

Title: Introduction to Effective Field Theory - Lecture 8C

Date: Nov 11, 2009 03:30 PM

URL: <http://pirsa.org/09110133>

Abstract:



Some issues that arise for Goldstone  
Bosons in nonrelativistic applications

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- May or may not break translation inv. (assume not).

May or " " " "  $T, P$  eg ferromagnet breaks  $T$   $\uparrow\uparrow\uparrow\uparrow$   
anti " " " " doesn't  $\uparrow\uparrow\uparrow$

Some issues that arise for Goldstone  
Bos. nonrelativistic applications

For the derivative interactions, systems like fluids,  
are not rotational invariant but sometimes  
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Some issues that arise for Goldstone  
in nonrelativistic applications

For  $\geq 2$  derivative interactions, systems like fluids,  
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Some issues that arise for Goldstone  
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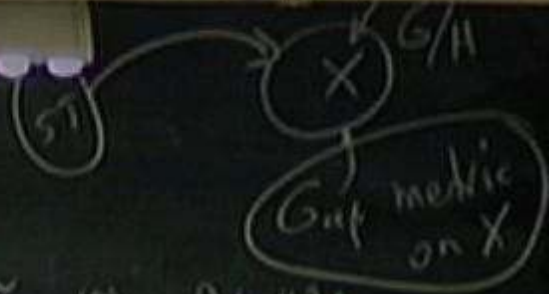
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eg  $\mathcal{L}_{kin} = -\frac{1}{2} G_{\mu\nu}(\theta) \partial_\mu \theta^\nu \partial^\mu \theta^\nu + \dots$

if  $\theta^\nu \rightarrow \tilde{\theta}^\nu = \theta^\nu + \xi^\nu(\theta)$  |  $\mathcal{L}_{kin} \rightarrow -\frac{1}{2} \tilde{G}_{\mu\nu}(\theta) \partial_\mu \theta^\nu \partial^\mu \theta^\nu$

where  $\tilde{G}_{\mu\nu} = G_{\mu\nu} + \left[ \xi^\gamma \partial_\gamma G_{\mu\nu} + G_{\mu\gamma} \partial_\nu \xi^\gamma + G_{\gamma\nu} \partial_\mu \xi^\gamma \right]$

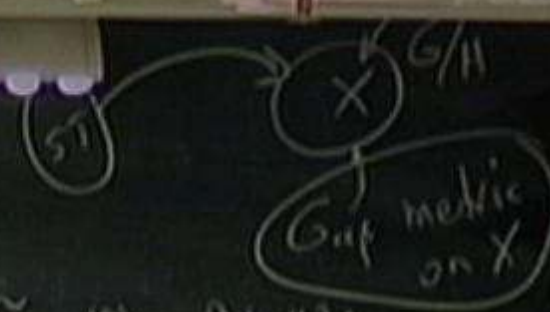


If T is unbroken (eg antiferromagnet) then

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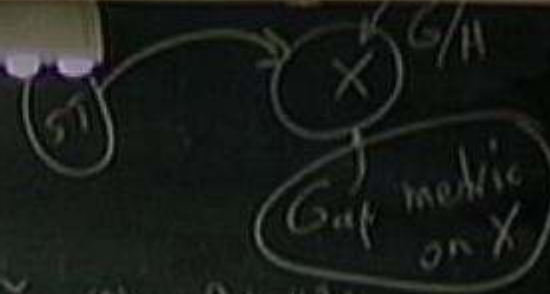
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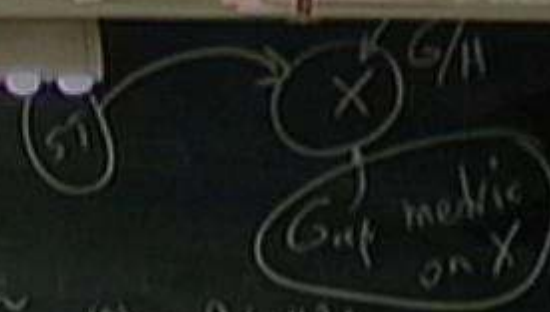
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 must go

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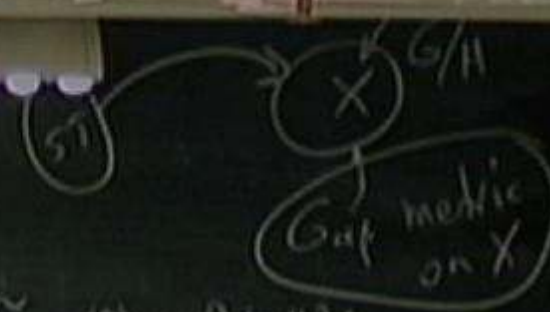
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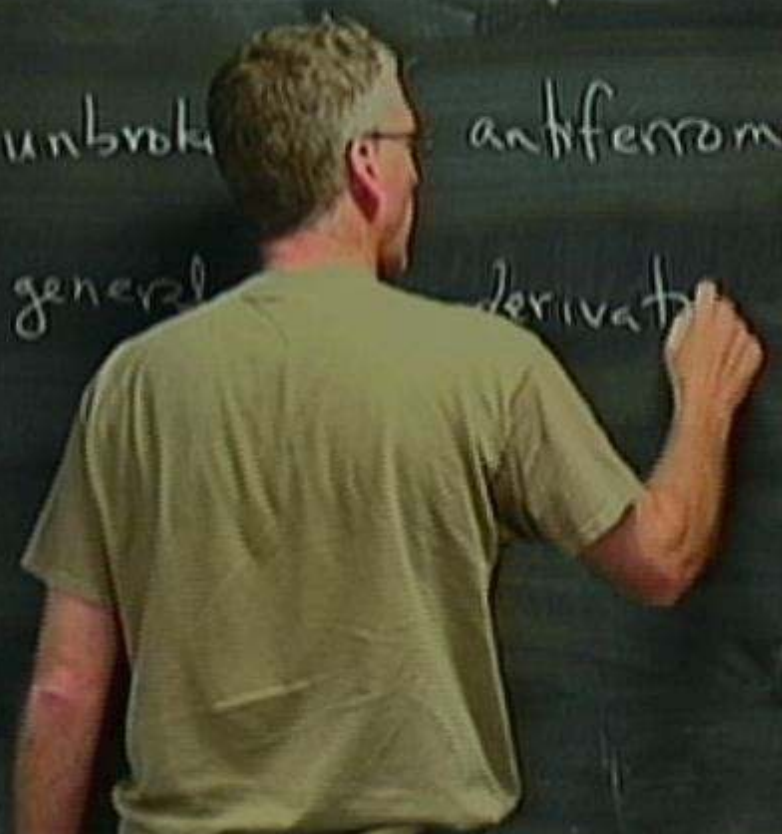
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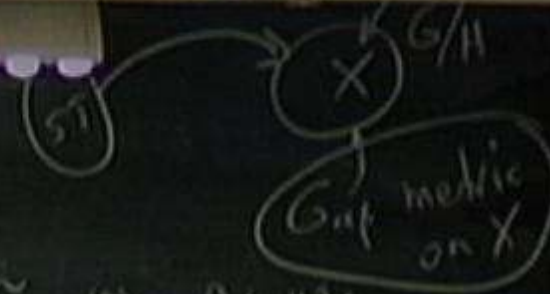
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If T is unbroken (antiferromagnet) then the most general derivative



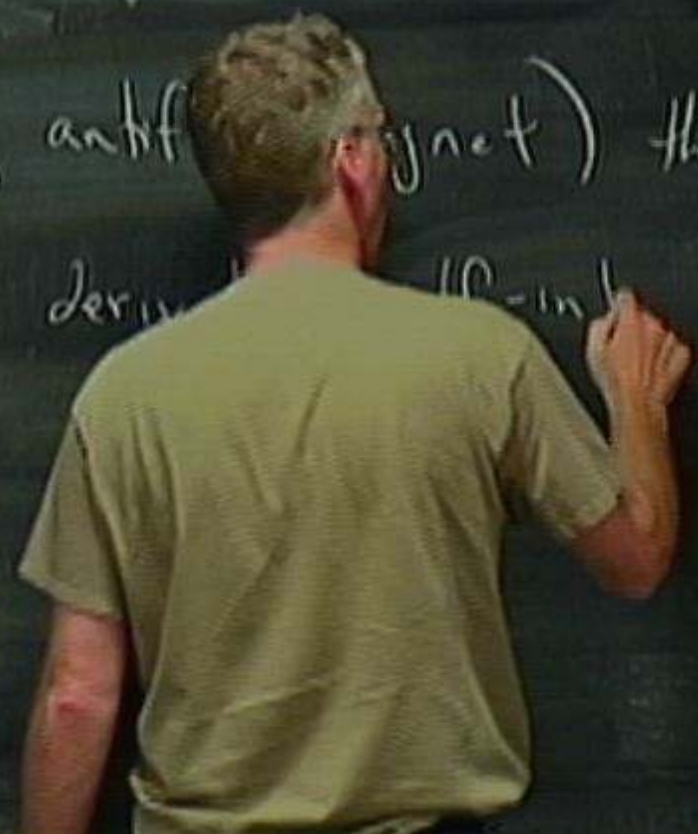
eg  $\mathcal{L}_{kin} = -\frac{1}{2} G_{\mu\nu}(\theta) \partial_\mu \theta^\nu \partial^\lambda \theta^\rho + \dots$



if  $\theta^\nu \rightarrow \tilde{\theta}^\nu = \theta^\nu + \xi^\nu(\theta)$   $\mathcal{L} \rightarrow -\frac{1}{2} \tilde{G}_{\mu\nu}(\theta) \partial_\mu \theta^\nu \partial^\lambda \theta^\rho$

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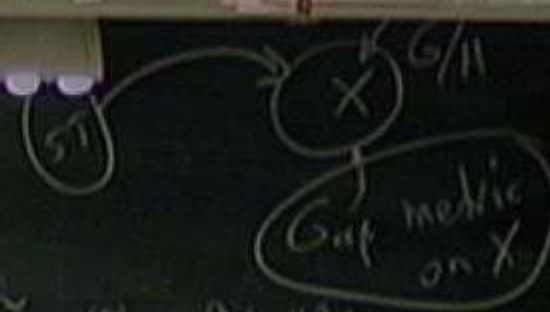
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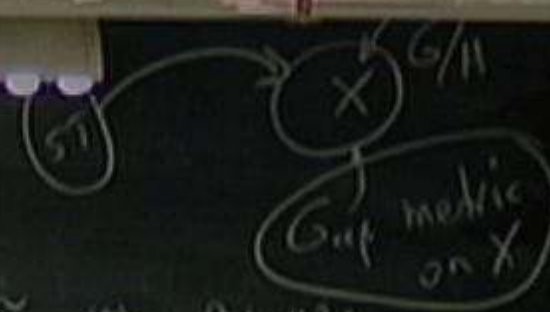
where  $\tilde{G}_{\mu\nu} = G_{\mu\nu} + \left[ \xi^\lambda \partial_\lambda G_{\mu\nu} + G_{\mu\lambda} \partial_\nu \xi^\lambda + G_{\lambda\nu} \partial_\mu \xi^\lambda \right]$



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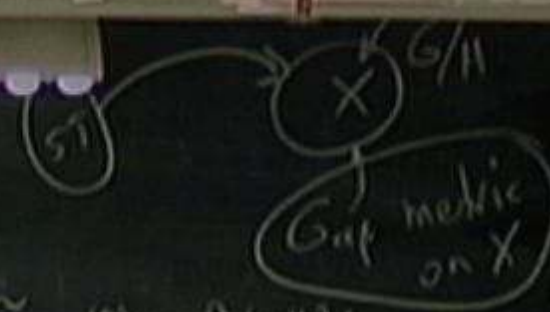


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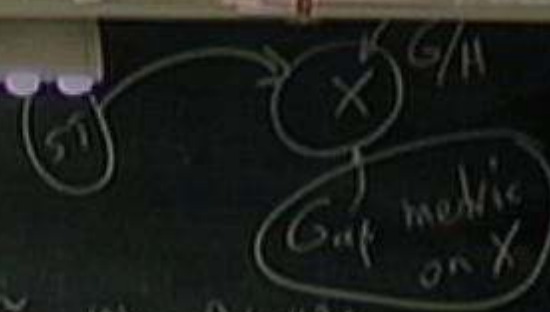


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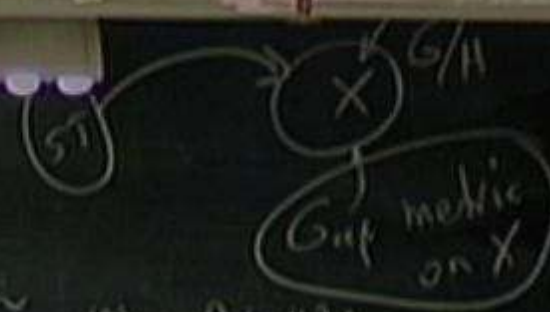


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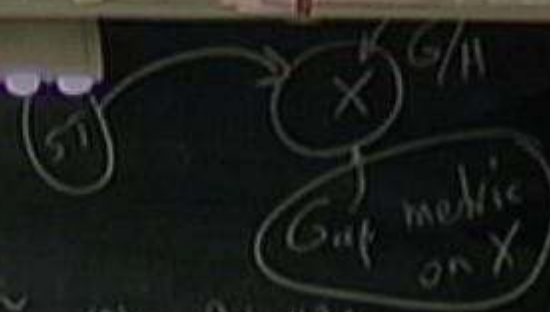


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If  $T$  is unbroken (eg antiferromagnet) then the most general  $\leq 2$  derivative self-interactions for Goldstones are

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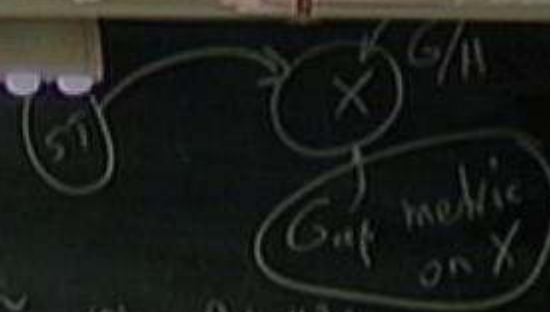


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If T is unbroken (ferromagnet) then the most general  $\leq 2$  derivative self-interactions for Goldstone bos

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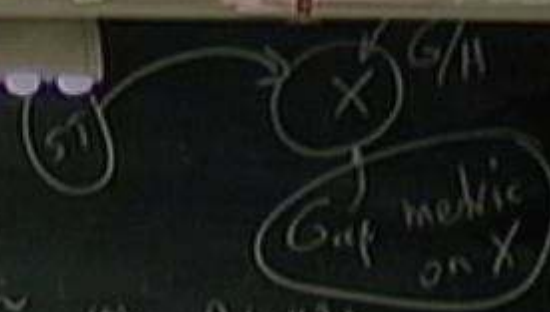
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If  $T$  unbroken (eg antiferromagnet) then the general  $\leq 2$  derivative self-interactions for bosons  $\phi \rightarrow H$  is:

eg  $\mathcal{L}_{kin} = -\frac{1}{2} G_{\mu\nu}(\theta) \partial_\mu \theta^\nu \partial^\mu \theta^\nu + \dots$

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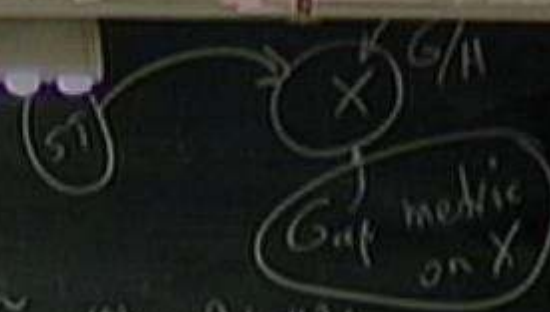
the most general  $\leq 2$  derivative self-interactions for Goldstone bosons  $G \rightarrow H$  are:

$\mathcal{L}_{\leq 2 \text{ deriv}}$

eg  $\mathcal{L}_{kin} = -\frac{1}{2} G_{\mu\nu}(\theta) \partial_\mu \theta^\nu \partial^\mu \theta^\nu + \dots$

if  $\theta^\nu \rightarrow \tilde{\theta}^\nu = \theta^\nu + \xi^\nu(\theta)$   $\mathcal{L} \rightarrow -\frac{1}{2} \tilde{G}_{\mu\nu}(\theta) \partial_\mu \tilde{\theta}^\nu \partial^\mu \tilde{\theta}^\nu$

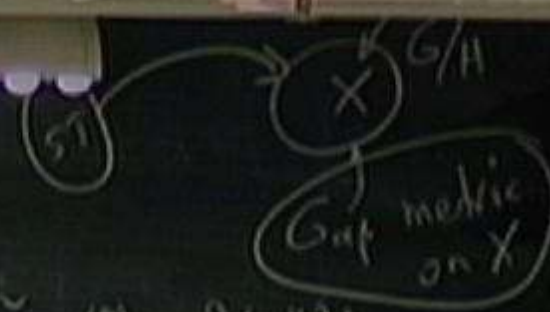
where  $\tilde{G}_{\mu\nu} = G_{\mu\nu} + \left[ \xi^\alpha \partial_\mu G_{\alpha\beta} + G_{\alpha\beta} \partial_\mu \xi^\alpha + G_{\mu\alpha} \partial_\beta \xi^\alpha \right]$



If  $T$  is unbroken (eg antiferromagnet) then the most general  $\leq 2$  derivative self-interactions for Goldstone bosons  $G \rightarrow H$  are:

$\mathcal{L} = \mathcal{L}_{2-deriv}$

eg  $\mathcal{L}_{kin} = -\frac{1}{2} G_{\mu\nu}(\theta) \partial_\mu \theta^\nu \partial^\mu \theta^\nu + \dots$



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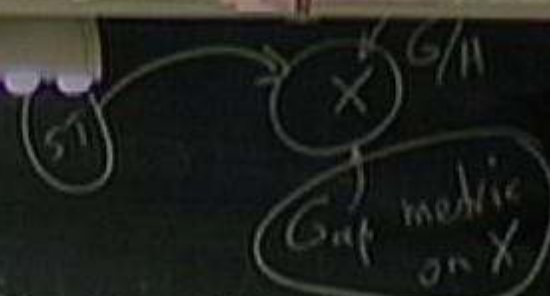
If T is unbroken (eg antiferromagnet) then the most general  $\leq 2$  derivative self-interactions Goldstone bosons  $G \rightarrow H$  are: [assuming rot, transl, inv]

$\mathcal{L}_{\leq 2-deriv} = -V_0 - \frac{F_0^2}{2} G_{\mu\nu}(\theta) \nabla^\mu \theta^\nu \cdot \nabla^\mu \theta^\nu + \frac{F_L^2}{2} G_{\mu\nu}(\theta) \dot{\theta}^\mu \dot{\theta}^\nu + \dots$

eg  $\mathcal{L}_{min} = -\frac{1}{2} G_{\mu\nu}(\theta) \partial_\mu \theta^\nu \partial^\mu \theta^\nu + \dots$

if  $\theta^\nu \rightarrow \tilde{\theta}^\nu = \theta^\nu + \xi^\nu(\theta)$   $\mathcal{L} \rightarrow -\frac{1}{2} \tilde{G}_{\mu\nu}(\theta) \partial_\mu \tilde{\theta}^\nu \partial^\mu \tilde{\theta}^\nu$

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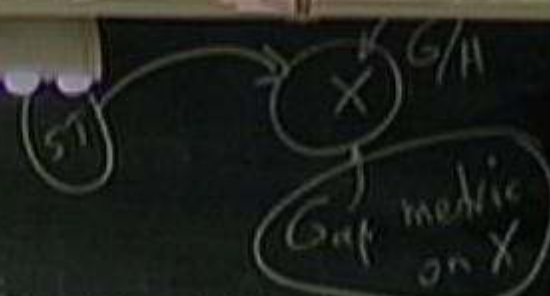
$\mathcal{L}_{(2+inv)} = -V_0 - \frac{F_0^2}{2} G_{\mu\nu}(\theta) \nabla^\mu \theta^\nu \cdot \nabla^\mu \theta^\nu + \frac{F_L^2}{2} G_{\mu\nu}(\theta) \partial^\mu \theta^\nu \partial^\mu \theta^\nu + \dots$

$\uparrow$   
 $\frac{1}{2} F^2 G_{\mu\nu}(\theta) \partial^\mu \theta^\nu \partial^\mu \theta^\nu \equiv \mathcal{L}_I$

eg  $\mathcal{L}_{kin} = -\frac{1}{2} G_{\mu\nu}(\theta) \partial_\mu \theta^\nu \partial^\mu \theta^\nu + \dots$

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$\uparrow$   
 $\partial_\mu \theta^\nu$

$-\frac{1}{2} F^2 G_{\mu\nu}(\theta) \partial_\mu \theta^\nu \partial^\mu \theta^\nu \text{ is LI}$

For the  $\leq 2$  derivative interactions, systems like fluids, solids needn't be rotational invariant but sometimes they are. (eg by "accident" for some lattice groups, or explicitly for fluids)

For linearized disturbances about  $\theta^{\nu} = \theta_0^{\nu} + \delta\theta^{\nu}$

For the  $\leq z$  derivative interactions, systems like fluids, solids needn't be rotational invariant but sometimes they are, (eg by "accident" for some lattice groups, or explicitly for fluids)

or linearized disturbances about  $\theta^{\alpha} = \theta_0^{\alpha} + \delta\theta^{\alpha}$

$$\ddot{\theta}^{\alpha} + \Gamma_{\beta\gamma}^{\alpha}(\theta) \dot{\theta}^{\beta} \dot{\theta}^{\gamma} - \left(\frac{F_s^2}{F_t^2}\right) \left[ \nabla^2 \theta^{\alpha} + \Gamma_{\beta\gamma}^{\alpha}(\theta) \nabla \theta^{\beta} \cdot \nabla \theta^{\gamma} \right] = 0$$

$$\Gamma_{\beta\gamma}^{\alpha} = \frac{1}{2} G^{\alpha\delta} (\partial_{\beta} G_{\delta\gamma} + \partial_{\gamma} G_{\delta\beta} - \partial_{\delta} G_{\beta\gamma})$$

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eg  $\mathcal{L}_{kin} = -\frac{1}{2} G_{\mu\nu}(\theta) \partial_\mu \theta^\nu \partial^\mu \theta^\nu + \dots$

(51)  
 $G_{\mu\nu}$  metric on  $X_\mu$

if  $\theta^\nu \rightarrow \tilde{\theta}^\nu = \theta^\nu + \xi^\nu$  then  $\mathcal{L}_{kin} \rightarrow -\frac{1}{2} \tilde{G}_{\mu\nu}(\theta) \partial_\mu \theta^\nu \partial^\mu \theta^\nu$

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If T is unbroken (eg antiferromagnet) then the most general  $\leq 2$  derivative self-interactions for Goldstone bosons  $\langle A \rangle = H$  are: [assuming rot, transl, inv]

$\mathcal{L}_{(2-der)} = -V_0 - \frac{F_1^2}{2} G_{\mu\nu}(\theta) \partial^\mu \theta^\nu \partial^\mu \theta^\nu + \frac{F_2^2}{2} G_{\mu\nu}(\theta) \partial^\mu \theta^\nu \partial^\rho \theta^\rho + \dots$

For the  $\leq 2$  derivative interactions, systems like fluids, solids needn't be rotational invariant but sometimes they are, (eg by 'accident' for some lattice groups, or explicitly for fluids)

For linearized disturbances about  $\theta^x = \theta_0^x + \delta\theta^x$

$$\ddot{\delta}^x + \Gamma_{pr}^x(\theta) \dot{\theta}^p \dot{\theta}^r - \left(\frac{F_s^2}{F_t^2}\right) \left[ \nabla^2 \delta^x + \Gamma_{pr}^d(\theta) \nabla \theta^p \cdot \nabla \theta^r \right] = 0$$

$$\Gamma_{pr}^x = \frac{1}{2} G^{\alpha\beta} (\partial_p G_{\beta r} + \partial_r G_{\beta p} - \partial_\beta G_{pr})$$

linearize:  $\ddot{\delta}^x - \left(\frac{F_s^2}{F_t^2}\right) \nabla^2 \delta^x = 0 \quad \omega^2 = \omega_s^2 \frac{p^2}{p_0^2} \quad \omega_s = \sqrt{\frac{F_s}{F_t}}$

Goldstone bosons  $\hookrightarrow H$  kye. [assuming rot., transl., inv.]

$$\mathcal{L}_{(2+inv)} = -V_0 - \frac{F^2}{2} G_{ab}(\theta) \nabla_0^\mu \theta^a \nabla_0^\mu \theta^b + \frac{F_L^2}{2} G_{ab}(\theta) \dot{\theta}^a \dot{\theta}^b + \dots$$

$\uparrow$   
 const.

$$- \frac{1}{2} F^2 G_{ab}(\theta) \partial_\mu \theta^a \partial^\mu \theta^b \quad \text{if LI}$$

$$\ddot{X}^\mu + \Gamma_{\nu\lambda}^\mu \dot{X}^\nu \dot{X}^\lambda = 0$$

Goldstone bosons  $\hookrightarrow H$   $\times \mathbb{R}^e$  [assuming rot., transl., inv.]

$$\mathcal{L}_{(2+1+1)} = -V_0 - \underbrace{\frac{F^2}{2} G_{ab}(\theta) \nabla_0^a \nabla_0^b + \frac{F^2}{2} G_{ab}(\theta) \dot{\theta}^a \dot{\theta}^b}_{-\frac{1}{2} F^2 G_{ab}(\theta) \dot{\theta}^a \dot{\theta}^b \text{ if LI}} + \dots$$

$\uparrow$   
 const.

$$\ddot{X}^\mu + \Gamma^\mu_{\nu\lambda} \dot{X}^\nu \dot{X}^\lambda = 0$$

Goldstone bosons  $\hookrightarrow H$   $\forall \xi \in [ \text{assuming rot., transl., inv} ]$

$$\mathcal{L}_{(2+inv)} = -V_0 - \underbrace{\frac{F^2}{2} G_{\alpha\beta}(\theta) \nabla^\alpha \theta^\beta \cdot \nabla^\alpha \theta^\beta + \frac{F_L^2}{2} G_{\alpha\beta}(\theta) \dot{\theta}^\alpha \dot{\theta}^\beta + \dots}_{-\frac{1}{2} F^2 G_{\alpha\beta}(\theta) \partial_\mu \theta^\alpha \partial^\mu \theta^\beta \text{ is LI}}$$

$\uparrow$   
 explicit

$$\ddot{X}^\mu + \Gamma^\mu_{\nu\lambda} \dot{X}^\nu \dot{X}^\lambda = 0$$



For the  $\leq \geq$  derivative interactions, systems like fluids, solids needn't be rotational invariant but sometimes they are, (eg by 'accident' for some lattice groups, or explicitly for fluids)

For linearized disturbances about  $\theta^x = \theta_0^x + \delta\theta^x$

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linearize:  $\ddot{\theta}^x - \left( \frac{F_s^2}{F_t^2} \right) \nabla^2 \theta^x = 0 \quad \omega^2 = c_s^2 p^2 \quad c_s = \frac{F_s}{F_t}$

Goldstone bosons  $\hookrightarrow H$   $\forall \epsilon$  [assuming rot, transl, inv]

$$\mathcal{L}_{(2+inv)} = -V_0 - \frac{F^2}{2} G_{\alpha\beta}(\theta) \nabla^\alpha \theta^\beta + \frac{F_L^2}{2} G_{\alpha\beta}(\theta) \dot{\theta}^\alpha \dot{\theta}^\beta + \dots$$

$\uparrow$   
 gauge

$-\frac{1}{2} F^2 G_{\alpha\beta}(\theta) \partial_\mu \theta^\alpha \partial^\mu \theta^\beta \quad \text{is LI}$

$$\ddot{X}^\mu + \Gamma^\mu_{\nu\lambda} \dot{X}^\nu \dot{X}^\lambda = 0$$

For the  $\leq 2$  derivative interactions, systems like fluids, solids needn't be rotational invariant but sometimes they are, (eg by "accident" for some lattice groups, or explicitly for fluids)

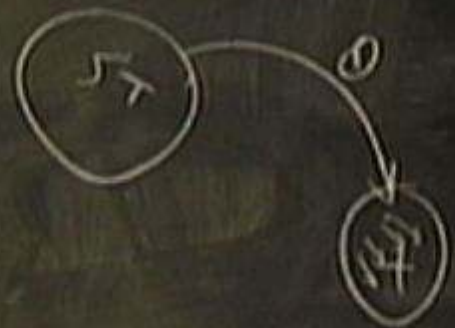
For an ferrimagnet,  $\Pi$  is broken + so a term linear in  $d/dt$  is allowed.

$$\mathcal{L}_{1-\text{der}} = -A \dot{\phi}^2$$

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$$-A \partial \partial^2$$



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ferrimagnet,  $\Pi$  is broken + so a term linear in  $d/dt$  is allowed.

$$P = -A \dot{\theta}^2$$



For the  $\leq 2$  derivative interactions, systems like fluids, solids needn't be rotational invariant but sometimes they are. (eg by "accident" for some lattice groups, or explicitly for fluids)

For an fermion system,  $\mathbb{T}$  is broken + so a term linear in  $\mathbf{p}$  is allowed.

$$\mathcal{L}_{1-der} =$$



dispersion relation  $\rightarrow \omega = c_s^2 p^{1/2}$

For the  $\leq 2$  derivative interactions, systems like fluids, solids needn't be rotational invariant but sometimes they are, (eg by 'accident' for some lattice groups, or explicitly for fluids)

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this changes dispersion relation to  $\omega^2 = c_s^2 p^2$

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May or may not break translation inv. (assume not)

May or " " " " T, P. eg ferromagnet breaks T ↑↑↑  
into " " " " doesn't " ↓↓↓

In some cases (eg  $SO(3) \rightarrow SO(2)$ ) no invariant such  
an  $\mathcal{L}_{1-d}$  is possible, because there is no such  
a vector field on  $X$ .

BUT  $G$  being a symmetry only requires  $S = \int dt d^3x \mathcal{L}$  to inv.

when is  $\delta \mathcal{L}_{1-d} = \frac{d}{dt} \Omega(t)$ ?

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when is  $\delta \mathcal{L}_{1-d} = \frac{d}{dt} \Omega(\theta)$ ?

$\uparrow$   
 $\delta \theta^\alpha$

$$-\frac{1}{2} F^2 (G_{\alpha\beta}(\theta)) \partial_\mu \theta^\alpha \partial^\mu \theta^\beta \quad \text{LJ}$$

$$\theta^\alpha \rightarrow \theta^\alpha + \delta \theta^\alpha$$

$$A_\alpha(\theta) \dot{\theta}^\alpha \rightarrow A_\alpha(\theta) (\delta \dot{\theta}^\alpha + \partial_\beta A_\alpha \delta \theta^\beta \dot{\theta}^\alpha)$$

$$= \frac{d}{dt} [A_\alpha \delta \theta^\alpha] + \left( -\partial_\beta A_\alpha \dot{\theta}^\beta + \partial_\alpha A_\beta \dot{\theta}^\beta \right)$$

$\frac{1}{2} F^2 (G_{\alpha\beta}(\theta) \dot{\theta}^\alpha \dot{\theta}^\beta) \quad \text{LJ}$

$$\theta^\alpha \rightarrow \theta^\alpha + \delta \theta^\alpha$$

$$\begin{aligned}
 A_\alpha(\theta) \dot{\theta}^\alpha &\rightarrow A_\alpha(\theta) (\delta \dot{\theta}^\alpha + \partial_\beta A_\alpha \delta \theta^\beta \dot{\theta}^\alpha) \\
 &= \frac{d}{dt} [A_\alpha \delta \theta^\alpha] + \underbrace{\left( -\partial_\beta A_\alpha \dot{\theta}^\beta + \partial_\alpha A_\beta \dot{\theta}^\beta \right)}
 \end{aligned}$$

$$A_\alpha \dot{\theta}^\alpha \rightarrow \tilde{A}_\alpha \dot{\theta}^\alpha \quad \tilde{A}'_\alpha = A_\alpha + \text{Lie deriv}$$

$$\begin{aligned}
 \mathcal{L}_{(2\text{-deriv})} &= -V_0 - \frac{F^2}{2} G_{\alpha\beta}(\theta) \nabla \theta^\alpha \cdot \nabla \theta^\beta + \frac{F^2}{2} G_{\alpha\beta}(\theta) \dot{\theta}^\alpha \dot{\theta}^\beta + \dots \\
 &\quad \uparrow \\
 &\quad \text{gauge}
 \end{aligned}$$

$-\frac{1}{2} F^2 G_{\alpha\beta}(\theta) \partial_\lambda \theta^\alpha \partial^\lambda \theta^\beta \quad \text{is LI}$

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$$A_\alpha(\theta) \dot{\theta}^\alpha \rightarrow A_\alpha(\theta) \delta \dot{\theta}^\alpha + \partial_\beta A_\alpha \delta \theta^\beta \dot{\theta}^\alpha$$

$$= \frac{d}{dt} [A_\alpha \delta \theta^\alpha] + \underbrace{\left( -\partial_\beta A_\alpha \dot{\theta}^\beta + \partial_\alpha A_\beta \dot{\theta}^\beta \right)}_{\text{Lie derivative of } A_\alpha} \delta \theta^\alpha$$

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For the  $\leq 2$  derivative interactions, systems like fluids, solids needn't be rotational invariant but sometimes they are, (eg by 'accident' for some lattice groups, or explicitly for fluids)

But if  $\delta \mathcal{L}_{int} = \mathcal{L} A_\alpha \delta \theta^\alpha + \frac{d}{dt} [ \dots ]$

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But if 
$$\delta \alpha_{1-d} = \mathcal{L} A_\alpha \delta \theta^\alpha + \frac{d}{dt} \left[ \dots \right]$$

$$\begin{aligned}
 \mathcal{L}_{(2\text{-deriv})} = & -V_0 - \frac{F^2}{2} G_{\alpha\beta}(\theta) \nabla\theta^\alpha \cdot \nabla\theta^\beta + \frac{F^2}{2} G_{\alpha\beta}(\theta) \dot{\theta}^\alpha \dot{\theta}^\beta + \dots \\
 & \uparrow \\
 & \text{scalar} \\
 & \underbrace{-\frac{1}{2}F^2 G_{\alpha\beta}(\theta) \partial_\mu\theta^\alpha \partial^\mu\theta^\beta}_{\text{LI}}
 \end{aligned}$$

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They are, (eg by "accident" for some lattice groups, or explicitly

$$\text{if } \delta \mathcal{L}_{1-d} = \mathcal{L} A_\alpha \dot{\theta}^\alpha = \frac{d}{dt} \left[ \underbrace{\Omega(\alpha)}_{\partial_\alpha \Omega \dot{\theta}^\alpha} \right]$$

$$\delta A_\alpha = \mathcal{L} A_\alpha = \partial_\alpha \Omega$$

$\alpha$  (2-deriv)  $- V_0 - \frac{1}{2} G_{ab}(\theta) \dot{\theta}^a \dot{\theta}^b + \frac{1}{2} G_{ab}(\theta) \theta^a \theta^b + \dots$   
 $\uparrow$   
 gauge

$-\frac{1}{2} F^2 G_{ab}(\theta) \partial_\mu \theta^a \partial^\mu \theta^b$  is LI

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$$\text{But if } \delta \mathcal{L}_{int} = \mathcal{L} A_\alpha \dot{\theta}^\alpha = \underbrace{\frac{d}{dt} [\Omega(\alpha)]}_{\partial_\alpha \Omega \dot{\theta}^\alpha}$$

$$\text{so } \delta A_\alpha = \mathcal{L} A_\alpha = \partial_\alpha \Omega$$

$\int$  Invariance only requires  $A_\alpha$  to be inv. up to a gauge transf'n.

$$\text{ie } F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \text{ to be inv.}$$

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So in the example of most interest  
 $SO(3) \rightarrow SO(2)$  where  $SO(3)/SO(2) = S^2$

Such an inv.  $F_{inv}$  exists.  $F_{inv} = \sin \theta$   
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So in the example of most interest

$$SO(3) \rightarrow SO(2)$$

where  $SO(3)/SO(2) = S^2$ ,

Such an inv.  $F_{inv}$  exists.

$$F_{\partial_x} = \sin \theta$$

$$F_{\partial_y} = c \cos \theta \dot{\theta}$$

$$A_{\partial_x} = (\cos \theta \pm 1)$$

For the  $\leq 2$  derivative interactions, systems like fluids, solids needn't be rotational invariant but sometimes they are. (eg by 'accident' for some lattice groups, or explicitly for fluids)

But if  $\delta \mathcal{L}_{int} = \mathcal{L} A_\mu \dot{\theta}^\mu = \underbrace{\frac{d}{dt} [\Omega(\theta)]}_{\partial_\alpha \Omega \dot{\theta}^\alpha}$

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Standard Model as an effective theory.

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the SM is by defn  
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Standard Model as an effective theory:

1) the SM is by defn  
the most general renormalizable theory  
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2) yet it doesn't describe some things: DM, DE?, grav. by  $(M_p)$   
neutrino masses,  $M_p \gg M_e \gg 10^3 \text{ eV}$

solids needn't be rotational invariant but sometimes they are. (eg by 'accident' for some lattice groups, or explicitly for fluids)

Standard Model as an effective theory:

- the SM is by def<sup>n</sup>
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  - 2) yet it doesn't describe some things: DM, DE?, grav. by  $(M_P)$   
 $\nu$ -masses,  $10^{16} M_P \gg 10^2 \text{ GeV (SM)}$

of the  $Z$  transformations  
 - May or may not break translation inv. (assume no)  
 May or " " " "  $T_{11}P$  eg ferromagnet breaks  $T$   
 anti " " " " doesn't

Field content:  $SU_3 \times SU_2 \times U_1$

$G_{\mu}^a$   $W_{\mu}^a$   $B_{\mu}$   $\leftarrow$   $\gamma$   $W^{\pm}$   $Z$ , gluons 8

fermions:  $\psi$

$\sum_k \psi_k, u_k, d_k$  3 generations

scalar:  $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$   $\rightarrow$  3 WBSG, 1 Higgs

$$\mathcal{L}_{(2-deriv)} = -V_0 - \frac{F_L^2}{2} G_{ab}(\theta) \nabla \theta^a \cdot \nabla \theta^b + \frac{F_L^2}{2} G_{ab}(\theta) \dot{\theta}^a \dot{\theta}^b + \dots$$

$\uparrow$   
 constant  
 $-\frac{1}{2} F^2 G_{ab}(\theta) \partial_\mu \theta^a \partial^\mu \theta^b$  is LI

expect  $\mathcal{L}_{eff} = \sqrt{-g} \sum_n \frac{c_n \mathcal{O}_n(\psi, A, \dots, \partial\psi)}{M^{d_n}} + M_P^2 R$   $M = \text{big mass}$

$$= \sqrt{-g} \left[ c_0 M_B^4 + \underbrace{c_2 M^2}_{M = \lambda V} \phi^\dagger \phi + (\log M) (\text{dim } 4) \right]$$

$V = 246 \text{ GeV}$   
 $\lambda \lesssim 4\pi$

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$$M = \lambda V$$

$$V = 246 \text{ GeV}$$

$$\lambda \approx 4\pi$$

$$\mathcal{D}_\mu \phi^\dagger \mathcal{D}^\mu \phi$$

$$b \rightarrow U \phi$$

$$\mathcal{L}_{(2+dim)} = -V_0 - \frac{F^2}{2} G_{\mu\nu}^2(\theta) \nabla\theta^\mu \cdot \nabla\theta^\nu + \frac{F_t^2}{2} G_{\mu\nu}(\theta) \dot{\theta}^\mu \dot{\theta}^\nu + \dots$$

$\underbrace{\hspace{10em}}_{-\frac{1}{2}F^2 G_{\mu\nu}(\theta) \partial_\mu\theta^\alpha \partial_\nu\theta^\beta \text{ is LI}}$

$\uparrow$   
 const.

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$\uparrow$   
 $(10^{-2} \text{ eV})^4$

$M = \lambda V$   
 $V = 246 \text{ GeV}$   
 $\lambda \lesssim 4\pi$

$\partial G_{\mu\nu}^2 \sim \tilde{G}^{\mu\nu}$

$$L_{eff} = \sqrt{-g} \sum_n \frac{c_n \mathcal{O}_n(\psi, A, \dots, \partial\psi)}{M^{d_n}} + M_P^2 R \quad (10^{16} \text{ GeV})^2 \quad M = \text{big mass}$$

$$= \sqrt{-g} \left[ c_0 M_B^4 + c_2 M^2 \boxed{\phi^\dagger \phi} + (\log M) (\text{dim } 4) \right]$$

$\uparrow$   
 $(10^{16} \text{ GeV})^4$   
 $M = \lambda V$   
 $V = 246 \text{ GeV}$   
 $\lambda \approx 4\pi$

$$+ \frac{1}{M} LLHH + \text{dim } 6$$

$$\begin{aligned}
 \mathcal{L}_{eff} &= \sqrt{-g} \sum_n \frac{c_n \mathcal{O}_n(\Psi, A, \dots, \partial\Psi)}{M^{d_n}} + M_{Pl}^2 R + (10^{16} \text{ GeV})^2 \\
 &= \sqrt{-g} \left[ c_0 M_{Pl}^4 + \underbrace{c_2 M^2}_{M=\lambda V} \boxed{\Phi^\dagger \Phi} + (\log M) \underbrace{(\text{dim } 4)}_{VVV} \right] \\
 &\quad + \frac{1}{M} \underbrace{LLHH}_{\checkmark} + (\text{dim } 6) \text{ ?}
 \end{aligned}$$

$M = \text{big mass}$   
 $\theta G_{\mu\nu}^2 \sim G_{\mu\nu}^2$   
 $\lambda \lesssim 4\pi$   
 $V = 246 \text{ GeV}$   
 $(10^{-2} \text{ eV})^4$