Title: Non-Equilibrium Systems (PHYS 606) - Lecture 6

Date: Nov 23, 2009 01:00 PM

URL: http://pirsa.org/09110112

Abstract:

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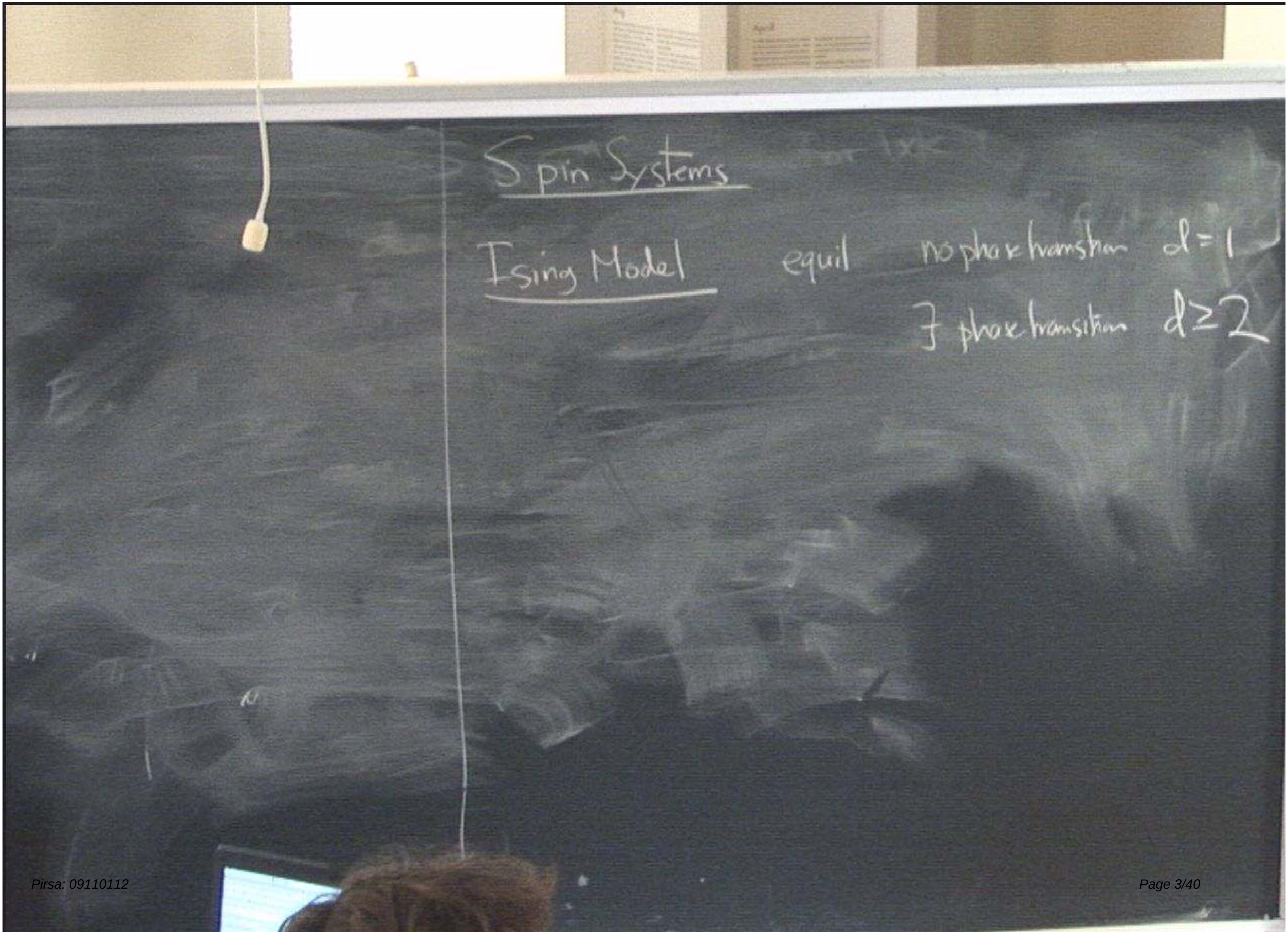
Spin Systems
Ising Model
equil
no phase transition d = 1
∃ phase transition d ≥ 2

Spin Systems

Ising Model equil no phase transition $d=1$

$\exists$ phase transition $d \geq 2$

$d=2$ soluble

$d>2$ not soluble $\rightarrow$ RG

Spin Systems

Ising Model equil no phase transition $d=1$

$\exists$ phase transition $d \geq 2$

$d=2$ soluble

$d>2$ not soluble $\to$ RG

$\Longrightarrow$ universality: exponents depend on (d, n)

space dim (d)

dim. order param (n)

non-equil spin systems

1st step: small deviations from equil. → Fluctuation-dissipation relations

2nd step: far from equilibrium

non-equil spin systems

1^{st} step: small deviations from equil. $\rightarrow$ Fluctuation-dissipation relations

2^{nd} step: far from equilibrium: start at $T = \infty$

suddenly "quench" to $T = 0$

$\Rightarrow$ coarsening

Central Dogma

- supercritical $T=\infty \to T_f > T_c$ dull
- critical $T=\infty \to T_f = T_c$
- subcritical

Central Dogma

- supercritical $T=\infty \rightarrow T_f > T_c$ dull
- critical $T=\infty \rightarrow T_f = T_c$ complex
- subcritical $T=\infty \rightarrow T_f < T_c$ simple

$$T_f = 0$$

Central Dogma

- supercritical	$T=\infty \to T_f > T_c$		dull
- critical	$T=\infty \to T_f = T_c$		complex
- subcritical	$T=\infty \to T_f < T_c$		simple

$$T_f < T_c \quad \Vert \quad \text{quenching to } T=0$$

Dynamic Scaling Hypothesis for quenching to $T=0$

- $\exists$ single growing length scale $\ell(t)$
- $\ell(t) \sim t^z$ power law
- z universal

Central Dogma

- supercritical $T=\infty \to T_f > T_c$ dull
- critical $T=\infty \to T_f = T_c$ complex
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$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \parallel$

Dynamic Scaling Hypothesis for quenching to $T=0$

- $\exists$ single growing length scale $\ell(t)$
- $\ell(t) \sim t^z$ power law
- z universal (often independent of (d,n), depends on conservation law)

Ising-Glauber Model

Qu: How to endow Ising model with dynamics?

Ising - Glauber Model

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Detailed Balance Principle:

Ising-Glauber Model

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Detailed Balance Principle:

state vector $\{S\} \equiv \{ \overset{\pm 1}{\overset{\shortparallel}{S_1}}, S_2 \ldots$

Ising-Glauber Model

Qu: How to endow Ising model with dynamics?

Detailed Balance Principle:

State vector $\{S\} \equiv \{S_1, S_2 \ldots$

± 1

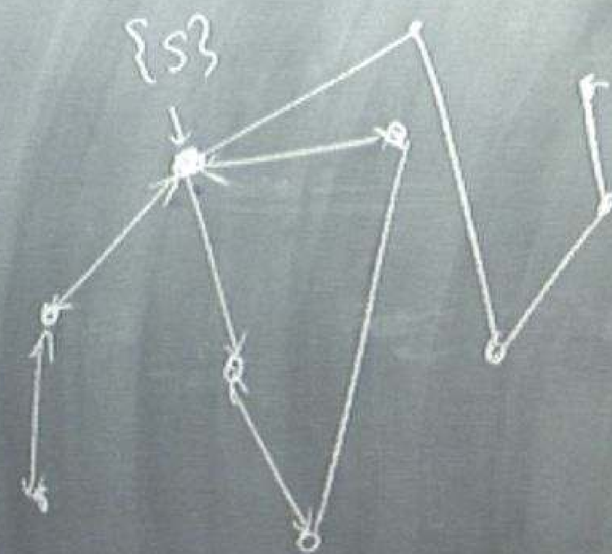
$\{S\}$

Ising-Glauber Model

Qu: How to endow Ising model with dynamics?

Detailed Balance Principle:

state vector $\{S\} \equiv \{S_1, S_2, \ldots\}$, $S_i = \pm 1$



$P(\{S\}, t)$ = prob of being in some pt in state space at time t

$s_N\}$ 2^N possible states

Master Eqn:

$$\frac{\partial P(\{s\},t)}{\partial t} = \underbrace{\sum_{s'} W(s' \to s)\, P(\{s\}',t)}_{\text{gain}}$$

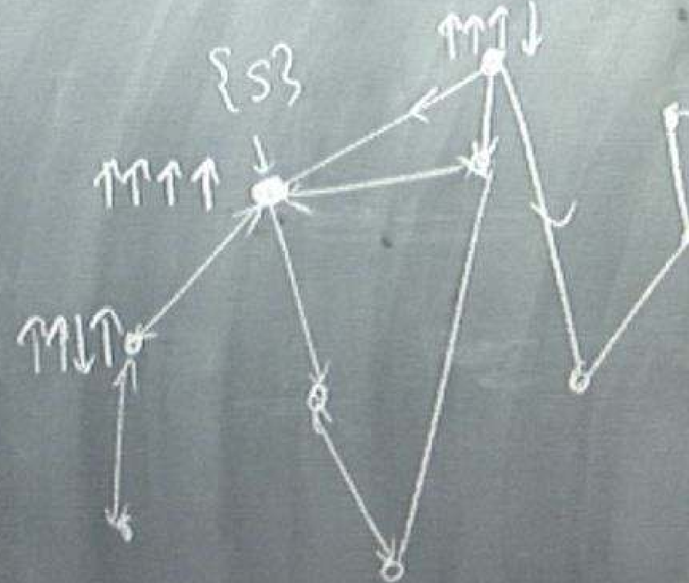
n conservation law)

Ising-Glauber Model

Qu: How to endow Ising model with dynamics?

Detailed Balance Principle:

state vector $\{S\} \equiv \{\overset{\pm 1}{\overset{\shortparallel}{S_1}}, S_2 \ldots$



$P(\{S\}, t)$ = prob of being in some pt in state space at time t

$s_N\}$ 2^N possible states

Master Eqn: (single spin flip dynamics)

$$\frac{\partial P(\{s\},t)}{\partial t} = \underbrace{\sum_{s_i'} W(s_i' \to s)\, P(\{s\}_i',t)}_{\text{gain}} - \sum_{s_i'} W(s \to s_i')\, P(\{s\},t)$$

n conservation law)

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Master Eqn: (single spin flip dynamics)

$$\frac{\partial P(\{s\},t)}{\partial t} = \underbrace{\sum_{s_i'} W(s_i' \to s)\, P(\{s\}_i',t)}_{\text{gain}} - \sum_{s_i'} W(s \to s_i')\, P(\{s\},t)$$

equil $\overset{\shortparallel}{0}$

$$\Rightarrow \sum_{s'} W(s' \to s)\, P(s',t) = \sum W(s \to s')\, P(s,t)$$

$\Leftrightarrow$ total flux in

= total flux out

$s_N\}$ 2^N possible states

Master Eqn: (single spin flip dynamics)

$$\frac{\partial P(\{s\},t)}{\partial t} = \underbrace{\sum_{s_i'} W(s_i' \to s)\, P(\{s\}_i',t)}_{\text{gain}} - \sum_{s_i'} W(s \to s_i')\, P(\{s\},t)$$

equil $\quad \overset{\shortparallel}{0}$

$$\Rightarrow \sum_{s'} W(s' \to s)\, P(s',t) = \sum W(s \to s')\, P(s,t) \iff \text{total flux in} = \text{total flux out}$$

D.B. flux on each bond is balanced

$s_N\}$ 2^N possible states

Master Eqn: (single spin flip dynamics)

$$\frac{\partial P(\{s\},t)}{\partial t} = \underbrace{\sum_{s_i'} W(s_i' \to s)\, P(\{s\}_i',t)}_{\text{gain}} - \sum_{s_i'} W(s \to s_i')\, P(\{s\},t)$$

equil $\overset{\shortparallel}{0}$

$$\Rightarrow \sum_{s'} W(s' \to s)\, P(s',t) = \sum W(s \to s')\, P(s,t)$$

$\Leftrightarrow$ total flux in = total flux out

D.B. : flux on each bond is balanced

$$\boxed{W(s' \to s)\, P(s',t) = W(s \to s')\, P(s,t)}$$

Ising-Glauber Model

$$D.B. \Rightarrow \frac{W(s \to s')}{W(s' \to s)} = \frac{P(s',t)}{P(s,t)} \underset{\text{equil}}{=}$$

Ising-Glauber Model

$$\mathcal{H} = -J \sum_{\langle ij \rangle}{}' s_i s_j$$

$$\text{D.B.} \Rightarrow \frac{W(s \to s_i')}{W(s_i' \to s)} = \frac{P(s',t)}{P(s,t)} \underset{\text{equil}}{=} \frac{e^{-\beta J \sum' s_i s_j}}{e^{+\beta J \sum' s_i s_j}}$$

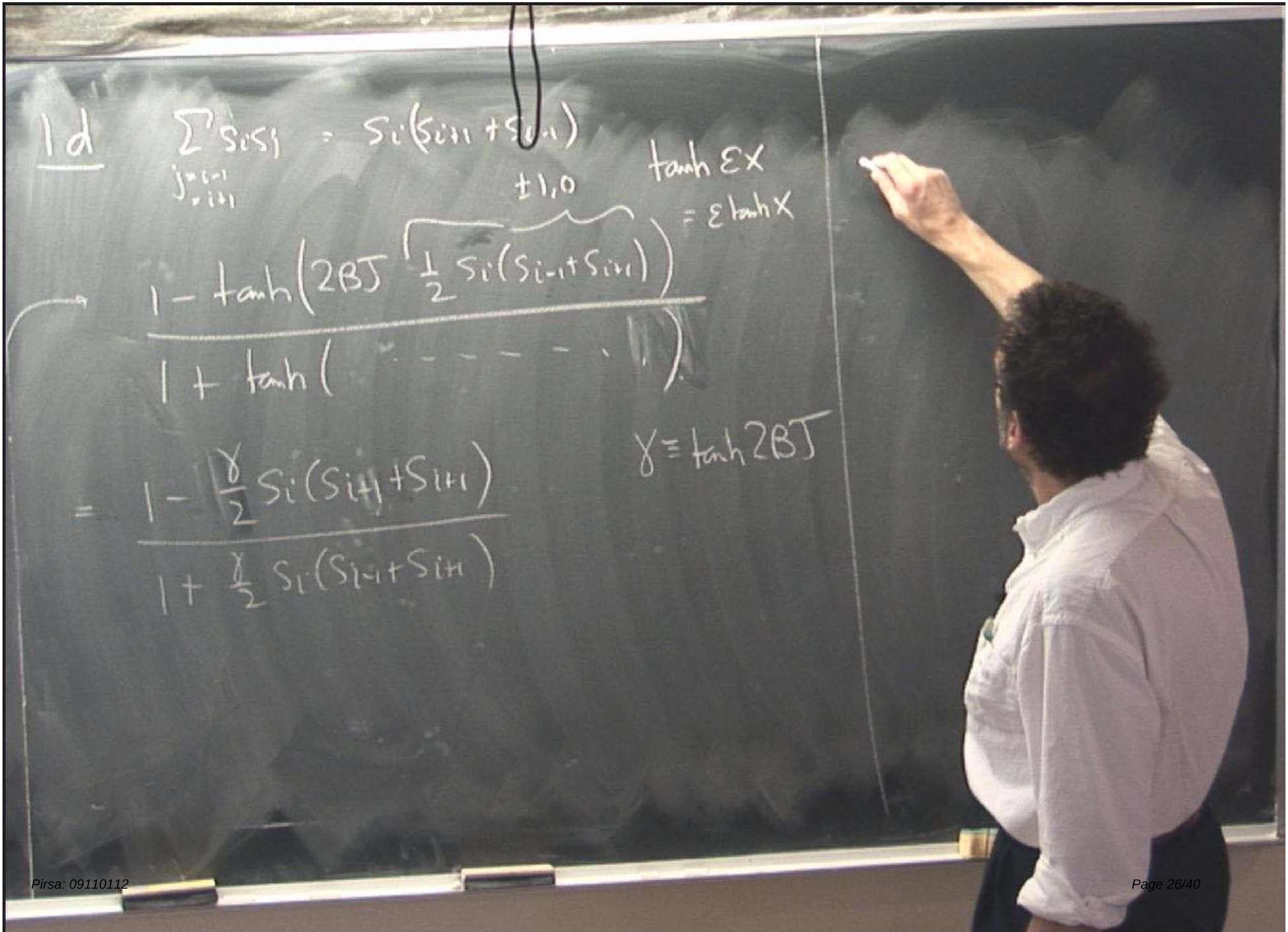
Ising - Glauber Model

$$\mathcal{H} = -J \sum_{\langle ij \rangle} s_i s_j$$

$$\text{D.B.} \Rightarrow \frac{W(s \to s_i')}{W(s_i' \to s)} = \frac{P(s',t)}{P(s,t)} \underset{\text{equil}}{=} \frac{e^{-\beta J \sum' s_i s_j}}{e^{+\beta J \sum' s_i s_j}}$$

$$= \frac{\cosh(\beta J \sum' s_i s_j) - \sinh(\beta J \sum' s_i s_j)}{\cosh(\beta J \sum' s_i s_j) + \sinh(\beta J \sum' s_i s_j)}$$

$$= \left(\frac{1 - \tanh \beta J \sum' s_i s_j}{1 + \tanh \beta J \sum' s_i s_j} \right)$$


1d
$\sum'_{j=i-1,\,i+1} s_i s_j = s_i(s_{i+1}+s_{i-1})$
$\pm 1, 0$
$\tanh \varepsilon x = \varepsilon \tanh x$
$\dfrac{1-\tanh\left(2\beta J\,\tfrac{1}{2}s_i(s_{i-1}+s_{i+1})\right)}{1+\tanh\left(\,-\,-\,-\,-\,-\,\right)}$
$= \dfrac{1-\tfrac{\gamma}{2}s_i(s_{i-1}+s_{i+1})}{1+\tfrac{\gamma}{2}s_i(s_{i-1}+s_{i+1})}$
$\gamma \equiv \tanh 2\beta J$

1d $\quad \sum_{j=i-1,\, i+1}' s_i s_j = s_i(s_{i+1}+s_{i-1})$

$\tanh \varepsilon x = \varepsilon \tanh x$

$$\frac{1-\tanh\Big(2\beta J\, \overbrace{\tfrac{1}{2} s_i(s_{i-1}+s_{i+1})}^{\pm 1,0}\Big)}{1+\tanh(\;\text{- - - - - -}\;)}$$

$\gamma \equiv \tanh 2\beta J$

$$= \frac{1-\frac{\gamma}{2} s_i(s_{i+1}+s_{i+1})}{1+\frac{\gamma}{2} s_i(s_{i-1}+s_{i+1})}$$

$W(s\to s')$

$= \frac{1}{2}\left[1-\frac{\gamma}{2} s_i(s_{i-1}+s_{i+1})\right]$

Ising-Glauber Model

Transition rate $W(s \to s_i') = \frac{1}{2}\left(1 - \frac{\gamma}{2} s_i (s_{i-1} + s_{i+1})\right)$

$$\begin{array}{ccc} i-1 & i & i+1 \\ \uparrow & \uparrow & \uparrow \end{array} \longrightarrow \begin{array}{ccc} \uparrow & \downarrow & \uparrow \end{array}$$

Ising - Glauber Model

Transition rate $W(s \to s_i') = \frac{1}{2}\left(1 - \frac{\gamma}{2} s_i (s_{i-1} + s_{i+1})\right)$

$i-1$	i	$i+1$			W
$\uparrow$	$\uparrow$	$\uparrow$	$\longrightarrow$	$\uparrow\downarrow\uparrow$	$\frac{1}{2}(1-\gamma)$
$\uparrow$	$\uparrow$	$\downarrow$	$\longrightarrow$	$\uparrow\downarrow\downarrow$	$\frac{1}{2}$
$\uparrow$	$\downarrow$	$\uparrow$	$\longrightarrow$	$\uparrow\uparrow\uparrow$	$\frac{1}{2}(1+\gamma)$

Ising-Glauber Model

$\gamma = \tanh 2\beta J$

Transition rate $W(s \to s_i') = \frac{1}{2}\left(1 - \frac{\gamma}{2} s_i (s_{i-1} + s_{i+1})\right)$

$i-1$	i	$i+1$			$W(\gamma)$	$\xrightarrow{\beta \to \infty} \gamma = 1$
$\uparrow$	$\uparrow$	$\uparrow$	$\longrightarrow$	$\uparrow\downarrow\uparrow$	$\frac{1}{2}(1-\gamma)$	0
$\uparrow$	$\uparrow$	$\downarrow$	$\longrightarrow$	$\uparrow\downarrow\downarrow$	$\frac{1}{2}$	$1/2$
$\uparrow$	$\downarrow$	$\uparrow$	$\longrightarrow$	$\uparrow\uparrow\uparrow$	$\frac{1}{2}(1+\gamma)$	1

$\gamma = \tanh 2\beta J$ Correlation functions

$S_j \equiv \langle s_j \rangle$; $S_{i,j} \equiv \langle s_i s_j \rangle$

$\gamma = 1$

0

1/2

1

$\langle \dot{s_j} \rangle = \langle -2 \; w_j s_j \rangle$

$= \langle -2 s_j \frac{1}{2} \left(1 - \frac{\gamma}{2} s_j (s_{j-1} + s_{j+1}) \right) \rangle$

$\dot{S}_j = - S_j +$

$\gamma = \tanh 2\beta J$ <u>correlation functions</u>

$s_j = \pm 1$

$s_j^2 = 1$

$S_j \equiv \langle s_j \rangle \quad ; \quad S_{i,j} \equiv \langle s_i s_j \rangle$

I.C. $S_j(t=0) = \delta_{j,0}$

$\gamma = 1$

0

$1/2$

1

$\langle \dot{s}_j \rangle = \langle -2 \; w_j s_j \rangle$

$= \langle -2 s_j \frac{1}{2}\left(1 - \frac{\gamma}{2} s_j (s_{j-1} + s_{j+1})\right) \rangle$

$$\boxed{\dot{S}_j = -S_j + \frac{\gamma}{2}(S_{j-1} + S_{j+1})}$$

$\gamma = \tanh 2\beta J$

correlation functions

$S_j \equiv \langle s_j \rangle \quad ; \quad S_{i,j} \equiv \langle s_i s_j \rangle$

$\gamma = 1$

0

$1/2$

1

$\langle \dot{s_j} \rangle = \langle -2 \, w_j s_j \rangle$

$= \langle -2 s_j \frac{1}{2} \left(1 - \frac{\gamma}{2} s_j (s_{j-1} + s_{j+1})\right) \rangle$

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$s_j = \pm 1$

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$$\boxed{S_j(t) = I_0(2\gamma t)\, e^{-2\gamma t}}$$

$\gamma = \tanh 2\beta J$ <u>correlation functions</u>

$S_j \equiv \langle s_j \rangle \; ; \; S_{i,j} \equiv \langle s_i s_j \rangle$

$\gamma = 1$

0

$1/2$

1

$\langle \dot{s_j} \rangle = \langle -2 \; w_j s_j \rangle$

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$$\boxed{S_j(t) = I_0(2\gamma t)\, e^{-2\gamma t}}$$

$T \to 0$

$S_j(t) = I_0(2t)\, e^{-2t}$

$S_j \equiv \langle s_j \rangle \quad ; \quad S_{i,j} \equiv \langle s_i s_j \rangle$

$$\langle \dot{s_j} \rangle = \langle -2 \; w_j s_j \rangle$$

$$= \langle -\cancel{2} s_j \tfrac{1}{\cancel{2}} \left(1 - \tfrac{\gamma}{2} s_j \left(s_{j-1} + s_{j+1}\right)\right)\rangle$$

$$\boxed{\dot{S}_j = -S_j + \frac{\gamma}{2}\left(S_{j-1} + S_{j+1}\right)}$$

$m = \frac{1}{N} \sum S_j \qquad \dot{m} = -m + \gamma m$

I.C. $S_j(t=0) = \delta_{j,0}$

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$T \to 0$

$$\boxed{S_j(t) = I_0(2t)\, e^{-2t}}$$

$\dot{m} = -(1-\gamma)m$

$\gamma = \tanh 2\beta J$ <u>correlation functions</u>

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$S_j \equiv \langle s_j \rangle \quad ; \quad S_{i,j} \equiv \langle s_i s_j \rangle$

I.C. $S_j(t=0) = \delta_{j,0}$

$\gamma = 1$

0

1/2

1

$\langle \dot{s_j} \rangle = \langle -2 \; w_j s_j \rangle$

$= \langle -2 s_j \frac{1}{2} \left(1 - \frac{\gamma}{2} s_j (s_{j-1} + s_{j+1})\right) \rangle$

$$\boxed{S_j(t) = I_0(2\gamma t)\, e^{-2t}}$$

$T \to 0$

$$\boxed{S_j(t) = I_0(2t)\, e^{-2t}}$$

$$\boxed{\dot{S}_j = -S_j + \frac{\gamma}{2}(S_{j-1} + S_{j+1})}$$

$\dot{m} = -(1-\gamma) m$

$m(t) = e^{-(1-\gamma)t}$

$m = \frac{1}{N} \sum S_j \qquad \dot{m} = -m + \gamma m$

$\gamma = \tanh 2\beta J$ Correlation functions

$S_j \equiv \langle s_j \rangle$; $S_{i,j} \equiv \langle s_i s_j \rangle$

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0

$1/2$

1

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$m = \frac{1}{N}\sum S_j \qquad \dot{m} = -m + \gamma m$

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I.C. $S_j(t=0) = \delta_{j,0}$

$$\boxed{S_j(t) = I_0(2\gamma t)\, e^{-2t}}$$

$T \to 0$

$$\boxed{S_j(t) = I_0(2t)\, e^{-2t}}$$

$\dot{m} = -(1-\gamma) m$

$m(t) = e^{-(1-\gamma)t}$ $\quad m \to 0 \;\; t \to \infty$

$\gamma < 1 \quad m \to 0$

$\gamma = 1 \quad m$ is conserved

Ising-Glauber Model

$$S_{i,j} = \langle s_i s_j \rangle$$

$$S_{i,i+1} = \langle s_i s_{i+1} \rangle$$

Ising - Glauber Model

$$S_{i,j} = \langle s_i s_j \rangle$$

$$S_{i,i+1} = \langle s_i s_{i+1} \rangle = +1 \text{ prob}(\uparrow\uparrow) + (-1) \text{ prob}(\uparrow\downarrow)$$

$$= +1\,(1-\rho) + (-1)\,\rho$$

$$\uparrow\uparrow\uparrow\uparrow\circ\downarrow\downarrow\downarrow$$

$i \quad i+1$

$$= \boxed{\begin{gathered} 1-2\rho = S_{i,i+1} \\ \rho = \tfrac{1}{2}\left(1 - S_{i,i+1}\right) \end{gathered}}$$

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