3 p_r}{r(r - 2m(r))}\,(\rho + p_r) + \frac{2}{r}(p_\perp - p_r) \qquad (33)$$

1. p_r and $p_\perp$ must be asymptotically vanishing;
2. p_r and $p_\perp$ must be finite at the horizon(s);
3. p_r and $p_\perp$ must be finite at the origin.

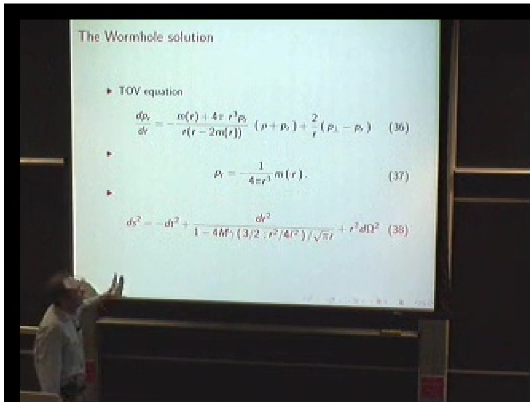
-

$$\rho(r) + p_r(r) \equiv -\ell\,(1 - 2m/r)\frac{d\rho}{dr} = \frac{1}{2\ell}\, r\,\rho\,(1 - 2m/r) \quad (34)$$

-

$$ds^2 = -e^{2\Phi(r)}(1 - 2m(r)/r)\,dt^2 + \frac{dr^2}{1 - 2m(r)/r} + r^2\,d\Omega^2$$

$$2\Phi(r) = -\frac{MG}{\ell}\left(1 - \frac{2}{\sqrt{\pi}}\gamma\,(3/2\,,r^2/4\ell^2)\right) \qquad (35)$$



The Wormhole solution

- TOV equation

$$\frac{dp_r}{dr} = -\frac{m(r) + 4\pi\ r^3 p_r}{r(r - 2m(r))}\ (\rho + p_r) + \frac{2}{r}(\,p_\perp - p_r\,) \qquad (36)$$

- $$p_r = -\frac{1}{4\pi r^3} m(\,r\,)\,. \qquad (37)$$

- $$ds^2 = -dt^2 + \frac{dr^2}{1 - 4M\gamma\,(\,3/2\,;\,r^2/4\ell^2\,)\,/\sqrt{\pi}r} + r^2 d\Omega^2 \quad (38)$$

The Reissner-Nordström geometry in the presence of ℓ

Properties of the charged source term

- the charge is diffused throughout a region of linear size ℓ

$$\rho_{el.}(\langle \mathbf{r} \rangle) = \frac{e}{(4\pi\ell^2)^{3/2}} \exp\left(-\langle \mathbf{r} \rangle^2 / 4\ell^2\right) \tag{39}$$

a "point-like object" when a minimal length is considered.

- We find the electric field to be:

$$E(r) = \frac{2Q}{\sqrt{\pi}\, r^2} \gamma\left(\frac{3}{2}; \frac{r^2}{4\ell^2}\right) \tag{40}$$

- $F^{\mu\nu} = \delta^{0[\mu|}\delta^{r|\nu]} E(r) \Rightarrow T_{el.}{}^{\mu}_{\nu}$

The Reissner-Nordström geometry in the presence of ℓ

The solution

- $ds^2 = g_{00}\, dt^2 - g_{00}^{-1}\, dr^2 - r^2 d\Omega^2$ with

$$g_{00} = 1 - \frac{4M}{r\sqrt{\pi}}\gamma + \frac{Q^2}{\pi\, r^2}\left[F(r) + \sqrt{2}\frac{r}{\ell}\gamma \right] \qquad (41)$$

- $M = \oint_\Sigma d\sigma^\mu \left(T^0_\mu|_{matt.} + T^0_\mu|_{el.} \right)$ where, Σ, is a $t = \mathrm{const.}$, closed three-surface.
- $F(r) \equiv \gamma^2\left(1/2\, , r^2/4\ell^2\right) - \frac{r}{\sqrt{2}\ell}\gamma\left(1/2\, , r^2/2\ell^2\right)$

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The Reissner-Nordström geometry in the presence of ℓ

The asymptotic behaviors

- small $r \Rightarrow F(r) \sim O(r^6)$
- again the "singularity" is cured by the vacuum fluctuation of the spacetime fabric

$$g_{00} = 1 - \frac{m_0}{3\sqrt{\pi}\,\ell^3}\, r^2 + O(r^4) \qquad (42)$$

where m_0 is "*bare mass*" only.

- at large distance, the asymptotic observer measures the *total mass-energy* M and the electric field in the usual way

$$g_{00} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \qquad (43)$$

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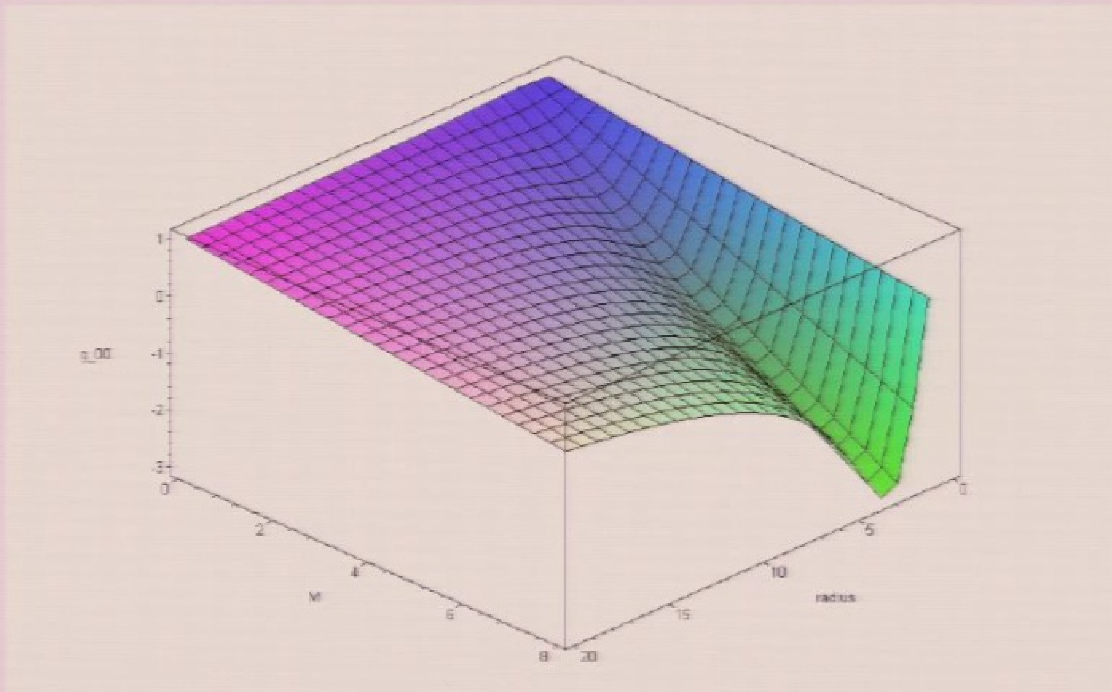
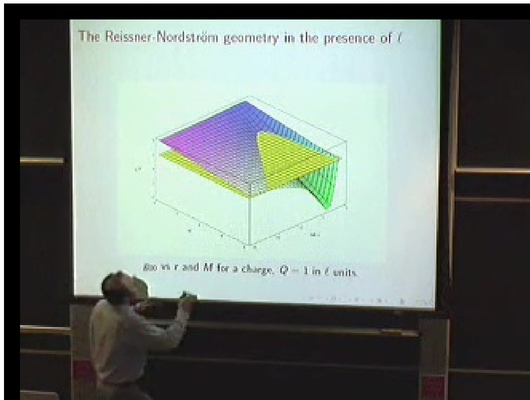
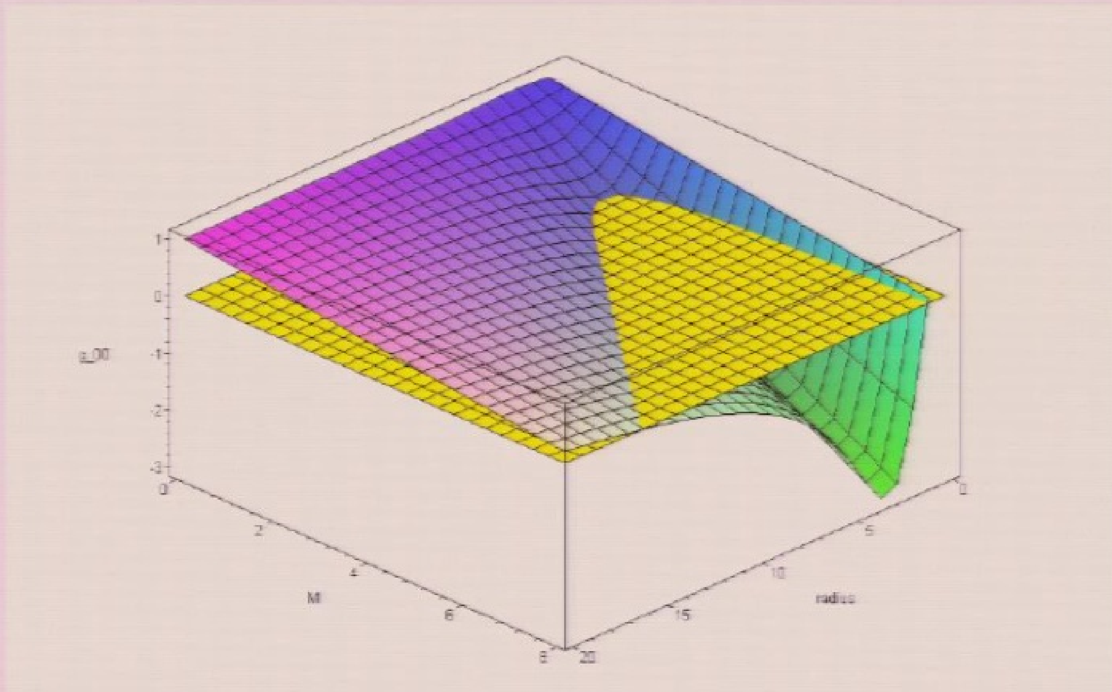
The Reissner-Nordström geometry in the presence of ℓ



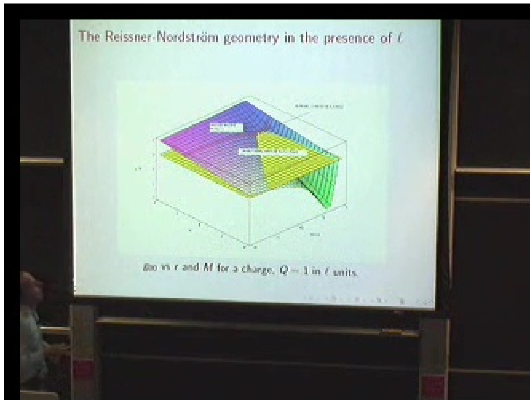
g_{00} vs r and M for a charge, $Q = 1$ in ℓ units.



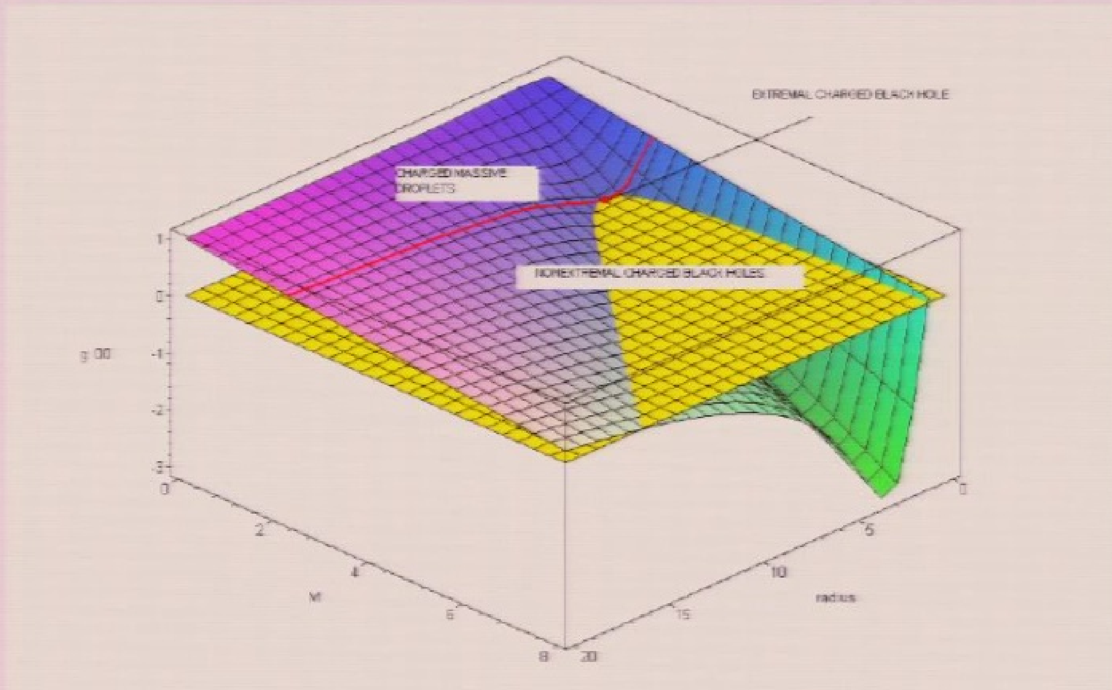
The Reissner-Nordström geometry in the presence of ℓ



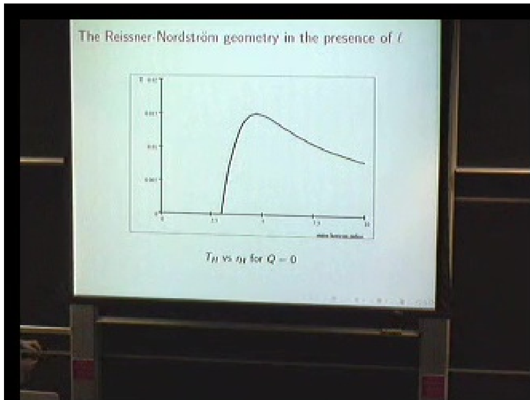
g_{00} vs r and M for a charge, $Q = 1$ in ℓ units.



The Reissner-Nordström geometry in the presence of ℓ



g_{00} vs r and M for a charge, $Q = 1$ in ℓ units.



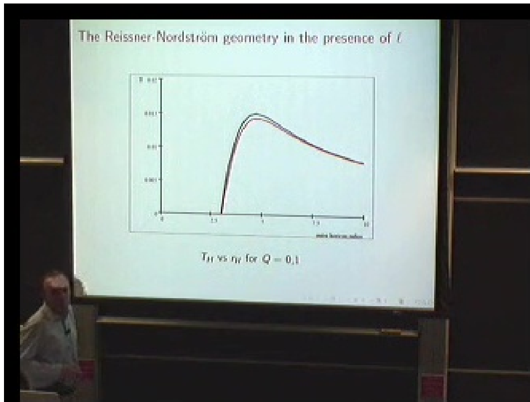
The Reissner-Nordström geometry in the presence of ℓ

The Hawking temperature

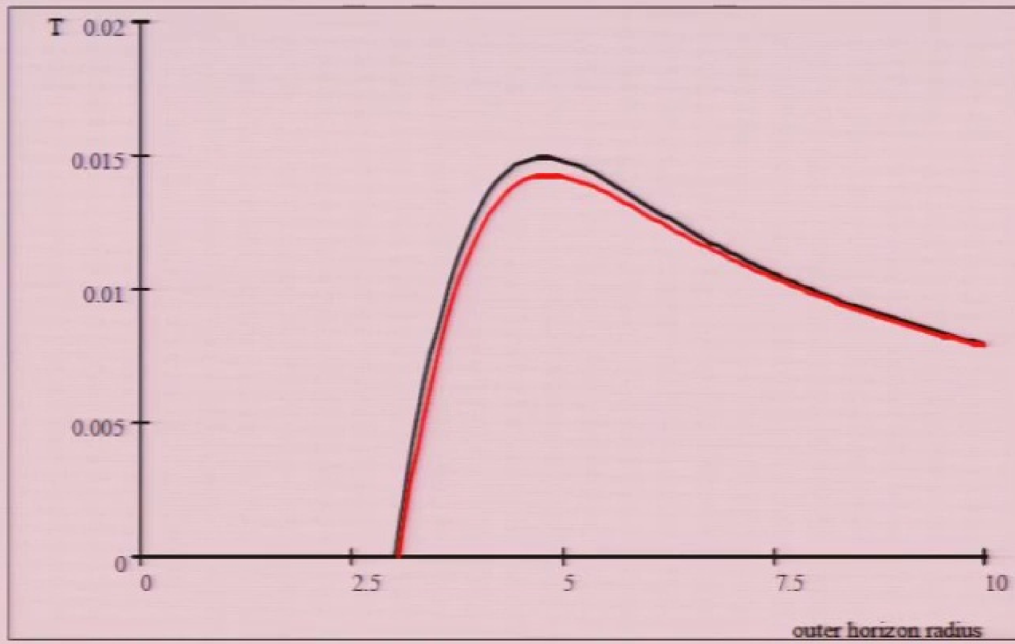
$$4\pi\, T_H = \frac{1}{r_+}\left[1 - \frac{r_+^3 \exp(-r_+^2/4\ell^2)}{4\ell^3 \gamma\left(3/2\,,r_+^2/4\ell^2\right)}\right] +$$

$$-\frac{4Q^2}{\pi r_+^3}\left[\gamma^2\left(3/2\,,r_+^2/4\ell^2\right) + \frac{r_+^3 \exp(-r_+^2/4\ell^2)}{16\,\ell^3 \gamma\left(3/2\,,r_+^2/4\ell^2\right)} F(r_+)\right]$$

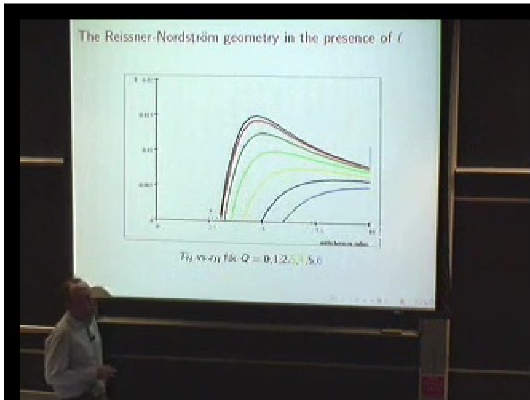
- again instead of growing indefinitely temperature reaches a maximum value and then drops to zero at the extremal BH
- the effect of charge is just to lower the maximum temperature



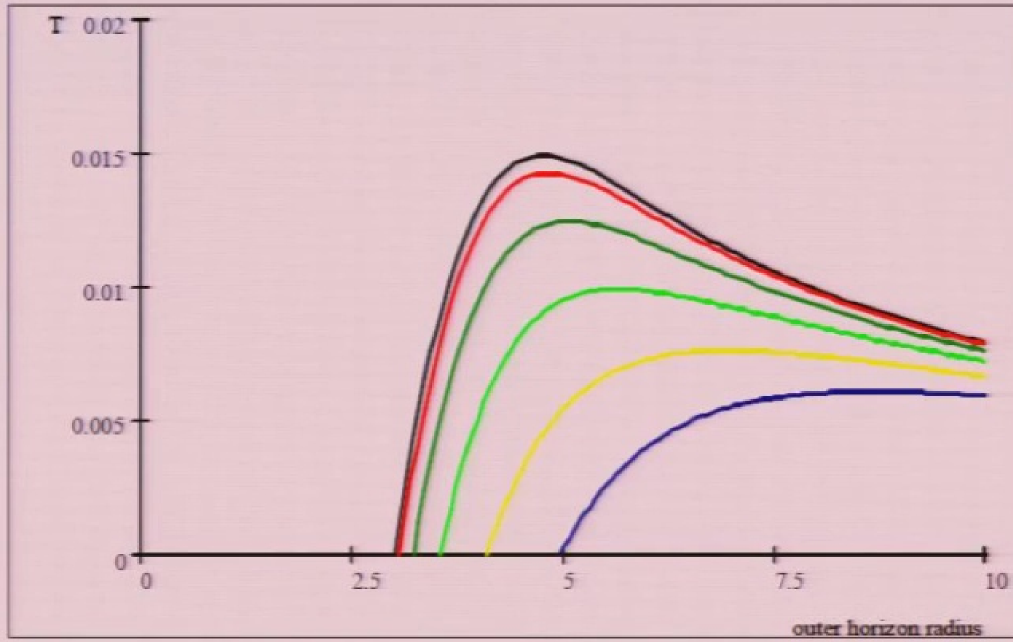
The Reissner-Nordström geometry in the presence of ℓ



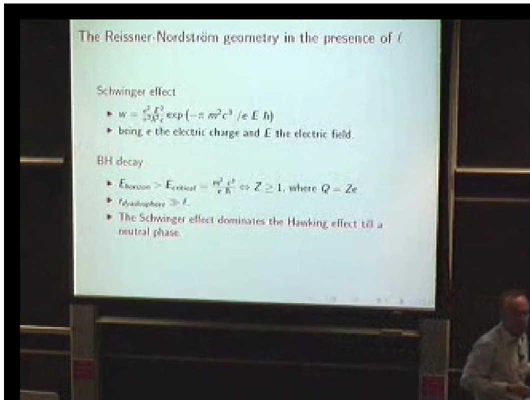
T_H vs r_H for $Q = 0,1$



The Reissner-Nordström geometry in the presence of ℓ



T_H vs r_H for $Q = 0,1,2,3,4,5$



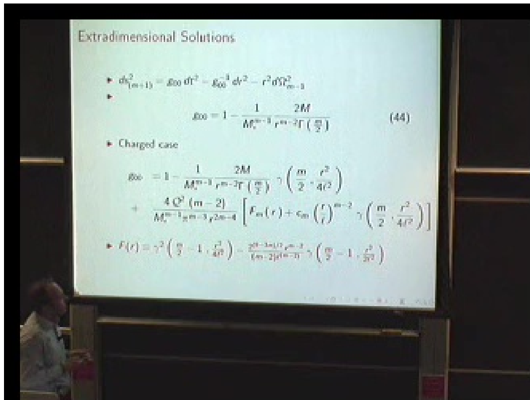
The Reissner-Nordström geometry in the presence of ℓ

Schwinger effect

- $w = \frac{e^2 \ E^2}{\pi^2 \hbar^2 c} \exp\left(-\pi \ m^2 c^3 \ / e \ E \ \hbar\right)$
- being e the electric charge and E the electric field.

BH decay

- $E_{horizon} > E_{critical} = \frac{m^2 \ c^3}{e \ \hbar} \Leftrightarrow Z \geq 1$, where $Q = Ze$
- $r_{dyadosphere} \gg \ell$.
- The Schwinger effect dominates the Hawking effect till a neutral phase.



Extradimensional Solutions

- $ds^2_{(m+1)} = g_{00}\, dt^2 - g_{00}^{-1}\, dr^2 - r^2 d\Omega^2_{m-1}$
- $$g_{00} = 1 - \frac{1}{M_*^{m-1}} \frac{2M}{r^{m-2}\Gamma\left(\frac{m}{2}\right)} \qquad (44)$$
- Charged case

$$\begin{aligned} g_{00} &= 1 - \frac{1}{M_*^{m-1}} \frac{2M}{r^{m-2}\Gamma\left(\frac{m}{2}\right)}\, \gamma\left(\frac{m}{2}, \frac{r^2}{4\ell^2}\right) \\ &+ \frac{4\,Q^2\,(m-2)}{M_*^{m-1}\pi^{m-3}\, r^{2m-4}} \left[F_m(r) + c_m \left(\frac{r}{\ell}\right)^{m-2} \gamma\left(\frac{m}{2}, \frac{r^2}{4\ell^2}\right)\right] \end{aligned}$$

- $F(r) \equiv \gamma^2\left(\frac{m}{2} - 1, \frac{r^2}{4\ell^2}\right) - \frac{2^{(8-3m)/2}\, r^{m-2}}{(m-2)\ell^{(m-2)}}\gamma\left(\frac{m}{2} - 1, \frac{r^2}{2\ell^2}\right)$

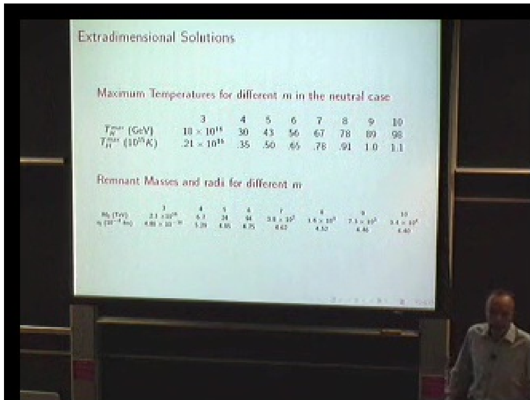
Extradimensional Solutions

Properties of the solutions

- Geometric and thermodynamic behavior equivalent to the 4d one.
  - $\Rightarrow$ there exists a mass threshold M_0 below which BH do not form.
  - $\Rightarrow$ there exists a zero temperature black hole remnant

BH remnants

- $1/\ell \sim M_* \sim 1$ TeV
- remnant cross section $\sigma_{BH} \simeq \pi r_0^2 \sim 10$ nb $\longrightarrow$ 1000 BHs per second at LHC.



Extradimensional Solutions

Maximum Temperatures for different m in the neutral case

	3	4	5	6	7	8	9	10
T_H^{max} (GeV)	18×10^{16}	30	43	56	67	78	89	98
T_H^{max} ($10^{15}K$)	$.21 \times 10^{16}$	.35	.50	.65	.78	.91	1.0	1.1

Remnant Masses and radii for different m

	3	4	5	6	7	8	9	10
M_0 (TeV)	2.3×10^{16}	6.7	24	94	3.8×10^2	1.6×10^3	7.3×10^3	3.4×10^4
r_0 (10^{-4} fm)	4.88×10^{-16}	5.29	4.95	4.75	4.62	4.52	4.46	4.40

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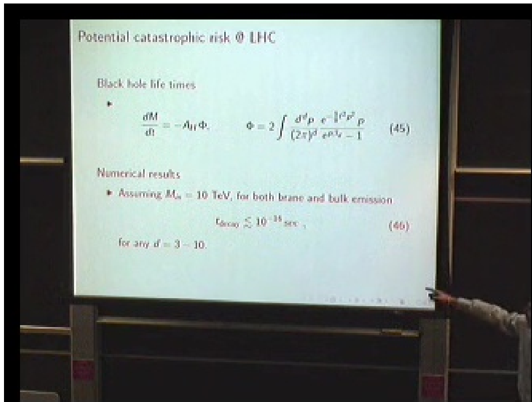


Potential catastrophic risk @ LHC

Black hole life times

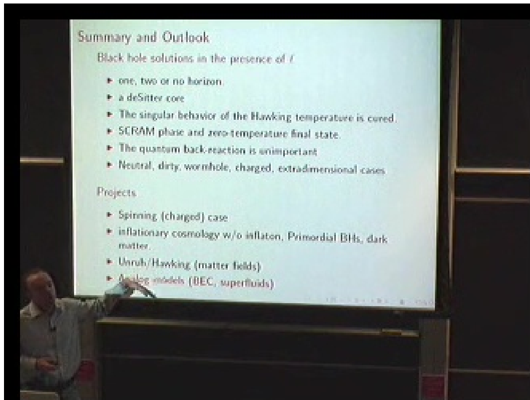
- $$\frac{dM}{dt} = -A_H\,\Phi, \qquad \Phi = 2\int \frac{d^d p}{(2\pi)^d}\frac{e^{-\frac{1}{8}\ell^2 p^2}\,p}{e^{p\beta_d}-1} \qquad (45)$$

Numerical results

- Assuming $M_{in} = 10$ TeV, for both brane and bulk emission

$$t_{\rm decay} \lesssim 10^{-16}\,{\rm sec}\;, \qquad (46)$$

for any $d = 3 - 10$.



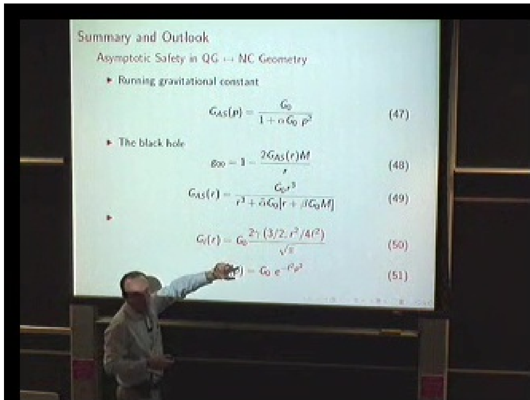
Summary and Outlook

Black hole solutions in the presence of ℓ

- one, two or no horizon.
- a deSitter core
- The singular behavior of the Hawking temperature is cured.
- SCRAM phase and zero-temperature final state.
- The quantum back-reaction is unimportant
- Neutral, dirty, wormhole, charged, extradimensional cases

Projects

- Spinning (charged) case
- inflationary cosmology w/o inflaton, Primordial BHs, dark matter.
- Unruh/Hawking (matter fields)
- Analog models (BEC, superfluids)



Summary and Outlook

Asymptotic Safety in QG ↔ NC Geometry

- Running gravitational constant

$$G_{AS}(p) = \frac{G_0}{1 + \alpha G_0 \, p^2} \tag{47}$$

- The black hole

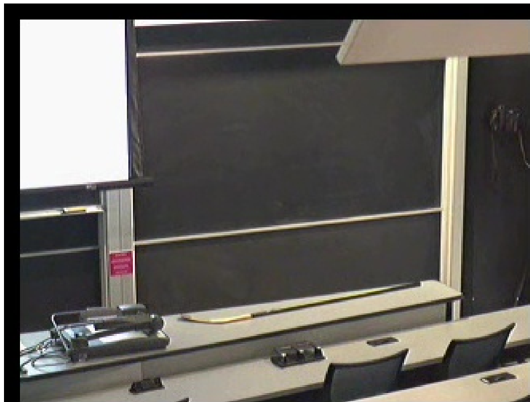
$$g_{00} = 1 - \frac{2G_{AS}(r)M}{r} \tag{48}$$

$$G_{AS}(r) = \frac{G_0 r^3}{r^3 + \tilde{\alpha} G_0 [r + \beta G_0 M]} \tag{49}$$

-

$$G_\ell(r) = G_0 \frac{2\gamma\left(3/2; r^2/4\ell^2\right)}{\sqrt{\pi}} \tag{50}$$

$$G_\ell(p) = G_0 \, e^{-\ell^2 p^2} \tag{51}$$



Summary and Outlook

LQG↔ NC Geometry

- LQBHs
- regular geometry
- zero temperature final state
- possible connection between LQG to LHC physics via NCBHs

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