Title: General Relativity for Cosmology - Lecture 18B

Date: Nov 19, 2009 05:30 PM

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Abstract:

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Title 12/4/2005

GR for cosmology, Achim Kempf, Fall 2009, Lecture 22

Evolution of Friedmann - Lemaître spacetime

Follow solution strategy developed in previous lecture:

- Use the physical laws of matter to determine the equation of state:

$$p = p(\rho)$$

Terminology:

Periods of time in which the eqn. of state can be approximated as:

$$p(\rho) = w\rho \quad \text{with} \quad w = \text{const}$$

are called cosmic epochs:

$$w = \begin{cases} 0 & \text{dust: "matter dominated" epoch} \\ 1/3 & \text{radiation: "radiation dominated"} \end{cases}$$

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Use $p(\rho)$ to solve (see previous lecture)

$$\frac{d}{da}\left(\rho a^3\right) = -3p(\rho)\, a^2$$

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to obtain $\rho(a)$. Namely, in each epoch: to solve.

$$\frac{d}{da}\left(\rho(a)\, a^3\right) = -3a^2 w\, \rho(a)$$

Solution:

$$\boxed{\rho(a) = \rho_0\, a^{-3(w+1)}}$$

Recall special cases:

dilution of matter, i.e. energy proportional to $\frac{1}{\text{Volume}} \sim \frac{1}{a^3}$

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dilution of energy $\sim \frac{1}{\text{Volume}}$ and energy loss due to wavelength stretching $\sim \frac{1}{a}$

vacuum energy due to cosmological constant is of course constant.

$$\rho(a) = \begin{cases} \rho_m\, a^{-3} & \text{in matter-dominated epoch } (w=0) \\ \rho_r\, a^{-4} & \text{in radiation-dominated epoch } (w=1/3) \\ \rho_\Lambda\, a^{0} & \text{in dark energy-dominated epoch } (w=-1) \end{cases}$$

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Now use $\rho(a)$ to turn the Friedmann eqn. into an ordinary differential equation for $a(t)$:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3} \rho(a)$$

(we omit the Λ term by agreeing to incorporate Λ in the definition of ρ, p.)

Observational evidence: the universe is spatially flat $K=0$ in a good approximation.

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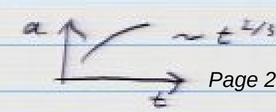

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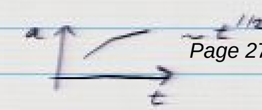

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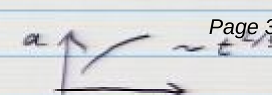

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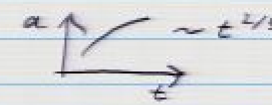

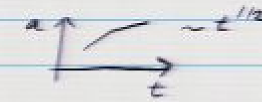

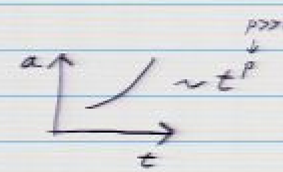

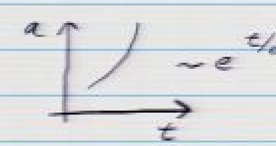

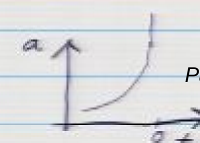

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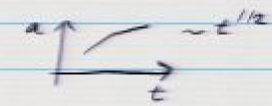

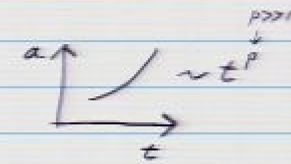

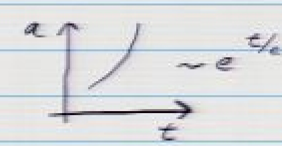

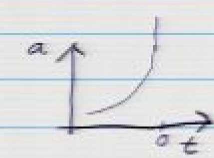

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Result for $k=0$:

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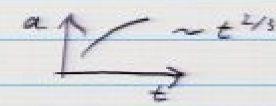

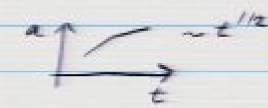

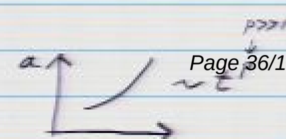

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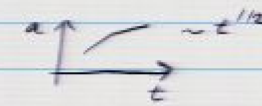

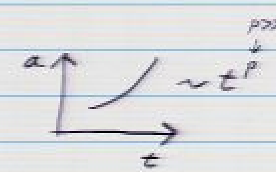

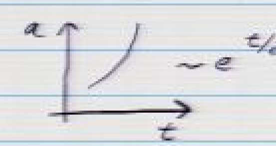

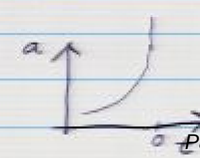

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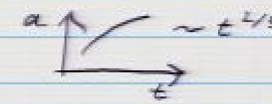

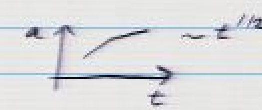

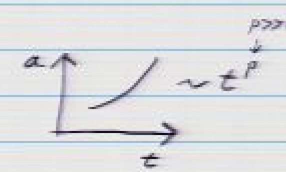

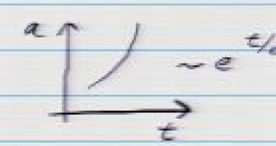

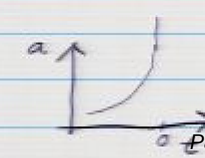

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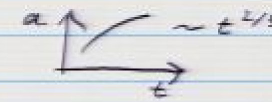

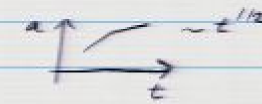

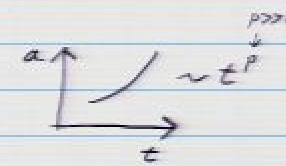

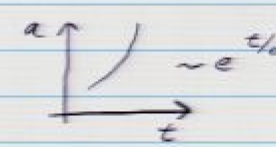

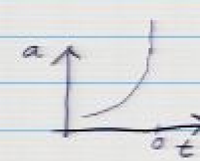

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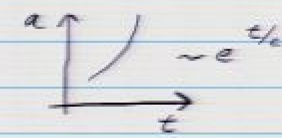

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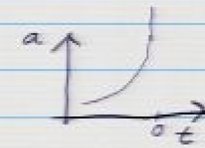

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see below for precise definition of $\rho_{critical}$

$\Lambda \simeq 0.7\, \rho_{critical}$ ("dark energy")

$\rho_{matter} \simeq 0.3\, \rho_{critical}$

$\rho_{dark\ matter} \simeq 0.9\, \rho_{matter}$

$\rho_{visible\ matter} \simeq 0.1\, \rho_{matter}$

short period of matter domination by "inflaton" particles which then decay leaving a hot soup of all sorts of particles

dark energy dominates | radiation dominates | matter dominates | dark energy dominates

$10^{-43}s$ … $10^{-32}s$ $10^{-11}s$ $10^{0}s$ $10^{13}s$ $10^{17}s$

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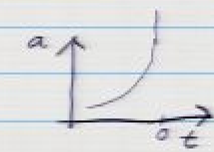

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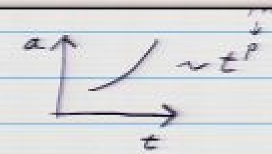

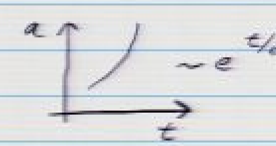

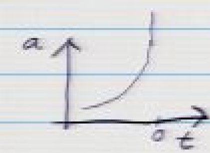

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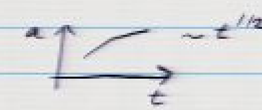

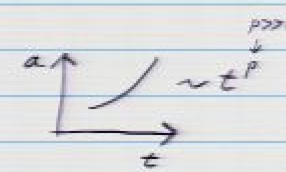

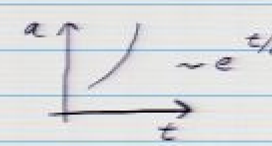

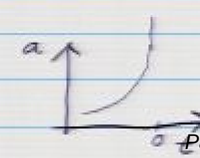

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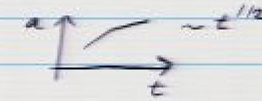

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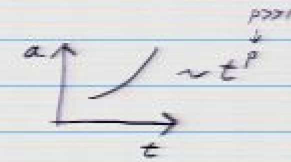

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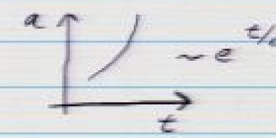

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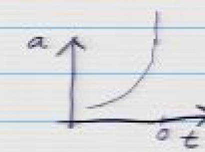

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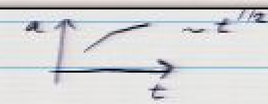

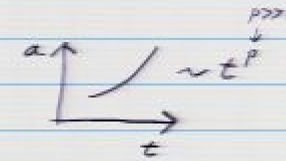

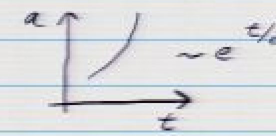

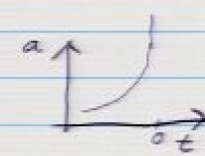

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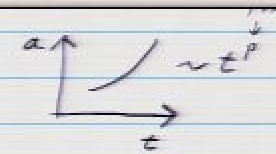
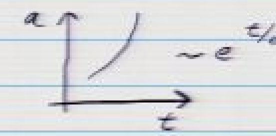
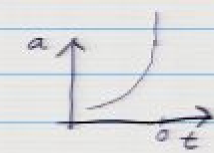
Most likely timeline:

short period of matter domination

$$a(t) = \begin{cases} (t/t_p)^p & \text{with } p \gg 1 \text{ in a so-called "power law epoch": } w = -1 + \frac{2}{3p} \text{ (verify)} \\ e^{t/t_c} & \text{in a totally dark energy dominated epoch: } w = -1. \\ (-t/t_s)^{-\varepsilon} & \text{in a "super-expanding" epoch with } w<-1 \text{ (i.e. } \varepsilon>0\text{) we would reach a big rip in finite time: at } t \to 0! \end{cases}$$







Definition: Any epoch in which $\ddot{a} > 0$, i.e., in which $w < -1/3$ (exercise: verify), is called an "inflationary epoch".

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see below for precise definition of $\rho_{critical}$

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short period of matter domination by "inflaton" particles which then decay leaving a hot soup of all sorts of particles

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is a good approximation.

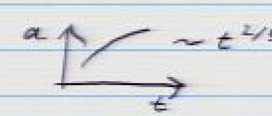
Result for $k=0$:

$$a(t) = \left(\frac{\pm t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

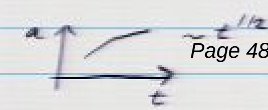
Note that, because $\dot{a}$ is squared in the Friedmann equation, there is always an expanding along with a contracting solution.

Consider the most important cases of expansions:

$(t/t_d)^{2/3}$ in a matter-dominated epoch: $w=0$



$(t/t_r)^{1/2}$ in a radiation-dominated epoch $w=1/3$



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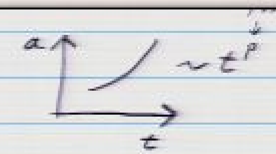

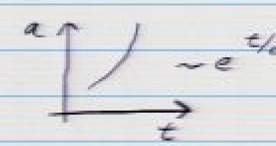

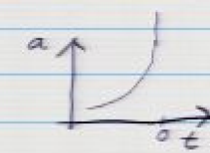

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.... ? 10^{-43} s — Inflation, $w = -1 + \varepsilon$ — 10^{-32} s — 10^{-11} s — 10^{0} s nuclei form — 10^{13} s atoms form & CMB released — 10^{17} s today — $\simeq \log(t)$

Planck era

Why inflation?

at this time, the temperature was so high that particle collisions occurred at a typical energy of $1\,\mathrm{TeV} = 1.6 \cdot 10^{-19} \cdot 10^{12}\,\mathrm{J}$ which is the maximal energy that accelerator experiments e.g. at CERN currently can produce.

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The flatness problem:

Reconsider the experimental finding of $K \approx 0$:

- Rewrite the Friedmann equation

$$3\left(\frac{\dot{a}}{a}\right)^2 + 3\,\frac{K}{a^2} = 8\pi G \rho + \Lambda$$

Hubble parameter (const. in space but not in time)

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$$\Rightarrow \quad \boxed{\frac{\Omega(t_0) - 1}{\Omega(t_1) - 1} = \left(\frac{t_0}{t_1}\right)^{2/3}}$$

- Given that $\Omega(t_1) - 1 = \mathcal{O}(1)$ today, at time t_1,
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Solution to this fine-tuning problem?

- Is there a type of epoch in which the universe evolves towards flatness, rather than away from it?

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After inflation, $V(\phi)$ becomes the kinetic and mass energy of all sorts of particles, thus making a hot primordial soup.

Speculation:

How did inflation start?
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In any spacetime a single quantum fluctuation of ϕ might elevate $V(\phi)$ locally so as to spawn a new universe!

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- ## The "horizon problem":

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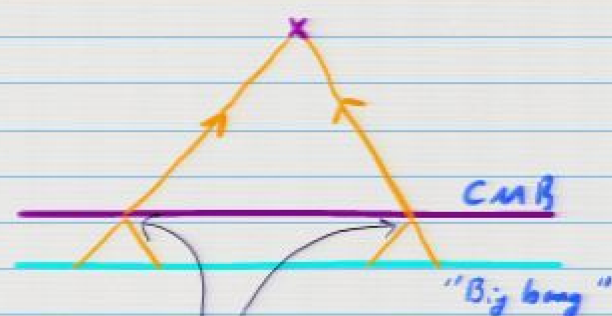

these two areas of the surface that emitted CMB photons do not have a common past. How come they are so similar?

Concretely: Only patches on the CMB sky of angular extent < 1 degree have a common past, if there was no inflation.

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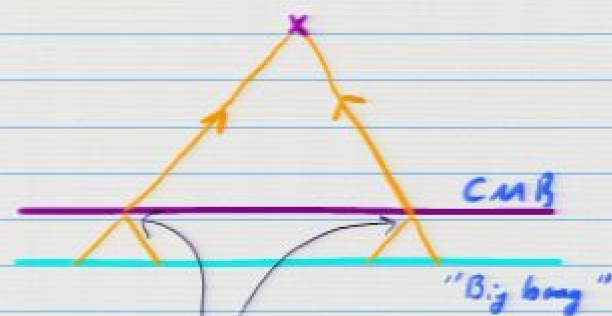

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- the occurrence and precise statistics of inhomogeneities in the universe!

- How? The quantum fluctuations of scalar fields (unlike those of spinor fields of, e.g., electrons and vector fields of, e.g., photons) are being amplified in an inflationary epoch, along with those of g.

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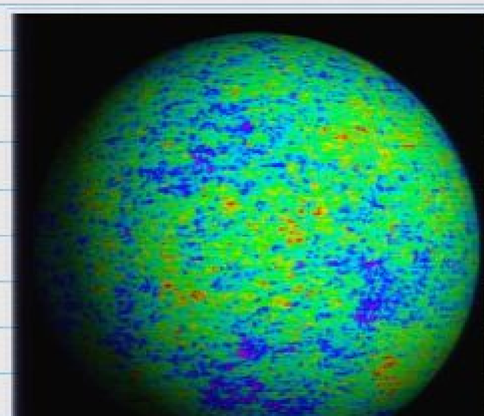
Actual temperature data:

($\Delta T \approx 10^{-6} K$ only!)



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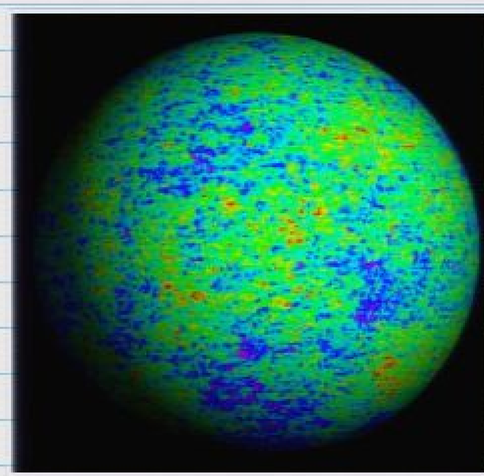

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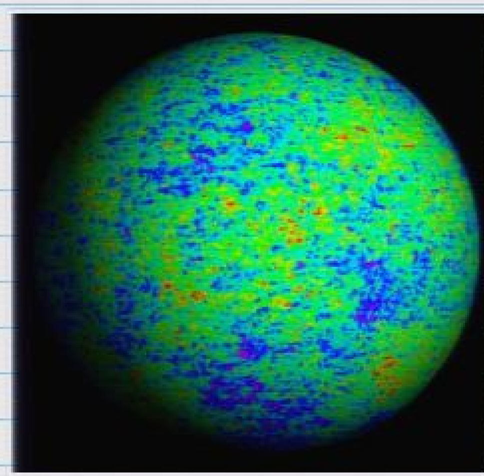

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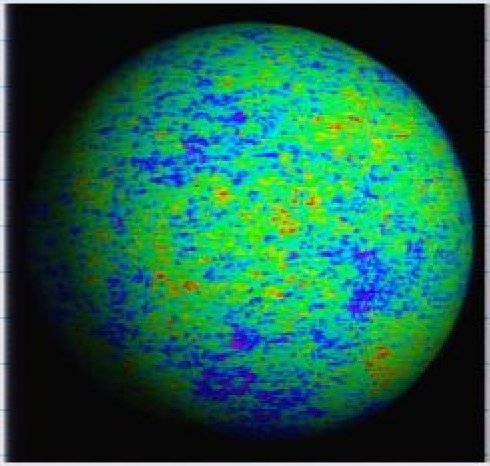

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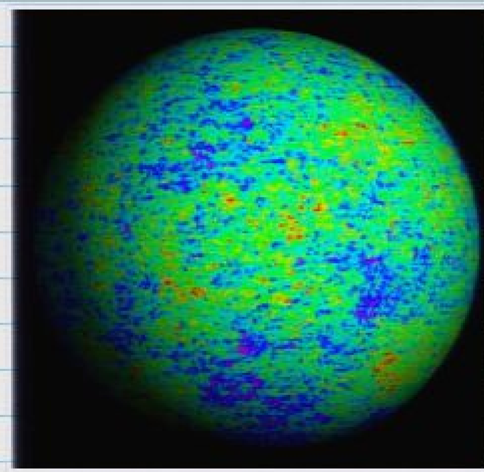

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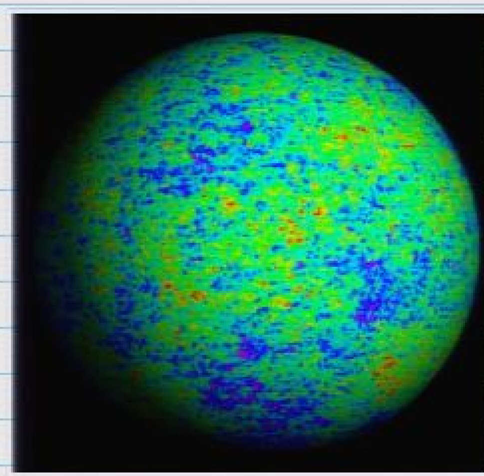

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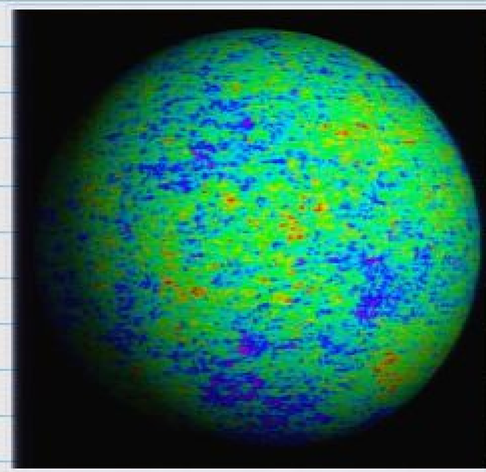

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Experiment

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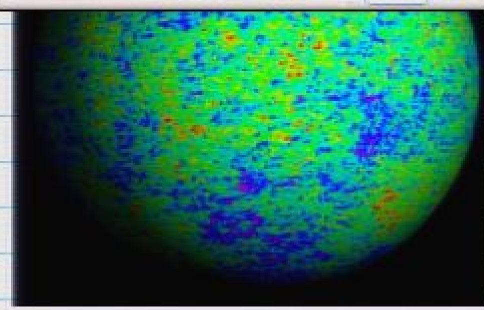

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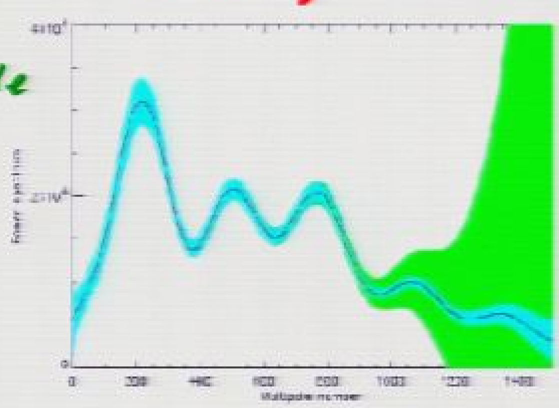

"angular wavelength^{-1}" ℓ

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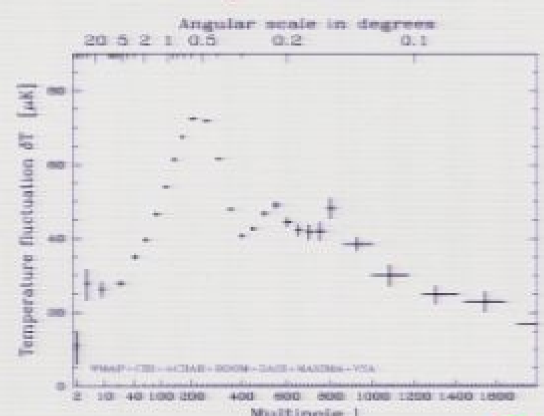

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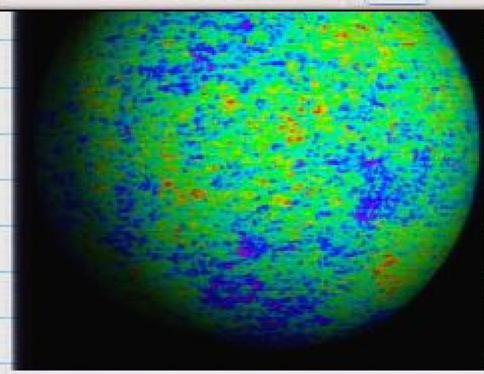

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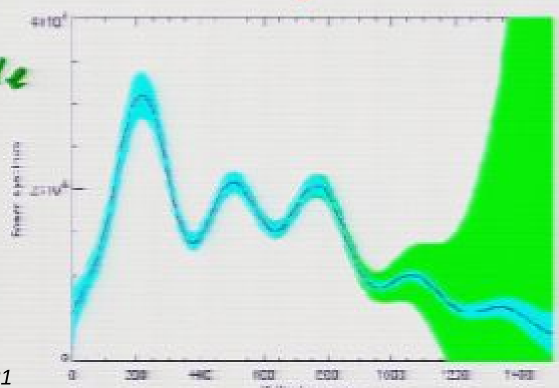

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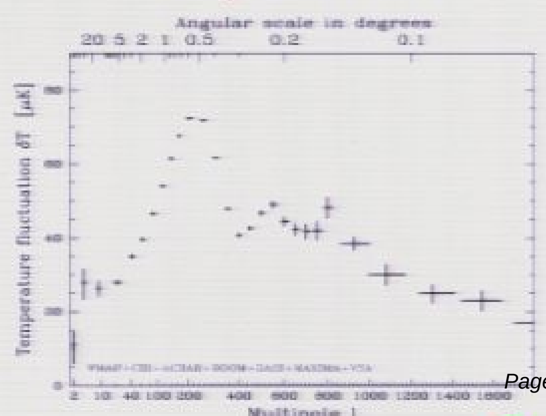

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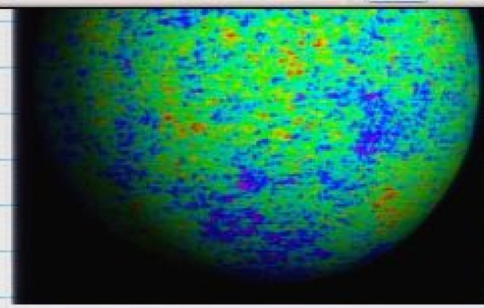

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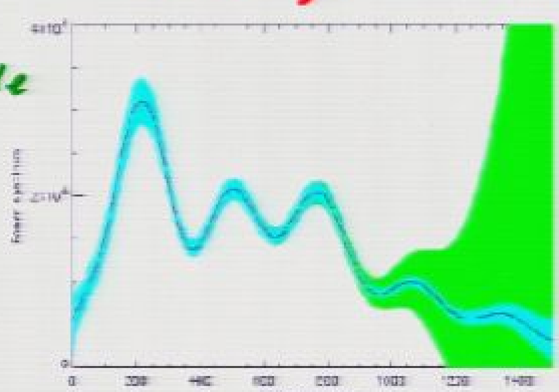
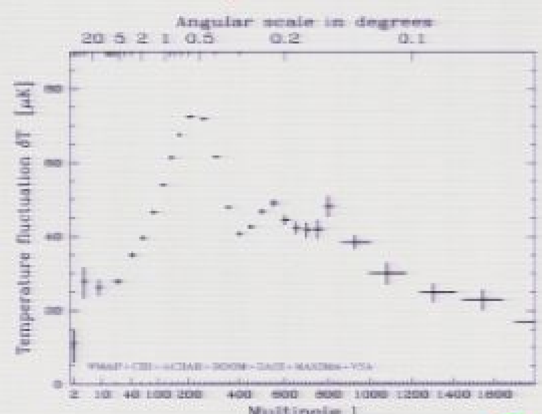
Theory

Amplitude



"angular wavelength^{-1}" ℓ

Experiment



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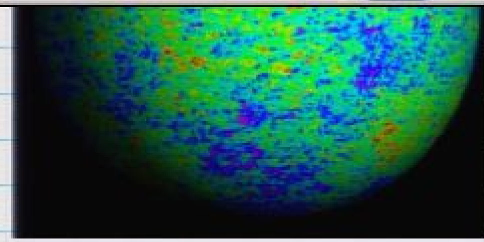
data :

$(\Delta T \simeq 10^{-6}\,K$ only!$)$



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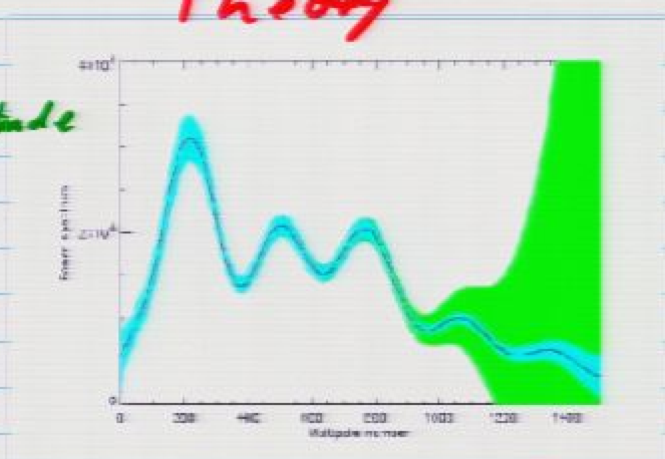

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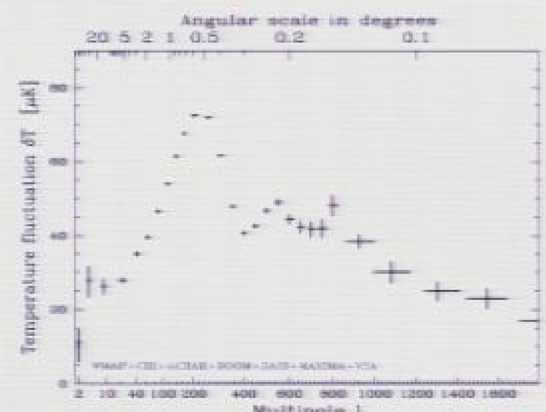

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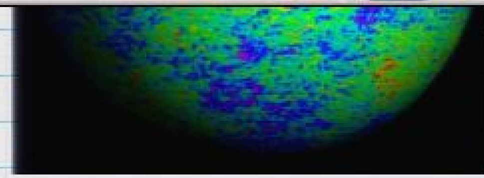

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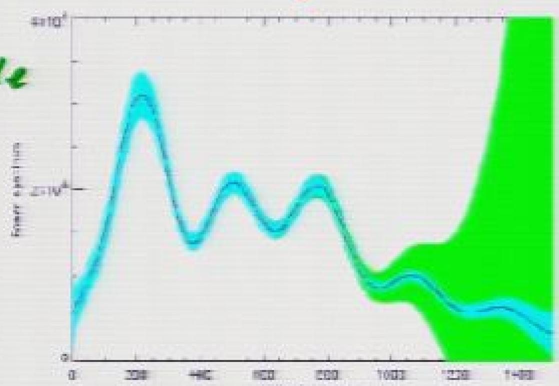

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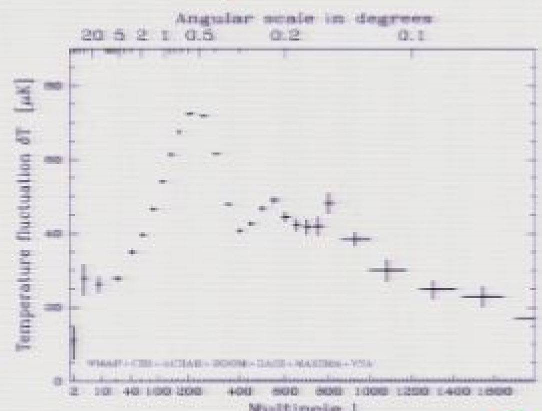

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Remark: A

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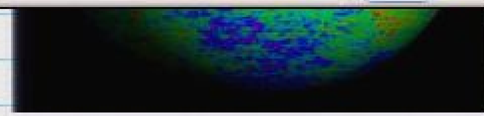

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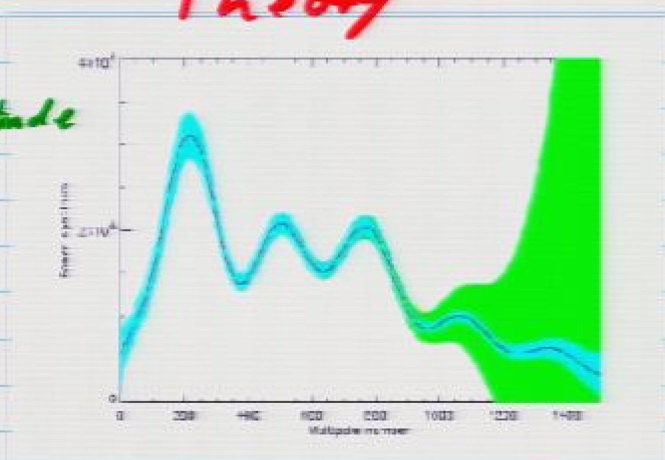

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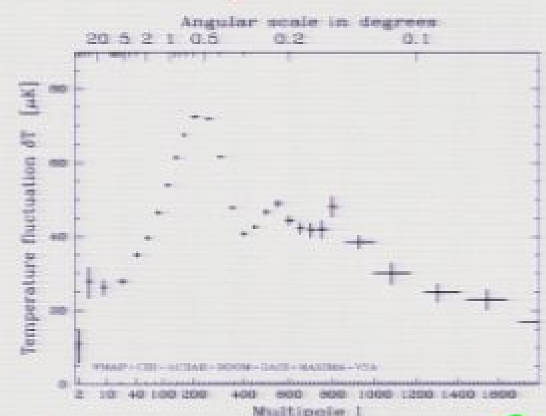

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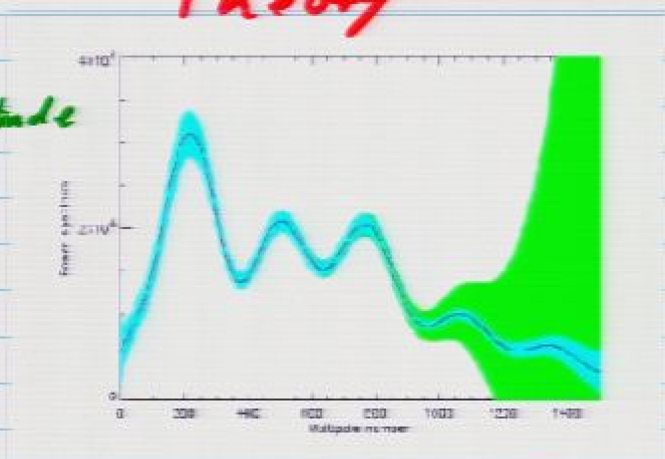

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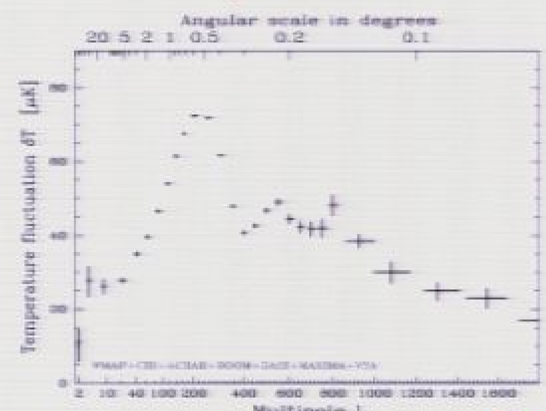

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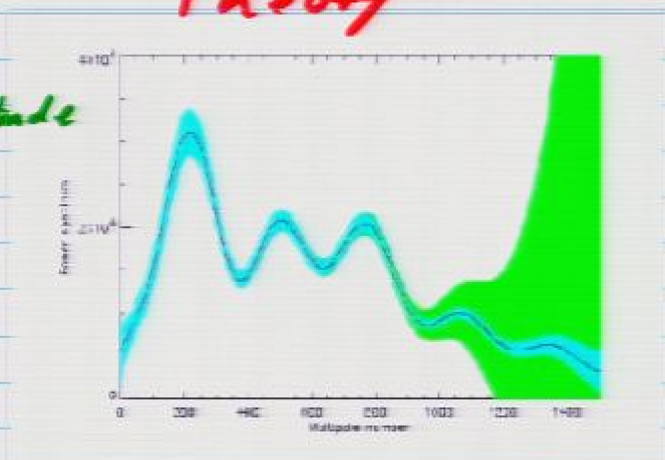

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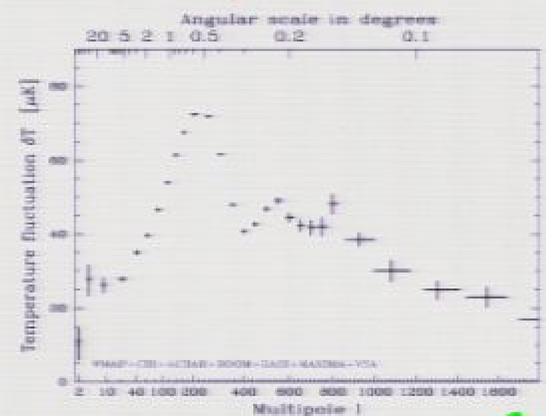

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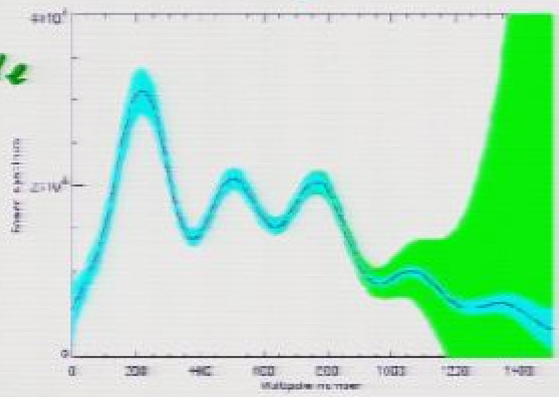

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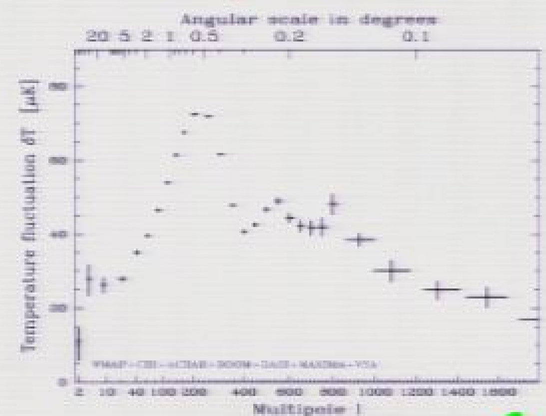

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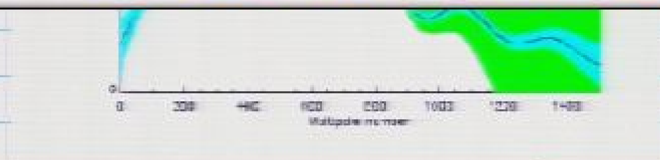

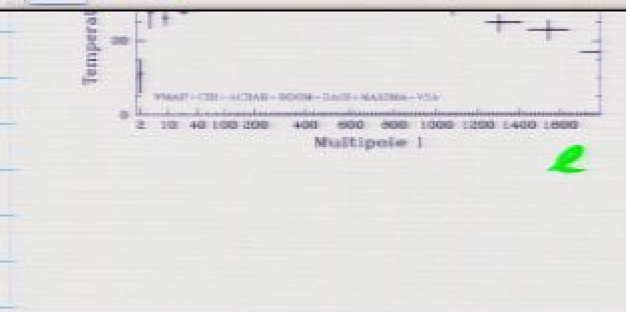

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These small inhomogeneities (presumably caused by quantum fluctuations) then explain the statistical distribution of galaxies:

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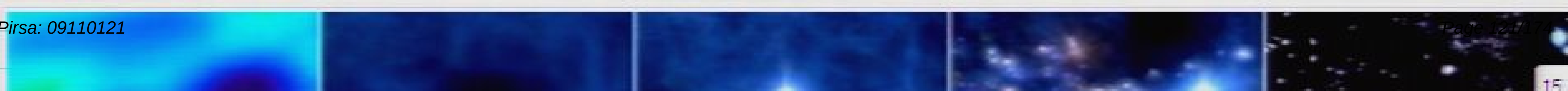
Visualization:



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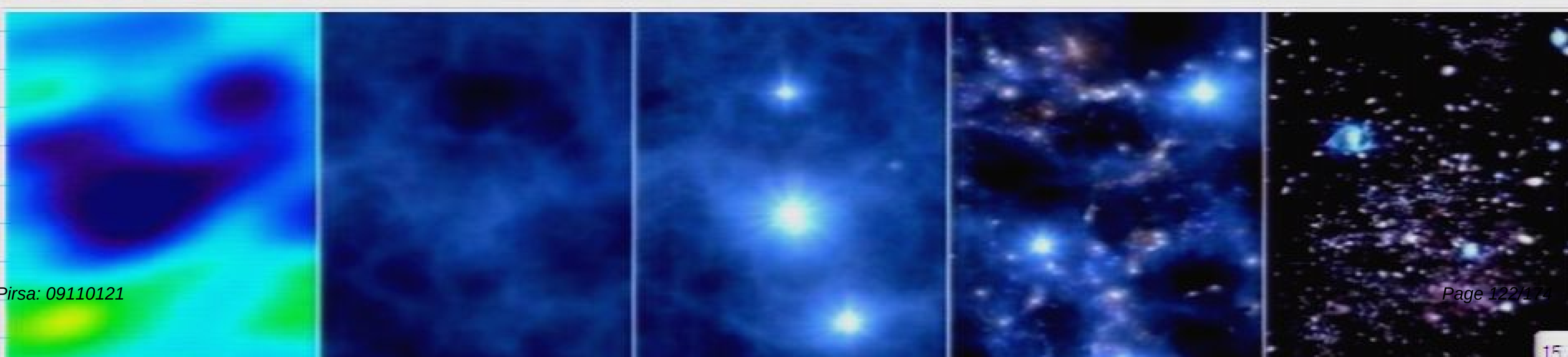

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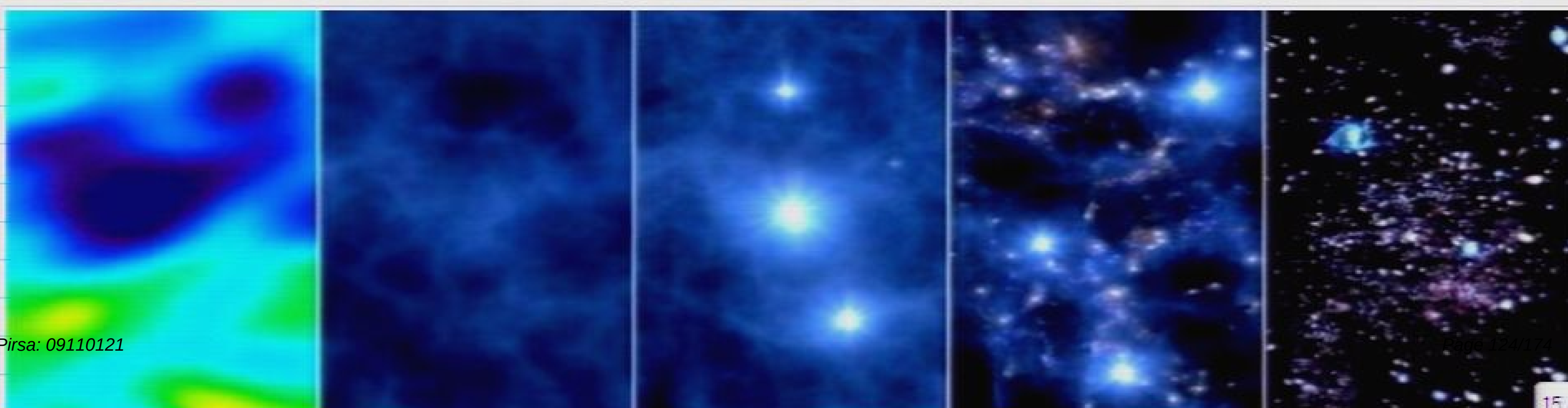

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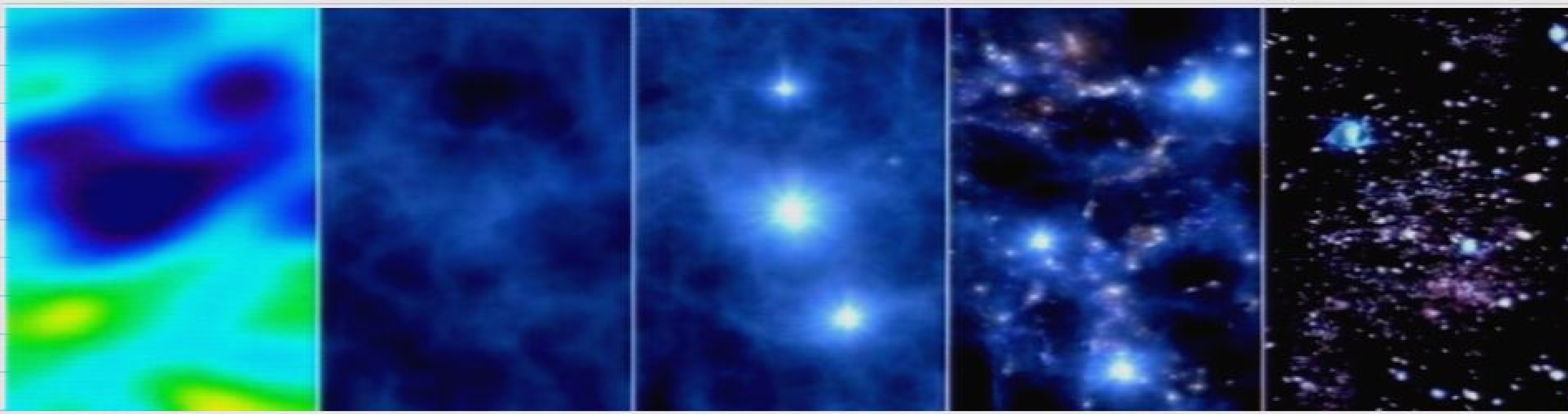

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The early history of the inhomogeneities is well-described

- Start with the action of gravity + inflaton field

$$S = \frac{1}{2}\int\left(-\phi\,\Box\phi - V(\phi)\right)\sqrt{g}\,d^4x - \frac{1}{16\pi G}\int R\sqrt{g}\,d^4x$$

- Introduce convenient variables:

In these coordinates, two galaxies always have the same numerical distance

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$$y^i := a^{-1}(t)\,x^i \qquad \text{("comoving" coordinates)}$$

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- Allow the field ϕ to fluctuate:

$$\phi(y,\tau) = \underbrace{\phi_0(\tau)}_{\text{homogeneous}} + \underbrace{\delta\phi(y,\tau)}_{\text{small to 1st order}}$$

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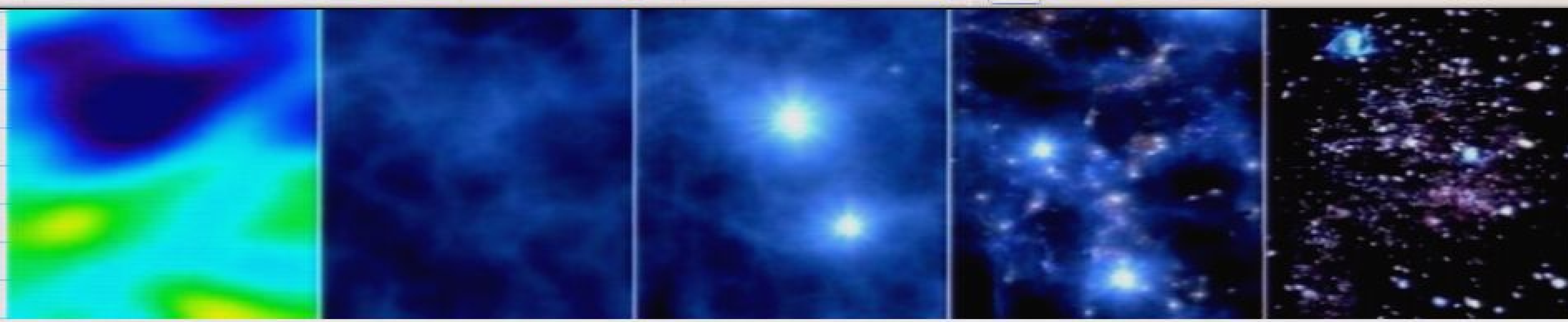

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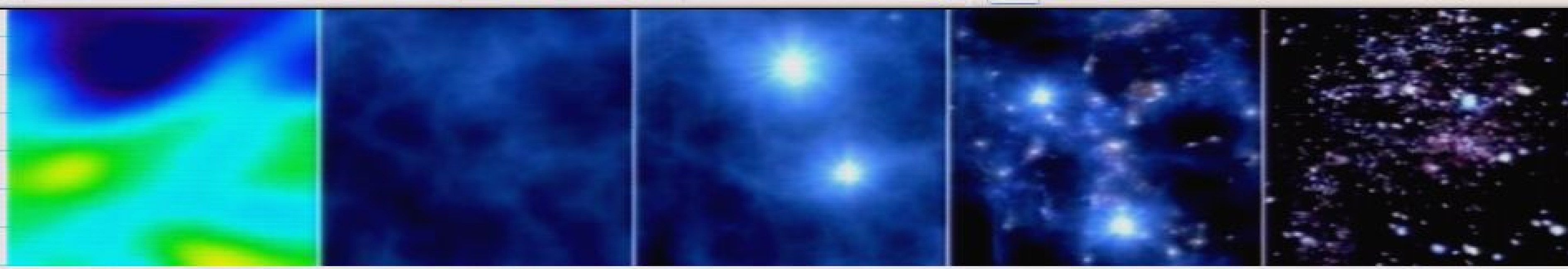

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$$ds^2 = \underbrace{ds_S^2}_{\text{contain "scalar"}} + \underbrace{ds_V^2}_{\text{"vector"}} + \underbrace{ds_T^2}_{\text{"tensor" fluctuations.}}$$

$$ds_S^2 = a^2(\tau)\left((1+2\Phi)\,d\tau^2 - 2\partial_i B\, dy^i d\tau - [(1-2\Psi)\delta_{ij} + 2\partial_i\partial_j E]\, dy^i dy^j\right)$$

$$ds_V^2 = a^2(\tau)\left(d\tau^2 + 2V_i\, dx^i d\tau - [\delta_{ij} + W_{i,j} + W_{j,i}]\, dx^i dx^j\right)$$

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- Here:
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The only gauge (i.e. cd.) invariant entity is the intrinsic curvature scalar:

$$R^{(3)} = -\frac{a'}{a}\frac{\delta\phi}{\phi_0'} - \Psi \qquad \left({}' \text{ means } \frac{d}{d\tau}\right)$$

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⇒ neglect here.

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Scalar perturbations:

The only gauge (i.e. cd.) invariant entity is the intrinsic curvature scalar:

$$R^{(1)} = -\frac{a'}{a}\frac{\delta\phi}{\phi_0'} - \Psi \qquad \left({}' \text{ means } \frac{d}{d\tau}\right)$$

It describes the (scalar) quantum fluctuations-induced CMB spectrum.

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- V_i, W_i are 3-vector perturbation functions
- h_{ij} is a 3-tensor perturbation function

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How to calculate the quantum fluctuations?

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- Use quantum field theory on curved space to calculate how

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Remark: See e.g. my own reseach on how Planck scale physics

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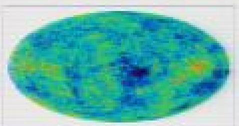
Next term:

Graduate Course: QFT for Cosmology

Quantum Field Theory for Cosmology



AMATH872 / PHYS785

Winter 2010

http://www.math.uwaterloo.ca/~akempf/QFT-for-Cosmology-W10.shtml

Instructor:	A. Kempf, (MC6071, ext. 35462)
Prerequisite:	AMATH 673, PHYS702 or consent of instructor. Some knowledge of General Relativity.
Time:	Tuesdays + Thursdays 4-5:30pm,
Venue:	Perimeter Institute's Bob room Video-linked to: * Univ. of Waterloo, Room MC6091 * Univ. of Guelph, Rozanski Room 106 * possibly other venues upon request (at 38.117.109.110 via H323)
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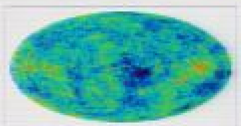

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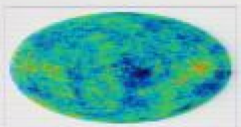

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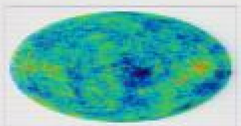

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This course begins with a thorough introduction to quantum field theory. Unlike the usual quantum field theory courses which aim at applications to particle physics, this course then focuses on those quantum field theoretic techniques that are important in the presence of gravity. In particular, this course introduces the properties of quantum fluctuations of fields

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Outline:

- From first to second quantization.
- Introduction to scalar quantum field theory.
- The Unruh effect.
- Canonical quantization in curved space-times.
- Path integral quantization of fields.
- Quantum fluctuations of scalar fields and of the metric.
- Applications: Inflationary cosmology and the origin of structure.
- (Time permitting:) Effective action and gravitational backreaction

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Lecture notes:

Detailed lecture notes will be made freely available here, usually a few days before each lecture.

In the meantime, here are the lecture notes of the previous teaching of this course, in W08:

Lecture notes of W08: 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23

Additional Literature:

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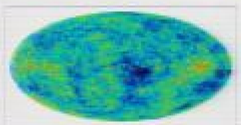

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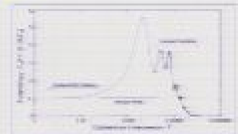

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