Title: Strong coupling problems in condensed matter and the AdS/CFT correspondence

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Abstract: I will survey some open problems posed by experiments on condensed matter systems, such as the high temperature superconductors. I will argue that their solutions require analyses of strong-coupling regimes which cannot be addressed by conventional field-theoretic means. I will describe insights drawn from the AdS/CFT correspondence, and discuss the connections to theories with simple gravity duals.

Strong coupling problems in condensed matter and the AdS/CFT correspondence

Reviews:

arXiv:0910.1139
arXiv:0901.4103

PHYSICS

VE RI TAS

Talk online: sachdev.physics.harvard.edu

Yejin Huh, Harvard

Max Metlitski, Harvard

Frederik Denef, Harvard
Sean Hartnoll, Harvard
Christopher Herzog, Princeton
Pavel Kovtun, Victoria
Dam Son, Washington

PHYSICS

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   Collisionless-to-hydrodynamic crossover of CFT3s

2. Quantum criticality of Dirac fermions
   "Vector" 1/N expansion

3. Quantum criticality of Fermi surfaces
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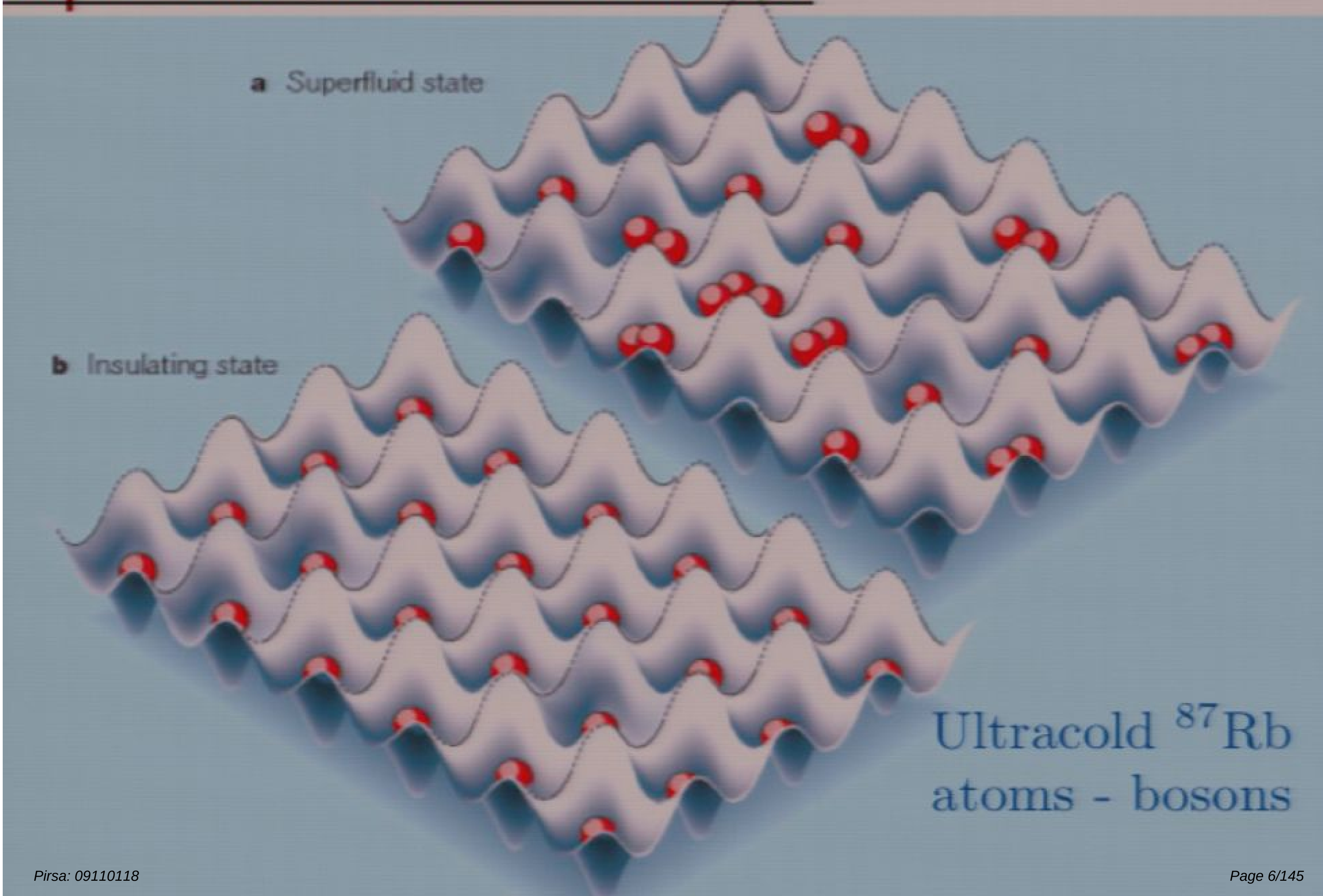
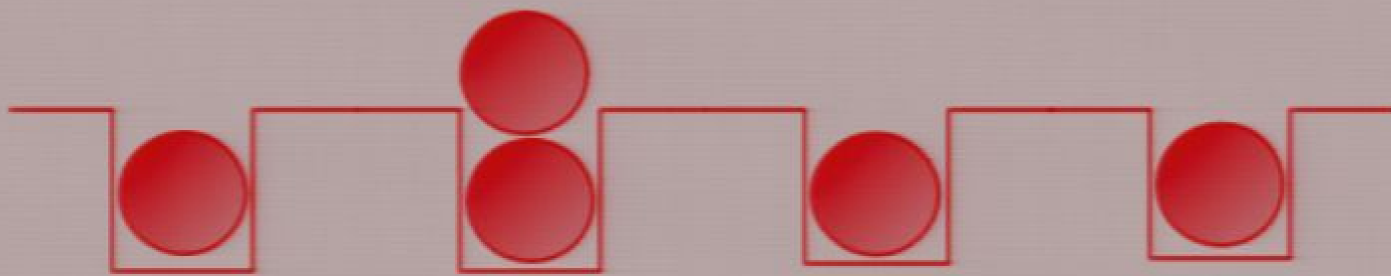
Superfluid-insulator transition



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Insulator (the vacuum) at large U

Excitations:



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Holes $\sim \psi$

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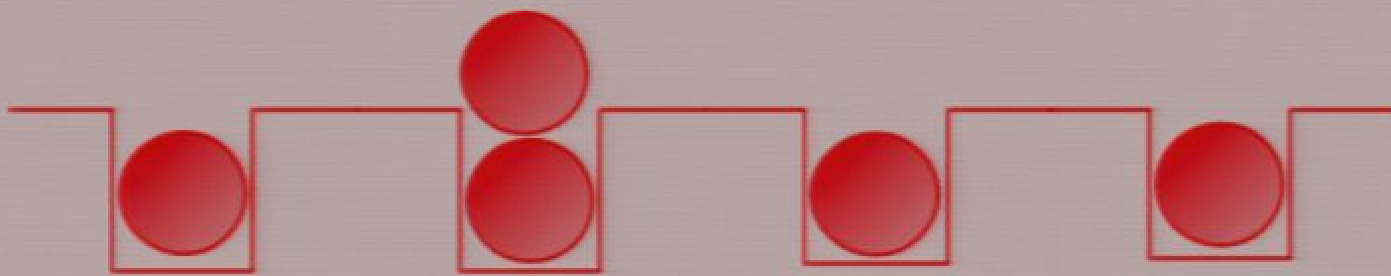

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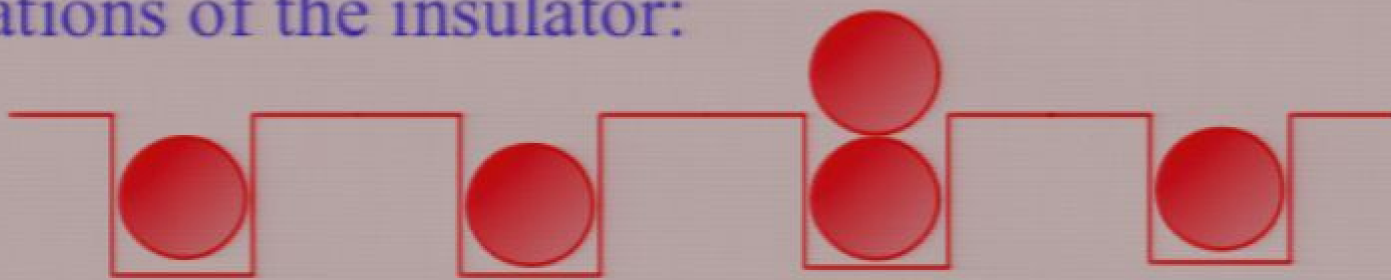
Excitations of the insulator:



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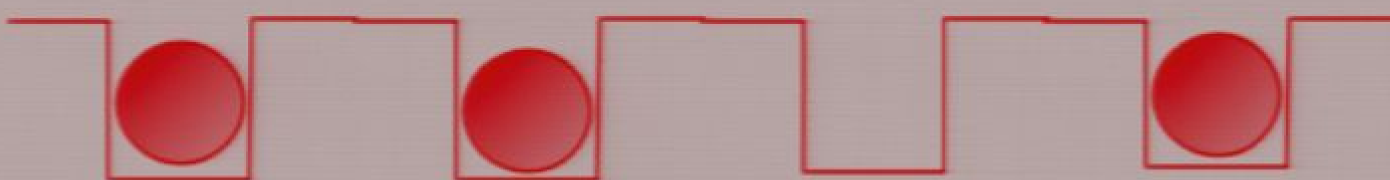

Holes ~ ψ

Density of particles = density of holes ⇒

"relativistic" field theory for ψ:

$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

Insulator $\Leftrightarrow$ $\langle \psi \rangle = 0$

Superfluid $\Leftrightarrow$ $\langle \psi \rangle \neq 0$

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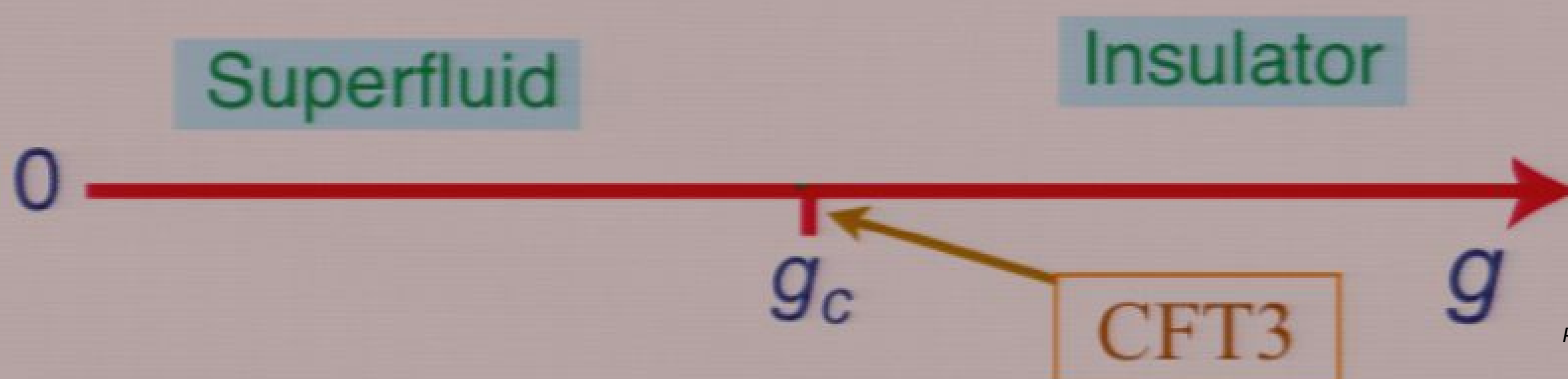

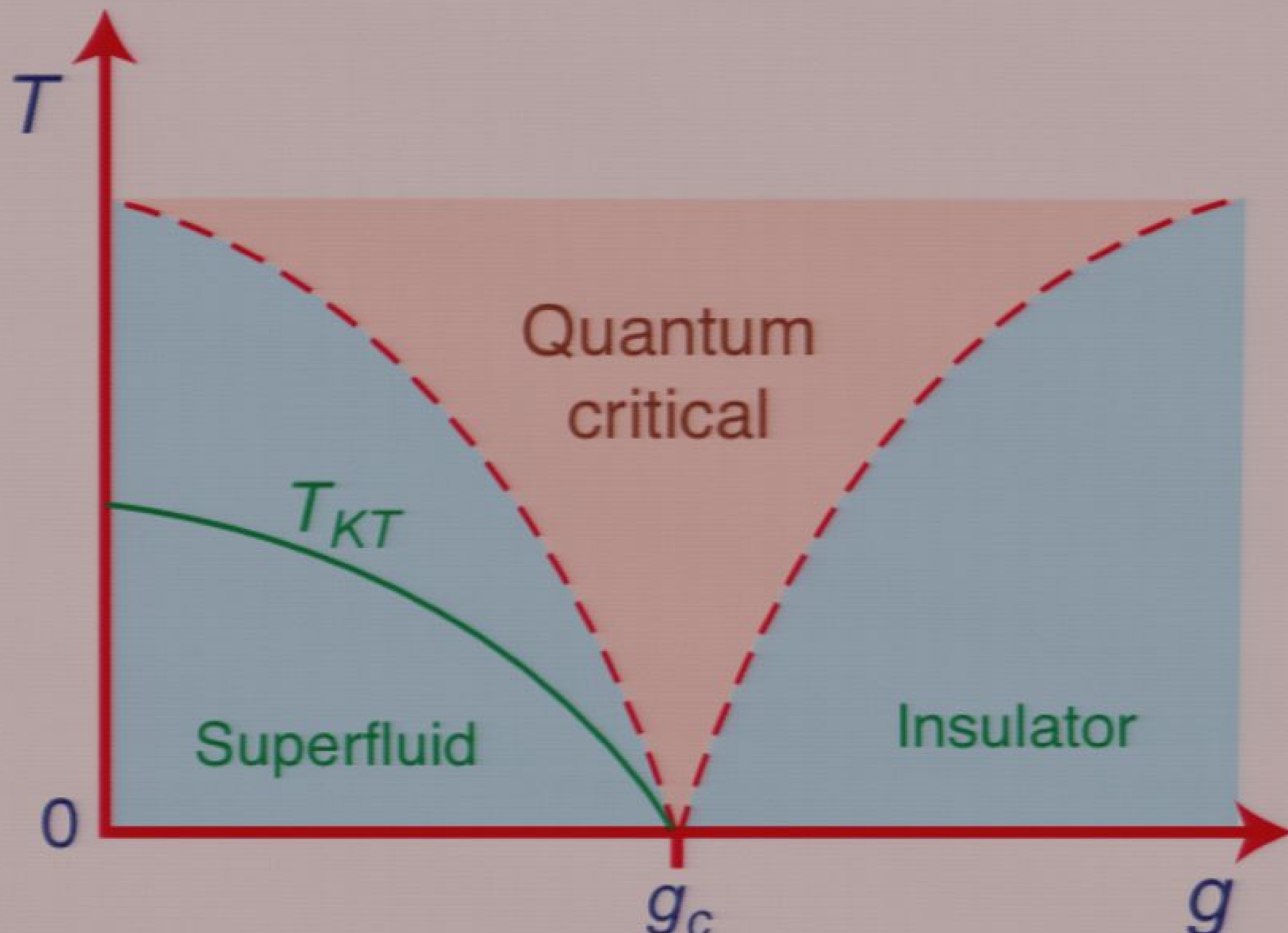

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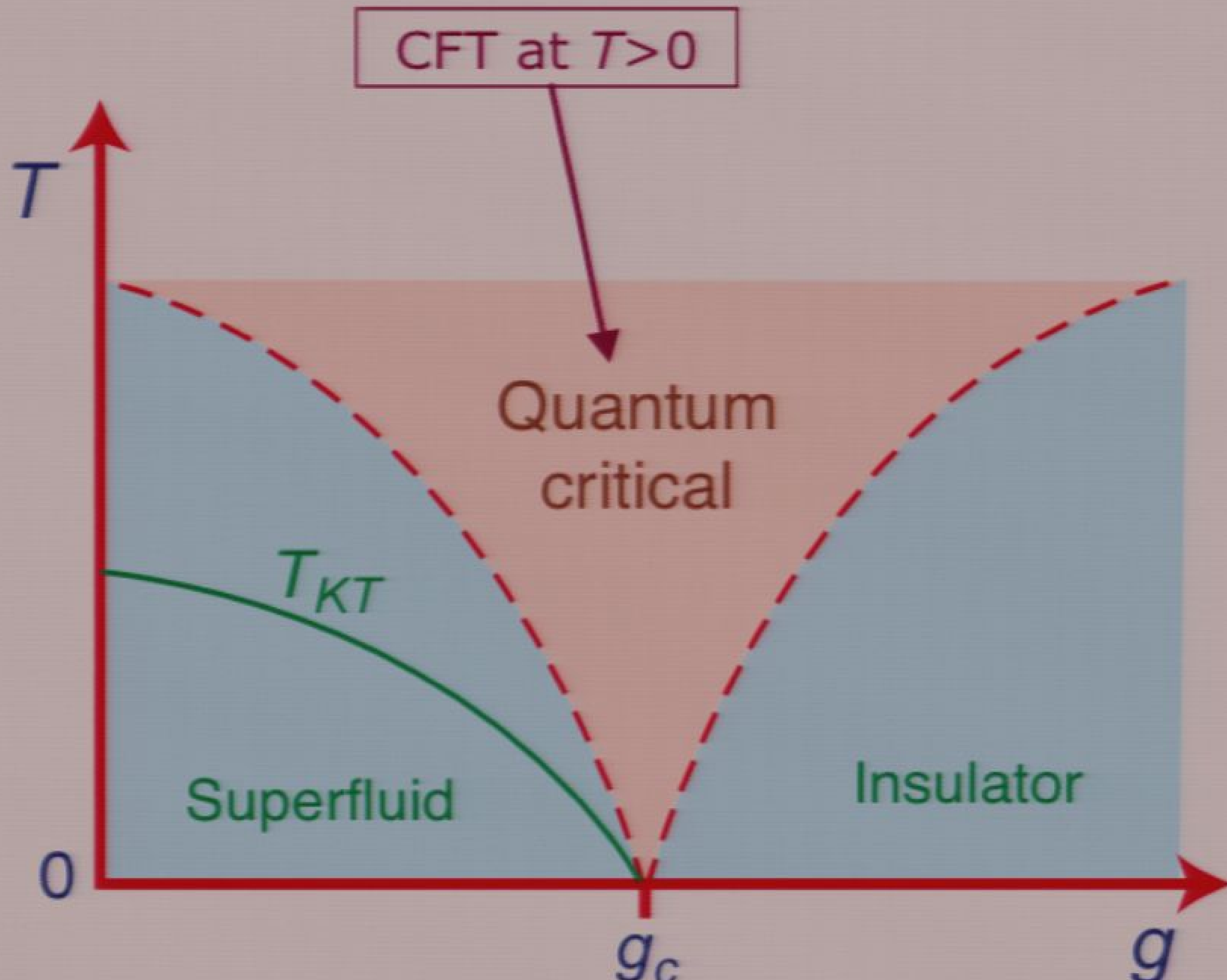

CFT at $T>0$
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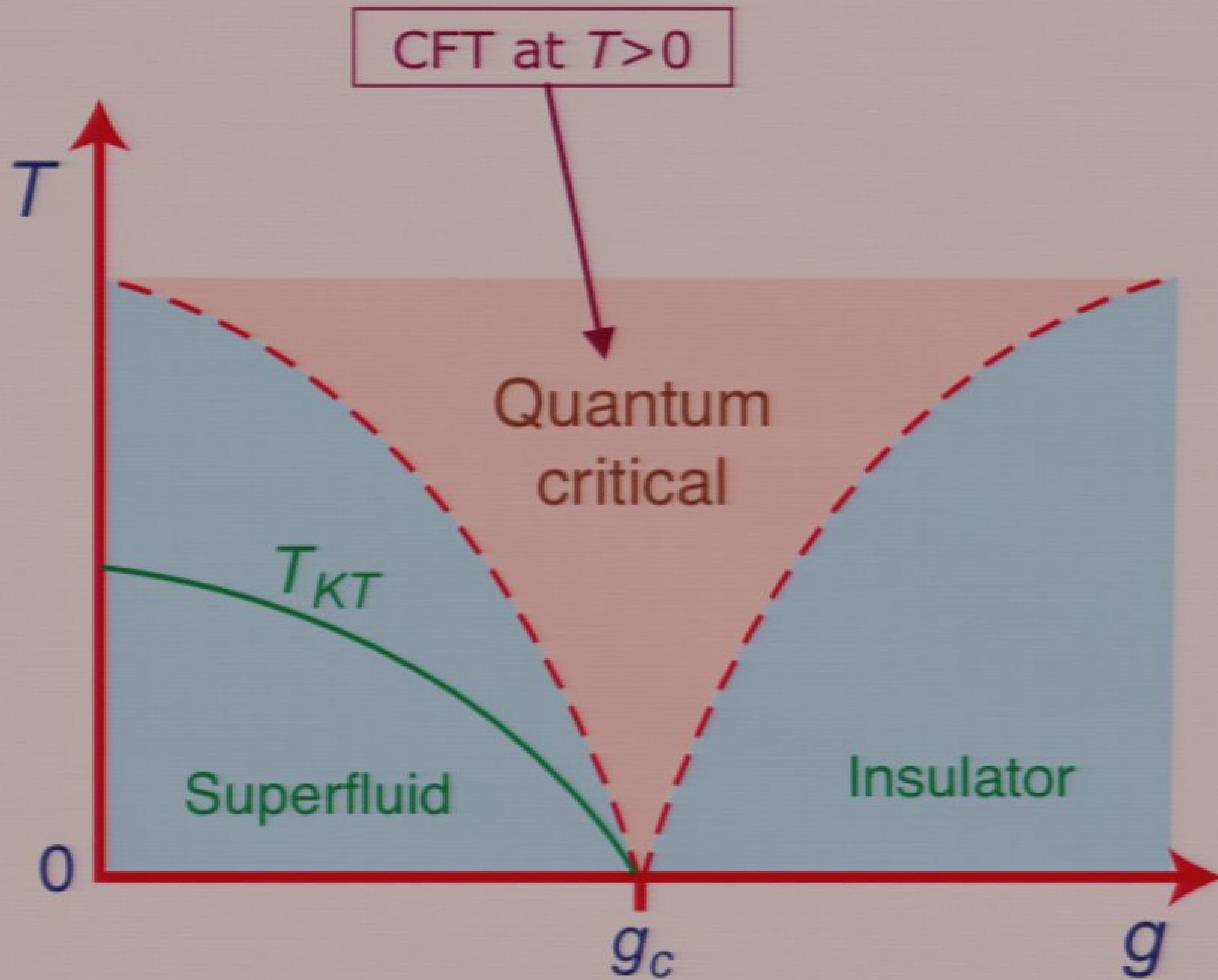
Quantum critical transport

Quantum *"perfect fluid"*
with shortest possible
relaxation time, τ_R

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$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Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

$$\sigma = \frac{4e^2}{h} \times [\text{Universal constant } \mathcal{O}(1) \ ]$$

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Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

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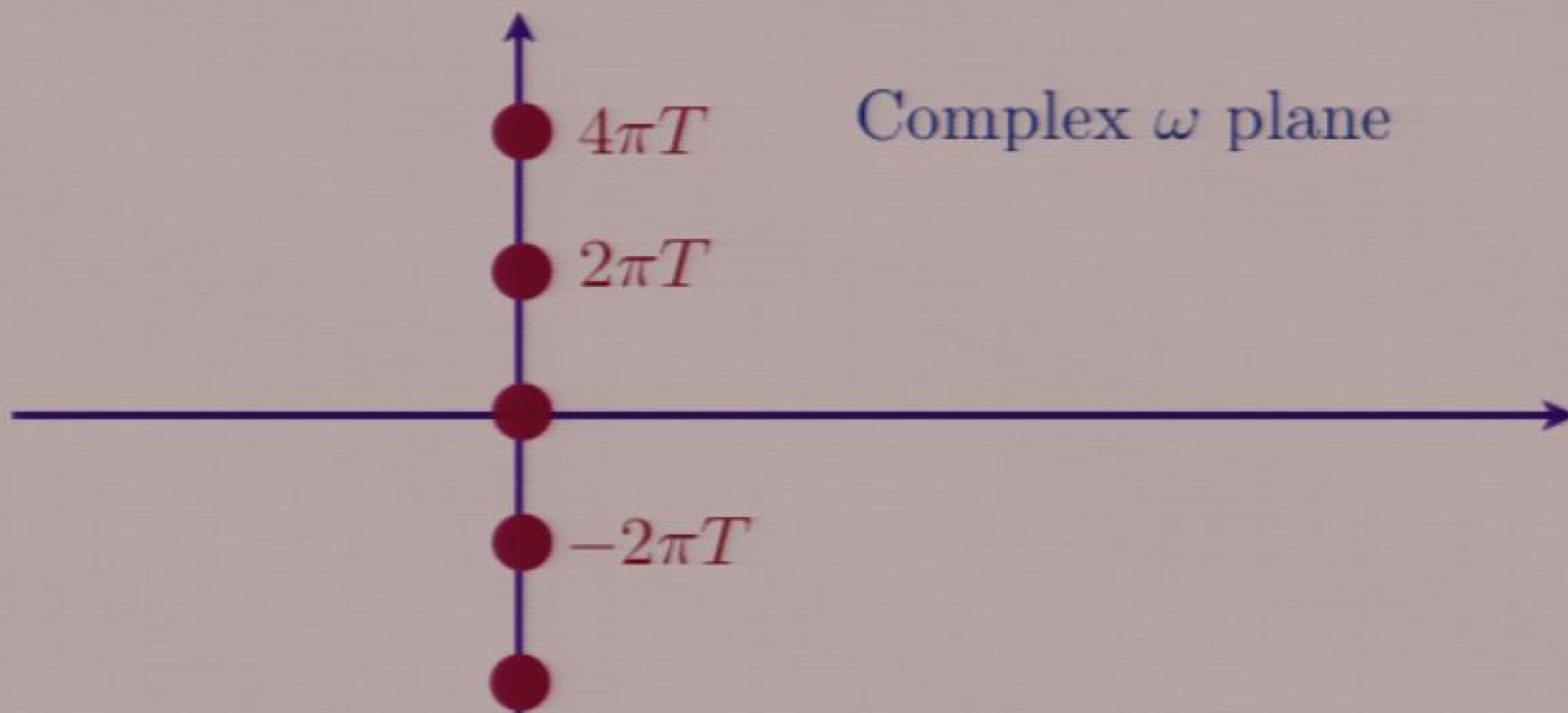

Direct $1/N$ or ϵ expansions for correlators at the Euclidean frequencies $\omega_n = 2\pi n T i$ (n integer) or in the conformal "collisionless" regime, $\hbar\omega \gg k_B T$.

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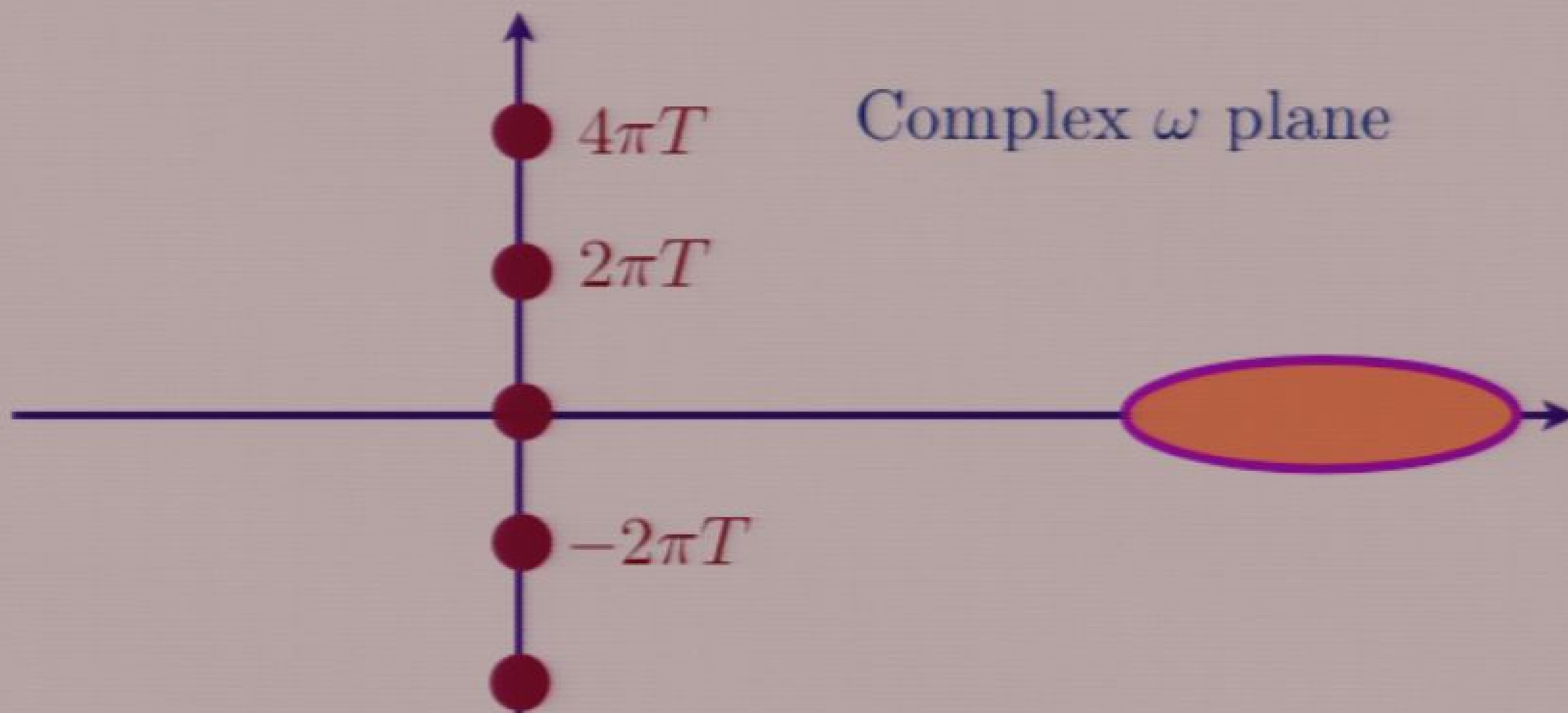

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Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For *all* CFT3s, at $\hbar\omega \gg k_B T$

$$\chi(k, \omega) = \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2 k^2 - \omega^2}} \; ; \; \sigma(\omega) = \frac{4e^2}{h} K$$

where K is a universal number characterizing the CFT3, and v is the velocity of "light".

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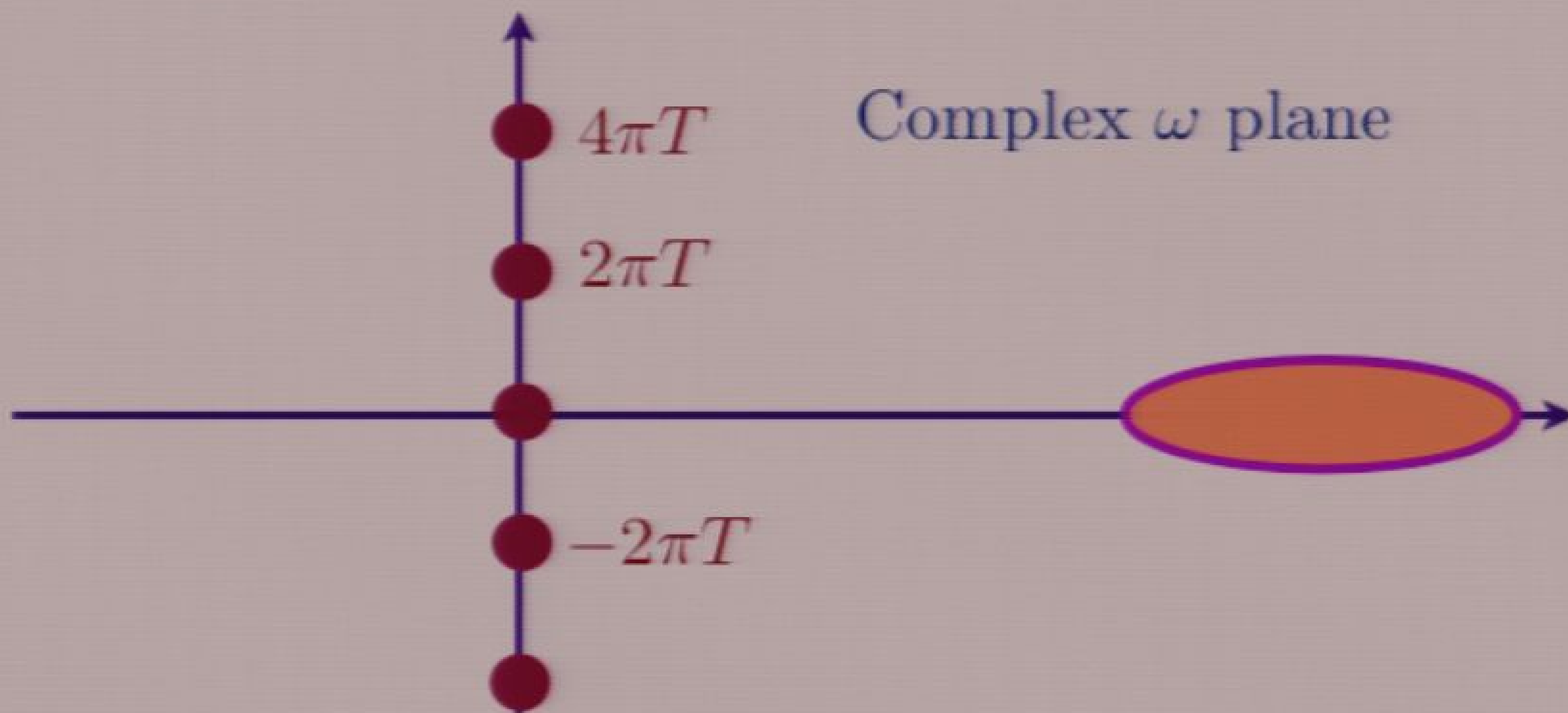

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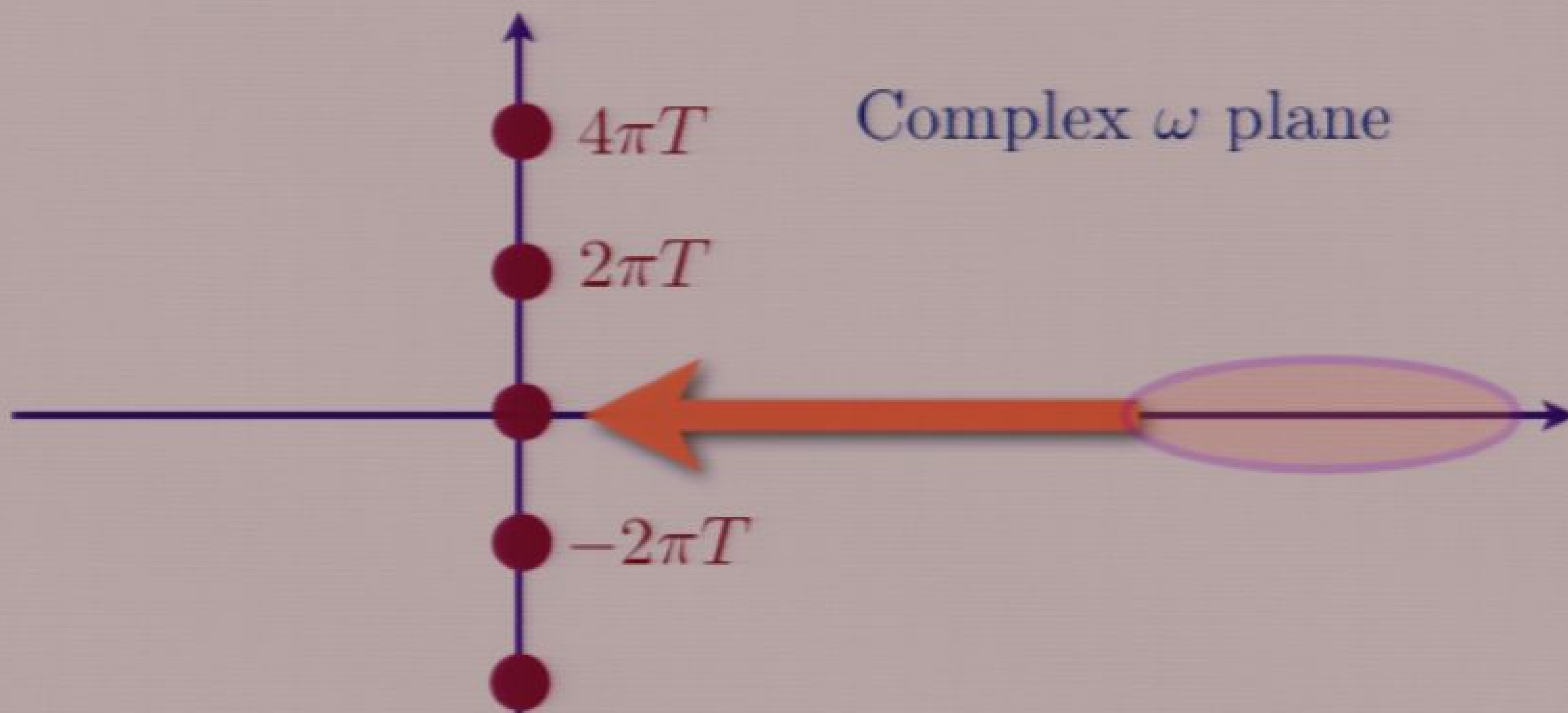

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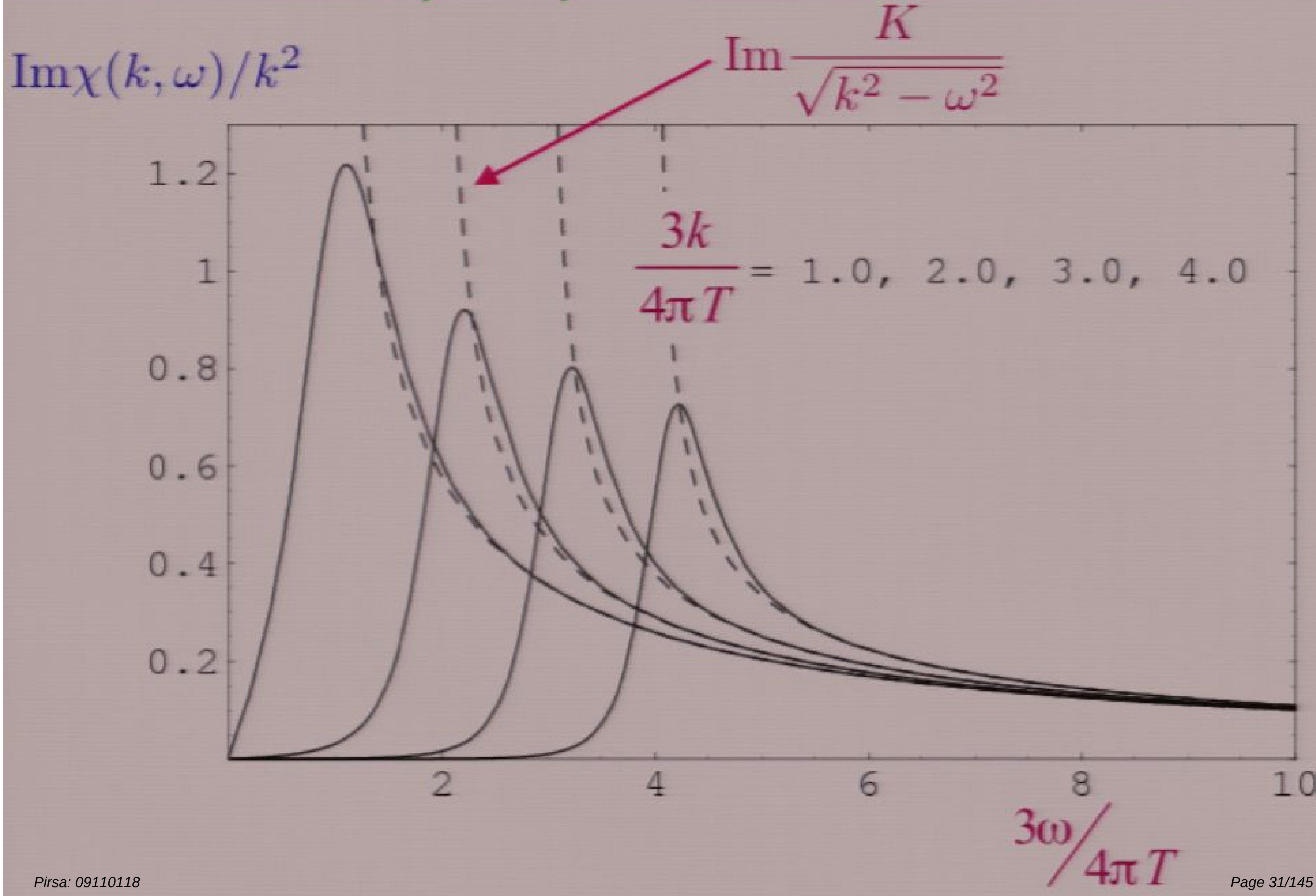
Collisionless to hydrodynamic crossover of SYM3



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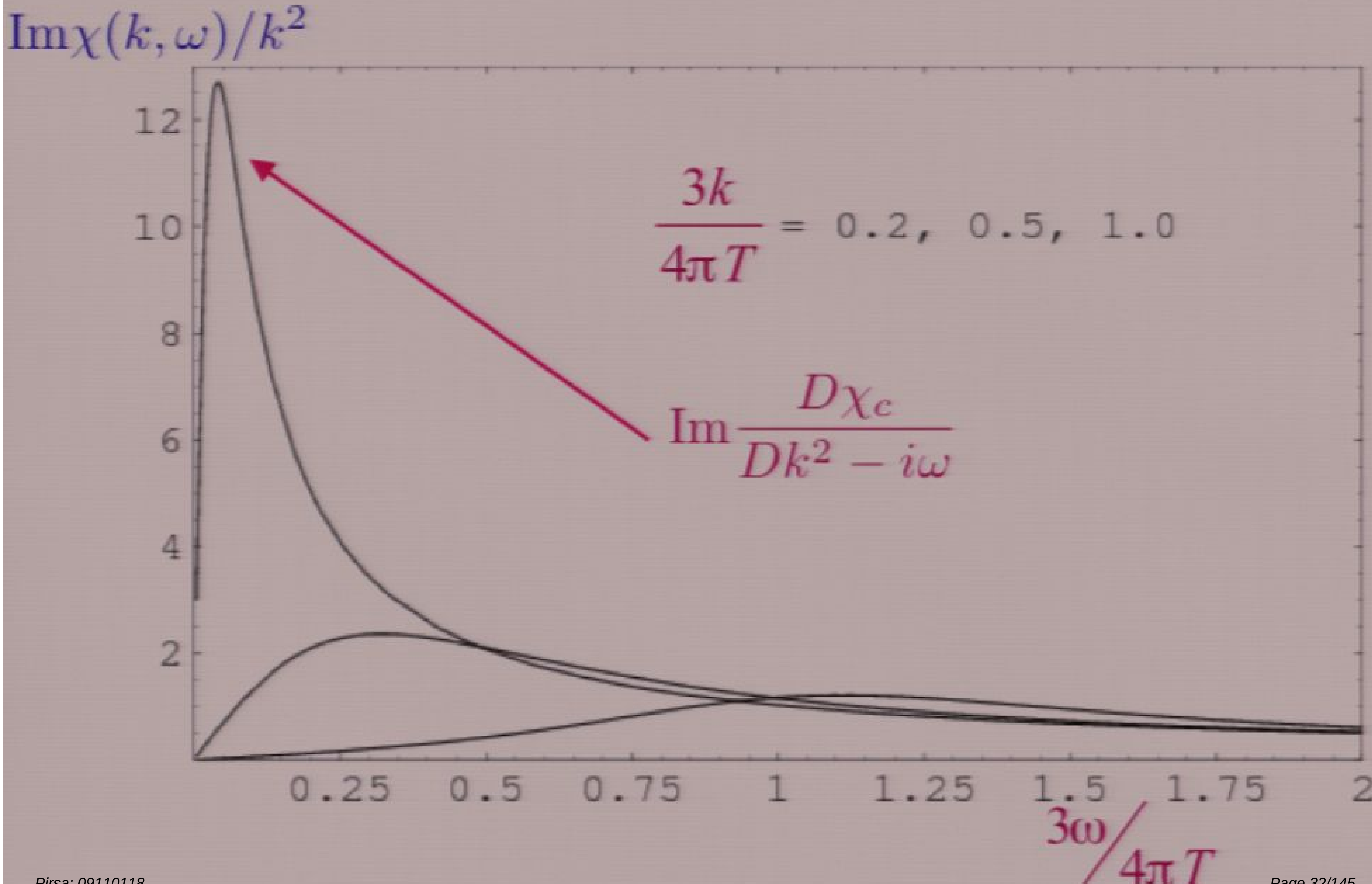

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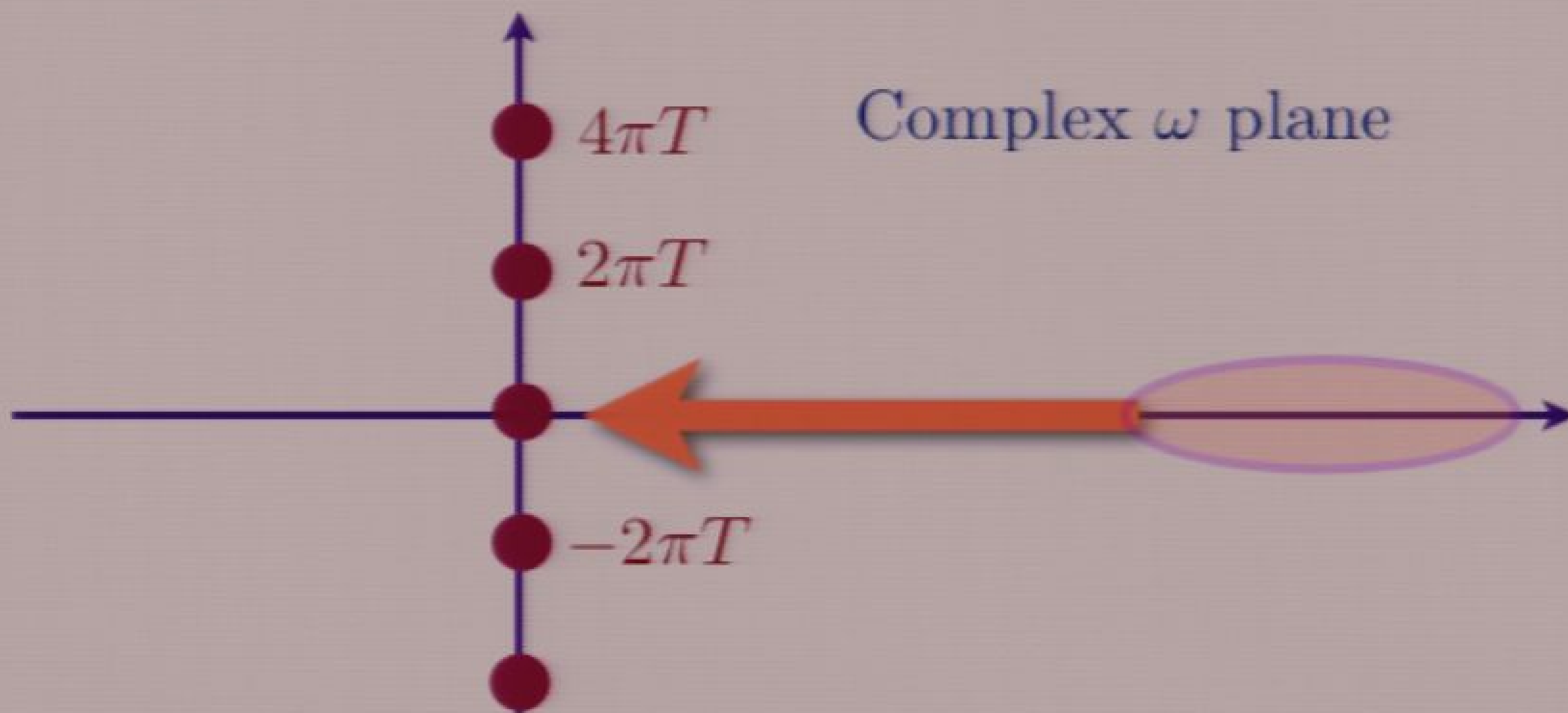

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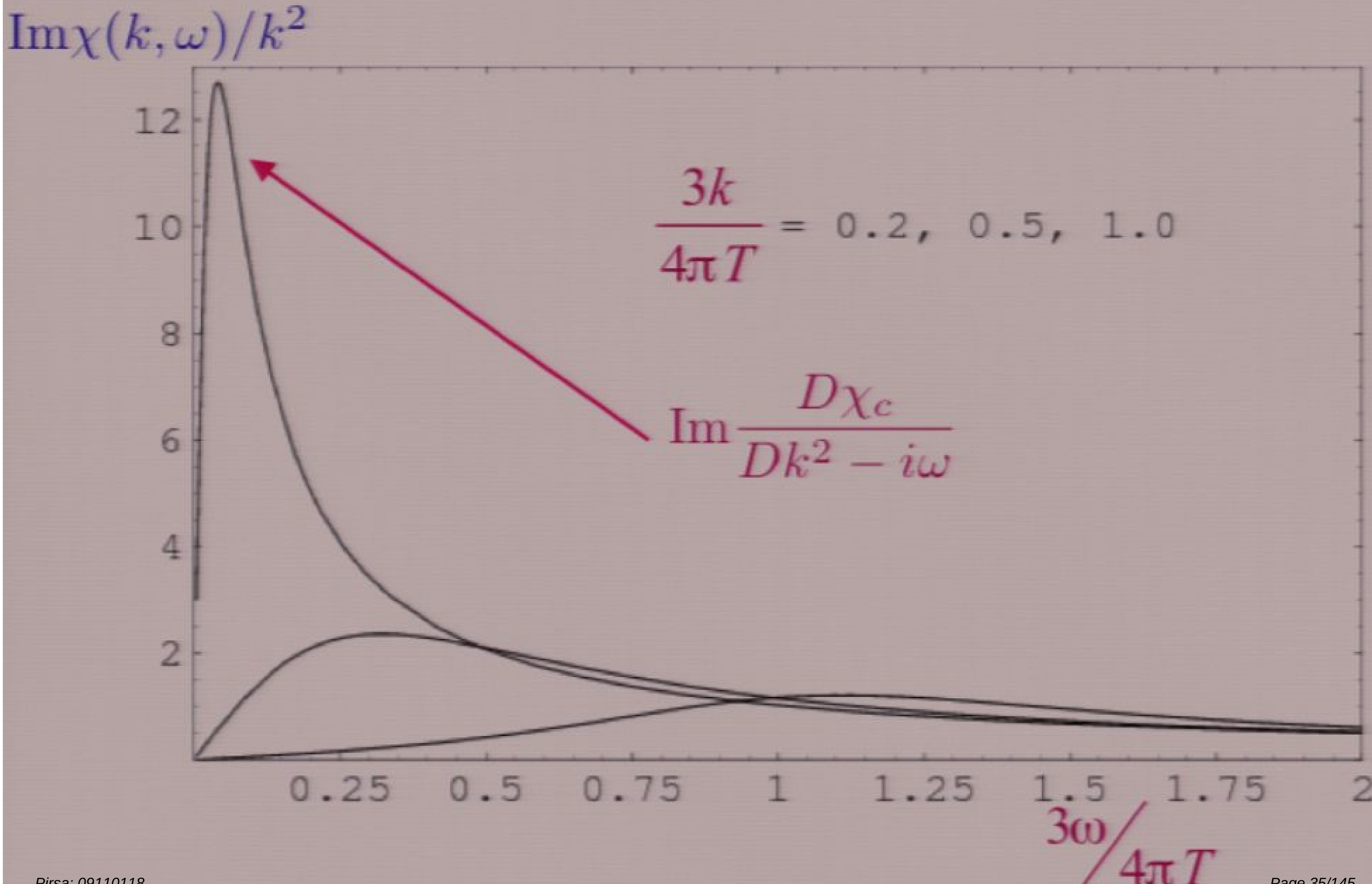

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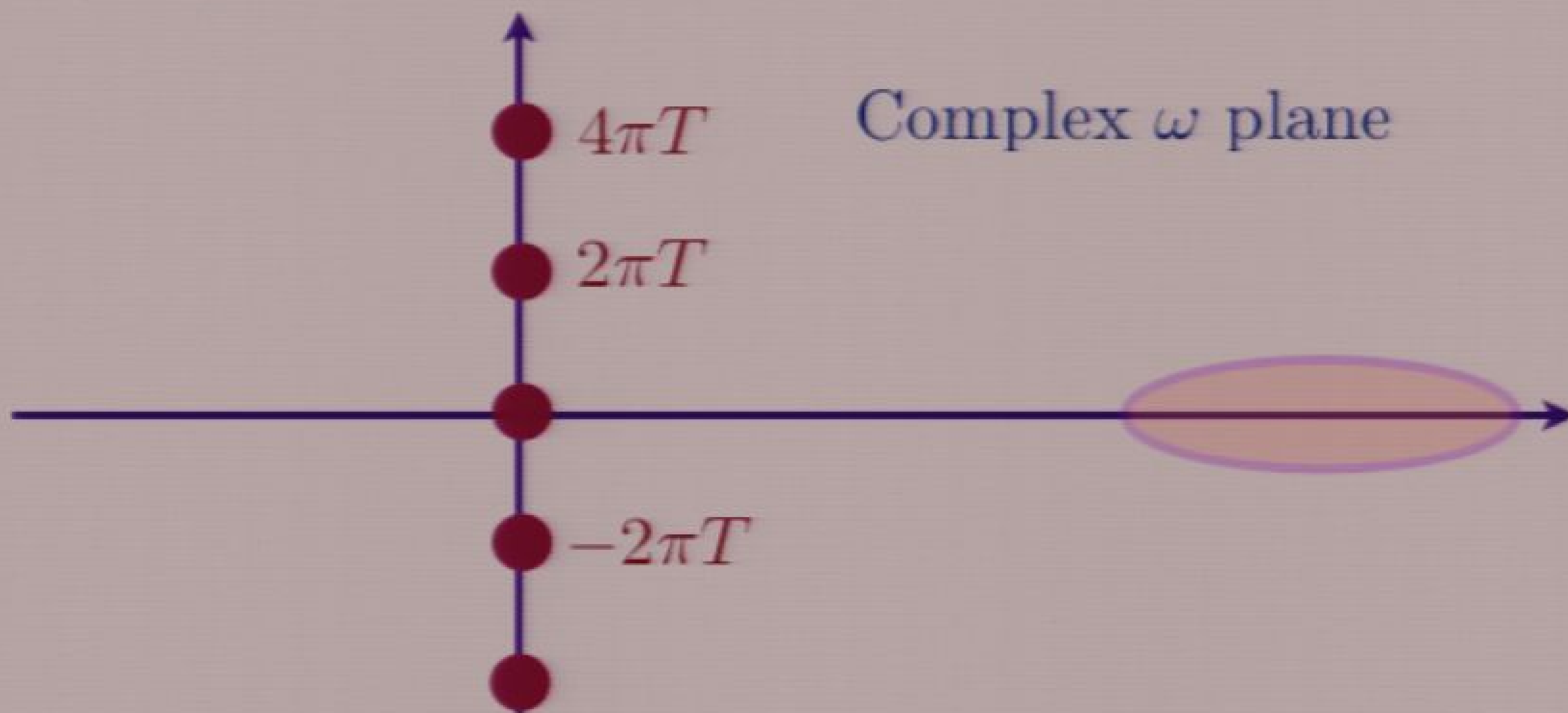

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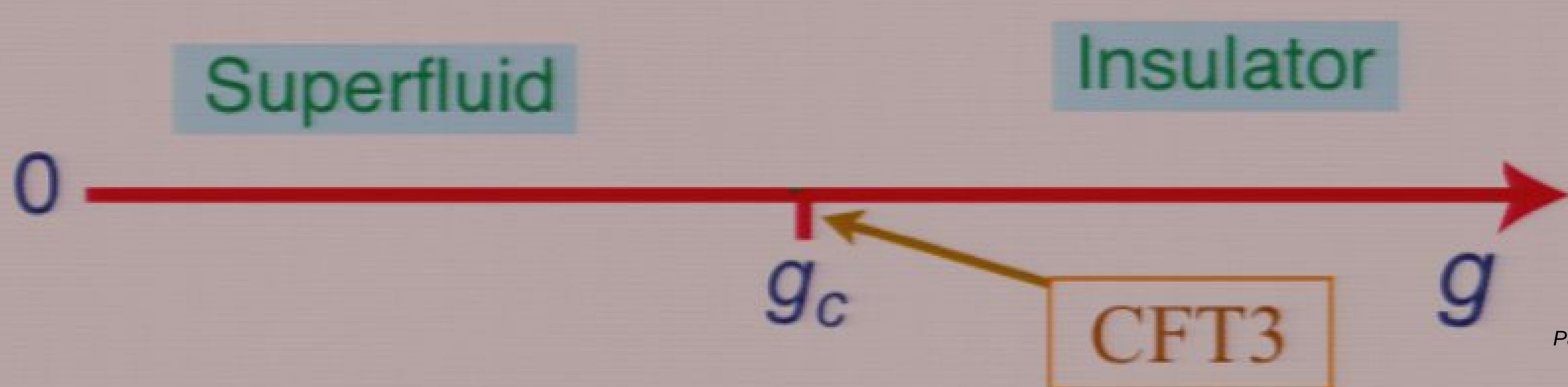

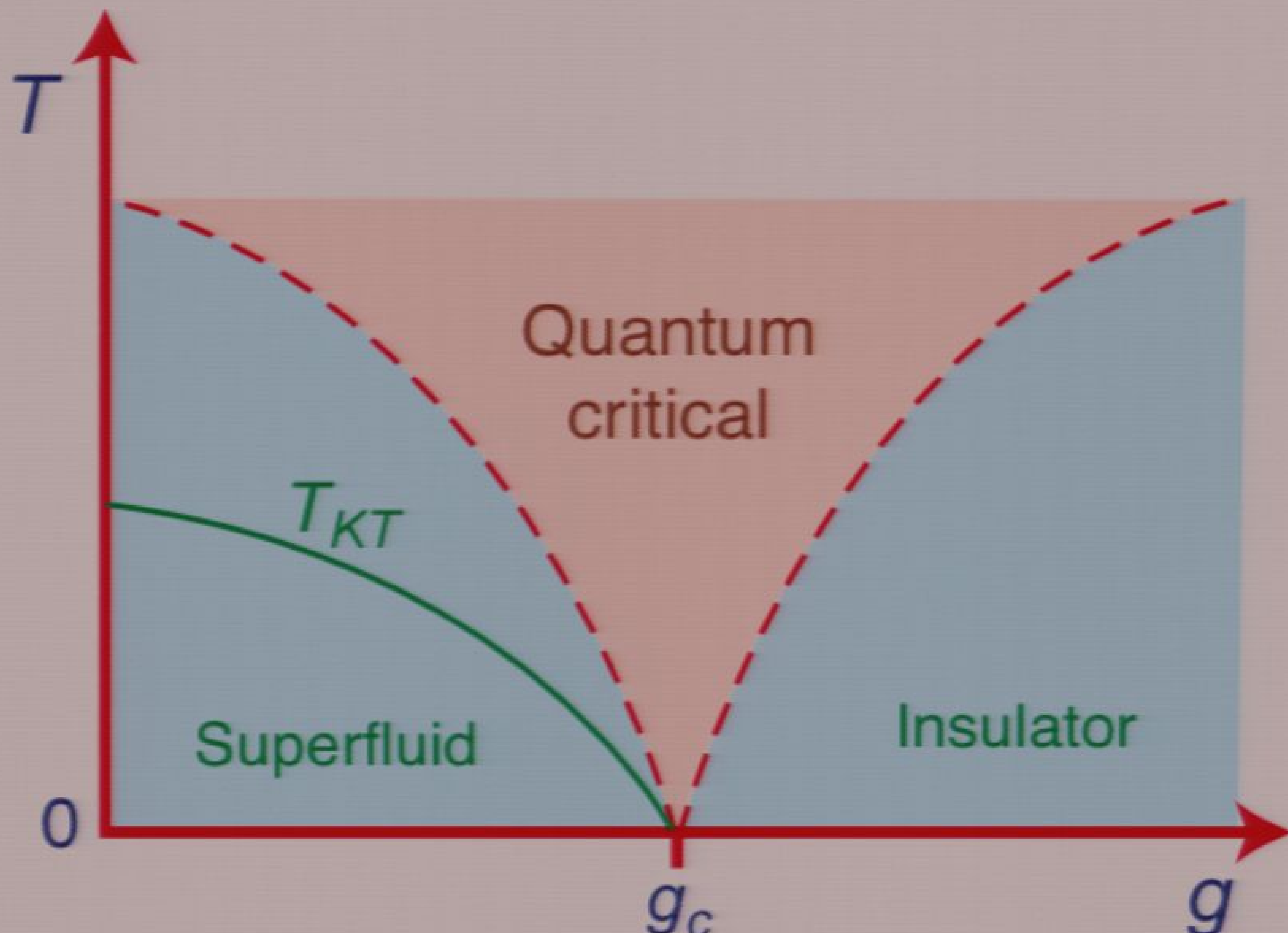
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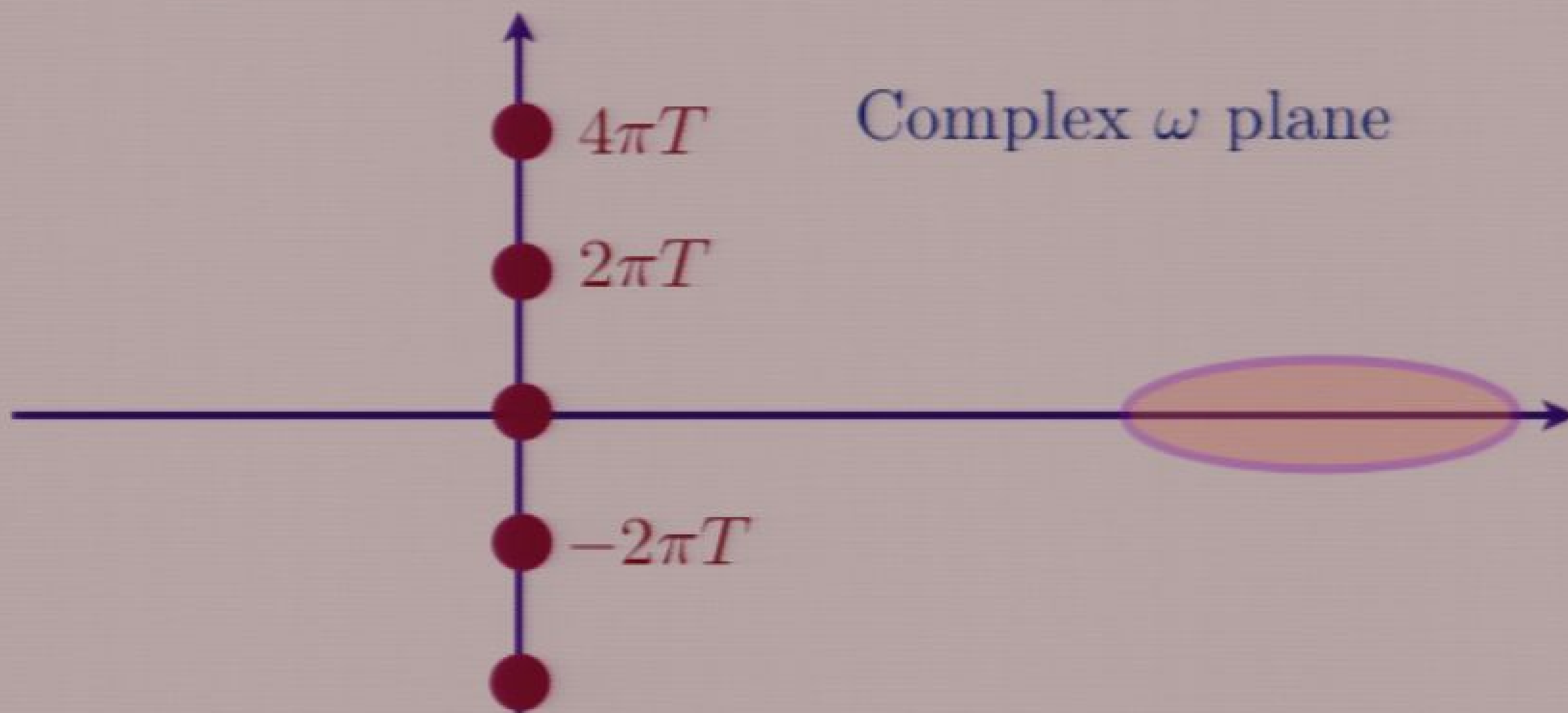

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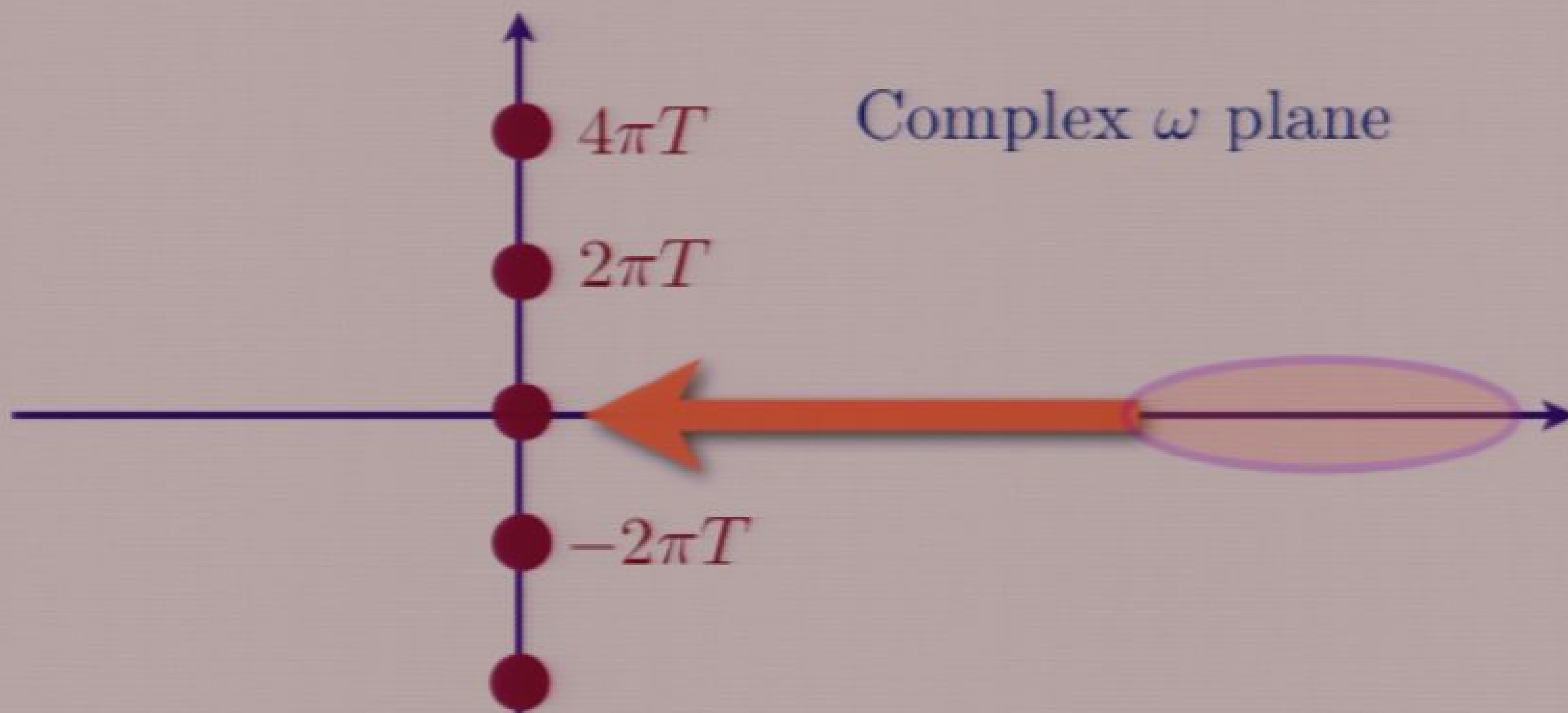

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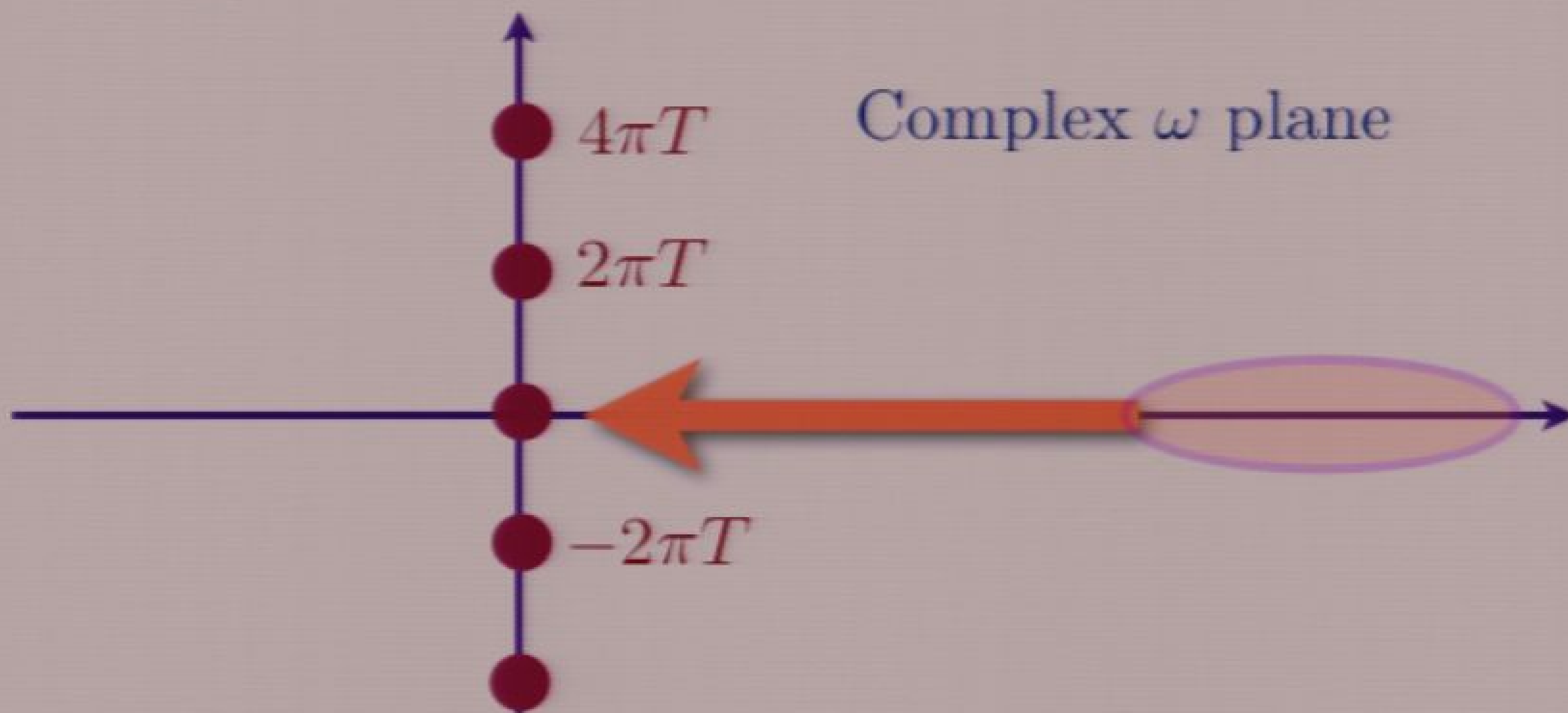

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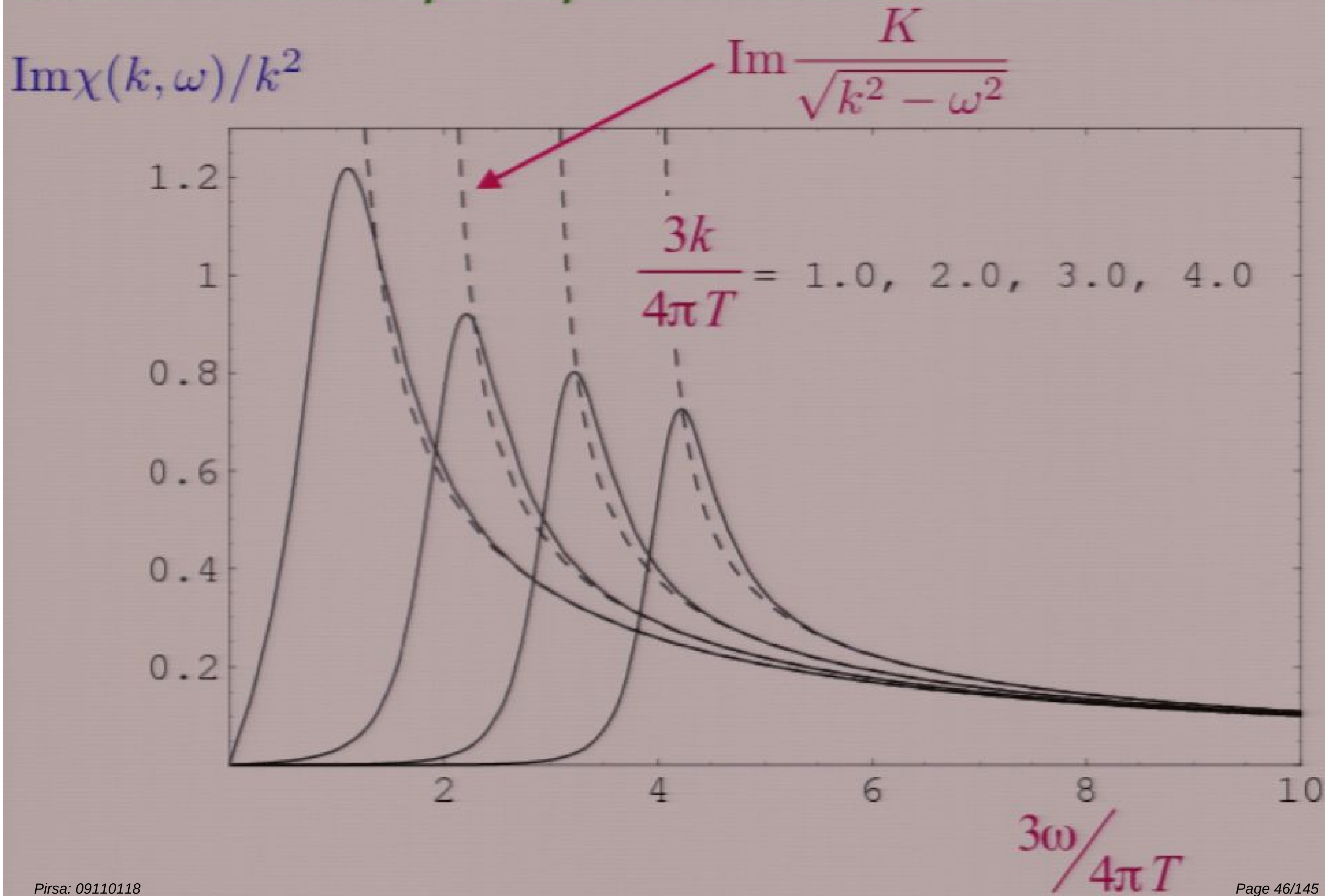

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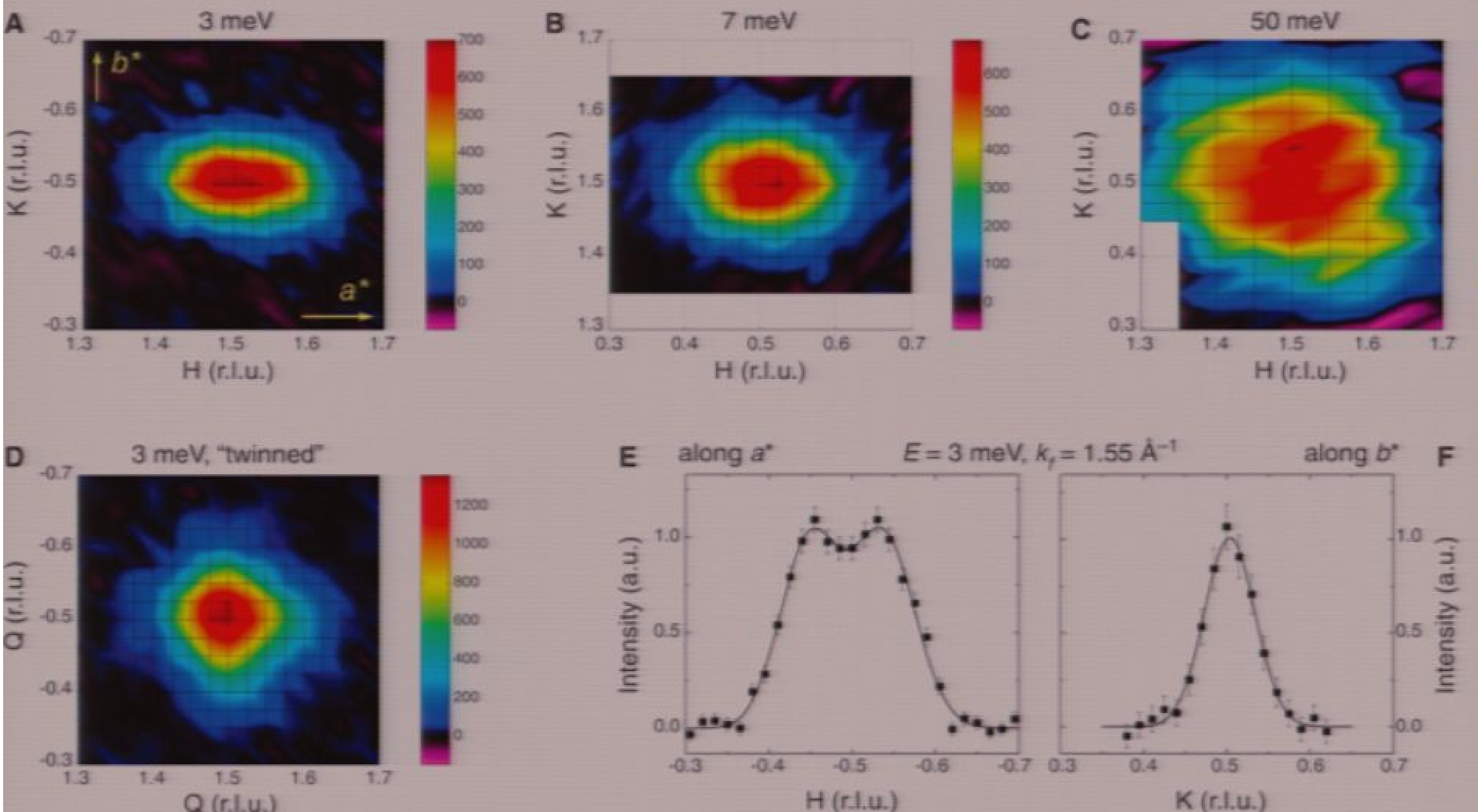

Nematic order in YBCO

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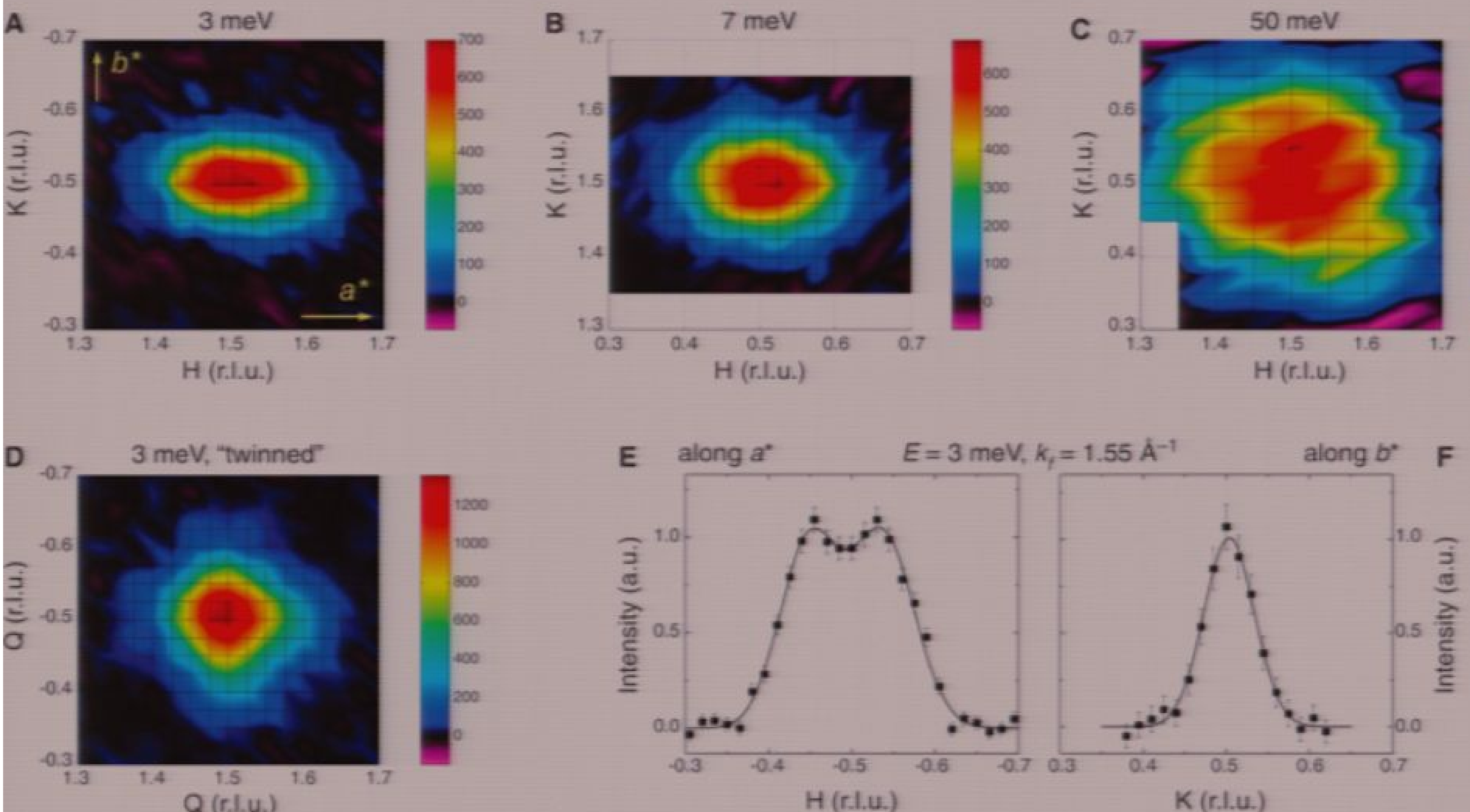

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ʒroken rotational symmetry
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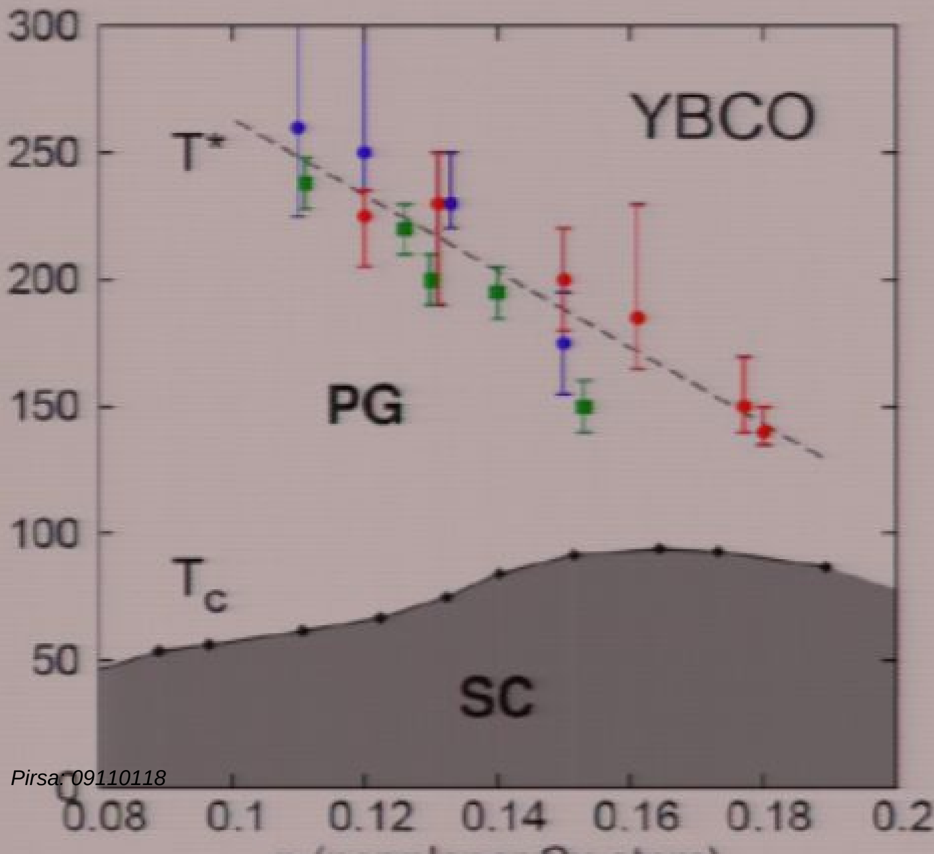
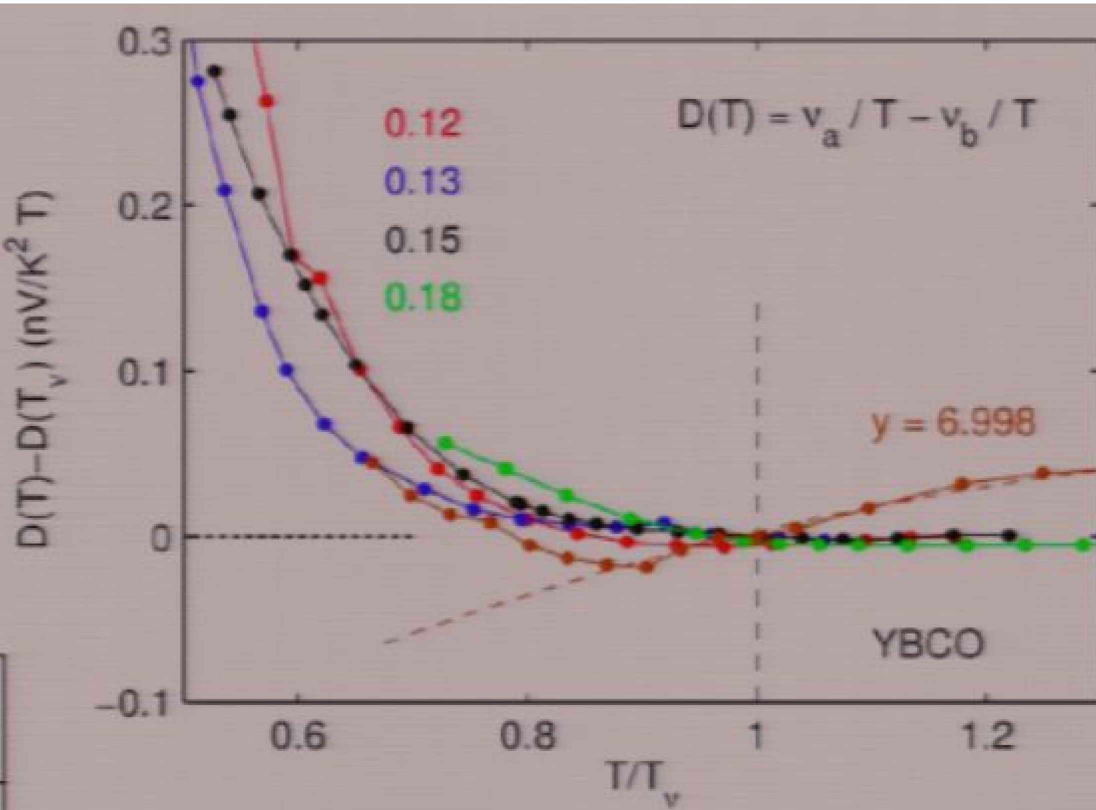
. Daou, J. Chang, David LeBoeuf, Olivier Cyr-
ʰhoiniere, Francis Laliberte, Nicolas Doiron-
eyraud, B. J. Ramshaw, Ruixing Liang,
.A. Bonn, W. N. Hardy, and Louis Taillefer
-Xiv: 0909.4430





S. A. Kivelson, E. Fradkin, and
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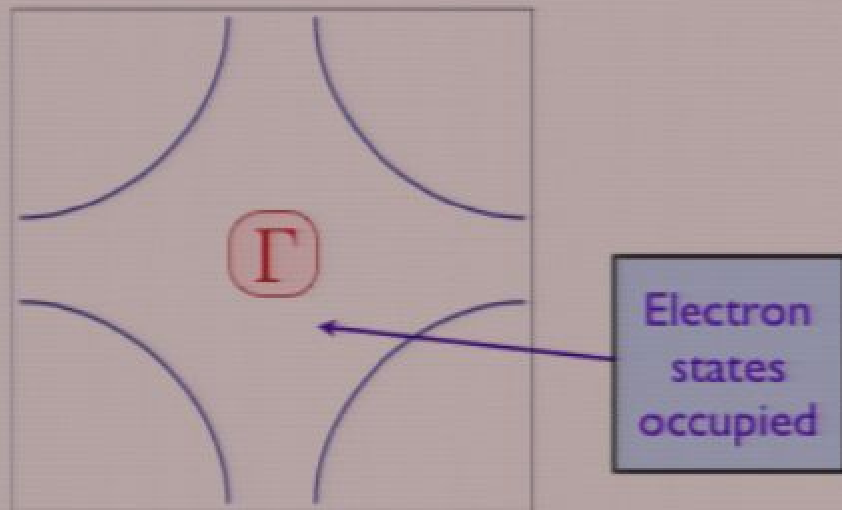
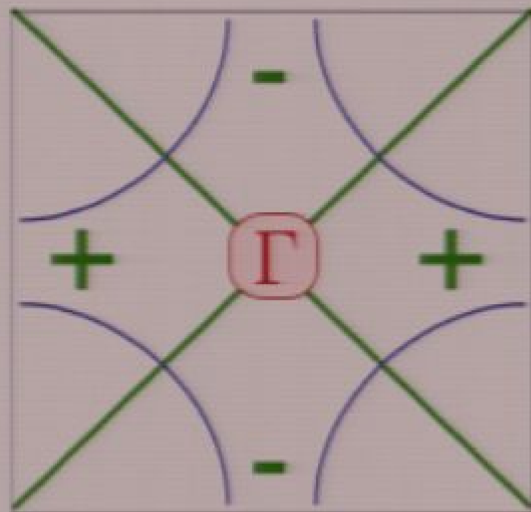
d-wave superconductivity in cuprates



$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

- Begin with free electrons.

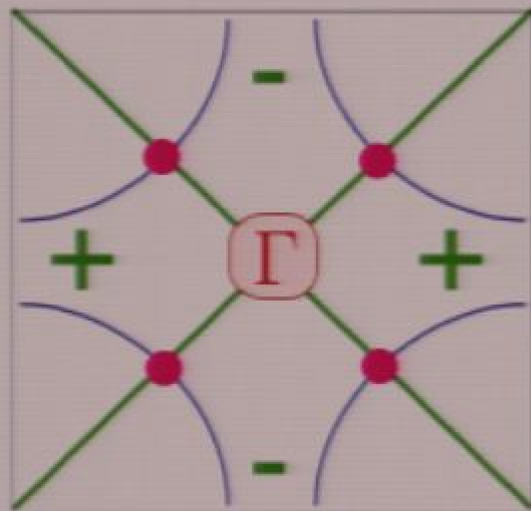
d-wave superconductivity in cuprates



$$H = \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \Delta_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} + \text{c.c.} \right)$$

- Begin with free electrons.
- Add d-wave pairing interaction $\Delta_k \sim \cos k_x - \cos k_y$ which vanishes along diagonals

d-wave superconductivity in cuprates



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- Begin with free electrons.
- Add d-wave pairing interaction Δ_k which vanishes along diagonals
- Obtain Bogoliubov quasiparticles with dispersion $\sqrt{\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$

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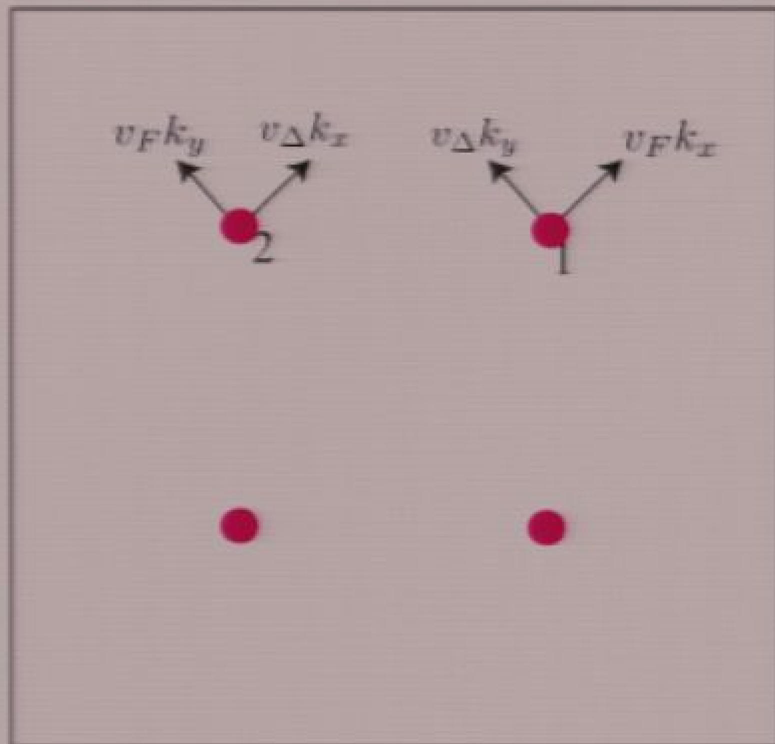


4 two-component Dirac fermions

$$S_\Psi = \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi^\dagger_{1a} \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x\right) \Psi_{1a}$$

$$+ \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi^\dagger_{2a} \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x\right) \Psi_{2a}$$

d-wave superconductivity in cuprates

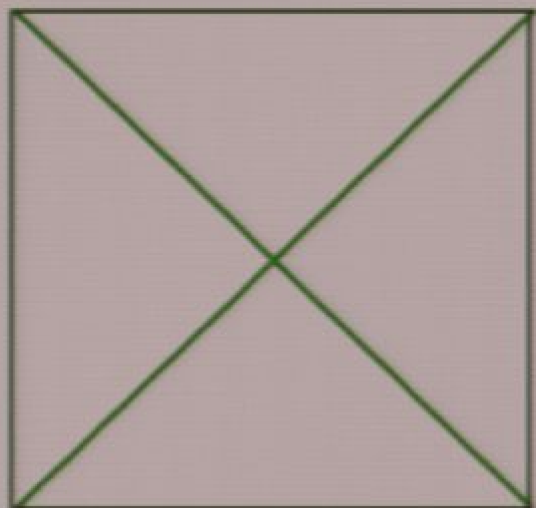
Now consider a discrete spontaneous symmetry breaking, with Ising symmetry, described by a real scalar field ϕ.
Two cases of experimental interest are:

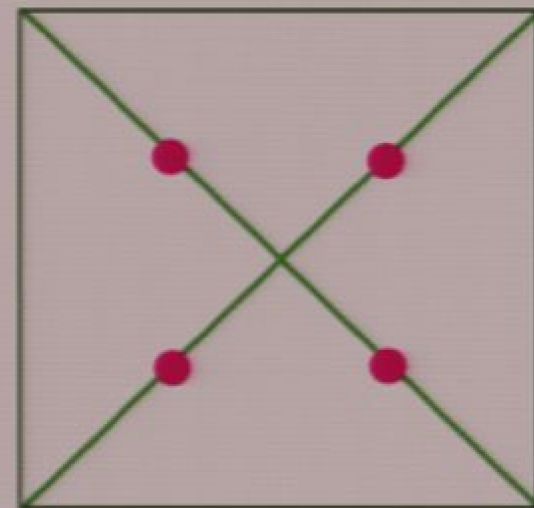
- Time-reversal symmetry breaking: leads to a $d_{x^2-y^2} + id_{xy}$ superconductor, in which the Dirac fermions are massive
- Break 4-fold lattice rotation symmetry to 2-fold lattice rotations: leads to a superconductor with **nematic** order.

We can write down the usual ϕ^4 theory for the scalar field:

$$S_\phi^0 = \int d^2x d\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla \phi)^2 + \frac{r}{2}\phi^2 + \frac{u_0}{24}\phi^4 \right]$$

Time-reversal symmetry breaking





$d_{x^2-y^2} \pm i d_{xy}$ superconductor

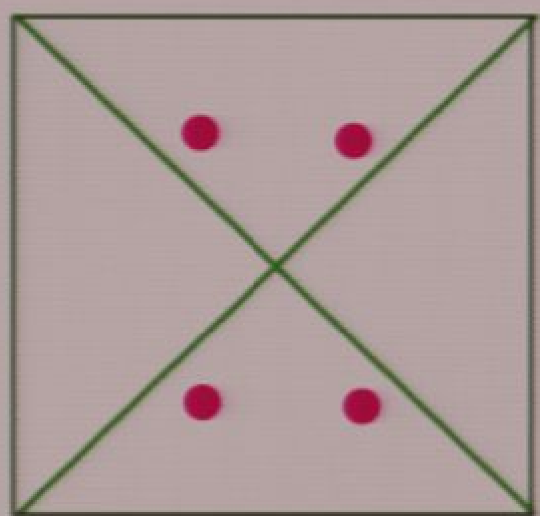
$\langle \phi \rangle \neq 0$

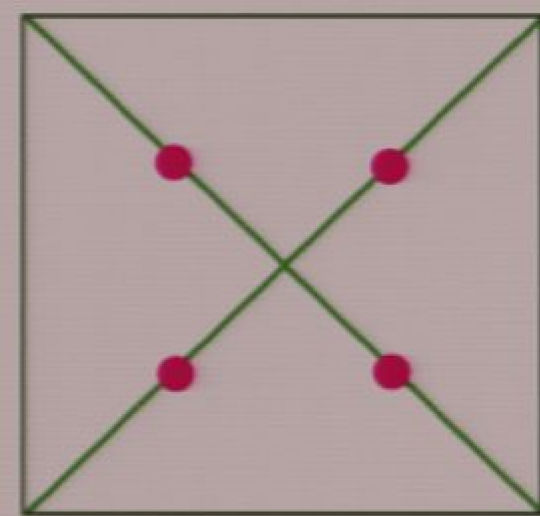
$d_{x^2-y^2}$ superconductor

$\langle \phi \rangle = 0$

r_c r

Lattice rotation symmetry breaking





$d_{x^2-y^2}$ superconductor
+ nematic order

$\langle \phi \rangle \neq 0$

$d_{x^2-y^2}$ superconductor

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r_c r

Ising order and Dirac fermions couple via a "Yukawa" term.

$$S_{\Psi\phi} = \int d^2x d\tau \left[\lambda_0 \phi \left(\Psi_{1a}^{\dagger} \tau^x \Psi_{1a} + \Psi_{2a}^{\dagger} \tau^x \Psi_{2a} \right) \right],$$

Nematic ordering

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M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

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Time reversal symmetry breaking

For the latter case *only*, with $v_F = v_\Delta = c$, theory reduces to relativistic Gross-Neveu model

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the scalar order parameter

$$S_\phi = \frac{N_f}{v_\Delta v_F}\Gamma\left[\lambda_0\phi(x,\tau);\frac{v_\Delta}{v_F}\right] + \frac{N_f}{2}\int d^2x d\tau\Big(r\phi^2(x,\tau)\Big) + \text{irrelevant terms}$$

where Γ is a non-local and non-analytic functional of ϕ.

The theory has only 2 couplings constants: r and v_Δ/v_F.

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There is a systematic expansion in powers of $1/N_f$ for renormalization group equations and all critical properties.

1. Quantum-critical transport

   Collisionless-to-hydrodynamic crossover of CFT3s

2. Quantum criticality of Dirac fermions

   "Vector" 1/N expansion

3. Quantum criticality of Fermi surfaces

   The genus expansion

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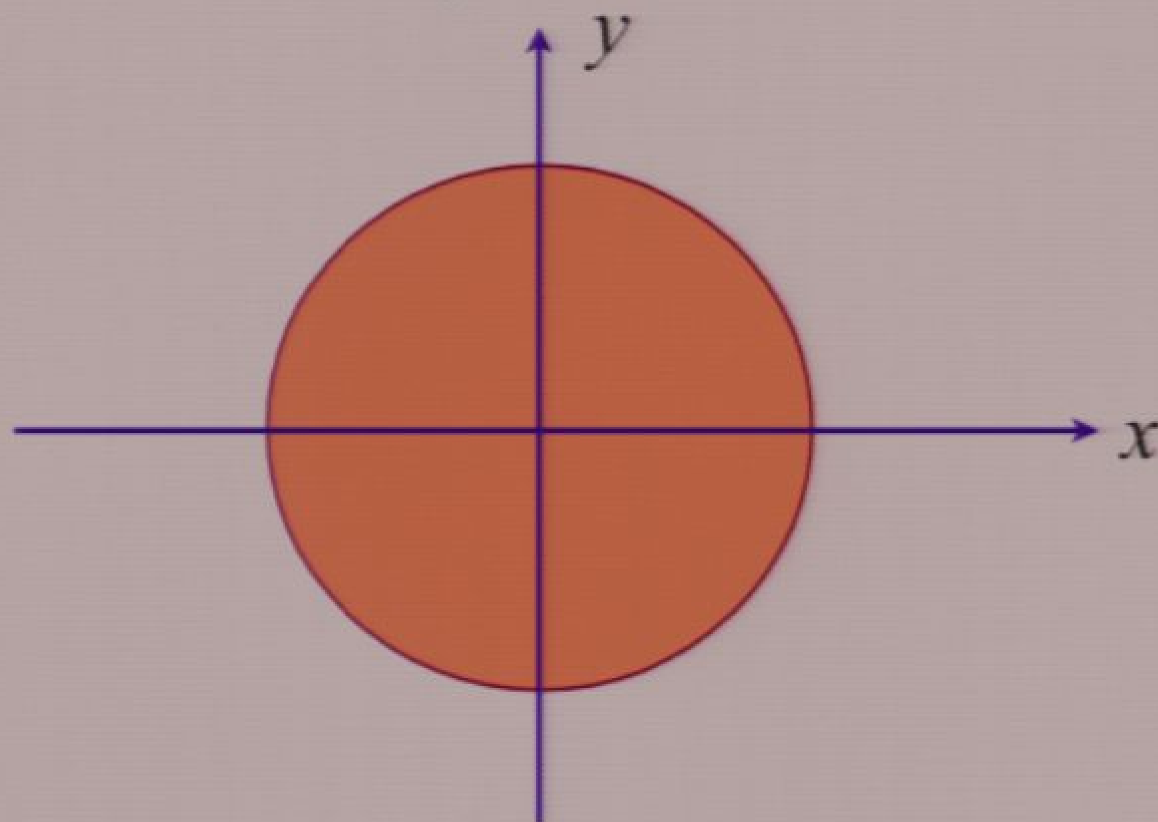
Quantum criticality of Pomeranchuk instability



Fermi surface with full square lattice symmetry

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$$
\begin{aligned}
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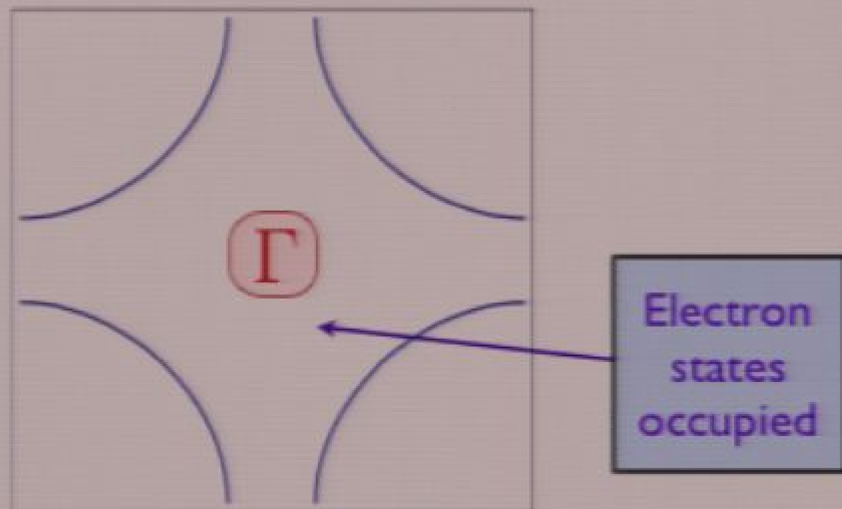
d-wave superconductivity in cuprates



$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

- Begin with free electrons.

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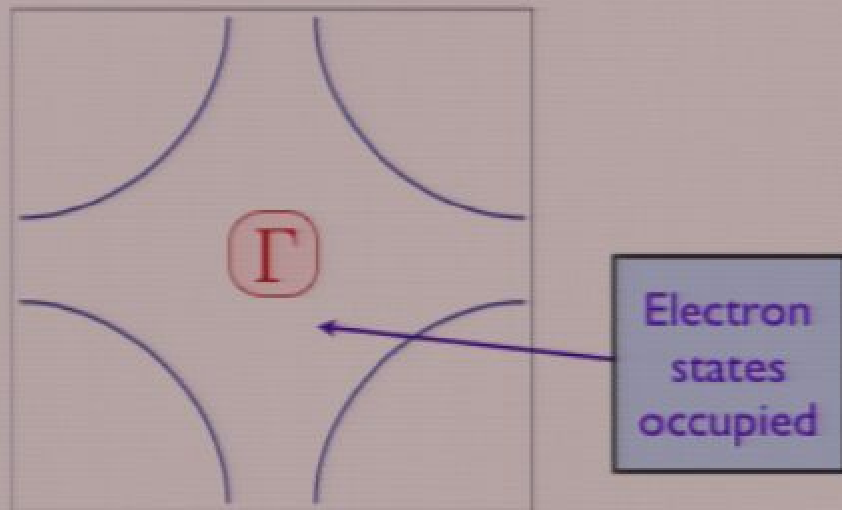

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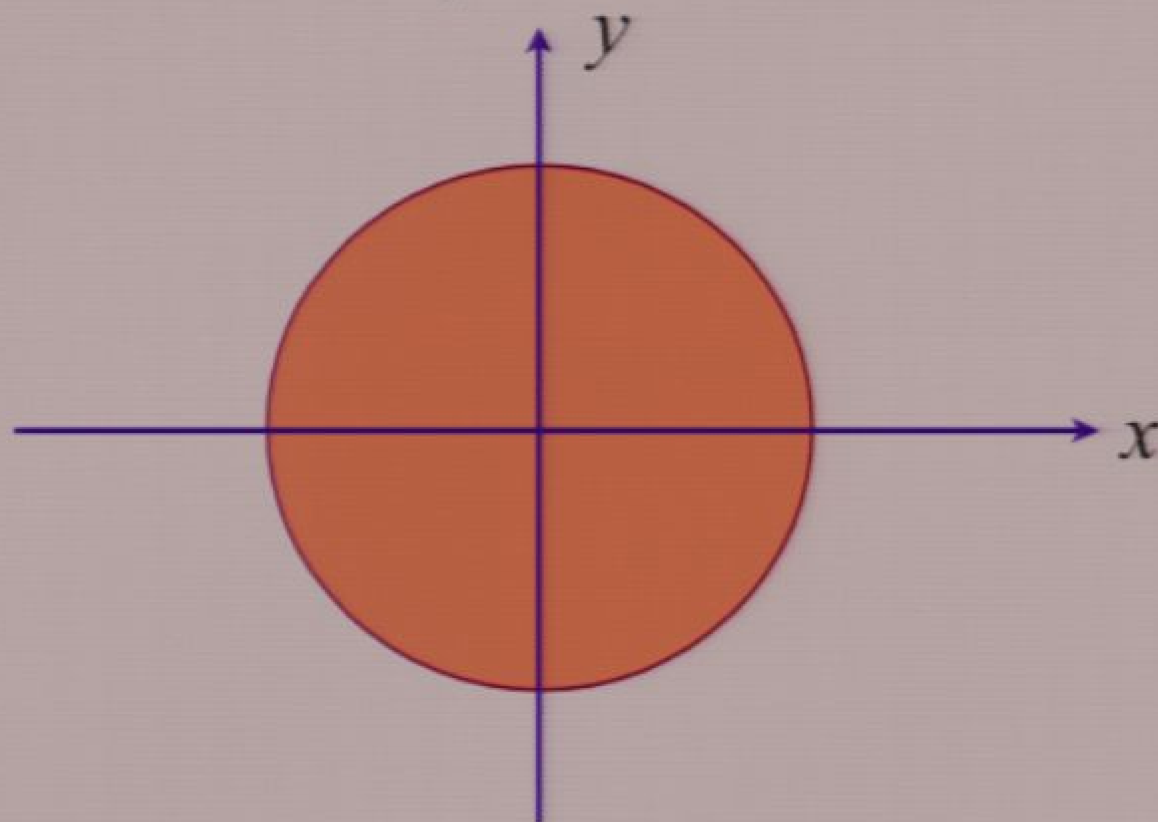
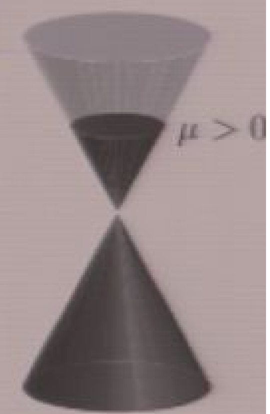
Quantum criticality of Pomeranchuk instability



Fermi surface with full square lattice symmetry

Electron Green's function in Fermi liquid (T=0)

$$G(k,\omega) = \frac{Z}{\omega - v_F(k-k_F) - i\omega^2 \mathcal{F}\left(\frac{k-k_F}{\omega}\right)} + \dots$$



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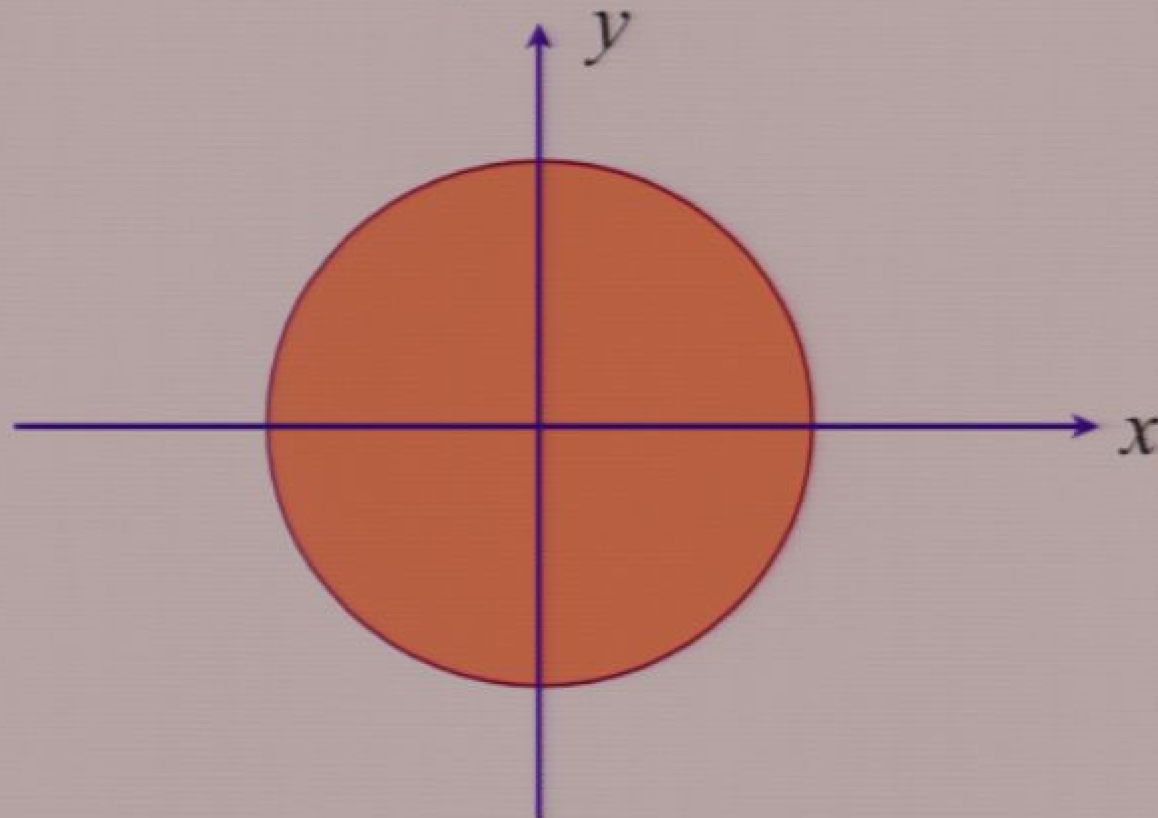

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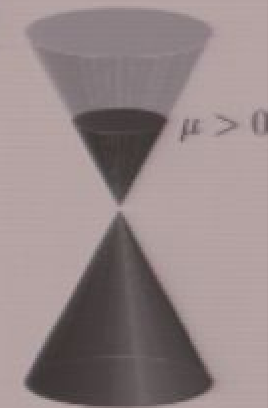

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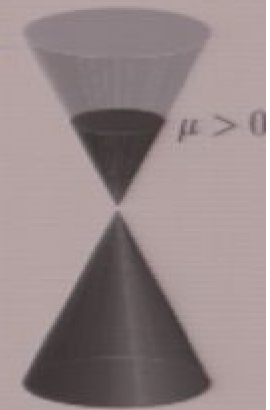
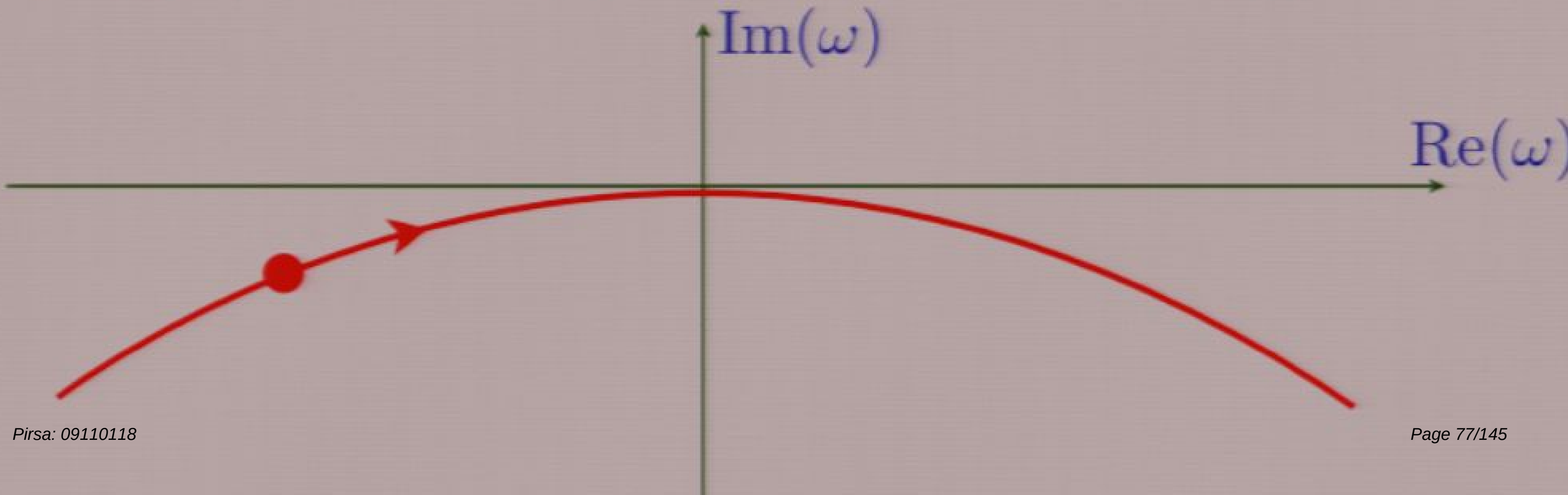


$$G(k,\omega) = \frac{Z}{\omega - v_F(k-k_F) - i\omega^2 \mathcal{F}\left(\frac{k-k_F}{\omega}\right)} + \ldots$$

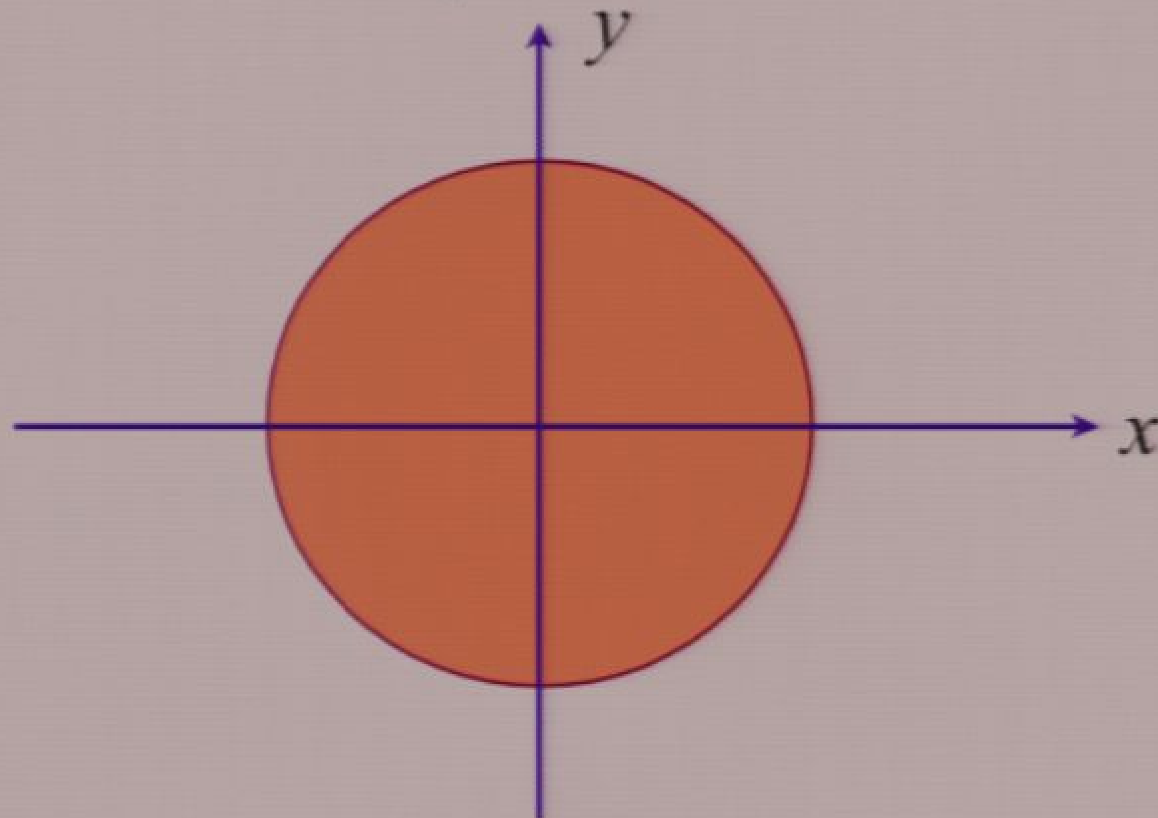
Green's function has a pole in the LHP at

$$\omega = v_F(k-k_F) - i\alpha(k-k_F)^2 + \ldots$$

Pole is at $\omega = 0$ precisely at $k = k_F$ *i.e.* on a sphere of radius k_F in momentum space. This is the *Fermi surface.*



Quantum criticality of Pomeranchuk instability



Fermi surface with full square lattice symmetry

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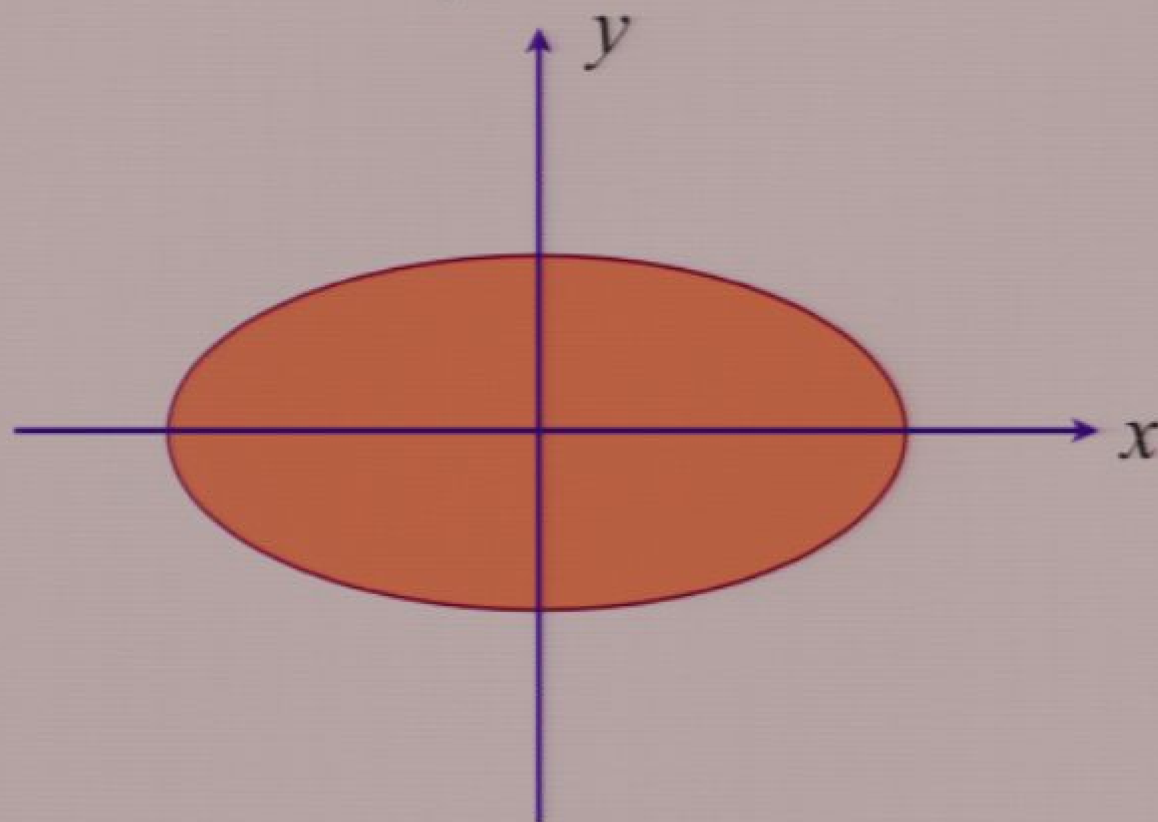
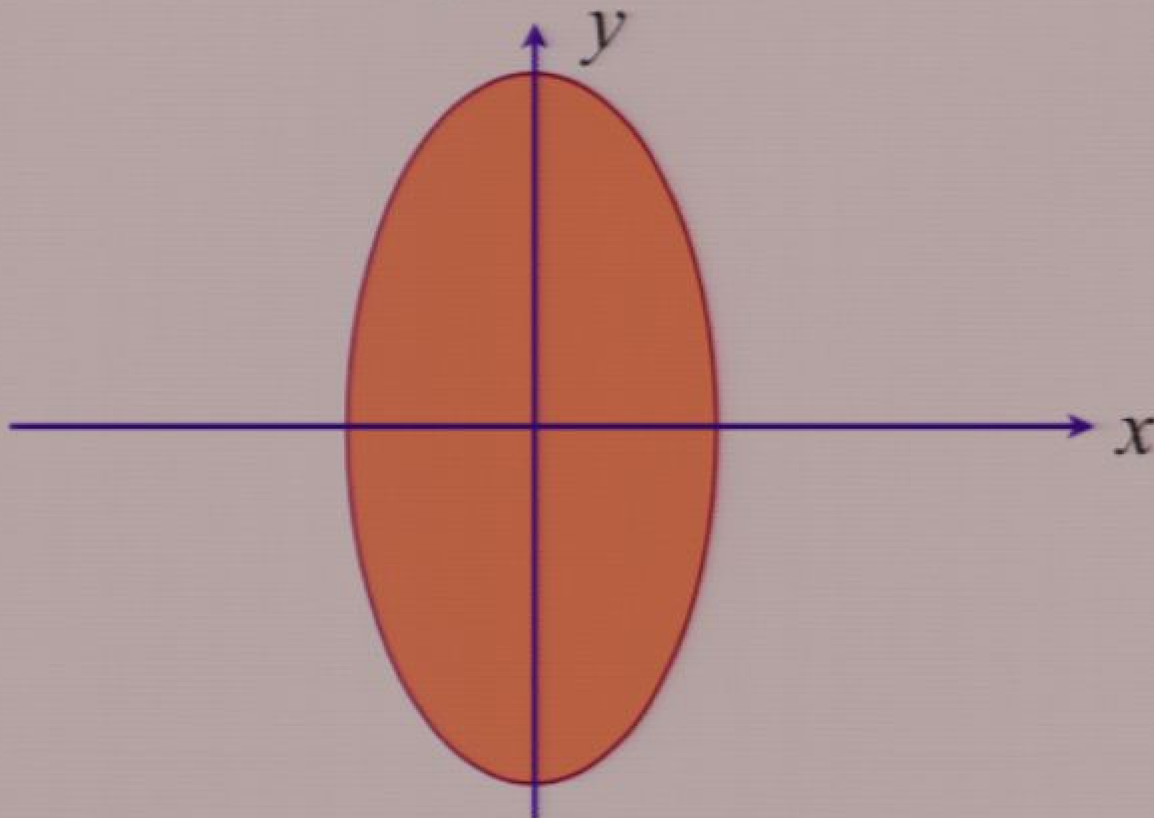


Spontaneous elongation along x direction:
Ising order parameter $\phi > 0$.

Quantum criticality of Pomeranchuk instability



Spontaneous elongation along y direction:
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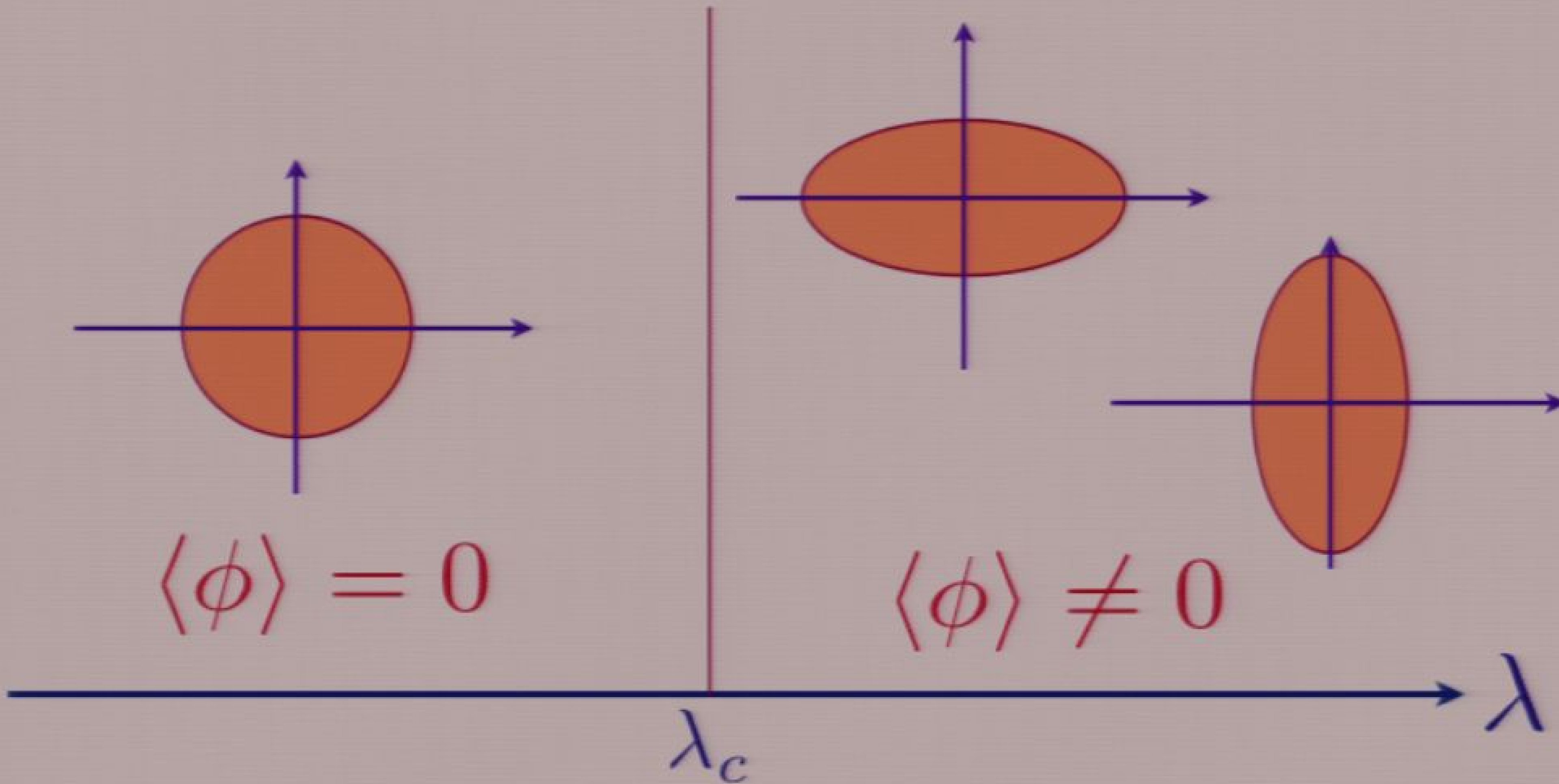
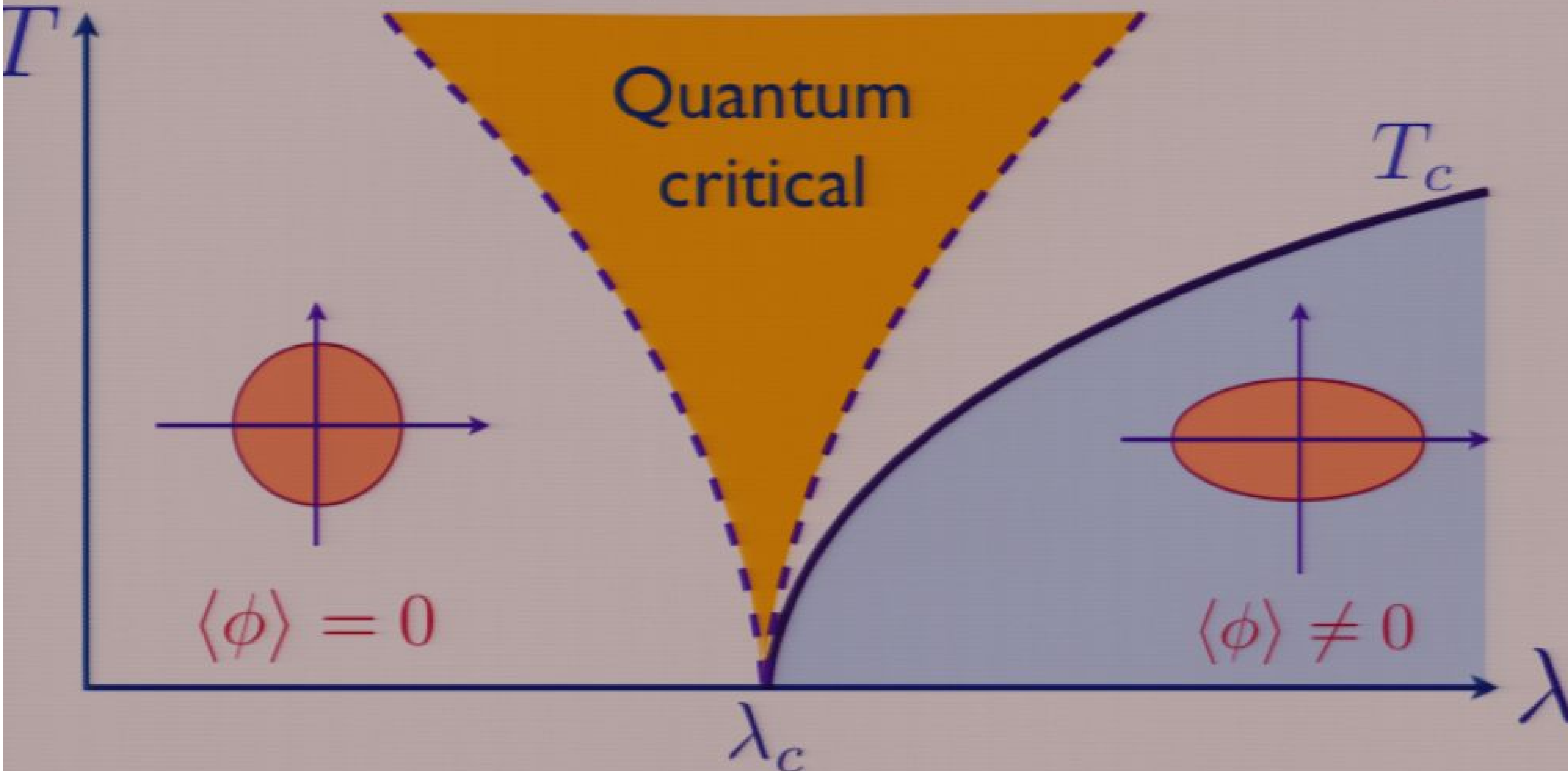
Quantum criticality of Pomeranchuk instability



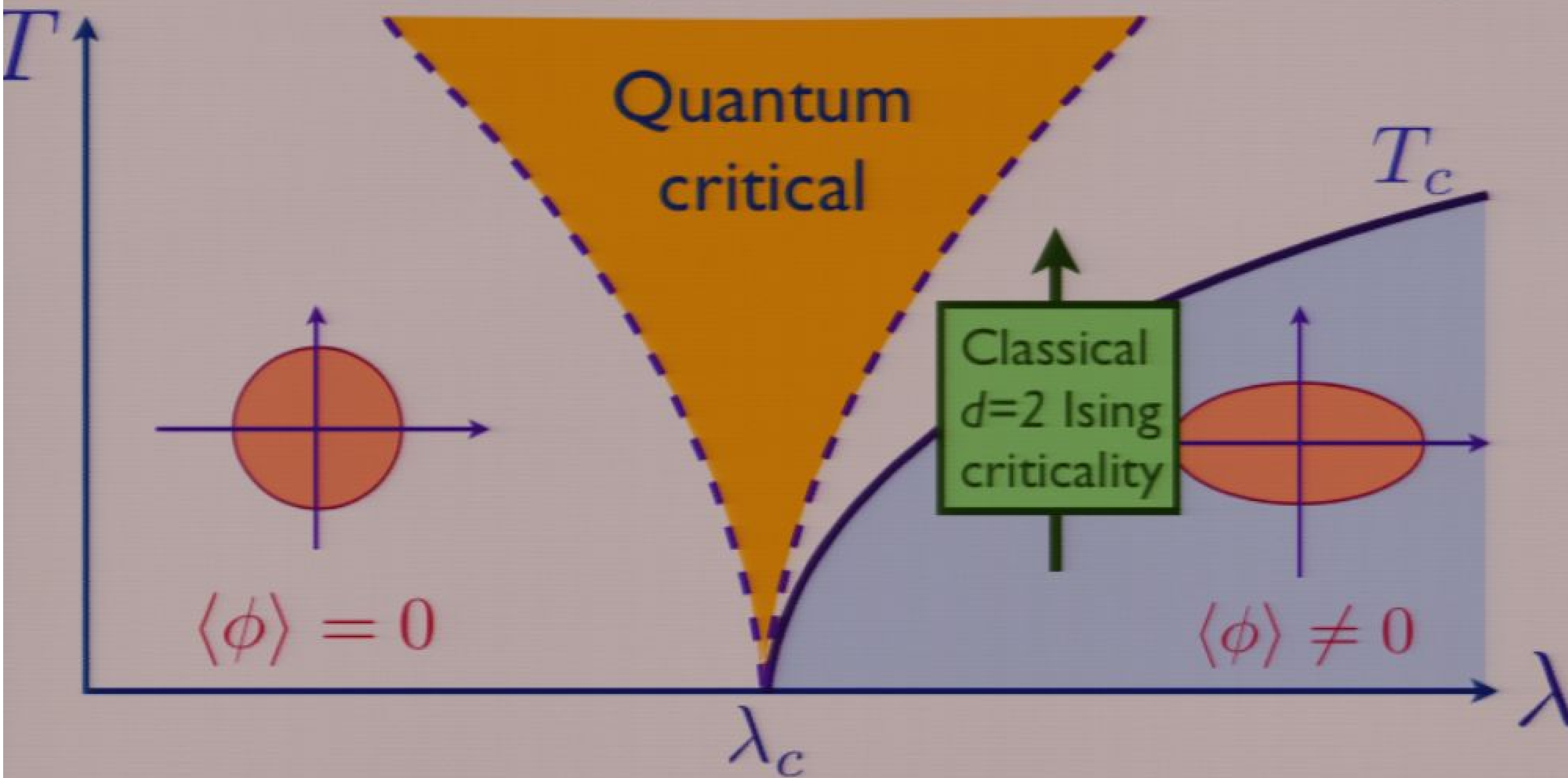
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Pomeranchuk instability



Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability



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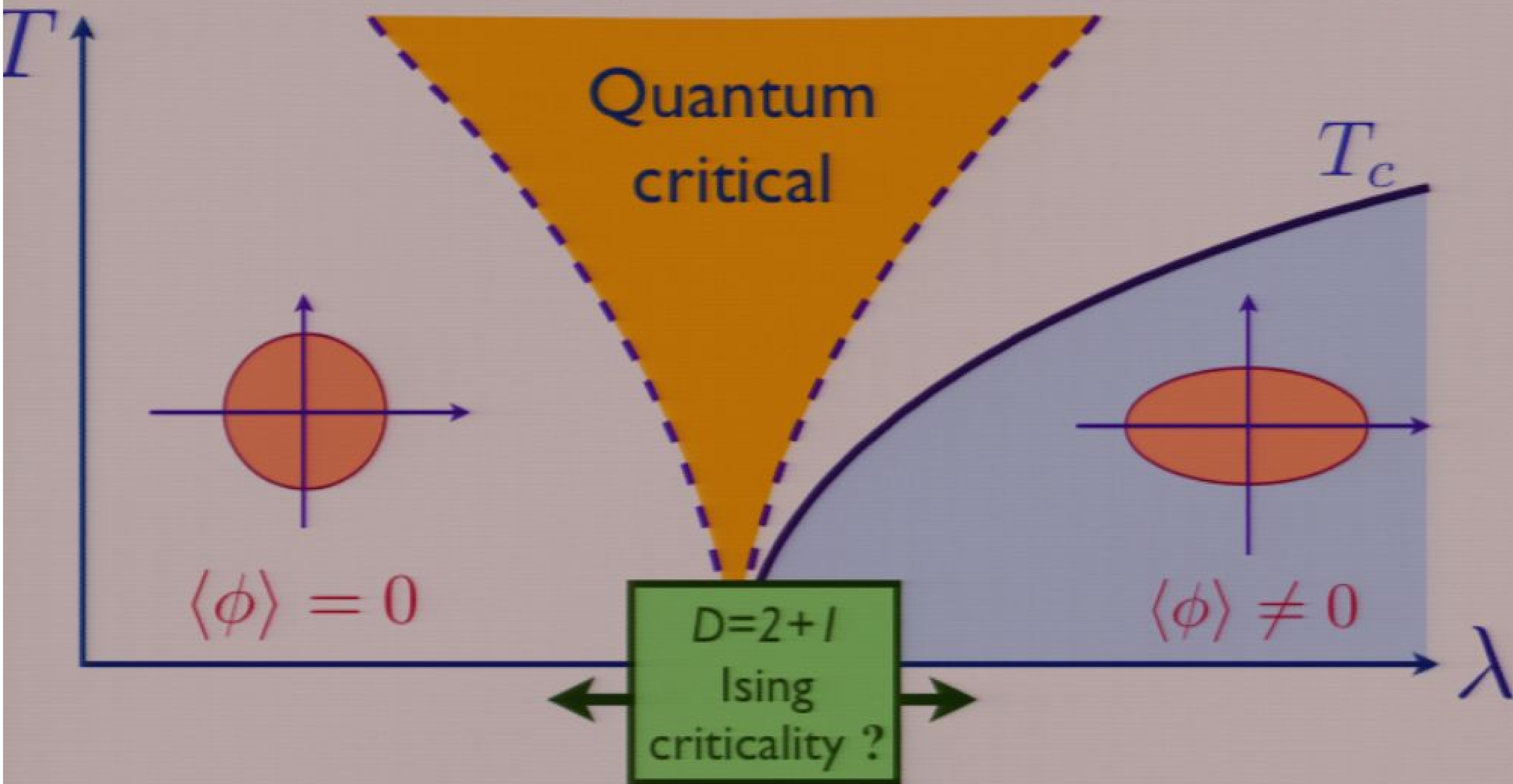

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Quantum criticality of Pomeranchuk instability

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

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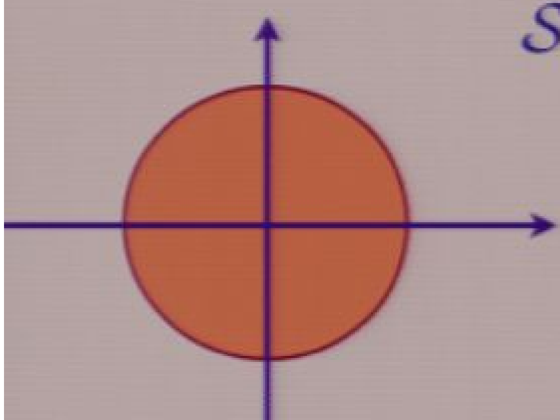
Effective action for electrons:



$$\mathcal{S}_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha}\right]$$

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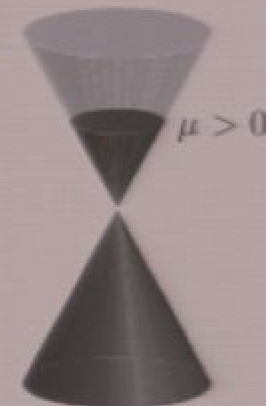

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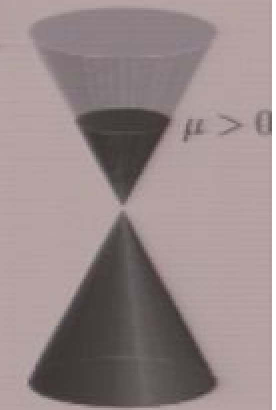
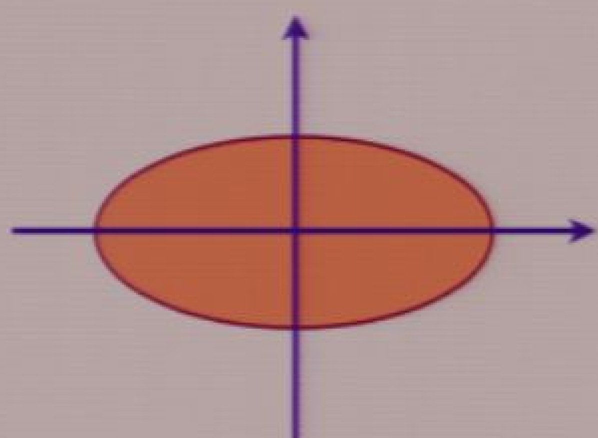


Quantum criticality of Pomeranchuk instability

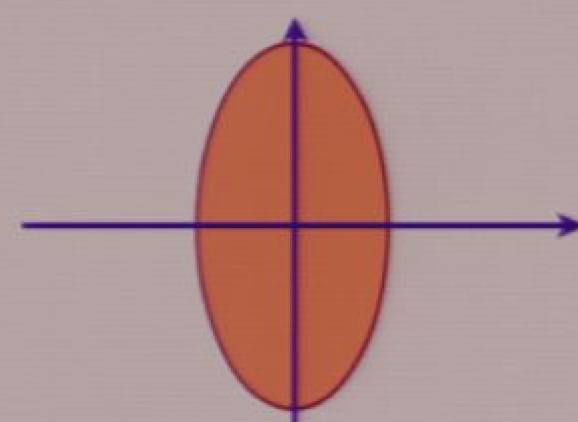
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau\, \phi \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} (\cos k_x - \cos k_y) c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

for spatially independent ϕ



$\langle \phi \rangle > 0$



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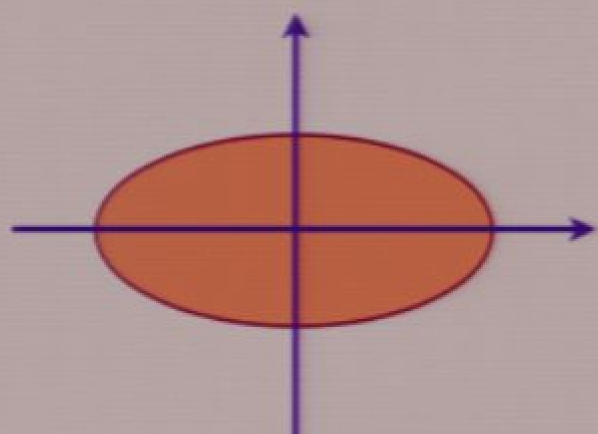

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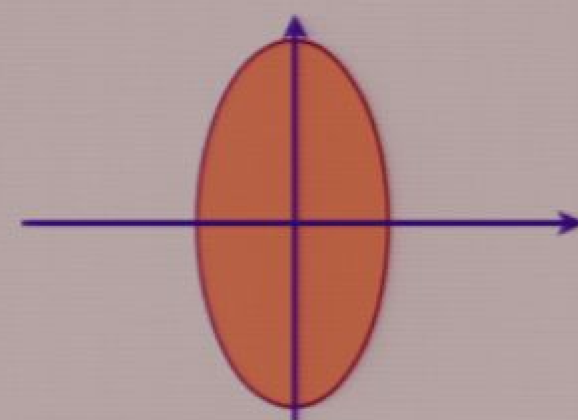

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Quantum criticality of Pomeranchuk instability

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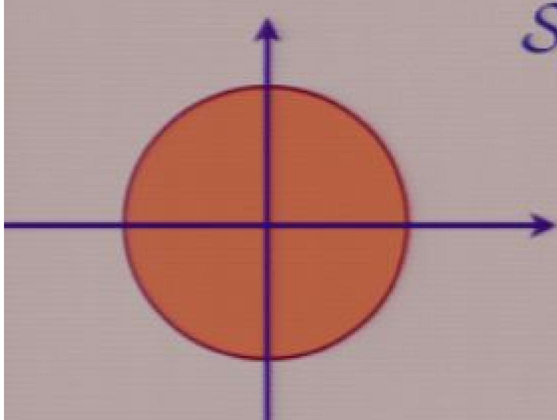

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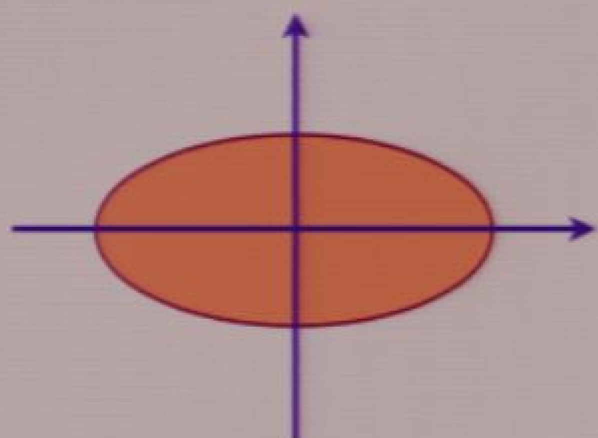

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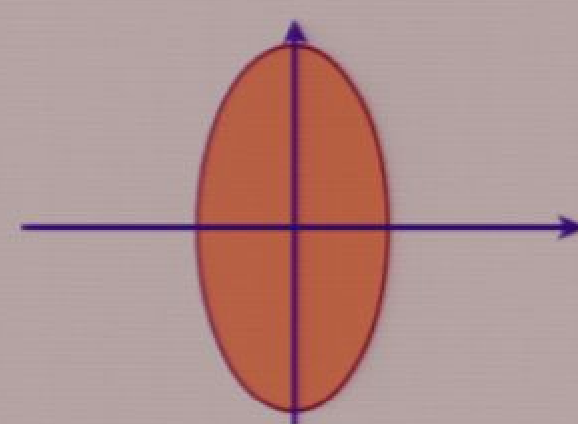

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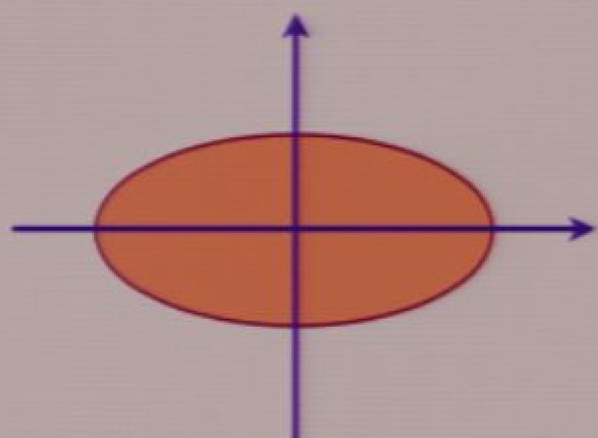

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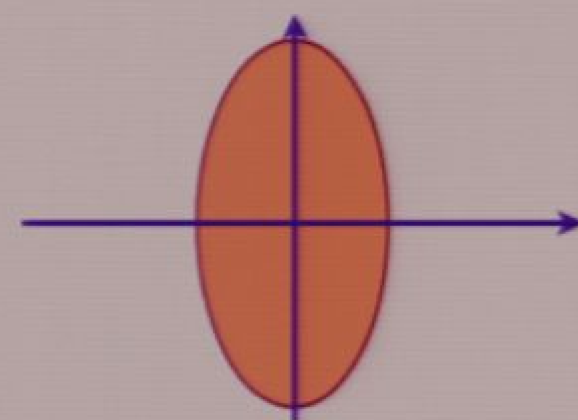

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Quantum critical field theory

$$\mathcal{Z} = \int \mathcal{D}\phi \mathcal{D}c_{i\alpha} \exp\left(-\mathcal{S}_\phi - \mathcal{S}_c - \mathcal{S}_{\phi c} \right)$$

Quantum criticality of Pomeranchuk instability

Hertz theory

Integrate out c_α fermions and obtain non-local corrections to ϕ action

$$\delta\mathcal{S}_\phi \sim N_f\gamma^2 \int \frac{d^2q}{4\pi^2} \int \frac{d\omega}{2\pi} |\phi(\mathbf{q},\omega)|^2 \left[\frac{|\omega|}{q} + q^2\right] + \ldots$$

This leads to a critical point with dynamic critical exponent $z = 3$ and quantum criticality controlled by the Gaussian fixed point.

Quantum criticality of Pomeranchuk instability

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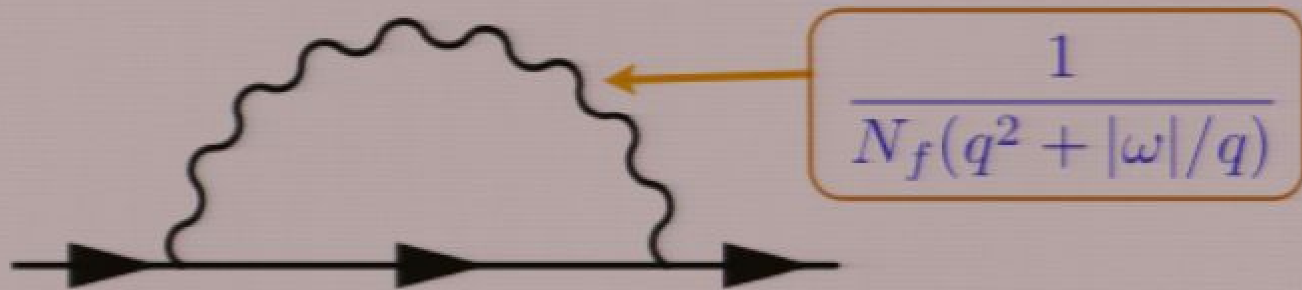

Self energy of c_α fermions to order $1/N_f$

$$\Sigma_c(k,\omega) \sim \frac{i}{N_f}\omega^{2/3}$$

This leads to the Green's function

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - \frac{i}{N_f}\omega^{2/3}}$$

Note that the order $1/N_f$ term is more singular in the infrared than the bare term; this leads to problems in the bare $1/N_f$ expansion in terms that are dominated by low frequency fermions.

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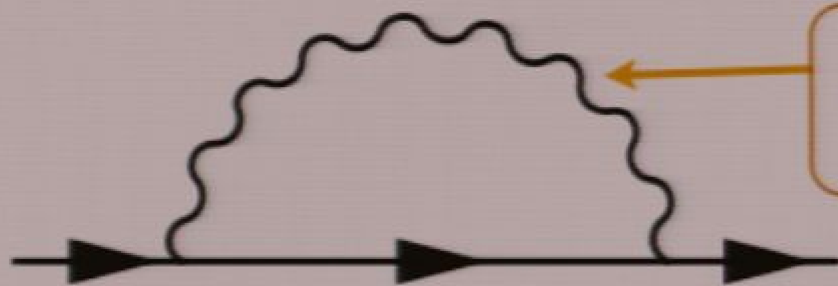
Quantum criticality of Pomeranchuk instability

Hertz theory



$$\frac{1}{N_f(q^2 + |\omega|/q)}$$

Self energy of c_α fermions to order $1/N_f$

$$\Sigma_c(k,\omega) \sim \frac{i}{N_f}\omega^{2/3}$$

This leads to the Green's function

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - \frac{i}{N_f}\omega^{2/3}}$$

Note that the order $1/N_f$ term is more singular in the infrared than the bare term; this leads to problems in the bare $1/N_f$ expansion in terms that are dominated by low frequency fermions.

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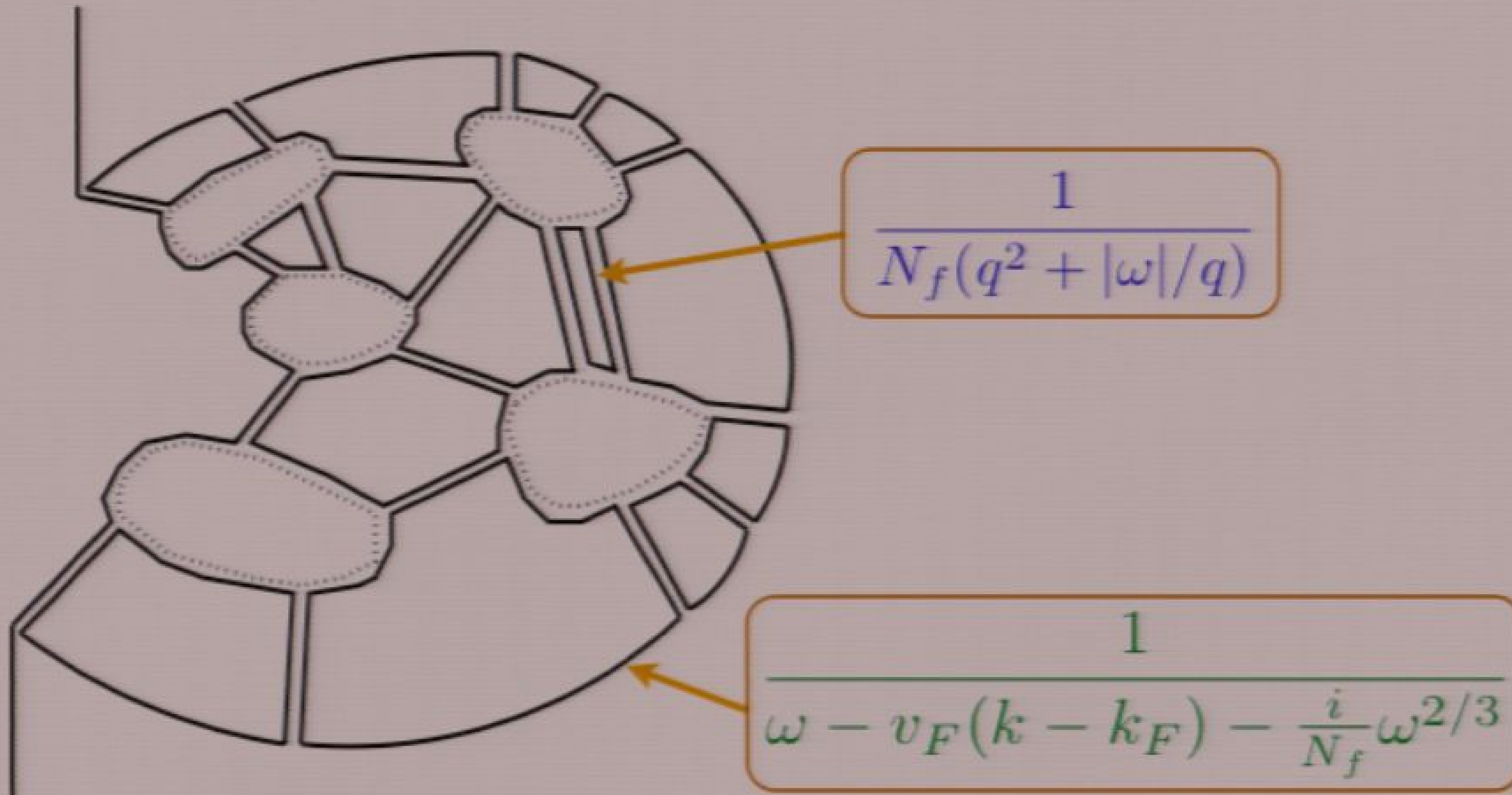
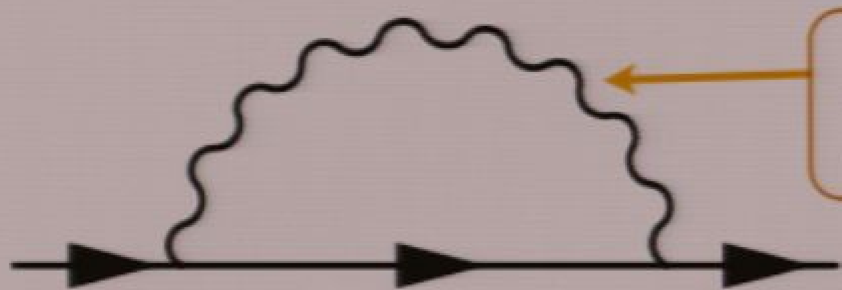


The infrared singularities of fermion particle-hole pairs are most severe on planar graphs: these all contribute at leading order in $1/N_f$.

Sung-Sik Lee, *Physical Review* B **80**, 165102 (2009)

Quantum criticality of Pomeranchuk instability

Hertz theory



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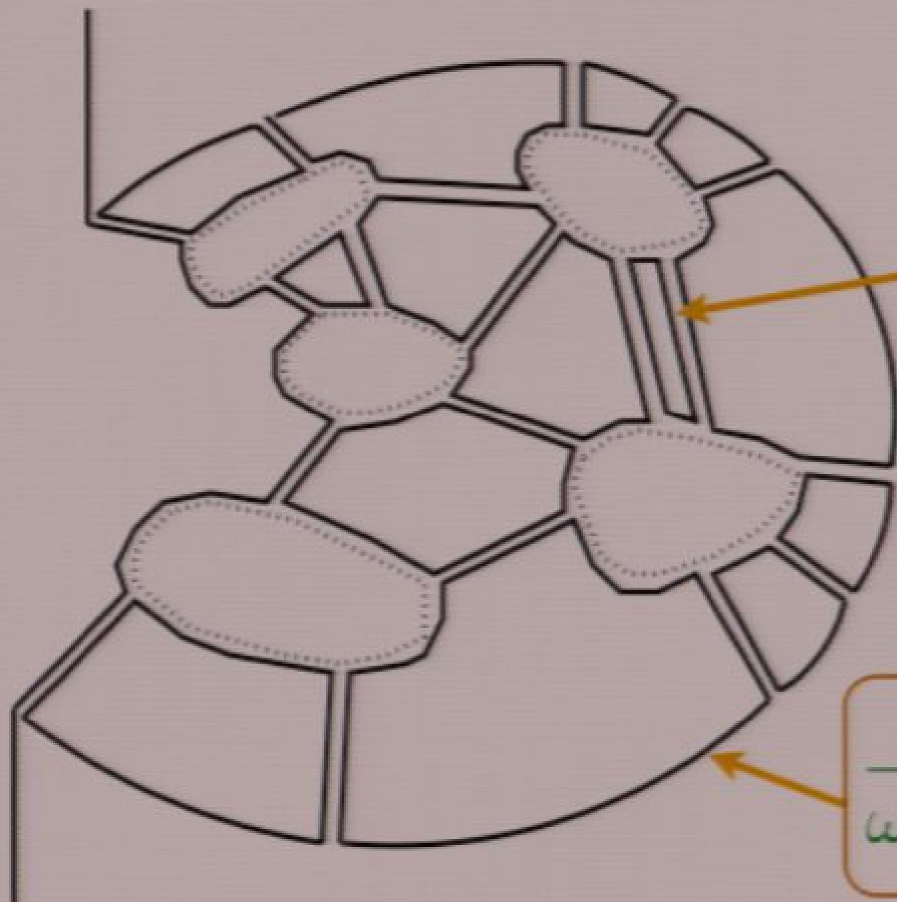

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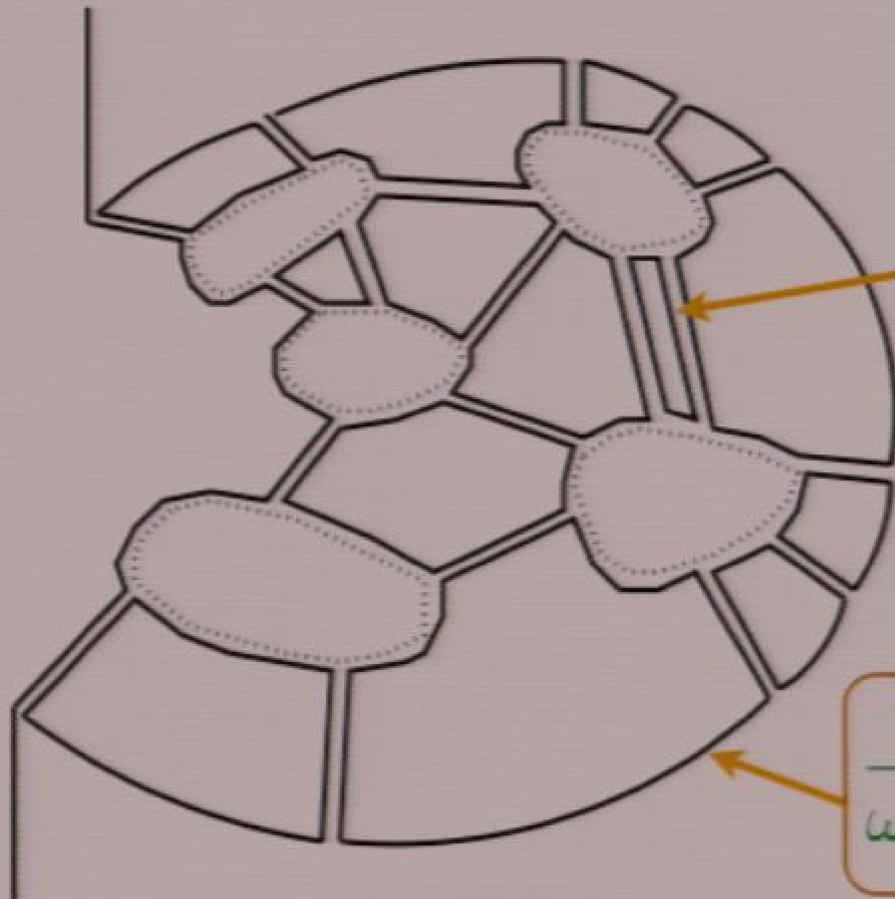

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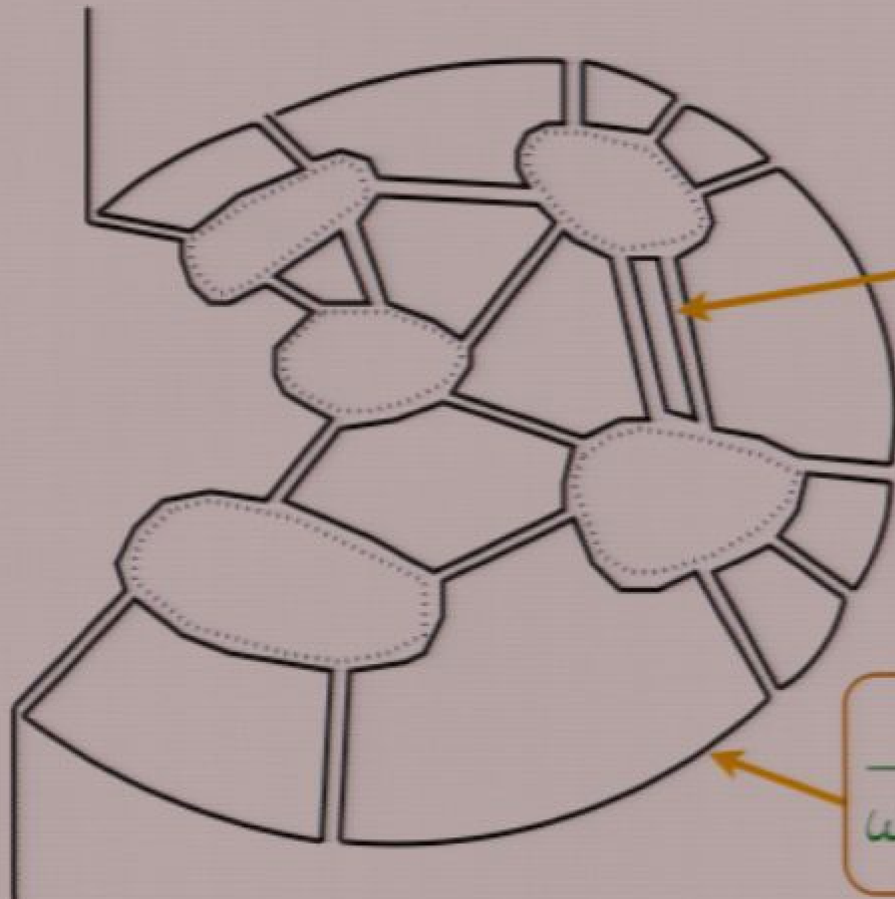
A string theory for the Fermi surface ?

Quantum criticality of Pomeranchuk instability



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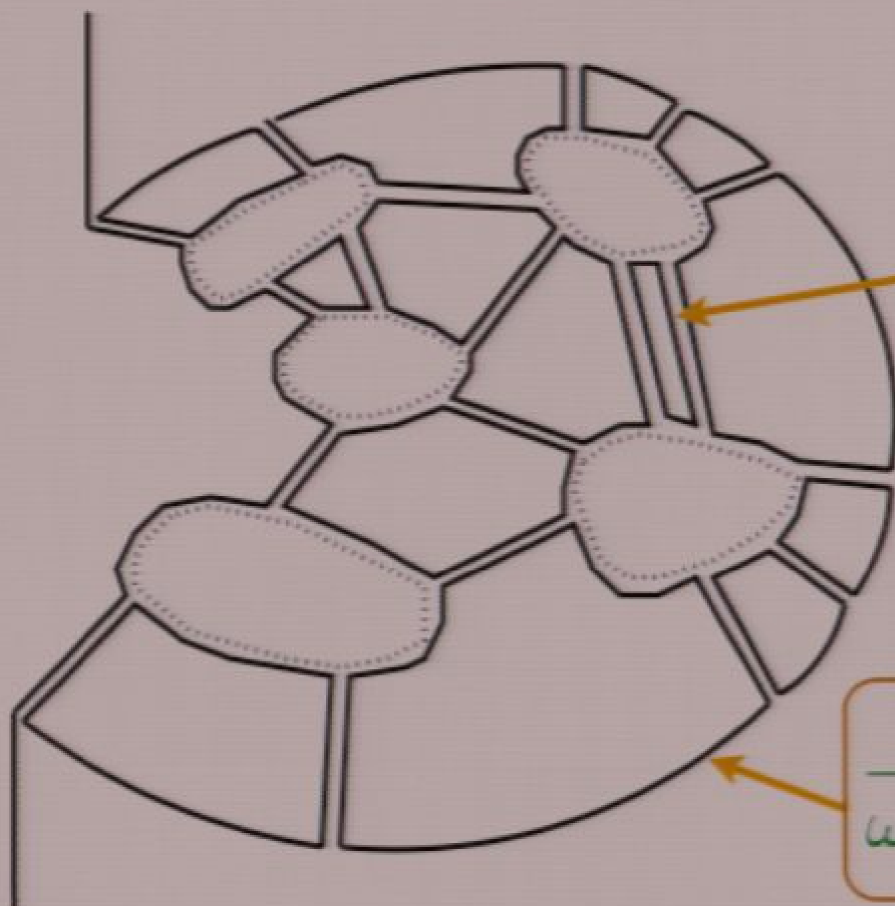

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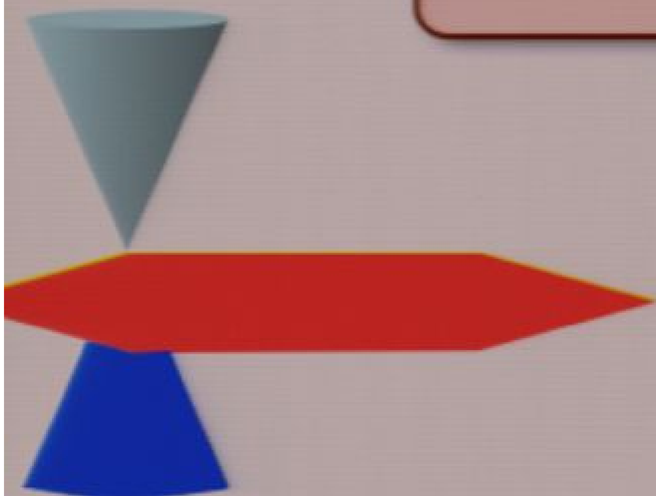
A string theory for the Fermi surface ?

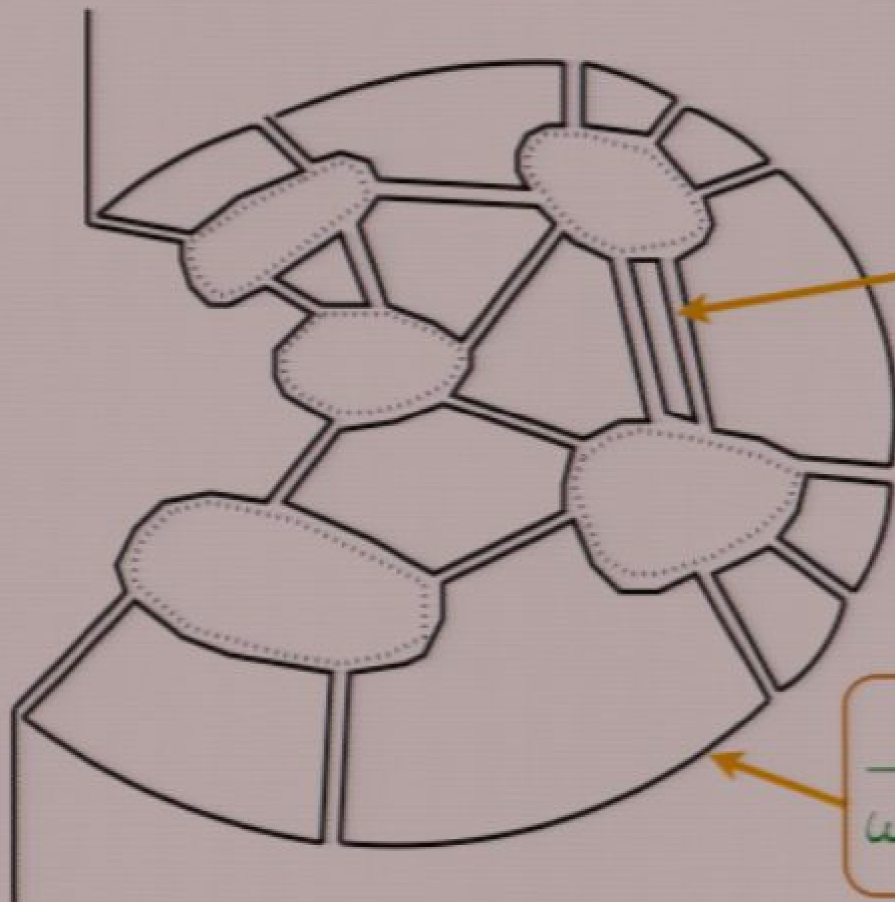
Conformal field theory
in 2+1 dimensions at $T = 0$



$\Updownarrow$

Einstein gravity
on AdS_4

Quantum criticality of Pomeranchuk instability



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Conformal field theory
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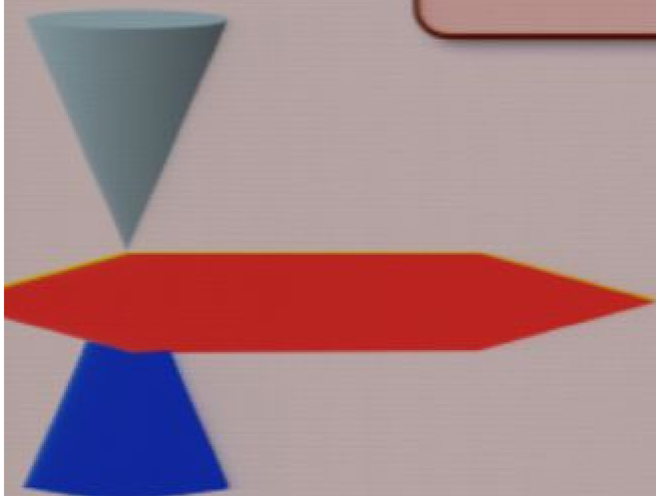

$\Updownarrow$

Einstein gravity on AdS_4
with a Schwarzschild
black hole

Conformal field theory
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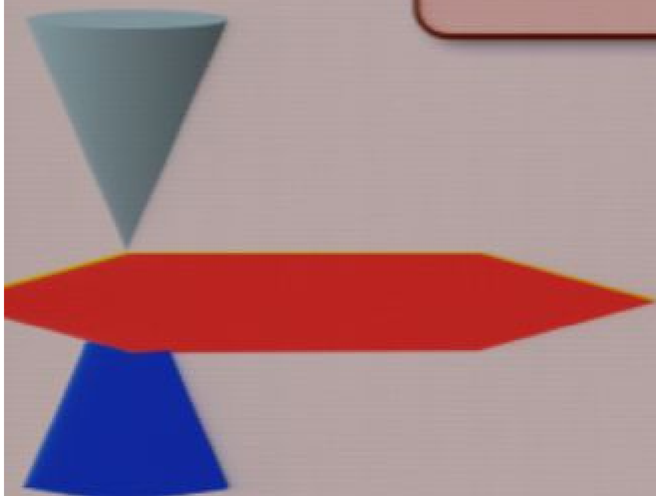

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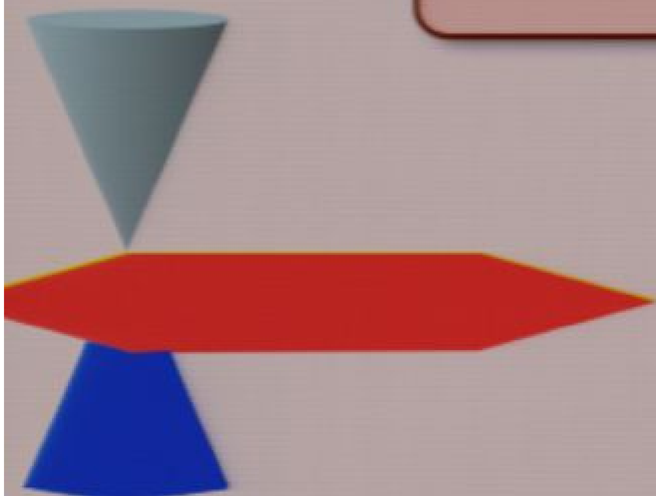
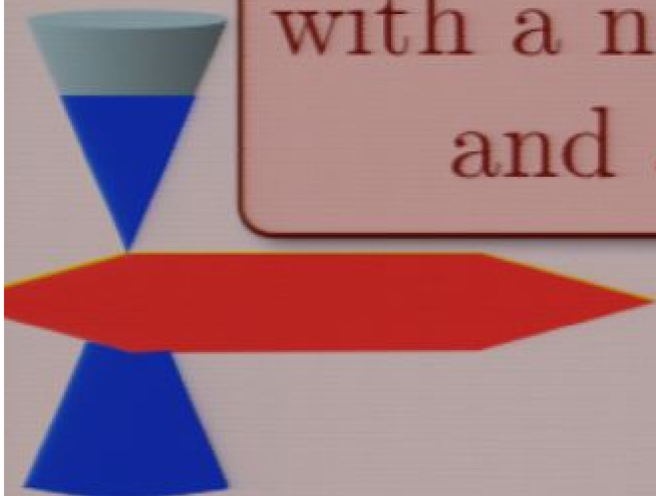


$\Updownarrow$

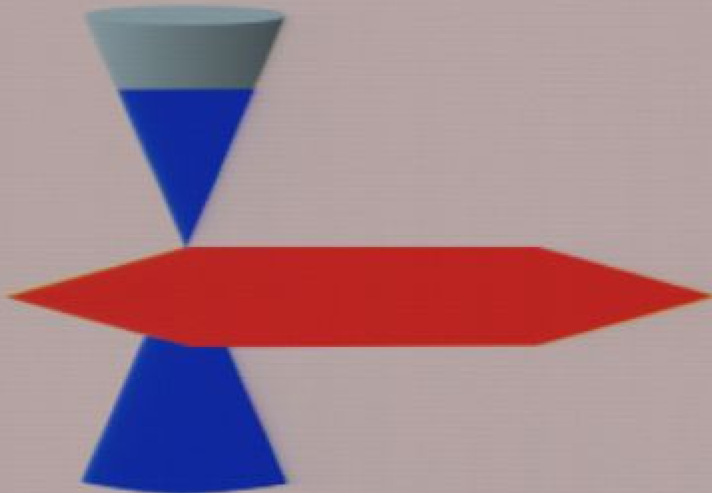
Einstein gravity on AdS_4
with a Schwarzschild
black hole

Conformal field theory
in 2+1 dimensions at $T > 0$,
with a non-zero chemical potential, μ
and applied magnetic field, B



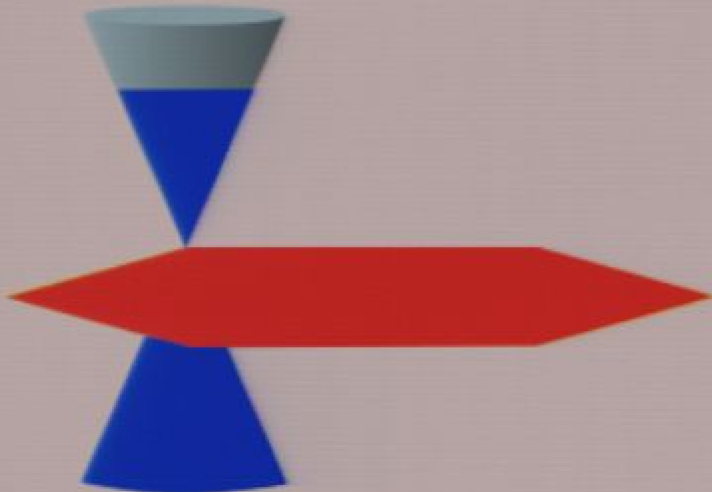
$\Updownarrow$

Einstein gravity on AdS_4
with a Reissner-Nordstrom
black hole carrying electric
and magnetic charges



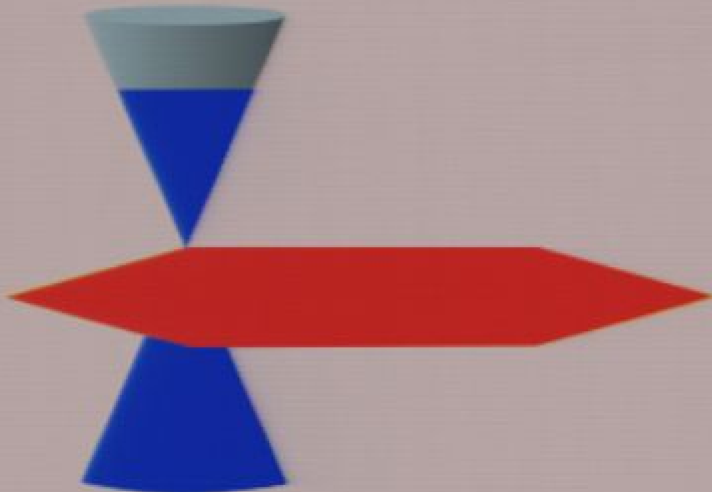
AdS_4-Reissner-Nordstrom black hole

$$
\begin{aligned}
ds^2 &= \frac{L^2}{r^2}\left(f(r)d\tau^2 + \frac{dr^2}{f(r)} + dx^2 + dy^2\right), \\
f(r) &= 1 - \left(1 + \frac{(r_+^2\mu^2 + r_+^4B^2)}{\gamma^2}\right)\left(\frac{r}{r_+}\right)^3 + \frac{(r_+^2\mu^2 + r_+^4B^2)}{\gamma^2}\left(\frac{r}{r_+}\right)^4, \\
A &= i\mu\left[1 - \frac{r}{r_+}\right]d\tau + Bx\,dy. \\
T &= \frac{1}{4\pi r_+}\left(3 - \frac{r_+^2\mu^2}{\gamma^2} - \frac{r_+^4B^2}{\gamma^2}\right).
\end{aligned}
$$



Examine free energy and Green's function of a probe particle

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



AdS$_4$-Reissner-Nordstrom black hole

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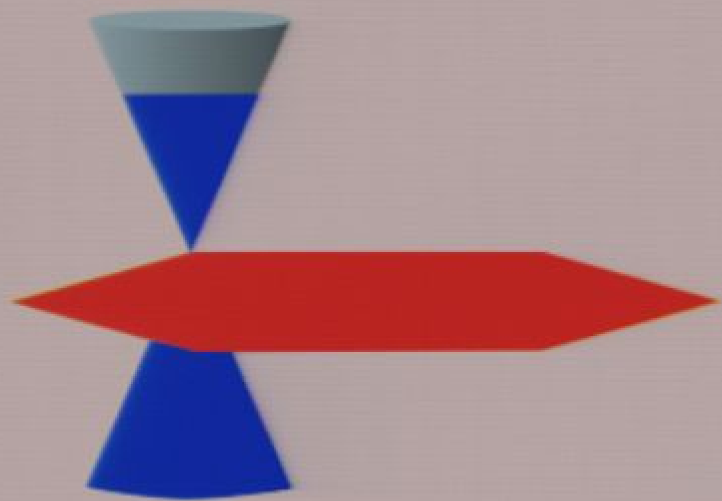

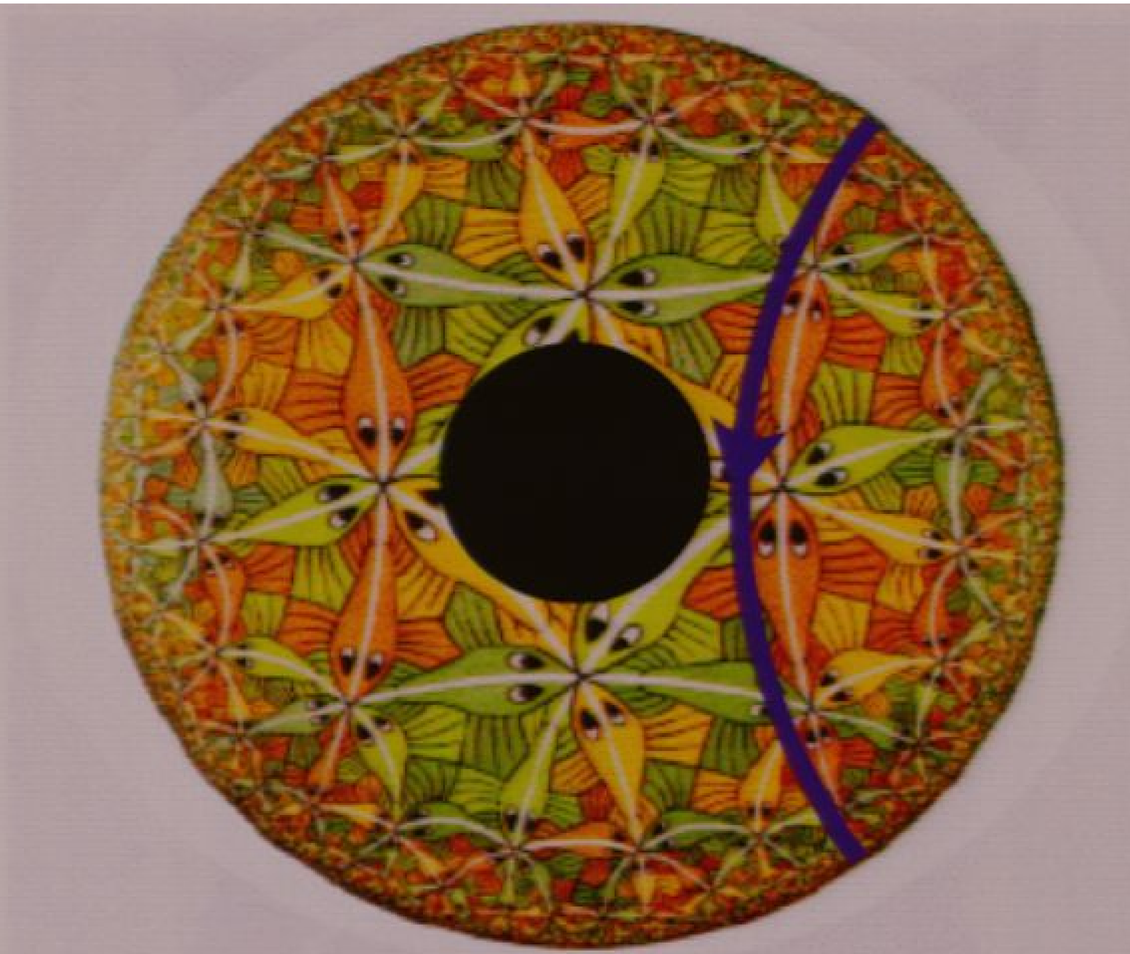

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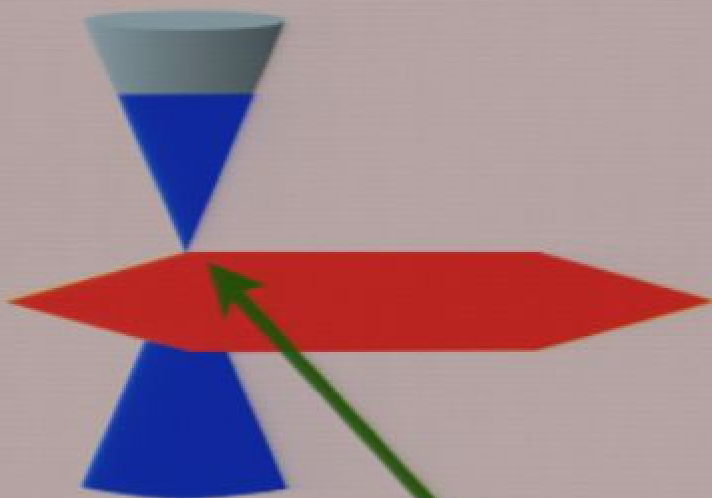
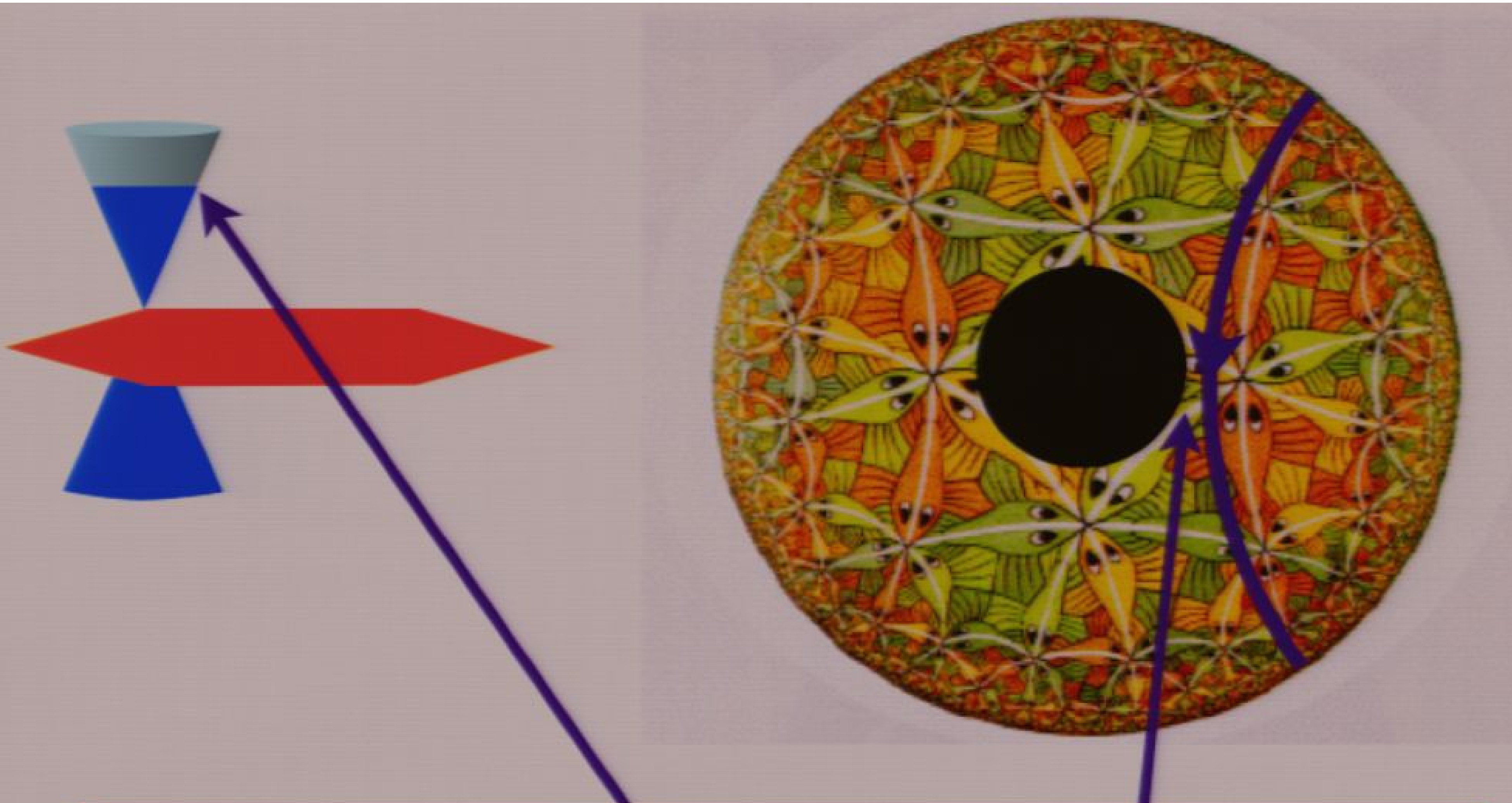


Short time behavior depends upon conformal AdS_4 geometry near boundary

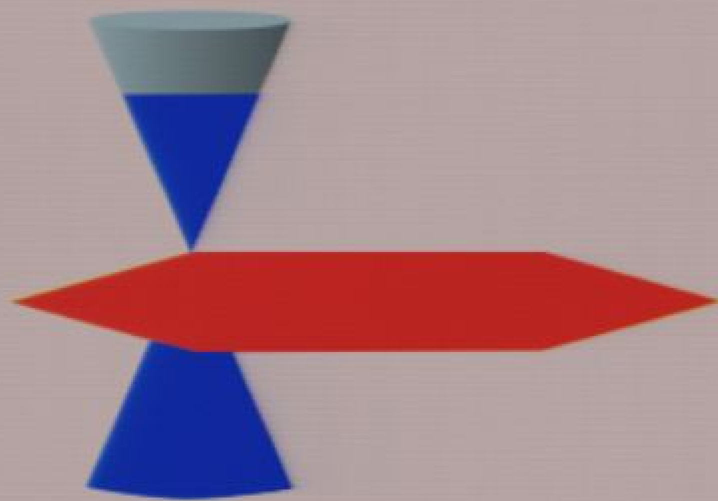
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

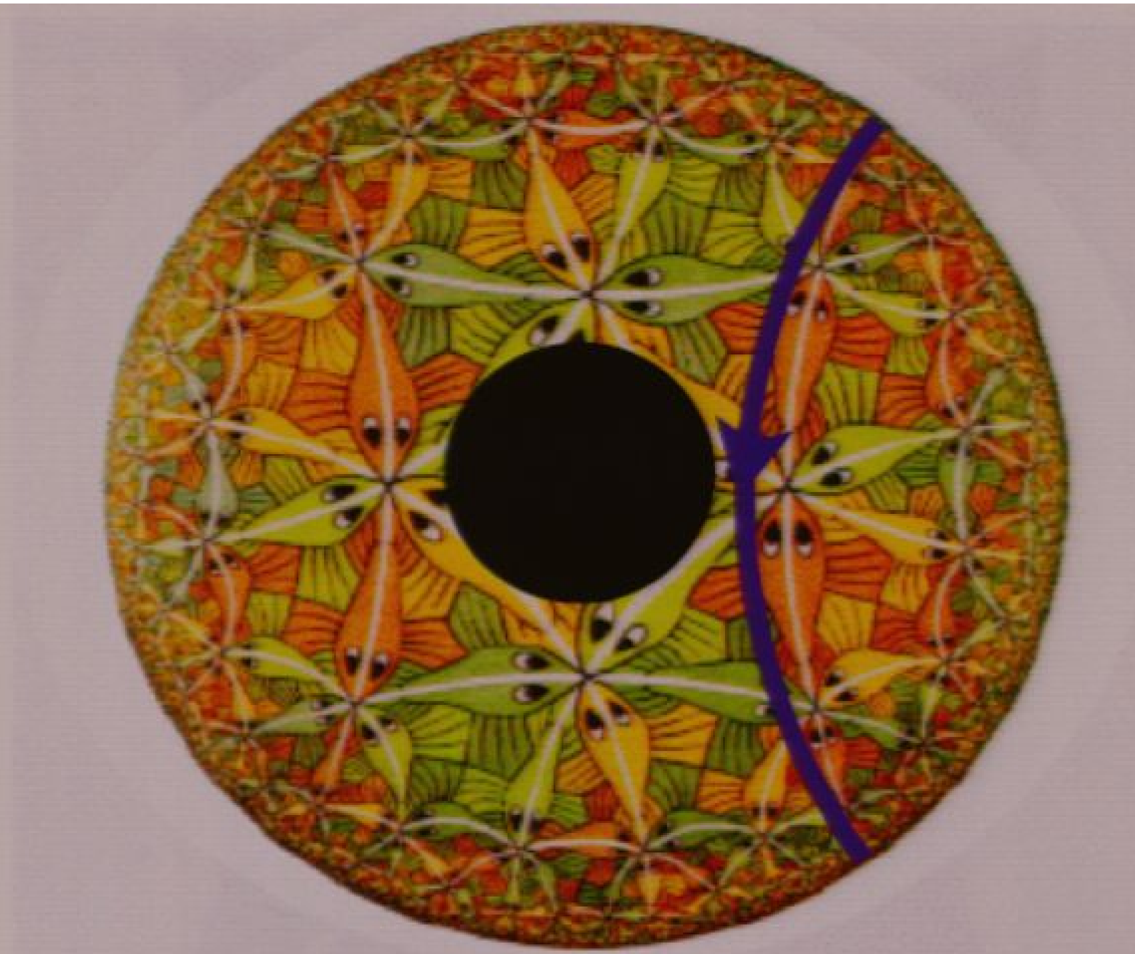
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



Long time behavior depends upon near-horizon geometry of black hole

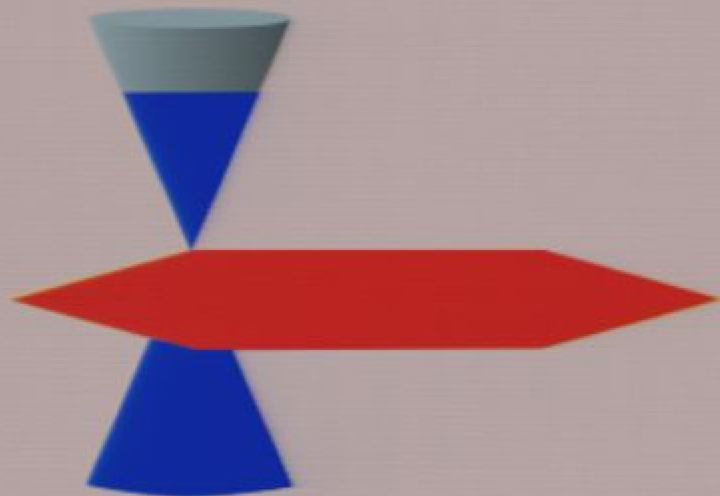
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788





Radial direction of gravity theory is measure of energy scale in CFT

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



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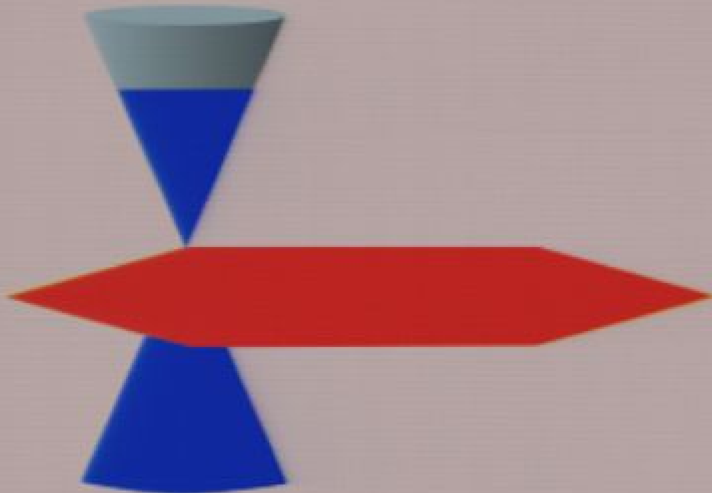
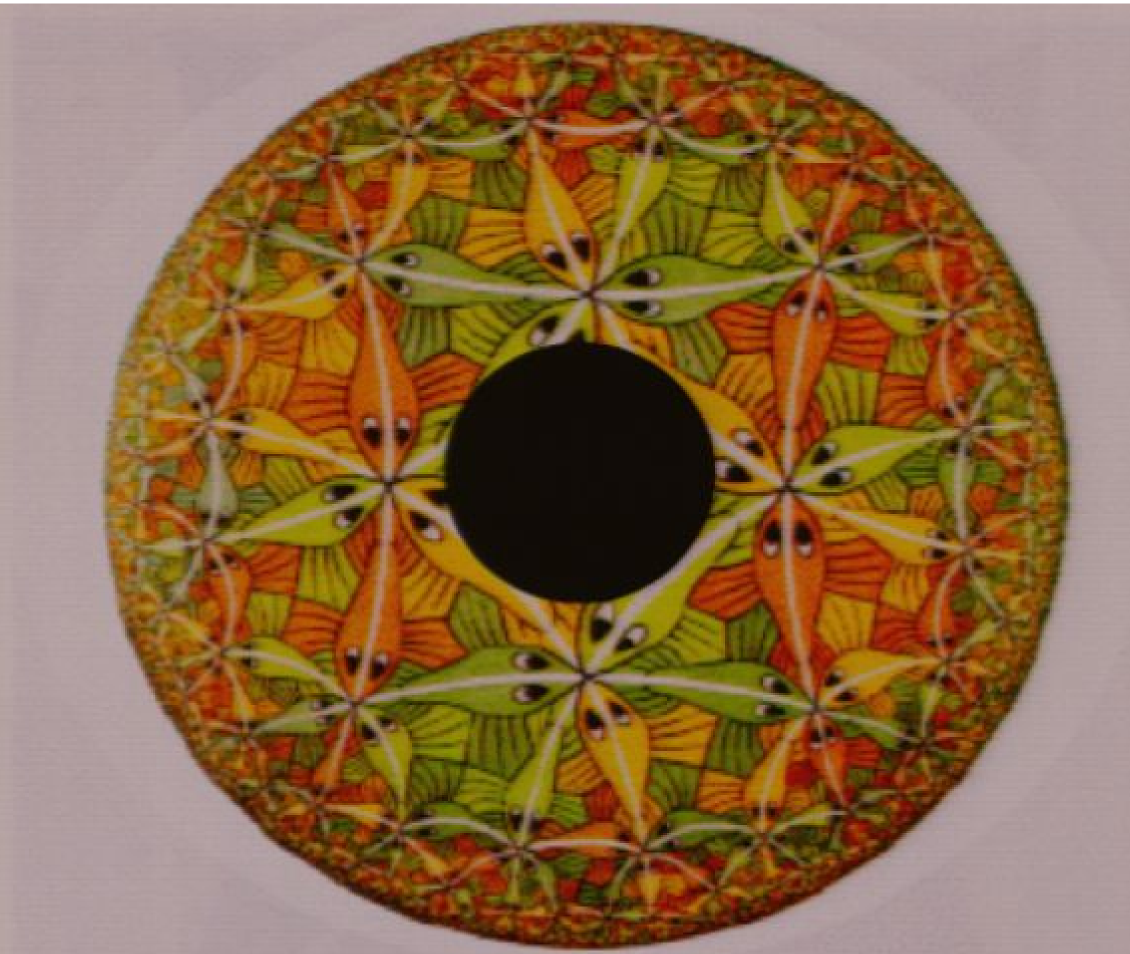
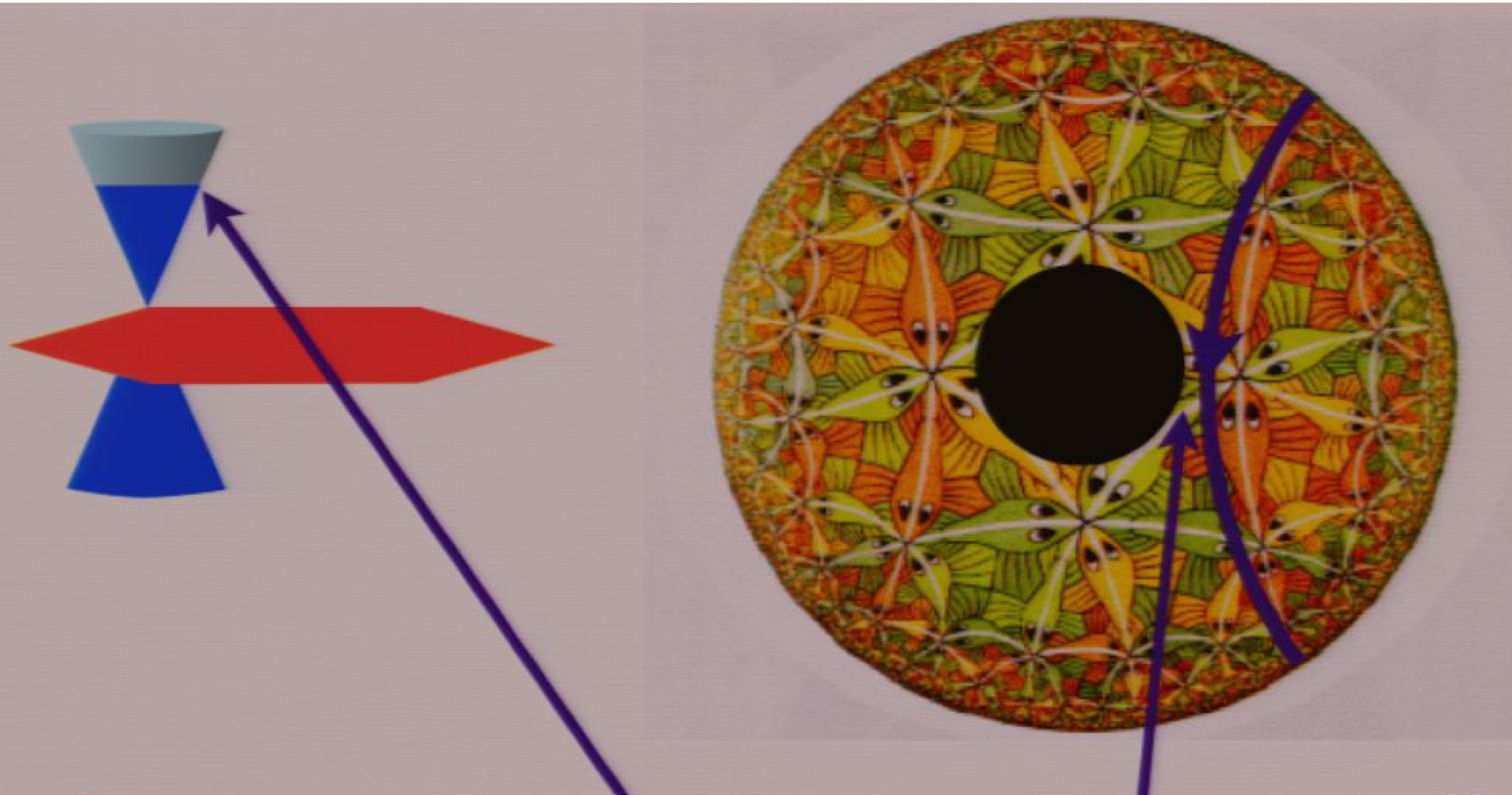


$AdS_2 \times R^2$ near-horizon geometry



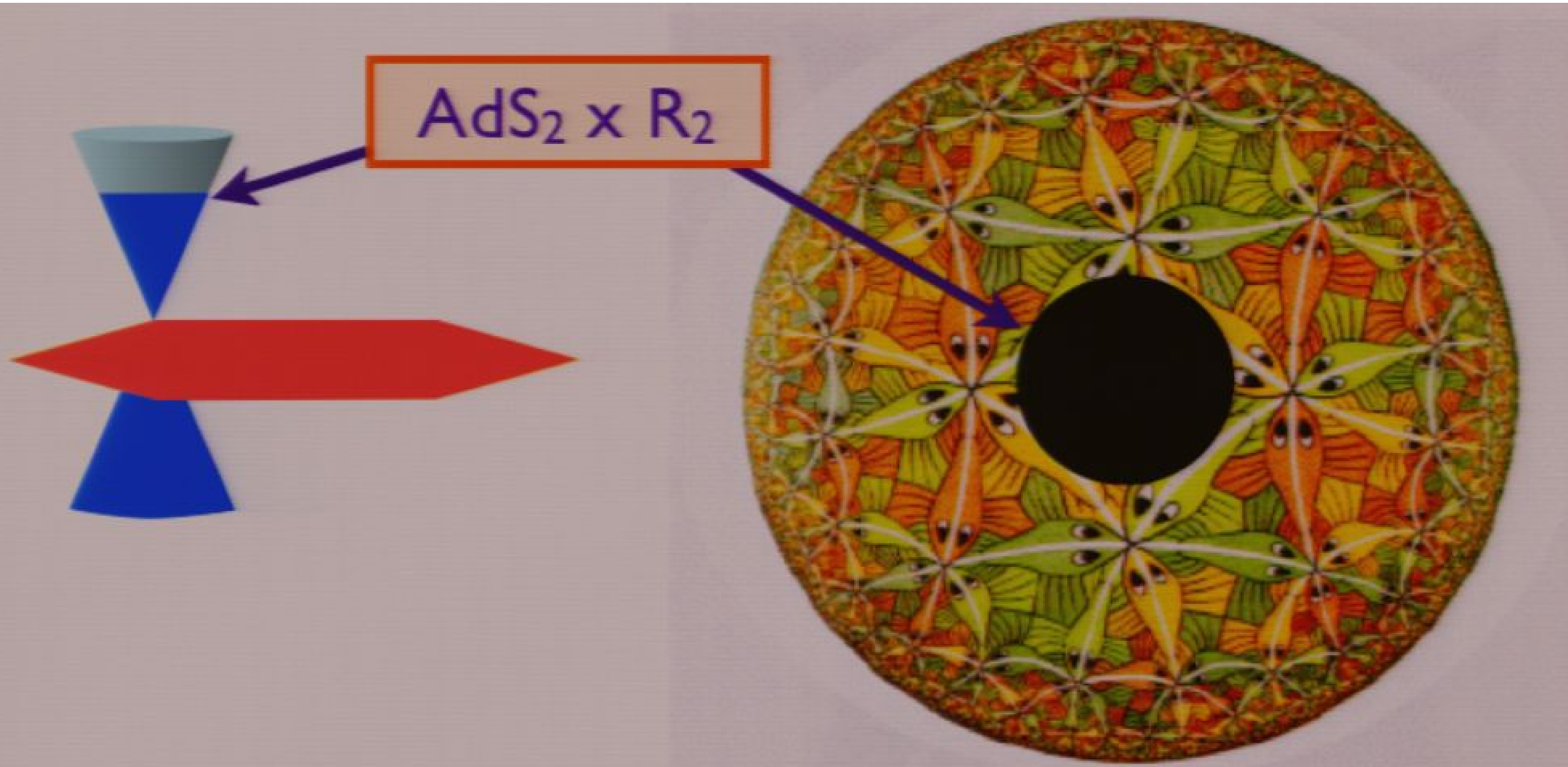
$$r - r_+ \sim \frac{1}{\zeta}$$

$$ds^2 = \frac{R^2}{\zeta^2}\left(-d\tau^2 + d\zeta^2\right) + \frac{r_+^2}{R^2}\left(dx^2 + dy^2\right)$$



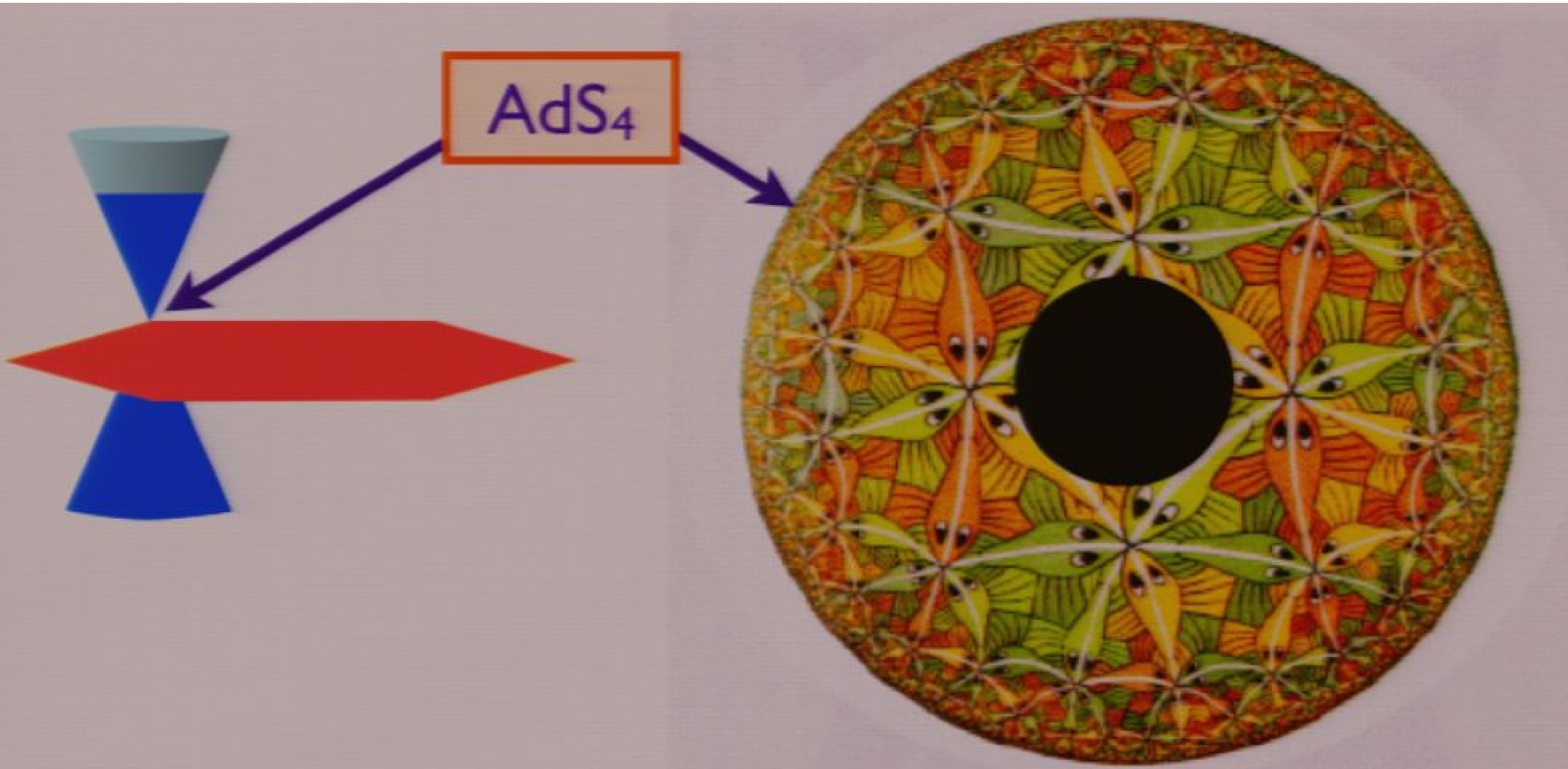
Infrared physics of Fermi surface is linked to the near horizon AdS_2 geometry of Reissner-Nordstrom black hole

T. Faulkner, H. Liu, J. McGreevy and D. Vegh, arXiv:0907.2694



Geometric interpretation of RG flow

T. Faulkner, H. Liu, J. McGreevy and D. Vegh, arXiv:0907.2694



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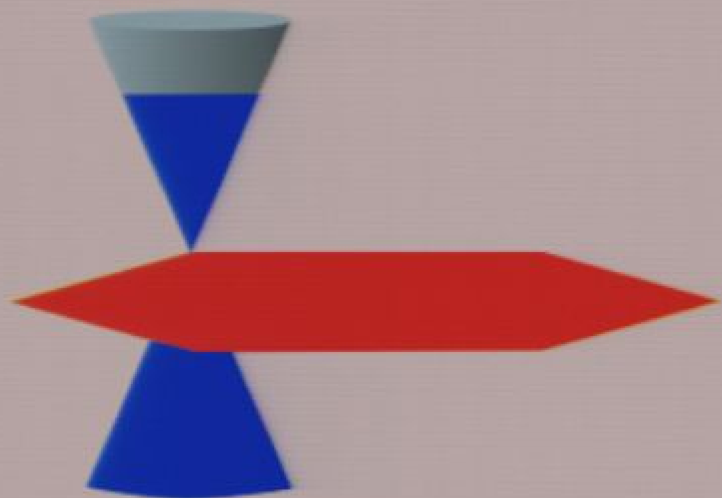

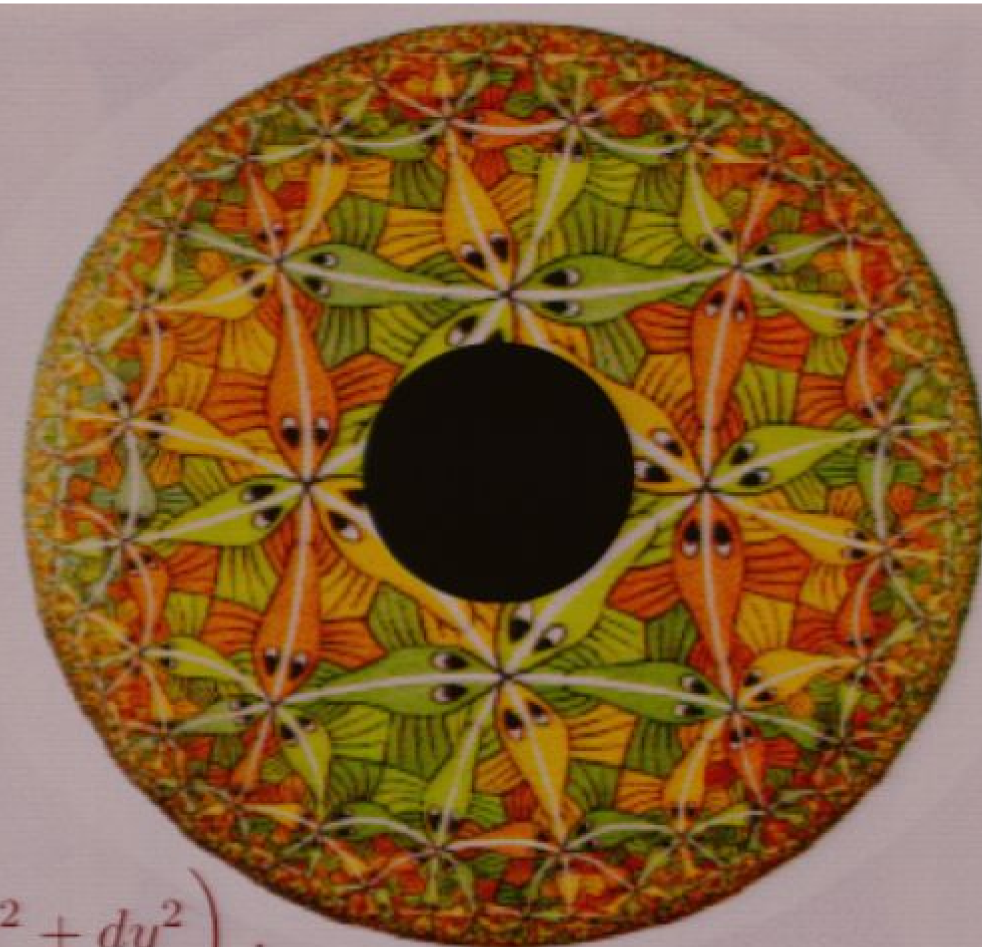

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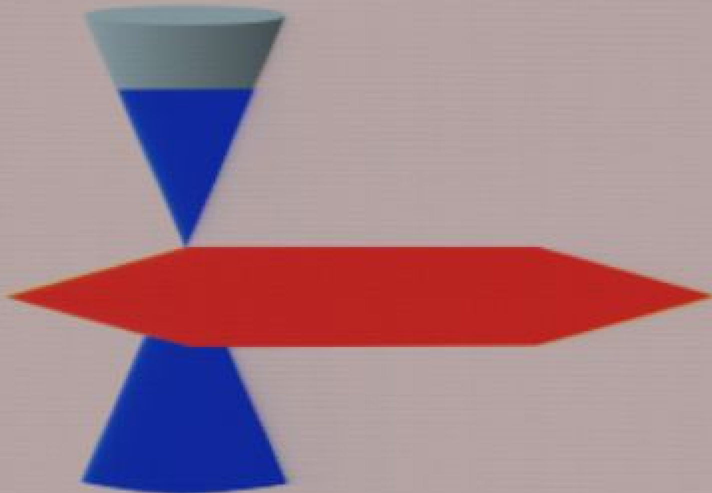
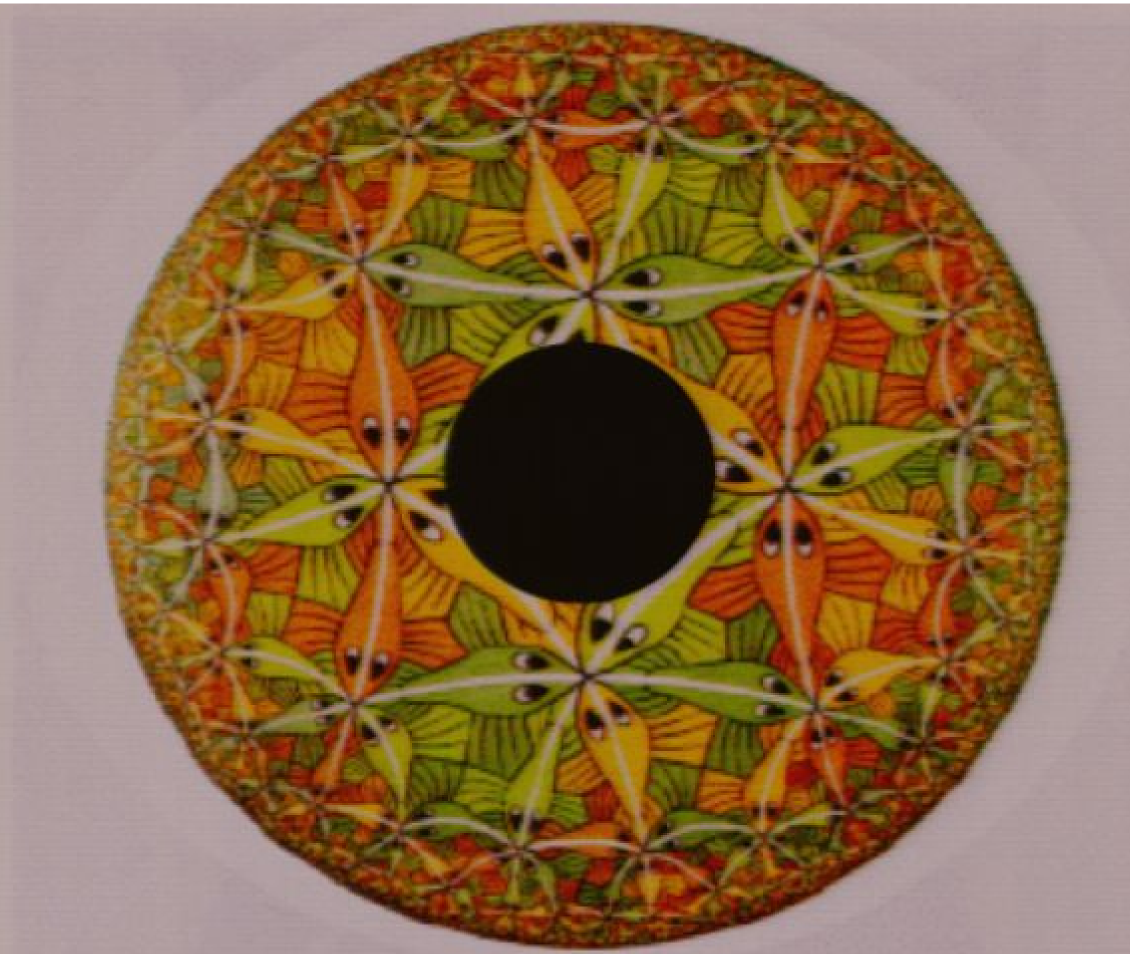


AdS$_2$ x R^2 near-horizon geometry



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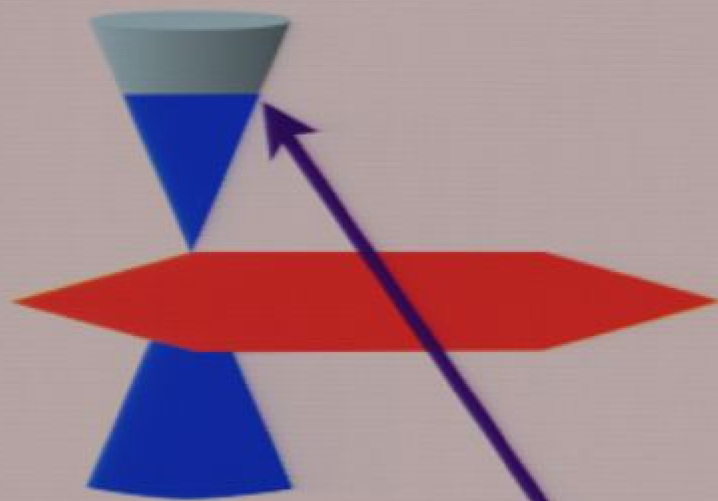

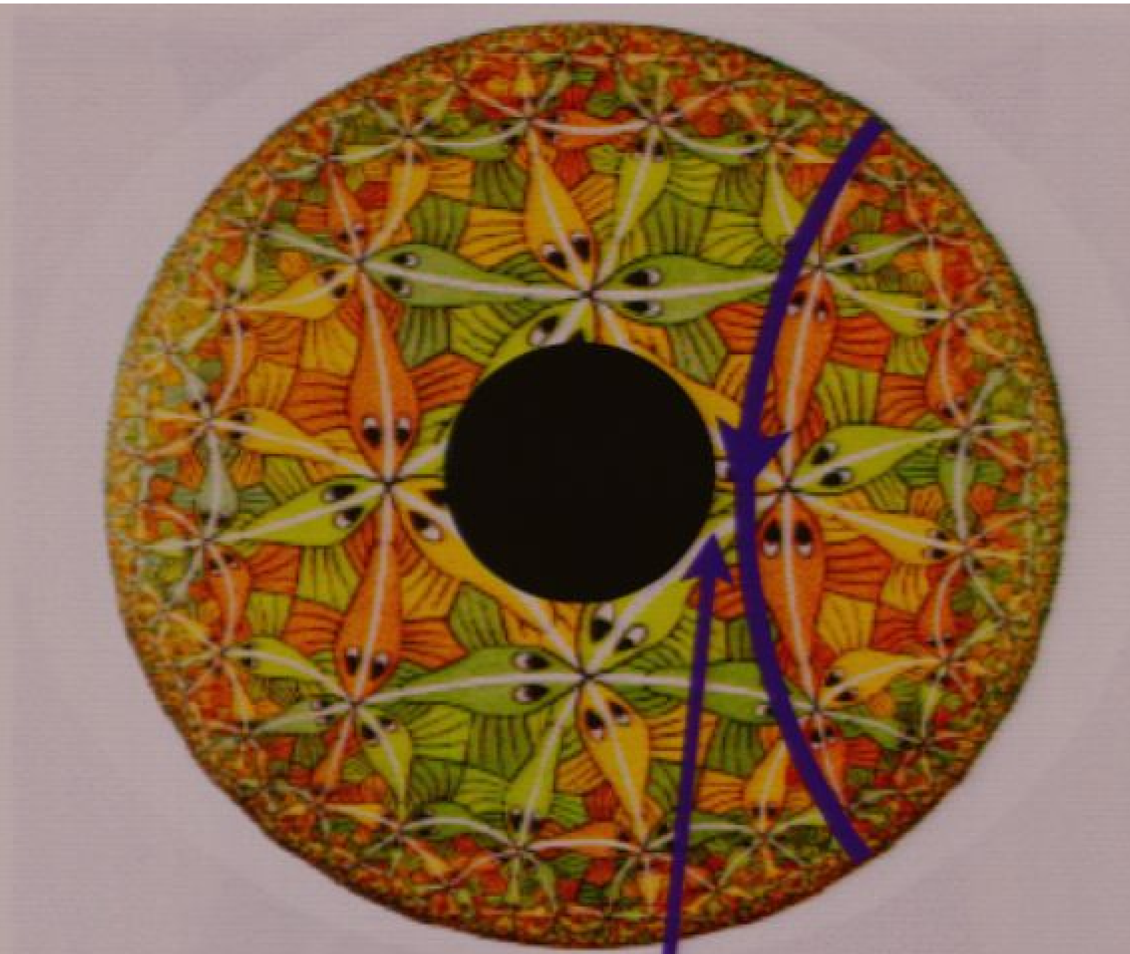

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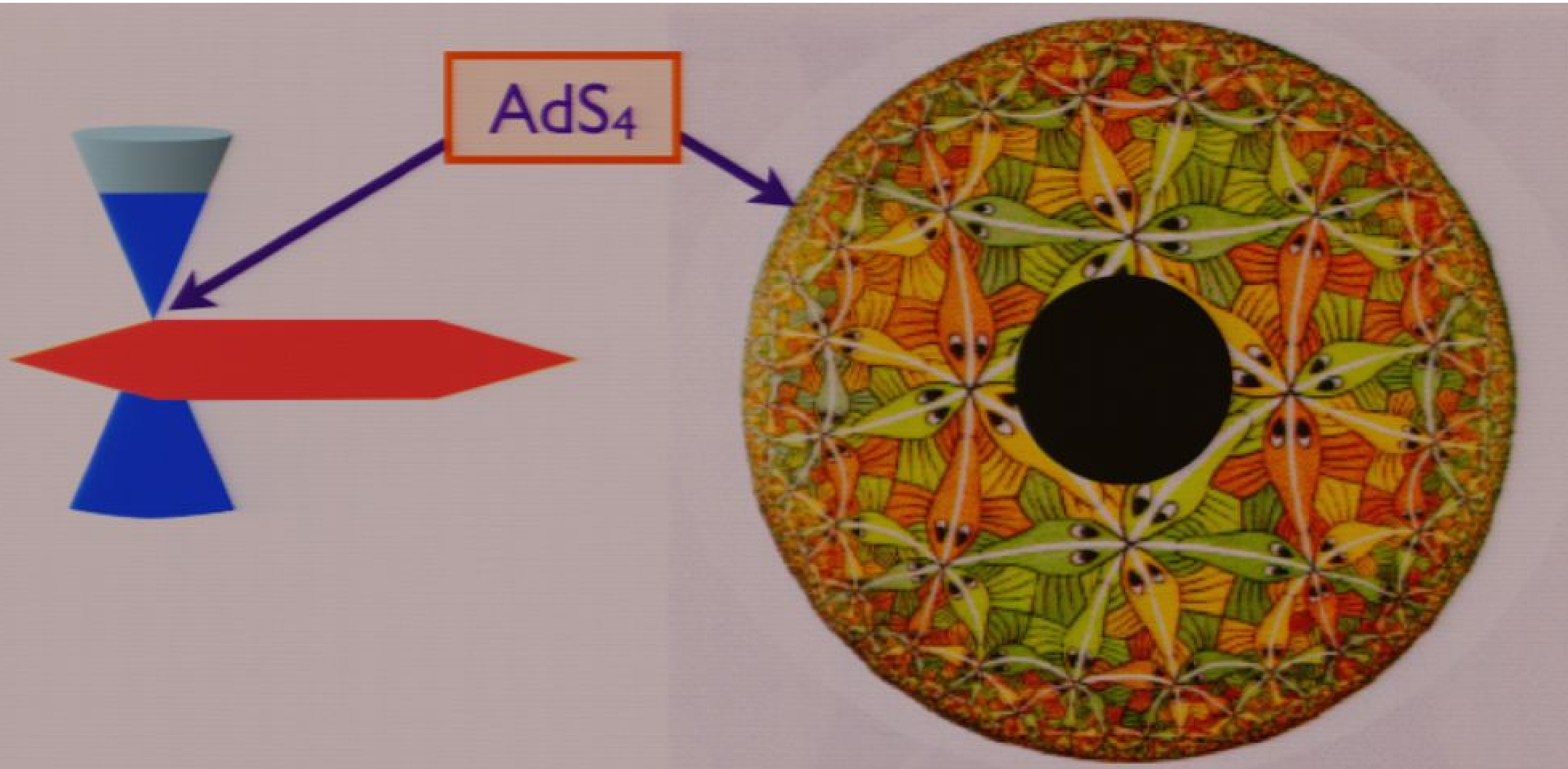

Geometric interpretation of RG flow

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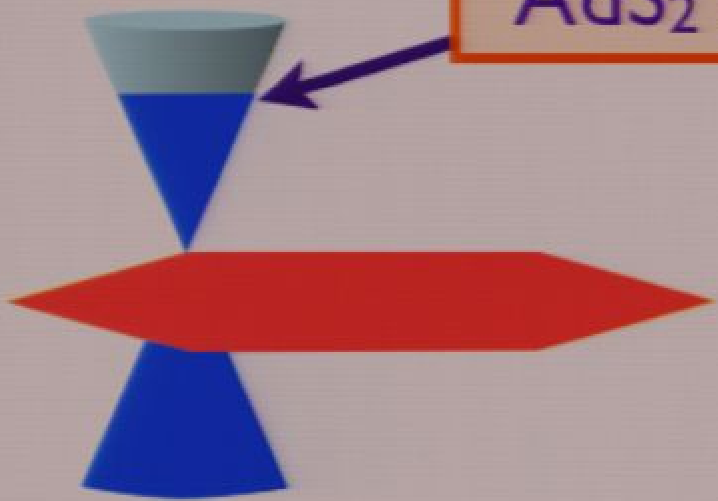
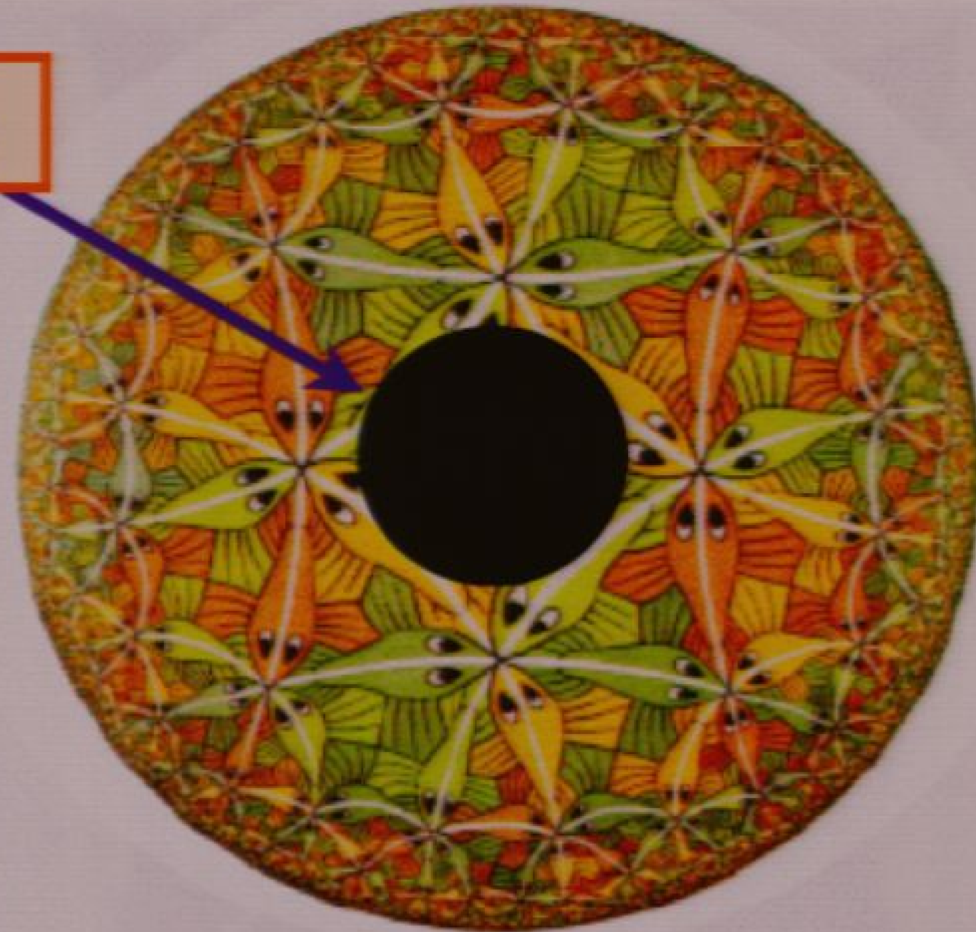
$AdS_2 \times R_2$





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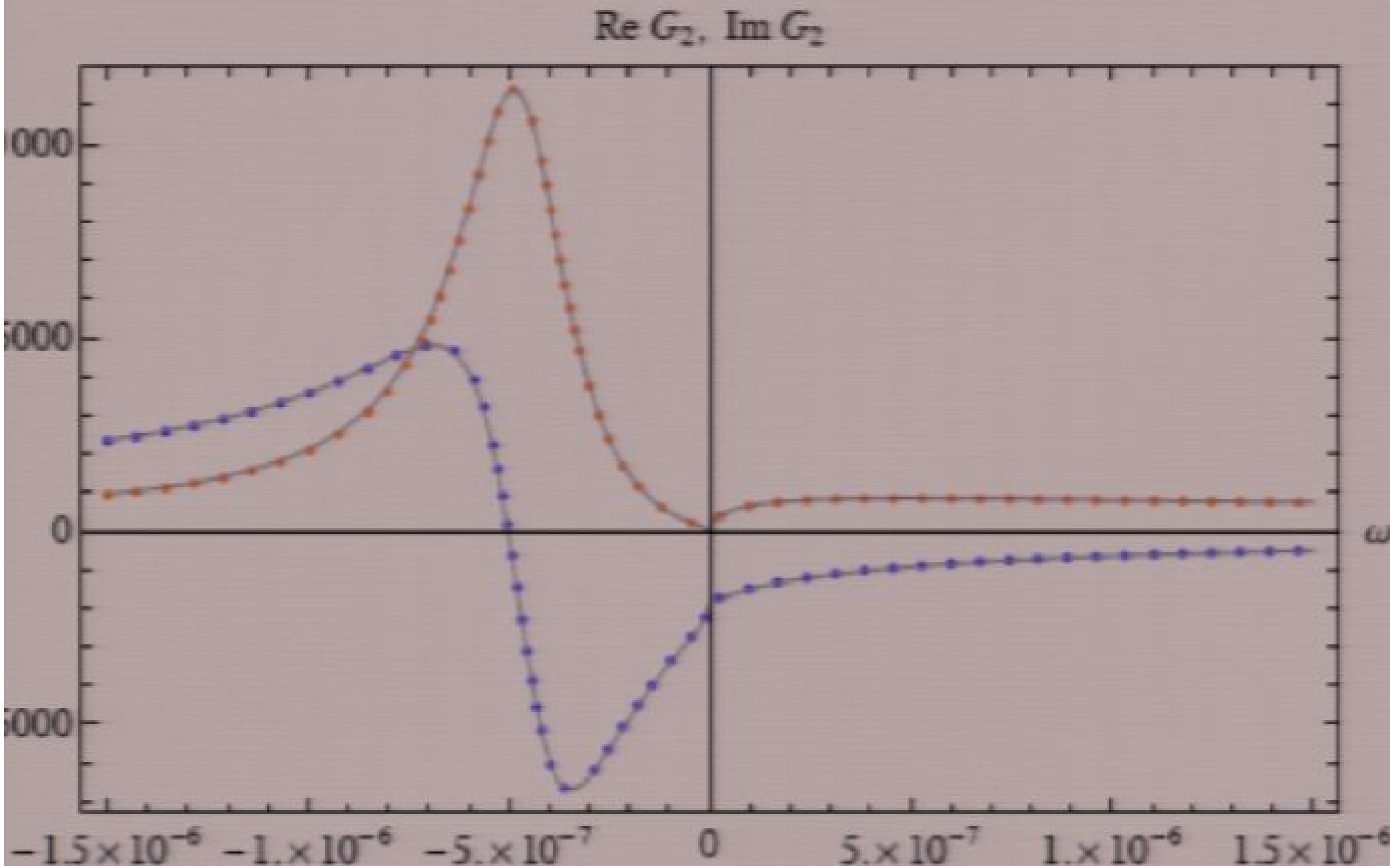
Green's function of a fermion

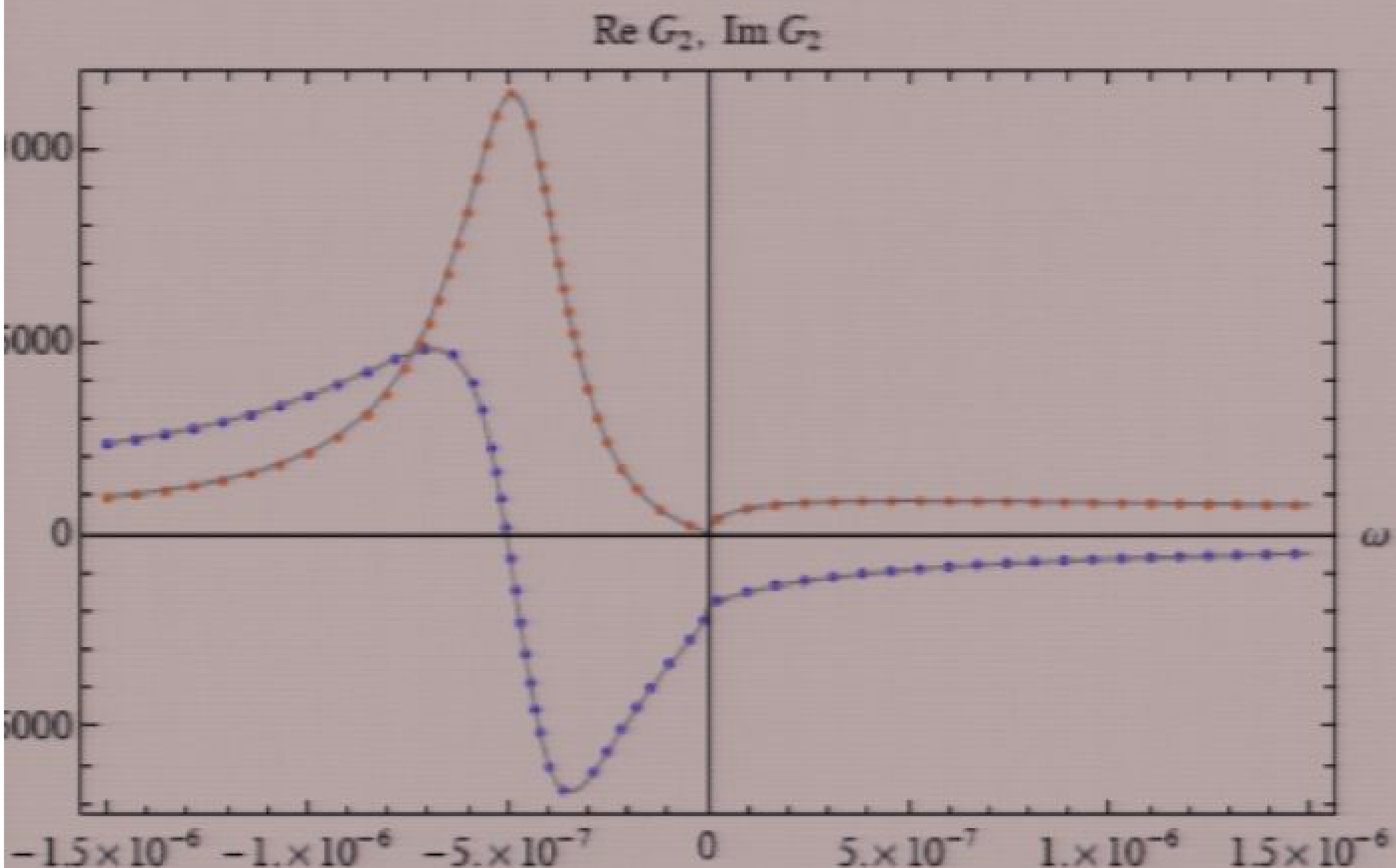


T. Faulkner, H. Liu, J. McGreevy, and D.Vegh, arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

See also M. Cubrovic, J Zaanen, and K. Schalm, arXiv:0904.1993

Green's function of a fermion



T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

Similar to non-Fermi liquid theories of Fermi surfaces coupled to gauge fields, and at quantum critical points

Free energy from gravity theory

The free energy is expressed as a sum over the "quasinormal frequencies", z_ℓ, of the black hole. Here ℓ represents any set of quantum numbers:

$$\mathcal{F}_{\text{boson}} = -T \sum_{\ell} \ln \left(\frac{|z_\ell|}{2\pi T} \left| \Gamma \left(\frac{i z_\ell}{2\pi T} \right) \right|^2 \right)$$

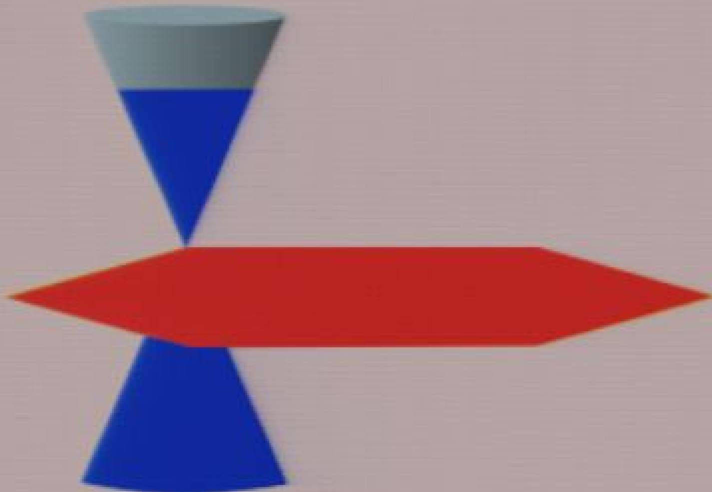
$$\mathcal{F}_{\text{fermion}} = T \sum_{\ell} \ln \left(\left| \Gamma \left(\frac{i z_\ell}{2\pi T} + \frac{1}{2} \right) \right|^2 \right)$$

Application of this formula shows that the fermions exhibit the dHvA quantum oscillations with expected period (2π/(Fermi surface ares)) in $1/B$, but with an amplitude corrected from the Fermi liquid formula of Lifshitz-Kosevich.

F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

Conclusions

The AdS/CFT offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density



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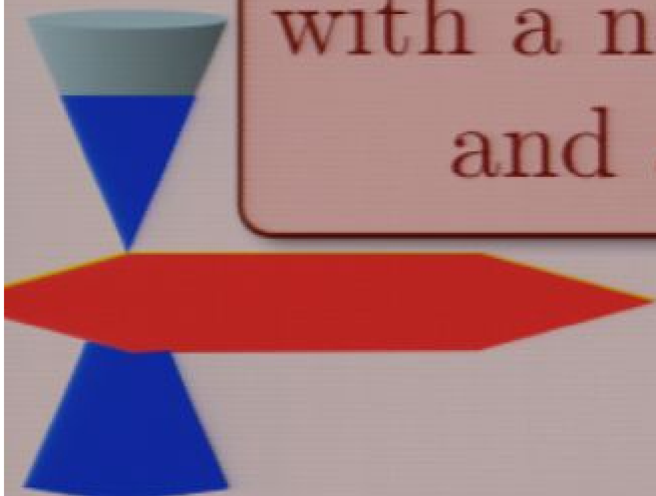
Conformal field theory
in 2+1 dimensions at $T > 0$,
with a non-zero chemical potential, μ
and applied magnetic field, B



$\updownarrow$

Einstein gravity on AdS_4
with a Reissner-Nordstrom
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Conformal field theory
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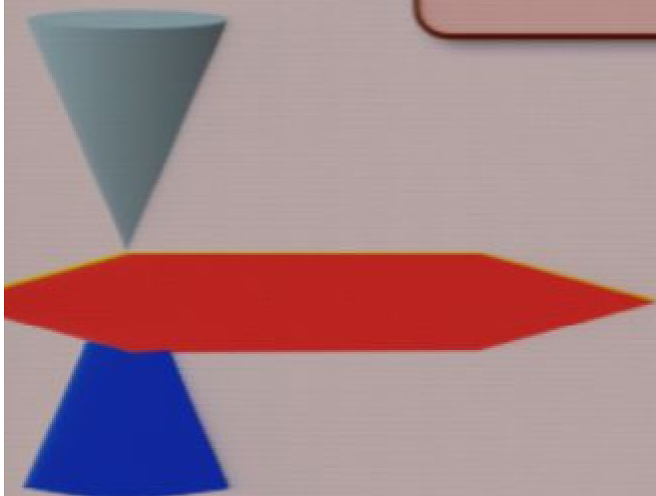

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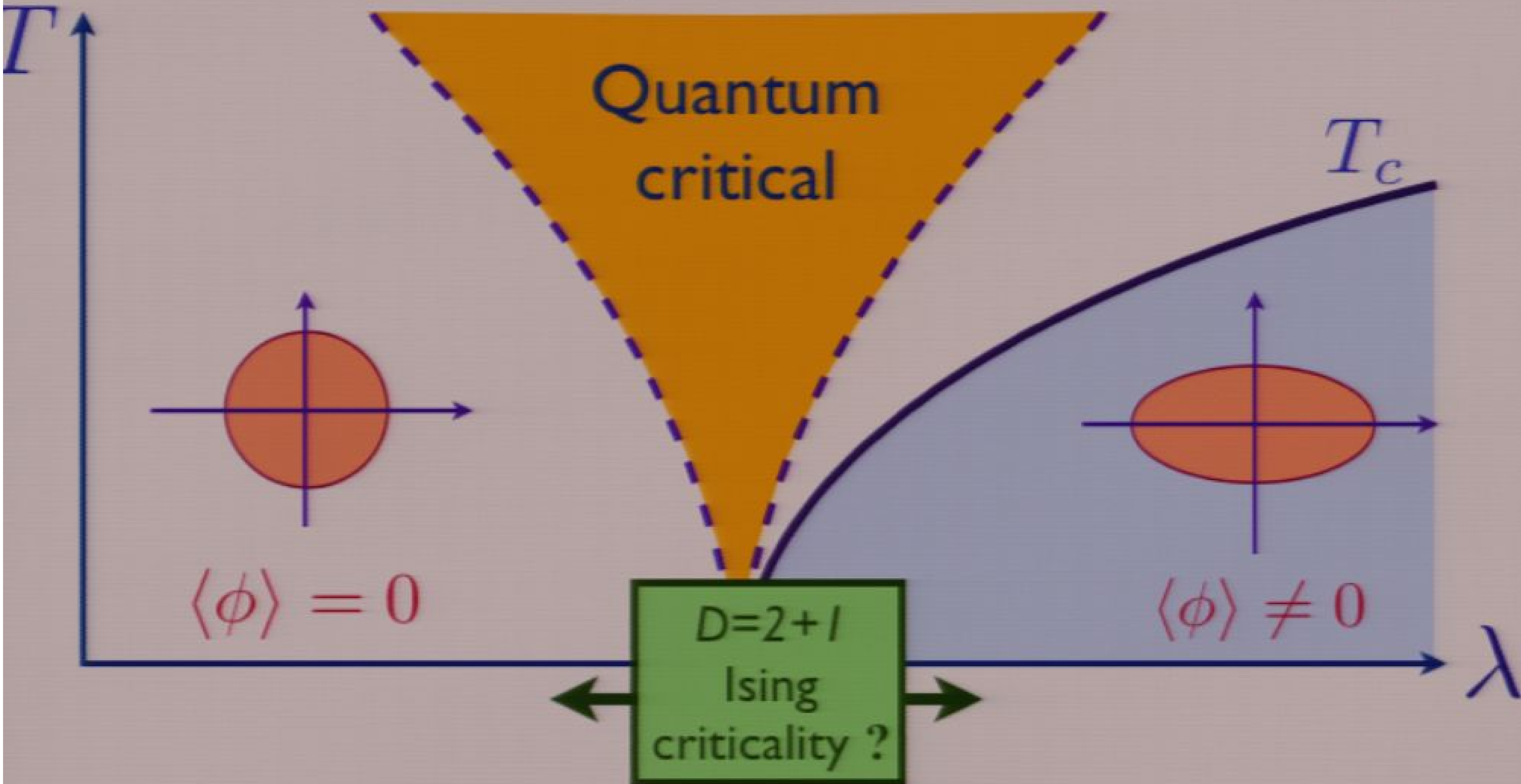
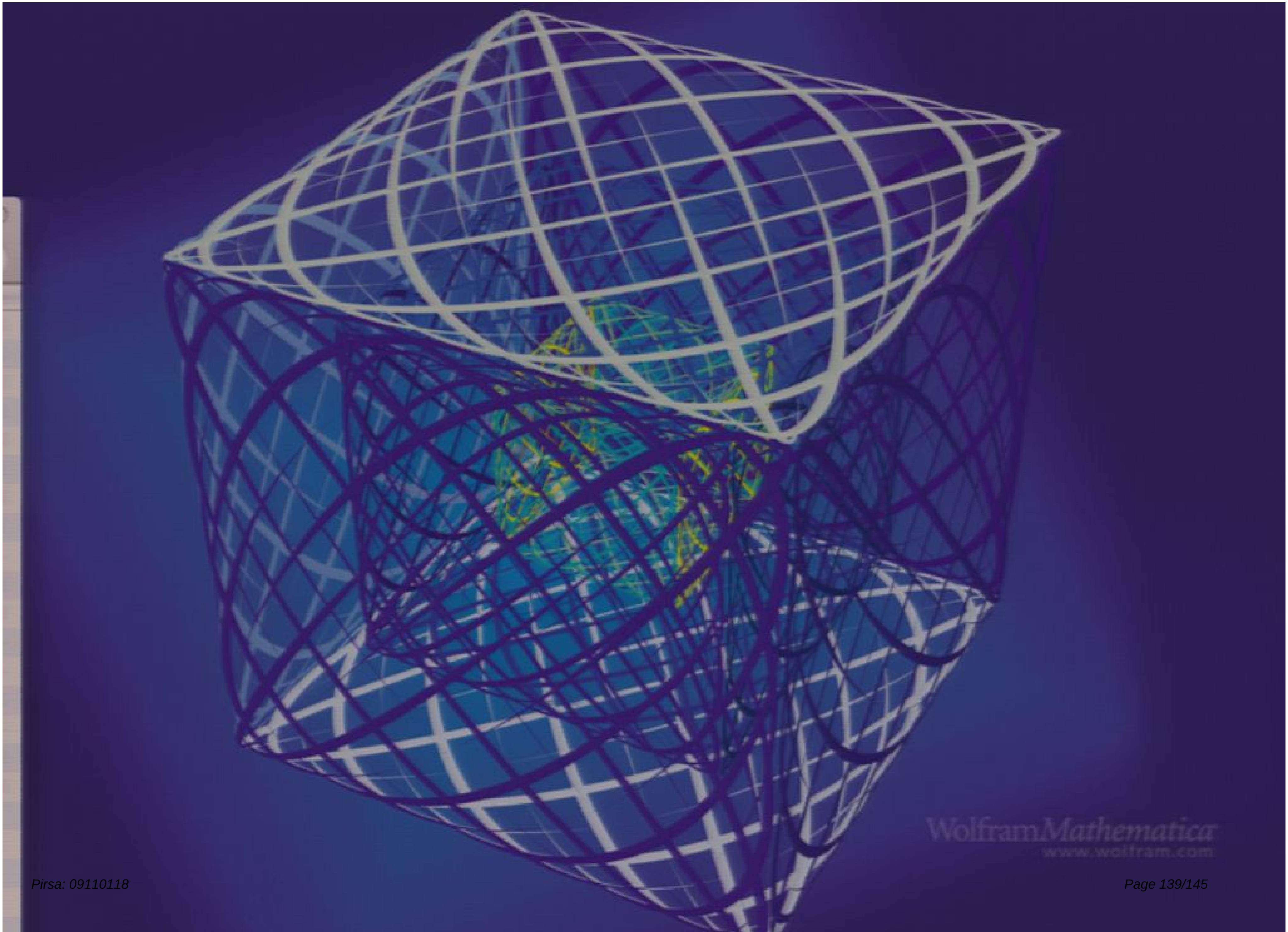
black hole

Quantum criticality of Pomeranchuk instability



Phase diagram as a function of T and λ


WolframMathematica
www.wolfram.com

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

$$\sigma = \frac{e^2}{h} \times [\text{Universal constant } \mathcal{O}(1) \ ]$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

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Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$
$$\sigma_{\text{Insulator}}(T \to 0) = 0$$
$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

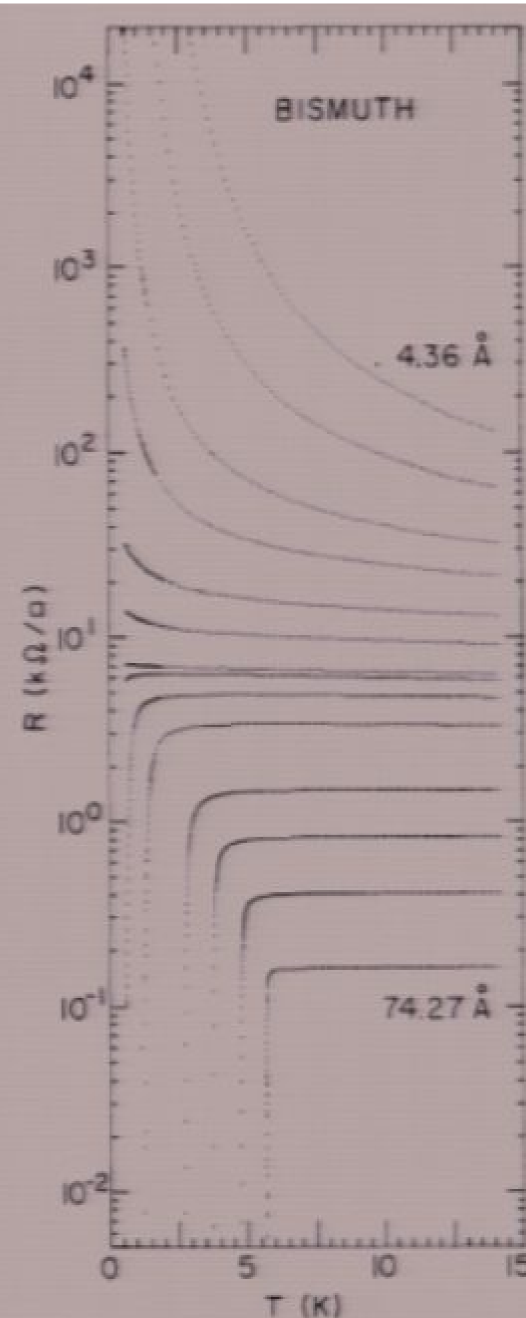

FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.