Title: Superconducting Dark Energy and Neutrino Oscillation

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Abstract:

SUPERCONDUCTING DARK ENERGY AND NEUTRINO OSCILLATIONS

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Discussion

WHAT IS THE CULPRIT?

"Quintessence" models of DE

Ratra, Peebles; Caldwell, Dave, Steinhardt; Wetterich

A classical, minimally-coupled scalar field evolves in a potential $V(\phi)$, while its energy density and pressure combine to produce a negative equation of state $w=p/\rho$.
Unlike the cosmological constant, the Quintessence field

admits fluctuations $\delta\phi$.

- **Fine-tuning problem**: in analogy with Λ, the need to tune initial values of potential to get the observed energy density and equation of state;

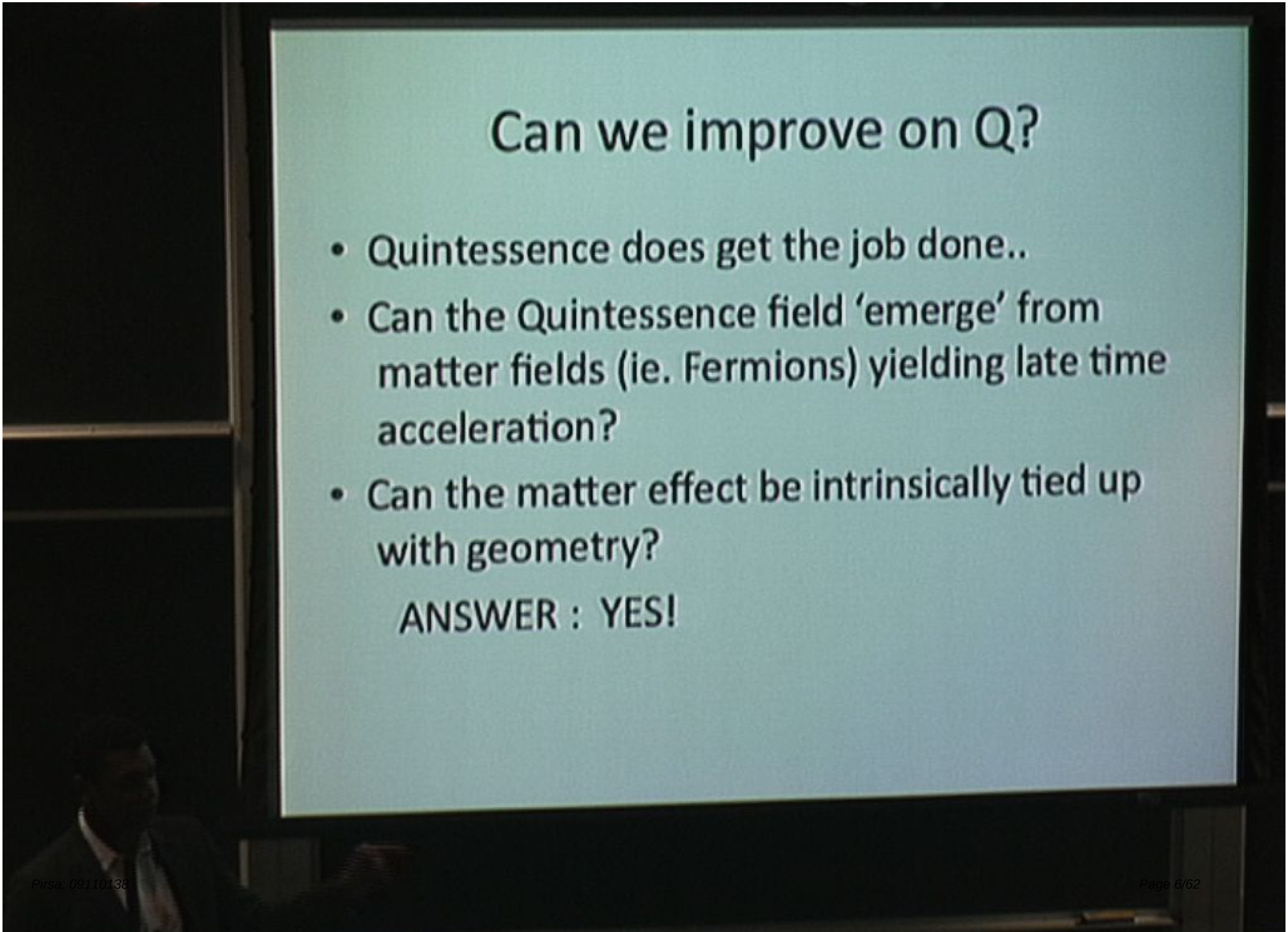
- **"Coincidence" problem**: why $\rho_m \sim \rho_\phi$ just today ?

$\Rightarrow$ Search for ATTRACTOR SOLUTIONS ("tracking fields")

Can we improve on Q?

- Quintessence does get the job done..
- Can the Quintessence field 'emerge' from matter fields (ie. Fermions) yielding late time acceleration?
- Can the matter effect be intrinsically tied up with geometry?

ANSWER : YES!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Neutrino Mixing and Oscillation

- If neutrinos are massive, then the **weak eigenstates** are not the same as the **mass eigenstates**:

PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix

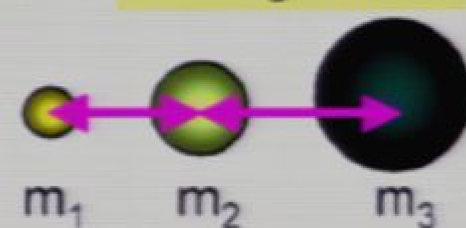
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Weak eigenstates „flavor eigenstates"

3 independent parameters + 1 complex phase

Mass eigenstates

$\theta_{12}, \theta_{23}, \theta_{13}$
$+ \delta$



$\Delta m^2{}_{12}, \Delta m^2{}_{23}$

What's Already Known

- Solar neutrinos/Reactor:
  - SNO & KamLAND have established ν oscillation
  - $\Delta m^2_{21} \approx 8 \times 10^{-5}$ eV^2, $\theta_{12} \approx 32°$
- Atmospheric neutrinos:

a — $\Delta m^2_{32} \approx (2.1 \pm 0.8) \times 10^{-3}$ eV^2, $\theta_{23} \approx 45°$

$\Delta m^2_{31} \approx \Delta m^2_{32} >> \Delta m^2_{21}$

θ_{12} and θ_{23} are large

Questions :

- Why is mass scale of neutrinos so small ?

$$L = \frac{1}{M}\bar{l} \cdot H \cdot \overline{H} \cdot l^c \Rightarrow m_\nu \approx \upsilon^2 / M \approx 10^{-3} - 10^{-4} eV$$

- about 10^{-3} eV ~ accidental or not ?

$$m_\phi \approx 10^{-33} eV \approx m_\nu^2 / M$$

- MaVaNs (Fardon, Nelson, Weiner)

- Does the scale of dark energy and neutrinos emerge from same underlying physics?

We Seek a New Path

A New Outlook:
Condensates in Cosmology

- Condensates are attractive because they can vanish in the UV (Debe scale)and IR.

- They can be purely long range (macroscopic) quantum phenomena.

- They are very common across a wide range of physical phenomena.

- In the context of inflation they could drive inflation and dissapear at the end (IR) leading to a natural graceful exit. Today we will address the CC problem with Fermionic condensates.

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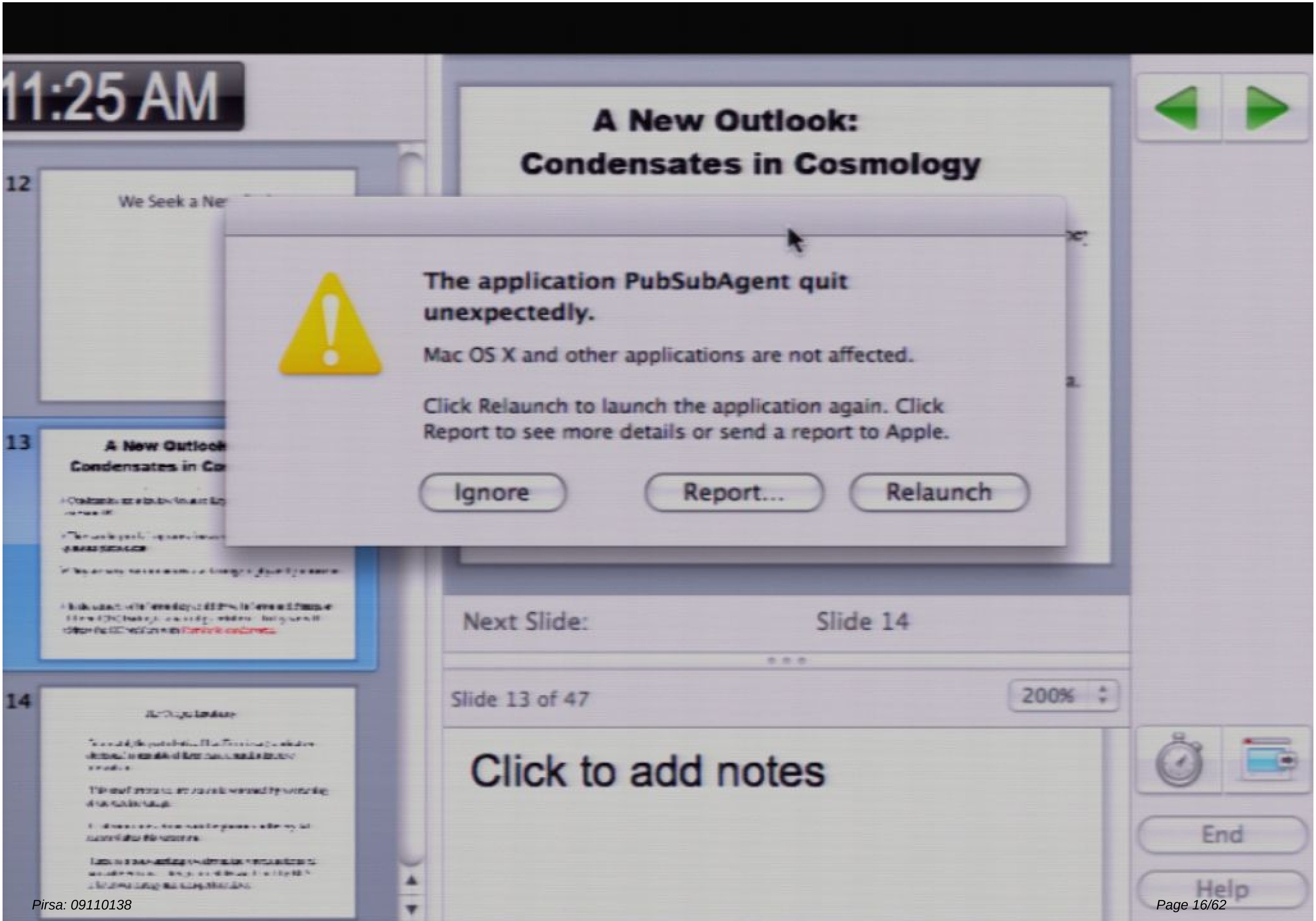
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The application PubSubAgent quit unexpectedly.
Mac OS X and other applications are not affected.
Click Relaunch to launch the application again. Click Report to see more details or send a report to Apple.
Ignore
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Next Slide:
Slide 14
Slide 13 of 47
200%
Click to add notes
End
Help

A New Outlook:

Cond ... y

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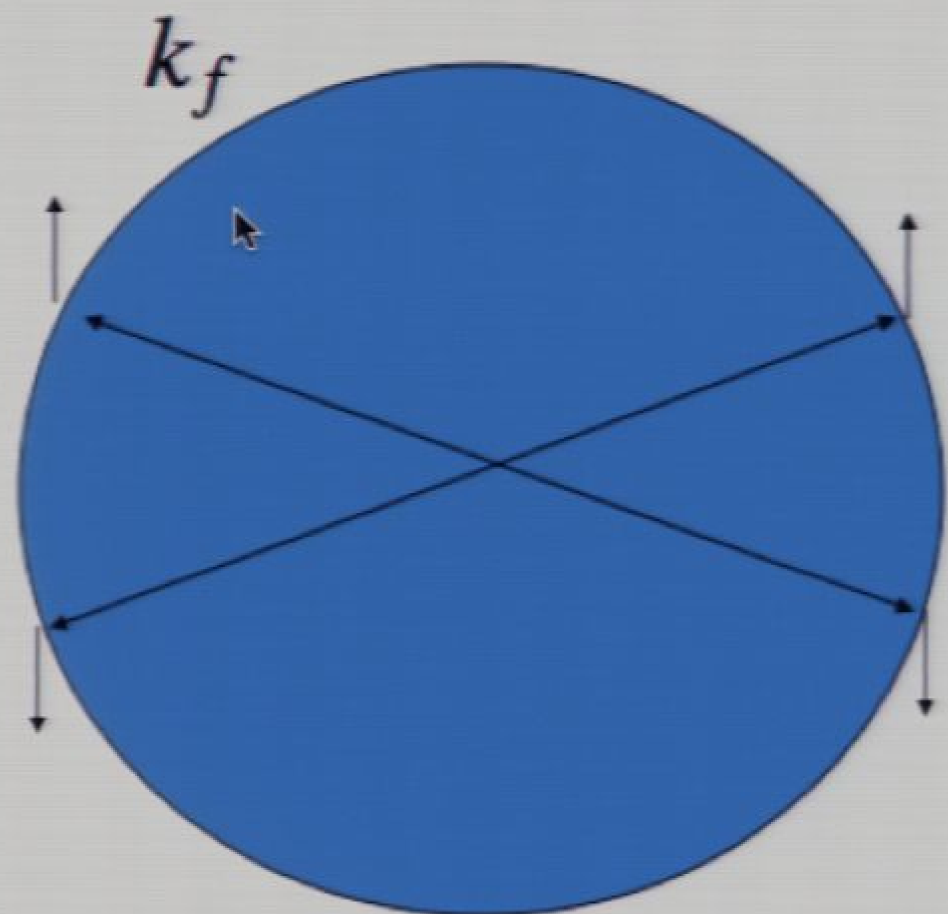
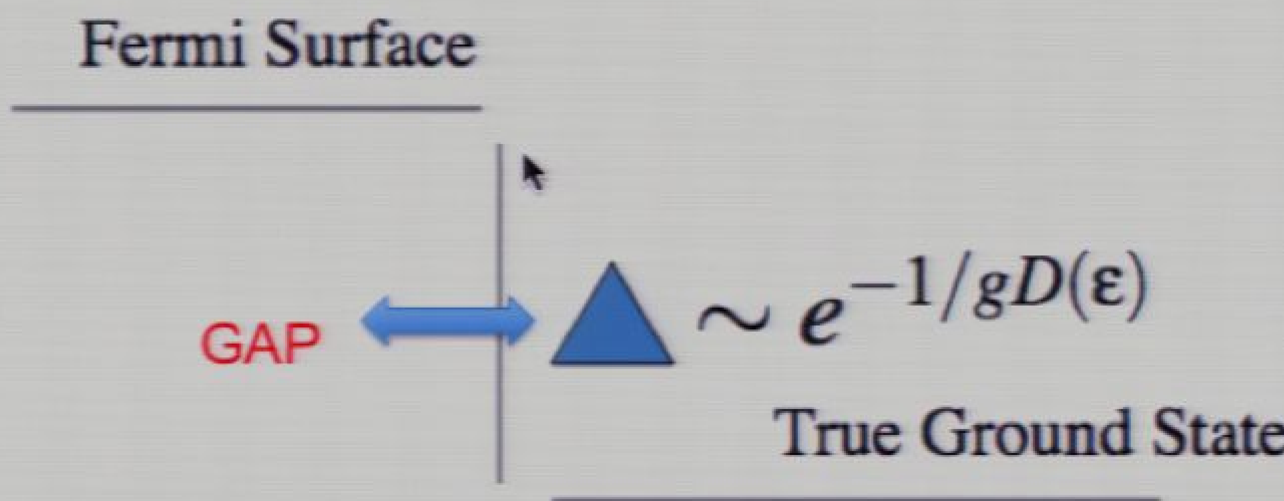
BCS Theory: Fundamentals

Key: Correlations of fermions across Fermi surface



SUPERCONDUCTIVITY



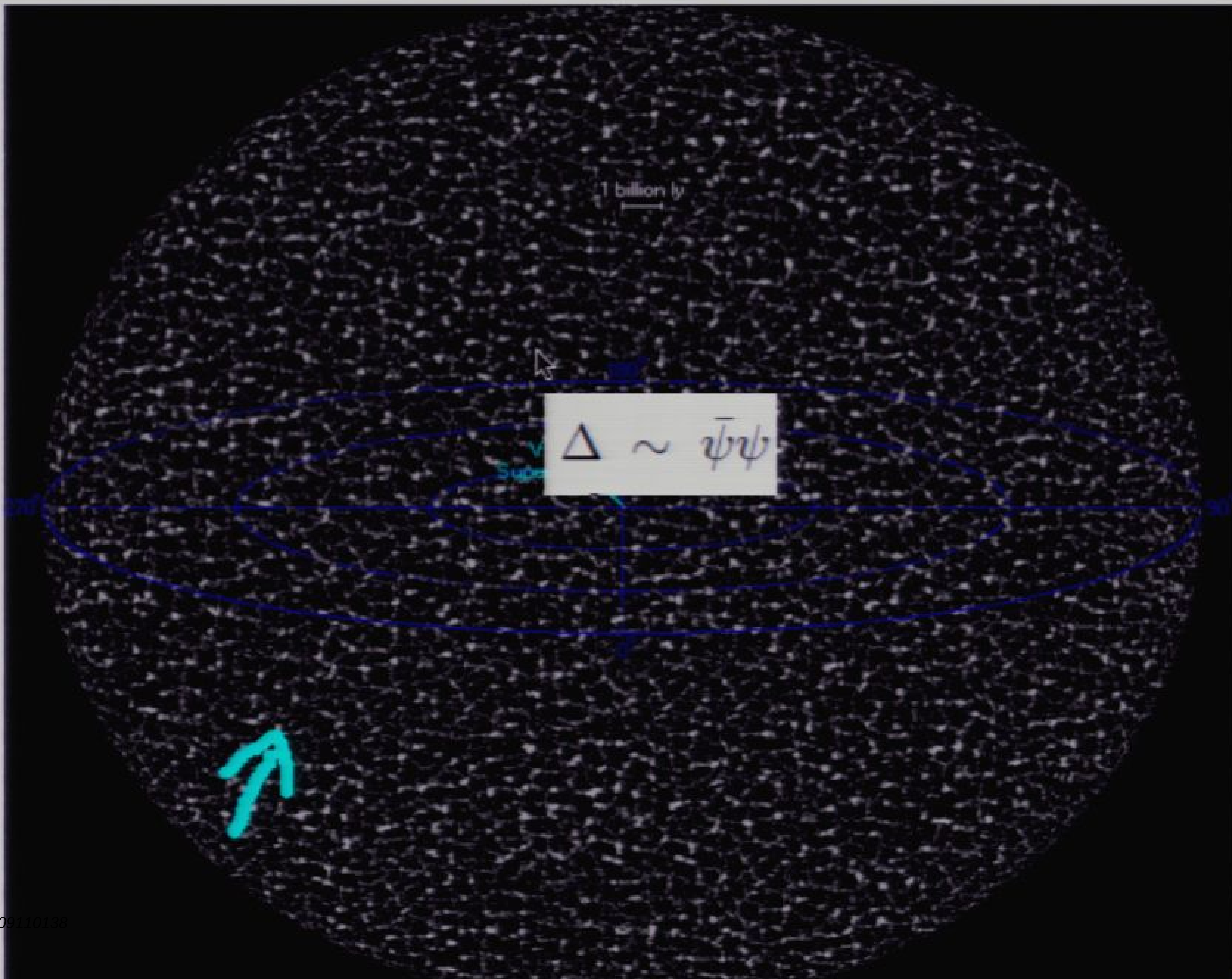
$$\Delta \sim e^{-1/gD(\varepsilon)}$$

$D(\varepsilon)$ =density of states

The perturbative vacuum (Fermi surface) is unstable, due to strongly correlated electron pairs.

There is a non-perturbative ground state with lower energy,

Fill The Universe up with Neutral Fermions



$$\Delta \sim \bar{\psi}\psi$$

The Theory

- Consider Fermions Covariantly coupled to GR.

$$S_{\mathrm{D}} = -\frac{\mathrm{i}}{2}\int_{\mathcal{M}} \mathrm{d}^4 x e \left(\bar{\psi}\gamma^I e_I{}^\mu \nabla_\mu \psi + \mathrm{c.c.}\right)$$

$$\nabla_\mu \Psi = \partial_\mu \Psi - \frac{1}{4} A_\mu^{IJ} \gamma_I \gamma_J \Psi$$

This is a generalization of the Dirac action on a curved manifold $\mathcal{M}$, e is the determinant of the gravitational field (vielbein, tetrad) $e_I{}^\mu$

$$S_{Tot} = \frac{1}{16\pi G}\left(\int d^4x\, e\, e_I^\mu\, e_J^\nu\, R_{\mu\nu}^{IJ} - \frac{1}{\gamma}\int d^4x\, e\, e_I^\mu\, e_J^\nu\, \tilde{R}_{\mu\nu}^{IJ} + \bar{\psi}\gamma^I e_I{}^\mu \nabla_\mu \psi + \text{c.c.}\right)$$

The equation of motion for the tetrad e_a^μ subject to a fermionic source solved in terms of a connection A_{IJ}^μ having two contributions, a torsion-free spin onnection for e_I^μ (as in the purely gravitational case) and a torsion term related to e axial fermion current.

$$A_\mu^{IJ} = \omega_\mu^{IJ} + C_\mu^{IJ}$$

Christoffel Connection

Contorsion Tensor

Gravitational Part of Action

$$S_H = \frac{1}{2\kappa}\int d^4x\, e\, e_I^\mu e_J^\nu P^{IJ}{}_{KL} F_{\mu\nu}^{KL}$$

Projection Matrix $$P^{IJ}{}_{KL} = \delta^I_{[K}\delta^J_{L]} - \frac{1}{2\gamma}\epsilon^{IJ}{}_{KL}$$

Metric Compatibility $$\frac{\delta S_H}{\delta A_\nu^{KL}} = -\frac{1}{\kappa} D_\mu \left(e\, e_I^{[\mu} e_J^{\nu]}\right) P^{IJ}{}_{KL}$$

Dirac Variation $$\frac{\delta S_D}{\delta A_\nu^{KL}} = -\frac{1}{8} e\bar{\Psi}\{\gamma_{[K}\gamma_{L]}, \gamma^I\} e_I^\nu \Psi$$

$$= \frac{e}{4}\epsilon^I{}_{KLM}(\bar{\Psi}\gamma_5\gamma^M\Psi) e_I^\nu$$

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Four Fermion Contd.

$$D_\mu \left(e\, e_I^{[\mu} e_J^{\nu]} \right) P^{IJ}{}_{KL} = \frac{\kappa e}{4} \epsilon^I{}_{KLM} j_a^M e_I^\nu$$

where j_a^M is the axial current given by $\bar{\Psi}\gamma_5\gamma^M\Psi$.

Writing the connection as $A_\mu^{IJ} = \omega_\mu^{IJ} + C_\mu^{IJ}$

$$C_\mu^{IJ} = \frac{\kappa}{4} \frac{\gamma^2}{\gamma^2+1} j_a^M \left\{ \epsilon_{MK}{}^{IJ} e_\mu^K - \frac{1}{2\gamma} \delta_M^{[J} e_\mu^{I]} \right\}$$

Finally…

- Substituting the Contortion Tensor into the total gravitational action yields:

$$S_{\rm int} = \frac{K}{2}\int d^4x\, e\left(\bar{\psi}\gamma_5\gamma_I\psi\right)\left(\bar{\psi}\gamma_5\gamma^I\psi\right)$$

Perez, Rovelli; Friedel, Minic; S.A

$$K = -3\pi G\,\frac{\gamma^2}{\gamma^2+1}$$

We have the desired weak, four fermion interaction

SURPRISE…Interaction is Attractive!

BCS CONDENSATION

We have to find the effective potential for the condensate.

We use the modern Path Integral approach. Advantage: Covariant

Fierz identity

$$(\bar{\psi}\gamma_5\gamma^I\psi)(\bar{\psi}\gamma_5\gamma_I\psi) = (\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5\psi)^2 + (\bar{\psi}\gamma^I\psi)(\bar{\psi}\gamma_I\psi)$$

$$S_{\text{int}} = \int d^4x\, e \left[\frac{(\bar{\psi}\psi)^2}{M^2}\right] = \int d^4x\, e \left[(\bar{\psi}\psi)\Delta - \frac{M^2}{4}\Delta^2\right] \equiv S_{\text{mass}} + S_{\text{tree}}$$

Δ Mediates the Four Fermion Interaction (Homogenous Torsion)

$$S_{\rm mass} = \int d^4x\, e(\bar\psi\psi)\Delta = \int d^4x\, a^3(\varepsilon^{\alpha\beta}\zeta_\beta\xi_\alpha + \varepsilon^{\dot\alpha\dot\beta}\xi^\dagger_{\dot\alpha}\zeta^\dagger_{\dot\beta})\Delta$$

t is clear that a non-zero value for the auxiliary field $\Delta \sim \bar\psi\psi$ would signal a (cosmological) BCS-like condensation.

$$\begin{aligned} S_{\rm fer} &\equiv S_{\rm D} + S_{\rm mass} \\ &= (2\pi)^4 \int d^4p \Big[\omega\xi^\dagger_{\mathbf{p},\omega}\xi_{\mathbf{p},\omega} - \xi^\dagger_{\mathbf{p},\omega}\bar\sigma^i p_i \xi_{\mathbf{p},\omega} + \omega\zeta_{-\mathbf{p},-\omega}\zeta^\dagger_{-\mathbf{p},-\omega} - \zeta_{-\mathbf{p},-\omega}\sigma^i p_i \zeta^\dagger_{-\mathbf{p},-\omega} \end{aligned}$$

Which can be rewritten in 4X4 matrix form

$$S_{\rm fer} = (2\pi)^4 \int d^4p\, (\xi^\dagger_{\mathbf{p},\omega}, \zeta_{-\mathbf{p},-\omega}) A_p \begin{pmatrix} \xi_{\mathbf{p},\omega} \\ \zeta^\dagger_{-\mathbf{p},-\omega} \end{pmatrix}$$

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where A_p is a 4×4 matrix given by

$$A_p = \begin{pmatrix} \omega - \bar{\sigma}^i p_i + \mu & \Delta \\ \Delta & \omega - \sigma^i p_i - \mu \end{pmatrix}$$

At this point we introduce a chemical potential μ in the action.

COMPUTE THE EFFECTIVE POTENTIAL

$$Z = \int [\mathcal{D}\Delta][\mathcal{D}\xi][\mathcal{D}\zeta]\mathrm{e}^{\mathrm{i}(S_{\mathrm{fer}}+S_{\mathrm{tree}})} \equiv \int [\mathcal{D}\Delta]\mathrm{e}^{\mathrm{i}S_{\mathrm{eff}}} \approx \mathrm{e}^{\mathrm{i}S_{\mathrm{eff}}}\Big|_{\mathrm{SP}}$$

$$S_{\mathrm{eff}} = S_{\mathrm{tree}} - \mathrm{i}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\ln(\det A_p)$$

Don't forget to renormalize...

After tedious calculations:

$$\rho_{\mathrm{gap}} = V_{\mathrm{min}} + \mu n$$ Energy dependent on number density and chem pot

$$= \frac{\Delta^2}{16\pi^2}\left[\Delta^2\left(N + \frac{3}{2} + \ln\Delta^2\right) - 4\mu^2(2N + 3 + 2\ln\Delta^2)\right]$$

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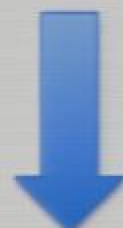

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Dynamical Equations

$$H^2 = \frac{8\pi}{3}(\rho_{\text{gap}} + \rho_{\text{m}})$$

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$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$

$\epsilon > 1$ corresponds to a decelerating universe, while $\epsilon < 1$ signals acceleration, if the universe expands.

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$$e^{-N/2} F(\gamma) M_{pl} = M^*$$

$$M^{*3} \Delta^{-2}$$

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Analytic Treatment: Early Times

$I \ll \mu, \Delta$ and $N \gg 1$ (later we motivate the last condition phenomenologically).
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$$\frac{\Delta^3}{2\sqrt{2}\pi^2}\left(N + 1 + \ln\Delta^2\right) = \frac{n_0}{a^3} \qquad \Delta \sim \frac{1}{a} \qquad \Rightarrow \qquad \rho_{\text{gap}} \sim -\frac{1}{a^4}$$

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COMPUTE THE EFFECTIVE POTENTIAL

$$Z = \int [\mathcal{D}\Delta][\mathcal{D}\xi][\mathcal{D}\zeta]\mathrm{e}^{\mathrm{i}(S_{\mathrm{fer}}+S_{\mathrm{tree}})} \equiv \int [\mathcal{D}\Delta]\mathrm{e}^{\mathrm{i}S_{\mathrm{eff}}} \approx \mathrm{e}^{\mathrm{i}S_{\mathrm{eff}}}\Big|_{\mathrm{SP}}$$

$$S_{\mathrm{eff}} = S_{\mathrm{tree}} - \mathrm{i}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\ln(\det A_p)$$

Don't forget to renormalize...

After tedious calculations:

$$\rho_{\mathrm{gap}} = V_{\mathrm{min}} + \mu n$$ Energy dependent on number density and chem pot

$$= \frac{\Delta^2}{16\pi^2}\left[\Delta^2\left(N + \frac{3}{2} + \ln\Delta^2\right) - 4\mu^2(2N + 3 + 2\ln\Delta^2)\right]$$

$$M^2 = \Delta^2(N+1) - 2\mu^2(N+2) + (\Delta^2 - 2\mu^2)\ln\Delta^2$$

$$e^{-N/2} F(\gamma) M_{pl} = M^* \qquad \Delta \ll \mu$$

$$M^{*2}\,\Delta^2$$

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$$\Delta << \mu$$

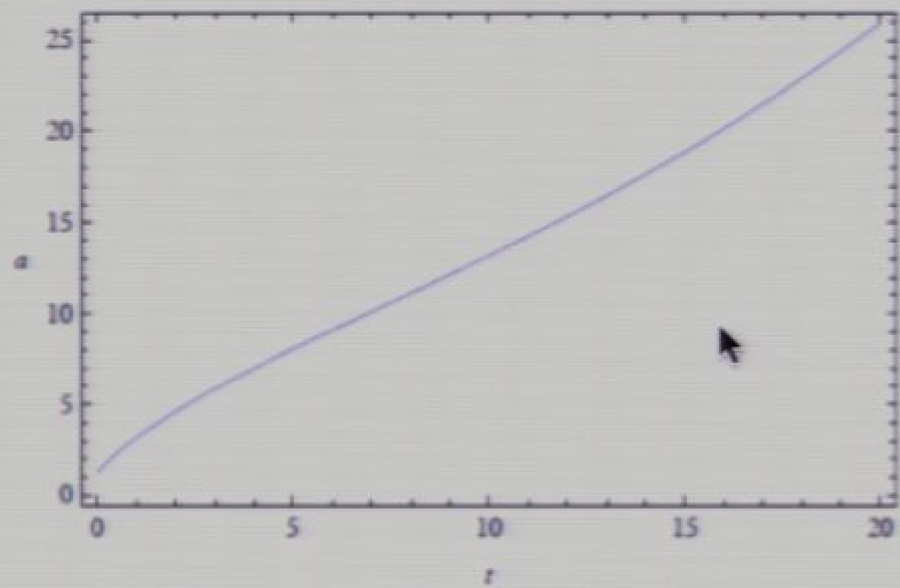
$$\Delta = e^{-\frac{M}{\mu}}$$

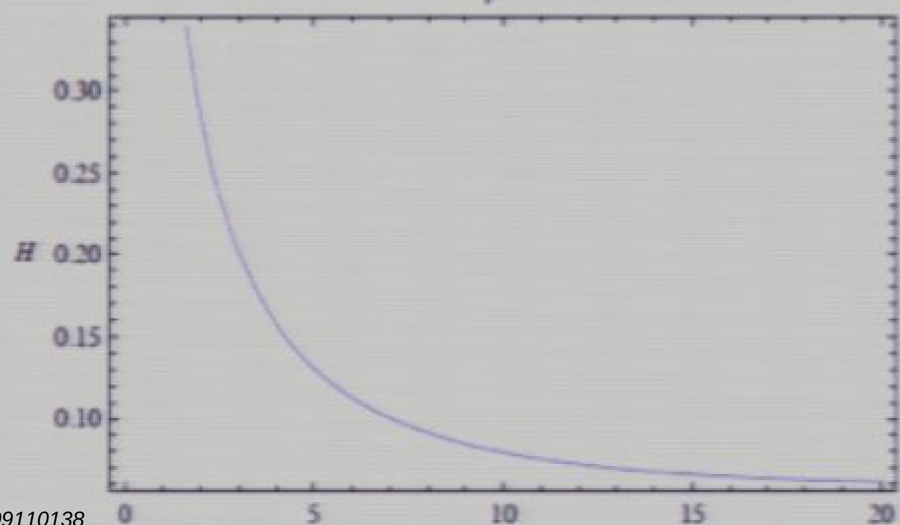
$$M^{*2} \Delta^2$$

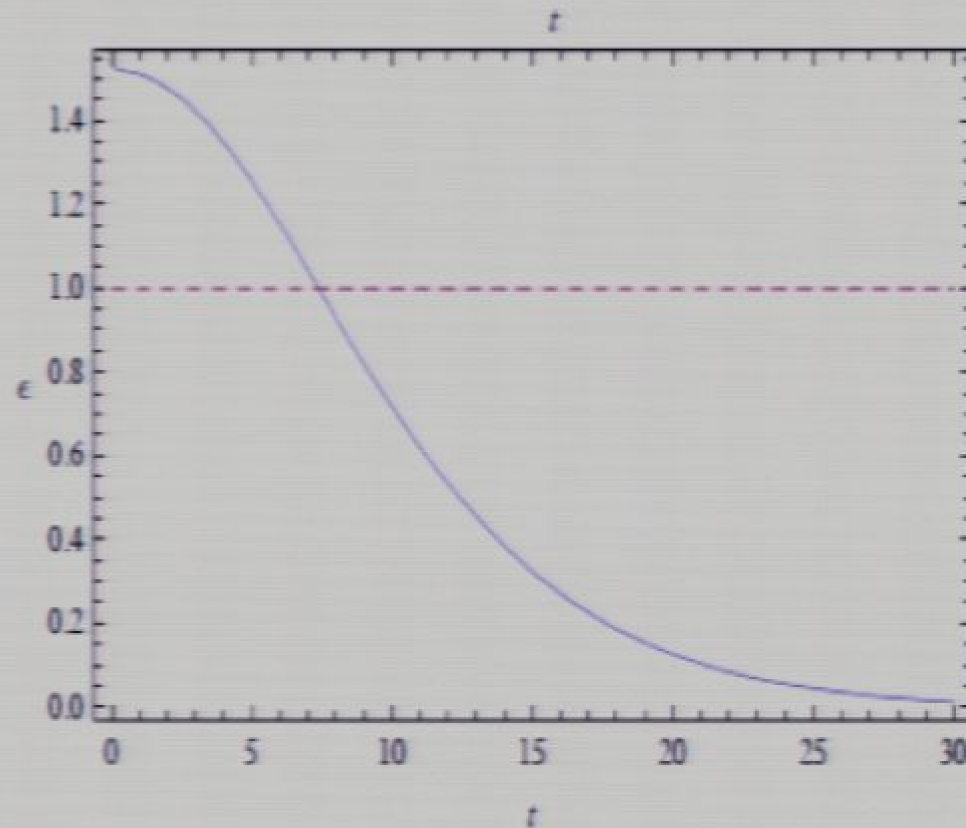
$$e \;\; f(\gamma) M_{pl} =$$

$$M^2 = C\Delta^2$$

Late Times: Do We Get Acceleration?

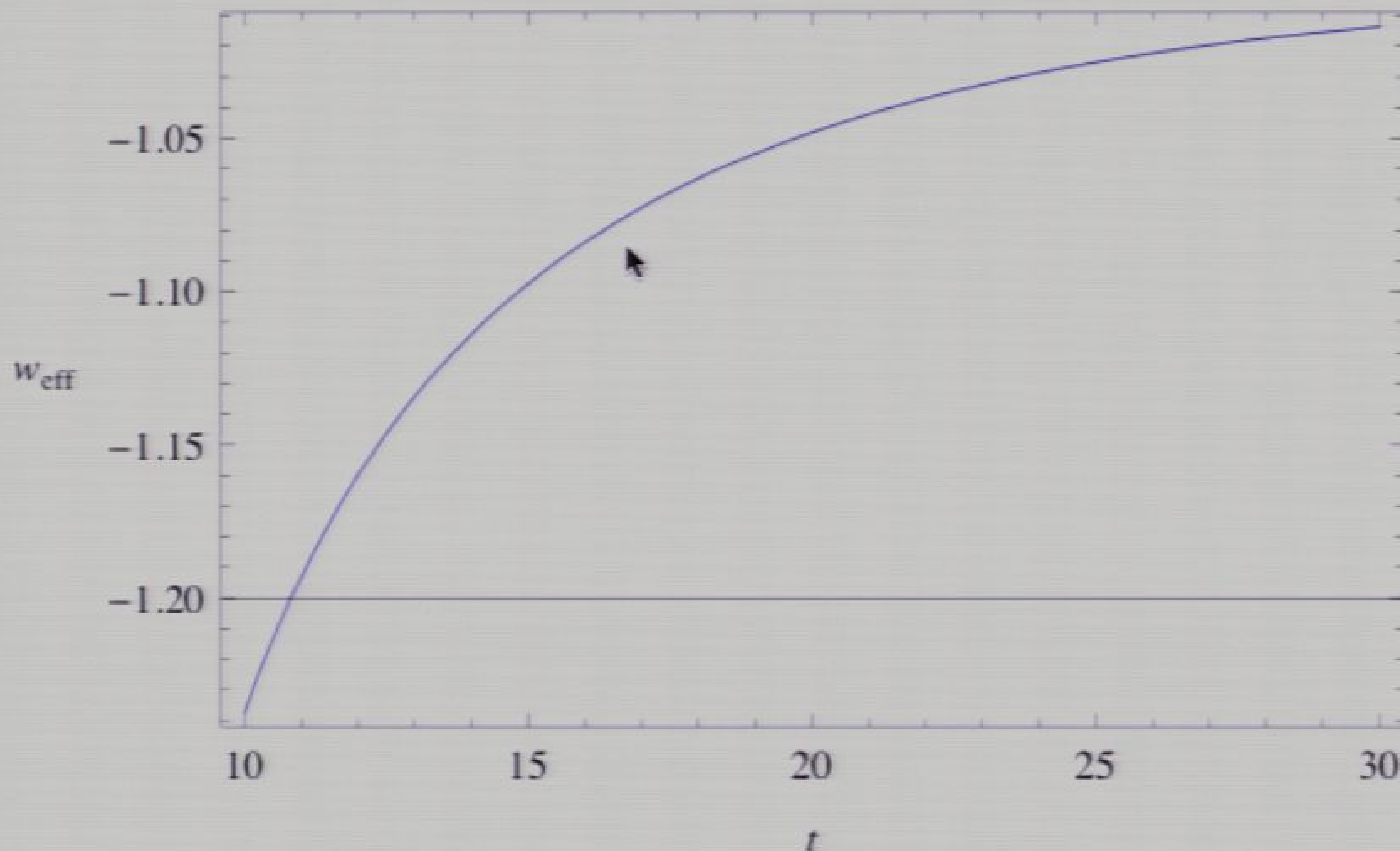






$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$

Equation of state parameter



Neutrino Oscillations

Resolve metric compatibility for 3-flavors

$$e^{\mu}_{I} C_{\mu JK} = \sum_{B} 4\pi G \frac{\gamma^2}{\gamma^2+1}\left(\frac{1}{2}\epsilon_{IJKL} J^{L}_{5B} - \frac{1}{\gamma}\eta_{I[J} J_{B5K]}\right)$$

$$\mathcal{L}_{\rm int} = \frac{1}{\rm M^2_{Pl}} \int \rm d^4x \, e \, \bar{\psi}_A \gamma_5 \gamma^I \psi_A \bar{\psi}_B \gamma_5 \gamma_I \psi_B$$

Flavors will get reshuffled due to a Fierz transformation:

$$\begin{aligned} \mathcal{O}_{\psi} &= \bar{\psi}_A \gamma_5 \gamma^I \psi_A \bar{\psi}_B \gamma_5 \gamma_I \psi_B = \bar{\psi}_A \psi_B \bar{\psi}_A \psi_B \\ &+ \bar{\psi}_A \gamma_5 \psi_B \bar{\psi}_A \gamma_5 \psi_B + \bar{\psi}_A \gamma^I \psi_B \bar{\psi}_A \gamma_I \psi_B \end{aligned}$$

Neutrino mass from condensate interaction

$$\mathcal{L} = \mathrm{i}\bar{\nu}_{\mathrm{A}}\partial_{\mu}\gamma^{\mu}\nu_{\mathrm{A}} + \bar{\nu}_{\mathrm{A}}\gamma^{\mu}\nu_{\mathrm{B}}\Delta_{\mu\mathrm{AB}} + (1+\gamma_5)\Delta_{\mathrm{AB}}\bar{\nu_{\mathrm{A}}}\nu_{\mathrm{B}}.$$

$$(1+\gamma_5)\Delta = m_{\Delta}$$

Field Equations:

$$(i\partial_{\mu} - \lambda_{AB}\Delta_{\mu})[\bar{\sigma}^{\mu}]^{\dot{a}a}\nu_{aB} - m^{*}_{\Delta}\bar{\nu}^{\dot{a}}_{B} = 0$$

$$(i\partial_{\mu} + \lambda_{AB}\Delta_{\mu})[\bar{\sigma}^{\mu}]_{\dot{a}a}\bar{\nu}^{\dot{a}B} - m_{\Delta}\nu_{aB} = 0$$

Neutrino Oscillations

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An ultrarelativistic neutrino of mass m, traveling a distance L along the z-direction will have a phase factor $p_\mu x^\mu = Et - \mathbf{px} \simeq (E - p_z)L$. We can simplify matters by realizing that, $E - p = \frac{(E^2 - p^2)}{E+p} \simeq \frac{m^2}{2E}$ and $E \simeq |p|$, leading to:

$$\nu(t,x)_e = cos\theta e^{im_1^2 L/2E} \nu_1 + sin\theta e^{-im_2^2 L/2E} \nu_2 \, .$$

Transform Dirac eq into flavor basis by Unitary transformation U

Neutrino Oscillations

- Consider two flavors

$$|\nu_e\rangle = cos\theta|\nu_1\rangle + sin\theta|\nu_2\rangle$$

In general flavor eigenstate is related to mass eigenstate by:

$$|\nu\rangle_A = U_{\alpha i}|\nu\rangle_i$$

If a neutrino propagates as a plane wave, then its time development is

$$\nu(t,x)_e = cos\theta e^{-ip_{1\mu}x^\mu}\nu_1 + sin\theta e^{-ip_{2\mu}x^\mu}\nu_2\,.$$

Final Result

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix} = \left[\frac{\Delta m^2_\Delta}{2E_\nu}\begin{pmatrix} sin^2\theta & cos\theta sin\theta \\ cos\theta sin\theta & cos^2\theta\end{pmatrix} + \begin{pmatrix}(\Delta + \sqrt{2}G_F N_e) & \Delta_{ex}/2 \\ \Delta^*_{ex}/2 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}$$

New Pediction

(see Ando, Mocociu, Kamionkowski)

OSCILLATION PROBABILITY

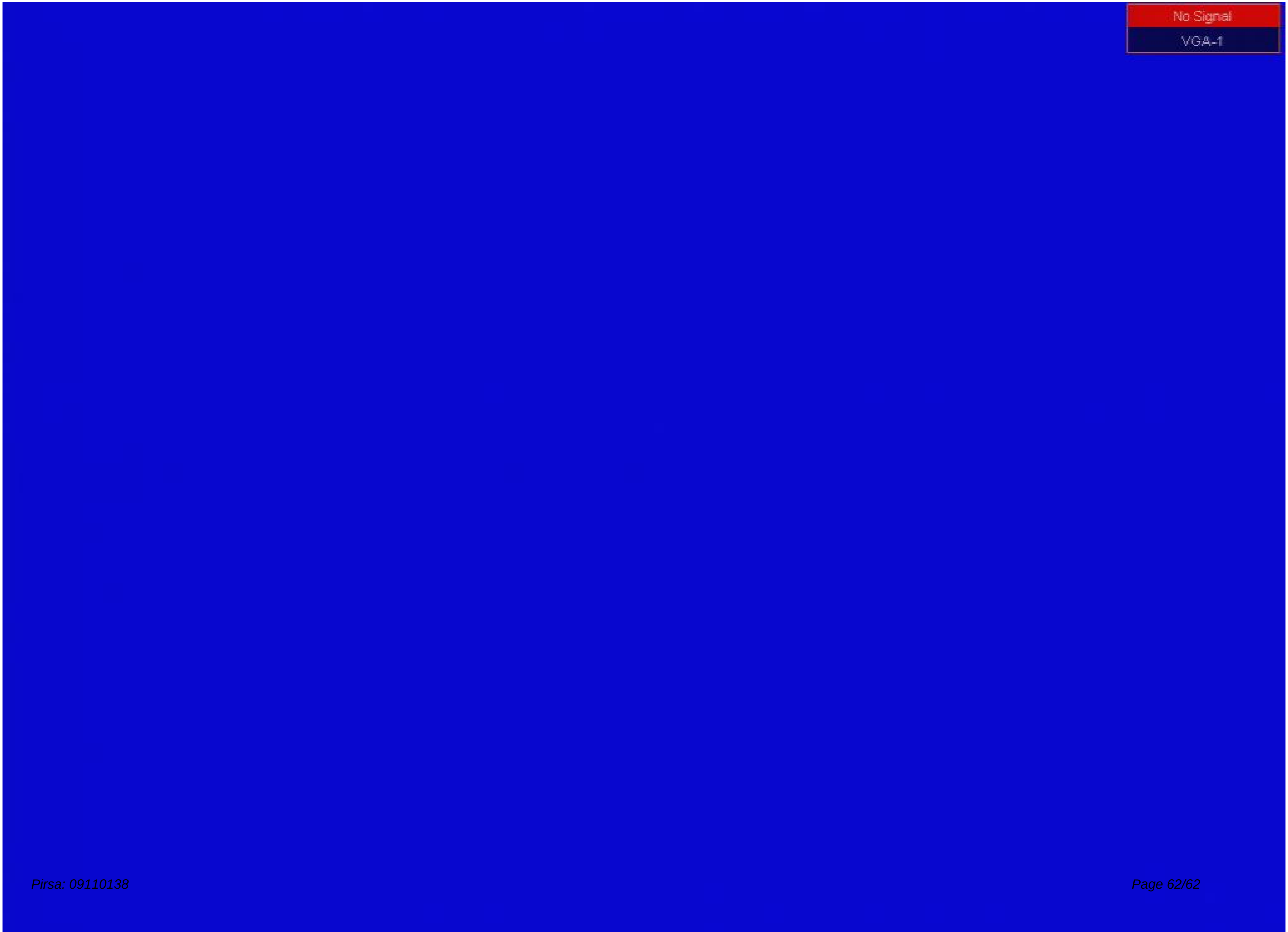
$$P_{ex} = P_{xe}\sin^2 2\theta_{\rm eff}\sin^2\left(\frac{\Delta_{\rm eff}}{2}L\right)$$

$$\Delta_{\rm eff} = \sqrt{(\tilde{\Delta} sin2\theta + \Delta_{ex})^2 + (\tilde{\Delta}\cos 2\theta - \Delta)^2}$$

$$\tilde{\Delta} \equiv \Delta m^2_\Delta/2E_\nu \longleftrightarrow 1 + \omega(t)_{\rm eff} = -\frac{\ln\rho(\Delta(t))_{\rm gap}}{\ln a(t)}$$

Conclusion

- The Dark Energy Problem requires new physics
- We propose gravitationally induced BCS condensate is the answer.
- The bonus is a singularity-free cosmology.
- If condensate is neutrino, get a connection between oscillations and scale of DE.
- On Cosmology side we need to look at effect of condensate on CMB (Polarization?, Lensing? Supernovae?)
- We still have to confront this mechanism with the main culprit, Inflation (work in progress S.A., A. Kosowsky)


No Signal
VGA-1