Title: The Reconstruction Problem in Asymptotically Safe Quantum Einstein Gravity

Date: Nov 06, 2009 04:30 PM

URL: http://pirsa.org/09110123

Abstract:

The Reconstruction Problem In Asymptotically Safe Quantum Einstein Gravity

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E.M., M. Reuter, Phys.Rev.D79:025008,2009. e-Print: arXiv:0811.3888 [hep-th]

Perimeter Institute, November 2009

Outline

- Motivation
- Bare vs Effective Action
- Quantum Einstein Gravity
- Summary

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Motivation

- A priori we do not have a path integral representation which gives rise to a given RG trajectory of the EAA.

$$\Gamma_k \to S_\Lambda \to H_\Lambda$$

- Use of Standard Techniques in QFT: Many properties are better described in a path integral setting (symmetries, implementation of constraints)
- Relation with other approaches: LQG, dynamical triangulations, where the bare action/Hamiltonian plays a central role.

Effective Average Action Γ_k

- Γ_k is an effective action with a built-in infrared cutoff.
- The Green functions depend on k
- No further Integrations (mean fields)
- A QFT is defined by complete RG solutions well defined $\forall k \in [0, \infty)$

Bare Action S_Λ

- S_Λ is a set of actions parametrized by Λ for the same system.
- Green functions independent of Λ after integration over low momenta
- To be used inside of a functional integral.
- A QFT is defined by S_Λ together with a regularized path integral (measure)

What is the/a path integral $\int \mathcal{D}_\Lambda e^{-S_\Lambda}$ such that it reproduces $k \mapsto \Gamma_k$ in the limit $\Lambda \to \infty$?

Claim:

- Using a UV regularization scheme, the information encoded in Γ_k is sufficient to determine S_Λ by deducing how the bare couplings must behave in the UV limit.

EAA with cutoff

For simplicity, take a scalar field χ on a d-dimensional Euclidean space. Let $-p^2$ be the eigenvalues of the Laplacian $\Box$.

- The UV cutoff Λ is implemented such that $|p| \equiv \sqrt{p^2} \leq \Lambda$
- The UV regulated $W_{k,\Lambda}$ is defined as

$$e^{W_{k,\Lambda}} = \int \mathcal{D}_\Lambda \chi e^{-S_\Lambda[\chi] - \Delta_k S[\chi] + \int J \cdot \chi} \tag{1}$$

Here $\chi = \sum_{|p| \in [0,\Lambda]} \chi_p e^{ixp}$ and $\mathcal{D}_\Lambda \chi = \prod_{|p| \in [0,\Lambda]} \int \mathrm{d}\chi_p M^{-[\chi_p]}$

- The EAA is defined now with a UV cutoff:

$$\Gamma_{k,\Lambda}[\phi] = \int \phi J - W_{k,\Lambda} - \Delta_k S[\phi] \tag{2}$$

where $\phi = \langle \chi \rangle$ is regularized as well

- The functional renormalization group equation (FRGE) is therefore

$$k\partial_k \Gamma_{k,\Lambda}[\phi] = \frac{1}{2}\mathrm{Tr}_\Lambda\left[\left(\Gamma^{(2)}_{k,\Lambda} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right] \tag{3}$$

where $\mathrm{Tr}_\Lambda = \mathrm{Tr}[\theta(\Lambda^2 - p^2)(\cdots)]$

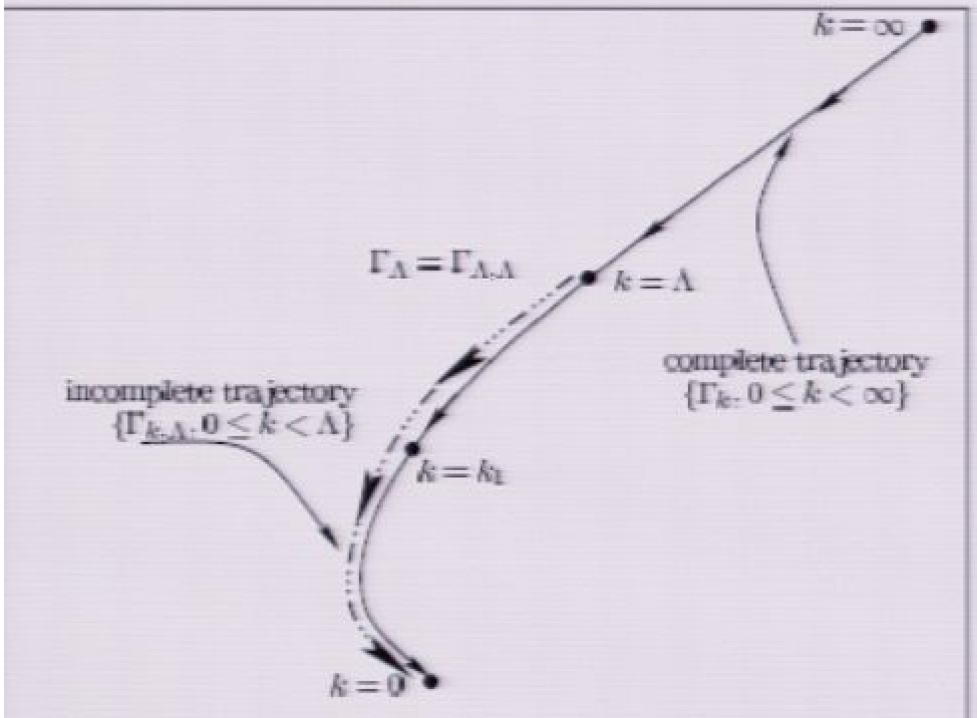
- the EAA also satisfies the integro differential equation

$$e^{-\Gamma_{k,\Lambda}[\phi]} = \int \mathcal{D}_\Lambda f \; e^{-S_{tot}} \tag{4}$$

with

$$S_{tot} = S_\Lambda[f+\phi] - \int f \frac{\delta}{\delta\phi}\Gamma_{k,\Lambda}[\phi] + \int f \mathcal{R}_k f$$

How are $\Gamma_{k,\Lambda}$ and Γ_k related?



Central Result

- Under certain conditions, a solution $\{\Gamma_{k,\Lambda}, \quad 0 \leq k < \Lambda\}$ is the restriction of a solution $\{\Gamma_k, \quad 0 \leq k < \infty\}$ to the FRGE without a UV cutoff:

$$\Gamma_{k,\Lambda} = \Gamma_k, \quad 0 \leq k < \Lambda \quad (5)$$

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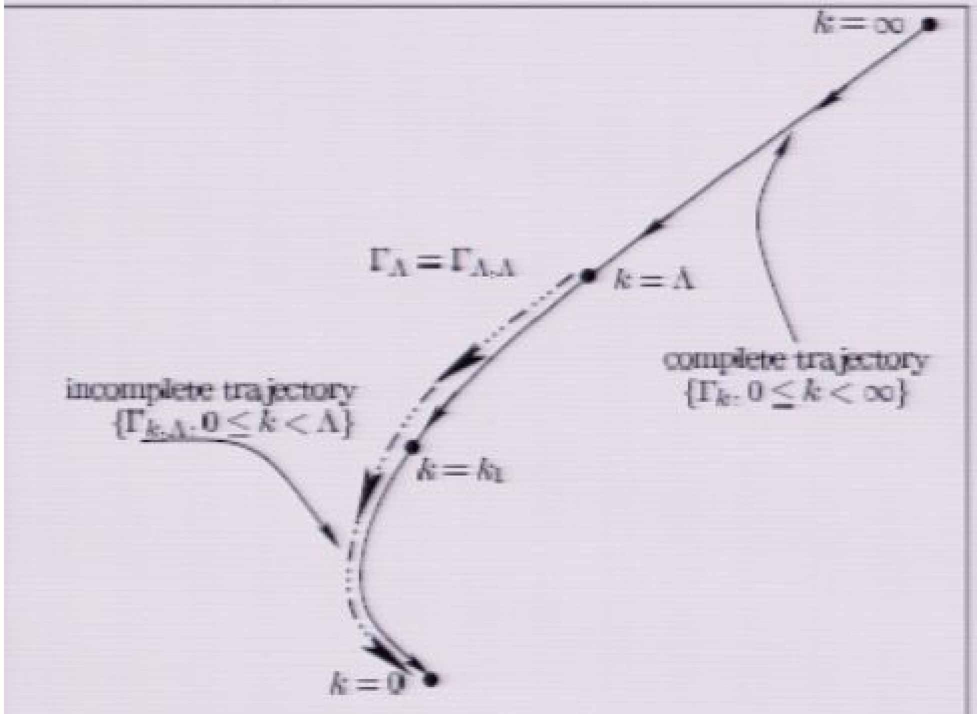

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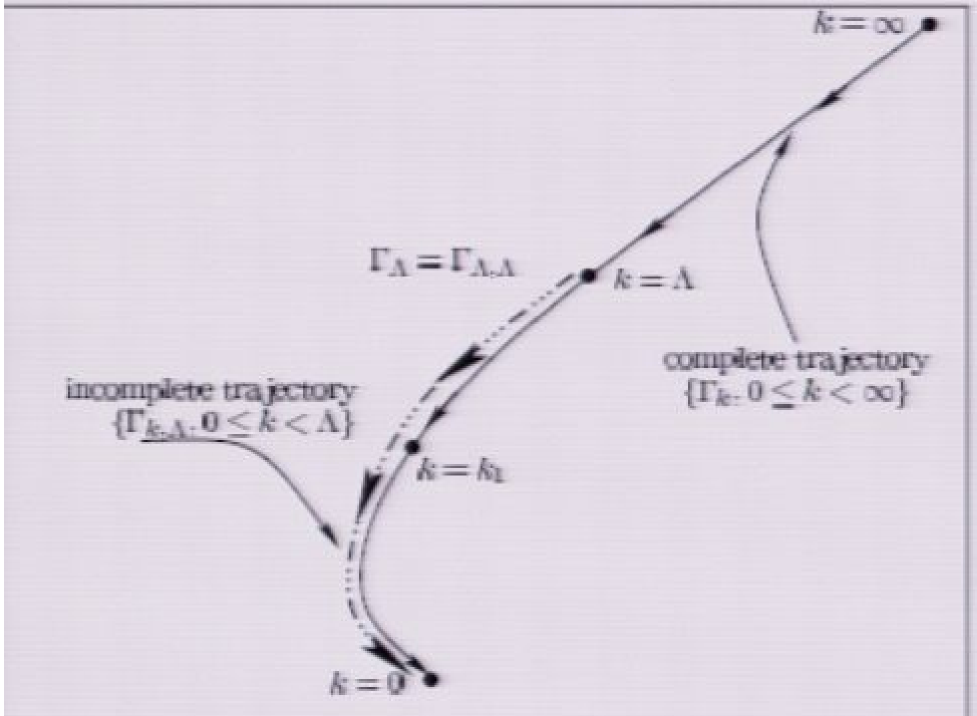

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How can we determine the corresponding Λ-dependence of S_Λ if we know a solution of the Λ-free flow equation?

- Solving eq.(4) for S_Λ we get the 1 loop formula

$$\Gamma_{k,\Lambda} - S_\Lambda = \frac{1}{2}\mathrm{Tr}_\Lambda\Big[\Big(S_\Lambda^{(2)} + \mathcal{R}_k\Big)M^{-2}\Big] \tag{6}$$

taking $k \to \Lambda$ we get the desired relation.

QEG and the Einstein-Hilbert truncation

We apply the strategy to QEG:

- Construct the EAA as usual with a UV regularized measure:

$$Z = \int \mathcal{D}_\Lambda h \, \mathcal{D}_\Lambda C \, \mathcal{D}_\Lambda \bar{C} \; e^{-S_\Lambda[h,C,\bar{C};\bar{g}] - \Delta_k S[h,C,\bar{C};\bar{g}]} \qquad (7)$$

- The UV cutoff is implemented by restricting the eigenvalues of the Laplacian $-\bar{D}^2$ to be smaller than Λ^2.
- Derivation of the EAA with UV cutoff follows the same ideas as before.

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Truncation→ Use Einstein Hilbert truncation for both Γ_k and S_Λ **and solve the 1-loop formula for the truncation**

$$\Gamma_k[g,\bar{g},\xi,\bar{\xi}] = -(16\pi G_k)-1\int\sqrt{g}(R(g)-2\bar{\lambda}_k) + S_{\mathrm{gh}}[g-\bar{g},\xi,\bar{\xi}] + S_{\mathrm{gf}}[g-\bar{g};\bar{g}]$$

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The map $(g, \lambda) \mapsto (\check{g}, \check{\lambda})$

$$\frac{1}{\check{g}}(3+2\check{\lambda}) - \frac{1}{g}(3+2\lambda) = \frac{3}{\pi}\frac{2-\check{\lambda}}{1-2\check{\lambda}}$$

$$\frac{\lambda}{g} - \frac{\check{\lambda}}{\check{g}} = \frac{1}{4\pi}\left[5\ln(1-2\check{\lambda}) - 5\ln\check{g} + Q\right] \qquad (9)$$

- Solved for $g > 0$ and $\lambda < 1/2$ and different values of Q. (correspond to different normalization of the measure)
- Bare trajectory has NGFP $(\check{g}_*(Q), \check{\lambda}_*(Q))$
- Flows are diffeomorphic near the NGFP $\rightarrow$ same critical exponents
- The "bare" GFP is located at the boundary of the domain $\rightarrow$ log-corrections to power low scaling.

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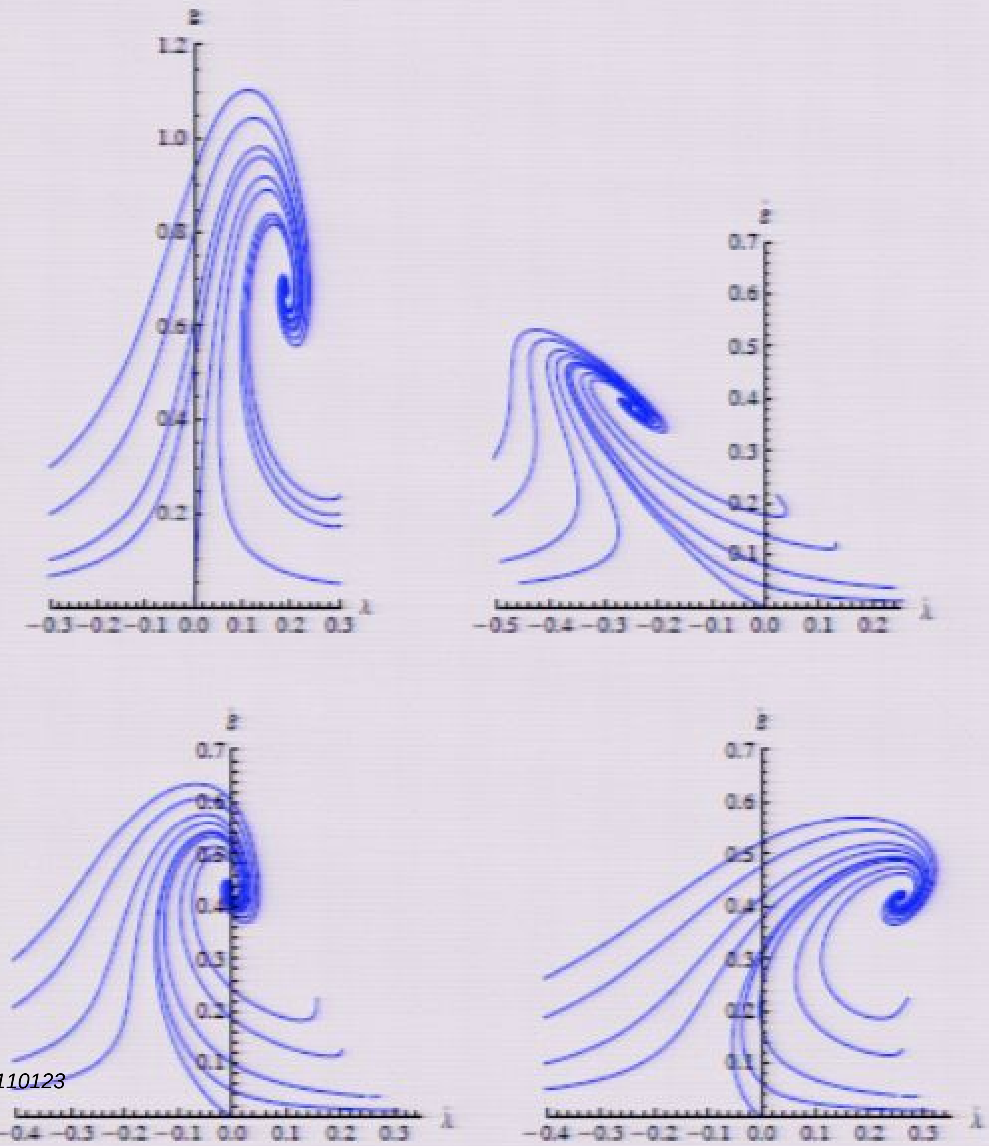

Phase portrait of the solutions

The diagram (a) shows the phase portrait of the effective RG flow on the (g, λ)-plane. The other diagrams are its image on the $(\check{g}, \check{\lambda})$-plane of bare parameters for three different values of Q, namely (b) $Q = +1$, (c) $Q = -0.1167$ where $\check{\lambda}_* = 0$, and (d) $Q = -1$, respectively.

Summary

- Reconstruction problem → given a UV regularization scheme and measure every solution of the FRGE gives rise to a bare action.
- Towards a Hamilton description → Identification of the d.o.f. we quantized.
- Contact with other approaches →(LQG, Causal dynamical calculations,....) correction term should be computed with the pertinent UV cutoff.