Title: Asymptotic safety in the nonlinear sigma models and gravity

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URL: http://pirsa.org/09110127

Abstract:

Asymptotic Safety in Gravity and Sigma Models

Asymptotic Safety, 30 years later

Waterloo, November 5-8, 2009

[A.Codello and R.Percacci, Phys. Lett. **B672** 280-283 (2009) arXiv:0810.0715 [hep-th]]

[R.Percacci and O.Z., arXiv:0910.0851 [hep-th]]

Two derivative terms

$$Z \int d^4x \, \mathrm{tr}(U^{-1}\partial_\mu U \, U^{-1}\partial^\mu U) = \frac{1}{2} Z \int d^4x \, \partial_\mu \varphi^\alpha \partial^\mu \varphi^\beta h_{\alpha\beta}(\varphi)$$

$$Z = \frac{1}{g^2}$$

$$Z \int d^4x \, \sqrt{g} g^{\mu\nu} R_{\lambda\mu}{}^{\lambda}{}_{\nu}$$

$$Z = \frac{1}{16\pi G}$$

$Z \approx \mathrm{mass}^2$, $g, \sqrt{G} \approx \mathrm{mass}^{-1}$ nonrenormalizable in $d = 4$

RESCALINGS

$$\varphi^\alpha = \frac{1}{\sqrt{Z}}\bar{\varphi}^\alpha$$

$$\frac{1}{2}\int d^4x\,\partial_\mu\bar{\varphi}^\alpha\partial^\mu\varphi^\beta h_{\alpha\beta}(\frac{\bar{\varphi}}{\sqrt{Z}})$$

$$g_{\mu\nu} = \frac{1}{Z}\bar{g}_{\mu\nu}$$

$$\int d^4x\,\sqrt{\bar{g}}\bar{g}^{\mu\nu}\bar{R}_{\lambda\mu}{}^{\lambda}{}_{\nu}$$

but G is still essential

Low Energy EFTs

χPT

$$S(U) = \int dx \, [g_2(U^{-1}\partial U)^2 + g_4(U^{-1}\partial U)^4 + g_6(U^{-1}\partial U)^6 + \ldots]$$

imilar for gravity

$$S(g_{\mu\nu}) = \int dx \, [g_0 + g_2 R + g_4 R^2 + g_6 R^3 + \ldots]$$

2 derivative truncation of FRGE

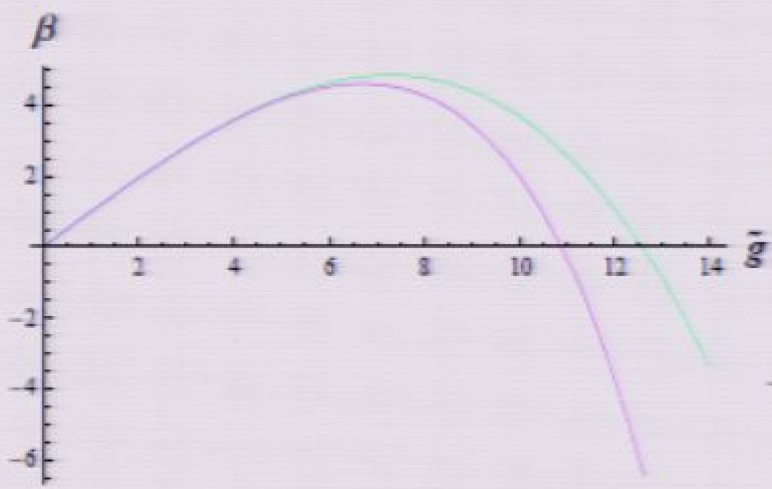
$$k\frac{\partial g_2}{\partial k} = B_1 k^2 \ ,$$

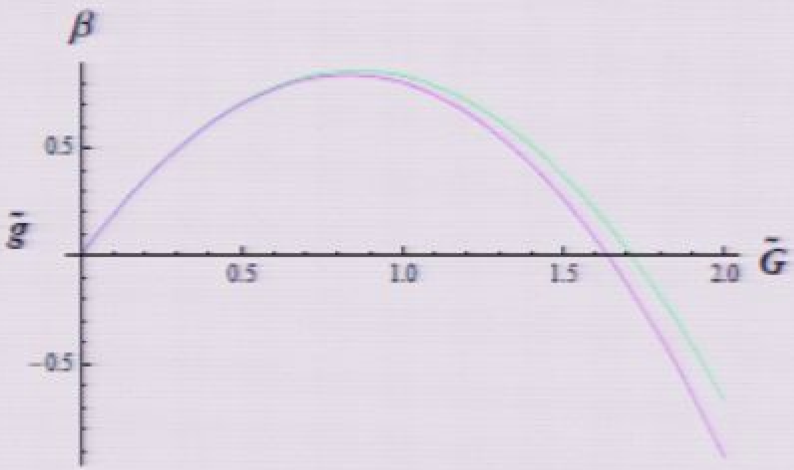
$$g_2 = 1/2g^2, \tilde{g} = kg$$

$$k\frac{\partial g^2}{\partial k} = -2B_1 g^4 k^2$$

$$k\frac{\partial \tilde{g}^2}{\partial k} = 2\tilde{g}^2 - 2B_1\tilde{g}^4 \ .$$

Beta functions





NLSM, 4 DERIVATIVE TRUNCATION

$$\frac{1}{2}\int d^4x \Big[\ \partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta h^{(2)}_{\alpha\beta}(\varphi) + \Box\varphi^\alpha\Box\varphi^\beta h^{(4)}_{\alpha\beta}(\varphi)$$

$$+\nabla_\mu\partial_\nu\varphi^\alpha\partial^\mu\varphi^\beta\partial^\nu\varphi^\gamma A_{\alpha\beta\gamma}(\varphi) + \partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta\partial_\nu\varphi^\gamma\partial^\nu\varphi^\delta T_{\alpha\beta\gamma\delta}(\varphi)\Big]\ .$$

FRGE 1-LOOP

$$k^2\partial_\mu\varphi^\gamma\partial^\mu\varphi^\delta\left(2R_{\gamma\delta}-2T^\alpha{}_{\alpha\gamma\delta}-T^\alpha{}_{\gamma\alpha\delta}\right)$$
$$-\frac{1}{6}\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta\partial_\nu\varphi^\gamma\partial^\nu\varphi^\delta R_{\alpha\gamma\epsilon\eta}R_{\beta\delta}{}^{\epsilon\eta}$$
$$+\frac{1}{2}h^{(2)}_{\alpha\beta}h^{(2)\alpha\beta}+\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta\left(T_\alpha{}^\gamma{}_\beta{}^\delta+2T_{\alpha\beta}{}^{\gamma\delta}-2R_\alpha{}^\gamma{}_\beta{}^\delta\right)h^{(2)}_{\gamma\delta}$$
$$+\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta\partial_\nu\varphi^\gamma\partial^\nu\varphi^\delta\Big[\frac{2}{3}T_{\alpha\epsilon\gamma\eta}T_\beta{}^{(\epsilon}{}_\delta{}^{\eta)}+\frac{1}{3}T_{\alpha\epsilon\beta\eta}T_\gamma{}^{(\eta}{}_\delta{}^{\epsilon)}+4T_{\alpha(\epsilon\gamma)\eta}T_\beta{}^{(\epsilon}{}_\delta{}^{\eta)}$$
$$-4R_{\alpha\epsilon\gamma\eta}T_\beta{}^{(\epsilon}{}_\delta{}^{\eta)}-2R_{\alpha\epsilon\beta\eta}T_\gamma{}^{(\eta}{}_\delta{}^{\epsilon)}+2R_{\alpha\epsilon\beta\eta}R_\gamma{}^{(\eta}{}_\delta{}^{\epsilon)}\Big]$$
$$+\Box\varphi^\alpha\Box\varphi^\beta R_{\alpha\beta}+\Box\varphi^\alpha\partial^\mu\varphi^\beta\partial_\mu\varphi^\gamma(2\nabla_\gamma R_{\alpha\beta}-\nabla_\alpha R_{\beta\gamma})$$
$$+\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta\left(h^{(2)}_{\alpha\gamma}R^\gamma{}_\beta-\frac{1}{2}\nabla_\gamma\nabla^\gamma h^{(2)}_{\alpha\beta}\right)+\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta\partial_\nu\varphi^\gamma\partial^\nu\varphi^\delta\times$$
$$\left(2R_\alpha{}^\epsilon T_{\epsilon\beta\gamma\delta}+2R_{\beta\delta}{}^{\epsilon\eta}T_{\alpha\eta\gamma\epsilon}-\frac{1}{2}\nabla_\epsilon\nabla^\epsilon T_{\alpha\beta\gamma\delta}-R_{\alpha\epsilon\beta\eta}R_\gamma{}^\epsilon{}_\delta{}^\eta\right)$$

NLSM, 4 DERIVATIVE TRUNCATION

$$\frac{1}{2}\int d^4x \Big[\; \partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta h^{(2)}_{\alpha\beta}(\varphi) + \Box\varphi^\alpha\Box\varphi^\beta h^{(4)}_{\alpha\beta}(\varphi)$$
$$+\nabla_\mu\partial_\nu\varphi^\alpha\partial^\mu\varphi^\beta\partial^\nu\varphi^\gamma A_{\alpha\beta\gamma}(\varphi) + \partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta\partial_\nu\varphi^\gamma\partial^\nu\varphi^\delta T_{\alpha\beta\gamma\delta}(\varphi)\Big] \; .$$

FRGE 1-LOOP

$$k^2\partial_\mu\varphi^\gamma\partial^\mu\varphi^\delta\left(2R_{\gamma\delta}-2T^\alpha{}_{\alpha\gamma\delta}-T^\alpha{}_{\gamma\alpha\delta}\right)$$

$$-\frac{1}{6}\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta\partial_\nu\varphi^\gamma\partial^\nu\varphi^\delta R_{\alpha\gamma\epsilon\eta}R_{\beta\delta}{}^{\epsilon\eta}$$

$$+\frac{1}{2}h^{(2)}_{\alpha\beta}h^{(2)\alpha\beta}+\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta\left(T_\alpha{}^\gamma{}_\beta{}^\delta+2T_{\alpha\beta}{}^{\gamma\delta}-2R_\alpha{}^\gamma{}_\beta{}^\delta\right)h^{(2)}_{\gamma\delta}$$

$$+\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta\partial_\nu\varphi^\gamma\partial^\nu\varphi^\delta\Big[\frac{2}{3}T_{\alpha\epsilon\gamma\eta}T_\beta{}^{(\epsilon}{}_\delta{}^{\eta)}+\frac{1}{3}T_{\alpha\epsilon\beta\eta}T_\gamma{}^{(\eta}{}_\delta{}^{\epsilon)}+4T_{\alpha(\epsilon\gamma)\eta}T_\beta{}^{(\epsilon}{}_\delta{}^{\eta)}$$

$$-4R_{\alpha\epsilon\gamma\eta}T_\beta{}^{(\epsilon}{}_\delta{}^{\eta)}-2R_{\alpha\epsilon\beta\eta}T_\gamma{}^{(\eta}{}_\delta{}^{\epsilon)}+2R_{\alpha\epsilon\beta\eta}R_\gamma{}^{(\eta}{}_\delta{}^{\epsilon)}\Big]$$

$$+\Box\varphi^\alpha\Box\varphi^\beta R_{\alpha\beta}+\Box\varphi^\alpha\partial^\mu\varphi^\beta\partial_\mu\varphi^\gamma(2\nabla_\gamma R_{\alpha\beta}-\nabla_\alpha R_{\beta\gamma})$$

$$+\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta\left(h^{(2)}_{\alpha\gamma}R^\gamma{}_\beta-\frac{1}{2}\nabla_\gamma\nabla^\gamma h^{(2)}_{\alpha\beta}\right)+\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta\partial_\nu\varphi^\gamma\partial^\nu\varphi^\delta\times$$

$$\left(2R_\alpha{}^\epsilon T_{\epsilon\beta\gamma\delta}+2R_{\beta\delta}{}^{\epsilon\eta}T_{\alpha\eta\gamma\epsilon}-\frac{1}{2}\nabla_\epsilon\nabla^\epsilon T_{\alpha\beta\gamma\delta}-R_{\alpha\epsilon\beta\eta}R_\gamma{}^\epsilon{}_\delta{}^\eta\right)$$

Spherical models S^n

$$R_{\alpha\beta\gamma\delta} = h_{\alpha\gamma}h_{\beta\delta} - h_{\alpha\delta}h_{\beta\gamma} \ ; \quad R_{\alpha\beta} = (n-1)h_{\alpha\beta} \ ; \quad R = n(n-1) \ .$$

$$h^{(2)}_{\alpha\beta} = \frac{1}{g^2}h_{\alpha\beta} \ ; \qquad h^{(4)}_{\alpha\beta} = \frac{1}{\lambda}h_{\alpha\beta} \ ;$$

$$T_{\alpha\beta\gamma\delta} = \frac{\ell_1}{2}\left(h_{\alpha\gamma}h_{\beta\delta} + h_{\alpha\delta}h_{\beta\gamma}\right) + \ell_2 h_{\alpha\beta}h_{\gamma\delta} \ .$$

SPHERICAL MODELS: BETA FUNCTIONS

$$\beta_\lambda = -\frac{n-1}{8\pi^2}\lambda^2$$

$$\beta_{f_1} = \frac{\lambda}{48\pi^2}\left((n+21)f_1^2 + 20f_2f_1 + 4f_2^2 + 6(n+3)f_1 + 24f_2 + 8\right)$$

$$\beta_{f_2} = \frac{\lambda}{8\pi^2}\left(\frac{n+15}{12}f_1^2 + \frac{3n+17}{3}f_1f_2 + \frac{6n+7}{3}f_2^2 - (n+3)f_1 - (3n+1)f_2 + n - \frac{7}{3}\right.$$

$$\beta_{\tilde{g}^2} = 2\tilde{g}^2 + \frac{\tilde{g}^4}{16\pi^2}\left((5+n)f_1 + (2+4n)f_2 + 4(1-n)\right) - \frac{\lambda\tilde{g}^2}{16\pi^2}\left((5+n)f_1 + (2+4n)f_2 + 2(1-n)\right)$$

Spherical models S^n

$$R_{\alpha\beta\gamma\delta} = h_{\alpha\gamma}h_{\beta\delta} - h_{\alpha\delta}h_{\beta\gamma}\ ; \quad R_{\alpha\beta} = (n-1)h_{\alpha\beta}\ ; \quad R = n(n-1)\ .$$

$$h^{(2)}_{\alpha\beta} = \frac{1}{g^2}h_{\alpha\beta}\ ; \qquad h^{(4)}_{\alpha\beta} = \frac{1}{\lambda}h_{\alpha\beta}\ ;$$

$$T_{\alpha\beta\gamma\delta} = \frac{\ell_1}{2}\left(h_{\alpha\gamma}h_{\beta\delta} + h_{\alpha\delta}h_{\beta\gamma}\right) + \ell_2 h_{\alpha\beta}h_{\gamma\delta}\ .$$

Spherical models: beta functions

$$\begin{aligned}
\beta_\lambda &= -\frac{n-1}{8\pi^2}\lambda^2 \\
\beta_{f_1} &= \frac{\lambda}{48\pi^2}\left((n+21)f_1^2 + 20 f_2 f_1 + 4 f_2^2 + 6(n+3) f_1 + 24 f_2 + 8\right) \\
\beta_{f_2} &= \frac{\lambda}{8\pi^2}\left(\frac{n+15}{12} f_1^2 + \frac{3n+17}{3} f_1 f_2 + \frac{6n+7}{3} f_2^2 - (n+3) f_1 - (3n+1) f_2 + n - \frac{7}{3}\right. \\
\beta_{\tilde{g}^2} &= 2\tilde{g}^2 + \frac{\tilde{g}^4}{16\pi^2}\left((5+n) f_1 + (2+4n) f_2 + 4(1-n)\right) \\
&\quad -\frac{\lambda \tilde{g}^2}{16\pi^2}\left((5+n) f_1 + (2+4n) f_2 + 2(1-n)\right)
\end{aligned}$$

Spherical models: fixed points

n	$\tilde{g}_*^{(III)}$	FP	$\tilde{g}_*$	f_{1*}	f_{2*}	θ_1	θ_2
3	8.886	NFP1	6.626	-0.693	0.453	0.094	-0.0121
3		NFP2	6.390	-1.042	0.615	0.103	0.0119
3		GFP1	0	-0.693	0.453	0.094	-0.0121
3		GFP2	0	-1.042	0.615	0.103	0.0119
4	7.255	NFP1	5.877	-0.479	0.398	0.105	-0.0412
4		NFP2	5.442	-1.555	0.852	0.132	0.0392
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$$\beta_{\tilde{g}^2} = 2\tilde{g}^2 + \frac{\tilde{g}^4}{16\pi^2}\left((5+n)f_1 + (2+4n)f_2 + 4(1-n)\right) - \frac{\lambda\tilde{g}^2}{16\pi^2}\left((5+n)f_1 + (2+4n)f_2 + 2(1-n)\right)$$

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\end{aligned}
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Chiral models $SU(N)$

$$\int d^4x \left[\frac{1}{2g^2} h_{\alpha\beta} \partial_\mu \varphi^\alpha \partial^\mu \varphi^\beta + \frac{1}{2\lambda} h_{\alpha\beta} \Box \varphi^\alpha \Box \varphi^\beta + \frac{1}{2} \partial_\mu \varphi^\alpha \partial^\mu \varphi^\beta \partial_\nu \varphi^\gamma \partial^\nu \varphi^\delta \sum_{i=1}^{4} \frac{f_i}{\lambda} T^{(i)}_{\alpha\beta\gamma\delta} \right]$$

$$T^{(1)}_{abcd} = \frac{1}{2}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) \ ; \quad T^{(2)}_{abcd} = \delta_{ab}\delta_{cd} \ ; \quad T^{(3)}_{abcd} = \frac{1}{2}(f_{ace} f_{bd}{}^{e} + f_{ade} f_{bc}{}^{e}) \ ;$$

$$T^{(4)}_{abcd} = \frac{1}{2}(d_{ace} d_{bd}{}^{e} + d_{ade} d_{bc}{}^{e}) \ ; \qquad T^{(5)}_{abcd} = d_{abe} d_{cd}{}^{e} \ .$$

$$\frac{2}{N} T^{(1)} - \frac{2}{N} T^{(2)} + T^{(3)} + T^{(4)} - T^{(5)} = 0 \ ,$$

For $N = 3$ also $T^{(2)} - T^{(3)} - 3T^{(4)} = 0$

Chiral model $N = 3$

$$\begin{aligned}
\beta_\lambda &= -\frac{3}{32\pi^2}\lambda^2 \\
\beta_{f_1} &= \frac{\lambda}{768\pi^2}\left[464 f_1^2 + 64 f_2^2 + 180 f_3^2 + 320 f_1 f_2 - 96 f_1 f_3 + 72 f_1 - 108 f_3 + 9\right] \\
\beta_{f_2} &= \frac{\lambda}{1536\pi^2}\Big[368 f_1^2 + 3520 f_2^2 + 180 f_3^2 + 2624 f_1 f_2 \\
&\quad + 480 f_1 f_3 + 1728 f_2 f_3 - 144 f_1 - 432 f_2 - 108 f_3 + 9\Big] \\
\beta_{f_3} &= \frac{\lambda}{32\pi^2}\left[2 f_3^2 + 16 f_1 f_3 + 8 f_2 f_3 - 4 f_1 - 4 f_2 - 3 f_3\right] \\
\beta_{\tilde{g}^2} &= 2\tilde{g}^2 + \frac{\tilde{g}^4}{16N\pi^2}(39 f_1 + 102 f_2 + 27 f_3 - 9) \\
&\quad - \frac{\lambda\tilde{g}^2}{16N\pi^2}(39 f_1 + 102 f_2 + 27 f_3 - 9/2)
\end{aligned}$$

Chiral model $N = 3$

$$
\begin{array}{lllll}
GFP1: & f_{1*} = -0.154\ ; & f_{2*} = 0.050\ ; & f_{3*} = 0.085\ ; & \tilde{g} = 0 \\
GFP2: & f_{1*} = -0.108\ ; & f_{2*} = 0.043\ ; & f_{3*} = 0.061\ ; & \tilde{g} = 0 \\
NFP1: & f_{1*} = -0.154\ ; & f_{2*} = 0.050\ ; & f_{3*} = 0.085\ ; & \tilde{g} = 11.17 \\
NFP2: & f_{1*} = -0.108\ ; & f_{2*} = 0.043\ ; & f_{3*} = 0.061\ ; & \tilde{g} = 11.50
\end{array}
\qquad (1)
$$

Chiral models $N > 3$

$$
\begin{aligned}
\beta_\lambda &= -\frac{N}{32\pi^2}\lambda^2 \\
\beta_{f_i} &= \frac{\lambda}{16\pi^2}P_i\left(f_j, N\right) \\
\beta_{\tilde{g}^2} &= 2\tilde{g}^2 + \frac{\tilde{g}^4}{16N\pi^2}\left(N(N^2+4)f_1 + 2N(2N^2-1)f_2 + 3N^2 f_3 + 5(N^2-4)f_4 - N^2\right) \\
&\quad -\frac{\lambda\tilde{g}^2}{16N\pi^2}\left(N(N^2+4)f_1 + 2N(2N^2-1)f_2 + 3N^2 f_3 + 5(N^2-4)f_4 - N^2/2\right)
\end{aligned}
$$

HIGHER DERIVATIVE GRAVITY

$$\Gamma_k = \int d^4x \sqrt{g} \left[2Z\Lambda - ZR + \frac{1}{2\lambda} C^2 + \frac{1}{\xi} R^2 + \frac{1}{\rho} E \right]$$

$$Z = \frac{1}{16\pi G}; \qquad \frac{1}{\xi} = -\frac{\omega}{3\lambda}; \qquad \frac{1}{\rho} = \frac{\theta}{\lambda}$$

K.S. Stelle, Phys. Rev. **D16**, 953 (1977).

J. Julve, M. Tonin, Nuovo Cim. **46B**, 137 (1978).

E.S. Fradkin, A.A. Tseytlin,
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A. Codello and R. Percacci, Phys.Rev.Lett. **97** 22 (2006)

Chiral models $N > 3$

$$
\begin{aligned}
\beta_\lambda &= -\frac{N}{32\pi^2}\lambda^2 \\
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\beta_{\tilde{g}^2} &= 2\tilde{g}^2 + \frac{\tilde{g}^4}{16N\pi^2}\left(N(N^2+4)f_1 + 2N(2N^2-1)f_2 + 3N^2 f_3 + 5(N^2-4)f_4 - N^2\right) \\
&\quad -\frac{\lambda\tilde{g}^2}{16N\pi^2}\left(N(N^2+4)f_1 + 2N(2N^2-1)f_2 + 3N^2 f_3 + 5(N^2-4)f_4 - N^2/2\right)
\end{aligned}
$$

Higher derivative gravity

$$\Gamma_k = \int d^4x \sqrt{g} \left[2Z\Lambda - ZR + \frac{1}{2\lambda} C^2 + \frac{1}{\xi} R^2 + \frac{1}{\rho} E \right]$$

$$Z = \frac{1}{16\pi G}\ ; \qquad \frac{1}{\xi} = -\frac{\omega}{3\lambda}\ ; \qquad \frac{1}{\rho} = \frac{\theta}{\lambda}$$

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Beta functions I

$$\beta_\lambda = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2$$

$$\beta_\xi = -\frac{1}{(4\pi)^2}\left(10\lambda^2 - 5\lambda\xi + \frac{5}{36}\right)$$

$$\beta_\rho = \frac{1}{(4\pi)^2}\frac{196}{45}\rho^2\lambda$$

$$\lambda(k) = \frac{\lambda_0}{1 + \lambda_0 \frac{1}{(4\pi)^2}\frac{133}{10}\log\left(\frac{k}{k_0}\right)}$$

$$\omega(k) \to \omega_* \approx -0.0228$$

$$\theta(k) \to \theta_* \approx 0.327$$

Beta functions II

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{1}{(4\pi)^2}\left[\frac{1+20\omega^2}{256\pi\tilde{G}\omega^2}\lambda^2 + \frac{1+86\omega+40\omega^2}{12\omega}\lambda\tilde{\Lambda}\right]$$

$$-\frac{1+10\omega^2}{64\pi^2\omega}\lambda + \frac{2\tilde{G}}{\pi} - q(\omega)\tilde{G}\tilde{\Lambda}$$

$$\beta_{\tilde{G}} = 2\tilde{G} - \frac{1}{(4\pi)^2}\frac{3+26\omega-40\omega^2}{12\omega}\lambda\tilde{G} - q(\omega)\tilde{G}^2$$

where $q(\omega) = (83 + 70\omega + 8\omega^2)/18\pi$

Flow in $\tilde{\Lambda}$–$\tilde{G}$ plane I

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{2\tilde{G}}{\pi} - q_* \tilde{G}\tilde{\Lambda}$$
$$\beta_{\tilde{G}} = 2\tilde{G} - q_* \tilde{G}^2$$

vhere $q_* = q(\omega_*) \approx 1.440$

$$\tilde{\Lambda}(t) = \frac{(2\pi\tilde{\Lambda}_0 - \tilde{G}_0(1-e^{4t}))e^{-2t}}{\pi(2 - q_*\tilde{G}_0(1-e^{2t}))} \; ; \quad \tilde{G}(t) = \frac{2\tilde{G}_0 e^{2t}}{2 - q_*\tilde{G}_0(1-e^{2t})}$$

$$\tilde{\Lambda}_* = \frac{1}{\pi q_*} \approx 0.221 \, , \qquad \tilde{G}_* = \frac{2}{q_*} \approx 1.389 \, .$$

Beyond 1 Loop

$$\tilde{\Lambda}_* = 0.11, \quad \tilde{G}_* = 0.865, \quad \lambda_* = -44.86, \quad \omega_* = 0.932\,.$$

$$\theta = 2.33 \pm 0.6i\,; \quad \theta_2 = 13.72\,; \quad \theta_3 = -7.00\,.$$

[D. Benedetti, P.F. Machado, F. Saueressig, arXiv:0901.2984]

To do list

- check higher orders of derivative expansion
- check gauge and Yukawa couplings