Title: CDT and Horava-Lifshitz gravity

Date: Nov 08, 2009 05:00 PM

URL: http://pirsa.org/09110131

Abstract:

CDT and Horava-Lifshitz gravity

J. Ambjørn[1, 3] A. Görlich[2] J. Jurkiewicz[2] S. Jordan[3]
R. Loll[3]

[1]Niels Bohr Institute, Copenhagen, Denmark

[2]Jagellonian University, Krakow, Poland

[3]University of Utrecht, The Netherlands

Gravity at a Lifshitz point
PI, November 8-10, 2009

QG main goal (at least in 80ties)

- Define the theory of QG
- Obtain the background geometry ($\langle g_{\mu\nu} \rangle$) we observe
- Study the fluctuations around the background geometry

What lattice gravity (dynamical triangulation, DT) offers:

- A non-perturbative QFT definition of QG using just standard QFT via the path integral.
- A background independent formulation.
- A path integral formulated directly as a sum over geometries (piecewise linear geometries as used, require no coordinates).

Virtues and drawbacks of DT

- V The Einstein-Hilbert action has a natural geometric realization on piecewise linear geometries (Regge).
- V The cut-off a is geometric (diffeomorphism invariant)
- D The formulation inherently Euclidean (Euclidean QG ? (action unbounded from below)).
- V The cut-off a automatically acts as a regularization of the unboundedness of Euclidean QG.
- D Gravity becomes ”emergent”: a subtle interplay between quantum measure and the action used.
- V Works beautifully when Euclidean QG is well defined: in 2d.

Main DT drawback: no interesting IR limit for $d > 2$.
That led to DT → CDT (causal dynamical triangulations)

CDT virtues and drawbacks

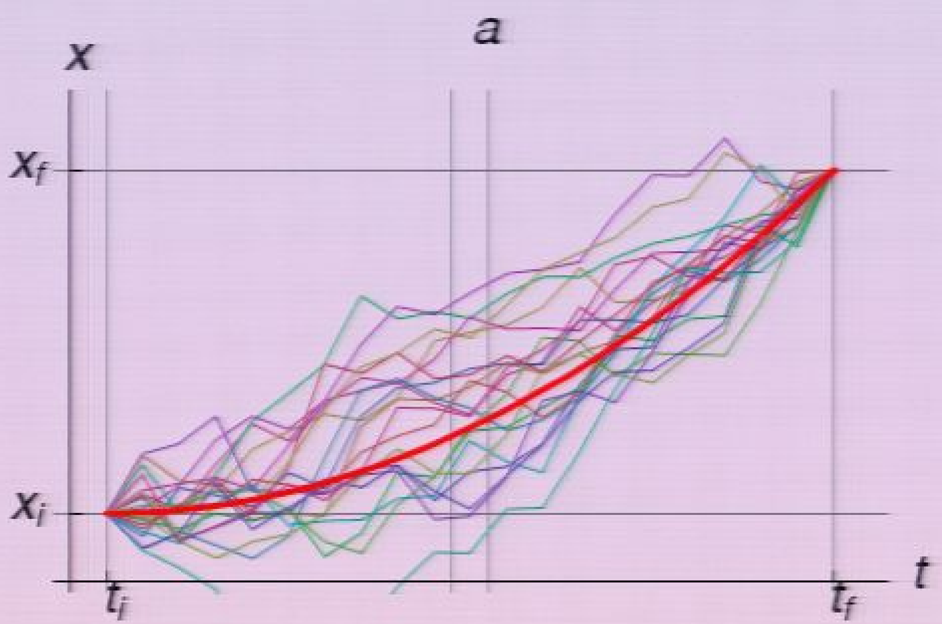
- V Path integral a sum over Lorentzian geometries.
- D One assumes the existence of a global time foliation.
- V Each configuration allows a rotation to Euclidean geometry, corresponding to $t \to t_4 = it$. One can then i.e. using Monte Carlo simulations. (The corresponding set of geometries will be different from the full set of Euclidean geometries).

Main CDT virtue: An interesting IR limit seems to exist.
Main questions: Is the theory UV complete and if so, what are the short distance properties of the theory.

Lattice gravity: causal dynamical triangulations

Basic tool: The path integral

Text-book example: non-relativistic particle in one dimension.



$$x(t) = \langle x(t) \rangle + y(t)$$
$$\langle |y| \rangle \propto \sqrt{\hbar / m\omega}$$

In QG we want $\langle x(t) \rangle$

$$\langle |y| \rangle \propto \sqrt{\hbar G}$$

Transition amplitude as a weighted sum over all possible trajectories. On the plot: time is discretized in steps a, trajectories are piecewise linear.

In a continuum limit $a \to 0$

$$G(\mathbf{x}_{\mathrm{i}}, \mathbf{x}_{\mathrm{f}}, t) := \int_{\text{trajectories: } \mathbf{x}_{\mathrm{i}} \to \mathbf{x}_{\mathrm{f}}} \mathrm{e}^{iS[\mathbf{x}(t)]}$$

where $S[\mathbf{x}(t)]$ is a classical action.

The QG amplitude between the two geometric states separated a proper time t

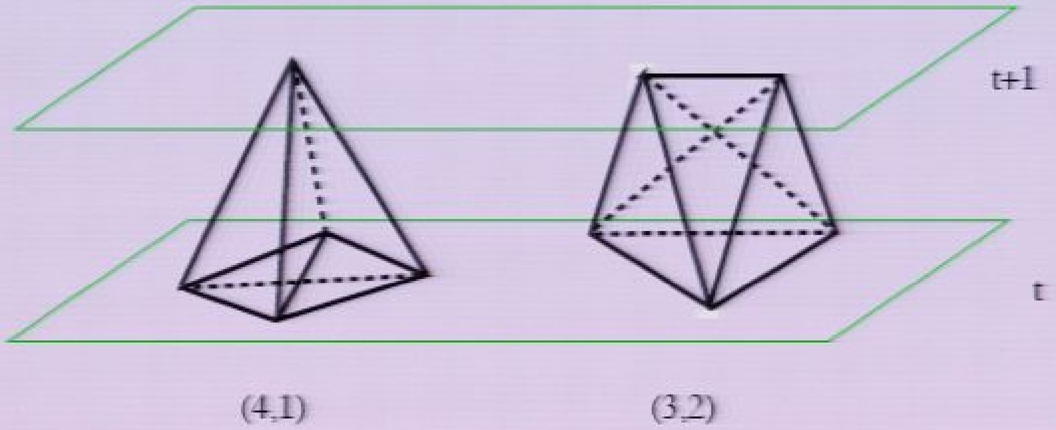
$$G(\mathbf{g}_{\mathrm{i}}, \mathbf{g}_{\mathrm{f}}, t) := \int_{\text{geometries: } \mathbf{g}_{\mathrm{i}} \to \mathbf{g}_{\mathrm{f}}} \mathrm{e}^{iS[\mathbf{g}_{\mu\nu}(t')]}$$

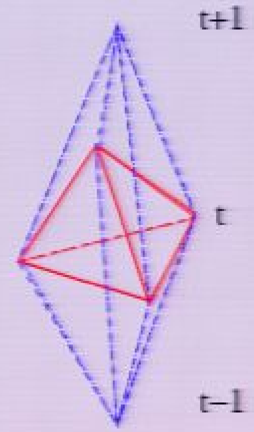
To define this path integral we need a geometric cut-off a and a definition of the class of geometries entering.

showcasing piecewise linear geometries via building blocks:

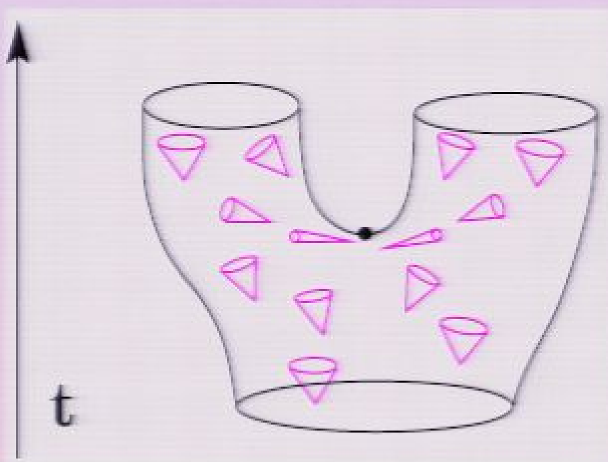
⟶

2d 3d 4d



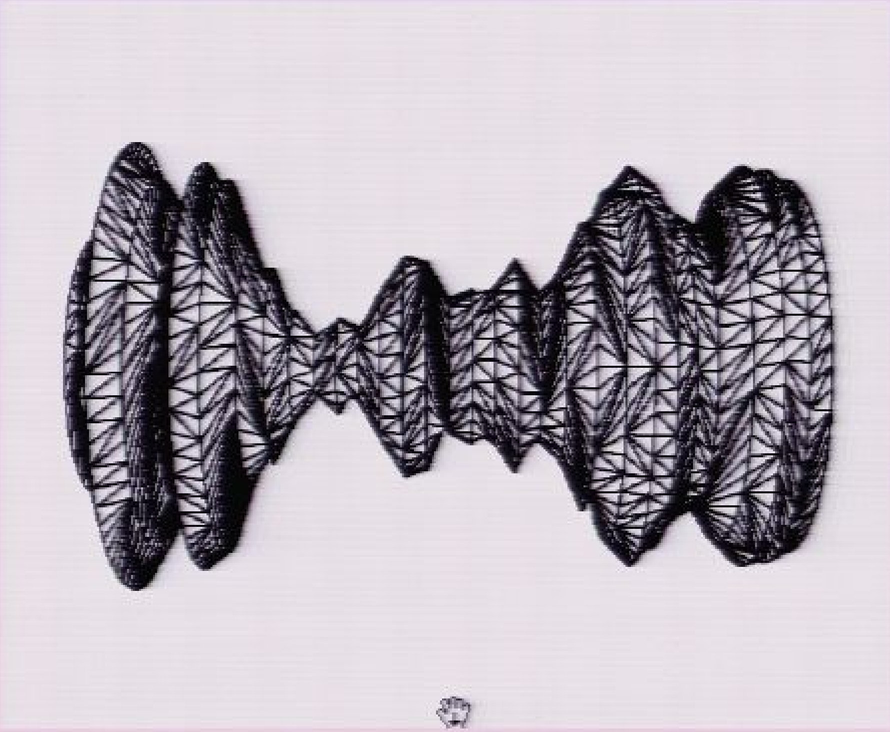


CDT slicing in proper time. Topology of space preserved.
Situation below not allowed.



$$G(\mathbf{g}_{\mathrm{i}}, \mathbf{g}_{\mathrm{f}}, t) := \int_{\text{geometries: } \mathbf{g}_{\mathrm{i}} \to \mathbf{g}_{\mathrm{f}}} \mathrm{e}^{iS[\mathbf{g}_{\mu\nu}(t')]}$$

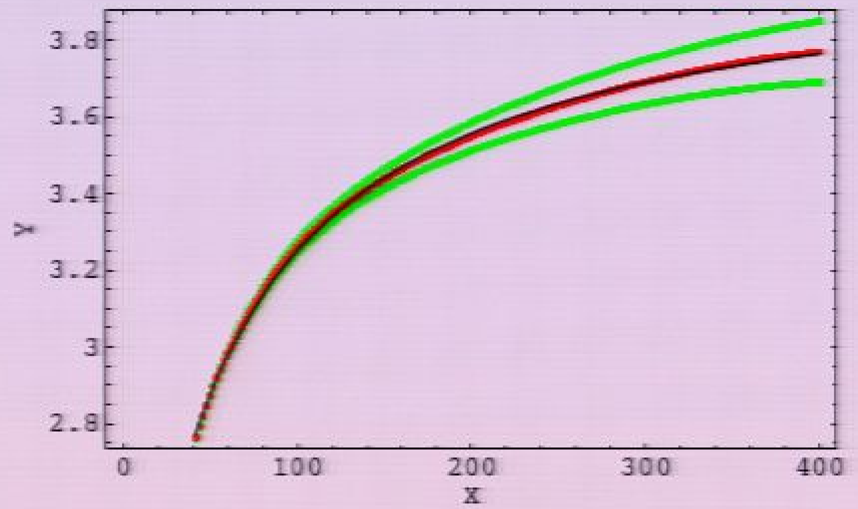
$$= \lim_{a \to 0} \sum_{T : T_i^{(3)} \to T_f^{(3)}} \frac{1}{C_T} \, \mathrm{e}^{iS_T}$$



Relation to the Horava model ?

The set-up is precisely as in the Horava model.

In addition the so-called spectral dimension in CDT and in the Horava model show the same characteristic behavior:



But the actions in the two models seemingly unrelated ?

We now have to choose a specific action ($a_t^2 = \tilde{\alpha} a_s^2, \;\; \tilde{\alpha} > 7/12$)

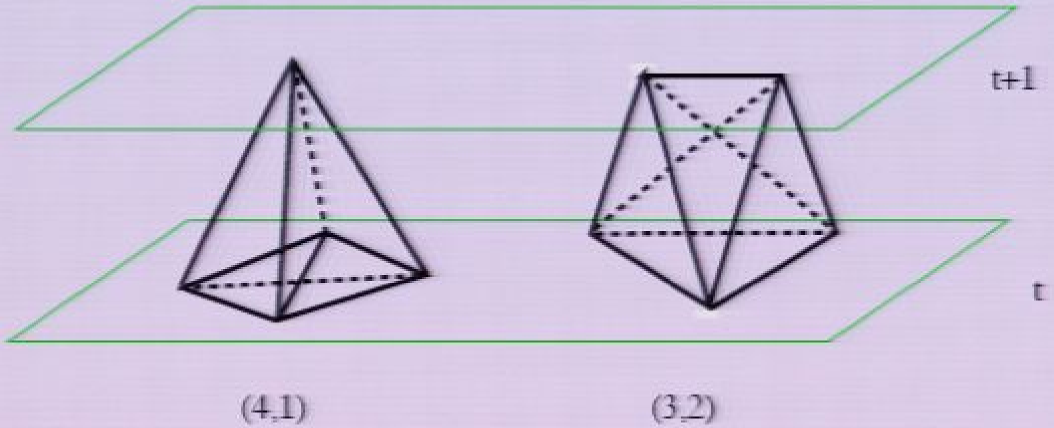
$$
\begin{aligned}
S_E = & -k^{(b)} \pi \sqrt{4\tilde{\alpha}-1} N_0 \\
& + N_4^{(4,1)} \left(k^{(b)} \sqrt{4\tilde{\alpha}-1} \left[-\frac{\pi}{2} - \frac{\sqrt{3}}{\sqrt{4\tilde{\alpha}-1}} \arcsin \frac{1}{2\sqrt{2}\sqrt{3\tilde{\alpha}-1}} \right. \right. \\
& \qquad \left. \left. + \frac{3}{2} \arccos \frac{2\tilde{\alpha}-1}{6\tilde{\alpha}-2} \right] + \lambda^{(b)} \frac{\sqrt{8\tilde{\alpha}-3}}{96} \right) \\
& + N_4^{(3,2)} \left(k^{(b)} \sqrt{4\tilde{\alpha}-1} \left[-\pi + \frac{\sqrt{3}}{4\sqrt{4\tilde{\alpha}-1}} \arccos \frac{6\tilde{\alpha}-5}{6\tilde{\alpha}-2} \right. \right. \\
& \qquad \left. + \frac{3}{4} \arccos \frac{4\tilde{\alpha}-3}{8\tilde{\alpha}-4} + \frac{3}{2} \arccos \frac{1}{2\sqrt{2}\sqrt{2\tilde{\alpha}-1}\sqrt{3\tilde{\alpha}-1}} \right] \\
& \qquad \left. + \lambda^{(b)} \frac{\sqrt{12\tilde{\alpha}-7}}{96} \right).
\end{aligned}
$$

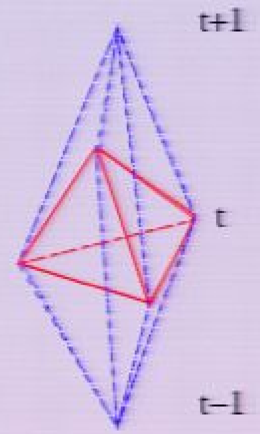
This expression can be summarized as

$$S_E = -\left(\kappa_0 + 6\Delta\right) N_0 + \kappa_4 \left(N_4^{(4,1)} + N_4^{(3,2)}\right) + \Delta \left(2N_4^{(4,1)} + N_4^{(3,2)}\right)$$

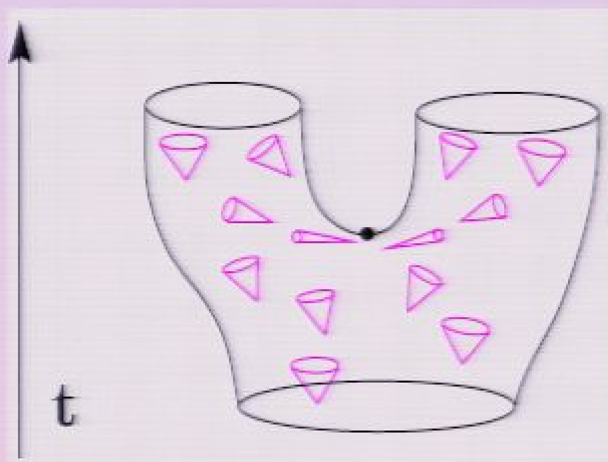
Δ is a function of $\tilde{\alpha}$ the asymmetry parameter between the space and lattice links. $\Delta = 0$ corresponds to $a_t = a_s$, i.e. $\tilde{a} = 1$.

In a given computer simulation $N_4 = N_4^{(4,1)} + N_4^{(3,2)}$ is kept fixed and thus effectively we have only two coupling constants: κ_0 and Δ.





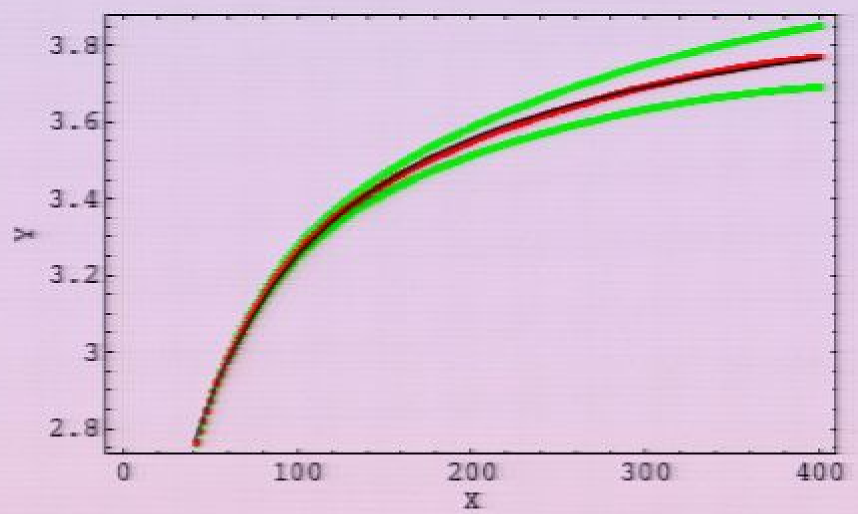
CDT slicing in proper time. Topology of space preserved.
Situation below not allowed.



Relation to the Horava model ?

The set-up is precisely as in the Horava model.

In addition the so-called spectral dimension in CDT and in the Horava model show the same characteristic behavior:



But the actions in the two models seemingly unrelated ?

We now have to choose a specific action ($a_t^2 = \tilde{\alpha} a_s^2, \ \ \tilde{\alpha} > 7/12$)

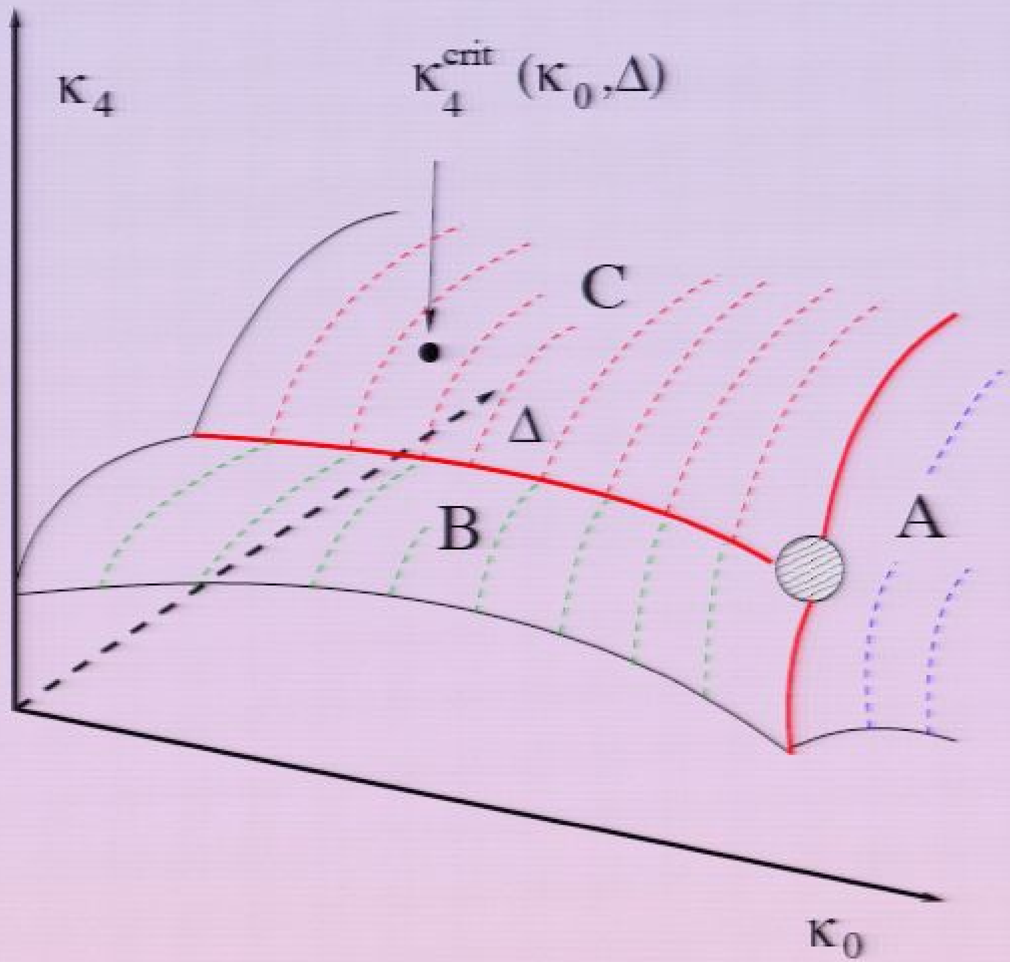
$$
\begin{aligned}
S_E = & -k^{(b)}\pi\sqrt{4\tilde{\alpha}-1}N_0 \\
& + N_4^{(4,1)}\left(k^{(b)}\sqrt{4\tilde{\alpha}-1}\left[-\frac{\pi}{2} - \frac{\sqrt{3}}{\sqrt{4\tilde{\alpha}-1}}\arcsin\frac{1}{2\sqrt{2}\sqrt{3\tilde{\alpha}-1}}\right.\right. \\
& \qquad \left.\left. + \frac{3}{2}\arccos\frac{2\tilde{\alpha}-1}{6\tilde{\alpha}-2}\right] + \lambda^{(b)}\frac{\sqrt{8\tilde{\alpha}-3}}{96}\right) \\
& + N_4^{(3,2)}\left(k^{(b)}\sqrt{4\tilde{\alpha}-1}\left[-\pi + \frac{\sqrt{3}}{4\sqrt{4\tilde{\alpha}-1}}\arccos\frac{6\tilde{\alpha}-5}{6\tilde{\alpha}-2}\right.\right. \\
& \qquad \left. + \frac{3}{4}\arccos\frac{4\tilde{\alpha}-3}{8\tilde{\alpha}-4} + \frac{3}{2}\arccos\frac{1}{2\sqrt{2}\sqrt{2\tilde{\alpha}-1}\sqrt{3\tilde{\alpha}-1}}\right] \\
& \qquad \left. + \lambda^{(b)}\frac{\sqrt{12\tilde{\alpha}-7}}{96}\right).
\end{aligned}
$$

This expression can be summarized as

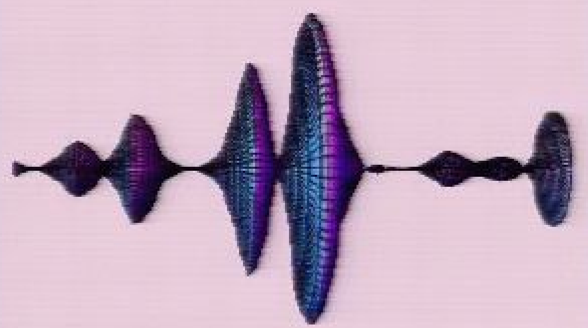
$$S_E = -\left(\kappa_0 + 6\Delta\right) N_0 + \kappa_4 \left(N_4^{(4,1)} + N_4^{(3,2)}\right) + \Delta\left(2N_4^{(4,1)} + N_4^{(3,2)}\right)$$

Δ is a function of $\tilde{\alpha}$ the asymmetry parameter between the space and lattice links. $\Delta = 0$ corresponds to $a_t = a_s$, i.e. $\tilde{a} = 1$.

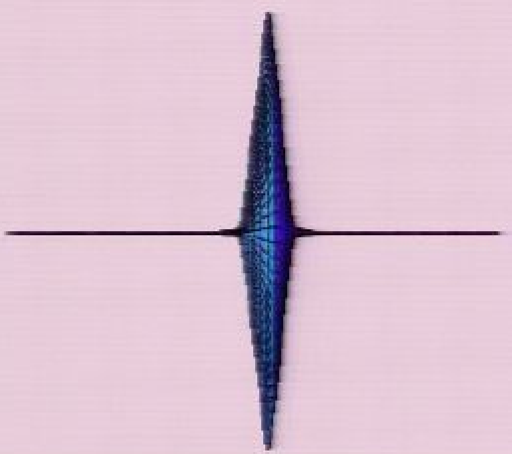
In a given computer simulation $N_4 = N_4^{(4,1)} + N_4^{(3,2)}$ is kept fixed and thus effectively we have only two coupling constants: κ_0 and Δ.



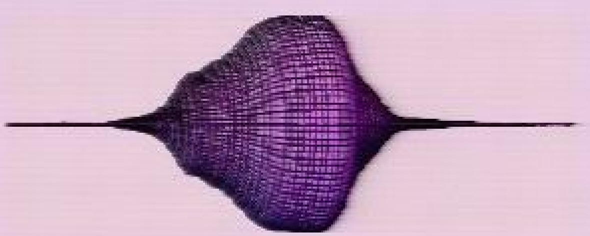
Asymmetry between space and time ? (like in Horava model)



- **Phase A**. Inhomogeneous in time. Dominance of the conformal factor for small bare $1/\kappa_0$. Recall conformal factor appears like $-\dot{\phi}^2(t)$

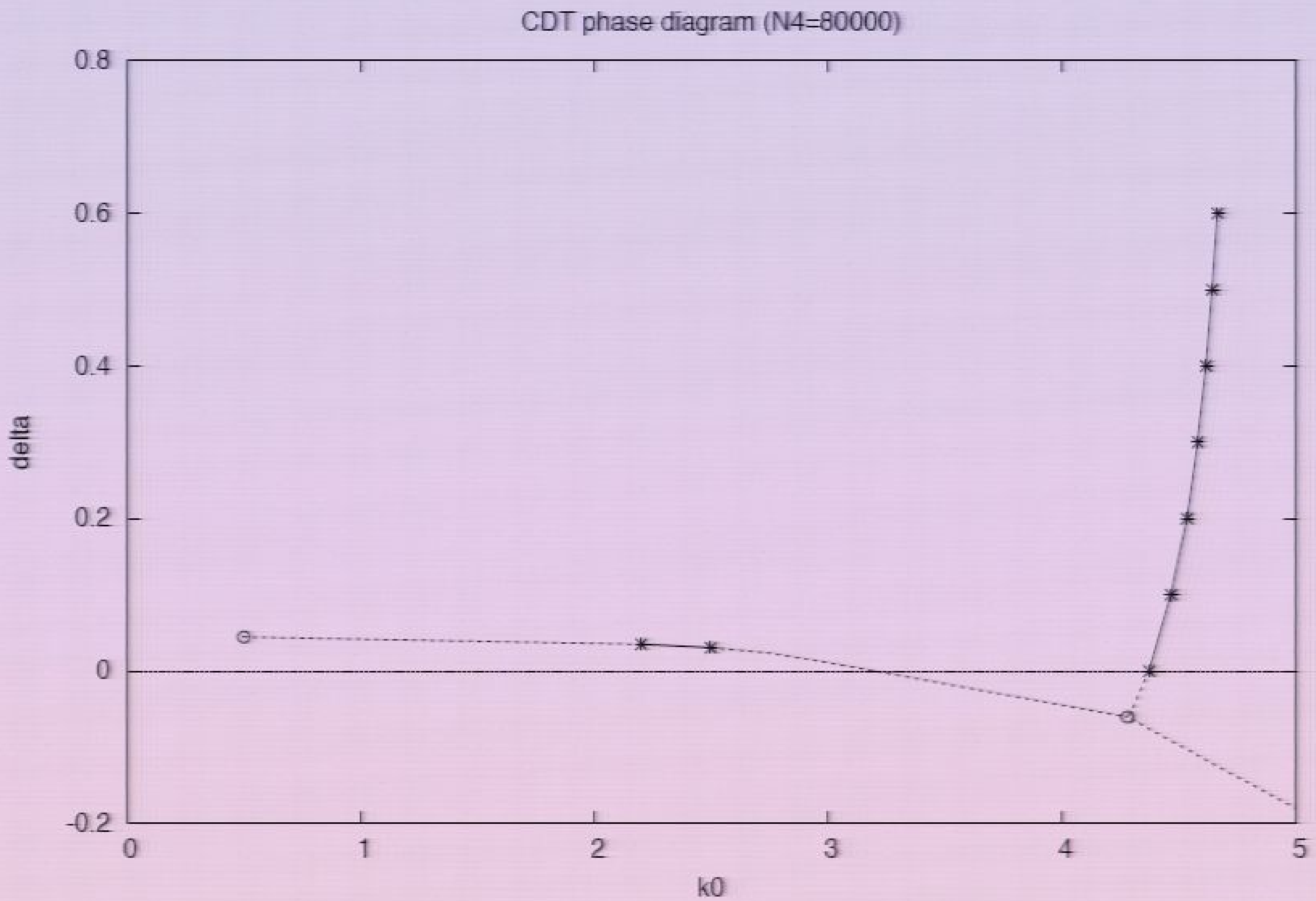


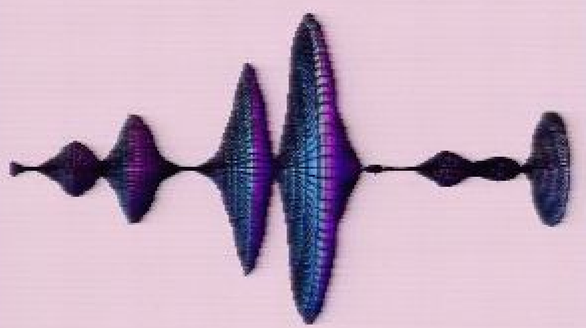
- **Phase B**. Inhomogeneous in space. Effective compactification into a 3d Euclidean DT, but in an "crumpled" inhomogeneous 3d space.



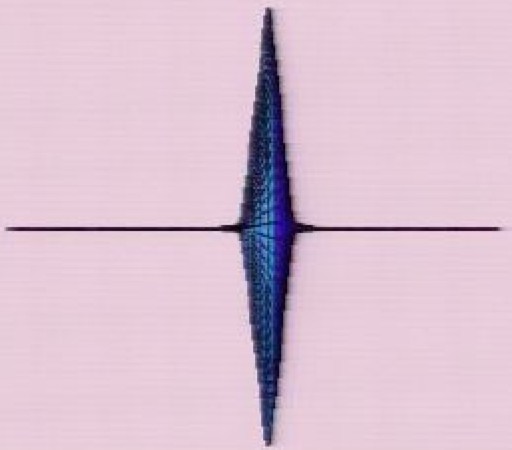
- **Phase C**. Extended de Sitter phase. $d_H = 4$. Lattice time extension depends on Δ but configurations identified by redefinition of a_t.

Phase diagram in $\kappa_0 - \Delta$ plane





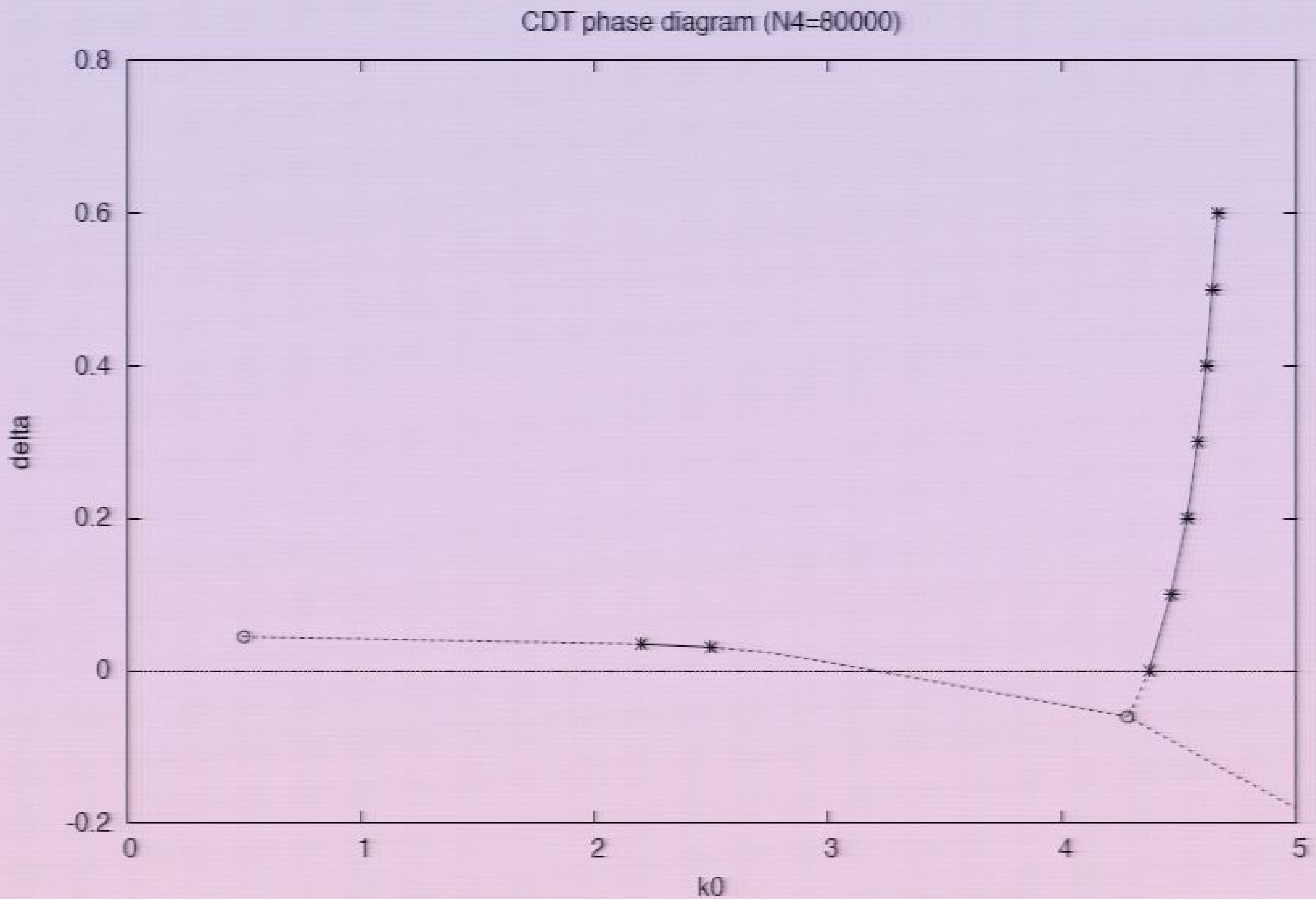
- **Phase A**. Inhomogeneous in time. Dominance of the conformal factor for small bare $1/\kappa_0$. Recall conformal factor appears like $-\dot{\phi}^2(t)$

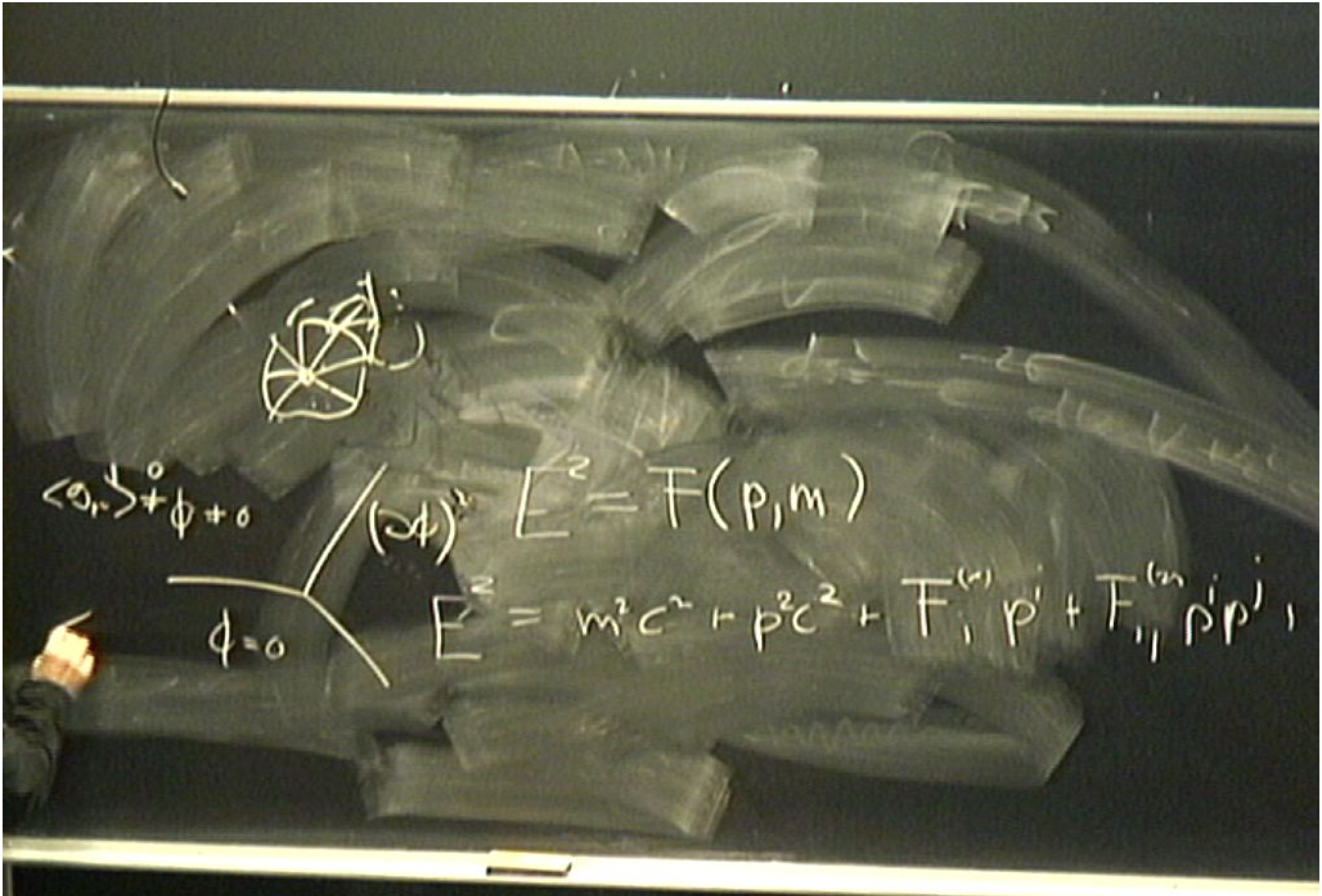


- **Phase B**. Inhomogeneous in space. Effective compactification into a 3d Euclidean DT, but in an "crumpled" inhomogeneous 3d space.

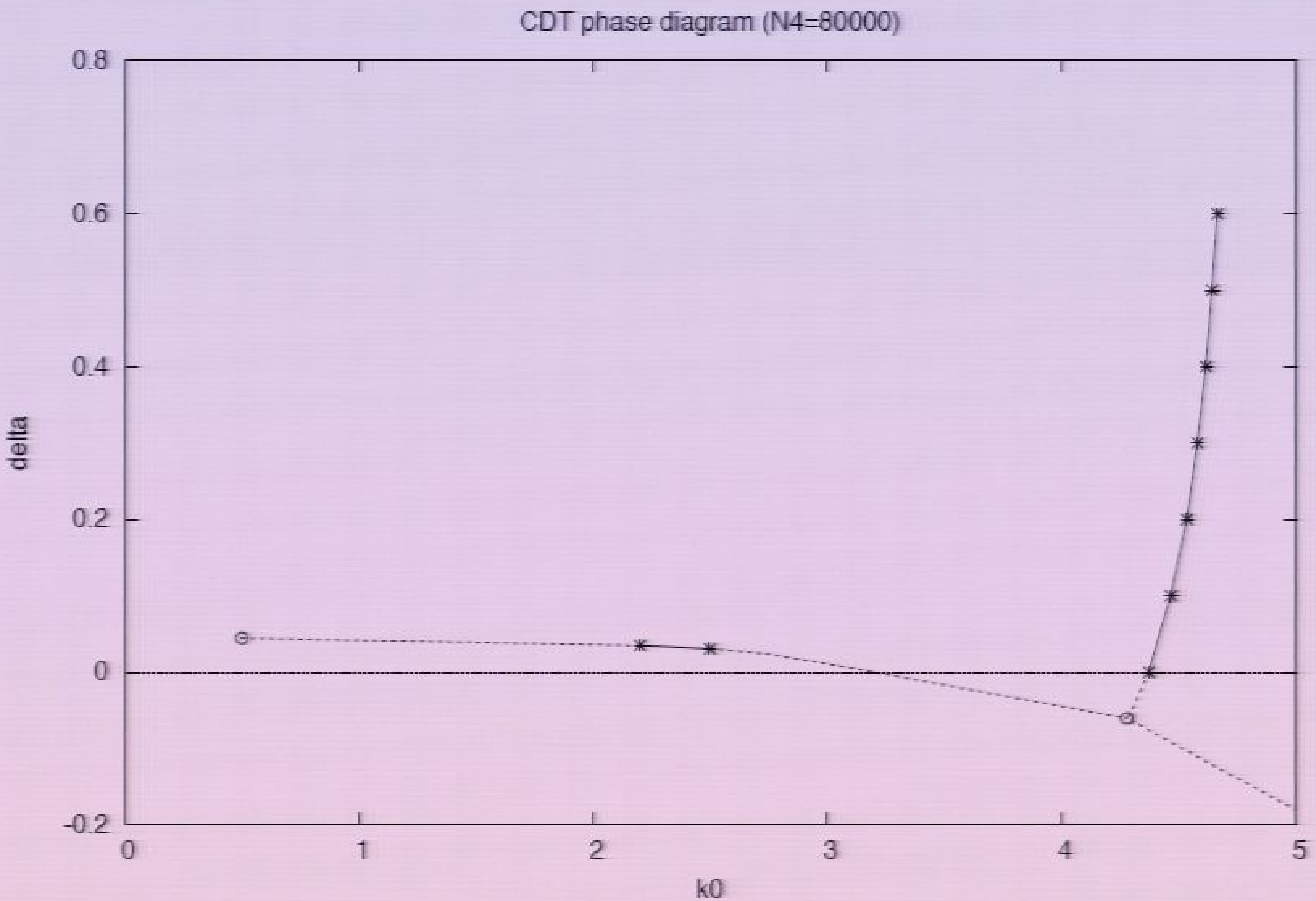
- **Phase C**. Extended de Sitter phase. $d_H = 4$. Lattice time extension depends on Δ but configurations identified by redefinition of a_t.

Phase diagram in $\kappa_0 - \Delta$ plane

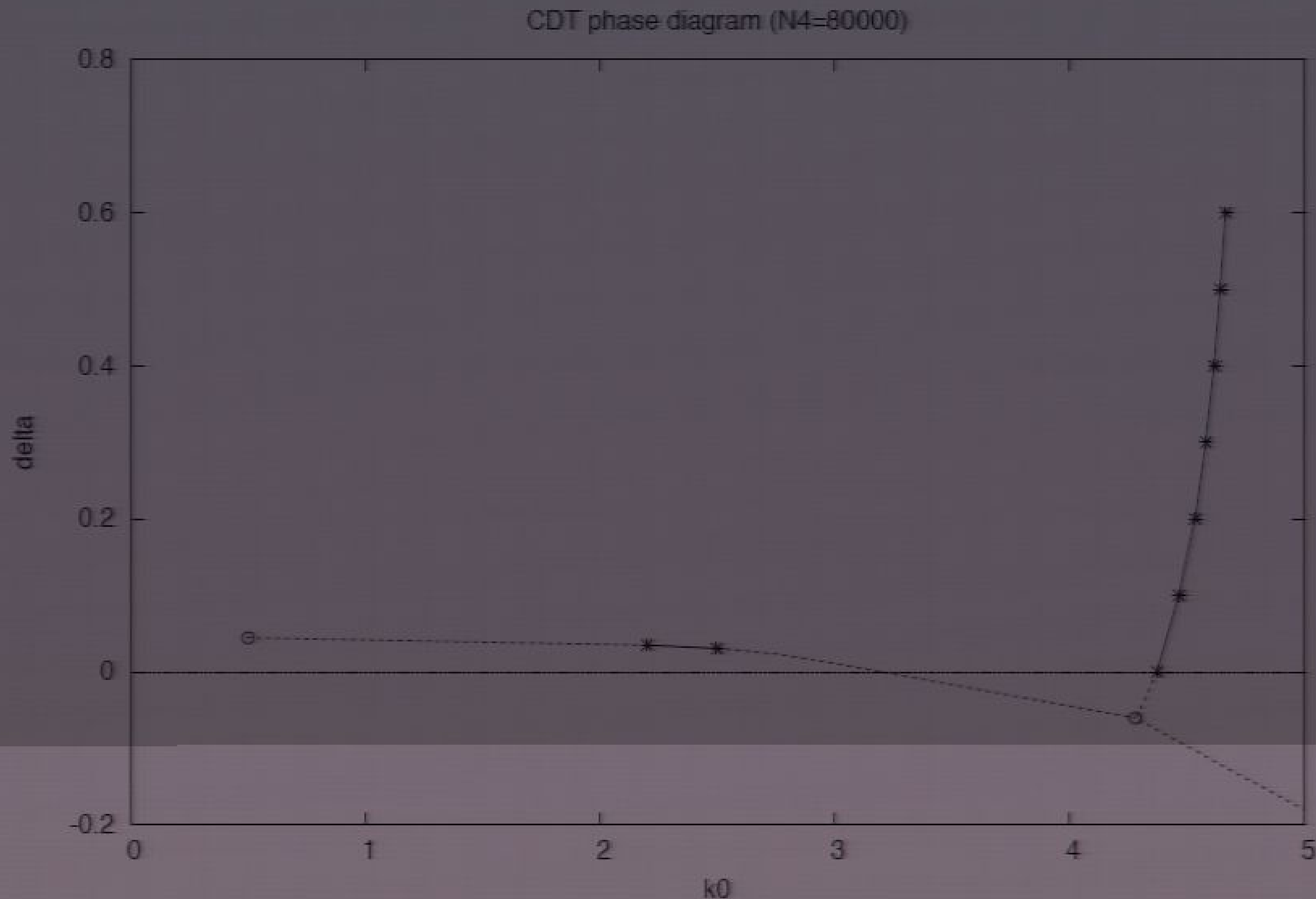



E² = F(p,m)
E² = m²c² + p²c² + F p + F p p
φ = 0

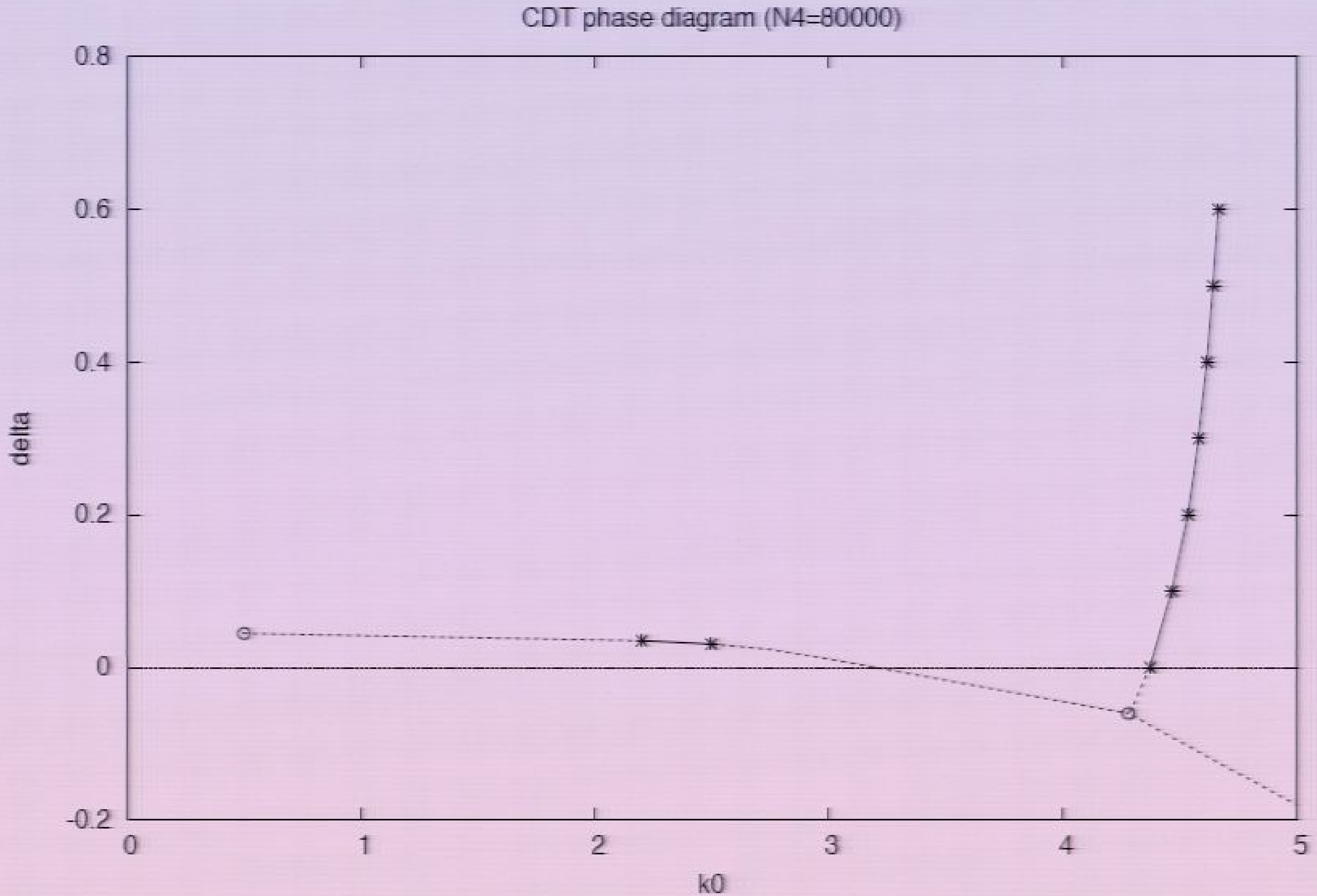
Phase diagram in $\kappa_0 - \Delta$ plane

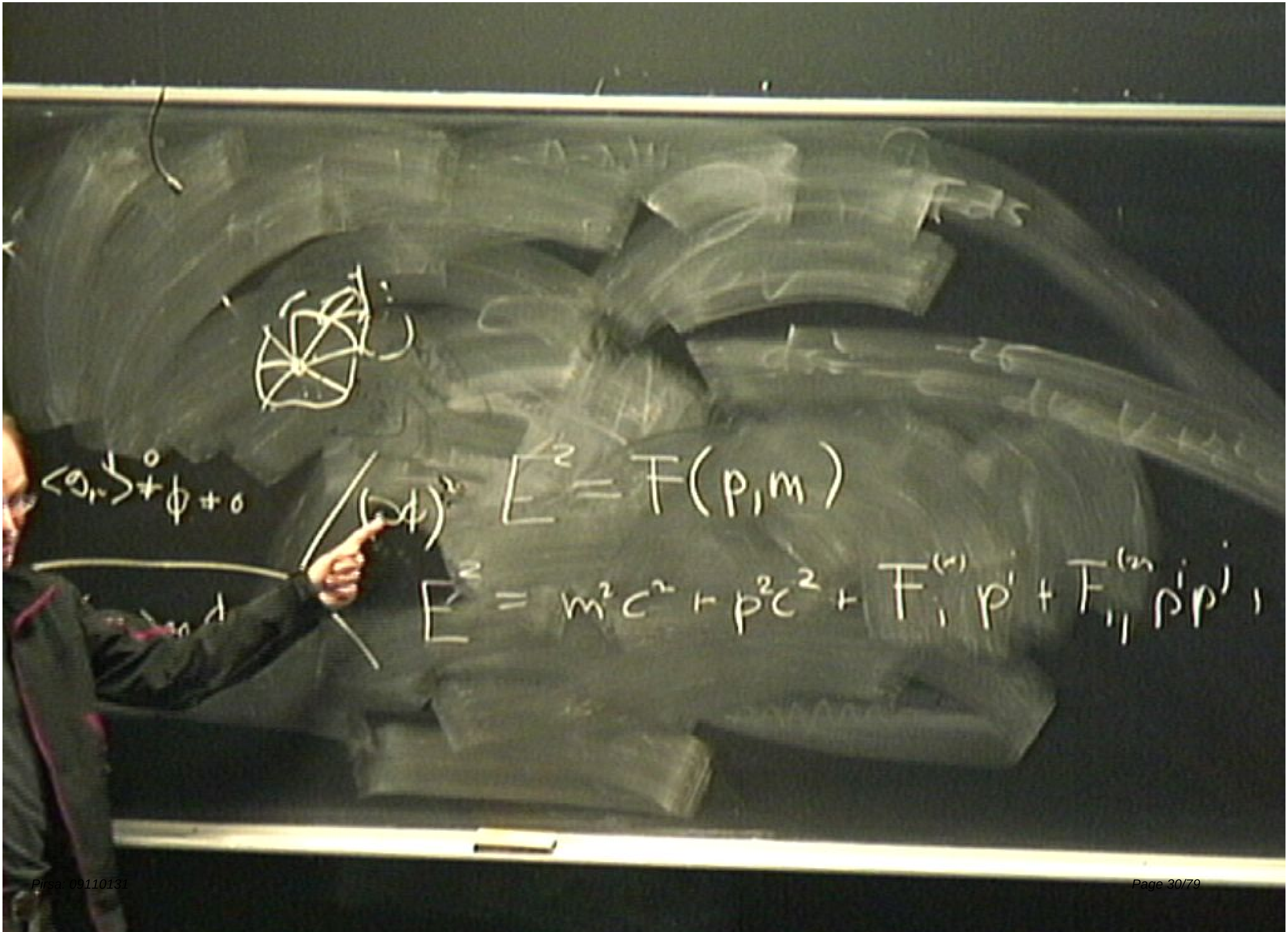


Phase diagram in $\kappa_0 - \Delta$ plane

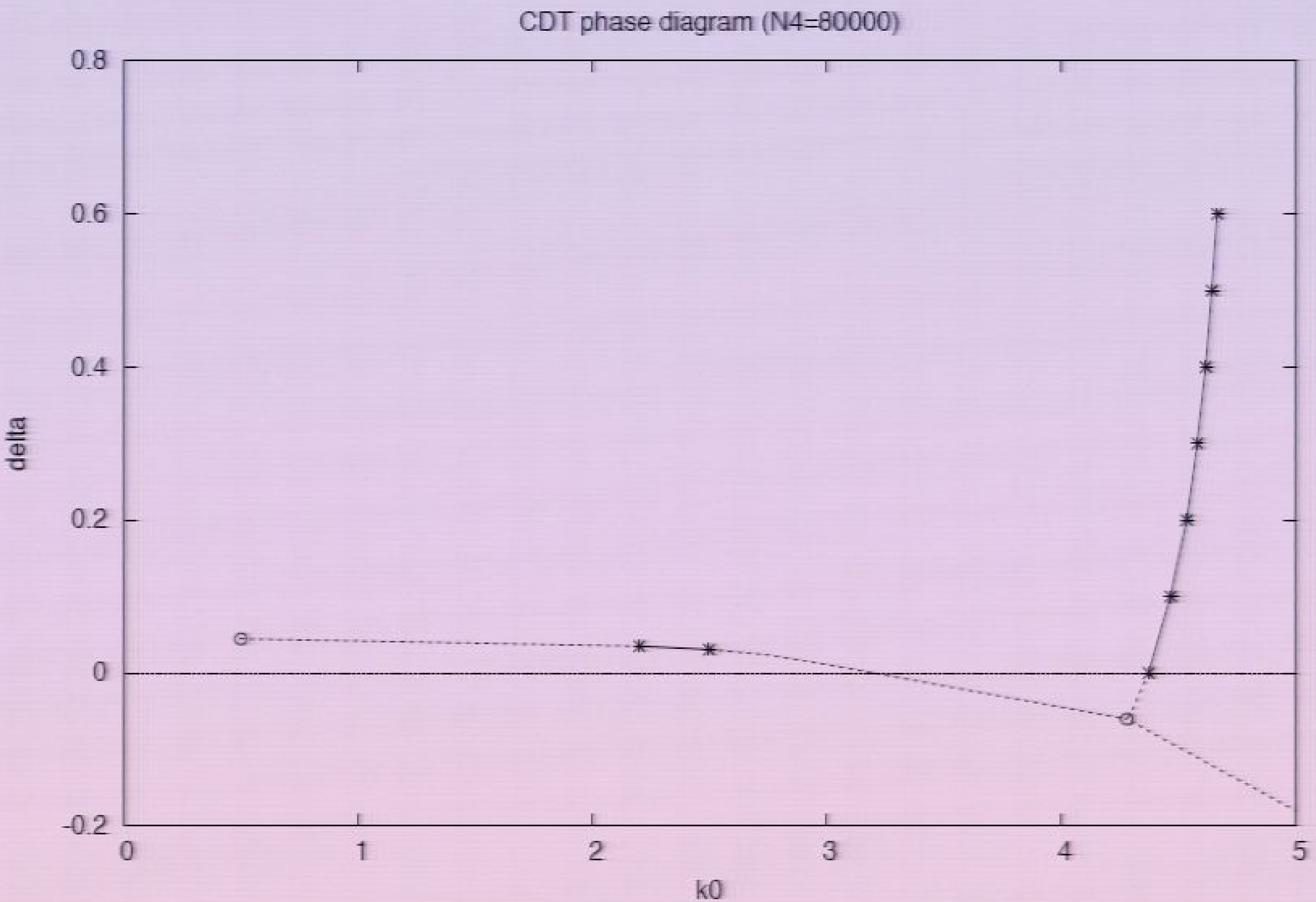


Phase diagram in $\kappa_0 - \Delta$ plane




E² = F(p,m)
E² = m²c² + p²c² + F_i^(1) p^i + F_ij^(2) p^i p^j

Phase diagram in $\kappa_0 - \Delta$ plane



1. order A-C phase transition

The transition from Euclidean deSitter space-time (phase C) to phase A (dominated by the conformal factor) is 1. order. Difficult to imagine to use it to define a UV completion of the IR deSitter behavior.

A naively defined lattice Horava model would presumably also have an unphysical A-phase, since also such model is unbounded in the Euclidean sector (wrong sign of the second order time derivative).

The transition line B-C

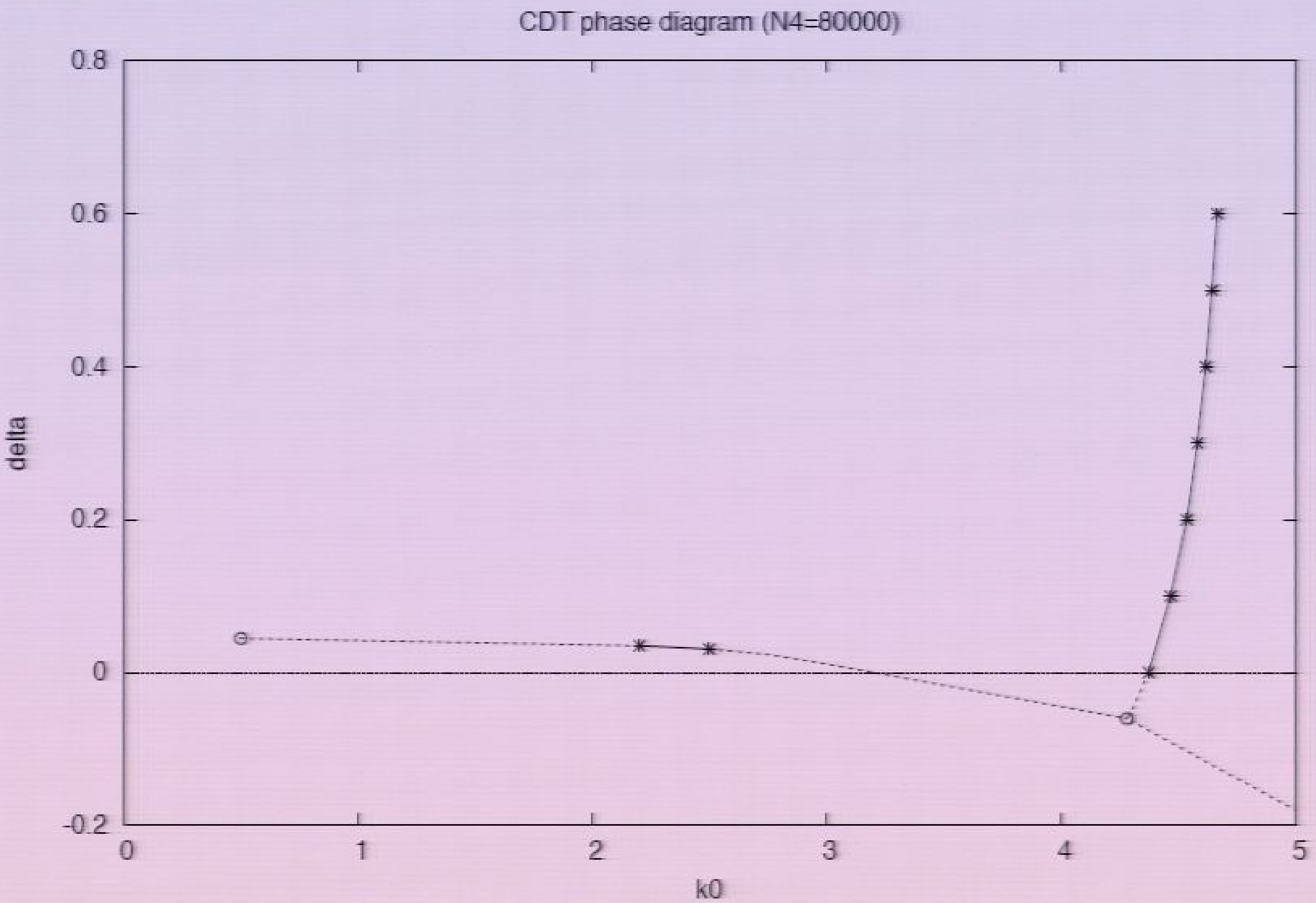
Could be 2. order phase transition line, verdict still up.

Can it serve as a Horava-Lifshitz UV completion of QG ?

Horava scaling:

$$V_3 = a^3(\Delta r)^3, \quad T = a^3 \Delta t \qquad \frac{\Delta t}{\Delta r} \propto (\Delta r)^2$$

Phase diagram in $\kappa_0 - \Delta$ plane



1. order A-C phase transition

The transition from Euclidean deSitter space-time (phase C) to phase A (dominated by the conformal factor) is 1. order. Difficult to imagine to use it to define a UV completion of the IR deSitter behavior.

A naively defined lattice Horava model would presumably also have an unphysical A-phase, since also such model is unbounded in the Euclidean sector (wrong sign of the second order time derivative).

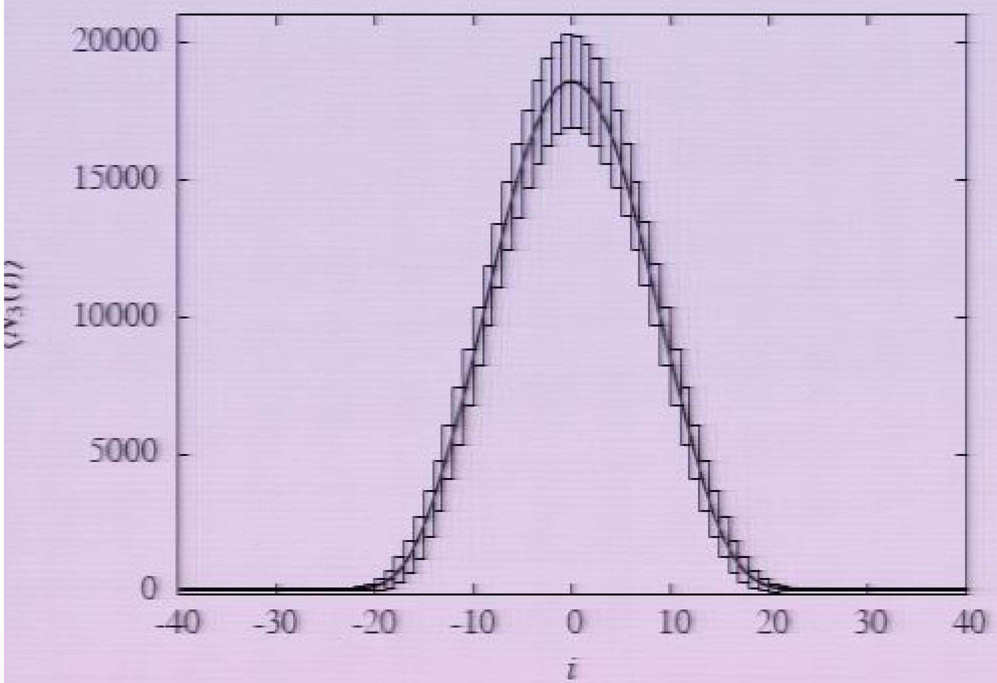
The transition line B-C

Could be 2. order phase transition line, verdict still up.

Can it serve as a Horava-Lifshitz UV completion of QG ?

Horava scaling:

$$V_3 = a^3(\Delta r)^3, \quad T = a^3 \Delta t \qquad \frac{\Delta t}{\Delta r} \propto (\Delta r)^2$$



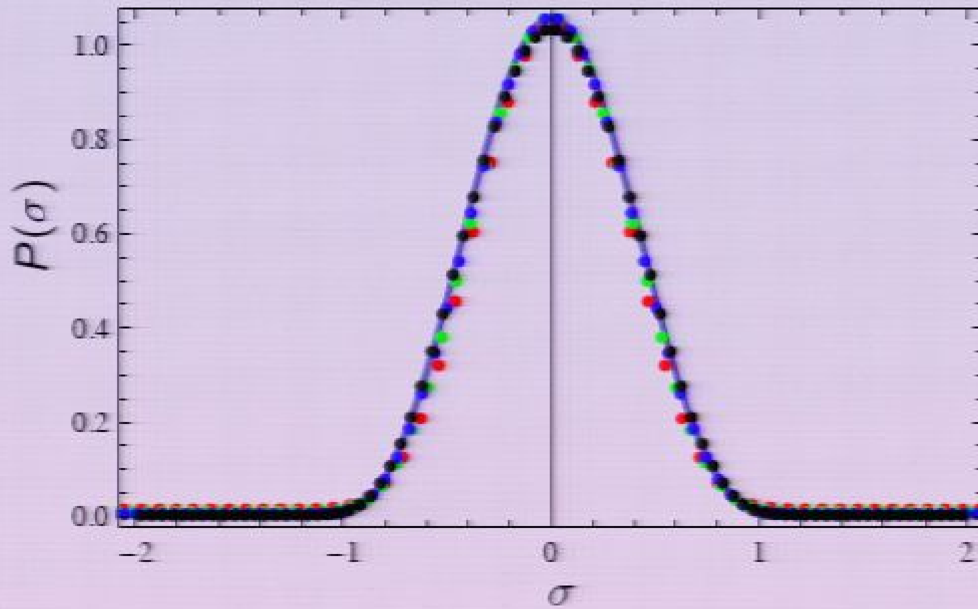


Figure: $N_4 = 22\text{k}, 45\text{k}, 91\text{k}, 182\text{k}$

$$\langle N_3(i)\rangle \propto N_4^{3/4} \cos^3\left(\frac{i}{s_0 N_4^{1/4}}\right)$$

$$\sigma \propto i/N_4^{1/d}$$
$$N_3(i) \propto N_4^{(d-1)/d} P(\sigma)$$

In phase C, deSitter space-time:

Best $d = 4$

Scenario 1: dimension of space is 3: $(\Delta r)^3 \sim N_3$

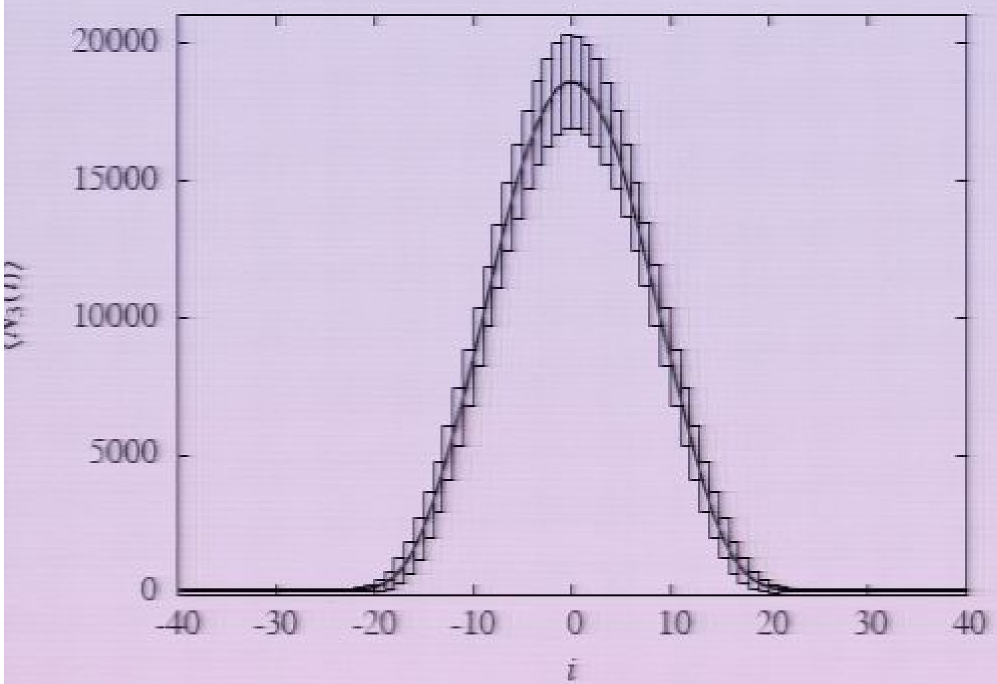
In Phase C, away from the B-C line: $\Delta t = s_0(\Delta) N_3^{1/3}$
If 2. order line: $s_0(\Delta) N_3^{1/3} \to \mathrm{const.}\, N_3^{\nu}$.

"Observations" $\nu \le 1/3$: $\frac{\Delta t}{\Delta r} \to 0$. no Horava scaling

Scenario 2: dimension of space is d_H: $(\Delta r)^{d_H} \sim N_3$

$$\frac{\Delta t}{\Delta r} \propto r^2 \quad \to \quad \frac{\Delta N_3^{\nu}}{N_3^{1/d_H}} \propto N_3^{2/d_H}$$

Horava scaling if $\nu = 3/d_H$



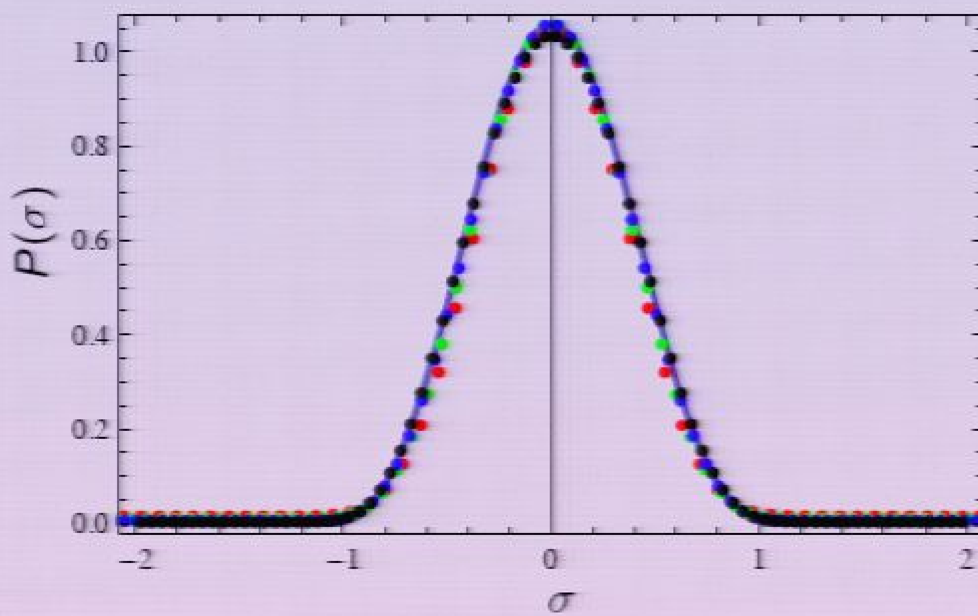


Figure: $N_4 = 22\text{k}, 45\text{k}, 91\text{k}, 182\text{k}$

$$\langle N_3(i) \rangle \propto N_4^{3/4} \cos^3\left(\frac{i}{s_0 N_4^{1/4}}\right)$$

$$\sigma \propto i/N_4^{1/d}$$

$$N_3(i) \propto N_4^{(d-1)/d} P(\sigma)$$

In phase C, deSitter space-time:

Best $d = 4$

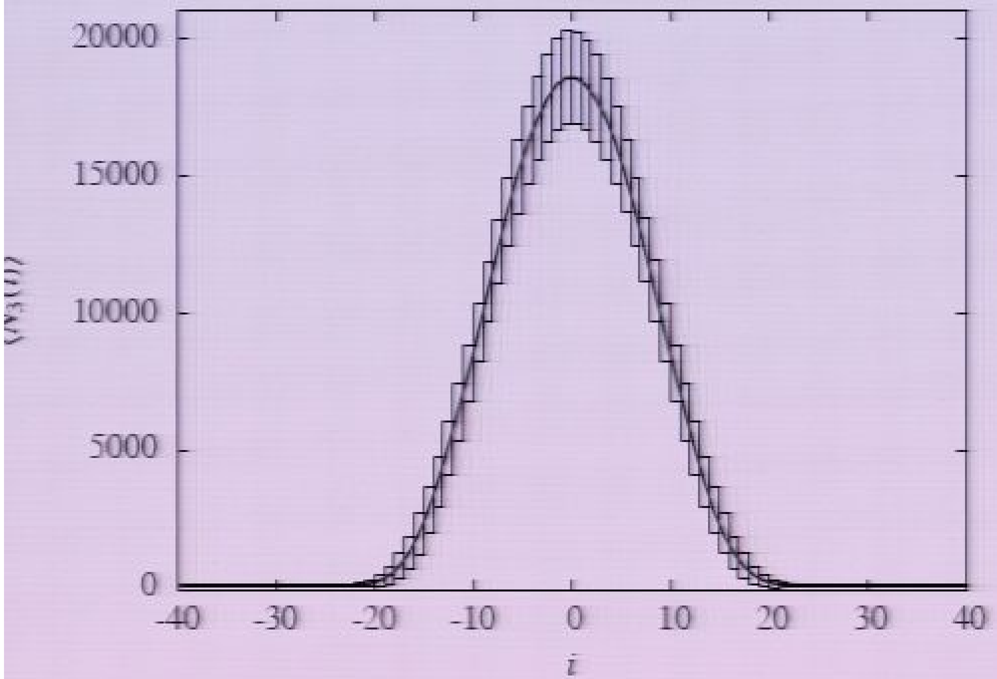
The transition line B-C

Could be 2. order phase transition line, verdict still up.

Can it serve as a Horava-Lifshitz UV completion of QG ?

Horava scaling:

$$V_3 = a^3 (\Delta r)^3, \quad T = a^3 \Delta t \qquad \frac{\Delta t}{\Delta r} \propto (\Delta r)^2$$



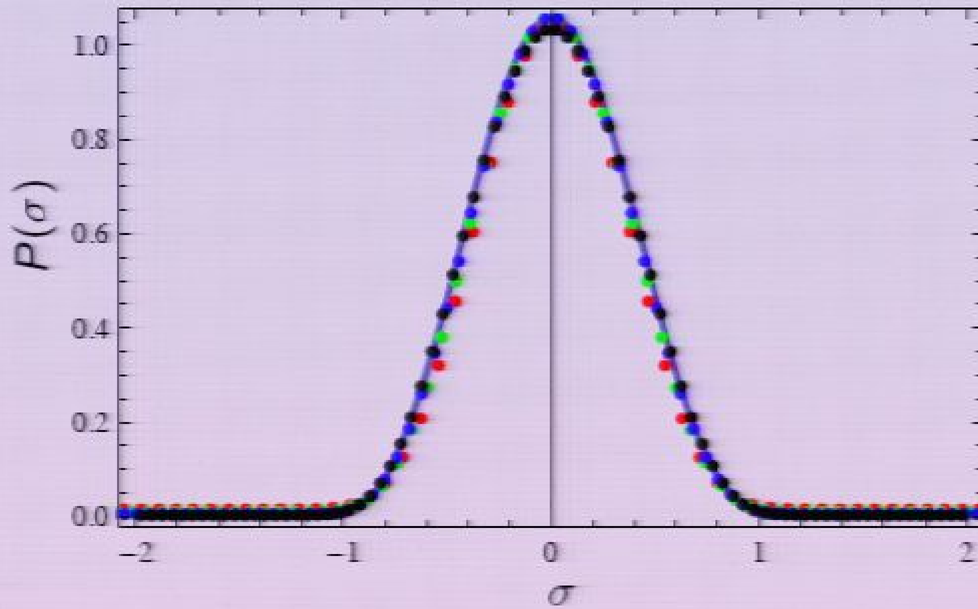


Figure: $N_4 = 22\text{k}, 45\text{k}, 91\text{k}, 182\text{k}$

$$\langle N_3(i) \rangle \propto N_4^{3/4} \cos^3 \left(\frac{i}{s_0 N_4^{1/4}} \right)$$

$$\begin{aligned} \sigma &\propto i/N_4^{1/d} \\ N_3(i) &\propto N_4^{(d-1)/d} P(\sigma) \end{aligned}$$

In phase C, deSitter space-time:

Best $d = 4$

Scenario 1: dimension of space is 3: $(\Delta r)^3 \sim N_3$

In Phase C, away from the B-C line: $\Delta t = s_0(\Delta) N_3^{1/3}$

If 2. order line: $s_0(\Delta) N_3^{1/3} \to \mathrm{const.} N_3^{\nu}$.

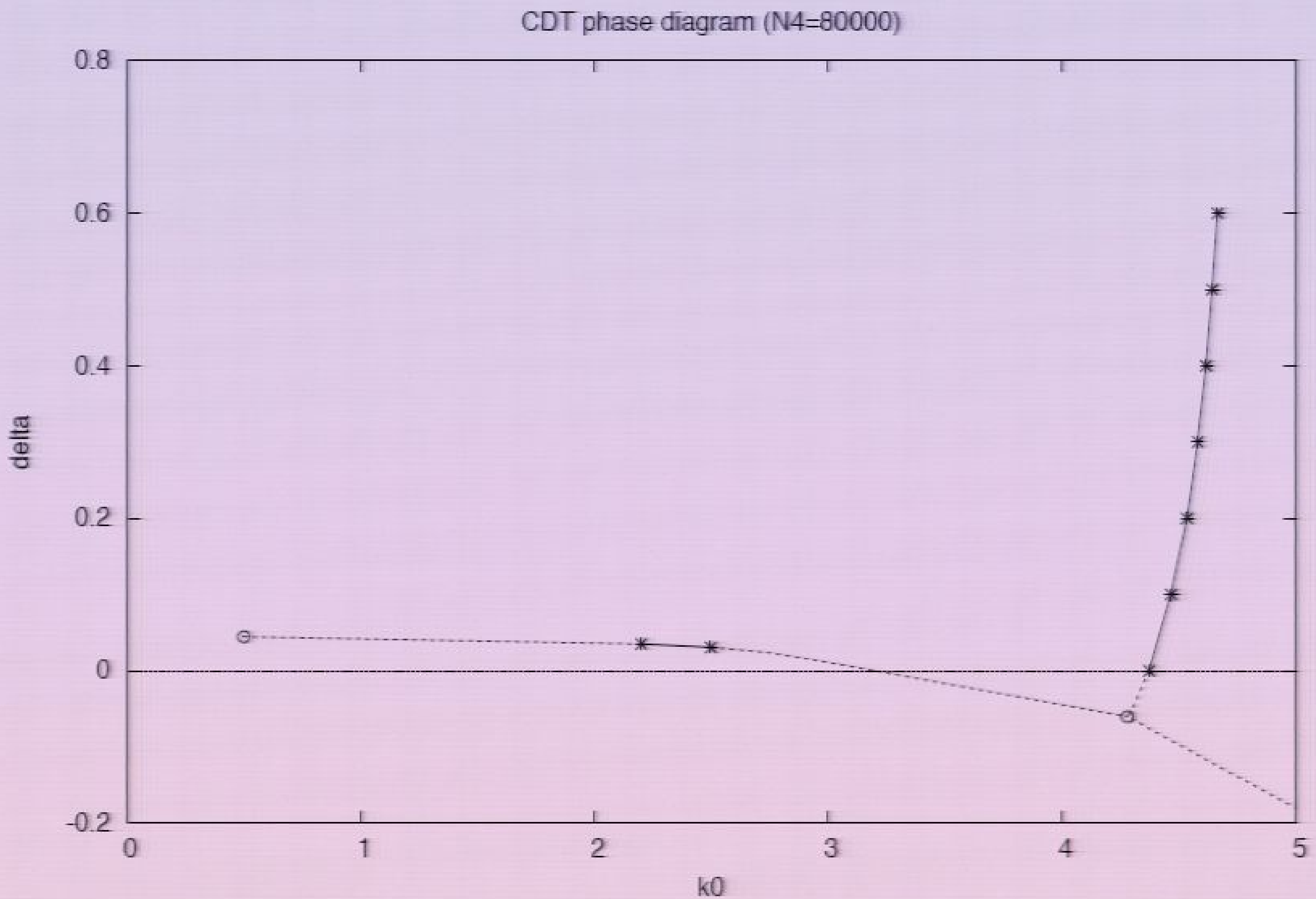
"Observations" $\nu \leq 1/3$: $\frac{\Delta t}{\Delta r} \to 0$. no Horava scaling

Scenario 2: dimension of space is d_H: $(\Delta r)^{d_H} \sim N_3$

$$\frac{\Delta t}{\Delta r} \propto r^2 \quad \to \quad \frac{\Delta N_3^{\nu}}{N_3^{1/d_H}} \propto N_3^{2/d_H}$$

Horava scaling if $\nu = 3/d_H$

Phase diagram in $\kappa_0 - \Delta$ plane



1. order A-C phase transition

The transition from Euclidean deSitter space-time (phase C) to phase A (dominated by the conformal factor) is 1. order. Difficult to imagine to use it to define a UV completion of the IR deSitter behavior.

A naively defined lattice Horava model would presumably also have an unphysical A-phase, since also such model is unbounded in the Euclidean sector (wrong sign of the second order time derivative).

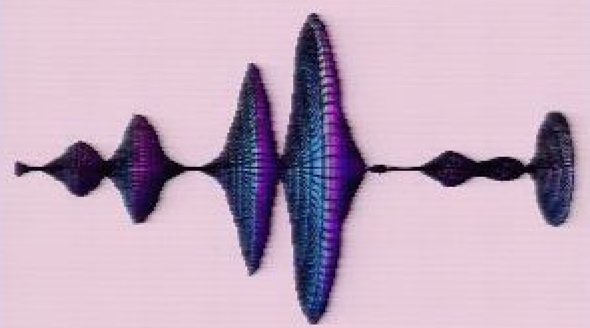
The transition line B-C

Could be 2. order phase transition line, verdict still up.

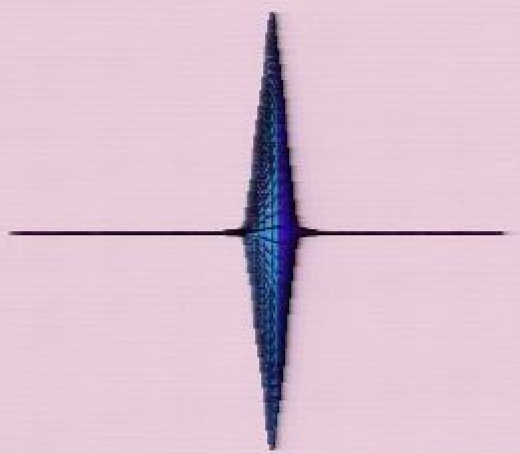
Can it serve as a Horava-Lifshitz UV completion of QG ?

Horava scaling:

$$V_3 = a^3(\Delta r)^3, \quad T = a^3 \Delta t \qquad \frac{\Delta t}{\Delta r} \propto (\Delta r)^2$$



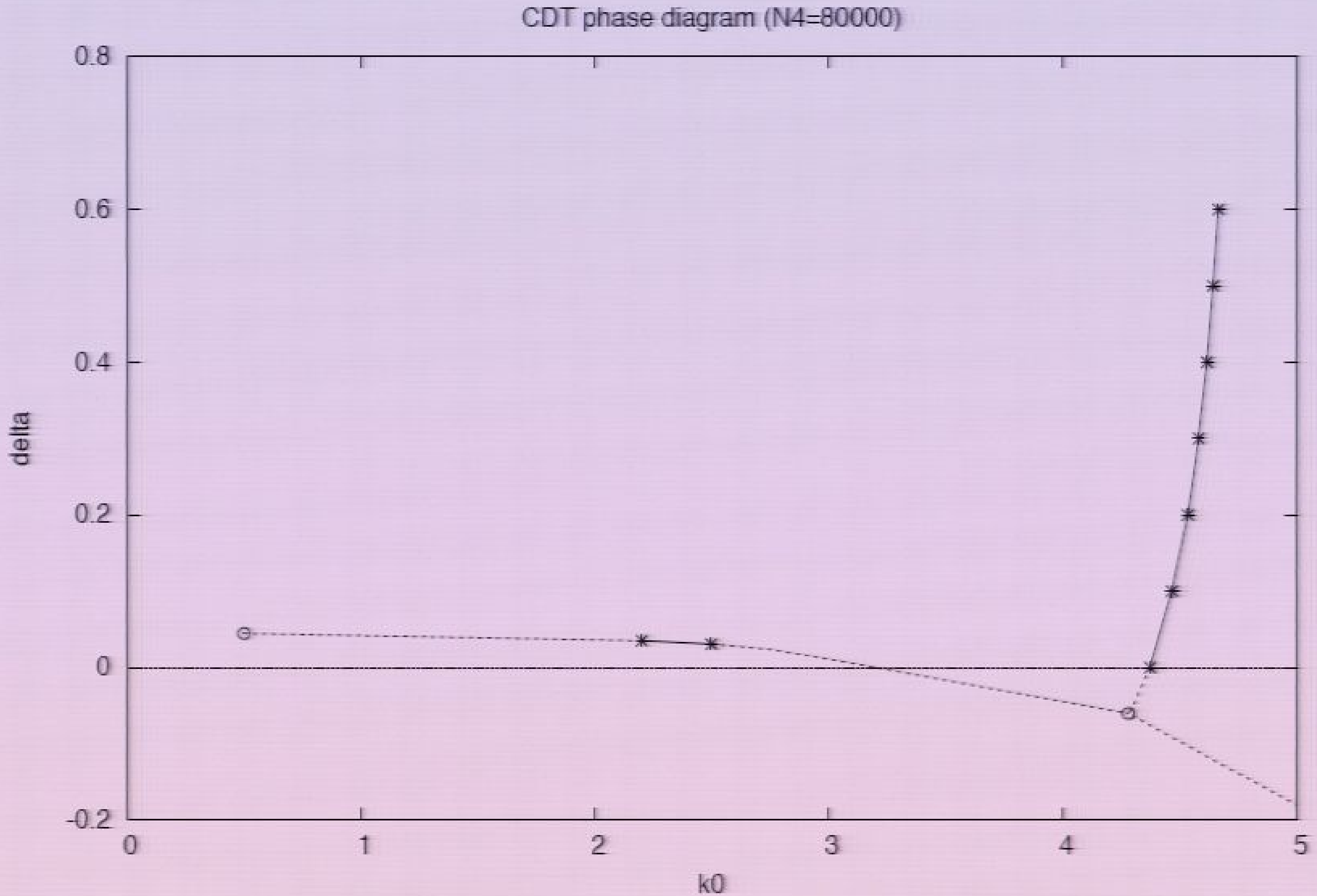
- **Phase A**. Inhomogeneous in time. Dominance of the conformal factor for small bare $1/\kappa_0$. Recall conformal factor appears like $-\dot{\phi}^2(t)$



- **Phase B**. Inhomogeneous in space. Effective compactification into a 3d Euclidean DT, but in an "crumpled" inhomogeneous 3d space.

- **Phase C**. Extended de Sitter phase. $d_H = 4$. Lattice time extension depends on Δ but configurations identified by redefinition of a_t.

Phase diagram in $\kappa_0 - \Delta$ plane



1. order A-C phase transition

The transition from Euclidean deSitter space-time (phase C) to phase A (dominated by the conformal factor) is 1. order. Difficult to imagine to use it to define a UV completion of the IR deSitter behavior.

A naively defined lattice Horava model would presumably also have an unphysical A-phase, since also such model is unbounded in the Euclidean sector (wrong sign of the second order time derivative).

The transition line B-C

Could be 2. order phase transition line, verdict still up.

Can it serve as a Horava-Lifshitz UV completion of QG ?

Horava scaling:

$$V_3 = a^3(\Delta r)^3, \quad T = a^3 \Delta t \qquad \frac{\Delta t}{\Delta r} \propto (\Delta r)^2$$

Scenario 1: dimension of space is 3: $(\Delta r)^3 \sim N_3$

In Phase C, away from the B-C line: $\Delta t = s_0(\Delta) N_3^{1/3}$

If 2. order line: $s_0(\Delta) N_3^{1/3} \to \text{const.} N_3^{\nu}$.

"Observations" $\nu \leq 1/3$: $\frac{\Delta t}{\Delta r} \to 0$. no Horava scaling

Scenario 2: dimension of space is d_H: $(\Delta r)^{d_H} \sim N_3$

$$\frac{\Delta t}{\Delta r} \propto r^2 \quad \to \quad \frac{\Delta N_3^{\nu}}{N_3^{1/d_H}} \propto N_3^{2/d_H}$$

Horava scaling if $\nu = 3/d_H$

Summary

- The set-up with a global time foliation is common to CDT and HL-gravity.
- The spectral dimension when measured in the deSitter phase of CDT varies from 4 (long distance) to 2 (short distance). Similar results were found by Horava, hinting that maybe the two theories have the same UV completion.
- We have argued that Horava scaling is a posibility along the B-C phase transition line if $\Delta t = N_3^{\nu}$ and $\nu = 3/d_H$ (and if it is second order.....)
- CDT is in principle ideally suited to study lattice HL gravity, given a suitable lattice action.

Scenario 1: dimension of space is 3: $(\Delta r)^3 \sim N_3$

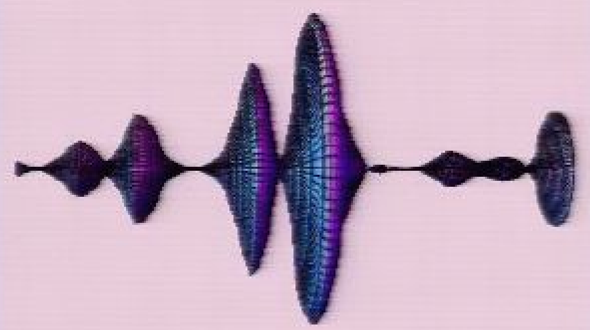
In Phase C, away from the B-C line: $\Delta t = s_0(\Delta) N_3^{1/3}$
If 2. order line: $s_0(\Delta) N_3^{1/3} \to \text{const.} N_3^{\nu}$.

"Observations" $\nu \leq 1/3$: $\frac{\Delta t}{\Delta r} \to 0$. no Horava scaling

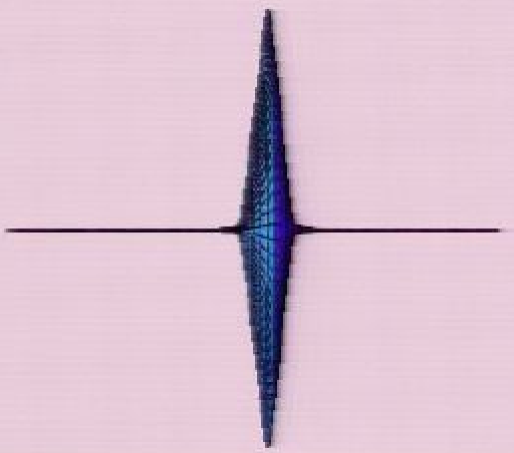
Scenario 2: dimension of space is d_H: $(\Delta r)^{d_H} \sim N_3$

$$\frac{\Delta t}{\Delta r} \propto r^2 \quad \to \quad \frac{\Delta N_3^{\nu}}{N_3^{1/d_H}} \propto N_3^{2/d_H}$$

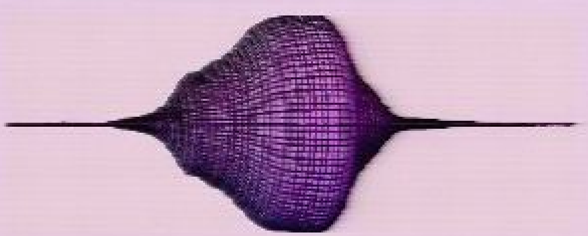
Horava scaling if $\nu = 3/d_H$



- **Phase A**. Inhomogeneous in time. Dominance of the conformal factor for small bare $1/\kappa_0$. Recall conformal factor appears like $-\dot{\phi}^2(t)$



- **Phase B**. Inhomogeneous in space. Effective compactification into a 3d Euclidean DT, but in an "crumpled" inhomogeneous 3d space.



- **Phase C**. Extended de Sitter phase. $d_H = 4$. Lattice time extension depends on Δ but configurations identified by redefinition of a_t.

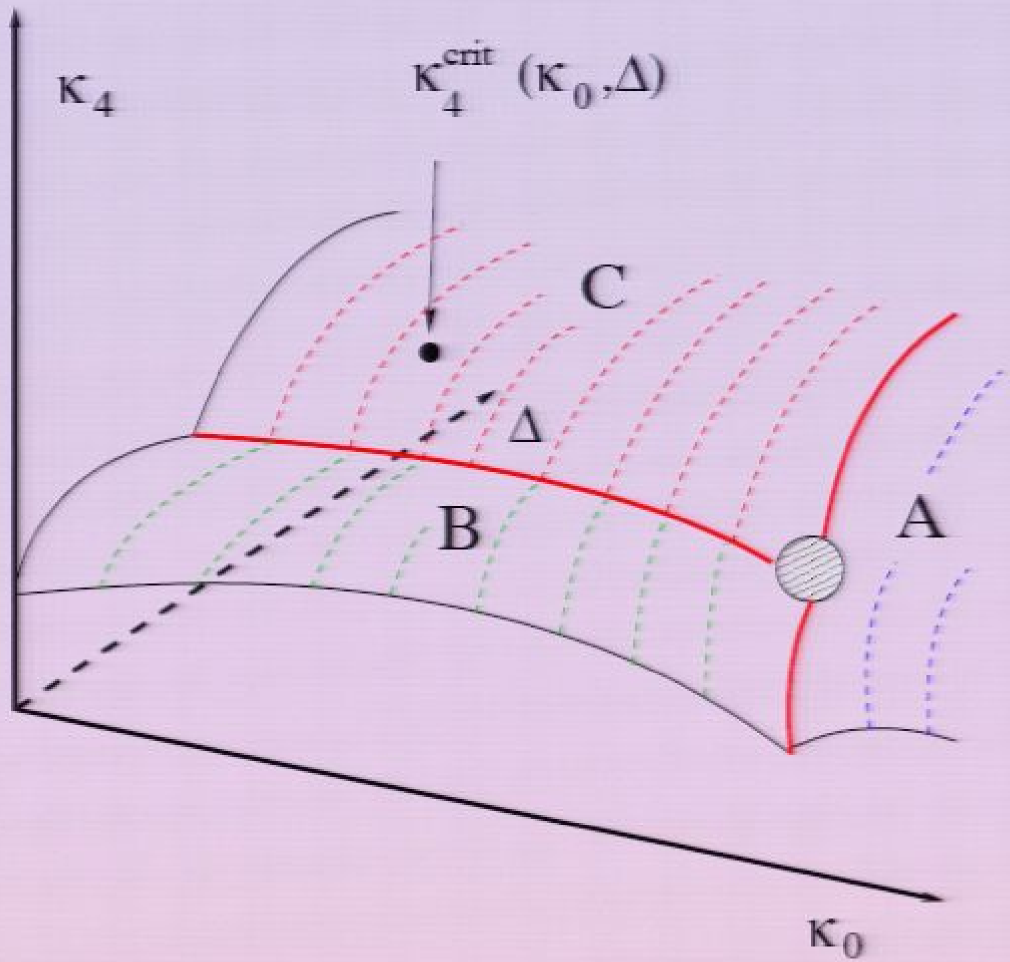
The transition line B-C

Could be 2. order phase transition line, verdict still up.

Can it serve as a Horava-Lifshitz UV completion of QG ?

Horava scaling:

$$V_3 = a^3(\Delta r)^3, \quad T = a^3 \Delta t \qquad \frac{\Delta t}{\Delta r} \propto (\Delta r)^2$$

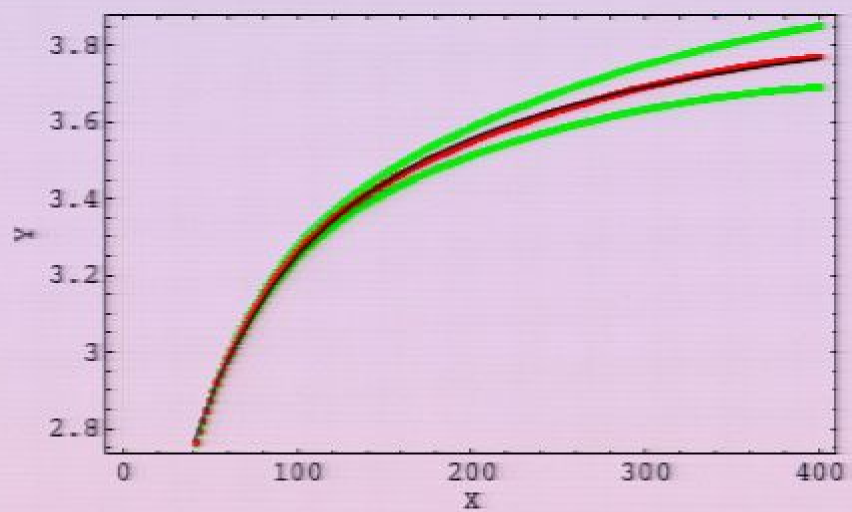


Asymmetry between space and time ? (like in Horava model)

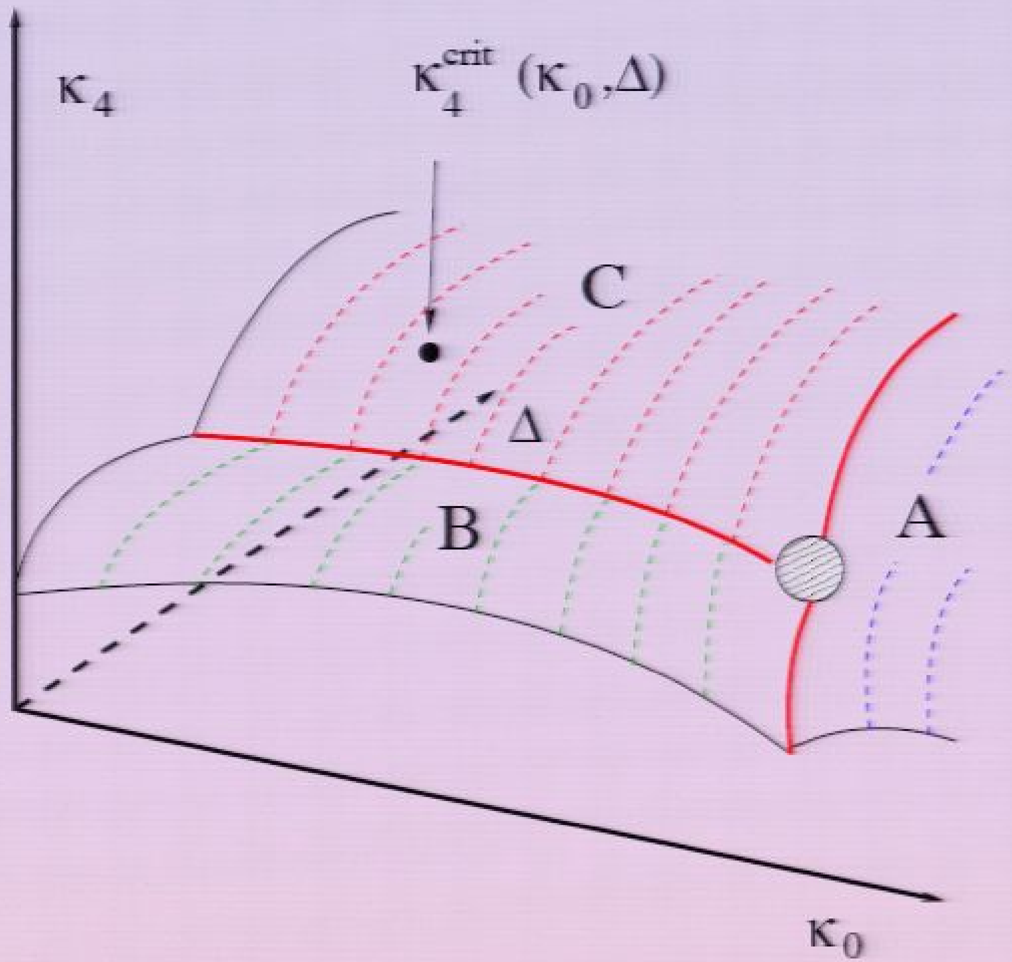
Relation to the Horava model ?

The set-up is precisely as in the Horava model.

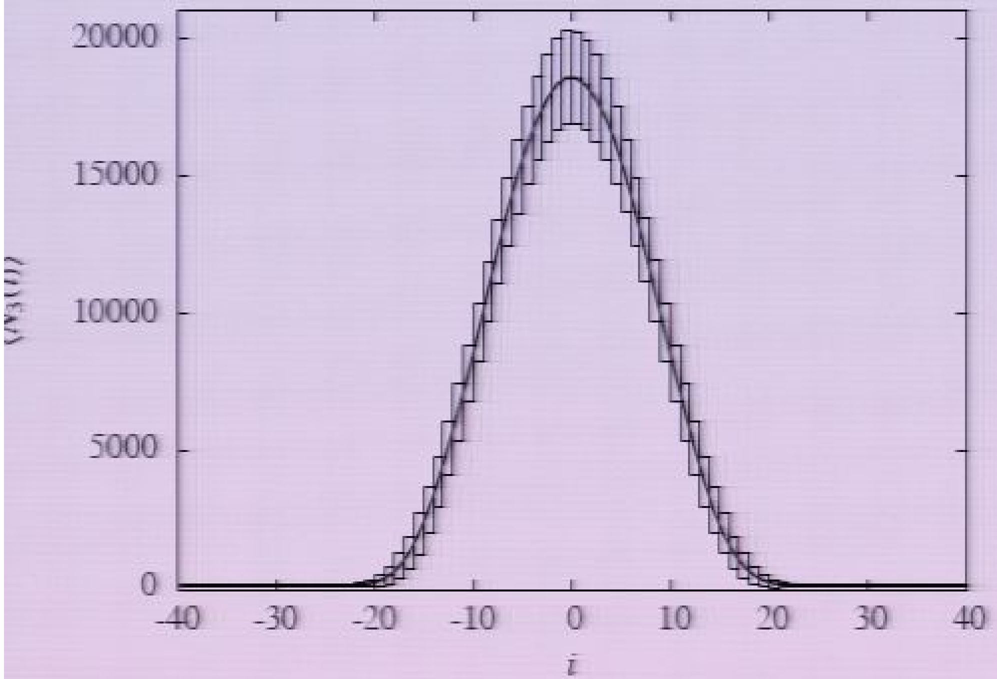
In addition the so-called spectral dimension in CDT and in the Horava model show the same characteristic behavior:



But the actions in the two models seemingly unrelated ?



Asymmetry between space and time ? (like in Horava model)



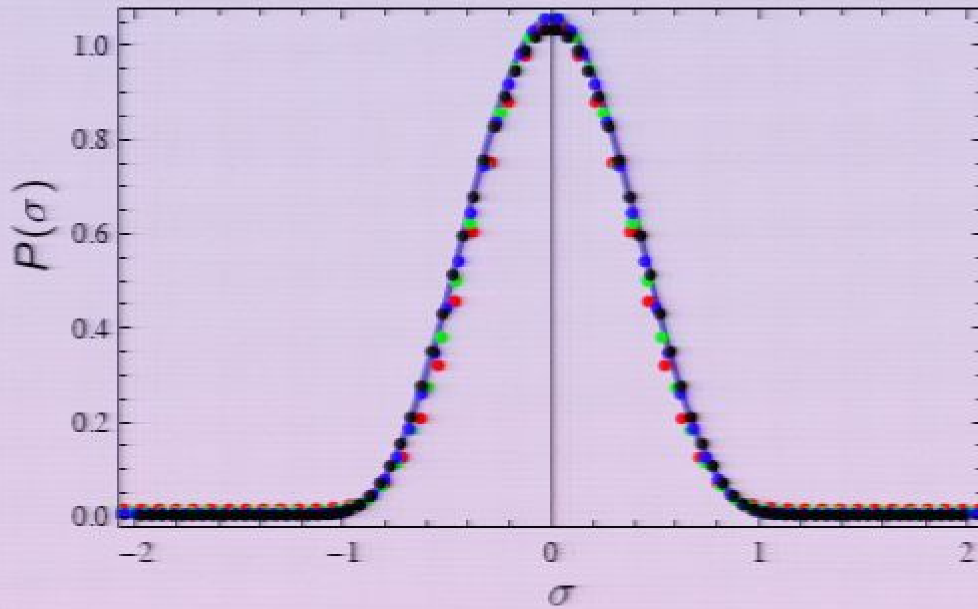


Figure: $N_4 = 22\text{k}, 45\text{k}, 91\text{k}, 182\text{k}$

$$\langle N_3(i)\rangle \propto N_4^{3/4}\cos^3\left(\frac{i}{s_0 N_4^{1/4}}\right)$$

$$\sigma \propto i/N_4^{1/d}$$

$$N_3(i) \propto N_4^{(d-1)/d} P(\sigma)$$

In phase C, deSitter space-time:

Best $d = 4$

Scenario 1: dimension of space is 3: $(\Delta r)^3 \sim N_3$

In Phase C, away from the B-C line: $\Delta t = s_0(\Delta) N_3^{1/3}$
If 2. order line: $s_0(\Delta) N_3^{1/3} \to \text{const.}\, N_3^{\nu}$.

"Observations" $\nu \leq 1/3$: $\frac{\Delta t}{\Delta r} \to 0$. no Horava scaling

Scenario 2: dimension of space is d_H: $(\Delta r)^{d_H} \sim N_3$

$$\frac{\Delta t}{\Delta r} \propto r^2 \quad \to \quad \frac{\Delta N_3^{\nu}}{N_3^{1/d_H}} \propto N_3^{2/d_H}$$

Horava scaling if $\nu = 3/d_H$

Minisuperspace model

The semiclassical distribution can be obtained from the minisuperspace effective action of Hartle and Hawking

$$S_{eff} = \frac{1}{24\pi G} \int dt \sqrt{g_{tt}} \left(\frac{g^{tt} \dot{V_3}^2(t)}{V_3(t)} + k_2 V_3^{1/3}(t) - \lambda V_3(t) \right),$$

The discretization of this action is (and we have reconstructed it from the date (the 3-volume–3-volume correlations))

$$S_{discr} = k_1 \sum_i \left(\frac{(N_3(i+1) - N_3(i))^2}{N_3(i)} + \tilde{k}_2 N_3^{1/3}(i) - \tilde{\lambda} N_3(i) \right),$$

$$G = \frac{a^2}{k_1} \frac{\sqrt{C_4}\, s_0^2}{3\sqrt{6}}.$$

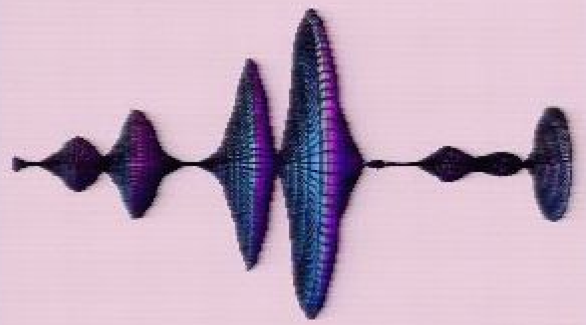
The transition line B-C

Could be 2. order phase transition line, verdict still up.

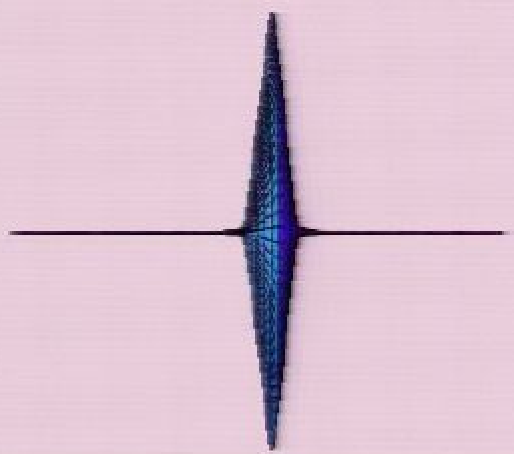
Can it serve as a Horava-Lifshitz UV completion of QG ?

Horava scaling:

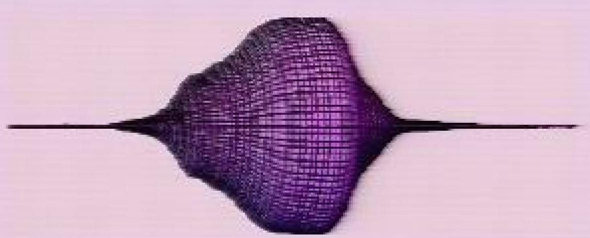
$$V_3 = a^3(\Delta r)^3, \quad T = a^3 \Delta t \qquad \frac{\Delta t}{\Delta r} \propto (\Delta r)^2$$



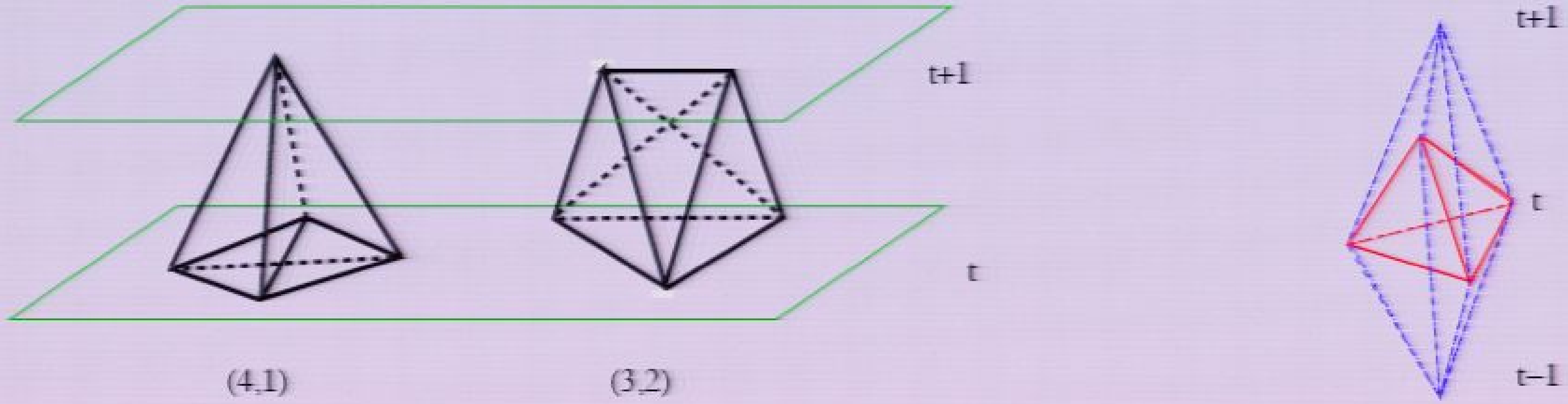
- **Phase A**. Inhomogeneous in time. Dominance of the conformal factor for small bare $1/\kappa_0$. Recall conformal factor appears like $-\dot{\phi}^2(t)$



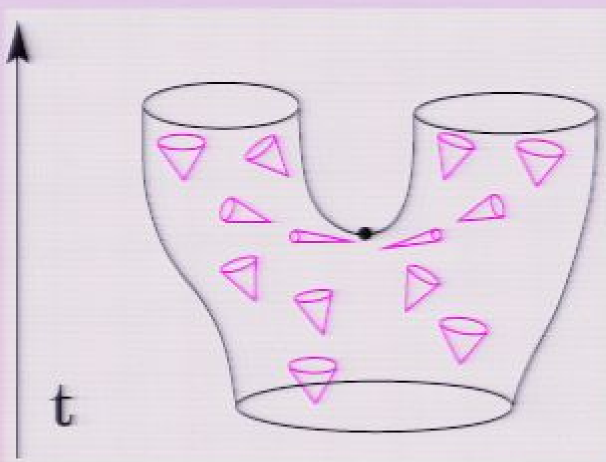
- **Phase B**. Inhomogeneous in space. Effective compactification into a 3d Euclidean DT, but in an "crumpled" inhomogeneous 3d space.



- **Phase C**. Extended de Sitter phase. $d_H = 4$. Lattice time extension depends on Δ but configurations identified by redefinition of a_t.

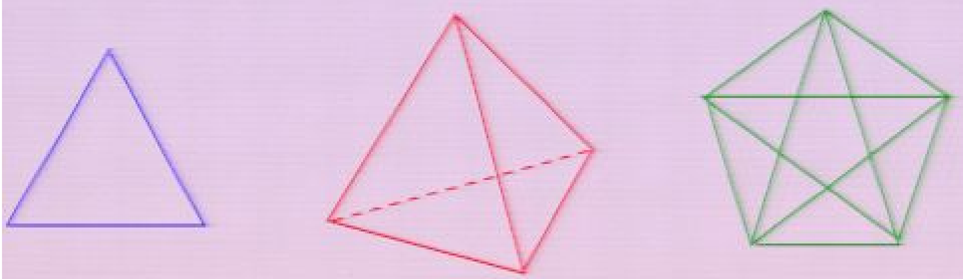


CDT slicing in proper time. Topology of space preserved.
Situation below not allowed.



showcasing piecewise linear geometries via building blocks:

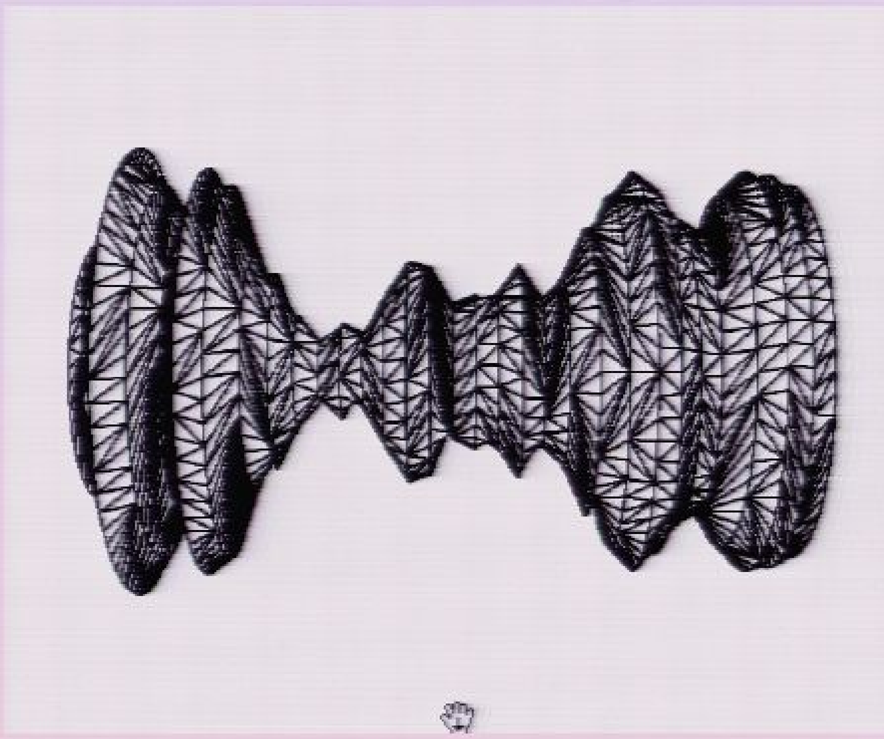
→



2d 3d 4d

$$G(\mathbf{g}_\mathrm{i}, \mathbf{g}_\mathrm{f}, t) \;:=\; \int_{\text{geometries: } \mathbf{g}_\mathrm{i} \to \mathbf{g}_\mathrm{f}} \mathrm{e}^{iS[\mathbf{g}_{\mu\nu}(t')]}$$

$$= \lim_{a \to 0} \sum_{T : T_i^{(3)} \to T_f^{(3)}} \frac{1}{C_T} \, \mathrm{e}^{iS_T}$$

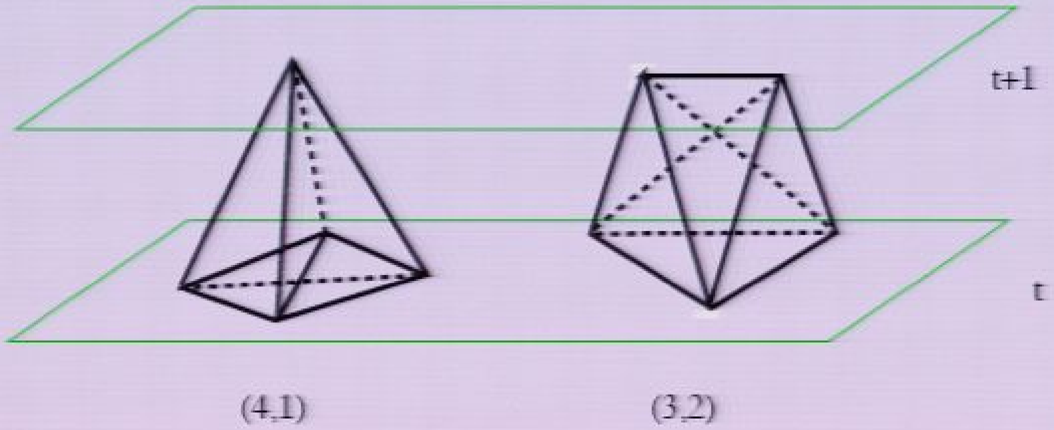


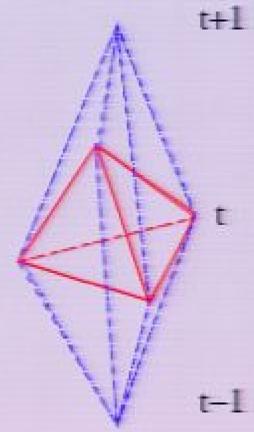
This expression can be summarized as

$$S_E = -\left(\kappa_0 + 6\Delta\right)N_0 + \kappa_4\left(N_4^{(4,1)} + N_4^{(3,2)}\right) + \Delta\left(2N_4^{(4,1)} + N_4^{(3,2)}\right)$$

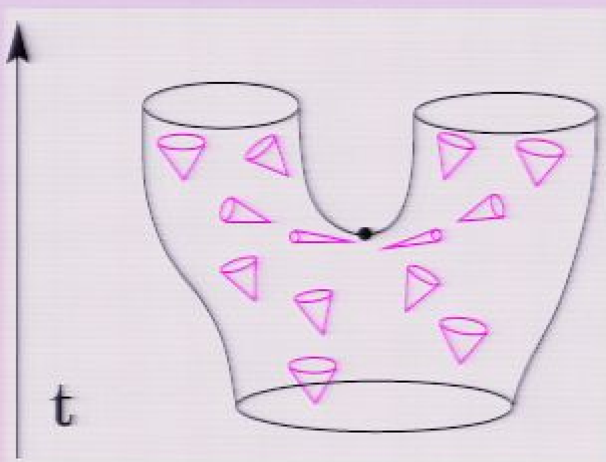
Δ is a function of $\tilde{\alpha}$ the asymmetry parameter between the space and lattice links. $\Delta = 0$ corresponds to $a_t = a_s$, i.e. $\tilde{a} = 1$.

In a given computer simulation $N_4 = N_4^{(4,1)} + N_4^{(3,2)}$ is kept fixed and thus effectively we have only two coupling constants: κ_0 and Δ.





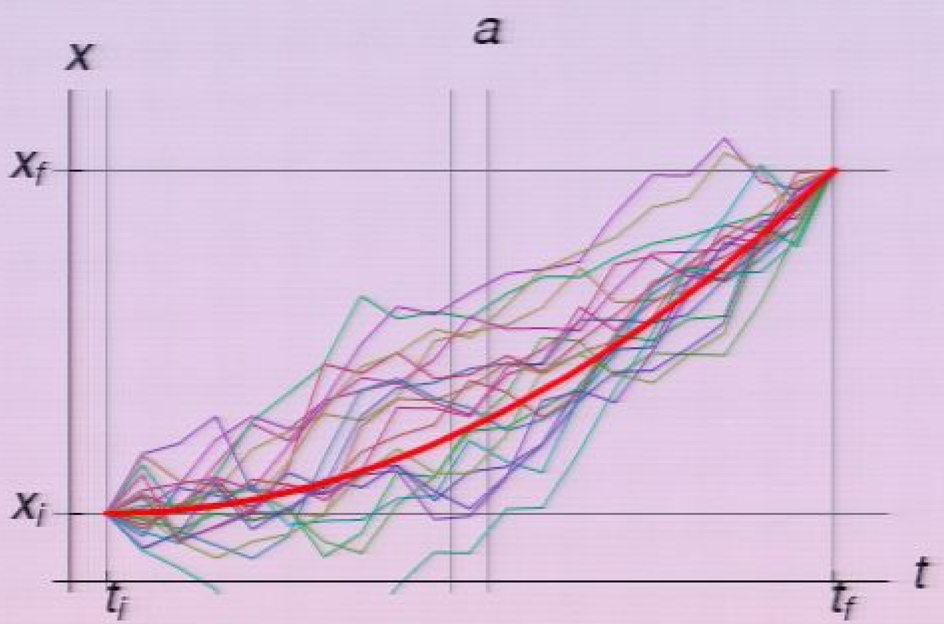
CDT slicing in proper time. Topology of space preserved.
Situation below not allowed.



Lattice gravity: causal dynamical triangulations

Basic tool: The path integral

Text-book example: non-relativistic particle in one dimension.



$$x(t) = \langle x(t)\rangle + y(t)$$
$$\langle |y| \rangle \propto \sqrt{\hbar/m\omega}$$

In QG we want $\langle x(t)\rangle$

$$\langle |y| \rangle \propto \sqrt{\hbar G}$$

Transition amplitude as a weighted sum over all possible trajectories. On the plot: time is discretized in steps a, trajectories are piecewise linear.

In a continuum limit $a \to 0$

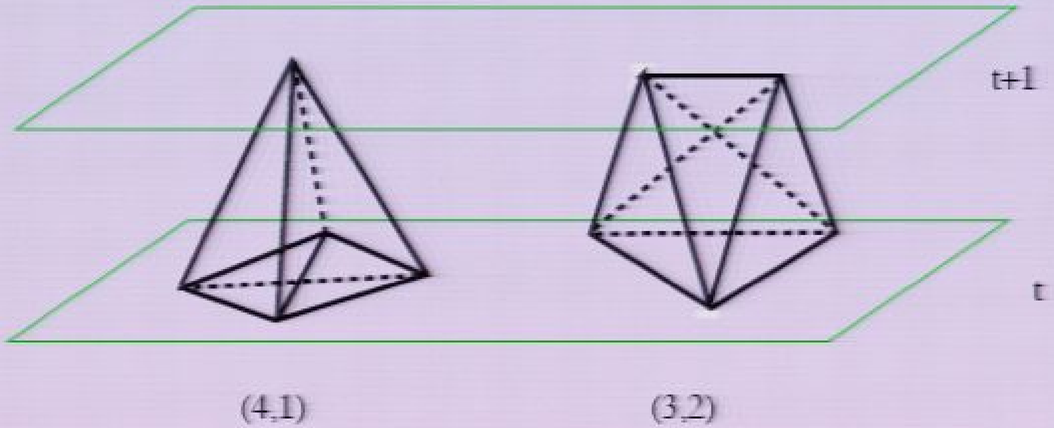
$$G(\mathbf{x}_\mathrm{i}, \mathbf{x}_\mathrm{f}, t) := \int_{\text{trajectories: } \mathbf{x}_\mathrm{i} \to \mathbf{x}_\mathrm{f}} e^{iS[\mathbf{x}(t)]}$$

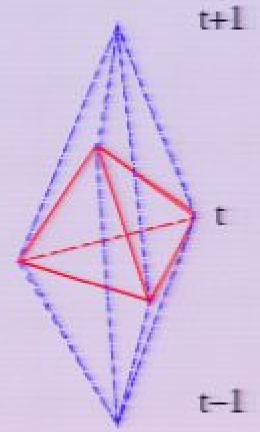
where $S[\mathbf{x}(t)]$ is a classical action.

The QG amplitude between the two geometric states separated a proper time t

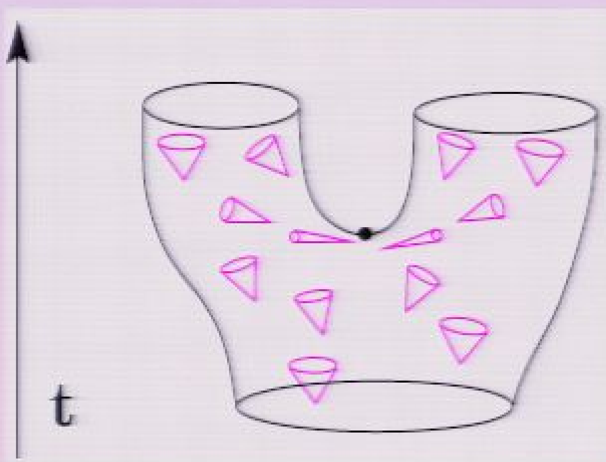
$$G(\mathbf{g}_\mathrm{i}, \mathbf{g}_\mathrm{f}, t) := \int_{\text{geometries: } \mathbf{g}_\mathrm{i} \to \mathbf{g}_\mathrm{f}} e^{iS[\mathbf{g}_{\mu\nu}(t')]}$$

To define this path integral we need a geometric cut-off a and a definition of the class of geometries entering.



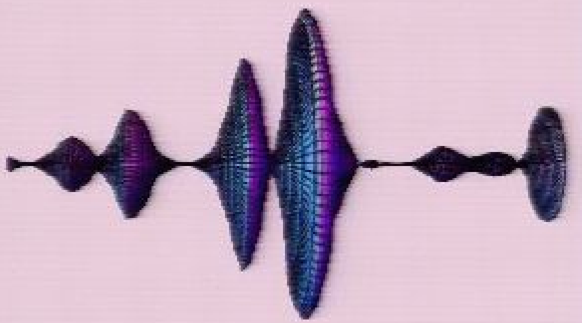


CDT slicing in proper time. Topology of space preserved.
Situation below not allowed.

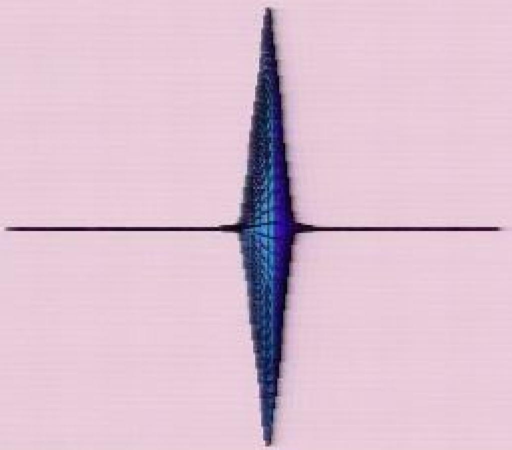


We now have to choose a specific action ($a_t^2 = \tilde{\alpha} a_s^2, \;\; \tilde{\alpha} > 7/12$)

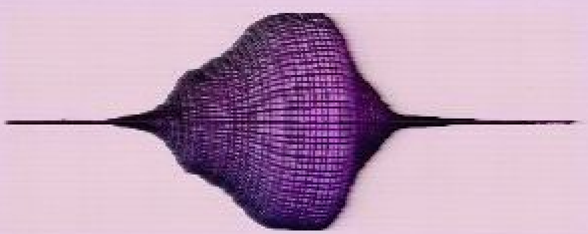
$$
\begin{aligned}
S_E = & -k^{(b)} \pi \sqrt{4\tilde{\alpha} - 1} N_0 \\
& + N_4^{(4,1)} \left(k^{(b)} \sqrt{4\tilde{\alpha} - 1} \left[-\frac{\pi}{2} - \frac{\sqrt{3}}{\sqrt{4\tilde{\alpha} - 1}} \arcsin \frac{1}{2\sqrt{2}\sqrt{3\tilde{\alpha} - 1}} \right. \right. \\
& \qquad \left. \left. + \frac{3}{2} \arccos \frac{2\tilde{\alpha} - 1}{6\tilde{\alpha} - 2} \right] + \lambda^{(b)} \frac{\sqrt{8\tilde{\alpha} - 3}}{96} \right) \\
& + N_4^{(3,2)} \left(k^{(b)} \sqrt{4\tilde{\alpha} - 1} \left[-\pi + \frac{\sqrt{3}}{4\sqrt{4\tilde{\alpha} - 1}} \arccos \frac{6\tilde{\alpha} - 5}{6\tilde{\alpha} - 2} \right. \right. \\
& \qquad \left. + \frac{3}{4} \arccos \frac{4\tilde{\alpha} - 3}{8\tilde{\alpha} - 4} + \frac{3}{2} \arccos \frac{1}{2\sqrt{2}\sqrt{2\tilde{\alpha} - 1}\sqrt{3\tilde{\alpha} - 1}} \right] \\
& \qquad \left. + \lambda^{(b)} \frac{\sqrt{12\tilde{\alpha} - 7}}{96} \right).
\end{aligned}
$$



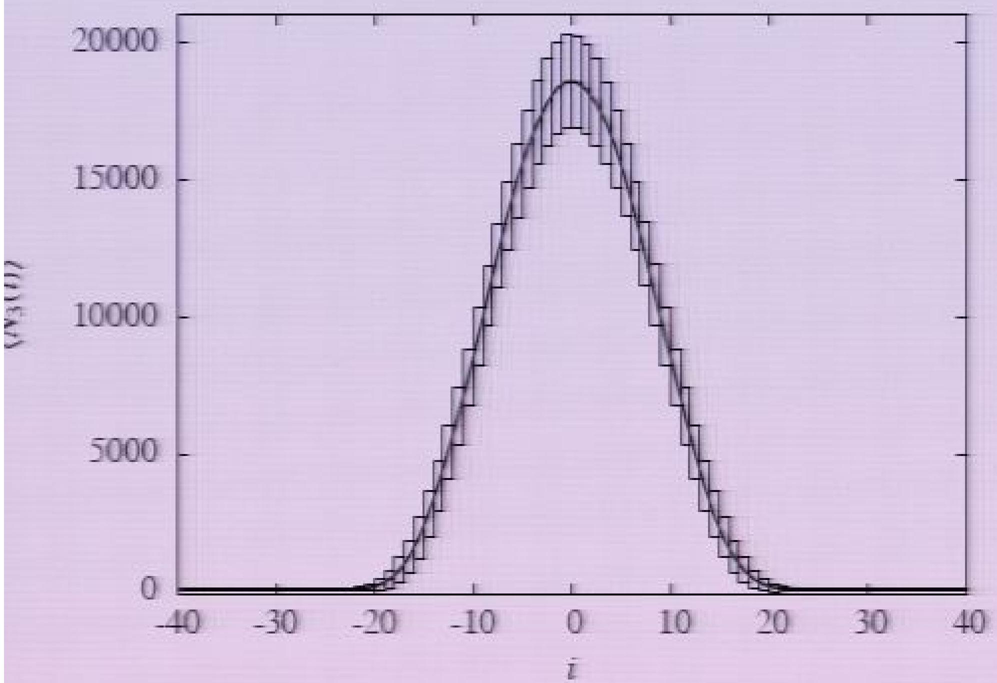
- **Phase A**. Inhomogeneous in time. Dominance of the conformal factor for small bare $1/\kappa_0$. Recall conformal factor appears like $-\dot{\phi}^2(t)$



- **Phase B**. Inhomogeneous in space. Effective compactification into a 3d Euclidean DT, but in an "crumpled" inhomogeneous 3d space.



- **Phase C**. Extended de Sitter phase. $d_H = 4$. Lattice time extension depends on Δ but configurations identified by redefinition of a_t.



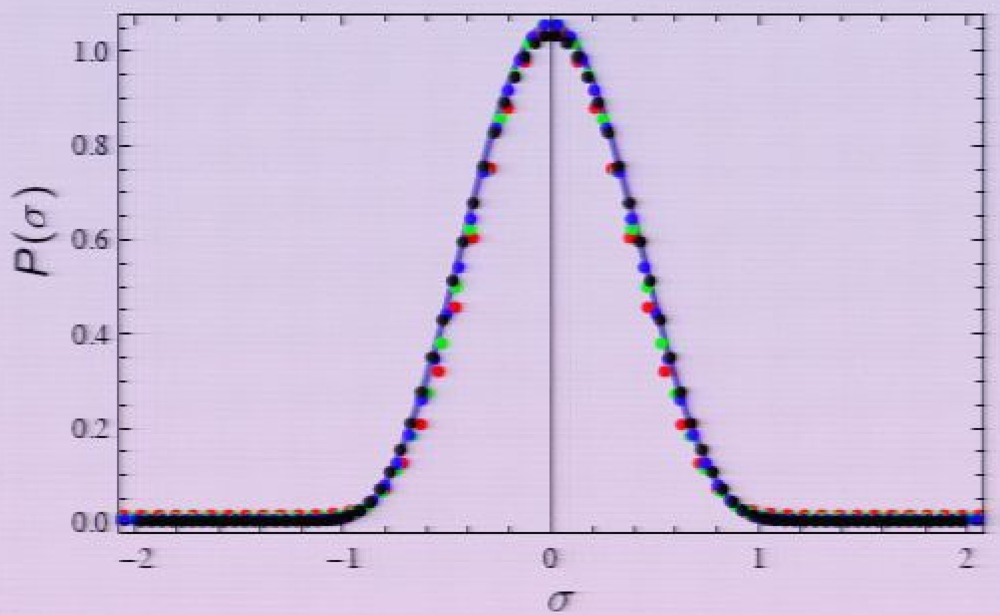


Figure: $N_4 = 22\text{k}, 45\text{k}, 91\text{k}, 182\text{k}$

$$\langle N_3(i)\rangle \propto N_4^{3/4}\cos^3\left(\frac{i}{s_0 N_4^{1/4}}\right)$$

$$\begin{aligned}\sigma &\propto i/N_4^{1/d}\\ N_3(i) &\propto N_4^{(d-1)/d}P(\sigma)\end{aligned}$$

In phase C, deSitter space-time:

Best $d = 4$

Minisuperspace model

The semiclassical distribution can be obtained from the minisuperspace effective action of Hartle and Hawking

$$S_{eff} = \frac{1}{24\pi G} \int dt \sqrt{g_{tt}} \left(\frac{g^{tt} \dot{V_3}^2(t)}{V_3(t)} + k_2 V_3^{1/3}(t) - \lambda V_3(t) \right),$$

The discretization of this action is (and we have reconstructed it from the date (the 3-volume–3-volume correlations))

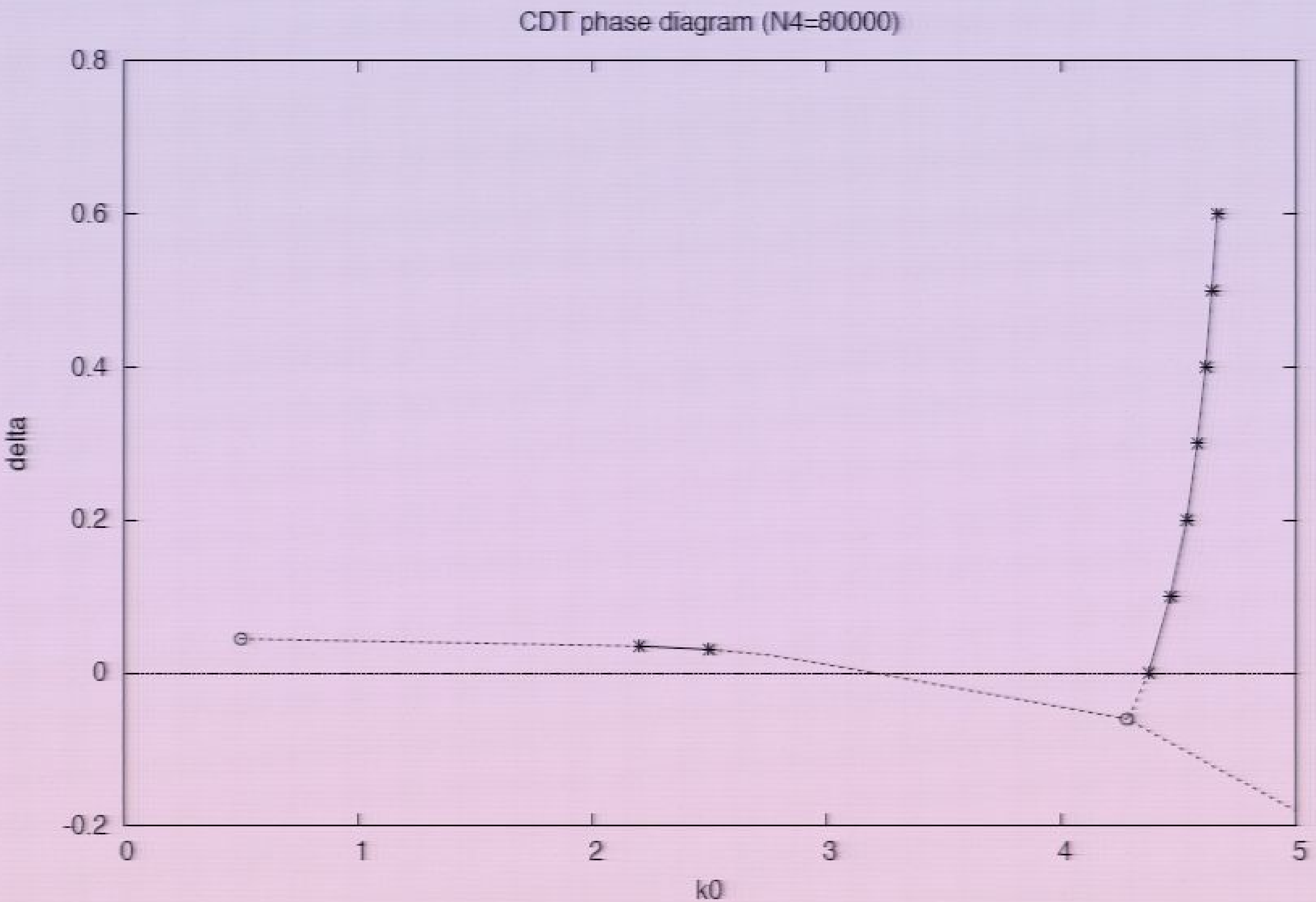
$$S_{discr} = k_1 \sum_i \left(\frac{(N_3(i+1) - N_3(i))^2}{N_3(i)} + \tilde{k}_2 N_3^{1/3}(i) - \tilde{\lambda} N_3(i) \right),$$

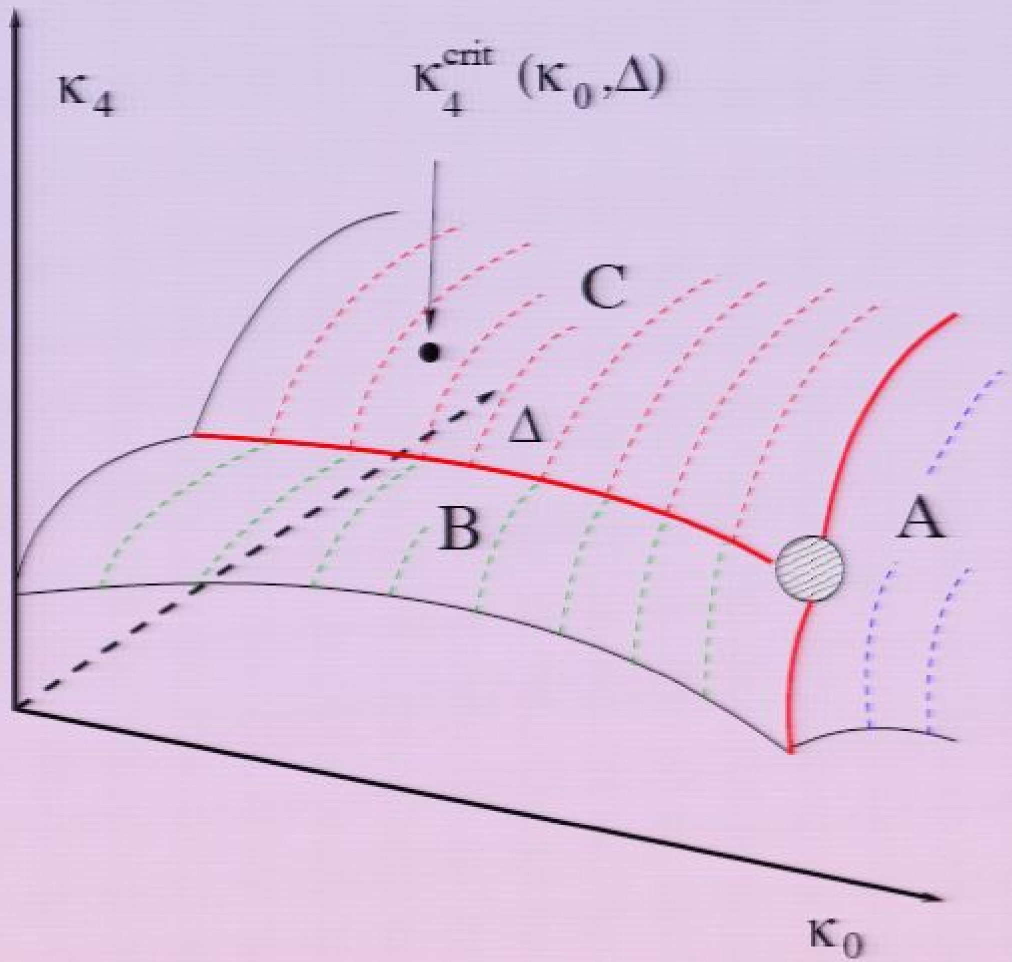
$$G = \frac{a^2}{k_1} \frac{\sqrt{C_4}\, s_0^2}{3\sqrt{6}}.$$

Summary

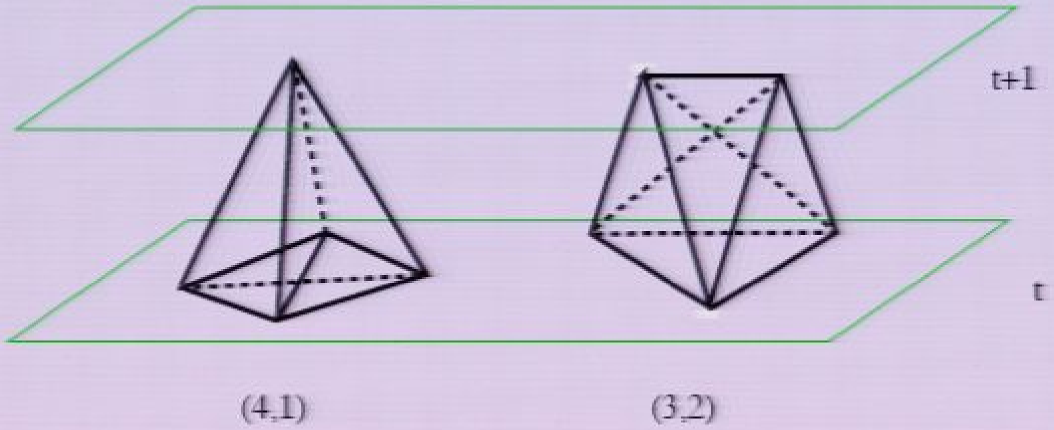
- The set-up with a global time foliation is common to CDT and HL-gravity.
- The spectral dimension when measured in the deSitter phase of CDT varies from 4 (long distance) to 2 (short distance). Similar results were found by Horava, hinting that maybe the two theories have the same UV completion.
- We have argued that Horava scaling is a posibility along the B-C phase transition line if $\Delta t = N_3^{\nu}$ and $\nu = 3/d_H$ (and if it is second order.....)
- CDT is in principle ideally suited to study lattice HL gravity, given a suitable lattice action.

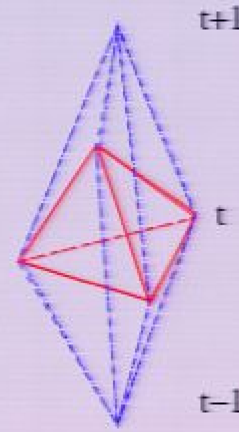
Phase diagram in $\kappa_0 - \Delta$ plane



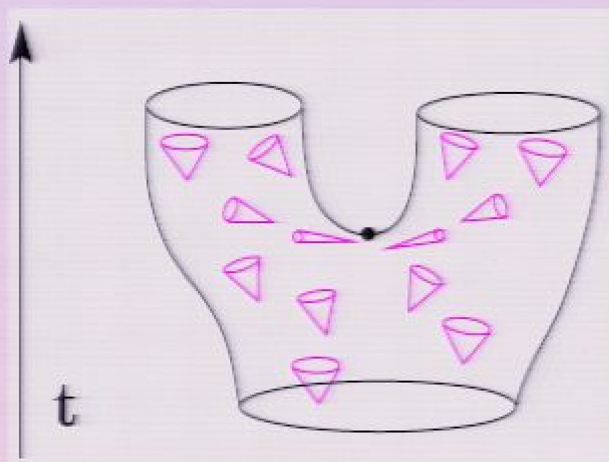


Asymmetry between space and time ? (like in Horava model)



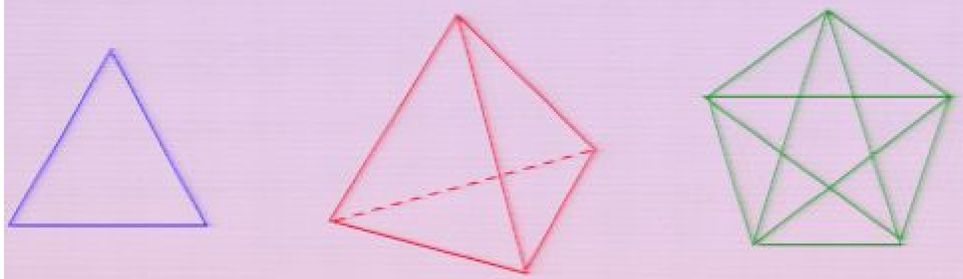


CDT slicing in proper time. Topology of space preserved.
Situation below not allowed.



showcasing piecewise linear geometries via building blocks:

⟶



2d 3d 4d