Title: Renormalization Group Flow in Scalar-Tensor Theories

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Abstract:

- Recent results on systems with gravitational and scalar fields nonminimally coupled

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  - Renormalization group flow approaches a UV fixed point
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  - Renormalization group flow approaches a UV fixed point
  - The UV critical surface has finite dimension
- Fixed point with vanishing matter interactions
- Recursive relations among critical exponents
- Five dimensional UV critical surface

Functional Renormalization Group Equation

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left(\frac{\delta^2\Gamma_k}{\delta\Phi\delta\Phi} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k$$

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- Solve, find fixed point, calculate stability matrix and its eigenvalues

$$\Gamma_k[\phi] = \sum_i g_i(k)\mathcal{O}^i[\phi] = \tilde{g}_i k^{d_i}\mathcal{O}^i[\phi]$$

$$k\frac{\partial}{\partial k}\tilde{g}_i(k) = \beta_i \,;\, \beta_i(\tilde{g}_*) = 0$$

$$k\frac{\partial}{\partial k}\tilde{g}_i(k) = M_{ij}\left(\tilde{g}_j(k) - \tilde{g}_j^*\right),\ M_{ij} = \frac{\partial\beta_i(\tilde{g})}{\partial\tilde{g}_j}||_{\tilde{g}=\tilde{g}^*}$$

Pure gravity

- Λ, R, $R^2 \Rightarrow$ fixed point exists, but all couplings are relevant
- $f(R) = \sum_{i=0}^{n} g_i R^i \Rightarrow$ three relevant directions up to $n = 8$
- Relied on heat-kernel expansion on de Sitter background
  $\Rightarrow$ different couplings of higher curvature invariants are combined
- Expansion on Einstein-space background
  $\Rightarrow$ can distinguish R^2 and C^2
  $\Rightarrow$ three dimensional UV critical surface

Gravity and matter

- Scalar, Dirac, and Maxwell matter fields minimally coupled
- Gravitational corrections to Yukawa and Yang-Mills couplings (on flat background)
- Nonminimally coupled scalar fields (scalar-tensor theory)

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- Einstein-Hilbert and scalar fields (G. Narain, R. Percacci)
- $f(R)$ and scalar fields (G. Narain, C. R.)

Scalar-Tensor theory

$$\Gamma_k[g,\phi] = \int d^d x \sqrt{g} \left\{ F(\phi^2, R) + \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right\} + S_{GF} + S_{gh}$$

$$F(\phi^2, R) = V_0(\phi^2) + V_1(\phi^2) R + \cdots + V_p(\phi^2) R^p = \sum_{a=0}^{p} V_a(\phi^2) R^a$$

- Expand around spherical background
- Arbitrary potential terms, analytic and Taylor expandable around $\phi^2 = 0$

Example beta function

$$F(\phi^2, R) = \lambda_0^0 + \lambda_2^0\phi^2 + \lambda_4^0\phi^4 + \lambda_0^1 R + \lambda_2^1\phi^2 R + \lambda_4^1\phi^4 R$$

$$\partial_t\tilde{\lambda}_2^0 = -2\tilde{\lambda}_2^0 + \frac{1}{48\pi^2}\left[-\frac{2\tilde{\lambda}_2^1}{\tilde{\lambda}_0^1} - \frac{9(1+2\tilde{\lambda}_2^1)}{2(\tilde{\lambda}_0^1 - \tilde{\lambda}_0^0)} + \frac{6\tilde{\lambda}_0^1(1+2\tilde{\lambda}_2^1)}{(\tilde{\lambda}_0^1 - \tilde{\lambda}_0^0)^2} \right.$$
$$\left. -\frac{3(3\tilde{\lambda}_0^0 - \tilde{\lambda}_0^1)(1+2\tilde{\lambda}_2^1)^2}{(1+2\tilde{\lambda}_2^0)(\tilde{\lambda}_0^1 - \tilde{\lambda}_0^0)^2} - \frac{9(1+2\tilde{\lambda}_2^1)^2}{2(1+2\tilde{\lambda}_2^0)^2(\tilde{\lambda}_0^1 - \tilde{\lambda}_0^0)} - \frac{18\tilde{\lambda}_4^0}{(1+2\tilde{\lambda}_2^0)^2}\right] + \dots$$

- Familiar beta function of the mass in ϕ^4 theory in flat space
- $1/(1+2\tilde{\lambda}_2^0)$: threshold effects for the contribution of scalar loops
- $\tilde{\lambda}_0^1 - \tilde{\lambda}_0^0 \to (1 - 2\tilde{\Lambda}) \approx 1$ for $\Lambda \ll k^2$

Gaussian matter fixed point

- Matter interactions can become weak in UV
- At Gaussian matter fixed point $\tilde{V}_a$ are $\tilde{\phi}^2$-independent

$$\tilde{V}_a^{(i)}(0) = 0 \text{ for } i \geq 1$$

- Can be shown to exist for arbitrary potential

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- For specific form

$$V_a(\phi^2)R^a = \sum_{i=0}^{q} \lambda_{2i}^a(k)\phi^{2i}R^a \Rightarrow \tilde{\lambda}_{2i}^a(k) \to 0 \text{ for } i \geq 1$$

Example stability matrix

$$F(\phi^2, R) = \lambda_0^0 + \lambda_2^0\phi^2 + \lambda_4^0\phi^4 + \lambda_0^1 R + \lambda_2^1\phi^2 R + \lambda_4^1\phi^4 R$$

$$(\tilde{\lambda}_0^0 \to 0.0065, \tilde{\lambda}_0^1 \to -0.022)$$

$$M|_{\text{GMFP}} = \begin{pmatrix} -0.580436 & 1.56941 & -0.0057625 & 0.00171023 & 0 & 0 \\ -5.90409 & -4.40523 & -0.0027738 & -0.0083214 & 0 & 0 \\ 0. & 0. & 1.41956 & 1.56941 & -0.034575 & 0.0102614 \\ 0. & 0. & -5.90409 & -2.40523 & -0.0166428 & -0.0499284 \\ 0. & 0. & 0. & 0. & 3.41956 & 1.56941 \\ 0. & 0. & 0. & 0. & -5.90409 & -0.405229 \end{pmatrix}$$

$$(-2.49 \pm 2.37i, -0.49 \pm 2.37i, 1.51 \pm 2.37i)$$

$$M_{12} = 2 \times 3 \times M_{01}$$

Linearized flow

$$M = \begin{pmatrix} M_{00} & M_{01} & 0 & 0 & \cdots \\ 0 & M_{11} & M_{12} & 0 & \ddots \\ 0 & 0 & M_{22} & M_{23} & \ddots \\ 0 & 0 & 0 & M_{33} & \ddots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \text{ with } M_{ij} = \begin{pmatrix} \frac{\partial \beta_{2i}^{(0)}}{\partial \tilde{\lambda}_{2j}^{(0)}} & \cdots & \frac{\partial \beta_{2i}^{(0)}}{\partial \tilde{\lambda}_{2j}^{(p)}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \beta_{2i}^{(p)}}{\partial \tilde{\lambda}_{2j}^{(0)}} & \cdots & \frac{\partial \beta_{2i}^{(p)}}{\partial \tilde{\lambda}_{2j}^{(p)}} \end{pmatrix}$$

$$M_{ii} = (d-2)i\,\mathbf{1} + M_{00} \; ; \qquad M_{i,i+1} = (i+1)(2i+1)M_{01}$$

$$\rho_{2i}^{(a)} = \rho_0^{(a)} + (d-2)\,i$$

- Gravitational corrections to the scaling exponents are always the same

Position of the fixed point × 1000

p	$\tilde{\lambda}^{(0)}_{0*}$	$\tilde{\lambda}^{(1)}_{0*}$	$\tilde{\lambda}^{(2)}_{0*}$	$\tilde{\lambda}^{(3)}_{0*}$	$\tilde{\lambda}^{(4)}_{0*}$	$\tilde{\lambda}^{(5)}_{0*}$	$\tilde{\lambda}^{(6)}_{0*}$	$\tilde{\lambda}^{(7)}_{0*}$	$\tilde{\lambda}^{(8)}_{0*}$
1	6.495	-21.579							
2	5.224	-16.197	1.834						
3	6.454	-20.756	1.071	-6.474					
4	6.354	-21.342	0.792	-6.807	-3.865				
5	6.355	-21.339	0.793	-6.793	-3.854	-0.024			
6	6.312	-21.669	0.586	-7.169	-5.576	-0.537	2.702		
7	6.318	-21.702	0.534	-6.469	-5.530	-1.979	2.761	2.565	
8	6.344	-21.489	0.678	-5.922	-4.574	-2.074	1.863	2.393	0.829

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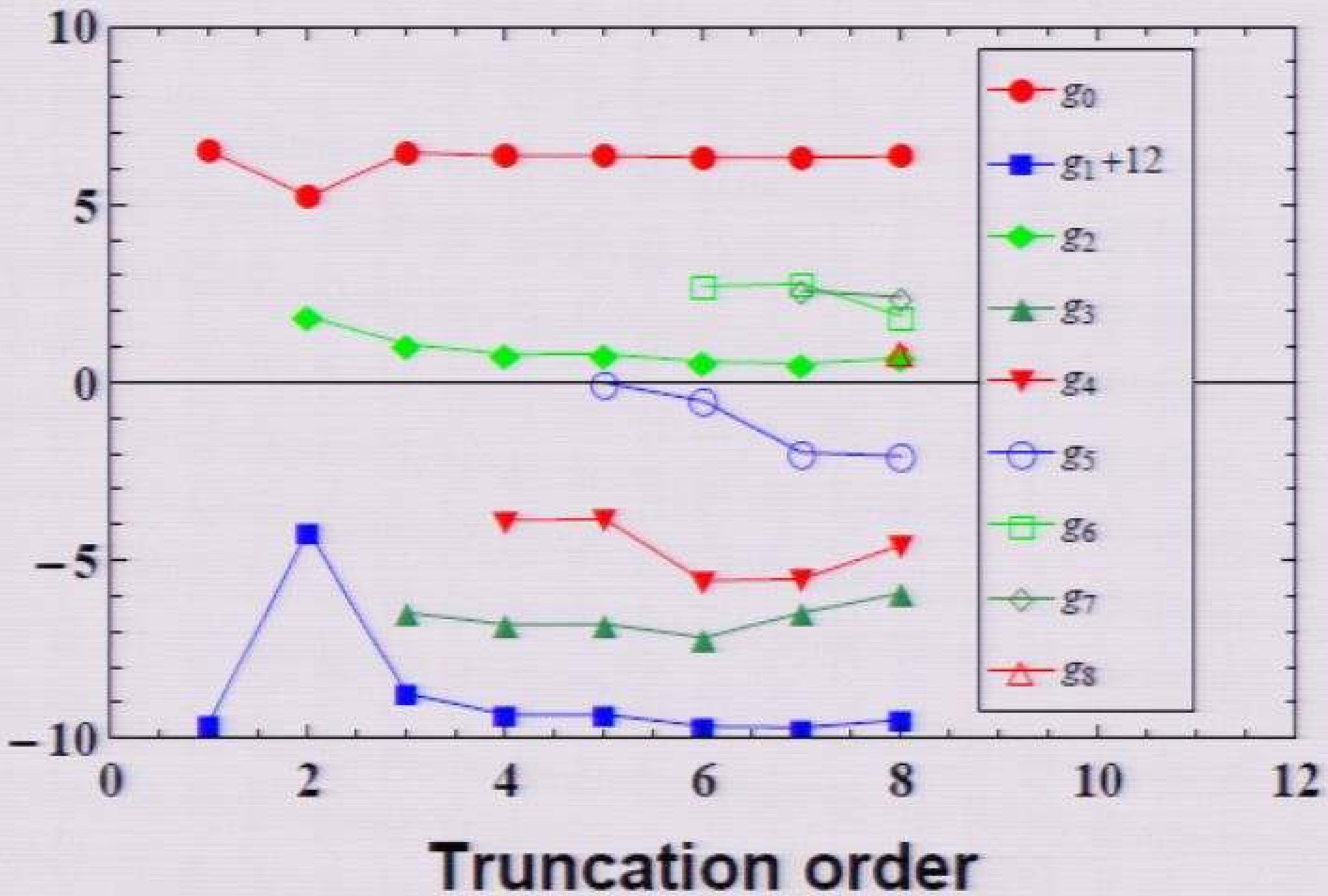
Critical exponents

p	ϑ'_0	ϑ''_0	$\vartheta^{(2)}_0$	$\vartheta^{(3)}_0$	$\vartheta^{(4)}_0$	$\vartheta^{(5)}_0$	$\vartheta^{(6)}_0$	$\vartheta^{(7)}_0$	$\vartheta^{(8)}_0$
1	2.493	2.368							
2	1.847	2.397	21.031						
3	3.077	2.524	2.033	-3.852					
4	3.261	2.772	1.670	-3.593	-5.182				
5	2.777	2.908	1.795	-4.176	-4.196	-6.764			
6	2.841	2.813	1.386	-4.000	-3.798	-5.947	-8.538		
7	2.930	2.964	1.312	-4.009	-2.760	-4.623	-7.459	-11.166	
8	2.331	2.902	1.570	-4.063	-0.673	-7.120	-7.323	-9.854	-11.611

Fixed point values × 1000



Position of the fixed point $\times$ 1000

p	$\tilde{\lambda}_{0*}^{(0)}$	$\tilde{\lambda}_{0*}^{(1)}$	$\tilde{\lambda}_{0*}^{(2)}$	$\tilde{\lambda}_{0*}^{(3)}$	$\tilde{\lambda}_{0*}^{(4)}$	$\tilde{\lambda}_{0*}^{(5)}$	$\tilde{\lambda}_{0*}^{(6)}$	$\tilde{\lambda}_{0*}^{(7)}$	$\tilde{\lambda}_{0*}^{(8)}$
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p	ϑ_0'	ϑ_0''	$\vartheta_0^{(2)}$	$\vartheta_0^{(3)}$	$\vartheta_0^{(4)}$	$\vartheta_0^{(5)}$	$\vartheta_0^{(6)}$	$\vartheta_0^{(7)}$	$\vartheta_0^{(8)}$
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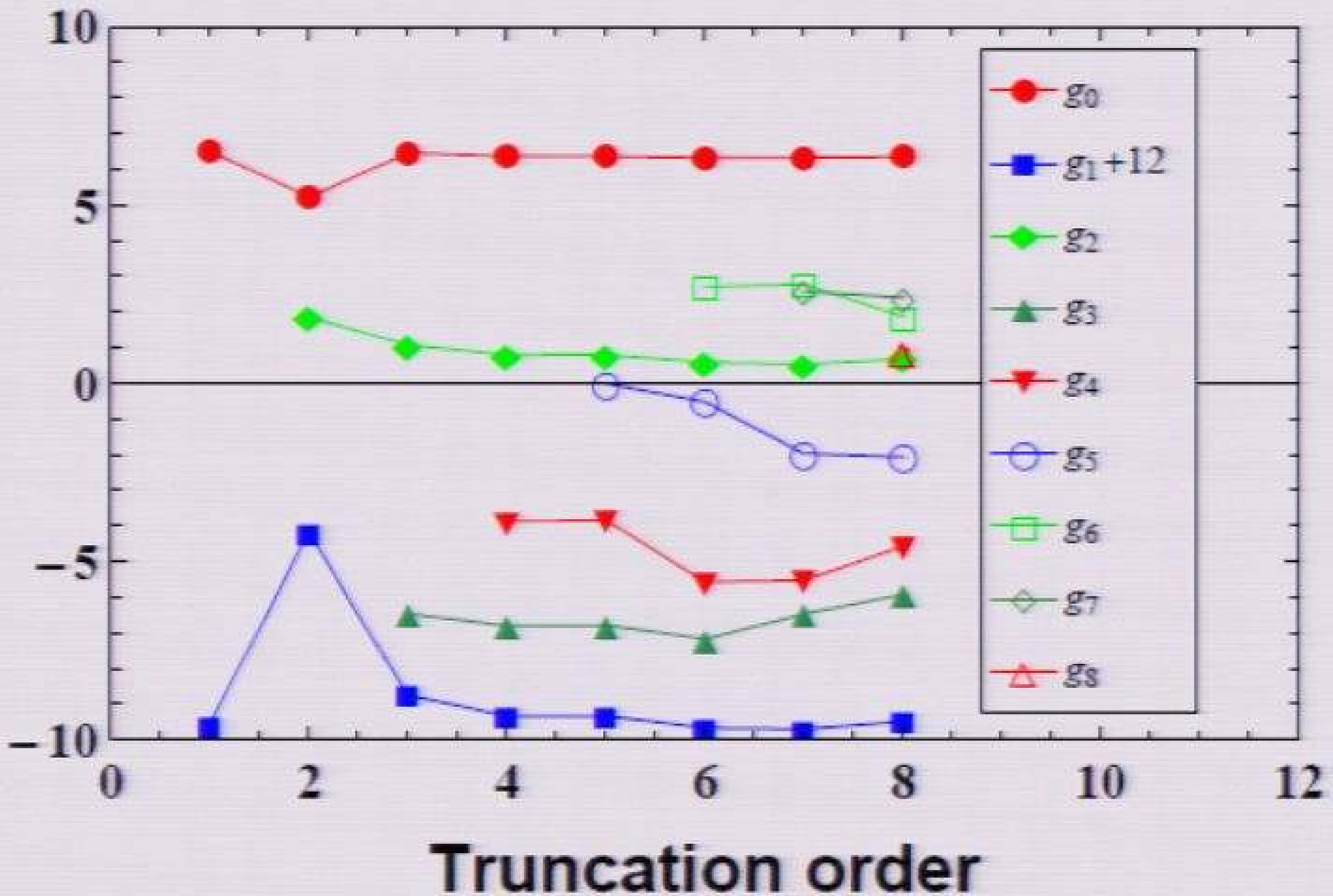

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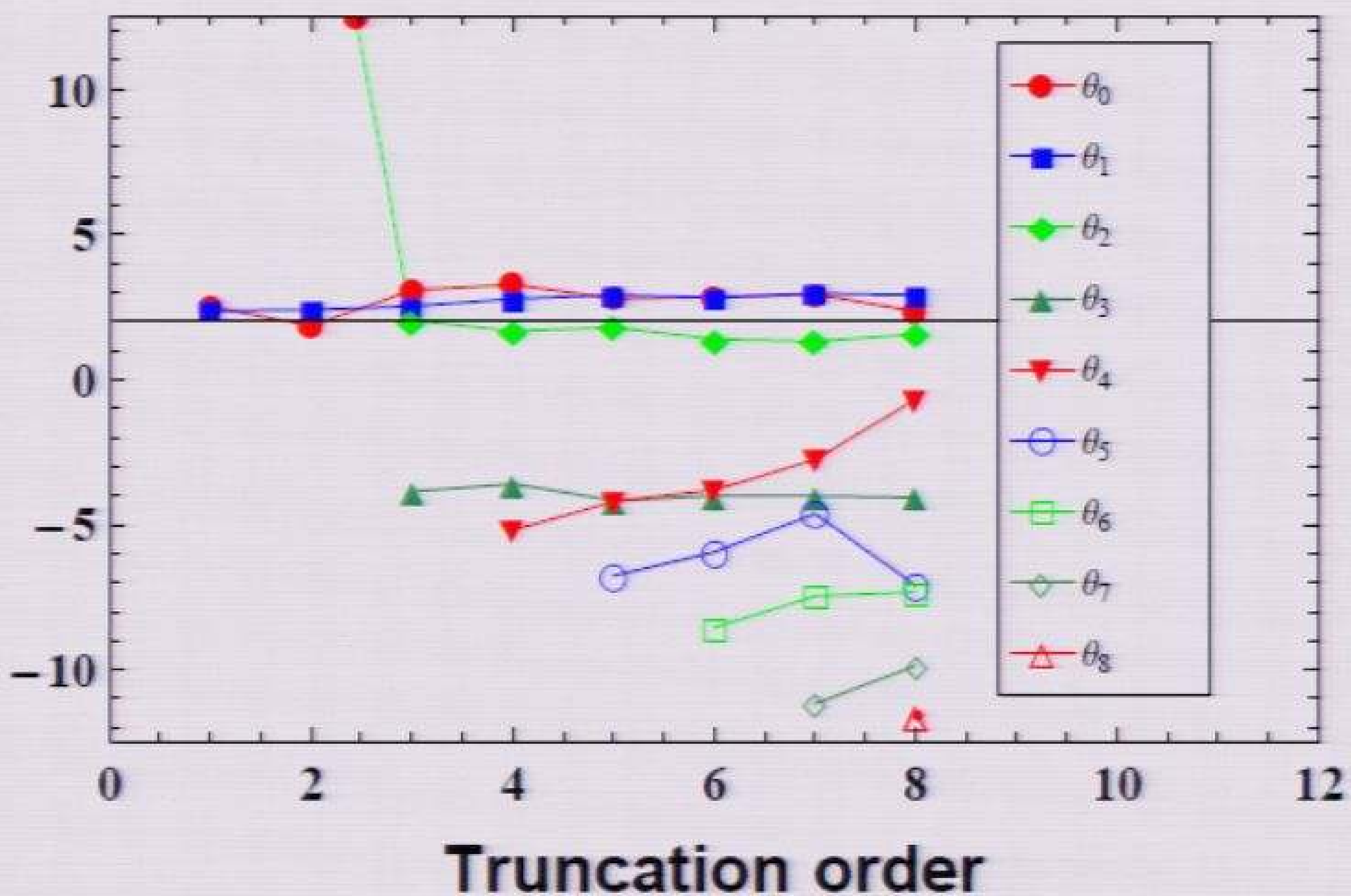

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- Determine the dimension of the UV critical surface of the full theory from the minimally coupled theory
- The UV critical surface remains finite dimensional