Title: Quantum Field Theory II (PHYS 603) - Special Lecture

Date: Nov 13, 2009 02:30 PM

URL: http://pirsa.org/09110135

Abstract:

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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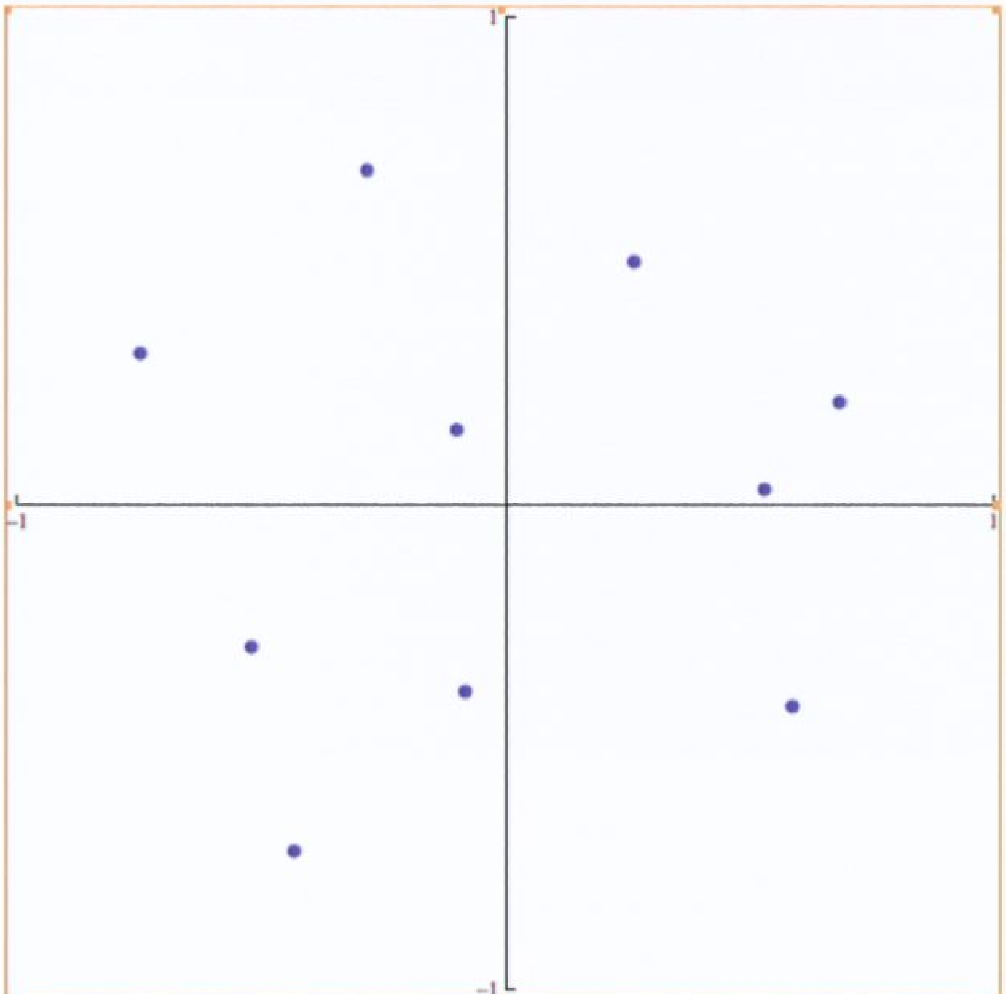
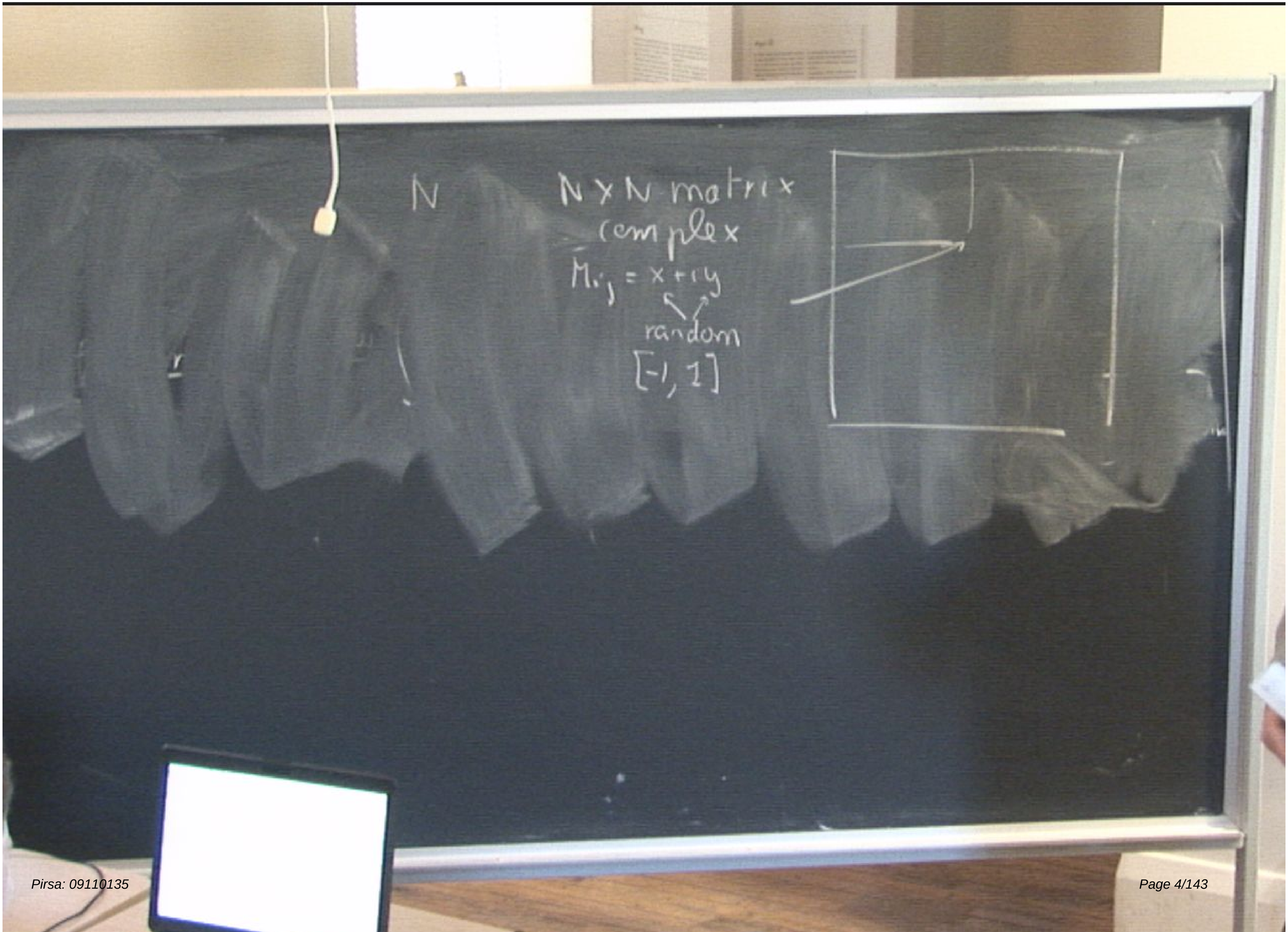
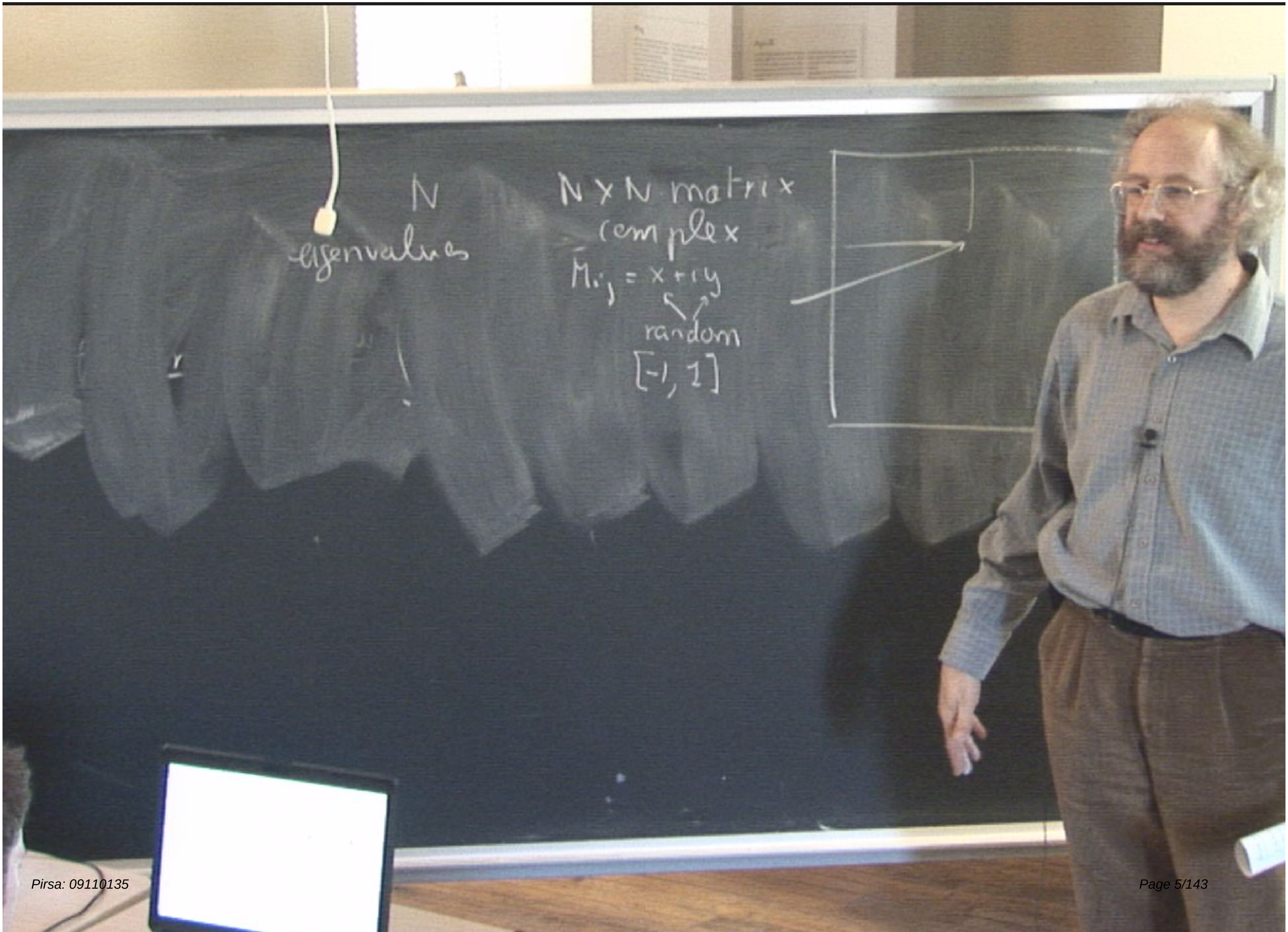
Random matrix PSI.nb

■ 1 random complex matrix with entries z=x+i y such that x & y in [-1, 1]

```
size = 10;
matrix = RandomComplex[{(-1 - I) / Sqrt[size], (1 + I) / Sqrt[size]}, {size, size}];
ev = Eigenvalues[matrix];
plot = ListPlot[Transpose[{Re[ev], Im[ev]}], PlotRange → {{-1, 1}, {-1, 1}}, AspectRatio → 1,
   PlotStyle → PointSize[Large], Ticks → {{-1, 0, 1}, {-1, 0, 1}}];
Show[plot]
```




N
N x N matrix
complex
Mij = x + iy
random
[-1, 1]


N
eigenvalues
N x N matrix
complex
M_ij = x + iy
random
[-1, 1]

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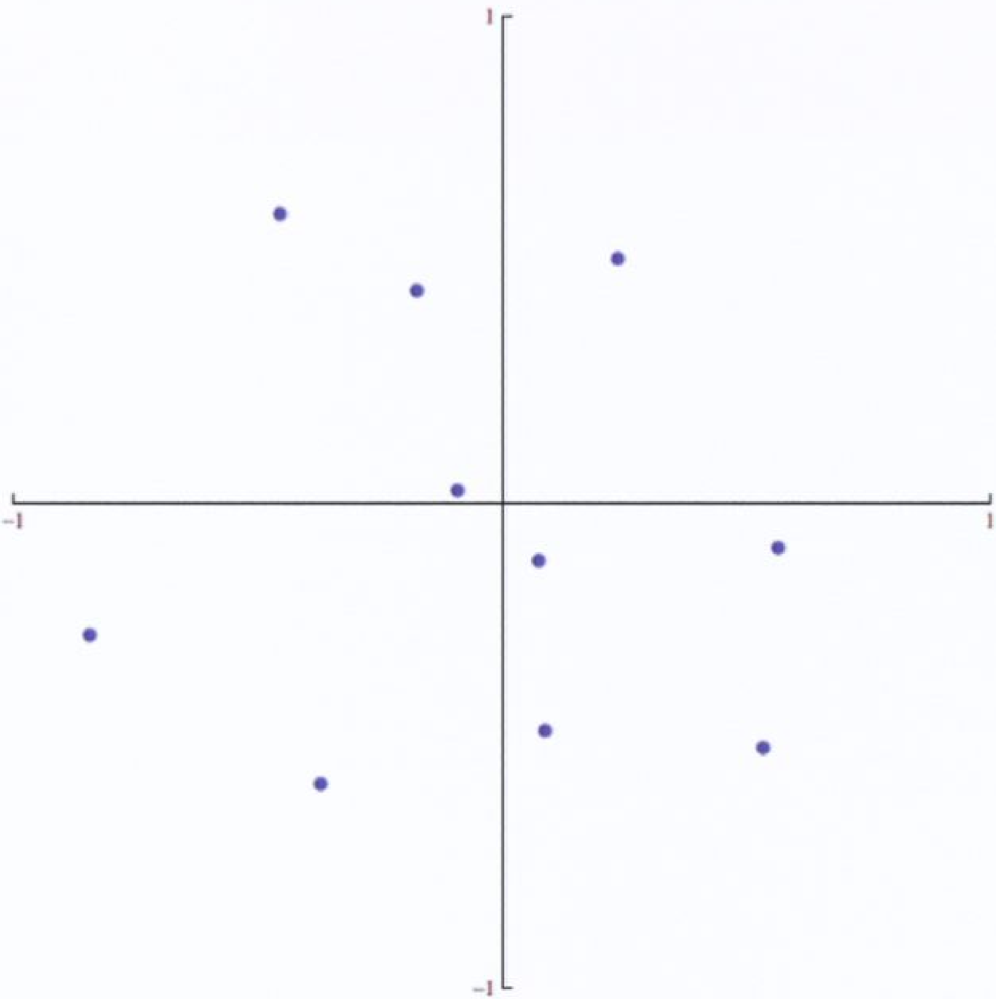
Random matrix PSI.nb

```
size = 10;
matrix = RandomComplex[{(-1 - I) / Sqrt[size], (1 + I) / Sqrt[size]}, {size, size}];
ev = Eigenvalues[matrix];
plot = ListPlot[Transpose[{Re[ev], Im[ev]}], PlotRange → {{-1, 1}, {-1, 1}}, AspectRatio → 1,
   PlotStyle → PointSize[Large], Ticks → {{-1, 0, 1}, {-1, 0, 1}}];
Show[plot]
```



/ishart matrices : eigenvalues distribution

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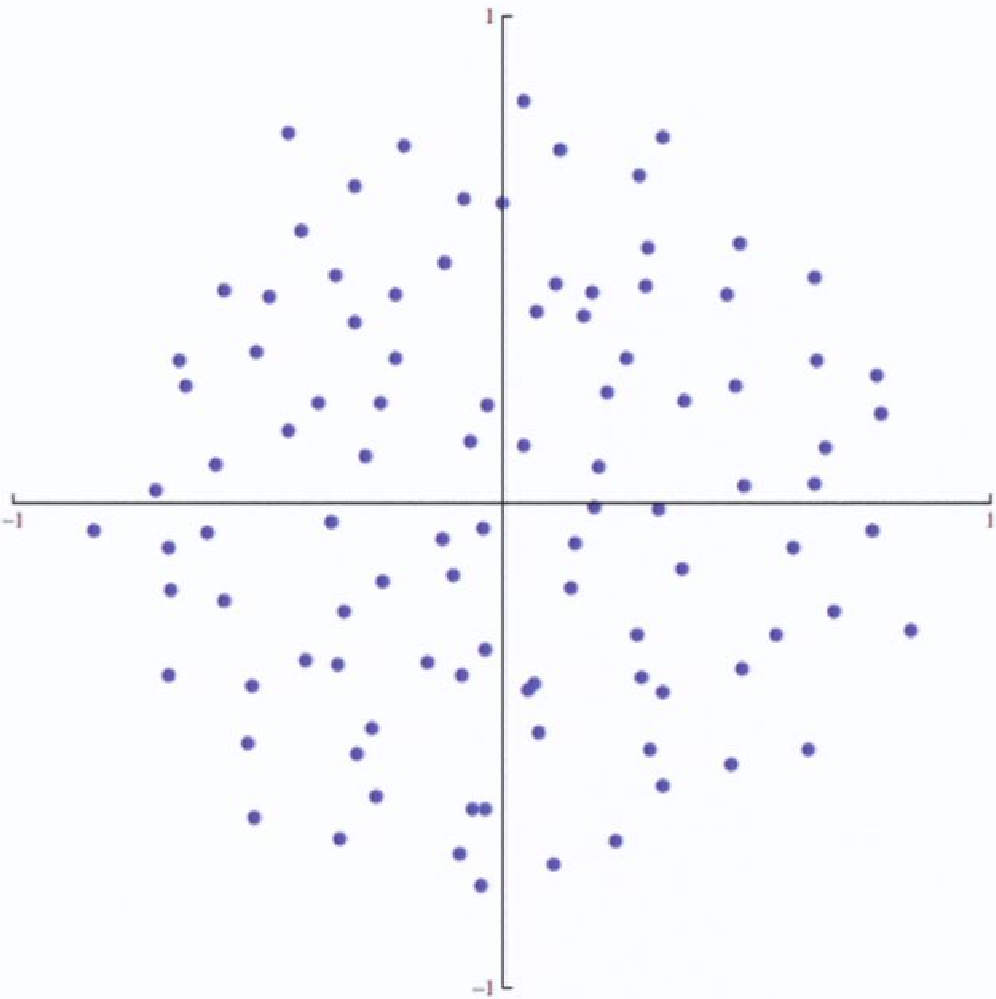
Random matrix PSI.nb

```
size = 100;
matrix = RandomComplex[{(-1 - I) / Sqrt[size], (1 + I) / Sqrt[size]}, {size, size}];
ev = Eigenvalues[matrix];
plot = ListPlot[Transpose[{Re[ev], Im[ev]}], PlotRange → {{-1, 1}, {-1, 1}}, AspectRatio → 1,
   PlotStyle → PointSize[Large], Ticks → {{-1, 0, 1}, {-1, 0, 1}}];
Show[plot]
```



Wishart matrices : eigenvalues distribution

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Random matrix PSI.nb

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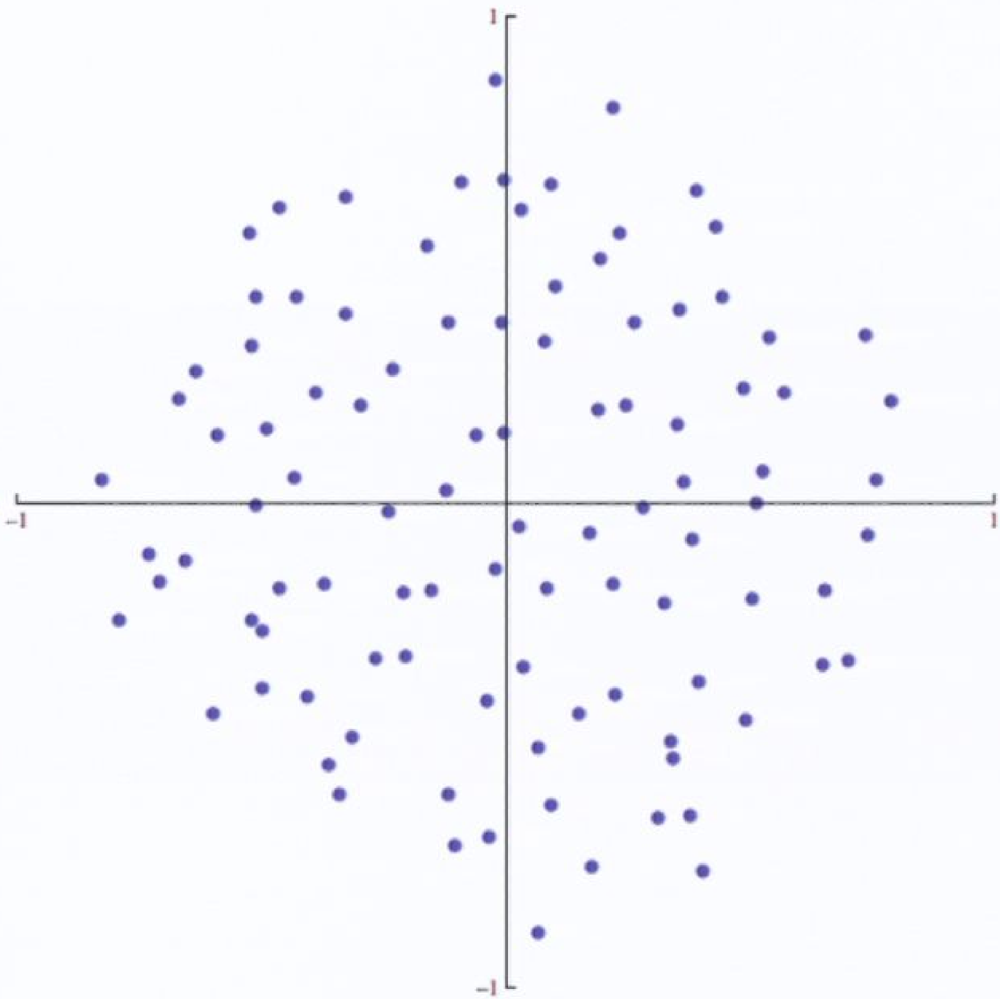
```



Vishart matrices : eigenvalues distribution

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Random matrix PSI.nb

...mbre circular ensemble : random complex matrices

- 1 random complex matrix with entries z=x+i y such that x & y in [-1, 1]

```
size = 100;
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plot = ListPlot[Transpose[{Re[ev], Im[ev]}], PlotRange → {{-1, 1}, {-1, 1}}, AspectRatio → 1,
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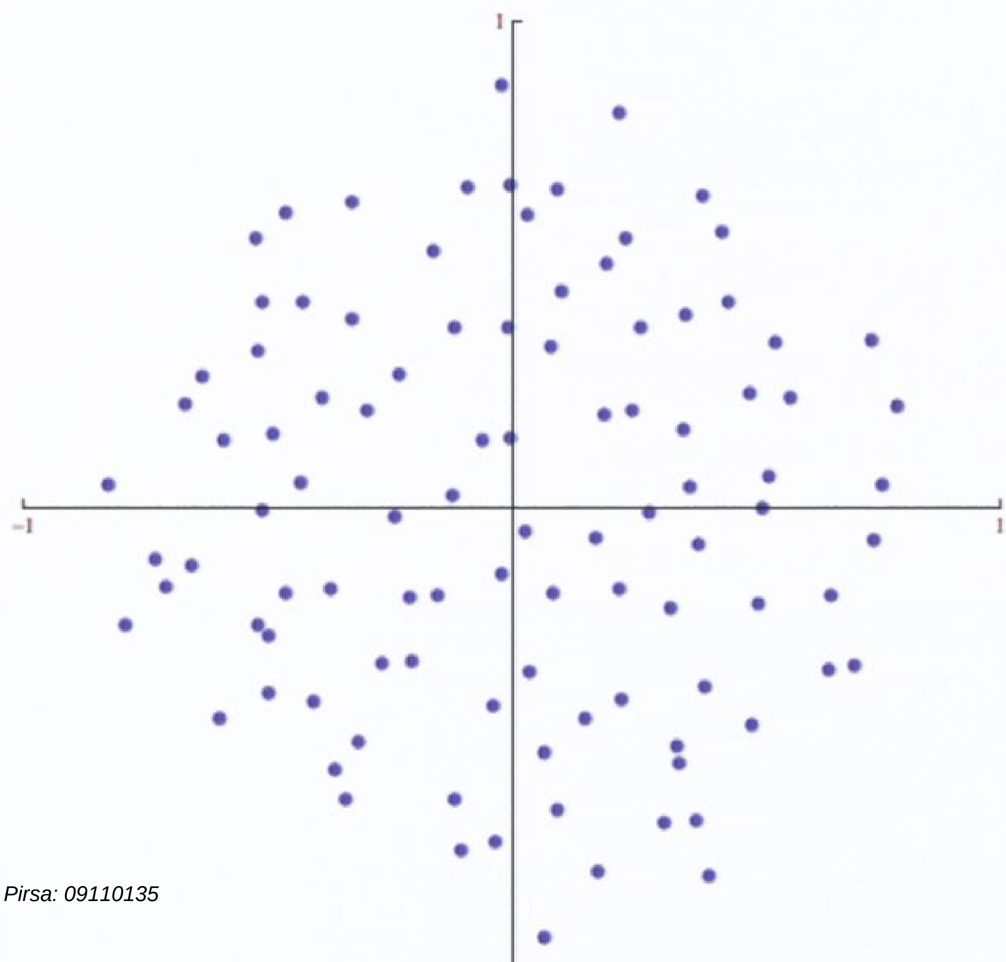
Show[plot]
```

38]=



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Running...Random matrix PSI.nb

...mble circular ensemble : random complex matrices

- 1 random complex matrix with entries z=x+i y such that x & y in [-1, 1]

```
size = 1000;
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ev = Eigenvalues[matrix];
plot = ListPlot[Transpose[{Re[ev], Im[ev]}], PlotRange → {{-1, 1}, {-1, 1}}, AspectRatio → 1,
   PlotStyle → PointSize[Large], Ticks → {{-1, 0, 1}, {-1, 0, 1}}];
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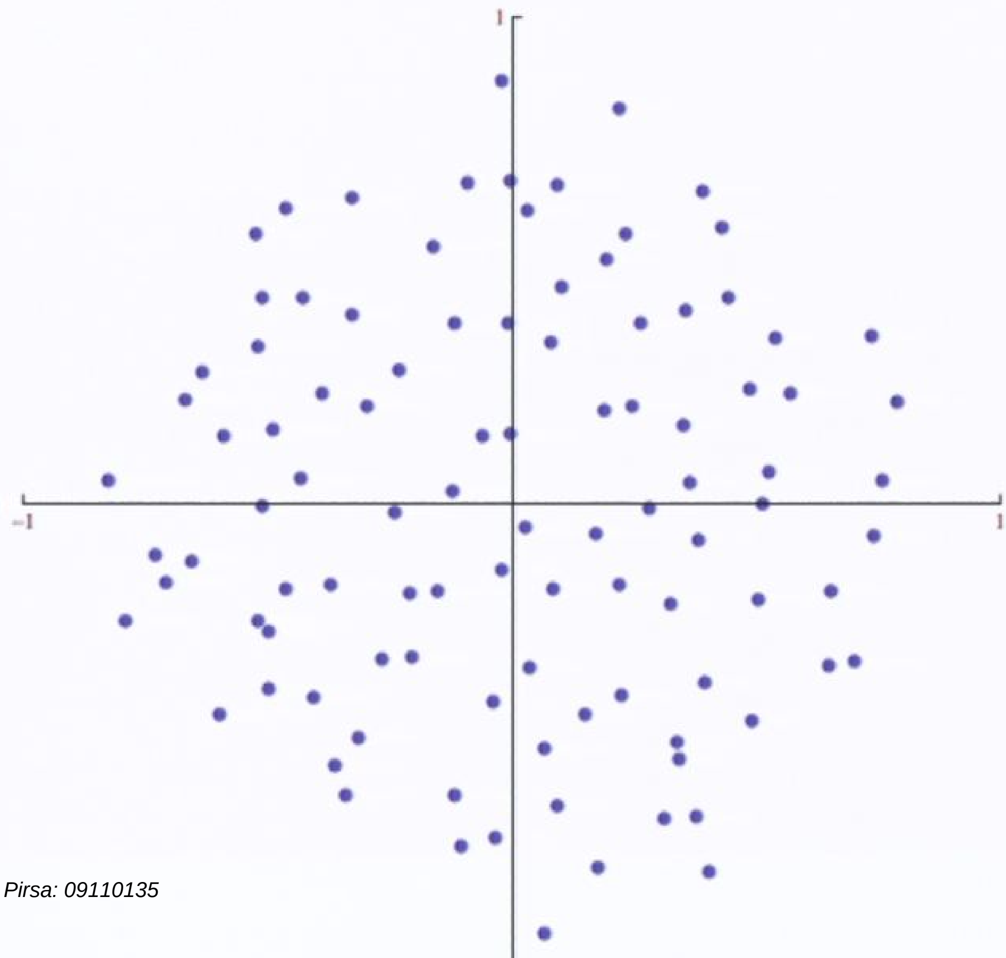
```



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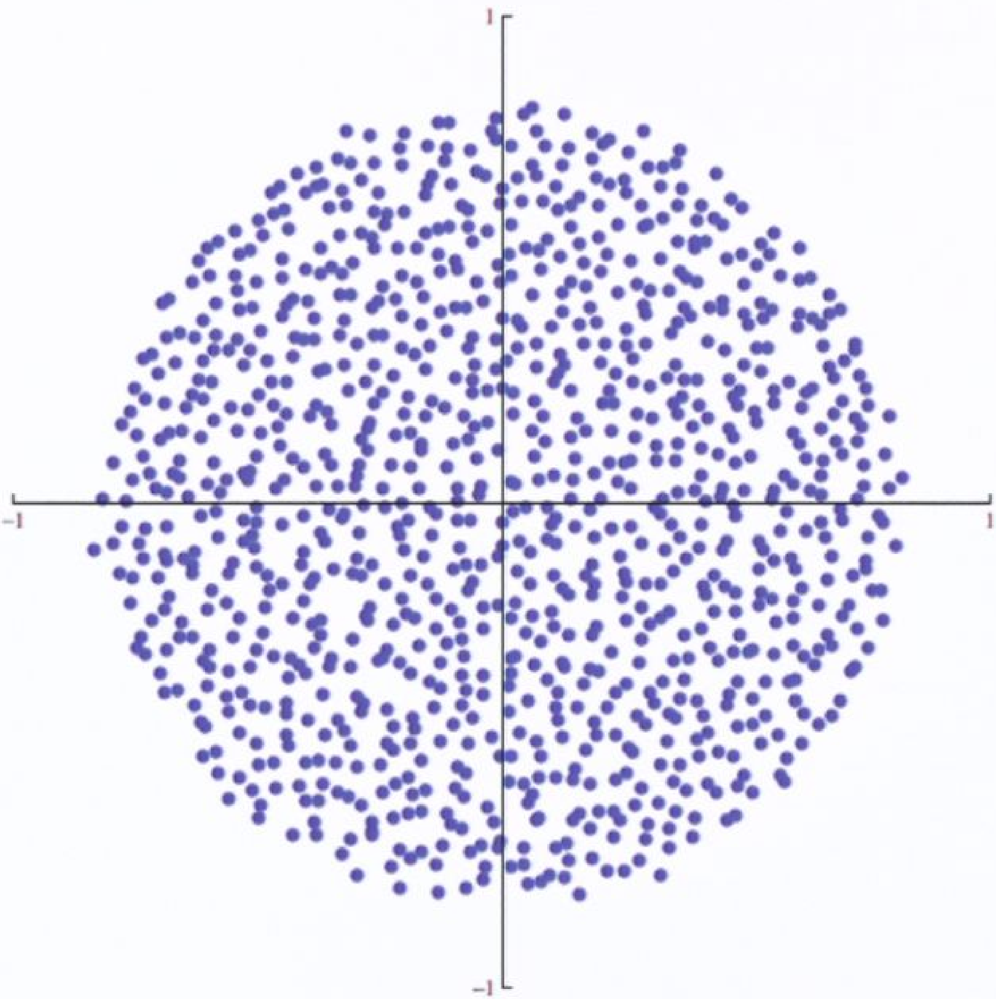

/ishart matrices : eigenvalues distribution

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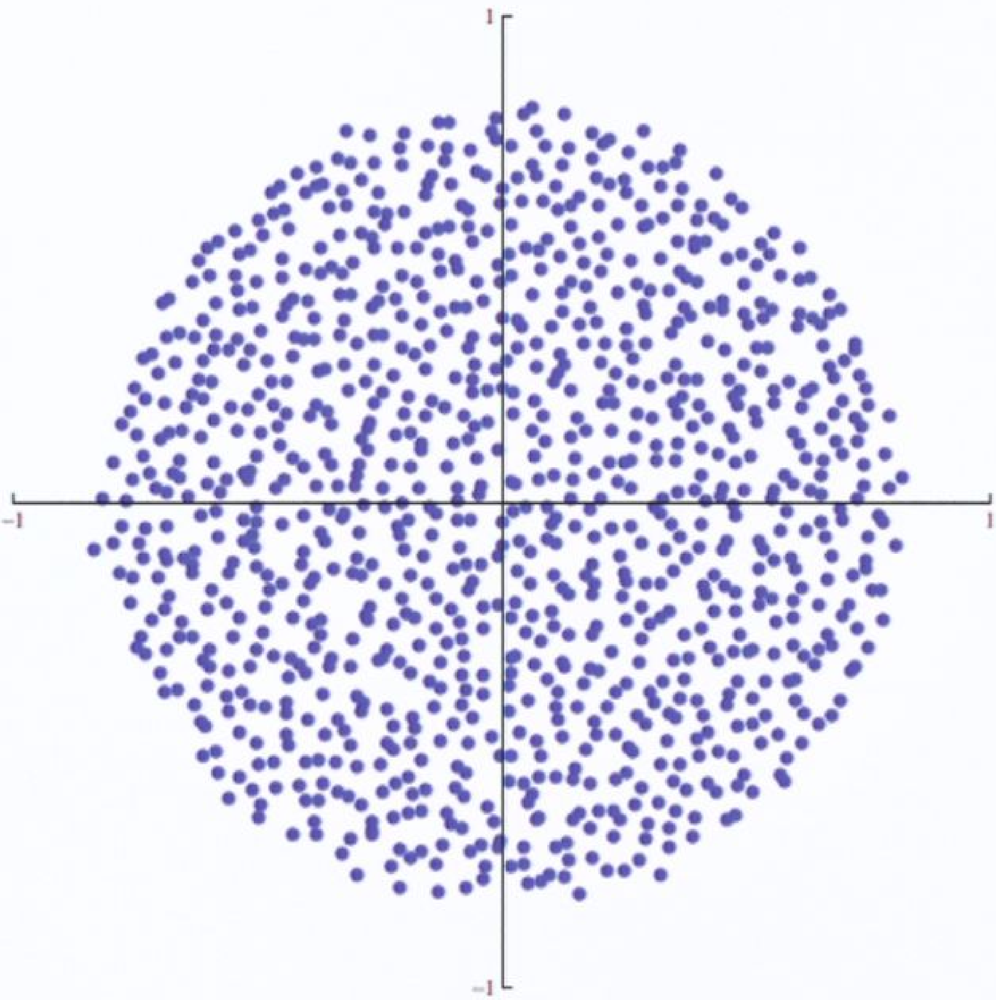
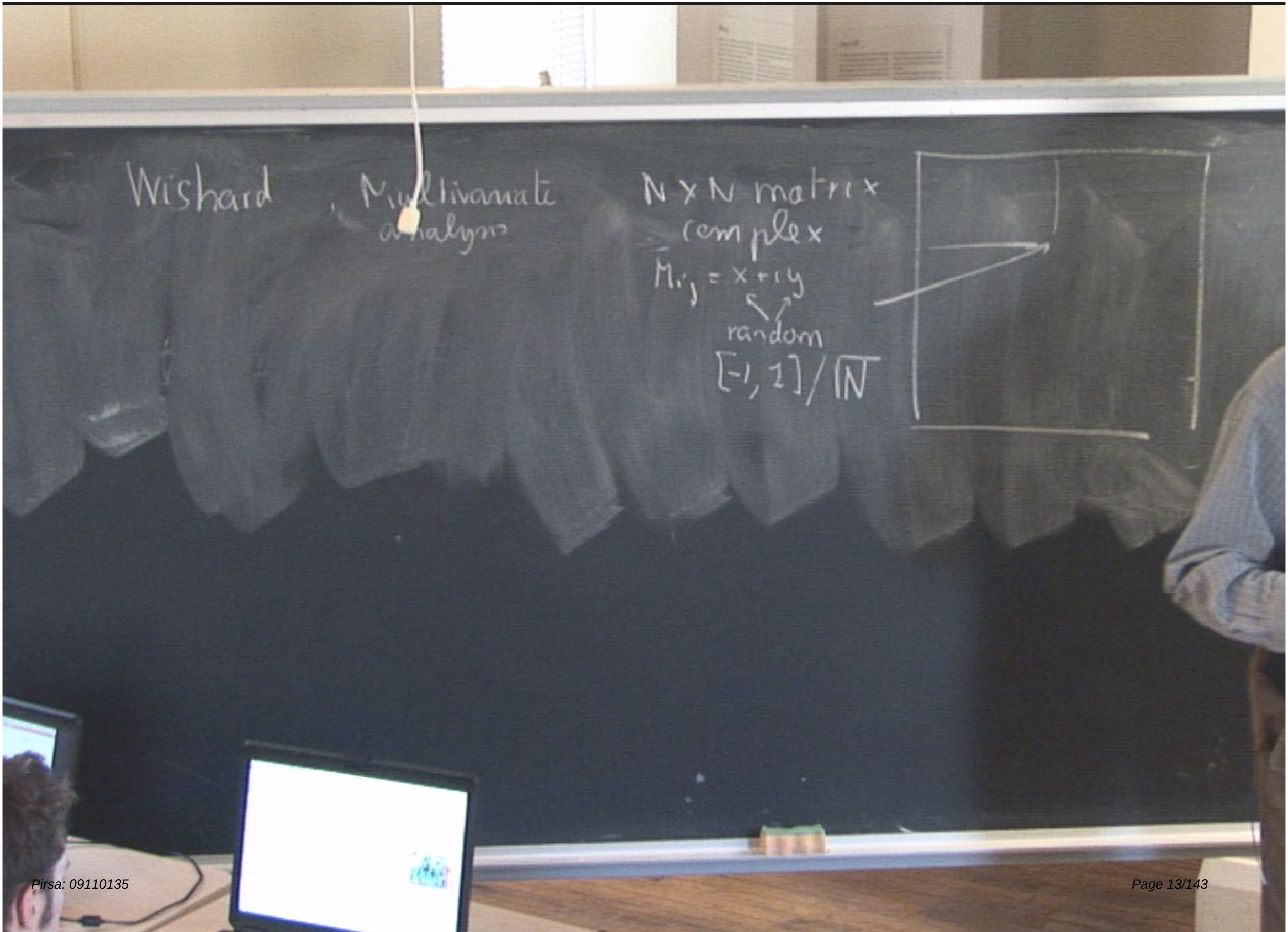
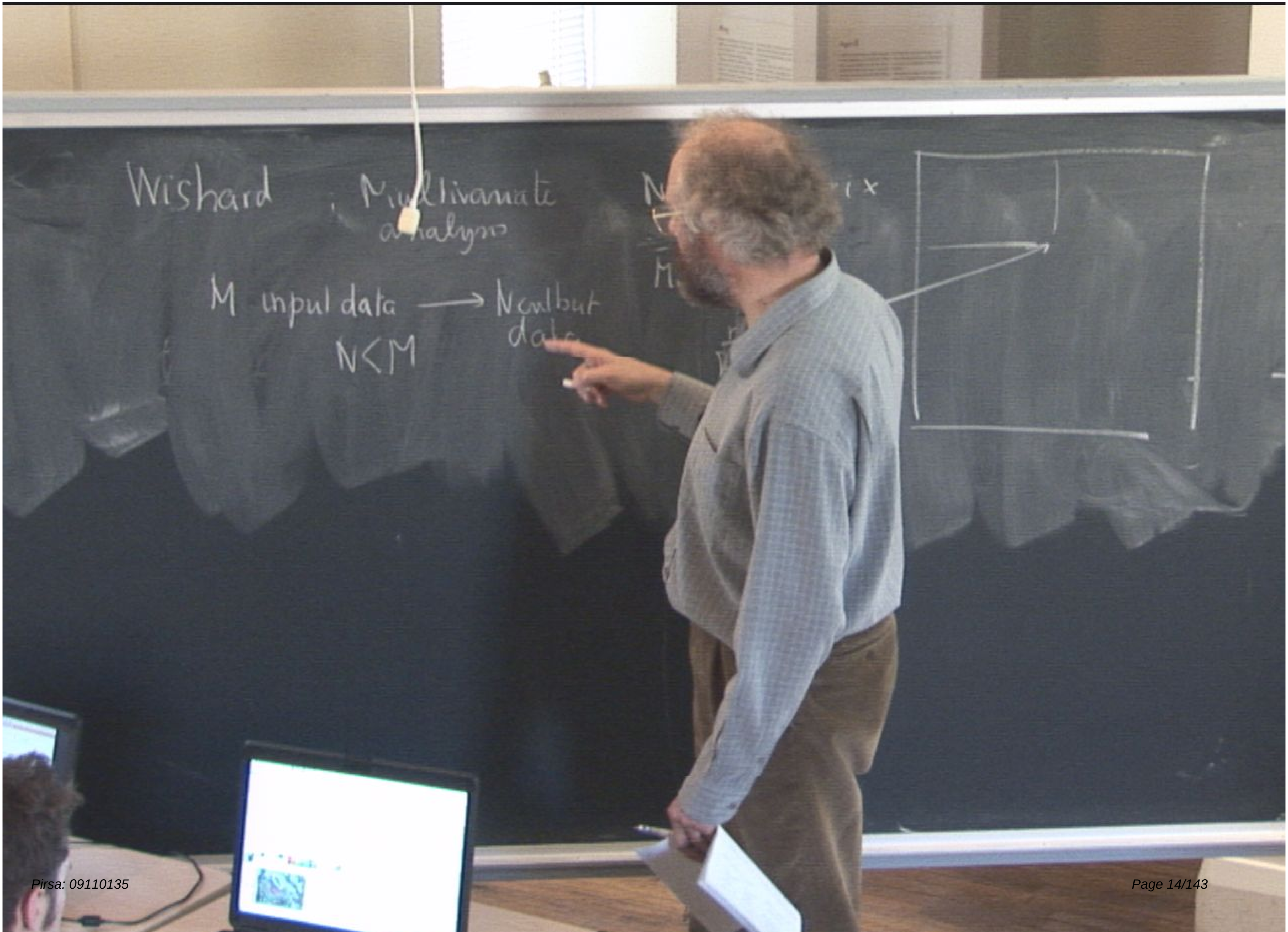
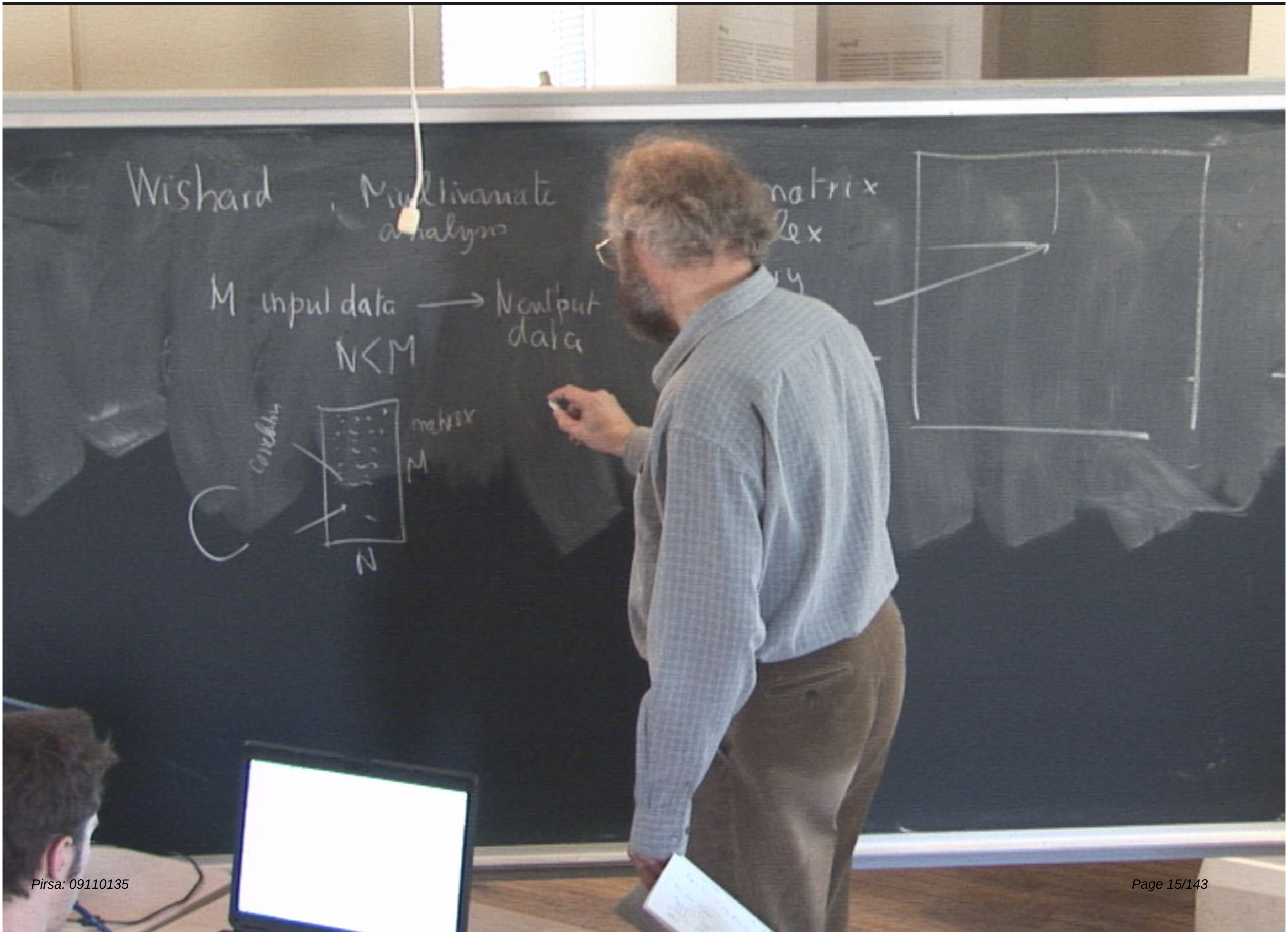
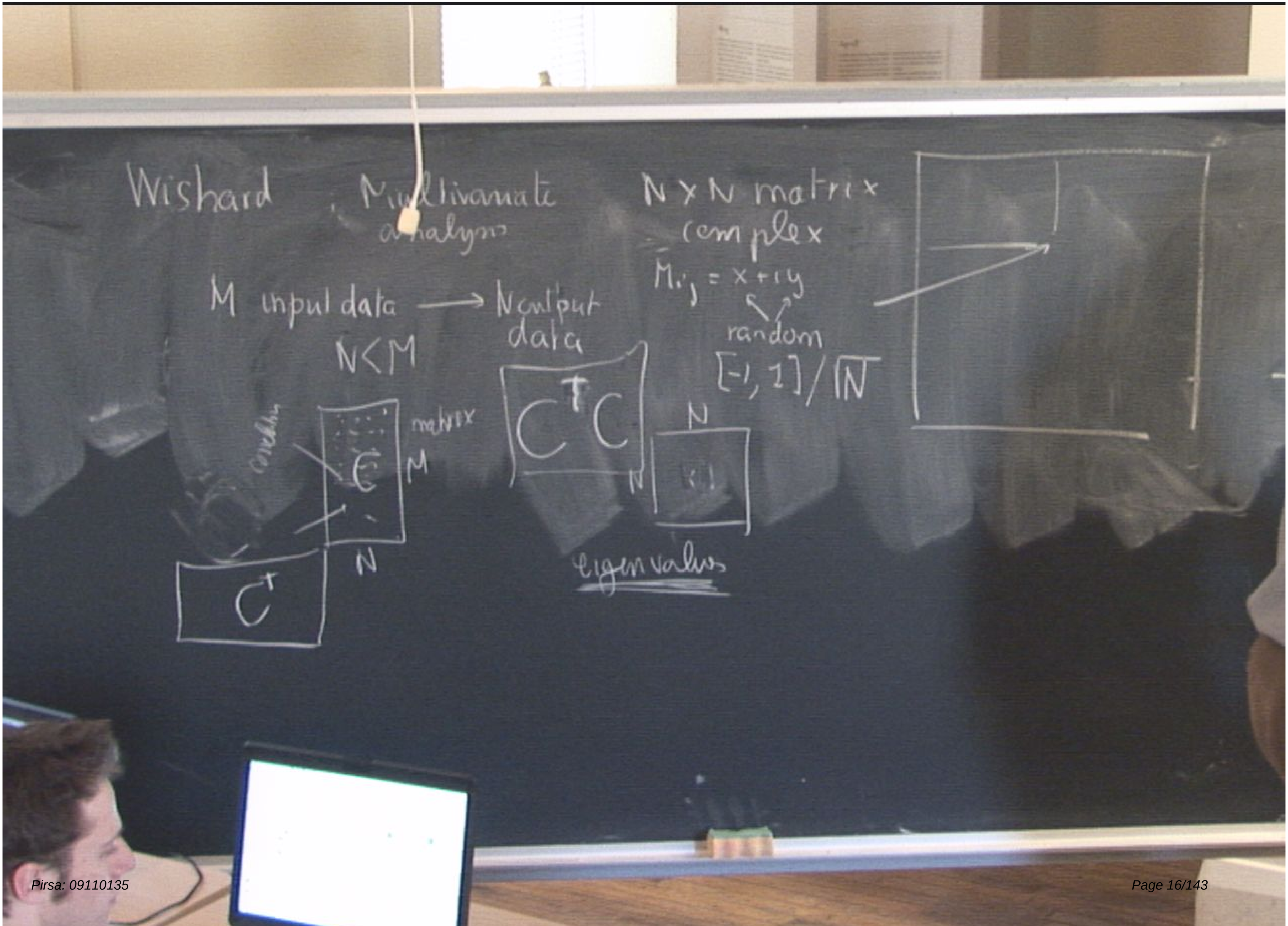


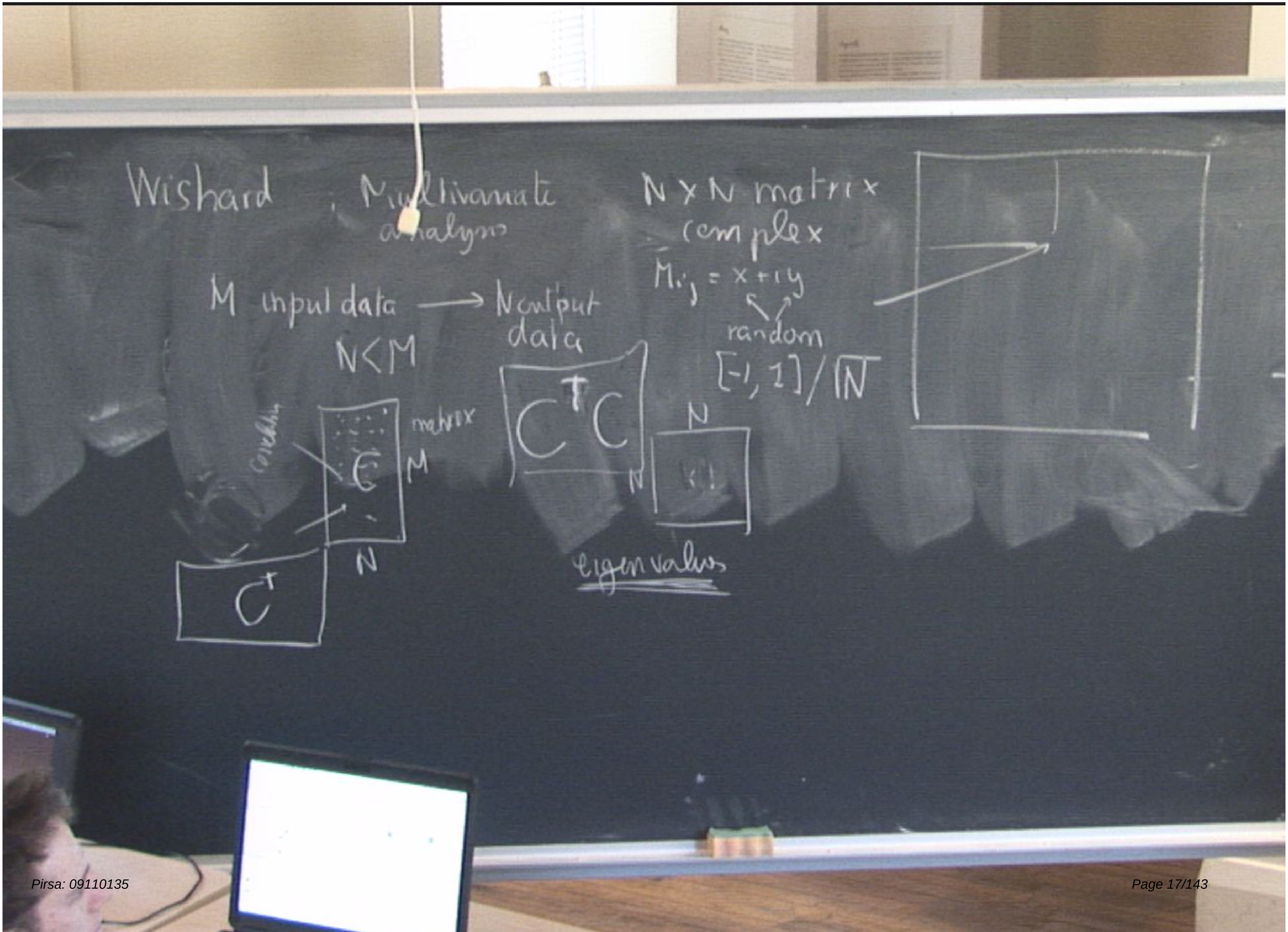
ishart matrices : eigenvalues distribution


Wishard
Multivariate analysis
N x N matrix
complex
M_ij = x + iy
random
[-1, 1]/√N


Wishard
Multivariate analysis
M input data → N output data
N<M


Wishard
Multivariate analysis
matrix
M input data → N output data
N<M
matrix
M
N


Wishard
Multivariate analysis
N x N matrix
complex
M_ij = x + iy
random
[-1, 1]/√N
M input data → N output data
N<M
matrix
M
N
C
C†
C†C
N
eigenvalues


Wishard
Multivariate analysis
N x N matrix
complex
M input data → N output data
N<M
random
[-1, 1]/√N
matrix
M
N
C†C
C†
eigenvalues

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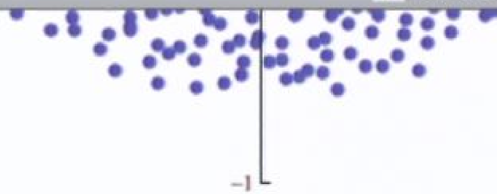
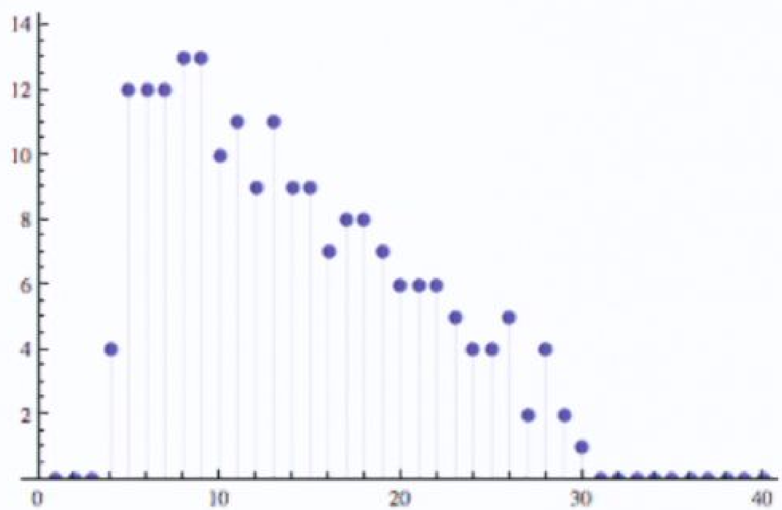
Random matrix PSI.nb



/ishart matrices : eigenvalues distribution

- eigenvalue distribution of R R† with R a random real n x m matrix with entries in [-1, 1]

```
size1 = 200;
size2 = 40;
matrixW = RandomReal[{-1, 1} / (size1 size2)^(1/4), {size1, size2}];
ev = Eigenvalues[matrixW.Transpose[matrixW]];
listc = BinCounts[ev, {0, 2, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```



- average over independent Wishart matrices

```
size2 = 800;
nsample = 100;
```

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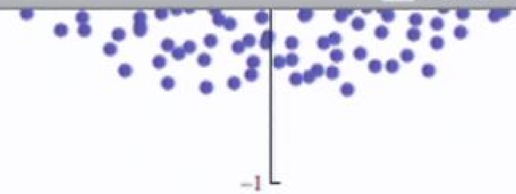
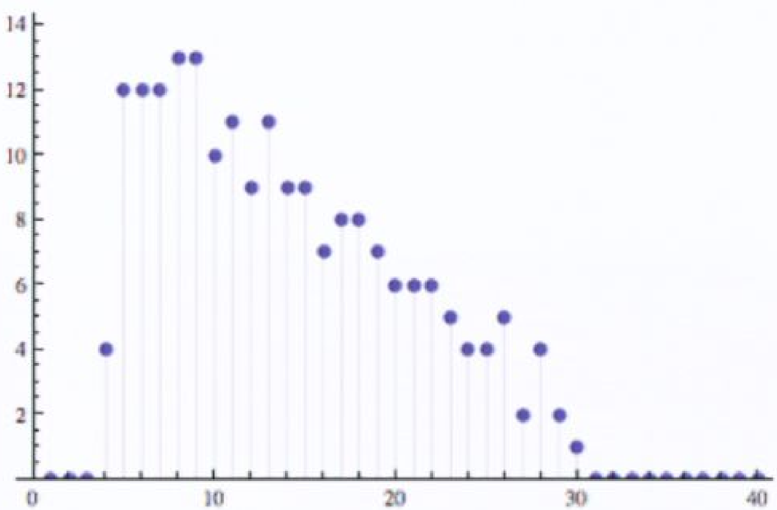
Random matrix PSI.nb



Wishart matrices : eigenvalues distribution

- eigenvalue distribution of R R† with R a random real n x m matrix with entries in [-1, 1]

```
size1 = 200;
size2 = 40;
matrixW = RandomReal[{-1, 1} / (size1 size2)^(1/4), {size1, size2}];
ev = Eigenvalues[matrixW.Transpose[matrixW]];
listc = BinCounts[ev, {0, 2, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```



- average over independent Wishart matrices

```
size1 = ...;
size2 = 800;
nsample = 100;
```

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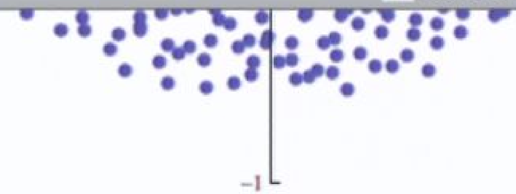
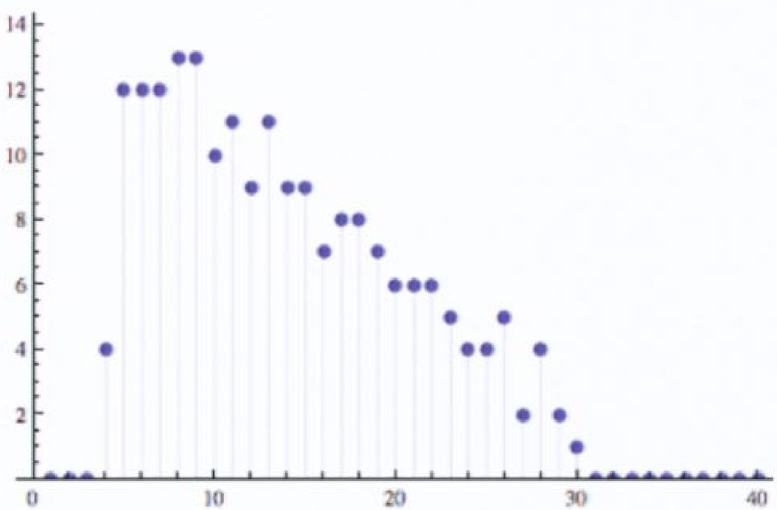
Random matrix PSI.nb



Wishart matrices : eigenvalues distribution

- eigenvalue distribution of R R† with R a random real n x m matrix with entries in [-1, 1]

```
size1 = 20;
size2 = 40;
matrixW = RandomReal[{-1, 1} / (size1 size2)^(1/4), {size1, size2}];
ev = Eigenvalues[matrixW.Transpose[matrixW]];
listc = BinCounts[ev, {0, 2, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```



- average over independent Wishart matrices

```
size1 = 200;
size2 = 800;
nsample = 100;
```

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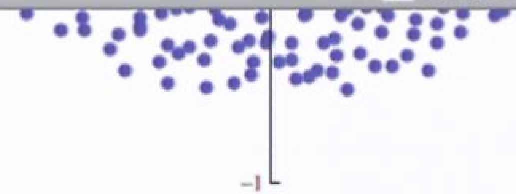
Random matrix PSI.nb

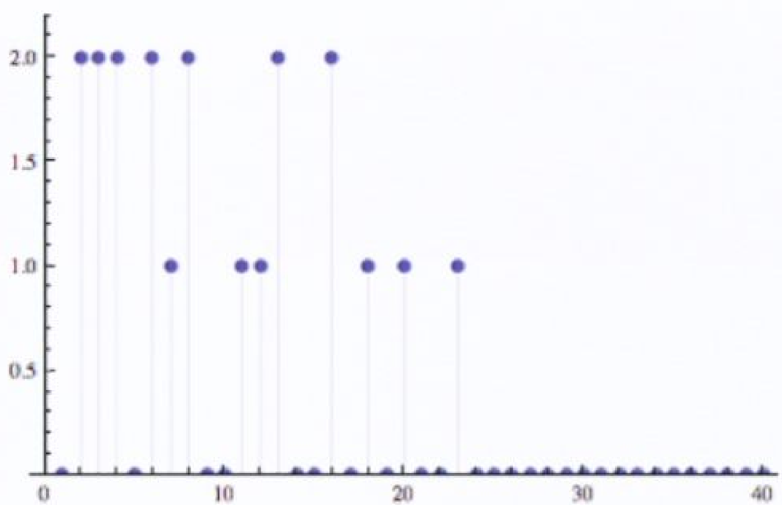


/ishart matrices : eigenvalues distribution

- eigenvalue distribution of R R† with R a random real n x m matrix with entries in [-1, 1]

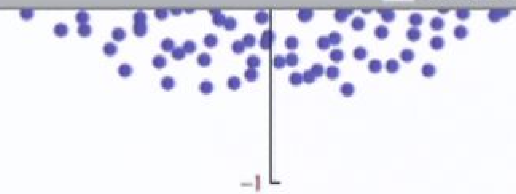
```
size1 = 20;
size2 = 40;
matrixW = RandomReal[{-1, 1} / (size1 size2)^(1/4), {size1, size2}];
ev = Eigenvalues[matrixW.Transpose[matrixW]];
listc = BinCounts[ev, {0, 2, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```

2



- average over independent Wishart matrices

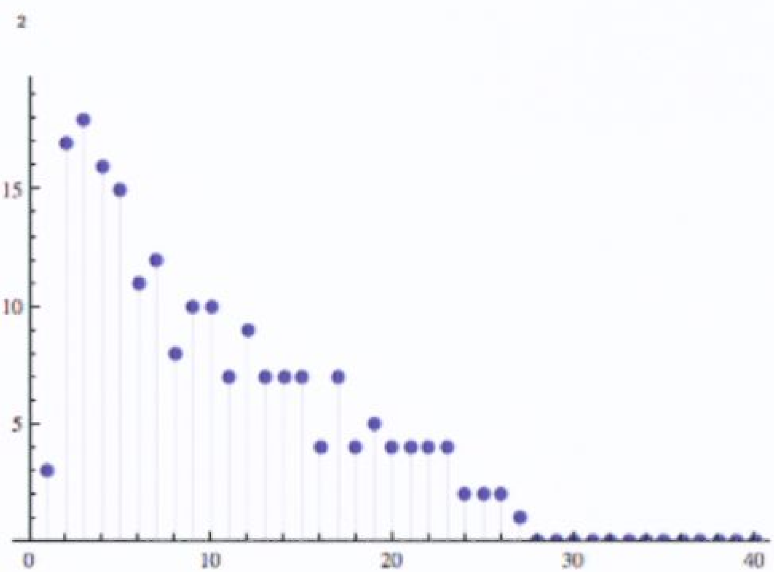

```
size1 = 200;
```



Wishart matrices : eigenvalues distribution

- eigenvalue distribution of R R† with R a random real n x m matrix with entries in [-1, 1]

```
size1 = 200;
size2 = 400;
matrixW = RandomReal[{-1, 1} / (size1 size2)^(1/4), {size1, size2}];
ev = Eigenvalues[matrixW.Transpose[matrixW]];
listc = BinCounts[ev, {0, 2, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```



- average over independent Wishart matrices

```
size1 = 200;
```

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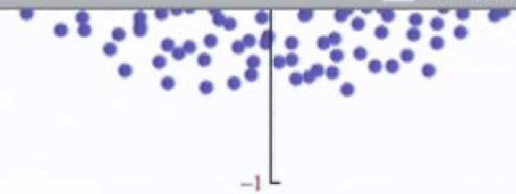
Random matrix PSI.nb



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```
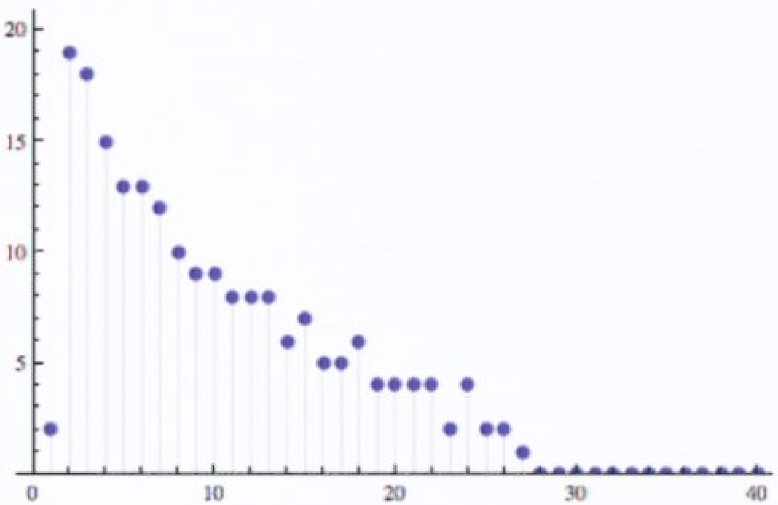

2



- average over independent Wishart matrices

```
size1 = 200;
```

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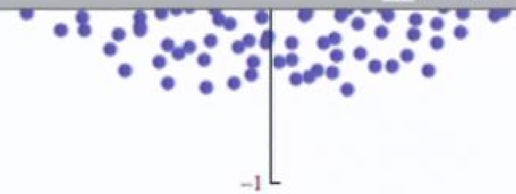
Random matrix PSI.nb



Wishart matrices : eigenvalues distribution

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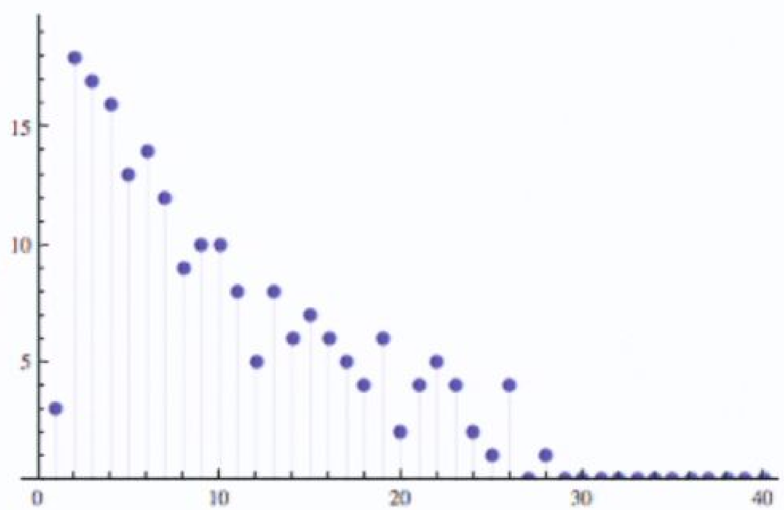
```

2



- average over independent Wishart matrices


```
size1 = 200;
```

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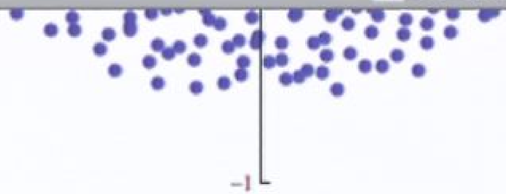
Random matrix PSI.nb



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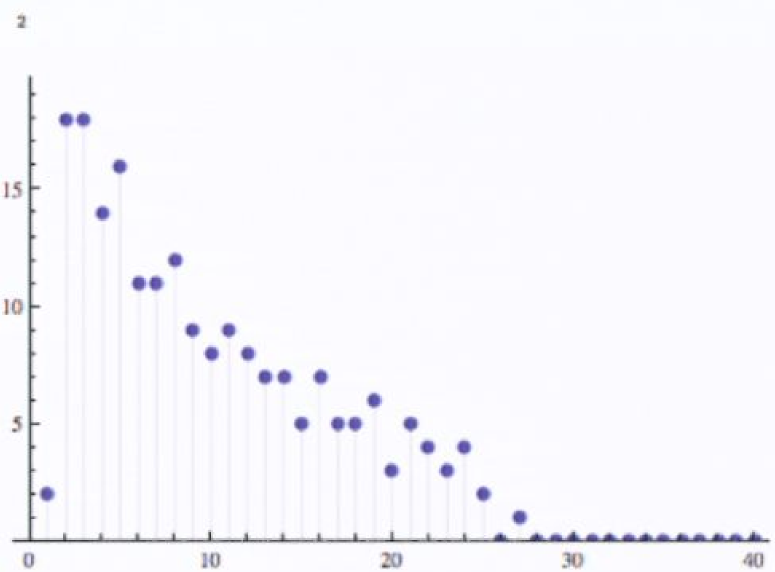

- average over independent Wishart matrices

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size1 = 200;
```

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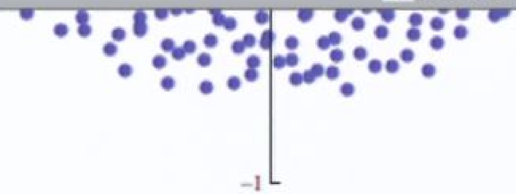
Random matrix PSI.nb



/ishart matrices : eigenvalues distribution

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```
size1 = 200;
size2 = 800;
matrixW = RandomReal[{-1, 1} / (size1 size2) ^ (1 / 4), {size1, size2}];
ev = Eigenvalues[matrixW.Transpose[matrixW]];
listc = BinCounts[ev, {0, 2, 1 / 20}];
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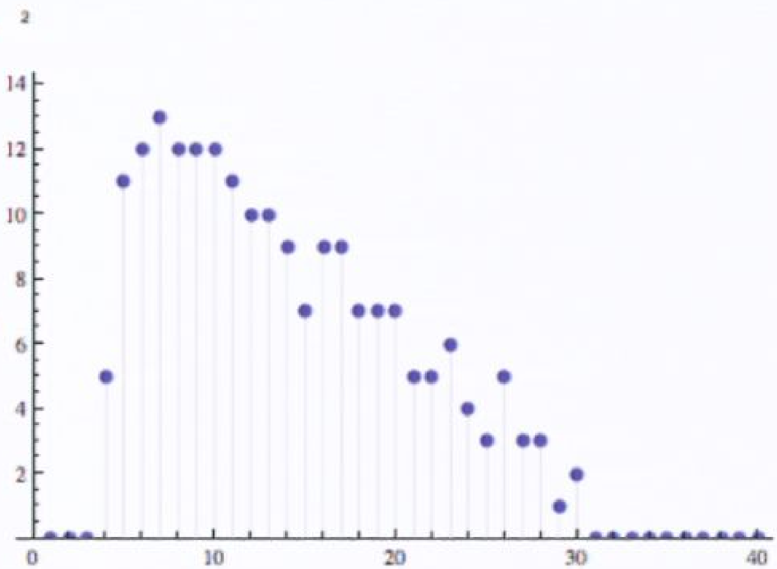

- ...er independent Wishart matrices

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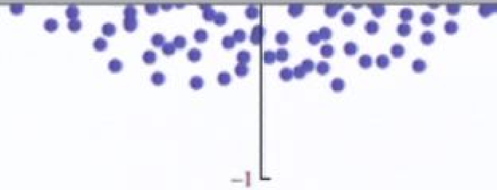
Random matrix PSI.nb



Wishart matrices : eigenvalues distribution

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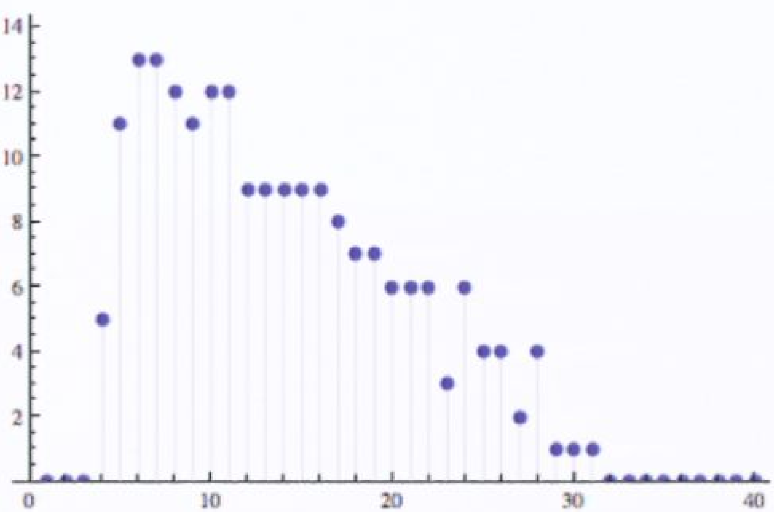

- ...er independent Wishart matrices

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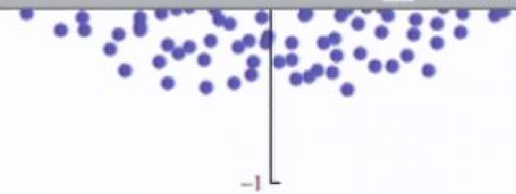
Random matrix PSI.nb



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```
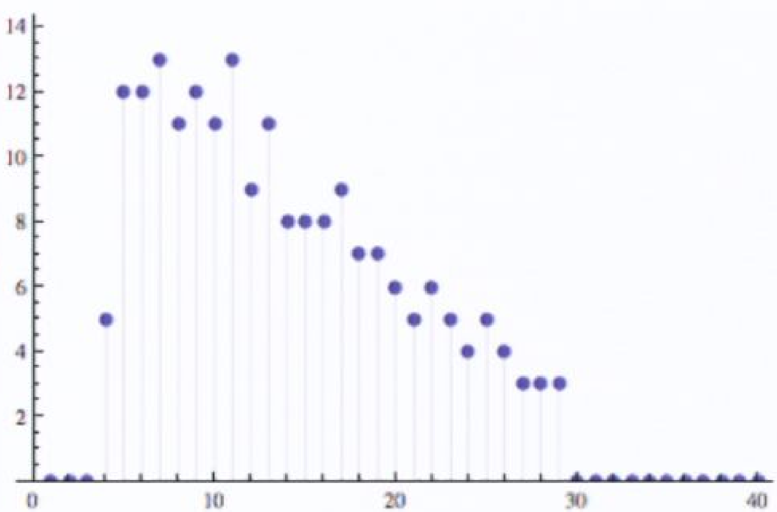

2



- ...er independent Wishart matrices

```
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```

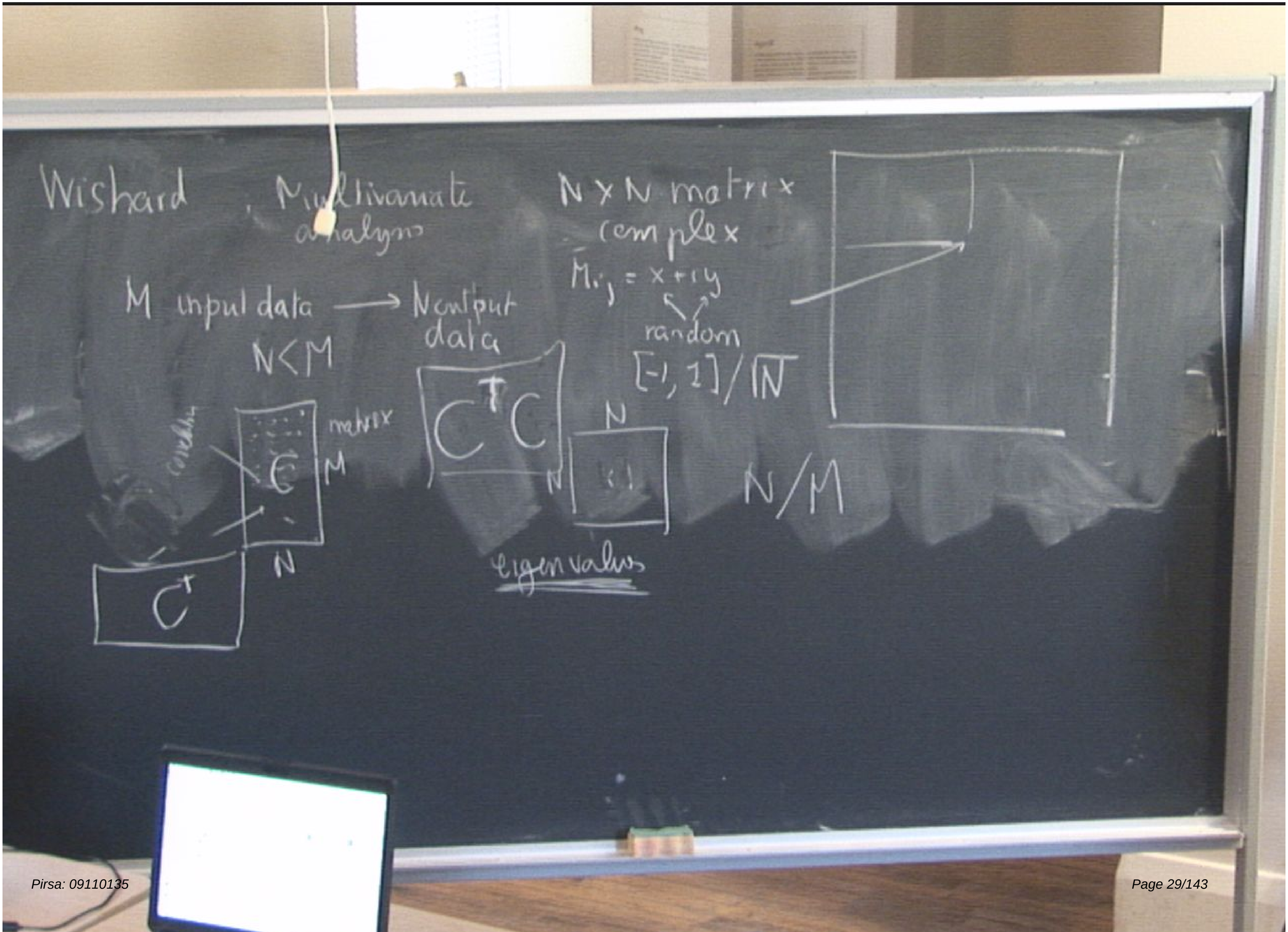

Wishard
Multivariate
analysis
N x N matrix
complex
$M_{ij} = x + iy$
random
$[-1, 1]/\sqrt{N}$
M input data → N output data
N<M
matrix
M
N
$C^{\dagger}C$
N
N
N/M
eigenvalues
$C^{\dagger}$

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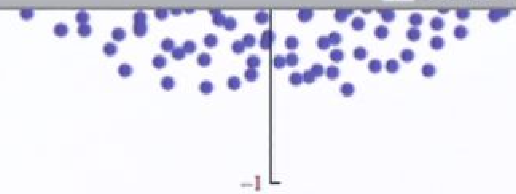
Random matrix PSI.nb

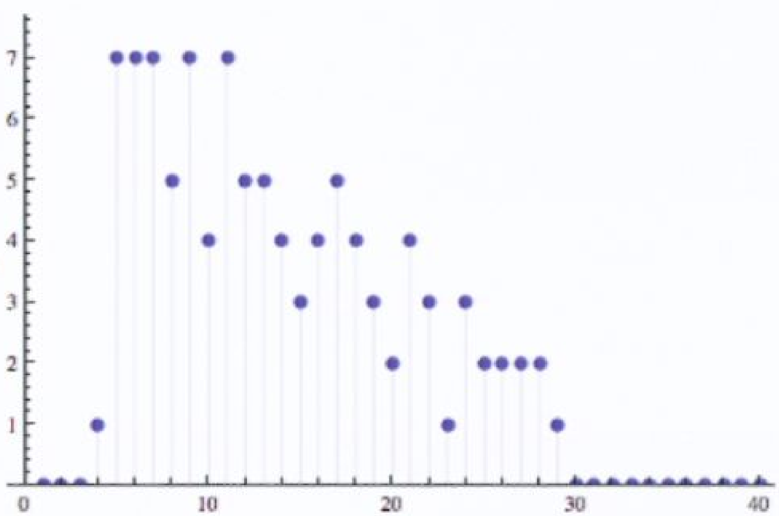


Wishart matrices : eigenvalues distribution

- eigenvalue distribution of R R† with R a random real n x m matrix with entries in [-1, 1]

```
size1 = 100;
size2 = 400;
matrixW = RandomReal[{-1, 1} / (size1 size2) ^ (1 / 4), {size1, size2}];
ev = Eigenvalues[matrixW.Transpose[matrixW]];
listc = BinCounts[ev, {0, 2, 1 / 20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```

2



- average over independent Wishart matrices

```
size1 = 200;
```

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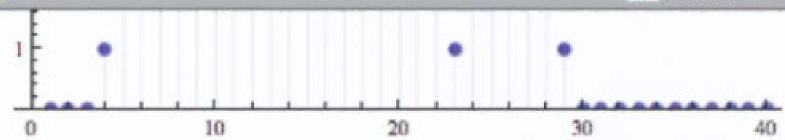
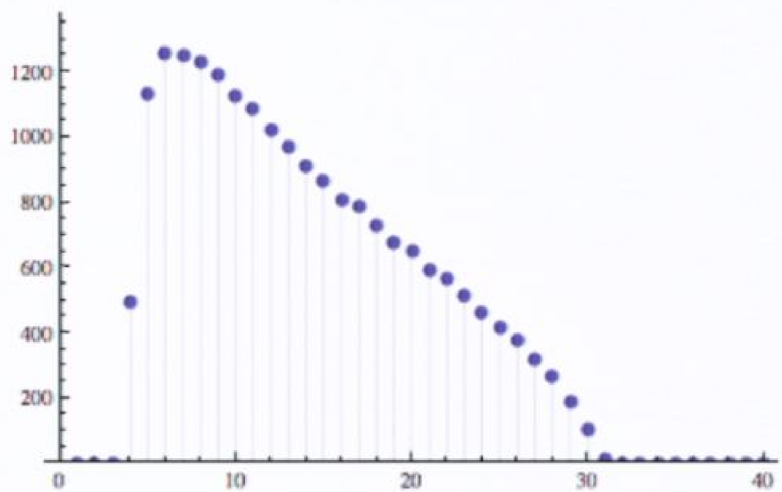
Random matrix PSI.nb



- average over independent Wishart matrices

```
size1 = 200;
size2 = 800;
nsample = 100;
listev = {};
Do[
  matrixW = RandomReal[{-1, 1} / (size1 size2)^(1/4), {size1, size2}];
  ev = Eigenvalues[matrixW.Transpose[matrixW]];
  listev = Join[ev, listev]
  , {nsample}];
listc = BinCounts[listev, {0, 2, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```



yson semicircle law

- 1 random symmetric matrix with entries in [-1, 1]

```
size = 1000;
```

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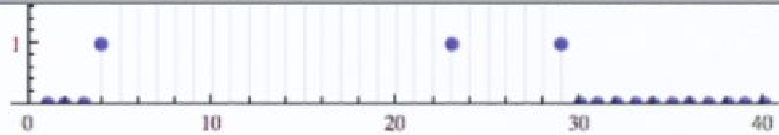
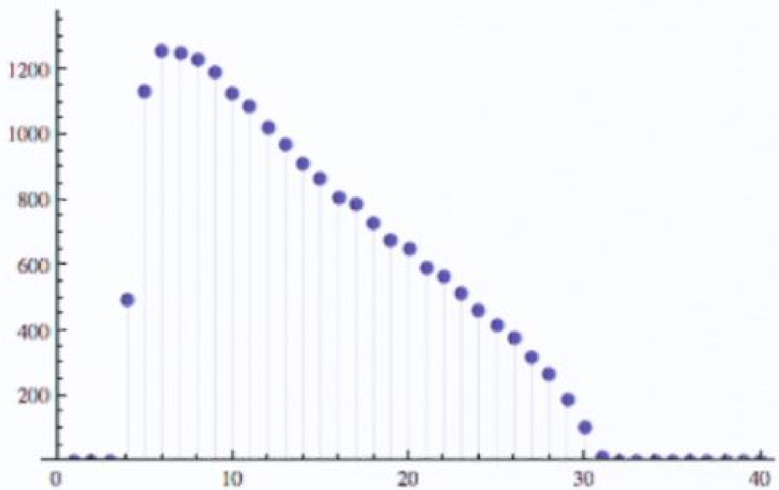
Random matrix PSI.nb



- average over independent Wishart matrices

```
size1 = 200;
size2 = 800;
nsample = 100;
listev = {};
Do[
  matrixW = RandomReal[{-1, 1} / (size1 size2)^(1/4), {size1, size2}];
  ev = Eigenvalues[matrixW.Transpose[matrixW]];
  listev = Join[ev, listev]
  , {nsample}];
listc = BinCounts[listev, {0, 2, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```



yson semicircle law

- 1 random symmetric matrix with entries in [-1, 1]

```
size = 1000;
```

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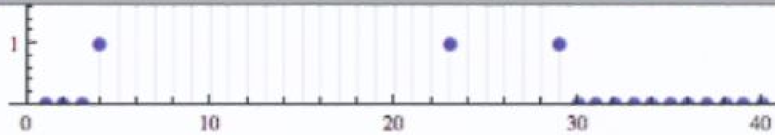
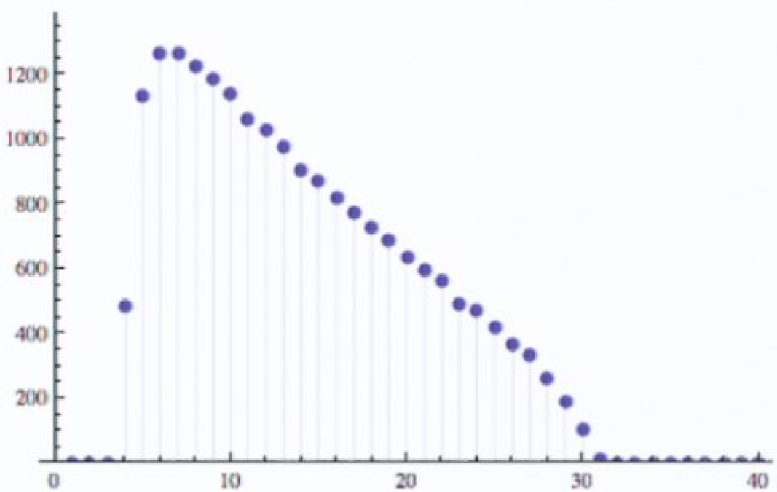
Random matrix PSI.nb



- average over independent Wishart matrices

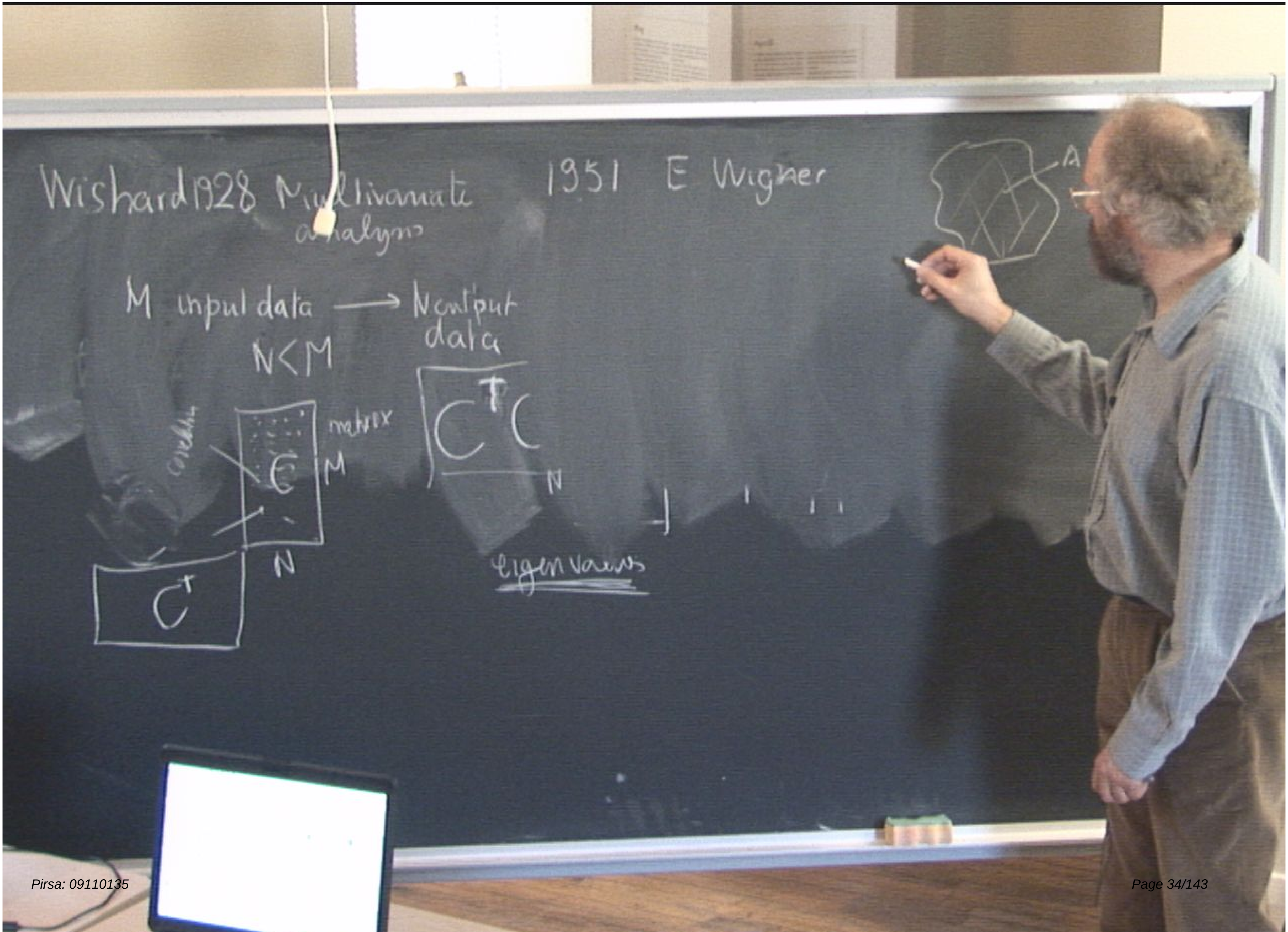
```
size1 = 200;
size2 = 800;
nsample = 100;
listev = {};
Do[
  matrixW = RandomReal[{-1, 1} / (size1 size2)^(1/4), {size1, size2}];
  ev = Eigenvalues[matrixW.Transpose[matrixW]];
  listev = Join[ev, listev]
  , {nsample}];
listc = BinCounts[listev, {0, 2, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```

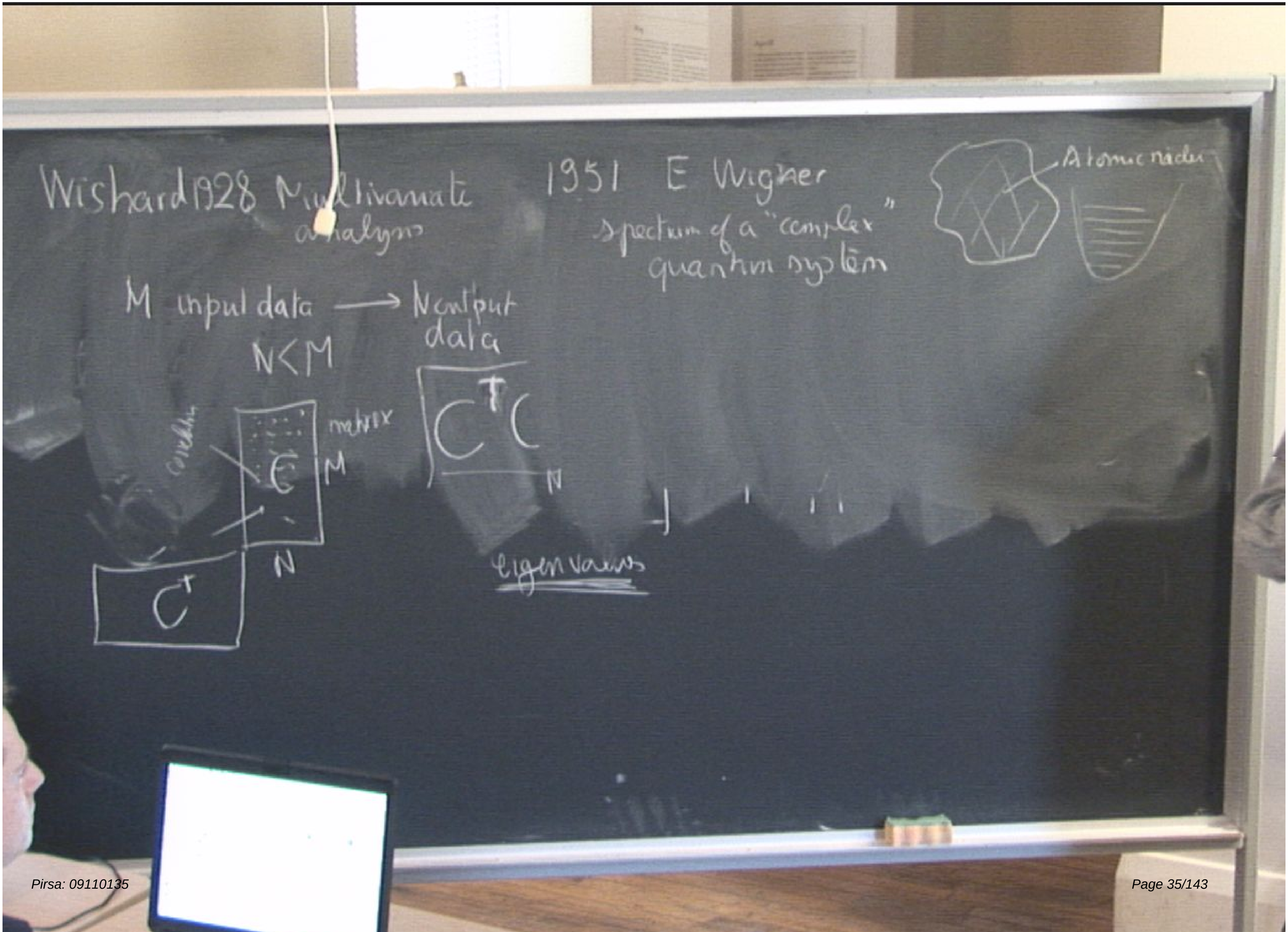


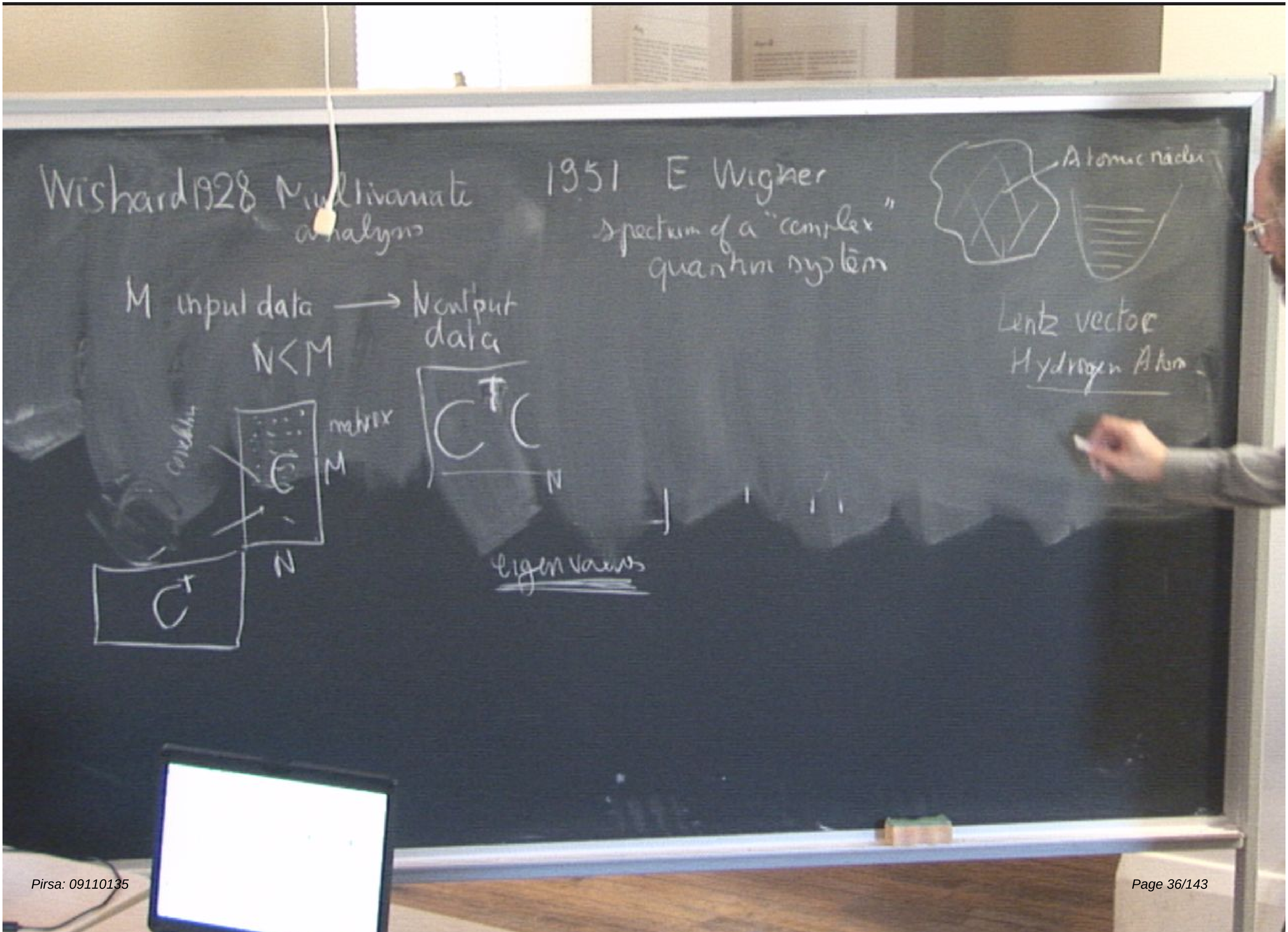
yson semicircle law

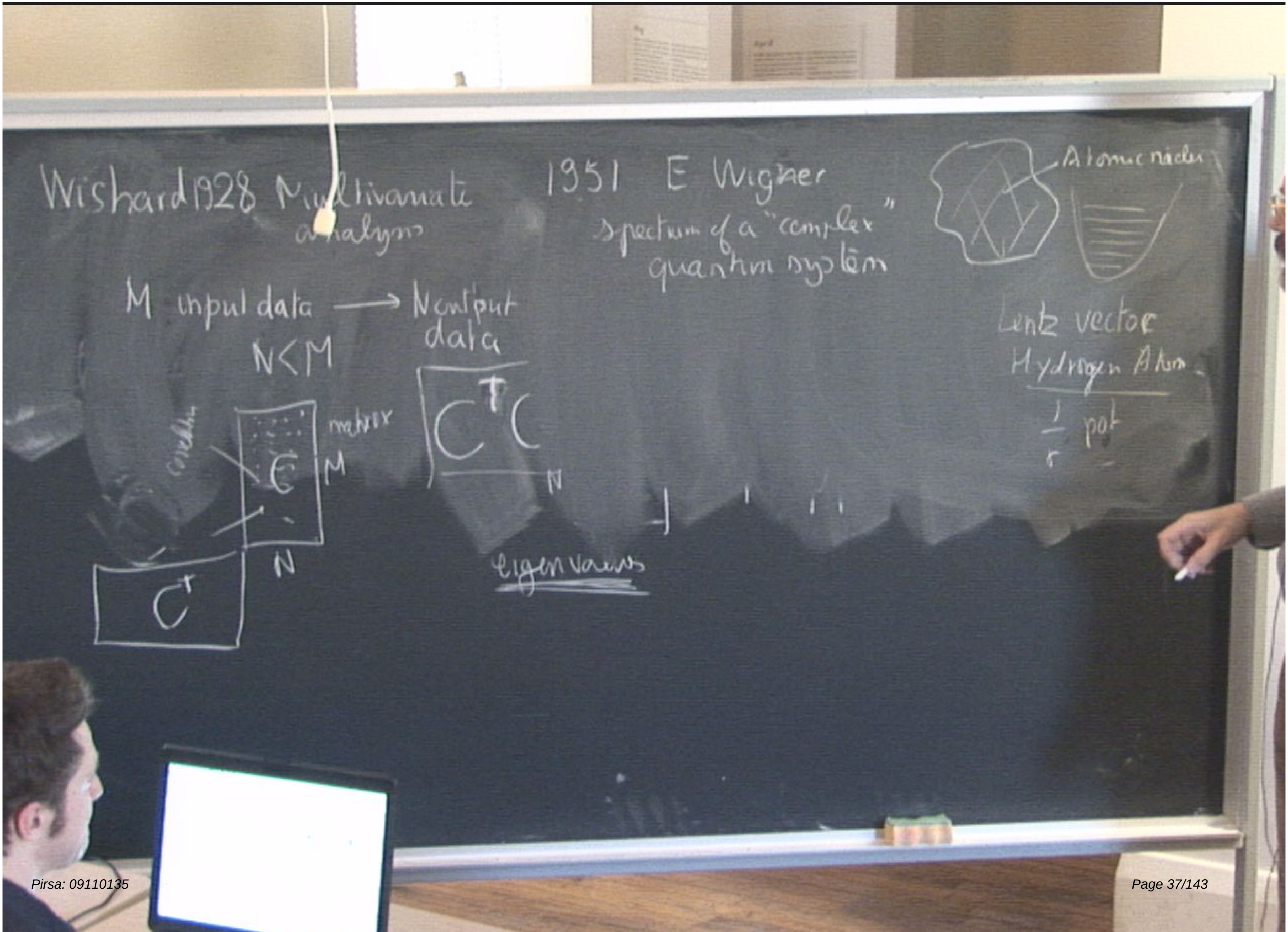
- 1 random symmetric matrix with entries in [-1, 1]

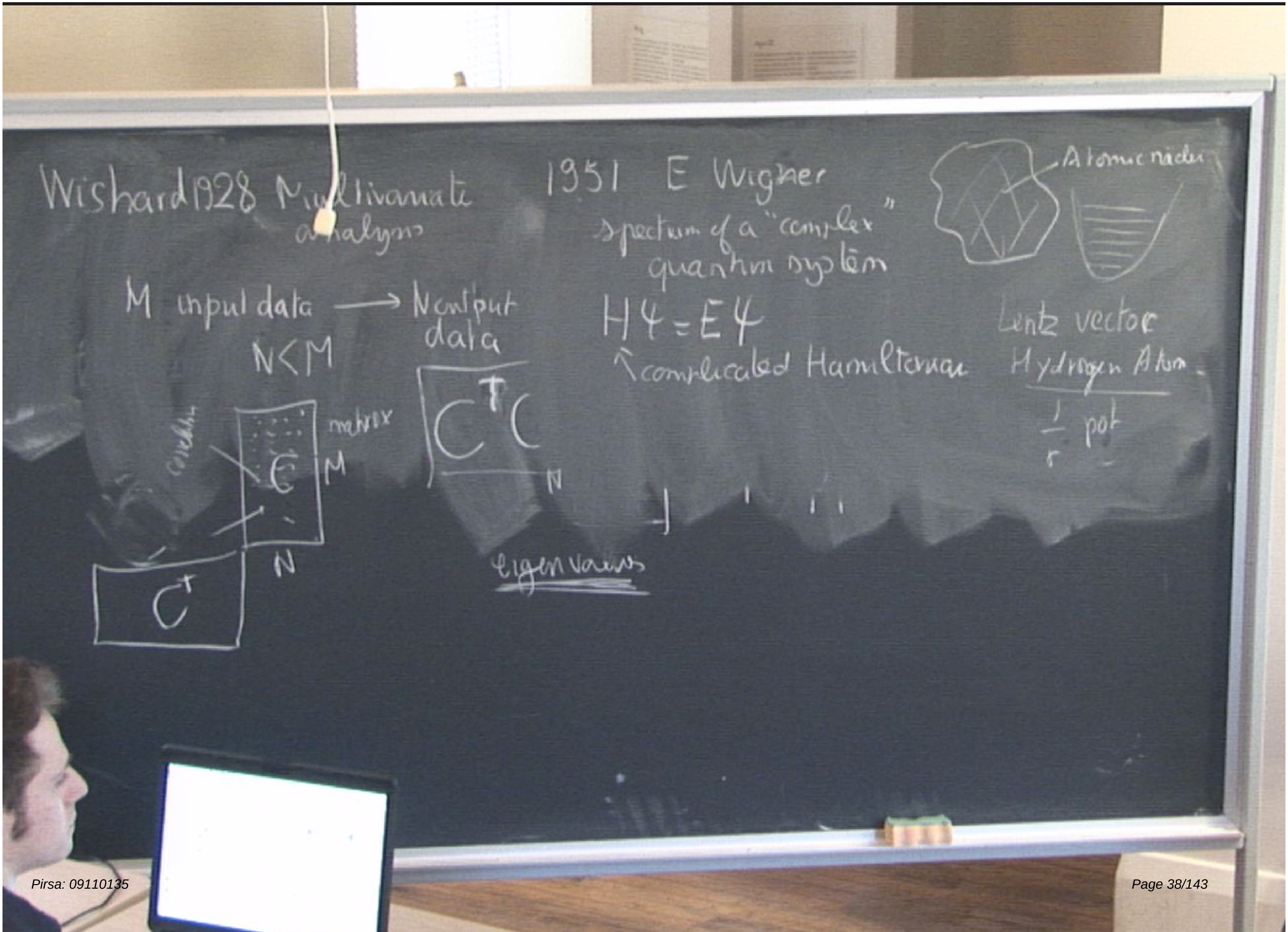
```
size = 1000;
```


Wishard 1928 Multivariate analysis
1951 E Wigner
M input data → N output data
N<M
matrix
C
M
N
C^T C
N
eigenvalues
C^T
A


Wishard 1928 Multivariate
analysis
1951 E Wigner
spectrum of a "complex"
quantum system
Atomic nuclei
M input data → N output data
N<M
matrix
C
M
N
C†
C†C
N
eigenvalues


Wishard 1928 Multivariate analysis
1951 E Wigner
spectrum of a "complex" quantum system
Atomic nuclei
M input data → N output data
N<M
matrix
C
M
N
C†
C†C
N
eigenvalues
Lentz vector
Hydrogen Atom


Wishard 1928 Multivariate analysis
M input data → N output data
N<M
matrix
C
M
N
C†
C†C
N
eigenvalues
1951 E Wigner
spectrum of a "complex" quantum system
Atomic nuclei
Lentz vector
Hydrogen Atom
1/r pot



Wishard 1928 Multivariate analysis
M input data → N output data
N<M
matrix
C
M
N
C†
C†C
N
1951 E Wigner
spectrum of a "complex" quantum system
HΨ = EΨ
complicated Hamiltonian
eigenvalues
Atomic nuclei
Lentz vector
Hydrogen Atom
1/r pot

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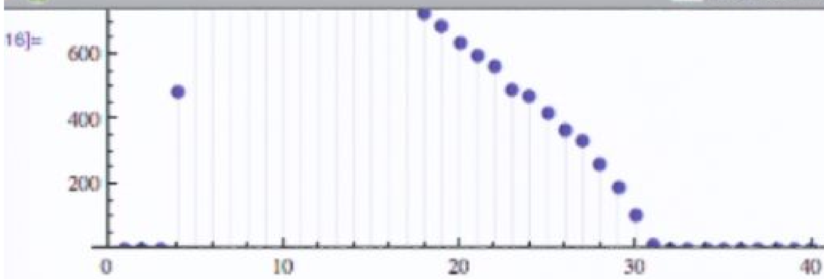
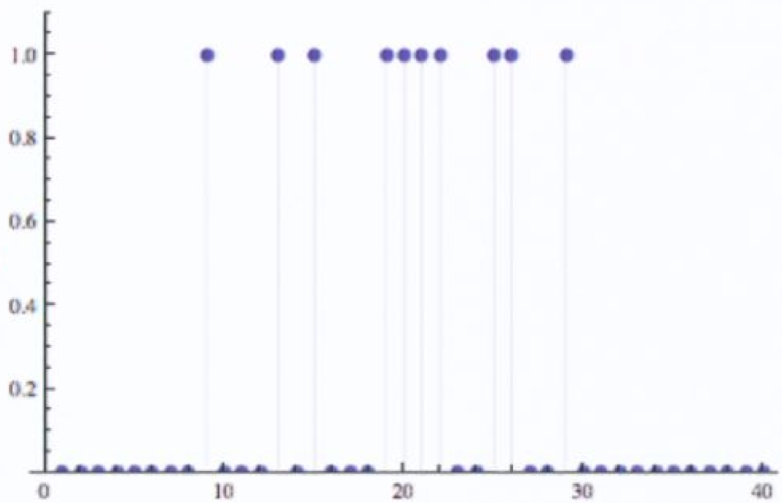
Random matrix PSI.nb



yson semicircle law

- 1 random symmetric matrix with entries in [-1, 1]

```
size = 10;
matrixD = RandomReal[{-1, 1} / (2 size)^(1/2), {size, size}];
matrixS = (matrixD + Transpose[matrixD]) / 2;
ev = Eigenvalues[matrixS];
listc = BinCounts[ev, {-1, 1, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```



- average over independent random symmetric matrix with entries in [-1, 1]

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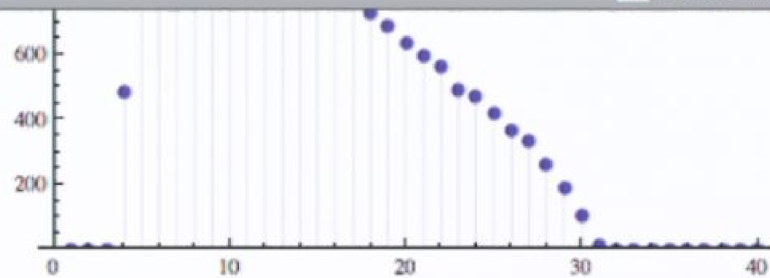
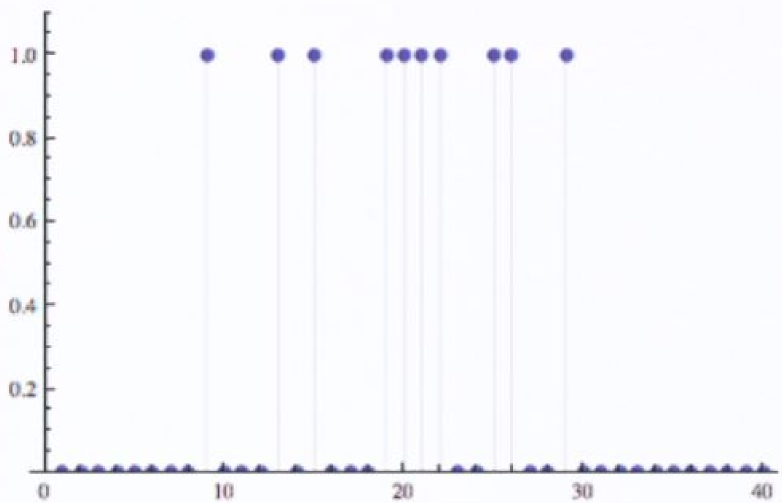
Random matrix PSI.nb



yson semicircle law

- 1 random symmetric matrix with entries in [-1, 1]

```
size = 10;
matrixD = RandomReal[{-1, 1} / (2 size)^(1/2), {size, size}];
matrixS = (matrixD + Transpose[matrixD]) / 2;
ev = Eigenvalues[matrixS];
listc = BinCounts[ev, {-1, 1, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```



- average over independent random symmetric matrix with entries in [-1, 1]

```
size = 400;
```

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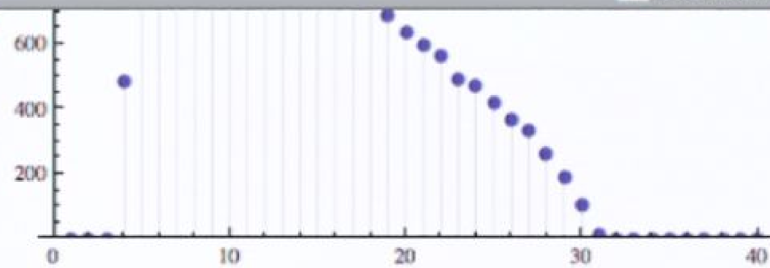
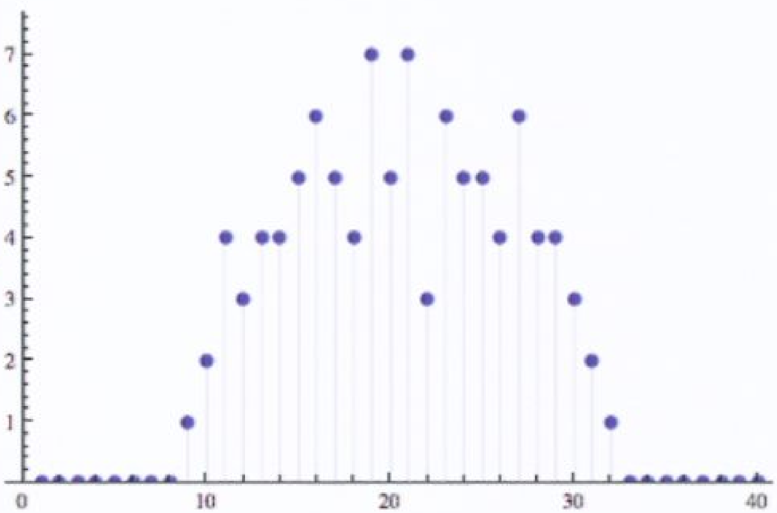
Random matrix PSI.nb



yson semicircle law

- 1 random symmetric matrix with entries in [-1, 1]

```
size = 100;
matrixD = RandomReal[{-1, 1} / (2 size) ^ (1 / 2), {size, size}];
matrixS = (matrixD + Transpose[matrixD]) / 2;
ev = Eigenvalues[matrixS];
listc = BinCounts[ev, {-1, 1, 1 / 20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```



- average over independent random symmetric matrix with entries in [-1, 1]

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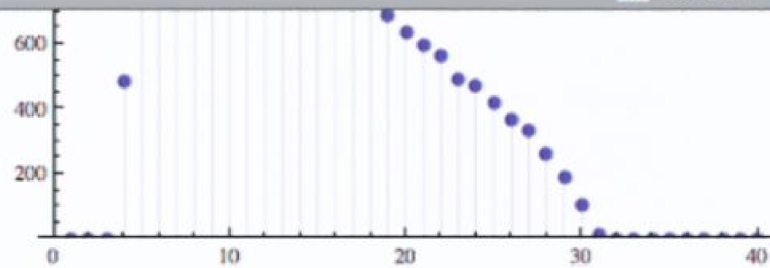
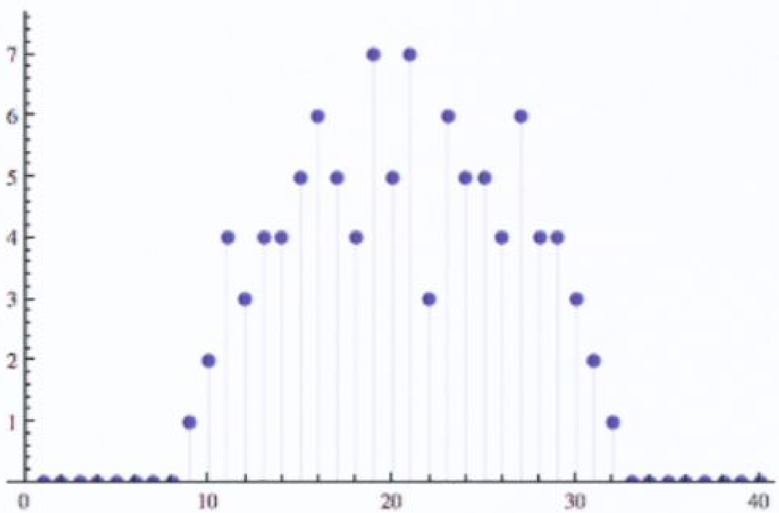
Random matrix PSI.nb



yson semicircle law

- 1 random symmetric matrix with entries in [-1, 1]

```
size = 100;
matrixD = RandomReal[{-1, 1} / (2 size)^(1/2), {size, size}];
matrixS = (matrixD + Transpose[matrixD]) / 2;
ev = Eigenvalues[matrixS];
listc = BinCounts[ev, {-1, 1, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```



- average over independent random symmetric matrix with entries in [-1, 1]

```
size = 400;
```

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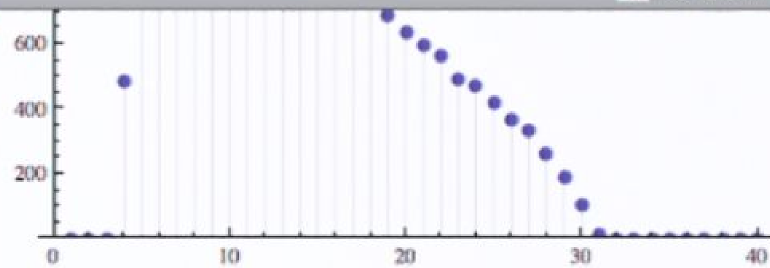
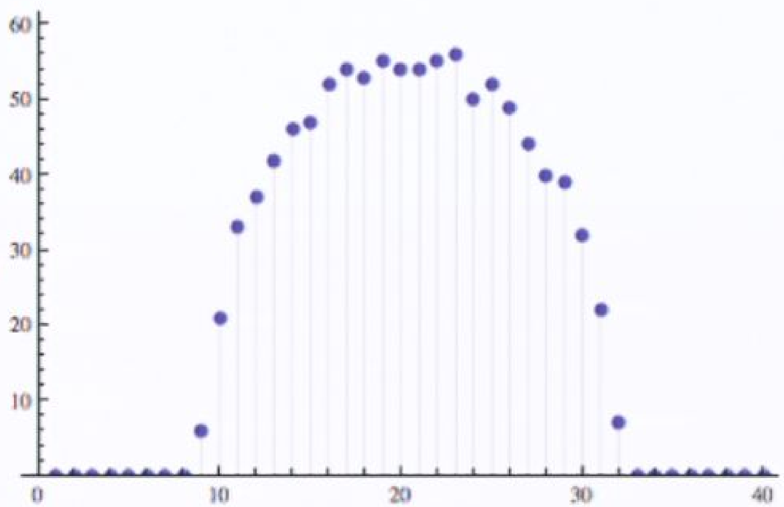
Random matrix PSI.nb



yson semicircle law

- 1 random symmetric matrix with entries in [-1, 1]

```
size = 1000;
matrixD = RandomReal[{-1, 1} / (2 size) ^ (1 / 2), {size, size}];
matrixS = (matrixD + Transpose[matrixD]) / 2;
ev = Eigenvalues[matrixS];
listc = BinCounts[ev, {-1, 1, 1 / 20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```



- average over independent random symmetric matrix with entries in [-1, 1]

```
size = 400;
```

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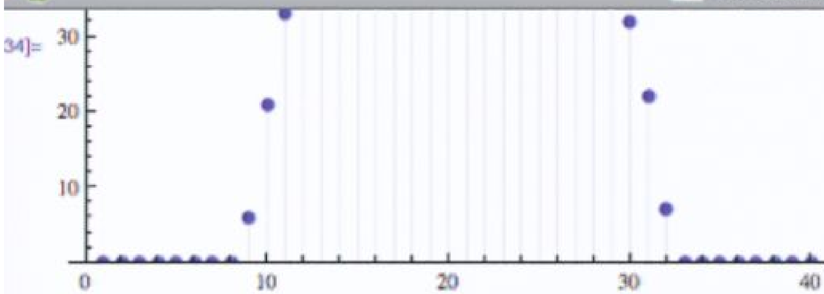
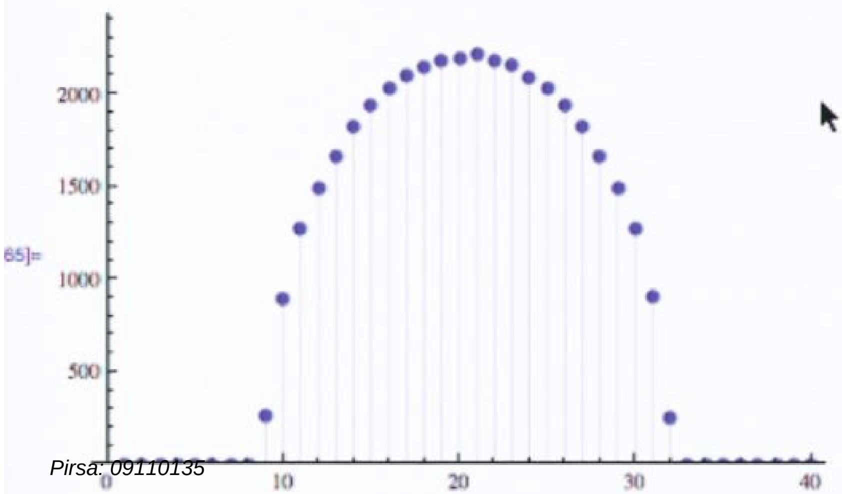
Random matrix PSI.nb



■ average over independent random symmetric matrix with entries in [-1, 1]

```
size = 400;
nsample = 100;
listev = {}
Do[
  matrixD = RandomReal[{-1, 1} / (2 size)^(1/2), {size, size}];
  matrixS = (matrixD + Transpose[matrixD]) / 2;
  ev = Eigenvalues[matrixS];
  listev = Join[ev, listev]
  , {nsample}];
listc = BinCounts[listev, {-1, 1, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```

```
{}
```



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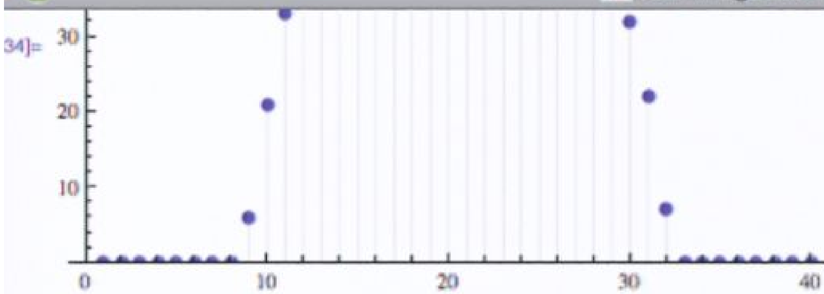
Running...Random matrix PSI.nb



- **average over independent random symmetric matrix with entries in [-1, 1]**

```
size = 400;
nsample = 100;
listev = {}
Do[
  matrixD = RandomReal[{-1, 1} / (2 size)^(1/2), {size, size}];
  matrixS = (matrixD + Transpose[matrixD]) / 2;
  ev = Eigenvalues[matrixS];
  listev = Join[ev, listev]
  , {nsample}];
listc = BinCounts[listev, {-1, 1, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```

37]= {}

- **average over independent random symmetric matrix with entries ± 1**

```
size = 400;
nsample = 100;
listev = {}
Do[
  matrixD = (RandomInteger[1, {size, size}] - .5) / (2 size)^(1/2);
  matrixS = (matrixD + Transpose[matrixD]) / 2;
  ev = Eigenvalues[matrixS];
  listev = Join[ev, listev]
  , {nsample}];
listc = BinCounts[listev, {-1, 1, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```

58]= {}

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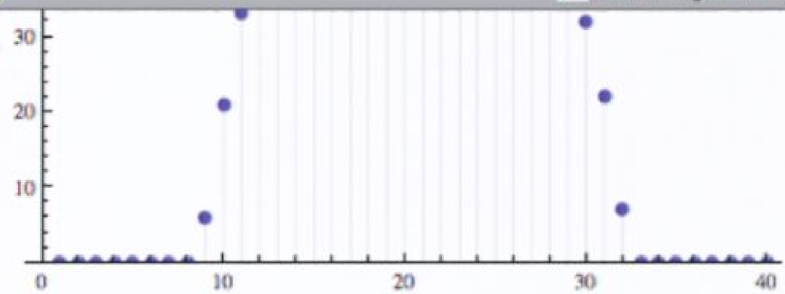
Running...Random matrix PSI.nb



- **average over independent random symmetric matrix with entries in [-1, 1]**

```
size = 400;
nsample = 100;
listev = {}
Do[
  matrixD = RandomReal[{-1, 1} / (2 size)^(1/2), {size, size}];
  matrixS = (matrixD + Transpose[matrixD]) / 2;
  ev = Eigenvalues[matrixS];
  listev = Join[ev, listev]
  , {nsample}];
listc = BinCounts[listev, {-1, 1, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```

```
{}
```

- **average over independent random symmetric matrix with entries ± 1**

```
size = 400;
nsample = 100;
listev = {}
Do[
  matrixD = (RandomInteger[1, {size, size}] - .5) / (2 size)^(1/2);
  matrixS = (matrixD + Transpose[matrixD]) / 2;
  ev = Eigenvalues[matrixS];
  listev = Join[ev, listev]
  , {nsample}];
listc = BinCounts[listev, {-1, 1, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```

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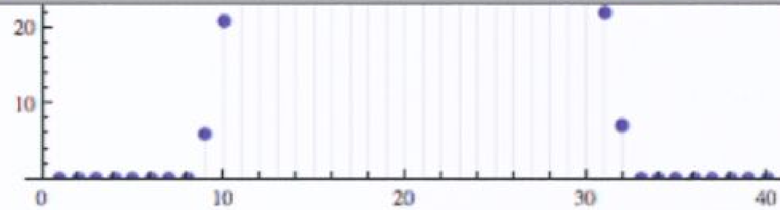
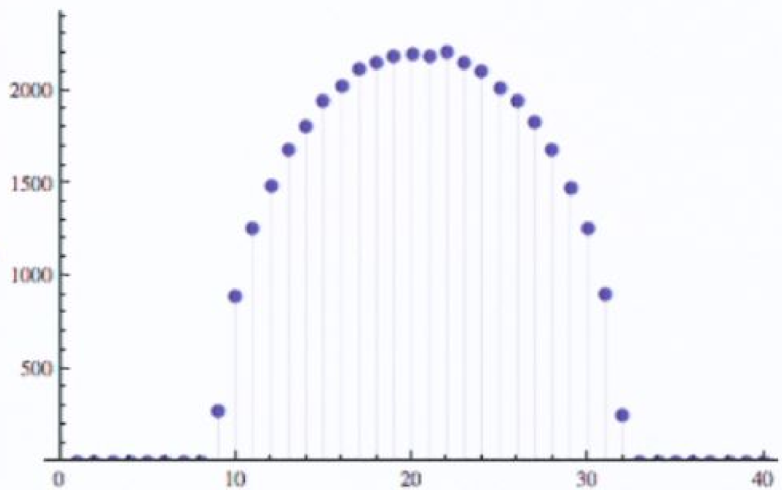
Random matrix PSI.nb



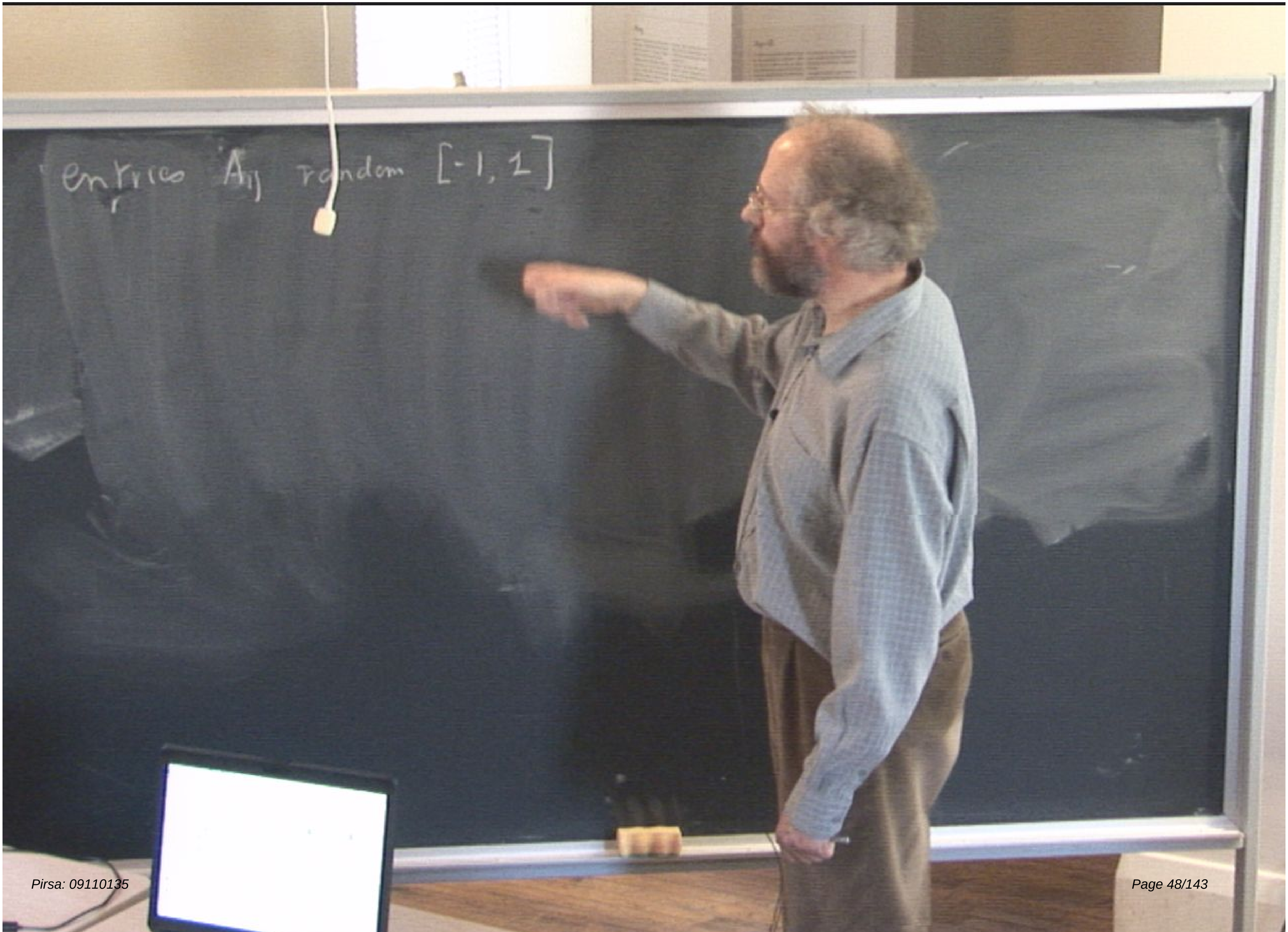
■ average over independent random symmetric matrix with entries in [-1, 1]

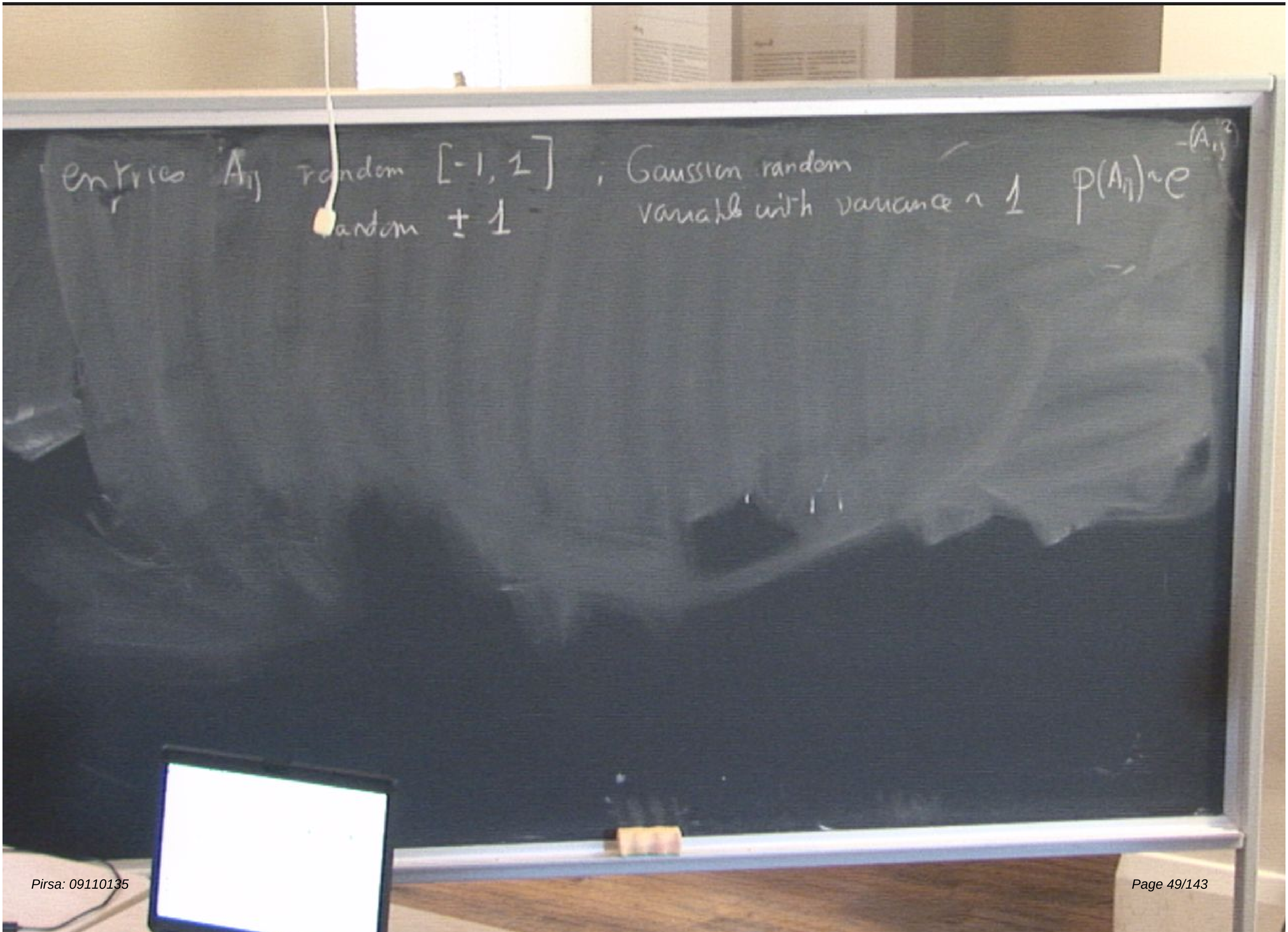
```
size = 400;
nsample = 100;
listev = {}
Do[
  matrixD = RandomReal[{-1, 1} / (2 size)^(1/2), {size, size}];
  matrixS = (matrixD + Transpose[matrixD]) / 2;
  ev = Eigenvalues[matrixS];
  listev = Join[ev, listev]
  , {nsample}];
listc = BinCounts[listev, {-1, 1, 1/20}];
ListPlot[listc, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → {0, Max[listc] 1.1}]
```

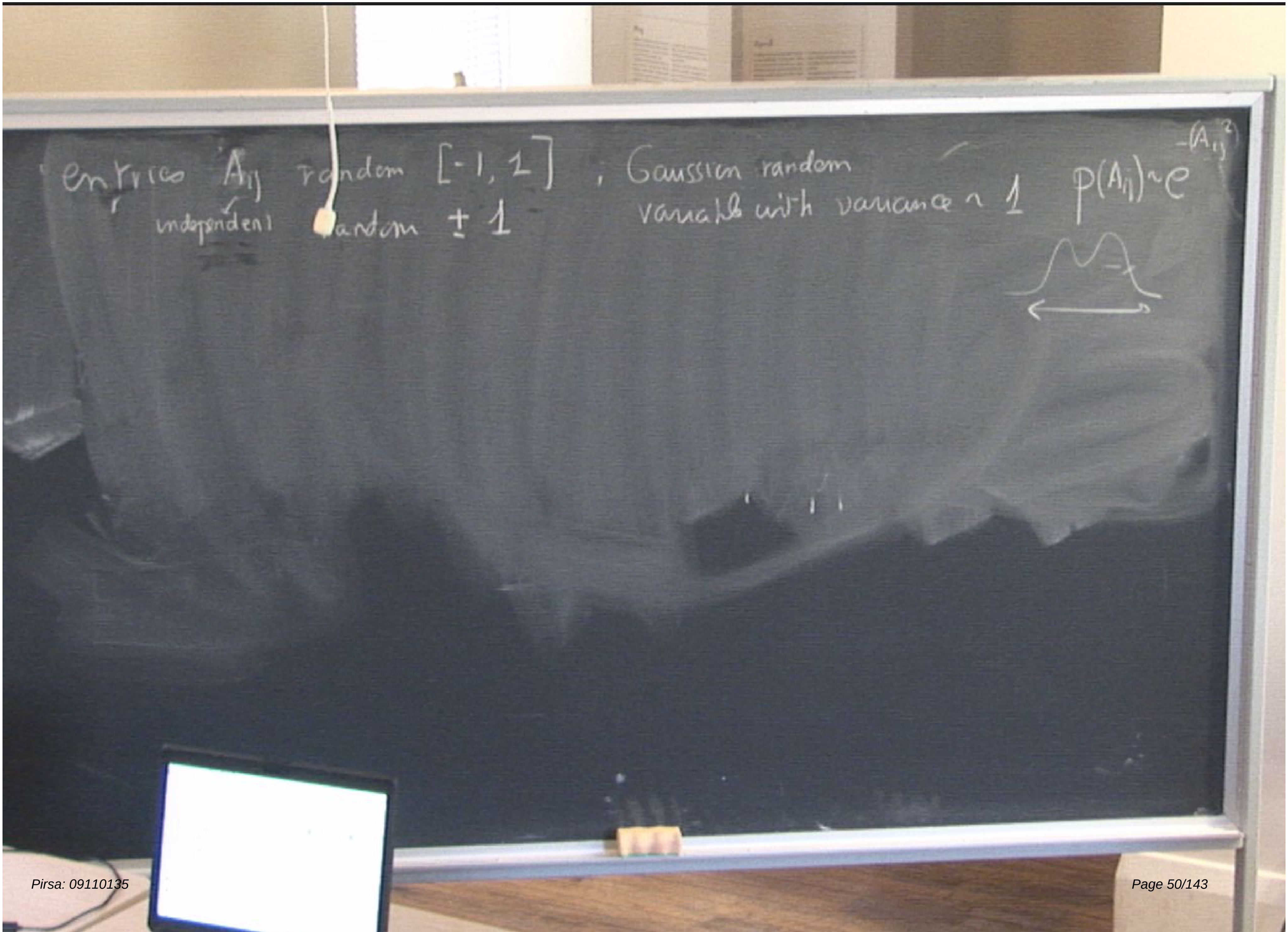
{}



■ average over independent random symmetric matrix with entries ± 1


entries A_{ij} random $[-1, 1]$


entries A_{ij} random $[-1, 1]$; Gaussian random
random ± 1
variable with variance ~ 1 $p(A_{ij}) \sim e^{-(A_{ij})^2}$

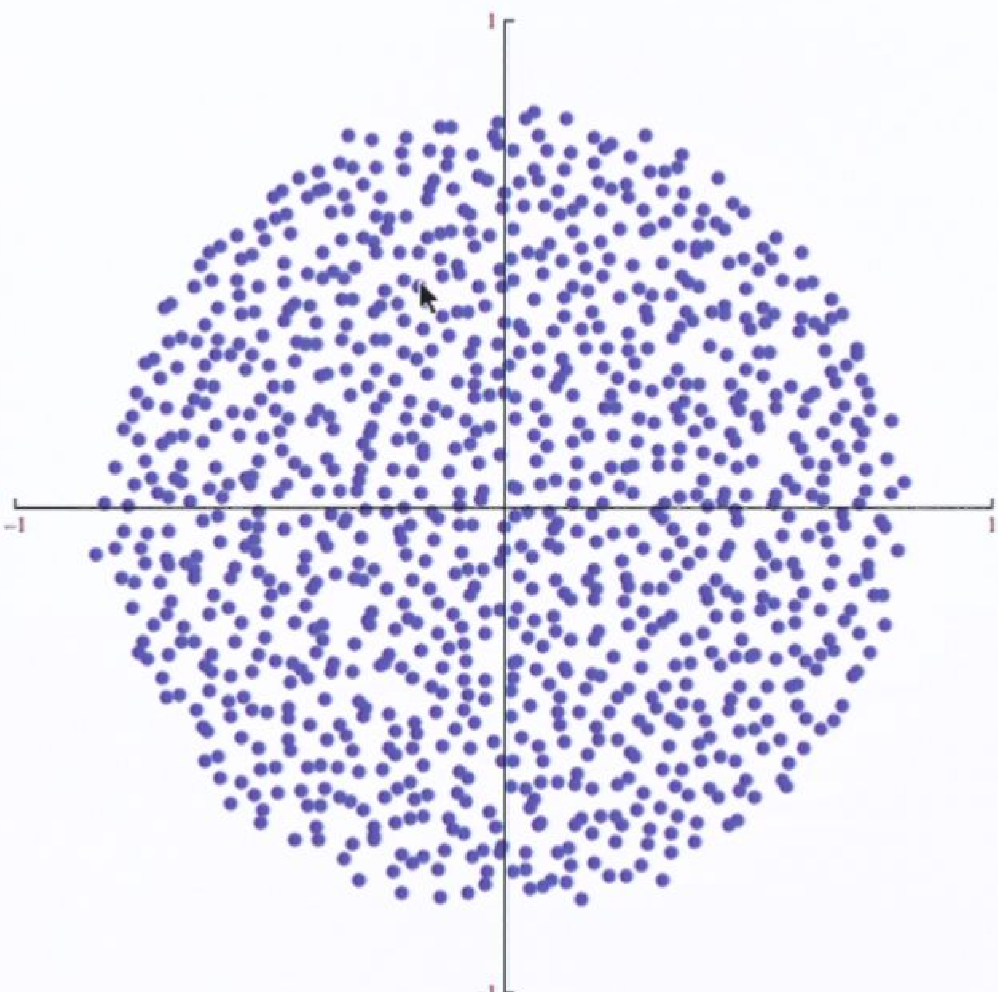

entries A_{ij} random $[-1, 1]$; Gaussian random
independent random ± 1
variable with variance ~ 1
$p(A_{ij}) \sim e^{-(A_{ij})^2}$

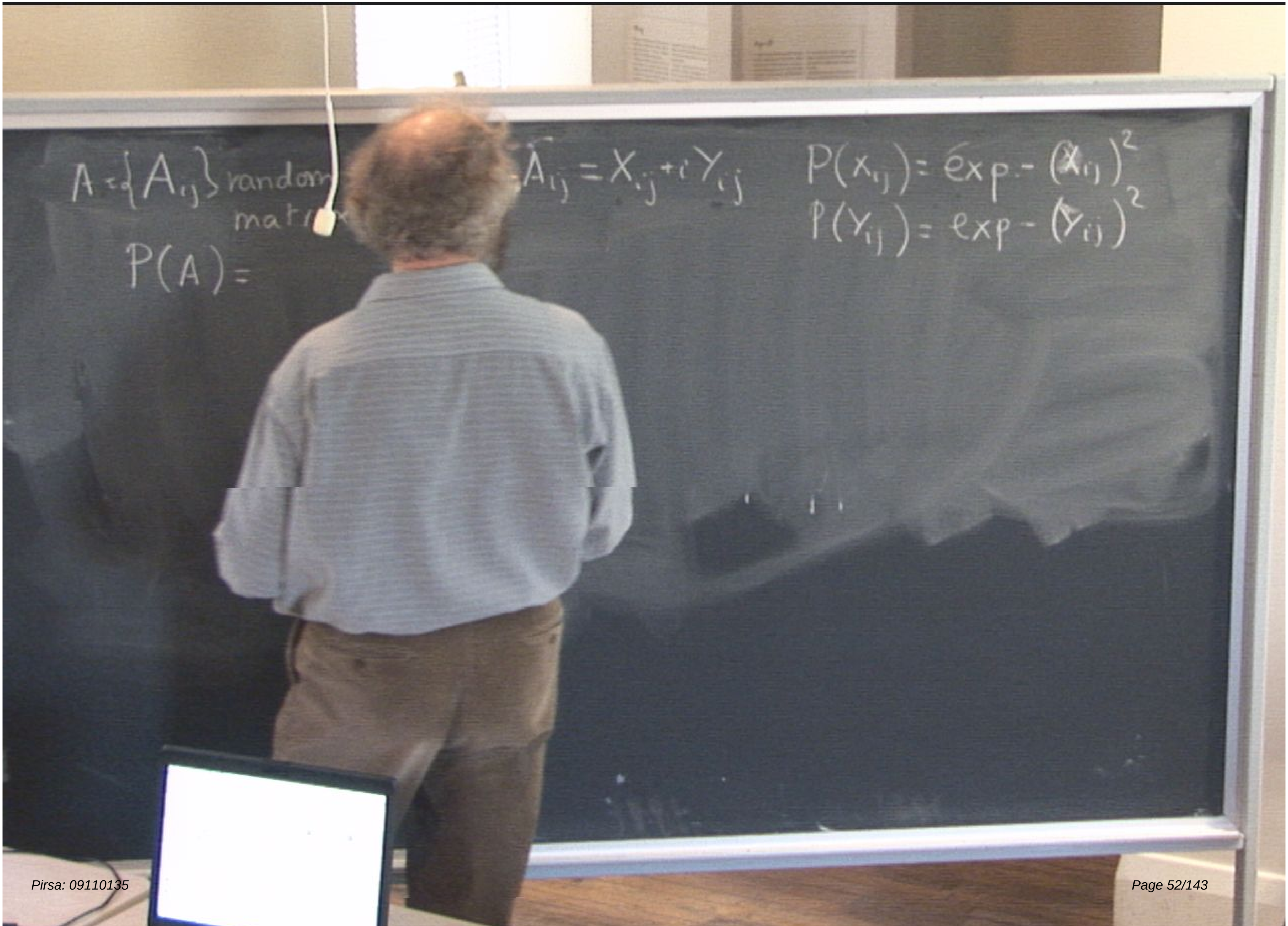
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help François David

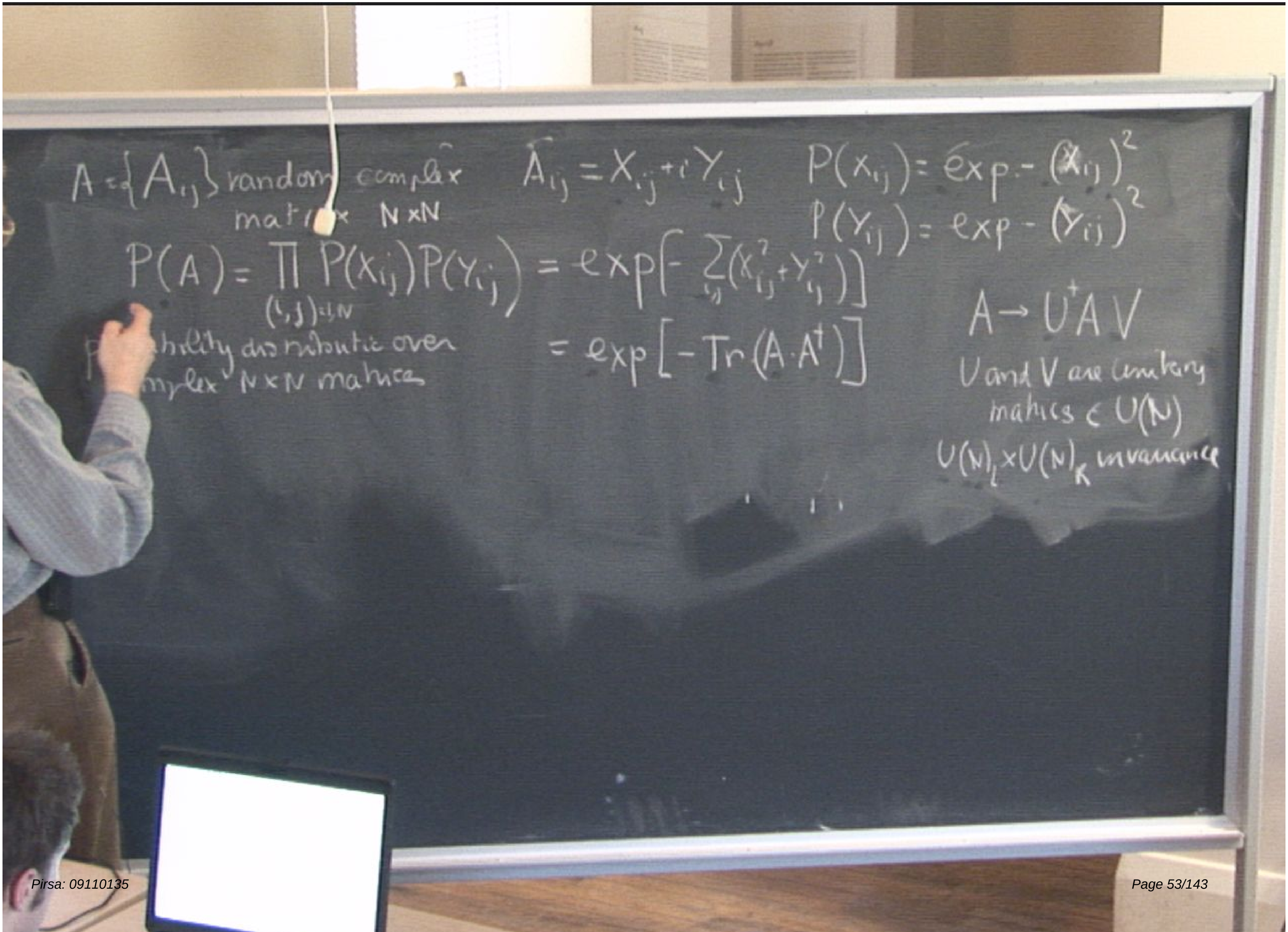
Random matrix PSI.nb

- 1 random complex matrix with entries z=x+i y such that x & y in [-1, 1]

```
size = 1000;
matrix = RandomComplex[{(-1 - I) / Sqrt[size], (1 + I) / Sqrt[size]}, {size, size}];
ev = Eigenvalues[matrix];
plot = ListPlot[Transpose[{Re[ev], Im[ev]}], PlotRange → {{-1, 1}, {-1, 1}}, AspectRatio → 1,
    PlotStyle → PointSize[Large], Ticks → {{-1, 0, 1}, {-1, 0, 1}}];
Show[plot]
```




A = {A_ij} random
P(A) =
A_ij = X_ij + i Y_ij
P(X_ij) = exp - (X_ij)^2
P(Y_ij) = exp - (Y_ij)^2


$A = \{A_{ij}\}$ random complex matrix $N \times N$
$A_{ij} = X_{ij} + i Y_{ij}$
$P(X_{ij}) = \exp - (X_{ij})^2$
$P(Y_{ij}) = \exp - (Y_{ij})^2$
$P(A) = \prod_{(i,j)=1,N} P(X_{ij}) P(Y_{ij}) = \exp\left[-\sum_{i,j}(X_{ij}^2 + Y_{ij}^2)\right]$
$= \exp\left[-\mathrm{Tr}(A \cdot A^\dagger)\right]$
probability distribution over complex $N \times N$ matrices
$A \to U^\dagger A V$
U and V are unitary matrices $\in U(N)$
$U(N)_L \times U(N)_R$ invariance

$A = \{A_{ij}\}$ random complex matrix $N \times N$

$$\hat{A}_{ij} = X_{ij} + i\,Y_{ij}$$

$$P(X_{ij}) = \exp - (X_{ij})^2$$

$$P(Y_{ij}) = \exp - (Y_{ij})^2$$

$$P(A) = \prod_{(i,j)=1,N} P(X_{ij})P(Y_{ij}) = \exp\left[-\sum_{i,j}(X_{ij}^2 + Y_{ij}^2)\right]$$

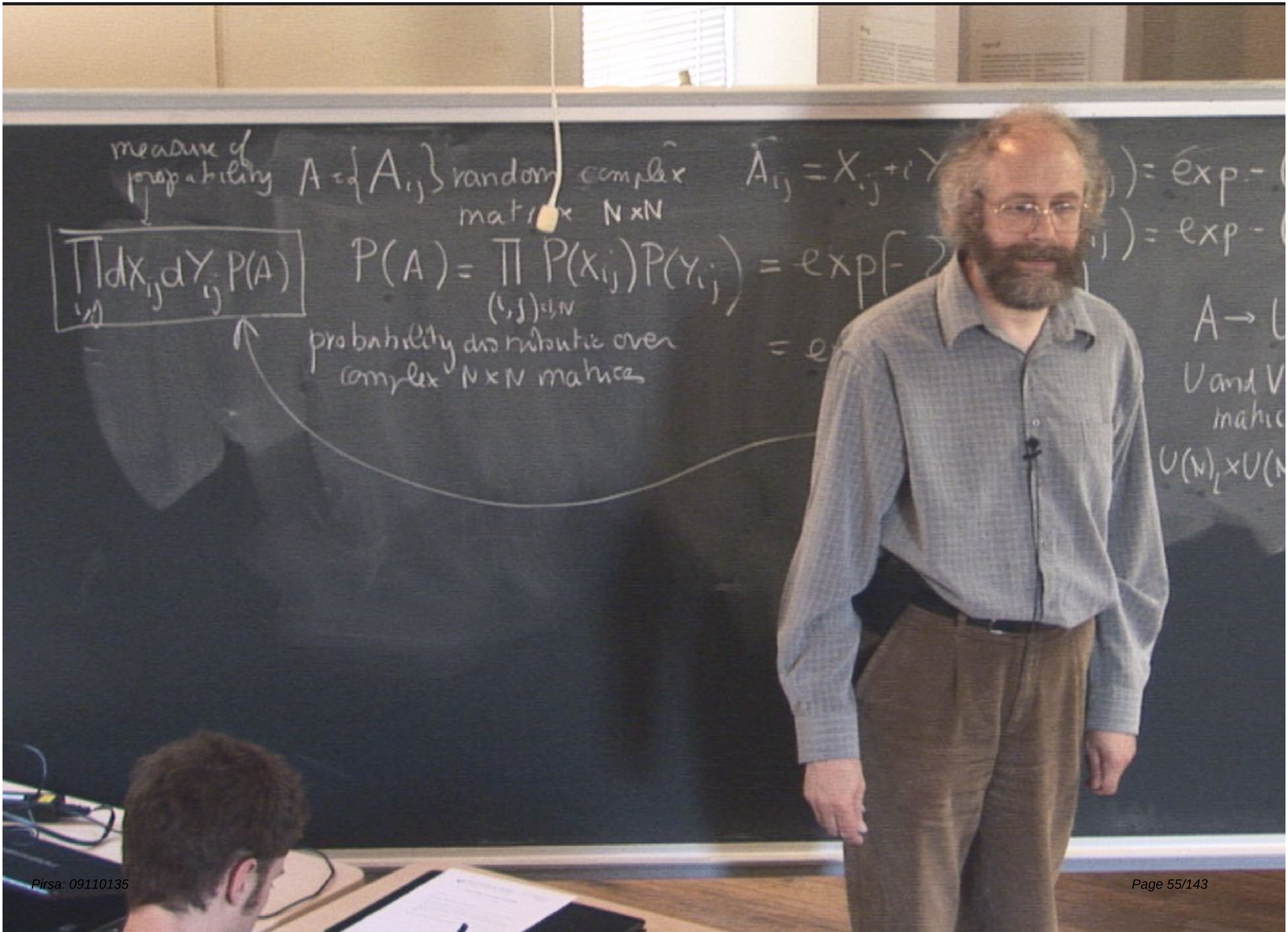
$$= \exp\left[-\mathrm{Tr}(A \cdot A^\dagger)\right]$$

probability distribution over complex $N \times N$ matrices

$$A \to U^\dagger A V$$

U and V are unitary matrices $\in U(N)$

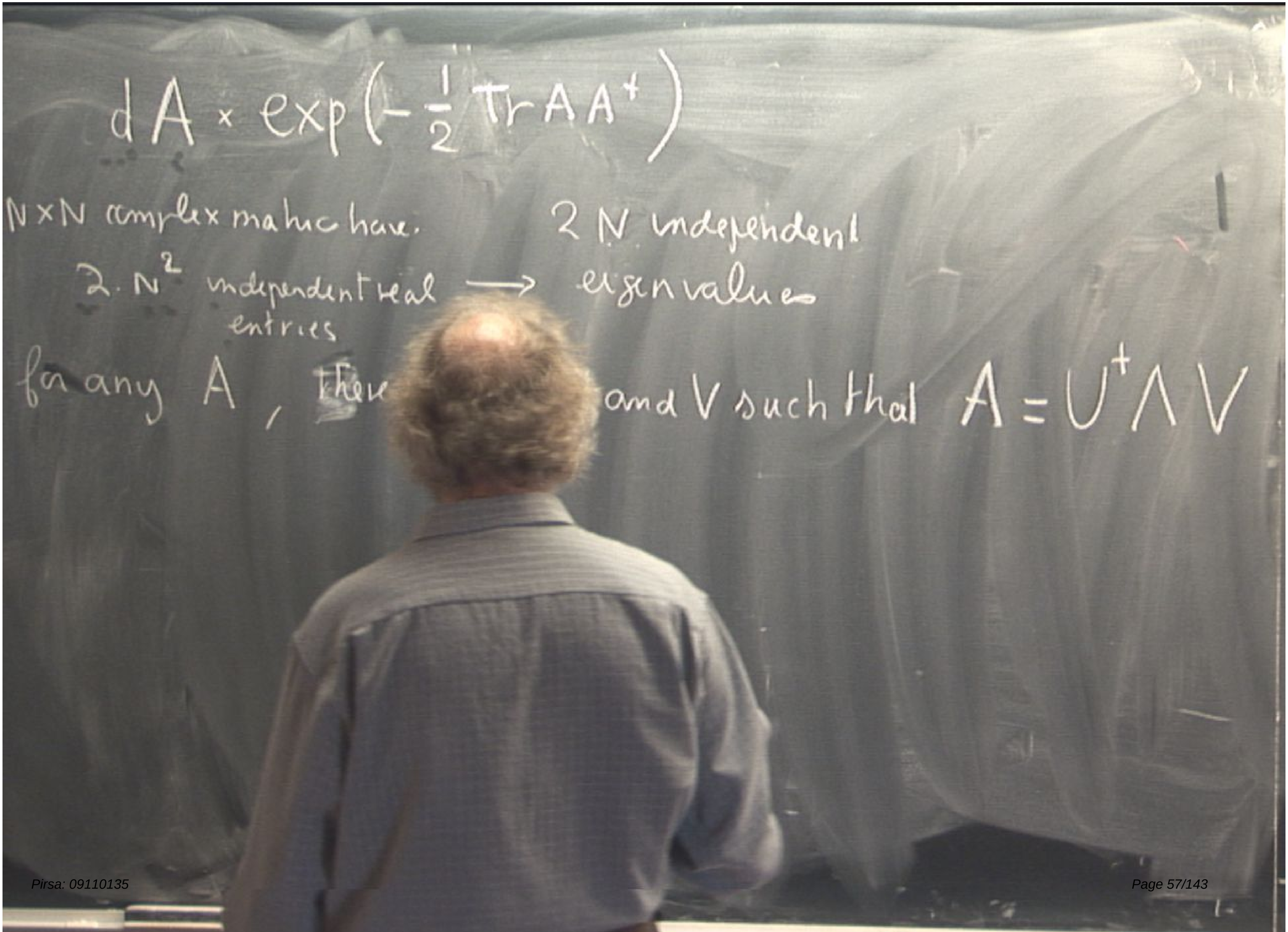
$U(N)_L \times U(N)_R$ invariance


measure of probability
A = {A_ij} random complex matrix N x N
Π dX_ij dY_ij P(A)
P(A) = Π P(X_ij) P(Y_ij) = exp(-
probability distribution over complex N x N matrices
U and V
U(N)_L x U(N

dA

$N \times N$ complex matrices have

$2 \cdot N^2$ independent real entries



$dA \times \exp\left(-\frac{1}{2}\,\mathrm{Tr}\,AA^{+}\right)$
N×N complex matrix have
2N independent
$2 \cdot N^2$ independent real → eigenvalues
entries
for any A, there
and V such that $A = U^{+}\Lambda V$

$$dA \times \exp\left(-\frac{1}{2}\,\mathrm{Tr}\,AA^{+}\right)$$

$N \times N$ complex matrices have $2 \cdot N^2$ independent real entries $\longrightarrow$ $2N$ independent eigenvalues

for any A, there exist a U and V such that $A = U^{+}\Lambda V$

$$\Lambda = \mathrm{diag}(\lambda_1, \dots \lambda_N) \to \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_N \end{pmatrix}$$

eigenvalues of A

$$dA \times \exp\left(-\frac{1}{2} \mathrm{Tr}\, A A^{+}\right) \qquad U(N)_L \times U(N)_R \text{ invariant}$$

$N \times N$ complex matrix have $\quad$ $2N$ independent

$2 \cdot N^2$ independent real entries $\longrightarrow$ eigenvalues

for any A, there exist a U and V such that $A = U^{+} \Lambda V$

$$\Lambda = \mathrm{diag}(\lambda_1, \dots \lambda_N) \to \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_N \end{pmatrix}$$

eigenvalues of A

$$dA \times \exp\left(-\frac{1}{2}\,\mathrm{Tr}\,AA^{\dagger}\right)$$

$U(N)_L \times U(N)_R$ invariance $\quad 2N^2 - 2N$

$N \times N$ complex matrices have $2 \cdot N^2$ independent real entries $\longrightarrow$ $2N$ independent eigenvalues

Fix a gauge condition

for any A, there exist a U and V such that $A = U^{\dagger} \Lambda V$

$$\Lambda = \mathrm{diag}(\lambda_1, \dots \lambda_N) = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_N \end{pmatrix}$$

eigenvalues of A

$2N^2 - 2N$ gauge conditions

$i \neq j$ $\quad \mathrm{Re}\, A_{ij} = 0$

dition $\quad \mathrm{Im}\, A_{ij} = 0$

$2N^2 - 2N$ gauge conditions

$i \neq j \quad \mathrm{Re}\, A_{ij} = 0$

...dition $\quad \mathrm{Im}\, A_{ij} = 0$

$$I = \int dA \prod_{i \neq j} \delta(\mathrm{Re}\, A_{ij}) \cdot \delta(\mathrm{Im}(A_{ij}))$$

$2N^2 - 2N$ gauge conditions

dition $\quad i \neq j \quad \mathrm{Re}\, A_{ij} = 0$

$\mathrm{Im}\, A_{ij} = 0$

$$I = \int dA \prod_{i \neq j} \delta(\mathrm{Re}\, A_{ij}) \cdot \delta(\mathrm{Im}(A_{ij}))\,(\text{F.P determinant}) \times e$$

$2N^2 - 2N$ gauge conditions

$i \neq j \quad \mathrm{Re}\, A_{ij} = 0$

dition $\quad \mathrm{Im}\, A_{ij} = 0$

$$I = \int dA \prod_{i \neq j} \delta(\mathrm{Re}\, A_{ij}) \cdot \delta(\mathrm{Im}(A_{ij}))\,(\text{F.P determinant}) \times e^{-\frac{1}{2}\mathrm{Tr}(AA^\dagger)}$$

$$= \int \prod_i dA_{ii}$$

$2N^2 - 2N$ gauge condition

$i \neq j \quad \mathrm{Re}\, A_{ij} = 0$

dition $\quad \mathrm{Im}\, A_{ij} = 0$

$$I = \int dA \prod_{i \neq j} \delta(\mathrm{Re}\, A_{ij}) \cdot \delta(\mathrm{Im}(A_{ij}))\,(\text{F.P determinant}) \times e^{-\frac{1}{2}\mathrm{Tr}(AA^{+})}$$

$$= \int \prod_i dA_{ii} \exp -\frac{1}{2} \sum_i |A_{ii}|^2$$

$2N^2 - 2N$ gauge conditions

dition

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$$= \int \prod_i dA_{ii} \exp\left(-\frac{1}{2}\sum_i |A_{ii}|^2\right) \qquad A_{ii} = \lambda_i \text{ eigenvalue of the matrix}$$

$2N^2 - 2N$ gauge condition

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$A_{ii} = \lambda_i$ eigenvalue of the matrix

F.P determinant

$2N^2 - 2N$ gauge conditions

$i \neq j \quad \mathrm{Re}\, A_{ij} = 0$

...dition $\quad \mathrm{Im}\, A_{ij} = 0$

$=1$

$$I = \int dA \prod_{i\neq j} \delta(\mathrm{Re}\, A_{ij}) \cdot \delta(\mathrm{Im}(A_{ij}))\, (\text{F.P determinant}) \times e^{-\frac{1}{2}\mathrm{Tr}(AA^{\dagger})}$$

$$= \int \prod_i dA_{ii} \exp\left(-\frac{1}{2}\sum_i |A_{ii}|^2\right)$$

$A_{ii} = \lambda_i$ eigenvalue of the matrix

$$\text{F.P determinant} = \prod_{i<j}$$

$2N^2 - 2N$ gauge conditions

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$A_{ii} = \lambda_i$ eigenvalue of the matrix

$$\text{F.P determinant} = \prod_{i<j} |\lambda_i - \lambda_j|$$

$2N^2 - 2N$ gauge conditions

dition $\quad i \neq j \quad \mathrm{Re}\, A_{ij} = 0$

$\mathrm{Im}\, A_{ij} = 0$

$$I = \int dA \prod_{i \neq j} \delta(\mathrm{Re}\, A_{ij}) \cdot \delta(\mathrm{Im}(A_{ij})) \,(\text{F.P determinant}) \times e^{-\frac{1}{2}\mathrm{Tr}(AA^\dagger)}$$

$$= \int \prod_i dA_{ii} \exp\left(-\frac{1}{2}\sum_i |A_{ii}|^2\right)$$

$A_{ii} = \lambda_i$ eigenvalue of the matrix

$$\text{F.P determinant} = \prod_{i<j} |\lambda_i - \lambda_j|^4$$

$2N^2 - 2N$ gauge conditions

dition $\quad i \neq j \quad \mathrm{Re}\, A_{ij} = 0$

$\mathrm{Im}\, A_{ij} = 0$

$$I = \int dA \prod_{i \neq j} \delta(\mathrm{Re}\, A_{ij}) \cdot \delta(\mathrm{Im}(A_{ij}))\, \Big(\text{F.P determinant} \Big) \times e^{-\frac{1}{2}\mathrm{Tr}(AA^{+})}$$

$$= \int \prod_i dA_{ii} \exp\Big(-\frac{1}{2}\sum_i |A_{ii}|^2\Big) \qquad A_{ii} = \lambda_i \text{ eigenvalue of the matrix}$$

$$\text{F.P determinant} = \prod_{i<j} |\lambda_i - \lambda_j|^4$$

$$= \int dA \times \exp\left(-\frac{1}{2} \mathrm{Tr} A A^{+}\right)$$

$$= \int_{\mathbb{C}^N} \prod_{i=1}^{N} d\lambda_i$$

$$I = \int dA \times \exp\left(-\frac{1}{2} \operatorname{Tr} A A^\dagger\right)$$

$$= \int_{\mathbb{C}^N} \prod_{i=1}^{N} d\lambda_i \, \exp\left[-\left[\sum_{i<j}\left(-\log|\lambda_i - \lambda_j| + \frac{1}{2}\sum_i |\lambda_i|^2\right)\right]\right]$$

$$I = \int dA \times \exp\left(-\frac{1}{2} \operatorname{Tr} A A^{\dagger}\right)$$

$$= \int_{\mathbb{C}^N} \prod_{i=1}^{N} d\lambda_i \, \exp\left[-\left[\sum_{i<j}\left(-\log|\lambda_i - \lambda_j| + \frac{1}{2}\sum_i |\lambda_i|^2\right)\right]\right]$$

$$I = \int dA \times \exp\left(-\frac{1}{2} \operatorname{Tr} A A^{\dagger}\right)$$

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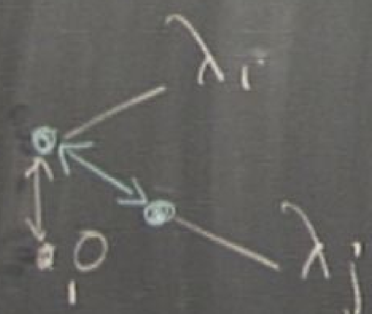
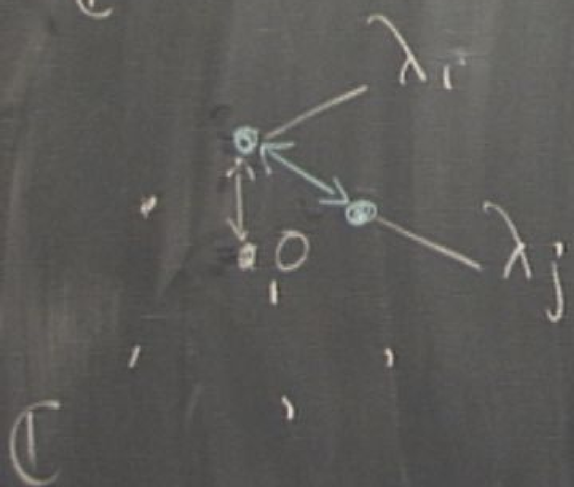


Partition function (statistical mecha
of a gas of N points with position λ
inside an harmonic potential

$$I = \int dA \times \exp\left(-\frac{1}{2} \operatorname{Tr} A A^{\dagger}\right)$$

$$= \int_{\mathbb{C}^N} \prod_{i=1}^{N} d\lambda_i \, \exp\left[-\left[\sum_{i<j}\left(-\log|\lambda_i - \lambda_j| + \frac{1}{2}\sum_i |\lambda_i|^2\right)\right]\right]$$

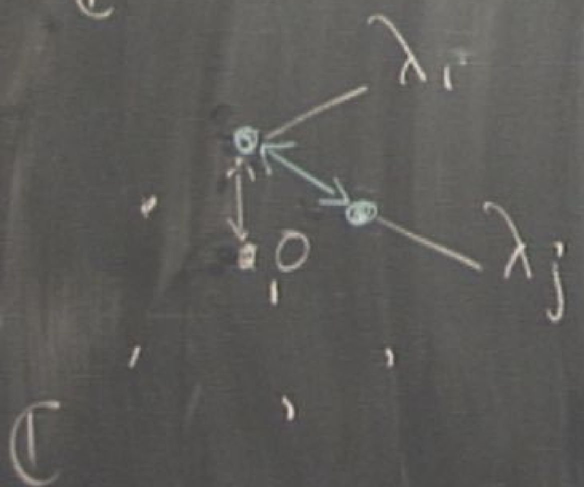


Partition function (statistical mecha
of a gas of N points with position λ
inside an harmonic potential

Coulomb potential in 2 dimensions

$$I = \int dA \times \exp\left(-\frac{1}{2}\,\mathrm{Tr}\,AA^{+}\right)$$

$$= \int_{\mathbb{C}^N} \prod_{i=1}^{N} d\lambda_i \; \exp\left[-\left[\sum_{i<j}\left(-\log|\lambda_i-\lambda_j| + \frac{1}{2}\sum_i |\lambda_i|^2\right)\right]\right]$$



Partition function (statistical mecha
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inside an harmonic potential

Coulomb potential in 2 dimensions

N charged particle

$$I = \int dA \times \exp\left(-\frac{1}{2} \mathrm{Tr} A A^\dagger\right)$$

$$= \int_{\mathbb{C}^N} \prod_{i=1}^{N} d\lambda_i \exp\left[-\left[\sum_{i<j}\left(-\log|\lambda_i - \lambda_j| + \frac{1}{2}\sum_i |\lambda_i|^2\right)\right]\right]$$

λ_j

Partition function (statistical mecha
of a gas of N points with position λ
inside an harmonic potential

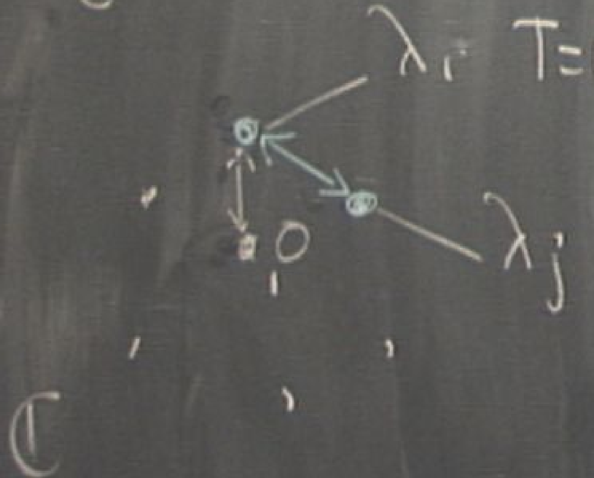
Coulomb potential in 2 dimensions

N charged particle ; Coulomb gas

Temperature normalized to 1

$$I = \int dA \times \exp\left(-\frac{1}{2} \operatorname{Tr} A A^{\dagger}\right)$$

$$= \int_{\mathbb{C}^N} \prod_{i=1}^{N} d\lambda_i \, \exp\left[-\left[\sum_{i<j}\left(-\log|\lambda_i - \lambda_j| + \frac{1}{2}\sum_i |\lambda_i|^2\right)\right]\right]$$



Partition function (statistical mecha
of a gas of N points with position λ
inside an harmonic potential

Coulomb potential in 2 dimens

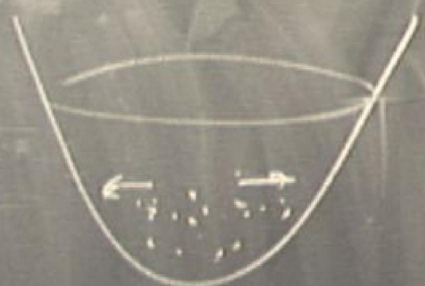
N charged particle ; Coulomb

Temperature normalized to 1

$2N^2 - 2N$ gauge conditions

$i \neq j \quad \mathrm{Re}\, A_{ij} = 0$

$\mathrm{Im}\, A_{ij} = 0$



$A_{ij}))\left(\text{F.P determinant}\right) \times e^{-\frac{1}{2}\mathrm{Tr}(AA^{+})}$

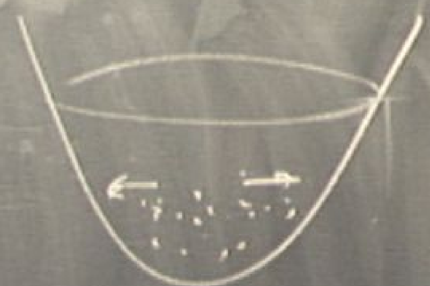
$A_{ii} = \lambda_i$ eigenvalue of the matrix

$|\lambda_i - \lambda_j|^{4\ (\text{or } 2\ ?)}$

$2N^2 - 2N$ gauge conditions

$i \neq j \quad \mathrm{Re}\, A_{ij} = 0$

$\mathrm{Im}\, A_{ij} = 0$

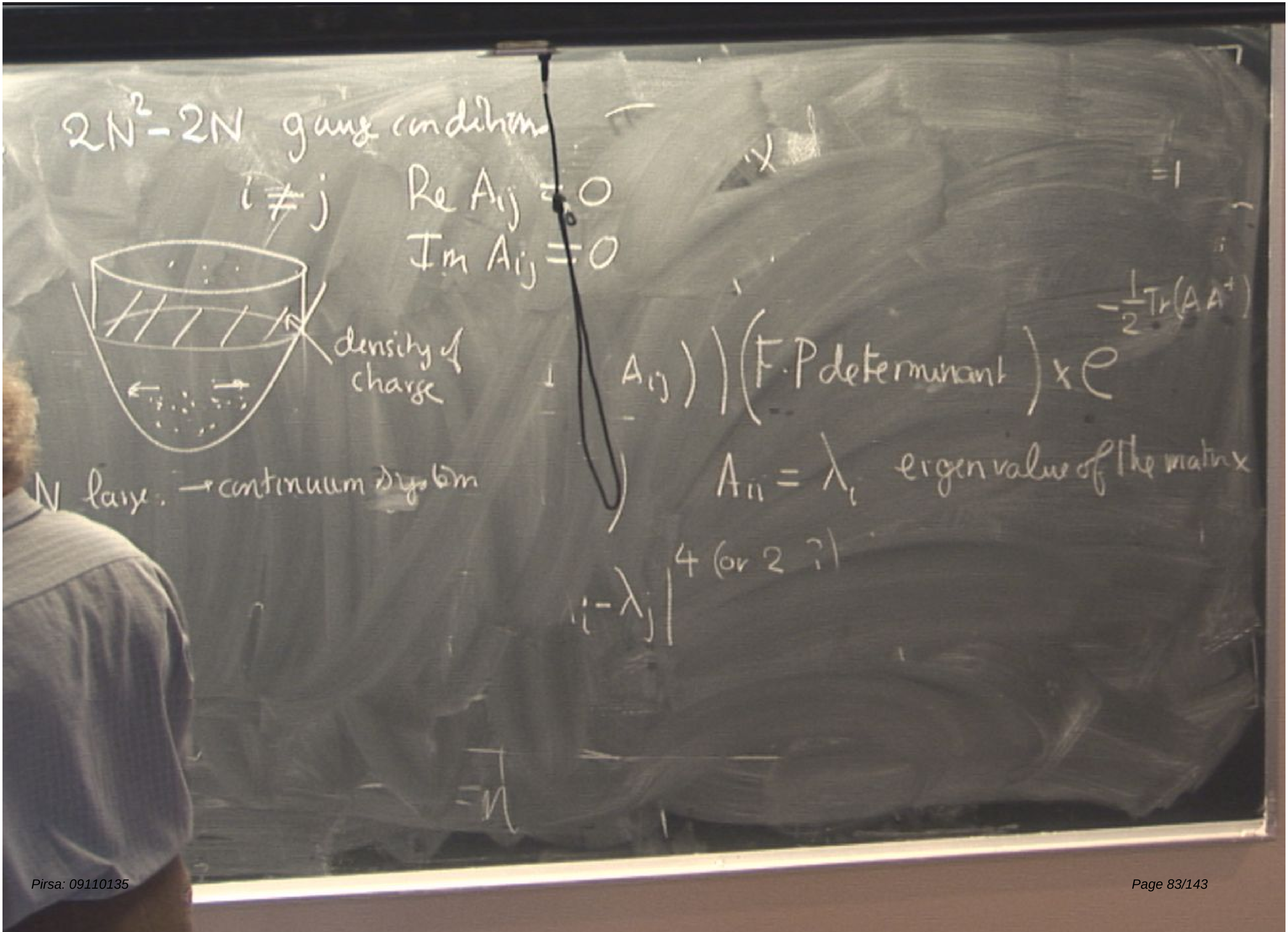


N large. → continuum system

$A_{ij}))\left(\text{F.P determinant}\right) \times e^{-\frac{1}{2}\mathrm{Tr}(AA^{+})}$

$A_{ii} = \lambda_i$ eigenvalue of the matrix

$\lambda_i - \lambda_j|^{4\ (\text{or } 2\ ?)}$

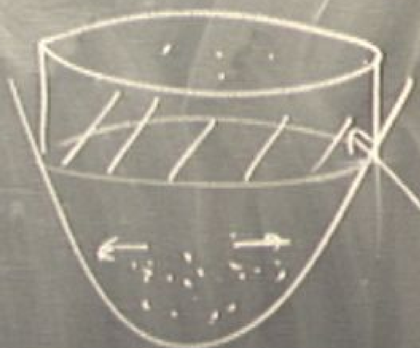


2N² - 2N gauge conditions
i ≠ j Re Aij = 0
Im Aij = 0
density of charge
N large: → continuum system
Aij)) (F.P determinant) x e^(-1/2 Tr(AA†))
Aii = λi eigenvalue of the matrix
λi - λj|^4 (or 2 ?)

$2N^2 - 2N$ gauge conditions

$i \neq j$ $\quad \mathrm{Re}\, A_{ij} = 0$

$\mathrm{Im}\, A_{ij} = 0$



density of charge

N large: → continuum system

$=1$

$$A_{ij})\,)\left(\text{F.P determinant}\right) \times e^{-\frac{1}{2}\mathrm{Tr}(AA^{\dagger})}$$

$A_{ii} = \lambda_i$ eigenvalue of the matrix

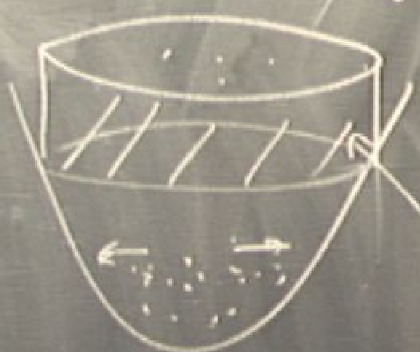
$$\lambda_i - \lambda_j|^{4\ (\text{or } 2\ ?)}$$

$=N$

$2N^2 - 2N$ gauge conditions

$i \neq j \quad \mathrm{Re}\, A_{ij} = 0$

$\mathrm{Im}\, A_{ij} = 0$



density of charge

N large → continuum system

electrostatics

$A_{ij})\,)\,\left(\text{F.P determinant}\right) \times e^{-\frac{1}{2}\mathrm{Tr}(AA^{+})}$

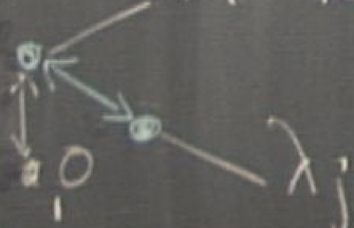
$) \quad A_{ii} = \lambda_i$ eigenvalue of the matrix

$\lambda_i - \lambda_j|^{4\ (\text{or } 2\ ?)}$

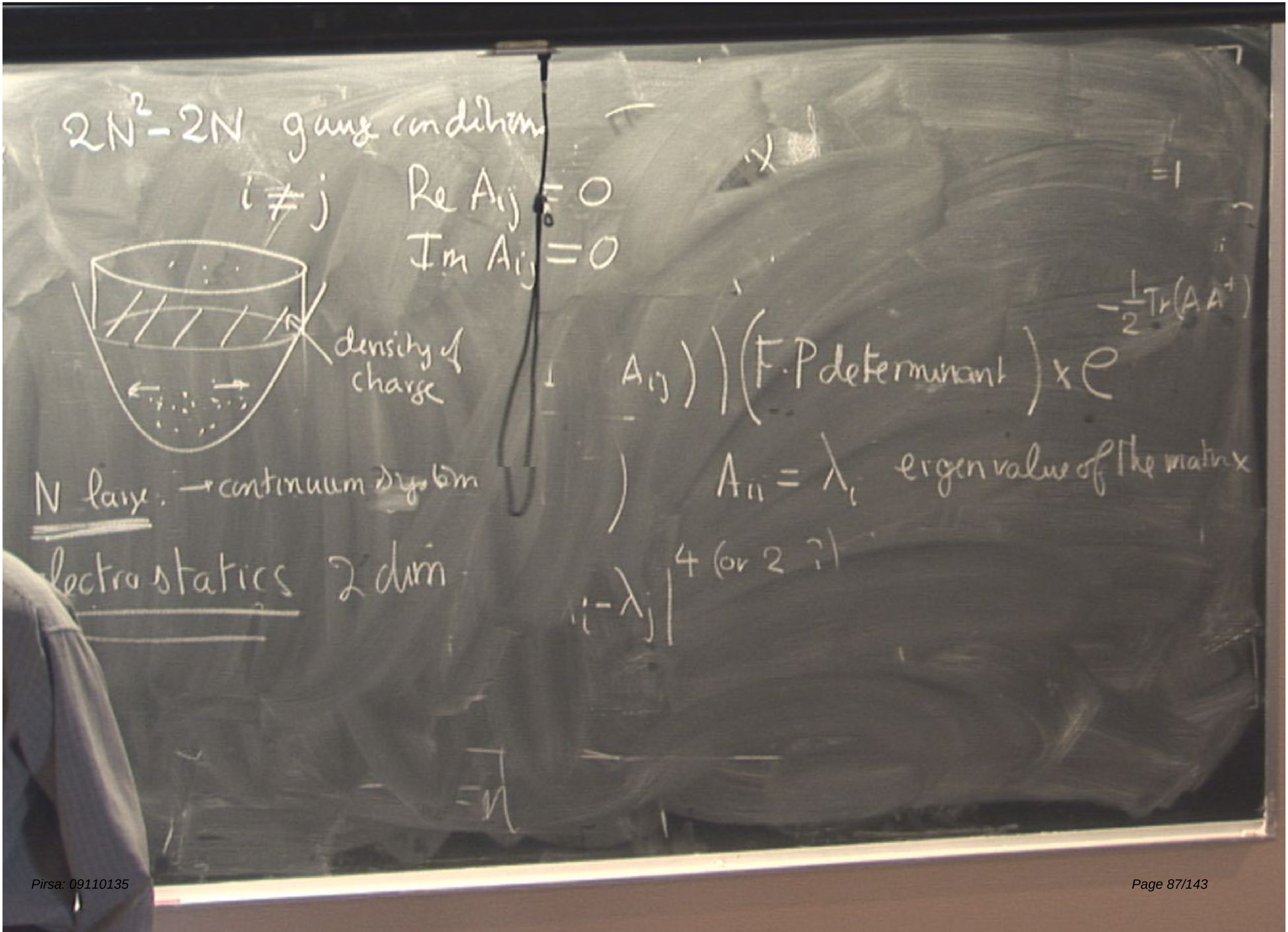
$$= \int dA \times \exp\left(-\frac{1}{2} \operatorname{Tr} A A^{\dagger}\right)$$

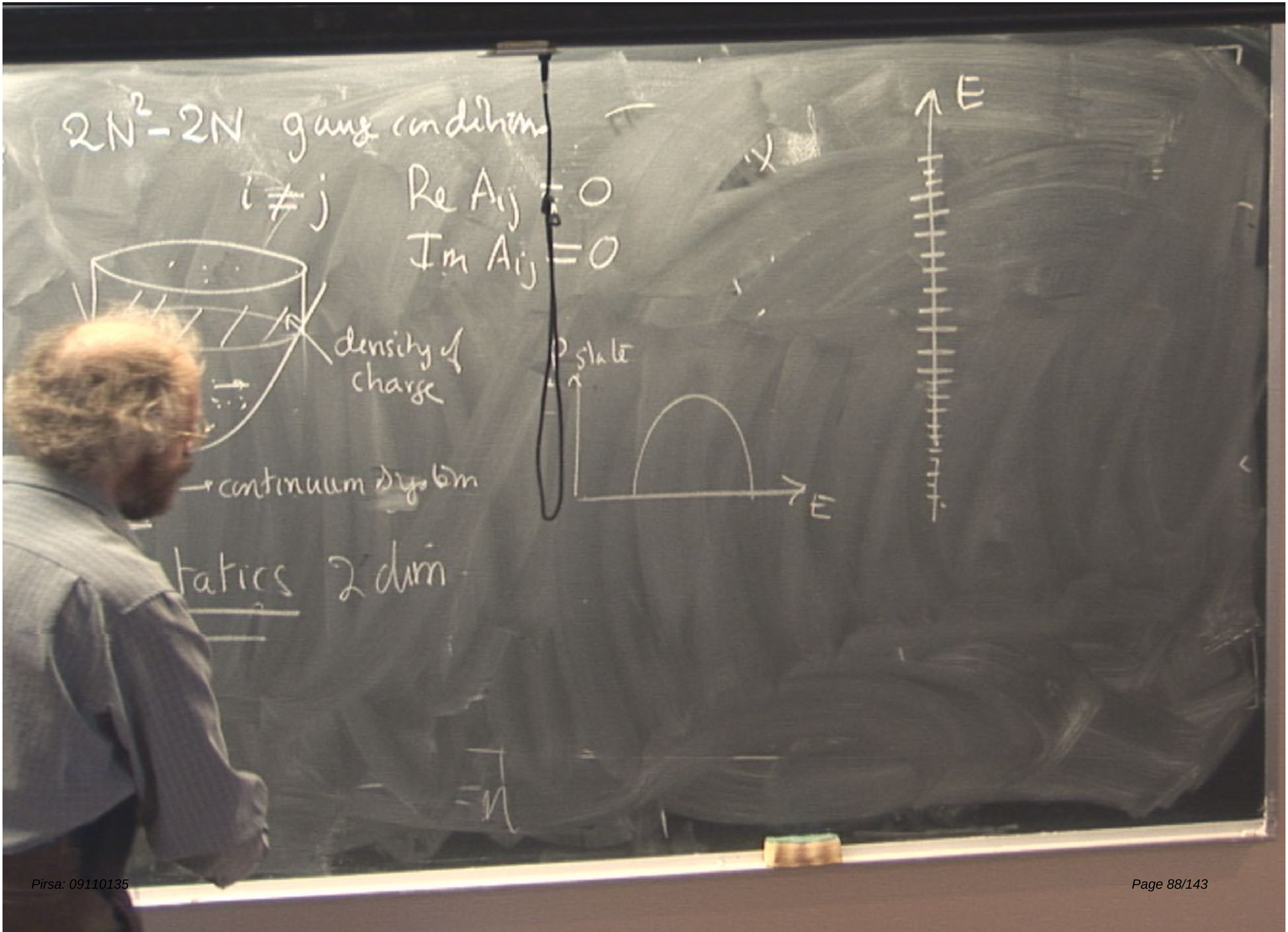
$$= \int \prod_{i=1}^{N} d\lambda_i \, \exp\left[-\left[\overbrace{\sum_{i<j} \left(-\log|\lambda_i - \lambda_j|\right)}^{\text{order } N^2} + \overbrace{\frac{N}{2} \sum_i |\lambda_i|^2}^{\text{order } N}\right)\right]$$

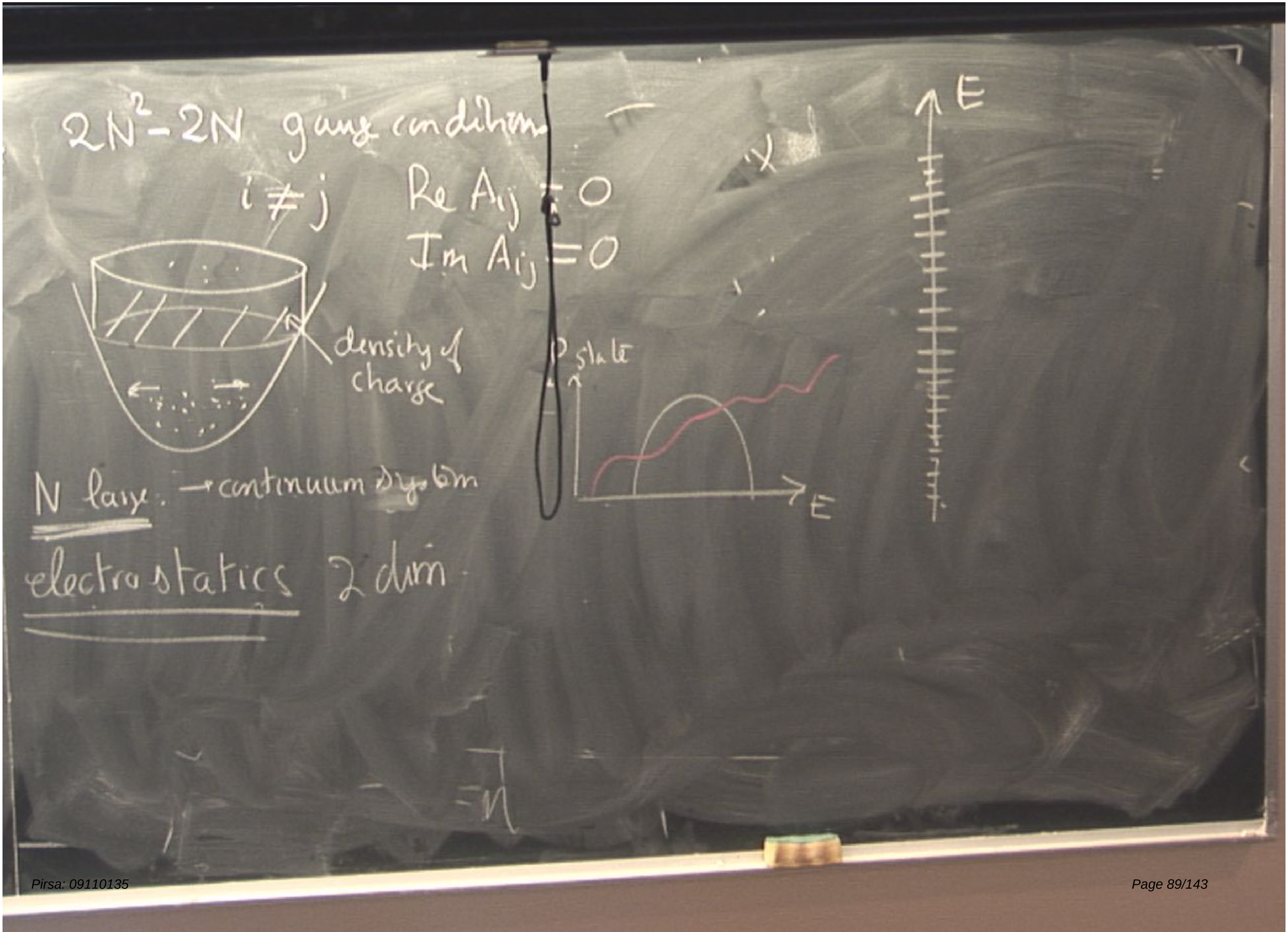
λ_i $T=1$

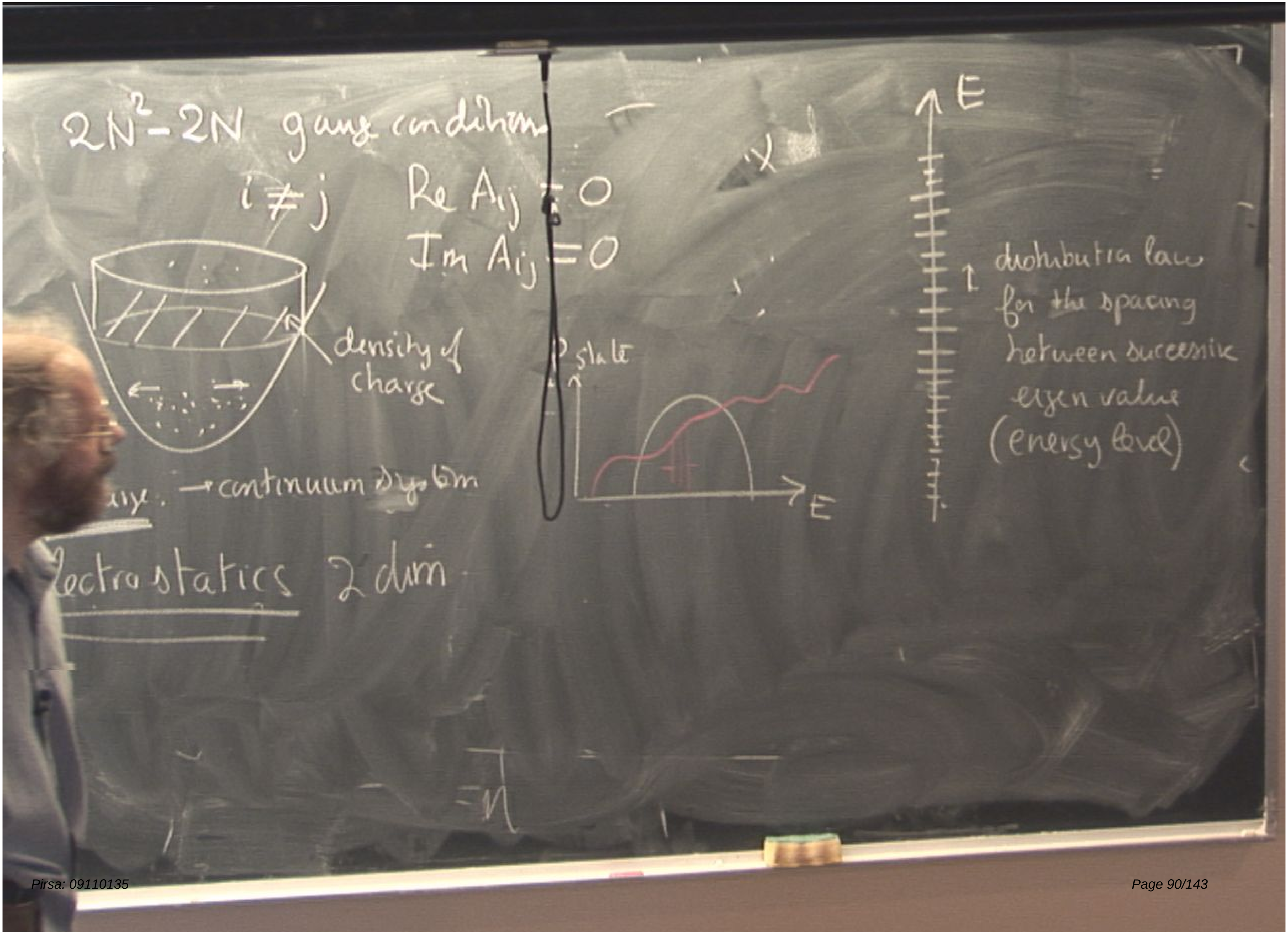


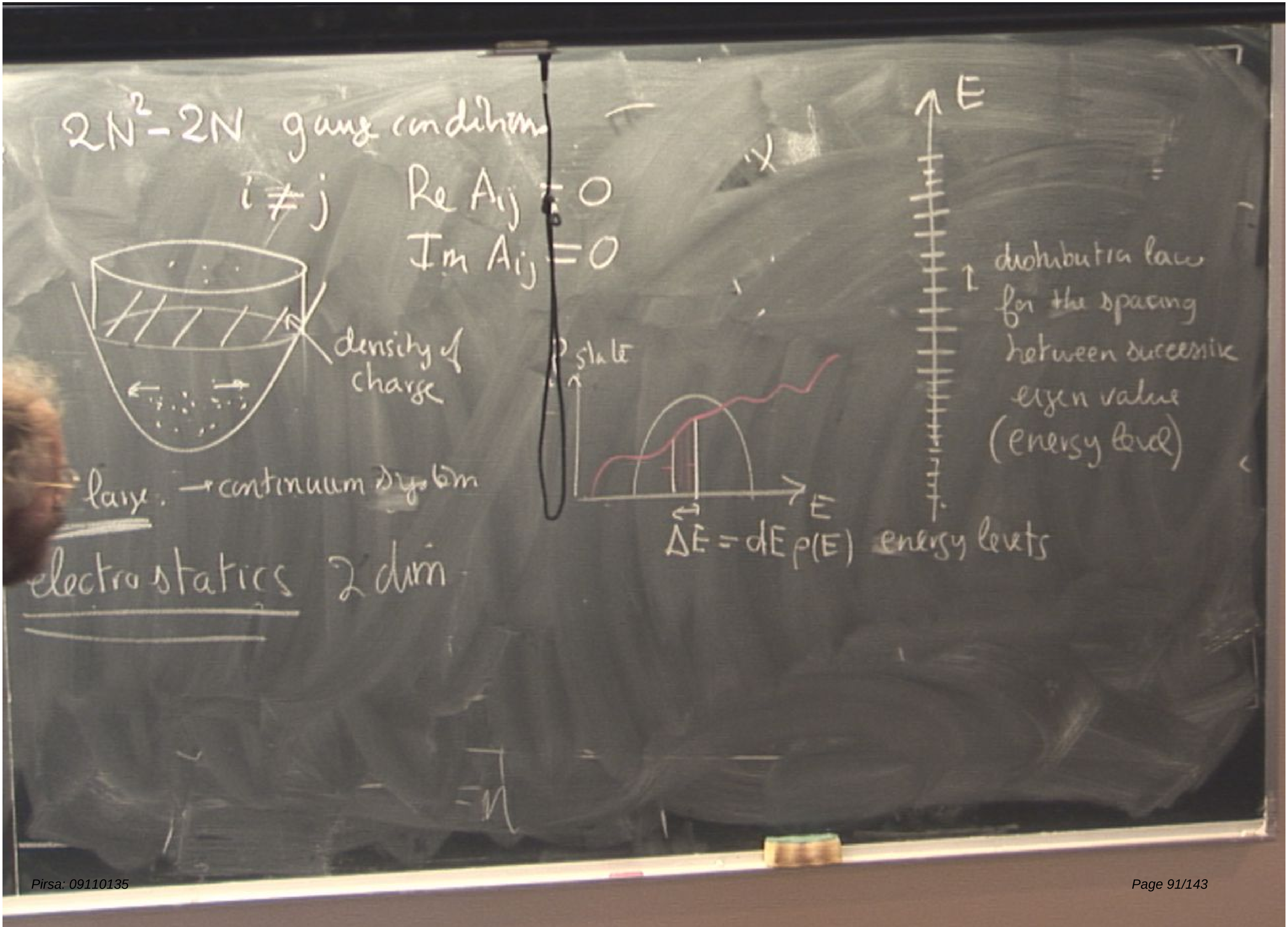
Partition function (statistical mechanics)
of a gas of N points with position λ_i.
inside an harmonic potential
Coulomb potential in 2 dimensions
N charged particle : Coulomb gas
Temperature normalized to 1

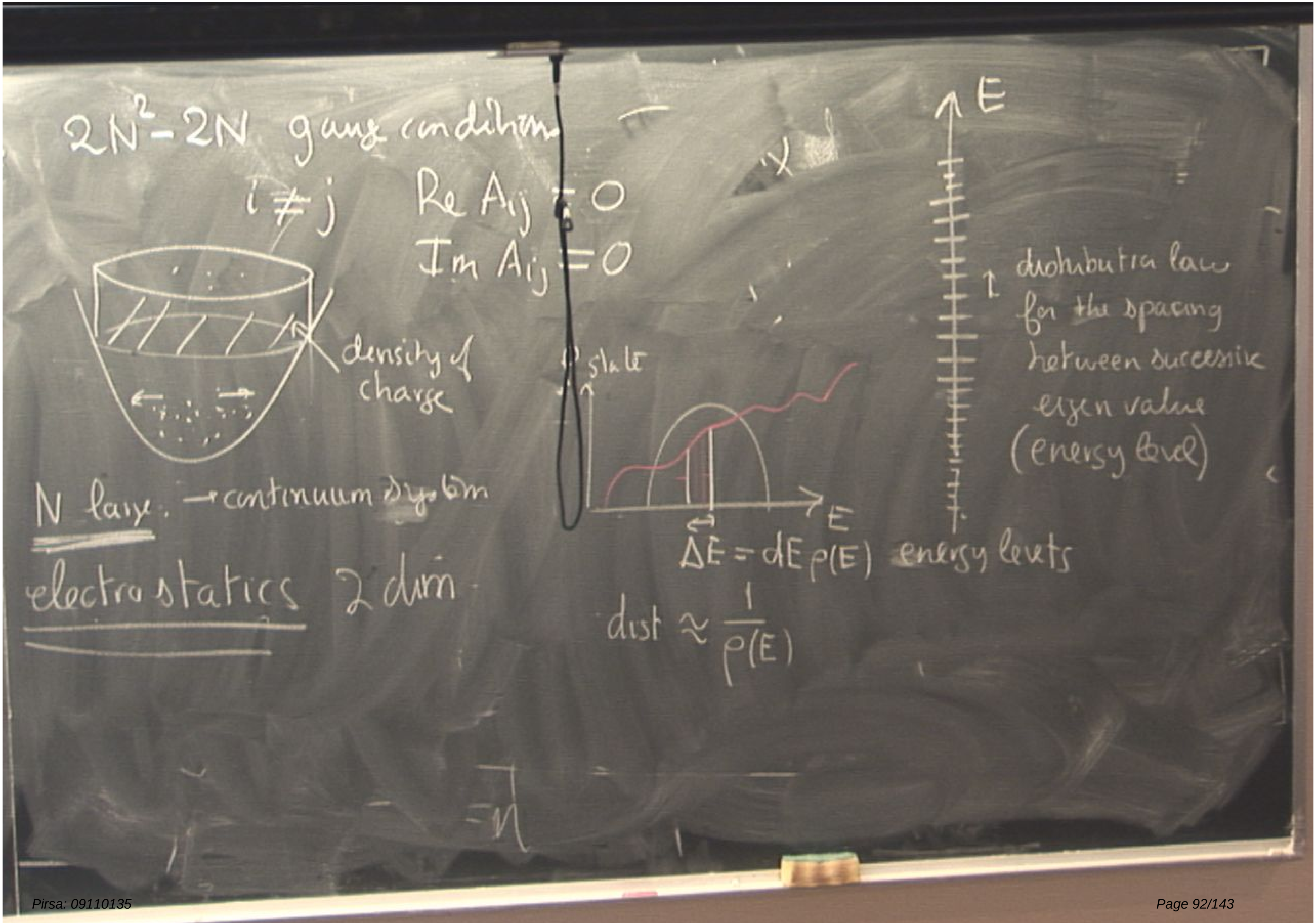

2N2 - 2N gauge conditions
i ≠ j Re Aij = 0
Im Aij = 0
density of charge
N large → continuum system
electrostatics 2 dim
Aij)) (F.P determinant) x e -1/2 Tr(AA†)
Aii = λi eigenvalue of the matrix
λi - λj|4 (or 2 ?)

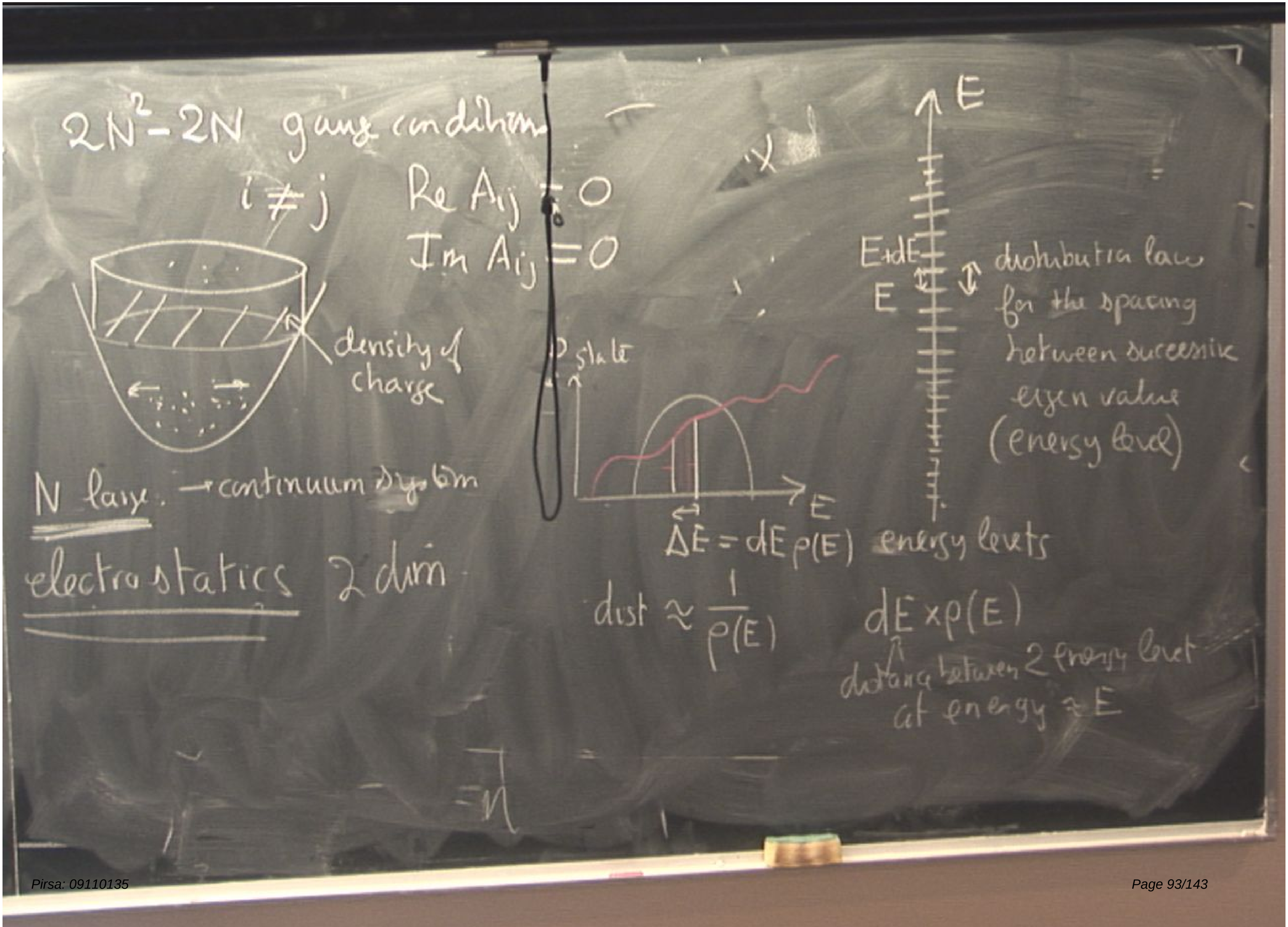

2N² - 2N gauge conditions
i ≠ j
Re A_ij = 0
Im A_ij = 0
density of charge
continuum system
ρ state
E
E
tatics 2 dim

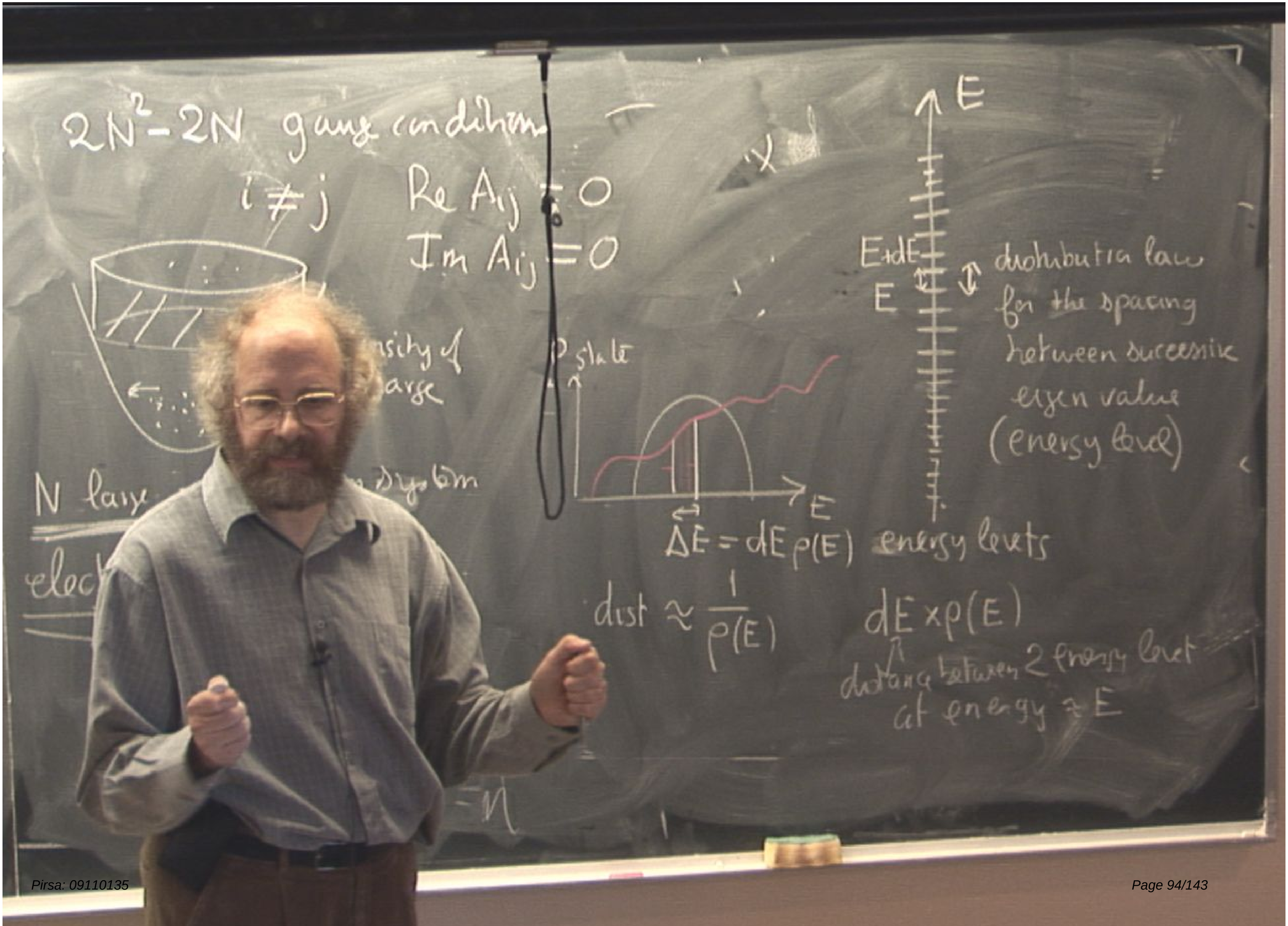

2N² - 2N gauge conditions
i ≠ j
Re A_ij = 0
Im A_ij = 0
density of charge
N large → continuum system
electrostatics 2 dim
ρ state
E
E


2N² - 2N gauge conditions
i ≠ j
Re Aij = 0
Im Aij = 0
density of
charge
→ continuum system
lectrostatics 2 dim
ρ state
E
E
distribution law
for the spacing
between successive
eigen value
(energy level)


$2N^2 - 2N$ gauge conditions
$i \neq j$ $\mathrm{Re}\, A_{ij} = 0$
$\mathrm{Im}\, A_{ij} = 0$
density of charge
large → continuum system
electrostatics 2 dim
ρ state
E
$\Delta E = dE\, \rho(E)$
energy levels
E
distribution law for the spacing between successive eigen value (energy level)


2N^2 - 2N gauge conditions
i ≠ j Re A_ij = 0
Im A_ij = 0
density of charge
N large → continuum system
electrostatics 2 dim
state
E
ΔE = dE ρ(E) energy levels
dist ≈ 1/ρ(E)
E
distribution law for the spacing between successive eigen value (energy level)


2N² - 2N gauge conditions
i ≠ j Re Aij = 0
Im Aij = 0
density of charge
N large → continuum system
electrostatics 2 dim
ρ state
E
ΔE = dE ρ(E) energy levels
dist ≈ 1/ρ(E)
dE × ρ(E)
distance between 2 energy level at energy ≈ E
E
E+dE
E
distribution law for the spacing between successive eigen value (energy level)


2N²-2N gauge conditions
i ≠ j Re Aij = 0
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N large
ρ state
E
ΔE = dE ρ(E) energy levels
dist ≈ 1/ρ(E)
dE × ρ(E)
distance between 2 energy level
at energy ≈ E
E
E+dE
E
distribution law
for the spacing
between successive
eigen value
(energy level)

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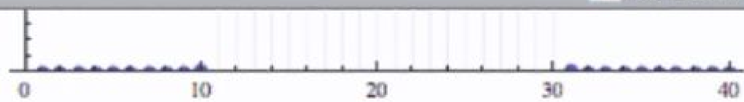
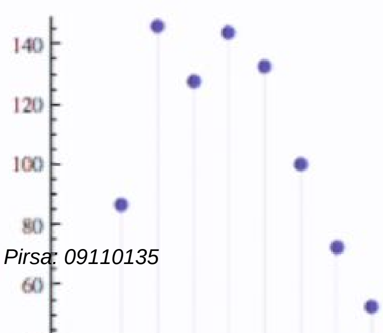
Random matrix PSI.nb



Vigner distribution: Distance between eigenvalues

- 1 random symmetric matrix with entries in [-1, 1]

```
size = 100;
matrixD = RandomReal[{-1, 1} / (2 size) ^ (1 / 2), {size, size}];
matrixS = (matrixD + Transpose[matrixD]) / 2;
ev = Sort[Eigenvalues[matrixS]];
(* Print[ev]; *)
listc = BinCounts[ev, {-1, 1, 1 / 20}];
listl = BinLists[ev, {-1, 1, 1 / 20}];
Table[Length[listl[[i]]], {i, 1, Length[listl]}];
listdist = {};
Do[
  If[listc[[i]] > 1,
   listtmp = Sort[listl[[i]]];
   (* Print["sousensemble de vp",listtmp]; *)
   disttmp = Table[(listtmp[[j + 1]] - listtmp[[j]]) listc[[i]] , {j, 1, listc[[i]] - 1}];
   (* Print[disttmp];  *)
   listdist = Join[listdist, disttmp];
  ],
  {i, 1, Length[listc]}];
listdist;
listcount = BinCounts[listdist, {0, 1 / 5, 1 / 100}];
ListPlot[listcount, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → Automatic]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help François David

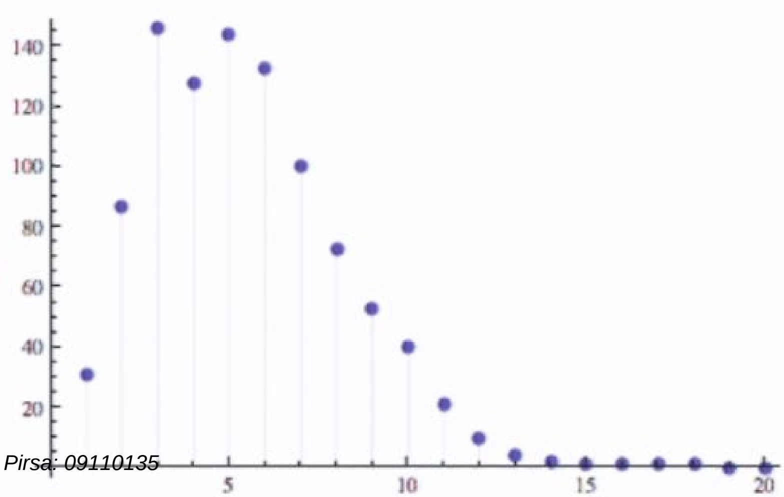
Random matrix PSI.nb

/igner distribution: Distance between eigenvalues

▪ 1 random symmetric matrix with entries in [-1, 1]

```
size = 100;
matrixD = RandomReal[{-1, 1} / (2 size)^(1/2), {size, size}];
matrixS = (matrixD + Transpose[matrixD]) / 2;
ev = Sort[Eigenvalues[matrixS]];
(* Print[ev]; *)
listc = BinCounts[ev, {-1, 1, 1/20}];
listl = BinLists[ev, {-1, 1, 1/20}];
Table[Length[listl[[i]]], {i, 1, Length[listl]}];
listdist = {};
Do[
  If[listc[[i]] > 1,
   listtmp = Sort[listl[[i]]];
   (* Print["sousensemble de vp",listtmp]; *)
   disttmp = Table[(listtmp[[j + 1]] - listtmp[[j]]) listc[[i]] , {j, 1, listc[[i]] - 1}];
   (* Print[disttmp];  *)
   listdist = Join[listdist, disttmp];
  ],
  {i, 1, Length[listc]}];
listdist;
listcount = BinCounts[listdist, {0, 1/5, 1/100}];
ListPlot[listcount, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → Automatic]
```

64]=

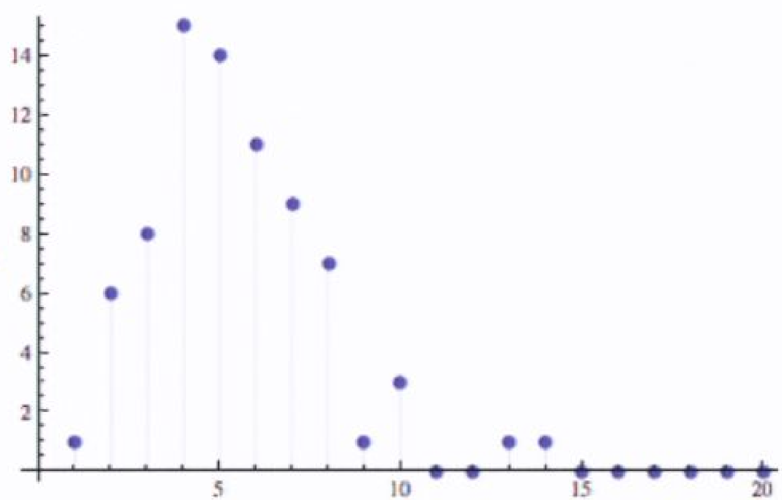


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help François David

Random matrix PSI.nb

■ random symmetric matrix with entries in [-1, 1]

```
size = 100;
matrixD = RandomReal[{-1, 1} / (2 size) ^ (1 / 2), {size, size}];
matrixS = (matrixD + Transpose[matrixD]) / 2;
ev = Sort[Eigenvalues[matrixS]];
(* Print[ev]; *)
listc = BinCounts[ev, {-1, 1, 1 / 20}];
listl = BinLists[ev, {-1, 1, 1 / 20}];
Table[Length[listl[[i]]], {i, 1, Length[listl]}];
listdist = {};
Do[
  If[listc[[i]] > 1,
   listtmp = Sort[listl[[i]]];
   (* Print["sousensemble de vp",listtmp]; *)
   disttmp = Table[(listtmp[[j + 1]] - listtmp[[j]]) listc[[i]] , {j, 1, listc[[i]] - 1}];
   (* Print[disttmp];  *)
   listdist = Join[listdist, disttmp];
  ],
  {i, 1, Length[listc]}];
listdist;
listcount = BinCounts[listdist, {0, 1 / 5, 1 / 100}];
ListPlot[listcount, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → Automatic]
```

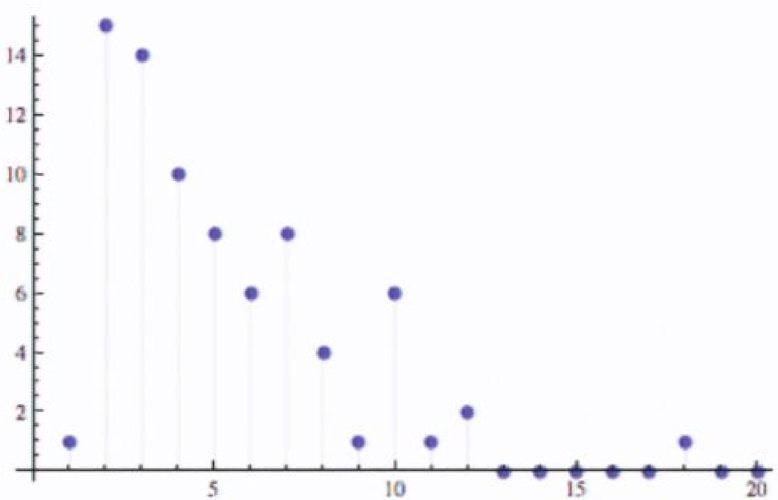


■ average over independent random symmetric matrix with entries in [-1, 1]

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help François David

Random matrix PSI.nb

```
size = 100;
matrixD = RandomReal[{-1, 1} / (2 size) ^ (1 / 2), {size, size}];
matrixS = (matrixD + Transpose[matrixD]) / 2;
ev = Sort[Eigenvalues[matrixS]];
(* Print[ev]; *)
listc = BinCounts[ev, {-1, 1, 1 / 20}];
listl = BinLists[ev, {-1, 1, 1 / 20}];
Table[Length[listl[[i]]], {i, 1, Length[listl]}];
listdist = {};
Do[
  If[listc[[i]] > 1,
   listtmp = Sort[listl[[i]]];
   (* Print["sousensemble de vp",listtmp]; *)
   disttmp = Table[(listtmp[[j + 1]] - listtmp[[j]]) listc[[i]] , {j, 1, listc[[i]] - 1}];
   (* Print[disttmp];  *)
   listdist = Join[listdist, disttmp];
  ],
  {i, 1, Length[listc]}];
listdist;
listcount = BinCounts[listdist, {0, 1 / 5, 1 / 100}];
ListPlot[listcount, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → Automatic]
```

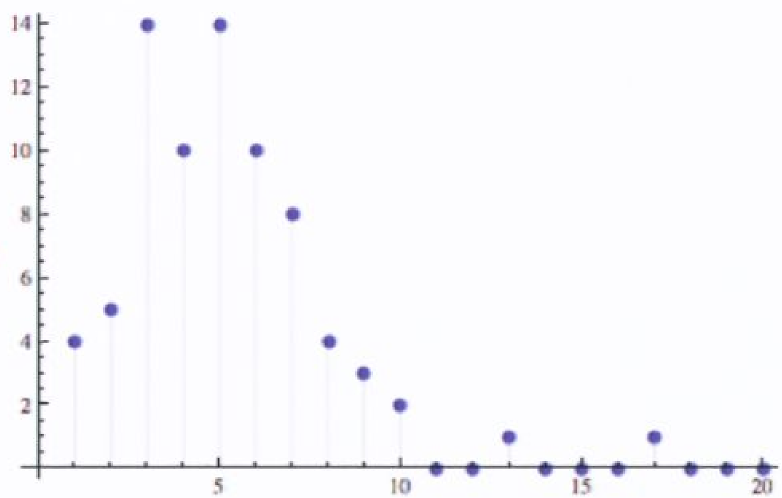


▪ average over independent random symmetric matrix with entries in [-1, 1]

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help François David

Random matrix PSI.nb

```
size = 100;
matrixD = RandomReal[{-1, 1} / (2 size)^(1/2), {size, size}];
matrixS = (matrixD + Transpose[matrixD]) / 2;
ev = Sort[Eigenvalues[matrixS]];
(* Print[ev]; *)
listc = BinCounts[ev, {-1, 1, 1/20}];
listl = BinLists[ev, {-1, 1, 1/20}];
Table[Length[listl[[i]]], {i, 1, Length[listl]}];
listdist = {};
Do[
  If[listc[[i]] > 1,
   listtmp = Sort[listl[[i]]];
   (* Print["sousensemble de vp",listtmp]; *)
   disttmp = Table[(listtmp[[j + 1]] - listtmp[[j]]) listc[[i]] , {j, 1, listc[[i]] - 1}];
   (* Print[disttmp];  *)
   listdist = Join[listdist, disttmp];
  ],
  {i, 1, Length[listc]}];
listdist;
listcount = BinCounts[listdist, {0, 1/5, 1/100}];
ListPlot[listcount, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → Automatic]
```



- average over independent random symmetric matrix with entries in [-1, 1]

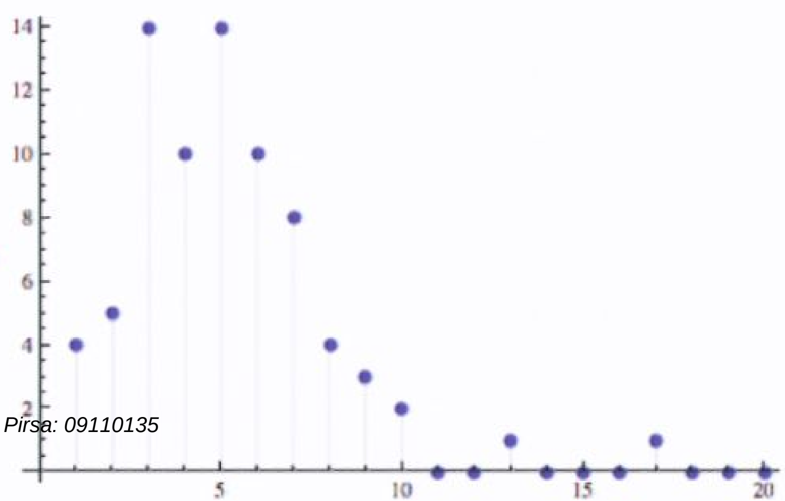
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help François David

Running...Random matrix PSI.nb

igner distribution: Distance between eigenvalues

- 1 random symmetric matrix with entries in [-1, 1]

```
size = 1000;
matrixD = RandomReal[{-1, 1} / (2 size) ^ (1 / 2), {size, size}];
matrixS = (matrixD + Transpose[matrixD]) / 2;
ev = Sort[Eigenvalues[matrixS]];
(* Print[ev]; *)
listc = BinCounts[ev, {-1, 1, 1 / 20}];
listl = BinLists[ev, {-1, 1, 1 / 20}];
Table[Length[listl[[i]]], {i, 1, Length[listl]}];
listdist = {};
Do[
  If[listc[[i]] > 1,
   listtmp = Sort[listl[[i]]];
   (* Print["sousensemble de vp",listtmp]; *)
   disttmp = Table[(listtmp[[j + 1]] - listtmp[[j]]) listc[[i]] , {j, 1, listc[[i]] - 1}];
   (* Print[disttmp];  *)
   listdist = Join[listdist, disttmp];
  ],
  {i, 1, Length[listc]}];
listdist;
listcount = BinCounts[listdist, {0, 1 / 5, 1 / 100}];
ListPlot[listcount, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → Automatic]
```

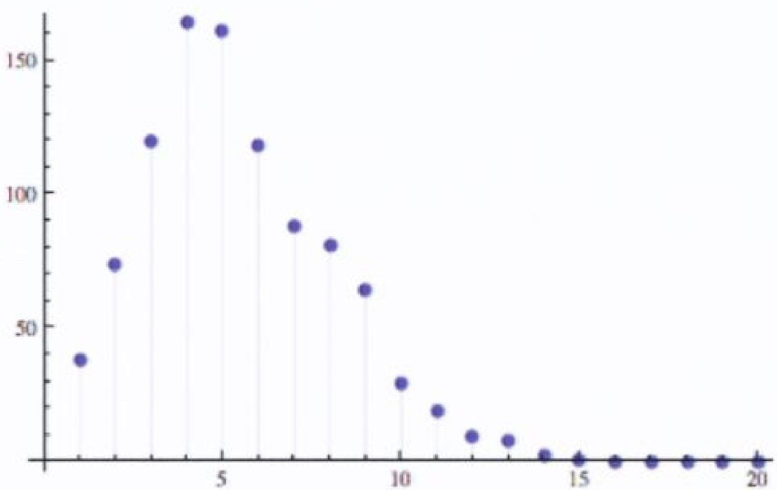


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help François David

Random matrix PSI.nb

■ 1 random symmetric matrix with entries in [-1, 1]

```
size = 1000;
matrixD = RandomReal[{-1, 1} / (2 size) ^ (1 / 2), {size, size}];
matrixS = (matrixD + Transpose[matrixD]) / 2;
ev = Sort[Eigenvalues[matrixS]];
(* Print[ev]; *)
listc = BinCounts[ev, {-1, 1, 1 / 20}];
listl = BinLists[ev, {-1, 1, 1 / 20}];
Table[Length[listl[[i]]], {i, 1, Length[listl]}];
listdist = {};
Do[
  If[listc[[i]] > 1,
   listtmp = Sort[listl[[i]]];
   (* Print["sousensemble de vp",listtmp]; *)
   disttmp = Table[(listtmp[[j + 1]] - listtmp[[j]]) listc[[i]] , {j, 1, listc[[i]] - 1}];
   (* Print[disttmp];  *)
   listdist = Join[listdist, disttmp];
  ],
  {i, 1, Length[listc]}];
listdist;
listcount = BinCounts[listdist, {0, 1 / 5, 1 / 100}];
ListPlot[listcount, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → Automatic]
```



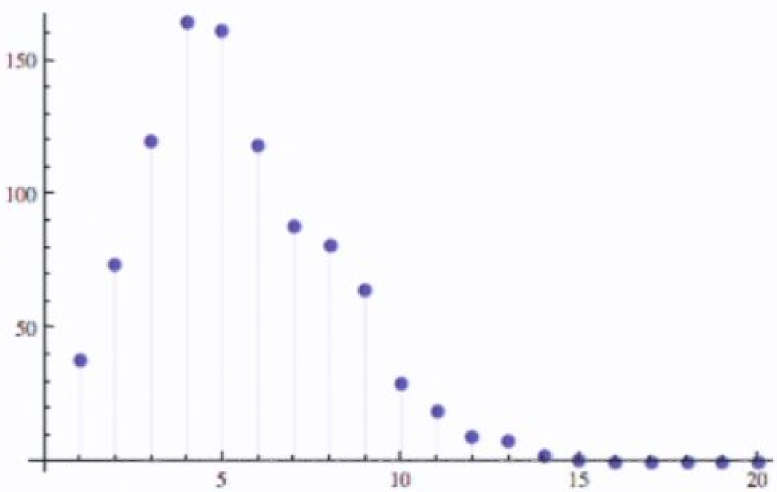
■ average over independent random symmetric matrix with entries in [-1, 1]

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help François David

Running...Random matrix PSI.nb

■ 1 random symmetric matrix with entries in [-1, 1]

```
size = 1000;
matrixD = RandomReal[{-1, 1} / (2 size) ^ (1 / 2), {size, size}];
matrixS = (matrixD + Transpose[matrixD]) / 2;
ev = Sort[Eigenvalues[matrixS]];
(* Print[ev]; *)
listc = BinCounts[ev, {-1, 1, 1 / 20}];
listl = BinLists[ev, {-1, 1, 1 / 20}];
Table[Length[listl[[i]]], {i, 1, Length[listl]}];
listdist = {};
Do[
  If[listc[[i]] > 1,
   listtmp = Sort[listl[[i]]];
   (* Print["sousensemble de vp",listtmp]; *)
   disttmp = Table[(listtmp[[j + 1]] - listtmp[[j]]) listc[[i]] , {j, 1, listc[[i]] - 1}];
   (* Print[disttmp];  *)
   listdist = Join[listdist, disttmp];
  ],
  {i, 1, Length[listc]}];
listdist;
listcount = BinCounts[listdist, {0, 1 / 5, 1 / 100}];
ListPlot[listcount, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → Automatic]
```

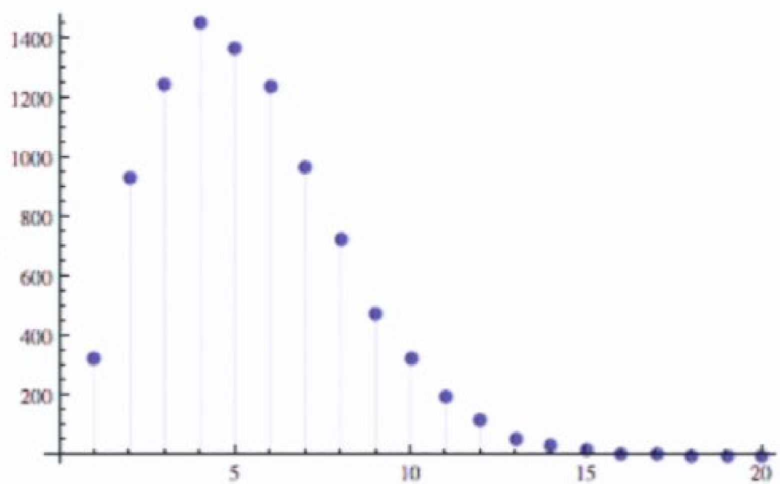


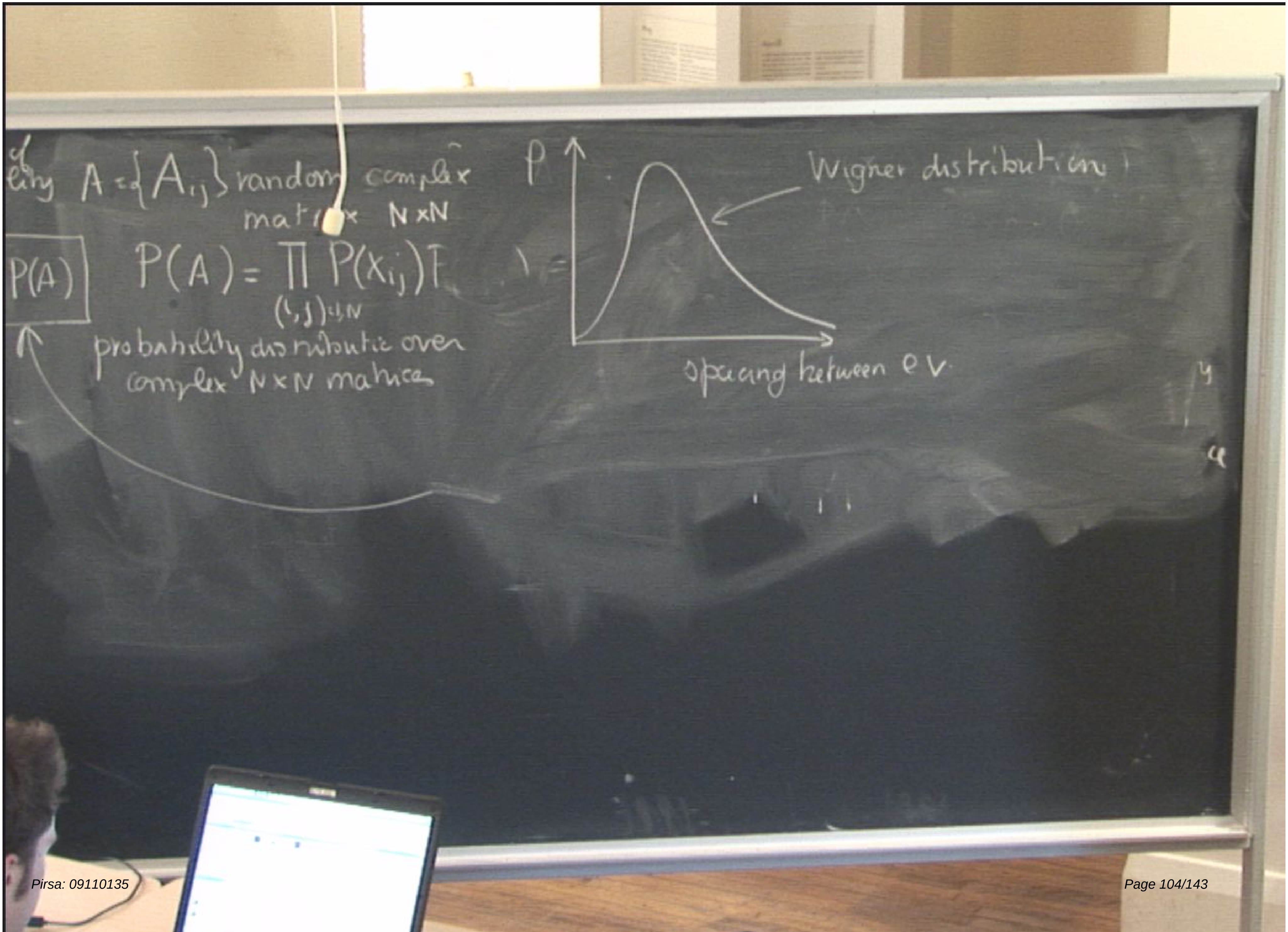
■ average over independent random symmetric matrix with entries in [-1, 1]

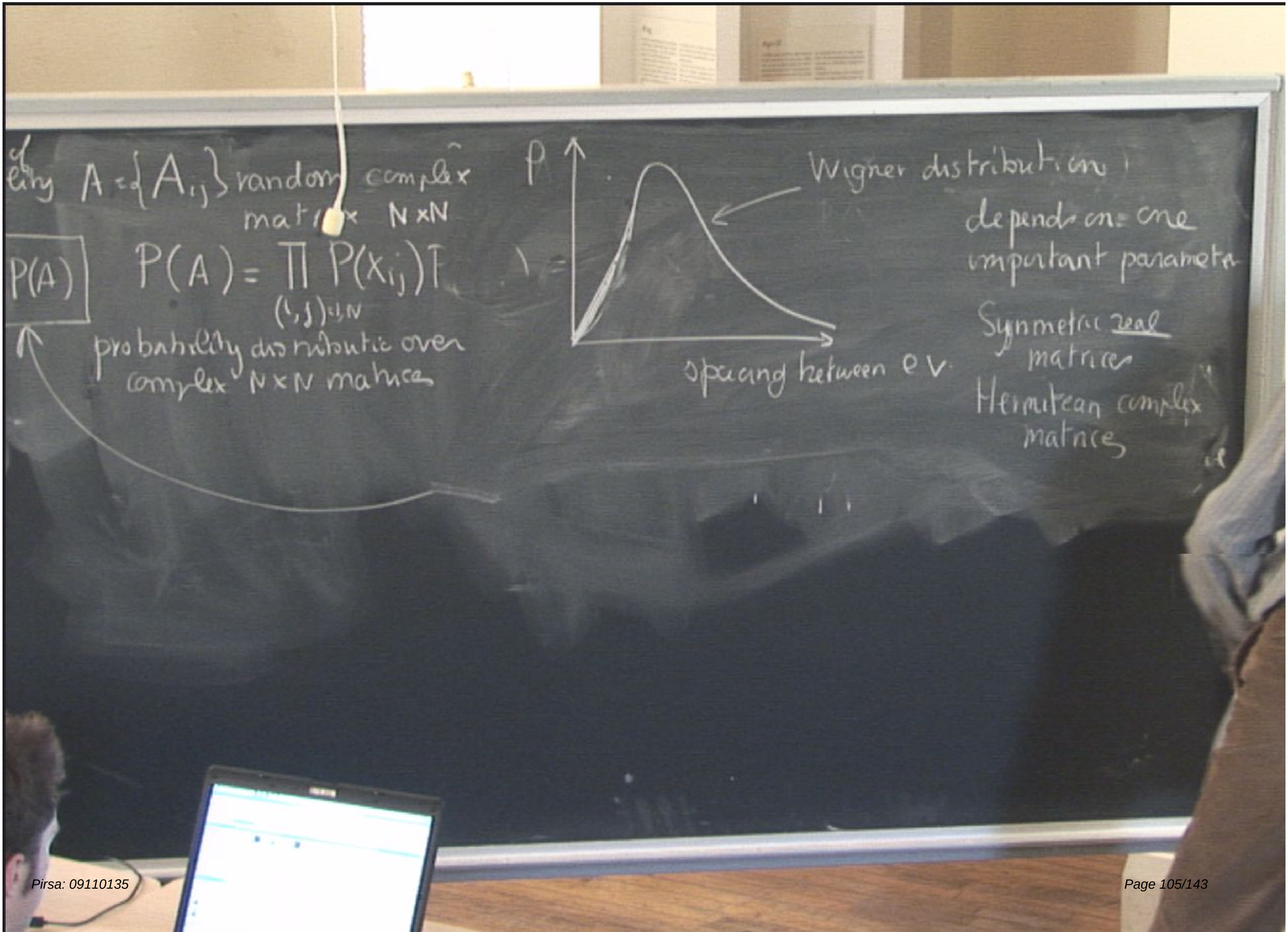
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help François David

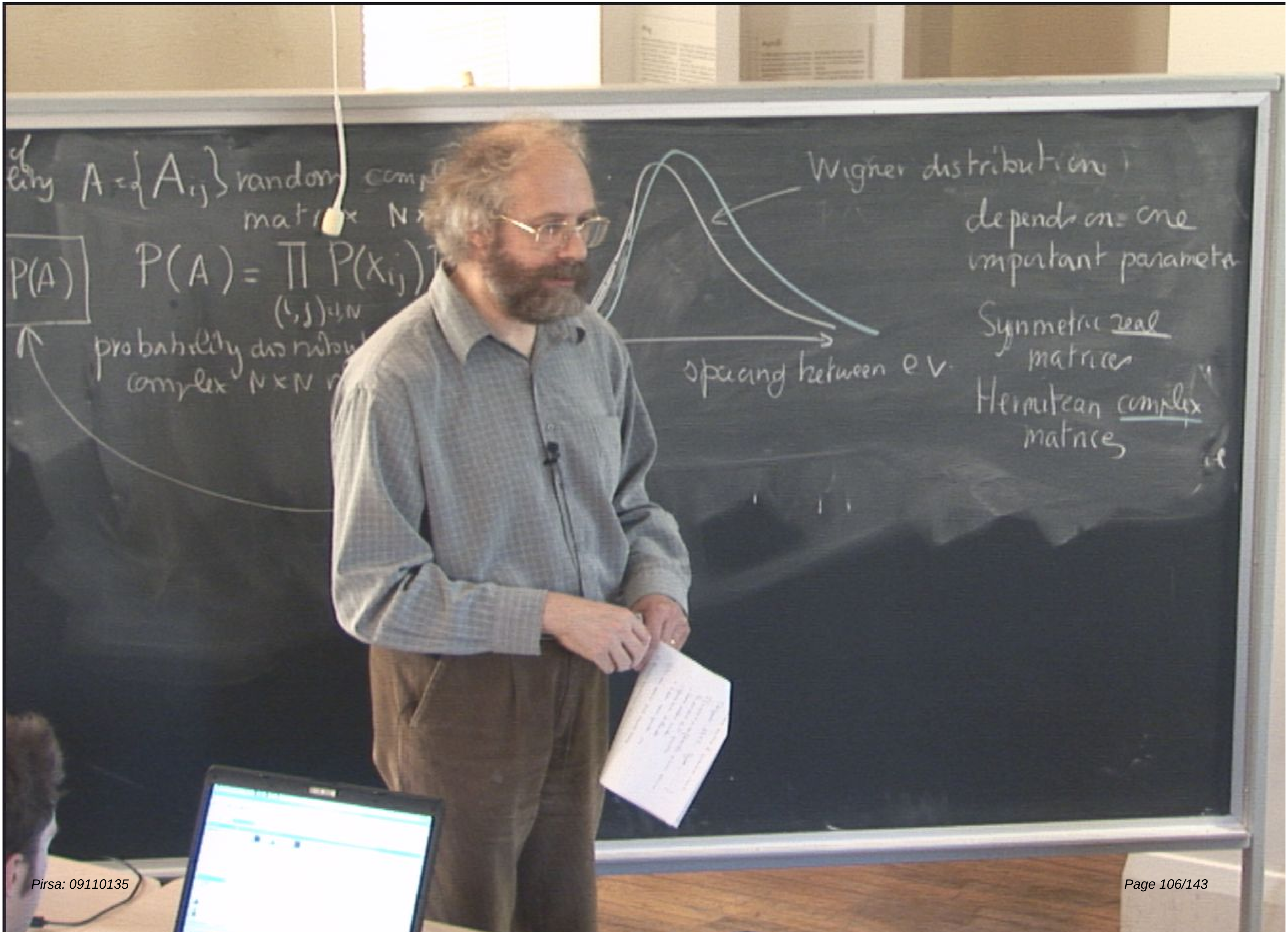
Random matrix PSI.nb

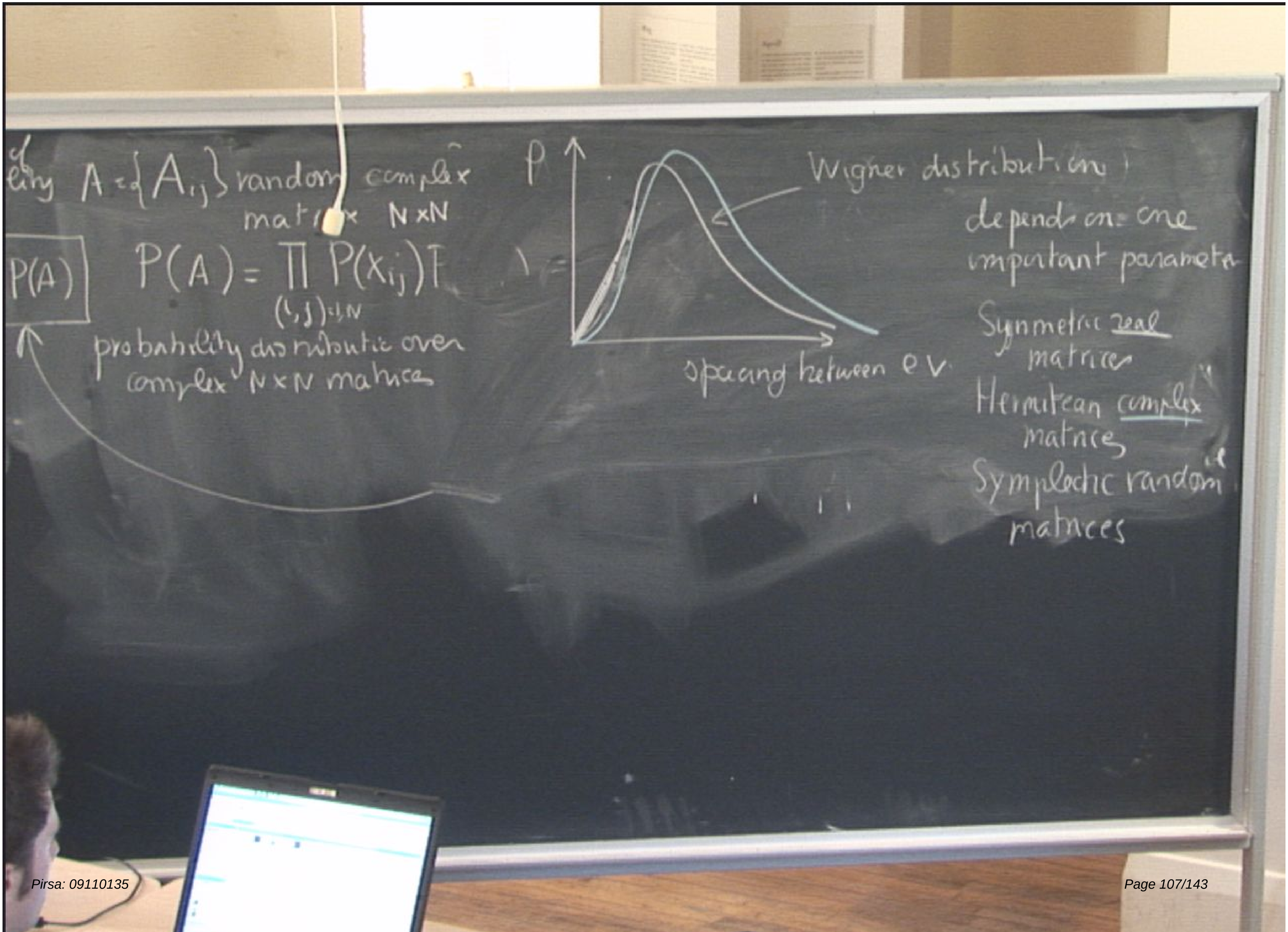
```
  (* Print[ev]; *)
  listc = BinCounts[ev, {-1, 1, 1/20}];
  listl = BinLists[ev, {-1, 1, 1/20}];
  Table[Length[listl[[i]]], {i, 1, Length[listl]}];
  listdist = {};
  Do[
   If[listc[[i]] > 1,
    listtmp = Sort[listl[[i]]];
    (* Print["sousensemble de vp",listtmp]; *)
    disttmp = Table[(listtmp[[j + 1]] - listtmp[[j]]) listc[[i]] , {j, 1, listc[[i]] - 1}];
    (* Print[disttmp];  *)
    listdist = Join[listdist, disttmp];
   ],
   {i, 1, Length[listc]}
  ];
  listdisttotal = Join[listdisttotal, listdist];
  , {nsample}];

listcount = BinCounts[listdisttotal, {0, 1/5, 1/100}];
ListPlot[listcount, Filling -> Axis, PlotStyle → PointSize[Large], PlotRange → Automatic]
```

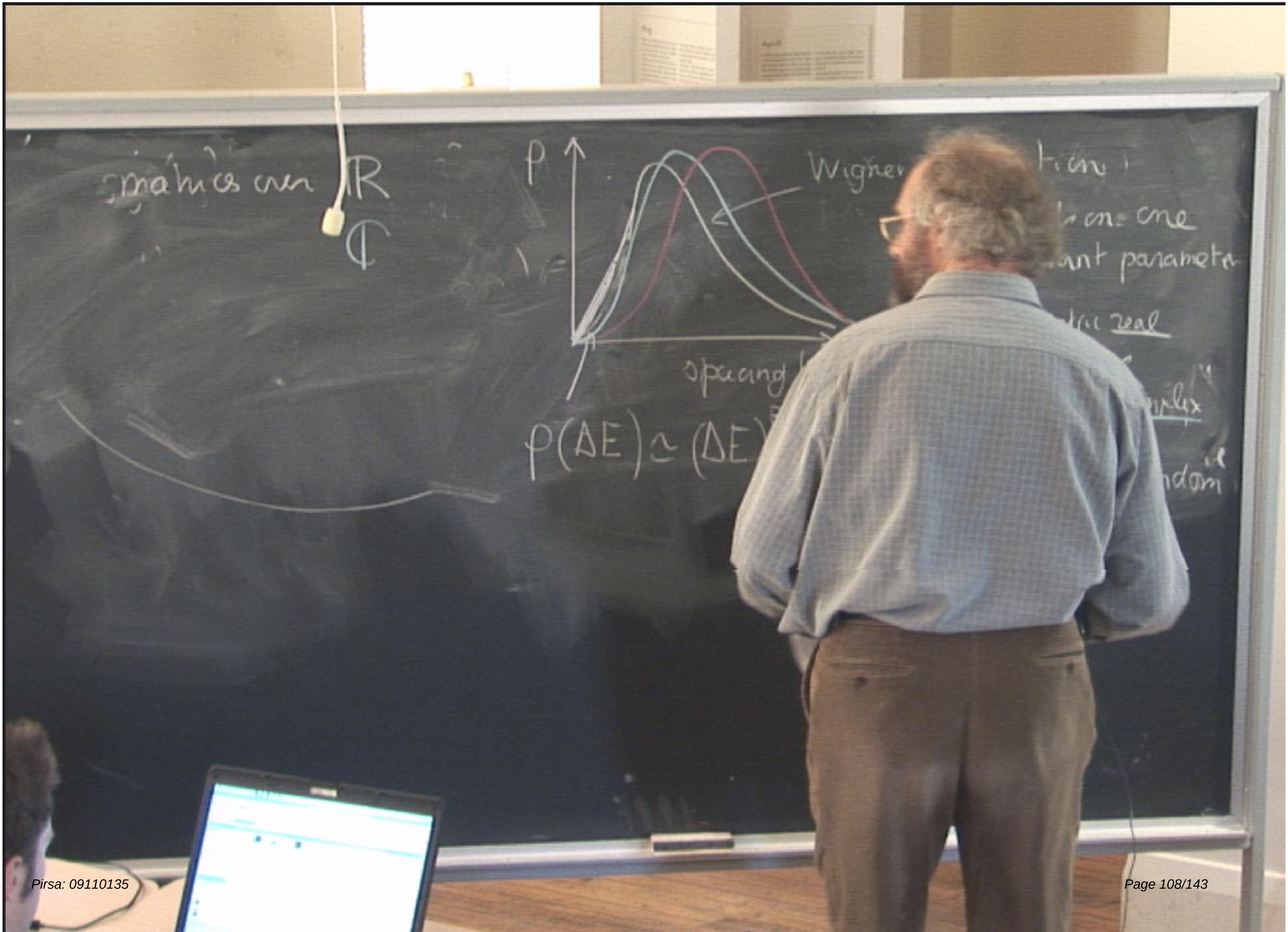



A = {A_ij} random complex matrix N×N
P(A)
P(A) = Π P(x_ij)
(i,j)=1,N
probability distribution over complex N×N matrices
P
Wigner distribution
spacing between e.v.

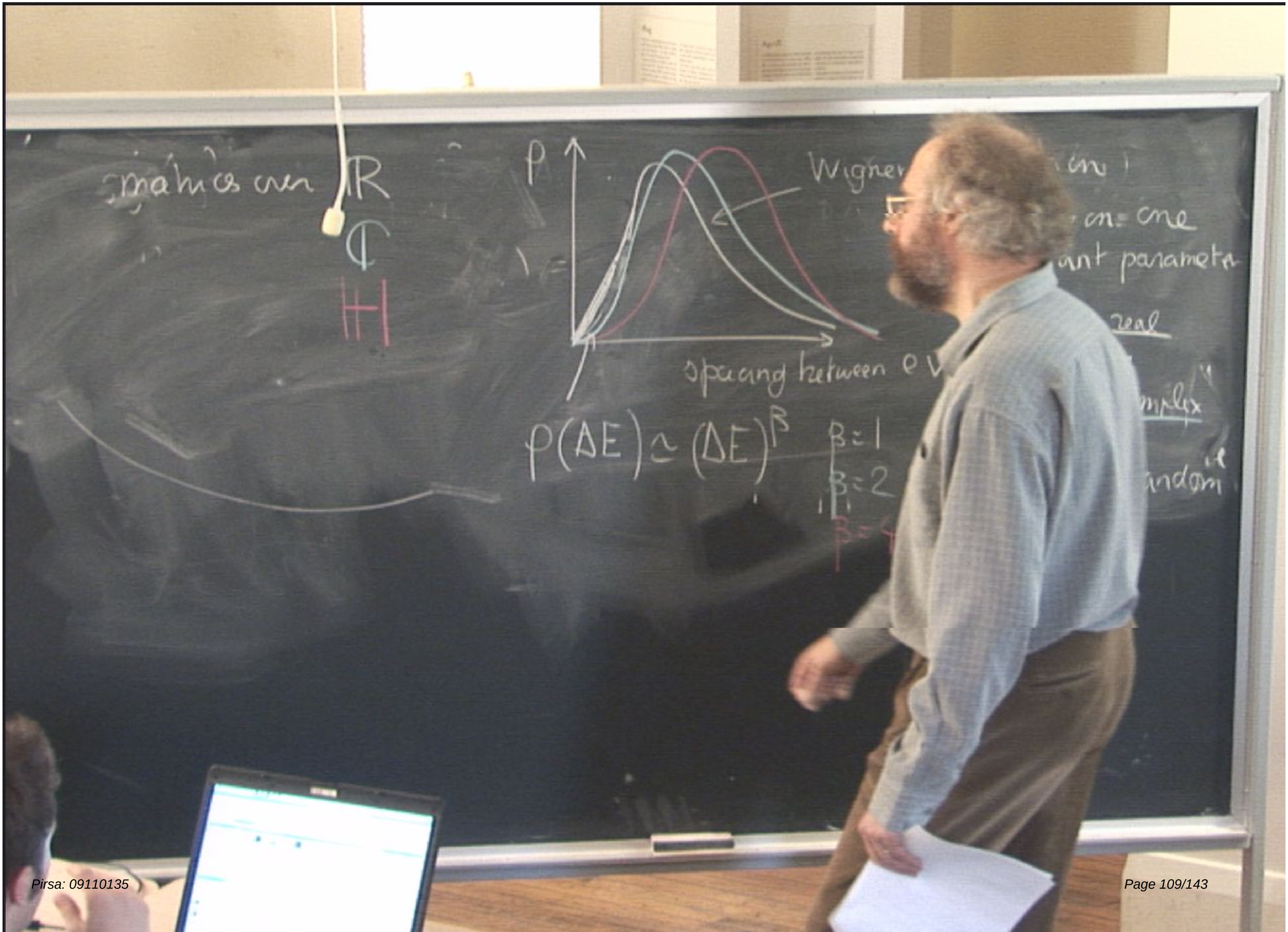

A = {A_ij} random complex
matrix N×N
P(A)
P(A) = Π P(x_ij)
probability distribution over
complex N×N matrices
P
Wigner distribution
spacing between e.v.
depends on one
important parameter
Symmetric real
matrices
Hermitean complex
matrices


A = {A_ij} random comp
matrix N×
P(A)
P(A) = Π P(X_ij)
probability distribut
complex N×N
Wigner distribution
spacing between e.v.
depends on one important paramete
Symmetric real matrices
Hermitean complex matrices

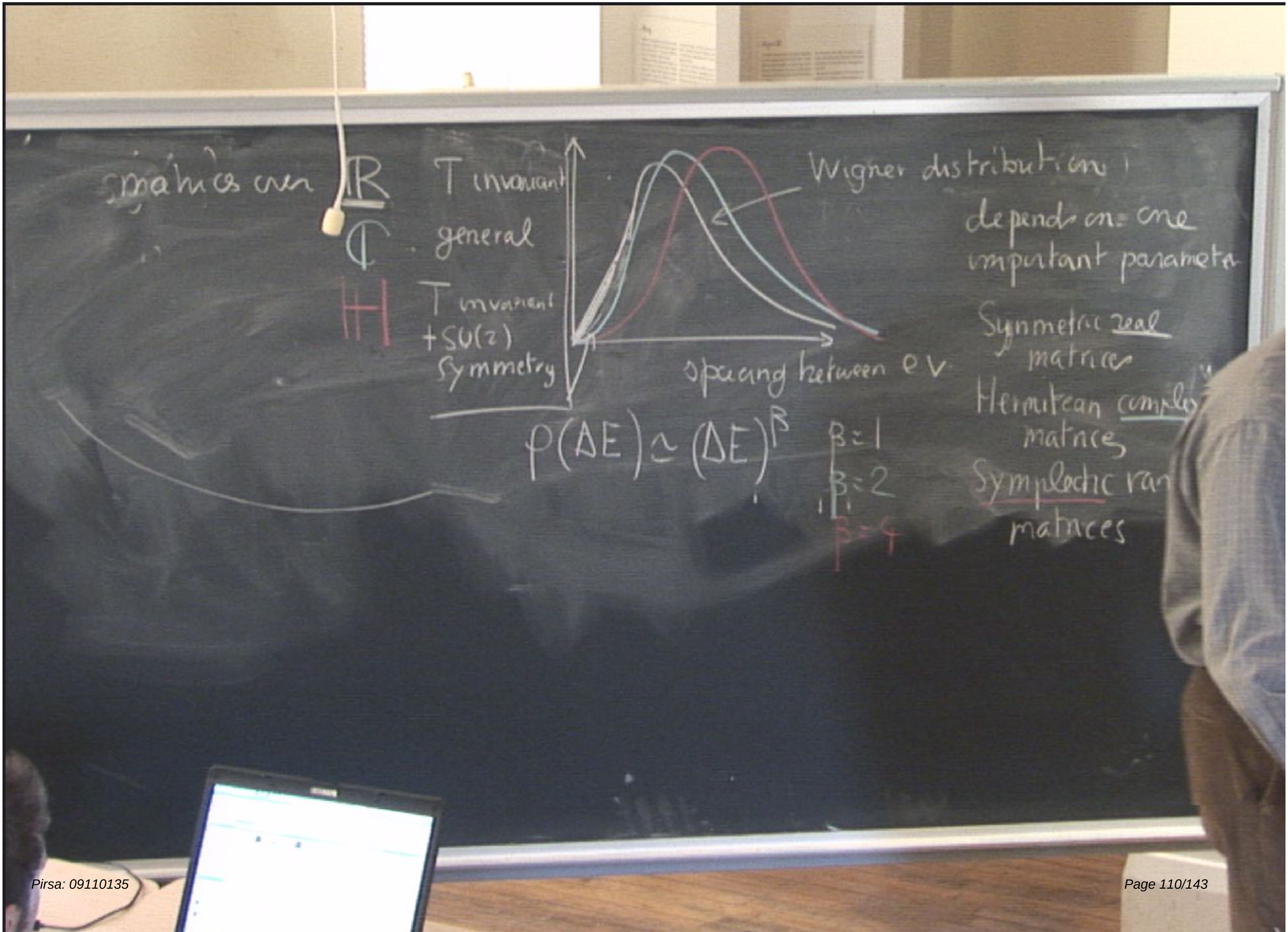

A = {A_ij} random complex matrix N×N
P(A) = ∏ P(x_ij)
probability distribution over complex N×N matrices
P(A)
P
spacing between e.v.
Wigner distribution
depends on one important parameter
Symmetric real matrices
Hermitean complex matrices
Symplectic random matrices



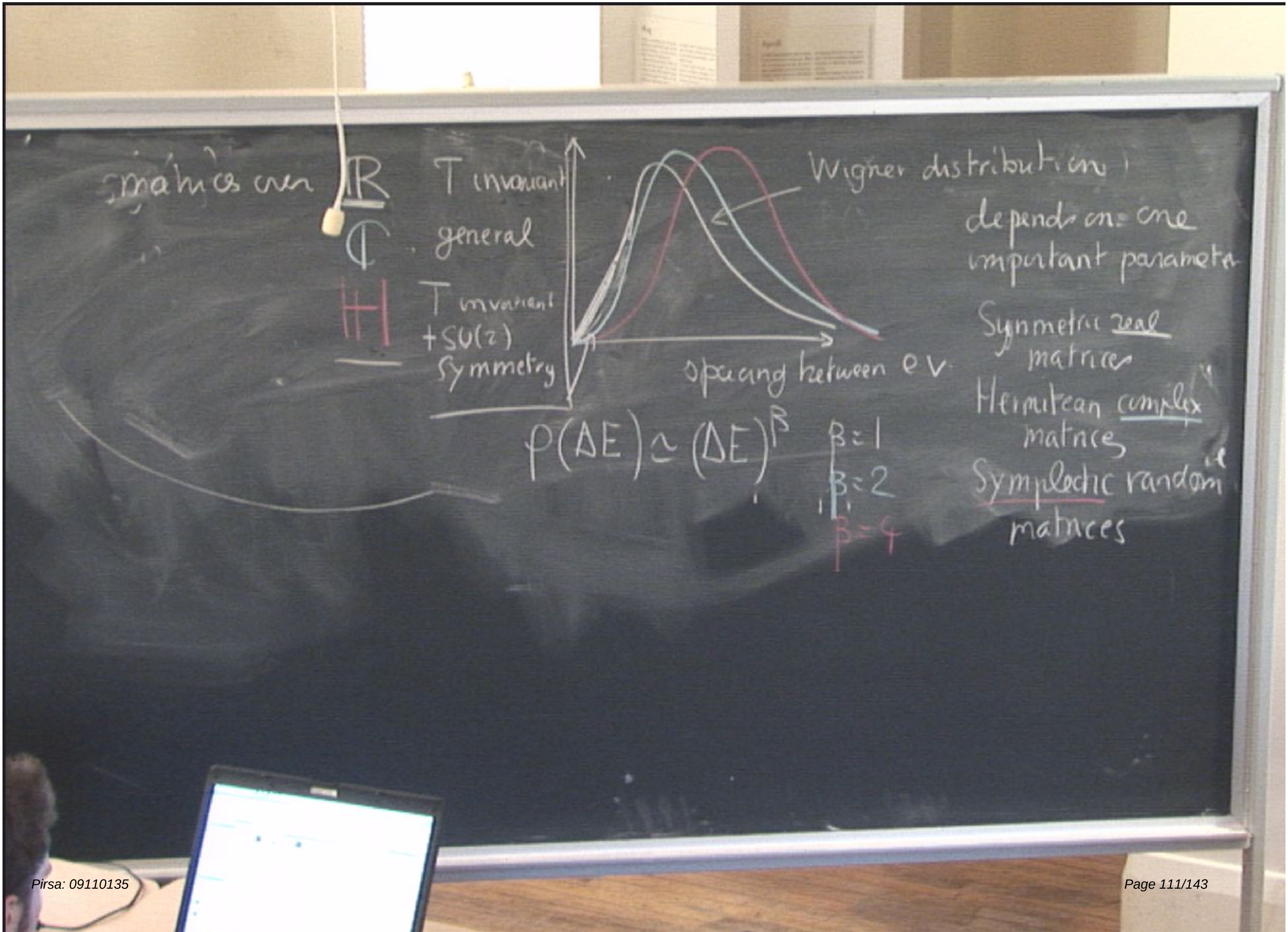
ρ
Wigner
spacing
$\rho(\Delta E) \simeq (\Delta E$



ρ
Wigner
spacing between ev
ρ(ΔE) ≃ (ΔE)^β
β=1
β=2
real



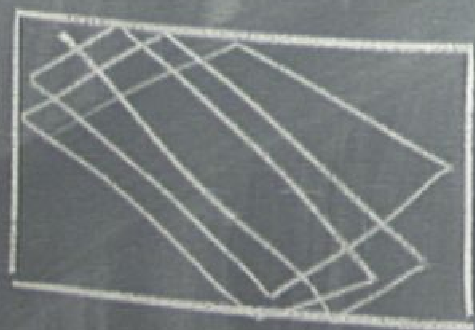
matrices over ℝ
T invariant
ℂ general
ℍ T invariant
+SU(2)
symmetry
Wigner distribution
spacing between e.v.
depends on one
important parameter
Symmetric real
matrices
Hermitean complex
matrices
Symplectic ran
matrices
ρ(ΔE) ≃ (ΔE)^β
β=1
β=2
β=4


matrices over
ℝ T invariant
ℂ general
ℍ T invariant
+SU(2)
symmetry
Wigner distribution
depends on one
important parameter
spacing between e.v.
Symmetric real
matrices
Hermitean complex
matrices
Symplectic random
matrices
$\rho(\Delta E) \simeq (\Delta E)^\beta$
$\beta=1$
$\beta=2$
$\beta=4$

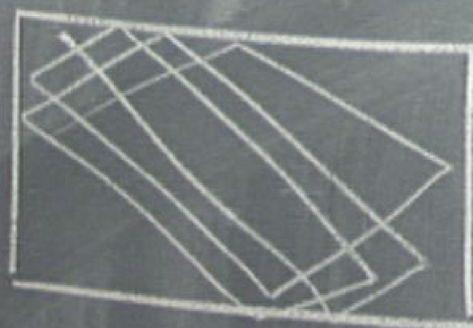
Energy level in some (Heavy) Nuclei

* Energy level in <u>some</u> (Heavy) Nuclei
* "Chaotic" Quantum systems

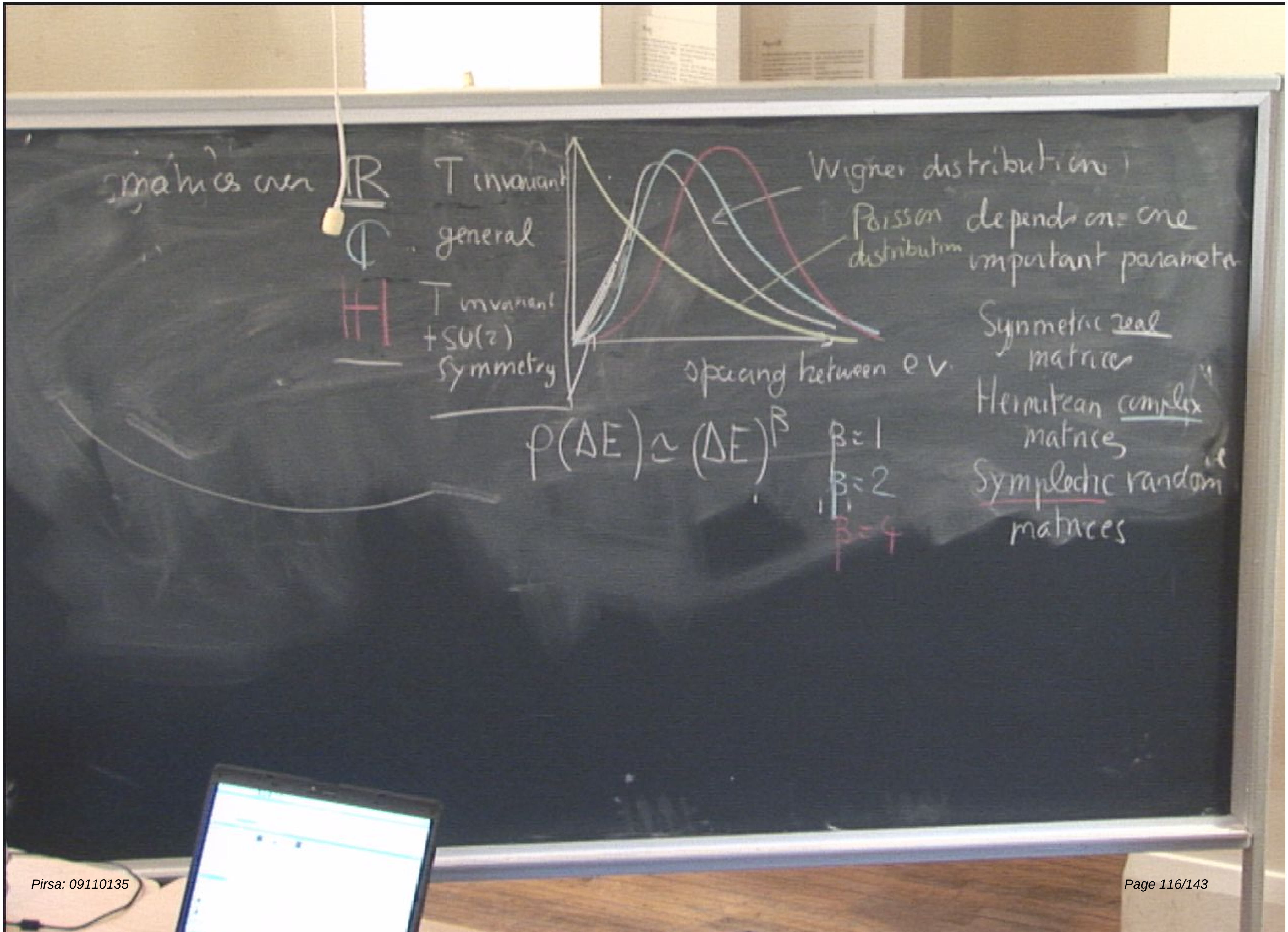
* Energy level in some (Heavy) Nuclei
* "Chaotic" quantum systems

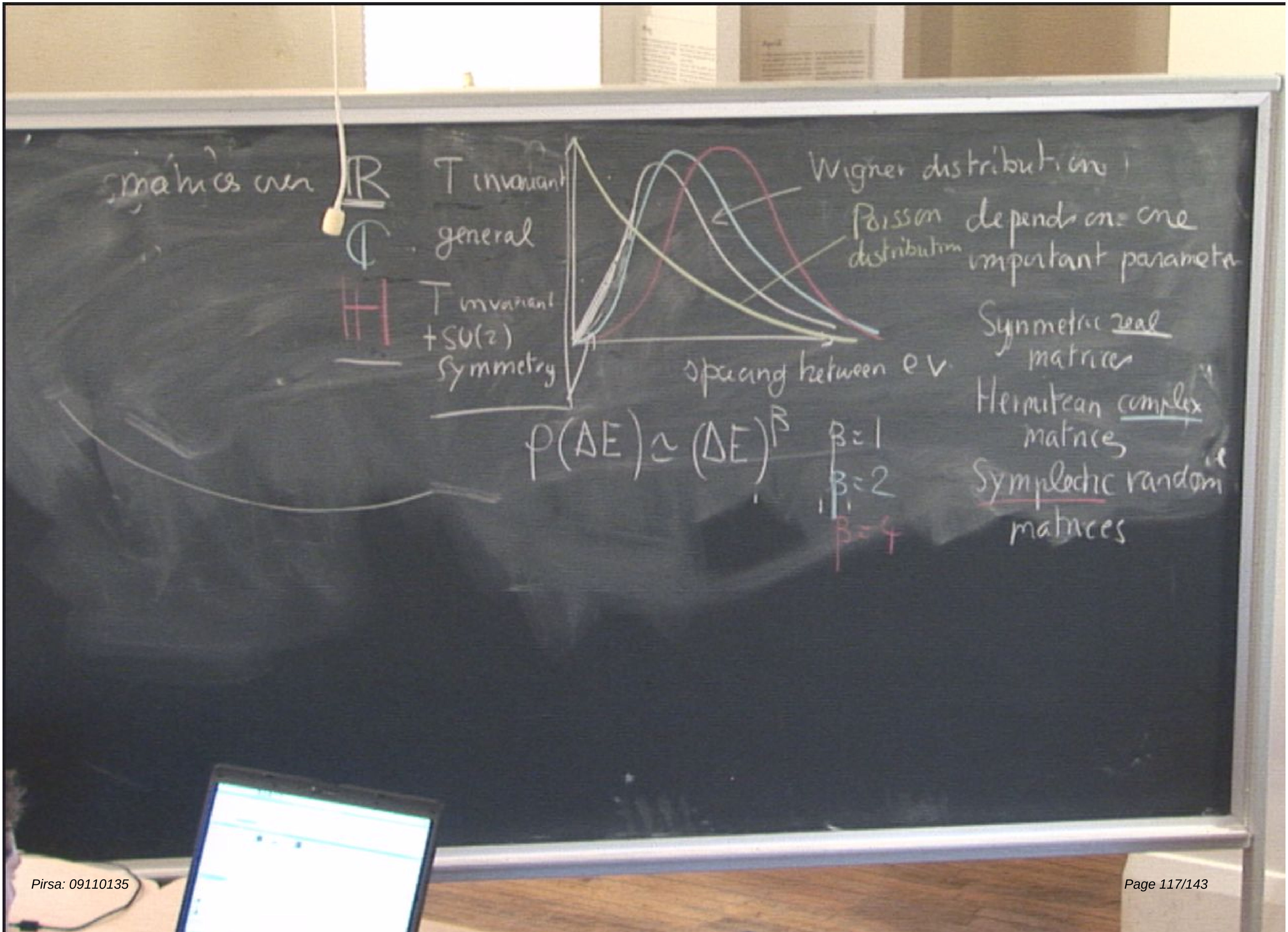


* Energy level in some (Heavy) Nuclei
* "Chaotic" quantum systems



$$E_{n,m} = n^2 a + m^2 b$$


matrices over ℝ
T invariant
ℂ
general
ℍ
T invariant
+SU(2)
symmetry
Wigner distribution
Poisson
distribution
depends on one
important parameter
spacing between e.v.
ρ(ΔE) ≃ (ΔE)^β
β=1
β=2
β=4
Symmetric real
matrices
Hermitean complex
matrices
Symplectic random
matrices


matrices over
ℝ T invariant
ℂ general
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+SU(2)
symmetry
Wigner distribution
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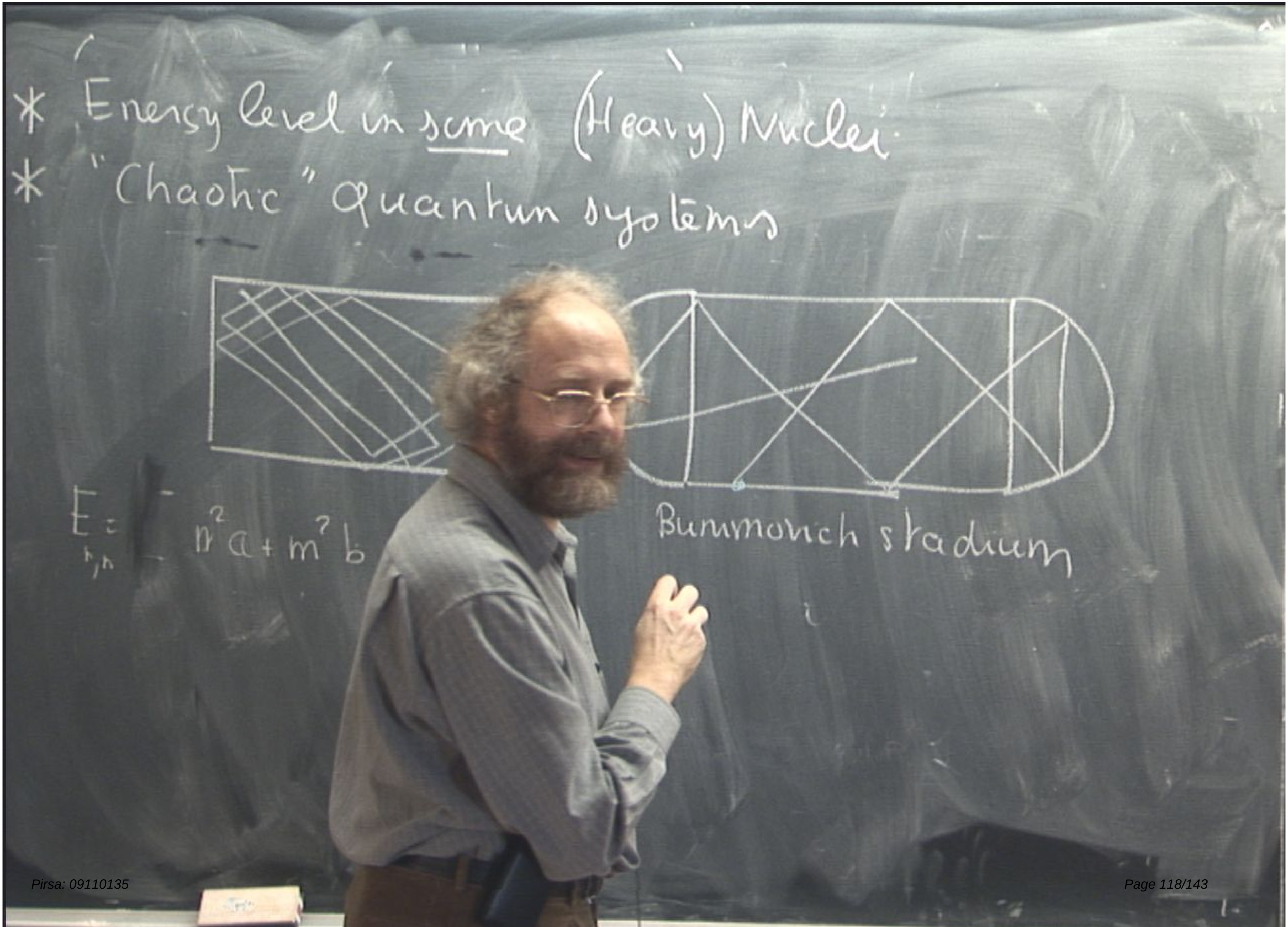
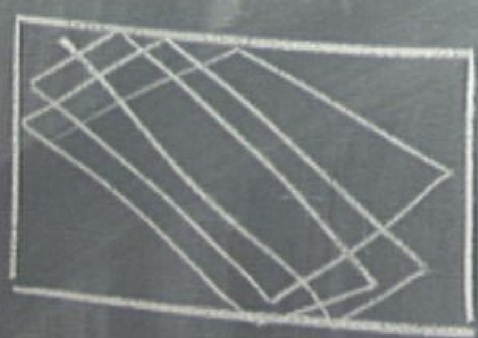
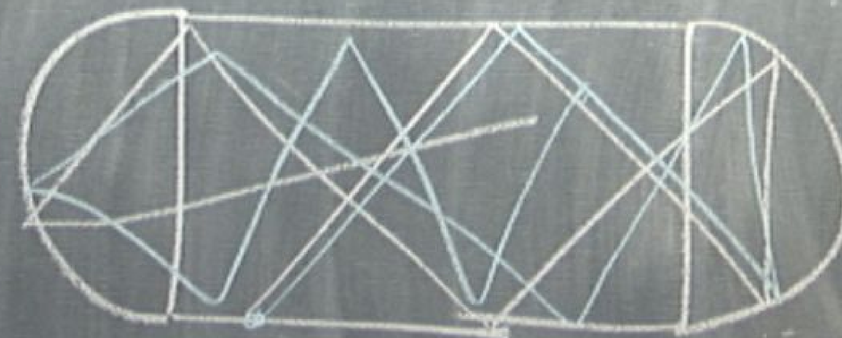
$\rho(\Delta E) \simeq (\Delta E)^\beta$
$\beta = 1$
$\beta = 2$
$\beta = 4$
Symmetric real
matrices
Hermitean complex
matrices
Symplectic random
matrices


* Energy level in some (Heavy) Nuclei
* "Chaotic" quantum systems
$E_{n,m} = n^2 a + m^2 b$
Bunimovich stadium

* Energy level in some (Heavy) Nuclei
* "Chaotic" Quantum systems

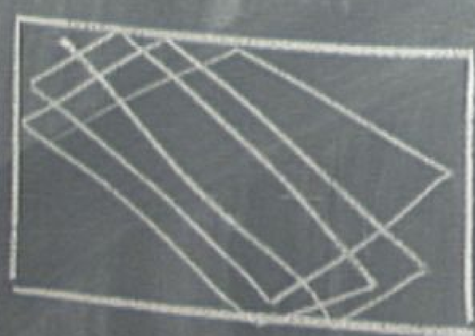


$E_{n,m} = n^2 a + m^2 b$



Bunimovich stadium

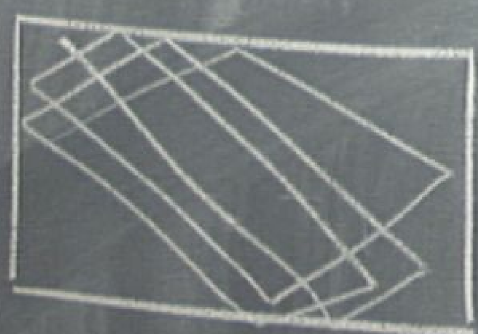
* Energy level in some (Heavy) Nuclei
* "Chaotic" quantum systems



$E_{n,m} = n^2 a + m^2 b$

Bunimovich stadium

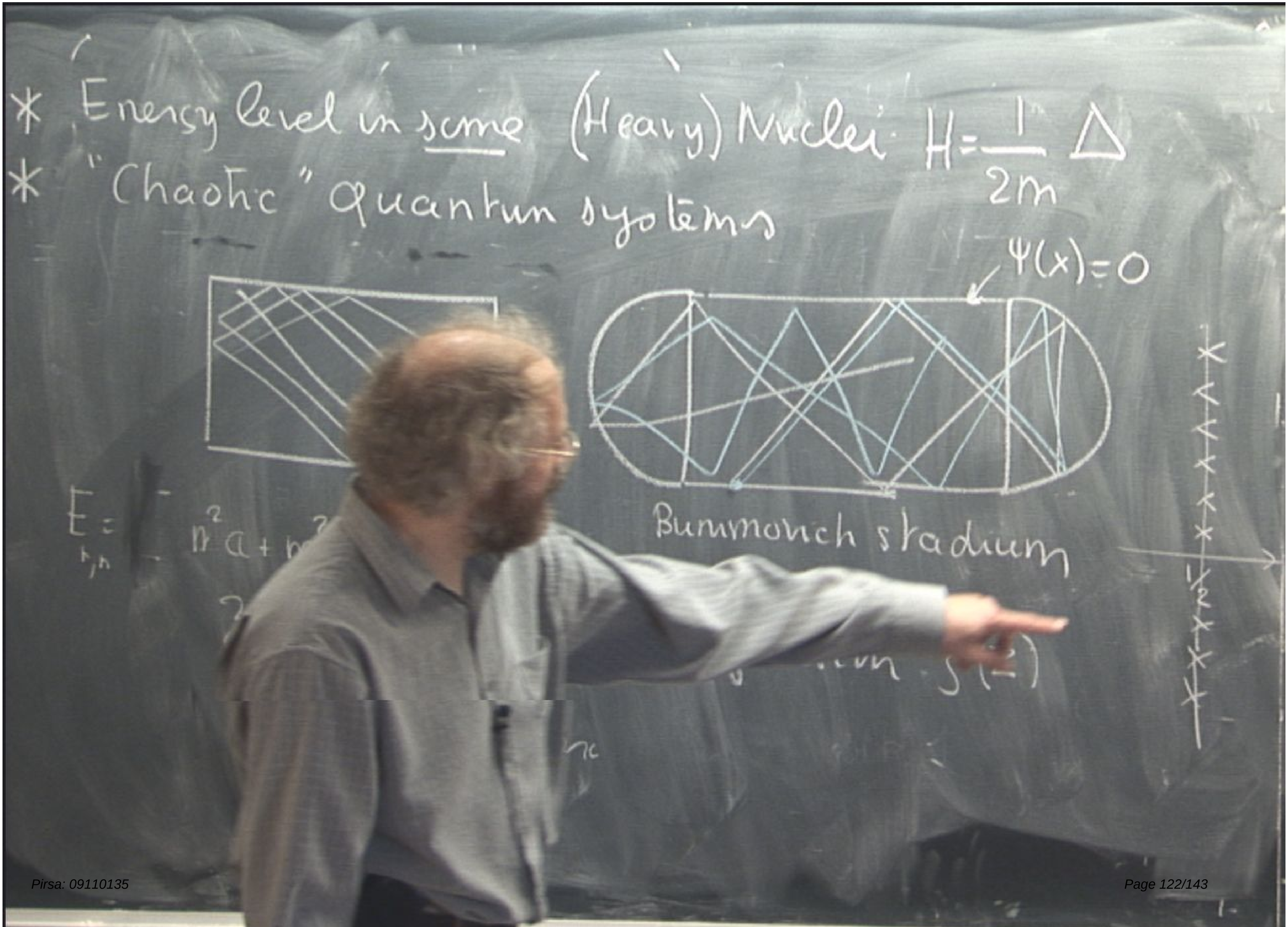
* Energy level in some (Heavy) Nuclei $H = \frac{1}{2m}\Delta$
* "Chaotic" quantum systems



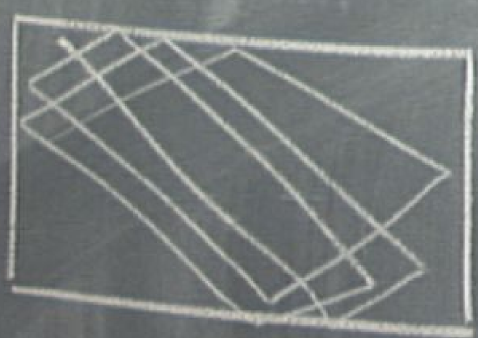
$\psi(x) = 0$

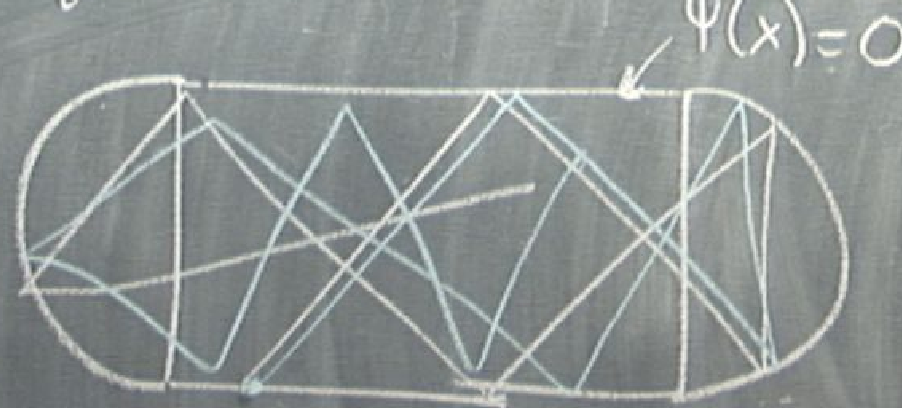
$E_{n,m} = n^2 a + m^2 b$

Bunimovich stadium


* Energy level in some (Heavy) Nuclei $H = \frac{1}{2m} \Delta$
* "Chaotic" quantum systems
$\psi(x) = 0$
Bunimovich stadium

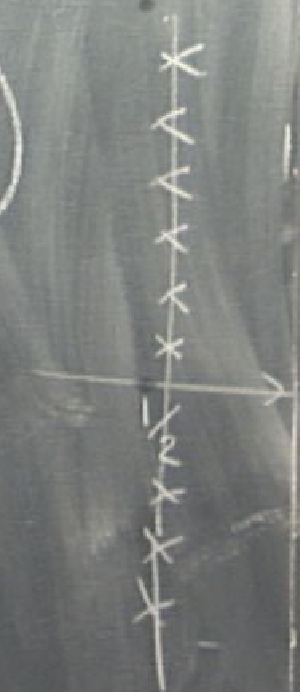
* Energy level in some (Heavy) Nuclei $H = \frac{1}{2m} \Delta$
* "Chaotic" quantum systems





$E_{n,m} = n^2 a + m^2 b$

Bunimovich stadium

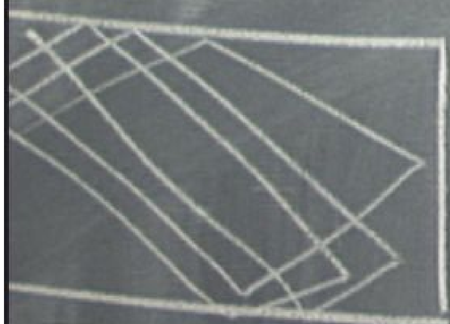


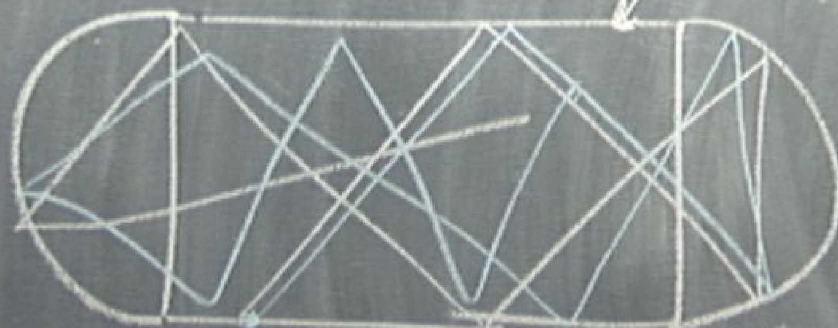
Zeros of the Riemann ζ function $\zeta(z)$

level in some (Heavy) Nuclei $H = -\frac{1}{2m}\Delta$

tic" Quantum systems

$\psi(x) = 0$



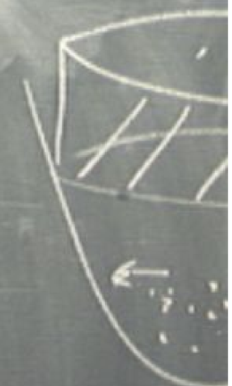


Bunimovich stadium

$a + m^2 b$

Zeros of the Riemann ζ function $\zeta(z)$

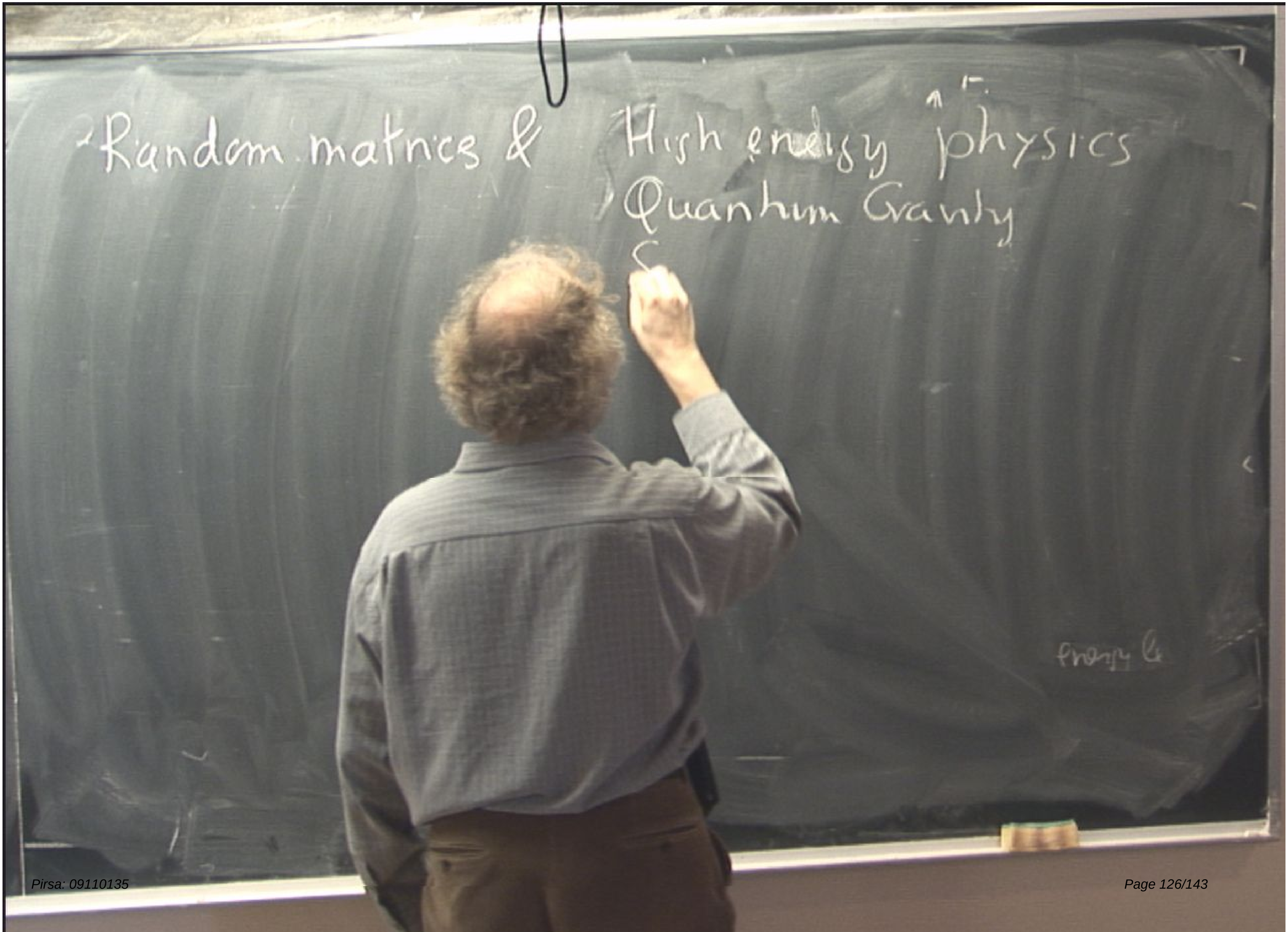
$2N^2 -$



N large

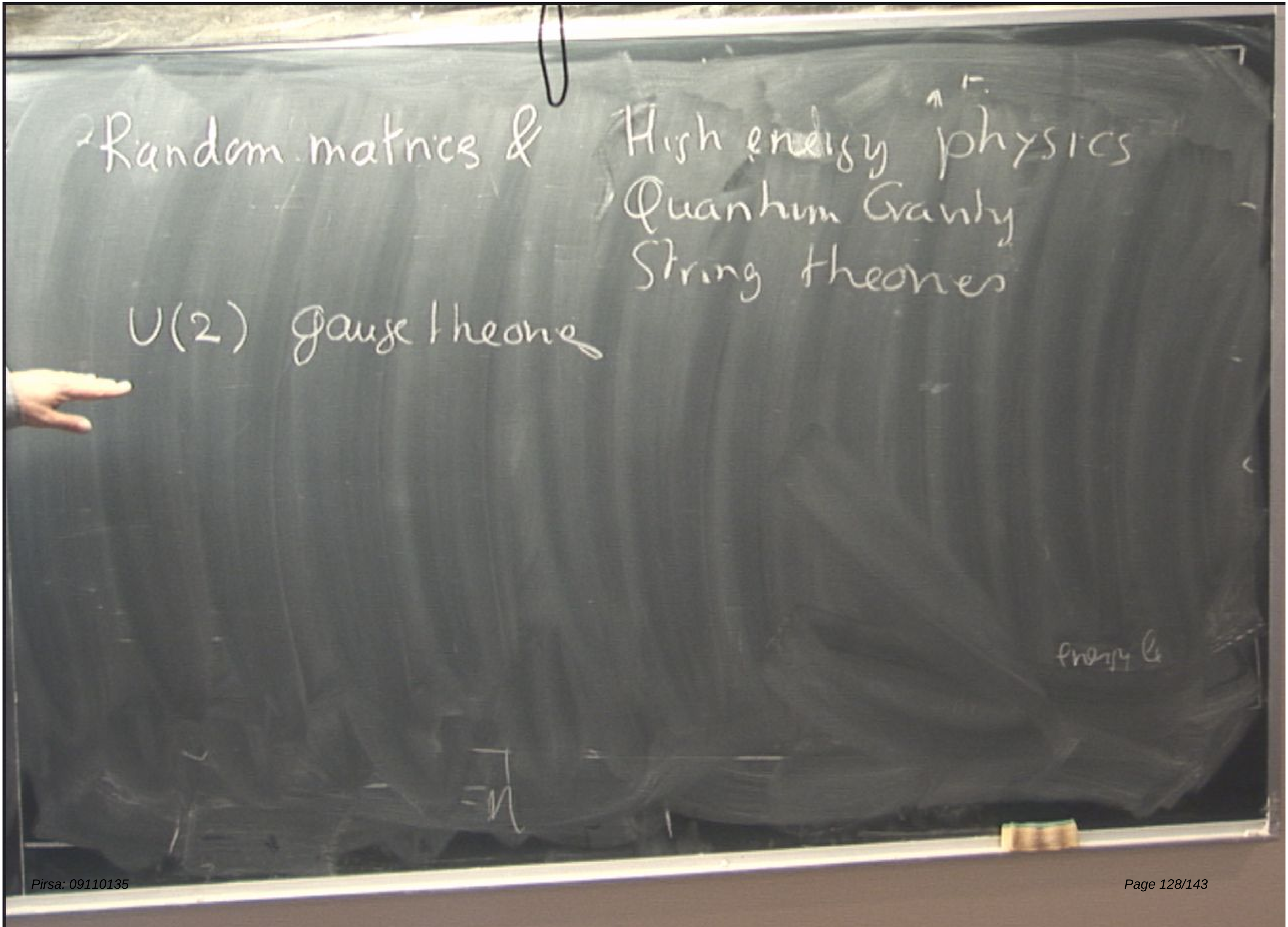
electro

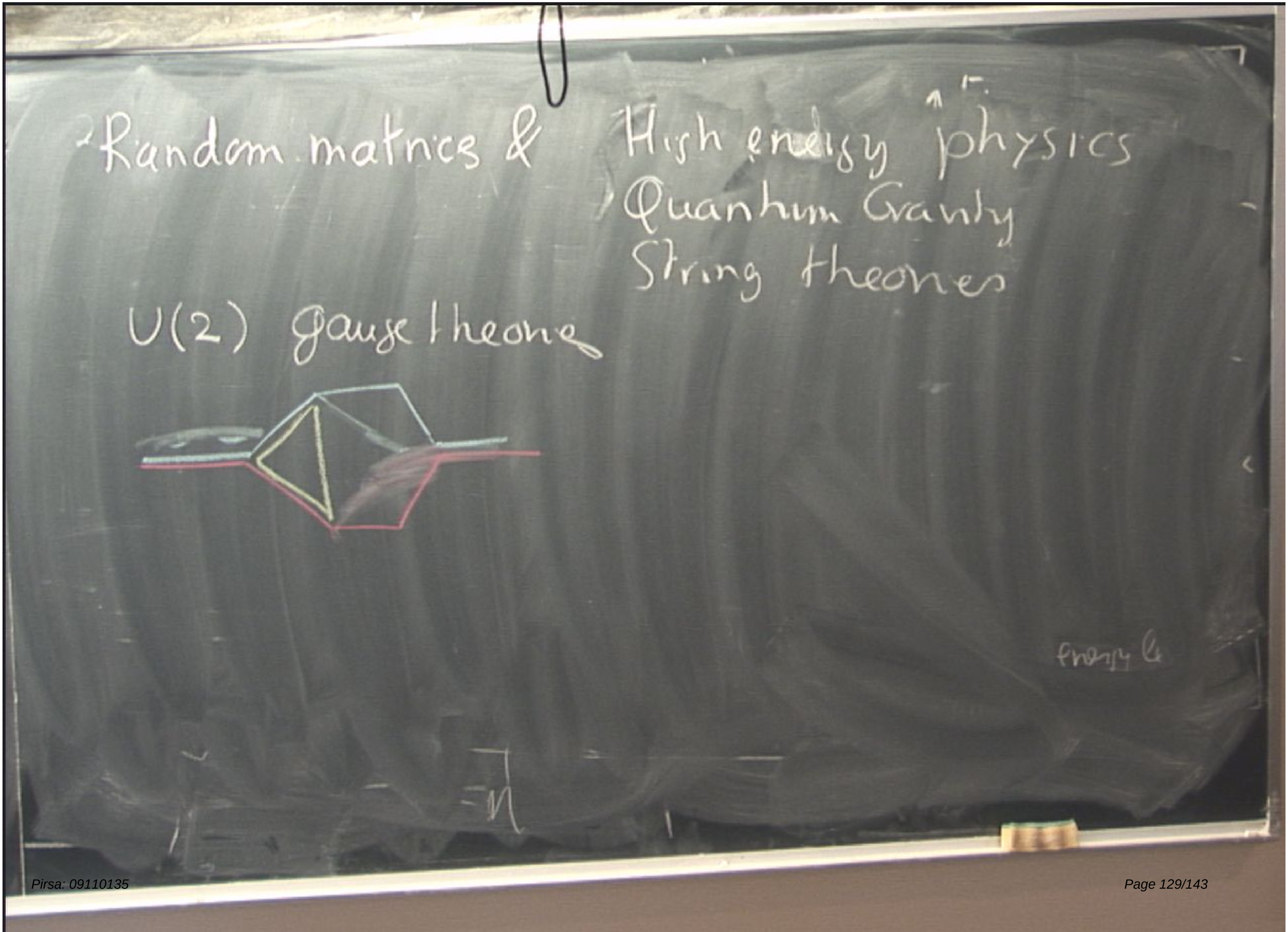

matrices over ℝ
T invariant
ℂ general
ℍ
T invariant
+ SU(2)
symmetry
Wigner distribution
Poisson distribution
depends on one important parameter
spacing between e.v.
Symmetric real matrices
Hermitean complex matrices
Symplectic random matrices
ρ(ΔE) ≃ (ΔE)^β
β=1
β=2
β=4


Random matrices &
High energy physics
Quantum Gravity

Random matrices & High energy physics
Quantum Gravity
String theories

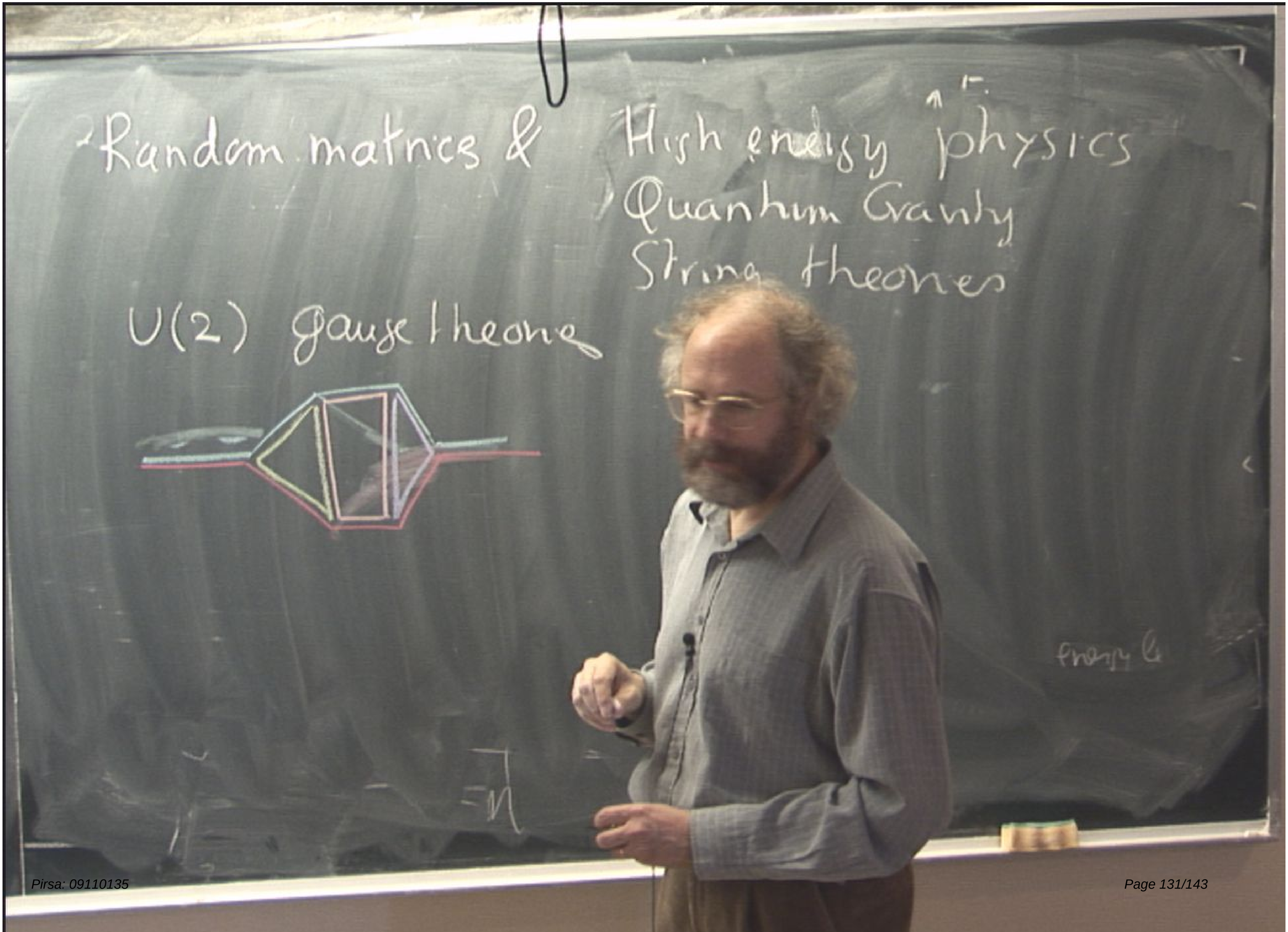
energy G

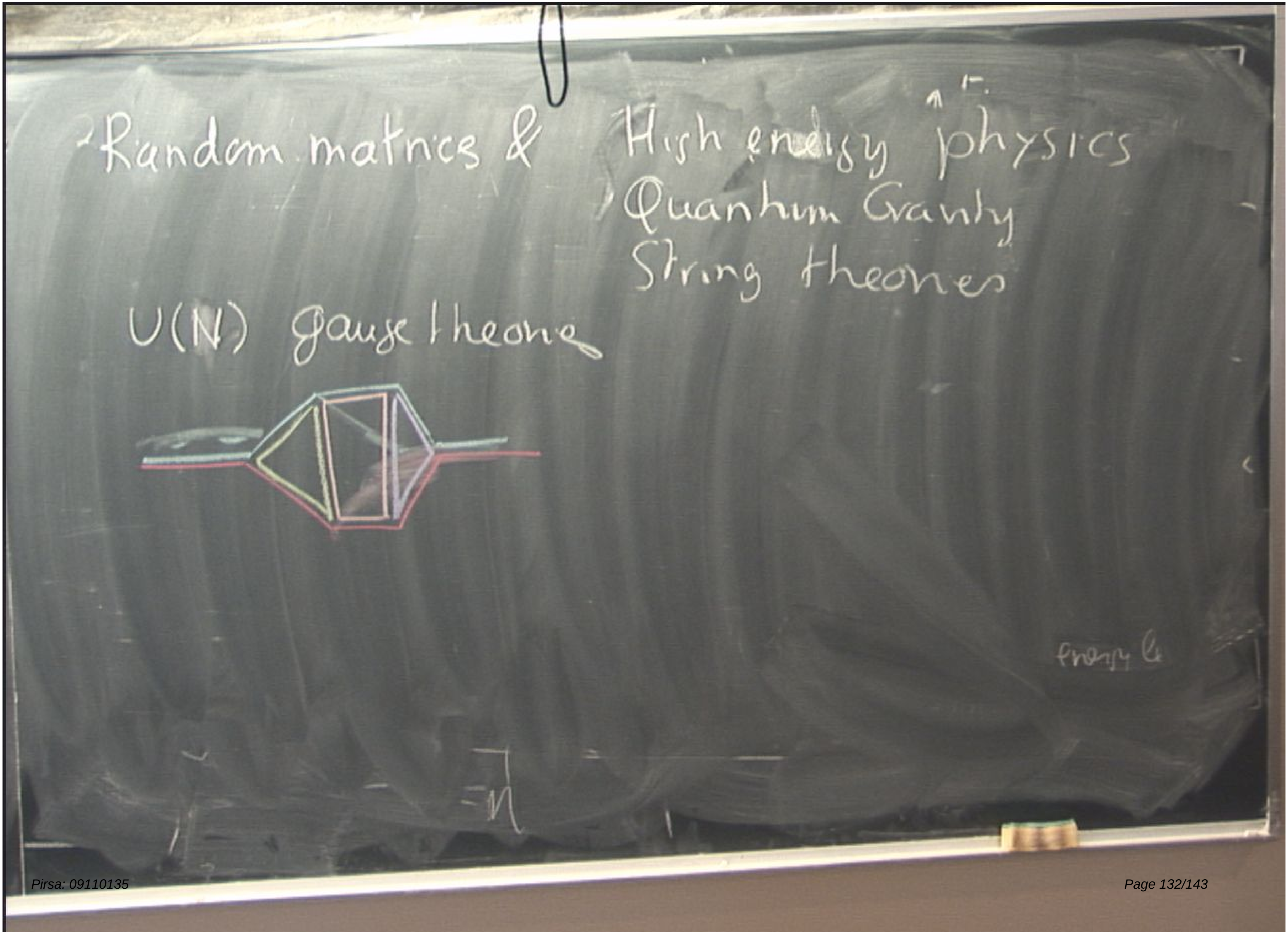

Random matrices &
High energy physics
Quantum Gravity
String theories
U(2) gauge theory


Random matrices &
High energy physics
Quantum Gravity
String theories
U(2) gauge theory

Random matrices & High energy physics
Quantum Gravity
String theories

U(2) gauge theorie


Random matrices & High energy physics
Quantum Gravity
String theories
U(2) gauge theory


Random matrices &
High energy physics
Quantum Gravity
String theories
U(N) gauge theorie

Random matrices & High energy physics

Quantum Gravity

String theories

U(N) gauge theory → N different colors

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String theories

U(N) gauge theory → N different colors

$N^3 g^6$

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U(N) gauge theory → N different colors

$N^3 g^6$

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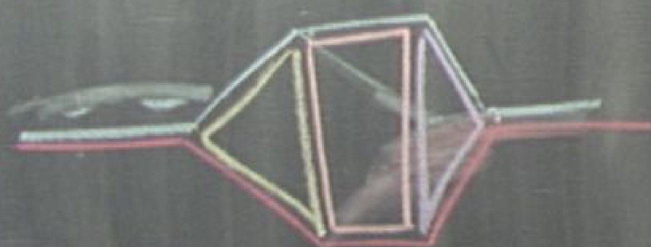
U(N) gauge theory $\rightarrow$ N different colors



$N^3 g^6$

$N g^6$

energy 4

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String theories

U(N) gauge theories → N different colors

planar

$N^3 g^6$ 't Hooft 74

$N \to \infty$ gN^2 finite

non planar

$N g^6$ the planar diagrams survive

Random matrices & High energy physics

Quantum Gravity

String theories

$U(N)$ gauge theory $\rightarrow$ N different colors

$N^3 g^6$ 't Hooft 74

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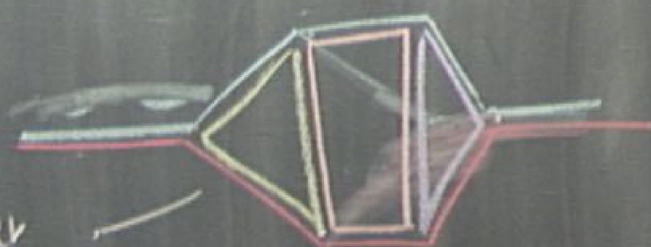
planar



non planar

$N g^6$ the planar diagrams survive

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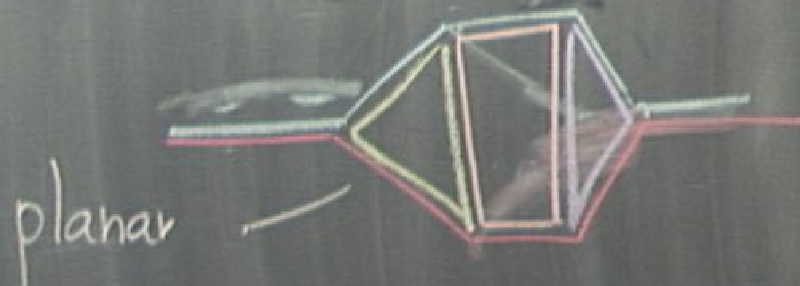
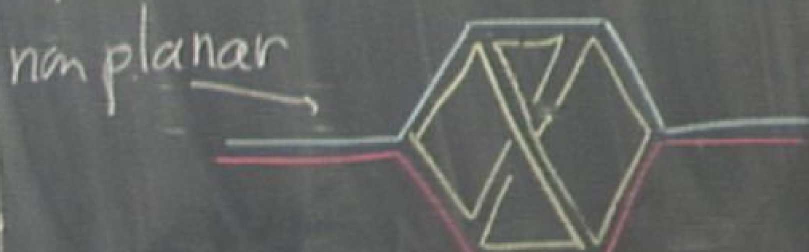
Quantum Gravity in 2D
String theories

U(N) gauge theories → N different colors

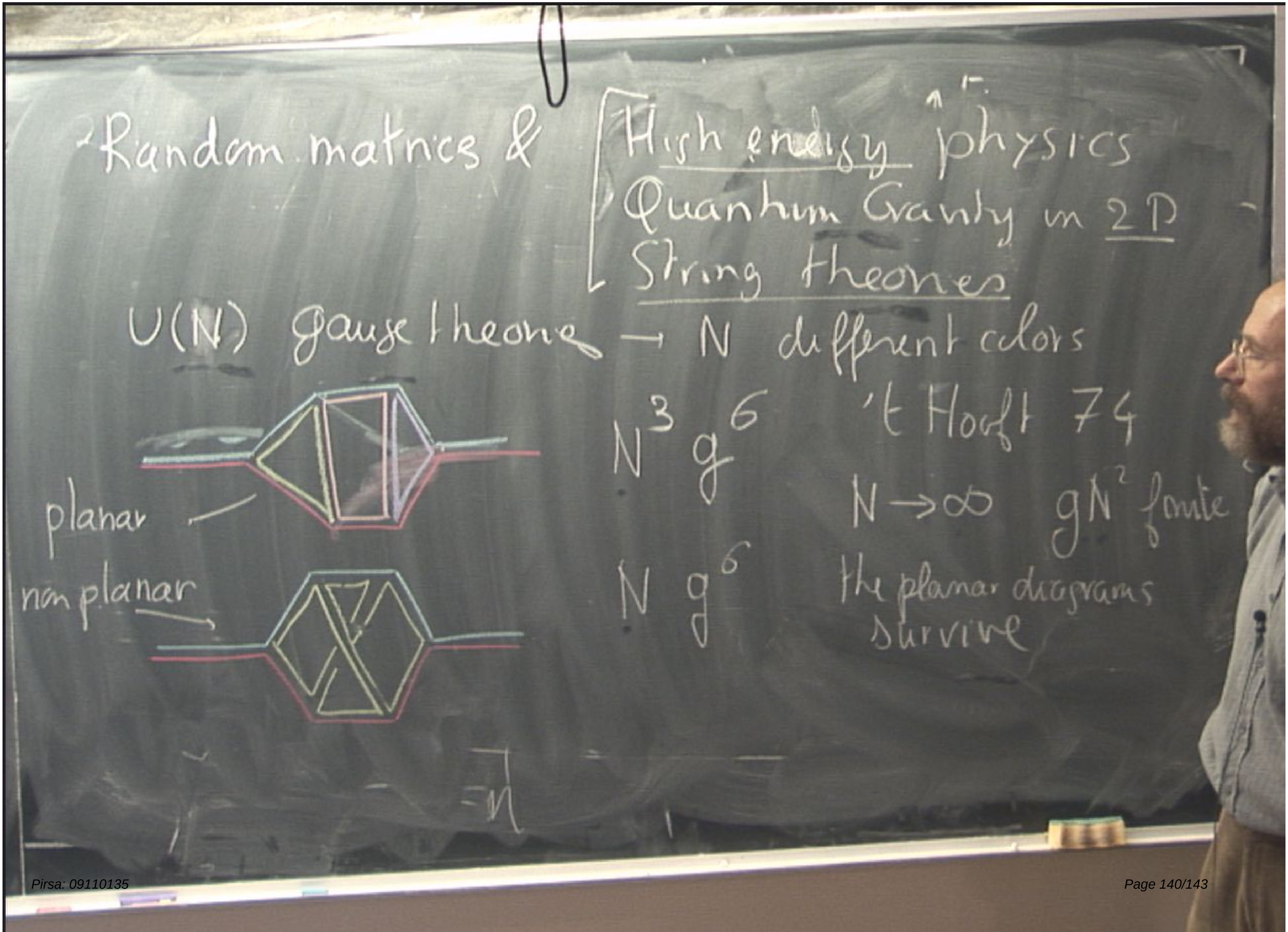


$N^3 g^6$ 't Hooft 74

$N \to \infty$ gN^2 finite



$N g^6$ the planar diagrams survive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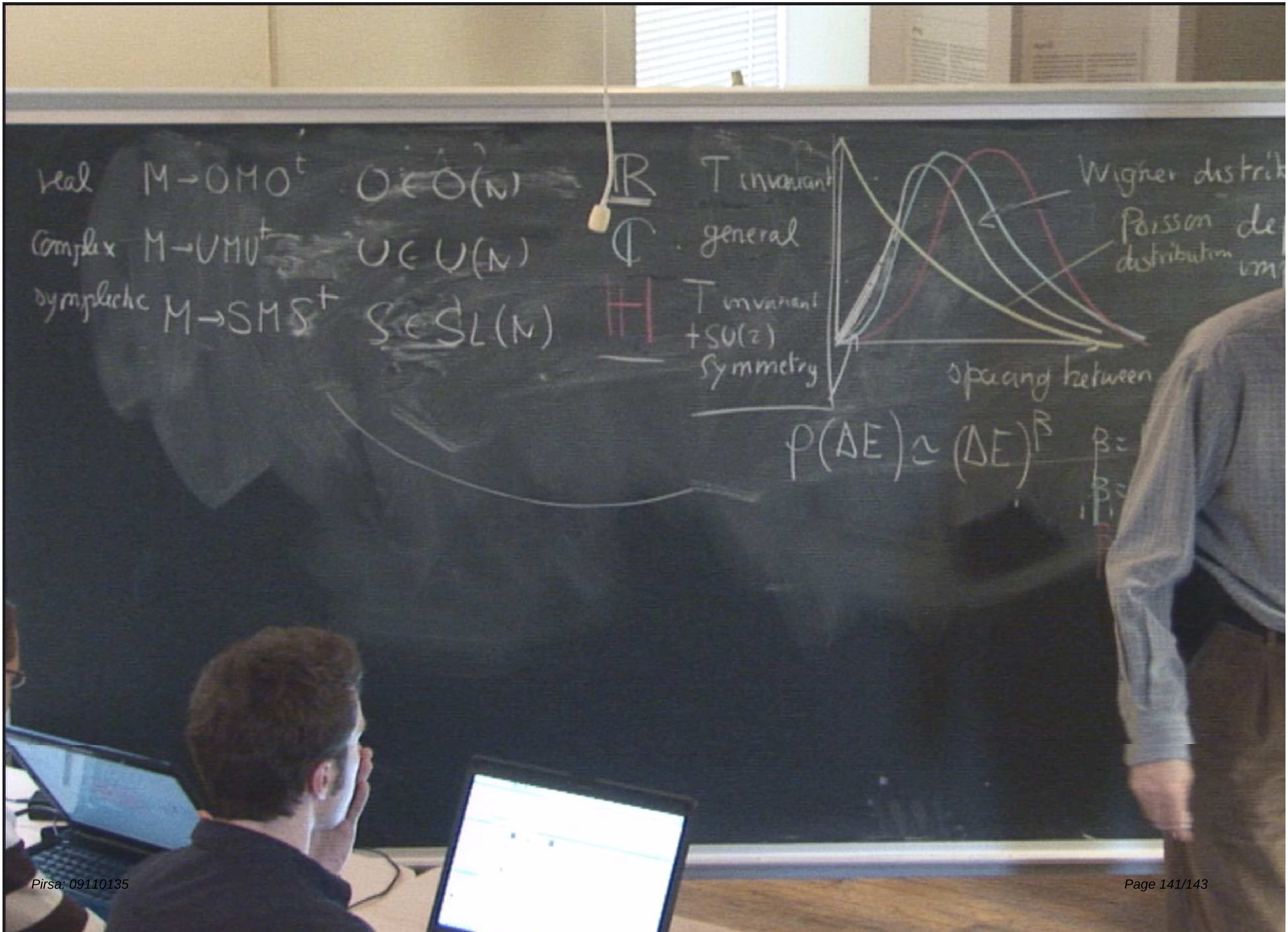
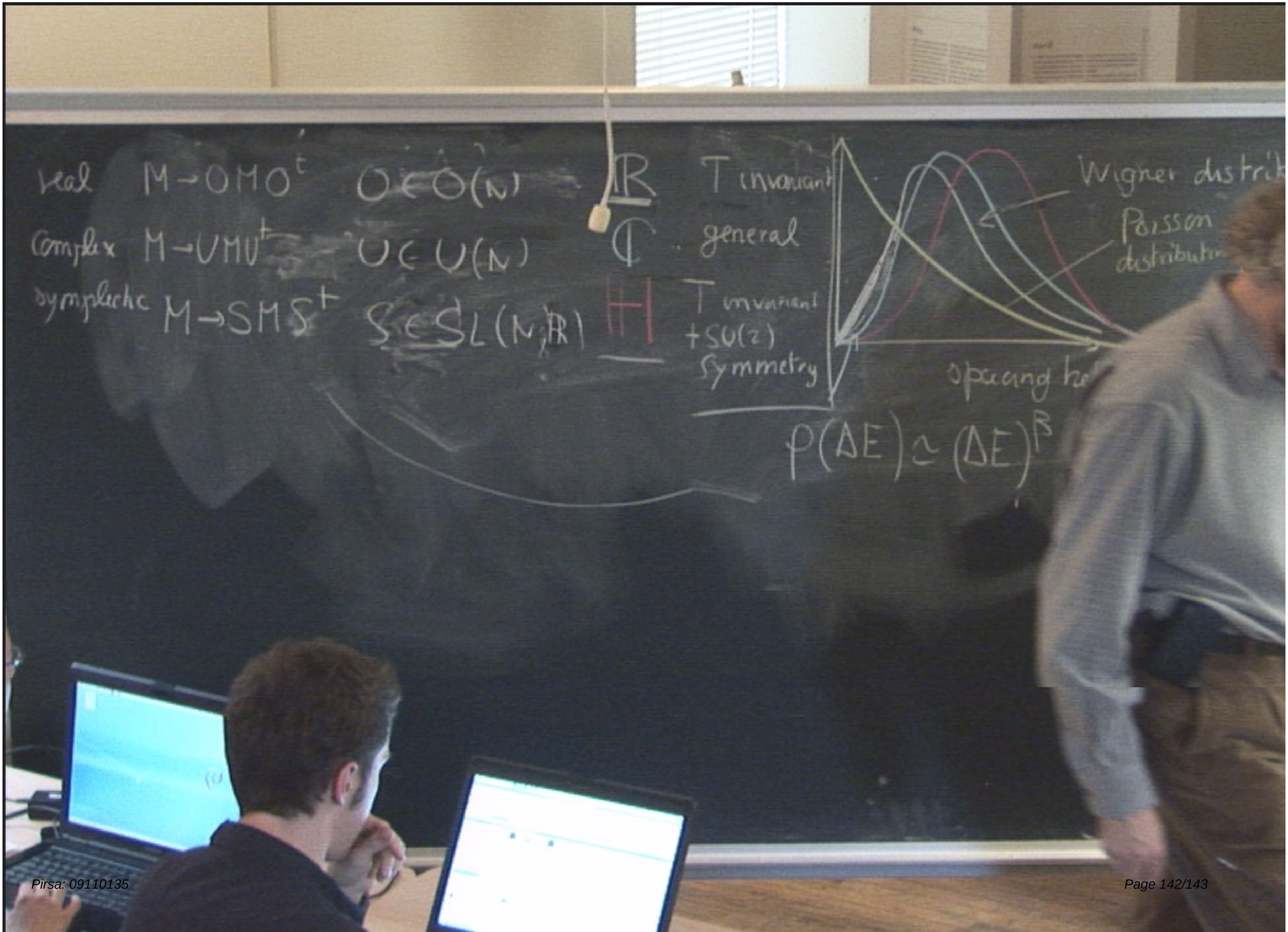
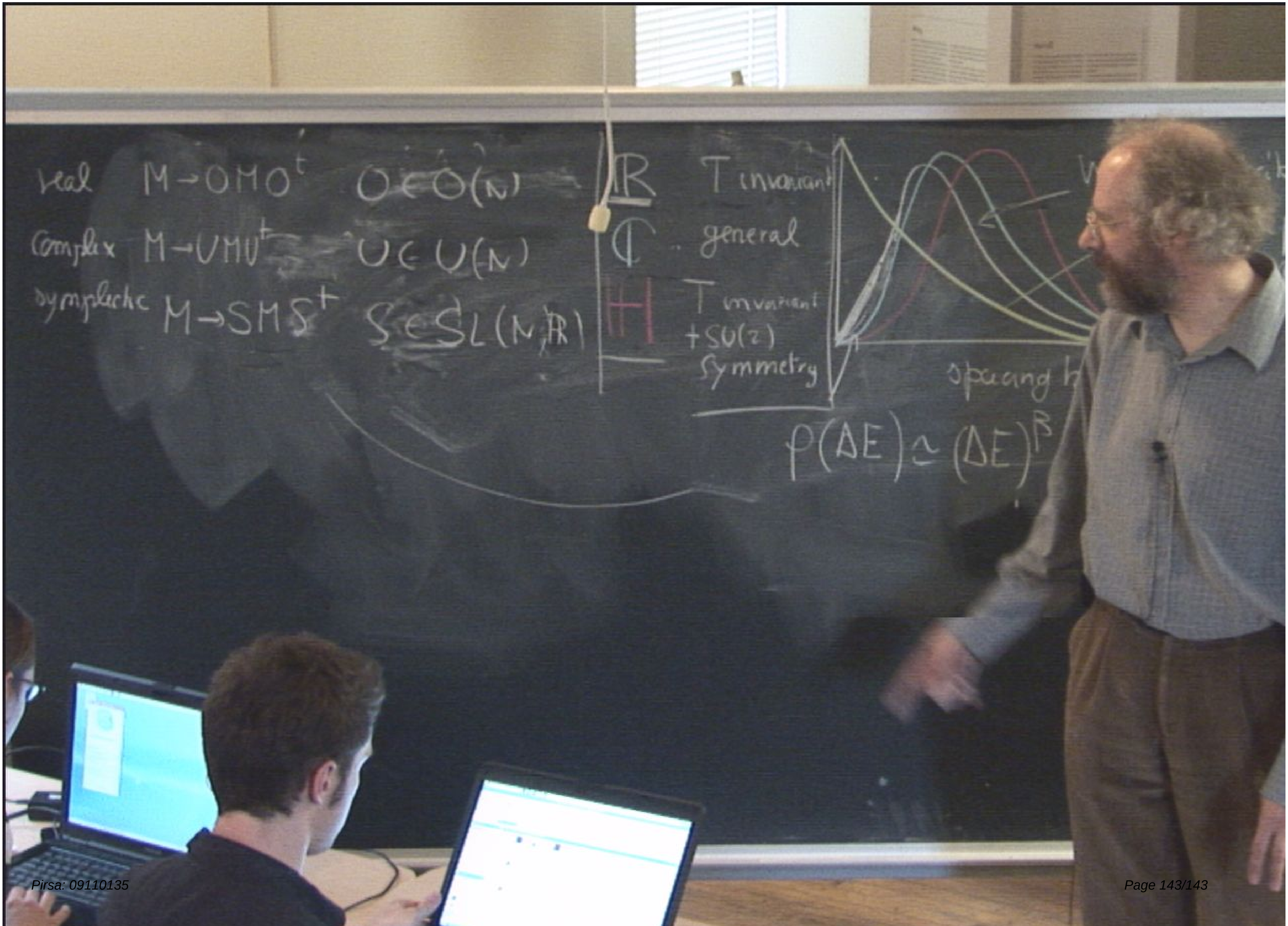

real M→OMO^t O∈O(N)
complex M→UMU^† U∈U(N)
symplectic M→SMS^† S∈SL(N)
ℝ T invariant
ℂ general
ℍ T invariant
+SU(2)
symmetry
Wigner distrib
Poisson distribution
spacing between
ρ(ΔE) ≃ (ΔE)^β


real M→OMO^t O∈O(N) ℝ T invariant
complex M→UMU^† U∈U(N) ℂ general
symplectic M→SMS^† S∈SL(N,ℝ) ℍ T invariant +SU(2) symmetry
Wigner distrib
Poisson distribution
spacing
ρ(ΔE) ≃ (ΔE)^β


real M→OMO^t O∈O(N)
complex M→UMU^† U∈U(N)
symplectic M→SMS^† S∈SL(N,R)
R T invariant
C general
H T invariant
+SU(2)
symmetry
spacing
ρ(ΔE) ≃ (ΔE)^β