Title: A Mechanism for Asymptotic Safety of Chiral Yukawa Systems

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Abstract:

A mechanism for asymptotic safety of chiral Yukawa systems

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Outline

1. *Triviality and the hierarchy problem*
2. *Asymptotic safety and the flow equation*
3. *Truncation for chiral Yukawa systems*
4. *Fixed-points of chiral Yukawa systems*
5. *Numerical example for Higgs/top mass prediction*

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- Both problems could be solved within the AS scenario.
- As a toy model for the SM we will investigate a class of chiral Yukawa systems

Triviality

- Higgs field is parametrized in terms of a bosonic field ϕ with a Lagrangian

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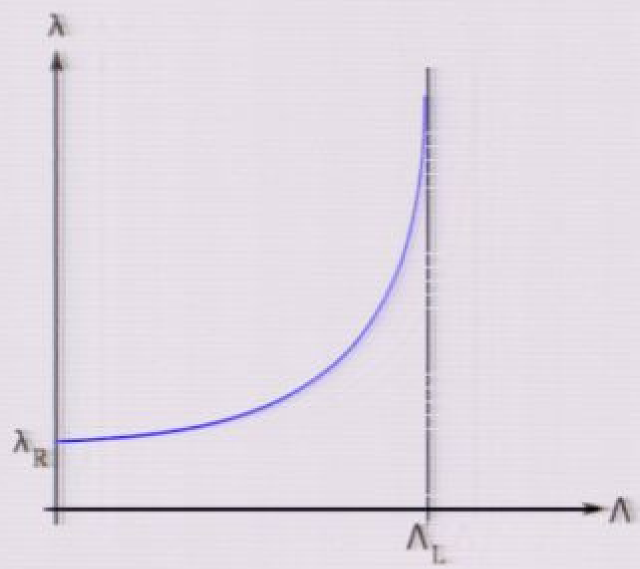

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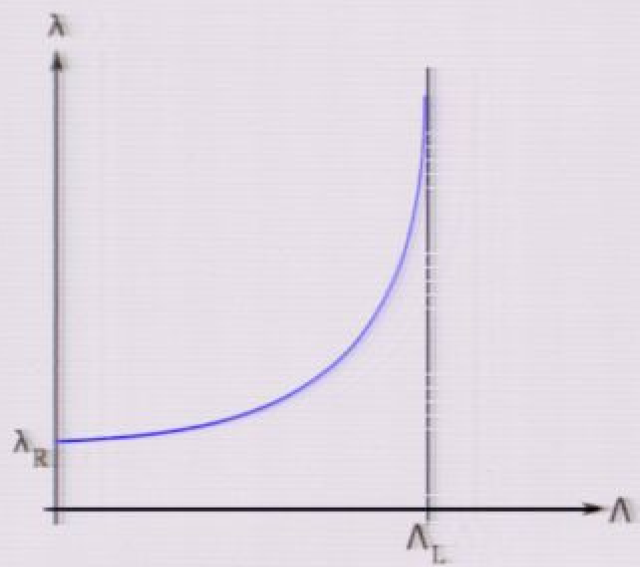

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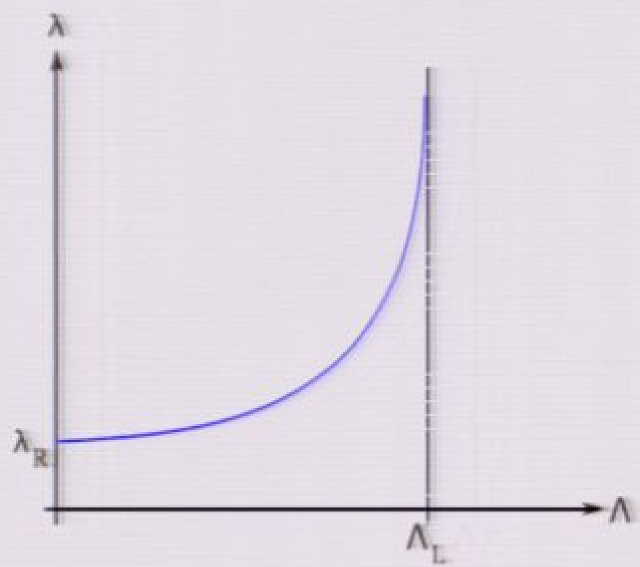

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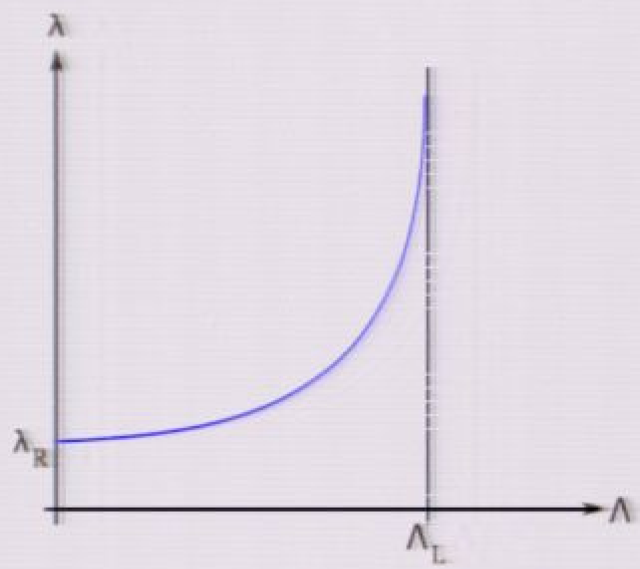

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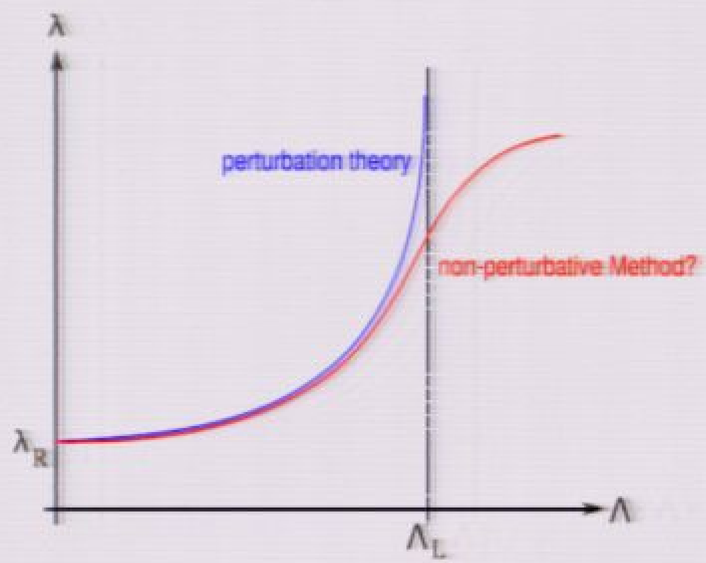

- Landau-pole indicates breakdown of perturbative QFT → new d.o.f.?
- Perturbation theory relies on an expansion around zero coupling.
- Near the Landau pole perturbation theory will loose its validity since λ grows large
- Need non-perturbative tool to study triviality & take into account fermions!

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- This seems to be "unnatural".

A hierarchy problem corresponds to the existence of a large critical exponent $\Theta_I > 0$ at a fixed point, e.g. in ϕ^4-theory we find at the GFP $\Theta = 2$.

Flow equation & asymptotic safety

Exact renormalization group equations (ERGE) derived from path-integral representation (Wetterich '93)

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\{[\Gamma_k^{(2)}[\Phi] + R_k]^{-1}(\partial_t R_k)\}, \quad \partial_t = k\frac{d}{dk}$$

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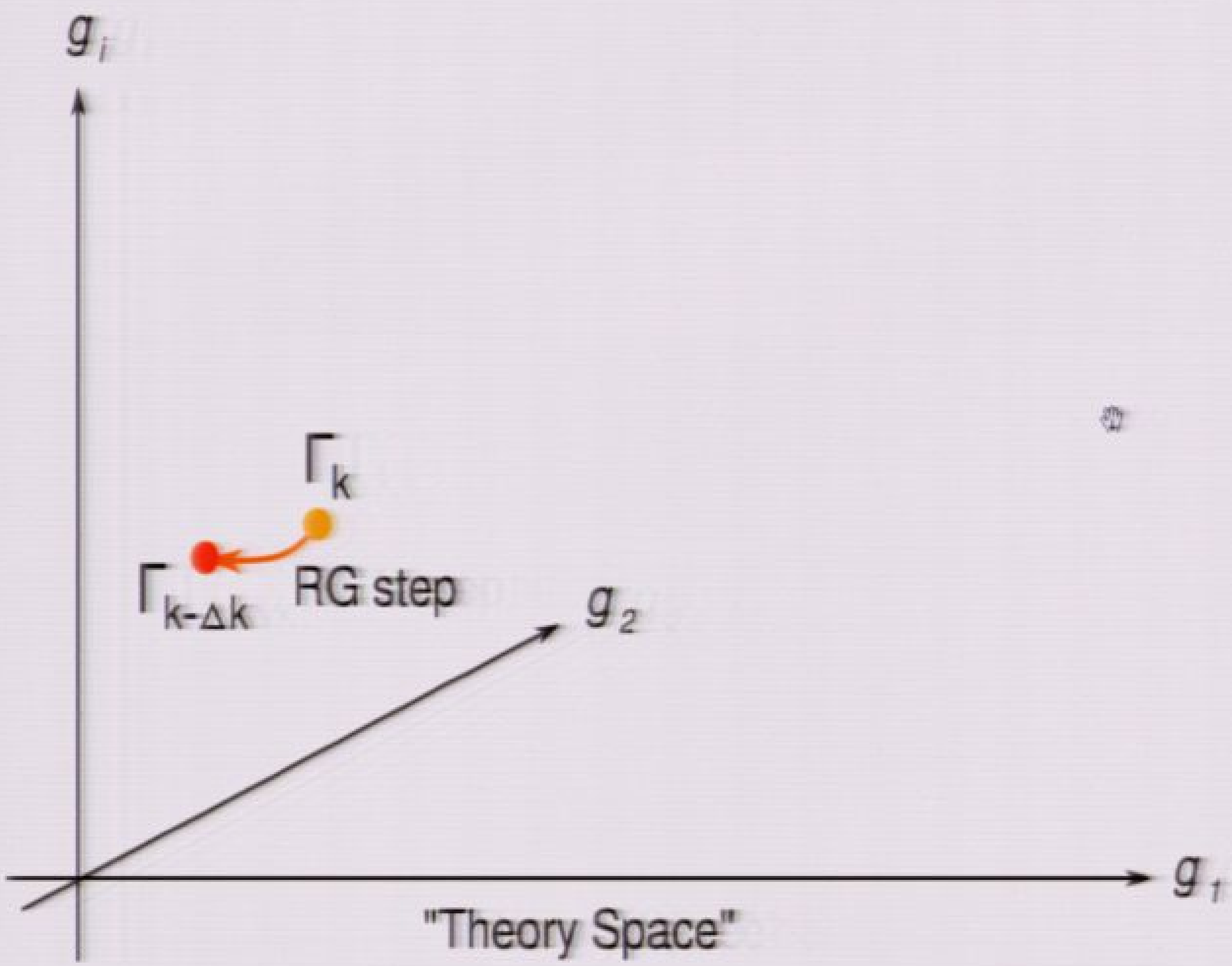
- Re $\Theta_I < 0$: irrelevant coupling (prediction for physical observable in the IR)

Asymptotic safety & theory space

Effective average action: $\Gamma_k[\chi] = \sum_i g_{i,k}\mathcal{O}_i$. Scale dependence: $\partial_t\Gamma_k[\chi] = \sum_i \beta_{i,k}\mathcal{O}_i$.



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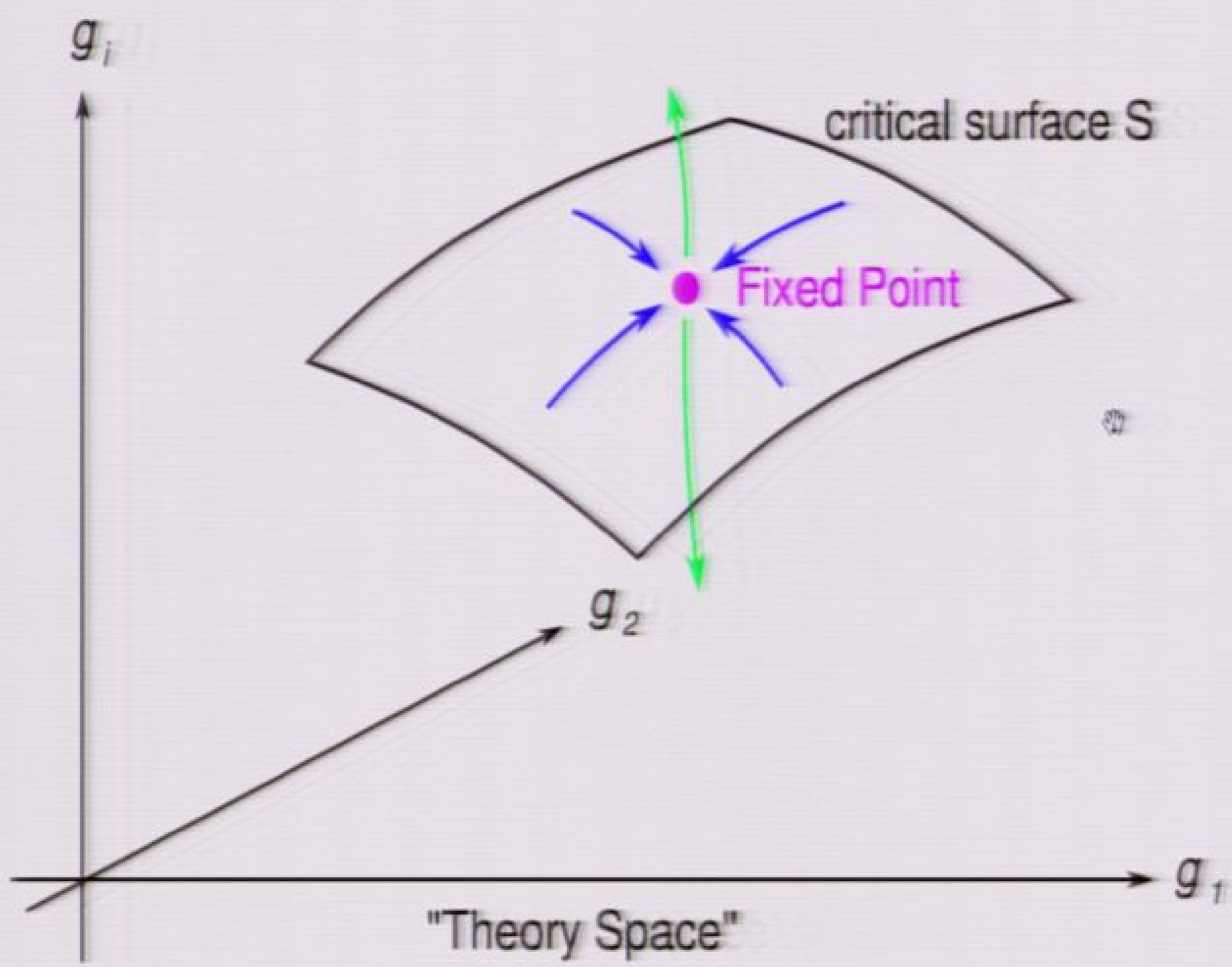

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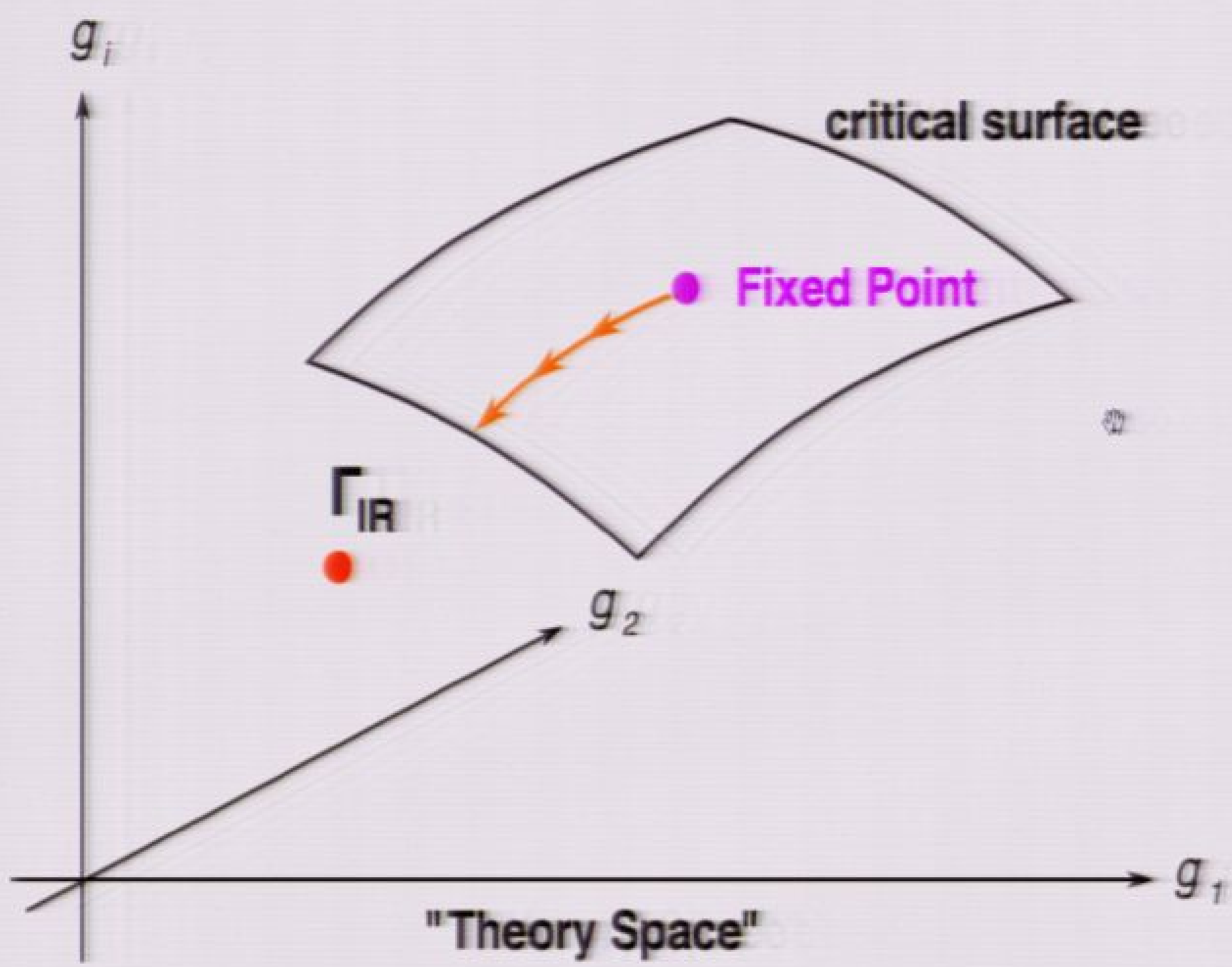
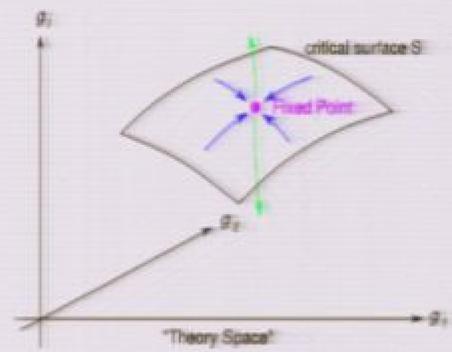


Triviality & hierarchy problem in the asymptotic safety scenario



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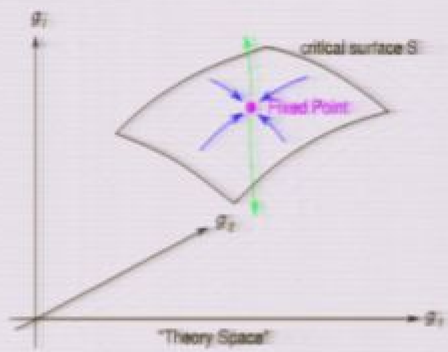

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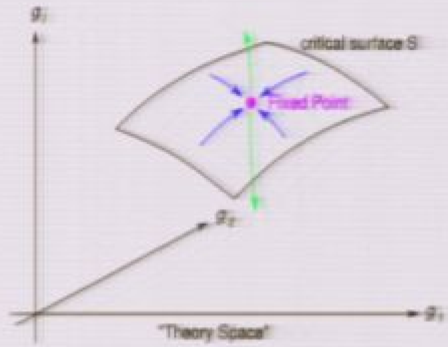

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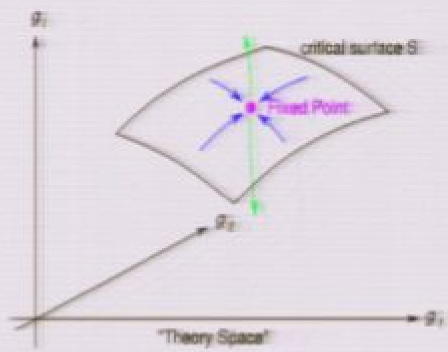

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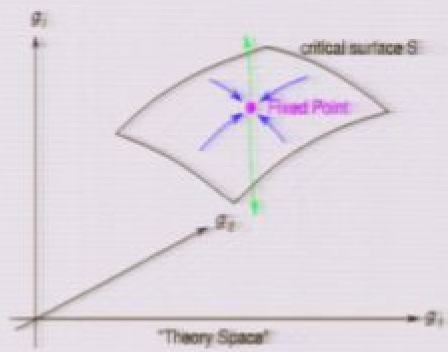

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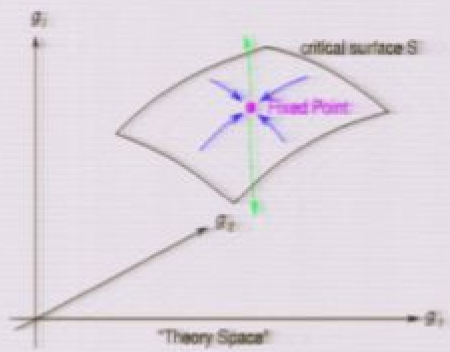

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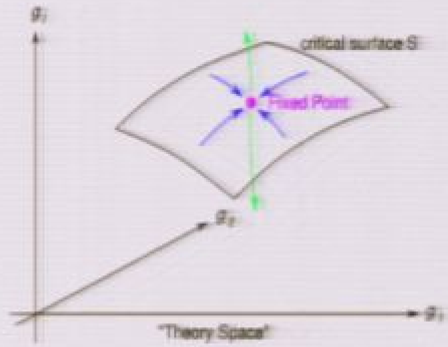

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- If all of them are small $\ll 1$ then then the hierarchy problem is solved.

Toy model - Chiral Yukawa system without gauge bosons

Derivative expansion, leading-order truncation

$$\Gamma_k = \int d^d x \Big\{ i(\bar{\psi}_L^a \partial\!\!\!/ \psi_L^a + \bar{\psi}_R \partial\!\!\!/ \psi_R) + (\partial_\mu \phi^{a\dagger})(\partial^\mu \phi^a) + U_k(\phi^{a\dagger}\phi^a) + \bar{h}_k \bar{\psi}_R \phi^{a\dagger} \psi_L^a - \bar{h}_k \bar{\psi}_L^a \phi^a \psi_R \Big\}$$

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- define $\rho = \phi^{a\dagger}\phi^a$.
- dimensionless quantities:
  $\tilde{\rho} = k^{2-d}\rho, \quad h^2 = k^{d-4}\bar{h}_k^2, \quad u(\tilde{\rho}) = k^{-d} U_k(\rho)|_{\rho = k^{d-2}\tilde{\rho}}$

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For the regime with spontaneously broken symmetry (SSB), we expand the effective potential about its minimum:
$\kappa := \tilde{\rho}_{\text{min}} > 0$,

$$u = \frac{\lambda_2}{2!}(\tilde{\rho} - \kappa)^2 + \frac{\lambda_3}{3!}(\tilde{\rho} - \kappa)^3 + \ldots$$

$\kappa, \lambda_{n_{\max}}, \lambda_2 > 0$.

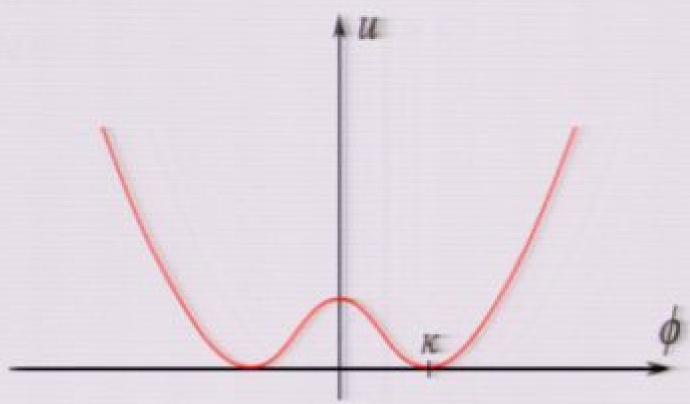
Toy model - Chiral Yukawa system without gauge bosons

Derivative expansion, leading-order truncation

$$\Gamma_k = \int d^dx \Big\{ i(\bar{\psi}_L^a \partial\!\!\!/ \psi_L^a + \bar{\psi}_R \partial\!\!\!/ \psi_R) + (\partial_\mu \phi^{a\dagger})(\partial^\mu \phi^a)$$
$$+ U_k(\phi^{a\dagger}\phi^a) + \bar{h}_k \bar{\psi}_R \phi^{a\dagger} \psi_L^a - \bar{h}_k \bar{\psi}_L^a \phi^a \psi_R \Big\}$$

- N_{L} left-handed fermions ψ_{L}^a
- one right-handed fermion ψ_{R}
- N_{L} complex bosons ϕ^a
- invariant under chiral $U(N_{\mathrm{L}})_{\mathrm{L}} \otimes U(1)_{\mathrm{R}}$ transformations
- define $\rho = \phi^{a\dagger}\phi^a$.
- dimensionless quantities:
  $\tilde{\rho} = k^{2-d}\rho, \quad h^2 = k^{d-4}\bar{h}_k^2, \quad u(\tilde{\rho}) = k^{-d} U_k(\rho)|_{\rho = k^{d-2}\tilde{\rho}}$

For the regime with spontaneously broken symmetry (SSB), we expand the effective potential about its minimum:
$\kappa := \tilde{\rho}_{\mathrm{min}} > 0$,



$$u = \frac{\lambda_2}{2!}(\tilde{\rho} - \kappa)^2 + \frac{\lambda_3}{3!}(\tilde{\rho} - \kappa)^3 + \ldots$$

$\kappa, \lambda_{n_{\max}}, \lambda_2 > 0.$

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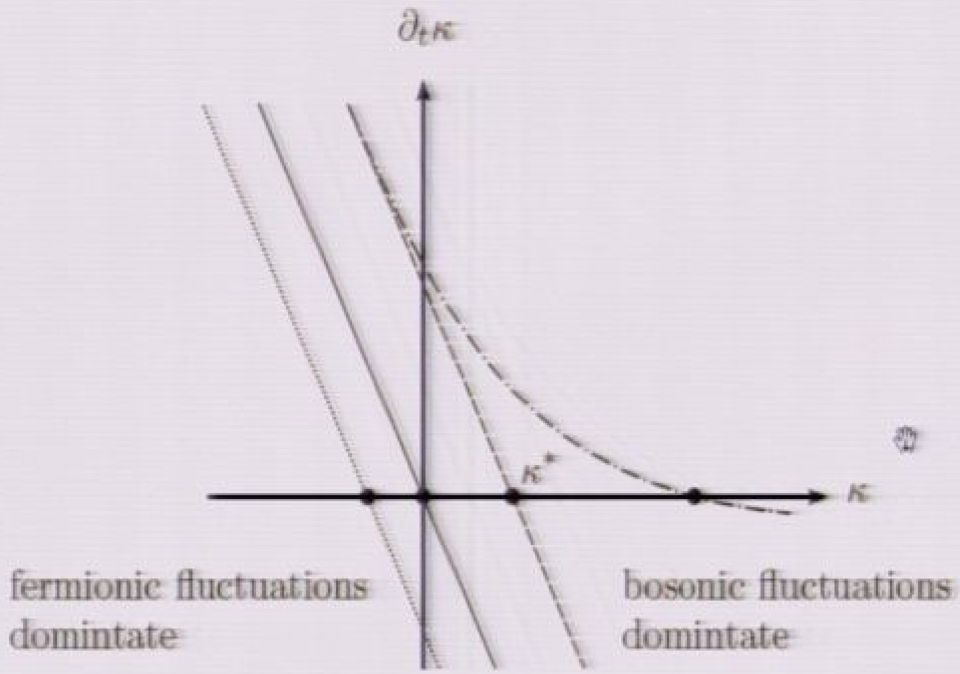

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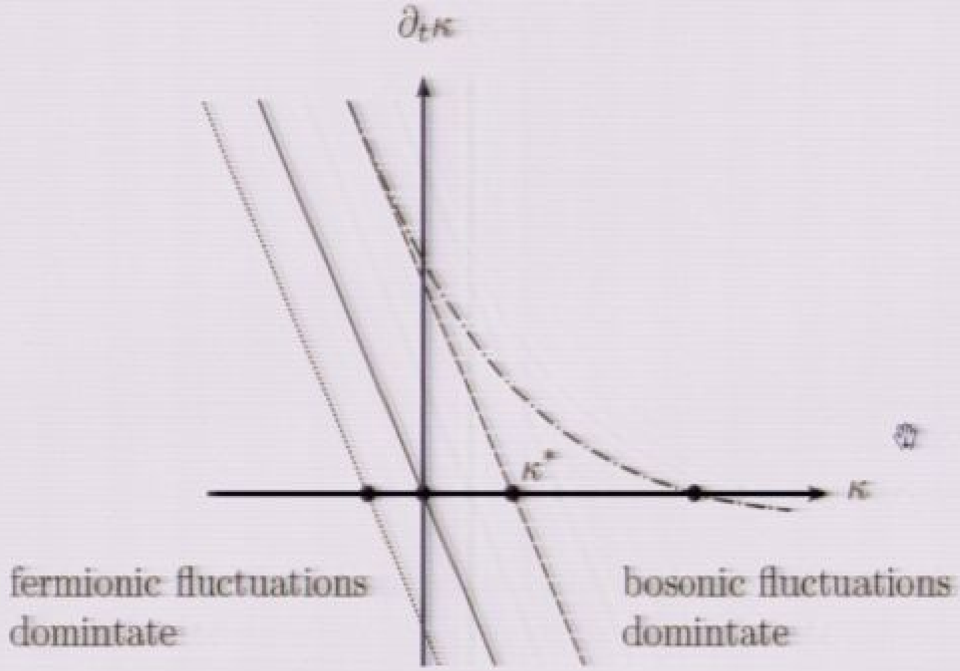

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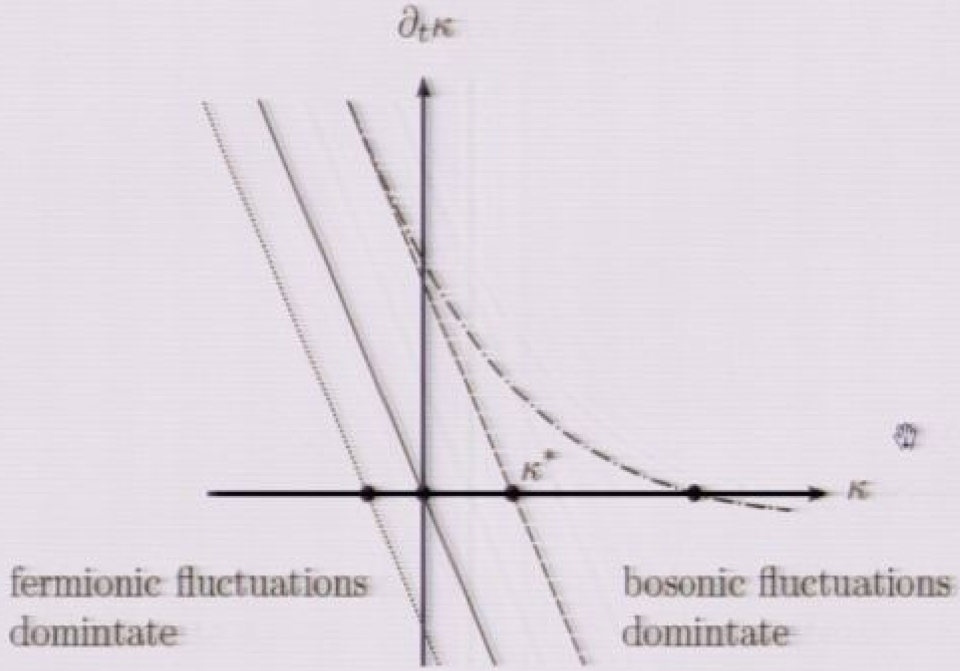

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- a suitable κ-dependence flattens the β-function near the fixed-point, which reduces the hierachy problem
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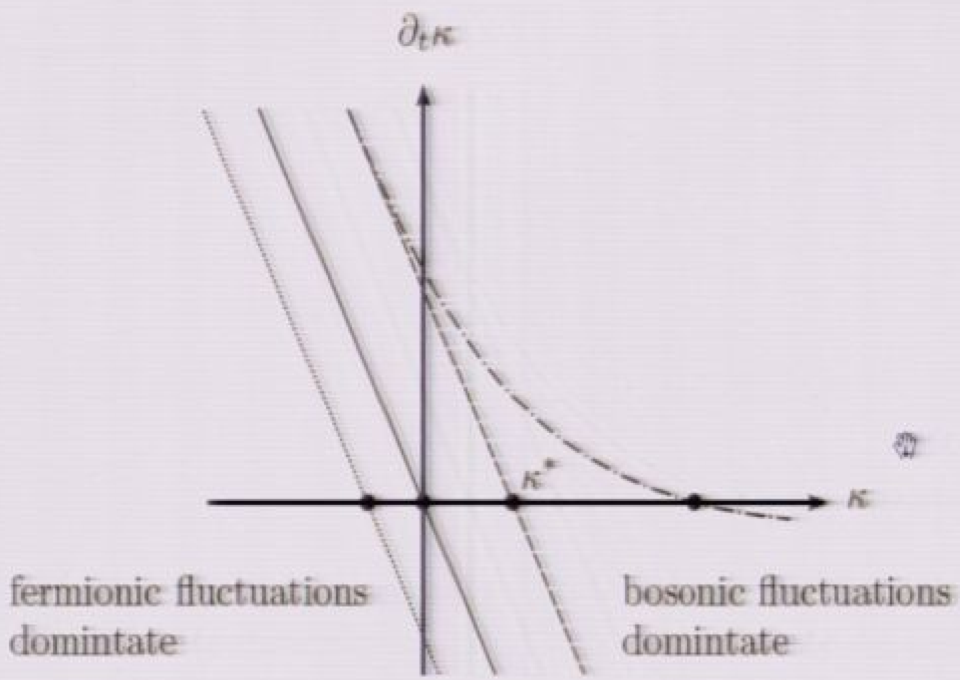

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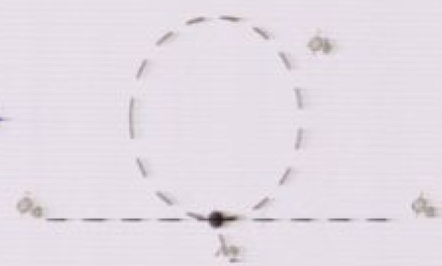
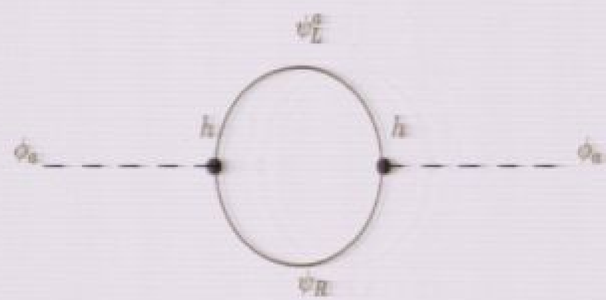
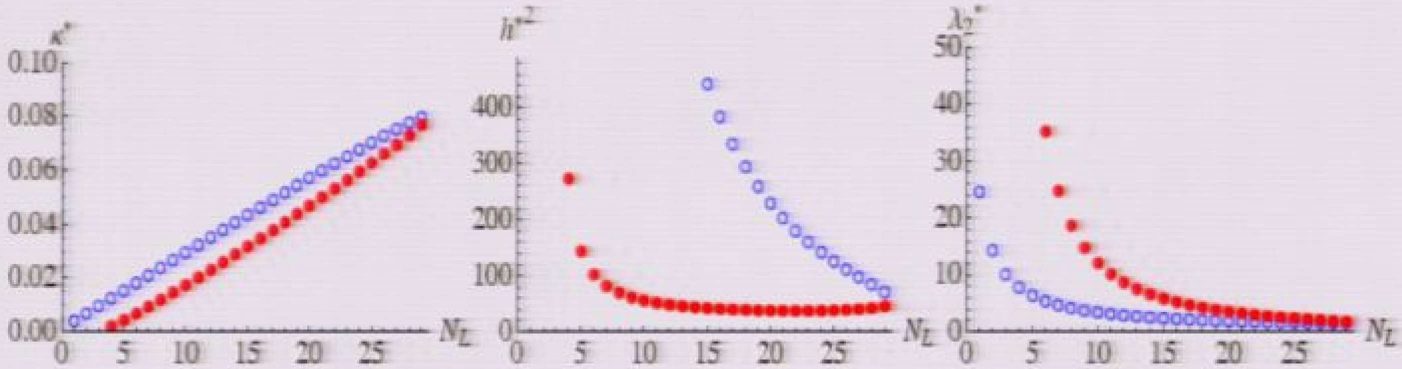
$\beta_\kappa = -2\kappa +$  $-$ 

Fixed-points and critical exponents

We find a NGFPs for $1 \leq N_L \leq 57$



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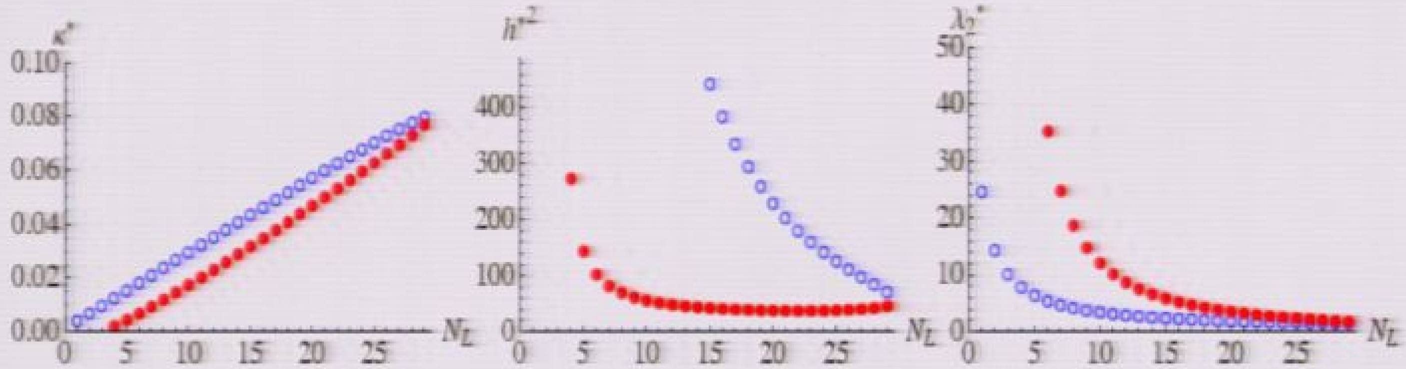

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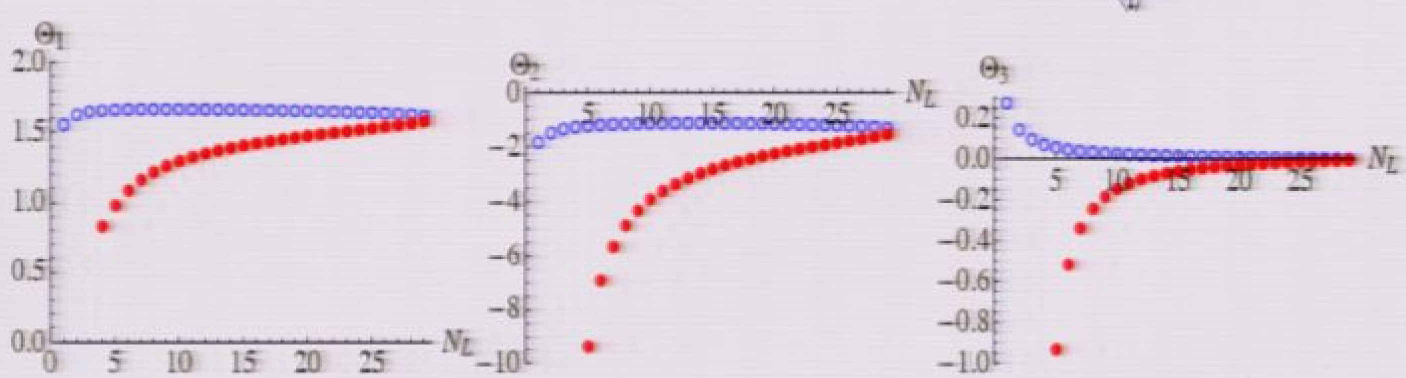

UV fixed-point regime for $N_{\rm L} = 10$

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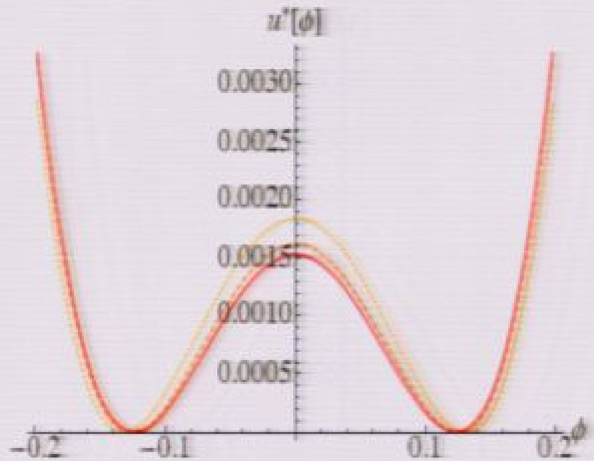

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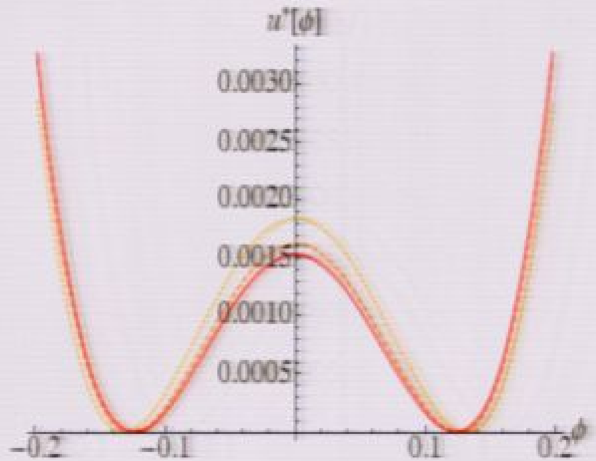

$$\text{FP}: \quad \kappa^* = 0.0152, \quad \lambda^* = 12.13, \quad h^{*2} = 57.41\,,$$
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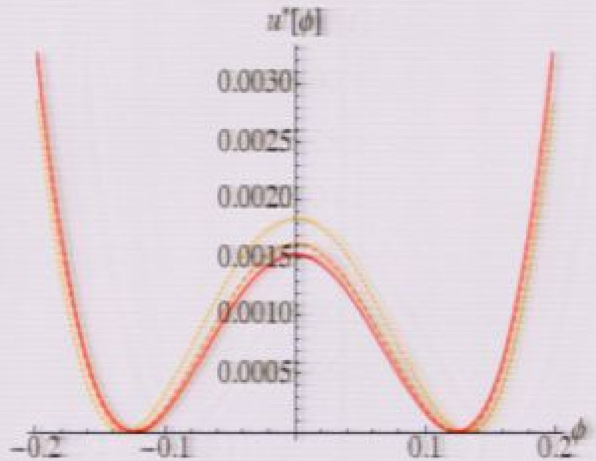

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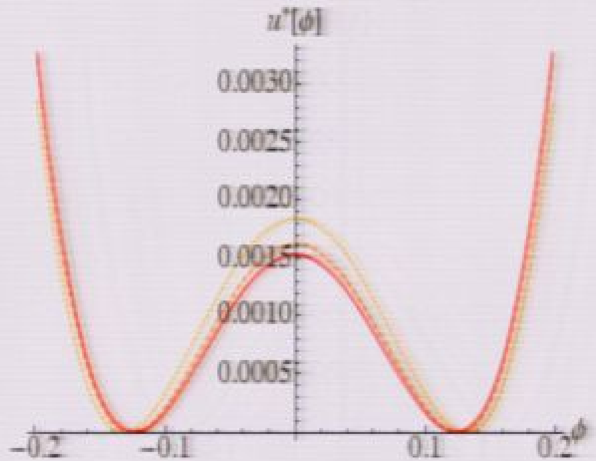

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- The real part of the relevant direction is 1.056 and not anymore 2 → Hierarchy problem weaker

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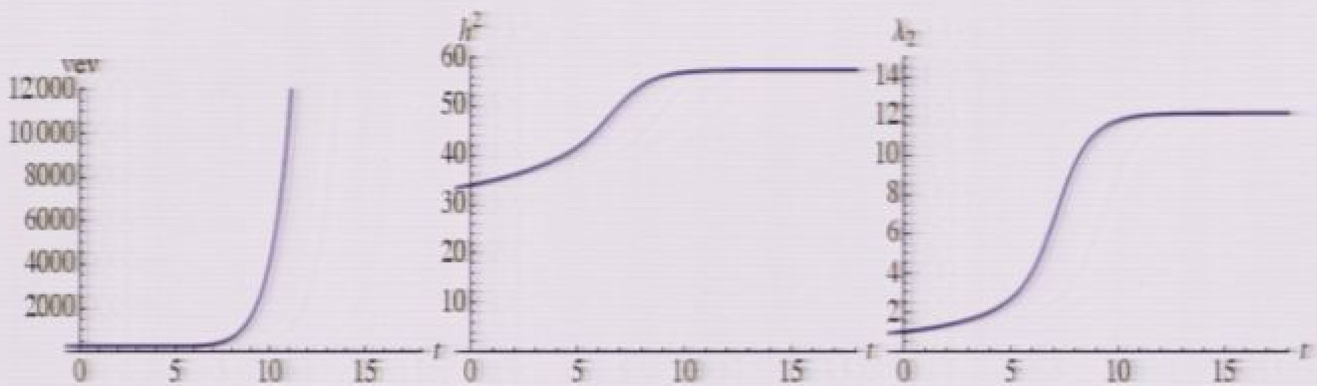

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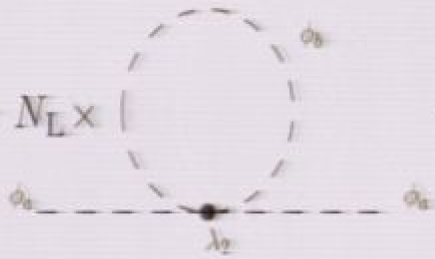

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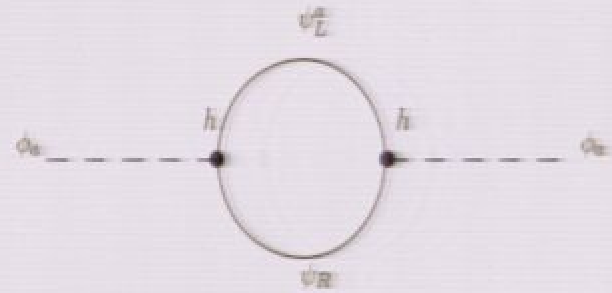

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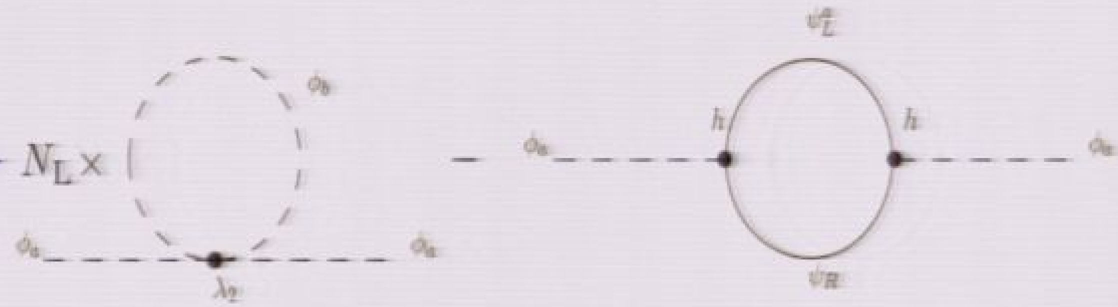

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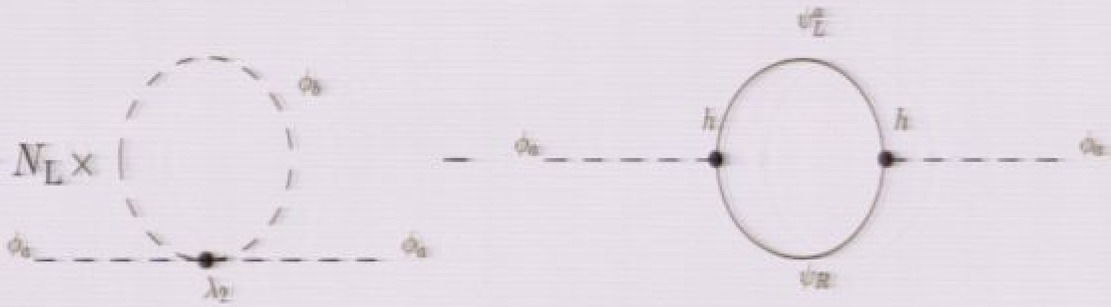

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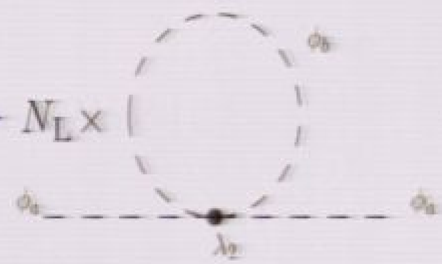
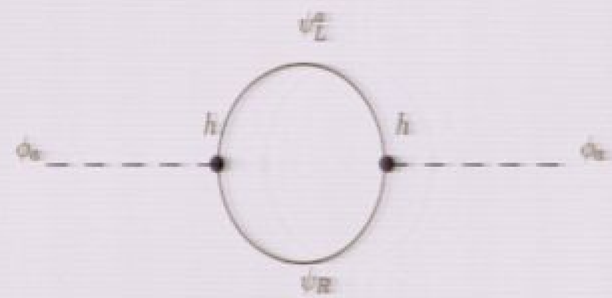
$\beta_\kappa = -2\kappa + N_{\mathrm{L}} \times \Big($  $-$  $\Big)$

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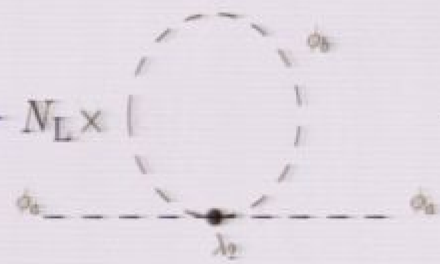

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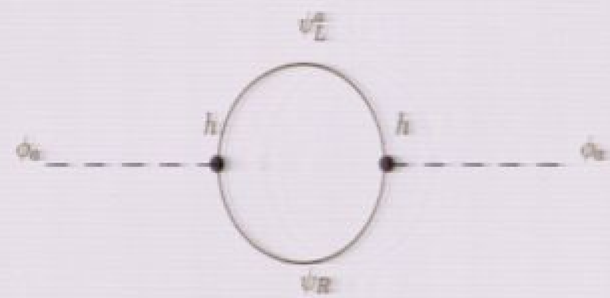

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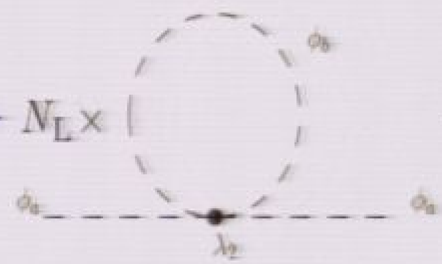
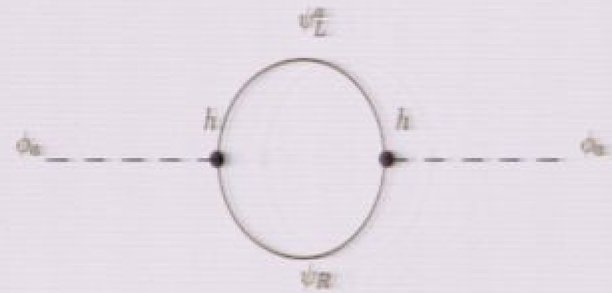
$\beta_\kappa = -2\kappa + N_{\mathrm{L}} \times$ (

$-$ 

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- Next step: Include $SU(N_{\mathrm{L}})$ gauge bosons (work in progress with H. Gies and S. Rechenberger)

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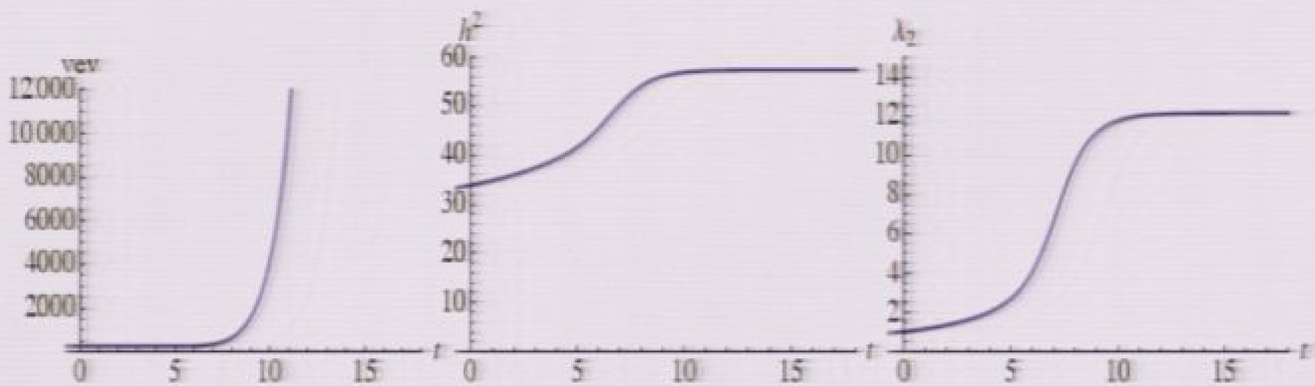

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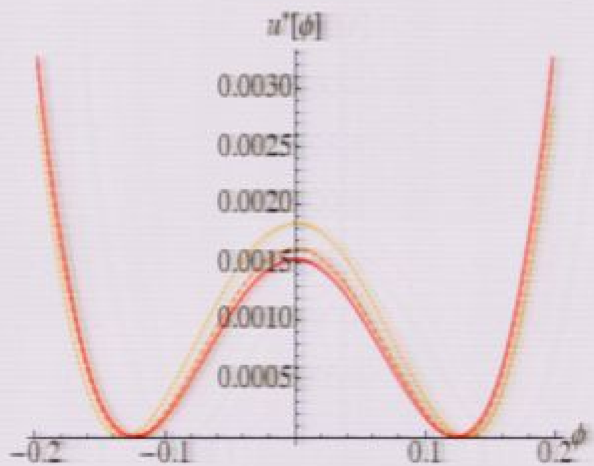

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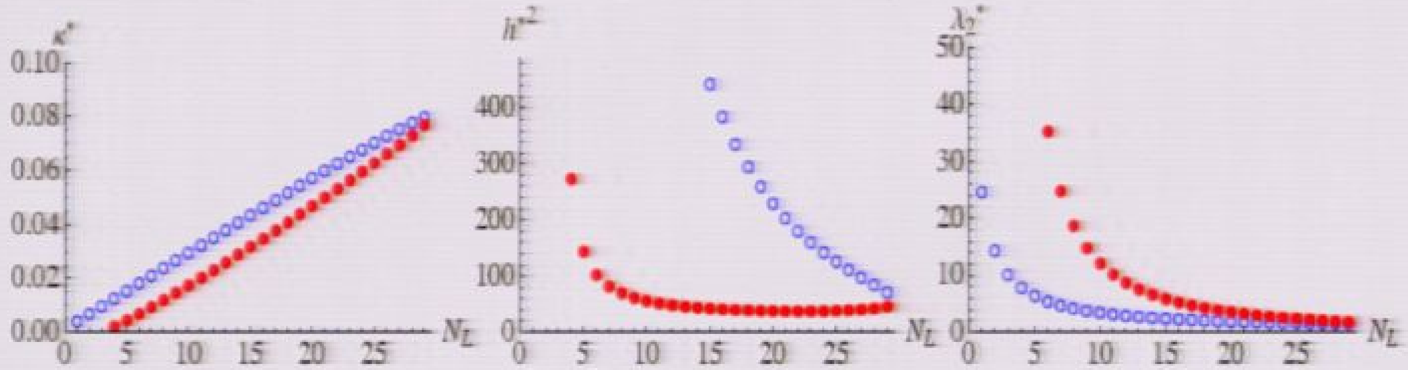

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