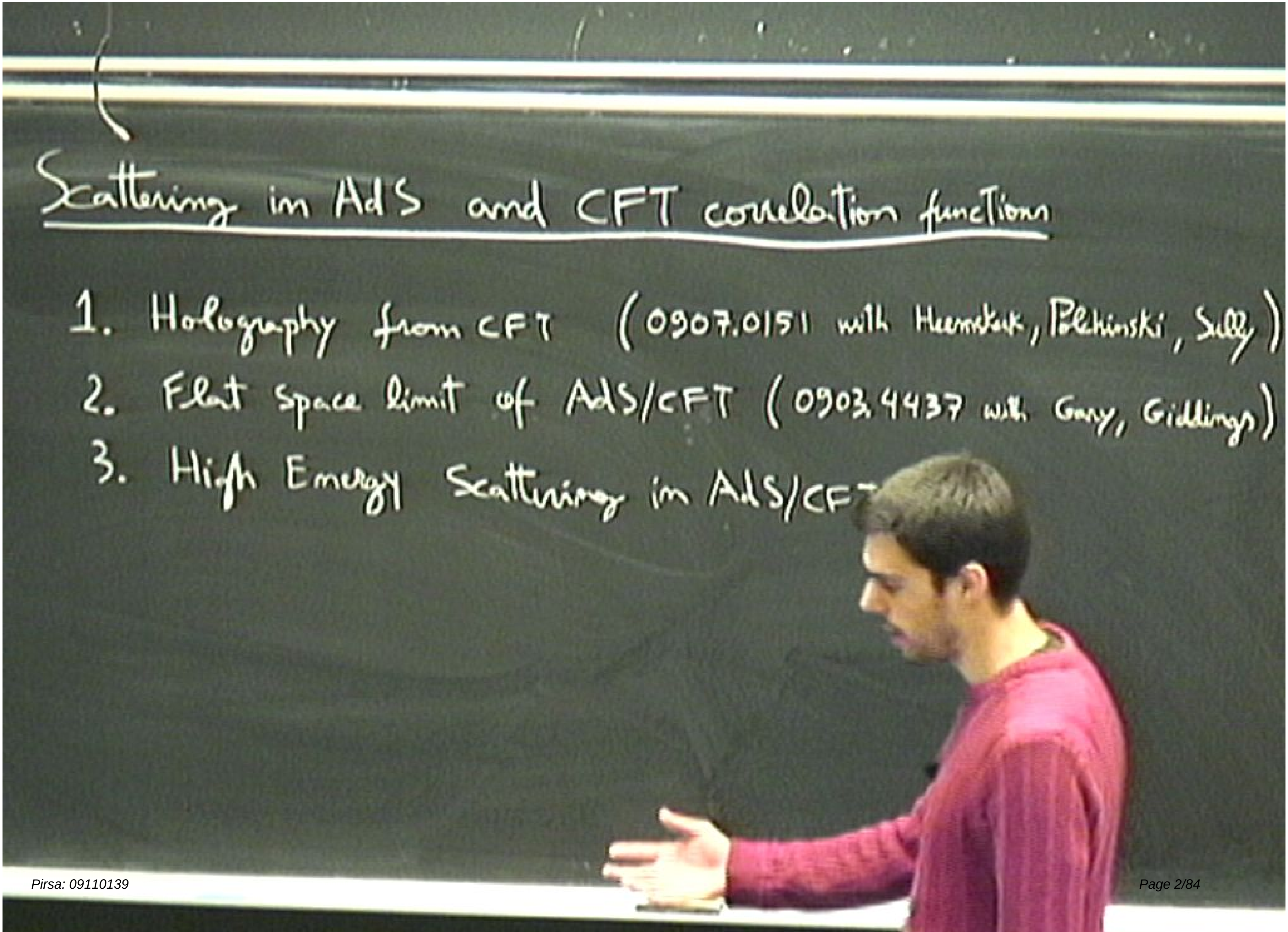
Title: Flat space Limit

Date: Nov 19, 2009 12:00 PM

URL: http://pirsa.org/09110139

Abstract: Using this limit we can extract the scattering amplitudes of the bulk theory in flat space from the correlation functions of the dual CFT. This gives a non-perturbative definition of flat space string theory scattering amplitudes as a limit of the correlation functions of SYM.


Scattering in AdS and CFT correlation functions
1. Holography from CFT (0907.0151 with Heemskerk, Polchinski, Sully)
2. Flat space limit of AdS/CFT (0903.4437 with Gary, Giddings)
3. High Energy Scattering in AdS/CF

Scattering in AdS and CFT correlation functions

1. Holography from CFT (0907.0151 with Heemskerk, Polchinski, Sully)
2. Flat space limit of AdS/CFT (0903.4437 with Gary, Giddings)
3. High Energy Scattering in AdS/CFT

$$\langle \mathcal{O}(y_1)\,\mathcal{O}(y_2)\,\mathcal{O}(y_3)\,\mathcal{O}(y_4)\rangle = \frac{\mathcal{A}(z,\bar{z})}{y_{13}^{2\Delta}\; y_{24}^{2\Delta}}$$

$$z\bar{z} = \frac{y_{13}^2\; y_{24}^2}{y_{12}^2\; y_{34}^2}$$

$$(1-z)(1-\bar{z}) = \frac{y_{14}^2\; y_{23}^2}{y_{12}^2\; y_{34}^2}$$

2. Flat Space limit of AdS/CFT

Requires sharp holography ($\ell_s, \ell_p \ll R \longrightarrow \infty$)

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Can we recover the flat space S-matrix from the CFT correlation functions?

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Yes!

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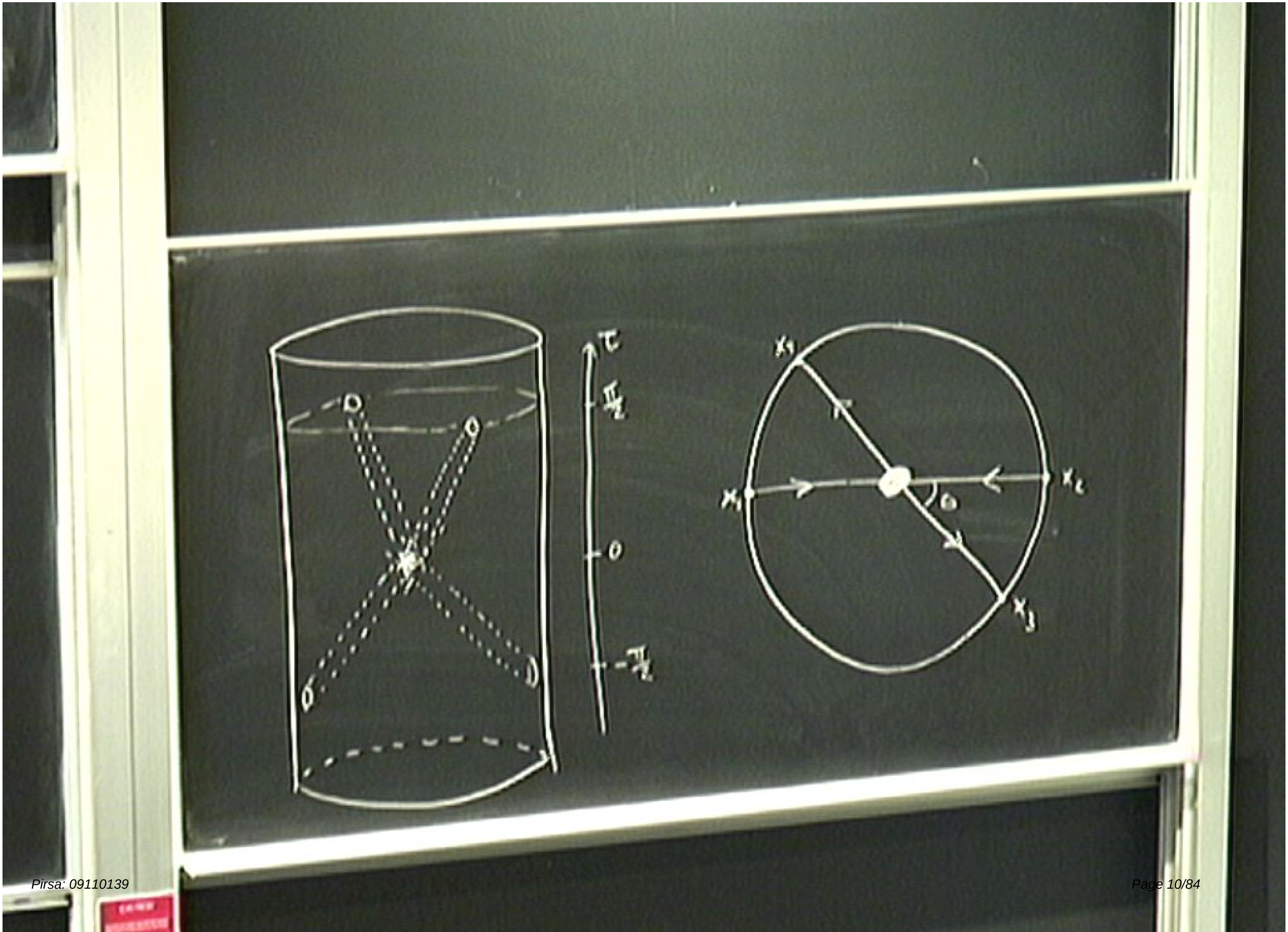
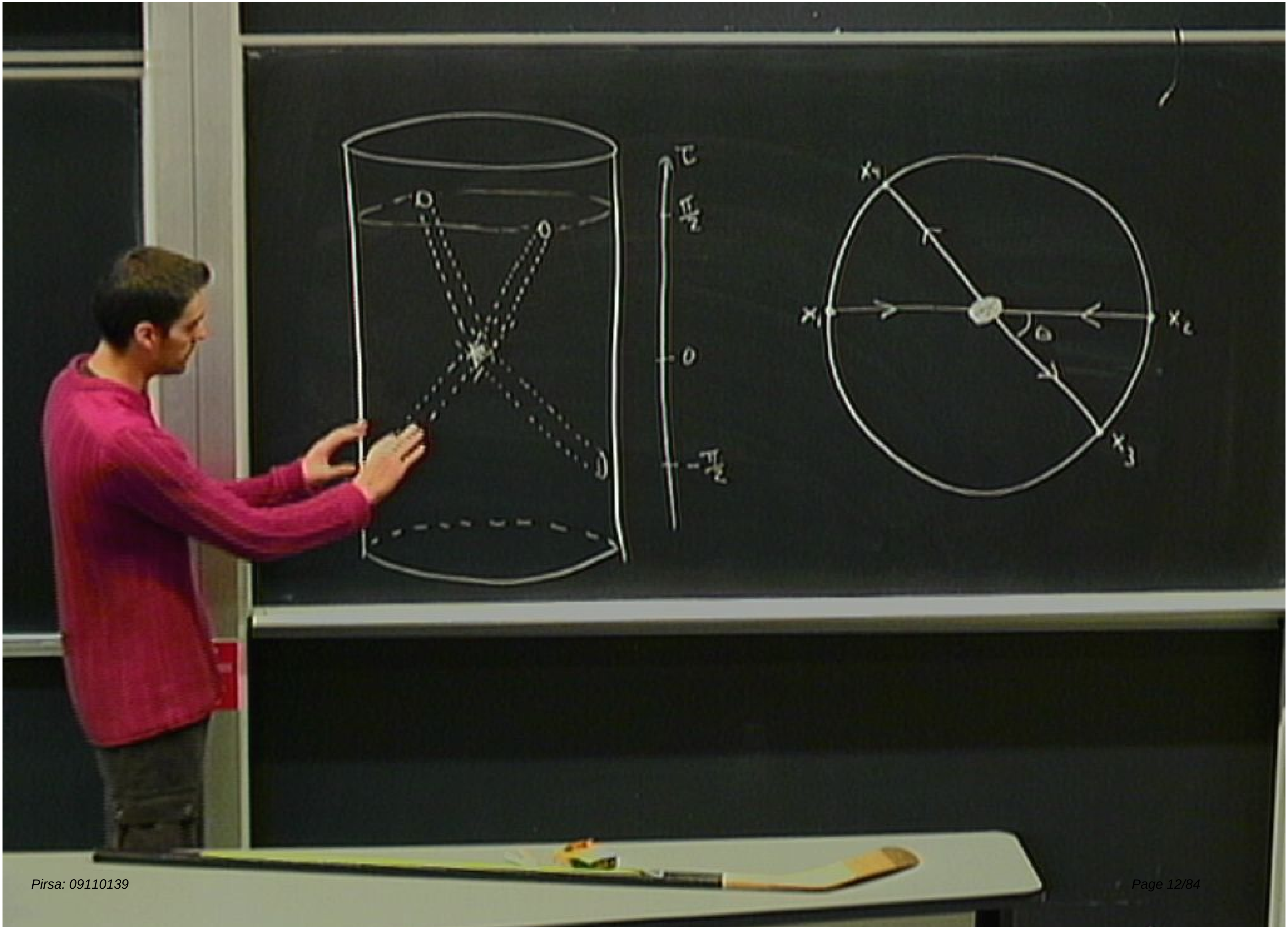

τ
π/2
0
-π/2
x1
x2
x3
x4
r
θ

Scattering experiment in AdS

bulk-to-bnd propagator

$$\Psi_i(\underset{\substack{\cap \\ \text{AdS}}}{w}) = \int_{\partial \text{AdS}} dy_i \; f_i(y) \; K(y_i, w)$$



τ
π/2
0
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x1
x2
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θ

Scattering experiment in AdS

bulk-to-bnd propagator

$$\underset{\substack{\Uparrow \\ \mathrm{AdS}}}{\Psi_i(w)} = \int_{\partial \mathrm{AdS}} dy_1 \, f_i(y) \, K(y_1, w) \underset{\substack{\Downarrow \\ w \ll R}}{\simeq} e^{i k_i \cdot w} F_i(w)$$

$$\frac{1}{E} \ll \Delta w \ll R$$

$$\int \prod_{i=1}^{4} dy_i \, f_i(y_i) \langle \mathcal{O}(y_1) \ldots \mathcal{O}(y_4) \rangle$$

Scattering experiment in AdS

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$$\frac{1}{E} \ll \Delta w \ll R \qquad \sum k_i = 0$$

$$\prod_{i=1}^{4} dy_i\, f_i(y_i) \langle \mathcal{O}(y_1) \ldots \mathcal{O}(y_4) \rangle \approx T(s,t) \int dw \prod_{i=1}^{4} F_i(w)$$

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$\int dw_i$ 

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Polchinski 99
Susskind 99

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Scattering experiment in AdS

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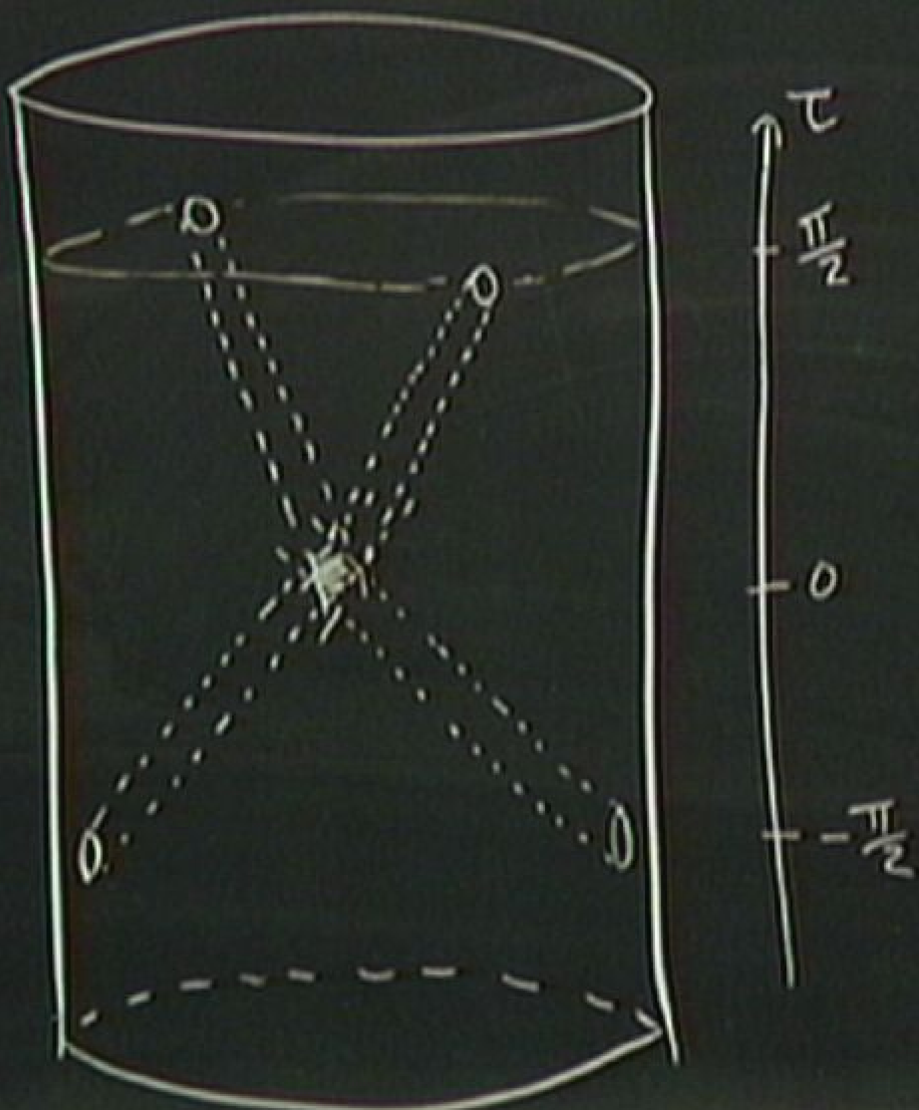
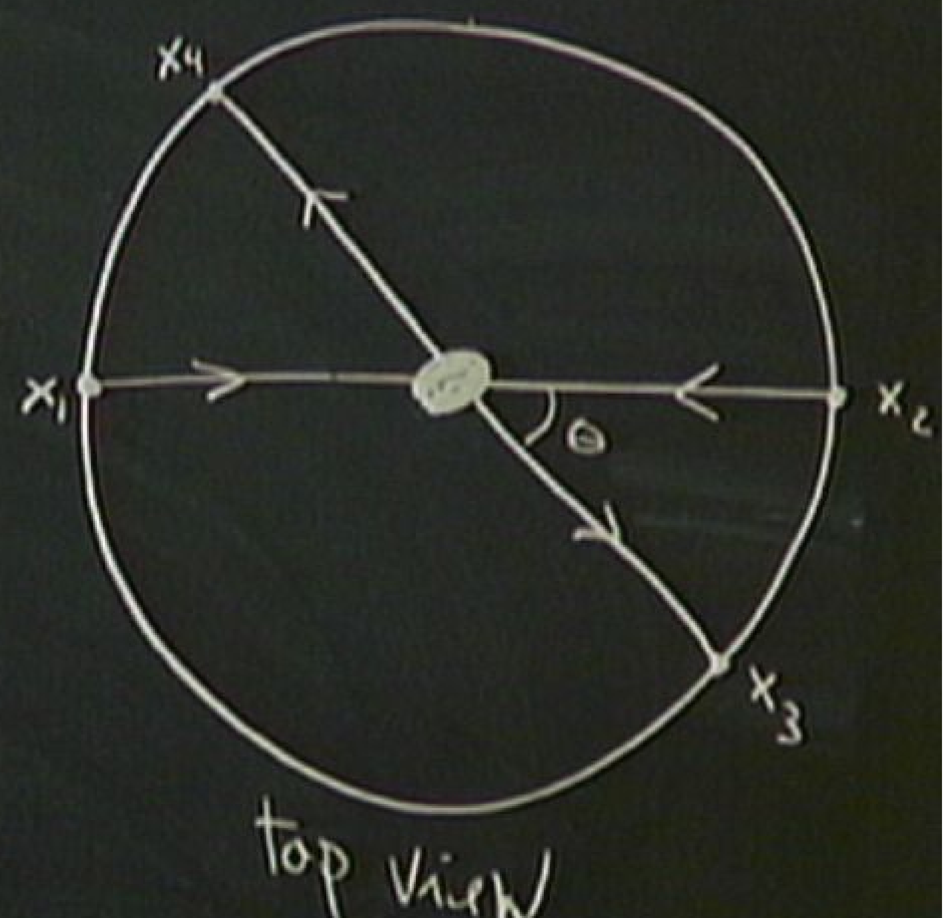


τ
π/2
0
−π/2



x4
x1
x2
x3
θ
top view

$\mathcal{A}$ has a singularity for $z = \bar{z}$

$\mathcal{A}$ has a singularity for $z = \bar{z} = \sin^2\frac{\theta}{2}$

A has a singularity for $z = \bar{z} = \sin^2 \frac{\theta}{2}$

$z = \sigma e^{\rho}$

$\bar{z} = \sigma e^{-\rho}$

$\mathcal{A}$ has a singularity for $z = \bar{z} = \sin^2\frac{\theta}{2} = \sigma$

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$$\mathcal{A}_1 \sim \frac{\mathcal{F}(\sigma)}{\rho^{4\Delta+2k-3}}$$

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$$z = \sigma e^{\rho}$$
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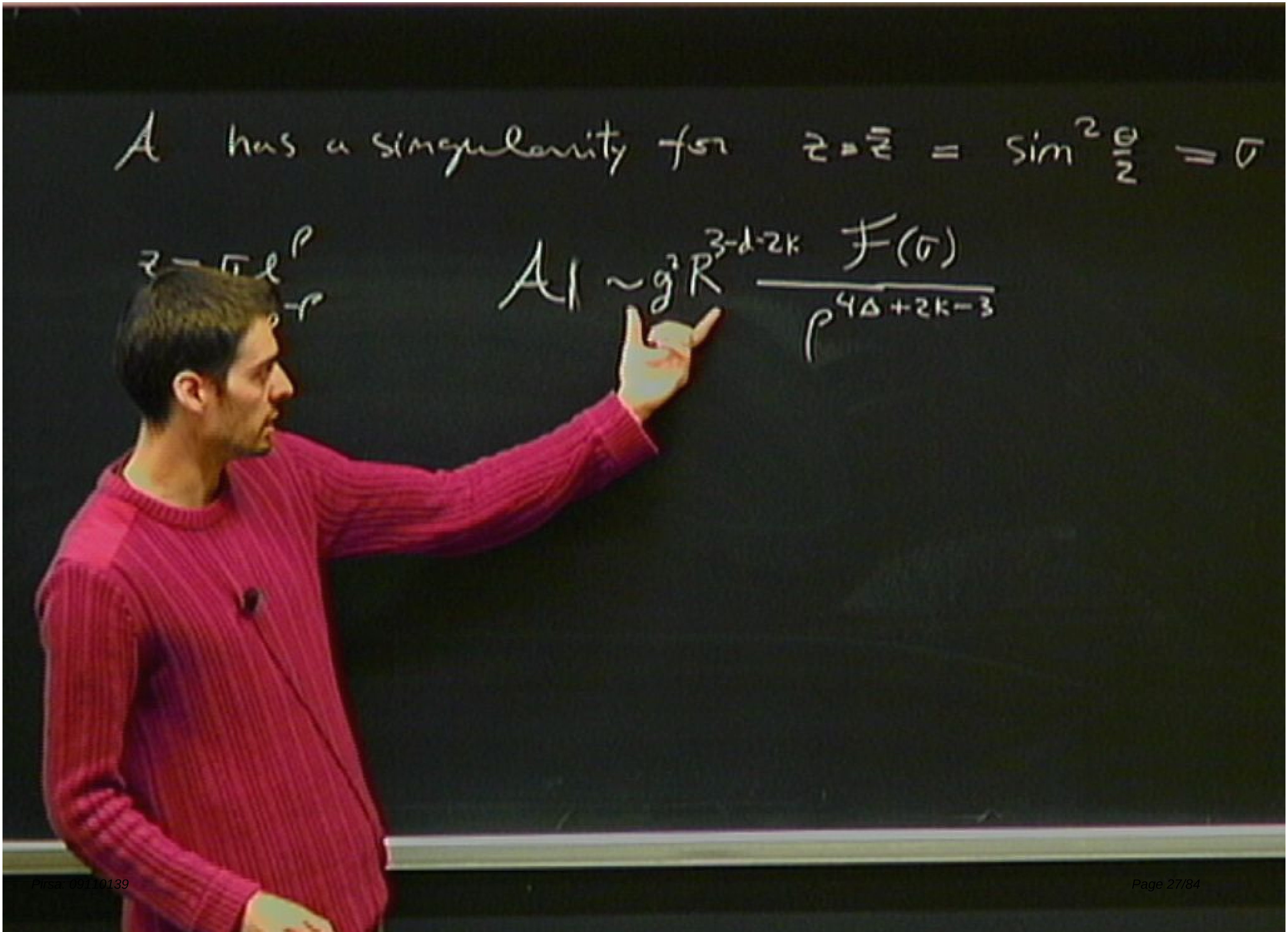
$$\mathcal{A}_1 \sim g^2 \frac{\mathcal{F}(\sigma)}{\rho^{4\Delta+2k-3}}$$


A has a singularity for $z = \bar{z} = \sin^2\frac{\theta}{2} = \sigma$
$A_1 \sim g^2 R^{3-d-2k} \frac{F(\sigma)}{\rho^{4\Delta+2k-3}}$

...gularity for $z = \bar{z} = \sin^2\frac{\theta}{2} = \sigma$

$$\mathcal{A} \sim g^2 R^{3-d-2k} \frac{\mathcal{F}(\sigma)}{[\text{illegible}]^{\Delta+2k-3}}$$

Example

$$g^2 \phi^2 (\nabla^2)^k \phi^2$$

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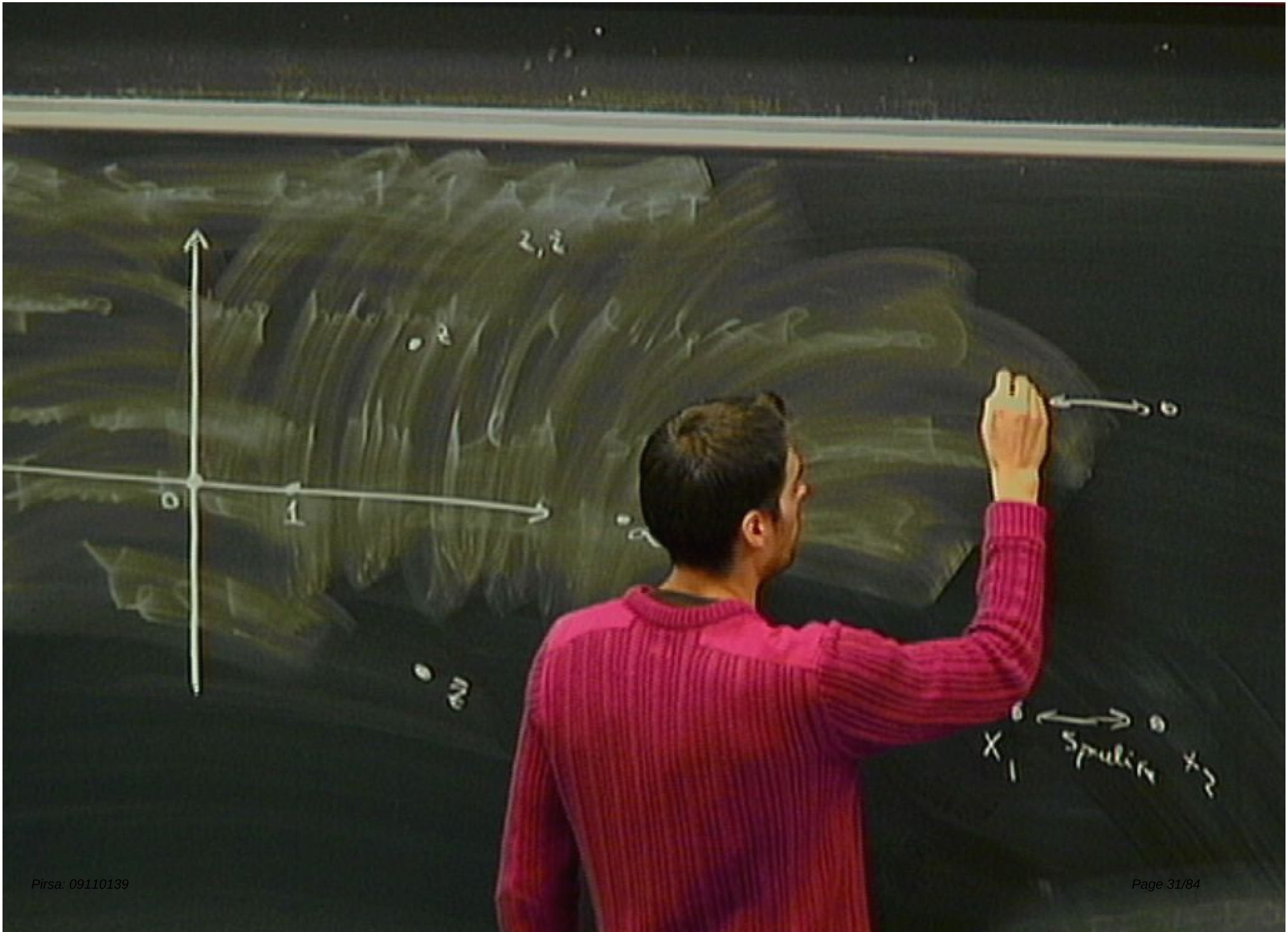
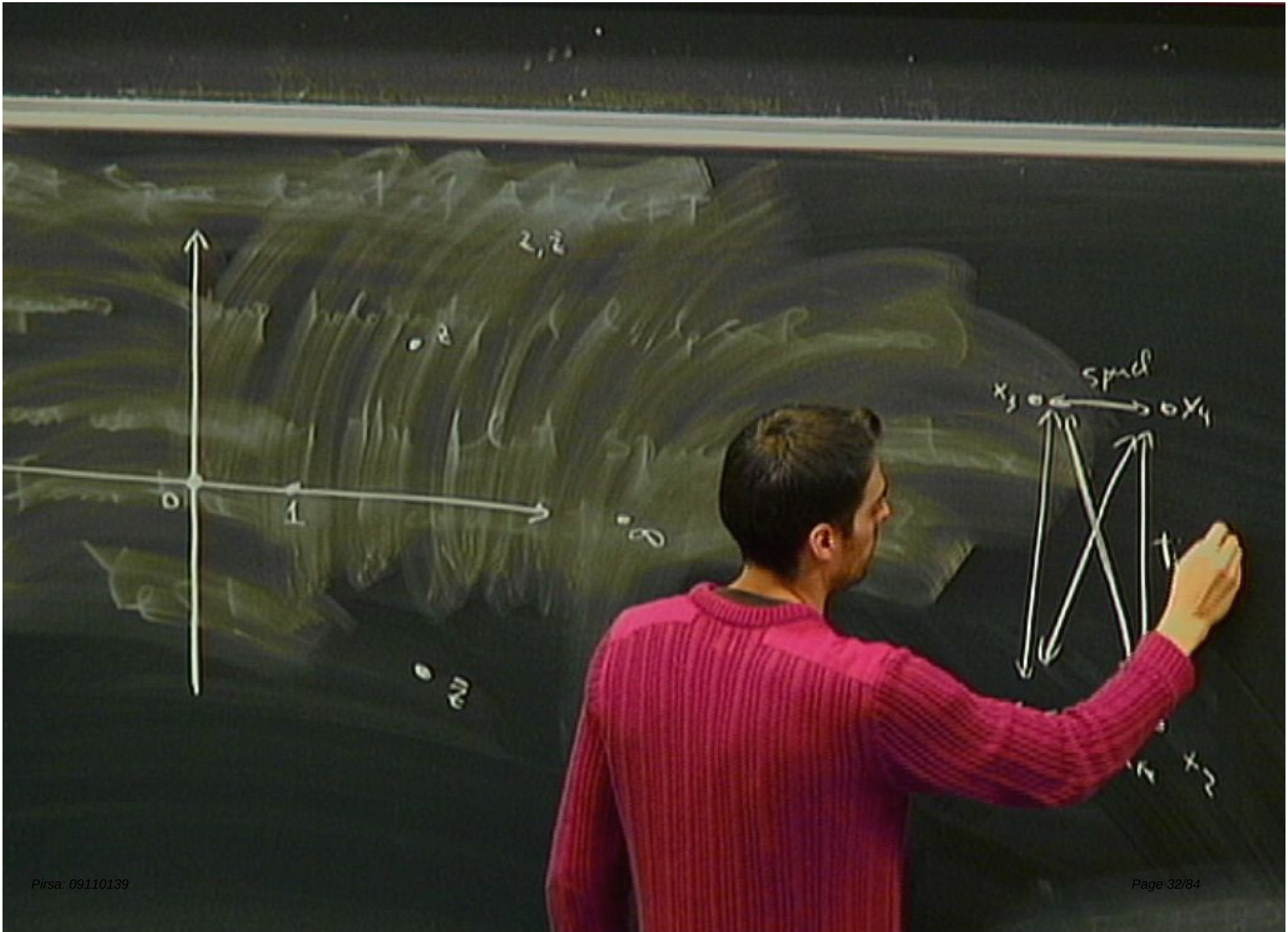
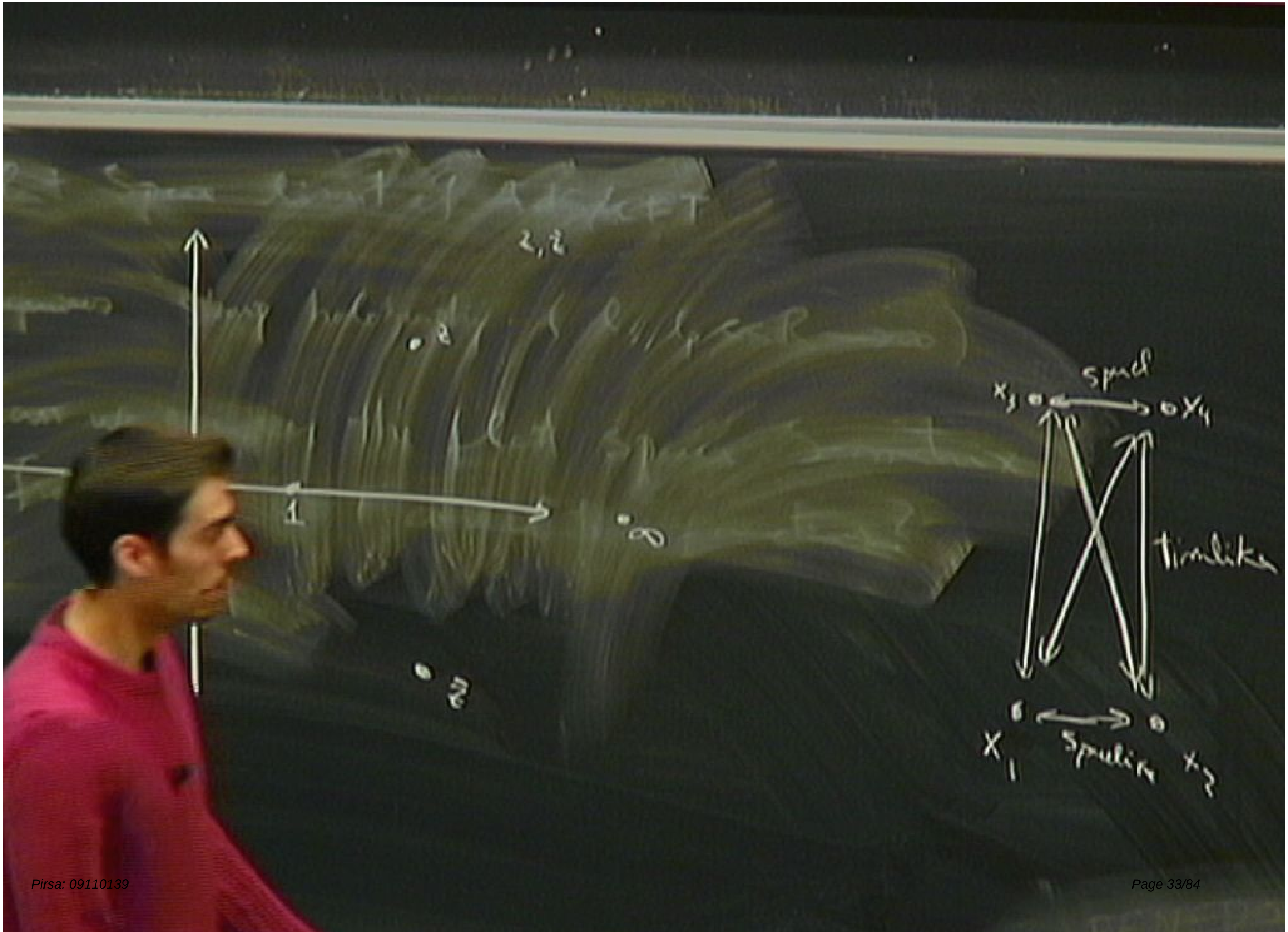
Example

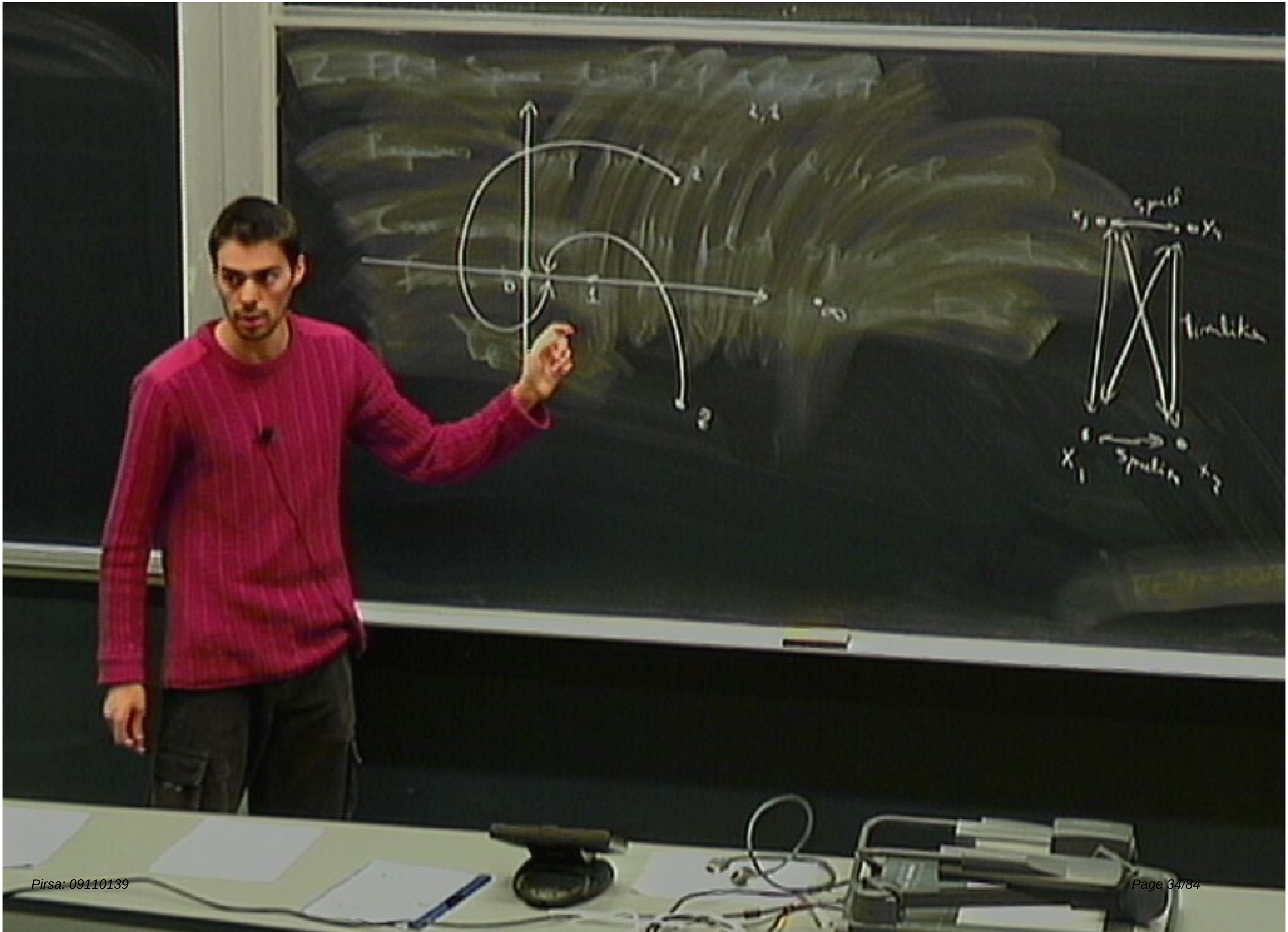
$g^2 \phi^2 (\nabla^2)^k \phi^2$

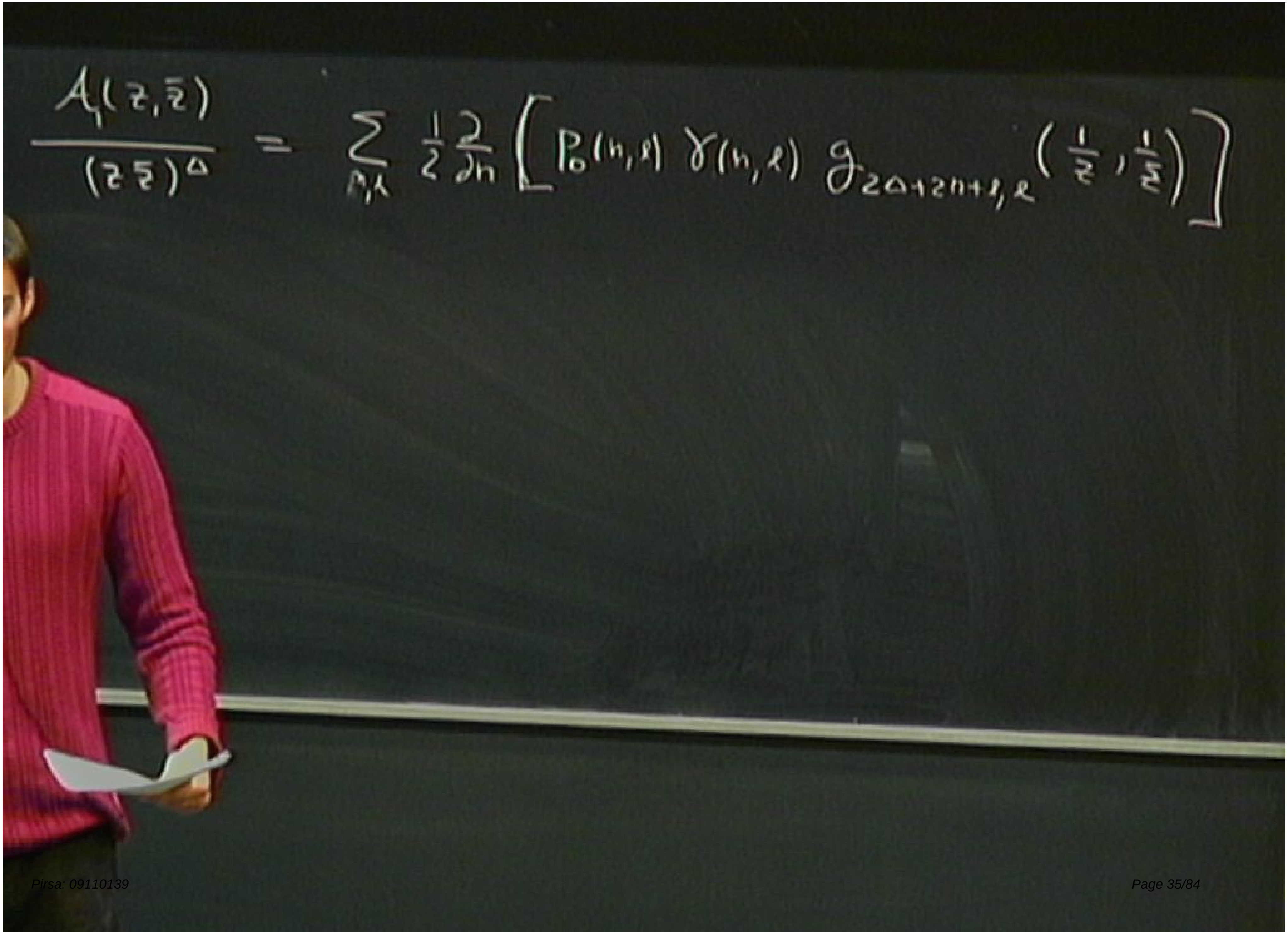
$$T(s,t) = g^2 s^k \frac{\mathcal{F}(\sigma)}{\sigma^{1-k}(1-\sigma)^{2\Delta-2+k}} \quad , \quad \sigma = \sin^2\frac{\theta}{2} = -\frac{t}{s}$$

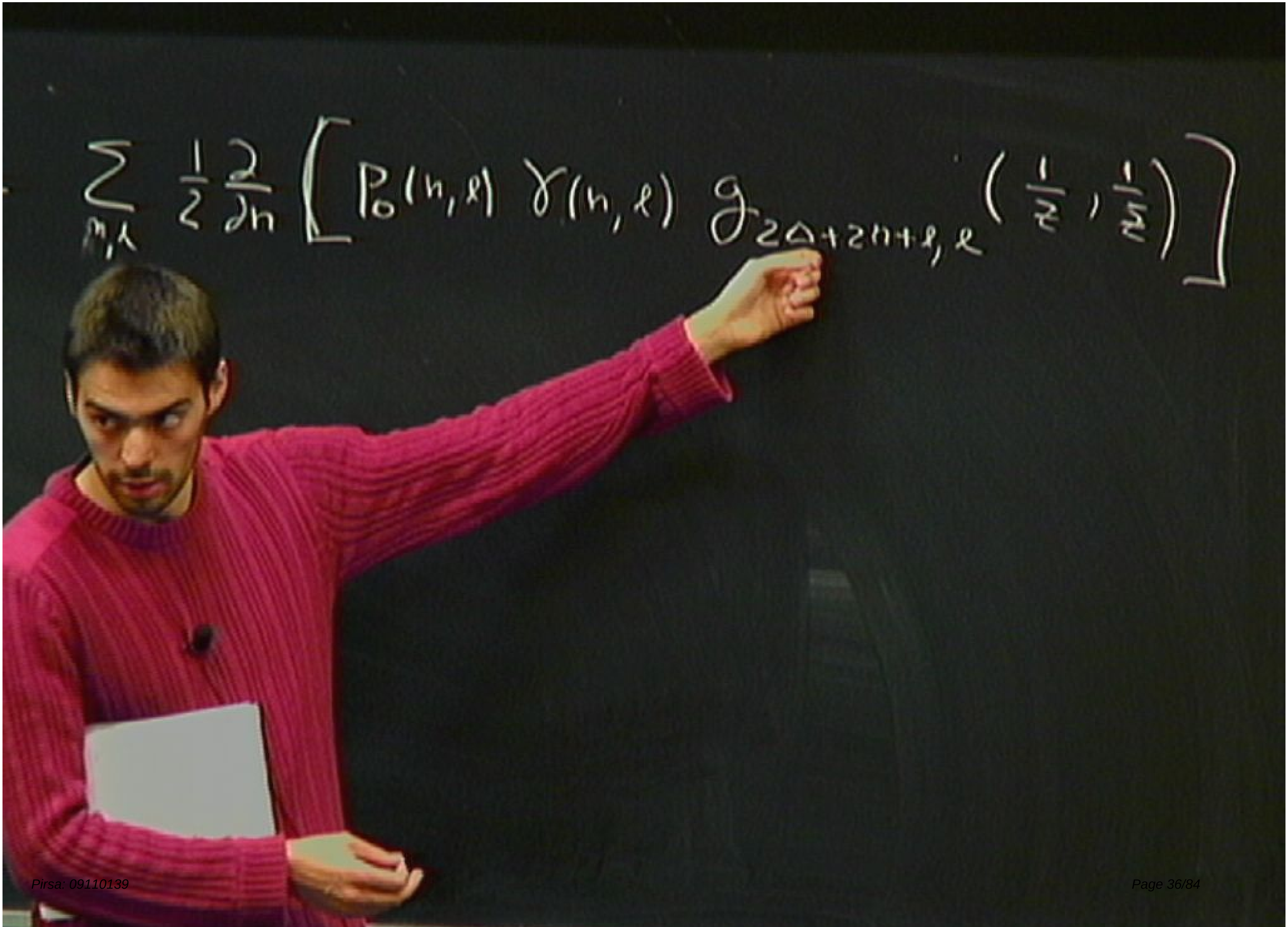

2,2
0
1
z
x_1
Spatial
x_2

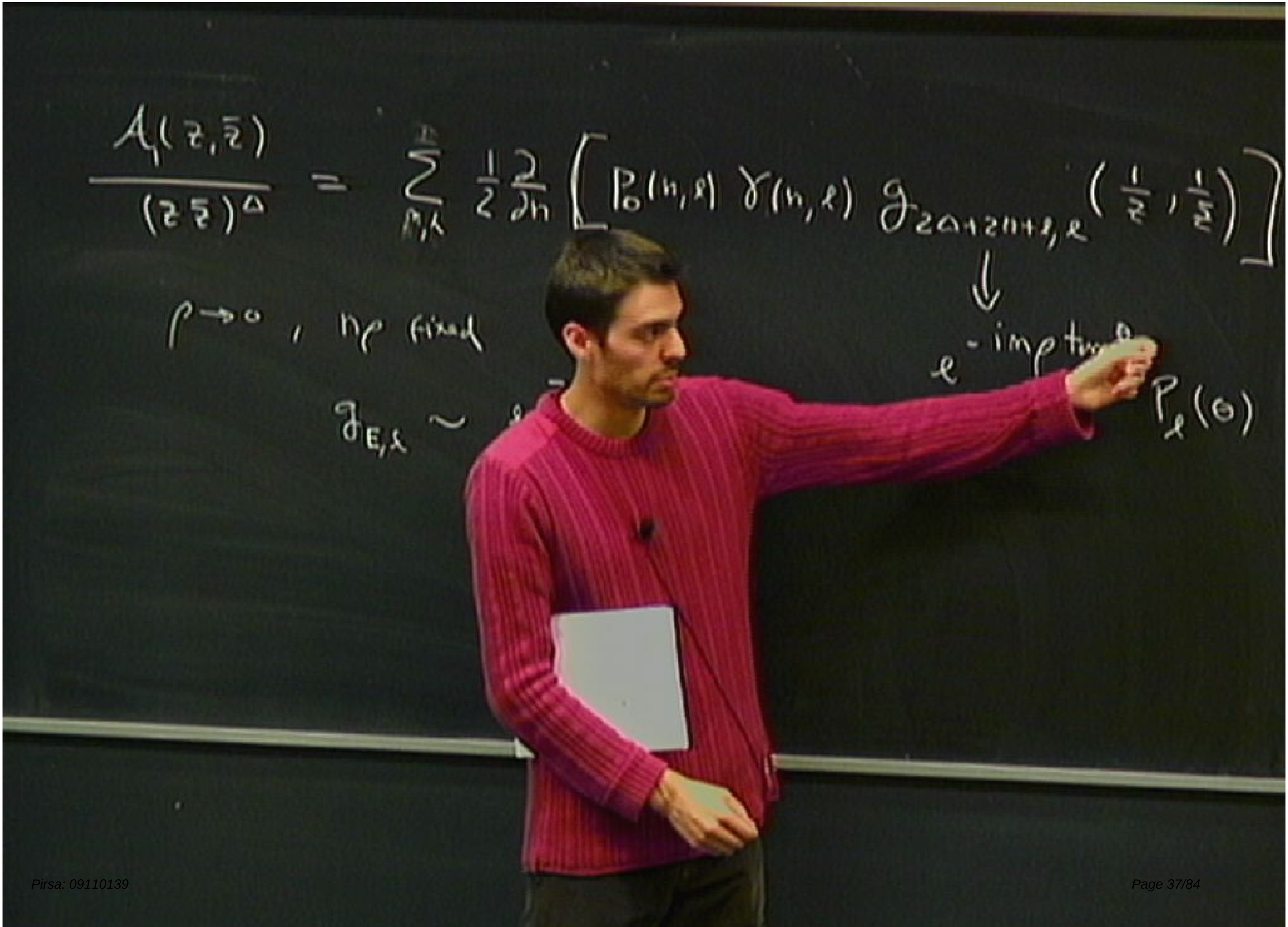

space
0
1
∞


2, z
z
1
∞
z̄
x3
x4
timelike
x1
x2


0
1
∞
spacelike
timelike
spacelike


$$\frac{A_1(z,\bar{z})}{(z\bar{z})^{\Delta}} = \sum_{n,l} \frac{1}{2}\frac{\partial}{\partial n}\left[P_0(n,l)\, \gamma(n,l)\, g_{2\Delta+2n+l,\,l}\left(\frac{1}{z},\frac{1}{\bar{z}}\right)\right]$$


$-\sum_{n,\ell} \frac{1}{2}\frac{\partial}{\partial n}\left[P_0(n,\ell)\,\gamma(n,\ell)\,g_{2\Delta+2n+\ell,\ell}\left(\frac{1}{z},\frac{1}{\bar z}\right)\right]$


$\frac{A_1(z,\bar z)}{(z\bar z)^{\Delta}} = \sum_{n,l}^{\infty} \frac{1}{2}\frac{\partial}{\partial n}\left[P_0(n,l)\,\gamma(n,l)\,g_{2\Delta+2n+l,l}\left(\frac{1}{z},\frac{1}{\bar z}\right)\right]$
$\rho \to 0$, $n\rho$ fixed
$P_l(\theta)$
$g_{E,l} \sim$

$$\frac{A_1(z,\bar{z})}{(z\bar{z})^{\Delta}} = \sum_{n,l}^{\infty} \frac{1}{2}\frac{\partial}{\partial n}\left[P_0(n,l)\,\gamma(n,l)\, g_{2\Delta+2n+\gamma,\,l}\left(\frac{1}{z},\frac{1}{\bar{z}}\right)\right]$$

$$\Big\downarrow$$

$$e^{-in\rho\tan\frac{\theta}{2}}\, P_l(\theta)$$

$\rho \to 0$, $n\rho$ fixed

$$g_{E,l} \sim e^{-i}$$

$$\frac{A_1(z,\bar{z})}{(z\bar{z})^{\Delta}} = \sum_{n,l}^{\infty} \frac{1}{2}\frac{\partial}{\partial n}\left[P_0(n,l)\,\gamma(n,l)\, \underset{\Big\downarrow}{g_{2\Delta+2n+l,\,l}}\left(\frac{1}{z},\frac{1}{\bar{z}}\right)\right]$$

$$e^{-imp\tan\frac{\theta}{2}}\;P_l(\theta)$$

$\rho \to 0$, $n\rho$ fixed

$$g_{E,l} \sim e^{-i}$$

$$\underset{\substack{\swarrow \\ \log n}}{\gamma(n,l)} \sim \mu_l\, n^{2k+d-3}$$

$$T(s,t) = s^k \sum_l \mu_l P_l(\theta)$$

$$\frac{A_1(z,\bar z)}{(z\bar z)^{\Delta}} = \sum_{n,\ell}^{\infty} \frac{1}{2}\frac{\partial}{\partial n}\left[P_0(n,\ell)\, \gamma(n,\ell)\, g_{2\Delta+2n+\ell,\ell}\left(\frac{1}{z},\frac{1}{\bar z}\right)\right]$$

$$\downarrow$$

$$e^{-in\rho\tan\frac{\theta}{2}} \quad P_\ell(\theta)$$

$\rho \to 0$, $n\rho$ fixed

$$g_{E,\ell} \sim \ell^{-i}$$

$$\gamma(n,\ell) \sim \mu_\ell\, n^{2k+d-3}$$

$$\downarrow$$

$\log n$

$$T(s,t) = s^k \sum_\ell \mu_\ell P_\ell(\theta)$$

$k = (\#\text{ of derivatives})/2$ in quartic vertex

$$\frac{A_1(z,\bar{z})}{(z\bar{z})^{\Delta}} = \sum_{n,\ell}^{\infty} \frac{1}{2}\frac{\partial}{\partial n}\left[P_0(n,\ell)\, \gamma(n,\ell)\, g_{2\Delta+2n+\ell,\ell}\left(\frac{1}{z},\frac{1}{\bar{z}}\right)\right]$$

$$g_{2\Delta+2n+\ell,\ell} \Downarrow e^{-in\rho \tan\frac{\theta}{2}} P_\ell(\theta)$$

$\rho \to 0$, $n\rho$ fixed

$$g_{E,\ell} \sim e^{-i}$$

$$\gamma(n,\ell) \sim \mu_\ell\, n^{2k+d-3}$$

$\gamma(n,\ell) \downarrow \log n$

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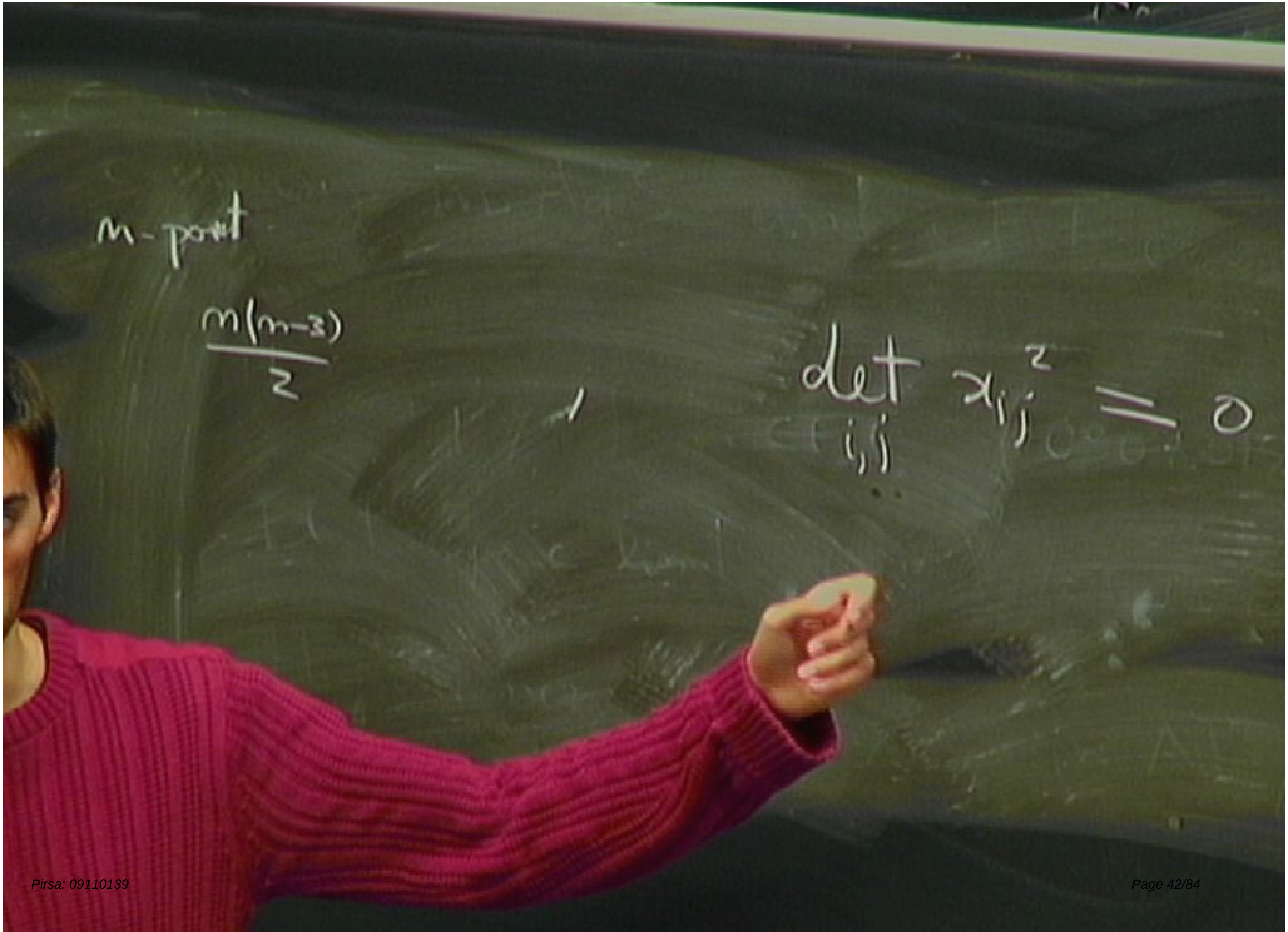

n-point
$\frac{n(n-3)}{2}$
$\det_{i,j} x_{i,j}^2 = 0$

m-point

$$\frac{m(m-3)}{2}$$

$$\det_{i,j} x_{ij}^2 = 0$$

$$T(s,t) = G_N\, s \sum_{k=0}^{\infty} (\alpha' s)^k f_k(\theta)$$

$$A \sim$$

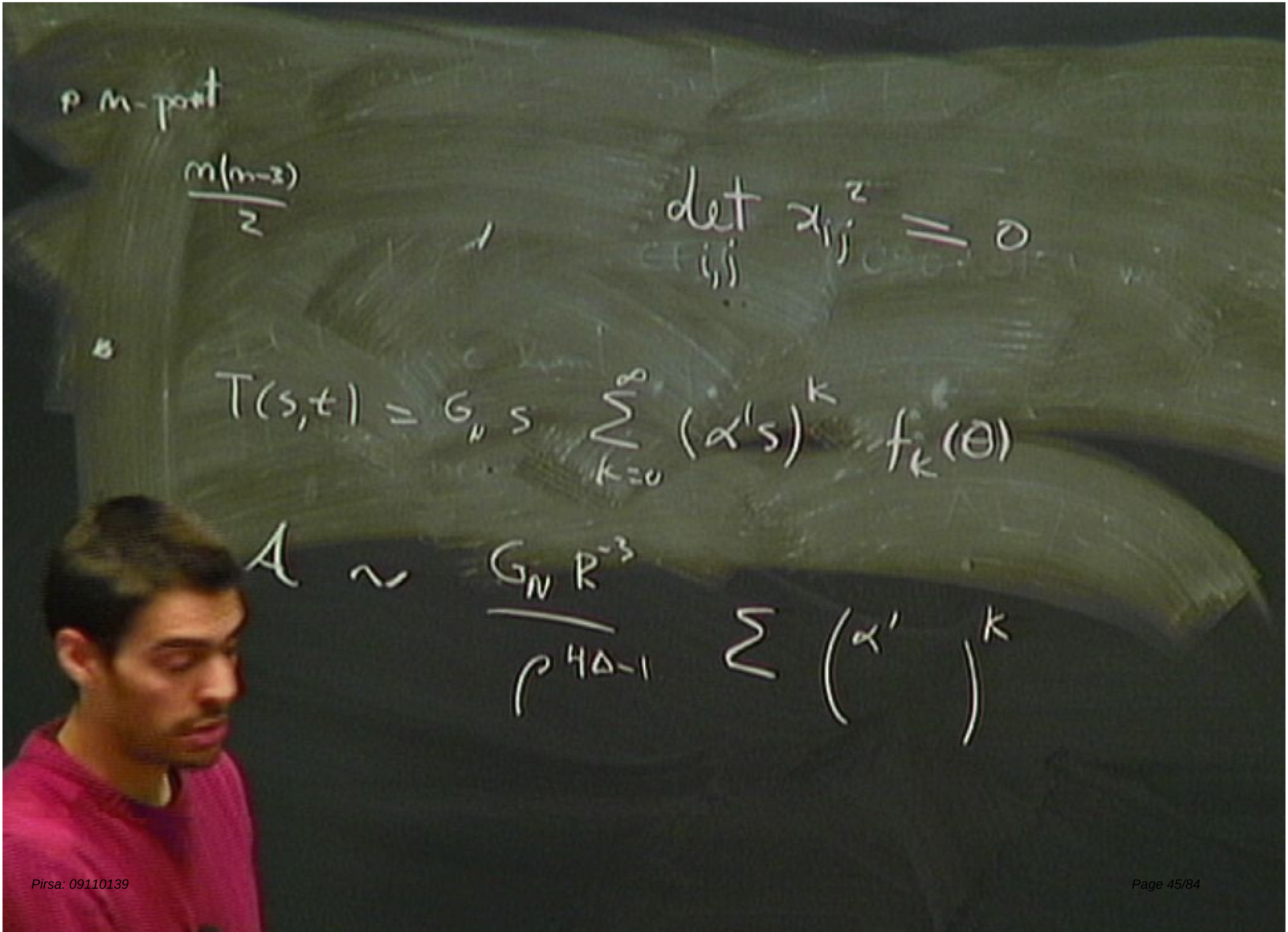
p m-point

$$\frac{m(m-3)}{2}$$

$$\det_{i,j} x_{ij}^2 = 0$$

$$T(s,t) = G_N\, s \sum_{k=0}^{\infty} (\alpha' s)^k f_k(\theta)$$

$$A \sim \frac{G_N R^{-3}}{\rho^{4\Delta-1}} \sum$$



n-point
$\frac{n(n-3)}{2}$
$\det_{i,j} x_{i,j}^2 = 0$
$T(s,t) = G_N\, s \sum_{k=0}^{\infty} (\alpha' s)^k f_k(\theta)$
$A \sim \frac{G_N R^{-3}}{\rho^{4\Delta-1}} \sum (\alpha' \;)^k$

m-point

$$\frac{m(m-3)}{2}$$

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$$A \sim \frac{G_N R^{-3}}{\rho^{4\Delta-1}} \sum_{n=0}^{\infty} \left(\frac{\alpha'}{R^2 \rho^2}\right)^n F_n(\theta)$$

• m-point

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$$\mathcal{A} \sim \frac{G_N R^{-3}}{\rho^{4\Delta-1}} \sum_{n=0}^{\infty} \left(\frac{\alpha'}{R^2\rho^2}\right)^n F_n(\sigma)$$

n-point

$\frac{n(n-3)}{2}$

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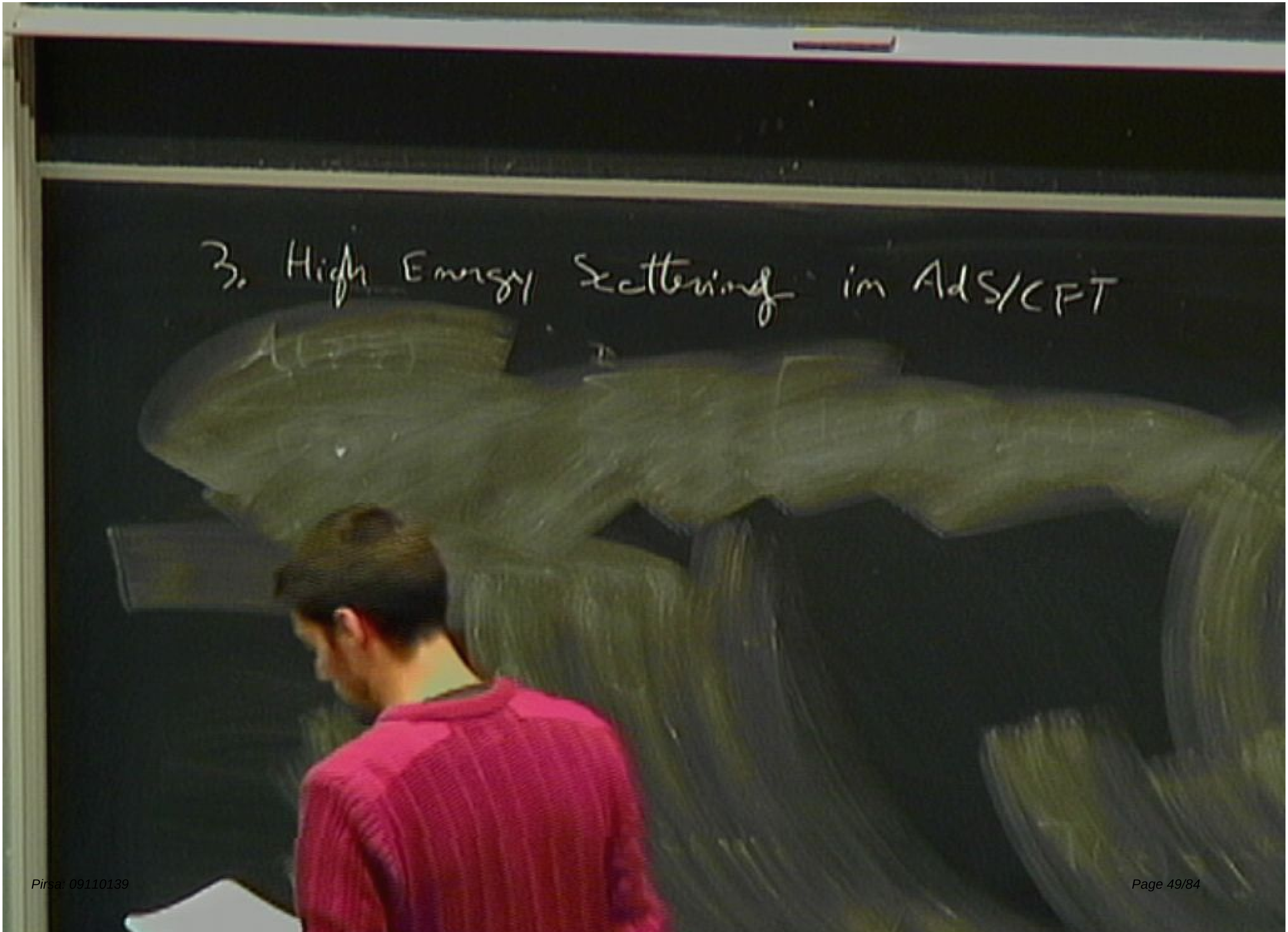
$$\mathcal{A} \sim \frac{G_N R^{-3}}{\rho^{4\Delta-1}} \sum_{n=0}^{\infty} \underbrace{\left(\frac{\alpha'}{R^2 \rho^2}\right)^n}_{\frac{1}{\sqrt{\lambda}\ \rho^2}} F_n(\theta)$$

$\rho \to 0$

$\lambda \to \infty$ $\quad \rho^2 \sqrt{\lambda}$ fixed


3. High Energy Scattering in AdS/CFT

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<u>Introduction</u>

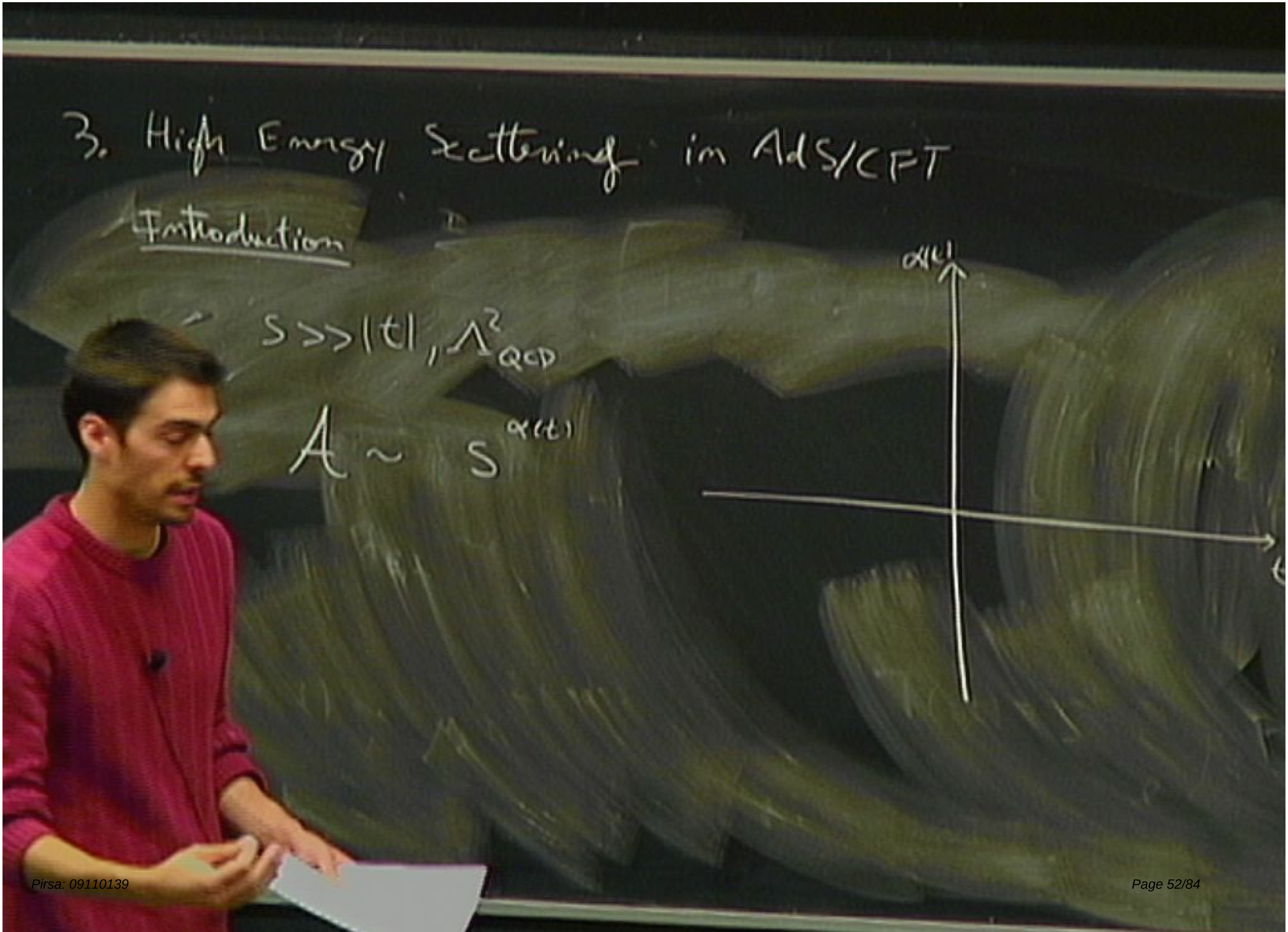
$s \gg |t|, \Lambda^2_{QCD}$

3. High Energy Scattering in AdS/CFT

Introduction

$s \gg |t|, \Lambda^2_{QCD}$

$$A \sim s^{\alpha(t)}$$


3. High Energy Scattering in AdS/CFT
Introduction
$s >> |t|, \Lambda^2_{QCD}$
$A \sim s^{\alpha(t)}$
$\alpha(t)$
t

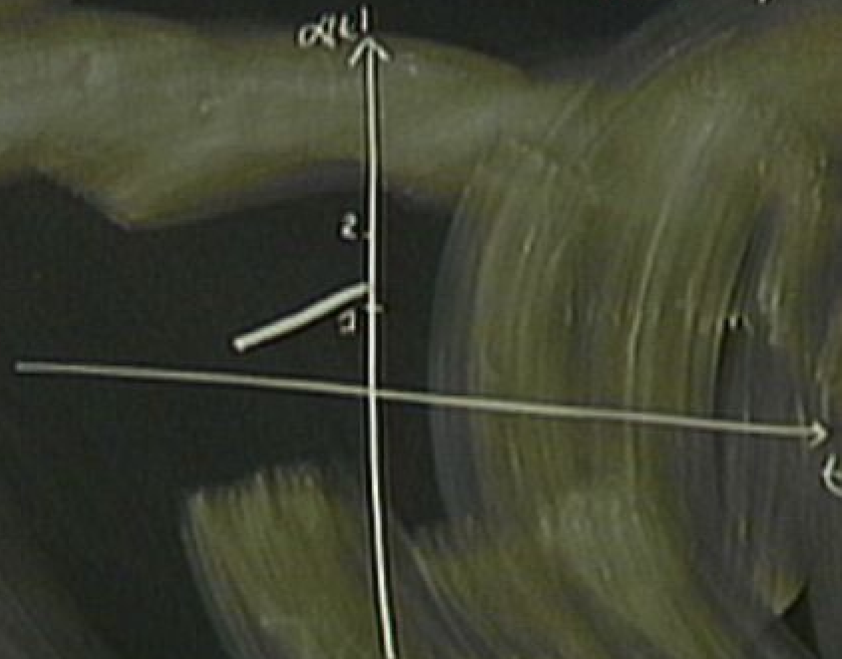
3. High Energy Scattering in AdS/CFT

Introduction

$$\alpha(t) \simeq 1{,}08 + 0{,}25\left(\frac{t}{\mathrm{GeV}}\right)$$

$s \gg |t|, \Lambda^2_{QCD}$

$$A \sim s^{\alpha(t)}$$



3. High Energy Scattering in AdS/CFT

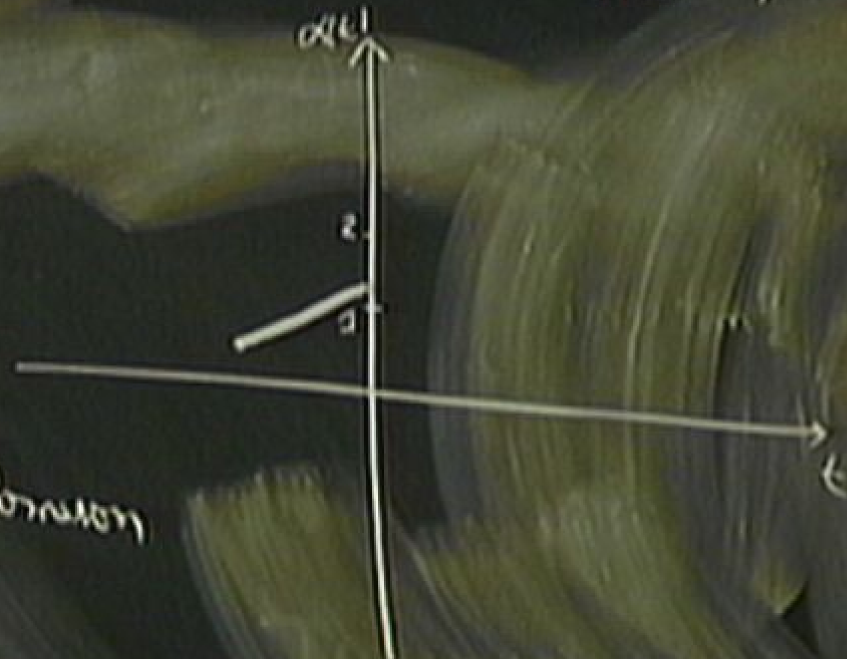
Introduction

$\alpha(t) \simeq 1{,}08 + 0{,}25 \left(\frac{t}{GeV}\right)$

$$s \gg |t|, \Lambda^2_{QCD}$$

$$A \sim s^{\alpha(t)}$$

$\sigma \sim s^{\alpha(0)-1}$, Soft Pomeron



3. High Energy Scattering in AdS/CFT

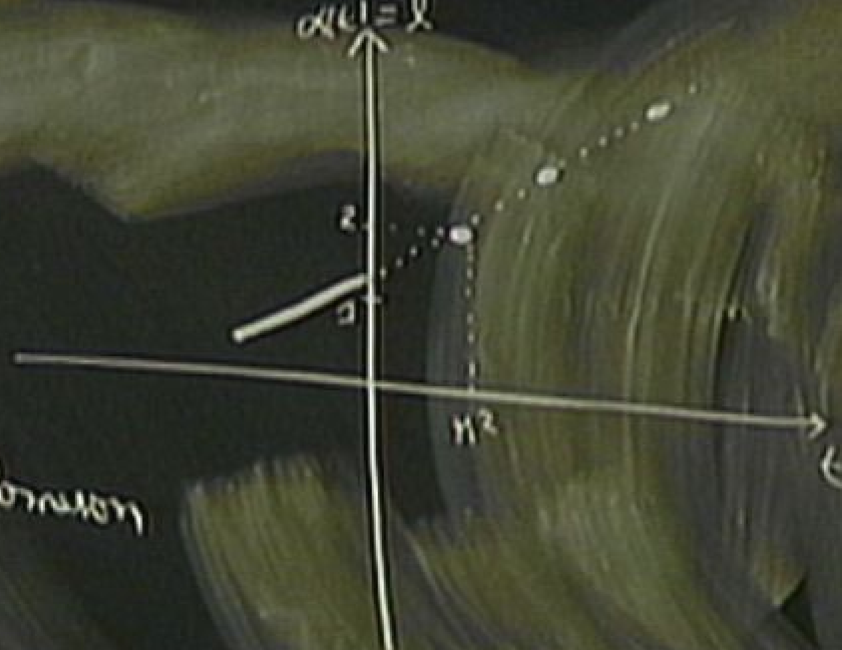
Introduction

• $s \gg |t|, \Lambda^2_{QCD}$

$$A \sim s^{\alpha(t)}$$

$\sigma \sim s^{\alpha(0)-1}$, Soft Pomeron

$$\alpha(t) \simeq 1{,}08 + 0{,}25\left(\frac{t}{GeV}\right)$$



3. High Energy Scattering in AdS/CFT

$$\alpha(t) \simeq 1{,}08 + 0{,}25\left(\frac{t}{\mathrm{GeV}}\right)$$

<u>Introduction</u>

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$$A \sim s^{\alpha(t)}$$

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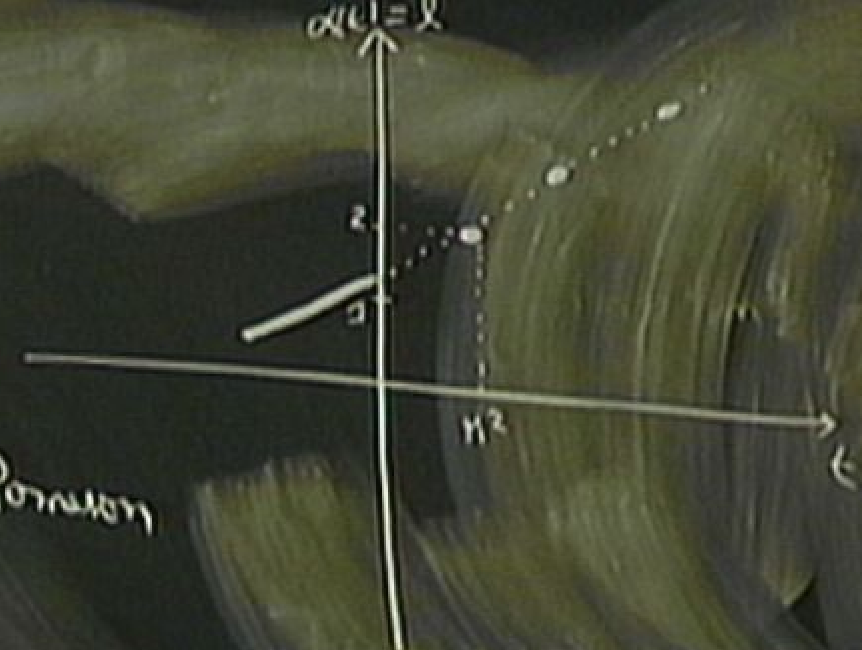
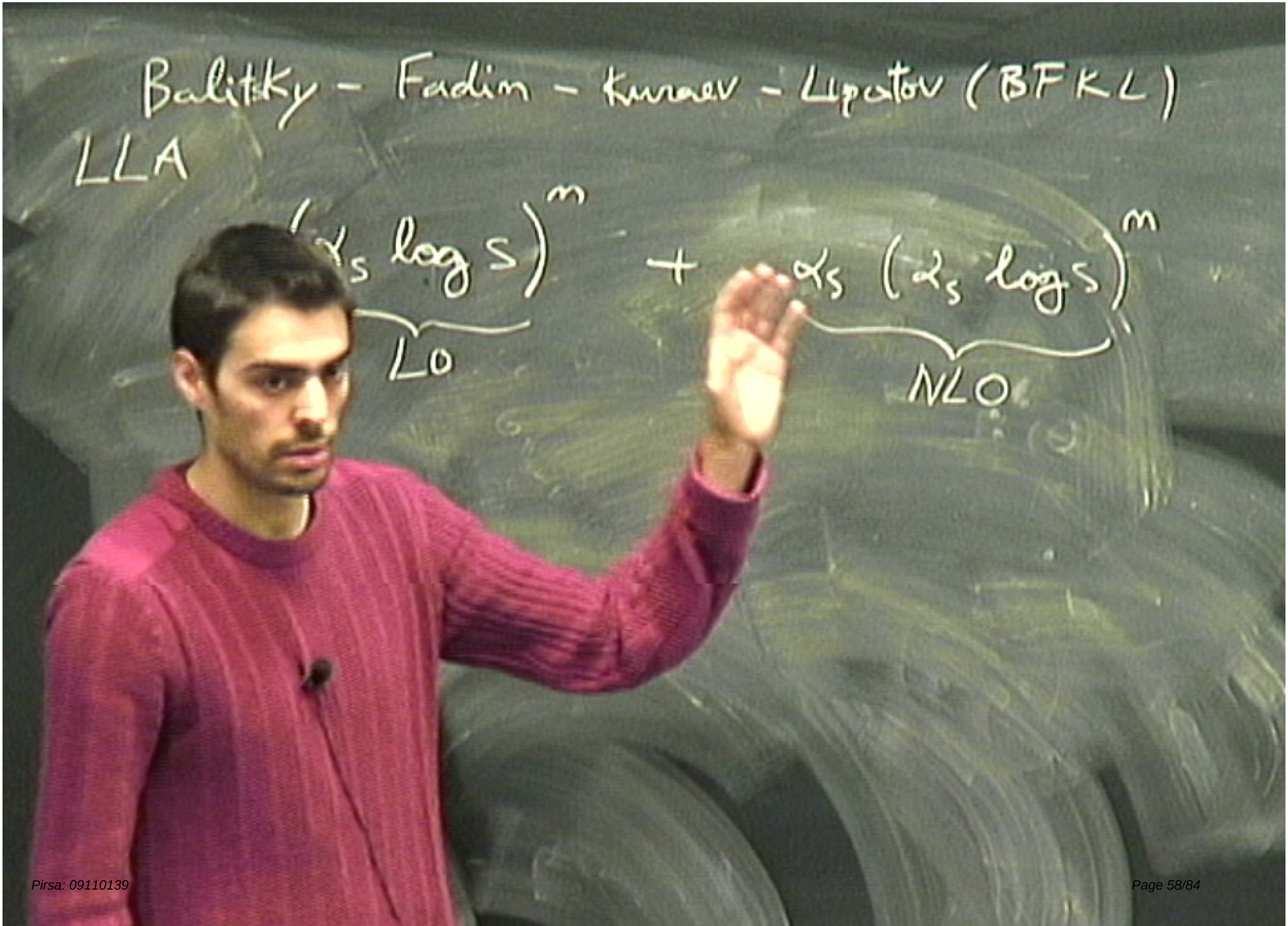


Balitsky - Fadin - Kuraev - Lipatov (BFKL)

α


Balitsky - Fadin - Kuraev - Lipatov (BFKL)
LLA
$\underbrace{(\alpha_s \log s)^n}_{LO} + \underbrace{\alpha_s (\alpha_s \log s)^n}_{NLO}$

Balitsky - Fadin - Kuraev - Lipatov (BFKL)

LLA

$$\underbrace{(\alpha_s \log s)^n}_{LO} + \underbrace{\alpha_s (\alpha_s \log s)^n}_{NLO} + \ldots$$

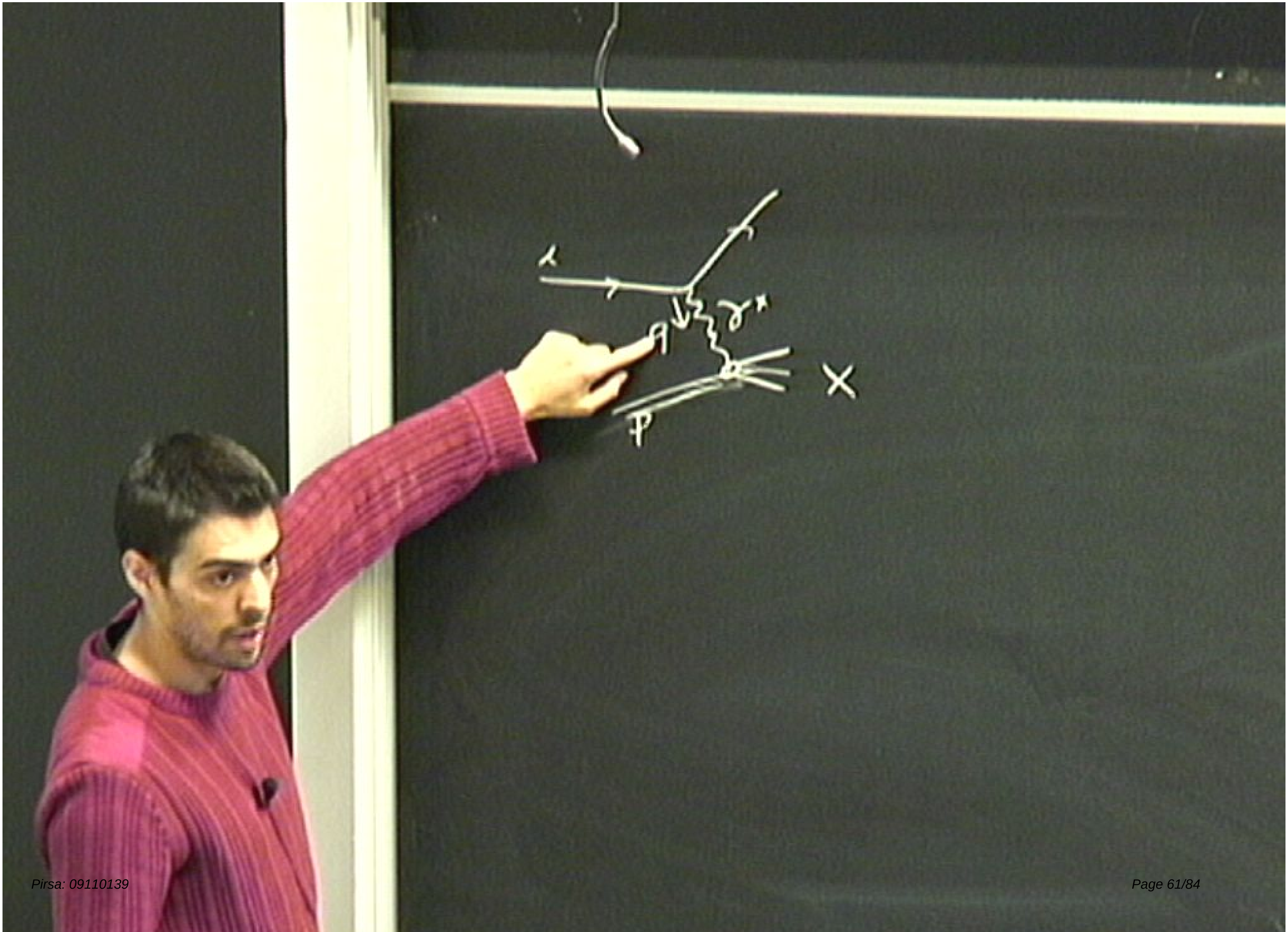
$$A \sim \frac{s^{j(0)}}{\sqrt{\ln s}}, \quad j(0) = 1 + \frac{4}{\pi} N \alpha_s \ln 2 + \ldots$$

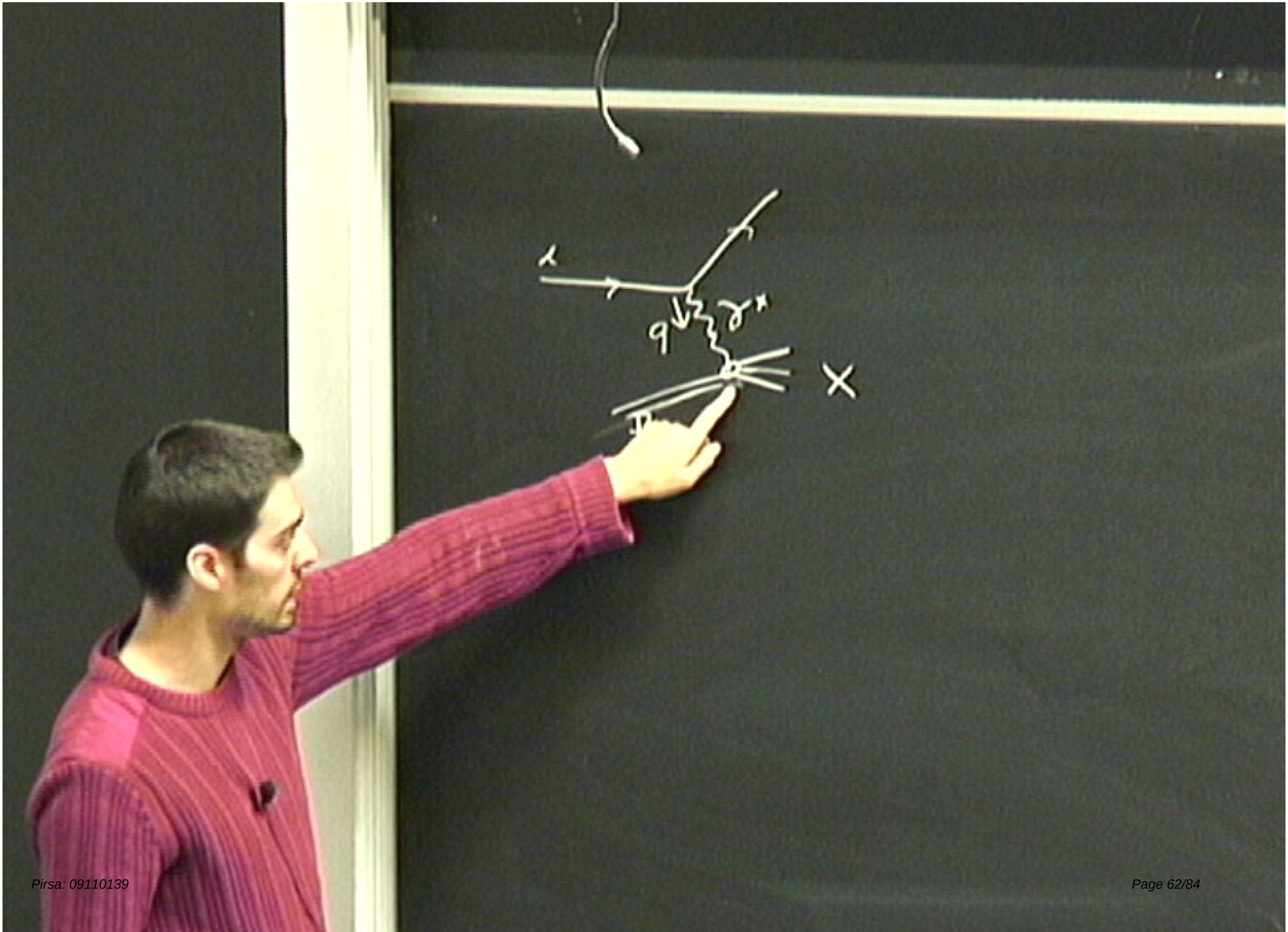
$\underbrace{\qquad}_{LO}$ $\qquad\qquad$ $\underbrace{\qquad}_{NLO}$

$$\lambda \sim \frac{s^{j(0)}}{\sqrt{\ln s}} \quad , \quad j(0) = 1 + \frac{4}{\pi} N\alpha_s \ln 2 + \dots$$

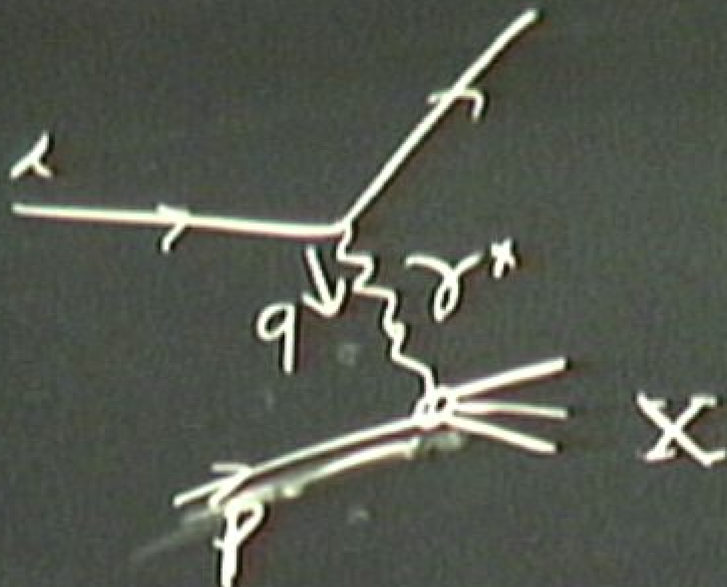
$\alpha_s(|t|)$

$$s \gg |t| \gg \Lambda^2_{QCD}$$


γ*
q
X
p


q
γ*
X

DIS



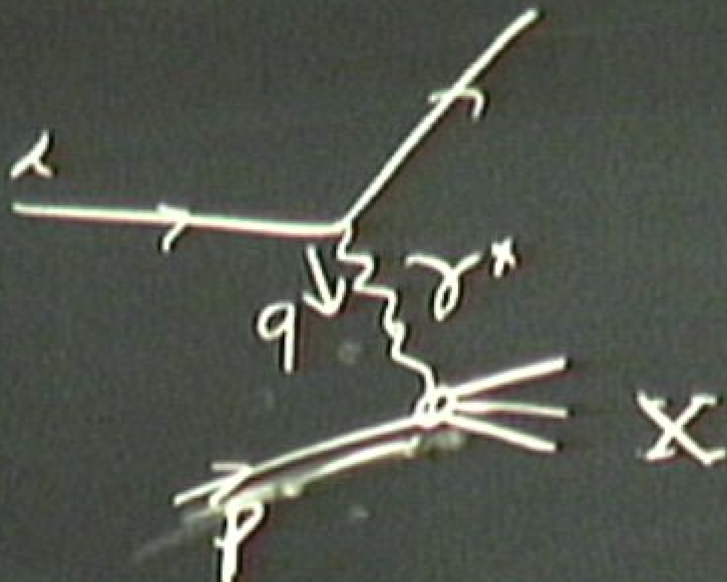
$$\alpha_s(q^2) \ll 1$$

$$x = -\frac{q^2}{2p\cdot q} \ll 1$$

↓

High-energy

DIS

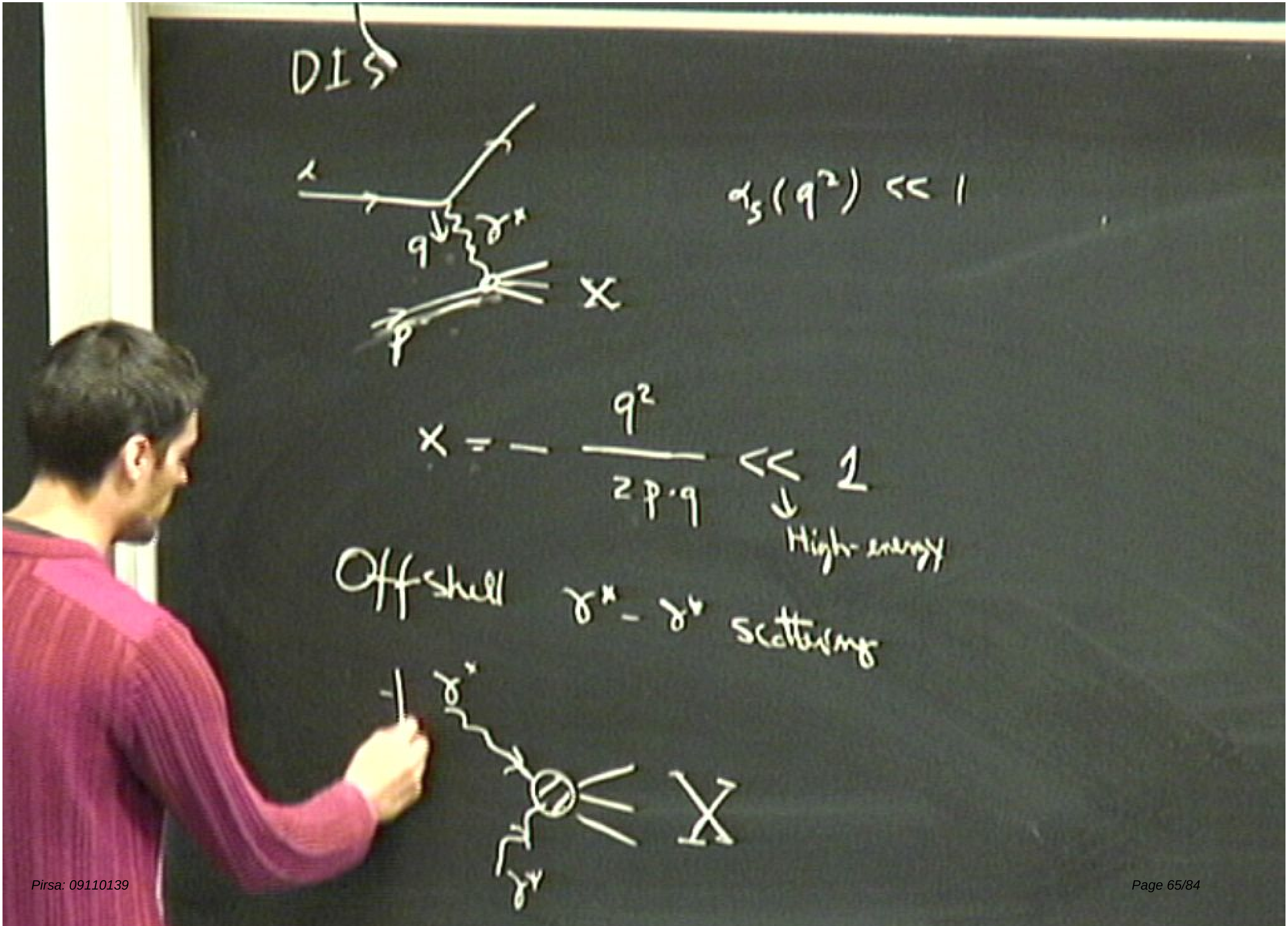


$$\alpha_s(q^2) \ll 1$$

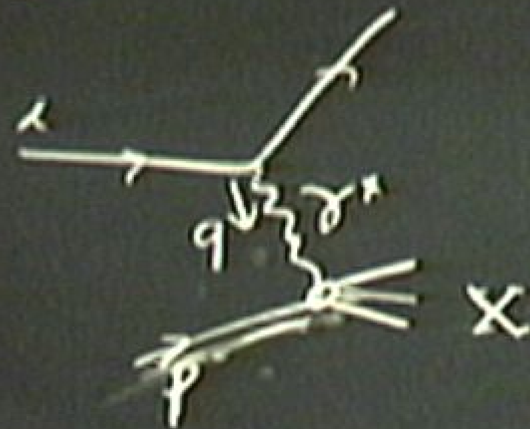
$$x = -\frac{q^2}{2p\cdot q} \ll 1$$

High-energy

Offshell γ^* – γ^* scattering


DIS
$\alpha_s(q^2) \ll 1$
q
γ^*
p
X
$x = -\frac{q^2}{2p\cdot q} \ll 1$
High-energy
Offshell γ^* - γ^* scattering
γ^*
X
γ^*

DIS

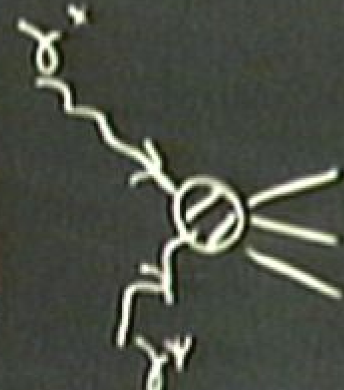


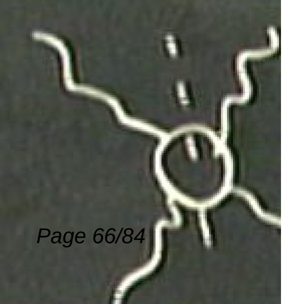
$\alpha_s(q^2) \ll 1$

$$x = -\frac{q^2}{2p\cdot q} \ll 1$$

↓ High-energy

Offshell $\gamma^* - \gamma^*$ scattering

$$\sum_X \Big|$$ 

$$X \Big|^2 = 2\,\mathrm{Im}$$ 

3. High Energy Scattering in AdS/CFT

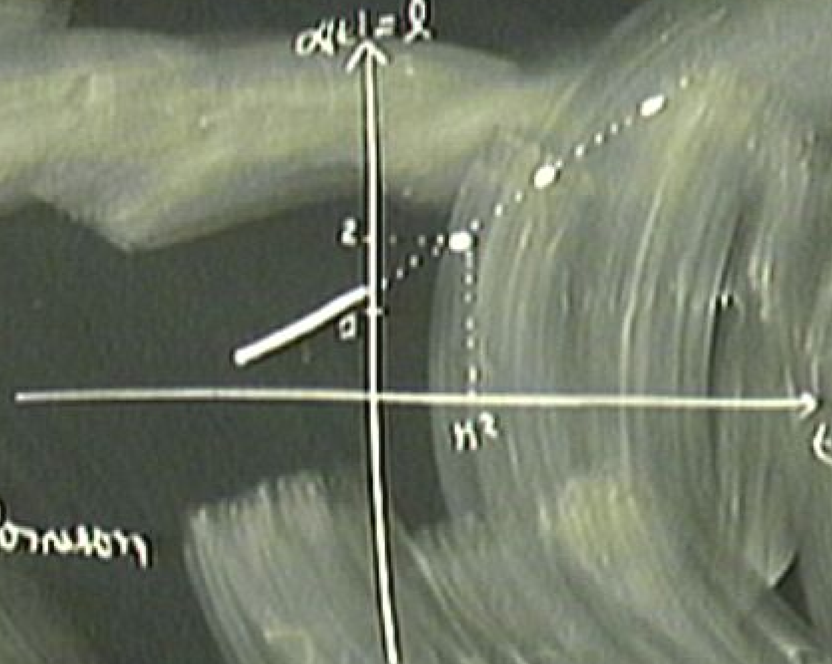
$\alpha(t) \simeq 1{,}08 + 0{,}25 \left(\frac{t}{\mathrm{GeV}}\right)$

Introduction

$s \gg |t|, \Lambda^2_{QCD}$

$$A \sim s^{\alpha(t)}$$

$\sigma \sim s^{\alpha(0)-1}$, Soft Pomeron



Balitsky - Fadin - Kuraev - Lipatov (BFKL)

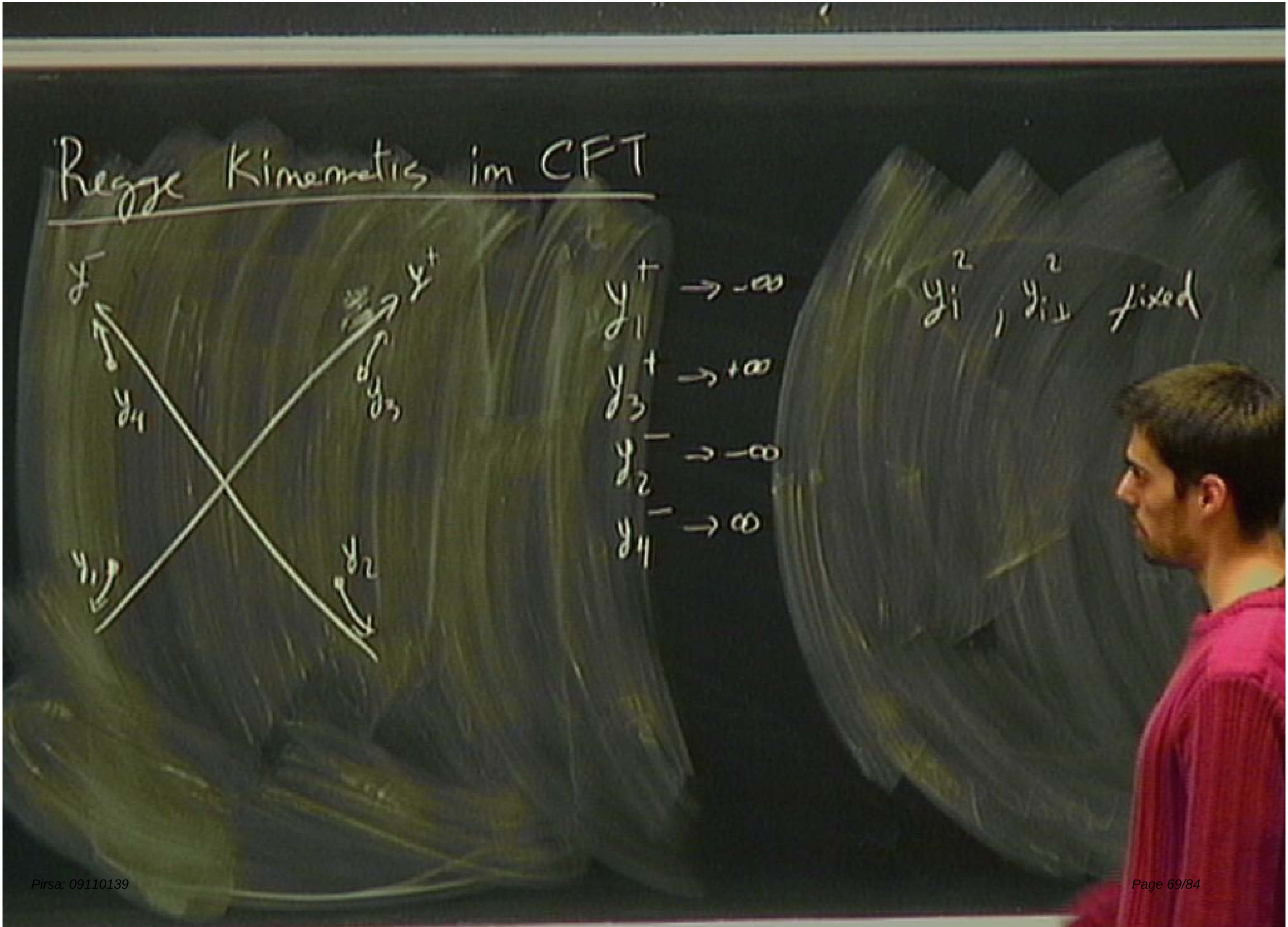
LLA

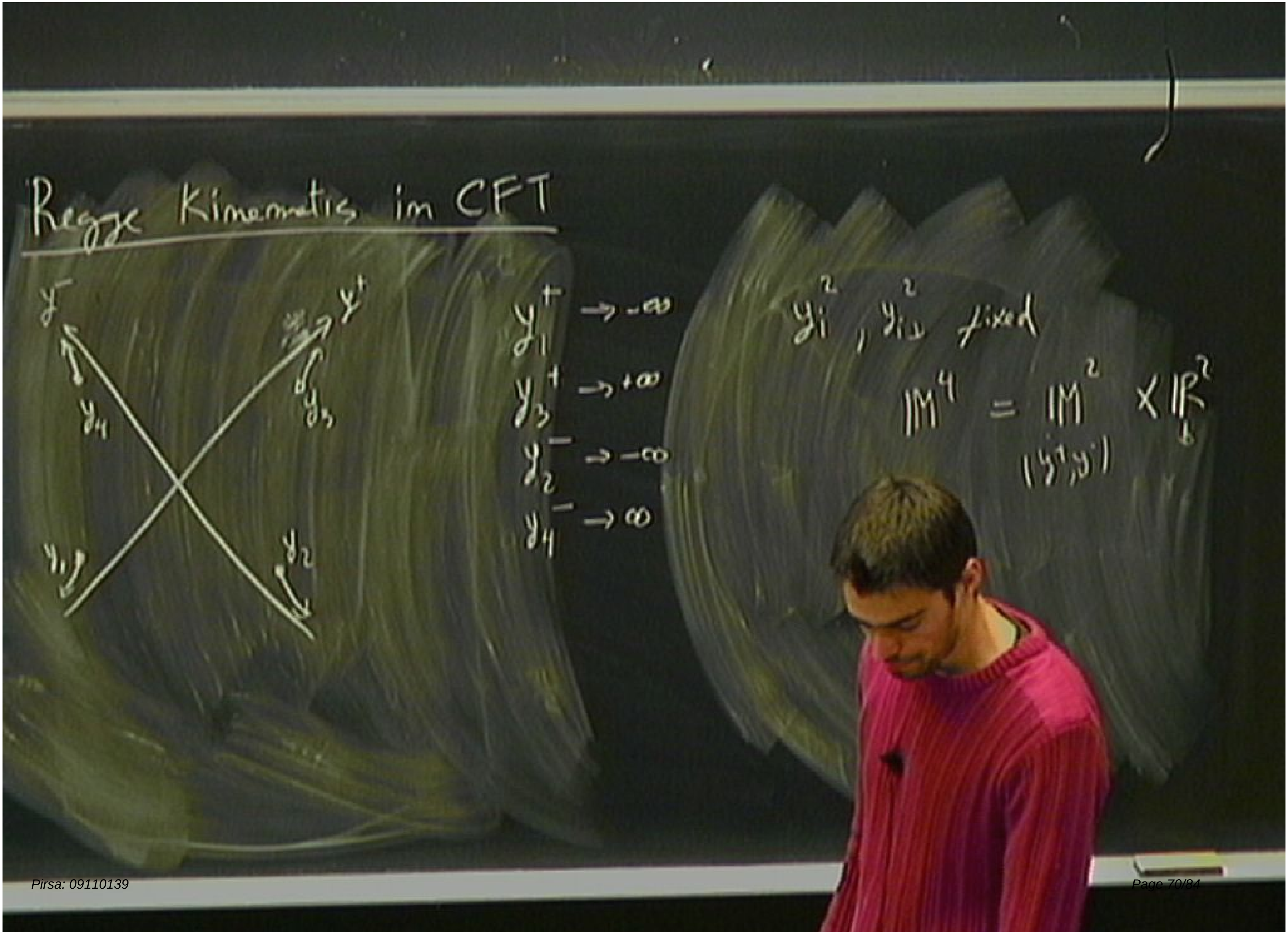
$$\underbrace{(\alpha_s \log s)^n}_{LO} + \underbrace{\alpha_s (\alpha_s \log s)^n}_{NLO} + \dots$$

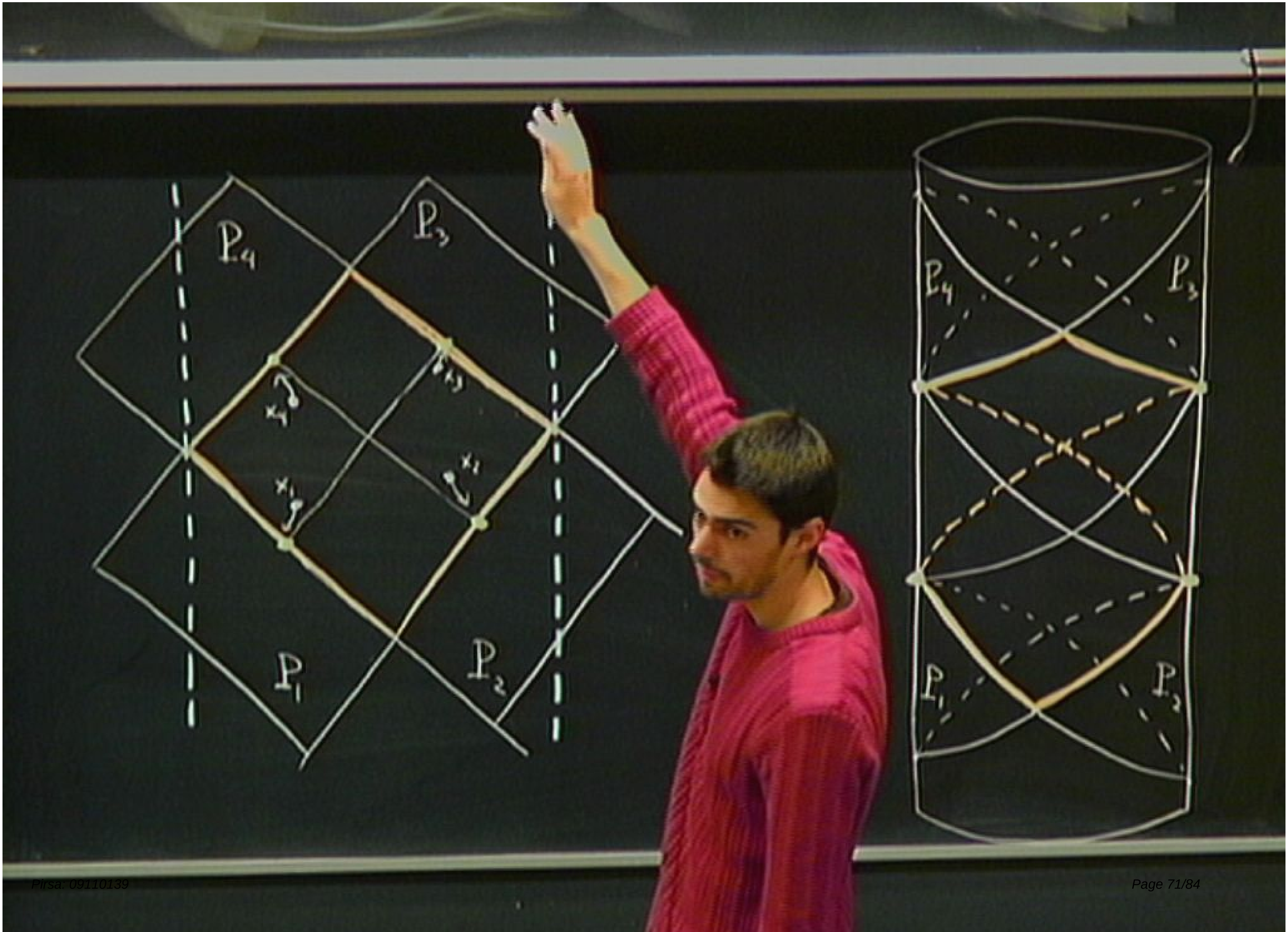
$$A \sim \frac{s^{j(0)}}{\sqrt{\ln s}} \, , \quad j(0) = 1 + \frac{4}{\pi} \underbrace{N \alpha_s}_{\lambda} \ln 2 + \dots$$

$\alpha_s(|t|)$

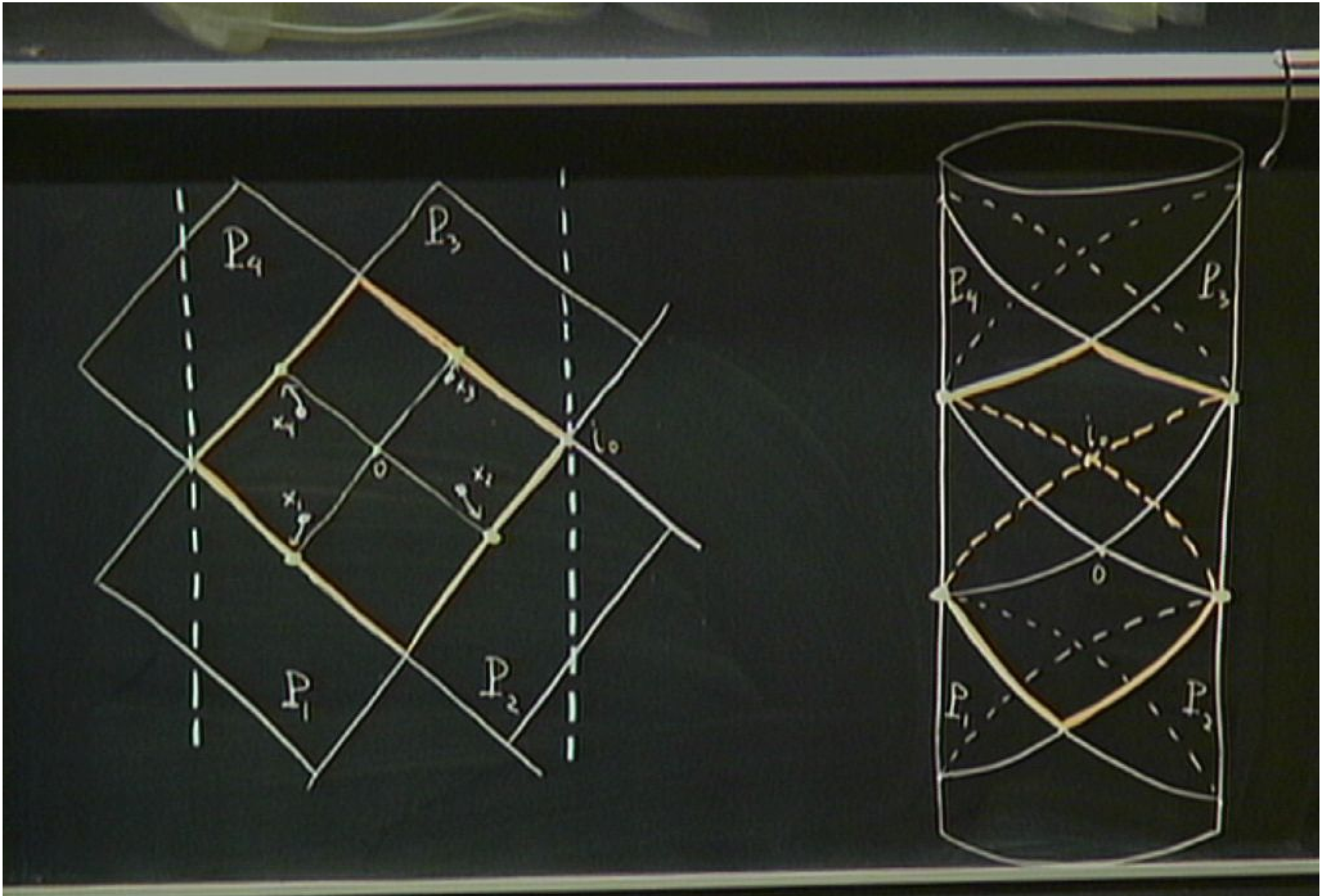
$$s \gg |t| \gg \Lambda^2_{QCD}$$


Regge Kinematics in CFT
y^-
y^+
y_4
y_3
y_1
y_2
$y_1^+ \to -\infty$
$y_3^+ \to +\infty$
$y_2^- \to -\infty$
$y_4^- \to \infty$
y_i^2, $y_{i\perp}^2$ fixed


Regge Kinematics in CFT
y-
y+
y4
y3
y1
y2
y1+ → -∞
y3+ → +∞
y2- → -∞
y4- → ∞
yi2, yi⊥2 fixed
M4 = M2 × R2
(y+, y-)



P_4
P_3
x_4
x_3
x_1
x_2
P_1
P_2
P_4
P_3
P_1
P_2


P4
P3
x4
x3
i0
0
x1
x2
P1
P2
P4
P3
i0
0
P1
P2

$$x_i = (x_i^+, x_i^-, x_{i\perp}) = -\frac{1}{y_i^+}(1, y_i^2, y_{i\perp}) \qquad \begin{array}{l} i=1,3 \\ i=2,4 \end{array}$$

$$-dy^+dy^- + dy_\perp^2 = \frac{1}{(x^+)^2}(-dx^+dx^- + dx_\perp^2)$$

Regge limit $\qquad x_i \to 0$

$$x_i = (x_i^+, x_i^-, x_{i\perp}) = -\frac{1}{y_i^+}(1, y_i^2, y_{i\perp})$$

$i = 1,3$
$i = 2,4$

$$-dy^+dy^- + dy_\perp^2 = \frac{1}{(x^+)^2}(-dx^+dx^- + dx_\perp^2)$$

Regge limit $\quad x_i \to 0$

$$A(y_i) = \pi \left|\frac{\partial y_i}{\partial x_i}\right|^{\Delta/4}$$

$$A(x_i)$$

$a \sim x_i$

$T: \quad x_{1,3} \to x_{1,3} + a \qquad x_{2,4} \Rightarrow x_{2,4}$

$a \sim x_i$

$T: \quad x_{1,3} \to x_{1,3} + a \qquad x_{2,4} \Rightarrow x_{2,4}$

$S:$

$a \sim x_i$

$T: \quad x_{1,3} \to x_{1,3} + a \qquad x_{2,4} \to x_{2,4} + O(a^2, x_i^2)$

$\quad \text{''} \to \text{''} + 0 \qquad \text{''} \to \text{''} + b$

$x = x_1 - x_3 \qquad \bar{x} = x_2 - x_4$

$a \sim x_i$

$T: \quad x_{1,3} \rightarrow x_{1,3} + a \qquad x_{2,4} \Rightarrow x_{2,4} + O(a^2, x_i^2)$

$S: \quad \text{''} \rightarrow \text{''} + 0 \qquad \text{''} \rightarrow \text{''} + b$

$x = x_1 - x_3 \qquad \bar{x} = x_2 - x_4$

$a \sim x_i$

$T: \quad x_{1,3} \to x_{1,3} + a \quad , \quad x_{2,4} \to x_{2,4} + O(a^2, x_i^2)$

$\quad : \quad '' \to '' + a \quad , \quad '' \to '' + b$

$x = x_1 - x_3 \qquad \bar{x} = x_2 - x_4$

$D: \quad x_{1,3} \to \lambda x_{1,3} \quad , \quad x_{2,4} \to \frac{1}{\lambda} x_{2,4}$

$a \sim x_i$

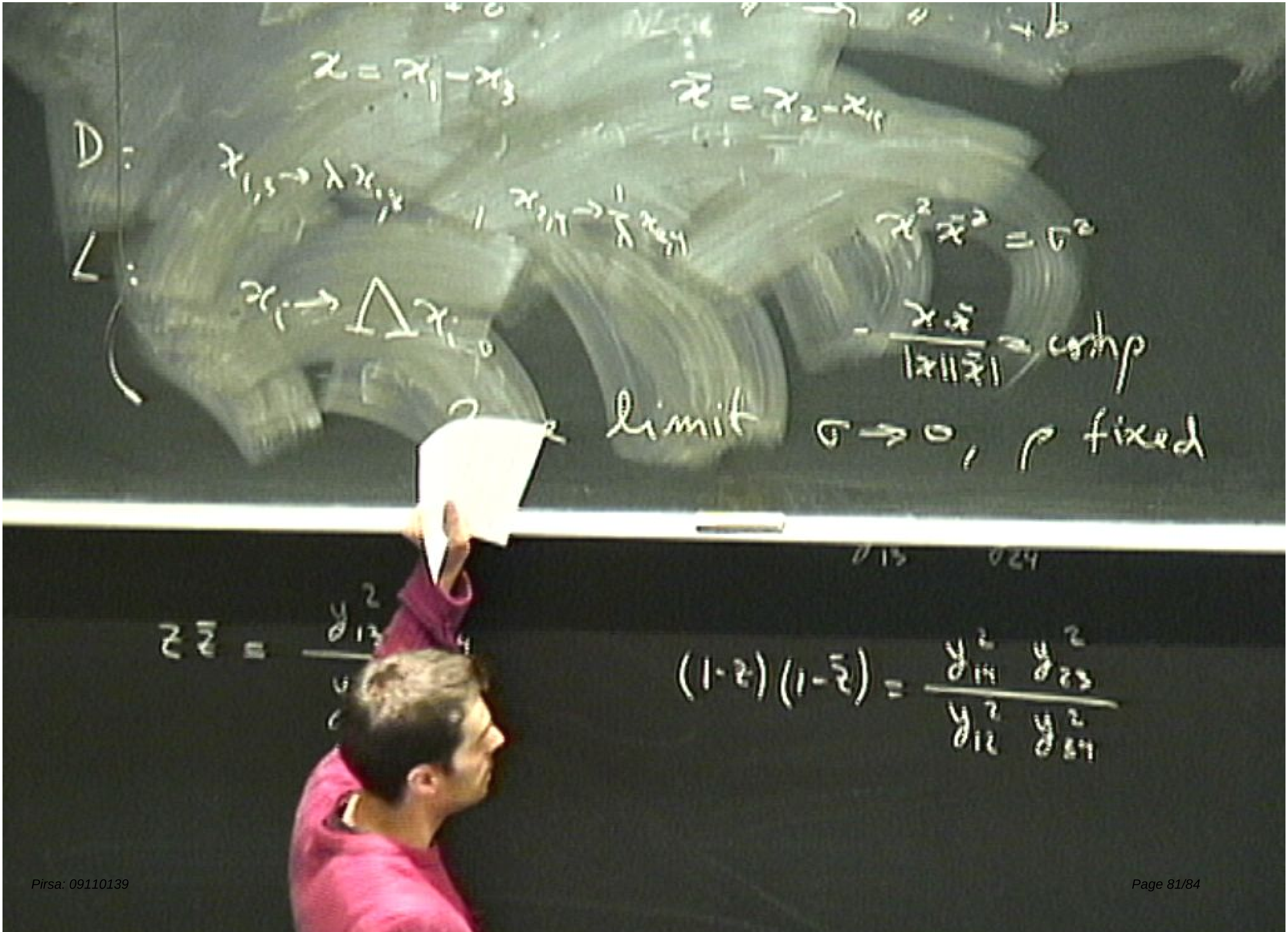
$T: \quad x_{1,3} \to x_{1,3} + a \qquad x_{2,4} \to x_{2,4} + O(a^2, x_i^2)$

$S: \quad \to + \qquad \to + b$

$x = x_1 - x_3 \qquad \bar{x} = x_2 - x_4$

$D: \quad x_{1,3} \to \lambda x_{1,3} \qquad x_{2,4} \to \frac{1}{\lambda} x_{2,4}$

$L: \quad x_i \to \Lambda x_i$


x = x1 - x3
x̄ = x2 - x4
D: x1,3 → λ x1,3 , x2,4 → 1/λ x2,4
x² x̄² = σ²
L: xi → Λ xi
x·x̄ / |x||x̄| = cosh ρ
limit σ → 0, ρ fixed
(1-z)(1-z̄) = y14² y23² / y12² y34²

$a \sim x_i$

$T:\quad x_{1,3} \to x_{1,3} \qquad x_{2,4} \to x_{2,4} + O(a^2, x_i^2)$

$S:\quad '' \to '' + a \qquad '' \to '' + b$

$x = x_1 - x_3 \qquad \bar{x} = x_2 - x_4$

$D:\quad x_{1,3} \to \lambda x_{1,3} \qquad x_{2,4} \to \frac{1}{\lambda} x_{2,4}$

$x^2 \bar{x}^2 = \sigma^2$

$L:\quad x_i \to \Lambda x_i$

$$-\frac{x \cdot \bar{x}}{|x||\bar{x}|} = \cosh\rho$$

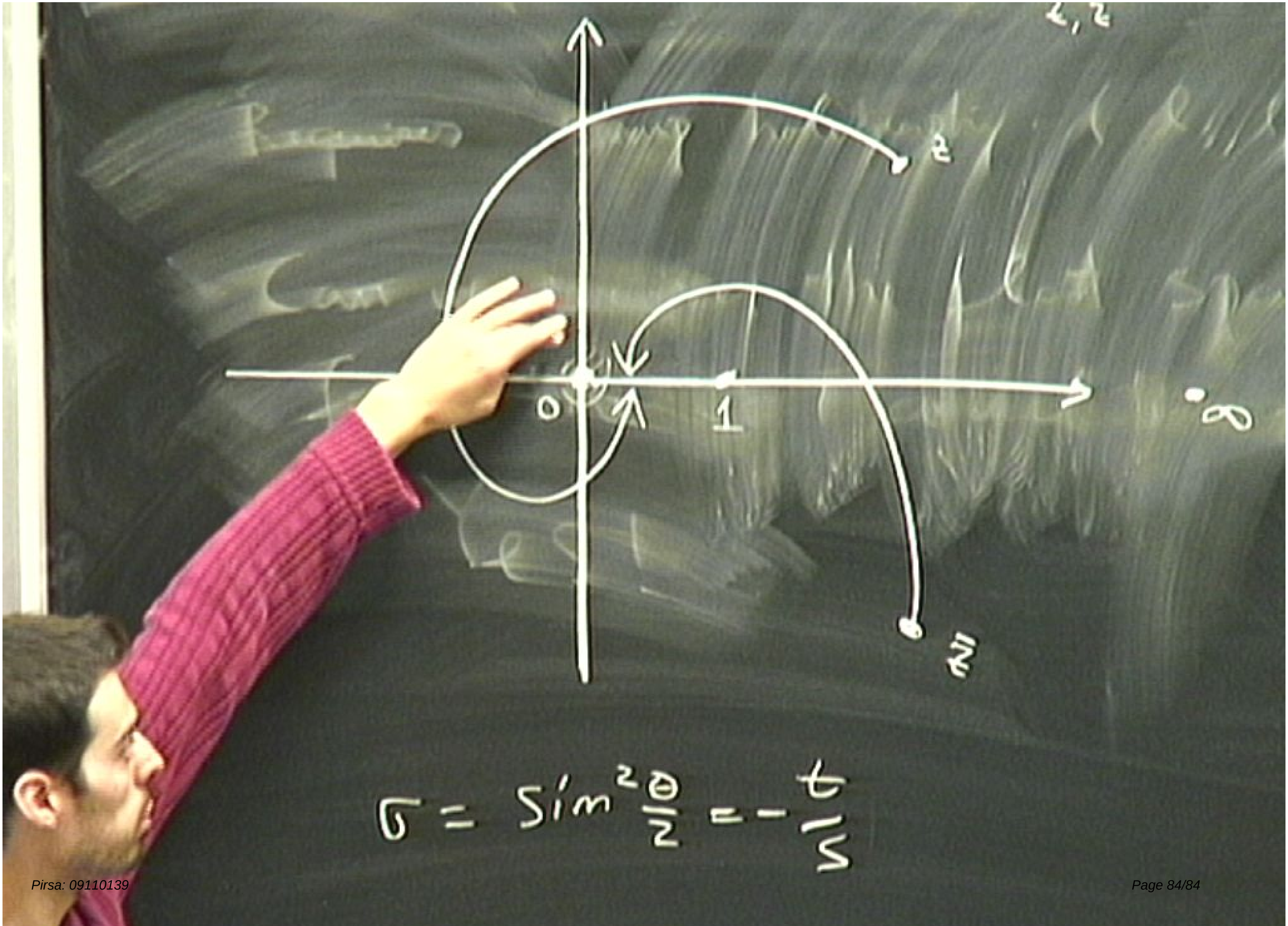
Regge limit $\sigma \to 0$, ρ fixed

Regge limit $\sigma \to 0$, ρ fixed

$$\langle \mathcal{O}(y_1)\, \mathcal{O}(y_2)\, \mathcal{O}(y_3)\, \mathcal{O}(y_4) \rangle = \frac{\mathcal{A}(z,\bar z)}{y_{13}^{2\Delta}\; y_{24}^{2\Delta}} = A(y_i)$$

$$z\bar z = \frac{y_{13}^2\, y_{24}^2}{y_{12}^2\, y_{34}^2} \qquad (1-z)(1-\bar z) = \frac{y_{14}^2\, y_{23}^2}{y_{12}^2\, y_{34}^2}$$

$$z\bar z = x^2 \bar x^2 \qquad z + \bar z = 2x\cdot\bar x$$


z
0
1
∞
z̄
σ = sin² θ/2 = − t/s